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# ALGEBRA

UMUMIY O'RTA TA'LIM MAKTABLARINING
9-SINFI UCHUN DARSLIK

Qayta ishlangan 4-nashri

*O'zbekiston Respublikasi Xalq ta'limi vazirligi tomonidan nashrga tavsiya etilgan*



„O'QITUVCHI" NASHRIYOT-MATBAA IJODIY UYI
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**Darslikdagi shartli belgilar:**

- ! – bilish muhim va eslab qolish foydali (yodlash shart emas) matn;
- △ – masalani yechish boshlandi;
- ▲ – masalani yechish tugadi;
- ○ – matematik tasdiqni asoslash yoki formulani keltirib chiqarish boshlandi;
- ● – asoslash yoki formulani keltirib chiqarish tugadi;
- ═ – yechilishi majburiy masalalarni ajratib turuvchi belgi;
- 33,34... – murakkabroq masala;
- ‖ – asosiy materialni ajratish;
- O‘zingizni tekshirib ko‘ring! – asosiy material bo‘yicha bilimni tekshirish uchun mustaqil ish;
- – amaliy-tatbiqiy va fanlararo bog‘liq masalalar;
- – tarixiy masalalar;
- – tarixiy ma’lumotlar.

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# 8-SINFDA O‘RGANILGAN MAVZULARNI TAKRORLASH

*Aziz o‘quvchi! 8-sinfda „Algebra“dan olgan bilimlaringizni yodga solish maqsadida Sizga bir nechta mashqlar taklif etamiz.*

**1.** 1) $y=2x+3$; 2) $y=-3x+4$; 3) $y=4x-1$; 4) $y=-2x-5$ funksiya grafigini chizing. Grafik qaysi choraklarda yotadi? Grafikning $Ox$ va $Oy$ o‘qlar bilan kesishish nuqtalari koordinatalarini ayting.

**2.** $y=kx+b$ funksiya grafigi $A(0;\ -7), B(2;\ 3)$ nuqtalardan o‘tadi. $k$ va $b$ ni toping.

**3.** To‘g‘ri chiziq $A(0;\ 5), B\ (1;\ 2)$ nuqtalardan o‘tadi. Shu to‘g‘ri chiziq tenglamasini yozing.

**4.** Tenglamalar sistemasini yeching:

1) $\begin{cases} 7x+4y=29; \\ 5x+2y=19; \end{cases}$ 2) $\begin{cases} 5x-4y=13; \\ 2x-y=4. \end{cases}$

**5.** 3 ta ot va 4 ta sigirga bir kunda 27 kg yem beriladi. Bir kunda 9 ta otga berilgan yem 5 ta sigirga berilgan yemdan 30 kg ko‘p. Bitta otga va bitta sigirga 1 kunda qancha yem beriladi?

**6.** Kitob va daftar birgalikda 5800 so‘m turadi. Kitob narxining 10% i daftar narxining 35% idan 220 so‘m qimmat. Kitob va daftar alohida-alohida necha so‘m turadi?

**7.** Tengsizlikni yeching:

1) $3(x-4)+5x<2x+3$; 2) $|5-2x|\le 3$; 3) $|3x-4|\ge 2$.

**8.** Tengsizliklar sistemasini yeching:

1) $\begin{cases} 4(2-x)>7-5x, \\ 15-4x<3; \end{cases}$ 2) $\begin{cases} 2(3-2x)>8-5x, \\ 10-x>2. \end{cases}$

**9.** $\frac{3x+4}{2}-\frac{1-x}{3}<\frac{7x-3}{2}-\frac{3-x}{3}$ tengsizlikning eng kichik butun yechimini toping.

**10.** Hisoblang:

1) $\sqrt{121\cdot 0{,}04\cdot 289}$; 2) $\sqrt{5\frac{1}{7}\cdot 3\frac{4}{7}}$; 3) $(\sqrt{32}+\sqrt{8})^2$.

**11.** Soddalashtiring:

1) $(8\sqrt{63}+3\sqrt{28}-5\sqrt{112}):2\sqrt{7}$;

2) $(15\sqrt{1{,}2}+\frac{1}{3}\sqrt{270}-2\sqrt{30}$;

3) $\frac{2}{\sqrt{11}+3}+\frac{7}{\sqrt{11}-2}$;

4) $\frac{4}{3-\sqrt{5}}+\frac{1}{2-\sqrt{5}}+\frac{3\sqrt{5}}{4}$.

Tenglamani yeching (**12–14**):

**12.** 1) $|7-x|=-7$; 2) $|x+6|=x+10$; 3) $\sqrt{(x-9)^2}=x-9$.

**13.** 1) $x^2-12x+11=0$; 2) $x^2-15x+56=0$;
3) $6x^2+7x-3=0$; 4) $16x^2+8x+1=0$.

**14.** 1) $x^4-10x^2+9=0$; 2) $10x^4+7x^2+1=0$.

**15.** 240 km masofani bir avtomobil ikkinchisiga qaraganda 1 soat tezroq bosib o‘tdi. Agar birinchi avtomobilning tezligi ikkinchisining tezligidan 20 km/h ortiq bo‘lsa,har bir avtomobilning tezligini toping.

**16.** 1) Ikki sonning ayirmasi 2,5 ga, kvadratlarining ayirmasi esa 10 ga teng. Shu sonlarni toping.

2) Yig‘indisi 1,4 ga,kvadratlarining yig‘indisi 1 ga teng bo‘lgan ikkita sonni toping.

**17.** $x^2-8x+3=0$ tenglamaning ildizlari $x_1$ va $x_2$ bo‘lsa, 1) $x_1^2+x_2^2$; 2) $x_1^3+x_2^3$; 3) $x_1^2x_2+x_1x_2^2$; 4) $x_1^2-x_2^2$ ni toping.

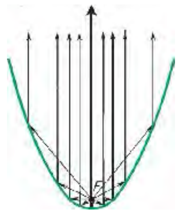
**18.** Sonni yuzdan birgacha yaxlitlang. Yaxlitlashning nisbiy xatoligini toping:

1) 6,7893; 2) 5,6409; 3) 0,9871; 4) 0,8245.

**19.** Sonni standart shaklda yozing:

1) 437,105; | 2) 91,352; | 3) 0,000 000 85; | 4) 0,000 079.

# I BOB. KVADRAT FUNKSIYA. KVADRAT TENGSIZLIKLAR



## 1-§. KVADRAT FUNKSIYANING TA'RIFI

Siz 8-sinfda $y=kx+b$ chiziqli funksiya va uning grafigi bilan tanishgansiz.

Fan va texnikaning turli sohalarida *kvadrat funksiyalar* deb ataladigan funksiyalar uchraydi. Misollar keltiramiz.

1) Tomoni $x$ bo'lgan kvadratning yuzi $y = x^2$ formula bo'yicha hisoblanadi.

2) Agar jism yuqoriga $v$ tezlik bilan otilgan bo'lsa, u holda $t$ vaqtda undan Yer sirtigacha bo'lgan masofa $s=-\frac{gt^2}{2}+vt+s_0$ formula bilan aniqlanadi, bunda $s_0$ –vaqtning $t=0$ boshlang'ich paytidagi jismdan Yer sirtigacha bo'lgan masofa.

Bu misollarda $y=ax^2+bx+c$ ko'rinishdagi funksiyalar qaraldi. Birinchi misolda $a-1, b-c-0$, o'zgaruvchilar esa $x$ va $y$ lar bo'ladi. Ikkinchi misolda $a=-\frac{g}{2}$, $b=v$, $c=s_0$, o'zgaruvchilar esa $t$ va $s$ harflari bilan belgilangan.

**!** ***Ta'rif.*** ***$y=a^2+bx+c$ funksiya kvadrat funksiya deyiladi, bunda $a$, $b$ va $c$ – berilgan haqiqiy sonlar, $a \neq 0$, $x$ – haqiqiy o'zgaruvchi.***

Masalan, quyidagi funksiyalar kvadrat funksiyalardir:

$y = x^2$, $\quad y--2x^2$, $\quad y-x^2-x$,

$y=x^2-5x+6$, $\quad y=-3x^2+\frac{1}{2}x$.

**1-masala.** $x = -2, x = 0, x = 3$ bo‘lganda

$$y(x) = x^2 - 5x + 6$$

funksiyaning qiymatini toping.

△ $y(-2) - (-2)^2 - 5 \cdot (-2) + 6 - 20;$

$y(0) - 0^2 - 5 \cdot 0 + 6 - 6;$

$y(3) - 3^2 - 5 \cdot 3 + 6 - 0.$ ▲

**2-masala.** $x$ ning qanday qiymatlarida $y - x^2 + 4x - 5$ kvadrat funksiya: 1) 7 ga; 2) −9 ga; 3) −8 ga; 4) 0 ga teng qiymatni qabul qiladi?

△ 1) Shartga ko‘ra $x^2 + 4x - 5 - 7$. Bu tenglamani yechib, quyidagini hosil qilamiz:

$$x^2 + 4x - 12 = 0,$$

$$x_{1,2} = -2 \pm \sqrt{4 + 12} = -2 \pm 4, \; x_1 = 2, \; x_2 = -6.$$

Demak, $y(2) = 7$ va $y(-6) = 7$.

2) Shartga ko‘ra $x^2 + 4x - 5 = -9$, bundan

$$x^2 + 4x + 4 = 0, \; (x + 2)^2 = 0, \quad x = -2.$$

3) Shartga ko‘ra $x^2 + 4x - 5 = -8$, bundan $x^2 + 4x + 3 = 0$. Bu tenglamani yechib, $x_1 = -3$, $x_2 = -1$ ekanini topamiz.

4) Shartga ko‘ra $x^2 + 4x - 5 = 0$, bundan $x_1 = 1$, $x_2 = -5$. ▲

Oxirgi holda $x$ ning $y = x^2 + 4x - 5$ funksiya 0 ga teng, ya’ni $y(1) = 0$ va $y(-5) = 0$ bo‘lgan qiymatlari topildi. $x$ ning bunday qiymatlari *kvadrat funksiyaning nollari* deyiladi.

**3-masala.** $y = x^2 - 3x$ funksiyaning nollarini toping.

△ $x^2 - 3x = 0$ tenglamani yechib, $x_1 = 0$, $x_2 = 3$ ekanini topamiz. ▲

## *Mashqlar*

**1.** (Og‘zaki.) Quyida ko‘rsatilgan funksiyalardan qaysilari kvadrat funksiya bo‘ladi:

1) $y = 2x^2 + x + 3;$ 2) $y = 3x^2 - 1;$ 3) $y = 5x + 1;$
4) $y = x^3 + 7x - 1;$ 5) $y = 4x^2;$ 6) $y = -3x^2 + 2x?$

**2.** $x$ ning shunday haqiqiy qiymatlarini topingki, $y = x^2 - x - 3$ kvadrat funksiya: 1) −1 ga; 2) −3 ga; 3) $-\frac{13}{4}$ ga; 4) −5 ga teng qiymat qabul qilsin.

**3.** $x$ ning qanday haqiqiy qiymatlarida $y = -4x^2 + 3x - 1$ kvadrat funksiya: 1) $-2$; 2) $-8$; 3) $-0{,}5$; 4) $-1$ ga teng qiymat qabul qiladi?

**4.** $-2$; $0$; $1$; $\sqrt{3}$ sonlaridan qaysilari quyidagi kvadrat funksiyaning nollari bo'ladi:
1) $y = x^2 + 2x$; 2) $y = x^2 + x$; 3) $y = x^2 - 3$;
4) $y = 5x^2 - 4x - 1$; 5) $y = x^2 - x$; 6) $y = x^2 + x - 2$?

**5.** Kvadrat funksiyaning nollarini toping:
1) $y = x^2 - x$; 2) $y = x^2 + 3$;
3) $y = 12x^2 - 17x + 6$; 4) $y = -6x^2 + 7x - 2$;
5) $y = 3x^2 - 5x + 8$; 6) $y = 2x^2 - 7x + 9$.

**6.** Agar $y = x^2 + px + q$ kvadrat funksiyaning $x_1$ va $x_2$ nollari ma'lum bo'lsa, $p$ va $q$ koeffitsiyentlarni toping:
1) $x_1 = 2$, $x_2 = 3$; 2) $x_1 = -4$, $x_2 = 1$;
3) $x_1 = -1$, $x_2 = -2$; 4) $x_1 = 5$, $x_2 = -3$.

**7.** $x$ ning $y = x^2 + 2x - 3$ va $y = 2x + 1$ funksiyalar teng qiymatlar qabul qiladigan qiymatlarini toping.

## 2-§. $y = x^2$ FUNKSIYA

$y = x^2$ funksiyani, ya'ni $a = 1$, $b = c = 0$ bo'lgandagi $y = ax^2 + bx + c$ kvadrat funksiyani qaraymiz. Bu funksiyaning grafigini yasash uchun uning qiymatlari jadvalini tuzamiz:

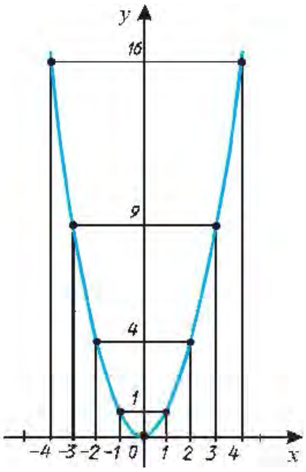
| $x$ | $-4$ | $-3$ | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3$ | $4$ |
|---|---|---|---|---|---|---|---|---|---|
| $y = x^2$ | 16 | 9 | 4 | 1 | 0 | 1 | 4 | 9 | 16 |

Jadvalda ko'rsatilgan nuqtalarni yasab va ularni silliq egri chiziq bilan tutashtirib, $y = x^2$ funksiyaning grafigini hosil qilamiz (1-rasm).

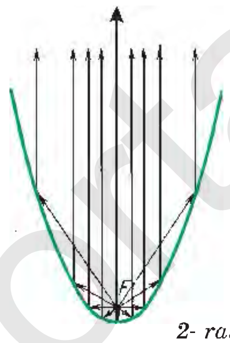
**$y = x^2$ funksiyaning grafigi bo'lgan egri chiziq *parabola* deyiladi.**

$y = x^2$ *funksiyaning xossalarini* qaraymiz.

1) $y = x^2$ funksiyaning qiymati $x \neq 0$ bo'lganda *musbat* va $x = 0$ bo'lganda *nolga* teng. Demak, $y = x^2$ parabola koordinatalar boshidan o'tadi, parabolaning qolgan nuqtalari esa abssissalar o'qidan



1- rasm.


2- rasm.

yuqorida yotadi. $y=x^2$ *parabola abssissalar o'qiga* (0; 0) *nuqtada urinadi,* deyiladi.

2) $y=x^2$ funksiyaning grafigi *ordinatalar o'qiga nisbatan simmetrik,* chunki $(-x)^2=x^2$. Masalan, $y(-3)=y(3)=9$ (1- rasm). Shunday qilib, ordinatalar o'qi *parabolaning simmetriya o'qi* bo'ladi. Parabolaning o'z simmetriya *o'qi* bilan kesishish nuqtasi *parabolaning uchi* deyiladi. $y=x^2$ parabola uchun koordinatalar boshi uning uchi bo'ladi.

3) $x\geq 0$ bo'lganda $x$ ning katta qiymatiga $y$ ning katta qiymati mos keladi. Masalan, $y(3)>y(2)$. $y=x^2$ funksiya $x\geq 0$ *oraliqda o'suvchi,* deyiladi (1- rasm).

$x\leq 0$ bo'lganda $x$ ning katta qiymatiga $y$ ning kichik qiymati mos keladi. Masalan, $y(-2)<y(-4)$. $y=x^2$ funksiya $x\leq 0$ *oraliqda kamayuvchi* deyiladi (1- rasm).

**Masala.** $y=x^2$ parabola bilan $y=x+6$ to'g'ri chiziqning kesishish nuqtalari koordinatalarini toping.

△ Kesishish nuqtalari $\begin{cases} y=x^2, \\ y=x+6 \end{cases}$

sistemaning yechimlari bo'ladi.

Bu sistemadan $x^2 = x + 6$, ya'ni $x^2 - x - 6 = 0$ ni hosil qilamiz, bundan $x_1 = 3, x_2 = -2$. $x_1$ va $x_2$ ning qiymatlarini sistemaning tenglamalaridan biriga qo'yib, $y_1 = 9, y_2 = 4$ ni topamiz.

**Javob:** (3; 9), (−2; 4). ▲

Parabola texnikada keng ko'lamda foydalaniladigan ko'pgina ajoyib xossalarga ega. Masalan, parabolaning simmetriya o'qida *parabolaning fokusi* deb ataladigan $F$ nuqta bor (2-rasm). Agar bu nuqtada yorug'lik manbai joylashgan bo'lsa, u holda paraboladan akslangan barcha yorug'lik nurlari parallel bo'ladi. Bu xossadan projektorlar, lokatorlar va boshqa asboblar tayyorlashda foydalaniladi.

$y = x^2$ parabolaning fokusi $\left(0; \frac{1}{4}\right)$ nuqta bo'ladi.

## *Mashqlar*

**8.** $y = x^2$ funksiyaning grafigini millimetrli qog'ozda yasang. Grafik bo'yicha:
1) $x = 0{,}8$; $x = 1{,}5$; $x = 1{,}9$; $x = -2{,}3$; $x = -1{,}5$ bo'lganda $y$ ning qiymatini taqriban toping;
2) agar $y = 2$; $y = 3$; $y = 4{,}5$; $y = 6{,}5$ bo'lsa, $x$ ning qiymatini taqriban toping.

**9.** $y = x^2$ funksiya grafigini yasamasdan: $A(2;\ 6)$, $B\ (-1;\ 1)$, $C(12;\ 144)$, $D(-3;\ -9)$ nuqtalardan qaysilari parabolaga tegishli bo'lishini aniqlang.

**10.** (Og'zaki.) $A(3;\ 9)$, $B(-5;\ 25)$, $C(4;\ 15)$, $D(\sqrt{3};\ 3)$ nuqtalarga ordinatalar o'qiga nisbatan simmetrik bo'lgan nuqtalarni toping. Bu nuqtalar $y = x^2$ funksiyaning grafigiga tegishli bo'ladimi?

**11.** (Og'zaki.) $y=x^2$ funksiyaning qiymatlarini

1) $x = 2{,}5$ va $x = 3\frac{1}{3}$; 2) $x = 0{,}4$ va $x = 0{,}3$;
3) $x = -0{,}2$ va $x = -0{,}1$; 4) $x = 4{,}1$ va $x = -5{,}2$

bo'lganda taqqoslang.

**12.** $y = x^2$ parabolaning:

1) $y = 25$; 2) $y = 5$; 3) $y = -x$;
4) $y = 2x$; 5) $y = 3 - 2x$; 6) $y = 2x - 1$

to'g'ri chiziq bilan kesishish nuqtalarining koordinatalarini toping.

**13.** $A$ nuqta $y = x^2$ parabola bilan
1) $y = -x - 6$, $A(-3;\ 9)$;   2) $y = 5x-6$, $A(2;\ 4)$
to‘g‘ri chiziqning kesishish nuqtasi bo‘ladimi?

**14.** Tasdiq to‘g‘rimi: $y = x^2$ funksiya:
1) $[1;\ 4]$ kesmada;   2) $(2;\ 5)$ intervalda;
3) $x > 3$ intervalda;   4) $[-3;\ 4]$ kesmada o‘sadi?

**15.** Bitta koordinata tekisligida $y = x^2$ parabola bilan $y = 3$ to‘g‘ri chiziqni yasang. $x$ ning qanday qiymatlarida parabolaning nuqtalari to‘g‘ri chiziqdan yuqorida bo‘ladi; pastda bo‘ladi?

**16.** $x$ ning qanday qiymatlarida $y = x^2$ funksiyaning qiymati:
1) 9 dan katta; 2) 25 dan katta emas; 3) 16 dan kichik emas; 4) 36 dan kichik bo‘ladi?

## 3- §. $y = ax^2$ FUNKSIYA

**1-masala.** $y = 2x^2$ funksiyaning grafigini yasang.
△ $y = 2x^2$ funksiyaning qiymatlar jadvalini tuzamiz:

| $x$ | $-3$ | $-2$ | $-1$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y = 2x^2$ | 18 | 8 | 2 | 0 | 2 | 8 | 18 |

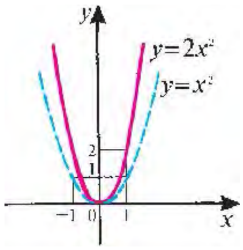
Topilgan nuqtalarni yasaymiz va ular orqali silliq egri chiziq o‘tkazamiz (3-rasm). ▲

$y = 2x^2$ va $y = x^2$ funksiyalarning grafiklarini taqqoslaymiz (3- rasm). $x$ ning aynan bir qiymatida $y = 2x^2$ funksiyaning qiymati $y - x^2$ funksiyaning qiymatidan 2 marta ortiq. Bu $y - 2x^2$ funksiya grafigining har bir nuqtasini $y = x^2$ funksiya grafigining xuddi shunday abssissali nuqtasining ordinatasini 2 marta orttirish bilan hosil qilish mumkinligini bildiradi.

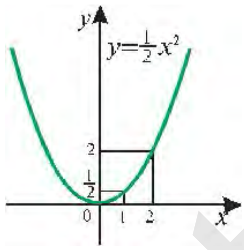
$y - 2x^2$ funksiyaning grafigi *$y - x^2$ funksiya grafigini $Ox$* o‘qidan *$Oy$* o‘qi bo‘yicha 2 marta *cho‘zish* bilan hosil qilinadi, deyiladi.

**2-masala.** $y = \frac{1}{2}x^2$ funksiyaning grafigini yasang.

△ $y = \frac{1}{2}x^2$ funksiyaning qiymatlar jadvalini tuzamiz:



*3- rasm.*



*4- rasm.*

| $x$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y=\frac{1}{2}x^2$ | 4,5 | 2 | 0,5 | 0 | 0,5 | 2 | 4,5 |

Topilgan nuqtalarni yasab, ular orqali silliq egri chiziq o‘tkazamiz (4- rasm). ▲

$y=\frac{1}{2}x^2$ va $y = x^2$ funksiyalarning grafiklarini taqqoslaymiz.

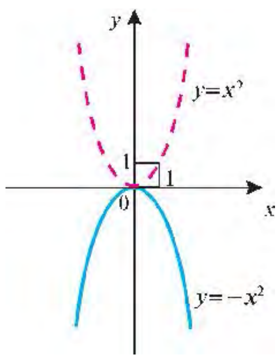
$y=\frac{1}{2}x^2$ funksiya grafigining har bir nuqtasini $y=x^2$ funksiya grafigining xuddi shunday abssissali nuqtasining ordinatasini 2 marta kamaytirish bilan hosil qilish mumkin.

$y=\frac{1}{2}x^2$ funksiyaning grafigi *$y=x^2$ funksiya grafigini Ox* o‘qiga *Oy* o‘qi bo‘yicha 2 marta *siqish* yo‘li bilan hosil qilinadi, deyiladi.

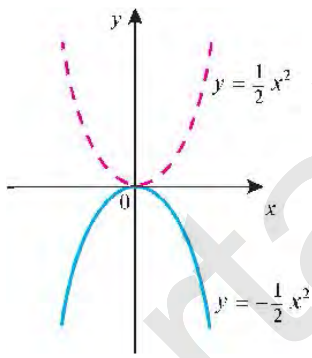
**3- masala.** $y = -x^2$ funksiyaning grafigini yasang.

△ $y=-x^2$ va $y=x^2$ funksiyalarni taqqoslaymiz. $x$ ning aynan bir qiymatida bu funksiyalarning qiymatlari modullari bo‘yicha teng va qarama-qarshi ishorali. Demak, $y=-x^2$ funksiyaning grafigini $y=x^2$ funksiya grafigini *Ox* o‘qiga nisbatan simmetrik ko‘chirish bilan hosil qilish mumkin (5- rasm). ▲

Shunga o‘xshash, $y=-\frac{1}{2}x^2$ funksiyaning grafigi *Ox* o‘qiga nisbatan $y=\frac{1}{2}x^2$ funksiya grafigiga simmetrikdir (6- rasm).




*5- rasm.*




*6- rasm.*

**$y = ax^2$ funksiyaning grafigi, bunda $a \neq 0$ ham *parabola* deb ataladi. $a > 0$ da parabolaning tarmoqlari *yuqoriga*, $a < 0$ da esa *pastga* yo'nalgan.**

$y = ax^2$ parabolaning fokusi $\left(0; \frac{1}{4a}\right)$ nuqtada joylashganligini ta'kidlaymiz.

$y = ax^2$ *funksiyaning asosiy xossalarini* sanab o'tamiz, bunda $a \neq 0$:

1) agar $a > 0$ bo'lsa, u holda $y = ax^2$ funksiya $x \neq 0$ bo'lganda musbat qiymatlar qabul qiladi;

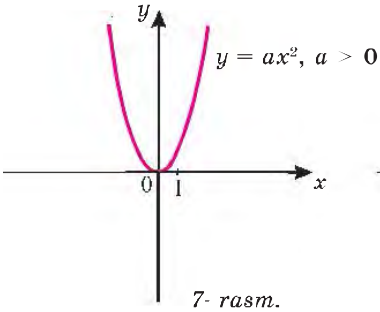
agar $a < 0$ bo'lsa, u holda $y = ax^2$ funksiya $x \neq 0$ bo'lganda manfiy qiymatlar qabul qiladi;

$y = ax^2$ funksiyaning qiymati faqat $x = 0$ bo'lgandagina 0 ga teng bo'ladi;

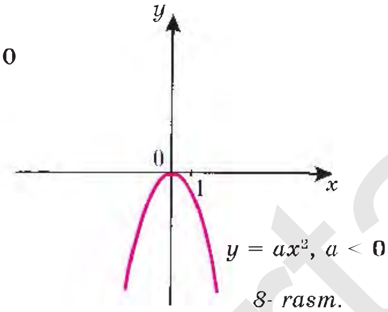
2) $y = ax^2$ parabola ordinatalar o'qiga nisbatan simmetrik bo'ladi;

3) agar $a > 0$ bo'lsa, u holda $y = ax^2$ funksiya $x > 0$ bo'lganda o'sadi va $x \leq 0$ bo'lganda kamayadi;

agar $a < 0$ bo'lsa, u holda $y = ax^2$ funksiya $x \geq 0$ bo'lganda kamayadi va $x \leq 0$ bo'lganda o'sadi.




7- *rasm.*




8- *rasm.*

Bu barcha xossalarni grafikdan bevosita ko‘rish mumkin (7- va 8-rasmlar).

### *Mashqlar*

**17.** Millimetrli qog‘ozda $y = 3x^2$ funksiyaning grafigini yasang. Grafik bo‘yicha:
1) $x = -2{,}8;\ -1{,}2;\ 1{,}5;\ 2{,}5$ bo‘lganda $y$ ning qiymatini toping;
2) agar $y = 9;\ 6;\ 2;\ 8;\ 1{,}3$ bo‘lsa, $x$ ning qiymatini taqriban toping.

**18.** (Og‘zaki.) Parabola tarmoqlarining yo‘nalishini aniqlang:

1) $y = 3x^2$; 2) $y = \frac{1}{3}x^2$; 3) $y = -4x^2$; 4) $y = -\frac{1}{3}x^2$.

**19.** Quyidagi funksiyalarning grafiklarini bitta koordinata tekisligida yasang:

1) $y = x^2$ va $y = 3x^2$; 2) $y = -x^2$ va $y = -3x^2$;

3) $y - 3x^2$ va $y - -3x^2$; 4) $y = \frac{1}{3}x^2$ va $y = -\frac{1}{3}x^2$.

Grafiklardan foydalanib, bu funksiyalardan qaysilari $x \geq 0$ oraliqda o‘suvchi ekanini aniqlang.

**20.** Quyidagi funksiyalar grafiklari kesishish nuqtalarining koordinatalarini toping:

1) $y = 2x^2$ va $y = 3x + 2$; 2) $y = -\frac{1}{2}x^2$ va $y = \frac{1}{2}x - 3$.

**21.** Funksiya $x \le 0$ oraliqda kamayuvchi bo‘ladimi:

1) $y = 4x^2$; 2) $y = -\frac{1}{4}x^3$; 3) $y = -5x^2$; 4) $y = -\frac{1}{5}x^2$?

**22.** $y = -2x^2$ funksiya:

1) $[-4; -2]$ kesmada; 3) $(3; 5)$ intervalda;
2) $[-5; 0]$ kesmada; 4) $(-3; 2)$ intervalda

o‘suvchi yoki kamayuvchi bo‘lishini aniqlang.

**23.** Tekis tezlanuvchan harakatda jism bosib o‘tgan yo‘l $s = \frac{at^2}{2}$ formula bilan hisoblanadi, bunda $s$ – yo‘l, metrlarda; $a$ – tezlanish, m/s² larda; $t$ – vaqt, sekundlarda o‘lchanadi. Agar jism 8 s da 96 m yo‘lni bosib o‘tgan bo‘lsa, $a$ tezlanishni toping.

## 4-§. $y = ax^2 + bx + c$ FUNKSIYA

**1-masala.** $y = x^2 - 2x + 3$ funksiyaning grafigini yasang va uni $y = x^2$ funksiya grafigi bilan taqqoslang.

△ $y = x^2 - 2x + 3$ funksiyaning qiymatlar jadvalini tuzamiz:

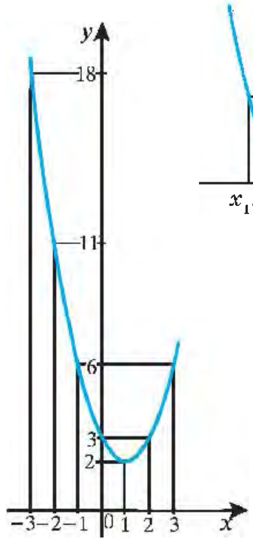
| $x$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y = x^2 - 2x + 3$ | 18 | 11 | 6 | 3 | 2 | 3 | 6 |

Topilgan nuqtalarni yasaymiz va ular orqali silliq egri chiziq o‘tkazamiz (9-rasm).

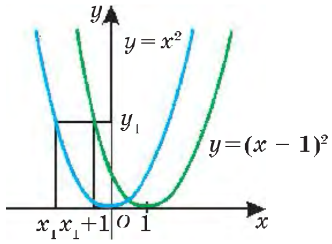
Grafiklarni taqqoslash uchun to‘la kvadratni ajratish usulidan foydalanib, $y = x^2 - 2x + 3$ formulaning shaklini almashtiramiz:

$$y = x^2 - 2x + 1 + 2 = (x - 1)^2 + 2.$$

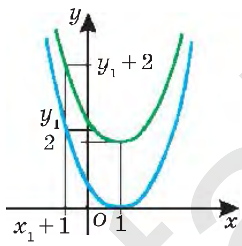
Avval $y = x^2$ va $y = (x - 1)^2$ funksiyalarning grafiklarini taqqoslaymiz. Agar $(x_1; y_1)$ nuqta $y = x^2$ parabolaning nuqtasi, ya’ni $y_1 = x_1^2$ bo‘lsa, u holda $(x_1 + 1; y_1)$ nuqta $y = (x - 1)^2$ funksiyaning grafigiga tegishli, chunki $((x_1+1)-1)^2 = x_1^2 = y_1$. Demak, $y = (x-1)^2$ funksiyaning grafigi $y = x^2$ paraboladan uni o‘ngga bir birlik *siljitish* (parallel ko‘chirish) natijasida hosil qilingan parabola bo‘ladi (10-rasm).



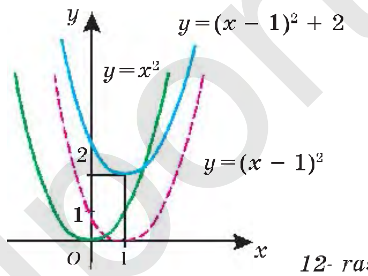

9- rasm.




10- rasm.




11- rasm.




12- rasm.

Endi $y=(x-1)^2$ va $y=(x-1)^2+2$ funksiyalarning grafiklarini taqqoslaymiz. $x$ ning har bir qiymatida $y=(x-1)^2+2$ funksiyaning qiymati $y-(x-1)^2$ funksiyaning mos qiymatidan 2 taga ortiq. Demak, $y=(x-1)^2+2$ funksiyaning grafigi $y=(x-1)^2$ parabolani ikki birlik yuqoriga siljitish bilan hosil qilingan paraboladir (11- rasm).

Shunday qilib, $y - x^2 - 2x + 3$ funksiyaning grafigi $y - x^2$ parabolani bir birlik o‘ngga va ikki birlik yuqoriga siljitish natijasida hosil qilingan parabola (12- rasm). $y = x^2 - 2x + 3$ parabolaning simmetriya o‘qi ordinatalar o‘qiga parallel va parabolaning uchi bo‘lgan (1; 2) nuqtadan o‘tgan to‘g‘ri chiziqdan iborat. ▲

*$y = a(x - x_0)^2 + y_0$ funksiyaning grafigi $y = ax^2$ parabolani:*

agar $x_0 > 0$ bo‘lsa, abssissalar o‘qi bo‘yicha o‘ngga $x_0$ ga, agar $x_0 < 0$ bo‘lsa, chapga $|x_0|$ ga siljitish;

agar $y_0 > 0$ bo‘lsa, ordinatalar o‘qi bo‘ylab yuqoriga $y_0$ ga, agar $y_0 < 0$ bo‘lsa, pastga $|y_0|$ ga siljitish yo‘li bilan hosil qilinadigan parabola bo‘lishi shunga o‘xshash isbot qilinadi.

*Istalgan* $y=ax^2+bx+c$ *kvadrat funksiyani* undan to‘la kvadratni ajratish yordamida

$$y=a\left(x+\frac{b}{2a}\right)^2-\frac{b^2-4ac}{4a},$$

ya’ni $y = a(x - x_0)^2 + y_0$ *kabi ko‘rinishda yozish mumkin*, bunda

$$x_0=-\frac{b}{2a},\ y_0=y(x_0)=\frac{-(b^2-4ac)}{4a}.$$

Shunday qilib, $y=ax^2+bx+c$ funksiyaning grafigi $y=ax^2$ parabolani koordinatalar o‘qlari bo‘ylab siljitishlar natijasida hosil bo‘ladigan parabola bo‘ladi. $y=ax^2+bx+c$ tenglik parabolaning tenglamasi deyiladi. $y=ax^2+bx+c$ parabola uchining $(x_0;\ y_0)$ koordinatalarini quyidagi formula bo‘yicha topish mumkin:

$$x_0=-\frac{b}{2a},\ y_0=y(x_0)=ax_0^2+bx_0+c.$$

$y=ax^2+bx+c$ parabolaning simmetriya o‘qi ordinatalar o‘qiga parallel va parabolaning uchidan o‘tuvchi to‘g‘ri chiziq bo‘ladi.

$y=ax^2+bx+c$ parabolaning tarmoqlari, agar $a>0$ bo‘lsa, yuqoriga yo‘nalgan, agar $a<0$ bo‘lsa, pastga yo‘nalgan bo‘ladi.

**2-masala.** $y=2x^2-x-3$ parabola uchining koordinatalarini toping.

△ Parabola uchining abssissasi:

$$x_0 - -\frac{b}{2a} - \frac{1}{4}.$$

Parabola uchining ordinatasi:

$$y_0=ax_0^2+bx_0+c=2\cdot\frac{1}{16}-\frac{1}{4}-3=-3\frac{1}{8}.$$

**Javob:** $\left(\frac{1}{4};\ -3\frac{1}{8}\right)$. ▲

**3-masala.** Agar parabolaning $(-2;\ 5)$ nuqta orqali o‘tishi va uning uchi $(-1;\ 2)$ nuqtada bo‘lishi ma’lum bo‘lsa, parabolaning tenglamasini yozing.

△ Parabolaning uchi $(-1;\ 2)$ nuqta bo‘lgani uchun parabolaning tenglamasini quyidagi ko‘rinishda yozish mumkin:

$$y = a(x + 1)^2 + 2.$$

Shartga ko‘ra $(-2; 5)$ nuqta parabolaga tegishli va

$$5 = a(-2 + 1)^2 + 2,$$

bundan $a = 3$.

Shunday qilib, parabola

$$y = 3(x + 1)^2 + 2 \quad \text{yoki} \quad y = 3x^2 + 6x + 5$$

tenglama bilan beriladi. ▲

## *Mashqlar*

Parabola uchining koordinatalarini toping (**24–26**):

**24.** (Og‘zaki.)
1) $y = (x - 3)^2 - 2$; 2) $y = (x + 4)^2 + 3$;
3) $y = 5(x + 2)^2 - 7$; 4) $y = -4(x - 1)^2 + 5$.

**25.** 1) $y = x^2 + 4x + 1$; 2) $y = x^2 - 6x - 7$;
3) $y = 2x^2 - 6x + 11$; 4) $y = -3x^2 + 18x - 7$.

**26.** 1) $y = x^2 + 2$; 2) $y = -x^2 - 5$; 3) $y = 3x^2 + 2x$;
4) $y = -4x^2 + x$; 5) $y = -3x^2 + x$; 6) $y = 2x^2 - x$.

**27.** $Ox$ o‘qida shunday nuqtani topingki, undan parabolaning simmetriya o‘qi o‘tsin:
1) $y = x^2 + 3$; 2) $y = (x + 2)^2$;
3) $y = -3(x + 2)^2 + 2$; 4) $y = (x - 2)^2 + 2$;
5) $y = x^2 + x + 1$; 6) $y = 2x^2 - 3x + 5$.

**28.** $y = x^2 - 10x$ parabolaning simmetriya o‘qi: 1) $(5; 10)$; 2) $(3; -8)$; 3) $(5; 0)$; 4) $(-5; 1)$ nuqtadan o‘tadimi?

**29.** Parabolaning koordinatalar o‘qlari bilan kesishish nuqtalarining koordinatalarini toping:
1) $y = x^2 - 3x + 2$; 2) $y = -2x^2 + 3x - 1$;
3) $y = 3x^2 - 7x + 12$; 4) $y = 3x^2 - 4x$.

---

**30.** Agar parabolaning $(-1; 6)$ nuqta orqali o‘tishi va uning uchi $(1; 2)$ nuqta ekani ma’lum bo‘lsa, parabolaning tenglamasini yozing.

**31.** (Og'zaki.) (1;−6) nuqta $y=-3x^2+4x-7$ parabolaga tegishli bo'ladimi? (−1; 8) nuqta-chi?

**32.** Agar (−1; 2) nuqta: 1) $y=kx^2+3x-4$; 2) $y=-2x^2+kx-6$ parabolaga tegishli bo'lsa, $k$ ning qiymatini toping.

**33.** $y=x^2$ parabola andazasi yordamida funksiyaning grafigini yasang:

1) $y=(x+2)^2$; 2) $y=(x-3)^2$; 3) $y=x^2-2$;
4) $y=-x^2+1$; 5) $y=-(x-1)^2-3$; 6) $y=(x+2)^2+1$.

**34.** $y=2x^2$ paraboladan uni:
1) $Ox$ o'qi bo'yicha 3 birlik o'ngga siljitish;
2) $Oy$ o'qi bo'yicha 4 birlik yuqoriga siljitish;
3) $Ox$ o'qi bo'yicha 2 birlik chapga va keyin $Oy$ o'qi bo'yicha bir birlik pastga siljitish;
4) $Ox$ o'qi bo'yicha 1,5 birlik o'ngga va keyin $Oy$ o'qi bo'yicha 3,5 birlik yuqoriga siljitish natijasida hosil bo'lgan parabolaning tenglamasini yozing.

## 5-§. KVADRAT FUNKSIYANING GRAFIGINI YASASH

**1-masala.** $y=x^2-4x+3$ funksiyaning grafigini yasang.

△ 1. Parabola uchining koordinatalarini hisoblaymiz:

$$x_0=-\frac{-4}{2}=2,$$
$$y_0=2^2-4\cdot 2+3=-1.$$

(2; −1) nuqtani yasaymiz.

2. (2; −1) nuqta orqali ordinatalar o'qiga parallel to'g'ri chiziq, ya'ni parabolaning simmetriya o'qini o'tkazamiz (13-*a* rasm).

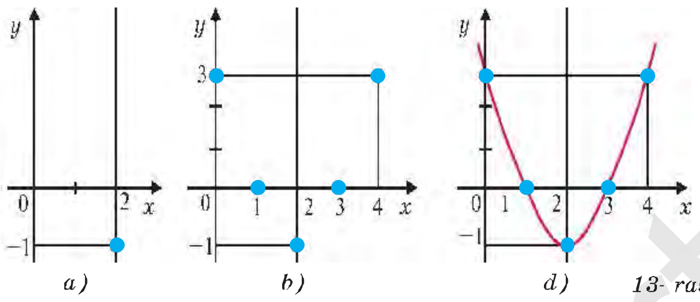
3. Ushbu

$$x^2-4x+3=0$$

tenglamani yechib, funksiyaning nollarini topamiz: $x_1=1$, $x_2=3$. (1; 0) va (3; 0) nuqtalarni yasaymiz (13-*b* rasm).

4. $Ox$ o'qida $x=2$ nuqtaga nisbatan simmetrik bo'lgan ikkita nuqtani, masalan, $x=0$ va $x=4$ nuqtalarni olamiz. Funksiyaning bu nuqtalardagi qiymatlarini hisoblaymiz: $y(0)=y(4)=3$.

(0; 3) va (4; 3) nuqtalarni yasaymiz (13-*b* rasm).



*13- rasm.*

5. Yasalgan nuqtalar orqali parabolani oʻtkazamiz (13-*d* rasm). ▲

Shu yoʻsinda *istalgan $y = ax^2 + bx + c$ kvadrat funksiyaning grafigini yasash mumkin:*

1. $x_0$, $y_0$ larni $x_0 = -\frac{b}{2a}$, $y_0 = y(x_0)$ formulalardan foydalanib hisoblab, parabolaning $(x_0;\ y_0)$ uchi yasaladi.

2. Parabolaning uchidan ordinatalar oʻqiga parallel toʻgʻri chiziq – parabolaning simmetriya oʻqi oʻtkaziladi.

3. Funksiyaning nollari (agar ular mavjud boʻlsa) topiladi va abssissalar oʻqida parabolaning mos nuqtalari yasaladi.

4. Parabolaning uning oʻqiga nisbatan simmetrik boʻlgan qandaydir ikkita nuqtasi yasaladi. Buning uchun $Ox$ oʻqida $x_0$ $(x_0 \neq 0)$ nuqtaga nisbatan simmetrik boʻlgan ikkita nuqta olish va funksiyaning mos qiymatlarini (bu qiymatlar bir xil) hisoblash kerak. Masalan, parabolaning abssissalari $x = 0$ va $x = 2x_0$ boʻlgan nuqtalarini (bu nuqtalarning ordinatalari $c$ ga teng) yasash mumkin.

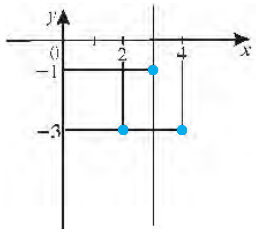
5. Yasalgan nuqtalar orqali parabola oʻtkaziladi. Grafikni yanada aniqroq yasash uchun parabolaning yana bir nechta nuqtasini topish foydali.

**2- masala.** $y = -2x^2 + 12x - 19$ funksiyaning grafigini yasang.

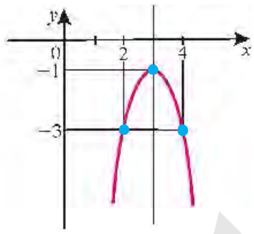
△ 1. Parabola uchining koordinatalarini hisoblaymiz:

$$x_0 = -\frac{12}{-4} = 3,\ y_0 = -2 \cdot 3^2 + 12 \cdot 3 - 19 = -1\,.$$

(3; −1) nuqtani – parabolaning uchini yasaymiz (14- rasm).



*14- rasm.*



*15- rasm.*

2. (3; −1) nuqta orqali parabolaning simmetriya o‘qini o‘tkazamiz (14- rasm).

3. $-2x^2 + 12x - 19 = 0$ tenglamani yechib, haqiqiy ildizlar yo‘qligiga va shuning uchun parabola $Ox$ o‘qini kesmasligiga ishonch hosil qilamiz.

4. $Ox$ o‘qida $x = 3$ nuqtaga nisbatan simmetrik bo‘lgan ikkita nuqtani, masalan, $x = 2$ va $x = 4$ nuqtalarni olamiz. Funksiyaning bu nuqtalardagi qiymatlarini hisoblaymiz:

$$y(2) = y(4) = -3.$$

(2; −3) va (4; −3) nuqtalarni yasaymiz (14- rasm).

5. Yasalgan nuqtalar orqali parabola o‘tkazamiz (15- rasm). ▲

**3- masala.** $y = -x^2 + x + 6$ funksiyaning grafigini yasang va shu funksiya qanday xossalarga ega ekanini aniqlang.

△ Funksiyaning grafigini yasash uchun uning nollarini topamiz: $-x^2 + x + 6 = 0$, bundan $x_1 = -2, x_1 = 3$. Parabola uchining koordinatalarini bunday topish mumkin:

$$x_0 = \frac{x_1 + x_2}{2} = \frac{-2+3}{2} = \frac{1}{2},$$

$$y_0 = y\left(\frac{1}{2}\right) = -\frac{1}{4} + \frac{1}{2} + 6 = 6\frac{1}{4}.$$

$a = -1 < 0$ bo‘lgani uchun parabolaning tarmoqlari pastga yo‘nalgan.

Parabolaning yana bir nechta nuqtasini topamiz: $y(-1) = 4$, $y(0) = 6$, $y(1) = 6$, $y(2) = 4$. Parabolani yasaymiz (16- rasm).

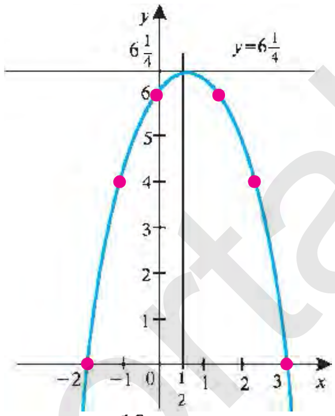
Grafik yordamida $y = -x^2 + x + 6$ *funksiyaning* quyidagi *xossalarini* hosil qilamiz:

1) $x$ ning istalgan qiymatlarida funksiyaning qiymatlari $6\frac{1}{4}$ ga teng yoki undan kichik;

2) $-2 < x < 3$ da funksiyaning qiymatlari musbat, $x < -2$ da va $x > 3$ da manfiy, $x = -2$ va $x = 3$ da nolga teng;

3) funksiya $x \leq \frac{1}{2}$ oraliqda o‘sadi, $x > \frac{1}{2}$ oraliqda kamayadi;

4) $x = \frac{1}{2}$ bo‘lganda funksiya $6\frac{1}{4}$ ga teng bo‘lgan eng katta qiymatini qabul qiladi;



*16- rasm.*

5) funksiyaning grafigi $x = \frac{1}{2}$ to‘g‘ri chiziqqa nisbatan simmetrik. ▲

$y = ax^2 + bx + c$ funksiya $x_0 = -\frac{b}{2a}$ nuqtada *eng kichik* yoki *eng katta qiymatlarni* qabul qiladi; bu $x_0$ nuqta parabola uchining abssissasidir.

Funksiyaning $x_0$ nuqtadagi qiymatini $y_0 = y(x_0)$ formula bo‘yicha topish mumkin. *Agar $a > 0$ bo‘lsa, u holda funksiya eng kichik qiymatga ega bo‘ladi, agar $a < 0$ bo‘lsa, u holda funksiya eng katta qiymatga ega bo‘ladi.*

Masalan, $y = x^2 - 4x + 3$ funksiya $x = 2$ bo‘lganda $-1$ ga teng bo‘lgan eng kichik qiymatini qabul qiladi (13- *d* rasm); $y = -2x^2 + 12x - 9$ funksiya $x = 3$ bo‘lganda $-1$ ga teng bo‘lgan eng katta qiymatini qabul qiladi (15- rasm).

**4-masala.** Ikkita musbat sonning yig‘indisi 6 ga teng. Agar ularning kvadratlari yig‘indisi eng kichik bo‘lsa, shu sonlarni toping. Shu sonlar kvadratlari yig‘indisining eng kichik qiymati qanday bo‘ladi?

△ Birinchi sonni $x$ harfi bilan belgilaymiz, bu holda ikkinchi son $6 - x$, ular kvadratlarining yig‘indisi esa $x^2 + (6 - x)^2$ bo‘ladi. Bu ifodaning shaklini almashtiramiz:

$$x^2 + (6 - x)^2 = x^2 + 36 - 12x + x^2 = 2x^2 - 12x + 36.$$

Masala $y = 2x^2 - 12x + 36$ funksiyaning eng kichik qiymatini topishga keltirildi. Shu parabola uchining koordinatalarini topamiz:

$$x_0 = -\frac{b}{2a} = -\frac{-12}{2 \cdot 2} = 3,\ y_0 = y(3) = 2 \cdot 9 - 12 \cdot 3 + 36 = 18.$$

Demak, $x = 3$ bo'lganda funksiya 18 ga teng eng kichik qiymatni qabul qiladi.

Shunday qilib, birinchi son 3 ga teng, ikkinchi son ham $6 - 3 = 3$ ga teng. Bu sonlar kvadratlari yig'indisining qiymati 18 ga teng. ▲

### *Mashqlar*

**35.** Parabola uchining koordinatalarini toping:

1) $y - x^2 - 4x - 5$; 2) $y - x^2 + 3x + 5$;
3) $y - -x^2 - 2x + 5$; 4) $y - -x^2 + 5x - 1$.

**36.** Parabolaning koordinata o'qlari bilan kesishish nuqtalarining koordinatalarini toping:

1) $y = x^2 - 3x + 5$; 2) $y = -2x^2 - 8x + 10$;
3) $y = -2x^2 + 6$; 4) $y = 7x^2 + 14$.

Funksiyaning grafigini yasang va grafik bo'yicha: 1) $x$ ning funksiyaning qiymatlari musbat, manfiy bo'ladigan qiymatlarini toping; 2) funksiyaning o'sish va kamayish oraliqlarini toping; 3) $x$ ning qanday qiymatlarida funksiya eng katta yoki eng kichik qiymatlar qabul qilishini aniqlang va ularni toping (**37–38**):

**37.** 1) $y = x^2 - 7x + 10$; 2) $y = -x^2 + x + 2$;
3) $y = -x^2 + 6x - 9$; 4) $y = x^2 + 4x + 5$.

**38.** 1) $y = 4x^2 + 4x - 3$; 2) $y = -3x^2 - 2x + 1$;
3) $y = -2x^2 + 3x + 2$; 4) $y = 3x^2 - 8x + 4$.

**39.** Kvadrat funksiyaning berilgan grafigi (17- rasm) bo'yicha uning xossalarini aniqlang.

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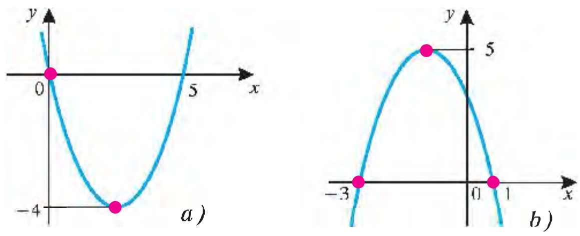
**40.** 15 sonini ikkita sonning yig'indisi shaklida shunday tasvirlangki, bu sonlarning ko'paytmasi eng katta bo'lsin.




*17- rasm.*

**41.** Ikki sonning yig'indisi 10 ga teng. Agar shu sonlar kublarining yig'indisi eng kichik bo'lsa, shu sonlarni toping.

**42.** Uy devorlariga yondashgan to'g'ri to'rtburchak shaklidagi maydonni uch tomonidan 12 m li panjara bilan o'rab olish talab etiladi. Maydonning o'lchamlari qanday bo'lganda uning yuzi eng katta bo'ladi?

**43.** Uchburchakda asosi bilan shu asosga tushirilgan balandlikning yig'indisi 14 sm ga teng. Shunday uchburchak 25 $sm^2$ ga teng yuzga ega bo'lishi mumkinmi?

**44.** Grafikni yasamasdan, $x$ ning qanday qiymatida funksiya eng katta (eng kichik) qiymatga ega bo'lishini aniqlang; shu qiymatni toping:

1) $y = x^2 - 6x + 13$; 2) $y = x^2 - 2x - 4$;
3) $y = -x^2 + 4x + 3$; 4) $y = 3x^2 - 6x + 1$.

**45.** Agar:

1) parabolaning tarmoqlari yuqoriga yo'nalgan, uning uchining abssissasi manfiy, ordinatasi esa musbat bo'lsa;
2) parabolaning tarmoqlari pastga yo'nalgan, uning uchining abssissa va ordinatasi manfiy bo'lsa, $y = ax^2 + bx + c$ parabola tenglamasi koeffitsiyentlarining ishoralarini aniqlang.

**46.** 5 m balandlikdan kamondan 50 m/s tezlik bilan yuqoriga vertikal ravishda nayza otildi. Nayzaning $t$ sekunddan keyin ko'tarilgan balandligi metrlarda $h = h(t) = 5 + 50t - \frac{gt^2}{2}$ formula bilan hisoblanadi, bunda $g \approx 10$ m/$s^2$. Nayza necha sekunddan keyin: 1) eng katta balandlikka erishadi va u qanday balandlik bo'ladi? 2) yerga tushadi?

## 6-§. KVADRAT TENGSIZLIK VA UNING YECHIMI

**1-masala.** To'g'ri to'rtburchakning tomonlari 2 dm va 3 dm ga teng. Uning har bir tomoni bir xil sondagi detsimetrlarga shunday orttirildiki, natijada to'g'ri to'rtburchakning yuzi 12 dm$^2$ dan ortiq bo'ldi. Har bir tomon qanday o'zgargan?

△ To'g'ri to'rtburchakning har bir tomoni $x$ detsimetrga orttirilgan bo'lsin. U holda yangi to'g'ri to'rtburchakning tomonlari $(2 + x)$ va $(3+x)$ detsimetrga, uning yuzi esa $(2+x)(3+x)$ kvadrat detsimetrga teng bo'ladi. Masala shartiga ko'ra $(2+x)(3+x)> 12$, bundan $x^2+5x+6 > 12$ yoki $x^2 + 5x - 6 > 0$.

Bu tengsizlikning chap qismini ko'paytuvchilarga ajratamiz:

$$(x + 6)(x - 1) > 0.$$

Masala shartiga ko'ra, $x > 0$ bo'lgani uchun $x + 6 > 0$.

Tengsizlikning ikkala qismini $x + 6$ musbat songa bo'lib, $x-1 > 0$, ya'ni $x > 1$ ni hosil qilamiz.

**Javob:** to'g'ri to'rtburchakning har bir tomoni 1 dm dan ko'proqqa orttirilgan. ▲

$x^2 + 5x - 6 > 0$ tengsizlikda $x$ bilan noma'lum son belgilangan. Bu– kvadrat tengsizlikka misol.

**!** ***Agar tengsizlikning chap qismida kvadrat funksiya, o'ng qismida esa nol tursa, bunday tengsizlik kvadrat tengsizlik deyiladi.***

Masalan,

$$2x^2 - 3x + 1 \geq 0, \qquad -3x^2 + 4x + 5 < 0$$

tengsizliklar kvadrat tengsizliklardir.

Bir noma'lumli *tengsizlikning yechimi* deb, noma'lumning shu tengsizlikni to'g'ri sonli tengsizlikka aylantiruvchi barcha qiymatlariga aytilishini eslatib o'tamiz.

*Tengsizlikni yechish* — uning barcha yechimlarini topish yoki ularning yo'qligini ko'rsatish demakdir.

**2-masala.** Tengsizlikni yeching:

$$x^2 - 5x + 6 > 0.$$

△ $x^2-5x+6=0$ kvadrat tenglama ikkita turli $x_1=2, x_2=3$ ildizga ega. Demak, $x^2-5x+6$ kvadrat uchhadni ko'paytuvchilarga ajratish mumkin:

$$x^2-5x+6=(x-2)(x-3).$$

Shuning uchun berilgan tengsizlikni bunday yozsa bo'ladi:

$$(x-2)(x-3)>0.$$

Agar ikkita ko'paytuvchi bir xil ishoraga ega bo'lsa, ularning ko'paytmasi musbat ekani ravshan.

1) Ikkala ko'paytuvchi musbat, ya'ni $x-2>0$ va $x-3>0$ bo'lgan holni qaraymiz.

Bu ikki tengsizlik quyidagi sistemani tashkil qiladi:

$$\begin{cases} x-2>0, \\ x-3>0. \end{cases}$$

Sistemani yechib, $\begin{cases} x>2, \\ x>3 \end{cases}$ ni hosil qilamiz, bundan $x>3$.

Demak, barcha $x>3$ sonlar $(x-2)(x-3)>0$ tengsizlikning yechimlari bo'ladi.

2) Endi ikkala ko'paytuvchi manfiy, ya'ni $x-2<0$ va $x-3<0$ bo'lgan holni qaraymiz.

Bu ikki tengsizlik quyidagi sistemani tashkil qiladi:

$$\begin{cases} x-2<0, \\ x-3<0. \end{cases}$$

Sistemani yechib, $\begin{cases} x<2, \\ x<3 \end{cases}$ ni hosil qilamiz, bundan $x<2$.

Demak, barcha $x<2$ sonlar ham $(x-2)(x-3)>0$ tengsizlikning yechimlari bo'ladi.

Shunday qilib, $(x-2)(x-3)>0$ tengsizlikning, demak, berilgan $x^2-5x+6>0$ tengsizlikning ham yechimlari $x<2$, shuningdek, $x>3$ sonlar bo'ladi.

**Javob:** $x<2, x>3$. ▲

**Umuman, agar $ax^2+bx+c=0$ kvadrat tenglama ikkita turli ildizga ega bo'lsa, u holda $ax^2+bx+c>0$ va $ax^2+bx+c<0$ kvadrat tengsizliklarni yechishni, kvadrat tengsizlikning chap qismini ko'paytuvchilarga ajratib, birinchi darajali tengsizliklar sistemasini yechishga keltirish mumkin.**

**3- masala.** $-3x^2 - 5x + 2 > 0$ tengsizlikni yeching.

△ Hisoblashlarni qulayroq olib borish uchun berilgan tengsizlikni birinchi koeffitsiyenti musbat bo‘lgan kvadrat tengsizliklar shaklida tasvirlaymiz. Buning uchun uning ikkala qismini $-1$ ga ko‘paytiramiz:

$$3x^2 + 5x - 2 < 0.$$

$3x^2+5x-2=0$ tenglamaning ildizlarini topamiz:

$$x_{1,2} = \frac{-5 \pm \sqrt{25+24}}{6} = \frac{-5 \pm 7}{6},$$

$$x_1 = \frac{1}{3}, \quad x_2 = -2.$$

Kvadrat uchhadni ko‘paytuvchilarga ajratib, quyidagini hosil qilamiz:

$$3\left(x-\frac{1}{3}\right)(x+2)<0.$$

Bundan ikkita sistemani olamiz:

$$\begin{cases} x-\frac{1}{3}>0, \\ x+2<0; \end{cases} \qquad \begin{cases} x-\frac{1}{3}<0, \\ x+2>0. \end{cases}$$

Birinchi sistemani bunday yozish mumkin:

$$\begin{cases} x>\frac{1}{3}, \\ x<-2, \end{cases}$$

bu sistema yechimlarga ega emasligi ko‘rinib turibdi.

Ikkinchi sistemani yechib, quyidagini topamiz:

$$\begin{cases} x<\frac{1}{3}, \\ x>-2, \end{cases}$$

bundan $-2 < x < \frac{1}{3}$.

Demak, $3(x-\frac{1}{3})(x+2)<0$ tengsizlikning, ya’ni $-3x^2-5x+2>0$ tengsizlikning yechimlari $\left(-2;\ \frac{1}{3}\right)$ intervaldagi barcha sonlar bo‘ladi.

**Javob:** $-2 < x < \frac{1}{3}$. ▲

## *Mashqlar*

**47.** (Og'zaki.) Quyidagi tengsizliklardan qaysilari kvadrat tengsizlik ekanini ko'rsating:

1) $x^2 - 4 > 0$; 2) $x^2 - 3x - 5 \le 0$; 3) $3x + 4 > 0$;
4) $4x - 5 < 0$; 5) $x^2 - 1 \le 0$; 6) $x^4 - 16 > 0$.

**48.** Tengsizlikni kvadrat tengsizlik ko'rinishiga keltiring:

1) $x^2 < 3x + 4$; 2) $3x^2 - 1 > x$;
3) $3x^2 < x^2 - 5x + 6$; 4) $2x(x + 1) < x + 5$.

**49.** (Og'zaki.) 0; –1; 2 sonlaridan qaysilari

1) $x^2 + 3x + 2 > 0$; 2) $-x^2 + 3{,}5x + 2 > 0$;
3) $x^2 - x - 2 \le 0$; 4) $-x^2 + x + \frac{3}{4} < 0$

tengsizlikning yechimlari bo'ladi?

Tengsizlikni yeching (**50–52**):

**50.** 1) $(x - 2)(x + 4) > 0$; 2) $(x - 11)(x - 3) < 0$;
3) $(x - 3)(x + 5) < 0$; 4) $(x + 7)(x + 1) > 0$.

**51.** 1) $x^2 - 4 < 0$; 2) $x^2 - 9 > 0$;
3) $x^2 + 3x < 0$; 4) $x^2 - 2x > 0$.

**52.** 1) $x^2 - 3x + 2 < 0$; 4) $x^2 + 2x - 3 > 0$;
2) $x^2 + x - 2 < 0$; 5) $2x^2 + 3x - 2 > 0$;
3) $x^2 - 2x - 3 > 0$; 6) $3x^2 + 2x - 1 > 0$.

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**53.** Tengsizlikni yeching:

1) $2\cdot\left(x - \frac{1}{3}\right)^2 > 0$; 2) $7\cdot\left(\frac{1}{6} - x\right)^2 \le 0$;
3) $3x^2 - 3 < x^2 - x$; 4) $(x - 1)(x + 3) > 5$.

**54.** Funksiyaning grafigini yasang. Grafik bo'yicha $x$ ning funksiya musbat qiymatlar; manfiy qiymatlar; nolga teng qiymat qabul qiladigan barcha qiymatlarini toping:

1) $y = 2x^2$; 2) $y = -(x + 1{,}5)^2$;
3) $y = 2x^2 - x + 2$; 4) $y = -3x^2 - x - 2$.

**55.** $x_1$ va $x_2$ sonlar (bunda $x_1 < x_2$) $y = ax^2 + bx + c$ funksiyaning nollari ekani ma'lum. Agar $x_0$ son $x_1$ va $x_2$ orasida yotsa, ya'ni $x_1 < x_0 < x_2$ bo'lsa, u holda $a(ax_0^2 + bx_0 + c) < 0$ tengsizlik bajarilishini isbotlang.

## 7-§. KVADRAT TENGSIZLIKNI KVADRAT FUNKSIYA GRAFIGI YORDAMIDA YECHISH

Kvadrat funksiya $y = ax^2 + bx + c$ (bunda $a \neq 0$) formula bilan berilishini eslatib o‘tamiz. Shuning uchun kvadrat tengsizlikni yechish kvadrat funksiyaning nollarini va kvadrat funksiya musbat yoki manfiy qiymatlar qabul qiladigan oraliqlarni izlashga keltiriladi.

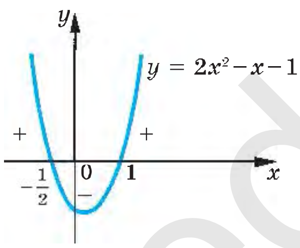
**1-masala.** Tengsizlikni grafik yordamida yeching:

$$2x^2 - x - 1 \leq 0.$$

△ $y = 2x^2 - x - 1$ kvadrat funksiyaning grafigi — tarmoqlari yuqoriga yo‘nalgan parabola.

Bu parabolaning $Ox$ o‘qi bilan kesishish nuqtalarini topamiz. Buning uchun $2x^2 - x - 1 - 0$ kvadrat tenglamani yechamiz. Bu tenglamaning ildizlari:

$$x_{1,2} = \frac{1 \pm \sqrt{1+8}}{4} = \frac{1=3}{4};\ x_1 = 1,\ x_2 = -\frac{1}{2}.$$




*18-rasm.*

Demak, parabola $Ox$ o‘qini $x = -\frac{1}{2}$ va $x = 1$ nuqtalarda kesadi (18-rasm).

$2x^2 - x - 1 \leq 0$ tengsizlikni $x$ ning funksiya nolga teng bo‘lgan yoki funksiyaning qiymatlari manfiy bo‘lgan qiymatlari qanoatlantiradi, ya’ni $x$ ning shunday qiymatlariki, bu qiymatlarda parabolaning nuqtalari $Ox$ o‘qida yoki shu o‘qdan pastda yotadi. 18-rasmdan ko‘rinib turibdiki, bu qiymatlar $\left[-\frac{1}{2}; 1\right]$ kesmadagi barcha sonlar bo‘ladi.

**Javob:** $-\frac{1}{2} \leq x \leq 1$. ▲

Bu funksiyaning grafigidan berilgan tengsizlikdan faqat ishorasi bilan farq qiladigan boshqa tengsizliklarni yechishda ham foydalanish mumkin. 18-rasmdan ko‘rinib turibdiki:

1) $2x^2 - x - 1 < 0$ tengsizlikning yechimlari $-\frac{1}{2} < x < 1$, ya’ni $\left(-\frac{1}{2}; 1\right)$ intervaldagi barcha sonlar;

2) $2x^2 - x - 1 > 0$ tengsizlikning yechimlari $x < -\frac{1}{2}$ va $x > 1$ oraliqlardagi barcha sonlar bo'ladi;

3) $2x^2 - x - 1 \geq 0$ tengsizlikning yechimlari $x \leq -\frac{1}{2}$ va $x \geq 1$ oraliqlardagi barcha sonlar bo'ladi.

**2-masala.** Tengsizlikni yeching:

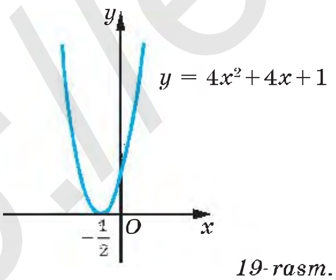
$$4x^2 + 4x + 1 > 0.$$

△ $y = 4x^2 + 4x + 1$ funksiya grafigining eskizini chizamiz. Bu parabolaning tarmoqlari yuqoriga yo'nalgan. $4x^2 + 4x + 1 = 0$ tenglama bitta $x = -\frac{1}{2}$ ildizga ega, shuning uchun parabola $Ox$ o'qiga $\left(-\frac{1}{2}; 0\right)$ nuqtada urinadi. Bu funksiyaning grafigi 19-rasmda tasvirlangan. Berilgan tengsizlikni yechish uchun $x$ ning qanday qiymatlarda funksiyaning qiymatlari musbat bo'lishini aniqlash kerak. Shunday qilib, $4x^2 + 4x + 1 > 0$ tengsizlikni $x$ ning parabolaning nuqtalari $Ox$ o'qidan yuqorida yotuvchi qiymatlari qanoatlantiradi. 19-rasmdan ko'rinib turibdiki, bunday qiymatlar $x = -0{,}5$ dan boshqa barcha haqiqiy sonlar bo'ladi.

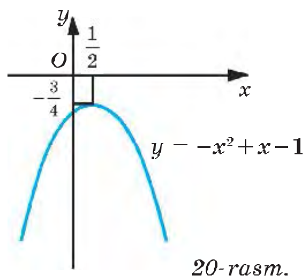
**Javob:** $x \neq -0{,}5$. ▲

19-rasmdan ko'rinib turibdiki:

1) $4x^2 + 4x + 1 \geq 0$ tengsizlikning yechimi barcha haqiqiy sonlar bo'ladi;




*19-rasm.*




*20-rasm.*

2) $4x^2 + 4x + 1 \leq 0$ tengsizlik bitta $x = -\frac{1}{2}$ yechimga ega;

3) $4x^2 + 4x + 1 < 0$ tengsizlik yechimlarga ega emas.

Agar $4x^2 + 4x + 1 = (2x + 1)^2$ ekani e'tiborga olinsa, bu tengsizliklarni og'zaki yechish mumkin.

**3-masala.** $-x^2 + x - 1 < 0$ tengsizlikni yeching.

△ $y = -x^2 + x - 1$ funksiya grafigining eskizini chizamiz. Bu parabolaning tarmoqlari pastga yo'nalgan. $-x^2 + x - 1 = 0$ tenglamaning haqiqiy ildizlari yo'q, shuning uchun parabola $Ox$ o'qini kesib o'tmaydi. Demak, bu parabola $Ox$ o'qidan pastda joylashgan (20-rasm). Bu barcha $x$ larda kvadrat funksiyaning qiymatlari manfiy, ya'ni $-x^2 + x - 1 < 0$ tengsizlik $x$ ning barcha haqiqiy qiymatlarida bajarilishini anglatadi. ▲

20-rasmdan yana $-x^2 + x - 1 < 0$ tengsizlikning yechimlari $x$ ning barcha haqiqiy qiymatlari bo'lishi, $-x^2 + x - 1 > 0$ va $-x^2 + x - 1 > 0$ tengsizliklar esa yechimlarga ega emasligi ko'rinib turibdi.

Shunday qilib, *kvadrat tengsizlikni grafik yordamida yechish uchun:*

1) kvadrat funksiya birinchi koeffitsiyentining ishorasi bo'yicha parabola tarmoqlarining yo'nalishini aniqlash;

2) tegishli kvadrat tenglamaning haqiqiy ildizlarini topish yoki ularning yo'qligini aniqlash;

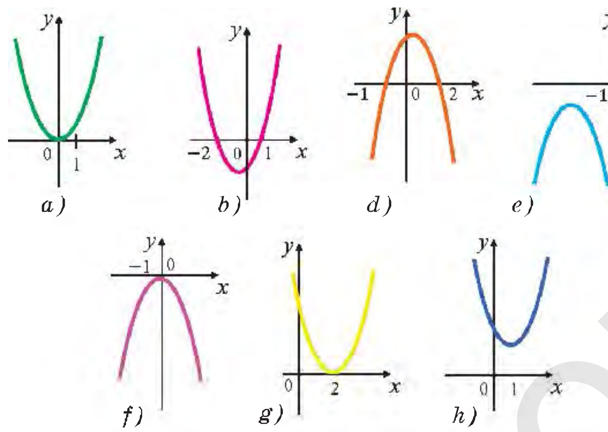
3) kvadrat funksiyaning $Ox$ o'qi bilan kesishish nuqtalari yoki urinish nuqtasidan (agar ular bo'lsa) foydalanib, kvadrat funksiya grafigining eskizini yasash;

4) grafik bo'yicha funksiya kerakli qiymatlarni qabul qiladigan oraliqlarni aniqlash kerak.

## *Mashqlar*

**56.** $y = x^2 + x - 6$ funksiyaning grafigini yasang. Grafik bo'yicha $x$ ning funksiya musbat qiymatlar; manfiy qiymatlar qabul qiladigan qiymatlarini toping.

**57.** (Og'zaki.) $y = ax^2 + bx + c$ funksiya grafigidan foydalanib (21-rasm), $x$ ning qanday qiymatlarida bu funksiya musbat qiymatlar, manfiy qiymatlar, nolga teng qiymat qabul qilishini ko'rsating.




*21-rasm.*

Kvadrat tengsizlikni yeching (**58–62**):

**58.** 1) $x^2 - 3x + 2 \le 0$; 2) $x^2 - 3x - 4 \ge 0$;
3) $-x^2 + 3x - 2 < 0$; 4) $-x^2 + 3x + 4 > 0$.

**59.** 1) $2x^2 + 7x - 4 < 0$; 2) $3x^2 - 5x - 2 > 0$;
3) $-2x^2 + x + 1 > 0$; 4) $-4x^2 + 3x + 1 < 0$.

**60.** 1) $x^2 - 6x + 9 > 0$; 2) $x^2 - 14x + 49 \le 0$;
3) $4x^2 - 4x + 1 \ge 0$; 4) $4x^2 - 20x + 25 < 0$.

**61.** 1) $x^2 - 4x + 6 > 0$; 2) $x^2 + 6x + 10 < 0$;
3) $x^2 + x + 2 > 0$; 4) $x^2 + 3x + 5 < 0$;
5) $2x^2 - 3x + 7 < 0$; 6) $4x^2 - 8x + 9 > 0$.

**62.** 1) $5 - x^2 \ge 0$; 2) $-x^2 + 7 < 0$;
3) $-2{,}1x^2 + 10{,}5x < 0$; 4) $-3{,}6x^2 - 7{,}2x < 0$.

**63.** (Og'zaki.) Tengsizlikni yeching:
1) $x^2 + 10 > 0$; 2) $x^2 + 9 < 0$;
3) $(x - 1)^2 + 1 > 0$; 4) $(x + 5)^2 + 3 < 0$;
5) $-(x + 1)^2 - 2 < 0$; 6) $-(x - 2)^2 - 4 > 0$.

Kvadrat tengsizlikni yeching (**64–66**):

**64.** 1) $4x^2 - 9 > 0$; 2) $9x^2 - 25 > 0$;
3) $x^2 - 3x + 2 > 0$; 4) $x^2 - 3x - 4 < 0$;
5) $2x^2 - 4x + 9 \le 0$; 6) $3x^2 + 2x + 4 \ge 0$.

**65.** 1) $2x^2 - 8x \le -8$; 2) $x^2 + 12x \ge -36$;
3) $9x^2 + 25 < 30x$; 4) $16x^2 + 1 > 8x$;
5) $2x^2 - x \ge 0$; 6) $3x^2 + x \le 0$.

**66.** 1) $x(x + 1) < 2(1 - 2x - x^2)$; 2) $x^2 + 2 < 3x - \frac{1}{8}x^2$;

3) $6x^2 + 1 \le 5x - \frac{1}{4}x^2$; 4) $2x(x - 1) < 3(x + 1)$.

**67.** $x$ ning funksiya noldan katta bo‘lmagan qiymatlarni qabul qiladigan barcha qiymatlarini toping:
1) $y = -x^2 + 6x - 9$; 2) $y = x^2 - 2x + 1$;

3) $y = -\frac{1}{2}x^2 - 3x - 4\frac{1}{2}$; 4) $y = -\frac{1}{3}x^2 - 4x - 12$.

**68.** 1) $x^2 - 2x + q > 0$ tengsizlikning $q > 1$ bo‘lgandagi yechimlari $x$ ning barcha haqiqiy qiymatlari bo‘lishini ko‘rsating;
2) $x^2 + 2x + q \le 0$ tengsizlik $q > 1$ bo‘lganda haqiqiy yechimlarga ega emasligini ko‘rsating.

**69.** $r$ ning $x^2 - (2 + r)x + 4 > 0$ tengsizlik $x$ ning barcha haqiqiy qiymatlarida bajariladigan barcha qiymatlarini toping.

## 8-§. INTERVALLAR USULI

Tengsizliklarni yechishda ko‘pincha intervallar usuli qo‘llaniladi. Bu usulni misollarda tushuntiramiz.

**1-masala.** $x$ ning qanday qiymatlarida $x^2 - 4x + 3$ kvadrat uchhad musbat qiymatlar, qanday qiymatlarida esa manfiy qiymatlar qabul qilishini aniqlang.

△ $x^2 - 4x + 3 = 0$ tenglamaning ildizlarini topamiz:

$$x_1 = 1,\ x_2 = 3.$$

Shuning uchun $x^2 - 4x + 3 = (x - 1)(x - 3)$.

$x = 1$ va $x = 3$ nuqtalar (22-rasm) son o‘qini uchta oraliqqa bo‘ladi:

$$x < 1, 1 < x < 3, x > 3.$$


*22-rasm.*

$1 < x < 3$ oraliq singari $x < 1, x > 3$ oraliqlar ham *intervallar* deyiladi.

