Identifying Neutral Technology Shocks[*]

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April 21, 2014

Preliminary and Incomplete

Abstract

The role of neutral technology shocks in driving business cycle fluctuations is hotly debated. Yet, we argue that there is no existing empirical methodology that allows to identify neutral shocks in the data in the presence of input heterogeneity in the aggregate production function. We develop a method that identifies a neutral technology shock based on the result that it is the only shock consistent with balanced growth properties. Monte Carlo simulations using benchmark business cycle models imply that the proposed method performs very well in small samples. We apply the method to assess the role of neutral technology in driving business cycle fluctuations in U.S. data.

JEL Classification: E32, E24

Keywords: Economic Shocks, Heterogeneity, Growth Accounting, Business Cycles

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1   Introduction

The objective of this paper is to propose a method to identify neutral labor-augmenting technology shocks in the data. Classic results, starting with Uzawa (1961), establish that these shocks drive the long-run economic behavior along the balanced growth path. They are also the key driving force inducing fluctuations in real business cycle (RBC) models pioneered by Kydland and Prescott (1982), Long and Plosser (1983), and play a quantitatively important role in New Keynesian models, e.g., Smets and Wouters (2007).[1] Moreover, the relationships between various economic variables and neutral technology shocks identified in the data are routinely used to assess model performance and to distinguish between competing models. For example, the empirical finding that aggregate hours worked fall in response to a technology shock called into question the usefulness of the RBC model for interpreting aggregate fluctuations.

However, the methods used in the literature to identify the technology shocks are not designed to measure *neutral* technology shocks. Consider, for example, the classic Solow residual accounting procedure. Suppose output is produced with effective labor input $L_t^e = G(L_1, ..., L_n, t)$ aggregating various labor inputs and, for simplicity, a single capital input according to the following constant returns to scale production function:

$$Y = F(K, Z\,G(L_1, ..., L_n, t)), \tag{1}$$

where Z represents the labor-augmenting neutral technology shock we are interested in identifying. Note that the labor aggregator is allowed to depend on time to capture non-neutral changes in technology, e.g., changes in relative productivity or substitutability of various labor inputs. Such changes are thought key for understanding various issues in macro and labor economics. For example, the vast literature on skill biased technical change rationalizes the simultaneous increase in supply and in the relative wages of college educated workers since the 1970s through the change in the relative productivity of these workers in aggregate production (e.g., Katz and Murphy (1992), Acemoglu (2002)).[2] Thus, as emphasized by Solow

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[1]In most models the production function is such that the labor augmenting or Harrod-neutral technology shocks are isomorphic to the Hicks-neutral shocks that do not affect the marginal rate of substitution between any factors of production.

[2]The alternative interpretation of the evidence in Krusell, Ohanian, Ríos-Rull, and Violante (2000) also relies on non-neutral change in the parameters governing relative productivity of the investment good sector.

(1957), one must allow for the possibility that the neutral technology parameter is only one of many technological parameters that can change over time. Differentiating the production function with respect to time and dividing by Y we obtain the Solow residual:

$$(1 - \omega_K)\frac{\dot{Z}}{Z} + \frac{\partial F/\partial t}{F} = \frac{\dot{Y}}{Y} - \omega_K\frac{\dot{K}}{K} - \sum_{j=1}^{n} \omega_{L_j}\frac{\dot{L_j}}{L_j}, \tag{2}$$

where, assuming that factors are paid their marginal products, ω_i represents income share of factor i. Clearly, as emphasized in the original Solow (1957) article, the residual equals neutral plus non-neutral technology changes. Hence its other name - the total factor productivity. As variables, such as total hours worked, may react either positively or negatively to a non-neutral shock, their response to an innovation in the Solow residual is difficult to interpret. Unfortunately, the growth accounting methodology is not designed to identify the contribution of only the neutral shock, to which models have a robust prediction regarding the response of endogenous variables. It also provides no possibility to ascertain the relative importance of neutral and biased technological innovations in driving aggregate economic dynamics.

An alternative approach to identifying neutral technology shocks is based on the assumption put forward by Gali (1999) that only technology shocks have a long-run effect on labor productivity (output per hour). He implemented this idea in a business cycle context using structural vector autoregressions (SVAR) identified with long-run restrictions following Blanchard and Quah (1989). Unfortunately, this approach is also not designed to identify neutral technology shocks. Denote by L the total sum of hours worked. Then, using (1), output per hour can be written as

$$log\left(\frac{Y_t}{L_t}\right) = log(Z_t) + log\left(\frac{L_t^e}{L_t}\right) + log\left(F\left(\frac{K_t}{Z_t L_t^e}, 1\right)\right). \tag{3}$$

Note that $\frac{K_t}{Z_t L_t^e}$ is stationary in most models (consistent with the stationary interest rate in the data). However, the long-run changes in productivity can be induced either by persistent technology shocks or by persistent changes in the effective labor input per hour worked. In particular, L^e/L could change in the long run either due to the persistent changes in worker composition (e.g., changes in female labor force participation and their distribution across occupations), changes in the effective units of labor supplied by various labor inputs (e.g., expecting longer careers, women invest more in human capital through on-the-job training) or changes in the production function parameters that govern the relative productivity or

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substitutability of various labor inputs (e.g., an increase in the relative productivity of females due to an increase in the demand for tasks in which they have a comparative advantage or due to directed changes in technology induced by an increase in their labor force participation). Any of these changes affecting labor productivity in the long run will be interpreted as a technology shock by this methodology. The existing literature provides no guidance on how the neutral technological changes can be isolated.[3]

These observations lead us to propose a method for estimating neutral technology shocks. To do so, we assume a constant returns to scale aggregate production function and exploit the rich implications of Uzawa's characterization of neutral technology on a balanced growth path. We do not assume the economy to be on the balanced growth path but instead use a weak conditional form of this assumption. We only require that the impulse responses to a permanent neutral technology shocks have the standard balanced growth properties in the long run. This is sufficient to identify the neutral technology shock because we are able to prove that no other shock (to non-neutral technology, preferences, etc.) satisfies these restrictions.

To implement this identification strategy we use a state-space model for a set of variables for which we know the long run effect of neutral technology. These macroeconomic variables can be represented as a sum of a neutral technology shock, which is treated as one of the unobserved components driving the system, and an unobserved state. For example, the log of the wage of workers of a particular type is written as the sum of the neutral technology shock and an unobserved component that is partially idiosyncratic to that worker type (in a competitive framework representing the derivative of the production function with respect to

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[3]The econometric issues underlying this approach have been intensely discussed in the literature (e.g., Faust and Leeper (1997), Chari, Kehoe, and McGrattan (2008), Christiano, Eichenbaum, and Vigfusson (2006), Fernandez-Villaverde, Rubio-Ramirez, Sargent, and Watson (2007)). Instead, we question the long-run restriction itself. Indeed, *any* shock that affects the composition of factors of production or their relative productivity in the long run will have a long-run effect on labor productivity and will be erroneously interpreted as a neutral technology shock by this methodology. Several related critiques of this approach appeared in the literature. Shea (1998) suggests that if low-productivity firms are destroyed in recessions, there might be a long run effect on productivity. Uhlig (2004) argues that permanently changing social attitudes to workplace, whereby workers substitute leisure activities at home with leisure activities at work, will result in mis-measurement of effective work hours and affect measured productivity in the long run. Francis and Ramey (2005, 2009) note that changes in capital taxes or low frequency movements in age composition of population also may have a long-run effect on labor productivity. Fisher (2006) imposes additional restrictions to separate neutral from investment-specific shocks.

that labor input).

We do not require orthogonality among the state variables, an assumption commonly used to identify these types of models although inconsistent with typical economic models. Instead we prove that the conditional balanced growth restrictions are sufficient to identify neutral technology shocks in the resulting system of equations collecting various macroeconomic time-series using filtering/smoothing techniques. Since we do not treat the technology shock as a residual, our method does not require to specify an explicit function that aggregates heterogeneous labor and capital inputs.[4] Instead, all this information is summarized in the unobserved states which we identify without the need to specify the structure behind the dynamics of these states. Moreover, our method does not require the parameters of this function to be invariant over time. The identification methodology relies on a testable assumption on the time series process for the neutral technology, e.g., AR(1) and other unobserved states, e.g., VAR(1). This process is only required to provide a good statistical approximation and does not have to be consistent with a structural model since we do not need to assign a structural interpretations to the other shocks affecting the economy.

To assess the small sample properties of the proposed method, we conduct a Monte Carlo study using samples drawn from estimated benchmark business cycle models. We consider the RBC and the New-Keynesian models with worker heterogeneity. We find that the proposed method is successful in identifying neutral technology shocks in the data generated by the models and does not confound neutral technology with other disturbances such as non-neutral technology, preference shifts or wage markup shocks.

The paper is organized as follows. In Section 2 we develop the method to recover neutral technology shocks and establish the sufficient conditions for identification. In Section 3 we illustrate the implementation and evaluate the performance of the proposed method in an estimated RBC model. In Section 4 we assess the performance of the proposed method in small samples drawn from an estimated medium scale DSGE model with multiple sources of real and nominal rigidities and numerous exogenous shocks. Finally, in Section 5 we apply our method in the data and estimate a quarterly technology series for the US. We also describe and

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[4]This is in contrast to attempts to identify neutral technology shocks by fully specifying the production function and all the associated inputs as in e.g., Nadiri and Prucha (2001) and Dupuy (2006). The data requirements underlying this approach seem prohibitive.

analyze the sequence of identified shocks and document its co-movement with other economic aggregates. Section 6 concludes.

