O‘ZBEKISTON RESPUBLIKASI OLIY VA O‘RTA MAXSUS TA’LIM VAZIRLIGI

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Oliy matematika fanining
KOMPLEKS SONLAR NAZARIYASI
bo‘limining o‘qitilishi bo‘yicha
ULUBIY KO‘TSATMA

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Mazkur ko‘rsatmada “Kompleks sonlar nazariyasi” bo‘limining o’qitilishi bo‘yicha amaliy va nazariy ma’lumotlar batafsil bayon qilingan. Bunda kompleks sonlar haqida tushunchalar, kompleks sonlar ustida amallar, uning trigonometrik shakli, geometrik ma’nosi keltirilgan. Kompleks sondan ildiz chiqarish va kompleks sonni darajaga ko‘tarish hamda Eyler formulasi keltirilgan. Bulardan tashqari ko‘phadni ko‘paytuvchilarga ajratish va Bezu teoremasi haqida ham tushunchalar berilgan. Kompleks o‘zgaruvchili ba’zi elementar funksiyalar haqida to‘xtalib o‘tilgan. Bu ko‘rsatmada talabalar bilimini tekshirish uchun yetarli miqdorda misollar va test savollari ham berilgan.

Ko’rsatma Oliy matematikaning “Kompleks sonlar nazariyasi” bo‘limini mustaqil o‘rganuvchilar uchun mo‘ljallangan.

Islom Karimov nomidagi Toshkent davlat texnika universiteti ilmiy-uslubiy kengashi qaroriga muvofiq chop etildi.

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Kirish

Istiqlolning dastlabki yillarida ta'lim O'zbekiston Respublikasining ijtimoiy taraqqiyotining ustivor yo'nalishi deb e'lon qilinishi va Birinchi Prezidentimizning bevosita tashabbuslari va bevosita raxnamoligida “Ta'lim to‘g‘risida”gi qonun hamda Kadrlar tayyorlash Milliy dasturining qabul qilinishi

Prezidentimiz SH.Mirziyoyev tomonidan taklif etilgan 2017-2021 yillarga mo‘ljallangan “Harakatlar strategiyasi”ni Oliy ta'limda xususan TDTUda O‘zbekiston Respublikasi Prezidentining 2017 yil 20 apreldagi “Oliy ta'lim tizimini yanada rivojlantirish chora-tadbirlari to‘g‘risida”gi PQ-2909-sonli qarorida belgilangan ustivor vazifalar bajarilishi rejasiga ko‘ra kafedrada bajariladigan ishlar rejasi tuzilib olingan.

I. KOMPLEKS SONLAR

1.1 Kompleks sonlar

Kompleks son deb

$$z = a + ib$$

Ifodaga aylantiriladi, bu yerda a va b haqiqiy sonlar, i-mavxum birlik, ushbu tengliklar bilan aniqlanadi:

$$i = \sqrt{-1} \text{ yoki } i^2 = 1$$

a-kompleks son z ning haqiqiy qismi, ib-mavhum qismi deyiladi. Ular bunday belgilanadi: $a = \mathrm{Re}, b = \mathrm{Im}\, z$. Agar $a = 0$ bo‘lsa, $0 + ib = ib$ sof mavhum son deyiladi; $b = 0$ agar bo‘lsa, haqiqiy son hosil bo‘ladi: $a + I * 0 = a$. Faqat mavhum qismining ishorasi bilan farq qiladigan ikki kompleks son: $z = a + ib$ va $z = a - ib$ bir-biriga qo‘shma deyiladi.

Ushbu ikki asosiy ta’rif qabul qilinadi.

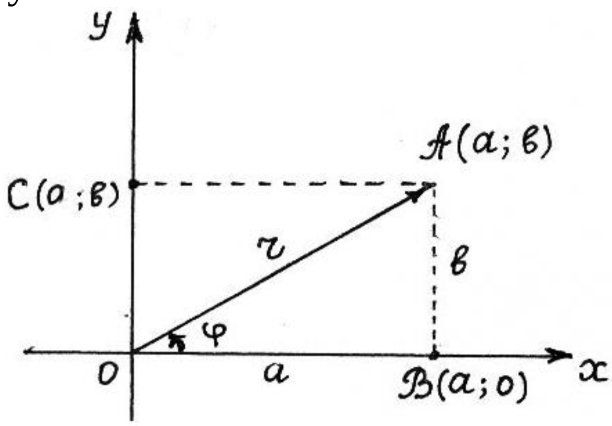
1. Agar $z_1 = a_1 + ib_1$ va $z_2 = a_2 + ib_2$ dan iborat ikki kompleks sonda $a_1 = a_2$; $b_1 = b_2$ bo‘lsa, ya’ni haqiqiy qismlar o‘zaro va mavhum qismlar o‘zaro teng bo‘lsa, bunday kompleks sonlar teng deyiladi.

2. Agar $a = 0,\ b = 0$ bo‘lsa, faqat shundagina kompleks son nolga teng bo‘ladi:

$$z = a + ib$$

1. Kompleks sonlarning geometrik tasviri.

Har qanday kompleks son $z = a + ib$ ni Oxy tekisligida koordinatalari a va b bo‘lgan $A(a,b)$ nuqta shaklida tasvirlash mumkin. Aksincha, Oxy tekisligidagi har qanday $M(a,b)$ nuqta $z = x + iy$ kompleks songa mos keladi. O‘zaro kompleks son tasvirlanadigan tekislik o‘zgaruvchi z ning kompleks tekisligi deyiladi.



O‘zgaruvchi z kompleks tekisligining Ox o‘qida yotuvchi nuqtalariga haqiqiy sonlar mos keladi $(b = 0)$. Oy o‘qida yotuvchi nuqtalar sof mavhum sonni tasvirlaydi, chunki bu holda $a = 0$. Shuning uchun kompleks sonlarni z ning klmpleks o‘zgaruvchi tekisligida tasvirlanganda Oy o‘q mavhum sonlar yoki mavhum o‘q, Ox o‘q esa haqiqiy o‘q deyiladi. $A(a,b)$ nuqtani koordinatalar boshi bilan tutashtirib, vektorni hosil qilamiz. Ba’zi hollarda $z = a + ib$ kompleks sonning geometrik tasviri AO vektor deb qabul qilish qulay bo‘ladi.

2. Kompleks sonning trigonometrik shakli.

Koordinatalar boshini qutb, Ox o‘qining musbat yo‘nalishini qutb deb olib, $A(a,b)$ nuqtaning qutb koordinatalarini φ va $r(r \geq 0)$, bilan belgilaymiz. Unda ushbu tengliklarni yozish mumkin:

$$a = r\cos\varphi, \qquad b = r\sin\varphi$$

Demak, kompleks son z ni bunday tasvirlash mumkin:

$$a + ib = r\cos\varphi + ir\sin\varphi$$

yoki

$$z = r(\cos\varphi + i\sin\varphi)$$

Bu tenglikning o‘ng tomonidagi ifodada $z = a + ib$ kompleks son yozuvining trigonometrik shakli deb ataladi.

z kompleks sonning modulini r va argumentini φ deb belgilaymiz; ular bunday ifodalanadi:

$$r = |z|, \ \varphi = \arg z$$

r va φ miqdorlar a va b orqali bunday ifodalanadi:

$$r = |z| \quad \varphi = Arctg\,\frac{b}{a}$$

Demak,

$$\left.\begin{array}{l} r = |z| = |a + ib| = \sqrt{a^2 + b^2} \\ \varphi = \arg z = \arg(a + ib) = Arctg\,\dfrac{b}{a} \end{array}\right\}$$

Kompleks sonning argumenti φ burchak Ox o‘qining musbat yo‘nalishidan soat strelkasi harakatiga teskari yo‘nalishda hisoblansa musbat, qarama-qarshi yo‘nalishda hisoblansa manfiy bo‘ladi. Ravshanki, argument bir qiymatli bo‘lmasdan, balki $2k\pi$ qo‘shiluvchiga (k-ixtiyoriy butun son) aniqlikda belgilanadi.

Izoh. Qo‘shma kompleks sonlar $z = a + ib$ va $z = a - ib$ teng modullarga ega: $|z| = |\bar{z}|$, argumentlarning absolyut qiymatlari teng, ammo ishoralari bilan farqlanadi: $\arg z = -\arg z$

Haqiqiy son A ni ham (3) shaklda yozish mumkin, ya’ni:

$A > 0$ bo‘lsa, $A = |A|(\cos 0 + i\sin 0)$,

$A < 0$ bo‘lsa, $A = |A|(\cos\pi + i\sin\pi)$

Nolga teng bo‘lgan kompleks sonning moduli nolga teng: $|0| = 0$. Nolning argumenti sifatida har qanday φ burchakni qabul qilish mumkin. Haqiqatdan har qanday φ burchak uchun ushbu tenglikni yozish mumkin:

$$0 = 0(\cos\varphi + i\sin\varphi)$$

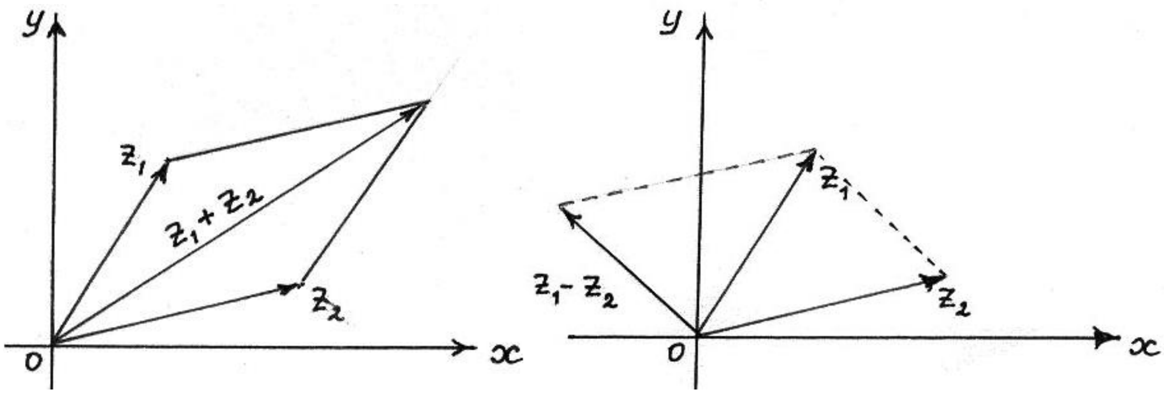
1. Kompleks sonlarni qo‘shish.

Ikki kompleks son $z_1 = a_1 + ib_1$ va $z_2 = a_2 + ib_2$ ning yig‘indisi deb ushbu

$$z_1 + z_2 = (a_1 + ib_1) + (a_2 + ib_2) = (a_1 + a_2) + i(b_1 + b_2)$$

Tenglik bilan aniqlangan kompleks songa aytiladi.

Bu formuladan vektorlar bilan tasvirlangan kompleks sonlarni qo‘shish-vektorlarni qo‘shish qoidasiga muvofiq bajarilishi kelib chiqadi.



a) **b)**

2. kompleks sonlarni ayirish.

Ikki $z_1 = a_1 + ib_1$ va $z_2 = a_2 + ib_2$ kompleks sonlarni ayirmasi deb shunday kompleks songa aytiladiki, unga z_2 kompleks sonni qo‘shganda z_1 kompleks son hosil bo‘ladi:

$$z_1 - z_2 = (a_1 + ib_1) - (a_2 + ib_2) = (a_1 - a_2) + i(b_1 - b_2)$$

Ikki kompleks son ayirmasining moduli shu sonlarni kompleks o‘zgaruvchilar tekisligida tasvirlovchi nuqtalar orasidagi masofaga teng:

$$|z_1 - z_2| = \sqrt{(a_1 - a_2)^2 + (b_1 - b_2)^2}$$

3. Kompleks sonlarni ko‘paytirish.

$z_1 = a_1 + ib_1$ va $z_2 = a_2 + ib_2$ kompleks sonlar ko‘paytmasi deb, ularni ikki xadlar singari algebra qoidasiga muvofiq, lekin

$$i^2 = -1, \quad i^3 = -i, \quad i^4 = (-i)i = -i^2 = 1, \quad i^5 = i \text{ va hokazo.}$$

umuman k butun bo‘lganda:

$$i^{4k} = -1, \quad i^{4k+1} = i, \quad i^{4k+2} = -1, \quad i^{4k+3} = -i$$

Ekanligini e’tiborga olib ko‘paytirganda hosil bo‘lgan kompleks songa aytiladi. Shu qoidaga asosan quyidagi ko‘paytmani hosil qilamiz:

$$z_1 z_2 = (a_1 + ib_1)(a_2 + ib_2) = a_1 a_2 + ib_1 a_2 + ia_1 b_2 + i^2 b_1 b_2$$

yoki

$$z_1 z_2 = (a_1 a_2 - b_1 b_2) + i(b_1 b_2 + a_1 b_2) \qquad (1)$$

Kompleks sonlar trigonometrik shaklda berilgan bo‘lsin:

$$z_1 = r_1(\cos\varphi_1 + i\sin\varphi_1), z_2 = r_2(\cos\varphi_2 + i\sin\varphi_2)$$

bu sonlarning ko‘paytmasini topamiz:

$$z_1 z_2 = r_1(\cos\varphi_1 + i\sin\varphi_1) r_2(\cos\varphi_2 + i\sin\varphi_2) = r_1 r_2[\cos\varphi_1\cos\varphi_2 + i\sin\varphi_1\cos\varphi_2 + i\cos\varphi_1\sin\varphi_2 + i^2\sin\varphi_1\sin\varphi_2]$$

$$= r_1 r_2[(\cos\varphi_1\cos\varphi_2 - \sin\varphi_1\sin\varphi_2) + i(\sin\varphi_1\cos\varphi_2 + \cos\varphi_1\sin\varphi_2)] = r_1 r_2[\cos(\varphi_1 + \varphi_2) + i\sin(\varphi_1 + \varphi_2)]$$

shunday qilib,

$$z_1 z_2 = r_1 r_2[\cos(\varphi_1 + \varphi_2) + i\sin(\varphi_1 + \varphi_2)]$$

Ya’ni ikki kompleks son ko‘paytmasi shunday kompleks sonki, uning moduli ko‘paytuvchilar modullarining ko‘paytmasiga teng, argumenti esa ko‘paytuvchilar argumentlarining yig‘indisiga teng.