Son o‘qi bo‘yicha o‘ngdan chapga harakat qilib, $x > 3$ intervalda $x^2 - 4x + 3 = (x - 1)(x - 3)$ uchhad musbat qiymatlar qabul qilishini ko‘ramiz, chunki bu holda ikkala $x - 1$ va $x - 3$ ko‘paytuvchi ham musbat.

Keyingi $1 < x < 3$ intervalda shu uchhad manfiy qiymatlar qabul qiladi va, shunday qilib, $x = 3$ nuqta orqali o‘tishda ishorasini o‘zgartiradi. Bu hol shuning uchun ham sodir bo‘ladiki, $(x - 1)(x - 3)$ ko‘paytmada $x = 3$ nuqta orqali o‘tishda $x - 1$ ko‘paytuvchi ishorasini o‘zgartirmaydi, ikkinchi $x - 3$ ko‘paytuvchi esa ishorasini o‘zgartiradi.

$x = 1$ nuqta orqali o‘tishda uchhad yana ishorasini o‘zgartiradi, chunki $(x - 1)(x - 3)$ ko‘paytmada birinchi $x - 1$ ko‘paytuvchi ishorasini o‘zgartiradi, ikkinchi $x - 3$ ko‘paytuvchi esa o‘zgartirmaydi.

Demak, son o‘qi bo‘yicha o‘ngdan chapga qarab harakat qilib bir intervaldan qo‘shni intervalga o‘ta borganda $(x - 1)(x - 3)$ ko‘paytmaning ishoralari almasha boradi.

Shunday qilib, $x^2 - 4x + 3$ kvadrat uchhadning ishorasi haqidagi masalani quyidagi usul bilan yechish mumkin.

$x^2 - 4x + 3 - 0$ tenglamaning ildizlarini son o‘qida belgilaymiz: $x_1 - 1, x_2 - 3$. Ular son o‘qini uchta intervalga ajratadi (22- rasm). $x > 3$ intervalda $x^2 - 4x + 3$ uchhadning musbat bo‘lishini aniqlab, uchhadning qolgan intervallardagi ishoralarini almasha boradigan tartibda belgilaymiz (23- rasm). 23- rasmdan ko‘rinib turibdiki, $x < 1$ va $x > 3$ bo‘lganda $x^2 - 4x + 3 > 0$, $1 < x < 3$ bo‘lganda esa $x^2 - 4x + 3 < 0$. ▲


*23-rasm.*

Qarab chiqilgan usul *intervallar usuli* deyiladi. Bu usuldan kvadrat tengsizliklarni va ba’zi tengsizliklarni yechishda foydalaniladi.

Masalan, 1- masalani yechganda biz aslida $x^2 - 4x + 3 > 0$ va $x^2 - 4x + 3 < 0$ tengsizliklarni intervallar usuli bilan yechdik.

**2-masala.** $x^3 - x < 0$ tengsizlikni yeching.

△ $x^3 - x$ ko‘phadni ko‘paytuvchilarga ajratamiz:

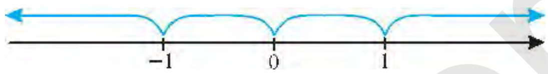
$$x^3 - x = x(x^2 - 1) = x(x - 1)(x + 1).$$

Demak, tengsizlikni bunday yozish mumkin:

$$(x + 1)x(x - 1) < 0.$$

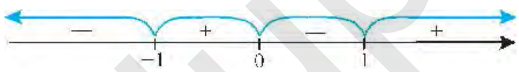
Son o‘qida $-1$, 0 va 1 nuqtalarni belgilaymiz. Bu nuqtalar son o‘qini to‘rtta intervalga ajratadi (24-rasm):

$$x < -1, -1 < x < 0, 0 < x < 1, x > 1.$$




*24-rasm.*

$x > 1$ bo‘lganda $(x + 1)x(x - 1)$ ko‘paytmaning hamma ko‘paytuvchilari musbat, shuning uchun $x > 1$ intervalda $(x+1)x(x-1) > 0$ bo‘ladi. Qo‘shni intervalga o‘tishda ko‘paytma ishorasining almashishini e’tiborga olib, har bir interval uchun $(x + 1)x(x - 1)$ ko‘paytmaning ishorasini topamiz (25-rasm).




*25-rasm.*

Shunday qilib, tengsizlikning yechimlari $x$ ning $x < -1$ va $0 < x < 1$ intervallardagi barcha qiymatlari bo‘ladi.

**Javob:** $x < -1, 0 < x < 1$. ▲

**3-masala.** $(x^2 - 9)(x + 3)(x - 2) > 0$ tengsizlikni yeching.

△ Berilgan tengsizlikni quyidagi ko‘rinishda yozish mumkin:

$$(x + 3)^2(x - 2)(x - 3) > 0. \qquad (1)$$

Barcha $x \neq -3$ da $(x + 3)^2 > 0$ bo‘lgani uchun $x \neq -3$ da (1) tengsizlikning yechimlari to‘plami

$$(x - 2)(x - 3) > 0 \qquad (2)$$

tengsizlik yechimlari to‘plami bilan ustma-ust tushadi.

$x = -3$ qiymat (1) tengsizlikning yechimi bo‘lmaydi, chunki $x = -3$ bo‘lganda tengsizlikning chap qismi 0 ga teng.

(2) tengsizlikni intervallar usuli bilan yechib, $x < 2, x > 3$ ni hosil qilamiz (26-rasm).


*26-rasm.*

$x=-3$ berilgan tengsizlikning yechimi bo‘lmasligini e’tiborga olib, oxirida javobni bunday yozamiz:

$$x<-3,\ -3<x<2,\ x>3.$$ ▲

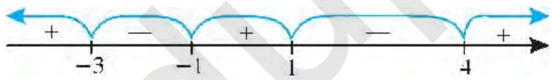
**4-masala.** Ushbu tengsizlikni yeching:

$$\frac{x^2+2x-3}{x^2-3x-4}\geq 0.$$

△ Kasrning surat va maxrajini ko‘paytuvchilarga ajratib quyidagini hosil qilamiz:

$$\frac{(x+3)(x-1)}{(x+1)(x-4)}\geq 0. \qquad (3)$$

Son o‘qida kasrning surat yoki maxraji nolga aylanadigan −3; −1; 1; 4 nuqtalarni belgilaymiz. Bu nuqtalar son to‘g‘ri chizig‘ini beshta intervalga ajratadi (27-rasm). $x>4$ bo‘lganda kasrning surat va maxrajidagi barcha ko‘paytuvchilar musbat va shuning uchun kasr musbat.




*27-rasm.*

Bir intervaldan keyingisiga o‘tishda kasr ishorasini o‘zgartiradi, shuning uchun kasrning ishoralarini 27- rasmdagidek qilib qo‘yish mumkin. $x=-3$ va $x=1$ qiymatlar (3) tengsizlikni qanoatlantiradi, $x=-1$ va $x=4$ bo‘lganda esa kasr ma’noga ega emas. Shunday qilib, berilgan tengsizlik quyidagi yechimlarga ega:

$$x\leq -3,\ -1<x\leq 1,\ x>4.$$ ▲

## *Mashqlar*

**70.** (Og‘zaki.) $x=5$ qiymat tengsizlikning yechimi bo‘lishini ko‘rsating:

1) $(x-1)(x-3)>0$;
2) $(x+2)(x+5)>0$;
3) $(x-7)(x-10)>0$;
4) $(x+1)(x-4)>0$.

Tengsizlikni intervallar usuli bilan yeching (**71–77**):

**71.** 1) $(x+2)(x-7) > 0$; 2) $(x+5)(x-8) < 0$;

3) $(x-2)\left(x+\frac{1}{2}\right) < 0$; 4) $(x+5)\left(x-3\frac{1}{2}\right) > 0$.

**72.** 1) $x^2+5x > 0$; 2) $x^2-9x > 0$; 3) $2x^2-x < 0$;
4) $x^2+3x < 0$; 5) $x^2+x-12 < 0$; 6) $x^2-2x-3 > 0$.

**73.** 1) $x^3-16x < 0$; 2) $4x^3-x > 0$;
3) $(x^2-1)(x+3) < 0$; 4) $(x^2-4)(x-5) > 0$.

**74.** 1) $(x-5)^2(x^2-25) > 0$; 2) $(x+7)^2(x^2-49) < 0$;
3) $(x-3)(x^2-9) < 0$; 4) $(x-4)(x^2-16) > 0$.

**75.** 1) $\frac{x-2}{x+5} > 0$; 2) $\frac{x-4}{x+3} < 0$; 3) $\frac{1,5-x}{3+x} \geq 0$;

4) $\frac{3,5+x}{x-7} \leq 0$; 5) $\frac{(2x+1)(x+2)}{x-3} < 0$; 6) $\frac{(x-3)(2x+4)}{x+1} \geq 0$.

**76.** 1) $\frac{x^2-2x-3}{(x-2)^2} \leq 0$; 2) $\frac{(x+4)^2}{2x^2-3x+1} \geq 0$; 3) $\frac{x^2-x}{x^2-4} > 0$; 4) $\frac{9x^2-4}{x-2x^2} < 0$.

**77.** 1) $(x^2-5x+6)(x^2-1) > 0$; 2) $(x+2)(x^2+x-12) > 0$;
3) $(x^2-7x+12)(x^2-x+2) \leq 0$; 4) $(x^2-3x-4)(x^2-2x-15) \leq 0$.

Tengsizlikni yeching (**78–80**):

**78.** 1) $\frac{x^2-x-12}{x-1} > 0$; 2) $\frac{x^2-4x-12}{x-2} < 0$; 3) $\frac{x^2+3x-10}{x^2-x-2} < 0$;

4) $\frac{x^2-3x-4}{x^2+x-6} > 0$; 5) $\frac{x^2+5x+6}{x+3} > 0$; 6) $\frac{x^2-8x+7}{x-1} < 0$.

**79.** 1) $\frac{x}{x-2} + \frac{3}{x} > \frac{3}{x-2}$; 2) $\frac{x^2}{x^2+3x} + \frac{2-x}{x+3} < \frac{5-x}{x}$.

**80.** 1) $\frac{x^2-7x-8}{x^2-64} < 0$; 2) $\frac{x^2+7x+10}{x^2-4} > 0$; 3) $\frac{5x^2-3x-2}{1-x^2} \geq 0$.

## 9- §. FUNKSIYANING ANIQLANISH SOHASI

Siz 7- sinfda funksiya tushunchasi bilan tanishgansiz. Shu tushunchani eslatib o‘tamiz.

> **!** **Agar sonlarning biror to‘plamidan olingan $x$ ning har bir qiymatiga $y$ son mos keltirilgan bo‘lsa, shu to‘plamda $y(x)$ *funksiya* berilgan deyiladi. Bunda $x$ *erkli o‘zgaruvchi* yoki *argument*, $y$ esa *erksiz o‘zgaruvchi* yoki *funksiya* deyiladi.**

Siz $y=kx+b$ chiziqli funksiya va $y=ax^2+bx+c$ kvadrat funksiya bilan tanishsiz.

Bu funksiyalar uchun argumentning qiymati istalgan haqiqiy son bo‘lishi mumkin.

Endi har bir nomanfiy $x$ songa $\sqrt{x}$ sonni mos qo‘yadigan funksiyani, ya’ni $y=\sqrt{x}$ funksiyani qaraymiz. Bu funksiya uchun argument faqat nomanfiy qiymatlarni qabul qilishi mumkin: $x \geq 0$. Bu holda funksiya barcha nomanfiy sonlar to‘plamida aniqlangan deyiladi va bu to‘plam $y=\sqrt{x}$ funksiyaning *aniqlanish sohasi* deb ataladi.

> Umuman, funksiyaning *aniqlanish sohasi* deb uning argumenti qabul qilinishi mumkin bo‘lgan barcha qiymatlar to‘plamiga aytiladi.

Masalan, $y=\frac{1}{x}$ formula bilan berilgan funksiya $x \neq 0$ da aniqlangan, ya’ni bu funksiyaning aniqlanish sohasi – noldan farqli barcha haqiqiy sonlar to‘plami.

> Agar funksiya formula bilan berilgan bo‘lsa, u holda funksiya argumentning berilgan formula ma’noga ega bo‘ladigan (ya’ni formulaning o‘ng qismida turgan ifodada ko‘rsatilgan hamma amallar bajariladigan) barcha qiymatlarida aniqlangan, deb hisoblash qabul qilingan.

Formula bilan berilgan funksiyaning aniqlanish sohasini topish – argumentning formula ma’noga ega bo‘ladigan barcha qiymatlarini topish demakdir.

**1- masala.** Funksiyaning aniqlanish sohasini toping:

1) $y(x)=2x^2+3x+5x$; 2) $y(x)=\sqrt{x-1}$;

3) $y(x)=\frac{1}{x+2}$; 4) $y(x)=\sqrt[4]{\frac{x+2}{x-2}}$.

△ 1) $2x^2+3x+5$ ifoda $x$ ning istalgan qiymatida ma'noga ega bo'lgani uchun, funksiya barcha $x$ larda aniqlangan.

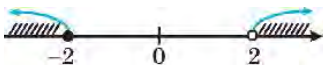
**Javob:** $x$ – istalgan son.

2) $\sqrt{x-1}$ ifoda $x-1\geq 0$ bo'lganda ma'noga ega, ya'ni funksiya $x\geq 1$ bo'lganda aniqlangan.

**Javob:** $x\geq 1$.

3) $\frac{1}{x+2}$ ifoda $x+2\neq 0$ bo'lganda ma'noga ega, ya'ni funksiya $x\neq -2$ bo'lganda aniqlangan.

**Javob:** $x\neq -2$.

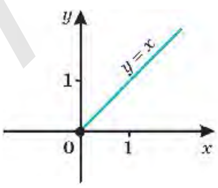


*28- rasm.*

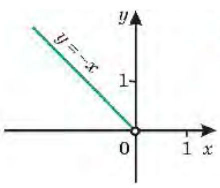
4) $\sqrt[4]{\frac{x+2}{x-2}}$ ifoda $\frac{x+2}{x-2}>0$ bo'lganda ma'noga ega. Bu tengsizlikni yechib, hosil qilamiz (28- rasm): $x\leq -2$ va $x>2$, ya'ni funksiya $x\leq -2$ va $x>2$ bo'lganda aniqlangan.

**Javob:** $x\leq -2$, $x>2$. ▲

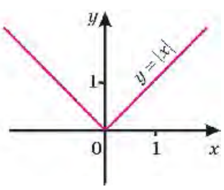
*Funksiyaning grafigi* deb koordinatalar tekisligining abssissalari shu funksiyaning aniqlanish sohasidan olingan erkli o'zgaruvchining qiymatlariga, ordinatalari esa funksiyaning mos qiymatlariga teng bo'lgan nuqtalar to'plamiga aytilishini eslatib o'tamiz.



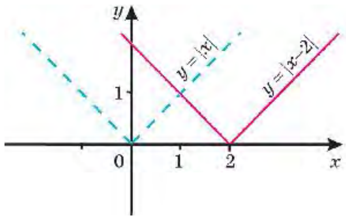
*29- rasm.*



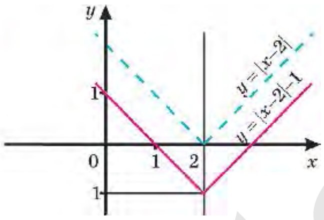
*30- rasm.*



*31- rasm.*




32- *rasm.*




33- *rasm.*

**2-masala.** $y=|x|$ funksiyaning aniqlanish sohasini toping va uning grafigini yasang.

△ Eslatib o'tamiz:

$$|x|=\begin{cases} x, \text{ agar } x\geq 0 \text{ bo‘lsa,} \\ -x, \text{ agar } x<0 \text{ bo‘lsa.} \end{cases}$$

Shunday qilib, $|x|$ ifoda istalgan haqiqiy $x$ da ma'noga ega, ya'ni $y=|x|$ funksiyaning aniqlanish sohasi barcha haqiqiy sonlar to'plamidan iborat.

Agar $x\geq 0$ bo'lsa, u holda $|x|=x$ bo'ladi va, shuning uchun, $x\geq 0$ bo'lganda $y=|x|$ funksiyaning grafigi birinchi koordinata burchagining bissektrisasi bo'ladi (29- rasm).

Agar $x<0$ bo'lsa, u holda $|x|=-x$ bo'ladi, demak, manfiy $x$ lar uchun $y=|x|$ funksiyaning grafigi ikkinchi koordinata burchagining bissektrisasi bo'ladi (30- rasm).

$y=|x|$ funksiyaning grafigi 31- rasmda tasvirlangan. ▲

Istalgan $x$ uchun $|-x|=|x|$. Shuning uchun $y=|x|$ funksiyaning grafigi ordinatalar o'qiga nisbatan simmetrik joylashgan.

**3-masala.** $y=|x-2|-1$ funksiyaning grafigini yasang.

△ $y=|x-2|$ funksiyaning grafigi $y=|x|$ funksiya grafigidan uni $Ox$ o'q bo'yicha 2 birlik o'ngga surish bilan hosil qilinadi (32- rasm).

$y=|x-2|-1$ funksiyaning grafigini hosil qilish uchun $y=|x-2|$ funksiyaning grafigini bir birlik pastga surish yetarli (33- rasm). ▲

## *Mashqlar*

**81.** Funksiya $y(x) = x^2 - 4x + 5$ formula bilan berilgan:

1) $y(-3)$, $y(-1)$, $y(0)$, $y(2)$ ni toping;

2) agar $y(x) - 1$, $y(x) - 5$, $y(x) - 10$, $y(x) - 17$ bo‘lsa, $x$ ning qiymatini toping.

**82.** Funksiya $y(x)=\frac{x-5}{x-1}$ formula bilan berilgan:

1) $y(-2)$, $y(0)$, $y(\frac{1}{2})$, $y(3)$ ni toping;

2) agar $y(x) = -3$, $y(x) = -2$, $y(x) = 13$, $y(x) = 19$ bo‘lsa, $x$ ning qiymatini toping.

Funksiyaning aniqlanish sohasini toping (**83–84**):

**83.** (Og‘zaki).

1) $y=4x^2-5x+1$; 2) $y=2-x-3x^2$; 3) $y=\frac{2x-3}{x-3}$;

4) $y=\frac{3}{5-x^2}$; 5) $y-\sqrt[4]{6-x}$; 6) $y=\sqrt{\frac{1}{x+7}}$.

**84.** 1) $y=\frac{2x}{x^2-2x-3}$; 2) $y=\sqrt[6]{x^2-7x+10}$;

3) $y=\sqrt[3]{3x^2-2x+5}$; 4) $y=\sqrt[6]{\frac{2x+4}{3-x}}$; 5) $y=\sqrt{\frac{3x-2}{4-x}}$.

**85.** Funksiya $y(x)=|2-x|-2$ formula bilan berilgan:

1) $y(-3)$, $y(-1)$, $y(1)$, $y(3)$ ni toping;

2) agar $y(x)=-2$, $y(x)=0$, $y(x)=2$, $y(x)=4$ bo‘lsa, $x$ ning qiymatini toping.

**86.** Funksiyaning aniqlanish sohasini toping:

1) $y=\sqrt{\frac{x-2}{x+3}}$; 2) $y=\sqrt[3]{\frac{1-x}{1-x}}$;

3) $y=\sqrt[4]{(x-1)(x-2)(x-3)}$; 4) $y=\sqrt{\frac{x^2-4}{x+1}}$;

5) $y=\sqrt{(x+1)(x-1)(x-4)}$; 6) $y=\sqrt[8]{\frac{x^2+4x-5}{x-2}}$;

**87.** (–2; 1) nuqta funksiya grafigiga tegishli bo‘ladimi:

1) $y = 3x^2 + 2x + 29$;  2) $y = |4 - 3x| - 9$;

3) $y = \frac{x^2+3}{x-1}$;  4) $y = \left|\sqrt{2-x} - 5\right| - 2$?

**88.** Funksiya grafigini yasang:

1) $y = |x+3| + 2$;  2) $y = -|x|$;  3) $y = 2|x| + 1$;

4) $y = 1 - |1 - x|$;  5) $y = |x| + |x - 2|$;  6) $y = |x+1| - |x|$.

**89.** $y = ax^2 + bx + c$ funksiya $A$ (0; 1), $B$ (1; 2), $C$ $(\frac{5}{6}; 1)$ nuqtalardan o‘tadi. 1) $a, b, c$ ni toping; 2) $x$ ning qanday qiymatlarida $y = 0$ bo‘ladi? 3) funksiya grafigini chizing.

## 10- §. FUNKSIYANING O‘SISHI VA KAMAYISHI

Siz $y = x$ va $y = x^2$ funksiyalar bilan tanishsiz. Bu funksiyalar *darajali funksiya*ning, ya’ni

$$y = x^r \qquad (1)$$

(bunda $r$ – berilgan son) funksiyaning xususiy hollaridir.

$r$ – natural son bo‘lsin, $r = n = 1, 2, 3, \ldots$ deylik. Bu holda natural ko‘rsatkichli darajali funksiya $y = x^n$ ni hosil qilamiz.

Bu funksiya barcha haqiqiy sonlar to‘plamida, ya’ni son o‘qining hamma yerida aniqlangan. Odatda, barcha haqiqiy sonlar to‘plami $\boldsymbol{R}$ harfi bilan belgilanadi. Shunday qilib, natural ko‘rsatkichli darajali funksiya $y = x^n$, $x \in \boldsymbol{R}$ uchun aniqlangan. Agar (1) da $r = -2k$, $k \subset \boldsymbol{N}$ bo‘lsa, u holda $y = x^{-2k} = \frac{1}{x^{2k}}$ funksiya hosil bo‘ladi. Bu funksiya $x$ ning noldan farqli barcha qiymatlarida aniqlangan. Uning grafigi $Oy$ o‘qqa nisbatan simmetrik. $r = -(2k-1)$, $k \in \boldsymbol{N}$ bo‘lsa, u holda $y = x^{-(2k-1)} = \frac{1}{x^{2k-1}}$ funksiyani olamiz. Uning xossalari sizga tanish $y = \frac{1}{x}$ funksiyaning xossalari kabi bo‘ladi. $p$ va $q$ – natural sonlar va $r = \frac{p}{q}$ – qisqarmas kasr bo‘lsin. $y = \sqrt[q]{x^p}$ funksiyaning aniqlanish sohasi $p$ va $q$ ning jufttoqligiga qarab turlicha bo'ladi. Masalan, $y=\sqrt[3]{x^2}$, $y=\sqrt[3]{x}$ funksiyalar ixtiyoriy $x \in \boldsymbol{R}$ da aniqlangan. $y=\sqrt[4]{x^3}$ funksiya esa $x$ ning nomanfiy, ya'ni $x \geq 0$ qiymatlarida aniqlangan.

8-sinf „Algebra" kursidan ma'lumki, har bir irratsional sonni chekli o'nli kasr bilan, ya'ni ratsional son bilan yaqinlashtirish mumkin. Amaliyotda irratsional sonlar ustida amallar ularning ratsional yaqinlashishlari yordamida bajariladi. Bu amallar shunday kiritiladiki, amallarning, tenglik va tengsizliklarning ratsional sonlar uchun xossalari irratsional sonlar uchun ham to'la saqlanadi.

$r_1, r_2, \ldots, r_k, \ldots$ ratsional sonlar $r$ irratsional sonning ratsional yaqinlashishlari bo'lsin. U holda $x$ musbat son bo'lganda, $x$ ning ratsional darajalari, ya'ni $x^{r_1}, x^{r_2}, \ldots, x^{r_k}, \ldots$ sonlar $x^r$ darajaning yaqinlashishlari bo'ladi. Bunday aniqlangan daraja *irratsional ko'rsatkichli daraja* deyiladi. Demak, $x > 0$ uchun daraja ko'rsatkichi ixtiyoriy $r$ bo'lgan $y = x^r$ funksiyani aniqlash mumkin.

Darajali funksiya $x$ ning (1) formula ma'noga ega bo'ladigan qiymatlari uchun aniqlangan. Masalan, $y = x$ va $y = x^2$ ($r = 1$ va $r = 2$) funksiyalarning aniqlanish sohasi barcha haqiqiy sonlar to'plami bo'ladi; $y=\frac{1}{x}$ ($r = 1$) funksiyaning aniqlanish sohasi nolga teng bo'lmagan barcha haqiqiy sonlar to'plami bo'ladi; $y=\sqrt{x}$ ($r=\frac{1}{2}$) funksiyaning aniqlanish sohasi barcha nomanfiy sonlar to'plamidan iborat.

**! Shuni eslatamizki, agar argumentning biror oraliqdan olingan katta qiymatiga funksiyaning katta qiymati mos kelsa, ya'ni shu oraliqqa tegishli istalgan $x_1$, $x_2$ uchun $x_2>x_1$ tengsizlikdan $y(x_2) > y(x_1)$ tengsizlik kelib chiqsa, $y(x)$ funksiya shu oraliqda *o'suvchi* funksiya deyiladi.**

**! Agar biror oraliqqa tegishli istalgan $x_1$, $x_2$ uchun $x_2>x_1$ tengsizlikdan $y(x_2) < y(x_1)$ kelib chiqsa, $y(x)$ funksiya shu oraliqda *kamayuvchi* funksiya deyiladi.**

Masalan, $y = x$ funksiya sonlar o'qida o'sadi. $y = x^2$ funksiya $x > 0$ oraliqda o'sadi, $x < 0$ oraliqda kamayadi.

*$y = x^r$ darajali funksiyaning o‘sishi yoki kamayishi daraja ko‘rsatkichining ishorasiga bog‘liq.*

**Agar $r > 0$ bo‘lsa, u holda $y = x^r$ darajali funksiya $x \geq 0$ oraliqda *o‘sadi.***

○ $x_2 > x_1 \geq 0$ bo‘lsin. $x_2 > x_1$ tengsizlikni musbat $r$ darajaga ko‘tarib, $x_2^r > x_1^r$ ni, ya’ni $y(x_2) > y(x_1)$ ni hosil qilamiz. ●

Masalan, $y = \sqrt{x}$ va $y = x^{\frac{3}{2}}$ funksiyalar $x \geq 0$ oraliqda o‘sadi. Bu funksiyalarning grafiklari 34-rasmda tasvirlangan. Shu rasmdan $y = \sqrt{x}$ funksiyaning grafigi $0 < x < 1$ oraliqda $y - x$ funksiyaning grafigidan yuqorida, $x > 1$ oraliqda esa $y = x$ funksiyaning grafigidan pastda yotishi ko‘rinib turibdi.

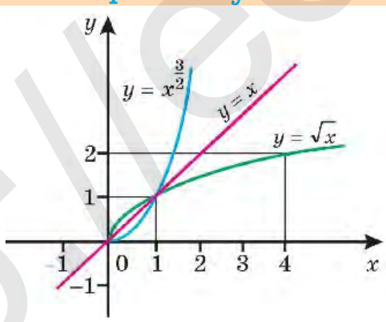
Agar $0 < r < 1$ bo‘lsa, $y = x^r$ funksiyaning grafigi xuddi shunday xossaga ega bo‘ladi.

$y = x^{\frac{3}{2}}$ funksiyaning grafigi $0 < x < 1$ oraliqda $y = x$ funksiya grafigidan pastda, $x > 1$ oraliqda esa $y = x$ funksiya grafigidan yuqorida yotadi.

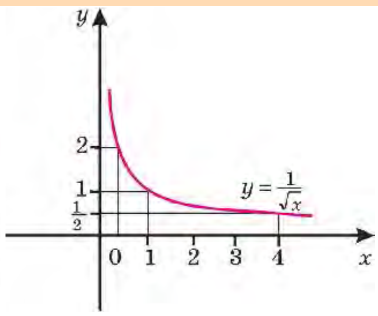
$r > 1$ bo‘lsa, $y - x^r$ funksiyaning grafigi xuddi shunday xossaga ega bo‘ladi.

Endi $r > 1$ bo‘lgan holni qaraymiz.

**Agar $r < 0$ bo‘lsa, u holda $y = x^r$ darajali funksiya $x > 0$ oraliqda *kamayadi.***




*34- rasm.*




*35- rasm.*

○ $x_2 > x_1 > 0$ bo'lsin. $x_2 > x_1$ tengsizlikni manfiy $r$ darajaga ko'tarib, chap va o'ng qismlari musbat bo'lgan tengsizliklarning xossasiga ko'ra $x_2^r > x_1^r$ ni, ya'ni $y(x_2) < y(x_1)$ ni hosil qilamiz. ●

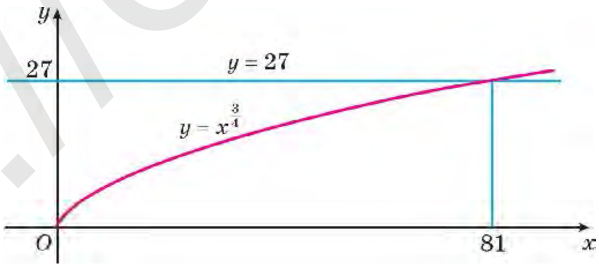
Masalan, $y = \frac{1}{\sqrt{x}}$, ya'ni $y = x^{-\frac{1}{2}}$ funksiya $x > 0$ oraliqda kamayadi. Bu funksiyaning grafigi 35- rasmda tasvirlangan.

**1-masala.** $x^{\frac{3}{4}} = 27$ tenglamani yeching.

△ $y = x^{\frac{3}{4}}$ funksiya $x \geq 0$ da aniqlangan. Shuning uchun berilgan tenglama faqat nomanfiy ildizlarga ega bo'lishi mumkin. Bunday ildizlardan biri: $x = 27^{\frac{4}{3}} = \left(\sqrt[3]{3^3}\right)^4 = 3^4 = 81$. Tenglamaning boshqa ildizlari yo'q, chunki $y = x^{\frac{3}{4}}$ funksiya $x \geq 0$ bo'lganda o'sadi va shuning uchun, agar $x > 81$ bo'lsa, u holda $x^{\frac{3}{4}} > 27$, agar $x < 81$ bo'lsa, u holda $x^{\frac{3}{4}} < 27$ bo'ladi (36- rasm). ▲

$x^r = b$ (bunda $r \neq 0, b > 0$) tenglamaning har doim musbat $x = b^{\frac{1}{r}}$ ildizga egaligi, shu bilan birga, bu ildizning yagonaligi shunga o'xshash isbotlanadi. Demak, $y = x^r$ (bunda $r > 0$) funksiya $x > 0$ bo'lganda *barcha musbat qiymatlarni qabul qiladi.*

Bu esa, masalan, $y = x^{\frac{3}{4}}$ (36- rasm) funksiyaning sekinlik bilan o'sishiga qaramasdan, uning grafigi $Ox$ o'qdan istalgancha uzoqla-




*36- rasm.*

shishini va $y - b$ to‘g‘ri chiziqni, $b$ ning qanday musbat son bo‘lishiga qaramasdan, kesishini bildiradi.

**2-masala.** $y - x + \frac{1}{x}$ funksiyaning $x > 1$ oraliqda o‘sishini isbotlang.

△ $x_2 > x_1 > 1$ bo‘lsin. $y(x_2) > y(x_1)$ ekanligini ko‘rsatamiz. $y(x_2) - y(x_1)$ ayirmani qaraymiz:

$$y(x_2) - y(x_1) = x_2 + \frac{1}{x_2} - (x_1 + \frac{1}{x_1}) = (x_2 - x_1)\frac{x_1 x_2 - 1}{x_1 x_2}.$$

$x_2 > x_1$, $x_1 > 1$, $x_2 > 1$ bo‘lgani uchun $x_2 - x_1 > 0$, $x_1 x_2 > 1$, $x_1 x_2 > 0$. Shuning uchun $y(x_2) - y(x_1) > 0$, ya’ni $y(x_2) > y(x_1)$. ▲

## *Mashqlar*

**90.** Funksiyaning grafigini yasang hamda o‘sish va kamayish oraliqlarini toping:

1) $y = 2x + 3$; 2) $y = 1 - 3x$; 3) $y = x^2 + 2$;
4) $y = 3 - x^2$; 5) $y = (1 - x)^2$; 6) $y = (2 + x)^2$.

**91.** (Og‘zaki). Funksiya $x > 0$ oraliqda o‘sadimi yoki kamayadimi:

1) $y = x^{\frac{3}{7}}$; 2) $y = x^{-\frac{3}{4}}$; 3) $y = x^{-\sqrt{2}}$; 4) $y = x^{\sqrt{3}}$?

**92.** $x > 0$ bo‘lganda funksiya grafigi eskizini chizing:

1) $y = x^{\frac{3}{2}}$; 2) $y = x^{\frac{2}{3}}$; 3) $y = x^{-\frac{3}{2}}$; 4) $y = x^{-\frac{2}{3}}$.

**93.** Tenglamaning musbat ildizini toping:

1) $x^{\frac{1}{2}} = 3$; 2) $x^{\frac{1}{4}} = 2$; 3) $x^{-\frac{1}{2}} = 3$; 4) $x^{-\frac{1}{4}} = 2$;
5) $x^{\frac{5}{6}} = 32$; 6) $x^{-\frac{4}{5}} = 81$; 7) $x^{-\frac{1}{3}} = 8$; 8) $x^{\frac{4}{5}} = 16$.

**94.** Millimetrli qog‘ozga $y = \sqrt[4]{x}$ funksiyaning grafigini chizing. Grafik bo‘yicha:

1) $y = 0{,}5$; 1; 4; 2,5 bo‘lganda $x$ ning qiymatlarini toping;

2) $\sqrt[4]{1{,}5}$; $\sqrt[4]{2}$; $\sqrt[4]{2{,}5}$; $\sqrt[4]{3}$ qiymatlarni taqriban toping.

**95.** Funksiyalar grafiklari kesishish nuqtalarining koordinatalarini toping:

1) $y=x^{\frac{4}{3}}$ va $y=625$; 2) $y=x^{\frac{6}{5}}$ va $y=64$;

3) $y=x^{\frac{3}{2}}$ va $y=216$; 4) $y=x^{\frac{7}{3}}$ va $y=128$.

**96.** 1) $y=x+\frac{1}{x}$ funksiyaning $0<x<1$ oraliqda kamayishini isbotlang;

2) $y-\frac{1}{x^2+1}$ funksiyaning $x\geq 0$ oraliqda kamayishini va $x\leq 0$ oraliqda o'sishini isbotlang;

3) $y=x^3-3x$ funksiyaning $x\geq -1$ va $x\geq 1$ oraliqlarda o'sishini va $-1\leq x\leq 1$ kesmada kamayishini isbotlang;

4) $y=x-2\sqrt{x}$ funksiyaning $x\geq 1$ oraliqda o'sishini va $0\leq x\leq 1$ kesmada kamayishini isbotlang.

**97.** Funksiya grafigini yasang hamda o'sish va kamayish oraliqlarini toping:

1) $y=\begin{cases} x+2, \text{ agar } x\leq -1 \text{ bo'lsa,} \\ x^2, \text{ agar } x>-1 \text{ bo'lsa;} \end{cases}$

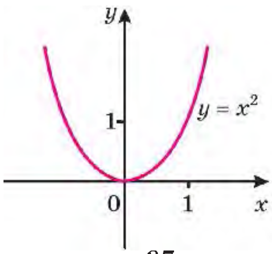
2) $y=\begin{cases} x^2, \text{ agar } x\leq 1 \text{ bo'lsa,} \\ 2-x^2, \text{ agar } x>1 \text{ bo'lsa;} \end{cases}$

3) $y=\begin{cases} -x-1, \text{ agar } x<-1 \text{ bo'lsa,} \\ -x^2+1, \text{ agar } x\geq -1 \text{ bo'lsa;} \end{cases}$

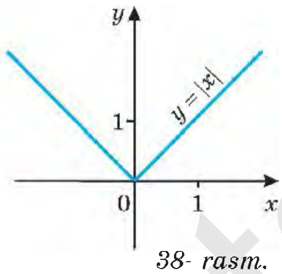
4) $y=\begin{cases} x^3, \text{ agar } x\leq 1 \text{ bo'lsa,} \\ -x^2+2x, \text{ agar } x\geq 1 \text{ bo'lsa.} \end{cases}$

## 11-§. FUNKSIYANING JUFTLIGI VA TOQLIGI

Siz $y = x^2$ va $y = |x|$ funksiyalarning grafiklari ordinatalar o'qiga nisbatan simmetrik (37 va 38- rasmlar) ekanligini bilasiz. Bunday funksiyalar *juft funksiyalar* deyiladi.




*37- rasm.*




*38- rasm.*

**Agar $y(x)$ funksiyaning aniqlanish sohasidan olingan istalgan $x$ uchun $y(-x)=y(x)$ bo‘lsa, bu funksiya *juft funksiya* deyiladi.**

Masalan, $y=x^4$ va $y=\frac{1}{x^2}$ funksiyalar juft funksiyalar, chunki istalgan $x$ uchun $(-x)^4=x^4$ va istalgan $x\neq 0$ uchun $\frac{1}{(-x)^2}=\frac{1}{x^2}$.

**1-masala.** $y=x^3$ funksiyaning grafigi koordinatalar boshiga nisbatan simmetrik ekanligini isbotlang va grafigini yasang.

△ 1) $y=x^3$ funksiyaning aniqlanish sohasi – barcha haqiqiy sonlar to‘plami.

2) $y=x^3$ funksiyaning qiymatlari $x>0$ bo‘lganda musbat, $x<0$ bo‘lganda manfiy, $x=0$ bo‘lganda nolga teng.

○ Aytaylik, $(x_0;\ y_0)$ nuqta $y=x^3$ funksiyaning grafigiga tegishli, ya’ni $y_0=x_0^3$ bo‘lsin. $(x_0;\ y_0)$ nuqtaga koordinatalar boshiga nisbatan simmetrik bo‘lgan nuqta $(-x_0;\ -y_0)$ koordinatalarga ega bo‘ladi. Bu nuqta ham $y=x^3$ funksiyaning grafigiga tegishli bo‘ladi, chunki $y_0=x_0^3$ to‘g‘ri tenglikning ikkala qismini $-1$ ga ko‘paytirib, hosil qilamiz: $-y_0=-x_0^3$ yoki $-y_0=(-x_0)^3$. ●

Bu xossa $y=x^3$ funksiyaning grafigini yasashga imkon beradi: avval grafik $x\geq 0$ uchun yasaladi, so‘ngra esa uni koordinatalar boshiga nisbatan simmetrik akslantiriladi.

3) $y=x^3$ funksiya aniqlanish sohasining hamma yerida o‘sadi. Bu musbat ko‘rsatkichli darajali funksiyaning $x\geq 0$ bo‘lganda

o‘sish xossasidan va grafikning koordinatalar boshiga nisbatan simmetrikligidan kelib chiqadi.

4) $x>0$ ning ba’zi qiymatlari (masalan, $x=0, 1, 2, 3$) uchun $y=x^3$ funksiyaning qiymatlari jadvalini tuzamiz, $x\geq 0$ bo‘lganda grafikning bir qismini yasaymiz va so‘ngra simmetriya yordamida grafikning $x$ ning manfiy qiymatlariga mos keluvchi qismini yasaymiz (39- rasm). ▲

Grafiklari koordinatalar boshiga nisbatan simmetrik bo‘lgan funksiyalar *toq* funksiyalar deyiladi. Shunday qilib, $y=x^3$ – toq funksiya.

**Agar $y(x)$ funksiyaning aniqlanish sohasidan olingan istalgan $x$ uchun**

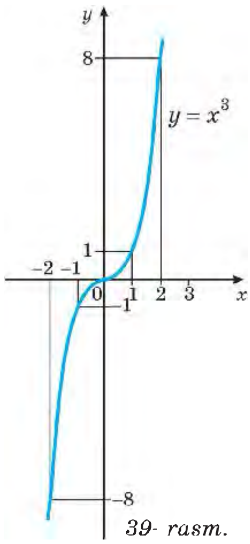
$$y(-x) = -y(x)$$

**bo‘lsa, bu funksiya *toq funksiya* deyiladi.**

Masalan, $y=x^5$, $y=\frac{1}{x^3}$ funksiyalar toq funksiyalardir, chunki istalgan $x$ uchun $(-x)^5=-x^5$ va istalgan $x\neq 0$ uchun $\frac{1}{(-x)^3}=-\frac{1}{x^3}$.

Juft va toq funksiyalarning *aniqlanish sohasi koordinatalar boshiga nisbatan simmetrik* ekanligini ta’kidlab o‘tamiz.

*Juftlik yoki toqlik xossalariga ega bo‘lmagan funksiyalar mavjud.* Masalan, $y=2x+1$ funksiyaning juft ham, toq ham emasligini ko‘rsatamiz. Agar bu funksiya juft bo‘lganida edi, u holda barcha $x$ uchun $2(-x)+1=2x+1$ tenglik bajarilgan bo‘lar edi; lekin, masalan, $x=1$ bo‘lganda bu tenglik noto‘g‘ri: $-1\neq 3$. Agar bu funksiya toq bo‘lganida edi, u holda barcha $x$ uchun $2(-x)+1=-(2x-1)$ tenglik bajarilgan bo‘lar edi; lekin masalan, $x=2$ bo‘lganda bu tenglik noto‘g‘ri: $-3\neq -5$.



*39- rasm.*

**2- masala.** $y=\sqrt[3]{x}$ funksiyaning grafigini yasang.

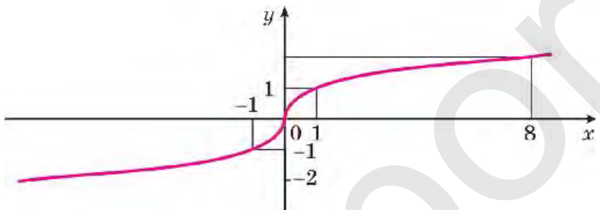
△ 1) Aniqlanish sohasi – barcha haqiqiy sonlar;

2) funksiya – toq, chunki istalgan $x$ uchun $\sqrt[3]{-x}=-\sqrt[3]{x}$;

3) $x \geq 0$ bo'lganda funksiya musbat ko'rsatkichli darajali funksiyaning xossasiga ko'ra o'sadi, chunki $x \geq 0$ bo'lganda $\sqrt[3]{x}=x^{\frac{1}{3}}$;

4) $x>0$ bo'lganda funksiyaning qiymati musbat; $y(0)=0$;

5) grafikka tegishli bir nechta, masalan, (0; 0), (1; 1), (8; 2) nuqtalarni topib, $x \geq 0$ ning qiymatlari uchun grafikning bir qismini yasaymiz va so'ngra simmetriya yordamida $x<0$ uchun grafikning ikkinchi qismini yasaymiz (40- rasm). ▲




*40- rasm.*

$y=\sqrt[3]{x}$ funksiya barcha $x$ lar uchun, $y=x^{\frac{1}{3}}$ funksiya esa faqat $x \geq 0$ uchun aniqlanganligini ta'kidlab o'tamiz.

## *Mashqlar*

Funksiya toq yoki juft bo'lishini aniqlang (**98–99**):

**98.** 1) $y=2x^4$; 2) $y=3x^5$; 3) $y=x^2+3$; 4) $y=x^3-2$.

**99.** 1) $y=x^{-4}$; 2) $y=x^{-3}$; 3) $y=x^4+x^2$; 4) $y=x^3+x^5$.

**100.** Funksiya grafigining eskizini chizing:

1) $y=x^4$; 2) $y=x^5$; 3) $y=-x^2+3$; 4) $y=\sqrt[5]{x}$.

**101.** Funksiya juft ham, toq ham emasligini ko'rsating:

1) $y=\frac{x+2}{x-3}$; 2) $y=\frac{x^2+x-1}{x+4}$; 3) $y=\frac{x-1}{x+1}$.

**102.** Funksiyaning juft yoki toq bo'lishini aniqlang:

1) $y=x^4+2x^2+3$; 2) $y=x^3-2x+1$; 3) $y=\frac{3}{x^3}+\sqrt[3]{x}$;

4) $y=x^4+|x|$; 5) $y=|x|+x^3$; 6) $y=\sqrt[3]{x-1}$.

**103.** Simmetriyadan foydalanib, juft funksiyaning grafigini yasang:

1) $y=x^2-2|x|+1$; 2) $y=x^2-2x$.

**104.** Simmetriyadan foydalanib, toq funksiyaning grafigini yasang:

1) $y=x|x|-2x$; 2) $y=x|x|+2x$.

**105.** Funksiyaning xossalarini aniqlang va uning grafigini yasang:

1) $y=\sqrt{x-5}$; 2) $y=\sqrt{x}+3$; 3) $y=x^4+2$; 4) $y=1-x^4$.

**106.** Funksiyaning grafigini yasang:

1) $y=\begin{cases} x^2, \text{ agar } x \geq 0 \text{ bo'lsa,} \\ x^3, \text{ agar } x<0 \text{ bo'lsa;} \end{cases}$ 2) $y=\begin{cases} x^3, \text{ agar } x>0 \text{ bo'lsa,} \\ x^2, \text{ agar } x \leq 0 \text{ bo'lsa;} \end{cases}$

3) $y=\begin{cases} -x^3, \text{ agar } x \leq 0 \text{ bo'lsa,} \\ -x^2, \text{ agar } x \geq 0 \text{ bo'lsa;} \end{cases}$ 4) $y=\begin{cases} x^4, \text{ agar } x \leq 1 \text{ bo'lsa,} \\ -x^2+2x, \text{ agar } x \geq 1 \text{ bo'lsa.} \end{cases}$

Argumentning qanday qiymatlarida funksiyaning qiymatlari musbat bo'lishini aniqlang. O'sish va kamayish oraliqlarini ko'rsating.

**107.** $y$ funksiya berilgan:

1) $y=x$; 2) $y=x^2$; 3) $y=x^2+x$; 4) $y=x^2-x$.

$x>0$ bo'lganda $y$ funksiyaning grafigini yasang. $x<0$ uchun shu funksiyalardan har birining grafigini shunday yasangki, yasalgan grafik: a) juft funksiyaning; b) toq funksiyaning grafigi bo'lsin. Hosil qilingan har bir funksiyani bitta formula bilan bering.

**108.** Funksiya grafigi simmetriya o'qining tenglamasini yozing:

1) $y=(x+1)^6$; 2) $y=x^6+1$; 3) $y=(x-1)^4$.

**109.** Funksiya grafigi simmetriya markazining koordinatalarini ko'rsating:

1) $y=x^3+1$; 2) $y=(x+1)^3$; 3) $y=x^5-1$.

## 12-§. DARAJA QATNASHGAN TENGSIZLIK VA TENGLAMALAR

Darajali funksiyaning xossalaridan har xil tenglama va tengsizliklarni yechishda foydalaniladi.

**1-masala.** $x^5 > 32$ tengsizlikni yeching.

△ $y=x^5$ funksiya $x$ ning barcha haqiqiy qiymatlarida aniqlangan va o‘sadi. $y(2)=32$ bo‘lgani uchun $x>2$ bo‘lganda $y(x)>32$ va $x>2$ bo‘lganda $y(x)>32$.

**Javob:** $x>2$. ▲

**2-masala.** $x^4 \leq 81$ tengsizlikni yeching.

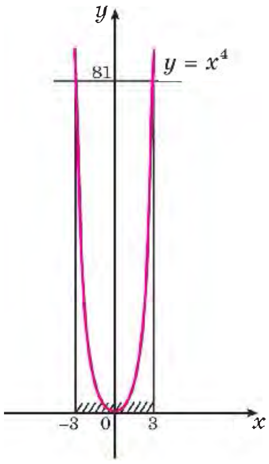
△ $y=x^4$ funksiya $x<0$ bo‘lganda kamayadi va $x>0$ bo‘lganda o‘sadi. $x^4=81$ tenglama ikkita haqiqiy ildizga ega: $x_1=-3$, $x_2=3$. Shuning uchun $x^4 \leq 81$ tengsizlik $x \leq 0$ bo‘lganda $-3 \leq x \leq 0$ yechimlarga va $x \geq 0$ bo‘lganda $0 \leq x \leq 3$ yechimlarga ega (41-rasm).

**Javob:** $-3 \leq x \leq 3$. ▲

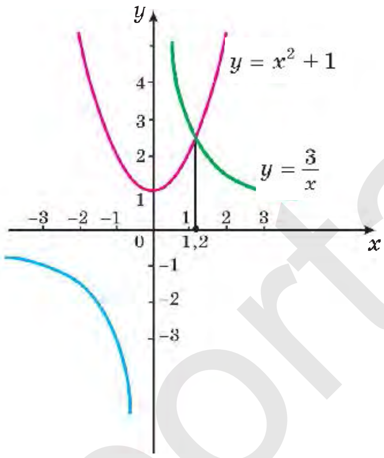
**3-masala.** Funksiyalarning grafiklari yordamida $\frac{3}{x}=x^2+1$ tenglamani yeching.

Bitta koordinatalar tekisligida $y=\frac{3}{x}$ va $y=x^2+1$ funksiyalarning grafiklarini yasaymiz (42- rasm).

△ $x<0$ bo‘lganda $\frac{3}{x}=x^2+1$ tenglama ildizlarga ega emas, chunki $\frac{3}{x}<0$, lekin $x^2+1>0$. $x>0$ bo‘lganda bu tenglama shu funksiyalar kesishish nuqtasining abssissasiga teng bo‘lgan bitta ildizga ega. 42-rasmdan ko‘rinib turibdiki, $x_1 \approx 1{,}2$. Tenglama boshqa musbat ildizlarga ega emas, chunki $x>x_1$ bo‘lganda $y=\frac{3}{x}$ funksiya kamayadi, $y=x_2+1$ funksiya esa o‘sadi va demak, funksiya-




*41-rasm.*




*42-rasm.*

larning grafiklari $x > x_1$ bo'lganda kesishmaydi. Xuddi shu sababga ko'ra ular $0 < x < x_1$ bo'lganda ham kesishmaydi.

**Javob:** $x_1 \approx 1{,}2$. ▲

**4-masala.** Tenglamani yeching:

$$\sqrt{2-x^2} = x. \qquad (1)$$

△ Aytaylik, $x$ – berilgan tenglamaning ildizi bo'lsin, ya'ni $x$ – shunday sonki, u (1) tenglamani to'g'ri tenglikka aylantiradi. Tenglamaning ikkala qismini kvadratga ko'tarib, hosil qilamiz:

$$2 - x^2 = x^2. \qquad (2)$$

Bundan $x^2 = 1$, $x_{1,2} = \pm 1$.

Demak, (1) tenglama ildizlarga ega, deb faraz qilib, biz bu ildizlar faqat 1 va –1 sonlari bo'lishi mumkinligini bilib oldik, endi bu sonlar (1) tenglamaning ildizlari bo'lish yoki bo'lmasligini tekshiramiz. $x = 1$ bo'lganda (1) tenglama to'g'ri tenglikka aylanadi: $\sqrt{2-1^2} = 1$. Shuning uchun $x = 1$ (1) tenglamaning ildizi.

$x=-1$ bo‘lganda (1) tenglamaning chap qismi $\sqrt{2-(-1)^2}=\sqrt{1}=1$ ga teng, o‘ng qismi esa $-1$ ga teng, ya’ni $x=-1$ (1) tenglamaning ildizi bo‘la olmaydi.

**Javob:** $x=1$. ▲

Qaralgan masalada (1) tenglama uning ikkala qismini kvadratga ko‘tarish yo‘li bilan yechiladi. Bunda (2) tenglama hosil bo‘ldi.

(1) tenglama faqat bitta ildizga ega: $x=1$, (2) tenglama esa ikkita ildizga ega: $x_{1,2}=\pm1$, ya’ni (1) tenglamadan (2) tenglamaga o‘tishda *chet ildizlar* deb ataluvchi ildizlar paydo bo‘ldi. Bu shuning uchun ham sodir bo‘ldiki, $x=-1$ bo‘lganda (1) tenglama $1=-1$ dan iborat noto‘g‘ri tenglikka aylandi, bu noto‘g‘ri tenglikning ikkala qismini kvadratga ko‘tarishda esa $1^2=(-1)^2$ dan iborat to‘g‘ri tenglik hosil bo‘ldi.

**!** **Tenglamaning ikkala qismini kvadratga ko‘tarishda chet ildizlar paydo bo‘lishi mumkin.**

**Tenglamani uning ikkala qismini kvadratga ko‘tarish bilan yechishda tekshirish o‘tkazish zarur.**

(1) tenglama – *irratsional tenglamaga* misol.

Yana irratsional tenglamalarga misollar keltiramiz:

$$\sqrt{3-2x}=1-x;\ \sqrt{x+1}=2-\sqrt{x-3}.$$

Bir nechta irratsional tenglamalarni yechaylik.

**5-masala.** Tenglamani yeching: $\sqrt{5-2x}=1-x$.

△ Tenglamaning ikkala qismini kvadratga ko‘taramiz:

$$5-2x=x^2-2x+1$$

yoki $x^2=4$, bundan $x_1=2$, $x_2=-2$. Topilgan ildizlarni tekshiramiz:

$x-2$ bo‘lganda berilgan tenglamaning chap qismi $\sqrt{5-2\cdot2}=1$ ga teng, o‘ng qismi $1-2=-1$ ga teng. $1\neq-1$ bo‘lganligi uchun $x=2$ berilgan tenglamaning ildizi bo‘la olmaydi.

$x=-2$ bo‘lganda tenglamaning chap qismi $\sqrt{5-2\cdot(-2)}=3$ ga teng, o‘ng qismi $1-(-2)=3$ ga teng. Demak, $x=-2$ berilgan tenglamaning ildizi.

**Javob:** $x=-2$. ▲

**6-masala.** Tenglamani yeching: $\sqrt{x-2}+3=0$.

△ Bu tenglamani $\sqrt{x-2}=-3$ ko‘rinishda yozib olaylik.

Arifmetik ildiz manfiy bo‘lishi mumkin emas, binobarin, bu tenglama ildizlarga ega emas.

**Javob:** Ildizlari yo‘q. ▲

**7-masala.** Tenglamani yeching: $\sqrt{x-1}+\sqrt{11-x}=4$.

△ Tenglamaning ikkala qismini kvadratga ko‘tarib,hosil qilamiz:

$$x-1+2\sqrt{x-1}\cdot\sqrt{11-x}+11-x=16.$$

O‘xshash hadlarni ixchamlab, tenglamani bunday ko‘rinishda yozamiz:

$$2\sqrt{x-1}\cdot\sqrt{11-x}=6 \text{ yoki } \sqrt{x-1}\cdot\sqrt{11-x}=3.$$

Oxirgi tenglamaning ikkala qismini kvadratga ko‘taraylik:

$$(x-1)(11-x)=9 \text{ yoki } x^2-12x+20=0,$$

bundan $x_1=2$, $x_2=10$. Tekshirish 2 va 10 sonlaridan har biri berilgan tenglamaning ildizi bo‘lishini ko‘rsatadi.

**Javob:** $x_1=2$, $x_2=10$. ▲

**8-masala.** Tengsizlikni yeching: $\sqrt{5-x}\le 7+x$.

△ Tengsizlik $x$ ning $-7\le x\le 5$ qiymatlarida ma’noga ega. Agar tengsizlik yechimga ega bo‘lsa, yechim shu $[-7;5]$ kesmaga tegishli bo‘ladi. Tengsizlikning har ikkala qismini kvadratga oshiramiz va ixchamlashdan so‘ng $x^2+15x+44\ge 0$ tengsizlikka kelamiz. Uning yechimi $x\le -11, x\ge -4$ ekani ravshan. Bu oraliqlarning $[-7;5]$ kesma bilan umumiy qismi $-4\le x\le 5$, ya’ni $[-4;5]$kesma bo‘ladi:

**Javob:** $-4\le x\le 5$. ▲

## *Mashqlar*

**110.** Tengsizlikni yeching:

1) $x^7>1$; 2) $x^3\le 27$; 3) $y^3\ge 64$; 4) $y^3<125$;

5) $x^4\le 16$; 6) $x^4>625$; 7) $x^5\le 243$; 8) $x^6\ge 64$.

**111.** 1) Agar kvadratning yuzi 361 $cm^2$ dan katta ekanligi ma'lum bo'lsa, uning tomoni qanday bo'lishi mumkin?
2) Agar kubning hajmi 343 $dm^3$ dan katta ekanligi ma'lum bo'lsa, uning qirrasi qanday bo'lishi mumkin?

**112.** (Og'zaki.) 7 soni tenglamaning ildizi bo'lishini ko'rsating:

1) $\sqrt{x-3}=2$; 2) $\sqrt{x^2-13}-\sqrt{2x-5}=3$; 3) $\sqrt{2x+11}=5$.

**113.** (Og'zaki.) Tenglamani yeching:

1) $\sqrt{x}=3$; 2) $\sqrt{x}=7$; 3) $\sqrt{2x-1}=0$; 4) $\sqrt{3x+2}=0$.

Tenglamani yeching (**114–117**):

**114.** 1) $\sqrt{x+1}=2$; 2) $\sqrt{x-1}=3$; 3) $\sqrt{1-2x}=4$;
4) $\sqrt{2x-1}=3$; 5) $\sqrt{3x+1}=10$; 6) $\sqrt{9-x}=4$.

**115.** 1) $\sqrt{x+1}-\sqrt{2x-3}$; 2) $\sqrt{x-2}-\sqrt{3x-6}$;
3) $\sqrt{x^2+24}=\sqrt{11x}$; 4) $\sqrt{x^2+4x}=\sqrt{14-x}$.

**116.** 1) $\sqrt{x+2}=x$; 2) $\sqrt{3x+4}=x$; 3) $\sqrt{20-x^2}=2x$;
4) $\sqrt{0,4-x^2}=3x$; 5) $\sqrt{4-x}=-\frac{x}{3}$; 6) $\sqrt{26-x^2}=5x$.

**117.** 1) $\sqrt{x^2-x-8}=x-2$; 2) $\sqrt{x^2+x-6}=x-1$.

**118.** Tengsizlikni yeching:
1) $(x-1)^3>1$; 2) $(x+5)^3>8$; 3) $(2x-3)^7\geq 1$;
4) $(3x-5)^7<1$; 5) $(3-x)^4>256$; 6) $(4-x)^4>81$.

**119.** Berilgan tenglama nima uchun ildizlarga ega emasligini tushuntiring:
1) $\sqrt{x}=-8$; 2) $\sqrt{x}+\sqrt{x-4}=-3$; 3) $\sqrt{-2-x^2}=12$;
4) $\sqrt{7x-x^2-63}=5$; 5) $\sqrt{x^2+7}=2$; 6) $\sqrt{x-2}=x$.

Tenglamani yeching (**120–122**):

**120.** 1) $\sqrt{x^2-4x+9}=2x-5$; 2) $\sqrt{x^2+3x+6}=3x+8$;

3) $2x=1+\sqrt{x^2+5}$; 4) $x+\sqrt{13-4x}=4$.

**121.** 1) $\sqrt{x+12}=2+\sqrt{x}$; 2) $\sqrt{4+x}+\sqrt{x}=4$.

**122.** 1) $\sqrt{2x+1}+\sqrt{3x+4}=3$; 2) $\sqrt{4x-3}+\sqrt{5x+4}=4$;

3) $\sqrt{x-7}-\sqrt{x+17}=-4$; 4) $\sqrt{x+4}-\sqrt{x-1}=1$.

**123.** $x$ ning qanday qiymatlarida funksiyalar bir xil qiymatlarni qabul qiladi:

1) $y=\sqrt{4+\sqrt{x}}$, $y=\sqrt{19-2\sqrt{x}}$; 2) $y=\sqrt{7+\sqrt{x}}$, $y=\sqrt{11-\sqrt{x}}$?

**124.** Tengsizlikni yeching:

1) $\sqrt{x-2}>3$; 2) $\sqrt{x-2}\le 1$; 3) $\sqrt{2-x}\ge x$;

4) $\sqrt{2-x}<x$; 5) $\sqrt{5x+11}>x+3$; 6) $\sqrt{x+3}\le x+1$.

## *I bobga doir mashqlar*

**125.** $x$ ning $y=2x^2-5x+3$ kvadrat funksiya: 1) 0 ga; 2) 1 ga; 3) 10 ga; 4) $-1$ ga teng qiymatlar qabul qiladigan qiymatini toping.

**126.** Tengsizlikni yeching:
1) $x^2\le 5$; 2) $x^2>36$; 3) $x^2\ge 9$; 4) $x^2<8$.

**127.** Parabolaning koordinata o'qlari bilan kesishish nuqtalari koordinatalarini toping:
1) $y=x^2+x-12$; 2) $y=-x^2+3x+10$;
3) $y=-8x^2-2x+1$; 4) $y=7x^2+4x-11$.

**128.** Parabola uchining koordinatalarini toping:
1) $y=x^2-4x-5$; 2) $y=-x^2-2x+3$;
3) $y=x^2-6x+10$; 4) $y=x^2+x+\frac{5}{4}$.

**129.** Funksiyaning grafigini yasang va grafik bo‘yicha uning xossalarini aniqlang:

1) $y = x^2 - 5x + 6$; 2) $y = x^2 + 10x + 30$;
3) $y = -x^2 - 6x - 8$; 4) $y = 2x^2 - 5x + 2$.

**130.** To‘g‘ri to‘rtburchakning perimetri 600 m. To‘g‘ri to‘rtburchakning yuzi eng katta bo‘lishi uchun uning asosi bilan balandligi qanday bo‘lishi kerak?

**131.** Agar $y = x^2 + px + q$ kvadrat funksiya:

1) $x = 0$ bo‘lganda 2 ga teng qiymatni, $x = 1$ bo‘lganda esa 3 ga teng qiymatni qabul qilsa, $p$ va $q$ koeffitsiyentlarni toping;
2) $x - 0$ bo‘lganda 0 ga teng qiymatni, $x - 2$ bo‘lganda esa 6 ga teng qiymatni qabul qilsa, $p$ va $q$ koeffitsiyentlarni toping.

**132.** $x$ ning qanday qiymatlarida funksiyalar teng qiymatlar qabul qiladi:

1) $y = x^2 + 3x + 2$ va $y = |7 - x|$;
2) $y = 3x^2 - 6x + 3$ va $y = |3x - 3|$?

Tengsizlikni yeching (**133–137**):

**133.** 1) $(x - 5,7)(x - 7,2) > 0$; 2) $(x - 2)(x - 4) > 0$;
3) $(x - 2,5)(3 - x) < 0$; 4) $(x - 3)(4 - x) < 0$.

**134.** 1) $x^2 > x$; 2) $x^2 > 36$; 3) $4 > x^2$; 4) $\frac{9}{16} > x^2$.

**135.** 1) $-2x^2 + 4x + 30 < 0$; 2) $-2x^2 + 9x - 4 > 0$;
3) $4x^2 + 3x - 1 < 0$; 4) $2x^2 + 3x - 2 < 0$.

**136.** 1) $x^2 - 3x + 8 > 0$; 2) $x^2 - 5x + 10 < 0$;
3) $2x^2 - 3x + 5 \geq 0$; 4) $3x^2 - 4x + 5 \leq 0$;
5) $-x^2 + 2x + 4 \leq 0$; 6) $-4x^2 + 7x - 5 \geq 0$.

**137.** 1) $(x - 2)(x^2 - 9) > 0$; 2) $(x^2 - 1)(x - 4) < 0$;

3) $\frac{(x+3)(x-5)}{x+1} \leq 0$; 4) $\frac{x-7}{(4-x)(2x+1)} \geq 0$; 5) $\frac{4x^2-4x-3}{x-3} > 0$;

6) $\frac{2x^2-3x-2}{x-1} < 0$; 7) $\frac{(x+1)(x-4)}{x^2-1} \geq 0$; 8) $\frac{x-1}{6x^2-7x-3} \leq 0$.

Tengsizlikni yeching (**138–139**):

**138.** 1) $\frac{1}{3}x \quad \frac{4}{9}x^2 > 1 - x$; 2) $\frac{1}{3}x(x+1) \le (x+1)^2$;

3) $x(1-x) > 1,5-x$; 4) $\frac{1}{3}x - \frac{4}{9} \ge x(x-1)$.

**139.** 1) $\frac{3x^2-5x-8}{2x^2-5x-3} > 0$; 2) $\frac{4x^2+x-3}{5x^2-9x-2} < 0$; 3) $\frac{2+7x-4x^2}{3x^2+2x-1} \le 0$;

4) $\frac{2+9x-5x^2}{3x^2-2x-1} \ge 0$; 5) $\frac{x^2-5x+6}{x^2+5x+6} > 0$; 6) $\frac{x^2+8x+7}{x^2+x-2} \le 0$.

**140.** Kater 4 soatdan ko‘p bo‘lmagan vaqt davomida daryo oqimi bo‘yicha 22,5 km yurishi va orqasiga qaytishi kerak. Agar daryo oqimining tezligi 3 km/h bo‘lsa, kater suvga nisbatan qanday tezlik bilan yurishi kerak?

**141.** Funksiyalarning grafiklarini bitta koordinata sistemasida yasang va $x$ ning qanday qiymatlarida bir funksiyaning qiymati ikkinchisinikidan katta (kichik) bo‘lishini aniqlang, natijani, tegishli tengsizlikni yechib, tekshiring:

1) $y=2x^2$, $y=2-3x$;
2) $y-x^2-2$, $y-1-2x$;
3) $y=x^2-5x+4$, $y=7-3x$;
4) $y=3x^2-2x+5$, $y=5x+3$.

Funksiyalar grafiklari kesishish nuqtalarining koordinatalarini toping (**142–143**):

**142.** 1) $y=x^2, y=x^3$; 2) $y=\frac{1}{x}, y=2x$; 3) $y=3x, y=\frac{3}{x}$.

**143.** 1) $y=\sqrt{x}, y=|x|$; 2) $y=\sqrt[3]{x}, y=\frac{1}{x}$; 3) $y=\sqrt{x}, y=x$.

**144.** Tengsizlikni yeching:

1) $x^4 \le 81$; 2) $x^5 > 32$; 3) $x^6 > 64$; 4) $x^5 \le -32$.

Tenglamani yeching (**145–146**):

**145.** 1) $\sqrt{3-x}=2$; 2) $\sqrt{3x+1}=7$; 3) $\sqrt{3-11x}=2x$.

**146.** 1) $\sqrt{2x-1}=x-2$; 2) $\sqrt{5x-1+3x^2}=3x$; 3) $\sqrt{2-2x}=x+3$.

**147.** Funksiyaning aniqlanish sohasini toping:

1) $y=\sqrt[5]{x^3+x-2}$; | 2) $x=\sqrt[3]{x^2+2x-15}$; | 3) $x=\sqrt[6]{6-x-x^2}$;

4) $y=\sqrt[4]{13x-22-x^2}$; | 5) $y=\sqrt{\dfrac{x^2+6x+5}{x+7}}$; | 6) $y=\sqrt{\dfrac{x^2-9}{x^2+8x+7}}$.

**148.** Funksiyaning ko'rsatilgan oraliqda o'sishi yoki kamayishini aniqlang:

1) $y=\dfrac{1}{(x-3)^2}$, $x>3$ oraliqda; 2) $y=\dfrac{1}{(x-2)^3}$, $x<2$ oraliqda;

3) $y=\sqrt[3]{x+1}$, $x\geq 0$ oraliqda; 4) $y=\dfrac{1}{\sqrt[3]{x+1}}$, $x<-1$ oraliqda.

### *O'ZINGIZNI TEKSHIRIB KO'RING!*

**1.** $y=-x^2+2x+3$ funksiya grafigi yordamida $x$ ning qanday qiymatida funksiyaning qiymati 3 ga teng bo'lishini toping.

**2.** $y=1-x^2$ funksiyaning grafigi bo'yicha $x$ ning funksiya musbat; manfiy qiymatlar qabul qiladigan qiymatlarini toping.

**3.** 1) $y=2x^2$; 2) $y=-3x^2$ funksiya qanday oraliqlarda o'sadi? Kamayadi? Shu funksiyaning grafigini yasang.

**4.** Tengsizlikni intervallar usuli bilan yeching:

1) $x(x-1)(x+2)\geq 0$; 2) $(x+1)(2-x)(x-3)\leq 0$.

**5.** Funksiyaning aniqlanish sohasini toping:

1) $y=\dfrac{8}{x-1}$; 2) $y=\sqrt{9-x^2}$; 3) $y=\sqrt{4-2x}$.

**6.** Tenglamani yeching:

1) $\sqrt{x-3}=5$; 2) $\sqrt{3-x-x^2}=x$; 3) $y=\sqrt{32-x^2}=x$.

**149.** Funksiyaning juft yoki toqligini aniqlang:

1) $y=x^6-3x^4+x^2-2$; 2) $y=x^5-x^3+x$;

3) $y=\frac{1}{(x-2)^2}+1$; 4) $y=x^7+x^5+1$.

**150.** Tengsizlikni yeching:

1) $(3x+1)^4>625$; 2) $(3x^2+5x)^5\leq 32$; 3) $(x^2-5x)^5>216$.

**151.** Tenglamani yeching:

1) $\sqrt{2x^2+5x-3}=x+1$; 2) $\sqrt{3x^2-4x+2}=x+4$;

3) $\sqrt{x+11}=1+\sqrt{x}$; 4) $\sqrt{x+19}=1+\sqrt{x}$.

**152.** Tengsizlikni yeching:

1) $\sqrt{x^2-8x}>3$; 2) $\sqrt{x^2-3x}<2$; 3) $\sqrt{3x-2}>x-2$;

4) $\sqrt{2x+1}\leq x-1$; 5) $\sqrt{3-x}>1-x$; 6) $\sqrt{4x-x^2}>4-x$.

### *I bobga doir sinov (test) mashqlari*

*Sinov mashqlarining har biriga 4 tadan „javob“ berilgan. 4 ta „javob“ning faqat bittasi to‘g‘ri, qolganlari esa noto‘g‘ri. O‘quvchilardan sinov mashqlarini bajarib yoki boshqa mulohazalar yordamida ana shu to‘g‘ri javobni topish (uni belgilash) talab qilinadi.*

**1.** $a$ ning shunday qiymatini topingki, $y=ax^2$ parabola bilan $y=5x+1$ to‘g‘ri chiziqning kesishish nuqtalaridan birining abssissasi $x=1$ bo‘lsin.

A) $a=6$; B) $a=-6$; C) $a=4$; D) $a=-4$.

Parabolaning koordinata o‘qlari bilan kesishish nuqtalarining koordinatalarini toping (**2–3**):

**2.** $y=x^2-2x+4$.

A) $(-1; 3)$; B) $(3; 1)$; C) $(1; 3)$; D) $(0; 4)$.

**3.** $y=6x^2-5x+1$.

A) $(\frac{1}{3}; 0), (\frac{1}{2}; 0), (0; 1)$; B) $(-\frac{1}{3}; 0), (-\frac{1}{2}; 0), (1; 0)$;

C) $(0; \frac{1}{3}), (0; \frac{1}{2}), (0; 1)$; D) $(\frac{1}{3}; 0), (-\frac{1}{2}; 0), (0; -1)$.

Parabola uchining koordinatalarini toping (**4–5**):

**4.** $y = x^2 - 4x$.

A) (0; 4); B) (4; 2); C) (2; −4); D) (−4; 2).

**5.** $y = x^2 + 6x + 5$.

A) (−3; −4); B) (−5; −1); C) (−1; −5); D) (3; 4).

**6.** Abssissalar o‘qini $x = 1$ va $x = 2$ nuqtalarda, ordinatalar o‘qini esa $y = \frac{1}{2}$ nuqtada kesib o‘tuvchi parabolaning tenglamasini yozing.

A) $y = \frac{1}{2}x^2 - \frac{3}{4}x + \frac{1}{2}$; B) $y = \frac{1}{4}x^2 - \frac{3}{4}x - \frac{1}{2}$;

C) $y = x^2 - 3x + 2$; D) to‘g‘ri javob berilmagan.

Parabola qaysi choraklarda joylashgan (**7–8**)?

**7.** $y = 3x^2 + 5x - 2$.

A) I, II, III; B) II, III, IV; C) I, III, IV; D) I, II, III, IV;

**8.** $y - -x^2 - 6x - 11$.

A) III, IV; B) I, II, III; C) II, III, IV; D) I, II.

**9.** Ikki musbat sonning yig‘indisi 160 ga teng. Agar shu sonlar kublarining yig‘indisi eng kichik bo‘lsa, shu sonlarni toping.

A) 95; 65; B) 155; 5; C) 75; 85; D) 80; 80.

**10.** $y = x^2 - 4x + 3$ funksiyaning eng kichik qiymatini toping.

A) −1; B) 1; C) 7; D) −8.

Tengsizlikni yeching (**11–15**):

**11.** $2x^2 - 8 \le 0$.

A) $-2 \le x \le 2$; B) $-2 \le x$; C) $x \ge 2$; D) $0 \le x \le 4$.

**12.** $3x^2 - 9 \ge 0$.

A) $x < \sqrt{3}$; B) $x > \sqrt{3}$; C) $x < -\sqrt{3}$, $x > \sqrt{3}$; D) $x \ge 3$.

**13.** $6x^2 + 5x - 6 > 0$.

A) $x > \frac{2}{3}$; B) $x < \frac{3}{2}$; C) $x < -\frac{3}{2}$, $x > \frac{2}{3}$; D) $-\frac{3}{2} < x < \frac{2}{3}$.

**14.** $\dfrac{x^2-7x+10}{x^2-3x-10} \le 0.$

A) $-2<x\le 2$; B) $-2< x <5$; C) $x\ne -2, x\ne 5$; D) $-2<x<0$.

**15.** $\dfrac{x^2+x}{-x^2+6x-8} \ge 0.$

A) $-2<x<3$; B) $x<-2;\ -1\le x\le 1,\ x>3$;

C) $-1\le x<3$; D) $x\ne -2,\ \ x\ne 3$.

**16.** $x^2+6x+5<0$ tengsizlikning barcha butun yechimlari yig‘indisini toping.

A) 10; B) 9; C) −9; D) −10.

**17.** $a$ ning qanday qiymatlarida $ax^2+4x+9a<0$ tengsizlik $x$ ning barcha qiymatlarida o‘rinli bo‘ladi?

A) $a<-\frac{2}{3}$; B) $a>\frac{2}{3}$; C) $a<-1$; D) $a>1$.

**18.** $a$ ning qanday qiymatida $ax^2-8x-2<0$ tengsizlik $x$ ning barcha qiymatlarida o‘rinli bo‘ladi?

A) $-8<a<8$; B) $a>8$; C) $a<8$; D) $a<-8$.

**19.** Funksiyaning aniqlanish sohasini toping: $y=\sqrt{-x^2+3x-2}$.

A) $1\le x\le 2$; B) $1<x<2$; C) $x\ge 2, x\le 1$; D) $-2\le x\le -1$.

**20.** Funksiyalarning qaysilari juft funksiya?

1) $y=x-\frac{1}{x}$; 2) $y-x^2+|x|$; 3) $y=-3+\frac{5}{x^4}$; 4) $y=x^2-\frac{3}{x}$.

A) 1, 2; B) 3, 4; C) 2, 3; D) 1, 4.

**21.** Funksiyalarning qaysilari toq funksiya?

1) $y=6x$; 2) $y=\sqrt[3]{x}$; 3) $y=4x+7$; 4) $y=2x^3-10$.

A) 1, 2; B) 2, 3; C) 3, 4; D) 1, 4.

## Amaliy-tatbiqiy va fanlararo bog‘liq masalalar

**1- masala.** Yengil avtomobil o‘zgarmas $v$ tezlik bilan harakatlanmoqda. Stop chizig‘igacha 50 metr qolganda svetoforning yashil chirog‘i o‘chib-yona boshladi. Shundan yarim sekund o‘tgandan keyin haydovchi tormozlanishni boshladi va stop chizig‘iga yetmasdan to‘xtadi. Yo‘l harakati qoidalaridan ma’lumki, $v_0 = 50$ km/h tezlik bilan harakat qilgan avtomobilning tormozlanish yo‘li $S_0 = 23{,}5$ m, bunda tormozlanish yo‘li deb tormozlanish boshlanishidan tugagunicha avtomobil bosib o‘tgan yo‘lga aytiladi. Avtomobilning svetofor o‘chib-yonishni boshlagandagi $v$ tezligini baholang.