2 Identifying Neutral Technology Shocks

In this section we propose a method to estimate Harrod-neutral technology shocks. Section 2.1 provides a characterization of these type of shocks. We prove that Harrod-neutral technical change is the only type of shock that can induce balanced growth on a set of macroeconomic variables. We next show how we can use this property to identify neutral technology shocks from the data using benchmark time series models. Section 2.2 present the time series model we use while Section 2.3 formally proves that the long run restrictions implied by balanced growth are sufficient to identify Harrod-neutral technology shocks. Section 2.4 discusses several issues related to the practical implementation of our approach.

2.1 Identification: Theory

In this section we build on this classic result and show how to use the insights from Uzawa's theorem (see Acemoglu (2009) for an excellent treatment) on technological progress in the long-run to identify Harrod-neutral technology shocks. Suppose that aggregate output Y_t is produced as follows

$$Y_t = F(K_{1,t}, \ldots, K_{J,t}, Z_t L_{1,t}, \ldots, Z_t L_{M,t}; \theta_t), \tag{4}$$

where $K_{j,t}$ represent capital input of type j, $L_{m,t}$ represents labor inputs of type m, Z_t is Harrod-neutral technology progress and θ_t is a vector collecting other non-neutral technological changes. We assume that F is constant return to scale in capital and labor inputs. Our methodology does not require any further restriction on the aggregate production function.

We make the following *conditional balanced growth assumption*, anticipating that the implementation of the identification methodology in the data will use a state-space model and thus identify the shock through its impulse response.

Assumption 1 (conditional balanced growth assumption). *A time T exists such that the impulse response of a variable X_t to a Harrod-neutral innovation ϵ_0^Z (to Z_0) of x percent*

at time 0,

$$IR_t^X(x) = E_0[X_t \mid \log(\epsilon_0^Z) = x] - E_0[X_t]$$

equals

$$IR_t^X(x) = (e^{g_X x} - 1)E_0[X_t]$$

for all $t \geq T$, where $g_X x$ is the percent increase in X. If $g_X = g$ for output, for all capital inputs, and for all types of investment and of consumption, then labor inputs $X = L_m$ do not respond in the long-run to a neutral shock, $g_X = g_{L_m} = 0$.

This assumption guarantees convergence of the impulse response for all variables. In addition, it assumes a linearity property of the long-run response to a neutral technology shock (i.e. a shock of size $2x$ has exactly twice the effect of a shock of size x). History dependence of the impulse response function is not ruled out in the short run, nor it is for all other economic shocks. fMoreover, this assumption tells us that if Harrod-neutral shocks induce a common trend in output, capital inputs, investments and consumption, then they do not influence labor inputs in the long run.

We now demonstrate that Harrod-neutral technology shocks are the only one that can induce a certain pattern of long run responses for a set of macroeconomic variables. This property will be then used to identify this technical change from aggregate data. Before stating the main theorem, though, we prove a useful result

Lemma 1. *Suppose the conditional balanced growth assumption holds for variables $X_t, X_{1,t}, \ldots, X_{H,t}$ where $X_t = \sum_{h=1}^{H} X_{h,t}$. The long-run response for variable X is g_X and for the components $X_h > 0$ equal to g_{X_h} . Then*

$$g_X = g_{X_1} = \cdots g_{X_h} = \cdots = g_{X_H}.$$

The proof is in Appendix I.1. Lemma 1 tells us that, for a shock to have a well defined long run effect on a variable X, it must have the same long run effect on its components.

Theorem 1. *Suppose the conditional balanced growth assumption holds. Then a permanent Harrod-neutral technological shock is the only shock with the following (balanced-growth) properties for some time T. An innovation which increases the level of the shock by x percent at time 0 implies for all $t \geq T$*

   - An increase in aggregate output Y by x percent, $IR_t^Y(x) = (e^x - 1)E_0[Y_t]$

- *An increase in investment I_j by x percent, $IR_t^{I_j}(x) = (e^x - 1)E_0[I_{j,t}]$*

- *An increase in capital K_j by x percent, $IR_t^{K_j}(x) = (e^x - 1)E_0[K_{j,t}]$*

- *An increase in aggregate consumption C by x percent, $IR_t^C(x) = (e^x - 1)E_0[C_t]$*

- *No effect on labor inputs L_m, $IR_t^{L_m}(x) = 0$*

- *No effect on the marginal product of capital F_{K_j}, $IR_t^{F_{K_j}}(x) = 0$*

- *An increase in the marginal product of labor F_{L_m} by x percent, $IR_t^{F_{L_m}}(x) = (e^x - 1)E_0[F_{L_m,t}]$*

The proof in Appendix I.2 follows the steps in the proof of Uzawa's theorem in Acemoglu (2009).

The conditions in the theorem rule out non-neutral technical change. For example, they rule out investment-specific shocks (Greenwood, Hercowitz, and Krusell, 1997), which are often modeled as a non-neutral shock to the technology for producing capital equipment. This shock has long run effects on the capital equipment to capital structures ratio and the capital output ratio, which is inconsistent with the above properties.

The theorem is not limited, however, to distinguishing between different types of technical change. Instead, it characterizes Harrod-neutral technology shocks and thus tells them part from any other economic shock, e.g. preference shocks, government expenditure shocks or wage mark-up shocks. The logic is as follows. If output and capital increase by the same percentage rate then constant returns to scale imply that effective labor input has to increase by the same percentage rate. Because labor inputs is assumed not to change in the long run, the productivity of labor has to increase by the same percentage term, i.e. it has to be a Harrod-neutral technological change.

2.2   Implementation: The State Space Model

In this section we show how we can implement the conditional balance growth restrictions of the previous section using a benchmark time series model. To this aim, we assume that we observe a vector time series $\mathbf{D_t}$, collecting growth rates of a set of macroeconomic variables. Without loss of generality, we write $\mathbf{D_t}$ as the sum of two components

$$\mathbf{D_t} = \Delta Z_t \mathbf{1_n} + \tilde{\mathbf{S}}_\mathbf{t}, \tag{5}$$

where ΔZ_t is the growth rate of the neutral technology series (in logs and $\mathbf{1_n}$ is the n-dimensional vector of ones), and $\tilde{\mathbf{S}}_\mathbf{t}$ is a vector of states. Both ΔZ_t and $\tilde{\mathbf{S}}_\mathbf{t}$ are unobserved. Clearly, any macroeconomic time-series can be written this way. Two examples for $\mathbf{D_t}$ with a clear economic interpretation, are output growth $\Delta \log(Y_t)$ and the growth rate of competitive wages for a worker of type m, $\Delta \log(W_{m,t})$:[5]

$$\Delta \log(Y_t) = \Delta Z_t + \Delta \log \left[F \left(\frac{K_{1,t}}{Z_t}, \ldots, \frac{K_{J,t}}{Z_t}, L_{1,t}, \ldots, L_{M,t}; \theta_\mathbf{t} \right) \right], \qquad (6)$$

$$\Delta \log(W_{m,t}) = \Delta Z_t + \Delta \log \left(\frac{\partial F}{\partial L_{m,t}} \right). \qquad (7)$$

Thus, for output the unobserved state $\tilde{\mathbf{S}}_\mathbf{t}$ is equal to $\Delta \log \left[F \left(\frac{K_{1,t}}{Z_t}, \ldots, \frac{K_{J,t}}{Z_t}, L_{1,t}, \ldots, L_{M,t}; \theta_\mathbf{t} \right) \right]$ and for wages the unobserved state is equal to $\Delta \log \left(\frac{\partial F}{\partial L_{m,t}} \right)$.

Since we treat the second component as an unobserved state variable, we do not have to make any assumptions on the shape of the production function. Instead our approach consists in restricting the time-series behavior of $\mathbf{S_t} = [\Delta Z_t, \tilde{\mathbf{S}}_\mathbf{t}]$ and in exploiting the factor structure of the system in (5). In particular, we propose to estimate the technology series $\{\Delta Z_t\}_{t=0}^T$ in a three steps procedure:

i) Assume a time series model for the behavior of $[\Delta Z_t, \tilde{\mathbf{S}}_\mathbf{t}]$, indexed by the vector of parameters $\mathbf{\Lambda}$.

ii) Estimate the parameters' vector $\mathbf{\Lambda}$.

iii) Conditional on the estimation of $\mathbf{\Lambda}$ and given a time series for $\mathbf{D_t}$, we estimate the realization of ΔZ_t using smoothing techniques.