1-izoh. $z = a + ib$ va $z = a - ib$ qo‘shma kompleks sonlar ko‘paytmasi (1) formulaga muvofiq bunday ifodalanadi:

$$z|\bar{z}| = a^2 + b^2,$$

yoki

$$z\bar{z} = |z|^2 = |\bar{z}|^2$$

Qo‘shma kompleks sonlar ko‘paytmasi ulardan har biri modulining kvadratiga teng.

4. Kompleks sonlarni bo‘lish.

Kompleks sonlarni bo‘lish ko‘paytirishga teskari amal kabi ta’riflanadi:

$$z_1 = a_1 + ib_1,\ z_2 = a_2 + ib_2,\ |z_2| = \sqrt{a_2^2 + b_2^2} \neq 0$$

deb faraz qilamiz. U holda $\frac{z_1}{z_2} = z$ shunday kompleks sonki, unda $z_1 = z_2 * z$ bo‘ladi.

Agar $\frac{a_1 + ib_1}{a_2 + ib_2} = x + iy$ bo‘lsa u holda $a_1 + ib_1 = (a_2 + ib_2)(x + iy)$ yoki $a_1 + ib_1 = (a_2 x - b_2 y) + i(a_2 y + b_2 x)$; x va y ushbu $a_1 = a_2 x - b_2 y,\ b_1 = b_2 x + a_2 y$ tenglamalar sistemasi bilan aniqlanadi. Bundan: $x = \frac{a_1 a_2 + b_1 b_2}{a_2^2 + b_2^2}$, $b = \frac{a_2 b_1 - a_1 b_2}{a_2^2 + b_2^2}$.

Nihoyat, ushbu formulani hosil qilamiz:

$$z = \frac{a_1 a_2 + b_1 b_2}{a_2^2 + b_2^2} + i\frac{a_2 b_1 - a_1 b_2}{a_2^2 + b_2^2}$$

Kompleks sonlarni bo‘lish amalda bunday bajariladi: $z_1 = a_1 + ib_1$ ni $z_2 = a_2 + ib_2$ga bo‘lish uchun bo‘linuvchi va bo‘luvchini bo‘luvchiga qo‘shma songa ko‘paytiramiz.

Unda bo‘luvchi haqiqiy son bo‘ladi; unga bo‘linuvchining haqiqiy va mavhum qismlarini bo‘lamiz:

$$\frac{a_1 + ib_1}{a_2 + ib_2} = \frac{(a_1 + ib_1)(a_2 - ib_2)}{(a_2 + ib_2)(a_2 - ib_2)} = \frac{(a_1a_2 + b_1b_2) + i(a_2b_1 - a_1b_2)}{a^2 + b^2} = \frac{a_1a_2 + b_1b_2}{a_2^2 + b_2^2} + i\frac{a_2b_1}{a_2^2 + b_2^2}$$

Kompleks sonlar trigonometrik shaklda

$$z_1 = r_1(\cos\varphi_1 + i\sin\varphi_1),$$
$$z_2 = r_2(\cos\varphi_2 + i\sin\varphi_2)$$

berilgan bo‘lsa, ushbuni hosil qilamiz:

$$\frac{z_1}{z_2} = \frac{r_1(\cos\varphi_1 + i\sin\varphi_1)}{r_2(\cos\varphi_2 - i\sin\varphi_2)} = \frac{r_1}{r_2}[\cos(\varphi_1 - \varphi_2) + i\sin(\varphi_1 - \varphi_2)]$$

Bu tenglikni tekshirish uchun bo‘luvchini bo‘linmaga ko‘paytirish kifoya:

$$r_2(\cos\varphi_2 + i\sin\varphi_2)\frac{r_1}{r_2}[\cos(\varphi_1 - \varphi_2) + i\sin(\varphi_1 - \varphi_2)] =$$

$$= r_2\frac{r_1}{r_2}[\cos(\varphi_2 + \varphi_1 - \varphi_2) + i\sin(\varphi_2 + \varphi_1 - \varphi_2)\ ] = r_1(\cos\varphi_1 + i\sin\varphi_1)$$

Shunday qilib, ikki kompleks son bo‘linmasining moduli bo‘linuvchi va bo‘luvchi modullarining bo‘linmasiga teng; bo‘linmaning argumenti bo‘linuvchi va bo‘luvchi argumentlarining ayirmasiga teng.

2-izoh. Kompleks sonlar ustida amallar bajarish qoidalari shuni ko‘rsatadiki, kompleks sonlarni qo‘shish, ayirish, ko‘paytirish, bo‘lish natijasida yana kompleks son hosil bo‘ladi.

3-izoh. Kompleks sonlar yig‘indisi, ayirmasi, ko‘paytmasi va bo‘linmasining ta’rifiga qaytib, u ifodalarda har bir kompleks son uning qo‘shmasi bilan almashtirilsa, shu amallarning natijalari ham unga qo‘shma kompleks sonlarga almashinishini tekshirib ko‘rish oson. Bundan ushbu teorema kelib chiqadi:

Teorema. Koeffitsentlari haqiqiy sonlar bo‘lgan ushbu

$$A_0x^n + A_1x^{n-1} + ... + A^n$$

ko‘phadda x o‘rniga $a+ib$son, so‘ngra unga qo‘shma son $a-ib$ qo‘yilsa, o‘rniga qo‘yish natijalari ham o‘zaro qo‘shma bo‘ladi.

Mustaqil yechish uchun misollar

Quyidagi kompleks sonning haqiqiy va mavhum qismi topilsin.

1) $z_1 = 1 + i\sqrt{3}$ $\qquad z_2 = 1 - i\sqrt{3}$

$z_1 \cdot z_2 = ?$ $z_1 + z_2 = ?$ $z_1 - z_2 = ?$ $\frac{z_1}{z_2} = ?$

2) $z_1 = 1 + i$ $z_2 = 1 - i$

$z_1 \cdot z_2 = ?$ $z_1 + z_2 = ?$ $z_1 - z_2 = ?$ $\frac{z_1}{z_2} = ?$

3) $z_1 = 1 + i$ $z_2 = 1 + i\sqrt{3}$

$z_1 \cdot z_2 = ?$ $z_1 + z_2 = ?$ $z_1 - z_2 = ?$ $\frac{z_1}{z_2} = ?$

4) $z_1 = 1 + i$ $z_2 = 1 - i$

$z_1 \cdot z_2 = ?$ $z_1 + z_2 = ?$ $z_1 - z_2 = ?$ $\frac{z_1}{z_2} = ?$

5) $z = \left(1 + i\sqrt{3}\right)^2$

6) $\frac{1}{1 - i}$

7) $z = 3$

8) $z = -2$

9) $z = -2i$

10) $z = 2 - 2i$

11) $z = 1 + i\sqrt{3}$

12) $z = -\sqrt{3} - i$

13) $z = -\sqrt{2} + i\sqrt{2}$

14) $z = (1 + i)^8$

15) $z = \frac{1}{\left(1 - i\sqrt{3}\right)^6}$

16) $z = \left(1 + i\sqrt{3}\right)^{15}$

17) $z = \left(\frac{1 - i}{1 + i}\right)^{10}$

18) $(-3 + 4i)^{1+i}$

19) $(-1)^{\sqrt{3}}$

20) 2^i

21) $(3 + 4i)^{1+i}$

22) e^i

23) 1^{-i}

24) 1^i

25) $(-i)^i$

26) $\left(\frac{1 - i}{\sqrt{2}}\right)^{1+i}$

Ko‘phadlarni ko‘paytuvchilarga ajrating.

1. $x^4 - 10x^3 + 38x^2 - 65x + 42$
2. $x^3 - 9x^2 + 18x + 28$
3. $x^4 + 4x^3 + 5x^2 + 4x + 4$
4. $x^3 - 5x^2 + 6x - 2$
5. $x^3 - 3x^2 + x + 1$
6. $x^4 + 2x^3 + 2x^2 + 2x + 1$
7. $x^3 - 5x^2 + 8x - 4$
8. $x^3 - 3x^2 + 4$

Kompleks sonni darajaga ko‘tarish.

Bundan oldingi paragrafdagi formuladan, agar n butun musbat son bo‘lsa, ushbu formula kelib chiqadi:

$$[r(\cos\varphi + i\sin\varphi)]^n = r^n(\cos\varphi + i\sin n\varphi)$$

Bu Muavr formulasi deb ataladi. Bundan ko‘rinadiki, kompleks sonni butun musbat drajaga ko‘tarishda modul shu darajaga ko‘tariladi, argument esa daraja ko‘rsatkichiga ko‘paytiriladi.

Endi Muavr formulasining yana bir tadbiqini qaraymiz. Bu formulada $r = 1$ deb faraz qilib,

$$(\cos\varphi + i\sin\varphi)^n = \cos n\varphi + i\sin n\varphi$$

Tenglikni hosil qilamiz. Chap tomonni Nyuton binomi formulasi bo‘yicha yoyib, haqiqiy va mavhum qismlarini tenglab, $\sin n\varphi$ va $\cos n\varphi$ ni $\sin\varphi$ va $\cos\varphi$ ning darajalari orqali ifoda qila olamiz.

Kompleks sondan ildiz chiqarish.

Kompleks sonning n-darajali ildizi deb n-darajaga ko‘targanda ildiz ostidagi songa teng bo‘ladigan kompleks songa aytiladi, ya’ni

$$\rho^n(\cos n\psi + i\sin n\psi) = r(\cos\varphi + i\sin\varphi)$$

bo‘lsa,

$$\sqrt[n]{r(\cos\varphi + i\sin\varphi)} = \rho(\cos\psi + i\sin\psi)$$

Teng kompleks sonlarning modullari teng bo‘lishi kerak, argumentlari esa 2π ga karrali songa farq qilishi mumkin bo‘lgani uchun

$$\rho^n = r, \qquad n\psi = \varphi + 2k\pi$$

bundan

$$\rho = \sqrt[n]{r}, \quad \psi = \frac{\varphi + 2k\pi}{n}$$

Bu yerda k- ixtiyoriy butun son, musbat r son ildizning arifmetik qiymati.

Demak,

$$\sqrt[n]{r(\cos\varphi + i\sin\varphi)} = \sqrt[n]{r}\left(\cos\frac{\varphi + 2k\pi}{n} + i\sin\frac{\varphi + 2k\pi}{n}\right)$$

k ga $0,1,2,...,n-1$ qiymatlari berib, ildizning n ta har xil qiymatlarini topamiz. Shunday qilib, kompleks sonning n-darajali ildizi n ta har xil qiymatga ega bo‘ladi.

1. Ikki hadli tenglamani yechish.

$$x^n = A$$

Shakldagi tenglama ikki hadli tenglama deyiladi. Bu tenglamaning ildizlarini topamiz.

Agar A haqiqiy musbat son bo‘lsa,

$$x=\sqrt[n]{A}\left(\cos\frac{2k\pi}{n}+i\sin\frac{2k\pi}{n}\right)\quad (k=0,1,2,\dots,n-1) \tag{2}$$

Qavs ichidagi ifoda 1 ning n-darajali ildizining hamma qiymatlarini beradi. Agar A haqiqiy manfiy son bo‘lsa,

$$x=\sqrt[n]{|A|}\left(\cos\frac{2k\pi}{n}+i\sin\frac{2k\pi}{n}\right)$$

Qavs ichidagi ifoda 1 ning n-darajali ildizining hamma qiymatlarini beradi. Agar A kompleks son bo‘lsa, x ning qiymatlari (2) formula bo‘yicha topiladi.

$z=x+iy$ bo‘lsin. Agar x va y haqiqiy o‘zgaruvchilar bo‘lsa, z kompleks o‘zgaruvchi deb ataladi. Kompleks o‘zgaruvchi z ning har bir qiymatiga xOy tekisligida ma’lum nuqta mos keladi.

Ta’rif. Kompleks o‘zgaruvchi z ning biror kompleks qiymatlar sohasida har bir qiymatiga boshqa ω kompleks miqdorning aniq qiymati mos kelsa, ω kompleks o‘zgaruvchi z ning funksiyasi bo‘ladi. Kompleks argumentning funksiyasi $\omega=f(z)$ yoki $\omega=\omega(z)$ bilan belgilanadi.

ω funksiyaning kompleks qiymatlari bunday belgilanadi:

$$e^{x+iy}=e^x(\cos y+i\sin y) \tag{*}$$

ya’ni

$$\omega(z)=e^x(\cos y+i\sin y)$$

Ko‘rsatkichli funksiyaning xossalari.

Agar z_1 va z_2 ikkita kompleks son bo‘lsa, unda

$$e^{z+z}=e^z e^z$$

Isbot.

$$z_1=x_1+iy_1,\qquad z_2=x_2+iy_2$$

bo‘lsin. U vaqtda

$$e^{z_1+z_2}=e^{(x_1+iy_1)+(x_2+iy_2)}=e^{(x_1+x_2)+i(y_1+y_2)}=e^{x_1}e^{x_2}\left[\cos(y_1+y_2)+i\sin(y_1+y_2)\right] \tag{3}$$

Ikkinchi tomondan, trigonometrik shakldagi ikki kompleks sonning ko‘paytmasi haqidagi teoremaga asosan:

$$e^{z_1}e^{z_2}=e^{x_1+iy_1}e^{x_2+iy_2}=e^{x_1}(\cos y_1+i\sin y_1)e^{x_2}(\cos y_2+i\sin y_2)=e^{x_1}e^{x_2}\left[\cos(y_1+y_2)+i\sin(y_1+y_2)\right]$$

(4)

(3) va (4) tengliklarda o‘ng tomonlar teng, demak, chap tomonlar ham teng bo‘ladi:

$$e^{z+z}=e^z e^z$$

$$e^{z-z_2}=\frac{e^{z_1}}{e^{z_2}} \tag{5}$$

formula ham isbotlanadi.