△ Svetofor o‘chib-yona boshlaganidan tormozlanish boshlanguncha avtomobil $0{,}5\ v$ metr masofani, keyin esa fizika kursidan ma’lum bo‘lgan $kv^2$ tormozlanish yo‘lini bosib o‘tadi, bunda

$$k = \frac{s_0}{v_0^2} = \frac{23{,}5}{13{,}88^2} \approx 0{,}12.$$

Demak, umumiy bosib o‘tilgan masofa, 50km/h tezlikning metr sekundlarda 13,88 m/s ekanligini hisobga olsak, 50 metrdan oshmasligi kerakligidan

$$0{,}5\,v + 0{,}12v^2 \le 50,$$

ya’ni

$$0{,}12v^2 + 0{,}5v - 50 \le 0. \qquad (1)$$

△ Bu tengsizlikni yechish uchun avval $0{,}12v^2 + 0{,}5v - 50$ uchhadning ildizlarini topamiz:

$$0{,}12v^2 + 0{,}5v - 50 = 0,$$

bundan

$$12v^2 + 50v - 5000 = 0.$$

Tenglamani yechamiz:

$$v_{1,2} = \frac{-50 \pm \sqrt{50^2 - 4 \cdot 12(-5000)}}{2 \cdot 12} = \frac{-25(1 = \sqrt{97})}{12},$$

bundan $v_1 = \dfrac{-25(1 + \sqrt{97})}{12}$ va $v_2 = \dfrac{25(\sqrt{97}\ \ 1)}{12}$.

U holda, (1) tengsizlikning yechimi $v_1 \le v \le v_2$ oraliqdagi sonlardan iborat. Lekin masalaning mohiyatiga ko‘ra, $v > 0$, demak, baholanayotgan $v$ tezlik $0 < v \le v_2$ oraliqdan tashqariga chiqib ketmasligi kerak, ya’ni $v \le \frac{-25(1=\sqrt{97}-1)}{12} \approx 18{,}43$ m/s yoki 66,35 $km/h$ dan oshmasligi kerak.

**Javob:** tezlik 66,35 $km/h$ dan oshmasligi kerak. ▲

**2- masala.** Bozorda ma’lum bir turdagi tovarlardan $n$ donasi bor va ular donasi $p$ pul birligida sotilmoqda, deylik. Monitoring shuni ko‘rsatdiki, ushbu tovarga bo‘lgan talab oshganda uning narxi oshadi va keltiriladigan shunday tovarlar soni $n = 40p$ formulaga ko‘ra o‘sadi. Ikkinchi tomondan, bozorga kirib kelib, xaridorlarga taklif etiladigan tovarlar soni $n$ o‘sa boshlashi bilan uning narxi teskari proporsional ravishda tushib borishi ma’lum:

$$p = \frac{150}{n-40}.$$

Bozorga kirib kelayotgan tovarlar soniga qo‘yiladigan shartni aniqlang.

△ Masalada so‘ralayotgan shartni aniqlash uchun taklif qilinayotgan narx $\frac{150}{n-40}$ talab bilan bog‘liq narx $\frac{n}{40}$ dan kam bo‘lmaslik shartidan foydalanamiz:

$$\frac{150}{n-40} \ge \frac{n}{40}.$$

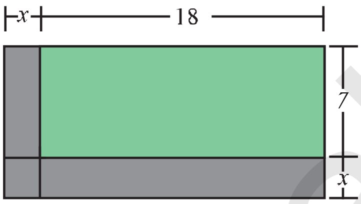
Bundan

$$n^2 - 40n - 6000 \le 0$$

tengsizlikni hosil qilamiz. Uning yechimlari $-60 \le n \le 100$. Masalaning mohiyatiga ko‘ra, bozorga kirib keladigan tovarlar soni $n$ natural son va u 100 dan oshmasligi kerak.

**Javob:** $n \le 100$. ▲

**3- masala.** Siz 7 metrga 18 metrli bog‘ingizning ikki tomonida toshdan yo‘lka qilmoqchisiz (43- rasm). Lekin Siz buning uchun 54 kvadrat metrdan ortiq bo‘lmaydigan joyni qoplay olishga yetadigan mablag‘ ajrata olasiz. Bunday yo‘lkaning eni ko‘pi bilan qanday bo‘lishi kerak?



*43-rasm*

△ Ravshanki, masalaning yechimini topish uchun umumiy yuz $(x + 18) \cdot (x + 7)$ kv.m dan bog‘ning yuzi $18 \cdot 7 = 126$ kv.m ni ayirib, natija yo‘lkaning yuzi 54 kv.m dan oshmasligini inobatga olishimiz kerak:

$$(x + 18) \cdot (x + 7) - 18 \cdot 7 \leq 54. \quad (1)$$

Bundan

$$x^2 + 25x - 54 \leq 0 \quad (2)$$

kvadrat tengsizlikni hosil qilamiz. $x^2+25x-54$ kvadrat uchhadning ildizlari $x_1 = -27$ va $x_2 = 2$ bo‘lgani uchun (2) tengsizlikning yechimlari $-27 \leq x \leq 2$ oraliqdagi sonlardan iborat bo‘ladi. Lekin masalaning mohiyatiga ko‘ra, yo‘lkaning eni $x$ manfiy son yoki nol bo‘la olmaydi. Shu sababli yo‘lkaning eni $0 < x \leq 2$ tengsizlikni qanoatlantiruvchi son bo‘la oladi. Demak, yo‘lkaning eni 2 metrdan oshmasligi kerak.

**Javob:** yo‘lkaning eni ko‘pi bilan 2 metr. ▲

## *Masalalar*

1. Yuk mashinasi $v$ o‘zgarmas tezlik bilan harakatlanmoqda. Stop chizig‘igacha 50 metr qolganda svetoforning yashil chirog‘i o‘chib-yona boshladi Shundan yarim sekund o‘tgandan keyin haydovchi tormozlanishni boshladi va stop chizig‘iga yetmasdan to‘xtadi. Yo‘l harakati qoidalaridan ma’lumki, $v_0 = 50$ km/h tezlik bilan harakat qilgan yuk mashinasining tormozlanish

yo‘li $S_0 = 28{,}9$ m. Yuk mashinasining svetofor o‘chib-yonishni boshlagandagi $v$ tezligini 0,01 aniqlikda baholang.

**2.** Bozorda ma’lum bir turdagi tovarlardan $n$ donasi bor va ular donasi $p$ pul birligida sotilmoqda, deylik. Monitoring shuni ko‘rsatdiki, ushbu tovarga bo‘lgan talab oshganda uning narxi oshadi va keltiriladigan shunday tovarlar soni $n = 60p$ formulaga ko‘ra o‘sadi. Ikkinchi tomondan, bozorga kirib kelib, xaridorlarga taklif etiladigan tovarlar soni $n$ o‘sa boshlashi bilan uning narxi teskari proporsional ravishda tushib borishi ma’lum:

$$p = \frac{60}{n - 40}.$$

Bozorga kirib kelayotgan tovar soniga qo‘yiladigan shartni aniqlang.

**3.** Kompaniya reklamaga umumiy $x$ (100 minglarda) so‘m sarflasin va uning natijasida $P$ foyda ko‘rsin deylik, bunda $P(x) = 20 + 40x - x^2$. Reklamaga qancha pul sarflansa, natijada foyda eng ko‘p bo‘ladi?

**4.** Mahsulot ishlab chiqaruvchi kichik korxonaning oylik foydasi $P = 250n - n^2$ (ming so‘mlarda) model bilan ifodalanadi deylik, bu yerda $n$ – ishlab chiqarilgan va sotilgan mahsulotlar soni. Eng katta foyda olish uchun kichik korxona oyiga nechta mahsulot ishlab chiqarishi va sotishi kerak?

**5.** Janubiy Amerikaning yomg‘irli o‘rmonlarining birida noyob turdagi hasharot topildi va atrof-muhitni o‘rganuvchi mutaxassis hasharotlarni himoyalangan hududga o‘tkazdi. O‘tkazilgandan keyin hasharotlar soni $t$ oyda

$$P(t) - 45(1 + 0{,}6t)(3 + 0{,}02t)$$

qonuniyat bilan oshib borgan bo‘lsa:

1) $t = 0$ da hasharotlar soni qancha bo‘lgan?
2) 10 yildan keyin ular soni qancha bo‘ladi?
3) Qachon ular soni 549 ta bo‘ladi?

## *Tarixiy ma'lumotlar*

**Abu Rayhon Beruniy (973–1048)**

„Funksiya" so'zi lotincha „*functio*" so'zidan olingan bo'lib, u „sodir bo'lish", „bajarish" degan ma'noni bildiradi. Funksiyaning dastlabki ta'riflari **G.Leybnis** (1646–1716), **I.Bernulli** (1667–1748), **N.I.Lobachevskiy** (1792–1856) asarlarida berilgan.

Funksiyaning hozirgi ta'rifini bilishmasa-da, qadimgi olimlar o'zgaruvchi miqdorlar orasida funksional bog'lanish bo'lishi lozimligini tushunishgan.

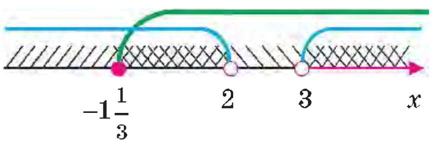
To'rt ming yildan avvalroq Bobil olimlari radiusi $r$ bo'lgan doira yuzi uchun – xatoligi sezilarli bo'lsa-da – $S=3r^2$ formulani chiqarishgan.

Sonning darajasi haqidagi ilk ma'lumotlar qadimgi bobilliklardan bizgacha yetib kelgan bitiklarda mavjud. Xususan, ularda natural sonlarning kvadratlari, kublari jadvallari berilgan.

Sonlarning kvadratlari, kublari jadvali, logarifmlar jadvali, trigonometrik jadvallar, kvadrat ildizlar jadvali miqdorlar orasidagi bog'lanishning jadval usulida berilishi, xolos.

Buyuk qomusiy olim **Abu Rayhon Beruniy** ham o'z asarlarida funksiya tushunchasidan, uning xossalaridan foydalangan. Abu Rayhon Beruniy mashhur „Qonuni Ma'sudiy" asarining 6-maqolasida argument va funksiyaning o'zgarish oraliqlari, funksiyaning ishoralari va eng katta, eng kichik qiymatlarini ta'riflaydi.

# II BOB. TENGLAMALAR VA TENGSIZLIKLAR SISTEMALARI




## 13-§. IKKINCHI DARAJALI TENGLAMA QATNASHGAN ENG SODDA SISTEMALARNI YECHISH

**1- masala.** To‘g‘ri burchakli uchburchakning gipotenuzasi $\sqrt{13}$ cm ga teng, uning yuzi esa 3 cm$^2$. Uchburchakning katetlarini toping.

△ Uchburchakning katetlari $x$ va $y$ santimetrga teng bo‘lsin. Pifagor teoremasi va to‘g‘ri burchakli uchburchakning yuzi formulasidan foydalanib, masala shartini bunday yozamiz:

$$\begin{cases} x^2 + y^2 = 13, \\ \frac{1}{2}xy = 3. \end{cases} \qquad (1)$$

Sistemaning birinchi tenglamasiga 4 ga ko‘paytirilgan ikkinchi tenglamasini qo‘shib, quyidagini hosil qilamiz:

$$x^2 + y^2 + 2xy = 25,$$

bundan $(x + y)^2 = 25$ yoki $x + y = \pm 5$. $x$ va $y$ lar musbat sonlar bo‘lgani uchun $x + y = 5$ bo‘ladi. Bu tenglamada $y$ ni $x$ orqali ifodalaymiz va (1) sistema tenglamalaridan biriga, masalan, ikkinchi tenglamaga qo‘yamiz:

$$y = 5 - x, \quad \frac{1}{2}x(5 - x) = 3.$$

Hosil qilingan tenglamani yechamiz:

$$5x - x^2 = 6,\ \ x^2 - 5x + 6 = 0,\ \ x_1 = 2,\ \ x_2 = 3.$$

Bu qiymatlarni $y - 5 - x$ formulaga qo‘yib, $y_1 = 3$, $y_2 = 2$ ni topamiz. Ikkala holda ham katetlardan biri 2 cm, ikkinchisi esa 3 cm.

**Javob:** 2 cm, 3 cm. ▲

**2-masala.** Tenglamalar sistemasini yeching:

$$\begin{cases} x + y = 3, \\ xy = -10. \end{cases}$$

△ Viyet teoremasiga teskari teoremaga ko‘ra, $x$ va $y$ sonlar

$$z^2 - 3z - 10 = 0$$

kvadrat tenglamaning ildizlari bo‘ladi. Bu tenglamani yechib, quyidagini hosil qilamiz: $z_1 = 5, z_2 = -2$. Demak, sistemaning yechimlari quyidagi sonlar juftliklari bo‘ladi: $x_1 = 5$, $y_1 = -2$ va $x_2 = -2$, $y_2 = 5$.

**Javob:** (5; −2), (−2; 5). ▲

**3-masala.** Tenglamalar sistemasini yeching:

$$\begin{cases} x^2 + 4xy - 2y^2 = -29, \\ 3x - y - 6 = 0. \end{cases}$$

△ Bu sistemani o‘rniga qo‘yish usuli bilan yechamiz:

$$y = 3x - 6,$$

$$x^2 + 4x(3x - 6) - 2(3x - 6)^2 = -29.$$

Bu tenglamani soddalashtirib, quyidagini hosil qilamiz: $5x^2 - 48x + 43 - 0$, bundan $x_1 - 1$, $x_2 - 8{,}6$. $x$ ning qiymatini $y = 3x - 6$ formulaga qo‘yib, $y_1 = -3$, $y_2 = 19{,}8$ ekanini topamiz.

**Javob:** (1; −3), (8,6; 19,8). ▲

**4-masala.** Tenglamalar sistemasini yeching:

$$\begin{cases} x^2 - y^2 = 16, \\ x - y = 2. \end{cases}$$

△ Sistemaning birinchi tenglamasini bunday yozamiz:

$$(x-y)(x+y)=16.$$

Bunga $x-y=2$ ni qo'yib, $x+y=8$ ni hosil qilamiz. Shunday qilib,

$$\begin{cases} x+y=8, \\ x-y=2. \end{cases}$$

Bu sistemani qo'shish usuli bilan yechib, $x=5$, $y=3$ ekanini topamiz.

**J a v o b :** (5; 3). ▲

### *Mashqlar*

**153.** Ikki noma'lumli birinchi darajali tenglamalar sistemasini yeching:

1) $\begin{cases} 2x-y=3, \\ 2y+x=14; \end{cases}$ 2) $\begin{cases} x+5y=9, \\ 3y-2x=-5; \end{cases}$

3) $\begin{cases} 3x+y+4=0, \\ 4y+8x-4=0; \end{cases}$ 4) $\begin{cases} 2x-3y+8=0, \\ 4x-2y+4=0. \end{cases}$

Tenglamalar sistemasini yeching (**154–158**):

**154.** 1) $\begin{cases} y=x+6, \\ x^2-4y=-3; \end{cases}$ 2) $\begin{cases} x=2-y, \\ y^2+x=32; \end{cases}$ 3) $\begin{cases} x+2y=1, \\ x+y^2=4; \end{cases}$

4) $\begin{cases} y-3x=2, \\ x^2-2y=3; \end{cases}$ 5) $\begin{cases} x=4-y, \\ x^2+y=4; \end{cases}$ 6) $\begin{cases} y-4x=5, \\ y^2+2x=-1. \end{cases}$

**155.** 1) $\begin{cases} x^2+xy=2, \\ y-3x=7; \end{cases}$ 2) $\begin{cases} x^2-xy-y^2=19, \\ x-y=7; \end{cases}$ 3) $\begin{cases} x+y=1, \\ x^2+y^2=5; \end{cases}$

4) $\begin{cases} x^2+y^2=17, \\ x-y=3; \end{cases}$ 5) $\begin{cases} x-y=2, \\ x^2-y^2=0; \end{cases}$ 6) $\begin{cases} x+y=0, \\ x^2+y^2=8. \end{cases}$

**156.** 1) $\begin{cases} x+y=5, \\ xy=6; \end{cases}$ 2) $\begin{cases} xy=7, \\ x+y=8; \end{cases}$ 3) $\begin{cases} x+y=12, \\ xy=11; \end{cases}$

4) $\begin{cases} x+y=-7, \\ xy=10; \end{cases}$ 5) $\begin{cases} xy=2, \\ x+y=3; \end{cases}$ 6) $\begin{cases} x+y=-11, \\ xy=18. \end{cases}$

**157.** 1) $\begin{cases} x-y=7, \\ x^2-y^2=14; \end{cases}$ 2) $\begin{cases} x+y=3, \\ x^2-y^2=15; \end{cases}$ 3) $\begin{cases} x^2-y^2=24, \\ x+y=4; \end{cases}$

4) $\begin{cases} x^2-y^2=8, \\ x-y=2; \end{cases}$ 5) $\begin{cases} x+y=-3, \\ x^2-y^2=-3; \end{cases}$ 6) $\begin{cases} x^2-y^2=7, \\ x+y=7. \end{cases}$

**158.** 1) $\begin{cases} x^2+y^2=17, \\ xy=4; \end{cases}$ 2) $\begin{cases} xy=10, \\ x^2+y^2=29; \end{cases}$ 3) $\begin{cases} xy=3, \\ x^2+y^2=10; \end{cases}$

4) $\begin{cases} xy=5, \\ x^2+y^2=26; \end{cases}$ 5) $\begin{cases} x^2+y^2=25, \\ xy=12; \end{cases}$ 6) $\begin{cases} x^2+y^2=50, \\ xy=7. \end{cases}$

**159.** Ikki sonning yig'indisi 18 ga, ularning ko'paytmasi esa 65 ga teng. Shu sonlarni toping.

**160.** Ikki sonning o'rta arifmetigi 20 ga, ularning o'rta geometrigi esa 12 ga teng. Shu sonlarni toping.

**161.** Tenglamalar sistemasini yeching (**161–163**):

1) $\begin{cases} x+2y=-3, \\ y^2-2x=3; \end{cases}$ 2) $\begin{cases} x+y=6, \\ xy=-7; \end{cases}$ 3) $\begin{cases} x^2-y^2=21, \\ x+y=7. \end{cases}$

**162.** 1) $\begin{cases} x-y=2, \\ xy=3; \end{cases}$ 2) $\begin{cases} x-y=3, \\ xy=4; \end{cases}$ 3) $\begin{cases} 2x^2-y^2=46, \\ xy=10. \end{cases}$

**163.** 1) $\begin{cases} (x-y)^2=4, \\ x+y=6; \end{cases}$ 2) $\begin{cases} x^2-y^2=0, \\ 4+xy=0; \end{cases}$ 3) $\begin{cases} x-y=4, \\ \frac{1}{x}+\frac{1}{y}=1. \end{cases}$

**164.** To‘g‘ri to‘rtburchak shaklidagi maydonni 1 km uzunlikdagi devor bilan o‘rab olish kerak. Agar maydonning yuzi 6 ha bo‘lsa, uning bo‘yi va eni qanday bo‘lishi kerak?

## 14-§. TENGLAMALAR SISTEMASINI YECHISHNING TURLI USULLARI

**1- masala.** Tenglamalar sistemasini yeching:

$$\begin{cases} x + y + 2xy = 10, \\ x + y - 2xy = -2. \end{cases}$$

△ Sistema tenglamalarini hadma-had qo‘shib, hosil qilamiz: $2x+2y=8$, bundan $y=4-x$. Bu ifodani sistemaning ixtiyoriy, masalan, ikkinchi tenglamasiga qo‘yamiz:

$$x + 4 - x - 2x(4-x) = -2,$$
$$4-8x+2x^2=-2, \quad 2x^2-8x+6=0,$$
$$x^2-4x+3=0, \quad x_1=1, x_2=3.$$

$y=4-x$ ekanligidan $y_1=3$, $y_2=1$.

**Javob.** (1; 3), (3; 1). ▲

**2- masala.** Tenglamalar sistemasini yeching:

$$\begin{cases} x - y^2 = 3, \\ xy^2 = 28. \end{cases}$$

△ Sistemaning birinchi tenglamasidan $y^2=x-3$. Bu ifodani sistemaning ikkinchi tenglamasiga qo‘yamiz:

$$x(x-3)=28, \quad x^2-3x-28=0.$$

Bundan $x_1 - 7, x_2 - -4$.

$y^2=x-3$ ekanligidan $y$ ning qiymatini topamiz:

1) Agarda $x=7$ bo‘lsa, u holda $y^2=7-3$, $y^2=4$, bundan $y=2$ yoki $y=-2$;

2) Agarda $x=-4$ bo'lsa, u holda $y^2=-4-3<0$, demak, haqiqiy ildizlar mavjud emas.

**Javob:** $(7;\ 2), (7;\ -2)$. ▲

Shuni aytib o'tish joizki, agarda birinchi tenglamada $x$ ni $y$ orqali ifodalab, ikkinchi tenglamaga qo'yilsa, bikvadrat tenglamani yechishga olib kelar edi.

**3- masala.** Tenglamalar sistemasini yeching:

$$\begin{cases} x+y=12, \\ \dfrac{1}{x}+\dfrac{1}{y}=\dfrac{3}{8}. \end{cases}$$

△ Agarda $(x;\ y)$ – sistemaning yechimi bo'lsa, u holda $x\neq 0$ va $y\neq 0$. Sistemaning ikkinchi tenglamasini quyidagicha yozamiz: $\dfrac{x+y}{xy}=\dfrac{3}{8}$.

Hosil bo'lgan tenglamaga $x+y=12$ qiymatni qo'yamiz: $\dfrac{12}{xy}-\dfrac{3}{8}$, bundan $xy=32$.

Berilgan sistemani yechish quyidagi sistemani yechishga keltirildi:

$$\begin{cases} x+y=12, \\ xy=32. \end{cases}$$

Viyet teoremasiga teskari teoremaga asosan hosil qilamiz: $x_1=4,\ y_1=8;\ x_2=8,\ y_2=4$.

**Javob:** $(4;\ 8), (8;\ 4)$. ▲

**4- masala.** Tenglamalar sistemasini yeching:

$$\begin{cases} x^3-y^3=7, \\ x^2y-xy^2=2. \end{cases}$$

△ Sistemaning ikkinchi tenglamasini $xy(x-y)=2$ ko'rinishda yozib olamiz. Ravshanki, $x\neq 0,\ y\neq 0$ va $x-y\neq 0$, u holda sistemaning birinchi tenglamasini ikkinchi tenglamaga bo'lib, hosil qilamiz:

$$\frac{x^3-y^3}{x^2y-xy^2}=\frac{7}{2};$$

$$\frac{(x-y)(x^2+xy+y^2)}{xy(x-y)}=\frac{7}{2};$$

$$2(x-y)(x^2+xy+y^2)=7xy,$$

$$2x^2-5xy+2y^2=0.$$

Hosil bo‘lgan tenglamani $x$ ga nisbatan kvadrat tenglama sifatida qarab, ildizlarini topamiz:

$$x_{1,2}=\frac{5y\pm\sqrt{25y^2-16y^2}}{4},$$

$$x_{1,2}=\frac{5y\pm 3y}{4}.$$

Bundan $x_1=2y$ yoki $x_2=\frac{y}{2}$.

Sistemaning ikkinchi tenglamasiga $x$ ning $y$ orqali topilgan ifodalarini qo‘yib, hosil qilamiz:

1) agarda $x=2y$ bo‘lsa, u holda $4y^3-2y^3=2$, bundan $y^3=1$ va $x=2$;

2) agarda $x=\frac{y}{2}$ bo‘lsa, u holda $\frac{y^3}{4}-\frac{y^2}{2}=2$, bundan $y^3=-8$, $y=-2$ va $x=-1$.

**Javob:** (2; 1), (−1; −2).

**5- masala.** Tenglamalar sistemasini yeching:

$$\begin{cases}x^2-2xy+4y^2=7,\\x^3+8y^3=35.\end{cases}$$

Kublar yig‘indisi formulasini qo‘llab, sistemaning ikkinchi tenglamasini quyidagi ko‘rinishda yozamiz:

$$(x+2y)(x^2-2xy+4y^2)=35.$$

Bu tenglamani sistemaning birinchi tenglamasiga bo‘lib, topamiz: $x+2y=5$.

Bu tenglamadan $2y$ ni $x$ orqali ifodalaymiz: $2y = 5 - x$ va sistemaning ikkinchi tenglamasiga qo‘yamiz:

$$x^3 + (5 - x)^3 = 35,$$
$$x^3 + 125 - 75x + 15x^2 - x^3 = 35,$$
$$15x^2 - 75x + 90 = 0,$$
$$x^2 - 5x + 6 = 0,$$
$$x_1 = 3, \ x_2 = 2.$$

Mos ravishda, $y$ ning qiymatlarini topamiz:

1) $2y=5-3$, bundan $y_1=1$, 2) $2y=5-2$, bundan $y_2 = \frac{3}{2}$.

**Javob:** $(3;\ 1)$, $(2;\ \frac{3}{2})$.

**6- masala.** Tenglamalar sistemasini yeching:

$$\begin{cases} x - y = 5, \\ \sqrt{\frac{x}{y}} - \sqrt{\frac{y}{x}} = \frac{5}{6}. \end{cases}$$

$\sqrt{\frac{x}{y}} = t$ deb belgilasak, $\sqrt{\frac{y}{x}} = \frac{1}{t}$, $t > 0$ bo‘ladi. U holda sistemaning ikkinchi tenglamasi $t - \frac{1}{t} - \frac{5}{6}$ ko‘rinishga keladi. Bu tenglamaning ikkala tomonini $t$ ga ko‘paytiramiz:

$$t^2 - \frac{5}{6}t - 1 = 0.$$

Bundan $t_{1,2} = \frac{5}{12} \pm \sqrt{\frac{25}{144} + 1} = \frac{5}{12} \pm \frac{13}{12}$, $t_1 = \frac{3}{2}$, $t_2 = -\frac{2}{3}$.

$t > 0$ bo‘lgani uchun $\sqrt{\frac{x}{y}} = \frac{3}{2}$ yoki $\frac{x}{y} - \frac{9}{4}$, bundan $x = \frac{9}{4}y$. $x$ uchun bu ifodani sistemaning birinchi tenglamasiga qo‘yib, hosil qilamiz: $\frac{9}{4}y - y = 5$, $\frac{5}{4}y = 5$, $y=4$, shu sababli $x=9$.

**Javob:** $(9;\ 4)$.

## *Mashqlar*

Tenglamalar sistemasini yeching (**165–175**):

**165.** 1) $\begin{cases} xy - x + y = 7, \\ xy + x - y = 13; \end{cases}$ 2) $\begin{cases} xy - 2(x + y) = 7, \\ xy + x + y = 29. \end{cases}$

**166.** 1) $\begin{cases} (x-1)(y-1) = 2, \\ x + y = 5; \end{cases}$ 2) $\begin{cases} (x-2)(y+1) = 1, \\ x - y = 3. \end{cases}$

**167.** 1) $\begin{cases} 2x + 3y = 3, \\ 4x^2 - 9y^2 = 27; \end{cases}$ 2) $\begin{cases} x - y = 5, \\ \sqrt{x} - \sqrt{y} = 1. \end{cases}$

**168.** 1) $\begin{cases} x^2 + y^2 = 34, \\ xy = 15; \end{cases}$ 2) $\begin{cases} x^2 + y^2 = 25, \\ xy = 12. \end{cases}$

**169.** 1) $\begin{cases} 2x - 3y = 1, \\ 2x^2 - xy - 3y^2 = 3; \end{cases}$ 2) $\begin{cases} x^3 + y^3 = 133; \\ x + y = 7; \end{cases}$

3) $\begin{cases} 2x^2 - 2xy^2 + x = -9, \\ 2y - 3x = 1; \end{cases}$ 4) $\begin{cases} x^2 + 6xy + 8yx^2 = 91, \\ x + 3y - 10 = 0. \end{cases}$

**170.** 1) $\begin{cases} x^2 + y^2 = 10, \\ xy = 3; \end{cases}$ 2) $\begin{cases} x^2 - xy + y^2 = 19, \\ xy = 15; \end{cases}$

3) $\begin{cases} x^2 + 4xy + y^2 = 94, \\ xy = 15; \end{cases}$ 4) $\begin{cases} x^2 - 6xy + y^2 = 8, \\ xy = 7. \end{cases}$

**171.** 1) $\begin{cases} \frac{1}{x} + \frac{1}{y} = 1, \\ x + y = 4; \end{cases}$ 2) $\begin{cases} \frac{x+y}{x-y} = \frac{3}{2}, \\ xy = 80; \end{cases}$

3) $\begin{cases} x - y = 3, \\ \frac{1}{x} - \frac{1}{y} = -0,3; \end{cases}$ 4) $\begin{cases} x + y = 9, \\ \frac{1}{x} + \frac{1}{y} = 0,5. \end{cases}$

**172.** 1) $\begin{cases} x^2 - y = 7, \\ x^2y = 18; \end{cases}$ 2) $\begin{cases} 2x^2 + y = 3, \\ x^2y - 1 = 0; \end{cases}$

3) $\begin{cases} x^2 - y^2 = 12, \\ x^2 + y^2 = 20; \end{cases}$ 4) $\begin{cases} x^2 - y^2 = 21, \\ x^2 + y^2 = 29. \end{cases}$

**173.** 1) $\begin{cases} x^3 + y^3 = 28, \\ xy^2 + x^2y = 12; \end{cases}$ 2) $\begin{cases} xy^2 + xy^3 = 10, \\ x + xy = 10. \end{cases}$

**174.** 1) $\begin{cases} x^3 + 27y^3 = 54, \\ x^2 - 3xy + 9y^2 = 9; \end{cases}$ 2) $\begin{cases} x^2 - xy + y^2 = 19, \\ x^2 + xy + y^2 = 49. \end{cases}$

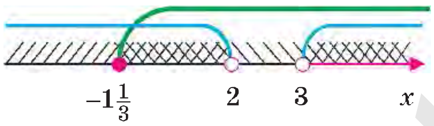
**175.** 1) $\begin{cases} x + y = 41, \\ \sqrt{\dfrac{x}{y}} + \sqrt{\dfrac{y}{x}} = \dfrac{41}{20}; \end{cases}$ 2) $\begin{cases} x + y = 10, \\ \sqrt{x} + \sqrt{y} = 4. \end{cases}$

## 15-§. IKKINCHI DARAJALI BIR NOMA'LUMLI TENGSIZLIKLAR SISTEMALARI

**1- masala.** Tengsizliklar sistemasini yeching:

$$\begin{cases} x^2 - 5x + 6 > 0, \\ 3x + 4 \ge 0. \end{cases}$$

△ Bu tengsizliklardan birinchisi kvadrat tengsizlik, ikkinchisi esa chiziqli tengsizlik. Birinchi tengsizlikning yechimlari 6-paragrafdagi 2-masalada ko'rsatilganidek $x<2$ va $x>3$ oraliqlardagi barcha sonlardan iborat. Ikkinchi tengsizlikning yechimlari esa $x \ge -1\frac{1}{3}$ oraliqdagi sonlardir. Bitta sonlar o'qida ham birinchi, ham ikkinchi tengsizlikarning yechimlari to'plamini tasvirlaylik. Ravshanki, sistemaning ikkala tengsizligini bir vaqtning o'zida qanoatlantiruvchi sonlar $-1\frac{1}{3} \le x < 2$ va $x < 3$ oraliqlardan iborat (44- rasm).




**44- rasm.**

**Javob:** $-1\frac{1}{3} \le x < 2,\ x > 3.$ ▲

**2- masala.** Tengsizlikni yeching:

$$|x^2 - x - 1| < 1.$$

△ $|x^2 - x - 1| < 1$ tengsizlik ikki tomonlama tengsizlikka tengkuchli ekanini bilamiz:

$$-1 < x^2 - x - 1 < 1.$$

Bu esa ikkita tengsizlikdan iborat sistemaga tengkuchli:

$$\begin{cases} x^2 - x - 1 < 1, \\ x^2 - x - 1 > -1 \end{cases}$$

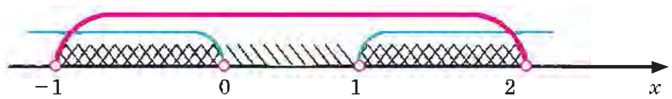
yoki

$$\begin{cases} x^2 - x - 2 < 0, \\ x^2 - x > 0. \end{cases}$$

Avval birinchi tengsizlikni yechamiz: $D = (-1)^2 - 4(-2) = 9 > 0$, demak, $x_1 = \frac{1-3}{2} = -1$, $x_2 = \frac{1+3}{2} = 2$. Bundan birinchi tengsizlikni qanoatlantiruvchi sonlar $-1 < x < 2$ oraliq ekanligi kelib chiqadi.

Ikkinchi tengsizlikni yechamiz: $x^2 - x = x(x-1) > 0$. Demak, bu tengsizlikning yechimi $x < 0$ va $x > 1$ oraliqlardagi barcha sonlardir.

Ikkala tengsizlikning yechimlarini bitta sonlar o‘qida tasvirlaymiz (45- rasm).



*45-rasm.*

Bundan sistemaning yechimi $-1<x<0$ va $1<x<2$ oraliqlarda yotgan barcha sonlardan iborat ekanligi kelib chiqadi.

**Javob:** $-1<x<0$, $1<x<2$. ▲

**3-masala.** Funksiyaning aniqlanish sohasini toping:

$$y=\sqrt{3x^2-x-14}+\sqrt{-x}.$$

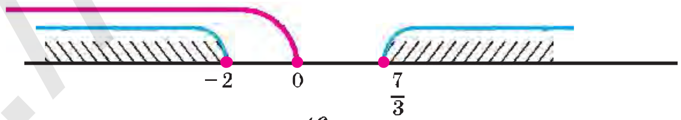
△ Kvadrat ildiz ostida turgan sonlar manfiy bo‘lmasligi shart bo‘lgani uchun berilgan funksiyaning aniqlanish sohasi quyidagi tengsizliklar sistemasining yechimidan iborat:

$$\begin{cases} 3x^2-x-14\geq 0, \\ -x\geq 0. \end{cases}$$

Avvaliga birinchi tengsizlikni yechamiz. $3x^2-x-14$ kvadrat uchhadning diskriminanti $D=(-1)^2-4\cdot3(-14)=169$, demak, $x_1=\frac{1-13}{6}=-2$, $x_2=\frac{1+13}{6}=\frac{7}{3}$. Shuning uchun va kvadrat uchhadning tarmoqlari yuqoriga yo‘nalgani sababli birinchi tengsizlikning yechimlari $x\leq-2$ va $x\geq\frac{7}{3}$ oraliqlardan iborat.

Ravshanki, ikkinchi tengsizlikni $-1$ ga ko‘paytirib, uning yechimlari $x\leq 0$ oraliqdan olingan barcha sonlardan iboratligini ko‘rish mumkin.

Birinchi va ikkinchi tengsizliklarning yechimlarini bitta sonlar o‘qida ifodalaymiz (46-rasm).



*46-rasm.*

Bundan sistemaning yechimi $x\leq-2$ ekanligi kelib chiqadi.

**Javob.** $x\leq-2$. ▲

## *Mashqlar*

**176.** Tengsizliklar sistemasini yeching:

1) $\begin{cases} 3x^2+5x-2<0, \\ 4x+9>0; \end{cases}$ 2) $\begin{cases} 3x^2+5x-2\le 0, \\ 2x+7<0. \end{cases}$

**177.** Tengsizlikni yeching:

1) $|x^2-6x|<27$; 2) $|x^2+6x|\le 27$;

3) $|x^2+4x|<12$; 4) $|x^2-4x|\le 12$.

Tengsizliklar sistemalarini yeching (**178–181**):

**178.** 1) $\begin{cases} x^2+x-6<0, \\ -x^2+2x+3\le 0; \end{cases}$ 2) $\begin{cases} x^2+x-6>0, \\ x^2+x+6>0. \end{cases}$

**179.** 1) $\begin{cases} x^2-3x+2\ge 0, \\ x^2-7x+12>0; \end{cases}$ 2) $\begin{cases} x^2+x-6<0, \\ x^2+x-2\ge 0. \end{cases}$

**180.** 1) $\begin{cases} 7x-x^2>0, \\ 36-x^2>0; \end{cases}$ 2) $\begin{cases} 8x+x^2<0, \\ 49-x^2>0. \end{cases}$

**181.** 1) $\begin{cases} -x^2+x+20\le 0, \\ x^2-x-2>0; \end{cases}$ 2) $\begin{cases} x^2+4x<0, \\ -x^2+x+2\ge 0. \end{cases}$

**182.** Funksiyaning aniqlanish sohasini toping:

1) $y=\sqrt{-x^2-6x-8}+\sqrt{\frac{1}{3}x+2}$, 2) $y=\sqrt{x-x^2}-\sqrt{-x^2+12x-35}$.

## 16-§. SODDA TENGSIZLIKLARNI ISBOTLASH

Tengsizliklarni isbotlashning turli usullari mavjud. Ulardan ba'zilarining qo'llanilishini ko'rib chiqamiz.

**1- masala.** Ikkita musbat *a* va *b* sonning o'rta arifmetigi shu sonlarning o'rta geometrigidan kichik emasligini isbotlang:

$$\frac{a+b}{2} \geq \sqrt{ab}. \qquad (1)$$

△ Tengsizlikni *bevosita ta'rifga asosan* isbotlaymiz, bunda $\frac{a+b}{2} - \sqrt{ab} \geq 0$ ekanini isbotlash talab etiladi.

Bu tengsizlik chap qismining shaklini almashtirib, quyidagini hosil qilamiz:

$$\frac{a+b}{2} - \sqrt{ab} = \frac{a+b-2\sqrt{ab}}{2} = \frac{(\sqrt{a}-\sqrt{b})^2}{2} \geq 0.$$

(1) munosabatda tenglik belgisi faqat $a=b$ bo'lgandagina to'g'ri bo'lishini ta'kidlaymiz. ▲

**2- masala.** Ikkita musbat $a$ va $b$ sonning o'rta geometrigi shu sonlarning o'rta garmonigidan kichik emasligini isbotlang:

$$\sqrt{ab} \geq \frac{2}{\frac{1}{a} - \frac{1}{b}}. \qquad (2)$$

△ Bu tengsizlikni *avval isbotlangan* (1) *tengsizlikdan foydalanib* hamda surati o'zgarmay, maxraji kichkinalashganda musbat kasr kattalashishidan foydalanib isbotlaymiz:

$$\frac{2}{\frac{1}{a} + \frac{1}{b}} = \frac{2ab}{a+b} = \frac{ab}{\frac{a+b}{2}} \leq \frac{ab}{\sqrt{ab}} = \sqrt{ab}. \ ▲$$

**3-masala.** Har qanday musbat $a$ son uchun tengsizlikni isbotlang:

$$a + \frac{1}{a} \geq 2. \qquad (3)$$

△ Bu tengsizlikni *teskarisini faraz qilish usuli* bilan isbotlaymiz. Bunda (3) tengsizlik $a$ ning biror-bir musbat qiymatida bajarilmasin deb faraz qilamiz, ya'ni

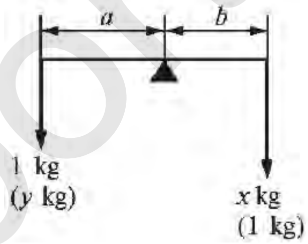
$$a+\frac{1}{a}<2$$

tengsizlik o'rinli bo'lsin. Tengsizlikning ikkala qismini $a$ ga ko'paytirib, hosil qilamiz:

$$a^2+1<2a,$$

ya'ni $a^2+1-2a<0$ yoki $(a-1)^2<0$, bu esa noto'g'ri tengsizlik, chunki har qanday haqiqiy sonning kvadrati (jumladan, $(a-1)^2$ ham) manfiy emas. Hosil bo'lgan qarama-qarshilikdan (3) tengsizlik har qanday musbat $a$ da to'g'ri tengsizlik ekanligi kelib chiqadi. ▲

**4- masala.** Sotuvchi olmalarni shayinli tarozida tortmoqda. Xaridor 1 kg olma oldi, so'ngra esa sotuvchidan tortishda olmalar bilan toshlarning o'rinlarini almashtirib tortishni iltimos qilib, yana 1 kg olma oldi. Agar tarozi rostlanmagan bo'lsa, kim zarar ko'radi?



*47- rasm.*

△ Aytaylik, tarozining yelkalari $a$ va $b$ ga teng bo'lsin (47- rasm). Rasmdan ko'rinib turibdiki, $a\neq b$. Birinchi marta tortishda xaridor $x$ kilogramm olma oldi. Fizika kursidan ma'lumki, $x\cdot b-1\cdot a$, bundan $x=\frac{a}{b}$. Ikkinchi marta tortishda xaridor $y$ kilogramm olma oldi. Muvozanatlik shartidan $y\cdot a=1\cdot b$, bundan $y=\frac{b}{a}$. Shunday qilib, $\frac{a}{b}+\frac{b}{a}$ kilogramm olma sotib olingan.

$\frac{a}{b}$ va $\frac{b}{a}$ sonlarning o'rta arifmetigi va o'rta geometrigi uchun tengsizlikdan foydalanib, quyidagini hosil qilamiz:

$$\frac{\frac{a}{b}+\frac{b}{a}}{2}>\sqrt{\frac{a}{b}\cdot\frac{b}{a}},$$

bundan $\frac{a}{b}+\frac{b}{a}>2.$

**Javob:** sotuvchi zarar ko‘radi. ▲

### *Mashqlar*

**183.** Ixtiyoriy haqiqiy $a$, $b$, $x$ larda tengsizliklarning o‘rinli ekanligini isbotlang:

1) $\frac{a^2+1}{2}\geq a;$ 2) $\frac{b^2+16}{4}\geq b;$ 3) $\frac{2x}{x^2+1}\leq 1;$ 4) $\frac{2x}{4x^2+9}\leq\frac{1}{6}.$

**184.** Agarda $ab>0$ bo‘lsa, tengsizlikni isbotlang:

1) $\frac{a}{b}+\frac{b}{a}\geq 2;$ 2) $(a+b)\left(\frac{1}{a}+\frac{1}{b}\right)\geq 4.$

**185.** Agarda $a\geq -1$, $a\neq 0$ bo‘lsa, tengsizlikni isbotlang:

$$\frac{4a^2+a+1}{4|a|}\geq\sqrt{a+1}.$$

**186.** $a\geq 0$, $b\geq 0$ va $a\neq b$ bo‘lsa, u holda $(\sqrt{a}+\sqrt{b})^2$ va $2\sqrt{2(a+b)\sqrt{ab}}$ larning qaysi biri katta?

**187.** Tengsizlikni isbotlang:

$$(a+1)(a+2)(a+3)(a+6)>96a^2,$$

bunda $a>0$.

**188.** Agarda $a>0$ bo‘lsa, tengsizlikni isbotlang:

$$\frac{a+4}{2}+\frac{a+9}{2}>5\sqrt{a}.$$

**189.** Agarda $a, b, c, d$ lar musbat sonlar bo‘lsa, u holda

$$\frac{a+c}{2}+\frac{b+d}{2}\geq\sqrt{(a+b)(c+d)}$$

tengsizlikni isbotlang.

**190.** Agarda $a \geq 0$, $b \geq 0$ va $c > 0$ bo‘lsa, u holda $\frac{ac^2 + b}{c} \geq 2\sqrt{ab}$ bo‘lishini isbotlang.

**191.** Agarda $a > 0$, $b > 0$ bo‘lsa, u holda $(a+b)\left(\frac{1}{a} + \frac{1}{b}\right) \geq 4$ tengsizlik o‘rinli bo‘lishini isbotlang.

**192.** Agarda $a > 0, b > 0$ va $c > 0$ bo‘lsa, u holda

$$\left(1 + \frac{a^2}{bc}\right)\left(1 + \frac{b^2}{ac}\right)\left(1 + \frac{c^2}{ab}\right) \geq 8$$

tengsizlik o‘rinli bo‘lishini isbotlang.

## *II bobga doir mashqlar*

**193.** Berilgan ifodani bir o‘zgaruvchi bo‘yicha kvadrat uchhad ko‘rinishida yozing:

1) $2y^2 - xy + 3$, agarda $y = 3x + 1$;
2) $2xy + 3x^2 - 7$, agarda $x = 2y + 1$.

**194.** Tenglamalar sistemasini o‘rniga qo‘yish usuli bilan yeching:

1) $\begin{cases} x + y = -1, \\ y^2 - 7x = 7; \end{cases}$ 2) $\begin{cases} x^2 - 3y = 13, \\ x - y = 3. \end{cases}$

**195.** Viyet teoremasiga teskari teoremani qo‘llab, tenglamalar sistemasini yeching:

1) $\begin{cases} x + y = -10, \\ xy = 21; \end{cases}$ 2) $\begin{cases} xy = -30, \\ x + y = 1; \end{cases}$

3) $\begin{cases} x + y = -6, \\ xy = -16; \end{cases}$ 4) $\begin{cases} x + y = 9, \\ xy = -10. \end{cases}$

Tenglamalar sistemasini yeching (**196–198**):

**196.** 1) $\begin{cases} x^2 - y^2 = 18, \\ x + y = 9; \end{cases}$ 2) $\begin{cases} x + y = 4, \\ x^2 - y^2 = 32; \end{cases}$

3) $\begin{cases} x = 7 + y, \\ x^2 = 56 + y^2; \end{cases}$ 4) $\begin{cases} y = x - 5, \\ x^2 = 10 + y^2. \end{cases}$

**197.** 1) $\begin{cases} y^2 + xy = 4, \\ x^2 + xy = -3; \end{cases}$ 2) $\begin{cases} xy + x^2 = 10, \\ xy + y^2 = 15; \end{cases}$

3) $\begin{cases} x - y = 7, \\ x^2 + y^2 = 9 - 2xy; \end{cases}$ 4) $\begin{cases} x + y = 8, \\ x^2 + y^2 = 16 + 2xy. \end{cases}$

**198.** 1) $\begin{cases} x^3 - y^3 = 9, \\ x - y = 3; \end{cases}$ 2) $\begin{cases} x^3 + y^3 = 26, \\ x + y = 2. \end{cases}$

Tenglamalar sistemasini yeching (**199–204**):

**199.** 1) $\begin{cases} (x+2)(y-3) = 1, \\ \dfrac{x+2}{y-3} = 1; \end{cases}$ 2) $\begin{cases} (y-3)(x+1) = 4, \\ \dfrac{x+1}{y-3} = 1. \end{cases}$

**200.** 1) $\begin{cases} \dfrac{x}{y} - \dfrac{y}{x} = \dfrac{5}{6}, \\ x^2 - y^2 = 5; \end{cases}$ 2) $\begin{cases} \dfrac{x}{y} - \dfrac{y}{x} = \dfrac{3}{2}, \\ x^2 - y^2 = 3; \end{cases}$

3) $\begin{cases} \dfrac{1}{x} + \dfrac{1}{y} = -\dfrac{1}{6}, \\ x - y = 5; \end{cases}$ 4) $\begin{cases} \dfrac{1}{x} - \dfrac{1}{y} = -\dfrac{5}{4}, \\ x + y = 3. \end{cases}$

**201.** 1) $\begin{cases} x - y^2 = 6, \\ xy^2 = 7; \end{cases}$ 2) $\begin{cases} y^2 + 1 = x, \\ xy^2 = 12; \end{cases}$

3) $\begin{cases} x^2 + y^2 = 58, \\ x^2 - y^2 = 40; \end{cases}$ 4) $\begin{cases} x^2 - y^2 = 32, \\ x^2 + y^2 = 40. \end{cases}$

**202.** 1) $\begin{cases} x^3 + y^3 = 26, \\ x + y = 2; \end{cases}$ 2) $\begin{cases} x = 3 + y, \\ x^3 - y^3 = 9; \end{cases}$

3) $\begin{cases} x^3 + y^3 = 2, \\ xy(x + y) = 2; \end{cases}$ 4) $\begin{cases} x^3 + 8y^3 = 16, \\ 2xy(x + 2y) = 16. \end{cases}$

**203.** 1) $\begin{cases} 2x^4 - 3x^2y = 36, \\ 3y^2 - 2x^2y = -9; \end{cases}$ 2) $\begin{cases} 3x^4 - 2x^2y = 24, \\ 2y^2 - 3x^2y = -6. \end{cases}$

**204.** 1) $\begin{cases} x + y = 10, \\ \sqrt{x} - \sqrt{y} = 2; \end{cases}$ 2) $\begin{cases} x + y = 5, \\ \sqrt{x} - \sqrt{y} = 1. \end{cases}$

**205.** 1) Ikki xonali son o'zining raqamlari yig'indisidan uch marotaba katta. Raqamlari yig'indisining kvadrati esa berilgan sondan uch marotaba katta. Shu sonni toping.

2) Ikki xonali son o'zining raqamlari yig'indisidan 4 marotaba katta. Raqamlari yig'indisining kvadrati esa berilgan sonning $\frac{3}{2}$ qismini tashkil etadi. Shu sonni toping.

**206.** 1) Ikkita kvadrat tomonlarining nisbati 5:4 kabi. Agarda har bir kvadratning tomonlari 2 cm ga kamaytirilsa, u holda hosil bo'lgan kvadratlar yuzlarining ayirmasi 2,8 $cm^2$ ga teng bo'ladi. Berilgan kvadratlarning tomonlarini toping.

2) To'g'ri to'rtburchak bo'yining eniga nisbati 3:2 kabi. Agarda ularni 1 cm dan kattalashtirsak, yangi hosil bo'lgan to'g'ri to'rtburchakning yuzi birinchi to'g'ri to'rtburchakning yuzidan 3 $cm^2$ ga katta bo'ladi. Birinchi to'g'ri to'rtburchakning bo'yi va enini toping.

**207.** Tengsizliklar sistemasini yeching:

1) $\begin{cases} x^2 + x - 6 < 0, \\ -2x^2 + 3x + 2 > 0; \end{cases}$ 2) $\begin{cases} x^2 + x - 6 > 0, \\ x^2 + 4x - 5 < 0; \end{cases}$

3) $\begin{cases} -3x^2 - 5x + 2 > 0, \\ -x^2 - 3x - 2 \geq 0; \end{cases}$ 4) $\begin{cases} -2x^2 - 2x + 4 \leq 0, \\ 3x^2 - 3x - 6 < 0. \end{cases}$

**208.** 1) Agarda $xy = 9$ va $x > 0$ ekani ma'lum bo'lsa, $x + y$ ning eng kichik qiymatini toping.

2) Agarda $ab = 8$ va $b > 0$ bo'lsa, u holda $2a+b$ ning eng kichik qiymatini toping.

**209.** Ifodaning eng kichik qiymatini toping:

1) $4x + \frac{81}{25x}$, $(x > 0)$; 2) $\frac{(x+3)(x+12)}{x}$, $x > 0$;

3) $\frac{4y^2 - 7y + 25}{y}$, $(y > 0)$; 4) $\frac{y^4 + y^2 + 1}{y^2 + 1}$.

**210.** Agarda $x+y=10$ va $x > 0$, $y > 0$ bo'lsa, u holda $xy$ ning eng katta qiymatini toping.

**211.** Agarda $2x+y=6$ va $x > 0$, $y > 0$ bo'lsa, u holda $xy$ ning eng katta qiymatini toping.

**212.** Tengsizlikni isbotlang:

$$a^2 + b^2 + c^2 \geq ab + ac + bc.$$

## *II bobga doir sinov (test) mashqlari*

**1.** Tenglamalar sistemasini yeching: $\begin{cases} x + y = 5, \\ xy = 4. \end{cases}$

A) $x = -4$, $y = -1$; B) $x = 1, y = -4$;
C) $x = 4, y = -1$; D) $(1; 4)$ va $(4; 1)$.

**2.** Tenglamalar sistemasini yeching: $\begin{cases} x + y = 4, \\ x^2 - y^2 = 8. \end{cases}$

A) $x = 3, y = 1$; B) $x = 5, y = -1$;
C) $x - 4, y - 0$; D) $x - 1, y - 3$.

**3.** Ikki sonning ayirmasi 3 ga, ularning ko'paytmasi 28 ga teng. Shu sonlarni toping.

A) 7 va 4; B) 5 va 2; C) 14 va 2; D) 11 va 8.

4. To‘g‘ri to‘rtburchakning perimetri 30 m ga, yuzi esa 56 $m^2$ga teng. Uning bo‘yi enidan necha metr uzun?

A) 1,2 m; B) 1 m; C) 2 m; D) 2,5 m.

5. 60 m masofani bir velosipedchi ikkinchisiga qaraganda 1 soat kechroq bosib o‘tdi. Agar birinchi velosipedchining tezligi ikkinchisining tezligidan 5 km/h kam bo‘lsa, har bir velosipedchining tezligini toping.

A) 20 km/h, 25 km/h; B) 10 km/h, 15 km/h;
C) 15 km/h, 20 km/h; D) 12 km/h, 17 km/h.

6. Tenglamalar sistemasini yeching: $\begin{cases} x+20y+10xy=40, \\ x+20y-10xy=-8. \end{cases}$

A) (0,6; 4) va (12; 0,2); B) (0,4; 6) va (0,12; 2);
C) (4; 0,6) va (12; 0,2); D) (4; 0,2) va (12; 0,6).

7. Tenglamalar sistemasini yeching: $\begin{cases} x-y^2=-3, \\ xy^2=54. \end{cases}$

A) (6; 4) va (4; 3); B) (−3; 6) va (6; −3);
C) (6; 3) va (3; −6); D) (6; 3) va (6; −3).

8. Tenglamalar sistemasini yeching:

$$\begin{cases} x-5y=-20, \\ \frac{5}{x}-\frac{5}{y}=2. \end{cases}$$

A) (−10; 5) va (2; 5); B) (−10; 2) va (5; 5);
C) (5; −10) va (−10; 2); D) (5; 5) va (−2; 10).

9. Tenglamalar sistemasini yeching:

$$\begin{cases} x^3-64y^3=56, \\ x^2y-4xy^2=4. \end{cases}$$

A) (4; $\frac{1}{2}$) va (−2; −1); B) (−2; $\frac{1}{2}$) va (4; −1);
C) (4; 1) va (−4; −2); D) (−2; −1) va (2; 1).

**10.** Tenglamalar sistemasini yeching:

$$\begin{cases} \sqrt{\dfrac{x-2}{y+5}} - \sqrt{\dfrac{y+5}{x-2}} = \dfrac{5}{6}, \\ x - y = 12. \end{cases}$$

A) (–1;12); B) (12;–1); C) (–1;11); D) (11;–1).

**11.** Tengsizliklar sistemasini yeching:

$$\begin{cases} 3x^2 + 10x - 8 < 0, \\ 2x + 9 \geq 0. \end{cases}$$

A) $-4 < x < \frac{2}{3}$; B) $-4,5 < x < \frac{2}{3}$; C) $x > -4,5$; D) $x < \frac{2}{3}$.

**12.** Tengsizlikni yeching: $|x^2 + x - 1| \leq 1$.

A) $-2 \leq x \leq 1,\ 2 < x \leq 3$; B) $-2 \leq x \leq -1,\ 0 \leq x \leq 1$;

C) $-1 \leq x \leq 0,\ 1 < x \leq 2$; D) $x \leq -2,\ x \geq 1$.

### Amaliy-tatbiqiy va fanlararo bog‘liq masalalar

**Masala.** Ikkita yuk mashinasi birgalikda ishlab, yukni 6 soatda tashishlari kerak edi. Ikkinchi mashina ish boshlanishiga kech qolgani sababli, u kelgunicha birinchi mashina butun yukning $\frac{3}{5}$ qismini tashib bo‘ldi. Yukning qolgan qismini faqat ikkinchi mashina tashidi va shu sababli yukni tashishga 12 soat vaqt ketdi. Yukni har bir mashinaning yolg‘iz o‘zi qancha vaqtda tashigan bo‘lar edi?

△ Yuk mashinalari tashishlari kerak bo‘lgan yukni bir deb qabul qilaylik. Butun yukni alohida o‘zi tashishi uchun birinchi mashina sarflaydigan vaqtni $x$ soat, ikkinchi mashina sarflaydigan vaqtni esa $y$ soat orqali belgilaylik. U holda bir soatda birinchi mashina yukning $\frac{1}{x}$ qismini, ikkinchisi esa $\frac{1}{y}$ qismini tashigan bo‘lar edi.

Birgalikda ishlab, ular bir soatda butun yukning $\left(\frac{1}{x}+\frac{1}{y}\right)$ qismini tashishgan bo‘lar edi va masalaning shartiga ko‘ra yukni 6 soatda tashishgan bo‘lar edi. Shu sababli, $\left(\frac{1}{x}-\frac{1}{y}\right)\cdot 6=1.$

Lekin aslida birinchi mashina, yukning $\frac{3}{5}$ qismini tashishga o‘z vaqtining $\frac{3}{5}$ qismini sarfladi, yukning qolgan qismini esa ikkinchi mashina tashidi va unga o‘z vaqtining $\frac{2}{5}$ qismini sarfladi. Bu holda butun yukni tashishga 12 soat ketganligini hisobga olsak, ikkinchi tenglamani hosil qilamiz:

$$\frac{3}{5}x+\frac{2}{5}y=12.$$

Masala quyidagi tenglamalar sistemasini yechishga keltirildi:

$$\begin{cases}\left(\frac{1}{x}+\frac{1}{y}\right)\cdot 6=1,\\ \frac{3}{5}x+\frac{2}{5}y=12.\end{cases}$$

Avvaliga sistemani soddalashtirib, keyin uni o‘rniga qo‘yish usuli bilan yechamiz:

$$\begin{cases}6x+6y=xy,\\ 3x+2y=60,\end{cases}$$

$$3x-60-2y,\ 120-4y+6y=(20-\frac{2}{3}y)y,$$

$$60+y=10y-\frac{1}{3}y^2,$$

bundan $y^2-27y+180=0,$

$$y_{1,2}=\frac{27}{2}\pm\sqrt{\frac{729}{4}-180}=\frac{27}{2}\pm\frac{3}{2},\ \ y_1=15,\ \ y_2=12.$$

$x = -20 - \frac{2}{3}y$ formuladan foydalanib, hosil qilamiz

$$x_1 = 10, \ x_2 = 12.$$

**Javob:** 10 soat va 15 soat – agarda mashinalarning yuk ko'tarish imkoniyatlari turlicha bo'lsa;

12 soat va 12 soat – agarda mashinalarning yuk ko'tarish imkoniyatlari bir xil bo'lsa. ▲

### *Masalalar*

**1.** 1) Birinchi tomosha zalida 420 ta, ikkinchi zalda esa 480 ta o'rindiq bor. Ikkinchi zalda birinchiga qaraganda 5 ta qator kam, lekin har bir qatorda birinchi zaldagi har bir qatordan 10 ta o'rindiq ko'proq. Birinchi zaldagi har bir qatorda nechta o'rindiq bor?

2) Qizil zalda 320 ta, ko'k zalda 360 ta o'rindiq bor. Qizil zalda ko'k zaldagiga qaraganda 2 ta qator ko'p, lekin har bir qatorida ko'k zalning har bir qatoridagiga qaraganda 4 tadan o'rindiq kam. Qizil zalda nechta qator bor?

**2.** 1) Ikkita nasos birgalikda ishlab 80 $m^3$ hajmli basseynni biror vaqtda to'ldirishadi. Agarda unumdorligini $1\frac{1}{3}$ marotaba oshirgan birinchi nasosning faqat o'zi ishlaganida basseynni to'ldirishga 2 soat ko'proq vaqt kerak bo'lar edi. Agarda faqatgina ikkinchi nasos o'z unumdorligini soatiga 1 $m^3$ ga kamaytirib ishlaganida basseynni to'ldirishga ketadigan vaqt $3\frac{1}{3}$ marotaba ko'proq bo'lar edi (ikkala nasos birgalikda ishlagandagi vaqtga nisbatan). Har bir nasosning unumdorligi qanday?

2) Malakalari bir xil turli sondagi ishchilardan tashkil topgan ikkita brigada detallar tayyorlashadi, bunda har bir ishchi ish kuni davomida 2 ta detal tayyorlaydi. Avvaliga faqat birinchi brigada ishlab 32 ta detal tayyorladi. Keyin ikkinchi brigadaning o'zi ishlab, yana 48 ta detal tayyorladi. Bu ishlarning hammasiga

4 kun vaqt ketdi. Shundan keyin birgalikda 6 kun ishlab, 240 ta detal tayyorlashdi. Har bir brigadada nechtadan ishchi bor?

**3.** 1) Mahsulotning yarmi 10% foyda bilan, ikkinchi yarmining yarmi 20% foyda bilan sotildi. Agarda hamma mahsulotni sotishdan tushgan umumiy foyda 12% ni taskil etgan bo'lsa, mahsulotning qolgan choragi necha foiz foydaga sotilgan?

2) Savdo firmasi do'konlarga tovarni qo'shimcha narx bilan yetkazib beradi: tovarlarning $\frac{3}{5}$ qismiga 5% ustama haq qo'yib, qolgan tovarlarning yarmiga 4% ustama haq qo'yib sotildi. Agarda hamma tovarlarga qo'yilgan ustama haq 7% ni tashkil qilgan bo'lsa, qolgan tovarlarning ikkinchi yarmiga foiz hisobida qanday ustama haq qo'yilgan?

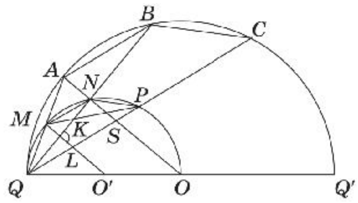
**4.** 1) Ikkita moddaning aralashmasi bor. Agarda bu aralashmaga ikkinchi moddadan 3 kg qo'shilsa, u holda uning aralashmadagi miqdori foizlarda ikki baravar ko'payadi, agarda boshlang'ich aralashmaga birinchi moddadan 3 kg qo'shilsa, u holda ikkinchi moddaning miqdori foiz hisobida ikki baravar kamayadi. Har bir moddaning boshlang'ich aralashmadagi massasini toping.

2) Ikkita suyuqlikning aralashmasi mavjud. Agarda bu aralashmaga birinchi suyuqlikdan 8 litr quyilsa, u holda uning aralashmadagi konsentratsiyasi ikki baravarga oshadi, agarda boshlang'ich aralashmaga ikkinchi suyuqlikdan 8 litr quyilsa, u holda birinchi suyuqlikning konsentratsiyasi bir yarim baravarga kamayadi. Har bir suyuqlikning aralashmadagi hajmini toping.

**5.** Samolyot $A$ dan $B$ gacha shamol yo'nalishida va $B$ dan $A$ ga shamolga qarshi uchib o'tdi, bunda shamolning tezligi o'zgarmadi. Boshqa safar samolyot shu marshrut bo'yicha reysni shamolsiz ob-havoda amalga oshirdi. Ikkala holda ham samolyot motorlari bir xil quvvatda ishladi. Qaysi holda umumiy parvozga kamroq vaqt ketdi?

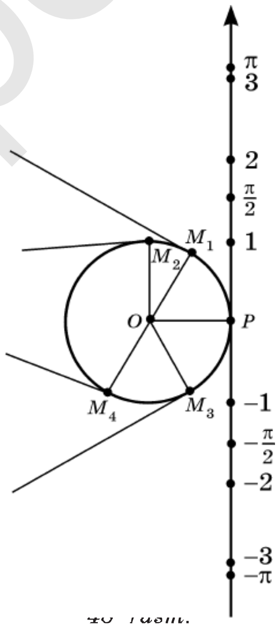
**6.** Ikkita traktorchi yer maydonini $p$ kunda hayday olishadi. Agarda birinchi traktorchi maydonning yarmini haydasa, keyin ikkinchi traktorchi qolgan qismini haydasa, u holda $q$ kun kerak bo'lar edi. $q \geq 2p$ ekanligini isbotlang.

# III BOB. TRIGONOMETRIYA ELEMENTLARI

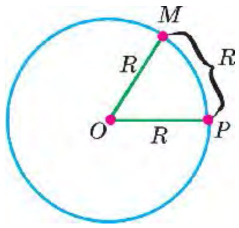



## 17-§. BURCHAKNING RADIAN O‘LCHOVI

Aytaylik, vertikal to‘g‘ri chiziq markazi $O$ nuqtada va radiusi 1 ga teng bo‘lgan aylanaga $P$ nuqtada urinsin (48-rasm). Bu to‘g‘ri chiziqni boshi $P$ nuqtada bo‘lgan son o‘qi deb, yuqoriga yo‘nalishni esa to‘g‘ri chiziqdagi musbat yo‘nalish deb hisoblaymiz. Son o‘qida uzunlik birligi sifatida aylananing radiusini olamiz. To‘g‘ri chiziqda bir nechta nuqtani belgilaylik: $\pm 1$, $\pm\frac{\pi}{2}$, $\pm 3$, $\pm\pi$ ($\pi$ – taqriban 3,14 ga teng bo‘lgan irratsional son ekanligini eslatib o‘tamiz). Bu to‘g‘ri chiziqni aylanadagi $P$ nuqtaga mahkamlangan cho‘zilmaydigan ip sifatida tasavvur qilib, uni fikran aylanaga o‘ray boshlaymiz. Bunda son (o‘qining) to‘g‘ri chizig‘ining, masalan, $1$, $\frac{\pi}{2}$, $-1$, $-2$ koordinatali nuqtalari aylananing, mos ravishda, shunday $M_1$, $M_2$, $M_3$, $M_4$ nuqtalariga o‘tadiki, $PM_1$ yoyning uzunligi 1 ga teng, $PM_2$ yoyning uzunligi $\frac{\pi}{2}$ ga teng va hokazo bo‘ladi.



48-rasm.



*49- rasm.*

Shunday qilib, *to'g'ri chiziqning har bir nuqtasiga aylananing biror nuqtasi mos keltiriladi.*

To'g'ri chiziqning koordinatasi 1 ga teng bo'lgan nuqtasiga $M_1$ nuqta mos keltirilgani uchun, $POM_1$ burchakni birlik burchak deb hisoblash va bu burchakning o'lchovi bilan boshqa burchaklarni o'lchash tabiiydir. Masalan, $POM_2$ burchakni $\frac{\pi}{2}$ ga teng, $POM_3$ burchakni −1 ga teng, $POM_4$ burchakni −2 ga teng deb hisoblash lozim. Burchaklarni o'lchashning bunday usuli matematika va fizikada keng qo'llaniladi. Bu holda burchaklar *radian o'lchovlarda o'lchanyapti* deyiladi, $POM_1$ esa **1** radian (1 rad) ga teng burchak deyiladi. Aylana $PM_1$ yoyining uzunligi radiusga teng ekanligini ta'kidlab o'tamiz (48- rasm).

Endi ixtiyoriy $R$ radiusli aylanani qaraymiz va unda uzunligi $R$ ga teng bo'lgan $PM$ yoyni va $POM$ burchakni belgilaymiz (49- rasm).

**! Uzunligi aylana radiusiga teng bo'lgan yoyga tiralgan markaziy burchak 1 *radian burchak* deyiladi.**

Bu holda 1 radian burchak uzunligi $R$ ga teng yoyni tortib turadi, deymiz.

1 rad burchakning gradus o'lchovini topaylik. 180° li markaziy burchak uzunligi $\pi R$ (yarimaylana) bo'lgan yoyni tortib turgani uchun uzunligi $R$ bo'lgan yoyni $\pi$ marta kichik burchak tortib turadi, ya'ni

$$1 \text{ rad} = \left(\frac{180}{\pi}\right)^{\circ}.$$

$\pi \approx 3,14$ bo'lgani uchun **1** rad ≈ **57,3°** bo'ladi.

Agar burchak $\alpha$ radiandan iborat bo'lsa, u holda uning gradus o'lchovi quyidagiga teng bo'ladi:

$$1 \text{ rad} = \left(\frac{180}{\pi}\alpha\right)^{\circ}. \quad \textbf{(1)}$$

**1-masala.** 1) $\pi$ rad; 2) $\frac{\pi}{2}$ rad; 3) $\frac{3\pi}{4}$ rad ga teng burchakning gradus o‘lchovini toping.

△ (1) formula bo‘yicha topamiz:

1) $\pi$ rad$=180°$; 2) $\frac{\pi}{2}$ rad$=90$ ; 3) . $\frac{3\pi}{4}$rad$=\left(\frac{180}{\pi}\cdot\frac{3\pi}{4}\right)^{\circ}=135$ .▲

1° li burchakning radian o‘lchovini topaylik. 180° li burchak $\pi$ rad ga teng bo‘lgani uchun

$$1 = \frac{\pi}{180}\text{rad}$$

bo‘ladi.

Agar burchak $\alpha$ gradusdan iborat bo‘lsa,u holda uning radian o‘lchovi

$$\alpha^{\circ} = \frac{\pi}{180}\alpha \text{ rad} \qquad (2)$$

ga teng bo‘ladi.

**2-masala.** 1) 45° ga teng burchakning; 2) 15° ga teng burchakning radian o‘lchovini toping.

△ (2) formula bo‘yicha topamiz:

1) $45^{\circ} = \frac{\pi}{180}\cdot 45 \text{ rad} = \frac{\pi}{4} \text{ rad}$; 2) $15^{\circ} = \frac{\pi}{180}\cdot 15 \text{ rad} = \frac{\pi}{12}\text{rad}$ . ▲

Ko‘proq uchrab turadigan burchaklarning gradus o‘lchovlarini va ularga mos radian o‘lchovlarini keltiramiz:

| Gradus | 0 | 30 | 45 | 60 | 90 | 180 |
|---|---|---|---|---|---|---|
| Radian | 0 | $\frac{\pi}{6}$ | $\frac{\pi}{4}$ | $\frac{\pi}{3}$ | $\frac{\pi}{2}$ | $\pi$ |

Odatda, burchakning o‘lchovi radianlarda berilsa, „rad“ nomi tushirib qoldiriladi.

Burchakning radian o‘lchovi aylana yoylarining uzunliklarini hisoblash uchun qulay. 1 radian burchak uzunligi $R$ radiusga teng yoyni tortib turgani uchun $\alpha$ radian burchak

$$l=\alpha R \qquad (3)$$

uzunlikdagi yoyni tortib turadi.

**3-masala.** Shahar kurantlari minut milining uchi radiusi $R \approx 0{,}8$ m bo‘lgan aylana bo‘ylab harakat qiladi. Bu milning uchi 15 min davomida qancha yo‘lni bosib o‘tadi?

△ Soat mili 15 min davomida $\frac{\pi}{2}$ radianga teng burchakka buriladi. (3) formula bo‘yicha $\alpha = \frac{\pi}{2}$ bo‘lganda topamiz:

$$l = \frac{\pi}{2} R \approx \frac{3{,}14}{2} \cdot 0{,}8 \text{ m} \approx 1{,}3 \text{ m}.$$

**Javob:** 1,3 m. ▲

(3) formula aylana radiusi $R=1$ bo‘lganda ayniqsa sodda ko‘rinishga ega bo‘ladi. Bu holda yoy uzunligi shu yoyni tortib turgan markaziy burchak kattaligiga teng, ya’ni $l = \alpha$ bo‘ladi. Radian o‘lchovning matematika, fizika, mexanika va boshqa fanlarda qo‘llanilishining qulayligi shu bilan izohlanadi.

**4-masala.** Radiusi $R$ bo‘lgan doiraviy sektor $\alpha$ rad burchakka ega. Shu sektorning yuzi

$$S = \frac{R^2}{2}\alpha$$

ga teng ekanligini isbotlang, bunda $0 < \alpha < \pi$.

△ $\pi$ rad li doiraviy sektor (yarimdoira)ning yuzi $\frac{\pi R^2}{2}$ ga teng. Shuning uchun 1 rad li sektorning yuzi $\pi$ marta kichik, ya’ni $\frac{\pi R^2}{2} : \pi$. Demak, $\alpha$ rad li sektorning yuzi $\frac{R^2}{2}\alpha$ ga teng. ▲

## *Mashqlar*

**213.** Graduslarda ifodalangan burchakning radian o‘lchovini toping:

1) $40°$; 2) $120°$; 3) $105°$; 4) $150°$;
5) $75°$; 6) $32°$; 7) $100°$; 8) $140°$.

**214.** Radianlarda ifodalangan burchakning gradus o‘lchovini toping:

1) $\frac{\pi}{6}$; 2) $\frac{\pi}{9}$; 3) $\frac{2}{3}\pi$; 4) $\frac{3}{4}\pi$; 5) 2;
6) 4; 7) 1,5; 8) 0,36; 9) $\frac{2\pi}{5}$; 10) 4,5.

**215.** Sonni 0,01 gacha aniqlikda yozing:

1) $\frac{\pi}{2}$;  2) $\frac{3}{2}\pi$;  3) $2\pi$;  4) $\frac{2}{3}\pi$;  5) $\frac{3\pi}{4}$.

**216.** Sonlarni taqqoslang:

1) $\frac{\pi}{2}$ va 2;  2) $2\pi$ va 6,7;  3) $\pi$ va $3\frac{1}{5}$;

4) $\frac{3}{2}\pi$ va 4,8;  5) $-\frac{\pi}{2}$ va $-\frac{3}{2}$;  6) $-\frac{3}{2}\pi$ va $-\sqrt{10}$.

**217.** (Og'zaki.) a) Teng tomonli uchburchak; b) teng yonli to'g'ri burchakli uchburchak; d) kvadrat; e) muntazam oltiburchak burchaklarining gradus va radian o'lchovlarini aniqlang.

**218.** Agar aylananing 0,36 m uzunlikdagi yoyini 0,9 radli markaziy burchak tortib tursa, aylana radiusini hisoblang.

**219.** Agar aylananing radiusi 1,5 cm ga teng bo'lsa, aylananing uzunligi 3 cm bo'lgan yoyini tortib turgan burchakning radian o'lchovini toping.

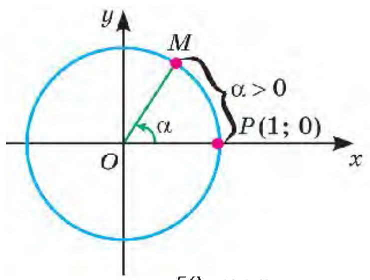
**220.** Doiraviy sektor yoyini $\frac{3\pi}{4}$ radli burchak tortib turadi. Agar doiraning radiusi 1 cm ga teng bo'lsa, sektorning yuzini toping.

**221.** Doiraning radiusi 2,5 cm ga teng, doiraviy sektorning yuzi esa 6,25 $cm^2$ ga teng. Shu doiraviy sektor yoyini tortib turgan burchakni toping.

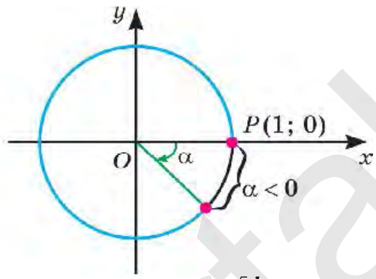
## 18-§. NUQTANI KOORDINATALAR BOSHI ATROFIDA BURISH

Avvalgi paragrafda son to'g'ri chizig'ining nuqtalari bilan aylana nuqtalari o'rtasida moslik o'rnatishning ko'rgazmali usulidan foydalanildi. Endi qanday qilib haqiqiy sonlar bilan aylananing nuqtalari o'rtasida aylana nuqtasini burish yordamida moslik o'rnatish mumkinligini ko'rsatamiz.

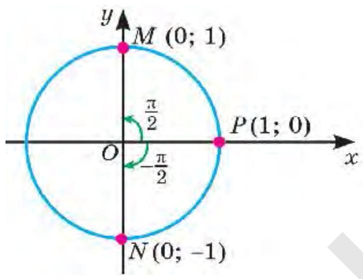
Koordinata tekisligida radiusi 1 ga teng va markazi koordinata boshida bo'lgan aylanani qaraymiz. U *birlik aylana* deyiladi. Birlik aylananing nuqtasini koordinata boshi atrofida $\alpha$ radian burchakka *burish tushunchasini* kiritamiz (bu yerda $\alpha$ – istalgan haqiqiy son).