For concreteness, suppose that ΔZ_t is an univariate AR(1) process with persistence parameter given by ϕ_{zz} and innovation variance given by r_{zz}^2 (which we normalize to one), while the unobserved states follow a VAR(1). None of the results discussed in this section depend on this parametrization, and richer dynamics can be allowed for by introducing additional lags and moving average terms. Under these assumptions we can express the dynamics of $\mathbf{D_t}$ in

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[5]Wages are competitive here for illustrative purposes only. Our method does not assume that wages are competitive.

state space form:

$$
\underbrace{\begin{bmatrix} \mathbf{D_t} \end{bmatrix}}_{\mathbf{D_t}}^{n \times 1} = \underbrace{\begin{bmatrix} \mathbf{1} & \mathbf{I} \end{bmatrix}}_{\mathbf{B}}^{n \times (n+1)} \underbrace{\begin{bmatrix} \Delta Z_t \\ \tilde{\mathbf{S}}_\mathbf{t} \end{bmatrix}}_{\mathbf{S_t}}^{(n+1) \times 1}
$$

$$
\underbrace{\begin{bmatrix} \Delta Z_t \\ \tilde{\mathbf{S}}_\mathbf{t} \end{bmatrix}}_{\mathbf{S_t}}^{(n+1) \times 1} = \underbrace{\begin{bmatrix} \phi_{zz} & \mathbf{0}' \\ \mathbf{\Phi}_{\tilde{\mathbf{S}}\mathbf{z}} & \mathbf{\Phi}_{\tilde{\mathbf{S}}\tilde{\mathbf{S}}} \end{bmatrix}}_{\mathbf{\Phi}}^{(n+1) \times (n+1)} \underbrace{\begin{bmatrix} \Delta Z_{t-1} \\ \tilde{\mathbf{S}}_{\mathbf{t-1}} \end{bmatrix}}_{\mathbf{S_{t-1}}}^{(n+1) \times 1} + \underbrace{\begin{bmatrix} r_{zz} & \mathbf{0}' \\ \mathbf{R}_{\tilde{\mathbf{S}}\mathbf{z}} & \mathbf{R}_{\tilde{\mathbf{S}}\tilde{\mathbf{S}}} \end{bmatrix}}_{\mathbf{R}}^{(n+1) \times (n+1)} \underbrace{\begin{bmatrix} e_{z,t} \\ \tilde{\mathbf{e}}_\mathbf{t} \end{bmatrix}}_{\mathbf{e_t}}^{n \times 1} \quad (8)
$$

$$
\mathbf{e_t} \overset{\text{i.i.d.}}{\sim} \mathcal{N}(\mathbf{0}_{n+1}, \mathbf{I}_{n+1})
$$

ΔZ_t is assumed to be an exogenous process in the above system. In particular, $\tilde{\mathbf{S}}_{\mathbf{t-1}}$ does not affect current technology once we condition on Z_{t-1}, this explaining the zeros in the transition matrix $\mathbf{\Phi}$. Moreover, the zero restrictions on the $\mathbf{R}$ matrix tell us that the first element of the $\mathbf{e_t}$ vector has to be interpreted as an innovation to technology. Notice that we allow for contemporaneous correlation among the innovations to ΔZ_t and $\tilde{\mathbf{S}}_\mathbf{t}$ since we do not restrict $\mathbf{R}_{\tilde{\mathbf{S}}\mathbf{z}}$ to be zero. This is particularly relevant in our application since technology shocks are likely to affect $\tilde{\mathbf{S}}_\mathbf{t}$.[6] Because of this correlation, the state space model is not identified without further restrictions. Fortunately, as we show in the next section, we can use Theorem 1 to impose a set of restrictions that are sufficient to identify the parameters of the model. In terms of notation, we will refer to $D_{j,t}$ as the jth element of the measurement vector $\mathbf{D_t}$ while to $S_{j,t}$ as the jth element of the state vector $\mathbf{S_t} = [\Delta Z_t, \tilde{\mathbf{S}}_\mathbf{t}]'$. We denote by $\mathbf{e_t}$ the vector $[e_{z,t}, \tilde{\mathbf{e}}_\mathbf{t}]'$.

2.3 Identification of the State Space Model

We include in the vector of observable variables, $\mathbf{D}_t$, the growth rates of output, investment and hours as well as of the wages of two groups of workers, (s)killed and (u)skilled:

$$
\mathbf{D}_t = (\Delta \log(Y_t), \Delta \log(I_t), \Delta \log(L_t), \Delta \log(W_{s,t}), \Delta \log(W_{u,t}))'. \quad (9)
$$

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[6]Both of the examples discussed earlier share this characteristic. A shock to technology affects labor inputs, this generating correlation between ΔZ_t and $\tilde{\mathbf{S}}_\mathbf{t}$.

From the discussion in Section 2.1 we know that these variables are sufficient to distinguish Harrod-neutral technology shocks from other economic disturbances. Clearly, one could incorporate in $\mathbf{D}_t$ more variables with known balanced growth restrictions: this would sharpen identification at the cost of increasing the complexity of the model.

We now formally define identifiability of the state space model

Definition 1. *Let $\mathbf{\Lambda}$ and $\hat{\mathbf{\Lambda}}$ be two parameterizations of the system in (8). These are observationally equivalent if $\mathbf{\Gamma_D}(\tau, \mathbf{\Lambda}) = \mathbf{\Gamma_D}(\tau, \hat{\mathbf{\Lambda}})$ for all $\tau \in \mathbb{N}$, where $\mathbf{\Gamma_D}(\tau, \mathbf{\Lambda})$ is the τth order autocovariance of $\mathbf{D_t}$ under $\mathbf{\Lambda}$.*

Definition 2. *The state space model in (8) is identifiable from the autocovariances of $\mathbf{D_t}$ at $\mathbf{\Lambda} = (\mathbf{\Phi}, \mathbf{R})$ if for any admissible parametrization $\hat{\mathbf{\Lambda}} = (\hat{\mathbf{\Phi}}, \hat{\mathbf{R}})$ we have that $\mathbf{\Lambda}$ and $\hat{\mathbf{\Lambda}}$ are observationally equivalent if and only if $\mathbf{\Phi} = \hat{\mathbf{\Phi}}$ and $\mathbf{R}\mathbf{R}' = \hat{\mathbf{R}}\hat{\mathbf{R}}'$*

In what follows, we show how the restrictions brought by the conditional balanced growth assumption are sufficient to guaranteee the identification of $\mathbf{\Lambda}$. Prior to that, we make an additional technical assumption

Assumption 2.   *i) The matrix $\mathbf{R}$ is invertible.*

ii) $(-1, 1, \ldots, 1)'$ is not an eigenvector with eigenvalue ϕ_{zz} of the matrix $\tilde{\mathbf{\Phi}}$.

In Appendix I.3 we prove that this assumption implies that the state space representation in (8) is minimal, i.e. the dimension of the state vector $\mathbf{S_t}$ can not be reduced. This assumption allows us to cast our problem within the literature of identification of minimal state space systems (Hannan and Diestler, 1988).

Lemma 2. *Let Assumption 2 hold. Then, the state space model in (8) is minimal.*

Given minimality of the state space in (8), lack of identification is known to be represented by linear transformations of the state vector through invertible matrices $\mathbf{T}$ and $\mathbf{U}$ with $\mathbf{U}\mathbf{U}' = \mathbf{I}$ (see Proposition 1-S in Komunjer and Ng (2011)). In fact, consider defining the state vector $\hat{\mathbf{S}}_\mathbf{t} = \mathbf{T}^{-1}\mathbf{S_t}$ and the innovation vector as $\hat{\mathbf{e}}_\mathbf{t} = \mathbf{U}^{-1}\mathbf{e_t}$. Then, one can rewrite the system in (8) as:

$$
\begin{aligned}
\tilde{\mathbf{D}}_\mathbf{t} &= \hat{\mathbf{B}}\hat{\mathbf{S}}_\mathbf{t} \\
\hat{\mathbf{S}}_\mathbf{t} &= \hat{\mathbf{\Phi}}\hat{\mathbf{S}}_\mathbf{t-1} + \hat{\mathbf{R}}\hat{\mathbf{e}}_\mathbf{t}
\end{aligned}
\tag{10}
$$

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where the new matrices $(\hat{\mathbf{B}}, \hat{\boldsymbol{\Phi}}, \hat{\mathbf{R}})$ are related to the original one as follows:

$$
\begin{aligned}
\hat{\mathbf{B}} &= \mathbf{BT} \\
\hat{\boldsymbol{\Phi}} &= \mathbf{T^{-1}\Phi T} \\
\hat{\mathbf{R}} &= \mathbf{T^{-1}RU}
\end{aligned}
\tag{11}
$$

Clearly, the observationally equivalent parametrization must satisfy the restrictions made on $(\mathbf{B}, \boldsymbol{\Phi}, \mathbf{R})$, narrowing the set of admissible $(\mathbf{T}, \mathbf{U})$ matrices. In what follows we provide a characterization of this set for the system in (8).

First of all, notice that since the matrix $\mathbf{B}$ is known, one needs to have $\hat{\mathbf{B}} = \mathbf{B}$. This implies that the matrix $\mathbf{T}$ has the form:

$$
\mathbf{T} = \begin{bmatrix} 1+\kappa_1 & -\kappa_2 & \dots & -\kappa_n \\ -\kappa_1 & 1+\kappa_2 & \dots & \kappa_n \\ \dots & \dots & \dots & \dots \\ -\kappa_1 & \kappa_2 & \dots & 1+\kappa_n \end{bmatrix} \qquad \mathbf{T^{-1}} = \begin{bmatrix} 1-\frac{\kappa_1}{\kappa} & \frac{\kappa_2}{\kappa} & \dots & \frac{\kappa_n}{\kappa} \\ \frac{\kappa_1}{\kappa} & 1-\frac{\kappa_2}{\kappa} & \dots & -\frac{\kappa_n}{\kappa} \\ \dots & \dots & \dots & \dots \\ \frac{\kappa_1}{\kappa} & \frac{\kappa_2}{\kappa} & \dots & 1-\frac{\kappa_n}{\kappa} \end{bmatrix}
\tag{12}
$$

for some scalars $\kappa_1, \dots, \kappa_n$ and for $\kappa = 1 + \left(\sum_{l=1}^{n} \kappa_l\right)$. What this means is that if $\kappa_1 = \kappa_2 = \kappa_3 = \dots \kappa_n = 0$ all the parameters of the system in (8) are identified and $\mathbf{T} = \mathbf{I}$: we are then able to identify correctly the parameters $\boldsymbol{\Phi}$ and $\boldsymbol{\Sigma} = \mathbf{RR'}$, only the ordering of $\tilde{\mathbf{e}}_\mathbf{t}$ would not be identified.