Agar m butun son bo‘lsa,

$$e^{(z)m} = e^{mz}$$

bo‘ladi. Agar $m > 0$ bo‘lsa, bu formula osongina hosil bo‘ladi; agar $m < 0$ bo‘lsa, bu formula (3) va (5) formulalarga asosan hosil qilinadi. Ushbu ayniyat

$$e^{z+2\pi i} = e^{z}$$

to‘g‘ridir.

Endi ushbu kompleks miqdorni qarab chiqamiz:

$$\omega = u(x) = iv(x)$$

bu yerda $u(x)$ va $v(x)$ haqiqiy o‘zgaruvchi x ning haqiqiy funksiyalaridir. Bu ω miqdor haqiqiy o‘zgaruvchining kompleks funksiyasidir.

Nazorat savollari

1. Kompleks son deb qanday ifodaga aytiladi?
2. Qanday ikkita kompleks son teng deyiladi?
3. Algebraik shaklda berilgan kompleks sonlarni qo‘shish va ayirish qanday qoidalarga asosan oshiriladi?
4. Algebraik shaklda berilgan kompleks sonlarni ko‘paytirish va bo‘lish qanday qoidalarga asosan oshiriladi?
5. To‘g‘ri burchakli koordinatalar sistemasida kompleks sonning tasviri qanday bo‘ladi?
6. To‘g‘ri burchakli koordinatalar sistemasida kompleks sonlar qanday qo‘shiladi?
7. Kopmleks sonning moduli va argumenti qanday aniqlanadi?
8. Kompleks son trigonometrik shaklda qanday tasvirlanadi?
9. Trigonometrik shaklda berilgan kompleks sonlarni qo‘shish, ayirish, ko‘paytirish va bo‘lish amallari qoidalarini ayting.
10. Trigonometrik shaklda berilgan kompleks sonlarni darajaga ko‘tarish qoidasini ta’riflab bering.

Eyler formulasi. Ko‘phadlar.

Agar (*) formulada $x = 0$ desak, unda

$$e^{iy} = \cos y + i \sin y \quad (6)$$

formulani hosil qilamiz. Bu ko‘rsatkichi mavhum son bo‘lgan ko‘rsatkichli funksiyani trigonometrik funksiyalar orqali ifodalovchi Eyler formulasidir. (6) formulada y ni y bilan almashtirib,

$$e^{-iy} = \cos y - i\sin y \qquad (7)$$

Formulani hosil qilamiz. (6) va (7) tengliklardan $\cos y$ va $\sin y$ ni topamiz:

$$\left.\begin{aligned} \cos y &= \frac{e^{iy}+e^{-iy}}{2} \\ \sin y &= \frac{e^{iy}-e^{iy}}{2i} \end{aligned}\right\}$$

So‘nggi formulalardan foydalanib, jumladan, $\cos\varphi$ va $\sin\varphi$ ni ularning har qanday butun musbat darajalarini va shu darajalar ko‘paytmasini karrali yoylarning sinuslari va kosinuslari orqali tasvirlashimiz mumkin.

Kompleks sonning ko‘rsatkichli shakli.

Kompleks sonni trigonometrik shaklda tasvirlaymiz:

$$z = r(\cos\varphi + i\sin\varphi)$$

Bu yerda r kompleks sonning moduli φ kompleks sonning argumenti. Eyler formulasiga ko‘ra

$$(\cos\varphi + i\sin\varphi) = e^{i\varphi}$$

Demak, har qanday kompleks sonni ushbu ko‘rsatkichli shaklda tasvirlash mumkin:

$$z = re^{i\varphi}$$

Ushbu kompleks sonlar berilgan bo‘lsin:

$$z_1 = r_1e^{i\varphi_1} \qquad z_2 = r_2e^{i\varphi_2}$$

bu holda

$$z_1z_2 = r_1e^{i\varphi_1}r_2e^{i\varphi_2} = r_1r_2e^{i(\varphi_1+\varphi_2)}$$

$$\frac{z_1}{z_2} = \frac{r_1e^{i\varphi_1}}{r_2e^{i\varphi_2}} = \frac{r_1}{r_2}e^{i(\varphi_1-\varphi_2)}$$

$$z^n = \left(re^{i\varphi}\right)^n = r^ne^{in\varphi}$$

$$\sqrt[n]{re^{i\varphi}} = \sqrt[n]{re^{i}}\,\frac{\varphi+2k\pi}{n} \qquad (k = 0,1,2,\ldots,n-1)$$

1.2 Ko‘phadni ko‘paytuvchilarga ajratish.

Ma’lumki, $f(x) = A_0x^n + A_1x^{n-1} + \ldots A_n$ funksiya, n butun musbat son bo‘lganda, ko‘phad yoki x ning butun ratsional funksiyasi deb ataladi; n soni ko‘phadning drajasi deyiladi. Bu yerda $A_0, A_1, \ldots, A_n$ koeffitsentlar haqiqiy yoki kompleks sonlar; erkli o‘zgaruvchi x ham haqiqiy, ham kompleks qiymatlar olishi mumkin.

O‘zgaruvchi x ning ko‘phadni nolga aylantiradigan qiymati ko‘phadning ildizi deb ataladi;

1-teorema. (Bezu teoremasi). Ko‘phad $f(x)$ ni $x-a$ ayirmaga bo‘lganda $f(a)$ ga teng qoldiq hosil bo‘ladi.

Isbot. $f(x)$ ni $x-a$ ga bo‘lganda bo‘linmada darajasi $f(x)$ darajasidan bitta kam bo‘lgan $f_1(x)$ ko‘phad hosil bo‘lib, qoldiq o‘zgarmas R bo‘ladi. Shunga ko‘ra bunday tenglikni yozish mumkin:

$$f(x)=(x-a)f_1(x)+R \tag{8}$$

bu tenglik x ning a ga teng bo‘lmagan hamma qiymatlari uchun to‘g‘ri. Endi x ni a ga intilishg majbur qilamiz. Unda (8) tenglik chap tomonining limiti $f(a)$ ga, o‘ng tomonining limiti R ga teng bo‘ladi. $f(x)$ va$(x-a)f_1(x)+R$. Funksiyalar o‘zaro teng bo‘lgani uchun ularning $x\to a$ dagi limitlari ham teng bo‘ladi, ya’ni $f(a)=R$.

Natija. Agar a ko‘phadning ildizi, ya’ni $f(a)=0$ bo‘lsa, $f(x)$ ko‘phad $x-a$ ga qoldiqsiz bo‘linadi, demak, ko‘paytma shaklida tasvirlanadi:

$$f(x)=(x-a)f_1(x)$$

bu yerda $f_1(x)$-ko‘phad.

2-teorema. Har qanday ratsional $f(x)$ funksiya eng kamida bitta haqiqiy yoki kompleks ildiziga egadir.

Bu teorema oliy algebrada isbot qilinadi. Bu yerda biz bu teoremani isbotsiz qabul qilamiz.

Algebraning asosiy teoremasidan foydalanib, ushbu teoremani isbot qilish oson:

3-teorema. n-darajali har qanday ko‘phad $x-a$ shakldagi n ta chiziqli ko‘paytuvchiga va x^n oldidagi koeffitsentga teng ko‘paytuvchiga ajraladi.

Isbot. $f(x)$ ni n- darajali ushbu

$$f(x)=A_0x^n+A_1x^{n-1}+...A_n$$

Ko‘phaddan iborat deb faraz qilaylik. Bu ko‘phad asosiy teoremaga ko‘ra eng kamida bitta ildizga ega; u ildizni a_1 bilan belgilaymiz. Unda ko‘phadni Bezu teoremasining natijasiga asosan bunday yozishimiz mumkin:

$$f(x)=(x-a_1)f_1(x)$$

bu yerda $f_1(x)\, n-1$ darajali ko‘phad; shuningdek $f_1(x)$ ham ildizga ega.

uni a_2 bilan belgilaymiz. Unda

$$f_1(x)=(x-a_1)f_2(x)$$

bu yerda $f_2(x)$ $n-2$ darajali ko‘phad. Shuning singari

$$f_2(x)=(x-a_3)f_3(x)$$

Chiziqli ko‘paytuvchilarga ajratish protsessini shunday davom ettirib, ushbu munosabatga kelib yetamiz:

$$f_{n-1}(x)=(x-a_n)f_n$$

bu yerda f_n nolinchi darajali ko‘phad, ya’ni tayin son. Bu son x^n oldidagi koeffitsentga teng, ya’ni $f_n=A_0$.

Hosil qilingan tengliklarga asosan, ushbu tenglikni yozishimiz mumkin:

$$f(x)=A_0(x-a_1)(x-a_2)...(x-a_n) \qquad (9)$$

Bu ajralmadan $a_1, a_2, ..., a_n$ sonlar ko‘phadning ildizlari ekanligi kelib chiqadi, chunki $x=a_1,\ x=a_2,\ ...,\ x=a_n$ o‘rniga qo‘yilganda tenglikning o‘ng tomoni, demak, chap tomoni ham nolga aylanadi.

4-teorema. Agar x ning $n+1$ ta $a_0, a_1, ..., a_n$ har xil qiymatlarida n-darajali ikki $\varphi_1(x)$ va $\varphi_2(x)$ ko‘phadning qiymatlari bir xil bo‘lsa, u ko‘phadlar bir-biriga aynan tengdir.

Isbot. Berilgan ko‘phadlar ayirmasini $f(x)$ bilan belgilaymiz:

$$f(x)=\varphi_1(x)\varphi_2(x)$$

Shartga muvofiq darajasi n dan yuqori bo‘lmagan, $a_1, a_2, ..., a_n$ nuqtalarda nolga aylanadigan ko‘phaddir. Demak, uni ushbu shaklda tasvirlash mumkin:

$$f(x)=A_0(x-a_1)(x-a_2)...(x-a_n)$$

Ammo shartga ko‘ra $f(x)$ ko‘phad a_0 nuqtada ham nolga aylanadi. Demak,

$$\varphi_1(x)-\varphi_2(x)\equiv 0 \quad \text{yoki} \quad \varphi_1(x)\equiv\varphi_2(x)$$

5-teorema. Agar $P(x)=A_0x^n+A_1x^{n-1}+...+A_{n-1}x+A_n$ ko‘phad aynan nolga teng bo‘lsa, uning hamma koeffitsentlari nolga teng.

Isbot. Bu ko‘phadni (9) formula bo‘yicha ko‘paytuvchilarga ajratamiz:

$$P(x)=A_0x^n+A_1x^{n-1}+...+A_{n-1}x+A_n=A_0(x-a_1)...(x-a_n)$$

Agar bu ko‘phad aynan nolga teng bo‘lsa, bu holda x ning $a_1, a_2, ..., a_n$ qiymatlaridan boshqa qiymatida ham nolga teng bo‘ladi. Lekin u vaqtda $x-a_1, ..., x-a_n$ ayirmalardan hech biri nolga teng emas, demak, $A_0=0$.

$A_1=0,\ A_2=0$ va hokazo ekanligini ham shu tariqa isbotlanadi.

6-teorema. Agar ikki ko‘phad bir-biriga aynan teng bo‘lsa, ko‘phadlardan birining koeffitsentlari ikkinchisining koeffitsentlariga tengdir.

Bu berilgan ko‘phadlarning ayirmasi aynan nolga teng ko‘phad bo‘lishidan kelib chiqadi. Demak, bundan oldingi teoremaga asosan, uning hamma koeffitsentlari nolga tengdir.

Agar n-darajali ko‘phadning chiziqli ko‘paytuvchilarga ajralmasi

$$f(x) = A_0(x-a_1)(x-a_2)...(x-a_n)$$

da ba’zi chiziqli ko‘paytuvchilar bir xil bo‘lsa, ularni birlashtirish mumkin va unda ko‘phadning ko‘paytuvchilarga yoyilmasi ushbu ko‘rinishda bo‘ladi.

$$f(x) = A_0(x-a_1)^{k_1}(x-a_2)^{k_2}...(x-a_m)^{k_m}$$

bunda

$$k_1 + k_2 + ... + k_m = n$$

Bu holda a_1 ildiz k_1 karrali ildiz, a_2 ildiz k_2 karrali ildiz va hokazo ataladi.

Har qanday n-darajali ko‘phad rosa n ta ildizga ega bo‘ladi.

Izoh.

$$f(x) = A_0x^n + A_1x^{n-1} + ...A_n$$

Ko‘phadning ildizlari haqida aytilganlarning hammasini ushbu

$$A\,x^n + A_1x^{n-1} + ...A_n = 0$$

Algebraik tenglamaning ildizlari terminlarida ifodalash mumkin.

Endi quyidagi teoremani isbot qilamiz:

Teorema. Agar a_1 son $f(x)$ ko‘phadning $k_1 > 1$ karrali ildizi bo‘lsa, shu son $f'(x)$ hosila uchun k_1-1 karrali ildiz bo‘ladi.

Isbot. Agar $x = a_1$ son ko‘phadning $k_1 > 1$ karrali ildizi bo‘lsa,

$$f(x) = (x-a_1)^{k_1}\varphi(x)$$

kelib chiqadi, bu yerda $\varphi(x) = (x-a_2)^{k_2}...(x-a_m)^{k_m}$ ko‘paytma $x = a_1$ bo‘lganda nolga aylanmaydi, ya’ni $\varphi(a_1) \neq 0$.

Differensiallaymiz:

$$f'(x) = k_1(x-a_1)^{k_1-1}\varphi(x) + (x-a_1)^{k_1}\varphi'(x) = (x-a_1)^{k_1-1}[k_1\varphi(x) + (x-a_1)\varphi'(x)]$$

Bunday belgilaymiz:

$$\psi(x) = k_1\varphi(x) + (x-a_1)\varphi'(x)$$

U vaqtda

$$f'(x) = k_1(x-a_1)^{k_1-1}\psi(x)$$

bunda

$$\psi(a_1) = k_1\varphi(a_1) + (a_1-a_1)\varphi'(a_1) = k_1\varphi(a_1) \neq 0$$

ya’ni $x = a_1$ son ko‘phadning $k_1 > 1$ karrali ildizidir. Teoremaning isbotidan $k_1 = 1$ bo‘lsa, a_1 son hosilaning ildizi emasligi kelib chiqadi.