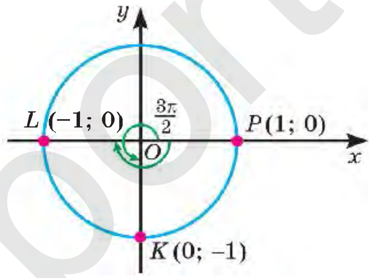

*50- rasm.*




*51- rasm.*




*52- rasm.*




*53- rasm.*

1. A y t a y l i k , $\alpha > 0$ b o ' l s i n . Nuqta birlik aylana bo'ylab $P$ nuqtadan soat mili yo'nalishiga qarama-qarshi harakat qilib, $\alpha$ uzunlikdagi yo'lni bosib o'tdi, deylik (50- rasm). Yo'lning oxirgi nuqtasini $M$ bilan belgilaymiz.

Bu holda $M$ nuqta $P$ nuqtani koordinata boshi atrofida $\alpha$ radian burchakka burish bilan hosil qilinadi, deb aytamiz.

2. A y t a y l i k , $\alpha < 0$ b o ' l s i n . Bu holda $\alpha$ radian burchakka burish harakat soat mili yo'nalishida sodir bo'lganligini va nuqta $|\alpha|$ uzunlikdagi yo'lni bosib o'tganligini bildiradi (51- rasm).

0 rad ga burish nuqta o'z o'rnida qolganligini anglatadi.

**Misollar:**

1) $P(1; 0)$ nuqtani $\frac{\pi}{2}$ rad burchakka burishda $(0; 1)$ koordinatali $M$ nuqta hosil qilinadi (52- rasm).

2) $P(1; 0)$ nuqtani $-\frac{\pi}{2}$ rad burchakka burishda $N(0; -1)$ nuqta hosil qilinadi (52- rasm).

3) $P(1;\ 0)$ nuqtani $\frac{3\pi}{2}$ rad burchakka burishda $K(0;\ -1)$ nuqta hosil qilinadi (53- rasm).

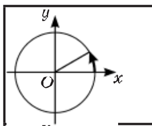
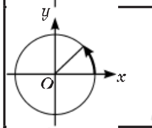
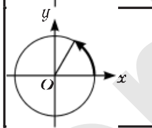
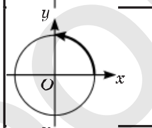
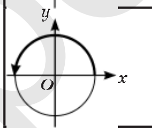
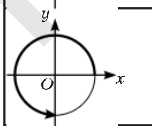
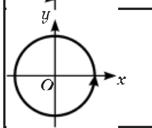
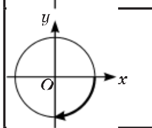
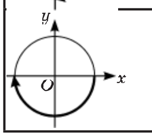
4) $P(1;\ 0)$ nuqtani $-\pi$ rad burchakka burishda $L(-1;\ 0)$ nuqta hosil qilinadi (53- rasm).

Geometriya kursida 0° dan 180° gacha bo'lgan burchaklar qaralgan. Birlik aylananing nuqtalarini koordinatalar boshi atrofida burishdan foydalanib, 180° dan katta burchaklarni, shuningdek, manfiy burchaklarni ham qarash mumkin. Burish burchagini graduslarda ham, radianlarda ham berish mumkin. Masalan, $P$ (1; 0) nuqtani $\frac{3\pi}{2}$ burchakka burish uni 270° ga burishni bildiradi; $-\frac{\pi}{2}$ burchakka burish $-90°$ ga burishdir.

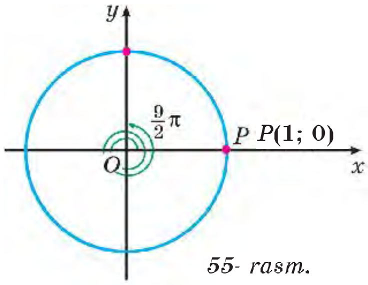
Ba'zi burchaklarni burishning radian va gradus o'lchovlari jadvalini keltiramiz (54- rasm).

$P(1;\ 0)$ nuqtani $2\pi$ ga, ya'ni 360° ga burishda nuqta dastlabki holatiga qaytishini ta'kidlab o'tamiz (jadvalga qarang). Shu nuqtani $-2\pi$ ga, ya'ni $-360°$ ga burishda u yana dastlabki holatiga qaytadi.

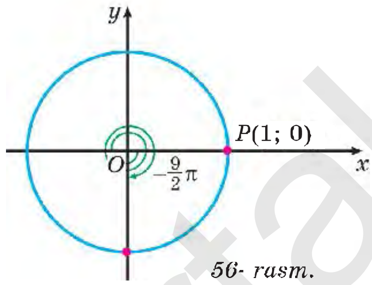
Nuqtani $2\pi$ dan katta burchakka va $-2\pi$ dan kichik burchakka burishga oid misollar qaraymiz. Masalan, $\frac{9\pi}{2} = 2 \cdot 2\pi - \frac{\pi}{2}$ burchakka burishda

| | | |
|---|---|---|
|  | $\frac{\pi}{6}$ | 30° |
|  | $\frac{\pi}{4}$ | 45° |
|  | $\frac{\pi}{3}$ | 60° |
|  | $\frac{\pi}{2}$ | 90° |
|  | $\pi$ | 180° |
|  | $\frac{3\pi}{2}$ | 270° |
|  | $2\pi$ | 360° |
|  | $-\frac{\pi}{2}$ | –90° |
|  | $-\pi$ | –180° |

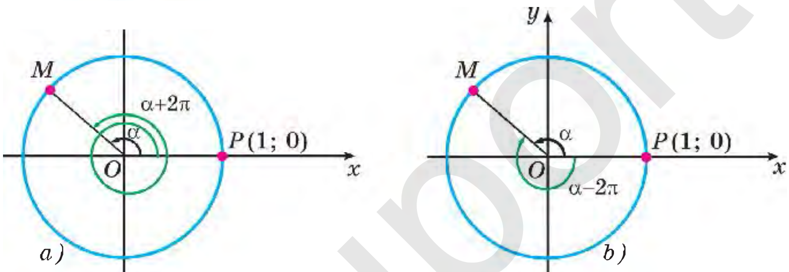
*54- rasm.*



55- rasm.



56- rasm.



57- rasm.

nuqta soat mili harakatiga qarama-qarshi ikkita to‘la aylanishni va yana $\frac{\pi}{2}$ yo‘lni bosib o‘tadi (55-rasm).

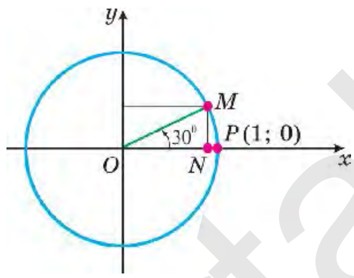
$-\frac{9\pi}{2}=2\cdot2\pi-\frac{\pi}{2}$ burchakka burishda nuqta soat mili harakati yo‘nalishida ikkita to‘la aylanadi va yana shu yo‘nalishda $\frac{\pi}{2}$ yo‘lni bosadi (56- rasm).

$P(1;\ 0)$ nuqtani $\frac{9\pi}{2}$ burchakka burishda $\frac{\pi}{2}$ burchakka burishdagi nuqtaning ayni o‘zi hosil bo‘lishini ta’kidlaymiz (55- rasm). $-\frac{9\pi}{2}$ burchakka burishda $-\frac{\pi}{2}$ burchakka burishdagi nuqtaning ayni o‘zi hosil bo‘ladi (56- rasm).

Umuman, agar $\alpha=\alpha_0+2\pi k$ (bunda $k$ – butun son) bo‘lsa, u holda $\alpha$ burchakka burishda $\alpha_0$ burchakka burishdagi nuqtaning ayni o‘zi hosil bo‘ladi.

Shunday qilib, har bir haqiqiy $\alpha$ songa birlik aylananing (1; 0) nuqtasini $\alpha$ rad burchakka burish bilan hosil qilinadigan birgina nuqtasi mos keladi.

Biroq, birlik aylananing ayni bir $M$ nuqtasiga ($P$(1; 0) nuqtani burishda $M$ nuqta hosil bo‘ladigan) cheksiz ko‘p $\alpha + 2\pi k$ haqiqiy sonlar mos keladi, $k$ – butun son (57- rasm).




*58- rasm.*

**1-masala.** $P$ (1; 0) nuqtani: 1) $7\pi$; 2) $\frac{5\pi}{2}$ burchakka burishdan hosil bo‘lgan nuqtaning koordinatalarini toping.

△ 1) $7\pi - \pi + 2\pi \cdot 3$ bo‘lgani uchun $7\pi$ ga burishda $\pi$ ga burishdagi nuqtaning o‘zi, ya’ni (−1; 0) koordinatali nuqta hosil bo‘ladi.

2) $-\frac{5\pi}{2} - -\frac{\pi}{2} - 2\pi$ bo‘lgani uchun $-\frac{5\pi}{2}$ ga burishda $-\frac{\pi}{2}$ ga burishdagi nuqtaning o‘zi, ya’ni (0; −1) koordinatali nuqta hosil bo‘ladi. ▲

**2-masala.** $\left(\frac{\sqrt{3}}{2}; \frac{1}{2}\right)$ nuqtani hosil qilish uchun (1; 0) nuqtani burish kerak bo‘lgan barcha burchaklarni yozing.

△ $NOM$ to‘g‘ri burchakli uchburchakdan (58- rasm) $NOM$ burchak $\frac{\pi}{6}$ ga tengligi kelib chiqadi, ya’ni mumkin bo‘lgan burish burchaklaridan biri $\frac{\pi}{6}$ ga teng. Shuning uchun $\left(\frac{\sqrt{3}}{2}; \frac{1}{2}\right)$ nuqtani hosil qilish uchun (1; 0) nuqtani burish kerak bo‘lgan barcha burchaklar bunday ifodalanadi: $\frac{\pi}{6} + 2\pi k$, bu yerda $k$ – istalgan butun son, ya’ni $k = 0; \pm 1; \pm 2; \ldots$ ▲

### *M a s h q l a r*

**222.** Birlik aylananing $P(1;\ 0)$ nuqtasini:

1) $90°$; 2) $-\pi$; 3) $180°$; 4) $\frac{\pi}{2}$; 5) $270°$; 6) $2\pi$

burchakka burish natijasida hosil boʻlgan nuqtalarining koordinatalarini toping.

**223.** Birlik aylanada $P(1;\ 0)$ nuqtani:

1) $\frac{\pi}{4}$; 2) $-\frac{\pi}{3}$; 3) $-\frac{2}{3}\pi$; 4) $\frac{3}{4}\pi$;

5) $\frac{\pi}{2}+2\pi$; 6) $-\pi-2\pi$; 7) $\frac{\pi}{4}-4\pi$; 8) $-\frac{\pi}{3}+6\pi$

burchakka burish natijasida hosil boʻlgan nuqtani belgilang.

**224.** $P(1;\ 0)$ nuqtani:

1) $2{,}1\pi$; 2) $2\frac{2}{3}\pi$; 3) $-\frac{13}{3}\pi$; 4) $-\frac{25}{4}\pi$; 5) $727°$; 6) $460°$

burchakka burish natijasida hosil boʻlgan nuqta joylashgan koordinatalar choragini aniqlang.

**225.** $P(1;\ 0)$ nuqtani:

1) $3\pi$; 2) $-\frac{7}{2}\pi$; 3) $-\frac{15}{2}\pi$; 4) $5\pi$;

5) $540°$; 6) $810°$; 7) $-\frac{9}{2}\pi$; 8) $450°$

burchakka burish natijasida hosil boʻlgan nuqtaning koordinatalarini toping.

**226.** 1) $(-1;\ 0)$; 2) $(1;\ 0)$; 3) $(0;\ 1)$; 4) $(0;\ -1)$ nuqtalarni hosil qilish uchun $P(1;\ 0)$ nuqtani burish kerak boʻlgan barcha burchaklarni yozing.

**227.** $P(1;\ 0)$ nuqtani berilgan:

1) 1; 2) 2,75; 3) 3,16; 4) 4,95; 5) 1,8

burchakka burish natijasida hosil boʻlgan nuqta joylashgan koordinatalar choragini toping.

**228.** Agar:

1) $a = 6{,}7\pi$; 2) $a = 9{,}8\pi$; 3) $a - 4\frac{1}{2}\pi$;

4) $a = 7\frac{1}{3}\pi$; 5) $a = \frac{11}{2}\pi$; 6) $a = \frac{17}{3}\pi$

boʻlsa, $a = x + 2\pi k$ tenglik bajariladigan $x$ sonni (bu yerda $0 \le x \le 2\pi$) va $k$ natural sonni toping.

**229.** Birlik aylanada $P(1;\ 0)$ nuqtani:

1) $\frac{\pi}{4} \pm 2\pi$; 2) $\frac{\pi}{3} \pm 2\pi$; 3) $\frac{2\pi}{3} \pm 6\pi$; 4) $\frac{3\pi}{4} \pm 8\pi$;

5) $4{,}5\pi$; 6) $5{,}5\pi$; 7) $-6\pi$; 8) $-7\pi$

burchakka burishdan hosil bo'lgan nuqtani yasang.

**230.** $P(1;\ 0)$ nuqtani:

1) $\frac{3\pi}{2} + 2\pi k$; 2) $\frac{5\pi}{2} + 2\pi k$; 3) $\frac{7\pi}{2} - 2\pi k$; 4) $-\frac{9\pi}{2} + 2\pi k$

burchakka (bu yerda $k$ – butun son) burishdan hosil bo'lgan nuqtaning koordinatalarini toping.

**231.** $(1;\ 0)$ nuqtani:

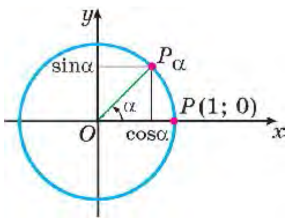
1) $\left(\frac{1}{2};\ \frac{\sqrt{3}}{2}\right)$; 2) $\left(\frac{\sqrt{3}}{2};\ \frac{1}{2}\right)$; 3) $\left(\frac{\sqrt{2}}{2};\ -\frac{\sqrt{2}}{2}\right)$; 4) $\left(-\frac{\sqrt{2}}{2};\ \frac{\sqrt{2}}{2}\right)$

koordinatali nuqta hosil qilish uchun burish kerak bo'lgan barcha burchaklarni yozing.

## 19-§. BURCHAKNING SINUSI, KOSINUSI, TANGENSI VA KOTANGENSI TA'RIFLARI

Geometriya kursida graduslarda ifodalangan burchakning sinusi, kosinusi va tangensi kiritilgan edi. Bu burchak 0° dan 180° gacha bo'lgan oraliqda qaralgan. Ixtiyoriy burchakning sinusi va kosinusi quyidagicha ta'riflanadi:

**1-ta'rif.** *$\alpha$ burchakning sinusi deb (1; 0) nuqtani koordinatalar boshi atrofida $\alpha$ burchakka burish natijasida hosil bo'lgan nuqtaning ordinatasiga aytiladi* **(sin$\alpha$ kabi belgilanadi).**



**2- ta'rif.** ***α burchakning kosinusi deb (1;0) nuqtani koordinatalar boshi atrofida α burchakka burish natijasida hosil bo'lgan nuqtaning abssissasiga aytiladi (cosα kabi belgilanadi).***

Bu ta'riflarda α burchak graduslarda, shuningdek,radianlarda ham ifodalanishi mumkin.

Masalan, (1; 0) nuqtani $\frac{\pi}{2}$ burchakka, ya'ni 90° ga burishda (0; 1) nuqta hosil qilinadi. (0; 1) nuqtaning ordinatasi 1 ga teng,shuning uchun

$$\sin\frac{\pi}{2} = \sin 90^\circ = 1;$$

bu nuqtaning abssissasi 0 ga teng, shuning uchun

$$\cos\frac{\pi}{2} = \cos 90^\circ = 0.$$

Burchak 0° dan 180° gacha oraliqda bo'lgan holda sinus va kosinuslarning ta'riflari geometriya kursidan ma'lum bo'lgan sinus va kosinus ta'riflari bilan mos tushishini ta'kidlaymiz.

Masalan,

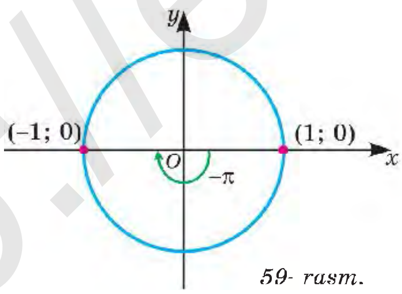
$$\sin\frac{\pi}{6} = \sin 30^\circ = \frac{1}{2}, \ \cos\pi = \cos 180^\circ = -1.$$

**1- masala.** $\sin(-\pi)$ va $\cos(-\pi)$ ni toping.

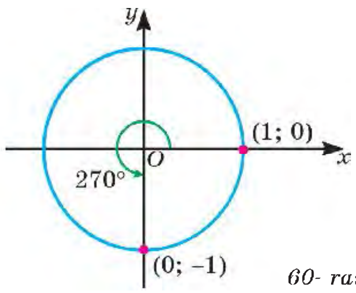
△ (1; 0) nuqtani $-\pi$ burchakka burganda u (−1; 0) nuqtaga o'tadi (59- rasm). Shuning uchun $\sin(-\pi) = 0$, $\cos(-\pi) = -1$. ▲

**2- masala.** sin 270° va cos 270° ni toping.

△ (1; 0) nuqtani 270° ga burganda u (0; −1) nuqtaga o'tadi (60- rasm). Shuning uchun $\cos 270^\circ = 0$, $\sin 270^\circ = -1$. ▲

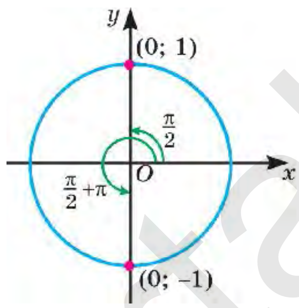


*59- rasm.*



*60- rasm.*

**3- masala.** $\sin t = 0$ tenglamani yeching.



*61- rasm.*

△ $\sin t = 0$ tenglamani yechish – bu sinusi nolga teng bo‘lgan barcha burchaklarni topish demakdir.

Birlik aylanada ordinatasi nolga teng bo‘lgan ikkita nuqta bor: (1; 0) va (–1; 0) (59-rasm). Bu nuqtalar (1; 0) nuqtani $0, \pi, 2\pi, 3\pi$ va hokazo, shuningdek, $-\pi, -2\pi, -3\pi$ va hokazo burchaklarga burish bilan hosil qilinadi.

Demak, $t = k\pi$ bo‘lganda (bunda $k$ – istalgan butun son) $\sin t = 0$ bo‘ladi. ▲

Butun sonlar to‘plami $\boldsymbol{Z}$ harfi bilan belgilanadi. $k$ son $\boldsymbol{Z}$ ga tegishli ekanligini belgilash uchun $k \in \boldsymbol{Z}$ yozuvdan foydalaniladi („$k$ son $Z$ ga tegishli“ deb o‘qiladi). Shuning uchun 3- masala javobini bunday yozish mumkin:

$$t = \pi k, k \in \boldsymbol{Z}.$$

**4- masala.** $\cos t - 0$ tenglamani yeching.

△ Birlik aylanada abssissasi nolga teng bo‘lgan ikkita nuqta bor: (0, 1) va (0; –1) (61- rasm).

Bu nuqtalar (1; 0) nuqtani $\frac{\pi}{2}$, $\frac{\pi}{2}+\pi$, $\frac{\pi}{2}+2\pi$ va hokazo, shuningdek, $\frac{\pi}{2}-\pi$, $\frac{\pi}{2}-2\pi$ va hokazo burchaklarga, ya’ni $\frac{\pi}{2}+k\pi$ (bunda $k \in \boldsymbol{Z}$) burchaklarga burish bilan hosil qilinadi.

**Javob:** $t = \frac{\pi}{2} + \pi k$, $k \in \boldsymbol{Z}$. ▲

**5- masala.** Tenglamani yeching: 1) $\sin t = 1$; 2) $\cos t = 1$.

△ 1) Birlik aylananing (0; 1) nuqtasi birga teng ordinataga ega. Bu nuqta (1; 0) nuqtani $\frac{\pi}{2} + 2\pi k, k \in \boldsymbol{Z}$ burchakka burish bilan hosil qilinadi.

2) (1; 0) nuqtani $2k\pi, k \in \boldsymbol{Z}$ burchakka burish bilan hosil qilingan nuqtaning abssissasi birga teng bo‘ladi.

**Javob:** $t = \frac{\pi}{2} + 2\pi k$ bo‘lganda $\sin t = 1$,

$t = 2\pi k$ bo‘lganda $\cos t = 1, k \in \boldsymbol{Z}$. ▲

**3-ta'rif.** ***α burchakning tangensi deb α burchak sinusining uning kosinusiga nisbatiga aytiladi*** **(tgα kabi belgilanadi).**

Shunday qilib, $\operatorname{tg}\alpha=\frac{\sin\alpha}{\cos\alpha}$.

Masalan, $\operatorname{tg}0^\circ=\frac{\sin 0^\circ}{\cos 0^\circ}=\frac{0}{1}=0$, $\operatorname{tg}\frac{\pi}{4}=\frac{\sin\frac{\pi}{4}}{\cos\frac{\pi}{4}}=\frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}=1$.

Ba'zan α *burchakning kotangensidan* foydalaniladi (ctgα kabi belgilanadi). U $\operatorname{ctg}\alpha=\frac{\cos\alpha}{\sin\alpha}$ formula bilan aniqlanadi.

Masalan, $\operatorname{ctg}270^\circ=\frac{\cos 270^\circ}{\sin 270^\circ}=\frac{0}{-1}=0$, $\operatorname{ctg}\frac{\pi}{4}=\frac{1}{\operatorname{tg}\frac{\pi}{4}}=\frac{1}{1}=1$.

sinα va cosα lar ixtiyoriy burchak uchun ta'riflanganligini, ularning qiymatlari esa −1 dan 1 gacha oraliqda ekanligini ta'kidlab o'tamiz; $\operatorname{tg}\alpha=\frac{\sin\alpha}{\cos\alpha}$ faqat $\cos\alpha\neq 0$ bo'lgan burchaklar uchun, ya'ni $\alpha\neq\frac{\pi}{2}+\pi k$, $k\in \boldsymbol{Z}$ dan boshqa ixtiyoriy burchaklar uchun aniqlangan.

Sinus, kosinus, tangens va kotangenslarning ko'proq uchrab turadigan qiymatlari jadvalini keltiramiz.

| α | 0 (0°) | $\frac{\pi}{6}$ (30°) | $\frac{\pi}{4}$ (45°) | $\frac{\pi}{3}$ (60°) | $\frac{\pi}{2}$ (90°) | π (180°) | $\frac{3}{2}\pi$ (270°) | 2π (360°) |
|---|---|---|---|---|---|---|---|---|
| sinα | 0 | $\frac{1}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{3}}{2}$ | 1 | 0 | −1 | 0 |
| cosα | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{1}{2}$ | 0 | −1 | 0 | 1 |
| tgα | 0 | $\frac{1}{\sqrt{3}}$ | 1 | $\sqrt{3}$ | Mavjud emas | 0 | Mavjud emas | 0 |
| ctgα | Mavjud emas | $\sqrt{3}$ | 1 | $\frac{1}{\sqrt{3}}$ | 0 | Mavjud emas | 0 | Mavjud emas |

**6- masala.** Hisoblang:

$$4\sin\frac{\pi}{6}+\sqrt{3}\cos\frac{\pi}{6}-\text{tg}\,\frac{\pi}{4}\,.$$

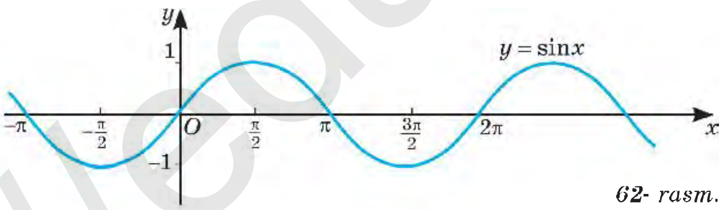
△ Jadvaldan foydalanib, hosil qilamiz:

$$4\sin\frac{\pi}{6}+\sqrt{3}\cos\frac{\pi}{6}-\text{tg}\,\frac{\pi}{4}-4\cdot\frac{1}{2}-\sqrt{3}\cdot\frac{\sqrt{3}}{2}-1-2{,}5\,.$$ ▲

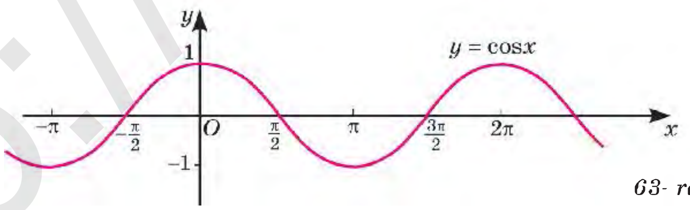
Sinus, kosinus, tangens va kotangenslarning bu jadvalga kirmagan burchaklar uchun qiymatlarini V.M.Bradisning to‘rt xonali matematik jadvallaridan, shuningdek, mikrokalkulator yordamida topish mumkin.

Agar har bir haqiqiy $x$ songa $\sin x$ son mos keltirilsa, u holda haqiqiy sonlar to‘plamida $y-\sin x$ funksiya berilgan bo‘ladi. $y=\cos x$, $y=\text{tg}\,x$ va $y=\text{ctg}\,x$ funksiyalar shunga o‘xshash aniqlanadi. $y=\cos x$ funksiya barcha $x\in \boldsymbol{R}$ da aniqlangan, $y=\text{tg}\,x$ funksiya $x\neq\frac{\pi}{2}-\pi k,\ k\in \boldsymbol{Z}$, $y=\text{ctg}\,x$ esa $x\neq\pi k$, $k\in \boldsymbol{Z}$ bo‘lganda aniqlangan. $y=\sin x$ va $y=\cos x$ funksiyalarning grafiklari 62- va 63- rasmlarda tasvirlangan.

$y=\sin x$, $y=\cos x$, $y=\text{tg}\,x$, $y=\text{ctg}\,x$ funksiyalar *trigonometrik* funksiyalar deyiladi.




*62- rasm.*




*63- rasm.*

## *Mashqlar*

**232.** Hisoblang:

1) $\sin\frac{3\pi}{4}$; 2) $\cos\frac{2\pi}{3}$; 3) $\mathrm{tg}\frac{5\pi}{6}$; 4) $\sin(-90^\circ)$;

5) $\cos(-180^\circ)$; 6) $\mathrm{tg}\left(-\frac{\pi}{4}\right)$; 7) $\cos(-135^\circ)$; 8) $\sin\left(-\frac{5\pi}{4}\right)$.

**233.** Agar:

1) $\sin\alpha=\frac{1}{2}$; 2) $\sin\alpha=-\frac{\sqrt{2}}{2}$; 3) $\cos\alpha=\frac{\sqrt{3}}{2}$;

4) $\cos\alpha=-\frac{1}{2}$; 5) $\sin\alpha=-0{,}6$; 6) $\cos\alpha=\frac{1}{3}$

bo'lsa, birlik aylanada $\alpha$ burchakka mos keluvchi nuqtani tasvirlang.

Hisoblang (**234–236**):

**234.** 1) $\sin\frac{\pi}{2}+\sin\frac{3\pi}{2}$; 2) $\sin\left(-\frac{\pi}{2}\right)+\cos\frac{\pi}{2}$; 3) $\sin\pi-\cos\pi$;

4) $\sin0-\cos2\pi$; 5) $\sin\pi+\sin1{,}5\pi$; 6) $\cos0-\cos\frac{3}{2}\pi$.

**235.** 1) $\mathrm{tg}\pi+\cos\pi$; 2) $\mathrm{tg}0^\circ-\mathrm{tg}180^\circ$; 3) $\mathrm{tg}\pi+\sin\pi$;

4) $\cos\pi-\mathrm{tg}2\pi$; 5) $\sin\frac{\pi}{4}-\cos\frac{\pi}{4}$; 6) $\mathrm{tg}\frac{\pi}{4}+\mathrm{ctg}\frac{\pi}{4}$.

**236.** 1) $3\sin\frac{\pi}{6}+2\cos\frac{\pi}{6}-\mathrm{tg}\frac{\pi}{3}$; 2) $5\sin\frac{\pi}{6}+3\mathrm{tg}\frac{\pi}{4}-\cos\frac{\pi}{4}-10\mathrm{tg}\frac{\pi}{4}$;

3) $\left(2\mathrm{tg}\frac{\pi}{6}-\mathrm{tg}\frac{\pi}{3}\right):\cos\frac{\pi}{6}$; 4) $\sin\frac{\pi}{3}\cos\frac{\pi}{6}-\mathrm{tg}\frac{\pi}{4}$.

**237.** Tenglamani yeching:

1) $2\sin x=0$; | 2) $\frac{1}{2}\cos x=0$; | 3) $\cos x-1=0$; | 4) $1-\sin x=0$.

**238.** (Og'zaki.) $\sin\alpha$ yoki $\cos\alpha$:

1) 0,49; 2) −0,875; 3) $-\sqrt{2}$; 4) $2-\sqrt{2}$; 5) $\sqrt{5}-1$

ga teng bo'lishi mumkinmi?

**239.** $\alpha$ ning berilgan qiymatida ifodaning qiymatini toping:

1) $2\sin\alpha+\sqrt{2}\cos\alpha$, bunda $\alpha=\frac{\pi}{4}$;

2) $0{,}5\cos\alpha-\sqrt{3}\sin\alpha$, bunda $\alpha=60^\circ$;

3) $\sin3\alpha-\cos2\alpha$, bunda $\alpha=\frac{\pi}{6}$;

4) $\cos\frac{\alpha}{2}+\sin\frac{\alpha}{3}$, bunda $\alpha=\frac{\pi}{2}$.

**240.** Tenglamani yeching:

1) $\sin x = -1$; 2) $\cos x = -1$; 3) $\sin 3x = 0$;
4) $\cos 0{,}5x = 0$; 5) $\cos 2x - 1 = 0$; 6) $1 - \cos 3x = 0$.

**241.** Tenglamani yeching:

1) $\sin(x+\pi) = -1$; 2) $\sin\frac{1}{2}(x-1) - 0$; 3) $\cos(x+\pi) = -1$;
4) $\cos 2(x+1) - 1 = 0$; 5) $\sin 3(x-2) = 0$; 6) $1 - \cos 3(x-1) = 0$.

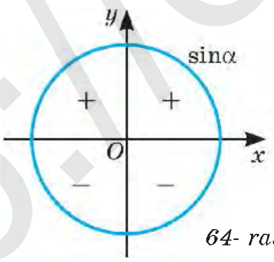
## 20-§. SINUS, KOSINUS VA TANGENSNING ISHORALARI

### 1. Sinus va kosinusning ishoralari

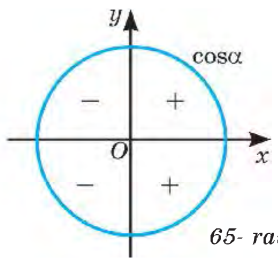
Aytaylik, (1; 0) nuqta birlik aylana bo‘yicha soat mili harakatiga qarama-qarshi harakat qilmoqda. Bu holda birinchi chorak (kvadrant)da joylashgan nuqtalarning ordinatalari va abssissalari musbat. Shuning uchun, agar $0 < \alpha < \frac{\pi}{2}$ bo‘lsa, $\sin\alpha > 0$ va $\cos\alpha > 0$ bo‘ladi (64, 65-rasmlar).

Ikkinchi chorakda joylashgan nuqtalar uchun ordinatalar musbat, abssissalar esa manfiy. Shuning uchun, agar $\frac{\pi}{2} < \alpha < \pi$ bo‘lsa, $\sin\alpha > 0$, $\cos\alpha < 0$ bo‘ladi (64, 65-rasmlar). Shunga o‘xshash, uchinchi chorakda $\sin\alpha < 0$, $\cos\alpha < 0$, to‘rtinchi chorakda esa $\sin\alpha < 0$, $\cos\alpha > 0$ (64, 65- rasmlar). Nuqtaning aylana bo‘yicha bundan keyingi harakatida sinus va kosinuslarning ishoralari nuqta qaysi chorakda turganligi bilan aniqlanadi.

Sinusning ishoralari 64-rasmda, kosinusning ishoralari esa 65- rasmda ko‘rsatilgan.




*64- rasm.*



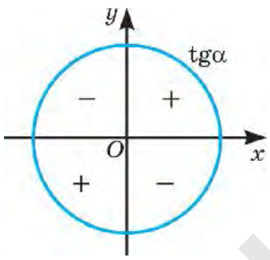

*65- rasm.*

Agar (1; 0) nuqta soat mili yo‘nalishida harakat qilsa, *u holda ham sinus va kosinusning ishoralari nuqta qaysi chorakda joylashganiga qarab aniqlanadi;* buni 64, 65- rasmlardan bilish ham mumkin.

**1-masala.** Burchak sinus va kosinuslarining ishoralarini aniqlang: 1) $\frac{3\pi}{4}$; 2) 745°; 3) $-\frac{5\pi}{7}$.

△ 1) $\frac{3\pi}{4}$ burchakka birlik aylananing ikkinchi choragida joylashgan nuqta mos keladi. Shuning uchun $\sin\frac{3\pi}{4}>0$, $\cos\frac{3\pi}{4}<0$.

2) $745°=2\cdot360°+25°$ bo‘lgani uchun (1;0) nuqtani 745° ga burishga birinchi chorakda joylashgan nuqta mos keladi. Shuning uchun $\sin745°>0$, $\cos745°>0$.



*66- rasm.*

3) $-\pi<-\frac{5\pi}{7}<-\frac{\pi}{2}$ bo‘lgani uchun (1;0) nuqtani $-\frac{5\pi}{7}$ burchakka burganda uchinchi chorakda joylashgan nuqta hosil qilinadi. Shuning uchun $\sin\left(-\frac{5\pi}{7}\right)<0$, $\cos\left(-\frac{5\pi}{7}\right)<0$. ▲

## 2. Tangensning ishoralari

Ta’rifga ko‘ra $\mathrm{tg}\alpha=\frac{\sin\alpha}{\cos\alpha}$. Shuning uchun, agar $\sin\alpha$ va $\cos\alpha$ bir xil ishoraga ega bo‘lsa, $\mathrm{tg}\alpha>0$, $\sin\alpha$ va $\cos\alpha$ qarama-qarshi ishoralarga ega bo‘lsa, $\mathrm{tg}\alpha<0$ bo‘ladi. Tangensning ishoralari 66-rasmda tasvirlangan.

$\mathrm{ctg}\alpha$ ning ishoralari $\mathrm{tg}\alpha$ ning ishoralari bilan bir xildir.

**2-masala.** Burchak tangensining ishoralarini aniqlang:

1) 260°; 2) 3.

△ 1) $180°<260°<270°$ bo‘lgani uchun $\mathrm{tg}260°>0$.

2) $\frac{\pi}{2}<3<\pi$ bo‘lgani uchun $\mathrm{tg}3<0$. ▲

## *Mashqlar*

**242.** Agar:

1) $\alpha=\frac{\pi}{6}$; 2) $\alpha=\frac{3\pi}{4}$; 3) $\alpha=210^\circ$; 4) $\alpha=-210^\circ$;

5) $\alpha=735^\circ$; 6) $\alpha=848^\circ$; 7) $\alpha=\frac{2\pi}{5}$; 8) $\alpha=\frac{5\pi}{8}$

bo'lsa, (1; 0) nuqtani $\alpha$ burchakka burishda hosil bo'lgan nuqta qaysi chorakda yotishini aniqlang.

**243.** Agar:

1) $\alpha=\frac{5\pi}{4}$; 2) $\alpha=\frac{5\pi}{6}$; 3) $\alpha=-\frac{5}{8}\pi$; 4) $\alpha=-\frac{4}{3}\pi$;

5) $\alpha=740^\circ$; 6) $\alpha=510^\circ$; 7) $\alpha=-\frac{7\pi}{4}$; 8) $\alpha=361^\circ$

bo'lsa, $\sin\alpha$ sonning ishorasini aniqlang.

**244.** Agar:

1) $\alpha=\frac{2}{3}\pi$; 2) $\alpha=\frac{7}{6}\pi$; 3) $\alpha=-\frac{3\pi}{4}$; 4) $\alpha=-\frac{2}{5}\pi$;

5) $\alpha=290^\circ$; 6) $\alpha=-150^\circ$; 7) $\alpha=\frac{6\pi}{5}$; 8) $\alpha=-100^\circ$

bo'lsa, $\cos\alpha$ sonning ishorasini aniqlang.

**245.** Agar:

1) $\alpha=\frac{5}{6}\pi$; 2) $\alpha=\frac{12}{5}\pi$; 3) $\alpha=-\frac{3}{5}\pi$; 4) $\alpha=-\frac{5}{4}\pi$;

5) $\alpha=190^\circ$; 6) $\alpha=283^\circ$; 7) $\alpha=172^\circ$; 8) $\alpha=200^\circ$

bo'lsa, $\mathrm{tg}\alpha$ va $\mathrm{ctg}\alpha$ sonlarning ishoralarini aniqlang.

**246.** Agar:

1) $\pi<\alpha<\frac{3\pi}{2}$; 2) $\frac{3\pi}{2}<\alpha<\frac{7\pi}{4}$; 3) $\frac{7}{4}\pi<\alpha<2\pi$;

4) $2\pi<\alpha<2{,}5\pi$; 5) $\frac{3\pi}{4}<\alpha<\pi$; 6) $1{,}5\pi<\alpha\leq1{,}8\pi$

bo'lsa, $\sin\alpha, \cos\alpha, \mathrm{tg}\alpha, \mathrm{ctg}\alpha$ sonlarning ishoralarini aniqlang.

**247.** Agar:

1) $\alpha=1$; 2) $\alpha=3$; 3) $\alpha=-3{,}4$; 4) $\alpha=-1{,}3$; 5) $\alpha=3{,}14$

bo'lsa, $\sin\alpha, \cos\alpha, \mathrm{tg}\alpha$ sonlarning ishoralarini aniqlang.

**248.** $0<\alpha<\frac{\pi}{2}$ bo'lsin. Sonning ishorasini aniqlang:

1) $\sin\left(\frac{\pi}{2}-\alpha\right)$; 2) $\cos\left(\frac{\pi}{2}+\alpha\right)$; 3) $\mathrm{tg}\left(\frac{3}{2}\pi-\alpha\right)$; 4) $\sin(\pi-\alpha)$;

5) $\cos(\alpha-\pi)$; 6) $\mathrm{tg}(\alpha-\pi)$; 7) $\cos\left(\alpha-\frac{\pi}{2}\right)$; 8) $\mathrm{ctg}\left(\alpha-\frac{\pi}{2}\right)$.

**249.** Sinus va kosinuslarning ishoralari bir xil (har xil) bo‘ladigan $\alpha$ sonning 0 dan $2\pi$ gacha oraliqda joylashgan barcha qiymatlarini toping.

**250.** Sonning ishorasini aniqlang:

1) $\sin\frac{2\pi}{3}\sin\frac{3\pi}{4}$; 2) $\cos\frac{2\pi}{3}\cos\frac{\pi}{6}$; 3) $\frac{\sin\frac{2\pi}{3}}{\cos\frac{3\pi}{4}}$; 4) $\operatorname{tg}\frac{5\pi}{4}+\sin\frac{\pi}{4}$.

**251.** Ifodalarning qiymatlarini taqqoslang:

1) $\sin 0{,}7$ va $\sin 4$; 2) $\cos 1{,}3$ va $\cos 2{,}3$.

**252.** Tenglamani yeching:

1) $\sin(5\pi+x)=1$; 2) $\cos(x-3\pi)=0$;

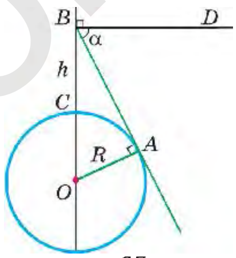
3) $\cos\left(\frac{5}{2}\pi+x\right)=-1$; 4) $\sin\left(\frac{9}{2}\pi+x\right)=-1$.

**253.** Agar:

1) $\sin\alpha+\cos\alpha=-1{,}4$; 2) $\sin\alpha-\cos\alpha=1{,}4$;
3) $\sin\alpha+\cos\alpha=1{,}4$; 4) $\cos\alpha-\sin\alpha=1{,}2$

bo‘lsa, $\alpha$ songa mos keluvchi nuqta qaysi chorakda joylashgan?

**254.** (*Beruniy masalasi.*) Tog‘ning balandligi $h-BC$ va $\alpha-\angle ABD$ burchak ma’lum bo‘lsa, Yer radiusi $R$ ni toping (67-rasm).




*67- rasm.*

## 21-§. AYNI BIR BURCHAKNING SINUSI, KOSINUSI VA TANGENSI ORASIDAGI MUNOSABATLAR

*Sinus bilan kosinus orasidagi munosabatni* aniqlaymiz.

Aytaylik, birlik aylananing $M(x; y)$ nuqtasi (1; 0) nuqtani $\alpha$ burchakka burish natijasida hosil qilingan bo‘lsin (68-rasm). U holda sinus va kosinusning ta’rifiga ko‘ra,

$$x = \cos\alpha,\ y = \sin\alpha$$

bo‘ladi.

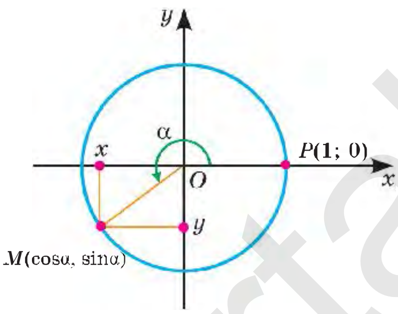
$M$ nuqta birlik aylanaga tegishli, shuning uchun uning $(x; y)$ koordinatalari $x^2 + y^2 = 1$ tenglamani qanoatlantiradi.

Demak,

$$\sin^2\alpha + \cos^2\alpha = 1. \quad (1)$$

(1) tenglik $\alpha$ ning istalgan qiymatida bajariladi va *asosiy trigonometrik ayniyat* deyiladi.

(1) tenglikdan $\sin\alpha$ ni $\cos\alpha$ orqali va, aksincha, $\cos\alpha$ ni $\sin\alpha$ orqali ifodalash mumkin:



*68- rasm.*

$$\sin\alpha = \pm\sqrt{1-\cos^2\alpha}\,, \quad (2)$$

$$\cos\alpha = \pm\sqrt{1-\sin^2\alpha}\,. \quad (3)$$

Bu formulalarda ildiz oldidagi ishora formulaning chap qismida turgan ifodaning ishorasi bilan aniqlanadi.

**1-masala.** Agar $\cos\alpha = -\frac{3}{5}$ va $\pi<\alpha<\frac{3\pi}{2}$ $\pi<\alpha<\frac{3\pi}{2}$ bo'lsa, $\sin\alpha$ ni hisoblang.

△ (2) formuladan foydalanamiz. $\pi<\alpha<\frac{3\pi}{2}$ bo'lgani uchun $\sin\alpha<0$ bo'ladi, shuning uchun (2) formulada ildiz oldiga „–" ishorasini qo'yish kerak:

$$\sin\alpha = -\sqrt{1-\cos^2\alpha} = -\sqrt{1-\frac{9}{25}} = -\frac{4}{5}\,. \ ▲$$

**2-masala.** Agar $\sin\alpha = \frac{1}{3}$ va $-\frac{\pi}{2}<\alpha<0$ bo'lsa, $\cos\alpha$ ni hisoblang.

△ $-\frac{\pi}{2}<\alpha<0$ bo'lgani uchun $\cos\alpha>0$ bo'ladi va shuning uchun (3) formulada ildiz oldiga „+" ishorasini qo'yish kerak:

$$\cos\alpha = \sqrt{1-\sin^2\alpha} = \sqrt{1-\frac{1}{9}} = \frac{2\sqrt{2}}{3}\,. \ ▲$$

Endi *tangens bilan kotangens orasidagi bog'lanishni* aniqlaymiz.

Tangens va kotangensning ta'rifiga ko'ra:

$$\mathrm{tg}\alpha = \frac{\sin\alpha}{\cos\alpha}, \ \mathrm{ctg}\alpha = \frac{\cos\alpha}{\sin\alpha}.$$

Bu tengliklarni ko'paytirib,

$$\boxed{\mathrm{tg}\alpha \cdot \mathrm{ctg}\alpha = 1} \qquad (4)$$

tenglikni hosil qilamiz. (4) tenglikdan $\mathrm{tg}\alpha$ ni $\mathrm{ctg}\alpha$ orqali, va aksincha, $\mathrm{ctg}\alpha$ ni $\mathrm{tg}\alpha$ orqali ifodalash mumkin:

$$\mathrm{tg}\alpha = \frac{1}{\mathrm{ctg}\alpha}, \qquad (5)$$

$$\mathrm{ctg}\alpha = \frac{1}{\mathrm{tg}\alpha}. \qquad (6)$$

(4)–(6) tengliklar $\alpha \neq \frac{\pi}{2}k, \ k \in \boldsymbol{Z}$ bo'lganda o'rinlidir.

**3-masala.** Agar $\mathrm{tg}\alpha = 13$ bo'lsa, $\mathrm{ctg}\alpha$ ni hisoblang.

△ (6) formula bo'yicha topamiz: $\mathrm{ctg}\alpha = \frac{1}{\mathrm{tg}\alpha} = \frac{1}{13}$. ▲

**4-masala.** Agar $\sin\alpha = 0{,}8$ va $\frac{\pi}{2} < \alpha < \pi$ bo'lsa, $\mathrm{tg}\alpha$ ni hisoblang.

△ (3) formula bo'yicha $\cos\alpha$ ni topamiz. $\frac{\pi}{2} < \alpha < \pi$ bo'lgani uchun $\cos\alpha < 0$ bo'ladi. Shuning uchun

$$\cos\alpha = -\sqrt{1 - \sin^2\alpha} = -\sqrt{1 - 0{,}64} = -0{,}6.$$

Demak, $\mathrm{tg}\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{0{,}8}{-0{,}6} = -\frac{4}{3}$. ▲

Asosiy trigonometrik ayniyatdan va tangensning ta'rifidan foydalanib, *tangens bilan kosinus orasidagi munosabatni* topamiz.

△ $\cos\alpha \neq 0$ deb faraz qilib, $\sin^2\alpha + \cos^2\alpha = 1$ tenglikning ikkala qismini $\cos^2\alpha$ ga bo'lamiz: $\frac{\cos^2\alpha - \sin^2\alpha}{\cos^2\alpha} = \frac{1}{\cos^2\alpha}$, bundan

$$\boxed{1+\mathrm{tg}^2\alpha=\frac{1}{\cos^2\alpha}}. \quad ▲ \qquad (7)$$

Agar $\cos\alpha \neq 0$ bo‘lsa, ya’ni $\alpha \neq \frac{\pi}{2}+\pi k,\ k\in \boldsymbol{Z}$ bo‘lsa, (7) formula to‘g‘ri bo‘ladi.

(7) formuladan tangensni kosinus va kosinusni tangens orqali ifodalash mumkin.

**5-masala.** Agar $\cos\alpha=-\frac{3}{5}$ va $\frac{\pi}{2}<\alpha<\pi$ bo‘lsa, $\mathrm{tg}\alpha$ ni hisoblang.

△ (7) formuladan hosil qilamiz:

$$\mathrm{tg}^2\alpha=\frac{1}{\cos^2\alpha}-1=\frac{1}{\left(-\frac{3}{5}\right)^2}-1=\frac{16}{9}.$$

Tangens ikkinchi chorakda manfiy, shuning uchun $\mathrm{tg}\alpha=-\frac{4}{3}$. ▲

**6-masala.** Agar $\mathrm{tg}\alpha=3$ va $\pi<\alpha<\frac{3\pi}{2}$ bo‘lsa, $\cos\alpha$ ni hisoblang.

△ (7) formuladan topamiz:

$$\cos^2\alpha=\frac{1}{1+\mathrm{tg}^2\alpha}=\frac{1}{10}.$$

$\pi<\alpha<\frac{3\pi}{2}$ bo‘lgani uchun $\cos\alpha<0$ va shuning uchun $\cos\alpha=-\sqrt{0,1}$. ▲

## *Mashqlar*

**255.** Agar:

1) $\cos\alpha=\frac{5}{13}$ va $\frac{3\pi}{2}<\alpha<2\pi$ bo‘lsa, $\sin\alpha$ va $\mathrm{tg}\alpha$ ni;
2) $\sin\alpha=0,8$ va $\frac{\pi}{2}<\alpha<\pi$ bo‘lsa, $\cos\alpha$ va $\mathrm{tg}\alpha$ ni;
3) $\cos\alpha=-\frac{3}{5}$ va $\frac{\pi}{2}<\alpha<\pi$ bo‘lsa, $\sin\alpha$, $\mathrm{tg}\alpha$ va $\mathrm{ctg}\alpha$ ni;
4) $\sin\alpha=-\frac{2}{5}$ va $\pi<\alpha<\frac{3\pi}{2}$ bo‘lsa, $\cos\alpha$, $\mathrm{tg}\alpha$ va $\mathrm{ctg}\alpha$ ni;
5) $\mathrm{tg}\alpha=\frac{15}{8}$ va $\pi<\alpha<\frac{3\pi}{2}$ bo‘lsa, $\sin\alpha$ va $\cos\alpha$ ni;
6) $\mathrm{ctg}\alpha=-3$ va $\frac{3\pi}{2}<\alpha<2\pi$ bo‘lsa, $\sin\alpha$ va $\cos\alpha$ ni hisoblang.

**256.** Asosiy trigonometrik ayniyat yordamida tengliklar bir vaqtda bajarilishi yoki bajarilmasligini aniqlang:

1) $\sin\alpha = 1$ va $\cos\alpha = 1$; 2) $\sin\alpha = 0$ va $\cos\alpha = -1$ ;

3) $\sin\alpha = -\frac{4}{5}$ va $\cos\alpha = -\frac{3}{5}$; 4) $\sin\alpha = \frac{1}{3}$ va $\cos\alpha = -\frac{1}{2}$.

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**257.** Tengliklar bir vaqtda bajarilishi mumkinmi:

1) $\sin\alpha = \frac{1}{5}$ va $\mathrm{tg}\alpha = \frac{1}{\sqrt{24}}$; 2) $\mathrm{ctg}\alpha = \frac{\sqrt{7}}{3}$ va $\cos\alpha = \frac{3}{4}$?

**258.** Aytaylik, to'g'ri burchakli uchburchakning burchaklaridan biri bo'lsin. Agar $\sin\alpha = \frac{2\sqrt{10}}{11}$ bo'lsa, $\cos\alpha$ va $\mathrm{tg}\alpha$ ni toping.

**259.** Teng yonli uchburchakning uchidagi burchagining tangensi $2\sqrt{2}$ ga teng. Shu burchakning kosinusini toping.

**260.** Agar $\cos^4\alpha - \sin^4\alpha = \frac{1}{8}$ bo'lsa, $\cos\alpha$ ni toping.

**261.** 1) $\sin\alpha = \frac{2\sqrt{3}}{5}$ bo'lsa, $\cos\alpha$ ni toping;

2) $\cos\alpha = -\frac{1}{\sqrt{5}}$ bo'lsa, $\sin\alpha$ ni toping.

**262.** $\mathrm{tg}\alpha = 2$ ekanligi ma'lum. Ifodaning qiymatini toping:

1) $\frac{\mathrm{ctg}\alpha + \mathrm{tg}\alpha}{\mathrm{ctg}\alpha - \mathrm{tg}\alpha}$; 2) $\frac{\sin\alpha - \cos\alpha}{\sin\alpha - \cos\alpha}$; 3) $\frac{2\sin\alpha + 3\cos\alpha}{3\sin\alpha - 5\cos\alpha}$;

4) $\frac{\sin^2\alpha - 2\cos^2\alpha}{\sin^2\alpha - \cos^2\alpha}$; 5) $\frac{3\sin\alpha - 2\cos\alpha}{4\sin\alpha + \cos\alpha}$; 6) $\frac{3\sin^2\alpha + \cos^2\alpha}{2\sin^2\alpha - \cos^2\alpha}$.

**263.** $\sin\alpha + \cos\alpha = \frac{1}{2}$ ekanligi ma'lum. 1) $\sin\alpha \cos\alpha$; 2) $\sin^3\alpha + \cos^3\alpha$ ifodalarning qiymatlarini toping.

**264.** Tenglamani yeching:

1) $2\sin x + \sin^2 x + \cos^2 x = 1$; 2) $\sin^2 x - 2 = \sin x - \cos^2 x$;

3) $2\cos^2 x - 1 = \cos x - 2\sin^2 x$; 4) $3 - \cos x = 3\cos^2 x + 3\sin^2 x$.

## 22-§. TRIGONOMETRIK AYNIYATLAR

**1-masala.** $\alpha \neq \pi k,\ k \in \boldsymbol{Z}$ bo‘lganda

$$1+\operatorname{ctg}^2\alpha=\frac{1}{\sin^2\alpha} \qquad (1)$$

tenglikning o‘rinli ekanligini isbotlang.

△ Kotangensning ta’rifiga ko‘ra $\operatorname{ctg}\alpha=\frac{\cos\alpha}{\sin\alpha}$ va shuning uchun

$$1+\operatorname{ctg}^2\alpha=1+\frac{\cos^2\alpha}{\sin^2\alpha}=\frac{\sin^2\alpha+\cos^2\alpha}{\sin^2\alpha}=\frac{1}{\sin^2\alpha}. \qquad (2)$$

Bu shakl almashtirishlar to‘g‘ri, chunki $\alpha \neq \pi k, k\in \boldsymbol{Z}$ bo‘lganda $\sin\alpha\neq 0$. ▲

(1) tenglik $\alpha$ ning mumkin bo‘lgan barcha (joiz) qiymatlari uchun o‘rinli, ya’ni uning chap va o‘ng qismlari ma’noga ega bo‘ladigan barcha qiymatlari uchun to‘g‘ri bo‘ladi. Bu kabi tengliklar *ayniyatlar* deyiladi, bunday tengliklarni isbotlashga doir masalalar ayniyatlarni isbotlashga doir masalalar deyiladi.

Kelgusida ayniyatlarni isbotlashda, agar masalaning shartida talab qilinmagan bo‘lsa, burchaklarning joiz qiymatlarini izlab o‘tirmaymiz.

**2-masala.** Ayniyatni isbotlang: $\cos^2\alpha = (1-\sin\alpha)(1+\sin\alpha)$.

△ $(1-\sin\alpha)(1+\sin\alpha) = 1-\sin^2\alpha = \cos^2\alpha$. ▲

**3-masala.** Ayniyatni isbotlang: $\frac{\cos\alpha}{1-\sin\alpha}=\frac{1+\sin\alpha}{\cos\alpha}$.

△ Bu ayniyatni isbotlash uchun uning chap va o‘ng qismlarining ayirmasi nolga teng ekanligini ko‘rsatamiz:

$$\frac{\cos\alpha}{1-\sin\alpha}-\frac{1+\sin\alpha}{\cos\alpha}=\frac{\cos^2\alpha-(1-\sin^2\alpha)}{\cos\alpha(1-\sin\alpha)}=\frac{\cos^2\alpha-\cos^2\alpha}{\cos\alpha(1-\sin\alpha)}=0. \ ▲$$

1–3- masalalarni yechishda *ayniyatlarni isbotlashning quyidagi usullaridan* foydalanildi: o‘ng qismida shakl almashtirib, uni chap qismiga tengligini ko‘rsatish; o‘ng va chap qismlarining ayirmasi nolga tengligini ko'rsatish. Ba'zan ayniyatlarni isbotlashda uning o'ng va chap qismlarining shaklini almashtirib bir xil ifodaga keltirish qulay.

**4-masala.** Ayniyatni isbotlang: $\frac{1-tg^2\alpha}{1+tg^2\alpha}=\cos^4\alpha-\sin^4\alpha$.

$$\triangle\ \frac{1-tg^2\alpha}{1+tg^2\alpha}=\frac{1-\frac{\sin^2\alpha}{\cos^2\alpha}}{1-\frac{\sin^2\alpha}{\cos^2\alpha}}=\frac{\cos^2\alpha-\sin^2\alpha}{\cos^2\alpha-\sin^2\alpha}=\cos^2\alpha-\sin^2\alpha.$$

$$\cos^4\alpha-\sin^4\alpha=(\cos^2\alpha-\sin^2\alpha)(\cos^2\alpha+\sin^2\alpha)=\cos^2\alpha-\sin^2\alpha.$$

Ayniyat isbotlandi, chunki uning chap va o'ng qismlari $\cos^2\alpha-\sin^2\alpha$ ga teng. ▲

**5-masala.** Ifodani soddalashtiring: $\frac{1}{tg\alpha-ctg\alpha}$.

$$\triangle\ \frac{1}{tg\alpha+ctg\alpha}=\frac{1}{\frac{\sin\alpha}{\cos\alpha}+\frac{\cos\alpha}{\sin\alpha}}=\frac{\sin\alpha\cos\alpha}{\sin^2\alpha+\cos^2\alpha}=\sin\alpha\cos\alpha.\ ▲$$

Trigonometrik ifodalarni soddalashtirishga doir masalalar yechishda, agar masalaning shartida talab qilinmagan bo'lsa, burchaklarning qabul qilishi mumkin bo'lgan joiz qiymatlarini topmaymiz.

## *Mashqlar*

**265.** Ayniyatni isbotlang:

1) $(1-\cos\alpha)(1+\cos\alpha)=\sin^2\alpha$; 2) $2-\sin^2\alpha-\cos^2\alpha=1$;

3) $\frac{\sin^2\alpha}{1-\sin^2\alpha}=tg^2\alpha$; 4) $\frac{\cos^2\alpha}{1-\cos^2\alpha}=ctg^2\alpha$;

5) $\frac{1}{1+tg^2\alpha}+\sin^2\alpha=1$; 6) $\frac{1}{1+ctg^2\alpha}+\cos^2\alpha=1$.

**266.** Ifodani soddalashtiring:

1) $\cos\alpha\cdot tg\alpha-2\sin\alpha$; 2) $\cos\alpha-\sin\alpha\cdot ctg\alpha$;

3) $\frac{\sin^2\alpha}{1+\cos\alpha}$; 4) $\frac{\cos^2\alpha}{1-\sin\alpha}$; 5) $\frac{tg\alpha\cdot\cos\alpha}{\sin^2\alpha}$.

**267.** Ifodani soddalashtiring va uning son qiymatini toping:

1) $\frac{\sin^2\alpha-1}{1-\cos^2\alpha}$, bunda $\alpha=\frac{\pi}{6}$; 2) $\frac{1}{\cos^2\alpha}-1$, bunda $\alpha=\frac{\pi}{3}$;

3) $\cos^2\alpha+\text{ctg}^2\alpha+\sin^2\alpha$, bunda $\alpha=\frac{\pi}{6}$;

4) $\cos^2\alpha+\text{tg}^2\alpha+\sin^2\alpha$, bunda $\alpha=\frac{\pi}{3}$.

**268.** Ayniyatni isbotlang:

1) $(1-\sin^2\alpha)(1+\text{tg}^2\alpha)=1$; 2) $\sin^2\alpha(1+\text{ctg}^2\alpha)-\cos^2\alpha=\sin^2\alpha$.

**269.** α ning barcha joiz qiymatlarida quyidagi ifoda ayni bir xil qiymatni qabul qilishini, ya'ni α ga bog'liq emasligini isbotlang:

1) $(1+\text{tg}^2\alpha)\cos^2\alpha$; 2) $\sin^2\alpha(1+\text{ctg}^2\alpha)$;

3) $\left(1+\text{tg}^2\alpha+\frac{1}{\sin^2\alpha}\right)\sin^2\alpha\cos^2\alpha$; 4) $\frac{1+\text{tg}^2\alpha}{1+\text{ctg}^2\alpha}-\text{tg}^2\alpha$.

**270.** Ayniyatni isbotlang:

1) $(1-\cos2\alpha)(1+\cos2\alpha)=\sin^2 2\alpha$; 2) $\frac{\sin\alpha-1}{\cos^2\alpha}=-\frac{1}{1-\sin\alpha}$;

3) $\cos^4\alpha-\sin^4\alpha=\cos^2\alpha-\sin^2\alpha$;

4) $(\sin^2\alpha-\cos^2\alpha)^2+2\cos^2\alpha\sin^2\alpha=\sin^2\alpha+\cos^2\alpha$;

5) $\frac{\sin\alpha}{1-\cos\alpha}+\frac{1+\cos\alpha}{\sin\alpha}=\frac{2}{\sin\alpha}$; 6) $\frac{\sin\alpha}{1-\cos\alpha}=\frac{1+\cos\alpha}{\sin\alpha}$;

7) $\frac{1}{1-\text{tg}^2\alpha}+\frac{1}{1+\text{ctg}^2\alpha}=1$; 8) $\text{tg}^2\alpha-\sin^2\alpha=\text{tg}^2\alpha\sin^2\alpha$.

**271.** Ifodani soddalashtiring va uning son qiymatini toping:

1) $\frac{(\sin\alpha-\cos\alpha)^2}{\sin^2\alpha}-(1+\text{ctg}^2\alpha)$, bunda $\alpha=\frac{\pi}{3}$;

2) $(1+\text{tg}^2\alpha)-\frac{(\sin\alpha-\cos\alpha)^2}{\cos^2\alpha}$, bunda $\alpha=\frac{\pi}{6}$.

**272.** Agar $\sin\alpha-\cos\alpha=0,6$ bo'lsa, $\sin\alpha\cos\alpha$ ning qiymatini toping.

**273.** Agar $\cos\alpha-\sin\alpha-0,2$ bo'lsa, $\cos^3\alpha-\sin^3\alpha$ ning qiymatini toping.

**274.** Tenglamani yeching:

1) $3\cos^2x-2\sin x=3-3\sin^2x$;

2) $\cos^2x-\sin^2x-2\sin x-1-2\sin^2x$.

## 23- §. α VA –α BURCHAKLARNING SINUSI, KOSINUSI, TANGENSI VA KOTANGENSI

Aytaylik, birlik aylananing $M_1$ va $M_2$ nuqtalari $P(1; 0)$ nuqtani, mos ravishda, α va –α burchaklarga burish natijasida hosil qilingan bo'lsin (69- rasm). U holda $Ox$ o'q $M_1OM_2$ burchakni teng ikkiga bo'ladi va shuning uchun $M_1$ va $M_2$ nuqtalar $Ox$ o'qqa nisbatan simmetrik joylashgan. Bu nuqtalarning abssissalari bir xil bo'ladi, ordinatalari esa faqat ishoralari bilan farq qiladi. $M_1$ nuqta (cosα; sinα) koordinatalarga, $M_2$ nuqta (cos(–α); sin(–α)) koordinatalarga ega. Shuning uchun

$$\sin(-\alpha) = -\sin\alpha, \cos(-\alpha) = \cos\alpha. \qquad (1)$$

Tangensning ta'rifidan foydalanib, hosil qilamiz:

$$\mathrm{tg}(-\alpha) = \frac{\sin(-\alpha)}{\cos(-\alpha)} = \frac{-\sin\alpha}{\cos\alpha} = -\mathrm{tg}\,\alpha.$$

Demak,

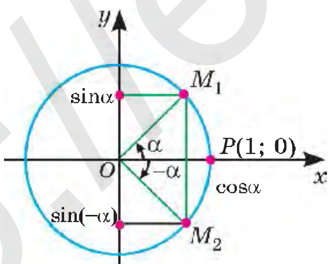
$$\mathrm{tg}(-\alpha) = -\mathrm{tg}\,\alpha. \qquad (2)$$

Shunga o'xshash,

$$\mathrm{ctg}(-\alpha) = -\mathrm{ctg}\,\alpha. \qquad (3)$$

(1) formula α ning istalgan qiymatida o'rinli bo'ladi, (2) formula esa $\alpha \neq \frac{\pi}{2} + \pi k,\ k \in$ bo'lganda o'rinlidir.

Agar $\alpha \neq \pi k,\ k \subset \mathbf{Z}$ bo'lsa, u holda $\mathrm{ctg}(-\alpha) = -\mathrm{ctg}\,\alpha$ bo'lishini ko'rsatish mumkin.



*69- rasm.*

(1)–(2) formulalar manfiy burchaklar uchun sinus, kosinus va tangensning qiymatlarini topishga imkon beradi.

Masalan:

$$\sin\left(-\frac{\pi}{6}\right) = -\sin\frac{\pi}{6} = -\frac{1}{2},$$

$$\cos\left(-\frac{\pi}{4}\right) = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2},$$

$$\mathrm{tg}\left(-\frac{\pi}{3}\right) = -\mathrm{tg}\,\frac{\pi}{3} = -\sqrt{3}.$$

### *Mashqlar*

**275.** Hisoblang:

1) $\cos(-\frac{\pi}{6})\sin(-\frac{\pi}{3})\;\;\text{tg}(-\frac{\pi}{4})$;     2) $\frac{1-\text{tg}^2(-30^\circ)}{1+\text{ctg}^2(-30^\circ)}$;

3) $2\sin(-\frac{\pi}{6})\cos(-\frac{\pi}{6})+\text{tg}(-\frac{\pi}{3})+\sin^2(-\frac{\pi}{4})$;

4) $\cos(-\pi)-\text{ctg}(-\frac{\pi}{2})-\sin(-\frac{3}{2}\pi)+\text{ctg}(-\frac{\pi}{4})$.

**276.** Ifodani soddalashtiring:

1) $\text{tg}(-\alpha)\cos\alpha+\sin\alpha$;     2) $\cos\alpha-\text{ctg}\alpha(-\sin\alpha)$;

3) $\frac{\cos(-\alpha)+\sin(-\alpha)}{\cos^2\alpha-\sin^2\alpha}$;     4) $\text{tg}(-\alpha)\text{ctg}(-\alpha)+\cos^2(-\alpha)+\sin^2\alpha$.

---

**277.** Ayniyatni isbotlang: $\frac{\cos^2\alpha-\sin^2\alpha}{\cos\alpha+\sin(-\alpha)}+\text{tg}(-\alpha)\cos(-\alpha)=\cos\alpha$.

**278.** Hisoblang:

1) $\frac{3-\sin^2\left(-\frac{\pi}{3}\right)-\cos^2\left(-\frac{\pi}{3}\right)}{2\cos\left(-\frac{\pi}{4}\right)}$;

2) $2\sin\left(-\frac{\pi}{6}\right)-3\text{ctg}\left(-\frac{\pi}{4}\right)+7{,}5\text{tg}(-\pi)+\frac{1}{8}\cos\left(-\frac{3}{2}\pi\right)$.

**279.** Soddalashtiring:

1) $\frac{\sin^3(-\alpha)+\cos^3(-\alpha)}{1-\sin(-\alpha)\cos(-\alpha)}$;     2) $\frac{1-(\sin\alpha-\cos(-\alpha))^2}{-\sin(-\alpha)}$.

## 24-§. QO'SHISH FORMULALARI

*Qo'shish formulalari* deb $\cos(\alpha \pm \beta)$ va $\sin(\alpha \pm \beta)$ larni $\alpha$ va $\beta$ burchaklarning sinus va kosinuslari orqali ifodalovchi formulalarga aytiladi.

**Teorema.** ***Ixtiyoriy*** $\alpha$ ***va*** $\beta$ ***uchun quyidagi tenglik o'rinli bo'ladi:***

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta. \qquad (1)$$

○ $M_0$ (1; 0) nuqtani koordinatalar boshi atrofida $\alpha, -\beta, \alpha + \beta$ radian burchaklarga burish natijasida, mos ravishda, $M_\alpha$, $M_{-\beta}$ va $M_{\alpha+\beta}$ nuqtalar hosil bo'ladi, deylik (70- rasm).

Sinus va kosinusning ta'rifiga ko'ra, bu nuqtalar quyidagi koordinatalarga ega:

$$M_\alpha(\cos\alpha;\ \sin\alpha),\quad M_{-\beta}(\cos(-\beta);\ \sin(-\beta)),$$
$$M_{\alpha+\beta}(\cos(\alpha + \beta);\ \sin(\alpha + \beta)).$$

$\angle M_0OM_{\alpha+\beta} = \angle M_{-\beta}OM_\alpha$ bo'lgani uchun $M_0OM_{\alpha-\beta}$ va $M_{-\beta}OM_\alpha$ teng yonli uchburchaklar teng va, demak, ularning $M_0M_{\alpha-\beta}$ va $M_{-\beta}M_\alpha$ asoslari ham teng. Shuning uchun

$$(M_0M_{\alpha+\beta})^2 = (M_{-\beta}M_\alpha)^2.$$

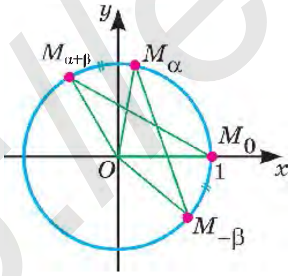
Geometriya kursidan ma'lum bo'lgan ikki nuqta orasidagi masofa formulasidan foydalanib, hosil qilamiz:

$$(1 - \cos(\alpha + \beta))^2 + (\sin(\alpha + \beta))^2 = (\cos(-\beta) - \cos\alpha)^2 + (\sin(-\beta) - \sin\alpha)^2.$$

23-§ dagi **(1)** formuladan foydalanib, bu tenglikning shaklini almashtiramiz:

$$1 - 2\cos(\alpha + \beta) + \cos^2(\alpha + \beta) + \sin^2(\alpha + \beta) =$$
$$= \cos^2\beta - 2\cos\beta\cos\alpha + \cos^2\alpha + \sin^2\beta + 2\sin\beta\sin\alpha + \sin^2\alpha.$$

Asosiy trigonometrik ayniyatdan foydalanib, hosil qilamiz:




*70- rasm.*

$$2 - 2\cos(\alpha + \beta) = 2 - 2\cos\alpha\cos\beta + 2\sin\alpha\sin\beta,$$

bundan $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$. ●

**1-masala.** $\cos75°$ ni hisoblang.

△ (1) formula bo'yicha topamiz:

$$\cos75° = \cos(45° + 30°) =$$
$$= \cos45°\cos30° - \sin45°\sin30° =$$
$$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{1}{2} = \frac{\sqrt{6}-\sqrt{2}}{4}.$$ ▲

(1) formulada $\beta$ ni $-\beta$ ga almashtirib, hosil qilamiz:

$$\cos(\alpha - \beta) = \cos\alpha\cos(-\beta) - \sin\alpha\sin(-\beta),$$

bundan

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta. \quad (2)$$

**2-masala.** $\cos15^\circ$ ni hisoblang.

△ (2) formulaga ko'ra, hosil qilamiz:

$$\cos15^\circ = \cos(45^\circ - 30^\circ) = \cos45^\circ\cos30^\circ + \sin45^\circ\sin30^\circ =$$

$$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\cdot\frac{1}{2} = \frac{\sqrt{6}+\sqrt{2}}{4}. \ ▲$$

**3-masala.** Ushbu formulalarni isbotlang:

$$\cos\left(\frac{\pi}{2} - \alpha\right) = \sin\alpha, \ \sin\left(\frac{\pi}{2} - \alpha\right) = \cos\alpha. \quad (3)$$

△ $\alpha = \frac{\pi}{2}$ bo'lganda (2) formulaga asosan:

$$\cos\left(\frac{\pi}{2} - \beta\right) = \cos\frac{\pi}{2}\cos\beta + \sin\frac{\pi}{2}\sin\beta = \sin\beta,$$

ya'ni

$$\cos\left(\frac{\pi}{2} - \beta\right) = \sin\beta. \quad (4)$$

Bu formulada $\beta$ ni $\alpha$ ga almashtirib, hosil qilamiz:

$$\cos\left(\frac{\pi}{2} - \alpha\right) = \sin\alpha.$$

(4) formulada $\beta = \frac{\pi}{2} - \alpha$ deb faraz qilsak:

$$\sin\left(\frac{\pi}{2} - \alpha\right) = \cos\alpha. \ ▲$$

(1)–(4) formulalardan foydalanib, *sinus uchun qo'shish formulasini* keltirib chiqaramiz:

$$\sin(\alpha + \beta) = \cos\left(\frac{\pi}{2} - (\alpha + \beta)\right) = \cos\left(\left(\frac{\pi}{2} - \alpha\right) - \beta\right) =$$

$$= \cos\left(\frac{\pi}{2} - \alpha\right)\cos\beta + \sin\left(\frac{\pi}{2} - \alpha\right)\sin\beta = \sin\alpha\cos\beta + \cos\alpha\sin\beta.$$

Shunday qilib,

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta. \qquad (5)$$

(5) formulada $\beta$ ni $-\beta$ ga almashtirib, hosil qilamiz:

$$\sin(\alpha - \beta) = \sin\alpha\cos(-\beta) + \cos\alpha\sin(-\beta),$$

bundan

$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta. \qquad (6)$$

**4-masala.** $\sin210^\circ$ ni hisoblang.

$$\triangle\ \sin210^\circ = \sin(180^\circ + 30^\circ) =$$

$$= \sin180^\circ\cos30^\circ + \cos180^\circ\sin30^\circ = 0\cdot\frac{\sqrt{3}}{2} + (-1)\cdot\frac{1}{2} = -\frac{1}{2}. \ \blacktriangle$$

**5-masala.** Hisoblang:

$$\sin\frac{8\pi}{7}\cos\frac{\pi}{7} - \sin\frac{\pi}{7}\cos\frac{8\pi}{7}.$$

$$\triangle\ \sin\frac{8\pi}{7}\cos\frac{\pi}{7} - \sin\frac{\pi}{7}\cos\frac{8\pi}{7} = \sin\left(\frac{8\pi}{7} - \frac{\pi}{7}\right) = \sin\pi = 0. \ \blacktriangle$$

**6-masala.** Tenglikni isbotlang:

$$\mathrm{tg}(\alpha + \beta) = \frac{\mathrm{tg}\alpha + \mathrm{tg}\beta}{1 - \mathrm{tg}\alpha\,\mathrm{tg}\beta}. \qquad (7)$$

$$\triangle\ \mathrm{tg}(\alpha + \beta) = \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} = \frac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\cos\alpha\cos\beta - \sin\alpha\sin\beta}.$$

Bu kasrning surat va maxrajini $\cos\alpha\cos\beta$ ga bo‘lib, (7) formulani hosil qilamiz. $\blacktriangle$

(7) formula hisoblashlarda foydali bo‘lishi mumkin.

Masalan, shu formula bo‘yicha topamiz:

$$\mathrm{tg}225^\circ = \mathrm{tg}(180^\circ + 45^\circ) = \frac{\mathrm{tg}180^\circ + \mathrm{tg}45^\circ}{1 - \mathrm{tg}180^\circ\,\mathrm{tg}45^\circ} = 1.$$

### *Mashqlar*

Qo‘shish formulalari yordamida hisoblang (**280–281**):

**280.** 1) $\cos135^\circ$; 2) $\cos120^\circ$; 3) $\cos150^\circ$; 4) $\cos240^\circ$.

**281.** 1) $\cos 57°30'\cos 27°30' + \sin 57°30'\sin 27°30'$;

2) $\cos 19°30'\cos 25°30' - \sin 19°30'\sin 25°30'$;

3) $\cos\frac{7\pi}{9}\cos\frac{11\pi}{9} - \sin\frac{7\pi}{9}\sin\frac{11\pi}{9}$;

4) $\cos\frac{8\pi}{7}\cos\frac{\pi}{7} + \sin\frac{8\pi}{7}\sin\frac{\pi}{7}$.