In general, we can easily verify that the state vector associated with the $\mathbf{T^{-1}}$ matrix becomes:

$$
\hat{\mathbf{S}}_\mathbf{t} = \begin{bmatrix} (1 - \frac{\kappa_1}{\kappa})Z_t + \sum_{l=2}^{n} \frac{\kappa_l}{\kappa} S_{l,t} \\ \frac{\kappa_1}{\kappa} Z_t + S_{2,t} - \sum_{l=2}^{n} \frac{\kappa_l}{\kappa} S_{l,t} \\ \dots \\ \frac{\kappa_1}{\kappa} Z_t + S_{n,t} - \sum_{l=2}^{n} \frac{\kappa_l}{\kappa} S_{l,t} \end{bmatrix}.
\tag{13}
$$

This parametrization needs to satisfy the restrictions on the transition equation in (8), namely that the first element of $\hat{\mathbf{S}}_\mathbf{t}$ follows an AR(1) with innovations given by the first element of $\hat{\mathbf{e}}_\mathbf{t}$. This cannot be ruled out given the assumptions made so far, i.e. without further restrictions, the system in (8) is not identified. This is where we use Theorem 1 which states that the balanced growth properties identify the neutral technology process.

The balanced growth restrictions for output, investment, hours, skilled and unskilled wages can be written as[7]

$$\begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \\ 1 \end{pmatrix} = \frac{1}{1-\rho_z} \mathbf{B}(\mathbf{I} - \boldsymbol{\Phi})^{-1} \mathbf{R}_{\mathbf{1:(n+1),1}}. \tag{14}$$

Thus, one can express the long run effect of neutral technology on the variables in $\mathbf{D}$ as a function of the parameters in the matrices $\boldsymbol{\Phi}$ and $\mathbf{R}$ and restrict it to be equal to 0 or 1. For example the first row of the restriction in (14) states that the the long-run response of output to a unit increase in $\varepsilon_{z,t}$ equals 1. Similarly rows $2, 4$ and 5 restrict the long-run response of investment, high skilled and low skilled wages to be of the same magnitude as well (again scaled by $\frac{1}{1-\rho_z}$). Row 3 requires the long-run response of hours to be 0.

As Theorem 1 shows, these long-run restrictions uniquely identify the neutral shock, that is $\mathbf{U}(1, 0, \ldots, 0)' = (1, 0, \ldots, 0)'$. This implies that the first column of $\mathbf{U}$ equals $(1, 0, \ldots, 0)'$ and using that $\mathbf{UU}' = \mathbf{I}$ then implies that the first row equals $(1, 0, \ldots, 0)$.

Theorem 2. [Identification] *Consider the state space model (8) with $\mathbf{D}$ including the logs of output, investment, hours worked, skilled and unskilled wages as in (9) and with balanced growth restrictions (14). Then the parameters $\boldsymbol{\Phi}$ and $\mathbf{RR}'$ are identified. In particular $\kappa_1 = \kappa_2 = \cdots = \kappa_n = 0$. Furthermore the neutral technology shock is identified, i.e.*

$$\mathbf{U} = \begin{bmatrix} 1 & 0 & \ldots & 0 \\ 0 & & & \\ & & \tilde{\mathbf{U}} & \\ 0 & & & \end{bmatrix}. \tag{15}$$

The proof is in Appendix I.4.

2.4 Discussion

In this section we discuss how we estimate the model, how to obtain impulse-responses and the choice of the time-series model that we use to implement our identification procedure. We

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[7]ΔZ_t follows an AR(1) with persistence parameter ρ_z. A one standard deviation error to the innovation (which we normalized to one) of the growth rate accumulates to a long-run change of $\frac{1}{1-\rho_z}$ in the level of z. As the balanced growth restrictions apply to changes in the level of z, the term $\frac{1}{1-\rho_z}$ multiplies the long-run effect on V.

also discuss how additional restrictions on the state space can be imposed.

2.4.1 Estimation

Because of the linear-gaussian structure of the state space model, we can evaluate the likelihood function using the Kalman filter. The model parameters are then estimated by maximum likelihood. Conditional on the estimated parameters, we can apply the Kalman smoother and obtain retrospective estimates of Harrod-neutral technical change, $\{p(\Delta Z_t | \mathbf{D^T})\}_{t=1}^{\mathbf{T}}$. See Durbin and Koopman (2001) for an extensive discussion of these methodologies.

2.4.2 Impulse Response Functions

Impulse Response Functions (IRFs) to a neutral technology shock for variables included in the data vector $\mathbf{D}_t$ can be easily computed using the estimated parameters and the state space model in (8). We may be also interested in computing IRFs for variables $\mathbf{x}_t$ that do not enter the measurement equation. In this case we proceed by using the estimated technology innovations of $\{e_{z,t}\}_{t=1}^T$. We project $\{e_{z,t}\}_{t=1}^T$ and its lags onto $\mathbf{x}_t$,

$$\mathbf{x}_t = \alpha + \beta(L)e_{z,t} + \varepsilon_t,$$

where $\beta(L)$ are polynomials in the lag operator and they represent the IRFs. OLS delivers consistent estimates of these parameters to the extent that $e_{z,t}$ is exogenous. This assumption is natural if we think of $\mathbf{x}_t$ as being generated by an underlying equilibrium model and we are willing to assume orthogonality of its structural shocks.

2.4.3 Choice of Time-Series Model

Our procedure requires to specify a parametric time series model for key macroeconomic variables. Because of its generality, we focus here on a linear state space model, but in principle our analysis could be carried using other linear or nonlinear time series models. As in the SVAR literature, we need to make several specitication choices regarding the number of macroeconomic time series to include in the model and the law of motion of the state variables.

The dimension of the state space $\mathbf{S_t}$ may be limited by the curse of dimensionality. First, the number of parameters increases in the lag length of the VAR for $\mathbf{S_t}$. This problem, common to the SVAR literature, can be partly circumvented with the use of shrinkage methods

that are becoming popular in applied time series econometrics (Del Negro and Schorfheide, 2010). However, because of the exogeneity restrictions on Z_t, we can adopt a more flexible specification for its law of motion without imposing much burden on the estimation. For example, suppose we assume a more general ARMA(p,q) for neutral technology. Then, the number of unknown parameters associated with the technology process equals $(n + 1)p + q$ with n being the dimension of $\mathbf{S_t}$. Second, given a DGP for the vector $\mathbf{S_t}$, the number of parameters to be estimated steeply increases in the number of variables in the measurement equation. For the example described in Section 3, the number of parameters to be estimated equals $2 + 2n(n + 1) + s(2 + n)$, where n is the dimension of the vector $\mathbf{D_t}$ and s is the dimension of $\mathbf{S_t}$. This limits the number of variables, and associated balanced growth restrictions, that can be allowed for.

The Monte Carlo exercise in the next section is supposed to shed lights on these issues. We will see that a parsimonious specification of the state space model considered in this section performs well when data are simulated from reasonably calibrated business cycle models.

2.4.4 Using Additional Theoretical Restrictions

The method proposed in this paper can easily accommodate additional restrictions implied by economic theory. While these restrictions are not strictly necessary, they may help sharpening identification of neutral technology shocks especially when dealing with short samples. A popular identification scheme in the SVAR literature are sign restrictions as in Uhlig (2005). These can be easily incorporated in our set up: for example, we could set $R_{\Delta w_j, z} > 0$ to restrict the neutral technology shock to have a positive impact effect on wages. Other types of information regarding the properties of neutral technology shocks can be easily implemented by appropriate restriction on the state space form. Aside from these identification schemes, the state space model considered here can incorporate external information without imposing excessive burden to the estimation. For example, suppose that we have a robust method to identify other types of structural structural shocks, say a government spending shock $\{e_{g,t}\}_{t=1}^T$ which we know a priori to be orthogonal to neutral technology shocks. Then, we could proceed in two steps: i) Add $\{e_{g,t}\}_{t=1}^T$ to the list of observables in the measurement equation; ii) add an additional state variables in $\mathbf{S}_t$ that selects one of the non-technology reduced form innovations; iii) restrict the matrix $\mathbf{R}$ so that $e_{z,t}$ and $e_{g,t}$ are orthogonal. Importantly, this does not

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result in additional parameters to be estimated, but it helps the identification of the neutral shock. See also Stock and Watson (2012) for a discussion of the role of external information ("instruments") for the identification of structural shocks in dynamic factor models.

3   An Example: A Simple RBC Model

We now illustrate the proposed procedure by means of an example. We study the basic RBC model with two types of labor, a useful benchmark due to its transparency and widespread use. We use this example to illustrate how our method for measuring neutral technology shocks can be applied in practice. Using data simulated from the calibrated model we study the relation between identified technology shocks and the true structural disturbances. In particular, we consider the small sample performance of our method and contrast it with the performance of an SVAR with long run restrictions on labor productivity and with Solow residuals. The transparency of the model allows us to isolate the reasons for the poor performance of the latter two methods in recovering neutral technology shocks.