Isbotlangan teoremadan yana ushbu natija chiqadi: a_1 son $f''(x)$ hosila uchun k_1-2 karrali ildiz, ..., $f^{(k_1-1)}(x)$ hosila uchun bir karrali ildiz bo‘lib, $f^{(k_1)}(x)$ hosila uchun ildiz bo‘lmaydi, ya’ni

$$f(a_1)=0,\ f'(a_1)=0,\ f''(a_1)=0,\dots,\ f^{(k_1-1)}(a_1)=0$$

lekin

$$f^{(k_1)}(a_1)\neq 0$$

formuladagi $a_1, a_2, \dots, a_n$ ildizlar haqiqiy va kompleks bo‘lishi mumkin.

Ushbu teorema o‘rinli.

Teorema. Agar koeffitsentlari haqiqiy bo‘lgan $f(x)$ ko‘phad $a+ib$ kompleks ildizga ega bo‘lsa, shu ko‘phad $a-ib$ qo‘shma kompleks ildizga ega bo‘ladi.

Isbot. Agar $f(x)$ ko‘phaddagi x o‘rniga $a+ib$ kompleks sonni qo‘yib, darajalarga ko‘tarishni bajarib, i ishtirok etmagan hadlarni ayrim va i ishtirok etgan hadlarni ayrim to‘plasak, ushbu ifodani hosil qilamiz:

$$f(a+ib)=M+iN$$

bu yerda M va N miqdorlar i qatnashmagan ifodalardir.

$a+ib$ ko‘phadning ildizi bo‘lgani uchun:

$$f(a+ib)=M+iN=0$$

bundan

$$M=0,\quad N=0$$

Endi ko‘phaddagi x o‘rniga $a-ib$ ifodani qo‘yamiz. Natijada $M+iN$ ga qo‘shma bo‘lgan $M-iN$ son chiqadi, ya’ni

$$f(a-ib)=M-iN$$

Yuqorida aytilgandek $M=0,\ N=0$ bo‘lgani uchun $f(a-ib)=0$, ya’ni $a-ib$ kompleks son ko‘phadning ildizidir.

Shunday qilib,

$$f(x)=A_0(x-a_1)(x-a_2)\dots(x-a_n)$$

Ajralmaga kompleks ildizlar o‘zining qo‘shmasi bilan juft holda kiradi.

Bir juft qo‘shma kompleks ildizlarga mos chiziqli ko‘paytuvchilarni ko‘paytirib, haqiqiy koeffitsentli kvadrat uchhad hosil qilamiz:

$$[x-(a+ib)][x-(a-ib)]=[(x-a)-ib][(x-a)+ib]=(x-a)^2+b^2=x^2-2ax+a^2+b^2=x^2+px+q$$

bu yerda $p=-2a,\ q=a^2+b^2$ haqiqiy sonlar.

Agar $a+ib$ son karrali ildiz bo‘lsa, unga qo‘shma bo‘lgan $a-ib$ son ham xuddi shunday karrali ildiz bo‘lishi kerak, ya’ni ko‘phad ajralmasida $x-(a+ib)$ chiziqli ko‘paytuvchi necha marta uchrasa, $x-(a-ib)$ chiziqli ko‘paytuvchi ham shuncha marta uchraydi.

Shunday qilib, haqiqiy koeffitsentli ko‘phad tegishli darajadagi chiziqli va kvadrat uchhad shaklidagi haqiqiy ko‘paytuvchilarga ajraladi, ya’ni

$$f(x) = A_0 (x - a_1)^{l_2} (x - a_2)^{l_2} \dots (x - a_r)^{l_e} \left(x^2 + p_1 x + q_1\right)^{k_1} \dots \left(x^2 + p_s x + q_s\right)^{k_l}$$

Bunda

$$k_1 + k_2 + \dots + k_r + 2l_1 + \dots + 2l_s = n$$

Nazorat savollari

1. Kompleks sonni ko‘rsatkichli shaklda tasvirlang.
2. $Z_1 = re^{ifg}$
3. Bezu teoremasini isbotlang.
4. n-darajali ko‘phad nechta ildizga ega?
5. Haqiqiy koeffitsentli ko‘phadning ildizlari kompleks bo‘lsa, ular o‘zaro qanday kompleks ildizlar deyiladi?

1.3 Test savollari

Quyidagi kompleks sonning haqiqiy va mavhum qismlarini topilsin.

1) $\frac{1}{1-i}$

A) $\operatorname{Re} z = \frac{1}{2}, \operatorname{Im} z = \frac{1}{2}$ B) $\operatorname{Re} z = -\frac{1}{2}, \operatorname{Im} z = \frac{3}{2}$

S) $\operatorname{Re} z = 0, \operatorname{Im} z = \frac{1}{2}$ D) $\operatorname{Im} z = -\frac{2}{3}, \operatorname{Re} z = 0$

2) $\frac{2}{1+i}$

A) $\operatorname{Re} z = 1, \operatorname{Im} z = -1$ B) $\operatorname{Re} z = \frac{1}{2}, \operatorname{Im} z = \frac{\sqrt{3}}{2}$

S) $\operatorname{Re} z = -1, \operatorname{Im} z = 2$ D) $\operatorname{Re} z = 1, \operatorname{Im} z = \frac{1}{2}$

3) $\frac{1-i}{1+i}$

A) $\operatorname{Re} z = 0, \operatorname{Im} z = -1$ B) $\operatorname{Re} z = 1, \operatorname{Im} z = -1$

S) $\operatorname{Re} z = 2, \operatorname{Im} z = -\frac{1}{2}$ D) $\operatorname{Re} z = -1, \operatorname{Im} z = \sqrt{3}$

4) $\frac{1}{i} - \frac{1}{1-i}$

A) $\operatorname{Re} z = -\frac{1}{2}, \operatorname{Im} z = -\frac{3}{2}$ B) $\operatorname{Re} z = -2, \operatorname{Im} z = \frac{1}{2}$

S) $\operatorname{Re} z = -\sqrt{3}, \operatorname{Im} z = -1$ D) $\operatorname{Re} z = 1, \operatorname{Im} z = -1$

5) $\dfrac{1}{\dfrac{1}{2}+i\dfrac{\sqrt{3}}{2}}$

A) $\operatorname{Re} z=\frac{1}{2}, \operatorname{Im} z=-\frac{\sqrt{3}}{2}$ B) $\operatorname{Re} z=-1, \operatorname{Im} z=\frac{1}{2}$

S) $\operatorname{Re} z=\frac{1}{2}, \operatorname{Im} z=1$ D) $\operatorname{Re} z=-1, \operatorname{Im} z=-\frac{\sqrt{3}}{2}$

6) $\frac{1}{1+i}+\frac{1}{1-i}$

A) $\operatorname{Re} z=1, \operatorname{Im} z=0$ B) $\operatorname{Re} z=1, \operatorname{Im} z=\frac{1}{2}$

S) $\operatorname{Re} z=\frac{1}{2}, \operatorname{Im} z=-\frac{1}{2}$ D) $\operatorname{Re} z=-1, \operatorname{Im} z=1$

7) $\frac{(1+i)(1-2i)}{1-i}$

A) $\operatorname{Re} z=2, \operatorname{Im} z=1$ B) $\operatorname{Re} z=-1, \operatorname{Im} z=2$

S) $\operatorname{Re} z=-2, \operatorname{Im} z=-1$ D) $\operatorname{Re} z=\frac{1}{2}, \operatorname{Im} z=0$

8) $\frac{1-i}{(1+i)(1-2i)}$

A) $\operatorname{Re} z=\frac{2}{5}, \operatorname{Im} z=-\frac{1}{5}$ B) $\operatorname{Re} z=2, \operatorname{Im} z=-1$

S) $\operatorname{Re} z=\frac{1}{2}, \operatorname{Im} z=1$ D) $\operatorname{Re} z=\frac{1}{2}, \operatorname{Im} z=-2$

9) $2i\frac{1-2i}{2i+1}$

A) $\operatorname{Re} z=-\frac{3}{5}, \operatorname{Im} z=\frac{6}{5}$ B) $\operatorname{Re} z=0, \operatorname{Im} z=-1$

S) $\operatorname{Re} z=-2, \operatorname{Im} z=1$ D) $\operatorname{Re} z=\frac{1}{2}, \operatorname{Im} z=-\frac{3}{5}$

10) $\frac{1+2i}{1-2i}-\frac{1}{2i}$

A) $\operatorname{Re} z=-\frac{3}{5}, \operatorname{Im} z=\frac{13}{10}$ B) $\operatorname{Re} z=2, \operatorname{Im} z=5$

S) $\operatorname{Re} z=-4, \operatorname{Im} z=3$ D) $\operatorname{Re} z=\frac{3}{5}, \operatorname{Im} z=5$

11) $\frac{1}{i}+\frac{1}{2i}-\frac{1}{3i}$

A) $\operatorname{Re} z=0, \operatorname{Im} z=-\frac{7}{6}$ B) $\operatorname{Re} z=3, \operatorname{Im} z=-\frac{5}{7}$

S) $\operatorname{Re} z=\frac{5}{6}, \operatorname{Im} z=2$ D) $\operatorname{Re} z=3, \operatorname{Im} z=7$

12) $\dfrac{i}{\dfrac{1}{i}+\dfrac{1-i}{1+i}}$

A) $\operatorname{Re} z = -\frac{1}{2}, \operatorname{Im} z = 0$ B) $\operatorname{Re} z = 2, \operatorname{Im} z = -1$

S) $\operatorname{Re} z = -5, \operatorname{Im} z = \frac{3}{5}$ D) $\operatorname{Re} z = 0, \operatorname{Im} z = \sqrt{5}$

13) $\dfrac{\frac{1}{i} - \frac{1-i}{1+i}}{i}$

A) $\operatorname{Re} z = 0, \operatorname{Im} z = 0$ B) $\operatorname{Re} z = 2, \operatorname{Im} z = -1$

S) $\operatorname{Re} z = -3, \operatorname{Im} z = 1$ D) $\operatorname{Re} z = 1, \operatorname{Im} z = \frac{1}{2}$

14) $\dfrac{2-3i}{(1+2i)3i}$

A) $\operatorname{Re} z = -\frac{7}{15}, \operatorname{Im} z = \frac{4}{15}$ B) $\operatorname{Re} z = -2, \operatorname{Im} z = 5$

S) $\operatorname{Re} z = -1, \operatorname{Im} z = 0$ D) $\operatorname{Re} z = 1, \operatorname{Im} z = 3$

15) $\left(\frac{1}{i} - \frac{1}{1-i}\right)\left(1 + i\sqrt{3}\right)$

A) $\operatorname{Re} z = \frac{3\sqrt{3}-1}{2}, \operatorname{Im} z = -\frac{3+\sqrt{3}}{2}$ B) $\operatorname{Re} z = \frac{\sqrt{3}}{2}, \operatorname{Im} z = 1$

S) $\operatorname{Re} z = 3, \operatorname{Im} z = -1$ D) $\operatorname{Re} z = \frac{\sqrt{3}}{3}, \operatorname{Im} z = 0$

16) $(1+i)(1-2i)(1-i)$

A) $\operatorname{Re} z = 2, \operatorname{Im} z = -4$ B) $\operatorname{Re} z = 0, \operatorname{Im} z = 2$

S) $\operatorname{Re} z = 0, \operatorname{Im} z = -3$ D) $\operatorname{Re} z = 2, \operatorname{Im} z = -1$

17) $\dfrac{\left(2 - \frac{1}{i}\right)i}{\left(1 - \frac{1}{i}\right)\left(2 + \frac{1}{i}\right)}$

A) $\operatorname{Re} z = -0{,}1, \operatorname{Im} z = 0{,}7$ B) $\operatorname{Re} z = -1, \operatorname{Im} z = 2$

S) $\operatorname{Re} z = -1, \operatorname{Im} z = 0{,}1$ D) $\operatorname{Re} z = 5, \operatorname{Im} z = 0{,}7$

18) $\dfrac{\left(2 - \frac{1}{i}\right)\left(2 + \frac{1}{i}\right)}{\left(2 - \frac{1}{i}\right)i}$

A) $\operatorname{Re} z = -\frac{1}{5}, \operatorname{Im} z = -\frac{7}{5}$ B) $\operatorname{Re} z = 2, \operatorname{Im} z = 1$

S) $\operatorname{Re} z = -1, \operatorname{Im} z = 21$ D) $\operatorname{Re} z = 0{,}3, \operatorname{Im} z = 1$

19) $\dfrac{(2i+1)i}{i-1}$

A) $\operatorname{Re} z = -\frac{3}{2}, \operatorname{Im} z = -\frac{3}{2}$ B) $\operatorname{Re} z = \frac{2}{3}, \operatorname{Im} z = 1$

S) $\operatorname{Re} z = \frac{3}{2}, \operatorname{Im} z = 0$ D) $\operatorname{Re} z = 1, \operatorname{Im} z = \frac{3}{2}$

20) $\frac{(1-i)(2-3i)}{i(1-i)}$

A) $\operatorname{Re} z = -3, \operatorname{Im} z = -2$ B) $\operatorname{Re} z = 1, \operatorname{Im} z = -2$

S) $\operatorname{Re} z = -2, \operatorname{Im} z = 1$ D) $\operatorname{Re} z = 3, \operatorname{Im} z = 1$

21) $(2+3i)(3-2i)$

A) $\operatorname{Re} z = 12, \operatorname{Im} z = 5$ B) $\operatorname{Re} z = 3, \operatorname{Im} z = -12$

S) $\operatorname{Re} z = -12, \operatorname{Im} z = 3$ D) $\operatorname{Re} z = 1, \operatorname{Im} z = 5$

22) $(a+bi)(a-bi)$

A) $\operatorname{Re} z = a^2 - b^2, \operatorname{Im} z = -2ab$ B) $\operatorname{Re} z = a^2 + b^2, \operatorname{Im} z = 2$

S) $\operatorname{Re} z = 2, \operatorname{Im} z = -2ab$ D) $\operatorname{Re} z = a-b, \operatorname{Im} z = a-b$

23) $(3-2i)^2$

A) $\operatorname{Re} z = 5, \operatorname{Im} z = -12$ B) $\operatorname{Re} z = 12, \operatorname{Im} z = 5$

S) $\operatorname{Re} z = 2, \operatorname{Im} z = 12$ D) $\operatorname{Re} z = 13, \operatorname{Im} z = 1$

24) $(1-i)^3$

A) $\operatorname{Re} z = -2, \operatorname{Im} z = 2$ B) $\operatorname{Re} z = 2, \operatorname{Im} z = 3$

S) $\operatorname{Re} z = -2, \operatorname{Im} z = 3$ D) $\operatorname{Re} z = 2, \operatorname{Im} z = -2$

25) $\frac{1+i}{1-i}$

A) $\operatorname{Re} z = 0, \operatorname{Im} z = 1$ B) $\operatorname{Re} z = 1, \operatorname{Im} z = 2$

S) $\operatorname{Re} z = 0, \operatorname{Im} z = 3$ D) $\operatorname{Re} z = 2, \operatorname{Im} z = 0$

26) $\frac{2i}{1+i}$

A) $\operatorname{Re} z = 1, \operatorname{Im} z = 1$ B) $\operatorname{Re} z = 2, \operatorname{Im} z = 2$

S) $\operatorname{Re} z = 0, \operatorname{Im} z = 1$ D) $\operatorname{Re} z = 4, \operatorname{Im} z = -1$

Quyidagi kompleks sonning moduli va argumenti topilsin.