**282.** 1) $\cos\left(\frac{\pi}{3}+\alpha\right)$, bunda $\sin\alpha = \frac{1}{\sqrt{3}}$ va $0 < \alpha < \frac{\pi}{2}$;

2) $\cos\left(\alpha - \frac{\pi}{4}\right)$, bunda $\cos\alpha = -\frac{1}{3}$ va $\frac{\pi}{2} < \alpha < \pi$.

Ifodani soddalashtiring (**283–284**):

**283.** 1) $\cos 3\alpha\cos\alpha - \sin\alpha\sin 3\alpha$; 2) $\cos 5\beta\cos 2\beta + \sin 5\beta\sin 2\beta$;

3) $\cos\left(\frac{\pi}{7}+\alpha\right)\cos\left(\frac{5\pi}{14}-\alpha\right) - \sin\left(\frac{\pi}{7}+\alpha\right)\sin\left(\frac{5\pi}{14}-\alpha\right)$;

4) $\cos\left(\frac{7\pi}{5}+\alpha\right)\cos\left(\frac{2\pi}{5}+\alpha\right) + \sin\left(\frac{7\pi}{5}+\alpha\right)\sin\left(\frac{2\pi}{5}+\alpha\right)$.

**284.** 1) $\cos(\alpha+\beta) + \cos\left(\frac{\pi}{2}-\alpha\right)\cos\left(\frac{\pi}{2}-\beta\right)$;

2) $\sin\left(\frac{\pi}{2}-\alpha\right)\sin\left(\frac{\pi}{2}-\beta\right) - \cos(\alpha-\beta)$.

Qo'shish formulalari yordamida hisoblang (**285–286**):

**285.** 1) $\sin 73°\cos 17° + \cos 73°\sin 17°$;

2) $\sin 73°\cos 13° - \cos 73°\sin 13°$;

3) $\sin\frac{5\pi}{12}\cos\frac{\pi}{12} + \sin\frac{\pi}{12}\cos\frac{5\pi}{12}$; 4) $\sin\frac{7\pi}{12}\cos\frac{\pi}{12} - \sin\frac{\pi}{12}\cos\frac{7\pi}{12}$.

**286.** 1) $\sin\left(\alpha+\frac{\pi}{6}\right)$, bunda $\cos\alpha = -\frac{3}{5}$ va $\pi < \alpha < \frac{3\pi}{2}$;

2) $\sin\left(\frac{\pi}{4}-\alpha\right)$, bunda $\sin\alpha = \frac{\sqrt{2}}{3}$ va $\frac{\pi}{2} < \alpha < \pi$.

**287.** Ifodani soddalashtiring:

1) $\sin(\alpha+\beta)+\sin(-\alpha)\cos(-\beta)$; 2) $\cos(-\alpha)\sin(-\beta)-\sin(\alpha-\beta)$;

3) $\cos\left(\frac{\pi}{2}-\alpha\right)\sin\left(\frac{\pi}{2}-\beta\right)-\sin(\alpha-\beta)$;

4) $\sin(\alpha+\beta)+\sin\left(\frac{\pi}{2}-\alpha\right)\sin(-\beta)$.

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**288.** Agar $\sin\alpha=-\frac{3}{5}$, $\frac{3}{2}\pi<\alpha<2\pi$ va $\sin\beta=\frac{8}{17}$, $0<\beta<\frac{\pi}{2}$ bo‘lsa, $\cos(\alpha+\beta)$ va $\cos(\alpha-\beta)$ ni hisoblang.

**289.** Agar $\cos\alpha=-0{,}8$, $\frac{\pi}{2}<\alpha<\pi$ va $\sin\beta=-\frac{12}{13}$, $\pi<\beta<\frac{3\pi}{2}$ bo‘lsa, $\sin(\alpha-\beta)$ ni hisoblang.

**290.** Ifodani soddalashtiring:

1) $\cos\left(\frac{2}{3}\pi-\alpha\right)+\cos\left(\alpha+\frac{\pi}{3}\right)$; 2) $\sin\left(\alpha+\frac{2}{3}\pi\right)-\sin\left(\frac{\pi}{3}-\alpha\right)$;

3) $\frac{2\cos\alpha\sin\beta+\sin(\alpha-\beta)}{2\cos\alpha\cos\beta-\cos(\alpha-\beta)}$; 4) $\frac{\cos\alpha\cos\beta-\cos(\alpha+\beta)}{\cos(\alpha-\beta)-\sin\alpha\sin\beta}$.

**291.** Ayniyatni isbotlang:

1) $\sin(\alpha-\beta)\sin(\alpha+\beta)=\sin^2\alpha-\sin^2\beta$;

2) $\cos(\alpha-\beta)\cos(\alpha+\beta)=\cos^2\alpha-\sin^2\beta$;

3) $\frac{\sqrt{2}\cos\alpha-2\cos\left(\frac{\pi}{4}-\alpha\right)}{2\sin\left(\frac{\pi}{4}+\alpha\right)-\sqrt{3}\sin\alpha}=-\sqrt{2}\mathrm{tg}\alpha$; 4) $\frac{\cos\alpha-2\cos\left(\frac{\pi}{3}+\alpha\right)}{2\sin\left(\alpha-\frac{\pi}{6}\right)-\sqrt{3}\sin\alpha}=-\sqrt{3}\mathrm{tg}\alpha$.

**292.** Ifodani soddalashtiring: 1) $\frac{\mathrm{tg}29^\circ+\mathrm{tg}31^\circ}{1-\mathrm{tg}29^\circ\mathrm{tg}31^\circ}$; 2) $\frac{\mathrm{tg}\frac{7}{16}\pi-\mathrm{tg}\frac{3}{16}\pi}{1+\mathrm{tg}\frac{7}{16}\pi\cdot\mathrm{tg}\frac{3}{16}\pi}$.

## 25-§. IKKILANGAN BURCHAKNING SINUSI VA KOSINUSI

Qo‘shish formulalaridan foydalanib, *ikkilangan burchakning sinusi va kosinusi formulalarini* keltirib chiqaramiz.

1) $\sin2\alpha=\sin(\alpha+\alpha)=\sin\alpha\cos\alpha+\sin\alpha\cos\alpha=2\sin\alpha\cos\alpha$.

Shunday qilib,

$$\sin 2\alpha = 2\sin\alpha\cos\alpha. \qquad (1)$$

**1-masala.** Agar $\sin\alpha = -0{,}6$ va $\pi < \alpha < \frac{3\pi}{2}$ bo'lsa, $\sin 2\alpha$ ni hisoblang.

△ (1) formula bo'yicha topamiz:

$$\sin 2\alpha = 2\sin\alpha\cos\alpha = 2 \cdot (-0{,}6) \cdot \cos\alpha = -1{,}2\cos\alpha.$$

$\pi < \alpha < \frac{3\pi}{2}$ bo'lgani uchun $\cos\alpha < 0$ bo'ladi va shuning uchun:

$$\cos\alpha = -\sqrt{1-\sin^2\alpha} = -\sqrt{1-0{,}36} = -0{,}8.$$

Demak, $\sin 2\alpha = -1{,}2 \cdot (-0{,}8) = 0{,}96$. ▲

2) $\cos 2\alpha = \cos(\alpha + \alpha) = \cos\alpha\cos\alpha - \sin\alpha\sin\alpha = \cos^2\alpha - \sin^2\alpha$.

Shunday qilib,

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha. \qquad (2)$$

**2-masala.** Agar $\cos\alpha = 0{,}3$ bo'lsa, $\cos 2\alpha$ ni hisoblang.

△ (2) formuladan va asosiy trigonometrik ayniyatdan foydalanib, quyidagini hosil qilamiz:

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = \cos^2\alpha - (1 - \cos^2\alpha) =$$
$$= 2\cos^2\alpha - 1 = 2 \cdot (0{,}3)^2 - 1 = -0{,}82. \ ▲$$

**3-masala.** Ifodani soddalashtiring: $\frac{\sin\alpha\cos\alpha}{1-2\sin^2\alpha}$.

△ $$\frac{\sin\alpha\cos\alpha}{1-2\sin^2\alpha} = \frac{2\sin\alpha\cos\alpha}{2(\sin^2\alpha+\cos^2\alpha-2\sin^2\alpha)} = \frac{\sin 2\alpha}{2(\cos^2\alpha-\sin^2\alpha)} =$$
$$= \frac{\sin 2\alpha}{2\cos 2\alpha} = \frac{1}{2}\operatorname{tg} 2\alpha. \ ▲$$

**4-masala.** Agar $\operatorname{tg}\alpha = \frac{1}{2}$ bo'lsa, $\operatorname{tg} 2\alpha$ ni hisoblang.

△ $$\operatorname{tg}(\alpha+\beta) = \frac{\operatorname{tg}\alpha+\operatorname{tg}\beta}{1-\operatorname{tg}\alpha\operatorname{tg}\beta}$$

formulada $\beta = \alpha$ deb faraz qilib (24-§ ga qarang), hosil qilamiz:

$$tg2\alpha = \frac{2tg\alpha}{1-tg^2\alpha}. \qquad (3)$$

Agar $tg\alpha = \frac{1}{2}$ bo‘lsa, u holda (3) formula bo‘yicha topamiz:

$$tg2\alpha = \frac{2\cdot\frac{1}{2}}{1-\left(\frac{1}{2}\right)^2} = \frac{4}{3}. \ ▲$$

## *Mashqlar*

Hisoblang (**293–294**):

**293.** 1) $2\sin15°\cos15°$; 2) $\cos^2 15° - \sin^2 15°$;
3) $(\cos75° - \sin75°)^2$; 4) $(\cos15° + \sin15°)^2$.

**294.** 1) $2\sin\frac{\pi}{8}\cos\frac{\pi}{8}$; 2) $\cos^2\frac{\pi}{8} - \sin^2\frac{\pi}{8}$;
3) $\sin\frac{\pi}{8}\cos\frac{\pi}{8} + \frac{1}{4}$; 4) $\frac{\sqrt{2}}{2} - \left(\cos\frac{\pi}{8} + \sin\frac{\pi}{8}\right)^2$.

**295.** Agar:
1) $\sin\alpha = \frac{3}{5}$ va $\frac{\pi}{2} < \alpha < \pi$; 2) $\cos\alpha = -\frac{4}{5}$ va $\pi < \alpha < \frac{3\pi}{2}$
bo‘lsa, $\sin2\alpha$ ni hisoblang.

**296.** Agar:
1) $\cos\alpha = \frac{4}{5}$; 2) $\sin\alpha = -\frac{3}{5}$ bo‘lsa, $\cos2\alpha$ ni hisoblang.

Ifodani soddalashtiring (**297–298**):

**297.** 1) $\sin\alpha\cos\alpha$; 2) $\cos\alpha\cos\left(\frac{\pi}{2} - \alpha\right)$;
3) $\cos4\alpha + \sin^2 2\alpha$; 4) $\sin2\alpha + (\sin\alpha - \cos\alpha)^2$.

**298.** 1) $\frac{\cos2\alpha+1}{2\cos\alpha}$; 2) $\frac{\sin2\alpha}{1-\cos^2\alpha}$; 3) $\frac{\sin^2\alpha}{(\sin\alpha+\cos\alpha)^2-1}$; 4) $\frac{1+\cos2\alpha}{1-\cos2\alpha}$.

**299.** Ayniyatni isbotlang:

1) $\sin 2\alpha = (\sin\alpha + \cos\alpha)^2 - 1$;  2) $(\sin\alpha - \cos\alpha)^2 = 1 - \sin 2\alpha$;

3) $\cos^4\alpha - \sin^4\alpha = \cos 2\alpha$;  4) $2\cos^2\alpha - \cos 2\alpha = 1$.

**300.** Agar:

1) $\sin\alpha + \cos\alpha = \frac{1}{2}$;  2) $\sin\alpha - \cos\alpha = -\frac{1}{3}$;  3) $\sin\alpha - \cos\alpha = \frac{1}{4}$

bo‘lsa, $\sin 2\alpha$ ni hisoblang.

**301.** Ayniyatni isbotlang:

1) $1 + \cos 2\alpha = 2\cos^2\alpha$;  2) $1 - \cos 2\alpha = 2\sin^2\alpha$.

**302.** Hisoblang:

1) $2\cos^2 15^\circ - 1$;  2) $1 - 2\sin^2 22{,}5^\circ$;

3) $2\cos^2\frac{\pi}{8} - 1$;  4) $1 - 2\sin^2\frac{\pi}{12}$.

**303.** Ifodani soddalashtiring:

1) $1 - 2\sin^2 5\alpha$;  2) $2\cos^2 3\alpha - 1$;  3) $\frac{1-\cos 2\alpha}{\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}$;

4) $\frac{2\cos^2\frac{\alpha}{2} - 1}{\sin 2\alpha}$;  5) $1 + \cos 4\alpha$;  6) $1 - 2\cos^2 5\alpha$.

**304.** Ayniyatni isbotlang:

1) $\frac{\cos 2\alpha}{\sin\alpha\cos\alpha - \sin^2\alpha} = \mathrm{ctg}\,\alpha - 1$;  2) $\frac{\sin 2\alpha - 2\cos\alpha}{\sin\alpha - \sin^2\alpha} = -2\,\mathrm{ctg}\,\alpha$;

3) $\mathrm{tg}\,\alpha(1 + \cos 2\alpha) = \sin 2\alpha$;  4) $\frac{1-\cos 2\alpha + \sin 2\alpha}{1+\cos 2\alpha + \sin 2\alpha} \cdot \mathrm{ctg}\,\alpha = 1$.

**305.** Agar $\mathrm{tg}\,\alpha = 0{,}6$ bo‘lsa, $\mathrm{tg}\,2\alpha$ ni hisoblang.

**306.** Hisoblang: 1) $\frac{2\,\mathrm{tg}\frac{\pi}{8}}{1-\mathrm{tg}^2\frac{\pi}{8}}$;  2) $\frac{6\,\mathrm{tg}\,15^\circ}{1-\mathrm{tg}^2 15^\circ}$;  3) $\frac{4\,\mathrm{tg}\,75^\circ}{1-\mathrm{tg}^2 75^\circ}$.

## 26-§. KELTIRISH FORMULALARI

Sinus, kosinus, tangens va kotangens qiymatlarining jadvallari $0^\circ$ dan $90^\circ$ gacha (yoki 0 dan $\frac{\pi}{2}$ gacha) burchaklar uchun tuziladi. Bu hol ularning boshqa burchaklar uchun qiymatlari o‘tkir burchaklar uchun qiymatlariga keltirilishi bilan izohlanadi.

**1-masala.** $\sin 870°$ va $\cos 870°$ ni hisoblang.

△ $870° - 2 \cdot 360° + 150°$. Shuning uchun $P(1;\ 0)$ nuqtani koordinatalar boshi atrofida $870°$ ga burganda nuqta ikkita to‘la aylanishni bajaradi va yana $150°$ burchakka buriladi, ya’ni $150°$ ga burishdagi $M$ nuqtaning xuddi o‘zi hosil bo‘ladi (71-rasm). Shuning uchun $\sin 870° - \sin 150°$ $\cos 870° - \cos 150°$.

$M$ nuqtaga $Oy$ o‘qqa nisbatan simmetrik bo‘lgan $M_1$ nuqtani yasaymiz (72- rasm). $M$ va $M_1$ nuqtalarning ordinatalari bir xil, abssissalari esa faqat ishoralari bilan farq qiladi. Shuning uchun

$$\sin 150° = \sin 30° = \frac{1}{2};\ \cos 150° = -\cos 30° = -\frac{\sqrt{3}}{2}.$$

**Javob:** $\sin 870° = \frac{1}{2}$, $\cos 870° = -\frac{\sqrt{3}}{2}$. ▲

1-masalani yechishda

$$\sin(2 \cdot 360° + 150°) = \sin 150°, \cos(2 \cdot 360° + 150°) = \cos 150°, \quad (1)$$

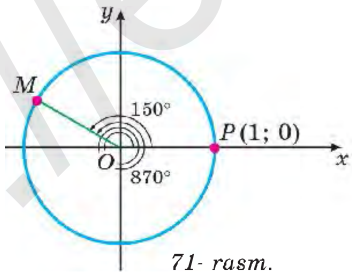
$$\sin(180° - 30°) = \sin 30°, \cos(180° - 30°) = -\cos 30° \quad (2)$$

tengliklardan foydalanildi.

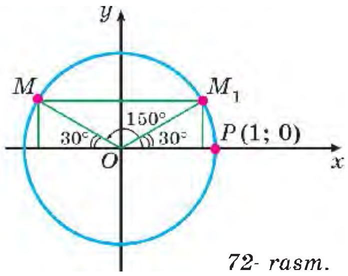
(1) tenglik to‘g‘ri tenglik, chunki $P(1;\ 0)$ nuqtani $\alpha + 2\pi k, k \in \mathbf{Z}$ burchakka burganda uni $\alpha$ burchakka burgandagi nuqtaning ayni o‘zi hosil bo‘ladi.

Shuning uchun ushbu formulalar to‘g‘ri bo‘ladi:

**!** $$\mathbf{\sin(\alpha + 2\pi k) = \sin\alpha,\ \cos(\alpha + 2\pi k) = \cos\alpha,\ k \in Z.} \quad \mathbf{(3)}$$



*71- rasm.*



*72- rasm.*

Xususan, $k = 1$ bo‘lganda:

$$\sin(\alpha + 2\pi) = \sin\alpha, \cos(\alpha + 2\pi) = \cos\alpha$$

tengliklar o‘rinlidir.

(2) tenglik

$$\sin(\pi - \alpha) = \sin\alpha, \; \cos(\pi - \alpha) = -\cos\alpha \qquad (4)$$

formulalarning xususiy holi sanaladi.

$\sin(\pi - \alpha) = \sin\alpha$ formulani isbot qilamiz.

○ Sinus uchun qo‘shish formulasini qo‘llab, hosil qilamiz:

$$\sin(\pi - \alpha) = \sin\pi\cos\alpha - \cos\pi\sin\alpha =$$

$$= 0 \cdot \cos\alpha - (-1) \cdot \sin\alpha = \sin\alpha. \; ●$$

(4) formulalarning ikkinchisi ham shunga o‘xshash isbot qilinadi. (4) formulalar *keltirish formulalari* deyiladi. (3) va (4) formulalar yordamida istalgan burchakning sinus va kosinusini hisoblashni ularning o‘tkir burchak uchun qiymatlarini hisoblashga keltirish mumkin.

**2-masala.** $\sin 930°$ ni hisoblang.

△ (3) formuladan foydalanib, hosil qilamiz:

$$\sin 930° = \sin(3 \cdot 360° - 150°) = \sin(-150°).$$

$\sin(-\alpha) = -\sin\alpha$ formula bo‘yicha $\sin(-150°) = -\sin 150°$ ni hosil qilamiz.

(4) formula bo‘yicha topamiz:

$$-\sin 150° = -\sin(180° - 30°) = -\sin 30° = -\frac{1}{2}.$$

**Javob:** $\sin 930° = -\frac{1}{2}$. ▲

**3-masala.** $\cos\frac{15\pi}{4}$ ni hisoblang.

$$△ \quad \cos\frac{15\pi}{4} = \cos(4\pi - \frac{\pi}{4}) = \cos(-\frac{\pi}{4}) = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}. \; ▲$$

Endi istalgan burchakning tangensini hisoblashni o‘tkir burchakning tangensini hisoblashga qanday keltirish mumkinligini ko‘rsatamiz.

(3) formuladan va tangensning ta’rifidan

$$\text{tg}(\alpha + 2\pi k) = \text{tg}\alpha, k \in Z$$

tenglik kelib chiqadi.

Bu tenglik va (4) formuladan foydalanib, hosil qilamiz:

$$\text{tg}(\alpha + \pi) = \text{tg}(\alpha + \pi - 2\pi) = \text{tg}(\alpha - \pi) = -\text{tg}(\pi - \alpha) =$$

$$= -\frac{\sin(\pi-\alpha)}{\cos(\pi-\alpha)} = -\frac{\sin\alpha}{-\cos\alpha} = \text{tg}\alpha.$$

Shuning uchun ushbu formula o‘rinli bo‘ladi:

**!** $$\text{tg}(\alpha + \pi k) = \text{tg}\alpha, k \in Z. \qquad (5)$$

**4-masala.** Hisoblang: 1) $\text{tg}\frac{11\pi}{3}$; 2) $\text{tg}\frac{13\pi}{4}$.

△ 1) $\text{tg}\frac{11\pi}{3} = \text{tg}(4\pi - \frac{\pi}{3}) = \text{tg}(-\frac{\pi}{3}) = -\text{tg}\frac{\pi}{3} = -\sqrt{3}$.

2) $\text{tg}\frac{13\pi}{4} = \text{tg}(3\pi + \frac{\pi}{4}) = \text{tg}\frac{\pi}{4} = 1$. ▲

24-§ da (3-masala)

**!** $$\sin(\frac{\pi}{2} - \alpha) = \cos\alpha, \cos(\frac{\pi}{2} - \alpha) = \sin\alpha$$

formulalar isbotlangan edi, ular ham *keltirish formulalari* deb ataladi. Bu formulalardan foydalanib, masalan, $\sin\frac{\pi}{6} = \cos\frac{\pi}{6}$, $\cos\frac{\pi}{3} = \sin\frac{\pi}{6}$ ni hosil qilamiz.

$x$ ning istalgan qiymati uchun $\sin(x+2\pi)=\sin x$, $\cos(x+2\pi)=\cos x$ tengliklar to‘g‘riligi ma’lum.

Bu tengliklardan ko‘rinadiki, argument $2\pi$ ga o‘zgarganda sinus va kosinusning qiymatlari davriy takrorlanadi. Bunday funksiyalar *davri* $2\pi$ *bo‘lgan davriy funksiyalar* deyiladi.

**Agar shunday $T \neq 0$ son mavjud bo‘lsaki, $y = f(x)$ funksiyaning aniqlanish sohasidagi istalgan $x$ uchun**

$$f(x - T) = f(x) = f(x + T)$$

**tenglik bajarilsa, $f(x)$ *davriy funksiya deb* ataladi. $T$ son $f(x)$ funksiyaning *davri* deyiladi.**

Bu ta’rifdan ko‘rinadiki, agar $x$ son $f(x)$ funksiyaning aniqlanish sohasiga tegishli bo‘lsa, u holda $x + T$, $x - T$ sonlar va, umuman, $x + Tn$, $n \in \boldsymbol{Z}$ sonlar ham shu davriy funksiyaning aniqlanish sohasiga tegishli va $f(x + Tn) = f(x)$, $n \in \boldsymbol{Z}$ bo‘ladi.

$2\pi$ soni $y$ — $\cos x$ funksiyaning *eng kichik musbat davri* ekanini ko‘rsatamiz.

○ $T > 0$ kosinusning davri bo‘lsin, ya’ni istalgan $x$ uchun $\cos(x+T) = \cos x$ tenglik bajariladi. $x = 0$ deb, $\cos T = 1$ ni hosil qilamiz. Bundan esa $T = 2\pi k$, $k \in \boldsymbol{Z}$. $T > 0$ bo‘lganidan $T$ quyidagi $2\pi, 4\pi, 6\pi, \ldots$ qiymatlarni qabul qila oladi va shuning uchun $T$ ning qiymati $2\pi$ dan kichik bo‘lishi mumkin emas. ●

$y = \sin x$ *funksiyaning eng kichik musbat davri ham* $2\pi$ *ga teng* ekanini isbotlash mumkin.

### *Mashqlar*

Hisoblang (**307–310**):

**307.** 1) $\sin\frac{13}{2}\pi$; 2) $\sin 17\pi$; 3) $\cos 7\pi$; 4) $\cos\frac{11}{2}\pi$;
5) $\sin 720°$; 6) $\cos 540°$; 7) $\sin 12{,}5\pi$; 8) $\cos 2025°$.

**308.** 1) $\cos 420°$; 2) $\mathrm{tg}\, 570°$; 3) $\sin 3630°$; 4) $\mathrm{ctg}\, 960°$;
5) $\sin\frac{13\pi}{6}$; 6) $\mathrm{tg}\frac{11}{6}\pi$; 7) $\mathrm{tg}\, 585°$; 8) $\mathrm{ctg}\frac{13\pi}{4}$.

**309.** 1) $\cos 150°$; 2) $\sin 135°$; 3) $\cos 120°$; 4) $\sin 315°$.

**310.** 1) $\operatorname{tg}\frac{5\pi}{4}$;  2) $\sin\frac{7\pi}{6}$;  3) $\cos\frac{5\pi}{3}$;

4) $\sin\left(-\frac{11\pi}{6}\right)$;  5) $\cos\left(-\frac{7\pi}{3}\right)$;  6) $\operatorname{tg}\left(-\frac{2\pi}{3}\right)$.

**311.** Ifodaning son qiymatini toping:

1) $\cos630° - \sin1470° - \operatorname{ctg}1125°$;

2) $\operatorname{tg}1800° - \sin495° + \cos945°$;

3) $\sin(-7\pi) - 2\cos\frac{13\pi}{3} - \operatorname{tg}\frac{7\pi}{4}$;

4) $\cos(-9\pi) + 2\sin\left(-\frac{49\pi}{6}\right) - \operatorname{ctg}\left(-\frac{21\pi}{4}\right)$.

**312.** Ifodani soddalashtiring:

1) $\cos^2(\pi - \alpha) + \sin^2(\alpha - \pi)$;

2) $\cos(\pi - \alpha)\cos(3\pi - \alpha) - \sin(\alpha - \pi)\sin(\alpha - 3\pi)$.

**313.** Hisoblang:

1) $\cos7230° + \sin900°$;  2) $\sin300° + \operatorname{tg}150°$;

3) $2\sin6{,}5\pi - \sqrt{3}\sin\frac{19\pi}{3}$;  4) $\sqrt{2}\cos4{,}25\pi - \frac{1}{\sqrt{3}}\cos\frac{61\pi}{6}$;

5) $\frac{\sin(-6{,}5\pi)+\operatorname{tg}(-7\pi)}{\cos(-7\pi)+\operatorname{ctg}(-16{,}25\pi)}$;  6) $\frac{\cos(-540°)+\sin480°}{\operatorname{tg}405°-\operatorname{ctg}330°}$.

**314.** Ifodani soddalashtiring:

1) $\frac{\sin\left(\frac{\pi}{2}-\alpha\right)+\sin(\pi-\alpha)}{\cos(\pi-\alpha)+\sin(2\pi-\alpha)}$;  2) $\frac{\cos(\pi-\alpha)+\cos\left(\frac{\pi}{2}-\alpha\right)}{\sin(\pi-\alpha)-\sin\left(\frac{\pi}{2}-\alpha\right)}$;

3) $\frac{\sin(\alpha-\pi)}{\operatorname{tg}(\alpha+\pi)}\cdot\frac{\operatorname{tg}(\pi-\alpha)}{\cos\left(\frac{\pi}{2}-\alpha\right)}$;  4) $\frac{\sin^2(\pi-\alpha)+\sin^2\left(\frac{\pi}{2}-\alpha\right)}{\sin(\pi-\alpha)}\cdot\operatorname{tg}(\pi-\alpha)$.

**315.** Uchburchakning ikkita ichki burchagi yig‘indisining sinusi uchinchi burchagining sinusiga tengligini isbotlang.

**316.** Ayniyatni isbotlang:

1) $\sin\left(\frac{\pi}{2}+\alpha\right)=\cos\alpha$;   2) $\cos\left(\frac{\pi}{2}+\alpha\right)=-\sin\alpha$;

3) $\cos\left(\frac{3}{2}\pi-\alpha\right)=-\sin\alpha$;   4) $\sin\left(\frac{3}{2}\pi-\alpha\right)=-\cos\alpha$.

**317.** Tenglamani yeching:

1) $\cos\left(\frac{\pi}{2}-x\right)=1$;   2) $\sin(\pi-x)=1$;   3) $\cos(x-\pi)=0$;

4) $\sin\left(x-\frac{\pi}{2}\right)=1$;   5) $\cos(\pi-2x)-1$;   6) $\sin\left(\frac{\pi}{2}+x\right)=0$.

## 27-§. SINUSLAR YIG'INDISI VA AYIRMASI. KOSINUSLAR YIG'INDISI VA AYIRMASI

**1-masala.** Ifodani soddalashtiring:

$$\left(\sin\left(\alpha+\frac{\pi}{12}\right)+\sin\left(\alpha-\frac{\pi}{12}\right)\right)\sin\frac{\pi}{12}.$$

△ Qo'shish formulasi va ikkilangan burchak sinusi formulasidan foydalanib, quyidagiga ega bo'lamiz:

$$\left(\sin\left(\alpha-\frac{\pi}{12}\right)+\sin\left(\alpha-\frac{\pi}{12}\right)\right)\sin\frac{\pi}{12}=$$

$$=\left(\sin\alpha\cos\frac{\pi}{12}+\cos\alpha\sin\frac{\pi}{12}+\sin\alpha\cos\frac{\pi}{12}-\cos\alpha\sin\frac{\pi}{12}\right)\sin\frac{\pi}{12}=$$

$$=2\sin\alpha\cos\frac{\pi}{12}\cdot\sin\frac{\pi}{12}=\sin\alpha\sin\frac{\pi}{6}=\frac{1}{2}\sin\alpha. \ ▲$$

Agar *sinuslar yig'indisi formulasi*

$$\boxed{\sin\alpha+\sin\beta=2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}} \qquad (1)$$

dan foydalanilsa, bu masalani soddaroq yechish mumkin. Shu formula yordamida quyidagini hosil qilamiz:

$$\left(\sin\left(\alpha+\frac{\pi}{12}\right)+\sin\left(\alpha-\frac{\pi}{12}\right)\right)\sin\frac{\pi}{12}=$$

$$=2\sin\alpha\cos\frac{\pi}{12}\cdot\sin\frac{\pi}{12}=\frac{1}{2}\sin\alpha.$$

Endi (1) formulaning o‘rinli ekanini isbotlaymiz.

○ $\frac{\alpha+\beta}{2}=x$, $\frac{\alpha-\beta}{2}=y$ belgilash kiritamiz. U holda $x+y=\alpha$, $x-y=\beta$ va shuning uchun $\sin\alpha+\sin\beta=\sin(x+y)+\sin(x-y)=\sin x\cos y+$ $+\cos x\sin y+\sin x\cos y-\cos x\sin y=2\sin x\cos y=2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$. ●

(1) formula bilan bir qatorda quyidagi *sinuslar ayirmasi formulasi, kosinuslar yig‘indisi* va *ayirmasi formulalaridan* ham foydalaniladi:

$$\sin\alpha-\sin\beta=2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}, \quad (2)$$

$$\cos\alpha+\cos\beta=2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}, \quad (3)$$

$$\cos\alpha-\cos\beta=-2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}. \quad (4)$$

(3) va (4) formulalar ham (1) formulaning isbotlanishiga o‘xshash isbotlanadi; (2) formula $\beta$ ni $-\beta$ ga almashtirish bilan (1) formuladan hosil qilinadi (*buni mustaqil isbotlang*).

**2-masala.** $\sin 75^\circ+\cos 75^\circ$ ni hisoblang.

△ $\sin 75^\circ+\cos 75^\circ=\sin 75^\circ+\sin 15^\circ=$

$$=2\sin\frac{75^\circ+15^\circ}{2}\cos\frac{75^\circ-15^\circ}{2}=2\sin 45^\circ\cos 30^\circ=2\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}=\frac{\sqrt{6}}{2}.$$ ▲

**3-masala.** $2\sin\alpha+\sqrt{3}$ ni ko‘paytmaga almashtiring.

△ $2\sin\alpha+\sqrt{3}=2\left(\sin\alpha+\frac{\sqrt{3}}{2}\right)=2\left(\sin\alpha+\sin\frac{\pi}{3}\right)=$

$$=4\sin\left(\frac{\alpha}{2}+\frac{\pi}{6}\right)\cos\left(\frac{\alpha}{2}-\frac{\pi}{6}\right).$$ ▲

**4-masala.** $\sin\alpha + \cos\alpha$ ifodaning eng kichik qiymati $\sqrt{2}$ ga, eng katta qiymati esa $\sqrt{2}$ ga teng ekanini isbotlang.

△ Berilgan ifodani ko'paytmaga almashtiramiz:

$$\sin\alpha+\cos\alpha=\sin\alpha+\sin\left(\frac{\pi}{2}-\alpha\right)=2\sin\frac{\pi}{4}\cos\left(\alpha-\frac{\pi}{4}\right)=\sqrt{2}\cos\left(\alpha-\frac{\pi}{4}\right).$$

Kosinusning eng kichik qiymati $-1$ ga, eng katta qiymati esa 1 ga teng bo'lgani uchun berilgan ifodaning eng kichik qiymati $\sqrt{2}\cdot(-1)=-\sqrt{2}$ ga, eng katta qiymati esa $\sqrt{2}\cdot 1=\sqrt{2}$ ga teng. ▲

## *Mashqlar*

**318.** Ifodani soddalashtiring:

1) $\sin(\frac{\pi}{3}+\alpha)+\sin(\frac{\pi}{3}-\alpha)$;
2) $\cos(\frac{\pi}{4}-\beta)-\cos(\frac{\pi}{4}+\beta)$;
3) $\sin^2(\frac{\pi}{4}+\alpha)-\sin^2(\frac{\pi}{4}-\alpha)$;
4) $\cos^2(\alpha-\frac{\pi}{4})-\cos^2(\alpha+\frac{\pi}{4})$.

**319.** Hisoblang:

1) $\cos105^\circ+\cos75^\circ$;
2) $\sin105^\circ-\sin75^\circ$;
3) $\cos\frac{11\pi}{12}+\cos\frac{5\pi}{12}$;
4) $\cos\frac{11\pi}{12}-\cos\frac{5\pi}{12}$;
5) $\sin\frac{7\pi}{12}-\cos\frac{\pi}{12}$;
6) $\sin105^\circ+\sin165^\circ$.

**320.** Ko'paytmaga almashtiring:

1) $1 + 2\sin\alpha$;
2) $1 - 2\sin\alpha$;
3) $1 + 2\cos\alpha$;
4) $1 + \sin\alpha$;
5) $1 - \cos\alpha$;
6) $1 + \cos\alpha$;

**321.** Ayniyatni isbotlang:

1) $\frac{\sin\alpha+\sin3\alpha}{\cos\alpha+\cos3\alpha}=\mathrm{tg}2\alpha$;
2) $\frac{\sin2\alpha+\sin4\alpha}{\cos2\alpha-\cos4\alpha}=\mathrm{ctg}\alpha$.

**322.** Ifodani soddalashtiring:

1) $\frac{2(\cos\alpha+\cos3\alpha)}{2\sin2\alpha+\sin4\alpha}$;
2) $\frac{1+\sin\alpha-\cos2\alpha-\sin3\alpha}{2\sin^2\alpha+\sin\alpha-1}$.

Ayniyatni isbotlang (**323–324**):

**323.** 1) $\cos^4\alpha - \sin^4\alpha + \sin 2\alpha = \sqrt{2}\cos\left(2\alpha - \frac{\pi}{4}\right)$;

2) $\cos\alpha + \cos\left(\frac{2\pi}{3} + \alpha\right) + \cos\left(\frac{2\pi}{3} - \alpha\right) = 0$.

**324.** 1) $\frac{\sin 2\alpha - \sin 5\alpha - \sin 3\alpha}{\cos\alpha + 1 - 2\sin^2 2\alpha} = 2\sin\alpha$;

2) $\frac{\sin\alpha - \sin 3\alpha + \sin 5\alpha + \sin 7\alpha}{\cos\alpha - \cos 3\alpha + \cos 5\alpha - \cos 7\alpha} = \text{ctg}\,\alpha$.

**325.** Ko‘paytma ko‘rinishida yozing:

1) $\cos 22^\circ + \cos 24^\circ + \cos 26^\circ + \cos 28^\circ$; 2) $\cos\frac{\pi}{12} + \cos\frac{\pi}{4} + \cos\frac{5\pi}{6}$.

**326.** $\text{tg}\,\alpha + \text{tg}\,\beta = \frac{\sin(\alpha+\beta)}{\cos\alpha \cdot \cos\beta}$ ayniyatni isbotlang va hisoblang:

1) $\text{tg}\,267^\circ + \text{tg}\,93^\circ$; 2) $\text{tg}\,\frac{5\pi}{12} + \text{tg}\,\frac{5\pi}{12}$; 3) $\text{tg}\,99^\circ + \text{tg}\,81^\circ$.

**327.** Ko‘paytuvchilarga ajrating:

1) $1 - \cos\alpha + \sin\alpha$; 2) $1 - 2\cos\alpha + \cos 2\alpha$;

3) $1 + \sin\alpha - \cos\alpha - \text{tg}\,\alpha$; 4) $1 + \sin\alpha + \cos\alpha + \text{tg}\,\alpha$.

## *III bobga doir mashqlar*

**328.** $0 < \alpha < \frac{\pi}{2}$ bo‘lsin. $P(1; 0)$ nuqtani:

1) $\frac{\pi}{2} - \alpha$; | 2) $\alpha - \pi$; | 3) $\frac{3\pi}{2} - \alpha$; | 4) $\frac{\pi}{2} + \alpha$; | 5) $\alpha - \frac{\pi}{2}$; | 6) $\pi - \alpha$

burchakka burish natijasida hosil bo‘lgan nuqta qaysi chorakda yotishini aniqlang.

**329.** Burchak sinusi va kosinusining qiymatini toping:

1) $3\pi$; 2) $4\pi$; 3) $3{,}5\pi$; 4) $\frac{5}{2}\pi$;

5) $\pi k$, $k \in \mathbf{Z}$; 6) $(2k+1)\pi$, $k \in \mathbf{Z}$; 7) $2k\pi$, $k \in \mathbf{Z}$; 8) $6{,}5\pi$.

**330.** Hisoblang:

1) $\sin 3\pi - \cos\frac{3\pi}{2}$;  2) $\cos 0 - \cos 3\pi + \cos 3{,}5$;

3) $\sin \pi k + \cos 2k\pi$, bunda $k$ – butun son;

4) $\cos\frac{(2k+1)\pi}{2} - \sin\frac{(4k+1)\pi}{2}$, bunda $k$ – butun son.

**331.** Toping:

1) agar $\sin\alpha = \frac{\sqrt{3}}{3}$ va $\frac{\pi}{2} < \alpha < \pi$ bo‘lsa, $\cos\alpha$ ni;

2) agar $\cos\alpha = -\frac{\sqrt{5}}{3}$ va $\pi < \alpha < \frac{3\pi}{2}$ bo‘lsa, $\text{tg}\,\alpha$ ni;

3) agar $\text{tg}\,\alpha = 2\sqrt{2}$ va $0 < \alpha < \frac{\pi}{2}$ bo‘lsa, $\sin\alpha$ ni;

4) agar $\text{ctg}\,\alpha = \sqrt{2}$ va $\pi < \alpha < \frac{3\pi}{2}$ $\pi < \alpha < \frac{3\pi}{2}$ bo‘lsa, $\sin\alpha$ ni.

**332.** Ayniyatni isbotlang:

1) $5\sin^2\alpha + \text{tg}\,\alpha\cos\alpha + 5\cos^2\alpha = 5 + \sin\alpha$;

2) $\text{ctg}\,\alpha\sin\alpha - 2\cos^2\alpha - 2\sin^2\alpha = \cos\alpha - 2$;

3) $\frac{3}{1\ \text{tg}^2\alpha} = 3\cos^2\alpha$;  4) $\frac{5}{1+\text{ctg}^2\alpha} = 5\sin^2\alpha$.

**333.** Ifodani soddalashtiring:

1) $2\sin(-\alpha)\cos\left(\frac{\pi}{2} - \alpha\right) - 2\cos(-\alpha)\sin\left(\frac{\pi}{2} - \alpha\right)$;

2) $3\sin(\pi - \alpha)\cos\left(\frac{\pi}{2} - \alpha\right) + 3\sin^2\left(\frac{\pi}{2} - \alpha\right)$;

3) $(1 - \text{tg}(-\alpha))(1 - \text{tg}(\pi + \alpha))\cos^2\alpha$;

4) $(1 + \text{tg}^2(-\alpha))\left(\frac{1}{1 + \text{ctg}^2(-\alpha)}\right)$.

**334.** Ifodani soddalashtiring va uning son qiymatini toping:

1) $\sin\left(\frac{3}{2}\pi - \alpha\right) + \sin\left(\frac{3}{2}\pi + \alpha\right)$, bunda $\cos\alpha = \frac{1}{4}$;

2) $\cos\left(\frac{\pi}{2} + \alpha\right) + \cos\left(\frac{3}{2}\pi - \alpha\right)$, bunda $\sin\alpha = \frac{1}{6}$.

**335.** Hisoblang:

1) $2\sin75^\circ\cos75^\circ$; 2) $\sin15^\circ$; 3) $\cos^2 75^\circ - \sin^2 75^\circ$;
4) $\sin75^\circ$; 5) $\cos75^\circ$; 6) $\sin135^\circ$.

### *O'ZINGIZNI TEKSHIRIB KO'RING!*

**1.** Agar: 1) $\sin\alpha = \frac{4}{5}$ va $\frac{\pi}{2} < \alpha < \pi$ bo'lsa, $\cos\alpha, \operatorname{tg}\alpha, \sin2\alpha$ ni,

2) $\cos\alpha = -0{,}6$ va $\pi < \alpha < \frac{3\pi}{2}$ bo'lsa, $\sin\alpha$, $\operatorname{ctg}\alpha, \cos2\alpha$ ni hisoblang.

**2.** Ifodaning qiymatini toping:

1) $4\cos\left(-\frac{\pi}{3}\right) - \operatorname{tg}\frac{\pi}{4} + 2\sin\left(-\frac{\pi}{6}\right) - \cos\pi$;

2) $\cos150^\circ$; 3) $\sin\frac{8\pi}{3}$; 4) $\operatorname{tg}\frac{5\pi}{3}$; 5) $\cos^2\frac{\pi}{8} - \sin^2\frac{\pi}{8}$.

**3.** (*G'iyosiddin Jamshid al-Koshiy masalasi.*)
$\sin3\alpha = 3\sin\alpha - 4\sin^3\alpha$ ekanini isbotlang.

**4.** Ayniyatni isbotlang:

1) $3 - \cos^2\alpha - \sin^2\alpha = 2$; 2) $1 - \sin\alpha\cos\alpha\operatorname{ctg}\alpha = \sin^2\alpha$.

**5.** Ifodani soddalashtiring:

1) $\sin(\alpha-\beta) - \sin\left(\frac{\pi}{2} - \alpha\right)\sin(-\beta)$; 2) $\sin^2\alpha + \cos2\alpha$;

3) $\operatorname{tg}(\pi - \alpha)\cos(\pi - \alpha) + \sin(4\pi + \alpha)$.

**336.** Ifodani soddalashtiring:

1) $\cos^2(\pi - \alpha) - \cos^2\left(\frac{\pi}{2} - \alpha\right)$; 2) $2\sin\left(\frac{\pi}{2} - \alpha\right)\cos\left(\frac{\pi}{2} - \alpha\right)$;

3) $\dfrac{\cos^2(2\pi+\alpha) - \sin^2(\alpha+2\pi)}{2\cos(\alpha+2\pi)\cos\left(\frac{\pi}{2}-\alpha\right)}$; 4) $\dfrac{2\sin(\pi-\alpha)\sin\left(\frac{\pi}{2}-\alpha\right)}{\sin^2\left(\alpha-\frac{\pi}{2}\right) - \sin^2(\alpha-\pi)}$.

Hisoblang (**337–338**):

**337.** 1) $\sin\frac{47\pi}{6}$; 2) $\mathrm{tg}\frac{25\pi}{4}$; 3) $\mathrm{ctg}\frac{27\pi}{4}$; 4) $\cos\frac{21\pi}{4}$.

**338.** 1) $\cos\frac{23\pi}{4}-\sin\frac{15\pi}{4}$; 2) $\sin\frac{25\pi}{3}-\mathrm{tg}\frac{10\pi}{3}$;

3) $3\cos3660° + \sin(-1560°)$; 4) $\cos(-945°) + \mathrm{tg}1035°$.

**339.** Sonlarni taqqoslang.

1) $\sin3$ va $\cos4$; 2) $\cos0$ va $\sin5$; 3) $\sin1$ va $\cos1$.

**340.** Sonning ishorasini aniqlang:

1) $\sin3{,}5\,\mathrm{tg}3{,}5$; 2) $\cos5{,}01\sin0{,}73$; 3) $\frac{\mathrm{tg}13}{\cos15}$;

4) $\sin1\cos2\,\mathrm{tg}3$; 5) $\sin2\cos2$; 6) $\mathrm{tg}1\cos1$.

**341.** Hisoblang:

1) $\sin\frac{\pi}{8}\cos\frac{3\pi}{8}+\sin\frac{3\pi}{8}\cos\frac{\pi}{8}$; 2) $\sin165°$; 3) $\sin105°$;

4) $\sin\frac{\pi}{12}$; 5) $1-2\sin^2195°$; 6) $2\cos^2\frac{3\pi}{8}-1$.

**342.** Ifodani soddalashtiring:

1) $(1+\mathrm{tg}(-\alpha))(1-\mathrm{ctg}(-\alpha))-\frac{\sin(-\alpha)}{\cos(-\alpha)}$; 2) $\frac{\mathrm{ctg}\alpha+\mathrm{tg}(-\alpha)}{\cos\alpha-\sin(-\alpha)}-\frac{\mathrm{tg}(-\alpha)}{\sin\alpha}$.

**343.** Berilgan: $\sin\alpha=\frac{\sqrt{5}}{3}$ va $\frac{\pi}{2}<\alpha<\pi$. $\cos\alpha, \mathrm{tg}\alpha, \mathrm{ctg}\alpha, \sin2\alpha, \cos2\alpha$ larning qiymatlarini hisoblang.

Ifodani soddalashtiring (**344–346**):

**344.** 1) $\cos^3\alpha\sin\alpha - \sin^3\alpha\cos\alpha$; 2) $\frac{\sin\alpha+\sin2\alpha}{1+\cos\alpha+\cos2\alpha}$.

**345.** 1) $\frac{\sin2\alpha-\sin2\alpha\cos2\alpha}{4\cos\alpha}$; 2) $\frac{2\cos^2 2\alpha}{\sin4\alpha\cos4\alpha+\sin4\alpha}$;

3) $\frac{\cos2\alpha+\sin2\alpha\cos2\alpha}{2\sin^2\alpha-1}$; 4) $\frac{(\cos\alpha-\sin\alpha)^2}{\sin2\alpha\cos2\alpha-\cos2\alpha}$.

**346.** 1) $\frac{\cos^2x}{1-\sin x}-\sin(\pi-x)$; 2) $\frac{\cos^2x}{1+\sin x}+\cos(1{,}5\pi+x)$;

3) $\frac{\sin^2x}{1+\cos x}-\sin(1{,}5\pi+x)$; 4) $\frac{\sin^2x}{1-\cos x}+\cos(3\pi-x)$.

**347.** 1) Agar $\mathrm{tg}\alpha = -\frac{3}{4}$ va $\mathrm{tg}\beta = 2{,}4$ bo‘lsa, $\mathrm{tg}(\alpha + \beta)$ ni;

2) agar $\mathrm{ctg}\alpha = \frac{4}{3}$ va $\mathrm{ctg}\beta = -1$ bo‘lsa, $\mathrm{ctg}(\alpha + \beta)$ ni hisoblang.

**348.** Ifodani soddalashtiring:

1) $2\sin\left(\frac{\pi}{4} + 2\alpha\right)\sin\left(\frac{\pi}{4} - 2\alpha\right)$; 2) $2\cos\left(\frac{\pi}{4} + 2\alpha\right)\cos\left(\frac{\pi}{4} - 2\alpha\right)$.

## *III bobga doir sinov (test) mashqlari*

**1.** $153^\circ$ ning radian o‘lchovini toping.

A) $\frac{17\pi}{20}$; B) $\frac{19\pi}{20}$; C) $17\pi$; D) $\frac{2\pi}{9}$.

**2.** $0{,}65\pi$ ning gradus o‘lchovini toping.

A) $11{,}7^\circ$; B) $117^\circ$; C) $116^\circ$; D) $118^\circ$.

**3.** Ko‘paytmalarning qaysi biri manfiy?

A) $\cos 314^\circ \sin 147^\circ$; B) $\mathrm{tg}200^\circ \mathrm{ctg}201^\circ$;
C) $\cos 163^\circ \cos 295^\circ$; D) $\sin 170^\circ \mathrm{ctg}250^\circ$.

**4.** Ko‘paytmaning qaysi biri musbat?

A) $\sin 2 \cos 2 \sin 1 \sin 1^\circ$; B) $\mathrm{tg}8^\circ \mathrm{ctg}8 \mathrm{ctg}10^\circ \mathrm{ctg}\sqrt{10}$;
C) $\sin 9^\circ \sin 9 \cos 9^\circ \cos 9$; D) $\cos 10^\circ \cos 10 \cos 11^\circ \cos\sqrt{11}$.

**5.** $\left(\frac{\sqrt{3}}{2}; \frac{1}{2}\right)$ nuqtaga tushish uchun (1; 0) nuqtani burish kerak bo‘lgan barcha burchaklarni toping?

A) $\frac{\pi}{6} + 2\pi k, k \in \mathbf{Z}$; B) $-\frac{\pi}{6} + \pi k, k \in \mathbf{Z}$;

C) $\frac{\pi}{6} + \pi k, k \in \mathbf{Z}$; D) $2\pi + \pi k, k \in \mathbf{Z}$.

**6.** (1; 0) nuqtani $\frac{5\pi}{2} + 2\pi k, k \in \mathbf{Z}$ burchakka burishdan hosil bo‘ladigan nuqtaning koordinatalarini toping.

A) (0; 1); B) (0; −1); C) (1; 0); D) (−1; 0).

**7.** Sonlarni o'sish tartibida yozing:

$a$ – sin1,57; $b$ – cos1,58; $c$ – sin3.

A) $a < c < b$; B) $b < c < a$; C) $c < a < b$; D) $b < a < c$.

**8.** Sonlarni kamayish tartibida yozing:

$a$ – cos2; $b$ – cos2°; $c$ – sin2; $d$ – sin2°.

A) $a > c > d > b$; B) $d > c > b > a$;
C) $b > c > d > a$; D) $c > d > b > a$.

**9.** Hisoblang: $\frac{\sin 136° \cdot \cos 46° - \sin 46° \cdot \cos 224°}{\sin 110° \cdot \cos 40° - \sin 20° \cdot \cos 50°}$.

A) cos40°; B) 0,5; C) sin44°; D) 2.

**10.** Hisoblang: $\frac{\sin 10° \cdot \sin 130° - \sin 100° \cdot \sin 220°}{\sin 27° \cdot \cos 23° - \sin 157° \cdot \cos 153°}$.

A) 1; B) –1; C) $\frac{\sqrt{3}}{2}$; D) $-\frac{\sqrt{3}}{2}$.

**11.** Hisoblang: $\cos(-225°) + \sin 675° + \text{tg}(-1035°)$.

A) 1; B) –1; C) $\sqrt{2}$; D) $-\frac{\sqrt{2}}{2}$.

**12.** $\sin\alpha = 0{,}6$ bo'lsa, $\text{tg}2\alpha$ ni toping $\left(0 < \alpha < \frac{\pi}{2}\right)$.

A) 3,42; B) $3\frac{3}{7}$; C) $\frac{7}{24}$; D) $-\frac{7}{24}$.

**13.** $\text{tg}\alpha$ – $\sqrt{5}$ bo'lsa, $\sin 2\alpha$ ni toping.

A) $\frac{3\sqrt{5}}{5}$; B) $-\frac{\sqrt{5}}{3}$; C) $\frac{\sqrt{5}}{3}$; D) $\sqrt{5}$.

**14.** $\text{tg}\alpha = \sqrt{7}$ bo'lsa, $\cos 2\alpha$ ni toping.

A) $\frac{4}{3}$; B) $-\frac{4}{3}$; C) $\frac{3}{4}$; D) $-\frac{3}{4}$.

**15.** Soddalashtiring: $\dfrac{\cos\left(\frac{\pi}{2}-\alpha\right)}{\sin(\pi-\alpha)}$.

A) $-1$; B) $1$; C) $0{,}5$; D) $-\frac{1}{2}$.

**16.** Soddalashtiring: $\dfrac{\sin 2\alpha+\sin(\pi-\alpha)\cdot\cos\alpha}{\sin\left(\frac{\pi}{2}-\alpha\right)}$.

A) $3\sin\alpha$; B) $\frac{1}{3}\sin\alpha$; C) $-\sin\alpha$; D) $\frac{1}{3}\cos\alpha$.

**17.** $\mathrm{tg}\alpha - \sqrt{7}$ bo‘lsa, $\dfrac{4\sin^4\alpha}{5\sin^2\alpha+15\cos^2\alpha}$ ni hisoblang.

A) $0{,}59$; B) $0{,}49$; C) $-0{,}49$; D) $0{,}2$.

**18.** $\cos\alpha+\sin\alpha=\frac{1}{3}$ bo‘lsa, $\sin^4\alpha+\cos^4\alpha$ ni toping.

A) $\frac{81}{49}$; B) $-\left(\frac{7}{9}\right)^2$; C) $\frac{49}{81}$; D) $-1\frac{32}{49}$.

**19.** Hisoblang: $\sin 100^\circ\cdot\cos 440^\circ+\sin 800^\circ\cdot\cos 460^\circ$.

A) $\frac{\sqrt{3}}{2}$; B) $1$; C) $-1$; D) $0$.

**20.** Soddalashtiring: $\dfrac{\sin 3\alpha}{\sin\alpha}+\dfrac{\cos 3\alpha}{\cos\alpha}$.

A) $4\cos 2\alpha$; B) $-2\sin 4\alpha$; C) $\sin 4\alpha$; D) $2\cos 2\alpha$.

**21.** $8x^2-6x+1=0$ tenglamaning ildizlari $\sin\alpha$ va $\sin\alpha$ bo‘lib, $\alpha, \beta$ lar I chorakda bo‘lsa, $\sin(\alpha+\beta)$ ni toping.

A) $\frac{\sqrt{3}(1+\sqrt{5})}{8}$; B) $\frac{\sqrt{2}(1+\sqrt{5})}{8}$; C) $\frac{\sqrt{3}(4-\sqrt{5})}{16}$; D) $\frac{\sqrt{3}(4-\sqrt{5})}{16}$.

**22.** $6x^2-5x+1=0$ tenglamaning ildizlari $\cos\alpha$ va $\cos\beta$ bo‘lib, $\alpha, \beta$ lar I chorakda bo‘lsa, $\cos(\alpha+\beta)$ ni toping.

A) $\frac{2\sqrt{6}-1}{6}$; B) $\frac{1-2\sqrt{6}}{6}$; C) $\frac{2\sqrt{6}-1}{7}$; D) $\frac{1-2\sqrt{6}}{5}$.

**23.** $x$ ni toping: $2(x+\sqrt{2})=\cos\left(\frac{\pi}{2}-2\alpha\right)+2\sin\left(\frac{3\pi}{2}+\alpha\right)\cdot\sin(\pi-\alpha)$.

A) $\frac{\sqrt{2}}{2}$; B) $\sqrt{2}$; C) $\sqrt{2}$; D) $2\sqrt{2}$.

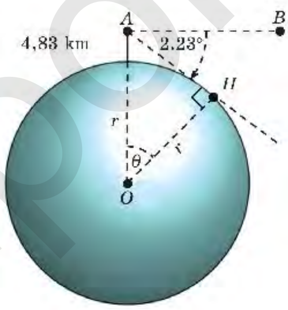
**24.** $x^2-7x+12=0$ tenglamaning ildizlari $\text{tg}\alpha$ va $\text{tg}\beta$ bo‘lsa, $\text{tg}(\alpha+\beta)$ ni toping:

A) 1; B) $\frac{7}{11}$; C) $\sqrt{3}$; D) $-\frac{7}{11}$.

## Amaliy-tatbiqiy va fanlararo bog‘liq masalalar

**Masala.** (Beruniy masalasi.) Kuzatuvchi dengiz sathidan 4,83 km balandlikdagi tog‘ning cho‘qqisida turib, okean gorizontiga og‘ish burchagi 2,23° ekanini o‘lchadi. Yerning radiusini toping.

O‘rta asrlarning buyuk qomusiy olimi Abu Rayhon Muhammad ibn Ahmad Beruniy (973–1048-y.) Yer sharining radiusini katta aniqlikda o‘lchagan bo‘lib, masalaning quyida keltirilgan yechish usuli unga tegishli.




*73-rasm.*

△ Yerni shar deb faraz qilamiz. $r$ orqali Yerning radiusini, $A$ orqali tog‘ning cho‘qqisini va $H$ orqali $A$ nuqtadan chiqqan to‘g‘ri chiziqda yotuvchi gorizont nuqtasini 73-rasmda ko‘rsatilgandek belgilaylik. $O$ nuqta Yerning markazi va $B$ nuqta $A$ nuqtadan chiquvchi va $\overline{OA}$ ga perpendikular bo‘lgan gorizontal chiziqning nuqtasi bo‘lsin. Burchak $\angle AOH$ ni 0 orqali belgilaylik.

$A$ nuqta dengiz sathidan 4,83 km balandlikda bo‘lgani uchun $OA - r + 4{,}83$. Bundan tashqari, $OH - r$. $\overline{AB}$ chiziq $\overline{OA}$ ga perpendikular bo‘lgani uchun $\angle OAB=90°$ va shu sababli $\angle OAH=90°-2{,}23°-87{,}77°$. Yer sathini rasmdagi kabi aylana sifatida qarasak, $\overline{AH}$ bu aylanaga urinma va, demak, $\overline{AH}$ va $\overline{OH}$ o‘zaro perpendikular bo‘ladi, natijada $\Delta OHA=90°$. $\angle OAH$ burchaklari yig‘indisi 180°

ekanligidan $\theta = 180^\circ - 90^\circ - 87{,}77^\circ = 2{,}23^\circ$. Demak, $\cos\theta = \frac{OH}{OA} = \frac{r}{r+4{,}83}$,

bundan $\frac{r}{r+4{,}83} = \cos 2{,}23^\circ$.

Bu tenglamani $r$ ga nisbatan yechamiz:

$$r = (r+4{,}83)\cos 2{,}23^\circ \Rightarrow r - r\cos 2{,}23^\circ = 4{,}83\cos 2{,}23^\circ \Rightarrow$$

$$\Rightarrow r = \frac{4{,}83\cos 2{,}23^\circ}{1-\cos 2{,}23^\circ} \Rightarrow r = 6372{,}91.$$

Shuni aytish joizki, hosil qilingan natija Yerning asl o'rtacha radiusi **6371** km ga juda yaqin.

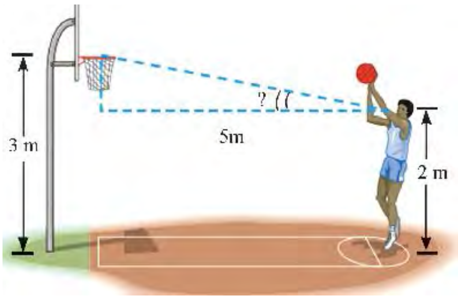
**Javob:** $r = 6372{,}91$ km. ▲

## *Masalalar*

*74- rasm.*

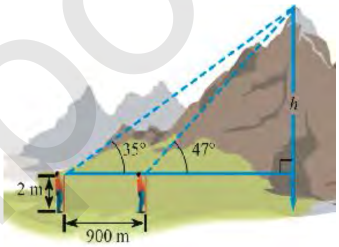
**1.** Kuzatuvchi Yer yo'ldoshi Yer yuzasidan $h$(km) masofada aylana bo'ylab harakat qilsin. Faraz qilaylik, $d$ yo'ldoshdan Yer sathining kuzatish mumkin bo'lgan oralig'ining uzunligi bo'lsin (74- rasm).
1) Markaziy burchak $\theta$ (radianlarda) va $h$ balandlikni bog'lovchi tenglamani toping;
2) kuzatilishi mumkin bo'lgan oraliqning $d$ uzunligi bilan $\theta$ ni bog'lovchi tenglamani toping;
3) $d$ va $h$ ni bog'lovchi tenglamani toping;
4) agarda $d = 4000$ km bo'lsa, Yer yo'ldoshi qanday balandlikda bo'lishi kerak?
5) agarda Yer yo'ldoshi 100 km balandlikda bo'lsa, $d$ qanday bo'ladi?

**2.** Basketbol savatidan 5 metr masofada turgan basketbolchining ko'zlari poldan 2 metr balandlik darajasida, bunda savatcha gardishi poldan 3 metr balandlikda (75-rasm). Uning ko'zlaridan savatcha gardishining markaziga qarash burchagi qanday?



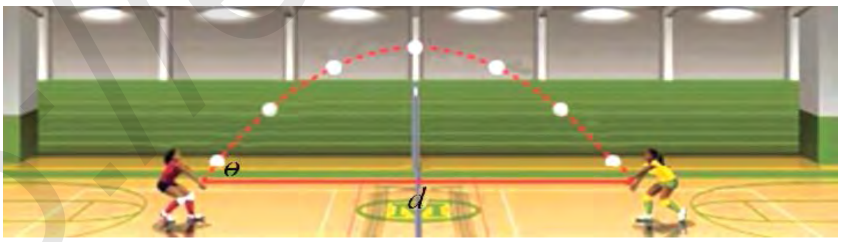

*75- rasm.*

3. Marksheyder (konlarni rejaga oluvchi va ulardan to'g'ri foydalanish bo'yicha mutaxassis) tog'ning balandligini o'lchash maqsadida oralaridagi masofa 900 metr bo'lgan ikkita nuqtadan ko'tarilish burchaklarini o'lchadi (76- rasm). Natijada birinchi burchak 47° va ikkinchisi 35° ekanligi aniqlandi. Agarda teodolit (burchakni o'lchovchi asbob)ning balandligi 2 metr bo'lsa, tog'ning balandligini toping.




*76- rasm.*

4. Voleybol o'yinida ko'tarilish burchagi $\theta$ va boshlang'ich tezligi $v$ m/s bilan otilgan to'p $d = \frac{v}{9,75}\sin 2(\theta)$ formulaga asosan $d$ gorizontal masofaga uchib boradi. Agarda $\theta = 60°$ va tezlik 12 m/s bo'lsa, $d$ ni toping (77- rasm).




*77- rasm.*

## *Tarixiy masalalar*

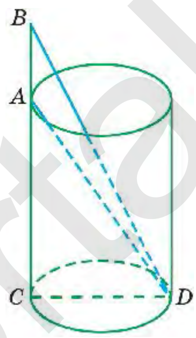
*Abu Rayhon Beruniy masalalari*

**1.** Quduq silindr shaklida bo‘lib, uning tubi quduq labidagi $A$ nuqtadan $\alpha$ burchak ostida, quduq devori davomidagi $B$ nuqtadan $\beta$ burchak ostida ko‘rinadi (78- rasm). Agar $AB = a$ bo‘lsa, quduqning chuqurligini toping:

Berilgan:

$\angle CAD - \alpha$, $\angle ABD - \beta$, $AB - a$.

Topish kerak: $AC = ?$



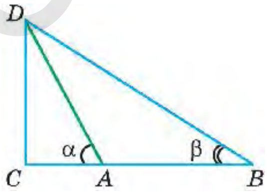
*78- rasm.*

**2.** Minora yerdagi $A$ nuqtadan $\alpha$ burchak ostida, $B$ nuqtadan esa $\beta$ burchak ostida ko‘rinadi (79- rasm). $AB = a$ bo‘lsa, minoraning balandligini toping.

Berilgan:

$\angle CAD - \alpha$, $\angle ABD - \beta$, $AB - a$.

Topish kerak: $CD - ?$



*79- rasm.*

*G‘iyosiddin Jamshid al-Koshiy masalasi*

**3.** Ixtiyoriy $\alpha$ burchak uchun

$$\sin\left(45^\circ \frac{\alpha}{2}\right) = \sqrt{\frac{1+\sin\alpha}{2}}$$

bo‘lishini isbotlang.

*Mashhur matematik Abulvafo Muhammad al-Buzjoniy (940–998) masalasi*

**4.** Ixtiyoriy $\alpha$ va $\beta$ uchun

$$\sin(\alpha-\beta) = \sqrt{\sin^2\alpha - \sin^2\alpha\sin^2\beta} - \sqrt{\sin^2\beta - \sin^2\alpha\sin^2\beta}$$

bo‘lishini isbotlang.

## *Tarixiy ma'lumotlar*


**Mirzo Ulug'bek (1394–1449)**

Matematikaning, xususan, trigonometriyaning rivojiga buyuk allomalar Muhammad al-Xorazmiy, Ahmad Farg'oniy, Abu Rayhon Beruniy, Mirzo Ulug'bek, Ali Qushchi, G'iyosiddin Jamshid al-Koshiy katta hissa qo'shganlar. Yulduzlarning osmon sferasidagi koordinatalarini aniqlash, sayyoralarning harakatlarini kuzatish, Oy va Quyosh tutilishini oldindan aytib berish va boshqa ilmiy, amaliy ahamiyatga molik masalalar aniq hisoblarni, bu hisoblarga asoslangan jadvallar tuzishni taqozo etar edi. Ana shunday astronomik (trigonometrik) jadvallar Sharqda „Zij"lar deb atalgan.

Muhammad al-Xorazmiy, Abu Rayhon Beruniy, Mirzo Ulug'bek kabi olimlarimizning matematik asarlari bilan birga „Zij"lari ham mashhur bo'lgan, ular lotin va boshqa tillarga tarjima qilingan, Yevropada matematikaning, astronomiyaning taraqqiyotiga salmoqli ta'sir o'tkazgan.

Beruniyning „Qonuni Ma'sudiy"asarida sinuslar jadvali 15 minut oraliq bilan, tangenslar jadvali $1^\circ$ oraliq bilan $10^{-8}$ gacha aniqlikda berilgan. Nihoyatda aniq „Zij"lardan biri Mirzo Ulug'bekning „Zij"i – „Ziji Ko'ragoniy"dir. Bunda sinuslar jadvali 1 minut oraliq bilan, tangenslar jadvali $0^\circ$ dan $45^\circ$ gacha 1 minut oraliq bilan, $46^\circ$ dan $90^\circ$ gacha esa 5 minut oraliq bilan $10^{-10}$ gacha aniqlikda berilgan.

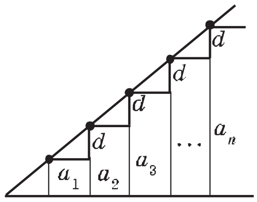
G'iyosiddin Jamshid al-Koshiy „Vatar va sinus haqida risola"sida $\sin 1^\circ$ ni verguldan so'ng 17 xona aniqligida hisoblaydi:

$$\sin 1^\circ = 0{,}017452406437283512...$$

Aylana uzunligi unga ichki va tashqi chizilgan muntazam $3 \cdot 2^n$ – ko'pburchaklar perimetrlarining o'rta arifmetigiga teng deb, $n = 28$ bo'lganda Jamshid al-Koshiy „Aylana haqida risola" asarida $2\pi$ uchun quyidagi natijani oldi:

$$2\pi = 6{,}2831853071795865...$$

# IV B O B. SONLI KETMA-KETLIKLAR. PROGRESSIYALAR




## 28-§. SONLI KETMA-KETLIKLAR

Kundalik amaliyotda turli buyumlarning joylashish tartibini ko‘rsatish uchun ularni nomerlashdan foydalaniladi. Masalan, har bir ko‘chada joylashgan uylar nomerlanadi. Kutubxonada kitob o‘quvchilarning abonementlari nomerlanadi va ularni berilgan nomerlar tartibida maxsus kartotekalarga joylashtiriladi.

Bankda omonatchining hisob raqami nomeri bo‘yicha undagi mablag‘ miqdorini ko‘rishi mumkin. Deylik, №1 hisob raqamida $a_1$ so‘m, №2 hisob raqamida $a_2$ so‘m va hokazo bo‘lsin. Natijada

$$a_1, a_2, a_3, \ldots, a_N$$

*sonli ketma-ketlikni* hosil qilamiz, bu yerda $N$ – barcha hisob raqamlarining soni. Bunda 1 dan $N$ gacha bo‘lgan har bir natural $n$ soniga $a_n$ soni mos qo‘yilgan.

Matematikada *cheksiz* sonli ketma-ketliklar o‘rganiladi:

$$a_1, a_2, a_3, \ldots, a_n, \ldots .$$

$a_1$ – soni ketma-ketlikning birinchi hadi, $a_2$ – ketma-ketlikning ikkinchi hadi, $a_3$ – ketma-ketlikning uchinchi hadi deyiladi va hokazo. $a_n$ – soni ketma-ketlikning $n$- (eninchi) hadi deb, natural $n$ soni esa uning nomeri deb ataladi. Masalan, natural sonlar kvadratlaridan iborat $1, 4, 9, 16, 25, \ldots, n^2, (n+1)^2, \ldots$ sonli ketma-ketlik uchun $a_1=1$ ketma-ketlikning birinchi hadi; $a_n=n^2$ ketma-ketlikning $n$- hadi; $a_{n+1}=(n+1)^2$ ketma-ketlikning $(n+1)$- hadi.

Sonli ketma-ketliklar ko'pincha umumiy $n$-hadining formulasi yordamida beriladi. Masalan, $a_n=\frac{1}{n}$ $(n=1, 2, 3, \ldots)$ formula yordamida $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots, \frac{1}{n}, \ldots$ sonli ketma-ketlik berilgan.

**1- masala.** Sonli ketma-ketlik $a_n=n(n-2)$ formula yordamida berilgan. Uning yuzinchi hadini hisoblang.

$$\triangle \; a_{100}=100 \cdot (100-2)=9800. \; \blacktriangle$$

**2-masala.** Sonli ketma-ketlik $a_n=2n+3$ formula yordamida berilgan. 1) Ketma-ketlikning 43 ga teng bo'lgan hadining nomerini aniqlang; 2) 50 soni ketma-ketlikning hadi bo'lishi yoki bo'lmasligini aniqlang.

△**1)** Shartga ko'ra $2n+3=43$, bundan $n=20$.

**2)** Agarda 50 soni ketma-ketlikning $n$- nomerli hadi bo'lsa, u holda $2n+3=50$, bundan $n=23{,}5$. Hosil bo'lgan $n$ ning qiymati natural son bo'lmagani uchun, u ketma-ketlik hadining nomeri bo'la olmaydi. Shu sababli, 50 soni ketma-ketlikning hadi emas. ▲

Ba'zida ketma-ketlik shunday formula orqali beriladiki, bunda uning biror nomerdan boshlab istalgan hadini undan oldingi bitta yoki bir nechta hadlari yordamida hisoblash mumkin bo'ladi. Ketma-ketlikning bunday berilish usuli *rekurrent* (lotincha *recuro* – qaytish) usuli deyiladi.

**3-masala.** Sonli ketma-ketlik $b_{n+2}=b_{n+1}+b_1$ rekurrent formula va $b_1=1$, $b_2=3$ shartlar yordamida berilgan. Bu ketma-ketlikning beshinchi hadini hisoblang.

△ $b_3=b_2+b_1=3+1=4$.

$b_4=b_3+b_2=4+3=7$.

$b_5=b_4+b_3=7+4=11$.

**Javob:** $b_5=11$. ▲

## *Mashqlar*

**349.** Natural sonlar kvadratlaridan iborat 1,4,9,16,25,...,$n^2$,$(n+1)^2$, ... sonli ketma-ketlik berilgan.

1) Ketma-ketlikning uchinchi, oltinchi, $n$-hadlarini ayting.

2) Ketma-ketlikning 4, 25, $n^2$, $(n+1)^2$ ga teng bo'lgan hadlarining nomerlarini ko'rsating.

**350.** $n$-hadining formulasi bilan berilgan ketma-ketlikning birinchi uchta hadini hisoblang:

1) $a_n = 2n + 3$; 2) $a_n = 2 + 3n$; 3) $a_n = 100 - 10n^2$;

4) $a_n = \frac{n-2}{3}$; 5) $a_n = \frac{1}{n}$; 6) $a_n = -n^3$.

**351.** (Og'zaki.) Sonli ketma-ketlik $x_n = n^2$ formula bilan berilgan. Ketma-ketlikning 100; 144; 225 ga teng bo'lgan hadlarining nomeri qanday? 48, 49, 169 sonlari shu ketma-ketlikning hadlari bo'ladimi?

**352.** Ketma-ketlik $a_n = n^2 - 2n - 6$ formula bilan berilgan.

1) −3; 2) 2; 3) 3; 4) 9

sonlari ketma-ketlikning hadlari bo'ladimi?

**353.** 1) $a_{n-1} = 3a_n + 1$; 2) $a_{n+1} = 5 - 2a_n$

rekurrent formula va $a_1 = 2$ shart bilan berilgan ketma-ketlikning dastlabki to'rtta hadini toping.

**354.** Sonli ketma-ketlik $n$-hadining formulasi $a_n = (n-1)(n+4)$ bilan berilgan. Agarda

1) $a_n = 150$; 2) $a_n = 104$ bo'lsa, $n$ ni toping.

**355.** $a_{n+1} = \sqrt{a_n}$ rekurrent formula va $a_1 = 256$ shart bilan berilgan ketma-ketlikning birinchi to'rtta hadini hisoblang.

**356.** $a_1 = 1$ shart va

1) $a_{n-1} = \sqrt{a_n^2 + 3}$; 2) $a_{n-1} = \sqrt{\frac{a_n^2}{3}}$

rekurrent formula bilan berilgan ketma-ketlikning dastlabki oltita hadini yozing.

**357.** Sonli ketma-ketlik $a_{n+2} - a_n^2 - a_{n+1}$ rekurrent formula va $a_1 - 2$, $a_2 = 3$, shart bilan berilgan. Ketma-ketlikning beshinchi hadini hisoblang.

**358.** $n$- hadining formulasi bilan berilgan sonli ketma-ketlikning, $(n+1)$-, $(n+2)$- va $(n+5)$- hadlarini yozing:

1) $a_n = -5n+4$; | 2) $a_n = 2(n-10)$; | 3) $a_n = 2\cdot 3^{n+1}$; | 4) $a_n = 7\cdot\left(\frac{1}{2}\right)^{n+2}$.

## 29- §. ARIFMETIK PROGRESSIYA

Quyidagi masalani ko'raylik.

**Masala.** O'quvchi sinovdan o'tish uchun tayyorgarlik ko'rib, har kuni 5 tadan sinov masalalarini yechishni rejalashtirdi. Har bir kun yechilishi rejalashtirilgan sinov masalalarining soni qanday o'zgarib boradi?

Rejalashtirilgan masalalar soni har bir kunga kelib quyidagicha o'zgarib boradi:

| 1- kun | 2- kun | 3- kun | 4- kun... |
|---|---|---|---|
| 5 ta | 10 ta | 15 ta | 20 ta ... |

Natijada quyidagi ketma-ketlikni hosil qilamiz:

$$5,\ 10,\ 15,\ 20,\ 25,\ \ldots .$$

$a_n$ orqali $n$- kunga kelib yechilishi lozim bo'lgan barcha masalalar sonini belgilaylik. Masalan:

$$a_1 = 5,\ a_2 = 10,\ a_3 = 15,\ \ldots .$$

Hosil qilingan

$$a_1,\ a_2,\ a_3,\ \ldots, a_n, \ldots$$

sonlar *sonli ketma-ketlik* deyiladi.

Bu ketma-ketlikda ikkinchisidan boshlab uning har bir hadi oldingi hadga ayni bir xil 5 sonini qo'shish natijasiga teng. Bunday ketma-ketlik *arifmetik progressiya* deyiladi.

**Ta'rif.** ***Agar $a_1, a_2, \dots, a_n, \dots$ sonli ketma-ketlikda barcha natural n lar uchun***

$$a_{n+1} = a_n + d$$

***(bunda d – biror son) tenglik bajarilsa, bunday ketma-ketlik arifmetik progressiya deyiladi.***

Bu formuladan $a_{n+1} - a_n = d$ ekanligi kelib chiqadi. $d$ son *arifmetik progressiyaning ayirmasi* deyiladi. Masalan,

1) Sonlarning $1, 2, 3, 4 \dots, n, \dots$ natural qatori arifmetik progressiyani tashkil qiladi. Bu progressiyaning ayirmasi $d = 1$.

