2. $\mathbf{R}$ to‘g‘ri chiziqda $\rho(x,y)=|arctgx-arctgy|$ akslantirish metrika bo‘lishini tekshiring. $\rho_1(x,y)=arctg|x-y|$ akslantirish $\mathbf{R}$ to‘plamda metrika bo‘ladimi?

3. $\mathbf{R}^n$ to‘plamda ushbu akslantirishlar metrika bo‘ladimi?

$$\rho_1(x,y)=\sum_{k=1}^{n}|x_k-y_k|; \qquad \rho_2(x,y)=\left|\max_{1\le k\le n}(x_k-y_k)\right|;$$

$$\rho_3(x,y)=\begin{cases}1, & agar \quad x\neq y\\ 0, & agar \quad x=y\end{cases}; \qquad \rho_4(x,y)=\sum_{k=1}^{n} sign|x_k-y_k| .$$

Quyidagi misollarda $\rho: X\times X\to\mathbf{R}$ akslantirish metrikaning qaysi shartini qanoatlantirmasligini aniqlang ().

7. $\rho(x,y)=(x-y)^2, \quad x,y\in\mathbf{R}$.

8. $\rho(x,y)=|x_1-y_1|+|x_2-y_2|, \quad x,y\in\mathbf{R}^3$.

9. $\rho(f,g)=|f(0)-g(0)|+|f(1)-g(1)|, \quad f,g\in C[a,b]$.

Quyidagi misollarda $\rho: X\times X\to\mathbf{R}$ akslantirish metrikaning qaysi shartir qanoatlantirmasligini aniqlang.

13. $\rho(x,y)=(x-y)^2, \quad x,y\in\mathbf{R}$.

14. $\rho(x,y)=|\sin x-\sin y|, \quad x,y\in\mathbf{R}$.

15. $\rho(x,y)=\begin{cases}0, & x=y\\ 1, & x<y\\ 2, & x>y\end{cases}, \quad x,y\in\mathbf{R}$.

16. $\rho(x,y)=\begin{cases}0, & x=y\\ EKUK(x,y), & x\neq y\end{cases}, \quad x,y\in\mathbf{N}$.

17. $\rho(x,y)=\begin{cases}0, & x=y\\ EKUB(x,y), & x\neq y\end{cases}, \quad x,y\in\mathbf{N}$.

18. $\rho(x,y)=|x_1-y_1|+|x_2-y_2|, \quad x,y\in\mathbf{R}^3$.

19. $\rho(f,g)=|f(0)-g(0)|+|f(1)-g(1)|, \quad f,g\in C[a,b]$.

20. $\rho(x,y)=|x_1-y_2|+|y_1-x_2|, \quad x,y\in\mathbf{R}^2$.

21. $\rho(x,y)=|x_1-y_1|+2|x_2-y_2|^2, \quad x,y\in\mathbf{R}^2$.

Quyidagi misollarda $\rho: X\times X\to\mathbf{R}$ akslantirishning metrika shartlarini qanoatlantirishini tekshiring.

2. $$\rho(x,y)=\sqrt{\sum_{i=1}^{n}|x_i-y_i|^2}, \quad x,y\in\mathbf{R}^n.$$

3. $$\rho_\infty(x,y)=\max_{1\le i\le n}|x_i-y_i|, \quad x,y\in\mathbf{R}^n.$$

4. $$\rho_p(x,y)=\sqrt[p]{\sum_{i=1}^{n}|x_i-y_i|^p}, \quad p\ge 1, \quad x,y\in\mathbf{R}^n.$$

10. $$\rho_1(f,g)=\int_a^b|f(x)-g(x)|dx, \quad f,g\in C[a,b].$$

11. $$\rho_2(f,g)=\sqrt{\int_a^b|f(x)-g(x)|^2 dx}, \quad f,g\in C[a,b].$$

12. $$\rho_p(f,g)=\sqrt[p]{\int_a^b|f(x)-g(x)|^p dx}, \quad p\ge 1, \quad f,g\in C[a,b].$$

6.1.7 Example. We can define another metric on $\mathbb{R}^2$ by putting

$$d^*(\langle a_1,a_2\rangle,\langle b_1,b_2\rangle)=\max\{|a_1-b_1|,|a_2-b_2|\}$$

where $\max\{x,y\}$ equals the larger of the two numbers $x$ and $y$.

6.1.8 Example. Yet another metric on $\mathbb{R}^2$ is given by

$$d_1(\langle a_1,a_2\rangle,\langle b_1,b_2\rangle)=|a_1-b_1|+|a_2-b_2|.$$

2. Let $d$ be a metric on a non-empty set $X$.

   (i) Show that the function $e$ defined by $e(a,b)=\min\{1,d(a,b)\}$ where $a,b\in X$, is also a metric on $X$.

3. (i) Let $d$ be a metric on a non-empty set $X$. Show that the function $e$ defined by

$$e(a,b) = \frac{d(a,b)}{1+d(a,b)}$$

where $a, b \in X$, is also a metric on $X$.

4. Let $d_1$ and $d_2$ be metrics on sets $X$ and $Y$ respectively. Prove that

(i) $d$ is a metric on $X \times Y$, where

$$d(\langle x_1, y_1\rangle, \langle x_2, y_2\rangle) = \max\{d_1(x_1,x_2), d_2(y_1,y_2)\}.$$

**5.** Bizga $X = R^n$ to‘plam va $d_2(x,y) = \max\limits_{i=\overline{1,n}}\{|x_i - y_i|\}$ funksiya berilgan bo‘lsin. Berilgan $d_2(x,y)$ - funksiya metrika ekanligi isbotlansin.

**6.** Barcha haqiqiy sonlar to‘plami $R$ ushbu $d(x,y) = \sqrt{|x-y|}$ metrika bilan metrik fazo bo‘ladimi?

**9.** Bizga $[0;1]$ kesmada uzluksiz funksiyalar to‘plami $C_{[0,1]}$ va $d_3(f,g) = \max\limits_{x\in[0;1]}|f(x) - g(x)|$ funksiya berilgan bo‘lsin. Bu $d_3(f,g)$ - funksiya metrika ekanligi ko‘rsatilsin.

**10.** Bizga $[0;1]$ kesmada uzluksiz funksiyalar to‘plami $C_{[0,1]}$ va $d_4(f,g) = \int_0^1 |f(t) - g(t)|dt$ funksiya berilgan bo‘lsin. Bu $d_4(f,g)$ - funksiya metrika ekanligi ko‘rsatilsin.

1. $(X,\rho)$ metrik fazo bo‘lsa, $X$ to‘plamda

$$\rho_1(x,y) = \frac{\rho(x,y)}{1+\rho(x,y)}; \qquad \rho_2(x,y) = \ln(1+\rho(x,y));$$

$$\rho_3(x,y) = e^{\rho(x,y)} - 1; \qquad \rho_4(x,y) = \min\{1; \rho(x,y)\}$$

akslantirishlarning har biri metrika bo‘lishini isbotlang.

**1.** Berilgan $(X,d)$ metrik fazo uchun quyidagi tengsizliklarni isbotlang:

a) $|d(x,z) - d(z,y)| \le d(x,y)$ bu yerda $x, y, z \in (X,d)$;

b) $d(x,Z) \le d(x,y) + d(Z,y)$, bu yerda $Z \subset (X,d)$;

c) $|d(x,Z) - d(Z,y)| \le d(x,y)$, bu yerda $Z \subset (X,d)$.