1) $z = 3i$

A) $|z| = 3,\ \arg z = \frac{\pi}{2}$ B) $|z| = 2,\ \arg z = \frac{\pi}{4}$

S) $|z| = -2,\ \arg z = 1$ D) $|z| = 4,\ \arg z = \frac{2}{3}$

2) $z = 1 + \cos\frac{\pi}{7} + i\sin\frac{\pi}{7}$

A) $|z| = 2\cos\frac{\pi}{14},\ \arg z = \frac{\pi}{14}$ B) $|z| = 3,\ \arg z = \frac{\pi}{14}$

S) $|z| = 1,\ \arg z = \sqrt{3}$ D) $|z| = 3,\ \arg z = \pi$

3) $z = \frac{1}{\sqrt{2}}(1-i)$

A) $|z|=1,\ \arg z=\frac{7\pi}{4}$ B) $|z|=2,\ \arg z=\frac{\pi}{4}$

S) $|z|=1,\ \arg z=\pi$ D) $|z|=2,\ \arg z=\frac{7\pi}{4}$

4) $z=-1+i\sqrt{3}$

A) $|z|=2,\ \arg z=\frac{2\pi}{3}$ B) $|z|=3,\ \arg z=\frac{\pi}{4}$

S) $|z|=1,\ \arg z=\sqrt{3}$ D) $|z|=1,\ \arg z=\frac{\pi}{4}$

5) $z=i$

A) $|z|=1,\ \arg z=\frac{\pi}{2}$ B) $|z|=1,\ \arg z=-\frac{\pi}{4}$

S) $|z|=2,\ \arg z=\frac{\pi}{3}$ D) $|z|=2,\ \arg z=\frac{\pi}{2}$

6) $z=-3$

A) $|z|=3,\ \arg z=\pi$ B) $|z|=2,\ \arg z=\frac{\pi}{2}$

S) $|z|=5,\ \arg z=\pi$ D) $|z|=2,\ \arg z=\pi$

7) $z=-\frac{1}{2}+i\frac{\sqrt{3}}{2}$

A) $|z|=1,\ \arg z=\frac{2\pi}{3}$ B) $|z|=-1,\ \arg z=\pi$

S) $|z|=1,\ \arg z=\frac{1}{2}$ D) $|z|=3,\ \arg z=\pi$

8) $z=-\frac{1}{2}-i\frac{\sqrt{3}}{2}$

A) $|z|=1,\ \arg z=\frac{4\pi}{3}$ B) $|z|=2,\ \arg z=\pi$

S) $|z|=2,\ \arg z=\frac{4\pi}{3}$ D) $|z|=1,\ \arg z=\sqrt{\pi}$

9) $z=\frac{1}{2}+i\frac{\sqrt{3}}{2}$

A) $|z|=1,\ \arg z=\frac{\pi}{3}$ B) $|z|=2,\ \arg z=\pi$

S) $|z|=1,\ \arg z=\frac{\pi}{5}$ D) $|z|=3,\ \arg z=\frac{\pi}{4}$

10) $z=\frac{1}{2}-i\frac{\sqrt{3}}{2}$

A) $|z|=1,\ \arg z=\frac{5\pi}{3}$ B) $|z|=3,\ \arg z=\pi$

S) $|z|=1,\ \arg z=2$ D) $|z|=1,\ \arg z=\frac{\pi}{7}$

11) $z=\frac{1-i}{1+i}$

A) $|z|=1,\ \arg z=\frac{3\pi}{2}$ B) $|z|=2,\ \arg z=\pi$

S) $|z|=5,\ \arg z=\frac{\pi}{2}$ D) $|z|=5,\ \arg z=\frac{3\pi}{2}$

12) $z=\frac{1+i}{1-i}$

A) $|z|=1,\ \arg z=\frac{\pi}{2}$ B) $|z|=3,\ \arg z=\frac{\pi}{3}$

S) $|z|=1,\ \arg z=\pi$ D) $|z|=1,\ \arg z=2$

13) $z=-\cos\frac{\pi}{7}+i\sin\frac{\pi}{7}$

A) $|z|=1,\ \arg z=\frac{6\pi}{7}$ B) $|z|=4,\ \arg z=\pi$

S) $|z|=2,\ \arg z=\frac{6\pi}{7}$ D) $|z|=1,\ \arg z=-\pi$

14) $z=bi,\quad b\neq 0$

A) $|z|=b,\ \arg z=\begin{cases}\frac{\pi}{2}\,agar, b>0\\ \frac{3\pi}{2}\,agar, b<0\end{cases}$ B) $|z|=a,\ \arg z=\frac{\pi}{2}$

S) $|z|=b,\ \arg z=a^2+b^2$ D) $|z|=b,\ \arg z=-a$

15) $z=-\cos\varphi-i\sin\varphi$

A) $|z|=1,\ \arg z=\pi+\varphi$ B) $|z|=3,\ \arg z=\varphi$

S) $|z|=4,\ \arg z=\pi$ D) $|z|=1,\ \arg z=-\pi$

16) $z=1-\cos\alpha+i\sin\alpha$

A) $|z|=2\sin\frac{\alpha}{2},\ \arg z=\frac{\pi}{2}-\frac{\alpha}{2}$ B) $|z|=2,\ \arg z=\frac{\pi}{2}$

S) $|z|=1,\ \arg z=\frac{\pi}{2}-\alpha$ D) $|z|=2,\ \arg z=2\sin\alpha$

17) $z=-\sin\alpha+i(1+\cos\alpha)$ $0<\alpha<\pi$

A) $|z|=2\cos\frac{\alpha}{2},\ \arg z=\frac{\pi}{2}+\frac{\alpha}{2}$ B) $|z|=1,\ \arg z=\frac{\pi}{2}$

S) $|z|=2,\ \arg z=\frac{\alpha}{2}$ D) $|z|=\sin\pi,\ \arg z=\pi$

18) $z=\left(1+i\sqrt{3}\right)^3$

A) $z=-8, |z|=8, \pi,\ \arg z=\pi$ B) $z=8, |z|=4, \pi,\ \arg z=\pi$

S) $z=2, |z|=8,,\ \arg z=1$ D) $z=6, |z|=8,\ \arg z=\frac{\pi}{2}$

19) $z=(-4+3i)^3$

A) $|z|=125,\ \arg z=\frac{3\pi}{2}+3arctg\frac{4}{3}$ B) $|z|=120,\ \arg z=\pi$

S) $|z| = -125,\ \arg z = \frac{3\pi}{2}$ D) $|z| = 125,\ \arg z = \pi$

20) $z = (1+i)^{10}$

A) $z = 32i, |z| = 32,,\ \arg z = \frac{\pi}{2}$ B) $z = 20, |z| = 32\ \arg z = \pi$

S) $z = 23, |z| = 32,,\ \arg z = -\frac{\pi}{2}$ D) $z = -32, |z| = 25\ \arg z = \pi$

21) $z = (1+i)^8 \left(1 - i\sqrt{3}\right)^{-6}$

A) $z = \frac{1}{4}, |z| = \frac{1}{4},\ \arg z = 0$ B) $z = -\frac{1}{4}, |z| = -\frac{1}{4},\ \arg z = 1$

S) $z = 2, |z| = 4\ \arg z = 2$ D) $z = 1, |z| = 2\ \arg z = -\pi$

22) $z = \left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)^{24}$

A) $z = 1, |z| = 1\ \arg z = 0$ B) $z = 2, |z| = 2\ \arg z = 1$

S) $z = 4, |z| = 4\ \arg z = 1$ D) $z = 1, |z| = -1\ \arg z = 0$

23) $z = \left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)^{10}$

A) $z = -\frac{1}{2} + i\frac{\sqrt{3}}{2}, |z| = 1,,\ \arg z = \frac{2\pi}{3}$ B) $z = \frac{1}{2}, |z| = 2\ \arg z = \pi$

S) $z = 1, |z| = 2\ \arg z = 1$ D) $z = 2, |z| = -2\ \arg z = \pi$

24) $z = \left(\sqrt{2} + i\sqrt{2}\right)^{25}$

A) $z = 2^{24}\sqrt{2}(1+i), |z| = 2^{24}\sqrt{2}, \arg z = \frac{\pi}{4}$ B) $z = 2^{24}, |z| = 3^{12}, \arg z = \pi$

S) $z = 21, |z| = 42 \arg z = \sqrt{3}$ D) $z = 32, |z| = 31, \arg z = \frac{\pi}{4}$

25) $z = \left(\frac{1 + i\sqrt{3}}{1 - i}\right)^{20}$

A) $z = 2^9\left(1 - i\sqrt{3}\right), |z| = 2^{10}, \arg z = \frac{5\pi}{3}$ B) $z = 25, |z| = 2^{10}, \arg z = \frac{\pi}{3}$

S) $z = 32, |z| = 2^{10}, \arg z = \pi$ D) $z = 25, |z| = 25, \arg z = 1$

26) $z = \frac{\left(1 + i\sqrt{3}\right)^{15}}{(1+i)^{10}}$

A) $z = 2^{10}i, |z| = 2^{10}, \arg z = \frac{\pi}{2}$ B) $z = 32, |z| = 32, \arg z = \frac{\pi}{2}$

S) $z = 2^5, |z| = 2^5, \arg z = \frac{\pi}{2}$ D) $z = 25, |z| = 25, \arg z = 1$

27) $z = -\sqrt{2} + i\sqrt{2}$

A) $2\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$ B) $4(\sin\pi + \cos\pi)$

S) $3\left(\cos\frac{\pi}{2}-i\sin\frac{\pi}{4}\right)$ D) $\cos\pi+i\sin\pi$

28) $z=\sin\alpha+i(1-\cos\alpha)$

A) $2\cos\frac{\alpha}{2}\left(\sin\frac{\alpha}{2}+i\sin\frac{\alpha}{2}\right)$ B) $2\left(\sin\frac{\pi}{2}+i\cos\frac{\pi}{2}\right)$

S) $\cos\pi+i\sin\pi$ D) $2\left(\sin\frac{\alpha}{2}+i\sin\frac{\alpha}{2}\right)$

Kompleks sondan ildiz chiqarish.