3.1   The Real Business Cycle Model with Heterogeneous Labor

We consider a frictionless RBC model with worker heterogeneity. Agents of type $j = \{u, s\}$ (unskilled of measure u and skilled of measure $1 - u$) value consumption, c_t, and dislike labor, h_t, according to a type-dependent utility function

$$U_j(c_t, h_t) = \log(c_t) - e^{A_t} b_j \frac{h_t^{1+\nu_j^{-1}}}{1 + \nu_j^{-1}}. \tag{16}$$

A_t is a shock to the disutility of labor parameterizing the labor wedge, commonly found to play an important role in business cycle accounting. We allow the elasticity of labor supply, ν_j, to differ across the two demographic groups. Because of this, aggregate productivity in our model will vary over the cycle due to endogenous changes in the skill compositions of the labor input. Firms in the economy have access to the production function

$$Y_t = K_t^\alpha (e^{Z_t} L_t^e)^{1-\alpha}, \tag{17}$$

where L_t^e, the effective labor input, is an aggregator of low and high-skilled labor

$$L_t^e = L_{s,t}^{\phi_t} L_{u,t}^{1-\phi_t}.$$

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Note that observed labor input (total hours) equals $L_t = L_{s,t} + L_{u,t}$, where unskilled labor input equals $L_{u,t} = uh_{u,t}$ and skilled labor input $L_{s,t} = (1-u)h_{s,t}$. The relative productivity of skilled workers, ϕ_t, changes over time. This is one source of non-neutral technical change in the model. The accumulation equation for investment is expressed as

$$K_{t+1} = (1-\delta)K_t + I_t q_t, \tag{18}$$

where q_t represents the current state of the technology for producing new capital goods, a second source of non-neural technical change. Capital depreciates in every period at rate δ. The resource constraint equals

$$Y_t = I_t q_t + C_t + g_t Y_t, \tag{19}$$

where g_t is the fraction of final good devoted to government spending.

The laws of motion for economic shocks are standard:[8]

$$\Delta Z_t = \gamma + \rho_z \Delta Z_{t-1} + \sigma_z \varepsilon_{z,t}, \tag{20}$$

$$A_t = \rho_a A_{t-1} + \sigma_a \varepsilon_{a,t}, \tag{21}$$

$$\log(\phi_t) = (1-\rho_\phi)\phi^* + \rho_\phi \log(\phi_{t-1}) + \sigma_\phi \varepsilon_{\phi,t}, \tag{22}$$

$$\log(g_t) = (1-\rho_g)g^* + \rho_g \log(g_{t-1}) + \sigma_g \varepsilon_{g,t}, \tag{23}$$

$$\log(q_t) = \rho_q \log(q_{t-1}) + \sigma_q \varepsilon_{\phi,t}. \tag{24}$$

Firms hire labor and rent capital from households at competitive factor prices, produce the final good and sell it to households in a competitive market. Households use labor and capital income to finance their consumption and saving choices. The equilibrium law of motion for the model's endogenous variables is defined by a set of conditions that describes the optimal behavior of agents, and the evolution of shocks. Since these equations are standard in the literature, we avoid repeating them here.

To ensure stationarity, certain model's endogenous variables need to be normalized. We have estimated the model with an unrestricted persistence of the preference shock process and

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[8]The only novel process here is the one for the skill-biased technical change. Although the specification we use permits $\phi_t > 1$, this event has almost zero measure in all our simulations. We could use a logistic function to preclude that. However, since we study a linearized version of the model, nothing would prevent the linearized shock to be larger than 1.

found that, to match the high persistence in hours worked, it is estimated to be unit root.[9] Given this, we restrict $\rho_a = 1$, and scale hours worked by a type j household by $e^{-\frac{\nu_j}{1+\nu_j}A_t}$, while the other model's variables by $e^{Z_t}e^{[\phi^*\frac{\nu_s}{1+\nu_s}+(1-\phi^*)\frac{\nu_u}{1+\nu_u}]A_t}$.

3.2 Identifying Neutral Technology Shocks: Setup

The choice of the variables added to the state vector is guided by the balanced growth restrictions. We therefore consider the growth rate of output, $\Delta \log(Y_t)$, the growth rate of wages of skilled workers, $\Delta \log(w_{s,t})$, and unskilled workers, $\Delta \log(w_{u,t})$, the growth rate of labor productivity, $\Delta \log(Y_t/L_t)$ and investment, $\Delta \log(I_t)$. As described earlier, we interpret each of these times series as the sum of two unobserved components: the growth rate in neutral technological component (common to all variables) and a residual component (specific to each variable). Thus, defining $\mathbf{D}_t = [\Delta \log(Y_t), \Delta \log(w_{s,t}), \Delta \log(w_{u,t}), \Delta \log(Y_t/L_t), \Delta \log(I_t)]'$ the vector of observables, and by $\mathbf{S}_t$ the vector collecting these unobserved components, we can write the measurement equation as

$$\mathbf{D}_t = \underbrace{\left[\begin{array}{cc} \mathbf{1} & \mathbf{I} \end{array}\right]}_{\mathbf{B}} \mathbf{S}_t. \tag{25}$$

Notice that, under this formulation, ΔZ_t is the first entry of the state vector. Next we must chose the model for the time series behavior of the state vector $\mathbf{S}_t$. In the Monte Carlo analysis, we will restrict to the simple VAR(1) model used in Section 2.2:

$$\mathbf{S}_t = \underbrace{\left[\begin{array}{cc} \rho_z & \mathbf{0}' \\ \mathbf{\Phi}_{\tilde{\mathbf{S}}\mathbf{z}} & \mathbf{\Phi}_{\tilde{\mathbf{S}}\tilde{\mathbf{S}}} \end{array}\right]}_{\mathbf{\Phi}} \mathbf{S}_{t-1} + \underbrace{\left[\begin{array}{cc} r_{zz} & \mathbf{0}' \\ \mathbf{R}_{\tilde{\mathbf{S}}\mathbf{z}} & \mathbf{R}_{\tilde{\mathbf{S}}\tilde{\mathbf{S}}} \end{array}\right]}_{\mathbf{R}} \left[\begin{array}{c} e_{z,t} \\ \tilde{\mathbf{e}}_t \end{array}\right]. \tag{26}$$

We restrict ΔZ_t to be exogenous with respect to the other unobserved states: this is achieved with the "zeros" restrictions on the matrix $\mathbf{\Phi}$ and $\mathbf{R}$. However, note that we are not ruling out correlation among the idiosyncratic states.

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[9]As in Gali (2005) and Chang, Doh, and Schrfheide (2007), among others.

3.2.1 Balanced Growth Restrictions

The balanced growth restrictions discussed in the earlier section can be easily implemented in this time series model. For example, consider a one standard innovation increase in neutral technology. Under balanced growth restrictions, we know that the effect of this shock on the level of output is equal to $\frac{\sigma_z}{1-\rho_z}$ in the long run. In the time series model described earlier, the long run effect of the first element of $\mathbf{S}_t$ (which we label neutral technological growth) on the level of output equals to:[10]

$$\lim_{m \to \infty} IR_{t+m}^{\log(Y)}(e_{z,t} = 1) = [1, 0, 0, 0, 0, 0]\mathbf{B}(\mathbf{I} - \boldsymbol{\Phi})^{-1}\mathbf{R}_{\mathbf{1}:(\mathbf{n}+\mathbf{k}),\mathbf{1}}.$$

Hence, the balanced growth restriction for output consist in equating the above expression to $\frac{\sigma_z}{1-\rho_z}$. Similar balanced growth restriction can be derived for the other variables. More specifically, we restrict output per hour and investment to have the same long run effect of output: this implies, among other things, that hours worked and the investment-output ratio are not affected in the long run by a neutral technology shock. Similarly, we restrict neutral technology shocks to have the same long run effect on the two wages (e.g., relative wages are not affected by a neutral technology shock in the long run).

From a technical point of view, these are restrictions on the $\boldsymbol{\Phi}$ and $\mathbf{R}$ matrices that, as discussed in the earlier section, are sufficient to identify the neutral technology shock.

3.3 Estimation and Results

The calibration is standard and described in Appendix III. We simulate 100 realizations from the calibrated model, with each sample being composed of 250 quarters. For each realization, we estimate the state space model discussed in Section 3.2, and we study the properties of the retrieved neutral technology innovations. In addition, we compute technology innovations using the Solow residual accounting and using SVAR with long-run restrictions as in Gali (1999).

We assess the accuracy of each procedure using the R^2 of the following linear regressions:

$$\varepsilon_{j,t}^{true} = \alpha + \beta \varepsilon_{z,t}^{\text{identified}} + \eta_t \qquad j \in \{z, a, \phi, g, q\}, \tag{27}$$

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[10]This expression derives from the fact that the long run effect of a shock on variable x equals the cumulative effect on its growth rates.

where $\{\varepsilon_{z,t}^{\text{identified}}\}$ are the technology shocks identified according to the procedure and $\{\varepsilon_{j,t}^{\text{true}}\}$ is structural innovation j in the model economy. These statistics have a clear interpretation. Indeed, a method that perfectly identifies the technology innovations would yield an $R^2 = 1$ with $\varepsilon_{z,t}^{true}$ as the dependent variable and an $R^2 = 0$ for the other structural innovations.[11] These R^2 are calculated for each of the Monte Carlo replications.

Table 1: **True vs. Identified Technology Shocks: RBC Model**

	$\varepsilon_{z,t}$	$\varepsilon_{a,t}$	$\varepsilon_{\phi,t}$	$\varepsilon_{g,t}$	$\varepsilon_{q,t}$
BHM	0.94	0.00	0.01	0.00	0.00
Gali	0.73	0.10	0.09	0.00	0.03
Solow	0.62	0.00	0.26	0.00	0.00

Notes: Each column contains R^2 from the regression of the structural innovation $\varepsilon_{j,t}$, $j \in \{z, a, \phi, g, q\}$ on technology shock $\varepsilon_{z,t}$, identified using the procedure in each row. Results are based on a Monte Carlo studies with 30 replications. BHM refers to the method proposed in this paper as specified in Section 3.2. Gali refers to the technology shock identified following the procedure in Gali (1999). The Solow residual is calculated applying Jorgenson correction for labor composition effects.