2) Butun manfiy sonlarning $-1, -2, -3, \dots, -n, \dots$ ketma-ketligi ayirmasi $d = -1$ bo'lgan arifmetik progressiyadir.

3) $3, 3, 3, \dots, 3, \dots$ ketma-ketlik ayirmasi $d = 0$ bo'lgan arifmetik progressiyadan iborat.

**1-masala.** $a_n = 1{,}5 + 3n$ formula bilan berilgan ketma-ketlik arifmetik progressiya bo'lishini isbotlang.

△ $a_{n+1} - a_n$ ayirma barcha $n$ uchun ayni bir xil ($n$ ga bog'liq emas) ekanligini ko'rsatish talab qilinadi.

Berilgan ketma-ketlikning $(n + 1)$- hadini yozamiz:

$$a_{n+1} = 1{,}5 + 3(n + 1).$$

Shuning uchun

$$a_{n+1} - a_n = 1{,}5 + 3(n + 1) - (1{,}5 + 3n) = 3.$$

Demak, $a_{n+1} - a_n$ ayirma $n$ ga bog'liq emas. ▲

Arifmetik progressiyaning ta'rifiga ko'ra $a_{n+1} = a_n + d$, $a_{n-1} = a_n - d$, bundan

$$a_n = \frac{a_{n-1} + a_{n+1}}{2}, n > 1.$$

**Shunday qilib, arifmetik progressiyaning ikkinchi hadidan boshlab har bir hadi unga qo'shni bo'lgan ikkita hadning o'rta arifmetigiga teng. „Arifmetik" progressiya degan nom shu bilan izohlanadi.**

Agar $a_1$ va $d$ berilgan bo'lsa, u holda arifmetik progressiyaning qolgan hadlarini $a_{n+1} - a_n + d$ formula bo'yicha hisoblash mumkinligini ta'kidlaymiz. Bunday usul bilan progressiyaning bir necha dastlabki hadini hisoblash qiyinchilik tug'dirmaydi; biroq, masalan, $a_{100}$ uchun talaygina hisoblashlar talab qilinadi. Odatda, buning uchun $n$-had formulasidan foydalaniladi.

Arifmetik progressiyaning ta'rifiga ko'ra

$$a_2 = a_1 + d,$$
$$a_3 = a_2 + d = a_1 + 2d,$$
$$a_4 = a_3 + d = a_1 + 3d \text{ va h.k.}$$

Umuman,

$$a_n = a_1 + (n - 1)d, \qquad (1)$$

chunki arifmetik progressiyaning $n$-hadi uning birinchi hadiga $d$ sonini $(n - 1)$ marta qo'shish natijasida hosil qilinadi.

(1) formula *arifmetik progressiyaning n-hadi formulasi* deyiladi.

**2-masala.** Agar $a_1 = -6$ va $d = 4$ bo'lsa, arifmetik progressiyaning yuzinchi hadini toping.

△ (1) formula bo'yicha: $a_{100} = -6 + (100 - 1) \cdot 4 = 390$. ▲

**3-masala.** 99 soni 3, 5, 7, 9, ... arifmetik progressiyaning hadi. Shu hadning nomerini toping.

△ Aytaylik, $n$ – izlangan nomer bo'lsin. $a_1 = 3$ va $d = 2$ bo'lgani uchun $a_n = a_1 + (n - 1)d$ formulaga ko'ra: $99 = 3 + (n - 1) \cdot 2$. Shuning uchun $99 = 3 + 2n - 2$; $98 = 2n$, $n = 49$.

**Javob:** $n = 49$. ▲

**4-masala.** Arifmetik progressiyada $a_8 = 130$ va $a_{12} = 166$. $n$-hadining formulasini toping.

△ (1) formuladan foydalanib, topamiz:

$$a_8 = a_1 + 7d, \quad a_{12} = a_1 + 11d.$$

$a_8$ va $a_{12}$ larning berilgan qiymatlarini qo'yib, $a_1$ va $d$ ga nisbatan tenglamalar sistemasini hosil qilamiz:

$$\begin{cases} a_1 + 7d = 130, \\ a_1 + 11d = 166. \end{cases}$$

Ikkinchi tenglamadan birinchi tenglamani ayirib, hosil qilamiz:

$$4d = 36, d = 9.$$

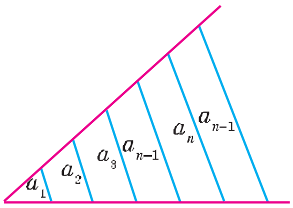
Demak, $a_1 = 130 - 7d = 130 - 63 = 67$.

Progressiya $n$-hadi formulasini yozamiz:

$$a_n = 67 + 9(n - 1) = 67 + 9n - 9 = 58 + 9n.$$

**J a v o b:** $a_n = 9n + 58$. ▲

**5-masala.** Burchakning bir tomonida uning uchidan boshlab teng kesmalar ajratiladi. Ularning oxirlaridan parallel to‘g‘ri chiziqlar o‘tkaziladi (80- rasm). Shu to‘g‘ri chiziqlarning burchak tomonlari orasidagi $a_1, a_2, a_3, \ldots$ kesmalarining uzunliklari arifmetik progressiya tashkil qilishini isbotlang.




*80- rasm.*

△ Asoslari $a_{n-1}$ va $a_{n+1}$ bo‘lgan trapetsiyada uning o‘rta chizig‘i $a_n$ ga teng. Shuning uchun

$$a_n = \frac{a_{n-1}+a_{n+1}}{2}.$$

Bundan $2a_n = a_{n-1} + a_{n-1}$ yoki

$$a_{n-1} - a_n = a_n - a_{n-1}.$$

Ketma-ketlikning har bir hadi bilan undan oldingi hadi ayirmasi ayni bir xil son bo‘lgani uchun bu ketma-ketlik arifmetik progressiya bo‘ladi. ▲

## *Mashqlar*

**359.** (Og‘zaki.) Arifmetik progressiyaning birinchi hadini va ayirmasini ayting:

1) 6, 8, 10, ...; 2) 7, 9, 11,...;
3) 25, 21, 17,...; 4) −12,−9,−6,....

**360.** Agar:

1) $a_1=2$ va $d=5$; 2) $a_1=-3$ va $d=2$; 3) $a_1=4$ va $d=-1$ bo‘lsa, arifmetik progressiyaning dastlabki beshta hadini yozing.

**361.** $n$-hadining formulasi bilan berilgan quyidagi ketma-ketlik arifmetik progressiya bo‘lishini isbotlang:

1) $a_n = 3 - 4n$; 2) $a_n = -5 + 2n$; 3) $a_n = 3(n + 1)$;
4) $a_n = 2(3 - n)$; 5) $a_n = 3 - 5n$; 6) $a_n = -7 + 3n$.

**362.** Arifmetik progressiyada:

1) agar $a_1 = 2$, $d = 3$ bo‘lsa, $a_{15}$ ni toping;

2) agar $a_1 = 3$, $d = 4$ bo‘lsa, $a_{20}$ ni toping;
3) agar $a_1 = -3$, $d = -2$ bo‘lsa, $a_{18}$ ni toping;
4) agar $a_1 = -2$, $d = -4$ bo‘lsa, $a_{11}$ ni toping.

**363.** Arifmetik progressiyaning $n$- hadi formulasini yozing:

1) 1, 6, 11, 16, ...; 2) 25, 21, 17,13,...;
3) −4, −6,−8, −10,...; 4) 1, −4, −9,−14,... .

**364.** −22 soni 44,38,32,... arifmetik progressiyaning hadi. Shu sonning nomerini toping.

**365.** 12 soni −18,−15,−12,... arifmetik progressiyaning hadi bo‘ladimi?

**366.** −59 soni 1,−5 ... arifmetik progressiyaning hadi. Uning nomerini toping. −46 soni shu progressiyaning hadi bo‘ladimi?

**367.** Agar arifmetik progressiyada:

1) $a_1 = 7, a_{16} = 67$; 2) $a_1 = -4, a_9 = 0$; 3) $a_2 = 8$, $a_{10} = 64$ bo‘lsa, uning ayirmasini toping.

**368.** Arifmetik progressiyaning ayirmasi 1,5 ga teng. Agar:
1) $a_9 = 12$; 2) $a_7 = -4$; 3) $a_{16} = 32{,}5$ bo‘lsa, $a_1$ ni toping.

**369.** Agar arifmetik progressiyada:
1) $d - -3$, $a_{11} - 20$; 2) $a_{21} - -10$, $a_{22} - -5{,}5$;
3) $a_3 = -1$, $a_9 = 17$ bo‘lsa, uning birinchi hadini toping.

**370.** Agar arifmetik progressiyada:
1) $a_3 = 13$, $a_6 = 22$; 2) $a_2 = -7$, $a_7 = 18$;
3) $a_7 = 11$, $a_{13} = 29$ bo‘lsa, uning $n$- hadi formulasini toping.

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**371.** $n$ ning qanday qiymatlarida 15,13,11,... arifmetik progressiyaning hadlari manfiy bo‘ladi?

**372.** Arifmetik progressiyada $a_1 = -10, d = 0{,}5$ bo‘lsa, $n$ ning qanday qiymatlarida $a_n < 2$ tengsizlik bajariladi?

**373.** Agar arifmetik progressiyada:

1) $a_8 - 126$, $a_{10} - 146$; 2) $a_8 - -64$, $a_{10} - -50$;

3) $a_8 = -7$, $a_{10} = 3$; 4) $a_8 = 0{,}5$, $a_{10} = -2{,}5$
bo‘lsa, uning to‘qqizinchi hadini va ayirmasini toping.

## 30-§. ARIFMETIK PROGRESSIYA DASTLABKI *n* TA HADINING YIG'INDISI

**1-masala.** 1 dan 100 gacha bo'lgan barcha natural sonlar yig'indisini toping.

△ Bu yig'indini ikki usul bilan yozamiz:

$$S = 1 + 2 + 3 + \ldots + 99 + 100,$$
$$S = 100 + 99 + 98 + \ldots + 2 + 1.$$

Bu tengliklarni hadlab qo'shamiz:

$$2S = \underbrace{101 + 101 + 101 + \ldots + 101 + 101}_{100 \text{ ta qo'shiluvchi}}$$

Shuning uchun $2S = 101 \cdot 100$, bundan $S = 101 \cdot 50 = 5050$. ▲

Endi ixtiyoriy

$$a_1, a_2, \ldots, a_n, \ldots$$

arifmetik progressiyani qaraymiz. $S_n$ – shu progressiya dastlabki $n$ ta hadining yig'indisi bo'lsin:

$$S_n = a_1 + a_2 + \ldots + a_{n-1} + a_n.$$

**Teorema. Arifmetik progressiya dastlabki $n$ ta hadining yig'indisi quyidagiga teng:**

$$S_n = \frac{a_1 + a_n}{2} n. \qquad \textbf{(1)}$$

○ $S_n$ ni ikki usul bilan yozib olamiz:

$$S_n = a_1 + a_2 + \ldots + a_{n-1} + a_n,$$
$$S_n = a_n + a_{n-1} + \ldots + a_2 + a_1.$$

Arifmetik progressiyaning ta'rifiga ko'ra, bu tengliklarni quyidagicha yozish mumkin:

$$S_n = a_1 + (a_1 + d) + (a_1 + 2d) + \ldots + (a_1 + (n-1)d), \qquad (2)$$
$$S_n = a_n + (a_n - d) + (a_n - 2d) + \ldots + (a_n - (n-1)d). \qquad (3)$$

(2) va (3) tengliklarni hadlab qo'shamiz:

$$2S_n = \underbrace{(a_1 + a_n) + (a_1 + a_n) + \ldots + (a_1 + a_n)}_{n \text{ ta qoshiluvchi}}$$

Demak, $2S_n = (a_1 + a_n)n$, bundan $S_n = \frac{a_1+a_n}{2}n$. ●

**2-masala.** Dastlabki $n$ ta natural son yig‘indisini toping.

△ Natural sonlarning

$$1,\ 2,\ 3,\ 4,\ 5,\ 6,\ \ldots,\ n,\ \ldots$$

ketma-ketligi ayirmasi $d = 1$ bo‘lgan arifmetik progressiyadir. $a_1 = 1$ va $a_n = n$ bo‘lgani uchun (1) formula bo‘yicha topamiz:

$$S_n = 1 + 2 + 3 + \ldots + n = \frac{1+n}{2} \cdot n.$$

Shunday qilib,

$$1+2+3+\ldots+n = \frac{n(n+1)}{2}. \ ▲$$

**3-masala.** Agar $38 + 35 + 32 + \ldots + (-7)$ yig‘indining qo‘shiluvchilari arifmetik progressiyaning ketma-ket hadlari bo‘lsa, shu yig‘indini toping.

△ Shartga ko‘ra, $a_1=38$, $d=-3$, $a_n=-7$. Endi $a_n=a_1+(n-1)d$ formulani qo‘llab, $-7=38+(n-1)(-3)$ ni hosil qilamiz, bundan $n=16$.

$S_n = \frac{a_1+a_n}{2}n$ formula bo‘yicha topamiz:

$$S_{16} = \frac{38-7}{2} \cdot 16 = 248. \ ▲$$

**4-masala.** Yig‘indi 153 ga teng bo‘lishi uchun 1 dan boshlab nechta ketma-ket natural sonlarni qo‘shish kerak?

△ Sonlarning natural qatori – ayirmasi $d = 1$ bo‘lgan arifmetik progressiya. Shartga ko‘ra $a_1 = 1, S_n = 153$. Dastlabki $n$ ta had yig‘indisi formulasini quyidagicha o‘zgartiramiz:

$$S_n = \frac{a_1+a_n}{2} \cdot n = \frac{a_1+a_1+(n-1)d}{2} \cdot n = \frac{2a_1+(n-1)d}{2} \cdot n.$$

Berilganlardan foydalanib, noma’lum $n$ ga nisbatan tenglama hosil qilamiz:

$$153 = \frac{2 \cdot 1+(n-1) \cdot 1}{2} \cdot n,$$

bundan

$$306 = 2n + (n - 1)n,\ n^2 + n - 306 = 0.$$

Bu tenglamani yechib, topamiz:

$$n_{1,2} = \frac{-1 \pm \sqrt{1+1224}}{2} = \frac{-1 \pm 35}{2},$$

$$n_1 = -18, n_2 = 17.$$

Qo‘shiluvchilar soni manfiy bo‘lishi mumkin emas, shuning uchun $n = 17$. ▲

## *Mashqlar*

**374.** Agar arifmetik progressiyada:

1) $a_1 = 1,\ a_n = 20,\ n = 50$; 3) $a_1 = -1,\ a_n = -40,\ n = 20$;
2) $a_1 = 1,\ a_n = 200,\ n = 100$; 4) $a_1 = 2,\ a_n = 100,\ n = 50$

bo‘lsa, uning dastlabki $n$ ta hadining yig‘indisini toping.

**375.** 2 dan 98 gacha bo‘lgan barcha natural sonlar yig‘indisini toping (98 ham yig‘indiga kiradi).

**376.** 1 dan 133 gacha bo‘lgan barcha toq sonlarning yig‘indisini toping (133 ham yig‘indiga kiradi).

**377.** Agar arifmetik progressiyada:

1) $a_1 = -5,\ d = 0,5$; 2) $a_1 - \frac{1}{2},\ d = -3$; 3) $a_1 = 36,\ d = -2,5$

bo‘lsa, uning dastlabki o‘n ikkita hadi yig‘indisini toping.

**378.** 1) agar $n = 11$ bo‘lsa, 9; 13; 17; ...;
2) agar $n = 12$ bo‘lsa, −16; −10; −4; ...

arifmetik progressiyaning dastlabki $n$ ta hadi yig‘indisini toping.

**379.** Agar:

1) $3 + 6 + 9 + \ldots + 273$; 2) $90 + 80 + 70 + \ldots + (-60)$

yig‘indining qo‘shiluvchilari arifmetik progressiyaning ketma-ket hadlari bo‘lsa, shu yig‘indini toping.

**380.** Barcha ikki xonali, barcha uch xonali sonlar yig‘indisini toping.

**381.** Arifmetik progressiya $n$-hadining formulasi bilan berilgan. Agar:

1) $a_n = 3n + 5$; 2) $a_n = 7 + 2n$ bo'lsa, $S_{50}$ ni toping.

**382.** Yig'indi 75 ga teng bo'lishi uchun 3 dan boshlab nechta ketma-ket natural sonni qo'shish kerak?

**383.** Agar arifmetik progressiyada:

1) $a_1 = 10, n = 14, S_{14} = 1050$; 2) $a_1 = 2\frac{1}{3}, n = 10, S_{10} = 90\frac{5}{6}$

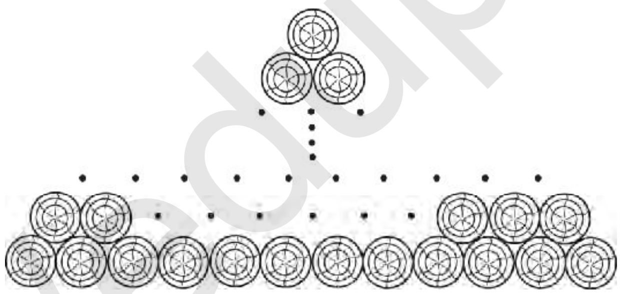
bo'lsa, $a_n$ va $d$ ni toping.

**384.** Agar arifmetik progressiyada:

1) $a_7 = 21$, $S_7 = 205$; | 2) $a_{11} = 92$, $S_{11} = 22$; | 3) $a_{20} = 65$, $S_{20} = 350$

bo'lsa, $a_1$ va $d$ ni toping.

**385.** Binobop to'sinlarni saqlashda ularni 81-rasmda ko'rsatilgandek taxlaydilar. Agar taxlamning asosida 12 ta to'sin turgan bo'lsa, bir taxlamda nechta to'sin bo'ladi?



*81- rasm.*

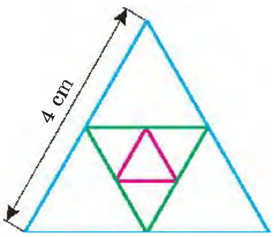
**386.** Arifmetik progressiyada $a_3 + a_9 = 8$. $S_{11}$ ni toping.

**387.** Agar arifmetik progressiyada $S_5 = 65$ va $S_{10} = 230$ bo'lsa, uning birinchi hadini va ayirmasini toping.

**388.** Arifmetik progressiya uchun $S_{12} = 3(S_8 - S_4)$ tenglik bajarilishini isbotlang.

## 31-§. GEOMETRIK PROGRESSIYA

Tomoni 4 cm bo‘lgan teng tomonli muntazam uchburchakni qaraymiz. Uchlari berilgan uchburchak tomonlarining o‘rtalaridan iborat bo‘lgan uchburchak yasaymiz (82-rasm). Uchburchak o‘rta chizig‘ining xossasiga ko‘ra ikkinchi uchburchakning tomoni 2 cm ga teng. Shunga o‘xshash yasashlarni davom ettirib, tomonlari 1, $\frac{1}{2}$, $\frac{1}{4}$ cm va hokazo bo‘lgan uchburchaklarni hosil qilamiz. Shu uchburchaklar tomonlarining uzunliklari ketma-ketligini yozamiz:



*82- rasm.*

$$4,\ 2,\ 1,\ \frac{1}{2},\ \frac{1}{4},\ \frac{1}{8},\ldots .$$

Bu ketma-ketlikda, ikkinchisidan boshlab, uning har bir hadi avvalgi hadni ayni bir xil $\frac{1}{2}$ songa ko‘paytirilganiga teng. Bunday ketma-ketliklar *geometrik progressiyalar* deyiladi.

**T a’ r i f.** *Agar*

$$b_1,\ b_2,\ b_3,\ \ldots,\ b_n,\ \ldots$$

*sonli ketma-ketlikda barcha natural n uchun*

$$b_{n+1} = b_n q$$

*tenglik bajarilsa, bunday ketma-ketlik geometrik progressiya deyiladi, bunda $b_n \neq 0$, q – nolga teng bo‘lmagan biror son.*

Bu formuladan $\frac{b_{n+1}}{b_n} = q$ ekanligi kelib chiqadi. $q$ son *geometrik progressiyaning maxraji* deyiladi.

**M i s o l l a r.**

1) 2,8,32,128,... – maxraji $q = 4$ bo‘lgan geometrik progressiya;

2) 1, $\frac{2}{3}$, $\frac{4}{9}$, $\frac{8}{27}$,... – maxraji $q = \frac{2}{3}$ bo‘lgan geometrik progressiya;

3) $-\frac{1}{12}, 1, -12, 144, \ldots$ – maxraji $q = -12$ bo'lgan geometrik progressiya;

4) 7, 7, 7, 7, ... – maxraji $q = 1$ bo'lgan geometrik progressiya.

**1- masala.** $b_n = 7^{2n}$ formula bilan berilgan ketma-ketlik geometrik progressiya bo'lishini isbotlang.

△ Barcha $n$ larda $b_n = 7^{2n} \neq 0$ ekanligini ta'kidlab o'tamiz. $\frac{b_{n+1}}{b_n}$ bo'linma barcha $n$ lar uchun $n$ ga bog'liq bo'lmagan ayni bir xil songa tengligini isbotlash talab qilinadi. Haqiqatan ham,

$$\frac{b_{n+1}}{b_n} = \frac{7^{2(n+1)}}{7^{2n}} = \frac{7^{2n+2}}{7^{2n}} = 49,$$

ya'ni $\frac{b_{n+1}}{b_n}$ bo'linma $n$ ga bog'liq emas. ▲

Geometrik progressiya ta'rifiga ko'ra

$$b_{n+1} = b_n q, \quad b_{n-1} = \frac{b_n}{q},$$

bundan

$$b_{n-1}^2 = b_{n-1} b_{n-1}, \ n > 1.$$

**Agar progressiyaning barcha hadlari musbat bo'lsa, u holda $b_n = \sqrt{b_{n-1} b_{n+1}}$ bo'ladi, ya'ni geometrik progressiyaning ikkinchisidan boshlab har bir hadi unga qo'shni bo'lgan ikkita hadning o'rta geometrigiga teng. „Geometrik" progressiya degan nom shu bilan izohlanadi.**

Agar $b_1$ va $q$ berilgan bo'lsa, u holda geometrik progressiyaning qolgan hadlarini $b_{n-1} = b_n q$ rekurrent formula bo'yicha hisoblash mumkinligini ta'kidlaymiz. Biroq, $n$ katta bo'lganda bu ko'p mehnat talab qiladi. Odatda, $n$-hadning formulasidan foydalaniladi.

Geometrik progressiyaning ta'rifiga ko'ra

$$b_2 = b_1 q,$$
$$b_3 = b_2 q = b_1 q^2,$$
$$b_4 = b_3 q = b_1 q^3 \text{ va h.k.}$$

Umuman,

$$b_n = b_1 q^{n-1}, \quad (1)$$

chunki geometrik progressiyaning $n$- hadi uning birinchi hadini $q$ songa $(n-1)$ marta ko'paytirish bilan hosil qilinadi.

(1) formula geometrik progressiya *n-hadi formulasi* deyiladi.

**2-masala.** Agar $b_1 = 81$ va $q = \frac{1}{3}$ bo'lsa, geometrik progressiyaning yettinchi hadini toping.

△ (1) formulaga ko'ra:

$$b_7 = 81 \cdot \left(\frac{1}{3}\right)^{7-1} = \frac{81}{3^6} = \frac{1}{9}. \ ▲$$

**3-masala.** 486 soni $2, 6, 18, \ldots$ geometrik progressiyaning hadi. Shu hadning nomerini toping.

△ Aytaylik, $n$ – izlangan nomer bo'lsin. $b_1 = 2, q = 3$ bo'lgani uchun $b_n = b_1 q^{n-1}$ formulaga ko'ra:

$$486 = 2 \cdot 3^{n-1},\ 243 = 3^{n-1},\ 3^5 = 3^{n-1},$$

bundan $n - 1 = 5$, $n = 6$. ▲

**4-masala.** Geometrik progressiyada $b_6 = 96$ va $b_8 = 384$. $n$- hadining formulasini toping.

△ $b_n = b_1 q^{n-1}$ formulaga ko'ra: $b_6 = b_1 q^5$, $b_8 = b_1 q^7$. $b_6$ va $b_8$ ning berilgan qiymatlarini qo'yib, hosil qilamiz: $96 = b_1 q^5, 384 = b_1 q^7$. Bu tengliklardan ikkinchisini birinchisiga bo'lamiz:

$$\frac{384}{96} = \frac{b_1 q^7}{b_1 q^5},$$

bundan $4 = q^2$ yoki $q^2 = 4$. Oxirgi tenglikdan $q = 2$ yoki $q = -2$ ekanini topamiz.

Progressiyaning birinchi hadini topish uchun $96 = b_1 q^5$ tenglikdan foydalanamiz:

1) $q = 2$ bo'lsin. U holda $96 = b_1 \cdot 2^5$, $96 = b_1 \cdot 32$, $b_1 = 3$.

Demak, $b_1 = 3$ va $q = 2$ bo'lganda $n$- hadning formulasi

$$b_n = 3 \cdot 2^{n-1}$$

bo'ladi.

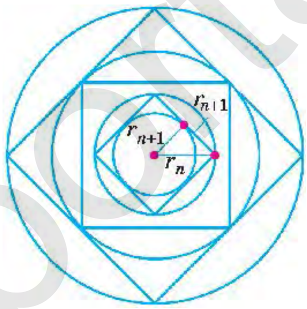
2) $q=-2$ bo‘lsin. U holda $96=b_1(-2)^5$, $96 = b_1(-32)$, $b_1= -3$. Demak, $b_1 = -3$ va $q = -2$ bo‘lganda, $n$-hadning formulasi

$$b_n = -3 \cdot (-2)^{n-1}$$

bo‘ladi.

**J a v o b :** $b_n=3\cdot 2^{n-1}$ yoki $b_n=-3\cdot(-2)^{n-1}$. ▲

**5- masala.** Aylanaga kvadrat ichki chizilgan, unga esa ikkinchi aylana ichki chizilgan. Ikkinchi aylanaga ikkinchi kvadrat ichki chizilgan, unga esa uchinchi aylana ichki chizilgan va hokazo (83- rasm). Aylanalarning radiuslari geometrik progressiya tashkil qilishini isbotlang.




*83- rasm.*

△ $n$- aylananing radiusi $r_n$ bo‘lsin. U holda Pifagor teoremasiga ko‘ra

$$r_{n+1}^2 + r_{n+1}^2 = r_n^2,$$

bundan,

$$r_{n-1}^2 = \frac{1}{2} r_n^2, \text{ ya'ni } r_{n-1} = \frac{1}{\sqrt{2}} r_n.$$

Demak, aylanalar radiuslarining ketma-ketligi maxraji $\frac{1}{\sqrt{2}}$ bo‘lgan geometrik progressiya tashkil qiladi. ▲

## *M a s h q l a r*

**389.** (Og‘zaki.) Ushbu geometrik progressiyaning birinchi hadi va maxraji nimaga teng:

1) 8, 16, 32, ... ; 2) −10, 20, −40, ... ;
3) 4, 2, 1, ... ; 4) −50, 10, −2, ... ?

**390.** Agar geometrik progressiyada:

1) $b_1 = 12$, $q = 2$; 2) $b_1 = -3$, $q = -4$; 3) $b_1 = 16$, $q = -2$

bo‘lsa, uning dastlabki beshta hadini yozing.

**391.** $n$-hadining formulasi bilan berilgan quyidagi ketma-ketlik geometrik progressiya bo‘lishini isbotlang:

1) $b_n = 3 \cdot 2^n$; 2) $b_n = 5^{n-3}$; 3) $b_n = (\frac{1}{3})^{n-2}$; 4) $b_n = \frac{1}{5^{n-1}}$.

**392.** Geometrik progressiyada:

1) $b_1 - 3$ va $q - 10$ bo‘lsa, $b_4$ ni;

2) $b_1 = 4$ va $q = \frac{1}{2}$ bo‘lsa, $b_7$ ni;

3) $b_1 = 1$ va $q = -2$ bo‘lsa, $b_5$ ni;

4) $b_1 = -3$ va $q = -\frac{1}{3}$ bo‘lsa, $b_6$ ni hisoblang.

**393.** Geometrik progressiya $n$-hadining formulasini yozing:

1) $4, 12, 36, \ldots;$ | 2) $3, 1, \frac{1}{3}, \ldots;$ | 3) $4, -1, \frac{1}{4}, \ldots;$

4) $3, -4, \frac{16}{3}, \ldots;$ | 5) $16, 8, 4, 2, \ldots;$ | 6) $-9, 3, -1, \frac{1}{3}, \ldots.$

**394.** Geometrik progressiyada tagiga chizilgan hadning nomerini toping:

1) $6, 12, 24, \ldots, \underline{192}, \ldots;$ 2) $4, 12, 36, \ldots, \underline{324}, \ldots;$

3) $625, 125, 25, \ldots, \underline{\frac{1}{25}};$ 4) $-1, 2, -4, \ldots, \underline{128}, \ldots.$

**395.** Agar geometrik progressiyada:

1) $b_1 = 2,\ b_5 = 162;$ 3) $b_1 = -128,\ b_7 = -2;$

2) $b_1 - 3,\ b_4 - 81;$ 4) $b_1 - 250,\ b_4 - -2$

bo‘lsa, uning maxrajini toping.

**396.** $2, 6, 18, \ldots$ geometrik progressiya berilgan.

1) shu progressiyaning sakkizinchi hadini hisoblang;

2) ketma-ketlikning 162 ga teng hadining nomerini toping.

**397.** Agar musbat hadli geometrik progressiyada:

1) $b_8 = \frac{1}{9},\ b_6 = 81;$ 2) $b_6 = 9,\ b_8 = 3;$ 3) $b_6 = 3,\ b_8 = \frac{1}{3}$

bo‘lsa, uning yettinchi hadini va maxrajini toping.

**398.** Agar geometrik progressiyada:

1) $b_4 = 9,\ b_6 = 20;$ | 2) $b_4 = 9,\ b_6 = 4;$ | 3) $b_1 = 320,\ b_6 = 204{,}8$

bo‘lsa, uning beshinchi va birinchi hadlarini toping.

**399.** Omonatchi jamg‘arma bankiga 2009- yilning 4- yanvar kuni 300000 so‘m pul qo‘ydi. Agar jamg‘arma banki yiliga jamg‘armaning 30% i miqdorida daromad bersa, omonatchining puli 2012- yilning 4- yanvariga borib qancha bo‘ladi?

**400.** Tomoni 4 cm bo‘lgan kvadrat berilgan. Uning tomonlarining o‘rtachalari ikkinchi kvadratning uchlari bo‘ladi. Ikkinchi kvadrat tomonlarining o‘rtalari uchinchi kvadratning uchlari bo‘ladi va hokazo. Shu kvadratlar yuzlarining ketma-ketligi geometrik progressiya tashkil qilishini isbotlang. Yettinchi kvadratning yuzini toping.

## 32-§. GEOMETRIK PROGRESSIYA DASTLABKI $n$ TA HADINING YIG‘INDISI

**1-masala.** Ushbu yig‘indini toping:

$$S = 1 + 3 + 3^2 + 3^3 + 3^4 + 3^5. \qquad (1)$$

△ Tenglikning ikkala qismini 3 ga ko‘paytiramiz:

$$3S = 3 + 3^2 + 3^3 + 3^4 + 3^5 + 3^6. \qquad (2)$$

(1) va (2) tengliklarni bunday yozib chiqamiz:

$$S = 1 + (3 + 3^2 + 3^3 + 3^4 + 3^5);$$
$$3S = (3 + 3^2 + 3^3 + 3^4 + 3^5) + 3^6.$$

Qavslarning ichida turgan ifodalar bir xil. Shuning uchun pastdagi tenglikdan yuqoridagi tenglikni ayirib, hosil qilamiz:

$$3S - S = 3^6 - 1, \quad 2S = 3^6 - 1,$$

$$S = \frac{3^6-1}{2} = \frac{729-1}{2} = 364. \ ▲$$

Endi maxraji $q \neq 1$ bo‘lgan ixtiyoriy $b_1$, $b_1q, \ldots, b_1q^n$, ... geometrik progressiyani qaraymiz. $S_n$ – shu progressiyaning dastlabki $n$ ta hadining yig‘indisi bo‘lsin:

$$S_n = b_1 + b_1q + b_1q^2 + \ldots + b_1q^{n-1}. \qquad (3)$$

**Teorema.** ***Maxraji $q \neq 1$ bo‘lgan geometrik progressiyaning dastlabki $n$ ta hadining yig‘indisi quyidagiga teng:***

$$S_n = \frac{b_1(1-q^n)}{1-q}. \qquad (4)$$

○ (3) tenglikning ikkala qismini $q$ ga ko‘paytiramiz:

$$qS_n = b_1q + b_1q^2 + b_1q^3 + ... + b_1q^n. \quad (5)$$

(3) va (5) tengliklarni, ulardagi bir xil qo‘shiluvchilarni ajratib, yozib chiqamiz:

$$S_n = b_1 + (b_1q + b_1q^2 + ... + b_1q^{n-1}),$$

$$qS_n - (b_1q + b_1q^2 + b_1q^3 + ... + b_1q^{n-1}) + b_1q^n.$$

Qavslarning ichida turgan ifodalar teng. Shuning uchun yuqoridagi tenglikdan pastdagisini ayirib, hosil qilamiz:

$$S_n - qS_n = b_1 - b_1q^n.$$

Bundan

$$S_n(1 - q) = b_1(1 - q^n), \quad S_n = \frac{b_1(1-q^n)}{1-q}.$$ ●

Agar $q = 1$ bo‘lsa, u holda

$$S_n = \underbrace{b_1 + b_1 + ... + b_1}_{n \text{ ta qo‘shiluvchi}} = b_1 n, \text{ ya’ni } S_n = b_1 n.$$

**2-masala.** 6, 2, $\frac{2}{3}$,... geometrik progressiya dastlabki beshta hadining yig‘indisini toping.

△ Bu progressiyada $b_1 = 6, q = \frac{1}{3}$. (4) formula bo‘yicha topamiz:

$$S_5 = \frac{6 \cdot \left(1 - \left(\frac{1}{3}\right)^5\right)}{1 - \frac{1}{3}} = \frac{6\left(1 - \frac{1}{243}\right)}{\frac{2}{3}} = \frac{6 \cdot 242 \cdot 3}{2 \cdot 243} = \frac{242}{27}.$$ ▲

**3-masala.** Maxraji $q = \frac{1}{2}$ bo‘lgan geometrik progressiyada dastlabki oltita hadning yig‘indisi 252 ga teng. Shu progressiyaning birinchi hadini toping.

△ (4) formuladan foydalanib, hosil qilamiz:

$$252 = \frac{b_1\left(1 - \frac{1}{2^6}\right)}{1 - \frac{1}{2}}.$$

Bundan $252 = 2b_1\left(1-\frac{1}{64}\right)$, $252 = \frac{b_1 \cdot 63}{32}$, $b_1 = 128$. ▲

**4-masala.** Geometrik progressiya dastlabki $n$ ta hadining yig'indisi $-93$ ga teng. Bu progressiyaning birinchi hadi $-3$ ga, maxraji esa 2 ga teng. $n$ ni toping.

△ (4) formuladan foydalanib, hosil qilamiz:

$$-93 = \frac{-3(1-2^n)}{1-2}.$$

Bundan $-31 = 1 - 2^n$, $2^n = 32$, $2^5 = 2^n$, $n = 5$. ▲

**5-masala.** 5, 15, 45, ..., 1215, ... – geometrik progressiya. $5 + 15 + 45 + ... + 1215$ yig'indini toping.

△ Bu progressiyada $b_1=5$, $q=3$, $b_n=1215$. Dastlabki $n$ ta had yig'indisi formulasini bunday almashtiramiz:

$$S_n = \frac{b_1(1-q^n)}{1-q} = \frac{b_1 - b_1q^{n-1}q}{1-q} = \frac{b_1 - b_nq}{1-q} = \frac{b_nq-b_1}{q-1}.$$

Masalaning shartidan foydalanib, topamiz:

$$S_n = \frac{1215 \cdot 3-5}{3-1} = \frac{3645-5}{2} = 1820. \blacktriangle$$

## *Mashqlar*

**401.** Agar geometrik progressiyada:

1) $b_1 = \frac{1}{2}, q = 2, n = 6$;  2) $b_1 = -2, q = \frac{1}{2}, n = 5$;

3) $b_1 = 1, q = -\frac{1}{3}, n = 4$;  4) $b_1 = -5, q = -\frac{2}{3}, n = 5$

bo'lsa, uning dastlabki $n$ ta hadining yig'indisini toping.

**402.** Geometrik progressiya dastlabki yettita hadining yig'indisini toping:

1) 5, 10, 20, ... ;  2) 2, 6, 18, ... ;  3) $\frac{1}{4}, \frac{1}{2}, 1, 2, ...$ .

**403.** Agar geometrik progressiyada:

1) $q = 2$, $S_7 = 635$ bo'lsa, $b_1$ va $b_7$ ni toping;

2) $q = -2$, $S_8 = 85$ bo'lsa, $b_1$ va $b_8$ ni toping.

**404.** Agar geometrik progressiyada:

1) $S_n = 189, b_1 = 3, q = 2;$

2) $S_n = 635, b_1 = 5, q = 2;$

3) $S_n = 170, b_1 = 256, q = -\frac{1}{2};$

4) $S_n = -99, b_1 = -9, q = -2$

bo‘lsa, uning hadlari soni $n$ ni toping.

**405.** Agar geometrik progressiyada:

1) $b_1 = 7,\ q = 3,\ S_n = 847$ bo‘lsa, $n$ va $b_n$ ni;

2) $b_1 = 8,\ q = 2,\ S_n = 4088$ bo‘lsa, $n$ va $b_n$ ni;

3) $b_1 = 2,\ b_n = 1458,\ S_n = 2186$ bo‘lsa, $n$ va $q$ ni;

4) $b_1 = 1,\ b_n = 2401,\ S_n = 2801$ bo‘lsa, $n$ va $q$ ni toping.

**406.** Agar sonlar yig‘indisining qo‘shiluvchilari geometrik progressiyaning ketma-ket hadlari bo‘lsa, shu yig‘indini toping:

1) $1 + 2 + 4 + \ldots + 128;$ 2) $1 + 3 + 9 + \ldots + 243;$

3) $-1 + 2 - 4 + \ldots + 128;$ 4) $5 - 15 + 45 - \ldots + 405.$

**407.** Agar geometrik progressiyada:

1) $b_2=15,\ b_3=25;$ | 2) $b_2=14,\ b_4=686,$ | 3) $b_2=15,\ b_4=375,$ $q > 0$ bo‘lsa, $b_5$ va $S_4$ ni toping.

**408.** Geometrik progressiya $n$-hadining formulasi bilan berilgan:

1) $b_n - 3\cdot 2^{n-1}$ bo‘lsa, $S_5$ ni toping;

2) $b_n = -2\cdot\left(\frac{1}{2}\right)^n$ bo‘lsa, $S_6$ ni toping.

**409.** Ayniyatni isbotlang:

$(x - 1)(x^{n-1} + x^{n-2} + \ldots + 1) = x^n - 1,$

bunda $n$ daraja ko‘rsatkichi va u 1 dan katta natural son.

**410.** Geometrik progressiyada:

1) $b_3 = 135, S_3 = 195$ bo‘lsa, $b_1$ va $q$ ni toping;

2) $b_1 = 12, S_3 = 372$ bo‘lsa, $q$ va $b_3$ ni toping.

**411.** Geometrik progressiyada:

1) $b_1 = 1$ va $b_3 + b_5 = 90$ bo‘lsa, $q$ ni;
2) $b_2 = 3$ va $b_4 + b_6 = 60$ bo‘lsa, $q$ ni;
3) $b_1 - b_3 = 15$ va $b_2 - b_4 = 30$ bo‘lsa, $S_{10}$ ni;
4) $b_3 - b_1 = 24$ va $b_5 - b_1 = 624$ bo‘lsa, $S_5$ ni toping.

## 33-§. CHEKSIZ KAMAYUVCHI GEOMETRIK PROGRESSIYA

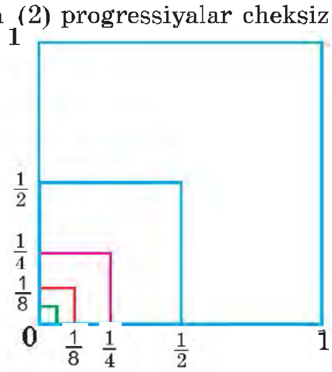
84- rasmda tasvirlangan kvadratlarni qaraymiz. Birinchi kvadratning tomoni 1 ga teng, ikkinchisiniki $\frac{1}{2}$ ga, uchinchisiniki esa $\frac{1}{2^2}$ ga teng va hokazo. Shunday qilib, kvadratning tomonlari maxraji $\frac{1}{2}$ bo‘lgan quyidagi geometrik progressiyani tashkil qiladi:

$$1,\ \frac{1}{2},\ \frac{1}{2^2},\ \frac{1}{2^3},\ \ldots,\ \frac{1}{2^{n-1}},\ \ldots . \qquad (1)$$

Bu kvadratlarning yuzlari esa maxraji $\frac{1}{4}$ bo‘lgan ushbu geometrik progressiyani tashkil qiladi:

$$1,\ \frac{1}{4},\ \frac{1}{4^2},\ \frac{1}{4^3},\ \ldots,\ \frac{1}{4^{n-1}},\ \ldots . \qquad (2)$$

84- rasmdan ko‘rinib turibdiki, kvadratlarning tomonlari va ularning yuzlari $n$ nomerning ortishi bilan borgan sari kamayib, nolga yaqinlasha boradi. Shuning uchun (1) va (2) progressiyalar cheksiz kamayuvchi progressiyalar deyiladi. Bu progressiyalarning maxrajlari birdan kichik ekanligini ta’kidlab o‘tamiz.



*84- rasm.*

Endi quyidagi geometrik progressiyani qaraymiz:

$$1,\ -\frac{1}{3},\ \frac{1}{3^2},\ -\frac{1}{3^3},\ \ldots,\ \frac{(-1)^{n-1}}{3^{n-1}},\ \ldots . \qquad (3)$$

Bu progressiyaning maxraji $q = -\frac{1}{3}$, hadlari esa $b_1 - 1$, $b_2 = -\frac{1}{3}$, $b_3 = \frac{1}{9}$, $b_4 = -\frac{1}{27}$ va hokazo.

$n$ nomerning ortishi bilan bu progressiyaning hadlari nolga yaqinlashadi. (3) progressiya ham *cheksiz kamayuvchi progressiya* deyiladi. Uning maxrajining moduli birdan kichik ekanligini ta'kidlab o'tamiz: $|q|<1$.

**Maxrajining moduli birdan kichik bo'lgan geometrik progressiya *cheksiz kamayuvchi* geometrik progressiya deyiladi.**

**1-masala.** $n$-hadining $b_n=\frac{3}{5^n}$ formulasi bilan berilgan geometrik progressiya cheksiz kamayuvchi bo'lishini isbotlang.

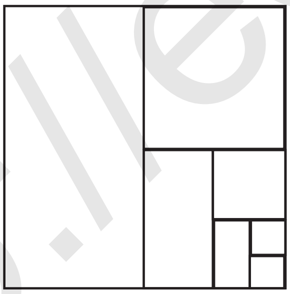
△ Shartga ko'ra $b_1=\frac{3}{5}$, $b_2=\frac{3}{5^2}=\frac{3}{25}$, bundan $q=\frac{b_2}{b_1}=\frac{1}{5}$. $|q|<1$ bo'lgani uchun berilgan geometrik progressiya cheksiz kamayuvchi bo'ladi. ▲

85-rasmda tomoni **1** bo'lgan kvadrat tasvirlangan. Uning yarmini shtrixlaymiz. So'ngra qolgan qismining yarmini shtrixlaymiz va hokazo. Shtrixlangan to'g'ri to'rtburchaklarning yuzlari quyidagi cheksiz kamayuvchi geometrik progressiyani tashkil qiladi:

$$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \dots .$$

Agar shunday yo'l bilan hosil qilingan barcha to'g'ri to'rtburchaklarni shtrixlab chiqsak, u holda butun kvadrat shtrix bilan qoplanadi. Hamma shtrixlangan to'g'ri to'rtburchaklar yuzlarining yig'indisini **1** ga teng deb hisoblash tabiiydir, ya'ni:

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\dots=1.$$



*85-rasm.*

Bu tenglikning chap qismida cheksiz sondagi qo'shiluvchilar yig'indisi turibdi. Dastlabki $n$ ta qo'shiluvchining yig'indisini qaraymiz:

$$S_n=\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\dots+\frac{1}{2^n}.$$

Geometrik progressiya dastlabki $n$ ta hadi yig'indisi formulasiga ko'ra:

$$S_n=\frac{1}{2}\cdot\frac{1-\left(\frac{1}{2}\right)^n}{1-\frac{1}{2}}=1-\frac{1}{2^n}.$$

Agar $n$ cheksiz o'sib borsa, u holda $\frac{1}{2^n}$ nolga istagancha yaqinlasha boradi (nolga intiladi). Bunday hol quyidagicha yoziladi:

$$n \to \infty \text{ da } \frac{1}{2^n} \to 0$$

(o'qilishi: $n$ cheksizlikka intilganda $\frac{1}{2^n}$ nolga intiladi) yoki

$$\lim_{n\to\infty} \frac{1}{2^n} = 0$$

(o'qilishi: $n$ cheksizlikka intilganda $\frac{1}{2^n}$ ketma-ketlikning limiti nolga teng).

Umuman, biror $a_n$ ketma-ketlik uchun $n\to\infty$ da $a_n - a \to 0$ bo'lsa, u holda $a_n$ ketma-ketlik $a$ songa intiladi ($a_n$ ketma-ketlikning $n\to\infty$ dagi limiti $a$ ga teng) deyiladi va bu $\lim\limits_{n\to\infty} a_n = a$ kabi yoziladi.

$n\to\infty$ da $\frac{1}{2^n} \to 0$ bo'lgani uchun $n\to\infty$ da $\left(1 - \frac{1}{2^n}\right) \to 1$, ya'ni $n\to\infty$ da $S_n \to 1$. Shuning uchun $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \ldots$ cheksiz yig'indi 1 ga teng deb hisoblanadi.

Endi ixtiyoriy cheksiz kamayuvchi geometrik progressiyani qaraymiz:

$$b_1, b_1q, b_1q^2, \ldots, b_1q^{n-1}, \ldots,$$

bunda $|q| < 1$.

***Cheksiz kamayuvchi geometrik progressiyaning yig'indisi deb n → ∞ da uning dastlabki n ta hadi yig'indisi intiladigan songa aytiladi.***

$S_n = \frac{b_1(1-q^n)}{1-q}$ formuladan foydalanamiz. Uni bunday yozamiz:

$$\boxed{S_n = \frac{b_1}{1-q} - \frac{b_1}{1-q} q^n.} \qquad (4)$$

Agar $n$ cheksiz o'ssa, $|q|<1$ bo'lgani uchun $q^n \to 0$. Shuning uchun $\frac{b_1}{1-q} \cdot q^n$ ham $n\to\infty$ da nolga intiladi. (4) formulada birinchi qo'shiluvchi $n$ ga bog'liq emas. Demak, $n\to\infty$ da $S_n$ yig'indi $\frac{b_1}{1-q}$ songa intiladi.

**Shunday qilib, cheksiz kamayuvchi geometrik progressiyaning $S$ yig'indisi quyidagiga teng:**

$$S = \frac{b_1}{1-q}. \qquad (5)$$

**Xususiy holda, $b_1 = 1$ bo'lganda, $S = \frac{1}{1-q}$ ni olamiz. Bu tenglik, odatda, ushbu ko'rinishda yoziladi:**

$$1 + q + q^2 + \ldots + q^{n-1} + \ldots = \frac{1}{1-q}.$$

Bu tenglik va (5) tenglik faqat $|q| < 1$ bo'lganda o'rinli bo'lishini ta'kidlab o'tamiz.

**2- masala.** $\frac{1}{2}, -\frac{1}{6}, \frac{1}{18}, -\frac{1}{54}, \ldots$ cheksiz kamayuvchi geometrik progressiya yig'indisini toping.

△ $b_1 = \frac{1}{2}$, $b_2 = -\frac{1}{6}$ bo'lgani uchun $q = \frac{b_2}{b_1} = -\frac{1}{3}$, $S = \frac{b_1}{1-q}$ formula bo'yicha:

$$S = \frac{\frac{1}{2}}{1-\left(-\frac{1}{3}\right)} = \frac{3}{8}. \ ▲$$

**3- masala.** Agar $b_3 = -1, q = \frac{1}{7}$ bo'lsa, cheksiz kamayuvchi geometrik progressiya yig'indisini toping.

△ $n = 3$ bo'lganda $b_n = b_1 q^{n-1}$ formulani qo'llasak, $-1 = b_1 \cdot \left(\frac{1}{7}\right)^{3-1}$, $-1 = b_1 \cdot \frac{1}{49}$ hosil bo'ladi, bundan $b_1 = -49$.

(5) formula bo'yicha $S$ yig'indini topamiz:

$$S = \frac{-49}{1-\frac{1}{7}} = -57\frac{1}{6}. \ ▲$$

**4- masala.** (5) formuladan foydalanib, $a$ – 0,(15) – 0,151515... cheksiz o'nli davriy kasrni oddiy kasr shaklida yozing.

△ Berilgan cheksiz kasr taqribiy qiymatlarining quyidagi ketma-ketligini tuzamiz:

$$a_1 = 0,15 = \frac{15}{100},$$

$$a_2 = 0{,}1515 = \frac{15}{100} + \frac{15}{100^2},$$

$$a_3 = 0{,}151515 = \frac{15}{100} + \frac{15}{100^2} + \frac{15}{100^3}.$$

Taqribiy qiymatlarni bunday yozish berilgan davriy kasrni cheksiz kamayuvchi geometrik progressiya yig‘indisi shaklida tasvirlash mumkinligini ko‘rsatadi:

$$a = \frac{15}{100} + \frac{15}{100^2} + \frac{15}{100^3} + \dots .$$

(5) formulaga ko‘ra:

$$a = \frac{\frac{15}{100}}{1 - \frac{1}{100}} = \frac{15}{99} = \frac{5}{33}. \ \blacktriangle$$

### *Mashqlar*

**412.** Ushbu geometrik progressiya cheksiz kamayuvchi bo‘lishini isbotlang:

1) $1, \frac{1}{2}, \frac{1}{4}, \dots;$  2) $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \dots;$  3) $-81, -27, -9, \dots;$

4) $-16, -8, -4, \dots;$  5) $3, 2, \frac{4}{3}, \frac{8}{9}, \dots;$  6) $8, 6, \frac{9}{2}, \frac{27}{8}, \dots .$

**413.** Agar geometrik progressiyada:

1) $b_1 = 40, \ b_2 = -20;$  2) $b_7 = 12, \ b_{11} = \frac{3}{4};$

3) $b_7 = -30, \ b_6 = 15;$  4) $b_5 = -9, \ b_9 = -\frac{1}{27}$

bo‘lsa, u cheksiz kamayuvchi bo‘ladimi? Shuni aniqlang.

**414.** Cheksiz kamayuvchi geometrik progressiya yig‘indisini toping:

1) $1, \frac{1}{3}, \frac{1}{9}, \dots;$  2) $6, 1, \frac{1}{6}, \dots;$  3) $-25, -5, -1, \dots;$

4) $-7, -1, -\frac{1}{7}, \dots;$  5) $128, 64, 2, \dots;$  6) $-81, -27, -9, \dots .$

**415.** Agar cheksiz kamayuvchi geometrik progressiyada:

1) $q = \frac{1}{2}, b_1 = \frac{1}{8};$  2) $q = -\frac{1}{3}, \ b_1 = 9;$

3) $q - \frac{1}{3}, b_5 - \frac{1}{81};$  4) $q = -\frac{1}{2}, \ b_4 = -\frac{1}{8}$

bo‘lsa, uning yig‘indisini toping.

**416.** $n$- hadining formulasi bilan berilgan quyidagi ketma-ketlik cheksiz kamayuvchi geometrik progressiya bo‘la oladimi?

1) $b_n = 3 \cdot (-2)^n$; 2) $b_n = -3 \cdot 4^n$; 3) $b_n = -2 \cdot \left(-\frac{1}{3}\right)^{n-1}$;

4) $b_n = 5 \cdot \left(-\frac{1}{2}\right)^{n-1}$; 5) $b_n = -2 \cdot (-3)^n$; 6) $b_n = 8 \cdot \left(-\frac{1}{4}\right)^{n-1}$.

**417.** Cheksiz kamayuvchi geometrik progressiya yig‘indisini toping:

1) $12, 4, \frac{4}{3}, \ldots$; 2) $100, -10, 1 \ldots$; 3) $98, 28, 8, \ldots$.

**418.** Agar cheksiz kamayuvchi geometrik progressiyada:

1) $q = \frac{1}{2}, b_5 = \frac{\sqrt{2}}{16}$; 2) $q = \frac{\sqrt{3}}{2}, b_4 = \frac{9}{8}$; 3) $q = \frac{\sqrt{2}}{2}, b_9 = 4$

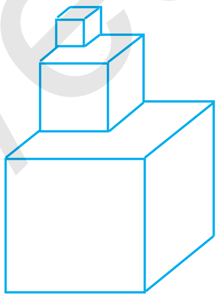
bo‘lsa, uning yig‘indisini toping.

**419.** Cheksiz kamayuvchi geometrik progressiyaning yig‘indisi 150 ga teng. Agar:

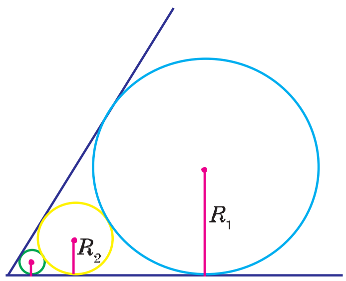
1) $q = \frac{1}{3}$ bo‘lsa, $b_1$ ni; 2) $b_1 = 75$; 3) $b_1 = 15$

bo‘lsa, $q$ ni toping.

**420.** Qirrasi $a$ bo‘lgan kubning ustiga qirrasi $\frac{a}{2}$ bo‘lgan kub qo‘yishdi, uning ustiga qirrasi $\frac{a}{4}$ bo‘lgan kub qo‘yishdi, so‘ngra uning ustiga qirrasi $\frac{a}{8}$ bo‘lgan kub qo‘yishdi va hokazo (86- rasm). Hosil bo‘lgan shaklning balandligini toping.



*86- rasm.*




*87- rasm.*

**421.** 60° li burchakka bir-biriga urinuvchi aylanalar ketma-ket ichki chizilgan (87- rasm). Birinchi aylananing radiusi $R_1$ ga teng. Qolgan aylanalarning $R_2$, $R_3, \ldots, R_n, \ldots$ radiuslarini toping va ular cheksiz kamayuvchi geometrik progressiya tashkil qilishini ko'rsating. $R_1 + 2(R_2 + R_3 + \ldots + R_n + \ldots)$ yig'indi birinchi aylananing markazidan burchakning uchigacha bo'lgan masofaga tengligini isbotlang.

**422.** Cheksiz davriy o'nli kasrni oddiy kasr shaklida yozing:

1) 0,(5); 2) 0,(9); 3) 0,(12); 4) 0,2(3); 5) 0,25(18).

### *IV bobga doir mashqlar*

**423.** Arifmetik progressiyaning ayirmasini toping, uning to'rtinchi va beshinchi hadlarini yozing:

1) $4,\ 4\frac{1}{3},\ 4\frac{2}{3},\ \ldots;$ 2) $3\frac{1}{2},\ 3,\ 2\frac{1}{2},\ \ldots;$

3) $1,\ 1+\sqrt{3},\ 1+2\sqrt{3},\ \ldots;$ 4) $\sqrt{2},\ \sqrt{2}-3,\ \sqrt{2}-6,\ \ldots.$

**424.** $n$- hadi $a_n = -2(1-n)$ formula bilan berilgan ketma-ketlik arifmetik progressiya bo'lishini isbotlang.

**425.** Agar arifmetik progressiyada:

1) $a_1 - 6,\ d = \frac{1}{2}$ bo'lsa, $a_5$ ni; 2) $a_1 = -3\frac{1}{3},\ d = -\frac{1}{3}$ bo'lsa, $a_7$ ni; 3) $a_1 = 4{,}8,\ d = 1{,}2$ bo'lsa, $a_{11}$ ni hisoblang.

**426.** Agar arifmetik progressiyada:

1) $a_1 = -1,\ a_2 = 1;$ 2) $a_1 = 3,\ a_2 = -3;$ 3) $a_3 = -2,\ a_5 = 6$ bo'lsa, uning dastlabki yigirmata hadining yig'indisini toping.

**427.** Agar arifmetik progressiyada:

1) $a_1 = -2,\ a_n = -60,\ n = 10;$ 2) $a_1 = \frac{1}{2},\ a_n = 25\frac{1}{2},\ n = 11$

bo'lsa, uning dastlabki $n$ ta hadining yig'indisini toping.

**428.** Agar:

1) $-38 + (-33) + (-28) + \ldots + 12;$
2) $-17 + (-14) + (-11) + \ldots + 13$

yig'indining qo'shiluvchilari arifmetik progressiyaning ketma-ket hadlari bo'lsa, shu yig'indini toping.

**429.** Geometrik progressiyaning maxrajini toping hamda uning to'rtinchi va beshinchi hadlarini yozing:

1) $3,\ 1,\ \frac{1}{3},\ \ldots;$ 2) $\frac{1}{4},\ -\frac{1}{8},\ \frac{1}{16},\ \ldots;$ 3) $3,\ \sqrt{3},\ 1,\ \ldots;$

4) $5,\ -5\sqrt{2},\ 10,\ \ldots;$ 5) $16, 4, 1, \ldots;$ 6) $8, -4, 2, \ldots$ .

**430.** Geometrik progressiyaning $n$-hadi formulasini yozing:

1) $-2,\ 4,\ -8,\ \ldots;$ 2) $-\frac{1}{2},\ 1,\ -2,\ \ldots;$ 3) $-27,\ -9,\ -3\ldots$ .

**431.** Agar geometrik progressiyada:

1) $b_1 = 2,\ q = 2,\ n = 6;$ 2) $b_1 = \frac{1}{8},\ q = 5,\ n = 4;$

3) $b_1 = -8,\ q - \frac{1}{2},\ n = 5$ bo'lsa, $b_n$ ni toping.

**432.** Agar geometrik progressiyada:

1) $b_1 = \frac{1}{2},\ q = -4,\ n = 5;$ 2) $b_1 = 2,\ q = -\frac{1}{2},\ n = 10;$

3) $b_1 = 10,\ q = 1,\ n = 6;$ 4) $b_1 = 5,\ q = -1,\ n = 9$

bo'lsa, uning dastlabki $n$ ta hadining yig'indisini toping.

**433.** Geometrik progressiyaning dastlabki $n$ ta hadining yig'indisini toping:

1) $128,\ 64,\ 31,\ \ldots,\ n = 6;$ 2) $162,\ 54,\ 18,\ \ldots,\ n = 5;$

3) $\frac{2}{3},\ \frac{1}{2},\ \frac{3}{8},\ \ldots,\ n = 5;$ 4) $\frac{3}{4},\ \frac{1}{2},\ \frac{1}{3},\ \ldots,\ n = 4.$

**434.** Berilgan geometrik progressiya cheksiz kamayuvchi ekanligini isbotlang va uning yig'indisini toping:

1) $-\frac{1}{2},\ -\frac{1}{4},\ -\frac{1}{8},\ \ldots;$ 2) $-1,\ \frac{1}{4},\ -\frac{1}{16},\ \ldots;$ 3) $7,\ 1,\ \frac{1}{7},\ \ldots$ .

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**435.** Agar arifmetik progressiyada $a_1 - 2\frac{1}{2}$ va $a_8 - 23\frac{1}{2}$ bo'lsa, uning ayirmasini toping.

**436.** Agar arifmetik progressiyada:

1) $a_1 = 5,\ a_3 = 15;$ 2) $a_3 = 8,\ a_5 = 2;$ 3) $a_2 = 18,\ a_4 = 14$

bo'lsa, uning dastlabki beshta hadini yozing.

**437.** $-10$ va 5 sonlari orasiga bitta sonni shunday qo'yingki, natijada arifmetik progressiyaning ketma-ket uchta hadi hosil bo'lsin.

**438.** Agar arifmetik progressiyada:

1) $a_{13} = 28,\ a_{20} = 38;$ 2) $a_{18} = -6,\ a_{20} = 6;$ 3) $a_6 = 10,\ a_{11} = 0$

bo'lsa, uning o'n to'qqizinchi va birinchi hadini toping.

### *O'ZINGIZNI TEKSHIRIB KO'RING!*

**1.** Arifmetik progressiyada: 1) $a_1 = 2$, $d = -3$; 2) $a_1 = -7$, $d = 2$ bo'lsa, $a_{10}$ ni va dastlabki o'nta hadning yig'indisini toping.

**2.** Geometrik progressiyada: 1) $b_1 = 4, q = \frac{1}{2}$; 2) $b_1 - \frac{1}{9}$, $q = 3$ bo'lsa, $b_6$ ni va dastlabki oltita hadning yig'indisini toping.

**3.** 1) $1, \frac{1}{3}, \frac{1}{9}, ...$; 2) $128, 32, 8...$, ketma-ketlik cheksiz kamayuvchi geometrik progressiya ekanligini isbotlang va uning barcha hadlari yig'indisini toping.

**439.** $x$ ning qanday qiymatlarida:

1) $3x, \frac{x-2}{2}, 2x - 1$; 2) $3x^2, 2, 11x$; 3) $x^2, 10x, 25$

sonlar arifmetik progressiyaning ketma-ket hadlari bo'ladi?

**440.** Quyidagi sonlar arifmetik progressiyaning ketma-ket uchta hadi bo'lishini ko'rsating:

1) $\sin(\alpha + \beta)$, $\sin\alpha\cos\beta$, $\sin(\alpha - \beta)$;
2) $\cos(\alpha + \beta)$, $\cos\alpha\cos\beta$, $\cos(\alpha - \beta)$;
3) $\cos2\alpha$, $\cos^2\alpha, 1$; 4) $\sin5\alpha$, $\sin3\alpha\cos2\alpha$, $\sin\alpha$.

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**441.** Yig'indi 252 ga teng bo'lishi uchun 5 dan boshlab nechta ketma-ket toq natural sonni qo'shish kerak?

**442.** Agar arifmetik progressiyada:

1) $a_1 - 40$, $n - 20$, $S_{20} - -40$; 2) $a_1 - \frac{1}{3}$, $n - 16$, $S_{16} - -10\frac{2}{3}$;
3) $a_1 = -4, n = 11$, $S_{11} = 231$ bo'lsa, $a_n$ va $d$ ni toping.

**443.** Geometrik progressiyada:

1) agar $b_1 = 4$ va $q = -1$ bo'lsa, $b_9$ ni hisoblang;
2) agar $b_1 = 1$ va $q = \sqrt{3}$ bo'lsa, $b_7$ ni hisoblang.

**444.** Agar geometrik progressiyada:

1) $b_2 = \frac{1}{2}$, $b_7 = 16$; 2) $b_3 = -3$, $b_6 = -81$;
3) $b_2 - 4$, $b_4 - 1$; 4) $b_4 - -\frac{1}{5}$, $b_6 - -\frac{1}{125}$

bo'lsa, uning beshinchi hadini toping.

**445.** 4 va 9 sonlari orasiga bitta musbat sonni shunday qo‘yingki, natijada geometrik progressiyaning ketma-ket uchta hadi hosil bo‘lsin.

**446.** Agar ketma-ketlik $n$- hadining:

1) $b_n = 5^{n+1}$; 2) $b_n = (-4)^{n+2}$; 3) $b_n = \frac{10}{7^n}$; 4) $b_n = -\frac{50}{3^{n+3}}$

formulasi bilan berilgan bo‘lsa, u cheksiz kamayuvchi geometrik progressiya bo‘la oladimi?

**447.** Agar geometrik progressiyada:

1) $b_2 = -81$, $S_2 = 162$; 2) $b_2 = 33$, $S_2 = 67$;
3) $b_1 + b_3 = 130$, $b_1 - b_3 = 120$; 4) $b_2 + b_4 = 68$, $b_2 - b_4 = 60$

bo‘lsa, u cheksiz kamayuvchi ekanligini ko‘rsating.

**448.** Dam oluvchi shifokor tavsiyasiga amal qilib, birinchi kuni Quyosh nurida 5 minut toblandi, keyingi har bir kunda esa toblanishni 5 minutdan oshirib bordi. Agar u toblanishni chorshanba kunidan boshlagan bo‘lsa, haftaning qaysi kuni uning Quyoshda toblanishi 40 minutga teng bo‘ladi?

**449.** Agar arifmetik progressiyada $a_1 + a_2 + a_3 = 15$ va $a_1 \cdot a_2 \cdot a_3 = 80$ bo‘lsa, uning birinchi hadi va ayirmasini toping.

**450.** Agar arifmetik progressiyada $a_1 + a_2 + a_3 = 0$ va $a_1^2 + a_2^2 + a_3^2 = 50$ bo‘lsa, uning birinchi hadi va ayirmasini toping.

**451.** Soat 1 da soat 1 marta, 2 da 2 marta, ..., 12 da 12 marta bong uradi. Soat mili navbatdagi har soatning yarmini ko‘rsatganda esa bir marta bong uradi. Bu soat bir sutkada necha marta bong uradi?

## *IV bobga doir sinov (test) mashqlari*

**1.** Arifmetik progressiyada $a_1 = 3$, $d = -2$. $S_{101}$ ni toping.
A) −9797; B) −9798; C) −7979; D) −2009.

**2.** Arifmetik progressiyada $d = 4$, $S_{50} = 5000$ bo‘lsa, $a_1$ ni toping.
A) −2; B) 2; C) 100; D) 1250.

**3.** Arifmetik progressiyada $a_1 = 1$, $a_{101} = 301$ bo‘lsa, $d$ ni toping.
A) 4; B) 2; C) 3; D) 3,5.

**4.** Arifmetik progressiyada $a_2 + a_9 = 20$ bo‘lsa, $S_{10}$ ni toping.
A) 90; B) 110; C) 200; D) 100.

**5.** 8 ga bo‘lganda 7 qoldiq beradigan ketma-ketlikning 5-hadini belgilang.
A) 47; B) 55; C) 39; D) 63.

**6.** 701 soni 1, 8, 15, 22,... progressiyaning nechanchi nomerli hadi?
A) 101; B) 100; C) 102; D) 99.

**7.** 1002, 999, 996, ... progressiyaning nechanchi nomerli hadidan boshlab, uning hadlari manfiy sonlar bo‘ladi?
A) 335; B) 336; C) 337; D) 334.

**8.** Arifmetik progressiyada $a_2 + a_6 = 44, a_5 - a_1 = 20$ bo‘lsa, $a_{100}$ ni toping.
A) 507; B) 495; C) 502; D) 595.

**9.** Arifmetik progressiyada $a_1=7$, $d=5$, $S_n=25450$ bo‘lsa, $n$ ni toping.
A) 99; B) 101; C) 10; D) 100.

**10.** Arifmetik progressiya $a_{12}+a_{15}=20$ bo‘lsa, $S_{26}$ ni toping.
A) 260; B) 270; C) 520; D) 130.

**11.** 1 va 11 sonlari orasida 99 ta shunday sonni joylashtiringki, ular bu sonlar bilan birgalikda arifmetik progressiya tashkil qilsin. Shu progressiya uchun $S_{50}$ ni toping.
A) $172\frac{1}{2}$; B) 495; C) 300; D) 178.

**12.** Arifmetik progressiyada $a_1=-20{,}7$, $d=1{,}8$ bo‘lsa, qaysi nomerli haddan boshlab progressiyaning barcha hadlari musbat bo‘ladi?
A) 18; B) 13; C) 12; D) 15.

**13.** 7 ga karrali dastlabki nechta natural sonni qo‘shganda 385 hosil bo‘ladi?
A) 12; B) 11; C) 10; D) 55.

**14.** Geometrik progressiyada $b_1 = 2, q = 3$ bo‘lsa, $S_6$ ni toping.
A) 1458; B) 729; C) 364; D) 728.

**15.** Geometrik progressiyada $q - \frac{1}{3}$, $S = 364$ bo‘lsa, $b_1$ ni toping.
A) $242\frac{2}{3}$; B) 81; C) $121\frac{1}{3}$; D) 240.

**16.** Geometrik progressiyada $S_4 = 10\frac{5}{8}$, $S_5 = 42\frac{5}{8}$, $b_1 = \frac{1}{8}$ bo‘lsa, $q$ ni toping:

A) 4; B) 2; C) 8; D) $\frac{1}{2}$.

**17.** Geometrik progressiyada 6 ta had bor. Dastlabki 3 ta hadining yig‘indisi 26 ga, keyingi 3 ta hadining yig‘indisi esa 702 ga teng. Progressiya maxrajini toping.

A) 4; B) 3; C) $\frac{1}{3}$; D) $2\sqrt{3}$.

**18.** Cheksiz kamayuvchi geometrik progressiyada $b_1 = \frac{1}{4}$, $S = 16$ bo‘lsa, $q$ ni toping.

A) $\frac{1}{2}$; B) $\frac{64}{65}$; C) $\frac{63}{64}$; D) $\frac{1}{4}$.

**19.** Geometrik progressiyada $q = \frac{\sqrt{3}}{2}$, $b_1 = 2 - \sqrt{3}$ bo‘lsa, $S$ ni toping.

A) $2+\sqrt{3}$; B) 3; C) $\frac{2\sqrt{3}}{3}$; D) 2.

## Amaliy-tatbiqiy va fanlararo bog‘liq masalalar

**1-masala.** Erkin tushayotgan jism birinchi sekundda 4,9 m, har bir keyingi sekundda esa oldingisiga qaraganda 9,8 m ga ko‘proq tushib boradi. Jism 4410 metr balandlikdan qancha vaqtda yerga tushadi.

△ Masalaning shartiga ko‘ra jism birinchi sekundda $a_1 - 4,9$, ikkinchi sekundda $a_2 - 4,9 + 9,8$, uchinchi sekundda $a_3 - a_2 + 9,8 - a_1 + 2\cdot 9,8$ va hokazo $n$- sekundda $a_n - a_{n-1} + 9,8 - a_1 + (n-1)9,8$ metr pastga tushadi, ya’ni har sekundda tushayotgan masofalar arifmetik progressiyani tashkil qiladi. Demak, jism $n$ sekundda yerga tushadi desak, arifmetik progressiyaning $n$ ta hadining yig‘indisi formulasiga asosan

$$4410 = a_1 + a_2 + a_3 + \ldots + a_n = \frac{2a_1 + (n-1)d}{2} \cdot n =$$
$$= \frac{2 \cdot 4,9 + (n-1)\cdot 9,8}{2} \cdot n.$$

Bundan $4,9n^2 = 4410$, $n^2 = 900$, $n = 30$ ni hosil qilamiz.

**Javob:** Jism 30 sekundda yerga tushadi. ▲

**2- masala.** Omonatchi $b$ so‘m mablag‘ini bankka yiliga $p\%$ dan qo‘ydi va $n$ yil o‘tgandan keyin hamma pulni qaytarib oldi. Agarda $b=4000000$, $p=8$ bo‘lsa, omonatchi ikki yildan keyin qancha pul olgan?

△ Boshlang‘ich qo‘yilgan mablag‘i $b$ so‘m bo‘lsa, bir yildan keyin omonatchining mablag‘i $b_1=b\cdot(1+\frac{p}{100})$ so‘m bo‘ladi. Keyingi yillar uchun quyidagi variantlardan biri bo‘lishi mumkin:

1) keyingi har yili protsent boshlang‘ich mablag‘ $b$ somdan hisoblanadi. Bunda ikkinchi yildan keyin $b_2=b+1+\frac{bp}{100}+\frac{bp}{100}=b\cdot\left(1+\frac{2p}{100}\right)$ so‘m va hokazo $n$- yildan keyin $b_n=b\cdot\left(1+\frac{np}{100}\right)$ so‘m bo‘ladi. Protsentni hisoblashning bu usuli *sodda protsent* deyiladi. Bunda, agar $b=4\,000000$, $p=8$, $n=2$ bo‘lsa, u holda $b_2=4\,000000\cdot 1{,}16=$ $=4\,640000$;

2) Keyingi har yili protsent oldingi yil yig‘ilgan mablag‘dan hisoblanadi. Ikki yildan keyin $b_2=b_1\cdot\left(1+\frac{p}{100}\right)=b\cdot\left(1+\frac{p}{100}\right)^2$ so‘m va hokazo $n$ yildan keyin $b_n=b_{n-1}\cdot\left(1+\frac{p}{100}\right)=b\cdot\left(1+\frac{p}{100}\right)^n$ bo‘ladi. Protsentni hisoblashning bu usuli *murakkab protsent* deyiladi. Bunda

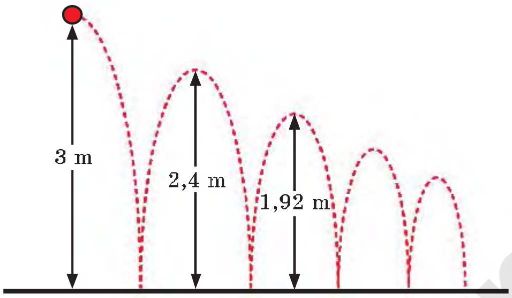
agar $b=4000000$, $=8$, $n=2$ bo‘lsa, u holda $b_2=4\ 000\ 000\cdot 1{,}08^2=$ $=4\ 665\ 600$ so‘m.

**Javob:** sodda protsent holida $b_n=b\cdot\left(1+\frac{np}{100}\right)$ so‘m; 4 640 000,

murakkab protsent holida $b_n=b\cdot\left(1+\frac{p}{100}\right)^n$ so‘m; 4665600 so‘m. ▲

## Masalalar

1. Erkin tushuvchi jism birinchi sekundda 4,9 m yo‘l bosadi, keyingi har bir sekundda esa oldingisidan 9,8 m ortiq yo‘l bosadi. Tushayotgan jism beshinchi sekundda qancha masofani bosib o‘tadi?
2. Sirkning sektorlaridan birida har bir keyingi qatorda oldingisiga qaraganda bittadan o‘rindiq ko‘proq. Agarda
   1) birinchi qatorda 8 ta o‘rindiq, qatorlar esa 22 ta;
   2) birinchi qatorda 10 ta o‘rindiq, qatorlar esa 21 ta bo‘lsa, shu sektorda nechta joy bor?
3. Sayyohlar daryo bo‘ylab 140 km yurishni rejalashtirishdi. Birinchi kun 5 km, har bir keyingi kun bo‘lsa, undan oldingi kunga nisbatan 2 km ko‘proq yurishadigan bo‘lsa, ular sayohatda necha kun bo‘lishadi?
4. Xamirturush hujayralari har bir hujayra ikkiga bo‘linishi orqali ko‘payadi. Agar boshlang‘ich holatda 6 ta hujayra bo‘lsa, 10 marta bo‘linishdan keyin hujayralar soni nechta bo‘ladi?
5. Kalendar yili davomida zavod xodimining oylik ish haqi har oy bir xil miqdorga oshirib borildi. Iyun, iyul, avgustda olgan oylik ish haqlarining umumiy miqdori 9 900 000 so‘m, sentabr, oktabr, noyabr uchun olingan ish haqlarining yig‘indisi esa 10 350 000 so‘m bo‘ldi. Xodimning yil davomida olgan umumiy ish haqini toping.
6. Havo vannasini olish yo‘li bilan davolanishda birinchi kuni davolanish 15 min davom etadi, keyingi har bir kunda uni 10 minutdan oshirib boriladi. Vanna olish ko‘pi bilan 1 soat-u 45 minut davom etishi uchun ko‘rsatilgan tartibda havo vannasini olish necha kun davom etadi?
7. Tashlangan elastik koptok yerga urilib yana tepaga chiqqanida har safar oldingi balandligining 80% iga ko‘tarilsa, u holda 3 metrdan tashlangan koptokning pastga va tepaga bosib o‘tgan umumiy vertikal masofalari yig‘indisini toping (88-rasm).