1) $z=\sqrt{1-i}$

A) $\pm\frac{\sqrt{2}}{2}\left(\sqrt{\sqrt{2}+1}-i\sqrt{\sqrt{2}-1}\right)$ B) $\frac{\sqrt{2}}{2}\left(\sqrt{2}-1\right)$

S) $\sqrt{2}+1$ D) $\frac{1}{2}+\frac{\sqrt{2}}{2}$

2) $z=\sqrt[3]{1}$

A) $-\frac{1}{2}\pm\frac{i\sqrt{3}}{2}$ B) $\frac{\sqrt{2}}{2}\left(\sqrt{2}-1\right)$

S) $\frac{\sqrt{3}}{2}+\frac{i}{2}$ D) $\frac{i}{2}-\frac{\sqrt{3}}{2}$

3) $z=\sqrt[4]{-1}$

A) $\pm\frac{\sqrt{2}}{2}(1+i)\pm\frac{\sqrt{2}}{2}(1-i)$ B) $\frac{\sqrt{2}}{2}(1+i)$

S) $\pm\frac{\sqrt{2}}{2}(1-i)$ D) $\frac{i}{2}-\frac{\sqrt{2}}{2}$

4) $z=\sqrt[3]{-2+2i}$

A) $\sqrt{2}\left[\cos\frac{\left(2k+\frac{3}{4}\right)\pi}{3}+i\sin\frac{\left(2k+\frac{3}{4}\right)\pi}{3}\right](k=0,1,2)$ B) $\frac{\sqrt{2}}{2}\left(\cos\frac{2k}{3}+i\sin\frac{k}{3}\right)$

S) $\frac{\sqrt{2}}{3}+\frac{i}{2}$ D) $\sqrt{8}$

5) $z=\sqrt[3]{i}$

A) $\pm\frac{\sqrt{3}}{2}+\frac{i}{2},-i$ B) $\frac{\sqrt{2}}{2}\pm i\frac{\sqrt{3}}{2}$

S) $\frac{1}{2}-\sqrt{2i}$ D) $\frac{1}{2}-\frac{\sqrt{2}}{3}$

6) $z=\sqrt[6]{-8}$

A) $\pm\frac{\sqrt{2}}{2}\left(\sqrt{3}+i\right),\pm\frac{\sqrt{2}}{2}\left(\sqrt{3}-i\right),\pm\sqrt{2}i$ B) $\frac{\sqrt{2}}{2}\left(\sqrt{3}-i\right)$

S) $\sqrt{3}\pm i$ D) $\frac{\sqrt{3}}{2}+i\frac{1}{2}$

7) $z=\sqrt{3+4i}$

A) $\pm(2+i)$ B) $\frac{1}{2}+i\frac{\sqrt{3}}{2}$

S) $\frac{\sqrt{2}}{2}+i\frac{1}{2}$ D) $2\pm 3i$

8) $z=\sqrt[5]{-4+3i}$

A) $\sqrt[5]{5}\left[\cos\frac{(2k+1)\pi-arctg\frac{3}{4}}{5}+i\sin\frac{(2k+1)\pi-arctg\frac{3}{4}}{5}\right]$ B) $\pm(2+i)$

S) ± 5 D) $\frac{1}{2}+i\frac{\sqrt{3}}{2}$

9) $z^2=i$

A) $z_1=\frac{1+i}{\sqrt{2}};\quad z_2=\frac{1}{2}+i\frac{\sqrt{3}}{2}$ B) $z_{1,2}=\pm(2+i)$

S) $z_1=\frac{1}{2}+i\frac{\sqrt{3}}{2}$ D) $z_{1,2}=\frac{1}{2}\pm i\frac{\sqrt{3}}{2}$

10) $z^7+i=0$

A) $z_k=e^{\frac{2k+1}{7}\pi\cdot i}(k=0,1,2,3,4,5,6)$ B) $z_k=\pm(2k+i),(k=0,1,2,3,4,5,6)$

S) $z_k=\frac{k}{2}+i\frac{\sqrt{3}}{2},(k=0,1,2,3,4,5,6)$ D) $z_k=\frac{k}{2}\pm i\frac{\sqrt{3}}{2},(k=0,1,2,3,4,5,6)$

11) $z^2=3-4i$

A) $z_1=2-i;\ z_2=-2+i$ B) $z_{1,2}=\pm(2+i)$

S) $z_1=\frac{1}{2}+i\frac{\sqrt{3}}{2}$ D) $z_{1,2}=\frac{1}{2}\pm i\frac{\sqrt{3}}{2}$

12) $z^8=1+i$

A) $z_k=\sqrt[16]{2e^{\frac{\pi i}{4}\left(k+\frac{1}{8}\right)}}(k=0,1,2,3,4,5,6,7)$ B) $z_k=\pm(2k+i),(k=0,1,2,3,4,5,6,7)$

S) $z_k=\frac{k}{2}+i\frac{\sqrt{3}}{2},(k=0,1,2,3,4,5,6,7)$ D) $z_k=\frac{k}{2}\pm i\frac{\sqrt{3}}{2},(k=0,1,2,3,4,5,6,7)$

13) $z^3=-1$

A) $z_1=-1;\quad z_2=\frac{1}{2}+i\frac{\sqrt{3}}{2};\ z_3=\frac{1}{2}-i\frac{\sqrt{3}}{2}$ B) $z_{1,2}=\pm(2+i);\ z_3=-1$

S) $z_1=\frac{1}{2}+i\frac{\sqrt{3}}{2};\ z_{2,3}=\pm i$ D) $z_{1,2}=\frac{1}{2}\pm i\frac{\sqrt{3}}{2};\ z_3=2$

14) $z^6=64$

A) $z_k=2\left(\frac{\sqrt{3}+i}{2}\right)^k+i\frac{\sqrt{3}}{2},(k=0,1,2,3,4,5)$ B) $z_k=\pm(2k+i)^k,(k=0,1,2,3,4,5)$

S) $z_k=\frac{k}{2}+i\frac{\sqrt{3}}{2},(k=0,1,2,3,4,5)$ D) $z_k=\frac{k}{2}\pm i\frac{\sqrt{3}}{2},(k=0,1,2,3,4,5)$

15) $(1+i)^{10}$

A) $32i$ B) $\frac{\sqrt{2}}{2}\left(\sqrt{2}-1\right)$ S) $\frac{\sqrt{3}}{2}+\frac{i}{2}$ D) $\frac{i}{2}-\frac{\sqrt{3}}{2}$

16) $\left(1-i\sqrt{3}\right)^6$

A) 64 B) $\frac{\sqrt{2}}{2}(1+i)$ S) $\pm\frac{\sqrt{2}}{2}(1-i)$ D) $\frac{i}{2}-\frac{\sqrt{2}}{2}$

17) $(-1+i)^5$

A) $4(1-i)$ B) $\frac{\sqrt{2}}{2}\left(\sqrt{2}-1\right)$ S) $\sqrt{2}+1$ D) $\frac{1}{2}+\frac{\sqrt{2}}{2}$

18) $\left(1+\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)^4$

A) $2\left(3+2\sqrt{2}\right)i$ B) $\frac{\sqrt{2}}{2}\pm i\frac{\sqrt{3}}{2}$ S) $\frac{1}{2}-\sqrt{2}i$ D) $\frac{1}{2}-\frac{\sqrt{2}}{3}$

19) $\left(\sqrt{3}+i\right)^3$

A) $8i$ B) $\frac{\sqrt{2}}{2}\left(\sqrt{3}-i\right)$ S) $\sqrt{3}\pm i$ D) $\frac{\sqrt{3}}{2}\pm i\frac{1}{2}$

20) $z=\sqrt[3]{1}$

A) $-\frac{1}{2}\pm\frac{i\sqrt{3}}{2}$ B) $\frac{\sqrt{2}}{2}\left(\sqrt{2}-1\right)$

S) $\frac{\sqrt{3}}{2}+\frac{i}{2}$ D) $\frac{i}{2}-\frac{\sqrt{3}}{2}$

21) $z=\sqrt[3]{i}$

A) $-i\frac{i\pm\sqrt{3}}{2}$ B) $\frac{\sqrt{2}}{2}\pm i$

S) $\frac{1}{2}-\sqrt{2}i$ D) $\frac{1}{2}-\frac{\sqrt{2}}{3}$

22) $z=\sqrt[6]{-1}$

A) $\pm i,\pm\frac{1}{2}\pm\frac{\sqrt{3}}{2}$ B) $\frac{\sqrt{2}}{2}\left(\sqrt{2}-1\right)$

S) $\frac{\sqrt{3}}{2}+\frac{i}{2}$ D) $\frac{i}{2}-\frac{\sqrt{3}}{2}$

23) $z=\sqrt[3]{-2+2i}$

A) $1+i,\ -1{,}36\pm 0{,}365i;\ 0{,}365-1{,}36i$ B) $\frac{1}{2}+i\frac{\sqrt{3}}{2},\ 0{,}365-1{,}36i$

S) $\frac{\sqrt{2}}{2}+i\frac{1}{2};\ -1{,}36\pm 0{,}365i$ D) $2\pm 3i;\ 1+i$

24) $z=\sqrt[3]{-1+i}$

A) $\sqrt[6]{2}(\cos\varphi+i\sin\varphi),\ \varphi=45^0,\ 165^0,\ 285^0$ B) $\frac{\sqrt{2}}{2}\left(\cos\frac{2\varphi}{3}+i\sin\frac{\varphi}{3}\right)$

S) $\frac{\sqrt{2}}{3}+\frac{i}{2}$ D) $\sqrt{8}(\cos\varphi+i\sin\varphi), \varphi=45^0,165^0,285^0$

25) $z=\sqrt[4]{3}\sqrt{-8+8i\sqrt{3}}$

A) $\pm 2(\sqrt{3}+i), \pm 2(-1+i\sqrt{3})$ B) $\frac{\sqrt{2}}{2}(1+i)$

S) $\pm\frac{\sqrt{2}}{2}(1-i)$ D) $\frac{i}{2}-\frac{\sqrt{2}}{2}$

26) $z=\sqrt{i}$

A) $\pm\frac{1+i}{\sqrt{2}}$ B) $\frac{\sqrt{2}}{2}(1+i)$

S) $\pm\frac{\sqrt{2}}{2}(1-i)$ D) $\frac{i}{2}-\frac{\sqrt{2}}{2}$

Quyidagi darajalarning barcha qiymatlarini toping.

1) $1^{\sqrt{2}}$

A) $\cos(2k\sqrt{2}\pi)+i\sin(2k\sqrt{2}\pi), k\in Z$ B) $\cos\pi+i\sin\pi$

S) $2\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$ D) $\sqrt{2}(\sin x+i\cos x)$

2) $\left(\frac{1+i}{\sqrt{2}}\right)^i$

A) $e^{\frac{\pi}{4}-2\pi k}, k\in Z$ B) $e^{\frac{\pi}{2}k}, k\in Z$

S) $e^{\pi k}, k\in Z$ D) $e^{-\pi k}, k\in Z$

3) $(-1)^i$

A) $e^{(2k+1)\pi}, k\in Z$ B) $e^{2k\pi}, k\in Z$

S) $e^{-2\pi}$ D) $e^{\cos\pi k}, k\in Z$

4) $(-2)^{\sqrt{2}}$

A) $2^{\sqrt{2}}\left[\cos(2k+1)\pi\sqrt{2}+i\sin(2k+1)\pi\sqrt{2}\right]$ B) $2^{\sqrt{2}}[\cos 2k\pi+i\sin k\pi]$

S) $2^2\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$ D) $2^3\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$

II. KOMPLEKS O‘ZGARUVCHILI FUNKSIYALAR

2.1 Kompleks argumentli elementar funksiyalar

Funksiyaning ta’rifi

Kompleks $z=x+iy$ sonning geometrik tasviri tekislikdagi (x,y) nuqtadan iborat ekanligi ma’lum. Har bir songa tekislikda bitta nuqta va, aksincha, tekislikdagi har bir nuqtaga bitta kompleks son mos keladi. Barcha kompleks sonlar to‘plami bilan tekislikning barcha nuqtalari orasida o‘zaro bir qiymatli moslik o‘rnatilgan. Bu tekislik kompleks tekislik yoki Z tekislik deb ataladi.

Kompleks tekislikning birorta E to‘plami berilgan bo‘lsin. Buning geometrik ma’nosi Z tekislikdagi E nuqtalar to‘plamidan iborat. Agar E to‘plam ochiq va bog’lamli bo‘lsa, bu to‘plam soha deb ataladi. Sohani G, B, D harflarining biri bilan belgilaymiz.

Ta’rif. Agar E to‘plamdan olingan har bir $z = x + iy$ kompleks songa biror qonun bo‘yicha aniq bir kompleks son mos kelsa, u holda E to‘plamda funksiya berilgan deyiladi va bu quyidagicha yoziladi:

$$w = f(z). \tag{10}$$

Bu ta’rifdan ko‘rinadiki, w funksiyani (10) ko‘rinishdan tashqari

$$w = u(x, y) + iv(x, y) \tag{11}$$

Odatda, w funksiyaning bir ko‘rinishidan ikkinchi ko‘rinishiga o‘tish mumkin bo‘lib, uning uchun doimo $z = x + iy$ ni esda saqlash tavsiya etiladi. Ma’lumki, $z = x + iy$ erkli o‘zgaruvchi, ya’ni argument, w esa erksiz o‘zgaruvchi, ya’ni funksiya deyiladi. $w = f(z)$ funksiyaning aniqlanish sohasi deb Z tekislikdagi shunday nuqtalar to‘plami E ga aytiladiki, bu to‘plamda olingan har qanday z_0 nuqtaga chekli $f(z_0)$ kompleks son mos keladi. (11) dagi u(x,y) berilgan f(z) funksiyaning haqiqiy qismi, v(x,y) esa mavhum qismi deyiladi.

Misollar.

1. $w = z^2$ funksiya berilgan. Uning aniqlanish sohasini toping.

Yechilishi. Bu misoldan ko‘rinadiki, $z = x + iy$ ga ixtiyoriy qiymat berish mumkin, chunki har qanday sonning kvadrati mavjud. Quyidagicha belgilaymiz: $f(z) = z^2$.

U holda, misol uchun

$$f(1+i) = (1+i)^2 = 1 + 2i + i^2, \qquad i^2 = -1, \qquad i = \sqrt{-1}; \qquad f(-i) = (-i)^2 = i^2 = -1;$$

$$f\left(\frac{1}{1-i}\right) = \left(\frac{1}{1-i}\right)^2 = \frac{1}{1-2i+i^2} = -\frac{1}{2i} = \frac{i}{2}.$$

Shunday qilib, $w = z^2$ funksiyaning qniqlanish sohasi tekislikning har qanday chegaralangan qismidan iborat ekan.

2. $f(z) = \frac{1}{z}$ funksiya berilgan bo‘lsa, tekislikning noldan farqli har qanday qismi bu funksiyaning aniqlanish sohasidir.

Masalan: $f\left(\frac{1}{i}\right) = \frac{1}{\frac{1}{i}} = i;\ f(-1+i) = \frac{1}{-1+i} = \frac{-1-i}{2} = -\frac{1+i}{2};$

$$f(3-5i) = \frac{1}{3-5i} = \frac{3+5i}{3^2+5^2} = \frac{3}{34} + i\frac{5}{34}.$$

3. $f(z)=\dfrac{z-i}{z+i}$ funksiya uchun $z=-i$ nuqta uning aniqlanish sohasiga kirmaydi. Boshqa nuqtalar sohaga kiradi. Masalan: $f(i)=\dfrac{i-i}{i+i}=0;\ f(-1)=\dfrac{-1-i}{-1+i}=\dfrac{1+i}{1-i}=i.$

4. $f(z)=z^2+z$ funksiya berilgan. Bu funksiyaning ba'zi xususiy qiymatlarini topaylik:

$$f(1+i)=(1+i)^2+(1+i)=1+3i;\ f(2-i)=(2-i)^2+(2-i)=5(1-i);$$
$$f(-1)=(-1)^2+(-1)=0;\quad f(i)=(i)^2+(i)=-1+i.$$

5. $\overline{G}\ w=z^2$ funksiyaning haqiqiy va mavhum qismlarini toping.