The results reported in Table 1 imply that the method proposed in this paper performs very well. The identified neutral technology shocks are closely related to the true neutral technology shocks used when simulating the model (median $R^2 = 0.94$), and are not systematically related to other structural shocks in the model. In contrast, the technology shocks identified using the other two methods are less closely related to the true neutral technology shocks and systematically pick up other structural disturbances. We now use this simple model to better understand the reasons for their shortcomings.

3.3.1 Using SVAR with long-run restrictions to identify technology shocks

The second row of Table 1 indicates that retrieving technology innovations using an SVAR with long run restrictions yields a median R^2 of only 0.73.[12] The reason is that an SVAR with

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[11]Note that these are theoretical benchmarks. Even if we observed the actual process for neutral technology, the R^2 of the $\varepsilon_{z,t}^{\text{true}}$ equation would be below 1 because of sampling errors in the estimation of (ρ_z, σ_z), which are needed to measure the innovations $\varepsilon_{z,t}^{\text{identified}}$.

[12]Specifically, we follow Gali (1999) and estimate a VAR(4) on the growth rate of labor productivity and hours worked, and identify technology innovations as the unique shock having a long run effect on labor productivity.

long run restrictions on labor productivity interprets any low frequency variation in labor productivity as a neutral technology shock. Indeed, labor productivity can be decomposed as

$$\log\left(\frac{Y_t}{L_t}\right) = Z_t + \alpha \log\left(\frac{K_t}{e^{Z_t}L_{e,t}}\right) + \log\left(\frac{L_{e,t}}{L_t}\right).$$

The idea underlying the use of an SVAR with long-run restrictions to identify z is that the second term $\alpha \log\left(\frac{K_t}{e^{Z_t}L_{e,t}}\right)$ is stationary and thus is not affected by any shock in the long-run. If labor is homogeneous, the third term $\log\left(\frac{L_{e,t}}{L_t}\right)$ is zero so that only neutral technology affects output per hour in the long-run.

If labor inputs are heterogeneous, the third term is not zero and will be moved by shocks other than Z_t. There are two key sources inducing such movements in the simple model studied in this section. Preference shocks induce changes in the share of hours worked by skilled and unskilled workers due to different labor supply elasticities of the two groups. This moves labor productivity at low frequencies and leads the SVAR procedure to erroneously interpret the innovations in preference shocks as technology shocks. This explains the R^2 of 0.1 in the regression of the preference shock on the technology shock identified using this method. In the next Section we will study a richer model with more shocks and will observe that *any* shock that induces persistent changes in labor composition will be interpreted as a technology shock in this context.

Even for a counterfactually constant share of hours worked by skilled and unskilled individuals, persistent changes in the relative productivity of skilled workers induced by the skill-biased shock ϕ_t will also induce low-frequency movements in the effective labor input per hour worked. This explains the R^2 of 0.09 in the regression of the skill-bias shock on the technology shock identified using this method. Thus, any such non-neutral shocks will also be identified as technology shocks by this methodology.

Finally, as is well known from the work of Fisher (2006), without additional restrictions this method confounds neutral and non-neutral investment-specific shocks.

3.3.2 The Solow Residual

The Solow residual explains on average only 62% of the variation in actual neutral technology because it is a composite of neutral and non-neutral technological change. This is clear form

Equation (2) in the Introduction, which specializes to

$$(1-\alpha)\frac{\dot{Z}}{Z} + \dot{\phi}(1-\alpha)\log\left(\frac{L_s}{L_e}\right) = \frac{\dot{Y}}{Y} - \alpha\frac{\dot{K}}{K} - \phi(1-\alpha)\frac{\dot{L_s}}{L_s} - (1-\phi)(1-\alpha)\frac{\dot{L_u}}{L_u}$$

in this model. Thus, the Solow residual picks up the non-neutral skill premium shock ϕ and explains on average 26% of its variation. The difference between how much of the skill premium shock is picked up by the Solow residual and how much is picked up by the SVAR with long run restrictions depends on the persistence of the skill premium shock. As the persistence of this shock is estimated to be less than one in this model, its contribution to the Solow residual - which is independent of this persistence - is larger.

Note that in contrast to the SVAR procedure, the Solow residual we compute does not pick up the effects of labor composition induced by the preference shocks. The reason is that when calculating the Solow residual we applied Jorgenson's correction for labor composition effects pioneered by Jorgenson and Griliches (1967). The key idea underlying this correction is to disaggregate the labor force into categories based on education, age, gender, etc. Then, in computing total effective labor input, each hour is weighted by the observed average wage of the group it belongs to, assumed to coincide with the marginal product of that labor input. Then, adding an additional worker with, say, a college degree would account for more of an increase in output than would adding a worker with a high school diploma. While this procedure corrects for pure changes in composition, in Appendix II we show that it does not correct for the biased changes in technology affecting the relative productivity of labor inputs. Of course, it was never intended to do so as the growth accounting literature was not interested in measuring neutral technological innovations.[13]

4   Monte Carlo Analysis using a New Keynesian Model

In this section we assess the performance of our method in a Monte Carlo study using a calibrated benchmark New Keynesian business cycle model. We use a medium-scale model

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[13]We computing the Solow residual we assumed that the parameter α is known and all inputs are observed. More realistically though, suppose that the aggregator L_t^e features richer heterogeneity, for example three groups l, m, h, $L_t^e = L_{h,t}^{\phi_{1,t}} L_{m,t}^{\phi_{2,t}} L_{l,t}^{1-\phi_{1,t}-\phi_{2,t}}$ but the researcher can distinguish only two groups. This misclassification worsens the ability of the Solow residual to identify technology shocks substantially whereas our methodology is immune to such misclassifications. The result in Table 1 assume a correct classification and thus the Solow residual performs better than it will likely do in real data where misclassification is present.

with price and wage rigidities, capital accumulation, investment adjustment costs, variable capital utilization, and habit formation. The model is based on Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007). We enrich this setting with labor heterogeneity. As in the RBC model analyzed in the previous section, workers can be of two types: low and high skilled. They are distinguished by their marginal productivity, defined through the production function in equation (17), and by their Frisch elasticity of labor supply $\nu_l \neq \nu_h$. Appendix IV contains the full description of the model. In addition to the economic shocks that were present in the RBC model, this model incorporates monetary policy shocks, price markup shocks, wage markup shocks and shocks to the discount factor of households. There are nine economic disturbances in total.

After calibrating the model to match the behavior of post-1984 U.S. business cycles, we apply our procedure on simulated data and compare our estimates with the true neutral technology series. The economic significance of deviations between the actual and estimated technology series is assessed by comparing our estimated impulse response functions to their theoretical counterparts. We repeat this exercise for technology series estimated using the SVAR with long run restrictions and the Solow residual accounting procedure. Finally, we perform various robustness checks by varying the parameter estimates of our benchmark calibration.

4.1   Calibration

Most of the model's parameters associated to preferences and technology are fixed to conventional values used in the literature. In particular, we use the estimates (posterior mean) reported by Schorfheide, Sill, and Kryshko (2010), who consider a version of the model studied here without wage markup shocks and labor heterogeneity. The parameters associated to labor heterogeneity come from our analysis of the RBC model, while those governing the economy's structural shocks are calibrated through moment matching. In particular, denote the parameters governing the structural shocks by θ, and let $\mathbf{m_T}$ be a vector of sample moments for selected time series of length $\mathbf{T}$ computed using US data. We denote by $\mathbf{m_T}(\theta)$ their model counterpart when the vector of structural parameter is θ. θ is chosen to minimize a weighted

distance between model and data moments:

$$\min_{\theta_2} \quad [\mathbf{m_T} - \hat{\mathbf{m}}(\theta)]'\mathbf{W_T}[\mathbf{m_T} - \hat{\mathbf{m}}(\theta)],$$

where $\mathbf{W_T}$ is a diagonal matrix whose nonzero elements are the inverse of the variance of the corresponding moment. The empirical moments included in the vector $\mathbf{m_T}$ are standard measures of cyclical variation and comovement for post 1984 quarterly US data. The time series used are the growth rate in GDP, private non-durable consumption, private nonresidential investment, total hours worked in the business sector, total hours of low and high skilled individuals in the business sector, nominal wages for these two demographic groups, labor productivity, and an inflation series constructed using the GDP deflator and the Federal Funds Rate. For each of these time series, we compute the sample standard deviation, the first order autocorrelation and the cross-correlation with GDP growth. We collect these sample moments in the vector $\mathbf{m_T}$. The associated model's moments are calculated via a Monte Carlo procedure. In particular, for each θ, we solve for the policy functions using first order perturbation. We next simulate a realization of length $\mathbf{T}$ for the model's counterparts of the above time series and calculate the vector $\hat{\mathbf{m}}_\mathbf{T}(\theta)$. We repeat this procedure $M = 300$ times, each time changing the seed used in the simulation. We then take the (component wise) median of $\hat{\mathbf{m}}(\theta)$ across the Monte Carlo replications.

Table A-3 summarizes the procedure used for the calibration of our model and reports numerical values for the structural parameters. Table A-4 reports the fit of our model in terms of the calibration targets. We can verify that the calibrated model is consistent along many dimensions with the behavior of aggregate time series at business cycle's frequencies, although certain features of the data are missed.