*88- rasm.*

## *Tarixiy masalalar*

1. *Beruniy masalasi.* Agar hadlari musbat geometrik progressiyaning: hadlari soni toq bo'lsa, u holda $b_{k-1}^2 = b_1 \cdot b_{2k+1}$; hadlari soni juft bo'lsa, $b_k \cdot b_{k-1} = b_1 \cdot b_{2k}$ bo'lishini isbotlang.
2. *Axmes papirusidan olingan masala (eramizdan oldingi 2000- yillar).* 10 o'lchov g'allani 10 kishi orasida shunday taqsimlanginki, bu kishilarning biri bilan undan keyingisi (yoki oldingisi) olgan g'allaning farqi $\frac{1}{8}$ o'lchovga teng bo'lsin.

## *Tarixiy ma'lumotlar*

„Qadimgi xalqlardan qolgan yodgorliklar" asarida Abu Rayhon Beruniy shaxmatning kashf etilishi haqidagi rivoyat bilan bog'liq birinchi hadi $b_1 = 1$ va maxraji $q = 2$ bo'lgan geometrik progressiyaning birinchi 64 ta hadining yig'indisini hisoblaydi; shaxmat taxtasidagi $k$- katakka mos sondan 1 soni ayirilsa, ayirma $k$- katakdan oldingi barcha kataklarga mos sonlar yig'indisiga teng bo'lishini ko'rsatadi, ya'ni

$$q^k - 1 = 1 + q + q^2 + \ldots + q^{k-1}$$

ekanini isbotlaydi.

# V BOB. EHTIMOLLIKLAR NAZARIYASI VA MATEMATIK STATISTIKA ELEMENTLARI

## 34-§ HODISALAR

Ehtimolliklar nazariyasi va matematik statistika tasodifiy hodisalar orasidagi bog'lanishlarni, qonuniyatlarni o'rganish va ulardan kelib chiqadigan xulosalarni amaliyot masalalarini yechishga qo'llashga bag'ishlangan fandir.

**1. Mumkin bo'lmagan, muqarrar va tasodifiy hodisalar.**

Hayotda hodisa deb ro'y beradigan yoki ro'y bermaydigan ixtiyoriy jarayonga aytiladi. Undan tashqari, insonlar tomonidan amalga oshiriladigan tajribalar yoki sinovlar, kuzatuvlar va o'lchash ishlarining natijalari ham hodisalardir. Barcha hodisalarni mumkin bo'lmagan, muqarrar va tasodifiy hodisalarga bo'lish mumkin.

***Mumkin bo'lmagan hodisa*** deb, berilgan sharoitlarda ro'y berishi mumkin bo'lmagan hodisaga aytiladi. Mumkin bo'lmagan hodisalarga misollar keltiraylik:

1) ko'lning suvi $+30°C$ da muzlaydi;

2) tomonlari 1 dan 6 gacha raqamlar bilan belgilangan o'yin kubigi tashlanganda 8 raqamining paydo bo'lishi.

***Muqarrar hodisa*** deb, berilgan sharoitlarda albatta ro'y berishi aniq bo'lgan hodisaga aytiladi. Masalan:1) qishdan keyin bahor keldi; 2) o'yin kubigini tashlaganda oltidan katta bo'lmagan (0 dan farqli) raqam tushdi.

***Tasodifiy hodisa*** deb, berilgan sharoitlarda ro'y berishi ham, ro'y bermasligi ham mumkin bo'lgan hodisaga aytiladi. Quyidagi hodisalar tasodifiy hodisalarga misol bo'la oladi: 1) 1 dan 50 gacha natural sonlar orasidan tasodifan tanlangan son 7 ga bo'linadi; 2) tashlangan tanga gerb tomoni bilan tushdi.

**2. Birgalikda bo'lishi mumkin va birgalikda bo'lmagan hodisalar.**

Berilgan shartlarda bir vaqtda ro'y berishi mumkin bo'lgan ikki hodisa birgalikda bo'lishi mumkin deyiladi, bir vaqtning o'zida ro'y bera olmaydigan hodisalar esa birgalikda bo'lmagan hodisalar deyiladi. Masalan, „quyosh chiqdi" va „kun sovuq" birgalikda bo'lishi mumkin hodisalar, „kun botdi" va „quyosh chiqdi" hodisalari esa birgalikda bo'lmagan hodisalardir. O'yin kubigi bilan bog'liq quyidagi hodisalarni qaraylik: 1) 3 ochko tushdi; 2) 4 ochko tushdi; 3) 3 ochkodan ko'proq tushdi; 4) uchga karrali ochko tushdi. Bu hodisalar ichida quyidagi uch juftlik birgalikda bo'lishi mumkin hodisalardir: 1- va 4- (3 soni uchga karrali bo'lgani uchun); 2- va 3- (4 ochko 3 ochkodan katta bo'lgani uchun); 3- va 4- (masalan, 6 ochko). Quyidagilar esa birgalikda bo'lmagan hodisalardir: 1- va 2- (bir vaqtning o'zida ikkita turli son tushihishi mumkin emas); 1- va 3- (3 ochkodan yuqori, ya'ni 4, 5, 6 ochkolari 3 ochko bilan bir vaqtda tusha olmaydi); 2- va 4- (4 soni 3 ga karrali emas).

**3. Teng imkoniyatli hodisalar.**

Quyidagicha hodisalar guruhlariga misollarni qaraylik:


*Gerb tomoni* *Raqamli tomoni*

*89-rasm.*

1) tangani bir marta tashlaganda „raqamli tomonining tushishi" va „gerbli tomonininig tushishi" (89-rasm);

2) o'yin kubigi bir marta tashlaganda „1 ochkoning tushishi" „2 ochkoning tushishi",..., „6 ochkoning tushishi";

3) bir tomoni ko‘k, qolgan tomonlari qizilga bo‘yalgan kubik tashlanganda „ko‘k tomoni yuqorida bo‘lib tushishi“ va „qizil tomoni yuqorida bo‘lib tushishi“;

4) ichida 10 ta oq va bitta qora shar bo‘lgan qutidan bitta shar olinganida uning „oq shar chiqishi“ va „qora shar chiqishi“.

1- va 2- misollarda hodisalardan birortasining ro‘y berishi uchun hodisalardan birida boshqasiga nisbatan biror-bir ustunlik bor deb bo‘lmaydi (tanga va kubiklar to‘g‘ri bo‘lsa albatta). Bunday hodisalar ***teng imkonyatli hodisalar*** deb ataladi.

3- va 4- misollarda teng imkoniyatli bo‘lmagan hodisalarga misollar ko‘rsatilgan. Haqiqatan ham, bo‘yalgan kubikning 5 ta tomoni qizil, bitta tomoni esa qora va, demak, qizil tomoni tushishi uchun imkoniyatlar qora tomoni tushishiga qaraganda ko‘proq. Shu kabi, oq sharlar chiqishi imkoniyatlari qora shar chiqishi imkoniyatidan ko‘proq.

### *Mashqlar*

Mashqlarda shartlar va bu shartlarda ro‘y berayotgan hodisalar tasvirlangan. Har bir hodisa uchun (og‘zaki) uning mumkin bo‘lmagan yoki muqarrar, yoki tasodifiy ekanligini aniqlang (**452–456**):

**452.** Maktabdagi o‘quvchilardan: 1) ikkitasining ismi bir xil; 2) hammasining bo‘yi bir xil.

**453.** Algebra darsligi tasodifiy ochilib, o‘ng betidagi uchinchi so‘z topildi. Bu so‘z: 1) „ehtimollik“ so‘zi; 2) „!“ belgisidan boshlanadi.

**454.** IX sinf (unda qizlar ham o‘g‘il bolalar ham bor) jurnalidagi ro‘yxatdan tasodifiy bir o‘quvchi tanlab olindi: 1) u qiz bola; 2) tanlangan o‘quvchining yoshi 16 da; 3) tanlangan o‘quvchi 15 oylik; 4) bu o‘quvchining yoshi 3 dan ortiq.

**455.** Bugun Samarqandda barometr normal atmosfera bosimini ko‘rsatmoqda. Bunda: 1) Samarqandda yashovchi ayolning qozonidagi suv $t = 70°C$ da qaynadi; 2) havo harorati $-5°C$ ga tushganda, ko‘lmakdagi suv yaxladi.

**456.** Ikkita o'yin kubigi tashlanmoqda: 1) birinchi kubikda 4 ochko, ikkinchisida esa 6 ochko tushdi; 2) ikkala kubikda tushgan ochkolar yig'indisi 1 ga teng; 3) ikkala kubikda tushgan ochkolar yig'indisi 14 ga teng; 4) ikkala kubikning har birida 5 ochkodan tushdi; 5) ikkala kubikda tushgan ochkolar yig'indisi 12 dan katta emas.

Berilgan hodisalar juftliklarining qaysilari birgalikda bo'lishini, qaysilari esa birgalikda bo'lmasligini ko'rsating(**457–459**):

**457.** Saodat va Shuhrat o'ynagan shashka o'yinida: 1) Saodat yutdi; Shuhrat yutqazdi; 2) Saodat yutqazdi; Shuhrat yutqazdi.

**458.** O'yin kubigi tashlandi. Uning yuqori tomoni: 1) 5 ochkoni; 3 ochkoni; 2) 1 ochkoni; toq ochkoni ko'rsatdi.

**459.** Domino to'plamidan bir domino donasi olindi, unda: 1) sonlaridan biri 4 dan katta, ikkinchisi 6 ga teng; 2) bitta son 5 dan kichik emas, ikkinchisi 5 dan katta emas; 3) sonlardan biri 5, ikkala son yig'indisi 12 ga teng; 4) ikkala son 4 dan katta, sonlarning yig'indisi 9 dan katta emas.

**460.** Quyidagi: 1) „qor yog'yapti"; 2) „osmonda birorta ham bulut yo'q"; 3) „havo harorati +37°C" hodisalaridan mumkin bo'lgan barcha juftliklarni tuzib, ular orasida birgalikda bo'lishi mumkin va birgalikda bo'lishi mumkin bo'lmagan hodisalar juftliklarini aniqlang.

**461.** Quyidagi hodisalar: 1) „bahor keldi"; 2) „dars jadvaliga ko'ra bugun 6 ta dars bo'ladi"; 3) „bugun 1-yanvar"; 4) „Toshkentdagi havo harorati +40°C" dan mumkin bo'lgan barcha juftliklarni tuzib, ular orasida birgalikda bo'lishi mumkin va birgalikda bo'lishi mumkin bo'lmagan hodisalar juftliklarini aniqlang.

**462.** To'rtta gugurt qutichasidan birining ichi bo'sh, qolganlarida gugurt donachalari bor. Tasodifiy ravishda tanlangan qutichalardan biri ochildi. „Gugurt qutichasining ichi bo'sh chiqdi" va „gugurt qutichasining ichi bo'sh emas" hodisalari teng imkoniyatli bo'ladimi?

**463.** O'yin kubigining: 1) 1 ta tomoni; 2) 2 ta tomoni yashilga, qolgan tomonlarilari esa qizilga bo'yaldi. „Yashil tomoni tushdi“ va „qizil tomoni tushdi“ hodisalari teng imkoniyatli bo‘ladimi?

**464.** Birdan oltigacha nomerlangan 6 ta oq, 6 ta qizil, 6 ta ko‘k, 6 ta sariq sharlar bir xaltaga solindi va aralashtirildi. Xaltadan tavakkaliga bitta shar olindi. Quyidagi hodisalar teng imkoniyatli bo‘ladimi: 1) „tanlangan shar oq“ va „tanlangan shar ko‘k“; 2) „tanlangan shar nomeri 5“ va „tanlangan shar nomeri 4“; 3) „tanlangan shar qizil va nomeri 2“ va „tanlangan shar sariq va nomeri 6“; 4) „tanlangan shar qizil“ va „tanlangan shar qizil emas“; 5) „tanlangan shar nomeri 2 dan katta emas“ va „tanlangan shar nomeri 2 dan katta“?

## 35-§ HODISANING EHTIMOLLIGI

Hayotda turli hodisalar bilan to‘qnashganda, ko‘p hollarda bu hodisalar ro‘y berishining ishonchlilik darajasiga baho beramiz. Bunda ba’zi hodisalar haqida „bunday bo‘lishi mumkin emas“ deb aytsak, boshqa bir hodisalar haqida „bu albatta ro‘y beradi“ yoki „bu hodisa ro‘y berishiga ishonch katta“ yoki „bu hodisa ro‘y berishiga ishonch kam“ deb aytamiz. Hodisalar ro‘y berishining ishonchlilik darajasini baholash ehtimollik tushunchasi bilan bog‘liq.

XVII asr fransuz olimlari Blez Paskal (1623 – 1662 ) va Pyer Ferma (1601 – 1665) orasida bir qator matematik masalalar bo‘yicha yozishgan xatlarida birinchi bor ehtimollik bilan bog‘liq masalalarni yechishning ilk bor umumiy yondashuvlari shakllandi. Blez Paskal 1654-yil 28-oktabrda Pyer Fermaga yozgan xatida, jumladan, quyidagicha mulohazalarni yuritadi:

„O‘yinchi kubikni tashlaganda qanday son tushishini bilmaydi. Lekin u 1, 2, 3, 4, 5 va 6 sonlari teng imkoniyatli ravishda tushishini biladi. Bundan tashqari, o‘yinchi tajriba (kubik tashlash) natijasida ko‘rsatilgan sonlardan birortasining tushishi bu muqarrar hodisa ekanligini ham biladi. Agarda biz muqarrar hodisaning ro‘y berish imkoniyatini 1 deb qabul qilsak, u holda shu sonlardan birining, masalan, 6 (xuddi shunday boshqa sonlarning ham) ning chiqishi 6 barobar kichik, ya’ni $\frac{1}{6}$ ga teng bo‘ladi“.

U yoki bu hodisaning muvaffaqiyatli ro'y berish imkoniyatini matematiklar ***hodisaning ehtimolligi*** deb nomlashdi va lotincha *probabilitas* – ehtimollik so'zining birinchi harfiga mos ravishda $P$ orqali belgilashdi.

Agarda $A$ orqali o'yin kubigi bir marta tashlanganda „5 ochko tushdi" hodisasi belgilansa, u holda $A$ hodisaning ehtimolligi $P(A)$ orqali belgilanadi, $P(A) = \frac{1}{6}$ ko'rinishda yoziladi va hodisaning ehtimolligi $\frac{1}{6}$ deb o'qiladi.

**1-masala.** Bir xil kartochkalarga 1 dan 20 gacha sonlar yozildi (har bir kartochkaga bittadan son yozildi). Kartochkalar stolga teskarisi bilan qo'yildi va aralashtirildi. Tasodifan olingan kartochkadagi sonning 7 bo'lishlik ehtimolligini toping.

△ Kartochkalar soni 20 ta va har bir kartochkaga 1 dan 20 gacha sonlar bir martadan yozilgani uchun tanlash natijasida 20 ta teng imkoniyatli hodisalar ro'y berishi mumkin (tajriba natijalari): 1) 1 soni chiqdi; 2) 2 soni chiqdi; ....; 20) 20 soni chiqdi.

Bunda „biror son chiqdi" hodisasi esa muqarrar hodisa. Bu muqarrar hodisaning ehtimolligi 1 ga teng va $A$ – „7 soni chiqdi" hodisasining ehtimolligi esa 20 marta kichik, ya'ni $P(A) = \frac{1}{20}$.

**Javob:** $\frac{1}{20}$ ▲.

Yuqorida qaralgan ***elementar hodisalardan*** tashqari murakkabroq hodisalarni ham o'rganish mumkin. Masalan, 1-masaladagi tanlangan kartochkadagi sonning tub son bo'lishligining ehtimolligini topish kerak bo'lsin. $A$ – „20 dan katta bo'lmagan tub sonning chiqishi" hodisasini qaraylik. Bu hodisa 8 ta holda (natijada) ro'y beradi – ya'ni 2, 3, 5, 7, 11, 13, 17, 19 tub sonlarning birortasi chiqqanda. Ushbu natijalar $A$ hodisa uchun *qulaylik tug'diruvchi imkoniyatlar* deb ataladi. Mumkin bo'lgan barcha natijalar (ular 20 ta) ichida 8 tasi qulaylik tug'diruvchi imkoniyatlardir, shu sababli $A$ hodisaning ehtimolligi

$$P(A) = \frac{8}{20} = \frac{2}{5}.$$

**Agarda biror tajribada $n$ ta teng imkoniyatli, o'zaro juftma-juft birgalikda bo'lmagan natija mavjud bo'lib, ulardan $m$ tasi $A$ hodisa uchun qulaylik tug'diruvchi imkoniyatlar bo'lsa, u holda $\frac{m}{n}$ nisbat *$A$ hodisa ro'y berishining ehtimolligi* deyiladi va quyidagicha yoziladi:**

$$P(A) = \frac{m}{n}. \qquad (1)$$

**2-masala.** O'yin kubigini bir marta tashlanganda toq sonli ochko chiqishining ehtimolligini toping.

△ $A$ – „toq sonli ochko chiqishi" hodisasiga qulaylik tug'diruvchi 3 ta natija (1 ning chiqishi, 3 ning chiqishi va 5 ochkoning chiqishi) mavjud, ya'ni $m = 3$. Teng imkoniyatli barcha natijalar soni esa $n = 6$, shu sababli

$$P(A) = \frac{m}{n} = \frac{3}{6} = \frac{1}{2}.$$

**Javob:** $\frac{1}{2}$. ▲

**3-masala.** Qutida 6 ta qizil va 4 ta ko'k shar bor. Ulardan biri tasodifan tanlanib, qutidan olindi. Olingan sharning qizil bo'lishlik ehtimolligini toping.

△ Tajribaning 10 ta teng imkoniyatli natijalari mavjud: 1- shar olindi, 2- shar olindi, ..., 10- shar olindi, ya'ni $n = 10$. Qulaylik tug'diruvchi natijalar soni esa $m = 6$ ta. Shu sababli

$$P(A) = \frac{m}{n} = \frac{6}{10} = \frac{3}{5}.$$

**Javob:** $\frac{3}{5}$. ▲

Muqarrar, mumkin bo'lmagan va tasodifiy hodisalarning ehtimolliklari haqida (1) formulaga asosan quyidagilarni aytish mumkin:

Agarda $A$ hodisa muqarrar ro'y beradigan hodisa bo'lsa, u holda barcha natijalar unga qulaylik tug'diruvchi bo'ladi, ya'ni $m = n$. U holda $P(A) = \frac{m}{n} = 1$.

Agarda $A$ hodisa ro‘y berishi mumkin bo‘lmagan hodisa bo‘lsa, u holda unga qulaylik tug‘diruvchi natijalar mavjud emas, ya’ni $m = 0$. Demak, bu holda $P(A) = \frac{0}{n} = 0$.

Agarda $A$ hodisa tasodifiy hodisa bo‘lsa, u holda unga qulaylik tug‘diruvchi natijalar uchun $0 < m < n$ shart bajariladi. Shu sababli, bunday hollarda $0 < P(A) = \frac{m}{n} < 1$.

### *Mashqlar*

**465.** Quyida keltirilgan hollarda ro‘y berishi mumkin bo‘lgan barcha elementar teng imkoniyatli hodisalarni sanab o‘ting: 1) tanga tashlash; 2) o‘yin kubigini tashlash; 3) yoqlarining rangi oq, qizil, sariq va ko‘k bo‘lgan tetraedrni tashlash; 4) sathi $A$, $B$, $C$, $D$, $E$ va $F$ orqali belgilangan 6 ta sektorga bo‘lingan ruletkaning strelkasini aylantirish.

**466.** Domino o‘yinining to‘liq komplektidan bitta donasi tasodifan olindi. Bu donaning:
1) 6 va 5 sonlari; 2) 0 va 1 sonlari; 3) bir xil sonlar; 4) har xil sonlar chiqishlik ehtimolligini toping.

**467.** Qutida 4 ta qizil va 5 ta ko‘k shar bor. Tasodifan bir shar olindi. Olingan sharning:
1) qizil; 2) ko‘k; 3) yashil; 4) qizil yoki ko‘k bo‘lishlik ehtimolligi qanday?

**468.** Qutida 3 ta ko‘k, 4 ta sariq, 5 ta qizil shar bor. Tasodifan bir shar olindi. Olingan sharning:
1) ko‘k; 2) sariq; 3) qizil; 4) ko‘k emas; 5) sariq emas; 6) qizil emaslik ehtimolligi qanday?

**469.** Bir xil kartochkalarga 1 dan 12 gacha sonlar yozildi (har bir kartochkaga bittadan son yozildi). Kartochkalar stolga teskarisi bilan qo‘yildi va aralashtirildi. Tasodifan olingan kartochkaning:
1) 5; 2) juft; 3) 3 ga karrali; 4) 4 ga karrali; 5) 5 ga bo‘linuvchi; 6) tub son bo‘lish ehtimolligi qanday?

**470.** Nigora dugonasining telefon nomerining oxirgi ikkita raqamini yoddan chiqarib qo‘ydi va uni tasodifan terdi. Nigora o‘z dugonasining telefoniga tushish ehtimolligi qanday?

**471.** Lotereyada 1000 ta chipta bo‘lib, undan 30 tasi yutuqli. Bitta chipta xarid qilindi. Xarid qilingan chipta:
1) yutuqli; 2) yutuqsiz bo‘lishlik ehtimolligi qanday?

**472.** Talaba imtihonga tayyorlanish jarayonida unda beriladigan 30 ta biletning bittasiga tayyorlanishga ulgurmadi. Imtihonda talabaga bilgan bileti tushishining ehtimolligi qanday?

**473.** Tanga 6 marotaba ketma-ket tashlanganda har safar gerb tomoni bilan tushdi. Tanga yana bir marotaba tashlansa, gerb tomoni bilan tushish ehtimolligi qanday?

**474.** 52 talik qartalar dastasidan bir qarta tasodifiy ravishda olindi. Ushbu qartaning
1) olti g‘ishtin; 2) sakkiz; 3) qizil tusdagi valet; 4) sonli chillik tusli; 5) toq sonli g‘ishtin tusli bo‘lishining ehtimolligi qanday?

## 36-§ TASODIFIY HODISANING NISBIY CHASTOTASI

Ehtimollikning oldingi paragrafda berilgan ta’rifi *ehtimollikning klassik ta’rifi* deyiladi. Klassik ta’rif sinov yoki tajribaning albatta o‘tkazilishini talab qilmaydi: hodisaning barcha teng imkoniyatli va qulaylik tug‘diruvchi natijalari nazariy jihatdan aniqlanadi.

Bunday ta’rifga ko‘ra tajribaning elementar teng imkoniyatli natijalari soni chekli va muayyan son bilan ifodalanadi. Lekin amaliyotda, ya’ni tabiatshunoslikda, iqtisodda, tibbiyotda, ishlab chiqarishda va boshqa sohalardagi tasodifiy jarayonlar o‘rganilayotganda tez-tez shunday sinovlar yoki tajribalar uchrab turadiki, ulardagi mumkin bo‘lgan natijalar soni qamrab olishning imkoni bo‘lmagan darajada ko‘p. Boshqa bir qator holatlarda tajribalarni amalda o‘tkazmaguncha natijalarning teng imkoniyatli bo‘lishini aniqlash qiyin yoki mumkin emas. Masalan, firma ishlab chiqargan ko‘plab lampochkalarni tekshirib ko‘rmaguncha „yaroqli“ yoki „yaroqsiz“ ligi teng imkoniyatli bo‘lish yoki bo‘lmasligini tasavvur qilish qiyin. Shu sababli, klassik ta’rif bilan bir qatorda, amaliyotda *ehtimollikning statistik ta’rif*idan ham foydalanishadi. Bu ta’rif bilan tanishish uchun *nisbiy chastota* tushunchasini kiritishimiz kerak bo‘ladi.

> **Berilgan tajribalar qatorida *A hodisaning nisbiy chastotasi* deb, ushbu hodisa ro‘y bergan tajribalar soni *M* ning o‘tkazilgan barcha tajribalar soni *N* ga nisbatiga aytiladi. Bunda *M* soni *A* hodisaning *chastotasi* deb ataladi.**

$A$ hodisaning nisbiy chastotasi $W(A)$ orqali belgilanadi. U holda ta’rifga ko‘ra

$$W(A) = \frac{M}{N}. \qquad (1)$$

**1-masala.** Sinfda 30 ta o‘quvchi bor. O‘tkazilgan nazorat ishidan 6 ta o‘quvchi 5 baho oldi. Sinfda o‘tkazilgan nazorat ishidan olingan a’lo baholarning nisbiy chastotasini toping.

△ $A$ – „5 baho olindi“ hodisasi bo‘lsa, bu hodisa 6 marta ro‘y berdi, ya’ni $M = 6$. Umumiy tajribalar soni $N = 30$, shu sababli

$$W(A) = \frac{6}{30} = \frac{1}{5}.$$

**Javob:** $\frac{1}{5}$. ▲

Fransuz tadqiqotchisi Byuffon (1707–1788) tangani 4040 marta tashlab ko‘rgan, shundan 2048 holatda tanga gerb tomoni bilan tushgan. Demak, bu holda ushbu tajribalar qatorida gerb tushishining nisbiy chastotasi $W(A) = \frac{2048}{4040} \approx 0,5069$ ga teng. Ingliz matematigi Karl Pirson esa tangani 24 000 marta tashlaganida gerb tomoni 12 012 marta tushgan. Demak, tanga tashlashning bu tajribalarida gerb tomoni tushishining nisbiy chastotasi $W(A) = \frac{12012}{24000} = 0,5005$ ga teng.

Bu ikkita holdagi natijani solishtirsak, nisbiy chastotalarning qiymati, umuman olganda, muayyan tajribalarga va ularning soniga qarab o‘zgarishi mumkinligini ko‘rishimiz mumkin.

Lekin tasodifiy hodisa nisbiy chastotasining asosiy xususiyati shundan iborat ekanki, tajribalar soni oshib borgani sari nisbiy chastota tobora barqarorlashib, biror son atrofida tebranib turar ekan. Shu son tasodifiy hodisaning *statistik ehtimolligi* sifatida qabul qilinadi. Masalan, tanga tashlashda bu son 0,5, ya’ni Byuffon tajribasidagi ham, Pirson tajribasidagi ham hosil bo‘lgan nisbiy chastotalar 0,5 ga juda yaqin sonlardir. Demak, tanga tashlanganda uning statistik ehtimolligi 0,5 ga teng.

Tanga tashlashga o‘xshash turli xil jarayonlarni o‘rganish bo‘yicha katta sondagi tajribalar turli tadqiqotchilar tomonidan o‘tkazilgan va ularning natijalari asosida shveysariyalik matematik olim Yakob Bernulli (1654–1705 ) *katta sonlar qonuni*ni asoslab berdi:

*Tajribalar soni katta bo‘lganda hodisaning nisbiy chastotasi $W(A)$ bu hodisaning ehtimolligi $P(A)$ dan amaliy jihatdan farq qilmasligini, ya’ni katta sonli tajribalarda $P(A) = W(A)$ ekanligi haqidagi dalilni muqarrar deb hisoblash mumkin.*

**2-masala.** Bir mamlakatda xorijdan kelgan sayyohlar va shu mamlakatning ichida sayohat qilgan mamlakat fuqarolari (ichki sayyohlar) haqida quyidagi ma’lumotlar berilgan bo‘lsin:

| Yillar | Sayyohlarning umumiy soni | |
|---|---|---|
| | Xorijiy sayyohlar soni | Ichki sayyohlar |
| 2014 | 610 623 | 403 989 |
| 2015 | 746 224 | 348 953 |
| 2016 | 822 558 | 316 897 |
| 2017 | 774 262 | 346 103 |
| 2018 | 811 314 | 351 028 |

Qaralayotgan yillarda mamlakat ichida sayohat qilgan mamlakat fuqarolari sonining nisbiy chastotasini toping.

Mamlakat ichida sayohat qilgan fuqarolar soni:
$M = 403989 + 348953 + 316897 + 346103 + 351028 = 1766970$, xorijlik sayyohlar soni esa: $610623 + 746224 + 822558 + 774262 + 811314 = 3764981$.

Umumiy sayyohlar soni: $N = 1766970 + 3764981 = 5531951$.

U holda,

$$W = \frac{M}{N} = \frac{1\,766\,970}{5\,531\,951} \approx 0,3194.$$

**Javob:** $W \approx 0,3194$.

## *Mashqlar*

**475.** Jadvalning oxirgi ustunini to'ldiring:

| Tartib raqami | Tajriba | Tajribalar soni ($N$) | $A$ hodisa | $A$ hodisaning chastotasi | $A$ hodisaning nisbiy chastotasi $\left(W(A)=\frac{M}{N}\right)$ |
|---|---|---|---|---|---|
| 1 | Tanga tashlash | 150 | Raqamli tomon tushishi | 78 | |
| 2 | Sportchi kamondan nishonga otyapti | 200 | Nishonga tegishi | 182 | |
| 3 | O'yin kubigi tashlanyapti | 400 | 4 tushishi | 67 | |

**476.** Bir shaharda 920 ta odamdan ishga qanday yetib borishlarini so'rashganda ularning: 350 tasi mashinada, 420 tasi jamoat transportida, 80 tasi velosipedda, 70 tasi piyoda borishlari ma'lum bo'lgan bo'lsa, 1) mashinada; 2) jamoat transportida; 3) velosipedda; 4) piyoda boruvchilar sonining nisbiy chastotasini toping.

**477.** Tayyorlangan 5000 ta qattiq diskdan 70 tasi yaroqsiz chiqdi. Yaroqsiz qattiq disk chiqishining nisbiy chastotasini topib, uni foizlarda ifodalang.

**478.** Yosh basketbolchilar guruhi to'pni savatga tushirish mashqlarini o'tkazishdi. Natijalar quyidagi jadvalda berilgan:

| | | | | | |
|---|---|---|---|---|---|
| Savatga otilgan to'plar soni ($N$) | 10 | 50 | 100 | 250 | 500 |
| Savatga tushgan to'plar chastotasi ($M$) | 6 | 32 | 68 | 155 | 320 |
| Savatga tushgan to'plar nisbiy chastotasi ($W$) | | | | | |

Jadvalning oxirgi satrini to'ldiring. To'plarning savatga tushishlik ehtimolligi $P$ ning qiymati haqida nima deyish mumkin (o'ndan birgacha aniqlikda)?

## 37-§ TASODIFIY MIQDORLAR

Statistika turli tasodifiy miqdorlar haqidagi ma'lumotlarni yig'ish, guruhlash, ma'lumotlarni jadvallar, diagrammalar, grafiklar va boshqa ko'rinishlarda ko'rgazmali tasvirlash hamda bu ma'lumotlarning tahlili bilan shug'ullanadigan fandir.

**!** ***Tasodifiy miqdor*** **deb, kuzatuvlar yoki tajribalarni o'tkazish davomida turli qiymatlarni tasodifiy ravishda qabul qilishi mumkin bo'lgan kattalikka aytiladi. Bunday miqdorlar haqida ularning qiymatlari tasodifga bog'liq deb aytishimiz mumkin.**

Masalan, koinotdan maktab hovlisiga tushayotgan kosmik zarrachalar soni, telefon stansiyasiga kelib tushayotgan qo'ng'iroqlar soni, piyoladagi choy molekulalarining tezligi, o'yin kubigini tashlaganda qanday raqam chiqishi va boshqalar tasodifiy miqdorlarga misol bo'la oladi.

**1-masala.** Ikkita o'yin kubigi tashlandi. Ikkita kubikdan tushadigan qanday ochkolar yig'indisi eng katta ehtimollik bilan bo'lishini aniqlash mumkinmi?

Har bir yig'indining paydo bo'lishlik ehtimolligini topamiz. Umumiy natijalar soni bu ikkita kubik tushishidan hosil bo'ladigan barcha yigindilar soni $6 \cdot 6 = 36$ ga teng. Yig'indi ochkolar jadvalini tuzamiz:

| 1- kubik | 2- kubik | | | | | |
|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| 6 | 7 | 8 | 9 | 10 | 11 | 12 |

Jadval yordamida har bir muayyan yig'indi uchun qulaylik tug'diruvchi natijalar soni $m$ ni aniqlaymiz:

$$m_2 = m_{12} = 1,\ m_3 = m_{11} = 2,\ m_4 = m_{10} = 3,$$
$$m_5 = m_9 = 4,\ m_6 = m_8 = 5,\ m_7 = 6.$$

Ikkita kubikni tashlaganda u yoki bu yig'indining hosil bo'lishlik ehtimolligini quyidagi jadval ko'rinishida ifodalash mumkin:

| Ochkolar yig'indisi | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Ehtimollik $\left(p = \frac{m}{n}\right)$ | $\frac{1}{36}$ | $\frac{2}{36}$ | $\frac{3}{36}$ | $\frac{4}{36}$ | $\frac{5}{36}$ | $\frac{6}{36}$ | $\frac{5}{36}$ | $\frac{4}{36}$ | $\frac{3}{36}$ | $\frac{2}{36}$ | $\frac{1}{36}$ |

Jadvaldan ochkolar yig'indisi 7 bo'lishligi eng katta ehtimollik $-\frac{6}{36} = \frac{1}{6}$ ga ega bo'lishi ko'rinib turibdi.

**Javob:** eng katta ehtimollikka ega bo'lgan ochkolar yig'indisi 7. ▲

1- masalada ikkita kubikni tashlagandagi ochkolar yig'indisi – *tasodifiy miqdor*. Uni $X$ orqali belgilaylik. U holda $X_1 = 2$, $X_2 = 3$, ..., $X_{10} = 11$, $X_{11} = 12$ sonlari $X$ tasodifiy miqdorning qiymatlaridir. $X$ ning har bir qiymatiga mos keluvchi $P_1$, $P_2$,..., $P_{10}$, $P_{11}$ ehtimolliklar qiymati quyidagi jadvalda ko'rsatilgan:

| $X$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $P$ | $\frac{1}{36}$ | $\frac{2}{36}$ | $\frac{3}{36}$ | $\frac{4}{36}$ | $\frac{5}{36}$ | $\frac{6}{36}$ | $\frac{5}{36}$ | $\frac{4}{36}$ | $\frac{3}{36}$ | $\frac{2}{36}$ | $\frac{1}{36}$ |

Bu jadval yordamida, masalan, $X$ miqdor bir xil ehtimollik bilan qanday qiymatlarni qabul qilishini; $X$ miqdorning qanday qiymati ko'proq ehtimollik bilan paydo bo'ladi va hokazo savollarga javobni osonlik bilan aniqlash mumkin. Bu jadval ikkita kubikni tashlagandagi ochkolar yig'indisidan iborat bo'lgan tasodifiy miqdor $X$ ning ehtimolliklar bo'yicha *taqsimot jadvali* deyiladi.

**Tasodifiy miqdor $X$ ning qiymatlarini va har bir qiymatni qabul qilish ehtimolligini ifodalovchi jadval *tasodifiy miqdorning ehtimolliklar bo'yicha taqsimot jadvali* deyiladi.**

Ehtimolliklar bo'yicha taqsimot jadvallari, ehtimolliklarni nazariy jihatdan hisoblash natijalari asosida tuziladi.

Amaliyotda, real tajribalar o'tkazilgandan keyin, tasodifiy miqdorlar qiymatlarining *chastotalar yoki nisbiy chastotalar bo'yicha taqsimot jadvallari* tuziladi. Undan keyin, yaqqolroq bo'lishi uchun, taqsimotlar jadvallari *diagramma* yoki *chastotalar poligoni* ko'rinishida tasvirlanadi. Ma'lumotlarni diagramma va chastotalar poligoni orqali tasvirlash bilan Siz 8-sinf Algebra kursida tanishgansiz.

**2- masala.** Kompaniyalarda ishlovchi xodimlar sonini o'rganish maqsadida 36 ta kompaniyadan ularda ishlaydiganlar soni bo'yicha ma'lumot olindi va ular quyidagi jadvalga kiritildi:

| | | | | | |
|---|---|---|---|---|---|
| 23 | 30 | 24 | 25 | 30 | 24 |
| 32 | 33 | 31 | 31 | 25 | 33 |
| 23 | 30 | 29 | 24 | 33 | 30 |
| 26 | 29 | 27 | 29 | 26 | 28 |
| 29 | 30 | 27 | 30 | 28 | 32 |
| 31 | 27 | 30 | 27 | 33 | 28 |

Bu ma'lumotlarni 1) chastotalar ($M$) va nisbiy chastotalar ($W$) bo'yicha taqsimotlar jadvali; 2) chastotalar poligoni yordamida tasvirlang.

△ 1) Jadvaldan ko'rinib turibdiki, xodimlar sonini $X$ orqali belgilasak, u tasodifiy miqdor bo'ladi. Bevosita jadvalni o'rganib, bu tasodifiy miqdorning qiymatlari 23 dan 33 gacha qiymatlarni qabul qilishini ko'ramiz va shu sonlarni jadvalda necha marta qatnashishini sanab, chastotalar bo'yicha taqsimot jadvalni tuzamiz:

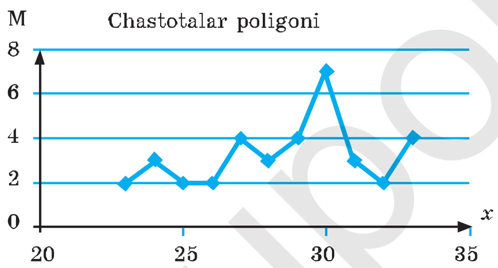
| $X$ | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $M$ | 2 | 3 | 2 | 2 | 4 | 3 | 4 | 7 | 3 | 2 | 4 |

Chastotalarning har birini kompaniyalar soni $N = 36$ ga bo‘lib, nisbiy chastotalar bo‘yicha taqsimot jadvalini hosil qilamiz:

| $X$ | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $W = \frac{M}{N}$ | 0,06 | 0,08 | 0,06 | 0,06 | 0,11 | 0,08 | 0,11 | 0,19 | 0,08 | 0,06 | 0,11 |

Bunda barcha chastotalar yig‘indisi $N = 36$ va barcha nisbiy chastotalar yig‘indisi esa **1** ga teng ekanligini eslatib o‘tamiz.

2) Kompaniyalar xodimlari soning chastotalar poligonini 90-rasmdan ko‘rish mumkin:




*90-rasm.*

Biror miqdorning barcha qiymatlari yig‘indisini topmoqchi bo‘lsak, L.Eyler tomonidan kiritilgan $\sum$ belgisidan foydalanamiz. Masalan, agarda $M$ chastota $M_1, M_2, ..., M_k$ qiymatlarni qabul qilsa, u holda quyidagicha belgilashdan foydalanamiz:

$$M_1 + M_2 + ... + M_k = \sum M.$$

Tasodifiy miqdorning barcha chastotalarining yig‘indisi tajribalar soni $N$ ga teng:

$$\sum M = N.$$

Har qanday tasodifiy miqdor uchun uning nisbiy chastotalarining yig‘indisi 1 ga teng.

$$\sum W = \sum\left(\frac{M}{N}\right) = \frac{M_1}{N} + \frac{M_2}{N} + \ldots + \frac{M_k}{N} =$$
$$= \frac{M_1 + M_2 + \ldots + M_k}{N} = \frac{\sum M}{N} = \frac{N}{N} = \mathbf{1}.$$

Ushbu paragrafda ko'rilgan tasodifiy miqdorlar bir-biridan ajralgan qiymatlarni qabul qiladi. Bunday miqdorlar *diskret* (lotin tilidagi *diskretus* – ajratilgan, uzilishli so'zidan) *miqdorlar* deb ataladi.

Agarda tasodifiy miqdor biror oraliqdagi barcha qiymatlarni qabul qilishi mumkin bo'lsa, u holda bunday miqdor *uzluksiz tasodifiy miqdor* deb ataladi. Uzluksiz tasodifiy miqdorlarga misol sifatida havo haroratining o'zgarishi, uydan maktabgacha borishga ketadigan vaqt, o'sayotgan terak daraxtining bo'yi, bekatda kutilayotgan avtobusning kelish vaqti va hokazoni keltirish mumkin.

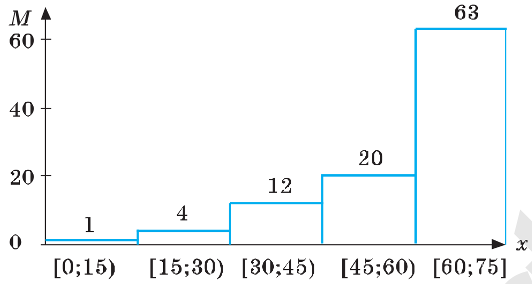
Uzluksiz tasodifiy miqdorlar cheksiz ko'p qiymatlar qabul qilsa-da, ularning taqsimotini berish mumkin. Buning uchun, uzluksiz miqdor qiymatlarining o'zgarish oralig'i qismlarga ajratiladi va tasodifiy kattalikning har bir qismga tushishining chastotalari (yoki ehtimolliklari) hisoblanadi.

Masalan, o'quvchi 100 kun sport zalida bo'lgani va har safar mashqlarga 1 soat-u 15 minutdan ortiq bo'lmagan vaqt sarflaganini yozib borgan bo'lsin. U holda sarflangan vaqtlarning minutlarda [0;75] oralig'ida bo'lishini e'tiborga olib, bu oraliqni, masalan, 5 ta teng vaqt oraliqlariga bo'lib, mashqlarga sarflangan vaqtlarning chastotalar jadvaliga kiritish mumkin:

| $T$ (minut) | [0; 15) | [15; 30) | [30; 45) | [45; 60) | [60; 75] |
|---|---|---|---|---|---|
| $M$ | 1 | 4 | 12 | 20 | 63 |

Bevosita chastotalar yig'indisini hisoblab, $\sum M = N = 100$ ekanligini ko'rish mumkin.

Ushbu jadvaldagi ma'lumotlarni *chastotalar gistogrammasi* – zinasimon shakl ko'rinishida tasvirlash mumkin (91-rasm). Bunda har bir zina asosi $h$ uzunlikka ega bo'lsa, u holda ustunning balandligini $\frac{M}{h}$, bu yerda $M$ bu $X$ tasodifiy kattalikning mos oraliqdagi chastotasi. U holda bunday ustunning yuzi $\frac{M}{h} \cdot h = M$ ga, gistogramma ostidagi shaklning yuzi esa $\sum M - N$ ga teng bo'ladi.

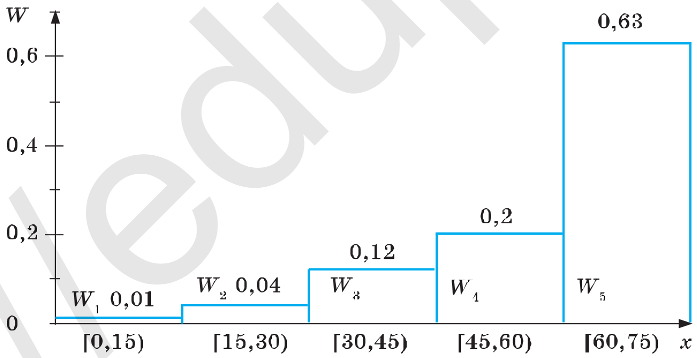



*91- rasm.*

Agarda chastotalar yordamida nisbiy chastotalar aniqlansa:

| $T$ (minut) | [0;15) | [15;30) | [30;45) | [45;60) | [60;75] |
|---|---|---|---|---|---|
| $W=\frac{M}{N}$ | 0,01 | 0,04 | 0,12 | 0,2 | 0,63 |

u holda ular yordamida chizilgan zinasimon shakl (92-rasm) tasodifiy kattalikning *nisbiy chastotalar bo'yicha gistogrammasi* deyiladi.




*92- rasm.*

Nisbiy chastotalar gistogrammasining har bir ustun ostidagi yuzi $W$ ning mos qiymatiga teng bo'ladi. U holda gistogramma ostidagi shaklning yuzi birga teng bo'ladi ($\sum W=1$).

## *Mashqlar*

**479.** 1) Oddiy o‘yin kubigi; 2) ikkita tomonida 1 ochko, ikkita tomonida 2 ochko, ikkita tomonida 3 ochko belgilangan kubik; 3) uchta tomonida 1 ochko, ikkita tomonida 2 ochko, bir tomonida 3 ochko belgilangan kubik; 4) ikki tomonida 1 ochko, uchta tomonida 2 ochko, bitta tomonida 3 ochko belgilangan kubik tashlanganda tushadigan „ochkolar soni“ – $X$ tasodifiy miqdor qiymatlarining $P$ ehtimolliklar bo‘yicha taqsimot jadvalini tuzing.

**480.** Stolga ikkita tanga tashlanyapti. Natija „gerb tomon“ tushsa shartli ravishda 0 sonli qiymat, natija „raqamli tomon“ tushsa 1 soni qiymatni beramiz. Tangalar tushganda berilgan sonli qiymatlar yig‘indisi – $X$ tasodifiy miqdorning $P$ ehtimolliklar bo‘yicha taqsimot jadvalini tuzing.

**481.** Yoqlari 1,2,3,4 sonlari bilan belgilangan ikkita tetraedr bir vaqtda stolga tashlanmoqda, bunda tetraedrlarning stolga tegib turgan yog‘idagi ochko hisobga olinadi. Ikkita tetraedrdan tushadigan qanday ochkolar: 1) yig‘indisining; 2) ko‘paytmasining eng katta ehtimollik bilan bo‘lishini aniqlash mumkinmi?

**482.** Ikkita o‘yin kubigi tashlandi. Ikkita kubikdan tushadigan ochkolar ko‘paytmasining ehtimolliklar bo‘yicha taqsimot jadvalini tuzing.

**483.** Kafening egasi tushlik vaqtida ovqatlanuvchilarga o‘z vaqtida xizmat qilish, shu vaqtda xizmat qiluvchilarning sonini to‘g‘ri belgilash va tayyorlanadigan taomlarga sarflanadigan xarajatlarni to‘g‘ri rejalashtirish maqsadida uning kafesida tushlik qiluvchilarning sonini 50 kun davomida jadvalga yozib bordi:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 20 | 27 | 23 | 27 | 26 | 18 | 22 | 25 | 26 | 23 |
| 23 | 25 | 28 | 26 | 23 | 22 | 21 | 19 | 21 | 29 |
| 30 | 27 | 26 | 30 | 29 | 22 | 18 | 29 | 22 | 26 |
| 28 | 27 | 29 | 27 | 22 | 29 | 26 | 27 | 21 | 19 |
| 25 | 29 | 29 | 21 | 18 | 26 | 20 | 24 | 19 | 27 |

Bu jadval yordamida kafeda tushlik qiluvchilar soni – $X$ tasodifiy miqdorning; 1) chastotalar ($M$) va nisbiy chastotalar ($W$) bo'yicha taqsimot jadvalini; 2) chastotalar poligonini tuzing.

**484.** Yopiq suv havzasiga suzishga kelgan o'g'il va qiz bolalarning soni besh oy davomida qayd qilinib, quyidagi jadval tuzildi:

| Oy | Suv havzasiga kelgan bolalar | |
|---|---|---|
| | Qiz bolalar | O'g'il bolalar |
| Aprel | 311 | 357 |
| May | 284 | 404 |
| Iyun | 278 | 417 |
| Iyul | 340 | 412 |
| Avgust | 322 | 406 |

Suv havzasiga kelgan o'g'il bolalar soni – $X$ tasodifiy miqdorning chastotasi, nisbiy chastotasini toping va chastotalar gistogrammasini tuzing.

**485.** Qiymatlari quyidagi telefon nomerlarida qatnashgan raqamlar bo'lgan $X$ tasodifiy miqdorning chastotalar bo'yicha taqsimot jadvalini tuzing:

1) 916549695, 939749596, 949039391, 913229296;

2) 945539391, 931179396, 913749193, 919149494.

**486.** Taqsimoti quyidagi jadvalda berilgan $X$ tasodifiy miqdorning chastotalar poligoni va nisbiy chastotalar poligonini tuzing:

1)

| X | 3 | 5 | 7 | 9 | 11 |
|---|---|---|---|---|---|
| M | 2 | 4 | 6 | 3 | 1 |

2)

| $X$ | 6 | 7 | 9 | 10 | 12 |
|---|---|---|---|---|---|
| $M$ | 5 | 4 | 7 | 3 | 6 |

**487.** Jadvalda 16 ta 9-sinf o'g'il bolalarining oyoq kiyimlari o'lchamlari yozilgan:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 38 | 38 | 39 | 39 | 39 | 40 | 40 | 41 |
| 41 | 41 | 41 | 41 | 42 | 42 | 42 | 43 |

9-sinf o'g'il bolalarining oyoq kiyimi o'lchami – $X$ tasodifiy miqdorning chastotalar bo'yicha va nisbiy chastotalar bo'yicha taqsimot jadvallarini tuzing.

## 38- §. TASODIFIY MIQDORLARNING SONLI XARAKTERISTIKALARI

Siz 8-sinf „Algebra“ kursining ma’lumotlar tahliliga bag‘ishlangan IV bobida bosh to‘plam, tanlanma, o‘rta qiymat, moda, mediana kabi tushunchalar bilan tanishgansiz. Xuddi shunday tushunchalarni tasodifiy miqdorlar uchun ham kiritish mumkin.

Statistikada ma’lumotlar to‘plami sifatida tasodifiy miqdorlarning sonli qiymatlari, ularning chastotalarini e’tiborga olgan holda qaraladi. Bunda tasodifiy miqdorlarning barcha qiymatlari *bosh to‘plam* deb ataladi, ularning tanlab olingan biror qismi esa *tanlanma* deb ataladi. Tanlanma *reprezentativ tanlanma* deyiladi, agarda tanlanmada tasodifiy miqdorning bosh to‘plamdagi va faqat undagi qiymatlari qatnashsa va undagi qiymatlar chastotalarining nisbati bosh to‘plamdagi kabi bo‘lsa.

**Misol.** $X$ tasodifiy miqdorning $M$ chastotalar bo‘yicha taqsimoti quyidagicha berilgan bo‘lsin:

| X | – 3 | 5 | 9 | 11 |
|---|---|---|---|---|
| M | 5000 | 2000 | 7000 | 3000 |

va bu tasodifiy miqdorning barcha qiymatlari (e’tibor bering, ularning soni 17000 ta) bosh to‘plam deb qabul qilingan bo‘lsin. Quyidagicha uchta tanlanmani qaraylik:

1-jadval

| $X$ | –3 | 5 | 9 | 11 |
|---|---|---|---|---|
| $M$ | 5 | 2 | 7 | 3 |

2-jadval

| $X$ | –3 | 9 | 11 |
|---|---|---|---|
| $M$ | 5 | 7 | 3 |

3-jadval

| $X$ | –3 | 5 | 9 | 11 |
|---|---|---|---|---|
| $M$ | 5 | 6 | 7 | 3 |

Taqsimoti 1-jadvalda berilgan tanlanma reprezentativ tanlanma, chunki unda ham $-3, 5, 9, 11$ qiymatlar va faqat shu qiymatlar qatnashyapti hamda bosh top‘lamda ham bu tanlanmada ham chastotalar nisbati bir xil: 5 000 : 2 000 : 7 000 : 3 000 – 5 :2:7:3.

Taqsimoti 2-jadvalda berilgan tanlanma reprezentativ tanlanma emas, chunki unda $X$ tasodifiy miqdorning 5 ga teng qiymati qatnashmayapti.

Taqsimoti 3-jadvalda berilgan tanlanma ham reprezentativ tanlanma emas, chunki unda chastotalar nisbati saqlanmagan: 5 000:2 000:7 000:3 000≠5:6:7:3.

Berilgan ma'lumotlarni, jumladan, tasodifiy miqdorlarning qiymatlarini ba'zan bitta son bilan tavsiflash yoki baholash mumkin. Bu son berilgan ma'lumotlar tarkibidagi sonlar yoki tasodifiy miqdorlar qiymatlari *markaziy tendensiyasining o'lchovi* ham deyiladi. Markaziy tendensiya o'lchovlariga misol sifatida moda, mediana va o'rta qiymat kabilarni keltirish mumkin.

Tasodifiy miqdorning qaralayotgan tanlamadagi chastotasi eng katta bo'lgan qiymati *moda* deb ataladi va $M_0$ deb belgilanadi.

Masalan, tanlanma 8, 9, 2, 4, 8, 6, 3 dan iborat bo'lsa, u holda uning modasi 8 ga teng. 5, 6, 11, 3, 3, 5 tanlanmaning modasi esa ikkita – $M_2 = 3$, $M_2 = 5$. Agarda 1, 3, 7, 20, 6, 11 tanlanmani qarasak, uning modasi yo'q.

Agarda tanlanma qiymatlarini o'sib borishi tartibida yozib olsak, u holda tanlanmani berilganlarning soni jihatidan teng ikkiga bo'luvchi son *mediana* deb ataladi va $M_e$ kabi belgilanadi. Agarda tartiblangan tanlanmada berilganlar soni toq bo'lsa, u holda mediana ularning o'rtasida turgan songa teng. Agarda tartiblangan tanlanmada berilganlar soni juft bo'lsa, u holda mediana o'rtada turgan ikkita sonning o'rta arifmetigiga teng.

**1-masala.** Tasodifiy miqdor qiymatlari tanlanmasining medianasini toping:

1) 8, 2, 0, 5, –5, 4, 8; 2) 8, 5, 3, 4, 7, 2.

△ 1) Tanlanma elementlarini o'sib borish tartibida joylashtiramiz: – 5, 0, 2, 5, 4, 8, 8. Berilganlar soni toq. 5 sonidan chapda va o'ngda uchtadan son bor, ya'ni 5 tanlanmaning o'rta soni, shu sababli $M_e$–5.

2) Berilgan 8, 5, 3, 4, 7, 2 tanlanma elementlarini o'sib borish tartibida yozamiz: 2, 3, 4, 5, 7, 8. Berilganlar soni juft. Tanlanmaning o'rtasida turgan sonlar: 4 va 5, shu sababli $M_e = \frac{4+5}{2} = 4{,}5$.

**Javob:** 1) 5; 2) 4,5. ▲

Tanlanmalarni o‘rganishda ahamiyatli bo‘lgan yana bir tushuncha – tanlanmaning kengligi tushunchasi bilan Siz 8- sinfda tanishgansiz. *Tanlanmaning kengligi* deb tasodifiy miqdorning eng katta qiymati bilan eng kichik qiymatining ayirmasiga aytiladi va u $R$ orqali belgilanadi.

Tanlamaning kengligi tasodifiy miqdor qiymatlarining qanchalik tarqoq ekanligini bildiradi.

**Misol.** 21, 27, 22, 8, 9, 15, 19, 21 va 190, 187, 198, 189, 195, 190 tanlanmalarning kengligini solishtiring.

1- tanlanmaning eng katta qiymati 27, eng kichik qiymati esa 8. Demak, 1-tanlanmaning kengligi $R_1$=27–8=19.

2- tanlanmaning eng katta qiymati 198, eng kichik qiymati esa 186. Natijada, 2-tanlanmaning kengligi $R_2$=198–186=12.

Demak, birinchi tanlanmaning qiymatlari ikkinchi tanlanmadagiga qaraganda tarqoq joylashgan.

Tasodifiy miqdor qiymatlarining *o‘rtacha qiymati (yoki o‘rta arifmetigi)* deb tanlanmadagi barcha sonlar yig‘indisining ularning soniga nisbatiga aytilishini eslatib o‘taylik. $X$ tasodifiy miqdor barcha qiymatlarining o‘rtachasi $\bar{X}$ orqali belgilanadi.

**2- masala.** Chastotalari bo‘yicha taqsimoti quyidagi jadvalda berilgan tasodifiy miqdor tanlanmasining o‘rtachasini toping:

*4-jadval*

| $X$ | 3 | 4 | 5 | 7 | 10 |
|---|---|---|---|---|---|
| $M$ | 3 | 1 | 2 | 1 | 3 |

$$\bar{X} = \frac{3\cdot3+4\cdot1+5\cdot2+7\cdot1+3\cdot10}{3+1+2-1+3} = \frac{9+4+10+7+30}{10} = 6.$$

**Javob:** 6.

Ehtimolliklari bo‘yicha taqsimoti ma’lum bo‘lgan tasodifiy miqdor tanlanmasini tavsiflovchi tushunchalardan yana biri bu *matematik kutilma* tushunchasidir.

Agarda $X$ tasodifiy miqdorning $X_1$, $X_2, \ldots, X_n$ qiymatlarni qabul qilish ehtimolliklari, mos ravishda, $P_1$, $P_1, \ldots P_n$ bo‘lsa, u holda

$$E = X_1P_1 + X_2P_2 + \ldots + X_nP_n \qquad (1)$$

soni $X$ tasodifiy miqdorning *matematik kutilmasi* deb ataladi.

Masalan, $X$ tasodifiy miqdorning ehtimolliklar bo‘yicha taqsimoti quyidagicha berilgan bo‘lsin:

| $X$ | 6 | 4 | 3 | 7 | 5 |
|---|---|---|---|---|---|
| $P$ | $\frac{1}{10}$ | $\frac{1}{5}$ | $\frac{2}{5}$ | $\frac{1}{5}$ | $\frac{1}{10}$ |

U holda bu tasodifiy miqdorning matematik kutilmasi:

$$E = 6 \cdot \frac{1}{10} + 4 \cdot \frac{1}{5} + 3 \cdot \frac{2}{5} + 7 \cdot \frac{1}{5} + 5 \cdot \frac{1}{10} = \frac{6+8+12+14+5}{10} = 4,5.$$

Tasodifiy miqdorning qiymati bilan tanlanmaning o‘rtachasi orasidagi ayirma *o‘rtachadan chetlanish* deb ataladi.

Masalan, tasodifiy miqdorning qiymati $X_1 = 35$, o‘rtachaning qiymati esa $\overline{X} = 32$ bo‘lsa, u holda $X_2$ ning o‘rtachadan chetlanishi $X_1 - \overline{X} = 35 - 32 = 3$.

Tanlanmaning barcha qiymatlarining o‘rtachadan chetlanishlari yig‘indisi nolga teng bo‘lishini ko‘rsatish oson:

$$(X_1 - \overline{X}) + (X_2 - \overline{X}) + \ldots + (X_n - \overline{X}) = (X_1 + X_2 + \ldots + X_n) - n \cdot \overline{X} =$$

$$= (X_1 + X_2 + \ldots + X_n) - n \cdot \frac{X_1 + X_2 + \ldots + X_n}{n} = 0.$$

Shu sababli, tasodifiy miqdor qiymatlarini tavsiflash uchun o‘rtacha chetlanishlar yig‘indisi o‘rniga o‘rtacha chetlanishlar kvadratlarining o‘rta arifmetigidan foydalaniladi. Bunday kattalik *dispersiya* (lotinchadan *dispersion* – sochilish, yoyilish) deb ataladi.

Agarda $X$ tasodifiy miqdor $N$ ta turli qiymatlarni qabul qilsa va uning o‘rtachasi $\overline{X}$ bo‘lsa, u holda uning dispersiyasi quyidagi formula yordamida topiladi:

$$D = \frac{(X_1 - \overline{X})^2 + (X_2 - \overline{X})^2 + \cdots - (X_n - \overline{X})^2}{N}. \qquad (2)$$

Demak, dispersiya – tasodifiy miqdor qiymatlarining o‘rtachadan chetlanishlar kvadratlarining o‘rta arifmetigiga teng.

Agarda $X$ tasodifiy miqdorning $X_1, X_2, \ldots, X_k$ qiymatlari mos ravishda, $M_1, M_2, \ldots, M_k$ chastotalar bilan takrorlansa, u holda uning dispersiyasini

$$D=\frac{(X_1-\bar{X})^2 M_1-(X_2-\bar{X})^2 M_2+\cdots+(X_k-\bar{X})^2 M_k}{M_1+M_2+\cdots M_k}. \qquad (3)$$

formula yordamida hisoblash mumkin, bunda

$$\bar{X}-\frac{X_1M_1+X_2M_2+\cdots+X_kM_k}{M_1+M_2+\cdots M_k}.$$

Masalan, 4-jadvaldagi tasodifiy miqdorning o'rtachasi $\bar{X}=6$ ekanligini aniqlagan edik. Endi shu miqdorning dispersiyasini hisoblaylik:

$$D=\frac{(X_1-\bar{X})^2 M_1+(X_2-\bar{X})^2 M_2-\cdots-(X_k-\bar{X})^2 M_k}{M_1+M_2+\cdots-M_k}=$$

$$-\frac{(3-6)^2\cdot 3+(4-6)^2\cdot 1+(5-6)^2\cdot 2+(7-6)^2\cdot 1+(10-6)^2\cdot 3}{3+1+2+1-3}-$$

$$=\frac{27+4+2+1+48}{10}=\frac{82}{10}=8{,}2.$$

Agarda tasodifiy miqdor biror o'lcham (masalan, santimetr) ga ega bo'lsa, u holda uning o'rtachasi $\bar{X}$ va o'rtachadan chetlanishi $X-\bar{X}$ ham $X$ miqdor bilan bir xil o'lcham (santimetr) ga ega. Chetlanishning kvadrati va dispersiya esa $X$ miqdor o'lchamining kvadrati (kvadrat santimetr) o'lchamiga ega. O'rtachadan chetlanishni baholash uchun $X$ tasodifiy miqdor bilan bir xil o'lchamga ega bo'lgan kattalikdan foydalanish qulay. Shu sababli, dispersiyadan olingan kvadrat ildiz, ya'ni $\sqrt{D}$ ning qiymatlaridan foydalaniladi.

Dispersiyadan olingan kvadrat ildiz *o'rtacha kvadrat chetlanish* deyiladi va $\sigma$ orqali belgilanadi, ya'ni $\sigma=\sqrt{D}$.

Masalan, 4-jadvaldagi tasodifiy miqdorning dispersiyasi $D=8{,}2$ ekanligini hisoblagan edik. Endi dispersiyaning shu qiymatidan kalkulator yordamida kvadrat ildiz olsak, o'rtacha kvadrat chetlanishni hosil qilamiz:

$$\sigma-\sqrt{D}=\sqrt{8{,}2}\approx 2{,}86.$$

Dispersiya va o‘rtacha kvadrat chetlanishni statistikada tasodifiy miqdor qiymatlarining o‘rta qiymat atrofidagi yoyilishining o‘lchamlari ham deb aytishadi.

### *Mashqlar*

**488.** Tasodifiy miqdor $X$ qiymatlarining bosh to‘plamdagi taqsimoti quyidagi jadvalda keltirilgan:

| $X$ | 8 | 9 | 11 | 15 | 16 |
|---|---|---|---|---|---|
| $M$ | 21 | 49 | 70 | 35 | 14 |

Berilgan bosh to‘plam uchun quyidagilardan qaysilari reprezentativ tanlanma bo‘ladi:

1)

| $X$ | 8 | 9 | 11 | 15 | 16 |
|---|---|---|---|---|---|
| $M$ | 3 | 7 | 10 | 5 | 4 |

2)

| $X$ | 8 | 9 | 15 | 16 |
|---|---|---|---|---|
| $M$ | 3 | 7 | 5 | 2 |

3)

| $X$ | 8 | 9 | 11 | 15 | 16 |
|---|---|---|---|---|---|
| $M$ | 3 | 7 | 10 | 5 | 2 |

4)

| $X$ | 8 | 9 | 11 | 15 | 16 |
|---|---|---|---|---|---|
| $M$ | 3 | 7 | 9 | 5 | 2 |

**489.** Tanlanmaning modasini toping:
1) 6, 17, 8, 9, 5, 8, 10; 2) 20, 11, 7, 5, 9, 11, 3;
3) 4, 6, 8, 4, 7, 6, 5; 4) 5, 7, 4, 3, 7, 2, 5.

**490.** Tanlanmaning medianasini toping:
1) 18, 13, 35, 19, 7; 2) 25, 16, 14, 21, 22;
3) 5, 2, 9, 14, 11; 4) 16, 7, 13, 9, 15.

**491.** Tanlanmaning kengligini toping:
1) 18, −4, 16, −3, 11, 5, 4, −5, 1, 3;
2) 26, 17, 4, 12, 2, 25, 19, 5, 6, 7.

**492.** Tanlanmaning o‘rtachasini toping:
1) 34, −10, 23, −18; 2) −3, 6, −19, −12, 1;
3) 0,5, 0,7, 0,4, 0,7, 0,6, 0,4; 4) 2,2, 2,3, 2,2, 1,8, 1,8, 2,3.

**493.** Tanlanmaning modasi, medianasi va o'rtachasini toping:
1) 4, −3, 2, 0, 3, −2; 2) 6, 5, −2, 4, −5, 0.

**494.** Chastotalari bo'yicha taqsimoti quyidagi jadvalda berilgan $X$ tasodifiy miqdor qiymatlari tanlanmasining o'rta arifmetigini toping:

1)

| $X$ | −3 | 0 | 1 | 4 |
|---|---|---|---|---|
| $M$ | 4 | 6 | 5 | 1 |

2)

| $X$ | −3 | 1 | 5 |
|---|---|---|---|
| $M$ | 5 | 6 | 3 |

3)

| $X$ | −5 | 2 | 3 |
|---|---|---|---|
| $M$ | 3 | 6 | 2 |

4)

| $X$ | −2 | 1 | 2 | 3 |
|---|---|---|---|---|
| $M$ | 5 | 4 | 3 | 2 |

**495.** Ehtimolliklari bo'yicha taqsimoti quyidagi jadvalda berilgan $X$ tasodifiy miqdor qiymatlarining matematik kutilmasini toping:

1)

| $X$ | −4 | −2 | 0 | 1 | 3 |
|---|---|---|---|---|---|
| $P$ | $\frac{3}{11}$ | $\frac{1}{11}$ | $\frac{5}{11}$ | $\frac{1}{11}$ | $\frac{1}{11}$ |

2)

| $X$ | −3 | −2 | 0 | 1 | 2 | 4 |
|---|---|---|---|---|---|---|
| $P$ | $\frac{1}{10}$ | $\frac{2}{10}$ | $\frac{3}{10}$ | $\frac{2}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ |

**496.** Tanlanmaning dispersiyasini toping
1) 9 cm, 11 cm, 8 cm, 10 cm; 2) 18 kg, 16 kg, 15 kg, 19 kg;
3) 8 s, 11 s, 8 s, 9 s, 9 s; 4) 1 m, 9 m, 4 m, 8 m, 8 m.

**497.** Chastotalari bo'yicha taqsimoti quyidagi jadvalda berilgan $X$ tasodifiy miqdor qiymatlari to'plamining dispersiyasini toping.

1)

| $X$ | 1 | 2 | 3 | 5 |
|---|---|---|---|---|
| $M$ | 2 | 3 | 3 | 2 |

2)

| $X$ | −2 | −1 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|
| $M$ | 1 | 3 | 2 | 1 | 2 | 1 |

**498.** Tanlanma elementlarining o'rta qiymatdan o'rta kvadrat chetlanishini hisoblang:
1) 4 g, 5 g, 8 g, 3 g, 5 g;
2) 9 cm, 12 cm, 7 cm, 10 cm, 12 cm.

**499.** Chastotalari bo'yicha taqsimoti berilgan $X$ tasodifiy miqdorning o'rta kvadrat chetlanishini toping:

1)

| X | −1 | 2 | 3 | 5 |
|---|---|---|---|---|
| $M$ | 3 | 2 | 2 | 1 |

2)

| X | −4 | −2 | 1 | 4 |
|---|---|---|---|---|
| $M$ | 1 | 4 | 3 | 2 |

## *V bobga doir mashqlar*

**500.** (Og'zaki.) Quyidagi tajribada ro'y berishi mumkin bo'lgan barcha elementar hodisalarni ayting: 1) tasodifiy ravishda yildagi oylar nomi aytiladi; 2) ikkita tanga tashlanib, tushayotgan tomonlari kuzatiladi; 3) birorta 50 dan kichik tub son aytiladi; 4) tasodifiy ravishda ikki xonali 3 ga karrali son aytiladi.

**501.** Qutida 4 ta qora, 5 ta qizil, 6 ta ko'k shar bor. Tasodifiy ravishda qutidan bitta shar olindi. Olingan shar: 1) qora; 2) qizil; 3) ko'k; 4) qora emas; 5) qizil emas; 6) ko'k emas; 7) yashil; 8) yoki qora, yoki qizil, yoki ko'k bo'lishlik ehtimolligini toping.

**502.** Tavakkaliga 1 dan 50 gacha bo'lgan natural son aytildi. Bu sonning: 1) 7; 2) 7 emas; 3) 7 ga karrali; 4) 10 ga karrali; 5) tub son emas; 6) 30 dan katta emas ekanligining ehtimolligini aniqlang.

**503.** Stolga o'yin kubigi bilan tanga tashlanyapti. Bunda 1) kubikda 5, tanga raqamli tomoni bilan; 2) kubikda chiqqan son tub, tanga gerb tomoni bilan tushishi ehtimolligini toping.

Tanlanmaning kengligi, modasi, medianasi va o'rtachasini toping (**504–507**):

**504.** 1) 2, 6, 6, 9, 11;
2) 4, 10, 13, 13, 19.

**505.** 1) −7, −7, −4, −4, 1, 3;
2) −3, −3, 1, 3, 10, 10 .

**506.** 1) 0,13,−5, −6, 14, −1, 11, −1, −8;
2) 5, −9, 14, 9, −5, −2, 0, 14, −5.

**507.** 1) −4, −14, 13, −6, 9, 14, 0, −6;
2) 15, −3, −9, 9, 13, −7, −3, 10.

**508.** Tanlanmaning dispersiyasi va o'rta kvadrat chetlanishini aniqlang:
1) 6, 11, 8, 9;
2) 9, 12, 8, 14;
3) 6, 3, 5, 4, 4;
4) 4, 3, 2, 2, 6;
5) 1, −2, 2, −3, 4;
6) −3, 3, −4, −2, 5.

**509.** Chastotalar bo'yicha taqsimoti bilan berilgan $Z$ tasodifiy miqdorning dispersiyasi va o'rta kvadrat chetlanishini toping:

1)

| $Z$ | –1 | 0 | 2 | 4 |
|---|---|---|---|---|
| $M$ | 2 | 1 | 3 | 1 |

2)

| $Z$ | –2 | 1 | 4 | 5 |
|---|---|---|---|---|
| $M$ | 1 | 2 | 3 | 1 |

**510.** Tanlanmalar dispersiyalarini solishtiring:
1) 4, 5, 7, 5, 9 va 6, 9, 7, 8; 2) –2, 2, 3 va –3, –1, 1, 3, 4.

**511.** Ehtimolliklar bo'yicha taqsimot jadvali:

1)

| $X$ | –2 | –1 | 2 | 3 |
|---|---|---|---|---|
| $P$ | 0,2 | 0,3 | 0,4 | 0,1 |

2)

| $X$ | –3 | –2 | 0 | 1 | 3 |
|---|---|---|---|---|---|
| $P$ | 0,2 | 0,2 | 0,3 | 0,2 | 0,1 |

bilan berilgan $X$ tasodifiy miqdorning matematik kutilmasini toping.

## *V bobga doir sinov (test) mashqlari*

**1.** Bir xil kartochkalarga 1 dan 15 gacha sonlar yozildi (har bir kartochkaga bittadan son yozildi). Kartochkalar stolga teskarisi bilan qo'yildi va aralashtirildi. Tasodifan olingan kartochkadagi sonning tub son bo'lishlik ehtimolligini toping.

A) $\frac{2}{5}$; B) $\frac{1}{5}$; C) $\frac{7}{15}$; D) $\frac{3}{5}$.

**2.** Qutida 3 ta oq va 7 ta qora shar bor. Ulardan biri tasodifan tanlanib qutidan olindi. Olingan sharning oq bo'lishlik ehtimolligini toping.

A) 0,5; B) 0,7; C) 0,3 D) 0,1.

**3.** Sinfdagi 27 ta o'quvchidan 15 tasi o'g'il bola. Sinfga bir o'g'il bola va ikki qiz bola kelib qo'shildi. Bunda o'g'il bolalar soni – X tasodifiy miqdorning nisbiy chastotasi qanchaga o'zgardi?

A) $\frac{1}{45}$ ga oshdi; B) $\frac{1}{45}$ ga kamaydi;

C) $\frac{2}{45}$ ga oshdi; D) $\frac{2}{45}$ ga kamaydi.

4. Tasodifiy miqdor qiymatlari tanlanmasining modasi bilan medianasining yig'indisini toping: 10, 4, 2, 7, −3, 6, 10;

A) 14; B) 17; C) 16; D) 13.

5. Tasodifiy miqdor qiymatlari tanlanmasining modasi bilan medianasining ko'paytmasini toping: 2, 0, 1, 4, −1, 2.

A) 2; B) 3; C) 0; D) 4.

6. Chastotalari bo'yicha taqsimoti quyidagi jadvalda berilgan $X$ tasodifiy miqdor tanlanmasining $X$ o'rtachasini toping:

| $X$ | −1 | 0 | 1 | 3 | 5 |
|---|---|---|---|---|---|
| $M$ | 2 | 1 | 3 | 1 | 2 |

A) $1\frac{5}{9}$; B) $1\frac{4}{9}$; C) $1\frac{1}{9}$; D) 1.

7. $X$ tasodifiy miqdorning ehtimolliklar bo'yicha taqsimotiga ko'ra matematik kutilmasini toping:

| $X$ | −1 | 2 | 3 | 5 | 7 |
|---|---|---|---|---|---|
| $M$ | $\frac{1}{9}$ | $\frac{2}{9}$ | $\frac{3}{9}$ | $\frac{2}{9}$ | $\frac{1}{9}$ |

A) $\frac{25}{9}$; B) $\frac{26}{9}$; C) $\frac{29}{9}$; D) $\frac{30}{9}$.

8. $X$ tasodifiy miqdorning chastotalar bo'yicha taqsimotiga ko'ra o'rta kvadrat chetlanishini toping:

| $X$ | −1 | 2 | 3 | 5 | 6 |
|---|---|---|---|---|---|
| $M$ | 1 | 3 | 2 | 2 | 1 |

A) 1; B) 1,5; C) 2; D) 2,5.

9. $X$ tasodifiy miqdorning ehtimolliklar bo'yicha taqsimotiga ko'ra dispersiyasini toping:

| $X$ | 2 | 3 | 5 | 7 |
|---|---|---|---|---|
| $P$ | 0,1 | 0,5 | 0,3 | 0,1 |

A) 2,9; B) 2,09; C) 2,99; D) 0,29.

## ✍ Amaliy-tatbiqiy va fanlararo bog'liq masalalar

**1-masala.** 5000 p.b. (pul birligi)da avtomobil, har biri 250 p.b. dan 4 ta televizor, har biri 200 p.b. dan 5 ta qo'l telefoni yutuqli lotereya o'ynalyapti. Hammasi bo'lib, 7 p.b. dan 1000 ta chipta sotilmoqda. Bitta chipta sotib olgan lotereya qatnashchisining toza yutug'ining taqsimot jadvalini tuzing va matematik kutilmasini hisoblang.