Yechilishi. $w=f(x+iy)=(x+iy)^2=x^2+2ixy+i^2y^2=x^2-y^2+i2xy,$ demak, funksiyaning haqiqiy va mavhum qismlari: $u=x^2-y^2$ va $v=2xy.$

6. $w=\dfrac{1}{z},(z\neq 0)$ funksiyaning haqiqiy va mavhum qismlarini toping.

Yechilishi. z o'rniga $z=x+iy$ ni qo'yib, maxrajni komplekslikdan ozod qilamiz:

$$w=\frac{1}{x+iy}=\frac{x-iy}{x^2+y^2}=\frac{x}{x^2+y^2}-i\frac{y}{x^2+y^2}.$$

Demak, $u=\dfrac{x}{x^2+y^2}$ va $v=-\dfrac{y}{x^2+y^2}.$

7. $w=\overline{z}-iz^2$ funksiya berilgan. u va v larni toping.

Yechilishi. $z=x+iy$ va $\overline{z}=x+iy$ lar o'zaro qo'shma sonlardir.

$$w=(x-iy)-i(x+iy)^2=(x-iy)-i\left(x^2+2ixy-y^2\right)=(x+2xy)-i\left(x^2-y^2+y\right),$$

demak, $u=x+2xy$ va $v=-\left(x^2-y^2+y\right)$.

Mustaqil yechish uchun misollar.

1. $f(z)=z^2$ funksiya berilgan. Ushbu xususiy qiymatlarni toping:
a) $f(2+3i)$; b) $f(-1-i)$ c) $f(a+ib)$.

2. $f(z)=\dfrac{z}{\overline{z}}$ funksiya berilgan. Quyidagilarni toping:
a) $f(1+i)$; b) $f(-i)$ c) $f(-2+4i)$.

3. $f(z)=(z+i)^2$ funksiya berilgan. Quyidagilarni toping:

a) $f(i)$; b) $f(-i)$ c) $f(7+6i)$.

4. $w=z^2+i$ funksiyaning haqiqiy va mavhum qismlarini ajrating.

5. $w=iz^2-\bar{z}$ funksiyaning haqiqiy va mavhum qismlarini ajrating.

6. $w=\frac{1+i\bar{z}}{1+z}$ funksiyaning haqiqiy va mavhum qismlarini ajrating.

7. $w=\frac{z}{\bar{z}}$ funksiyaning haqiqiy va mavhum qismlarini ajrating.

2.2 Funksiyaning aniqlanish sohasi

Berilgan har qanday

$w=u+iv=f(z)$

funksiya aniqlanish sohasi G ga ega. Ma'lumki, agar chegara ham sohaga tegishli bo'lsa, u yopiq soha deyilib, $\bar{G}$ ko'rinishida yoziladi. Masala yechishda soha berilgan bo'ladi yoki funksiyaning o'ziga qarab sohani aniqlash talab qilinadi.

Misollar.

1. Ushbu $|z-\alpha|<R$ tengsizlik qanday sohani bildirishini aniqlang, bu yerda $z=x+iy$ va $\alpha=a+ib$.

Yechilishi. Ma'lumki,

$|z-\alpha|=|(x-a)+i(y-b)|=\sqrt{(x-a)^2+(y-b)^2}<R$ $z-\alpha=(x+iy)-(a+ib)=(x-a)+i(y-b)$,

yoki

$$(x-a)^2+(y-b)^2<R^2.$$

Bu markazi (a,b) nuqtada joylashgan R radiusli aylananing ichki nuqtalari to'plamidan, ya'ni doiradan iborat. Agar $\alpha=0, R=1$ bo'lsa, uni birlik doira deyiladi va :

$$|z|<1.$$

2. Ushbu $\operatorname{Re} z\geq 1$ tengsizlikni qanoatlantiruvchi nuqtalar to'plami qanday to'plamni aniqlaydi?

Yechilishi. Ma'lumki, $z=x+iy$, $\operatorname{Re} z=x\geq 1.$ Bu esa $x=1$ to'g'ri chiziq va uning o'ng tomonida yotuvchi nuqtalar to'plamidir.

3. Ushbu $Jmz<2$ tengsizlikni qanoatlantiruvchi nuqtalar to'plami qanday to'plamni aniqlaydi?

Yechilishi. Ma’lumki, $Jmz = Jm(x+iy) = y < 2.$ Bu esa $y = 2$ to‘g’ri chiziqning ostida yotuvchi nuqtalar sohasidan iborat.

4. Ushbu $-1 < Jmz < 2$ tengsizlikni qanoatlantiruvchi nuqtalar to‘plami qanday sohani tashkil etadi?

Yechilishi. 2-misolga ko‘ra $-1 < y < 2.$ Bu esa $y = -1$ va $y = 2$ to‘g’ri chiziqlar orasidagi nuqtalar to‘plamadir.

5. Ushbu $\left|z^2 - 1\right| \geq a^2,\, a > 0$ tengsizlikni qanoatlantiruvchi nuqtalar to‘plamini aniqlang.

Yechilishi. $z^2 - 1 = (x+iy)^2 - 1 = \left(x^2 - y^2 - 1\right) + 2ixy;$

$\left|z^2 - 1\right| = \sqrt{\left(x^2 - y^2 - 1\right) + (2xy)^2} = \sqrt{\left[\left(x^2 - y^2\right) - 1\right]^2 + 4x^2y^2} \geq a^2$ yoki bu yerdan

$\left(x^2 + y^2\right)^2 - 2\left(x^2 - y^2\right) \geq a^4 - 1,$ bu esa Bernulli lemniskatasi va uning tashqarisidan iborat.

6. Ushbu $4 \leq |z-1| + |z+1| \leq 8$ tengsizliklarni qanoatlantiruvchi nuqtalar to‘plamini aniqlang.

Yechilishi. $|z-1| = |(x-1) + iy| = \sqrt{(x-1)^2 + y^2},$ $|z+1| = |(x+1) + iy| = \sqrt{(x+1)^2 + y^2}.$ U holda

$4 \leq \sqrt{(x-1)^2 + y^2} + \sqrt{(x+1)^2 + y^2} \leq 8.$ Radikallar yig’indisini bir marta 4 ga, ikkinchi marta 8 ga tenglab olib, so‘ngra irrasionallikdan ozod qilsak, quyidagi ellipslarni hosil qilamiz:

$\frac{x^2}{4} + \frac{y^2}{3} = 1$ va $\frac{x^2}{16} + \frac{y^2}{15} = 1.$ Demak, izlanayotgan G to‘plam bu ellipslar orasidagi xalqadan iborat.

Mashqlar. Quyidagi tengsizliklarni qanoatlantiruvchi nuqtalar to‘plamini aniqlang.

1. $\operatorname{Re} z \leq -1$ tengsizlik qanday sohani bildiradi?
2. $Jmz > 1.$
3. $-1 \leq \operatorname{Re} z \leq 1.$
4. $1 \leq Jmz \leq 2.$
5. $1 \leq |z + 2 + i| \leq 2.$
6. $|z-1| < |z-i|.$

7. $|z-a|<|1-a\bar{z}|$, a-haqiqiy son bo‘lib, $|a|\neq 1.$

8. $Jm\left(\frac{1}{z}\right)<-\frac{1}{2}.$

9. $\left|\frac{z-3}{z-2}\right|\geq 1.$

2.3 Ba’zi egri chiziqlar

Tekislikdagi egri chiziq turli tenglamalar orqali ifoda qilinishi mumkin. Birgina egri chiziqning o‘zini dekart koordinatalari sistemasida, qutb koordinatalar sistemasida, parametric shaklda yoki vektor orqali berish mumkin. Ana shu egri chiziqni kompleks shakldagi tenglama orqali ham ifoda qilish mumkin. Misol uchun, agar tekislikdagi egri chiziqning ushbu

$$x=x(t), y=y(t), \left(t_0\leq t\leq T\right)$$

parametrik tenglamalari berilgan bo‘lsa, uni $z=x+iy=x(t)+iy(t)=z(t),$ ya’ni

$z=z(t),\ \left(t_0\leq t\leq T\right)$ ko‘rinishda yozish mumkin.

Agar tekislikdagi chiziqning dekart koordinatalari sistemasidagi

$$y=f(x),\ \left(a\leq x\leq b\right)$$

tenglamasi berilgan bo‘lsa, $z=x+iy$ va $\bar{z}=x-iy$ lardan foydalanib $x=\frac{z+\bar{z}}{2}$ va $y=\frac{z-\bar{z}}{2}$ qiymatlarni eltib qo‘yiladi: $\frac{z-\bar{z}}{2i}=f\left(\frac{z+\bar{z}}{2}\right).$

Misollar.

1. Ellipsning dekart koordinatalar sistemasidagi tenglamasi berilgan:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1.$$

Bu tenglamani kompleks ko‘rinishga keltiring.

Yechilishi. x, y larning yuqoridagi z va $\bar{z}$ lar orqali ifodasi qo‘yilsa, ellipsning kompleks shakldagi tenglamasi $\frac{\left(z+\bar{z}\right)^2}{4a^2}-\frac{\left(z-\bar{z}\right)^2}{4b^2}=1$ ko‘rinishga ega bo‘ladi.

Agar o‘sha ellips ushbu $x = a\cos t,\ y = b\sin t, (0 \le t \le 2\pi)$ parametrik tenglamalar orqali berilgan bo‘lsa, uning kompleks shakldagi tenglamasi $z = a\cos t + ib\sin t,\quad (0 \le t \le 2\pi)$ ko‘rinishni oladi. Xususiy holda, agar $b = a = r$ bo‘lsa, ellipsdan aylana hosil bo‘lib, uning tenglamasi Eyler formulasiga muvofiq ixcham $z = r(\cos t + i\sin t) = re^{it}, (0 \le t \le 2\pi)$ ko‘rinishga keladi. Bu aylananing birinchi xil tenglamasini quyidagicha yozish mumkin: $\left(z+\bar{z}\right)^2 - \left(z-\bar{z}\right)^2 = (2r)^2$. bu yerda $2r$ - aylana diametri.

2. Ushbu $\operatorname{Re} z^2 = a^2$ tenglamani qanoatlantiruvchi nuqtalar to‘plami qanday chiziqni aniqlaydi?

Yechilishi. $z^2 = (x+iy)^2 = x^2 - y^2 + 2ixy, \operatorname{Re} z^2 = x^2 - y^2 = a^2.$ Bu tengyoqli giperbolaning tenglamasidir.

3. Ushbu $|z-i| = |z+2|$ tenglama qanday chiziqni ifodalaydi?

Yechilishi. Ma’lumki,

$$|z-i| = |x + i(y-1)| = \sqrt{x^2 + (y-1)^2},$$

$$|z+2| = |(x+2) + iy| = \sqrt{(x+2)^2 + y^2}.$$

Bunga asosan:

$\sqrt{x^2 + (y-1)^2} = \sqrt{(x+2)^2 + y^2}$, bu tenglikning ikki tomonini kvadratga ko‘tarib ixchamlashtirilsa, ushbu $4x + 2y + 3 = 0$ to‘g’ri chiziq tenglamasi kelib chiqadi. Bu chiziq $z_0 = -2$ va $z_1 = -i$ nuqtalarni tutashtiruvchi kesmaning o‘rtasidan o‘tadigan perpendikulyar ekanligini ko‘rsatish mumkin.

4. Ushbu $\arg(z-i) = \frac{\pi}{4}$ tenglama qanday chiziqni ifodalaydi?

Yechilishi. Ma’lumki,

$z - i = x + iy - i = x + i(y-1), tg\varphi = \frac{y-1}{x}.$ Berilgan misolda $\varphi = \frac{\pi}{4}.$

Demak, $\frac{y-1}{x} = tg\frac{\pi}{4} = 1$ yoki $y - x - 1 = 0.$ Bu esa $z_0 = i$ nuqtadan o‘tib, Ox o‘qning musbat tomoni bilan $\frac{\pi}{4}$ burchak hosil qiladigan to'g’ri chiziqdir.

5. Ushbu $2z\bar{z}+(2+i)z+(2-i)\bar{z}=2$ tenglama qanday chiziqni ifodalaydi?

Yechilishi. Ma’lumki,

$$2z\bar{z}=2(x+iy)(x-iy)=2\left(x^2+y^2\right),$$

$$(2+i)z+(2-i)\bar{z}=2(z+\bar{z})+i\left(z-\bar{z}\right)=4x-2y.$$

Shunga asosan berilgan tenglamaning chap tomoni quyidagi

$$2\left(x^2+y^2\right)+4x-2y=2$$

Yoki

$$\left(x+1\right)^2+\left(y-\frac{1}{2}\right)^2=\frac{9}{4}$$

aylana tenglamasini ifodalaydi.

6. Ushbu $|z|-3Jmz=6$ tenglama qanday chiziqni ifodalaydi?

Yechilishi. Ma’lumki, $|z|=|x+iy|=\sqrt{x^2+y^2}$, $Jmz=Jm(x+iy)=y.$ Shu sababli $\sqrt{x^2+y^2}=3(2+y),$ buni irrasionallikdan ozod qilib, ixchamlashtirgandan so‘ng giperbola tenglamasi $\dfrac{\left(y+\frac{9}{4}\right)^2}{\left(\frac{3}{4}\right)^2}-\dfrac{x^2}{\left(\frac{3\sqrt{2}}{2}\right)^2}=1$ kelib chiqadi. Demak, giperbolani aniqlar ekan.