4.2 Identifying Technology Shocks in Model-Generated Data

Suppose that data on output, capital and hours worked etc. have been generated from the New Keynesian model described above and assume that a researcher identifies technology using the methodology proposed in this paper (i.e., estimates the state space model discussed in Section 3.2), as the Solow residual or using a SVAR with long run restrictions on labor productivity. Is the researcher correctly backing-out the actual realization of technology shocks in

the economy? To answer this question, we perform a simple exercise. Given the parametrization of our model in Table A-3, we simulate $M = 300$ realizations of length $T = 250$ for the model's variables, and calculate the series of "technology" innovations identified using the three methods.[14]

In order to assess the accuracy of each procedure, as in Section 3.3, we consider the R^2 of the following linear regressions:

$$\varepsilon_{j,t}^{true} = \alpha + \beta\varepsilon_{z,t}^{\text{identified}} + \eta_t \qquad\qquad j \in \{z, a, \phi, g, q, \beta, r, p, w\},$$

where $\{\varepsilon_{z,t}^{\text{identified}}\}$ are the identified technology shocks and $\{\varepsilon_{j,t}^{\text{true}}\}$ is structural innovation j in the model economy. The results are presented in Table 2.

As in our analysis of the simple RBC model, we find that the shocks identified using the SVAR with long run restrictions or as the Solow residuals have little structural interpretation, whereas the proposed method is recovering neutral technology shocks very well. Indeed, the median R^2 of our method equals 0.93. For comparison, retrieving technology innovations using SVAR with long run restrictions yields a median R^2 of 0.52, while the Solow residual explains on average 23% of the variation in actual neutral technology. As discussed in Section 3.3, these two methods are not well suited to identify neutral technology shocks in models with heterogeneous inputs. Indeed, the SVAR with long run restrictions on labor productivity interprets any low frequency variation in labor productivity as a neutral technology shock, while the Solow residual is a composite of neutral and non-neutral skill-biased technical change. This generates biased estimates of the neutral technology shock, as is clear from Table 2. The SVAR procedure systematically picks up changes in the composition of the labor force induced by preference and other shocks, which drive labor productivity at low frequencies in the model. The Solow residual, instead, explains on average 42% of the variation in the skill premium shock. The procedure proposed in this paper is not subject to these problems and provides a correct identification of the neutral technology shock.

Notice also that the correlation between the Solow residual and the Long-Run shock to productivity is quite high (0.58) and of similar magnitude to the empirical one reported by

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[14]When calculating the Solow residual we use the true parameter α rather than estimating it. Moreover, we assume that the level of capital utilization is observed by the researcher. Therefore, the only source of discrepancy between neutral technology shocks and the solow residual is coming from the time variation in the non nutral technological parameter.

Table 2: **True vs. Identified Technology Shocks: NK Model**

Method	$\varepsilon_{z,t}$	$\varepsilon_{a,t}$	$\varepsilon_{\phi,t}$	$\varepsilon_{g,t}$	$\varepsilon_{q,t}$	$\varepsilon_{\beta,t}$	$\varepsilon_{r,t}$	$\varepsilon_{p,t}$	$\varepsilon_{w,t}$
BHM	0.93	0.00	0.01	0.00	0.00	0.00	0.00	0.01	0.00
Gali	0.52	0.08	0.14	0.00	0.03	0.02	0.00	0.00	0.04
Solow	0.23	0.00	0.42	0.00	0.00	0.00	0.00	0.00	0.00

Notes: Each column contains R^2 from the regression of the structural innovation $\varepsilon_{j,t}$, $j \in \{z, a, \phi, g, q, \beta, r, p, w\}$ on technology shock $\varepsilon_{z,t}$, identified using the procedure in each row. Results are based on a Monte Carlo studies with 300 replications. BHM refers to the method proposed in this paper as specified in Section 3.2. Gali refers to the technology shock identified following the procedure in Gali (1999). The Solow residual is calculated applying Jorgenson correction for labor composition effects.

Gali (2004). Table 3 suggests, therefore, that a high correlation between these two series is not necessarily a sign of the robustness for either one of the two procedures.

4.3   Impulse Response Functions

In the previous section we assessed the quality of our method as well as of the other two methods (Gali and Solow) by considering the correlation between the true technology series and the identified ones. While indicative of the various biases induced by the three methods, these correlations do not provide information on the economic importance of these biases. In this Section we complement this evidence by computing impulse responses to identified technology shocks. For all three identified technology series we compute the impulse response of key model variables - output, consumption, investment, hours, relative wages of skilled and unskilled and inflation - and compare it to the impulse response for the true technology series. Figure 1 shows the results. The response of each of these variables is reported in a separate row of the figure. The three columns report results for, respectively, our method, SVAR with long run restriction on labor productivity and the Solow residual. In each panel, the dashed line reports the true impulse response while the solid line the estimated one, with the shaded area marking the 90% confidence interval for the estimated impulse response.[15]

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[15]The estimated impulse response and their confidence interval are constructed as via a Monte Carlo simulation. Specifically, for $n = 1 : N$, we i) apply the three procedures on time series simulated from the model; ii) collect the series of estimated technology innovations for the three procedures; iii) compute impulse response as described in Section 2.4.2. The figure reports the pointwise median and 90% confidence interval across these Monte Carlo simulations.

Figure 1: **Impulse Responses to Identified Technology Shocks**
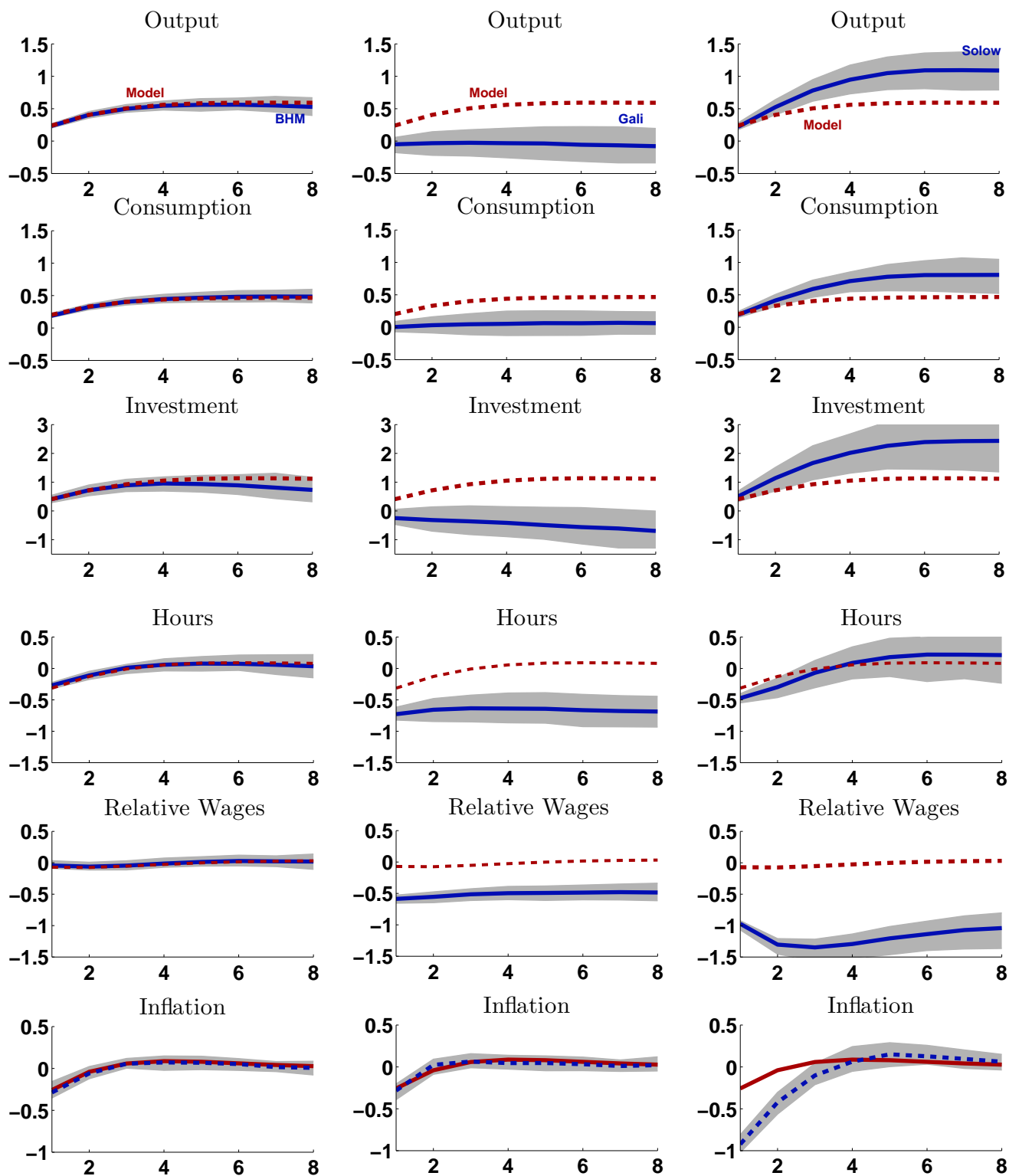
As already suggested by the high correlation between our identified technology series and the true technology series, we find in the first column of the figure that the true and estimated impulse response are very similar as well. With the exception of investment, we can verify that our estimated impulse response functions track very closely their model counterpart. Moreover, the true impulse response always fall in the 90% confidence interval of our estimator. This is clearly not the case for the SVAR approach: the estimates of the the response of the model's variables to a neutral technology shock are, in fact, very imprecise. From our previous discussion we know that SVARs with long run restrictions on labor productivity misinterpret low frequency variation in labor supply with a neutral technology shock. Specifically, a decline in labor supply moves measured output per worker up, and it is interpreted by this procedure as a technology improvements. Not surprisingly, the response of hours to this innovation is biased downward relative to its response to the true technology shock. Because of that, the response of output, consumption and investment is also biased downward: in our numerical simulations, a researcher using SVARs with long run restrictions would conclude that neutral technology shocks are unimportant for business cycle fluctuations, as these variables hardly move conditional on an increase in the identified neutral shock. The response to the Solow residual for real variables are more in line with the true impulse response functions. This reflects the fact that, under our parametrization, skill premium shocks are fairly unimportant for business cycle dynamics. The pattern, though, is that the a positive skill bias shock raises the Solow residual. This shock, in the model, lowers worked hours, increase output and its components and lowers inflation as it decreases firms' marginal costs. These biases can be observed by comparing the true and estimated impulse response functions in the third column of the figure.