△ $X$ – bitta chiptaga tushgan toza yutuq bo'lsa, u holda uning qiymati:

bitta ham yutuq chiqmasa, $0 - 7 = -7$;

qo'l telefoni yutilgan bo'lsa, $200 - 7 = 193$;

televizor yutilgan bo'lsa, $250 - 7 = 243$;

avtomobil yutilgan bo'lsa, $5\,000 - 7 = 4\,993$

pul birligida bo'ladi. 1000 ta chiptadan 990 tasiga yutuq chiqmasligini va yutuqlar soni $5 + 4 + 1 = 10$ ekanligini hisobga olib, ehtimollikning klassik ta'rifiga ko'ra hosil qilamiz:

$X$ – tasodifiy miqdor

$-7$ qiymatni qabul qilish ehtimolligi $\frac{990}{1000} = 0{,}990$;

193 qiymatni qabul qilish ehtimolligi esa $\frac{5}{1000} = 0{,}005$;

243 qiymatni qabul qilish ehtimolligi $\frac{4}{1000} = 0{,}004$;

4 993 qiymatni qabul qilish ehtimolligi $\frac{1}{1000} = 0{,}001$.

Demak, $X$ – tasodifiy miqdorning ehtimolliklar bo'yicha taqsimot jadvali quyidagicha bo'ladi:

| $X$ | −7 | 193 | 243 | 4 993 |
|---|---|---|---|---|
| $P$ | 0,990 | 0,005 | 0,004 | 0,001 |

Taqsimot jadvali asosida matematik kutilmani hisoblash mumkin:

$E = (-7) \cdot 0{,}990 + 193 \cdot 0{,}005 + 243 \cdot 0{,}004 - 4\,993 \cdot 0{,}001 = 0,$

ya'ni o'rtacha yutuq nolga teng. Hosil bo'lgan natija,lotereya biletlarini sotishdan tushgan hamma pul yutuqlarga ketishini anglatadi.

**Javob:** taqsimot jadvali:

| $X$ | −7 | 193 | 243 | 4993 |
|---|---|---|---|---|
| $P$ | 0,990 | 0,005 | 0,004 | 0,001 |

va matematik kutilma $E = 0$.▲

**2-masala.** Bir firmaga tarjimonlik ishiga ikkita nomzod harakat qilmoqda. Ularga bir xil sinov muddati belgilandi va 125 betlik bir xil matn tarjimaga berildi. Ularning har kuni necha bet matn tarjima qilganliklari quyidagi jadvalda berilgan:

| Haftaning kunlari | Kunlik tarjima qilingan betlar soni | |
|---|---|---|
| | 1- nomzod ($X$) | 2- nomzod ($Y$) |
| Dushanba | 24 | 25 |
| Seshanba | 26 | 31 |
| Chorshanba | 25 | 27 |
| Payshanba | 23 | 22 |
| Juma | 27 | 20 |

Ish beruvchi jadvaldagi ma'lumotlarni tahlil qilgan holda, nomzodlarning qaysi birini ishga olishni afzal ko'radi?

△ Nomzodlarning har biri 5 kunda 125 betdan tarjima qilishdi, demak, ikkala nomzodning ham o'rtacha mehnat unumdorligi bir xil:

$$X = Y = \frac{125}{5} = 25 \text{ (bet/kun)}.$$

Ikkala tasodifiy miqdor $X$ va $Y$ ning ham modasi yo'q, medianalari esa bir xil (25 va 25). Nomzodlardan qaysi birini ishga olish maqsadga muvofiq ekan? Bu holda nomzodlar mehnat unumdorliklarining *barqarorligini* solishtirish orqali amalga oshirish mumkin. Buni esa chetlanishlar kvadratlarining yig'indilarini yoki dispersiyalarni solishtirish orqali amalga oshirsa bo'ladi:

| Haftaning kunlari | Tasodifiy miqdorning qiymati | | O'rtachadan chetlanish $\overline{X} = \overline{Y} = 25$ | | Chetlanishlar kvadratlari | |
|---|---|---|---|---|---|---|
| | $X$ | $Y$ | $X-\overline{X}$ | $Y-\overline{Y}$ | $(X-\overline{X})^2$ | $(Y-\overline{Y})^2$ |
| Dushanba | 24 | 25 | −1 | 0 | 1 | 0 |
| Seshanba | 26 | 31 | 1 | 6 | 1 | 36 |
| Chorshanba | 25 | 27 | 0 | 2 | 0 | 4 |
| Payshanba | 23 | 22 | −2 | −3 | 4 | 9 |
| Juma | 27 | 20 | 2 | −5 | 4 | 25 |
| Jami | 125 | 125 | 0 | 0 | 10 | 74 |

Ko'rinib turibdiki, chetlanishlar kvadratlarining yig'indisi $X$ uchun **10**, $Y$ uchun esa **74**, yoki dispersiyalarni hisoblasak:

$$D(X)=\frac{(X_1-\overline{X})^2+(X_2-\overline{X})^2+\cdots+(X_5-\overline{X})^2}{5}=\frac{10}{5}=2.$$

$$D(Y)=\frac{(Y_1-\overline{Y})^2+(Y_2-\overline{Y})^2+\cdots+(Y_5-\overline{Y})^2}{5}=\frac{74}{5}=14{,}8.$$

Demak, $X$ tasodifiy miqdorning dispersiyasi $Y$ tasodifiy miqdorning dispersiyasidan kichik. Amaliy jihatdan bu natija ikkinchi nomzodning mehnat unumdorligi barqaror emasligini ko'rsatadi: ba'zi kunlari u imkoniyatlaridan to'laligicha foydalanmasdan ishladi, boshqa kunlari esa imkoniyat darajasidan ko'proq ishlashga harakat qildi, bu esa, albatta, bajarilayotgan ishning sifatiga salbiy ta'sir qilishi mumkin. Ko'rinib turibdiki, natijada ish beruvchi birinchi nomzodni ishga olishni afzal ko'radi.

**Javob:** ish beruvchi birinchi nomzodni ishga olishni afzal ko'radi.

**3-masala.** Ikkita kamonchi nishonga kamon o'qidan otganda oladigan ochkolari – $X$ va $Y$ tasodifiy miqdorlarning ehtimolliklar bo'yicha taqsimot jadvali ma'lum:

1-kamonchi uchun

| $X$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $P$ | 0,15 | 0,11 | 0,04 | 0,05 | 0,04 | 0,10 | 0,10 | 0,04 | 0,05 | 0,12 | 0,20 |

va 2- kamonchi uchun

| $Y$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $P$ | 0,01 | 0,03 | 0,05 | 0,09 | 0,11 | 0,24 | 0,21 | 0,10 | 0,10 | 0,04 | 0,02 |

Kamonchilardan qaysi biri kamondan nishonga yaxshiroq otadi?

△ Ravshanki, kamonchilardan qaysi birining nishonga tegadigan o‘rtacha ochkosi ko‘proq bo‘lsa, shunisini yaxshi nishonga oluvchi deyish mumkin. Shu sababli, $X$ va $Y$ tasodifiy miqdorlarning matematik kutilmasini hisoblaymiz:

$$E(X) = 0 \cdot 0{,}15 + 1 \cdot 0{,}10 + 2 \cdot 0{,}04 + \dots + 9 \cdot 0{,}12 + 10 \cdot 0{,}20 = 5{,}36,$$

$$E(Y) = 0 \cdot 0{,}01 + 1 \cdot 0{,}03 + 2 \cdot 0{,}05 + \dots + 9 \cdot 0{,}04 + 10 \cdot 0{,}02 = 5{,}36,$$

ya’ni, ikkala kamonchining ham nishonga tegadigan ochkolari o‘rtacha bir xil.

Endi $X$ va $Y$ larning dispersiya va o‘rta kvadrat chetlanishlarini hisoblab ko‘raylik:

$$D(X) = (0 - 5{,}36)^2 \cdot 0{,}15 + (1 - 5{,}36)^2 \cdot 0{,}11 + \dots +$$
$$+ (10 - 5{,}36)^2 \cdot 0{,}20 - 13{,}6,$$
$$\sigma(X) = \sqrt{D(X)} = 3{,}69;$$

$$D(Y) = (0 - 5{,}36)^2 \cdot 0{,}01 + (1 - 5{,}36)^2 \cdot 0{,}03 + \dots +$$
$$+ (10 - 5{,}36)^2 \cdot 0{,}02 = 4{,}17,$$
$$\sigma(Y) = \sqrt{D(Y)} = 2{,}04.$$

Shunday qilib, nishonga tegadigan ochkolarning o‘rta qiymatlari teng $E(X) = E(Y)$ bo‘lsa-da, ikkinchi kamonchi uchun dispersiya birinchi kamonchiga qaraganda kichikroq: $D(Y) < D(X)$, ya’ni ikkinchi kamonchining nishonga tegadigan ochkolarining „markaz“ ($E(Y) = 5{,}36$) atrofida joylashish tarqoqligi birinchi kamonchiga nisbatan kichikroq. Boshqacha aytganda, uning natijalari birinchi kamonchining natijalariga qaraganda 5,36 dan uzoqroqqa ketib qolmagan. Demak, u birinchi kamonchiga qaraganda yuqoriroq natijalarga erishishi uchun nishonga yaxshiroq poylab, $E(Y)$ ni o‘ngroqqa (yuqoriroqqa) siljitishga harakat qilishi kerak.

**Javob:** kamonchilardan birinchisi nishonga yaxshiroq otadi. ▲

**4- masala.** Musobaqalar davrida futbol jamoasi o‘yinchilari tomonidan raqib darvozasiga kiritgan to‘plari soni $X$ ning chastotalar bo‘yicha taqsimoti berilgan:

| $X$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $Y$ | 3 | 3 | 2 | 1 | 1 |

Barcha kiritilgan to‘plar sonining o‘rtacha qiymatdan o‘rta kvadrat chetlanishini hisoblang.

△ Avval o‘rtachani hisoblaymiz:

$$\bar{X} = \frac{X_1M_1 + X_2M_2 - \cdots + X_5M_5}{M_1 - M_2 - \cdots + M_5} =$$

$$= \frac{0\cdot 3 + 1\cdot 3 + 2\cdot 2 + 3\cdot 1 + 4\cdot 1}{3+3+2+1+1} = \frac{0+3+4+3+4}{10} = \frac{14}{10} = 1{,}4.$$

Keyingi hisoblash natijalari quyidagi jadvalda keltirilgan:

| $X$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $M$ | 3 | 3 | 2 | 1 | 1 |
| $X-\bar{X}$ | –1,4 | –0,4 | 0,6 | 1,6 | 2,6 |
| $(X-\bar{X})^2$ | 1,96 | 0,16 | 0,36 | 2,56 | 6,76 |
| $(X-\bar{X})^2\cdot M$ | 5,88 | 0,48 | 0,72 | 2,56 | 6,76 |

U holda, dispersiya va o‘rta kvadrat chetlanish quyidagicha hisoblanadi:

$$D = \frac{(X_1-\bar{X})^2M_1 + (X_2-\bar{X})^2M_2 + \cdots + (X_5-\bar{X})^2M_5}{M_1+M_2+\cdots+M_5} =$$

$$= \frac{5{,}88+0{,}48-0{,}72+2{,}56+6{,}76}{10} = \frac{16{,}4}{10} = 1{,}64,$$

$$\sigma = \sqrt{D} = \sqrt{1{,}64} \approx 1{,}28.$$

**Javob:** $\sigma \approx 128$. ▲

### *Mashqlar*

1. Ko‘p yillik statistik ma’lumotlar asosida 4 ta farzandli oilalardagi o‘g‘il bolalar soni – $X$ tasodifiy miqdorning taqsimot qonuni quyidagi jadvalda berilgan bo‘lsa, uning matematik kutilmasi va dispersiyasini hisoblang.

| $X$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $P$ | 0,055 | 0,235 | 0,375 | 0,265 | 0,070 |

2. Ikki gimnastchining sport musobaqasidagi chiqishiga 9 ta hakam 10 balli tizimda qo'ygan ballari quyidagi jadvalda berilgan:

| Gimnastchining nomeri | Hakamning nomeri va qo'ygan ballari | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 1 | 8,7 | 8,8 | 8,9 | 8,9 | 8,7 | 9,2 | 8,9 | 9,6 | 8,8 |
| 2 | 9,0 | 9,1 | 9,0 | 8,8 | 8,5 | 8,9 | 9,0 | 9,0 | 9,1 |

Har bir gimnastchi olgan ballarini, mos ravishda, $X$ va $Y$ tasodifiy miqdorlar deb qaralsa, ularning matematik kutilmasini va dispersiya hamda o'rta kvadrat chetlanishlarini hisoblang va solishtiring.

3. Xaridorlarning oyoq kiyimlarga bo'lgan talabini o'rganayotgan talaba ikkita do'konda har kuni sotilgan oyoq kiyimlar sonini 25 kun davomida yozib bordi. Agarda $X_1$ birinchi do'konda, $X_2$ ikkinchi do'konda sotilgan oyoq kiyimlar soni bo'lsa, u holda quyidagi jadvallarda keltirilgan ma'lumotlarga asosan $X_1$ va $X_2$ tasodifiy miqdorlarning matematik kutilmasi va o'rta kvadrat chetlanishini hisoblang. Olingan natijalarni taqqoslab, do'konlardagi oyoq kiyim sotilishini solishtiring.

| $X_1$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $Y$ | 2 | 7 | 4 | 7 | 2 | 3 |

| $X_2$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $Y$ | 3 | 5 | 4 | 7 | 5 | 1 |

4. Silindr ko'rinishidagi po'latdan yasalgan g'o'lachalar partiyasidan olingan yigirmata g'o'lacha asoslarining $d$ diametrlari ikkita turli o'lchov asboblari yordamida o'lchandi. Birinchi o'lchov asbobi yordamida (1 mm gacha aniqlikda) olingan natijalar chapdagi, ikkinchisida olingan natijalar esa o'ngdagi jadvalda keltirilgan:

| $d_1$ | 58 | 59 | 60 | 61 | 62 |
|---|---|---|---|---|---|
| $M_1$ | 2 | 4 | 8 | 4 | 2 |

| $d_2$ | 59 | 60 | 61 | 62 |
|---|---|---|---|---|
| $M_2$ | 4 | 10 | 4 | 2 |

$d_1$ va $d_2$ tasodifiy miqdorlarning dispersiyalarini taqqoslang.

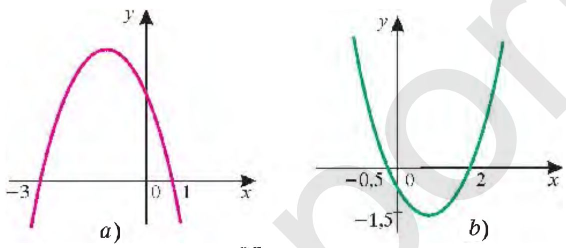
## IX SINF „ALGEBRA“ KURSINI TAKRORLASH UCHUN MASHQLAR

**512.** Funksiyaning grafigini yasang:

1) $y = x^2 + 6x - 9$;  2) $y = x^2 - \frac{7}{2}$;  3) $y = x^2 - 12x + 4$;

4) $y = x^2 + 3x - 1$;  5) $y = x^2 + x$;  6) $y = x^2 - x$.

**513.** (Og‘zaki.) $y = ax^2 + bx + c$ funksiya grafigidan foydalanib (93- rasm), uning xossalarini aniqlang.




*93- rasm.*

**514.** Funksiyaning grafigini yasang va xossalarini aniqlang:

1) $y = -2x^2 - 8x - 8$;  2) $y = 3x^2 + 12x + 16$;

3) $y = 2x^2 - 12x + 19$;  4) $y = 3 + 2x - x^2$.

**515.** Funksiyaning grafigini bitta koordinata tekisligida yasang:

1) $y = \frac{1}{3}x^2$ va $y = -\frac{1}{3}x^2$;  2) $y = 3x^2$ va $y = 3x^2 - 2$.

Tengsizlikni yeching (**516–519**):

**516.** 1) $(x-5)(x+3) > 0$;  2) $(x+15)(x+4) < 0$.

**517.** 1) $x^2 + 3x > 0$;  2) $x^2 \quad x\sqrt{5} < 0$;  3) $x^2 - 16 \le 0$;

4) $x^2 - 3 > 0$;  5) $x^2 - 4x \le 0$;  6) $x^2 - 7 \ge 0$.

**518.** 1) $x^2 - 8x + 7 > 0$;  2) $x^2 + 3x - 54 < 0$;

3) $\frac{1}{2}x^2 + 0,5x - 1 > 0$;  4) $5x^2 + 9,5x - 1 < 0$.

**519.** 1) $x^2 - 6x + 9 > 0;$ 2) $x^2 - 24x + 144 \leq 0;$

3) $\frac{1}{2}x^2 - 4x + 8 < 0;$ 4) $\frac{1}{3}x^2 - 4x + 12 \geq 0.$

Tengsizlikni intervallar usuli bilan yeching (**520–522**):

**520.** 1) $(x+3)(x-4) > 0;$ 2) $\left(x - \frac{1}{2}\right)(x - 0,7) < 0;$

3) $(x-2,3)(x+3,7) < 0;$ 4) $(x+2)(x-1) \leq 0.$

**521.** 1) $(x+2)(x-1) \geq 0;$ 2) $(x+2)(x-1)^2 \leq 0;$

3) $(x+2)(x-1)^2 > 0;$ 4) $(2-x)(x+3x)^2 \geq 0.$

**522.** 1) $\frac{3-x}{2+x} > 0;$ 2) $\frac{0,5+x}{x-2} < 0;$ 3) $\frac{(x-1)(x+2)}{x} < 0;$

**523.** Trapetsiyaning yuzi 19,22 $cm^2$ dan ortiq. Uning o‘rta chizig‘i balandligidan ikki marta katta. Trapetsiyaning o‘rta chizig‘ini va balandligini toping.

**524.** Parallelogrammning tomoni shu tomonga tushirilgan balandlikdan 2 cm ortiq. Agar parallelogrammning yuzi 15 $cm^2$ dan ortiq bo‘lsa, shu tomonning uzunligini toping.

**525.** Tengsizlikni intervallar usuli bilan yeching:

1) $(x+2)(x+5)(x-1)(x+4) > 0;$ 2) $\frac{3x-1}{3x+1} + \frac{x-3}{x+3} \geq 2.$

**526.** Agar $x^2+px+q$ kvadrat uchhad $x=0$ bo‘lganda $-14$ ga teng qiymatni, $x=-2$ bo‘lganda esa $-20$ ga teng qiymatni qabul qilsa, shu kvadrat uchhadning $p$ va $q$ koeffitsiyentlarini toping.

**527.** Agar $y = x^2 + px + q$ parabola:

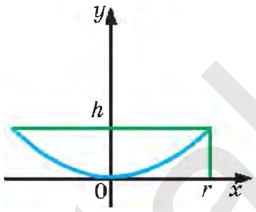
1) abssissalar o‘qini $x = \frac{1}{2}$ va $x = \frac{2}{3}$ nuqtalarda kessa;

2) abssissalar o‘qi bilan $x = -7$ nuqtada urinsa;

3) abssissalar o‘qini $x = 2$ va ordinatalar o‘qini $y = -1$ nuqtada kesib o‘tsa, $p - q$ ni toping.

**528.** Agar parabola abssissalar o‘qini 5 nuqtada kessa va uning uchi $\left(2\frac{3}{4}; 10\frac{1}{8}\right)$ nuqta bo‘lsa, shu parabolaning tenglamasini yozing.

**529.** Teleskopning (reflektorning) qaytaruvchi ko‘zgusi o‘q kesimi bo‘yicha parabola shakliga ega (94- rasm). Shu parabolaning tenglamasini yozing.




*94- rasm.*

**530.** Agar $y=ax^2+bx+c$ kvadrat funksiyaning grafigi:
1) $A(-1;\ 0)$, $B(3;\ 0)$ va $C(0;\ -6)$ nuqtalardan o‘tsa;
2) $K(-2;0)$, $L(1;0)$, $M(0;\ 2)$ nuqtalardan o‘tsa, uning koeffitsiyentlarini toping.

**531.** Istalgan nomanfiy $a$ va $b$ sonlar uchun
1) $a^2+b^2 \le (a+b)^2$;  2) $a^3+b^3 \le (a+b)^3$
tengsizlikning to‘g‘ri bo‘lishini isbotlang.

**532.** Funksiyaning grafigini yasang:

1) $y=\sqrt{x^2}$;  2) $y=|x-1|$;
3) $y \quad \sqrt{x^2-6x+9}$;  4) $y \quad \sqrt{x^2+4x-4}$.

**533.** Tenglamaning haqiqiy ildizlarini toping:
1) $x^2-|x|-2=0$;  2) $x^2-4|x|+3=0$;  3) $|x^2-x|=2$;
4) $|x^2+x|=1$;  5) $|x^2-2|=2$;  6) $|x^2-26|=10$.

**534.** Ildiz chiqaring:

1) $\sqrt[5]{7\frac{19}{32}}$;  2) $\sqrt{5\frac{4}{9}}$;  3) $\sqrt[3]{\frac{8b^6}{343a^9}}$, $a \ne 0$;  4) $\sqrt[4]{\frac{16x^8}{81y^4}}$, $y>0$.

**535.** Soddalashtiring:

1) $(3\sqrt{20}+7\sqrt{15}-\sqrt{5}):\sqrt{5}$;  2) $(\sqrt[3]{7}-\sqrt[3]{14}+\sqrt[3]{56}):\sqrt[3]{7}$;

3) $2\sqrt{\frac{3}{2}} + \sqrt{6} \quad 3\sqrt{\frac{2}{3}}$;  4) $7\sqrt{1\frac{3}{4}} \quad \sqrt{7}+0{,}5\sqrt{343}$.

**536.** Ifodalarning qiymatlarini taqqoslang:

1) $\left(\frac{\sqrt{5}}{3}\right)^{-1/3}$ va $\left(\frac{\sqrt{5}}{3}\right)^{-1/2}$;  2) $(2\sqrt{0{,}5})^{0{,}3}$ va $(2\sqrt{0{,}5})^{0{,}37}$.

**537.** Ifodani soddalashtiring:

1) $\frac{\sqrt[6]{a\sqrt[3]{a^{-1}}}}{a^{\frac{2}{9}}}$;  2) $\frac{\sqrt[4]{x^3\sqrt[3]{x}}}{x^{\frac{1}{3}}}$;  3) $(16a^{-1})^{-\frac{3}{4}}$;  4) $(27b^{-6})^{\frac{2}{3}}$.

**538.** Ildiz belgisi ostidan ko‘paytuvchini chiqaring:

1) $\sqrt{9a^2b}$, bunda $a < 0$, $b > 0$; 2) $\sqrt{25a^2b^3}$, bunda $a > 0$, $b > 0$;

**539.** Ko‘paytuvchini ildiz belgisi ostiga kiriting:

1) $x\sqrt{5}$, bunda $x > 0$; 2) $x\sqrt{3}$, bunda $x < 0$;

3) $-a\sqrt{3}$, bunda $a \geq 0$; 4) $-a\sqrt{5}$, bunda $a < 0$.

**540.** $y = -\frac{25}{x}$ funksiyaning grafigiga:

1) $A(\sqrt{5};\ \ 5\sqrt{5})$; 2) $B(\ \ 5\sqrt{2}; 5\sqrt{2})$; 3) $C(0{,}1;\ 250)$

nuqta tegishli bo‘lish yoki bo‘lmasligini aniqlang.

**541.** $y = \sqrt{1-2x}$ funksiya grafigiga: 1) $C\left(\frac{1}{4};\frac{\sqrt{2}}{2}\right)$; 2) $D\left(-\frac{1}{2};1\right)$;

$E\ (-4;\ 3)$ nuqta tegishli bo‘lish yoki bo‘lmasligini aniqlang.

**542.** Funksiyaning grafigini yasang:

1) $y = x^2 + 6x + 10$; 2) $y = -x^2 - 7x - 6$.

**543.** $P(1;\ 0)$ nuqtani: 1) $A(0;\ 1)$; 2) $B)\ (0;\ -1)$; 3) $C(-1;\ 0)$; 4) $D(1;\ 0)$ nuqtaga o‘tkazadigan bir necha burish burchaklarini ko‘rsating.

**544.** Hisoblang: 1) $\dfrac{\sin\frac{\pi}{4} + \cos\frac{\pi}{3} - \mathrm{tg}\frac{\pi}{3}}{\mathrm{ctg}\frac{\pi}{6} - \sin\frac{\pi}{6} - \cos\frac{\pi}{4}}$; 2) $\dfrac{\cos\frac{\pi}{4} - \sin\frac{\pi}{6} - \mathrm{tg}\frac{\pi}{4}}{\mathrm{ctg}\frac{\pi}{4} - \cos\frac{\pi}{3} - \sin\frac{\pi}{4}}$.

**545.** Sonning musbat yoki manfiy ekanligini aniqlang:

1) $\sin\frac{\pi}{5}\sin\frac{4\pi}{5}\cos\frac{\pi}{6}$; 2) $\sin\alpha\cos(\pi+\alpha)\mathrm{tg}\alpha$, $0 < \alpha < \frac{\pi}{2}$.

**546.** Berilgan: $\sin\alpha = 0{,}6$, $\sin\beta = -0{,}28$, $0 < \alpha < \frac{\pi}{2}, \pi < \beta < \frac{3\pi}{2}$.

Hisoblang: 1) $\cos(\alpha-\beta)$; 2) $\sin(\alpha+\beta)$; 3) $\cos(\alpha+\beta)$.

**547.** Ko‘paytuvchilarga ajrating:

1) $\sin2\alpha - 2\sin\alpha$; 2) $\sin\alpha + \sin\frac{\alpha}{2}$;

3) $\cos\alpha - \sin2\alpha$; 4) $1 - \sin2\alpha - \cos^2\alpha$.

**548.** Agar 1) $\cos\frac{\alpha}{2} = \ \frac{8}{17}$ va $\sin\frac{\alpha}{2} < 0$; 2) $\sin\frac{\alpha}{2} = \ \frac{5}{13}$ va $\cos\frac{\alpha}{2} < 0$ bo‘lsa $\sin\alpha, \cos\alpha, \mathrm{tg}\alpha$ ni hisoblang.

**549.** Agar

1) $a_1=10, d=6, n=23$; 2) $a_1=42,\ d=\frac{1}{2},\ n=12$;

3) $a_1=0, d=-2, n=7$; 4) $a_1=\frac{1}{3},\ d=\frac{2}{3},\ n=18$

bo'lsa, arifmetik progressiyaning $n$- hadini va dastlabki $n$ ta hadining yig'indisini hisoblang.

**550.** Agar $a_1 = 2, a_n = 120, n= 20$ bo'lsa, arifmetik progressiyaning dastlabki $n$ ta hadi yig'indisini toping.

**551.** $n$- hadi $a_n \quad \frac{1-2n}{3}$ formula bilan berilgan ketma-ketlik arifmetik progressiya bo'lishini isbotlang.

**552.** Agar geometrik progressiya uchun

1) $b_1 = 5$ va $q = -10$ bo'lsa, $b_4$ ni toping;

2) $b_4 = -5000$ va $q = -10$ bo'lsa, $b_1$ ni toping.

**553.** Agar:

1) $b_1=3, q=2,\ n=5$; 2) $b_1 = 1,\ q = 5,\ n = 4$

bo'lsa, geometrik progressiyaning $n$- hadini va dastlabki $n$ ta hadi yig'indisini hisoblang.

**554.** Agar: 1) $b_1=\frac{1}{4}, q=2, n=6$; 2) $b_1 = \frac{1}{5}, q = -5,\ n = 5$ bo'lsa, geometrik progressiya dastlabki $n$ ta hadining yig'indisini toping.

**555.** Cheksiz kamayuvchi geometrik progressiya yig'indisini toping.

1) $6,\ 4,\ \frac{8}{3},\ldots$; 2) $5,\ -1,\ \frac{1}{5},\ldots$; 3) $1,\ -\frac{1}{4},\ \frac{1}{16},\ \ldots$ .

**556.** Ildiz belgisi ostidan ko'paytuvchini chiqaring:

1) $\sqrt{20a^4b}$, bunda $a < 0,\ b > 0$; 2) $\sqrt{(a-1)^2}$, bunda $a < 1$;

**557.** Ifodani soddalashtiring:

1) $\frac{\sqrt{(a\ \ b)^2}}{a-b}$, bunda $a > b$; 2) $\frac{\sqrt{(a-b)^2}}{a-b}$, bunda $b > a$.

**558.** Maxrajdagi irratsionallikni yo'qoting:

1) $\frac{1}{2+\sqrt[3]{3}}$; 2) $\frac{1}{\sqrt{a}-\sqrt[4]{b}}$; 3) $\frac{1}{\sqrt[3]{3}\ \ \sqrt[3]{2}}$; 4) $\frac{2}{\sqrt{5}+\sqrt{5}}$.

**559.** Ifodani soddalashtiring:

1) $\frac{\sqrt{ab}\,\sqrt[4]{a}}{(a\ \ 2)\sqrt[4]{a\ \ ^{1}b^{2}}} - \frac{a^2-4}{a^2\ \ 4}$;  2) $\left(\frac{\sqrt{a}}{b+\sqrt{ab}} - \frac{\sqrt{a}}{b-\sqrt{ab}}\right)\cdot\frac{b-a}{2\sqrt{ab}}$.

**560.** Tenglamani yeching:

1) $\sqrt{x\ \ 2} = 4$;  2) $\sqrt{x+3} = 8$;  3) $\sqrt{2x+1} = \sqrt{x\ \ 1}$.

**561.** Ifodani soddalashtiring:

1) $\frac{tg^2\alpha}{1+ctg^2\alpha}$;  2) $\frac{1+ctg^2\alpha}{ctg^2\alpha}$;  3) $\frac{tg\alpha - tg\beta}{ctg\alpha + ctg\beta}$;

4) $(tg\alpha + ctg\alpha)^2 - (tg\alpha - ctg\alpha)^2$;  5) $(\sin\alpha - \cos\alpha)^2 + 2\sin\alpha\cos\alpha$.

**562.** Tenglamani yeching:

1) $1 - \cos x - 2\sin\frac{x}{2} = 0$;  2) $1 + \cos 2x + 2\cos x = 0$.

**563.** Ayniyatni isbotlang:

1) $\frac{tg(\alpha-\beta)+tg\beta}{tg(\alpha+\beta)-tg\beta} = \frac{\cos(\alpha-\beta)}{\cos(\alpha-\beta)}$;  2) $\frac{\sin(\alpha+\beta)+\sin(\alpha-\beta)}{\cos(\alpha+\beta)+\cos(\alpha-\beta)} = tg\alpha$.

**564.** Ayniyatni isbotlang:

1) $1 + \sin\alpha = 2\cos^2\left(\frac{\pi}{4}\ \ \frac{\alpha}{2}\right)$;  2) $1\ \ \sin\alpha = 2\sin^2\left(\frac{\pi}{4}\ \ \frac{\alpha}{2}\right)$.

**565.** Arifmetik progressiyada $a_1 + a_5 = \frac{5}{3}$; $a_3 a_4 = \frac{65}{72}$. Progressiyaning dastlabki o‘n yettita hadining yig‘indisini toping.

**566.** Geometrik progressiyada $q - 3$, $S_6 - 1\,820$ bo‘lsa, $b_1$ va $b_5$ ni toping.

**567.** Cheksiz kamayuvchi geometrik progressiyaning yig‘indisi $\frac{8}{5}$ ga teng, ikkinchi hadi $-\frac{1}{2}$ ga teng. Uchinchi hadini toping.

Ifodani soddalashtiring:

**568.** 1) $\sqrt{5+\sqrt{21}}$;  2) $\sqrt{4+\sqrt{7}}$;  3) $\sqrt{5+2\sqrt{6}}$;  4) $\sqrt{8\ \ 2\sqrt{15}}$.

**569.** Agar: 1) $tg\frac{\alpha}{2} = -2{,}4$; 2) $\sin\frac{\alpha}{2} = \frac{12}{13}$ bo‘lsa, $\sin\alpha$ va $\cos\alpha$ ni hisoblang.

# JAVOBLAR

**2.** 2) $x_1 - 0, x_2 - 1$; 4) $x$ ning berilgan funksiyaning qiymati –5 ga teng bo‘ladigan haqiqiy qiymatlari yo‘q. **3.** 2) $x_1 - 1\frac{3}{4}$. $x_2 = -1$; 4) $x_1 = 0$, $x_2 - \frac{3}{4}$. **4.** 2) 0; 4) 1. **5.** 2) nollari yo‘q; 4) $x_1 = \frac{2}{3}, x_2 = \frac{1}{2}$; 6) nollari yo‘q. **6.** 2) $p = 3, q = -4$; 4) $p = -2$, $q = -15$. **7.** $x_{1,2} = \pm 2$. **9.** $B$ va $C$. **12.** 2) $(\sqrt{5}; 5), (-\sqrt{5}; 5)$; 4) (0; 0), (2; 4); 6) (1; 1). **13.** 2) Ha. **14.** 2) Ha; 4) yo‘q; **16.** 1) $x < -3, x > 3$; 2) $-5 < \leq x \leq 5$; 3) $x \leq -4, x \geq 4$; 4) $-6 < x < 6$. **20.** 2) (–3; –4,5), (2; –2). **21.** 2) Ha; 4) yo‘q. **22.** 1) O‘suvchi; 2) kamayuvchi; 3) o‘suvchi; 4) o‘suvchi ham, kamayuvchi ham bo‘lmaydi. **23.** 3 m/s². **26.** 2) (0; –5); 4) $\left(\frac{1}{8}; \frac{1}{16}\right)$. **27.** 2) $x = -2$; 4) $x = 2$; 6) $x - \frac{3}{4}$. **28.** 2) Yo‘q; 4) yo‘q. **29.** 2) (1; 0), (0,5; 0), (0; –1); 4) (0; 0), $\left(\frac{4}{3}; 0\right)$.

**30.** $y = x^2 - 2x + 3$. **32.** 2) $k = -10$. **34.** 1) $y = 2(x - 3)^2$; 2) $y = 2x^2 - 4$; 3) $y = 2(x + - 2)^2 - 1$; 4) $y = 2(x - 1,5)^2 + 3,5$. **35.** 2) $\left(-\frac{3}{2}; \frac{11}{4}\right)$; 4) $\left(\frac{5}{2}; \frac{21}{4}\right)$. **36.** 2) (1;0), (–5; 0), (0; 10); 4) (0; 14). **40.** 7,5+7,5. **41.** 5 va 5. **42.** Devorga parallel tomon 6 m; qolgan tomonlari 3 m dan. **43.** Yo‘q. **44.** 2) $x - 1$ da $y - -5$ eng kichik qiymat; 4) $x = 1$ da $y = -2$ eng kichik qiymat. **45.** 1) $a > 0, b > 0, c > 0$; 2) $a < 0, b > 0, c < 0$. **46.** 1) 5 s dan keyin eng katta balandlik 130 m ga teng; 2) $(5 + \sqrt{26})s$. **48.** 2) $3x^2 - x - 1 > 0$; 4) $2x^2 + x - 5 < 0$. **50.** 2) $3 < x < 11$; 4) $x < -7, x > -1$. **51.** 2) $x < -3, x > 3$; 4) $x < 0, x > 2$. **52.** 2) $-2 < x < 1$; 4) $x < -3, x > 1$; 6) $x < -1, x > \frac{1}{3}$. **53.** 2) $x = \frac{1}{6}$; 4) $x < -4, x > 2$. **56.** Musbat qiymatlar $x < -3, x > 2$ oraliqlarda, manfiy qiymatlar $-3 < < x < 2$ intervalda. **58.** 2) $x \leq -1, x \geq 4$; 4) $-1 < x < 4$. **59.** 2) $x < -\frac{1}{3}$, $x > 2$; 4) $x \leq -0,25$; $x \geq 1$. **60.** 2) $x = 7$; 4) yechimlari yo‘q. **61.** 2) Yechimlari yo‘q; 4) yechimlari yo‘q; 6) $x$ – istalgan haqiqiy son. **62.** 2) $x < -\sqrt{7}$, $x > \sqrt{7}$; 4) $x < -2$; $x > 0$. **64.** 2) $x < -\frac{5}{3}$ $x > \frac{5}{3}$; 4) $-1 < x < 4$; 6) $x$ – istalgan haqiqiy son. **65.** 2) $x$ – istalgan haqiqiy son; 4) $x \neq \frac{1}{4}$; 6) $-\frac{1}{3} \leq x \leq 0$. **66.** 2) Yechimlari yo‘q; 4) $-0,5 < x < 3$. **67.** 2) $x = 1$; 4) $x$ – istalgan haqiqiy son. **69.** $-6 < r < 2$. **71.** 2) $-5 < x < 8$; 4) $x < -5$, $x > 3\frac{1}{2}$. **72.** 2) $x < 0, x > 9$; 4) $-3 < x < 0$; 6) $x < -1, x > 3$. **73.** 2) $-\frac{1}{2} < x <$

$< 0,\ x > \frac{1}{2}$; 4) $-2 < x < 2, x > 5$. **74.** 2) $-7 < x < 7$; 4) $-4 < x < 4,\ x > 4$. **75.** $-3 < x < 4$; 4) $-3{,}5 \le x < 7$; 6) $-2 \le x < -1, x \ge 3$. **76.** 2) $x < 0{,}5,\ x > 1$; 4) $x < -\frac{2}{3}, 0 < x < \frac{1}{2}, x > \frac{2}{3}$. **77.** 2) $-4 < x < -2, x > 3$; 4) $-3 \le x \le -1, 4 \le x \le 5$. **78.** 2) $x < -2, 2 < x < 6$; 4) $x < -3, -1 \le x < 2, x \ge 4$. **79.** 2) $-\sqrt{15} < x < -3, 0 < x < \sqrt{15}$. **80.** 1) $-8 < x < -1$; 2) $x < -5,\ x > 2$; 3) $-1 < x \le -\frac{2}{5}$. **81.** 2) $x = 2$ da $y - 1$; $x - 0$ va $x - 4$ da $y - 5$; $x - -1$ va $x - 5$ da $y - 10$; $x - -2$ va $x - 6$ da $y = 17$. **82.** 1) $y(-2) = -1, y(0) = -5,\ y\left(\frac{1}{2}\right) = -11, y(3) = 4$; 2) $x = -\frac{1}{2}$ da $y = -3$; $x = -1$ da $y = -2$; $x = \frac{3}{2}$ da $y = 13$; $x = \frac{4}{3}$ da $y = 19$. **84.** 2) $x \le 2, x \ge 5$; 4) $-2 \le x < 3$. **85.** 1) $y(-3) = 3, y(-1) = 1, y(1) = -1, y(3) = 1$; 2) $x = 2$ da $y = -2$; $x = 0$ va $x = 4$ da $y = 0$; $x = -2$ va $x = 6$ da $y = 2$; $x = -4$ va $x = 8$ da $y = 4$. **86.** 2) $x \ne -1$; 5) $-1 \le x \le 1, x \ge 4$; 6) $-5 \le x \le 1, x > 2$. **87.** 2) Ha; 4 ha. **93.** 2) $x - 16$; 4) $x \quad \frac{1}{16}$; 6) $x = \frac{1}{243}$. **95.** 2) $x - 32$; 4) $x - 8$. **98.** 2) toq; 4) juft ham, toq ham bo'lmaydi. **99.** 2) toq; 4) toq. **108.** 2) $x = 0$. **109.** 2) $(-1; 0)$. **110.** 2) $x \le 3$; 4) $y < 5$; 6) $x < -5, x > 5$. **111.** 2) Kubning qirrasi 7 dm dan ortiq. **114.** 2) $x = 10$; 4) $x = 5$. **115.** 2) $x = 2$; 4) $x = 2$; $x = -7$. **116.** 2) $x = 4$; 4) $x = 0{,}2$. **117.** $x = \frac{7}{3}$. **118.** 2) $x > -3$; 4) $x < 2$; 6) $x < 1, x > 7$. **120.** 2) $x = -2$; 4) $x_1 = 1$; $x_2 = 3$. **121.** 2) $x = 2{,}25$. **122.** 2) $x = 1$; 4) $x = 5$. **123.** 2) $x = 4$. **124.** 2) $2 \le x \le 3$; 4) $1 < x \le 2$; 6) $x \ge 1$. **125.** 2) $x_1 = 2, x_2 = 0{,}5$; 4) $x$ ning bunday qiymati yo'q. **126.** 2) $x < -6,\ x > 6$. **127.** 2) $(5; 0), (-2; 0), (0; 10)$; 4) $(1; 0), \left(-\frac{11}{7}; 0\right), (0; -11)$. **128.** 2) $(-1; 4)$; 4) $\left(-\frac{1}{2}; 1\right)$. **130.** 150 m va 150 m. **131.** 2) $p = 1, q = 0$. **132.** 1) $x_1 = 1,\ x_2 = -5$; 2) $x_1 - 0, x_2 - 1, x_3 - 2$. **133.** 2) $x < 2, x > 4$; 4) $x < 3, x > 4$. **134.** 2) $x < -6,\ x > 6$; 4) $-\frac{3}{4} < x < \frac{3}{4}$. **135.** 2) $x < \frac{1}{2}, x > 4$; 4) $-2 < x < \frac{1}{2}$. **136.** 2) Yechimlari yo'q; 4) yechimlari yo'q; 6) yechimlari yo'q. **137.** 2) $x < -1, 1 < x < 4$; 4) $x < -\frac{1}{2}$, $4 < x \le 7$; 6) $x \ge 2, -\frac{1}{2} \le x < 1$. **138.** 2) $x \le -\frac{3}{2},\ x \ge -1$; 4) $x - \frac{2}{3}$. **139.** 2) $-1 < x < -\frac{1}{5}, \frac{3}{4} < x < 2$; 4) $-\frac{1}{3} < x < -\frac{1}{5}, \frac{1}{2} < x < 2$. **140.** 12 km/h dan kam emas. **142.** 2) $\left(-\frac{1}{\sqrt{2}};\ -\sqrt{2}\right), \left(-\frac{1}{\sqrt{2}};\ \sqrt{2}\right)$. **143.** 2) $(-1; -1)$; $(1; 1)$. **144.** 2) $x > 2$; 4)

$x \le -2$. **145.** 2) $x = 16$. **146.** 2) $x_1 = \frac{1}{2}$, $x_2 = \frac{1}{3}$. **147.** 2) $x$ – istalgan son; 4) $2 \le \le x \le 11$; 6) $x < -7$, $-3 \le x < -1$, $x \ge 3$. **148.** 2) kamayadi; 4) kamayadi. **149.** 2) toq; 4) juft ham, toq ham bo‘lmaydi. **150.** 2) $-2 \le x \le \frac{1}{3}$. **151.** 2) $x_1 = -1$, $x_2 = 7$; 4) $x = 81$. **152.** 1) $x < -1, x > 9$; 2) $-1 < x \le 0, 3 \le x < 4$; 3) $\frac{2}{3} \le x < 6$; 4) $x \ge 4$. **153.** 2) (4; 1); 4) (0,5; 3). **154.** 2) (7; –5), (–4; 6); 4) (–1; –1), (7; 23). **155.** 2) (4; –3); (17;10); 4) (4; 1), (–1; –4). **156.** 2) (1; 7), (7; 1); 4) (–2; –5), (–5; –2). **157.** 2) (4; –1); 4) (3; 1). **158.** 2) (2; 5), (5; 2), (–2; –5),(–5; –2); 4) (1; 5), (5; 1), (–1; –5), (–5; –1). **159.** 5 va 13. **160.** 4 va 36. **161.** 2) (7; –1), (–1;7). **163.** 1) (4; 1) (–1; –4); 2) (2; 4), (4; 2); 3) (2; 2). **164.** 300 m, 200 m. **165.** 2) (4; 5) va (5; 4). **166.** 2) (1; –2) va (3; 0). **167.** 2) (9; 4). **168.** 2) (3; 4),(4; 3),(–3; –4), (–4; –3). **169.** 2) (2; 5) va (5; 2); 4) (1; 3) va (19; –3). **170.** 2) (3; 5), (5; 3), (–3; –5), (–5; –3); 4) (1; 7), (7; 1), (–1; –7), (–7; –1). **171.** 2) (20; 4) va (–20; –4); 4) (3; 6) va (6; 3). **172.** 2) (–1; 1) (1; 1) $\left(-\frac{\sqrt{2}}{2};2\right)$, 2) $\left(\frac{\sqrt{2}}{2};2\right)$; 4) (–5; –2), (–5; 2), (5; –2). **173.** 2) (5; 1). **174.** 2) (–5; –1), (–3; –5),(3; 5),(5; 3), **175.** (1; 9) va (9; 1). **176.** 2) sistema yechimga ega emas. **177.** 2) $-9 \le x \le 3$; 4) $-6 \le x \le 2$. **178.** 2. $-\infty < x < -3$ va $2 < x < +\infty$. **179.** $-3 < x \le -2$ va $1 \le x \le 2$. **180.** $-7 < x < 0$. **181.** $-1 \le x \le 0$. **182.** 2) Ш. **194.** (–1; –4) va (4; 4); 2) (2; –2) va (9; 5). **195.** 2) (–5; 6) va (6; –5); 4) (–1; 10) va (10; –1). **196.** 2) (6; –2); 4) (3,5; –1,5). **197.** 2) (–2; –3) va (2; 3); 4) (2; 6) va (6; 2). **198.** 2) (–1; 3) va (3; –1). **199.** 2) (–3; 1) va (1; 5). **200.** 2) (–2; 1) va (2;1); 4) (–1;4) va (24; 0,6). **201.** 2) $(4; -\sqrt{3})$ va $(4; \sqrt{3})$; 4) (–6; –2), (–6; 2), (6; –2), (6; 2). **202.** 2) (1; –2) va (2; –1); 4) (2; 1). **203.** 2) $\left(-2\sqrt[4]{\frac{3}{5}};\sqrt{\frac{3}{5}}\right)$ va $\left(-2\sqrt[4]{\frac{3}{5}};\sqrt{\frac{3}{5}}\right)$. **204.** 2) (4; 1); 4) (100; 4). **205.** 2) 24. **206.** 2) Bo‘yi 1,2 cm va eni 0,8 cm. **207.** 2) $-5 < x < -3$; 4) $1 \le x \le 2$. **208.** 2) 8. **209.** 2) 27; 4) 1. **213.** 2) $\frac{2\pi}{3}$; 4) $\frac{5\pi}{6}$; 6) $\frac{8\pi}{45}$; 8) $\frac{7\pi}{9}$. **214.** 2) $20^\circ$; 4) $135^\circ$; 6) $\left(\frac{720}{\pi}\right)^\circ$; 8) $\left(\frac{324}{4\pi}\right)^\circ$. **215.** 2) 4,71; 4) 2,09. **216.** 2) $2\varpi < 6,7$; 4) $\frac{3\pi}{2} < 4,8$; 6) $-\frac{3\pi}{2} < -\sqrt{10}$. **218.** 0,4 m. **219.** 2 rad. **220.** $\frac{3\pi}{8}$ cm$^2$. **221.** 2 rad. **222.** 2) (–1; 0); 4) (0; –1); 6) (1; 0). **224.** 2) ikkinchi chorak; 4) to‘rtinchi chorak; 6) ikkinchi chorak. **225.** 2) (0; 1); 4) (–1; 0); 6) (0; 1). **226.** 2) $2\pi k, k = 0, \pm 1, \pm 2, \ldots$; 4) $\frac{\pi}{2} + 2\pi k$, $k - 0$, $\perp 1$, $\perp 2$, … . **227.** 2) ikkinchi chorak; 4) to‘rtinchi chorak. **228.**

2) $x - 1{,}8\pi, k - 4$; 4) $x = \frac{4}{3}\pi$, $k - 3$; 6) $x = \frac{5}{3}\pi$ $k - 2$. **230.** 2) (0; 1); 4) (0; −1). **231.** 2) $\frac{\pi}{6} + 2\pi k$, $k = 0, \pm1, \pm2, \ldots$; 4) $\frac{3\pi}{4} + 2\pi k$, $k = 0, \pm1, \pm2, \ldots$. **232.** 2) $-\frac{1}{2}$; 4) −1; 6) −1; 8) $\frac{1}{\sqrt{2}}$. **234.** 2) −1; 4) −1; 6) 1. **235.** 2) 0; 4) −1. **236.** 2) $\frac{\sqrt{2}\ \ 9}{2}$; 4) $-\frac{1}{4}$. **237.** 2) $x = \frac{\pi}{2} + \pi k$, $k = 0, +1, +2, \ldots$; 4) $x - \frac{\pi}{2} + 2\pi k$, $k = 0, +1, +2, \ldots$. **239.** 2) $-\frac{5}{4}$; 4) $\frac{1-\sqrt{2}}{2}$. **240.** 2) $x - \pi - 2\pi k$, $k - 0, \pm1, \pm2, \ldots$; 4) $x - \pi + 2\pi k$, $k - 0, \perp1, \perp2, \ldots$; 6) $x = \frac{2}{3}k\pi$, $k - 0, \perp1, \perp2, \ldots$. **241.** 2) $x - 2\pi k - 1$, $k - 0, \perp1, \perp2, \ldots$; 4) $x = k\pi - 1$, $k = 0, \pm1, =2, \ldots$; 6) $x = \frac{2\pi k}{3} + 1$, $k = 0, \pm1, \pm2, \ldots$. **242.** 2) ikkinchi chorak; 4) ikkinchi chorak; 6) ikkinchi chorak. **243.** 2) musbat; 4) musbat; 6) musbat. **244.** 2) manfiy; 4) manfiy; 6) musbat. **245.** 2) musbat, musbat; 4) manfiy, manfiy; 6) manfiy, manfiy; 8) musbat, musbat. **246.** 2) $\sin\alpha < 0$, $\cos\alpha > 0$, $\mathrm{tg}\alpha < 0$, $\mathrm{ctg}\alpha < 0$; 4) $\sin\alpha > 0$, $\cos\alpha > 0$, $\mathrm{tg}\alpha > 0$, $\mathrm{ctg}\alpha > 0$. **247.** 2) $\sin 3 > 0$, $\cos 3 < 0$, $\mathrm{tg}3 < 0$; 4) $\sin(-1{,}3) < 0$, $\cos(-1{,}3) > 0$, $\mathrm{tg}(-1{,}3) < 0$. **248.** 2) manfiy; 4) musbat; 6) musbat; 8) manfiy. **249.** Agar $0 < \alpha < \frac{\pi}{2}$ yoki $\pi < \alpha < \frac{3\pi}{2}$ bo'lsa, $\sin\alpha$ va $\cos\alpha$ sonlarining ishoralari mos tushadi; agar $\frac{\pi}{2} < \alpha < \pi$ yoki $\frac{3\pi}{2} < \alpha < 2\pi$ bo'lsa, $\sin\alpha$ va $\cos\alpha$ sonlari qarama-qarshi ishoralarga ega. **250.** 2) manfiy; 4) musbat. **251.** 2) $\cos 1{,}3 > \cos 2{,}3$. **252.** 2) $x - \frac{\pi}{2} - k\pi$, $k = 0, \pm1, \pm2, \ldots$; 4) $x = \pi + 2k\pi$, $k = 0, \pm1, \pm2, \ldots$. **253.** 2) ikkinchi chorak. **254.** $\frac{h\cos\alpha}{1-\cos\alpha}$. **255.** 2) $\cos\alpha = -\frac{3}{5}$, $\mathrm{tg}\alpha = -\frac{4}{3}$; 4) $\cos\alpha = \frac{\sqrt{21}}{5}$, $\mathrm{tg}\alpha = \frac{2}{\sqrt{21}}$, $\mathrm{ctg}\alpha - \frac{\sqrt{21}}{2}$; 6) $\sin\alpha = -\frac{1}{\sqrt{10}}$, $\cos\alpha = \frac{3}{\sqrt{10}}$. **256.** 2) bajariladi; 4) bajarilmaydi. **257.** 2) bajarilmaydi. **258.** $\cos\alpha = \frac{9}{11}$, $\mathrm{tg}\alpha - \frac{2\sqrt{10}}{9}$. **259.** $\frac{1}{3}$. **260.** $\cos\alpha = \pm\frac{3}{4}$. **261.** $\sin\alpha - \perp\frac{2}{\sqrt{5}}$. **262.** 2) $\frac{1}{3}$; 4) 2. **263.** 1) $-\frac{3}{8}$; 2) $\frac{11}{16}$. **264.** 1) $x = \pi k$, $k = 0, \pm1, \pm2, \ldots$; 2) $x - -\frac{\pi}{2} - 2\pi k$, $k = 0, \pm1, \pm2, \ldots$; 3) $x = 2\pi k$, $k = 0, \pm1, =2, \ldots$; 4) $\frac{\pi}{2} + \pi k$, $k = 0, \pm1, \pm2, \ldots$. **266.** 1) 0; 4) $1 + \sin\alpha$. **267.**

2) 3; 4) 4. **271.** 2) $\frac{2}{\sqrt{3}}$. **272.** $\frac{8}{25}$. **273.** $\frac{37}{125}$. **274.** 1) $x - \pi k, k - 0, \pm 1, \pm 2, \ldots$; 2) $x - \frac{\pi}{2} + 2\pi k$, $k = 0, \pm 1, \pm 2, \ldots$. **275.** 2) $\frac{1}{3}$; 4) –3. **276.** 2) $2\cos\alpha$; 4) 2. **278.** 2) 2. **279.** 2) $-2\cos\alpha$. **280.** 2) $-\frac{1}{2}$; 4) $-\frac{1}{2}$. **281.** 2) $\frac{1}{\sqrt{2}}$; 4) –1. **282.** 2) $\frac{4\ \sqrt{2}}{6}$.

**283.** 2) $\cos 3\beta$; 4) –1. **284.** $-\sin\alpha \cdot \sin\beta$. **285.** 2) $\frac{\sqrt{3}}{2}$; 4) 1. **286.** 2) $-\frac{2+\sqrt{14}}{6}$. **287.** 2) $-\sin\alpha \cdot \cos\beta$; 4) $\sin\alpha \cdot \cos\beta$. **288.** $\cos(\alpha + \beta) = \frac{84}{85}$; $\cos(\alpha - \beta) - \frac{36}{85}$. **289.** 2) $-\frac{63}{65}$. **290.** 2) 0; 4) $\mathrm{tg}\alpha \cdot \mathrm{tg}\beta$. **293.** 2) $\frac{\sqrt{3}}{2}$; 4) $\frac{3}{2}$. **294.** 2) $\frac{1}{\sqrt{2}}$; 4) –1. **295.** 2) $\frac{24}{25}$. **296.** 2) $\frac{7}{25}$. **297.** 2) $\frac{1}{2}\sin 2\alpha$; 4) 1. **298.** 2) $2\mathrm{ctg}\alpha$; 4) $\mathrm{ctg}^2\alpha$. **300.** 2) $\frac{8}{9}$. **302.** 2) $\frac{1}{\sqrt{2}}$; 4) $\frac{\sqrt{3}}{2}$. **303.** 2) $\cos 6\alpha$; 4) $\frac{1}{2\sin\alpha}$. **305.** $\frac{15}{8}$. **306.** 2) $\sqrt{3}$. **307.** 2) 0; 4) 0; 6) –1. **308.** 2) $\frac{1}{\sqrt{3}}$; 4) $\frac{1}{\sqrt{3}}$; 6) $-\frac{1}{\sqrt{3}}$. **309.** 2) $\frac{1}{\sqrt{2}}$; 4) $-\frac{1}{\sqrt{2}}$. **310.** 2) $-\frac{1}{2}$; 4) $\frac{1}{2}$; 6) $\sqrt{3}$. **311.** 2) $-\sqrt{2}$; 4) –1. **312.** 2) $\cos 2\alpha$. **313.** 2) $-\frac{5\sqrt{3}}{6}$; 4) $\frac{1}{2}$; 6) $\frac{5-3\sqrt{3}}{4}$.

**314.** 2) 1; 4) $-\frac{1}{\cos\alpha}$. **317.** 2) $x - \frac{\pi}{2} + 2\pi k$, $k = 0, +1, +2, \ldots$; 4) $x = \pi + 2\pi k$, $k - 0, \pm 1, \pm 2, \ldots$. **318.** 2) $\sqrt{2}\sin\beta$; 4) $\sin 2\alpha$. **319.** 2) 0; 4) $-\frac{\sqrt{6}}{2}$; 6) $\frac{\sqrt{6}}{2}$. **320.** 2) $4\sin\left(\frac{\pi}{12} - \frac{\alpha}{2}\right)\cos\left(\frac{\pi}{12}\ \ \frac{\alpha}{2}\right)$; 4) $2\sin\left(\frac{\pi}{4} - \frac{\alpha}{2}\right)\cos\left(\frac{\pi}{4} - \frac{\alpha}{2}\right)$. **322.** 2) $2\sin\alpha$. **325.** 2) $2\sqrt{3}\sin\frac{5\pi}{24}\sin\frac{\pi}{8}$. **326.** 2) 0. **327.** 2) $2\cos\alpha(\cos\alpha - 1)$; 4) $(\sin\alpha + \cos\alpha) \cdot \left(1 - \frac{1}{\cos\alpha}\right)$.

**328.** 2) uchinchi chorak; 4) ikkinchi chorak; 6) ikkinchi chorak. **329.** 2) 0; 1; 4) 1; 0; 6) 0; –1. **330.** 2) 2; 4) –1. **331.** 2) $\frac{2}{\sqrt{5}}$; 4) $-\frac{1}{\sqrt{3}}$. **333.** 2) 3; 4) $\mathrm{tg}^2\alpha$.

**334.** 2) $-\frac{1}{3}$. **335.** 2) $-\frac{\sqrt{3}}{2}$; 4) $\frac{\sqrt{2}(\sqrt{3}+1)}{4}$. **336.** 2) $\sin 2\alpha$; 4) $\mathrm{tg}2\alpha$. **337.** 2) 1; 4)

$-\frac{1}{\sqrt{2}}$. **338.** 2) $-\frac{\sqrt{3}}{2}$; 4) $-1-\frac{1}{\sqrt{2}}$. **339.** 2) cos0 > sin5. **340.** 2) Musbat; 4) manfiy. **341.** 2) $\frac{\sqrt{2}(\sqrt{3}\ \ 1)}{4}$; 4) $\frac{\sqrt{6}-\sqrt{2}}{4}$; 6) $\frac{1}{\sqrt{2}}$. **342.** 2) $\frac{1}{\sin\alpha}$. **343.** $\cos\alpha - \ \frac{2}{3}$; $\mathrm{tg}\alpha=-\frac{\sqrt{5}}{2}$; $\mathrm{tg}\alpha=-\frac{\sqrt{5}}{2}$; $\sin2\alpha=-\frac{4\sqrt{5}}{9}$; $\cos2\alpha=-\frac{1}{9}$. **344.** 2) tgα. **345.** 2) $\frac{1}{\sin4\alpha}$; 4) $-\frac{1}{\cos2\alpha}$. **346.** 2) 1; 4) 1. **347.** 2) –7. **348.** 2) cos4α. **350.** 2) 5,8,11; 4) $-\frac{1}{3}, 0, \frac{1}{3}$ 6) –1,–8,–27. **352.** 2) Bo‘ladi; 4) bo‘ladi. **354.** 2) $n=9$. **360.** 2) –3, –1,1,3,5. **362.** 2) 79; 4) –42. **363.** 2) $a_n = 29 - 4n$; 4) $a_n = 6 - 5n$. **364.** 12. **365.** Ha,$n = 11$. **366.** $n = 11$,yo‘q. **367.** 2) 0,5. **368.** 2) –13. **369.** 2) –100. **370.** 2) $a_n = 5n - 17$. **371.** $n \geq 9$. **372.** $n < 25$. **373.** 2) $a_9 = -57, d = 7$; 4) $a_9 = -1, d = -15$. **374.** 30. **375.** 60. **376.** 2) 10050; 4) 2550. **377.** 4850. **378.** 4480. **379.** 2) –192. **380.** 2) 204. **381.** 2) 240. **382.** 4905; 494550. **383.** 2) 2900. **384.** 10. **385.** 2) $a_{10}=15\frac{5}{6}$ $d=\frac{3}{2}$. **386.** 2) $a_1=-88, d=18$. **387.** 78 ta to‘sin. **388.** 44. **389.** $a_1=5$, $d = 4$. **392.** 2) –3,12,–48,192,–768. **394.** 2) $\frac{1}{16}$; 4) $\frac{1}{81}$. **395.** 2) $b_n=3\cdot\left(\frac{1}{3}\right)^{n-1}$; 4) $b_n=3\cdot\left(-\frac{4}{3}\right)^{n-1}$. **396.** 2) 5; 4) 8. **397.** 2) 3; 4) $-\frac{1}{5}$. **398.** $b_8 = 2374, n = 5$. **399.** $b_7 - 3\sqrt{3}$, $q - \frac{1}{\sqrt{3}}$. . **400.** $b_5 - 6$, $b_1 = 30\frac{3}{8}$ yoki $b_5 - -6, b_1\ \ -30\frac{3}{8}$. **401.** 659100 so‘m. **402.** 0,25 cm². **403.** 2) $-\frac{31}{8}$; 4) $-\frac{275}{81}$; 6) –400. **404.** 2) 2186. **405.** 2) $b\ = -1, b_8 = 128$. **406.** 2) $n = 7$; 4) $n = 5$. **407.** 2) $n = 9, b_9 = 2048$; 4) $n = 5$, $q=7$. **408.** 2) 364; 4) 305. **409.** 2) $b_5 = 4802, S_4 = 800$. **410.** 2) $1\frac{31}{32}$. . **412.** 2) $q= 5, b_3 = 300$ yoki $q = -6, b_3 = 432$. **413.** 2) $q = 2$ yoki $q = -2$; 4) $S_5 = 781$ yoki $S_5= 521$. **415.** 2) ha; 4) ha. **416.** 2) 7,2; 4) $-8\frac{1}{6}$. . **417.** 2) $\frac{27}{4}$; 4) $\frac{2}{3}$. **418.** 2) yo‘q; 4) ha. **419.** 2) $90\frac{10}{11}$. **420.** 2) $6+4\sqrt{3}$. **421.** 2) $\frac{1}{2}$. **422.** $2a$. **423.** $R_n - \frac{1}{3^{n\ \ 1}}\cdot R_1$.

**424.** 2) 1; 4) $\frac{7}{30}$. **425.** 2) $d - -\frac{1}{2}$, $a_4 = 2$, $a_5 = 1\frac{1}{2}$; 4) $d = -3$, $a_4 - \sqrt{2} - 9$, $a_5 = \sqrt{2} - 12$. **427.** $-5\frac{1}{3}$. **428.** 2) –1080. **429.** 143. **430.** 2) –22. **431.** 2) $q = -\frac{1}{2}$, $b_4 = -\frac{1}{32}$, $b_5 \quad \frac{1}{64}$; 4) $q = -\sqrt{2}$, $b_4 - \quad 10\sqrt{2}$, $b_5 = 20$. **432.** 2) $b_n = -0{,}5 \cdot (-2)^{n-1}$. **433.** 2) $b_n = \frac{125}{8}$. **434.** 2) $S_{10} = 1\frac{85}{256}$; 4) $S_9 = 5$. **435.** 2) 242; 4) $\frac{65}{36}$. **436.** 2) $-\frac{4}{5}$. **437.** $24\frac{41}{74}$. **438.** 2) 14,11,8,5,2. **439.** $-\frac{5}{2}$. **440.** 2) $a_{19} = 0, a_1 = -108$. **441.** 2) $x_1 = \frac{1}{3}$; 4) $x_2 = -4$. **443.** 14. **444.** 2) $a_{16} = -1\frac{2}{3}$, $d - -\frac{2}{15}$. **445.** 2) 27. **446.** 2) –27; 4) $\pm\frac{1}{25}$. **447.** 6. **448.** 2) Yo'q; 4) ha. **450.** Chorshanba kuni. **451.** $a_1 = 8$ $d = -3$ yoki $a_1 = 2, d = 3$. **452.** $a_1 = 5, d = -5$ yoki $a_1 = -5, d = 5$. **453.** 180 marta. **453.** 2) Mumkin bo'lmagan. **454.** 2) Tasodifiy; 4) muqarrar. **457.** 2) Birgalikda bo'lmagan. **462.** Teng imkoniyatli emas. **466.** 2) $\frac{1}{28}$; 4) $\frac{3}{4}$. **467.** 2) $\frac{5}{9}$; 4) 1. **468.** 2) $\frac{1}{3}$; 4) $\frac{3}{4}$; 6) $\frac{7}{12}$. **469.** 2) $\frac{1}{2}$; 4) $\frac{1}{4}$; 6) $\frac{5}{12}$. **470.** 0,01. **471.** 2) 0,97. **472.** $\frac{29}{30}$. **473.** $\frac{1}{2}$; **474.** 2) $\frac{1}{13}$; 2) $\frac{9}{52}$. **476.** 2) $\frac{21}{46}$; 4) $\frac{7}{92}$. **477.** 1,4%. **482.** 2) Mumkin,4 ochko. **488.** 3 tanlanma. **489.** 2) 11; 4) 5 va 7. **490.** 2) 21; 4) 13. **491.** 2) 24. **492.** 2) –5,4; 4) 2,1. **494.** 2) $\frac{3}{7}$; 4) $\frac{3}{7}$. **495.** 2) 0,1. **496.** 2) 2,5 $kg^2$,4) $6m^2$. **502.** 2) 0,98; 4) 0,1; 6) 0,6. **503.** 2) 0,25. **505.** 2) 13,–3 va 10,2 3. **511.** 2) –0,5. **516.** 2) $-15 < x < 2$; 4) $x \le 12, x \ge 12$. **517.** 2) $0 < x < \sqrt{5}$;; 4) $x < -\sqrt{3}$; $x > \sqrt{3}$. **518.** 2) $-9 < x < 6$; 4) $-2 < x < 0{,}1$; 6) $x \le \frac{1}{8}$, $x \ge 2$. **519.** 2) $x = -12$; 4) $x$ – istalgan haqiqiy son; 6) yechimlari yo'q. **520.** 2) $-0{,}7 < x < \frac{1}{2}$; 2) $-2 \le x \le 1$. **521.** 2) $x \le -2, x = 1$; 4) $x \le -\frac{1}{3}$, $0 \le x \le 2$. **522.** 2) $-0{,}5 \le x < 2$. **523.** Balandlik 3,1 cm dan ortiq, o'rta chiziq 6,2 cm dan ortiq. **524.** 5 cm ortiq. **525.** 2)

$x < -7, -1 \le x \le 2$; 4) $-1 \le x < \frac{1}{3}., x > \frac{1}{3}$. **526.** $p = 5, q = -14$. **527.** 2) $p = 14, q - 49$. **528.** $y - -2x^2 + 11x - 5$. **529.** $y - \frac{n}{r^2}x^2$. **530.** 2) $a - -1, b - -1, c - 2$.

**531.** *Ko'rsatma.* 1) $\frac{a}{b} = A^3, \frac{b}{c} = B^3, \frac{c}{a} - C^3$ kabi belgilab va $ABC - 1$ tenglikni hisobga olib, berilgan tengsizlikni $A^3 + B^3 + C^3 \ge 3ABC$ ko'rinishda yozing, uni $(A + B + C)(A^2 + B^2 - C^2 - AB - AC - BC) \ge 0$ ko'rinishda almashtiring. ($A^2 + B^2 + C^2 \ge AB + AC + BC$ tengsizlik ushbu $A^2 + B^2 \ge 2AB$, $A^2 + C^2 \ge 2AC$, $B^2 + C^2 \ge 2BC$ tengsizliklarni qo'shish bilan hosil qilinadi; 2) o'rta arifmetik va o'rta geometrik miqdorlarga doir tengsizliklarni qo'shing: $\frac{bc}{a} + \frac{ac}{b} \ge 2c$, $\frac{ac}{b} + \frac{ab}{c} \ge 2a$, $\frac{ab}{c} + \frac{bc}{a} \ge 2b$; 3) tengsizlikning chap qismidan o'ng qismini ayiring va hosil bo'lgan kasrning suratini bunday ko'rinishda yozing: $(a + b)(a - b)^2 + (b + c)(b - c)^2 + (a + c)(a - c)^2$; 1) $x_{1,2} = \pm 2$; 2) $x_{1,2} = \pm 1$; 3) $x_{3,4} = \pm 3$; 3) $x_1 = -1, x_2 = 2$; 4) $x_{1,2} = \frac{-1 \pm \sqrt{5}}{2}$; 5) $x_1 = 0, x_{2,3} = \pm 2$; 6) $x_{1,2} = \pm 4, x_{3,4} = \pm 6$. **534.** 2) $2\frac{1}{3}$; 4) $\frac{2x^2}{3y}$.

**535.** 2) $3 - \sqrt[3]{2}$; 4) $6\sqrt{7}$. **536.** 2) $(2\sqrt{0,5})^{0,3} < (2\sqrt{0,5})^{0,37}$. **537.** 2) $\sqrt{x}$; 4) $9b^{-4}$. **538.** 2) $5ab\sqrt{b}$. **539.** 2) $-\sqrt{3x^2}$; 4) $\sqrt{5a^2}$. **540.** 2) Yo'q. **541.** 2) Yo'q. **544.** $-1$. **545.** 2) Manfiy. **548.** 2) $-0,8$. **547.** 2) $2\sin\frac{3\alpha}{4}\cos\frac{\alpha}{4}$; 4) $\sin\alpha(\sin\alpha - 2\cos\alpha)$. **548.** $\sin\alpha = \frac{240}{289}$, $\cos\alpha - -\frac{161}{289}$, $\text{tg}\alpha = -\frac{240}{161}$. **549.** 2) $a_{12} = 47,5$, $S_{12} = 537$; 4) $a_{18} - 11\frac{2}{3}$, $S_{18} = 108$.

**550.** 1220. **552.** 2) $b_1 = 5$. **553.** 2) $b_4 = 125, S_4 = 156$; 4) $b_4 = 81, S_5 = 61$. **554.** $15\frac{3}{4}$. **555.** 2) $4\frac{1}{6}$; 4) 1; 6) $-\frac{5}{4}(1 + \sqrt{5})$.. **557.** 2) $-1$; 4) $-\frac{1}{x}$. **558.** 2) $\frac{(a + \sqrt{b})(\sqrt{a} + \sqrt[4]{b})}{a^2 - b}$; 4) $0,1(5 - \sqrt{5})5 + \sqrt{5}$. **559.** 2) $-\frac{\sqrt{a}}{b}$; 4) $\sqrt{a} + \sqrt{b}$. **560.** 2) $x = 61$. **561.** 2) $\frac{1}{\cos^2\alpha}$.

**562.** 2) $x = \frac{\pi}{2} + \pi n$, $x = \pi + 2n$, $n \in \mathbf{Z}$. **565.** $39\frac{2}{3}$. **566.** $b_1 = 5, b_6 = 405$. **567.** $\frac{1}{8}$.

**561.** 8,13,18 yoki 20,13,6. **568.** 1) $\frac{\sqrt{3} + \sqrt{7}}{\sqrt{2}}$; 2) $\frac{1 + \sqrt{7}}{\sqrt{2}}$. **569.** $\sin\alpha = -\frac{120}{169}$, $\cos\alpha = -\frac{119}{169}$.

## „O‘zingizni tekshirib ko‘ring“ topshiriqlariga javoblar

**I bob.** **1.** $x_1 = 0, x_2 = 2$. **2.** $-1 < x < 1$ bo‘lganda $y > 0$; $x < -1$ bo‘lganda $y < 0$; $x > 1$. **3.** 1) $x > 0$ bo‘lganda funksiya o‘sadi; $x < 0$ bo‘lganda funksiya kamayadi. **4.** 1) $x \geq 1$; $-2 \leq x \leq 0$. **5.** 1) $x \neq 1$; 2) $-3 \leq x \leq 3$. **6.** 1) $x = 28$; 2) $x = 1$.

**III bob.** **1.** 1) $\cos\pm = \frac{3}{5}$, $\operatorname{tg}\alpha = \frac{4}{3}$, $\sin 2\alpha - -\frac{24}{25}$. **2.** 1) 1; 2) $\frac{\sqrt{3}}{2}$; 3) $\frac{\sqrt{3}}{2}$; 4) $\sqrt{3}$; 5) $\frac{\sqrt{2}}{2}$. **5.** 1) $\sin\alpha\cos\beta$; 2) $\cos^2\alpha$; 3) $2\sin\alpha$.

**IV bob.** **1.** 1) $a_{10} = -25, S_{10} = -115$. **2.** 1) $b_6 = \frac{1}{8}$, $S_6 = 7\frac{7}{8}$. **3.** 1) $q = \frac{1}{3}$, $S = 1{,}5$.

## Amaliy-tatbiqiy va fanlararo bog‘liq masalalarga javoblar

**I bob.** **1.** Tezlik 60,01 km/h dan oshmasligi kerak. **2.** $n \leq 30$. **3.** 2 mln. 10 m. **4.** 125 ta. **5.** 1) 135 ta; 2) 17739 ta; 3) $\approx$ 4,9 oyda.

**II bob.** **1.** 2) 20 ta qator. **2.** Birinchi brigadada 8 ta, ikkinchisida 12 ta ishchi. **3.** 2) 16%. **4.** 2) 4 $l$ va 12 $l$. **5.** Shamolsiz ob-havoda.

**III bob.** **1.** 4) $\approx$335,42 km; 5) $\approx$2243,3 km. **2.** $\approx 11{,}3^{\circ}$. **3.** 1818 m. **4.** $\approx$12,8 m.

**IV bob.** **1.** 420. **2.** 10 km. **3.** 3072. **4.** 39 300000 so‘m. **5.** 27 metr.

**V bob.** **1.** $E(X) = 26$, $D(X) = 0{,}9964$. 2. $E(X) \approx 8{,}94$, $E(Y) \approx 8{,}93$, $D(X) \approx 0{,}07$, $D(Y) \approx 0{,}03$, $G(X) \approx 0{,}071$, $\sigma(Y) \approx 0{,}76$. 3. $E(X_1) - E(X_2) - 3{,}36$, $\sigma(X_1) \approx 1{,}47$, $\sigma(X_2) \approx 1{,}41$. 4. $E(d_1) = 60, D(d_1) = 1{,}2$, $E(d_2) = 60{,}02$, $D(d_2) = 0{,}76$.

## MUNDARIJA

## IV bob. SONLI KETMA-KETLIKLAR. PROGRESSIYALAR



## V bob. EHTIMOLLIKLAR NAZARIYASI VA MATEMATIK STATISTIKA ELEMENTLARI

**Alimov Sh.A.**

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# ALGEBRA

Umumiy o'rta ta'lim maktablarining
9-sinfi uchun darslik

Qayta ishlangan 4-nashri

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*Toshkent – 2019*

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## Ijaraga beriladigan darslik holatini ko'rsatuvchi jadval

| T/r | O'quvchining ismi va familiyasi | O'quv yili | Darslikning olingandagi holati | Sinf rahbarining imzosi | Darslikning topshirilgandagi holati | Sinf rahbarining imzosi |
|---|---|---|---|---|---|---|
| 1 | | | | | | |
| 2 | | | | | | |
| 3 | | | | | | |
| 4 | | | | | | |
| 5 | | | | | | |
| 6 | | | | | | |

**Darslik ijaraga berilib, o'quv yili yakunida qaytarib olinganda yuqoridagi jadval sinf rahbari tomonidan quyidagi baholash mezonlariga asosan to'ldiriladi:**

| | |
|---|---|
| Yangi | Darslikning birinchi marotaba foydalanishga berilgandagi holati. |
| Yaxshi | Muqova butun, darslikning asosiy qismidan ajralmagan. Barcha varaqlari mavjud, yirtilmagan, ko'chmagan, betlarida yozuv va chiziqlar yo'q. |
| Qoniqarli | Muqova ezilgan, birmuncha chizilib, chetlari yedirilgan, darslikning asosiy qismidan ajralish holati bor, foydalanuvchi tomonidan qoniqarli ta'mirlangan. Ko'chgan varaqlari qayta ta'mirlangan, ayrim betlariga chizilgan. |
| Qoniqarsiz | Muqovaga chizilgan, yirtilgan, asosiy qismidan ajralgan yoki butunlay yo'q, qoniqarsiz ta'mirlangan. Betlari yirtilgan, varaqlari yetishmaydi, chizib, bo'yab tashlangan. Darslikni tiklab bo'lmaydi. |