7. To‘g’ri chiziqning ushbu $Ax+By+C=0$ tenglamasini kompleks shaklga keltiring.

Yechilishi. Ma’lumki, $z=x+iy$ va $\bar{z}=x-iy;$ $x=\dfrac{z+\bar{z}}{2}$ va $y=\dfrac{z-\bar{z}}{2}.$

Bularni berilgan tenglamaga qo‘yib chiqamiz, u holda

$A\dfrac{z+\bar{z}}{2}+B\dfrac{z-\bar{z}}{2i}+C=0$ yoki $\left(A+iB\right)\bar{z}+\left(A-iB\right)z+2C=0.$ $A+iB=\alpha$ va $A-iB=\bar{\alpha}$ deb belgilab, ixchamlashtirishdan so‘ng: $\alpha\bar{z}+\bar{\alpha}z+2C=0.$

Namunaviy variantlar

1. Kompleks sonlar ustida amallarni bajaring.

1.	$(1+4i)\cdot(2-3i)+\dfrac{2i(5+2i)}{1+2i}$	2.	$\dfrac{(2-6i)\cdot i}{-4+2i}-(1-i)^2$
3.	$\dfrac{5+i}{-1-2i}+\dfrac{2+3i}{i}$	4.	$\dfrac{(1-5i)\cdot(2+i)}{-1+i}-i^{7}(2-3i)$
5.	$(2-i)^2+\dfrac{3+i}{1-2i}$	6.	$\dfrac{4-5i^3}{1+i}-3i(5+2i)$
7.	$\dfrac{(1-2i)(1+i)}{3-i}-2i(2-i)$	8.	$\dfrac{5+3i}{1+3i}-i(2+3i)$
9.	$(3-2i)^2+\dfrac{9-8i}{4+2i}-i^5$	10.	$(-1+i)\cdot(3+2i)+\dfrac{i(6-4i)}{2+2i}$
11.	$5-3i+\dfrac{i^3(2-i)}{2+i}$	12.	$(4-i)^2+\dfrac{1+8i^3}{4-2i}$
13.	$\dfrac{(1-2i)^2}{3+i}-1+i$	14.	$\dfrac{5i+2i^6}{1-i}-3+2i$
15.	$\dfrac{i^5(6-i)}{-2+i}-2+3i$	16.	$\dfrac{(1+2i)\cdot(3-i)}{2-i}-i(5+3i)$
17.	$\dfrac{i}{-1+3i}-1+4i^5$	18.	$\dfrac{(1-i)\cdot(5+i)}{-3+i}-i^{3}(1+i)$
19.	$\dfrac{(1+5i)\cdot(1-i)}{-1+2i}-3i$	20.	$\dfrac{2+4i}{1-3i}-i^3(1+3i)$

2. Tenglamaning haqiqiy yechimini toping.

	$(2-i)^2 x+(3-2i)\,y=-2i$
	$(5+2i)\,x+(1-3i)\,y=x+y+8-5i$
	$(1+4i)\,x+(5-2i)\,y=(3+i)x-(2+3i)y+3+7i$
	$(3+5i)\,x+(1-2i)\,y=(3-4i)i$
	$(5+i)^2 x-y=(1+i)\,x+9i$
	$(2+i)\,ix+(4-i)\,y=y+5i$
	$(5+i)\,x+(4-2i)\,y=ix-(2+i)y+4+i$
	$(2-i)\,x+(-5+2i)\,y=1-i$
	$(1+3i)\,x+(2-i)^2\,y=(-1-4i)i$
	$(3-i)\,x+(2+2i)\,y=(1+2i)x-iy$
	$(2+3i)\,x+(1-i)\,y=1+9i$
	$(3-2i)\,x+(1+4i)\,y=5+6i$
	$(6-i)\,x+(3+2i)\,y=x-13i+13$
	$(5-2i)\,x+(1+4i)\,y=7+6i$
	$(-4+i)\,x+(3-2i)\,y=-7+3i$
	$\dfrac{2+i}{i}\,x-(4+2i)\,y=3+4i$
	$(5+i)\,x-(1+i)\,y=-7-3i$
	$(2+i)\,x+(3-2i)\,y=(1-i)x+(4+i)\,y$
	$(7-i)\,x+(-2+4i)\,y=11+x$
	$x+(-1+3i)\,y=1-6i$

3. Sohani aniqlang.

1	$\operatorname{Im}(\bar{z})>-1$	2.	$-1\le \operatorname{Re}(z)\le 3$

3.	$0 < \arg z < \frac{\pi}{2}$	4.	$1 \le \lvert z-3 \rvert \le 3$
5.	$\lvert z-i \rvert < 5$	6.	$\lvert z \rvert^2 = (\mathrm{Re}\, z)^2 + 9$
7.	$\mathrm{Im}(z-4i) > 0$	8.	$\lvert z+4i \rvert < 4$
9.	$\mathrm{Re}(z \cdot i) > 3$	10.	$\mathrm{Im}(z^2) \le 2$
11.	$\frac{\pi}{2} \le \frac{\pi}{2} + \arg z \le \pi$	12.	$\mathrm{Re}(z+2) \ge 0$
13.	$\mathrm{Re}(z-2) \le 1$	14.	$\mathrm{Im}(z \cdot i) < -1$
15.	$2 < \mathrm{Im}(z) \le 4$	16.	$\mathrm{Re}(z^2) = 0$
17.	$\frac{\pi}{4} < \arg z < \frac{\pi}{2}$	18.	$\mathrm{Im}(z+i) \le 3$
19.	$\lvert z+1-2i \rvert \le 2$	20.	$\lvert z \rvert > (\mathrm{Re}\, z)^2 - 4$

4. z_1 va z_2 kompleks sonlarni trigonometrik va eksponensial ko‘rinishda ifodalang

1	$z_1 = 2 + 2\sqrt{3}i$, $z_2 = 3 - 3i$	2.	$z_1 = -4\sqrt{3} + 4i$, $z_2 = 0{,}5 + 0{,}5i$
3.	$z_1 = -3 + 3i$, $z_2 = \sqrt{3} + i$	4.	$z_1 = -7 + 7\sqrt{3}i$, $z_2 = 3\sqrt{3} + 3i$
5.	$z_1 = -\sqrt{3} - i$, $z_2 = -5i$	6.	$z_1 = 4 - 4\sqrt{3}i$, $z_2 = 0{,}5i$

7.	$z_1 = -2 - 2i$, $z_2 = 1 + i\sqrt{3}$	8.	$z_1 = 6\sqrt{3} + 6i$, $z_2 = -\sqrt{2} - \sqrt{2}\,i$
9.	$z_1 = -3 - 3\sqrt{3}\,i$, $z_2 = -2i$	10.	$z_1 = -2 + 2\sqrt{3}\,i$, $z_2 = -0{,}5i$
11.	$z_1 = -\frac{1}{4} + \frac{\sqrt{3}}{4}i$, $z_2 = 2\sqrt{3} + 2i$	12.	$z_1 = 4\sqrt{3} + 4i$, $z_2 = \sqrt{2} - \sqrt{2}\,i$
13.	$z_1 = 1 - \sqrt{3}\,i$, $z_2 = 4 + 4i$	14.	$z_1 = 5 + 5\sqrt{3}\,i$, $z_2 = -2\sqrt{3} + 2i$
15.	$z_1 = 2 - 2\sqrt{3}\,i$, $z_2 = \sqrt{2} + \sqrt{2}\,i$	16.	$z_1 = -2\sqrt{3} + 2i$, $z_2 = 4i$
17.	$z_1 = -2 - 2\sqrt{3}\,i$, $z_2 = 6\sqrt{3} + 6i$	18.	$z_1 = \sqrt{3} - i$, $z_2 = 4 + 4i$
19.	$z_1 = -3 - 3i$, $z_2 = 4\sqrt{3} + 4i$	20.	$z_1 = -3 + 3\sqrt{3}\,i$, $z_2 = 3\sqrt{3} - 3i$

5.4-misol uchun quyidagilarni hisoblang.

1	$z_1^5 \cdot z_2$, $\frac{z_1}{z_2}$, $\sqrt[4]{z_2}$	2.	$z_1 \cdot z_2$, $\frac{z_1^5}{z_2}$, $\sqrt[3]{z_2}$
3.	$z_1 \cdot z_2^5$, $\frac{z_1}{z_2}$, $\sqrt[3]{z_2^5}$	4.	$z_1 \cdot z_2$, $\frac{z_1}{z_2^5}$, $\sqrt[3]{z_1}$
5.	$z_1^7 \cdot z_2$, $\frac{z_1}{z_2}$, $\sqrt[4]{z_2}$	6.	$z_1 \cdot z_2$, $\frac{z_1^4}{z_2}$, $\sqrt[4]{z_1}$
7.	$z_1 \cdot z_2$, $\frac{z_1^8}{z_2}$, $\sqrt[4]{z_2}$	8.	$z_1^5 \cdot z_2$, $\frac{z_1}{z_2}$, $\sqrt[4]{z_1^5}$

9.	$z_1\cdot z_2,\ \frac{z_1^3}{z_2},\ \sqrt[5]{z_2}$	10.	$z_1\cdot z_2,\ \frac{z_1}{z_2},\ \sqrt[4]{z_1^3}$
11.	$z_1^5\cdot z_2,\ \frac{z_1}{z_2},\ \sqrt[3]{z_2}$	12.	$z_1\cdot z_2,\ \frac{z_1}{z_2^3},\ \sqrt[5]{z_1}$
13.	$z_1\cdot z_2^6,\ \frac{z_1}{z_2},\ \sqrt[4]{z_1}$	14.	$z_1\cdot z_2^7,\ \frac{z_1}{z_2},\ \sqrt[3]{z_2}$
15.	$z_1\cdot z_2,\ \frac{z_1^5}{z_2},\ \sqrt[4]{z_1^5}$	16.	$z_1^3\cdot z_2,\ \frac{z_1}{z_2},\ \sqrt[4]{z_1^3}$
17.	$z_1\cdot z_2^5,\ \frac{z_1}{z_2},\ \sqrt[3]{z_1}$	18.	$z_1\cdot z_2,\ \frac{z_1}{z_2^7},\ \sqrt[4]{z_1}$
19.	$z_1^5\cdot z_2,\ \frac{z_1}{z_2},\ \sqrt[3]{z_2}$	20.	$z_1\cdot z_2,\ z_1 : z_2^3,\ \sqrt[4]{z_2^3}$

6. Quyidagi berilganlarga asosan analitik $f(z)=u+i\,v$ kompleks o‘zgaruvchili funksiyani tuzing.

1.	$u=x^3-3xy^2,\quad f(0)=i;$	2.	$u=x^2-y^2+2x,\ f(i)=-1+2i$
3.	$u=2e^x\cos y,\quad f(0)=2;$	4.	$u=\mathrm{arctg}\frac{y}{x},\quad x>0,\quad f(1)=0$;
5.	$u=x^2-y^2+5x;$	6.	$v=4-2y;$
7.	$u=\frac{y}{y^2+x^2};$	8.	$u=3+x^2-y^2;$
9.	$v=x^2-y^2+2;$	10.	$v=e^x\sin y;$
1	$u=x^2-y^2+xy,\quad f(0)=0;$	1	$u=x^3-3xy^2,\quad f(0)=0;$

1.		2.	
13.	$u=\frac{y}{y^2+x^2}, \quad f(2)=0;$	14.	$v=e^x\cos y, \quad f(0)=1;$
15.	$u=x^2-y^2+11, \ f(0)=11-7i$	16.	$u=2x^3-6xy^2, \quad f(0)=0;$
17.	$v=\frac{-y}{x^2+y^2}, \quad f(1)=1;$	18.	$v=2xy+5y, \quad f(0)=0;$
19.	$u=x^2-y^2, \quad f(0)=0;$	20.	$v=e^x\cos y, \quad f(0)=i;$

7. Chegirmalar yordamida integrallarni hisoblang.

1.	$\int\limits_{\lvert z\rvert=2}\frac{z^2+z-1}{z^2(z-1)}dz;$	2.	$\int\limits_{\lvert z\rvert=\frac{1}{2}}\frac{dz}{z^3-z^5};$
3.	$\int\limits_{\lvert z+i\rvert=1}\frac{z^2}{\left(z^2+1\right)^2}dz;$	4.	$\int\limits_{\lvert z+i\rvert=0,7}\frac{dz}{z\left(1+z^2\right)};$
5.	$\int\limits_{\lvert z-2\rvert=\frac{3}{2}}\frac{e^{iz}}{z(z-\pi)}dz;$	6.	$\int\limits_{\lvert z\rvert=1}\frac{e^z}{z^2\left(z^2-9\right)}dz;$
7.	$\int\limits_{\lvert z-2\rvert=\frac{1}{2}}\frac{z}{(z-1)(z-2)^2}dz;$	8.	$\int\limits_{\lvert z\rvert=1}\frac{e^{2z}}{\left(z-\frac{\pi}{4}i\right)}dz;$
9.	$\int\limits_{\lvert z-1-i\rvert=2}\frac{dz}{(z-1)^2\left(z^2+1\right)^2};$	10.	$\int\limits_{\lvert z\rvert=3}\frac{z-1}{z^2+4}dz;$

11.	$\int\limits_{\lvert z\rvert=2} \operatorname{tg} z\, dz;$	12.	$\int\limits_{\lvert z\rvert=\frac{3}{2}} \frac{dz}{(1+z)^2(z+2)};$
13.	$\int\limits_{\lvert z\rvert=2} \frac{\sin z}{z\left(z-\frac{\pi}{2}\right)} dz;$	14.	$\int\limits_{\lvert z\rvert=1} \frac{dz}{\sin z};$
15.	$\int\limits_{\lvert z\rvert=2} \frac{z-1}{z^2+1} dz;$	16.	$\int\limits_{\lvert z\rvert=1} \frac{z^3}{\sin z} dz;$
17.	$\int\limits_{\lvert z\rvert=2} \frac{\sin z}{\left(z-\frac{\pi}{2}\right)^2} dz;$	18.	$\int\limits_{\lvert z\rvert=r} \sin\frac{1}{z}\, dz;$
19.	$\int\limits_{\lvert z\rvert=1} \frac{z^3}{2z^4+1} dz;$	20.	$\int\limits_{x^2+y^2=2x} \frac{dz}{z^4+1};$

Adabiyotlar:

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Mundarija