Beside the average behavior, the figure also documents that our method significantly improves in the precision of estimates for the impulse response. Confidence interval are, in fact, significantly tighter relative to the other two approaches. This is the result that the technology series we identify is, on average, less noisy with respect to the other methods.

4.4 Sensitivity

We now assess whether these results are sensitive to the particular parameterization we used. To do so we vary the value of each potentially relevant estimated parameter to the upper or lower boundary of its 95% confidence interval. For each resulting parameterization we report the R^2 of the following linear regressions:

$$\varepsilon_{z,t}^{true} = \alpha + \beta \varepsilon_{z,t}^{\text{identified}} + \eta_t,$$

where $\{\varepsilon_{z,t}^{\text{identified}}\}$ are the identified technology shocks and $\{\varepsilon_{z,t}^{\text{true}}\}$ is structural neutral technology innovation. The results are presented in Table 3.

Table 3: **Sensitivity to Parameters in New Keynesian Model**

	BHM		Gali		Solow	
Parameter	down	up	down	up	down	up
$\theta_w \in \{0.15, 0.35\}$	0.95	0.87	0.46	0.54	0.23	0.23
$h \in \{0.58, 0.72\}$	0.92	0.92	0.53	0.50	0.23	0.23
$\gamma_u \in \{0.14, 0.41\}$	0.93	0.93	0.55	0.50	0.23	0.23
$\kappa \in \{0.84, 3.91\}$	0.93	0.93	0.52	0.50	0.23	0.23
$\rho_w \in \{0.02, 0.04\}$	0.93	0.90	0.52	0.46	0.23	0.23
$\rho_q \in \{0.31, 0.46\}$	0.93	0.93	0.52	0.52	0.23	0.23
$\rho_\phi \in \{0.04, 0.12\}$	0.93	0.93	0.52	0.46	0.23	0.24
$\sigma_w \in \{0.27, 0.62\}$	0.93	0.93	0.50	0.53	0.23	0.23
$100 \times \sigma_p \in \{0.09, 0.21\}$	0.93	0.92	0.51	0.51	0.23	0.23

Notes: Each column contains the R^2 from the regression of the structural neutral technology innovation $\varepsilon_{z,t}$ on the technology shock $\varepsilon_{z,t}$, identified using the procedure in the respective column. The column "down" refers to lowering the respective parameter to the lower bound of its condifence band and the column "up" refers to the increase to the upper bound. Results are based on a Monte Carlo study with 300 replications. BHM refers to the method proposed in this paper as specified in Section 3.2. Gali refers to the technology shock identified following the procedure in Gali (1999). The Solow residual is calculated applying Jorgenson correction for labor composition effects.

Table 3 shows results for those parameters where we obtained a different R^2 from either increasing it to the lower or upper bound of its confidence interval. In addition we report results for "technology" parameters such as κ (capital adjustment), γ_u (capital utilization), the persistence of the price of new investment ρ_q and of the skill shock, ρ_ϕ that may be thought of easily confoundable with neutral technology. We find that this is not the case. This

conclusion remains also if we for example increase the persistence of the price of investment q to $\rho_q = 0.99$. Similarly changing the parameters governing the stickiness of prices and wages, σ_w, σ_p, ρ_w and θ_w does not alter our conclusions. Although the R^2 slightly moves, we checked that this change is inconsequential for the impulse responses to the neutral technology identified using our proposed method.

5   Technology Shocks in U.S. Data

In the next draft we will apply our method to identify neutral technology shocks in U.S. data. In particular, we will describe and analyze the sequence of identified shocks and document its co-movement with other economic aggregates. Finally, we will hopefully be able to provide conclusive answers to some of the classic questions in macroeconomics.

6   Conclusion

Standard methods for identifying technology shocks in the data do not identify neutral technology in models with heterogeneous inputs. In particular, the presence of worker heterogeneity invalidates the key identification assumption in Gali (1999) because not only technology, but virtually all persistent shocks have a long run effect on productivity in such models. The identification of neutral technology shocks using the Solow residual accounting procedure is also biased if the effects of factor heterogeneity and non-neutral technical changes are not explicitly accounted for.

Yet, most models have clear predictions for the dynamic responses of variables to neutral technology shocks only. Thus, to evaluate such models it is desirable to be able to separate neutral technology shocks from the multitude of other shocks in the data and to compare the conditional response of variables to these neutral shocks in the data to the responses implied by the models. As existing measures of technology in the data confound neutral technology with non-neutral technology shocks or even with non-technology shocks, such a comparison would not be informative on the empirical performance of a model.

In this paper we therefore propose a method to identify neutral technology in the data. We use Uzawa's classic characterization on balanced growth, to show that imposing balanced

growth properties on long-run impulse responses uniquely identifies neutral technology shocks. We implement this identification in the data using an identified state-space model and establish in Monte Carlo simulations that neutral technology is very well recovered in business cycle models including the New Keynesian one. In particular small samples do not lead our methodology to confound neutral technology neither with non-neutral technology shocks nor with non-technology shocks, such as wage markup shocks or preference shifts.

References

ACEMOGLU, D. (2002): "Technical Change, Inequality, and the Labor Market," *Journal of Economic Literature*, 40(1), 7–72.

——— (2009): *Introduction to Modern Economic Growth.* Princeton Univ. Press.

BASU, S., J. G. FERNALD, AND M. S. KIMBALL (2006): "Are Technology Improvements Contractionary?," *The American Economic Review*, 96(5), 1418–1448.

BLANCHARD, O. J., AND D. QUAH (1989): "The Dynamic Effects of Aggregate Demand and Supply Disturbances," *American Economic Review*, 79(4), 655–673.

CHANG, Y., T. DOH, AND F. SCHRFHEIDE (2007): "Non-stationary Hours in a DSGE Model," *Journal of Money, Credit and Banking*, 39(6), 1357–1373.

CHARI, V., P. J. KEHOE, AND E. R. MCGRATTAN (2008): "Are structural VARs with long-run restrictions useful in developing business cycle theory?," *Journal of Monetary Economics*, 55(8), 1337–1352.

CHRISTIANO, L. J., M. EICHENBAUM, AND C. L. EVANS (2005): "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy*, 113(1), 1–45.

CHRISTIANO, L. J., M. EICHENBAUM, AND R. J. VIGFUSSON (2006): "Alternative Procedures for Estimating Vector Autoregressions Identified with Long-run Restrictions," *Journal of the European Economic Association*, 4(2-3), 475–483.

DEL NEGRO, M., AND F. SCHORFHEIDE (2010): "Bayesian Macroeconometrics," Working Paper, University of Pennsylvania.

DIEWERT, W. E. (1976): "Exact and Superlative Index Numbers," *Journal of Econometrics*, 4(2), 115–145.

DUPUY, A. (2006): "Hicks Neutral Technical Change Revisited: CES Production Function and Information of General Order," *B.E. Journals in Macroeconomics*, 6(2), 1–26.

DURBIN, J., AND S. KOOPMAN (2001): *Time Series Analysis by State-Space Methods.* Oxford University Press, Oxford, UK.

FAUST, J., AND E. LEEPER (1997): "When Do Long-Run Identifying Restrictions Give Reliable Results?," *Journal of Business and Economic Statistics*, 15(3), 345–353.

FERNANDEZ-VILLAVERDE, J., J. F. RUBIO-RAMIREZ, T. J. SARGENT, AND M. W. WATSON (2007): "ABCs (and Ds) of Understanding VARs," *American Economic Review*, 97(3), 1021–1026.

FISHER, J. D. M. (2006): "The Dynamic Effects of Neutral and Investment-Specific Technology Shocks," *Journal of Political Economy*, 114(3), 413–451.

FRANCIS, N., AND V. A. RAMEY (2005): "Is the Technology-Driven Real Business Cycle Hypothesis Dead? Shocks and Aggregate Fluctuations Revisited," *Journal of Monetary Economics*, 52(8), 1379–1399.

——— (2009): "Measures of per Capita Hours and Their Implications for the Technology-Hours Debate," *Journal of Money, Credit and Banking*, 41(6), 1071–1097.

GALI, J. (1999): "Technology, Employment, and the Business Cycle: Do Technology Shocks Explain Aggregate Fluctuations?," *American Economic Review*, 89(1), 249–271.

——— (2004): "On the Role of Technology Shock as a Source of Business Cycles: Some New Evidence," *Journal of the European Economic Association*, 2(1-2), 372–380.

——— (2005): "Trends in Hours, Balanced Growth, and the Role of Technology in the Business Cycle," *Federal Reserve Bank of St. Louis Review*, 87(4), 459–486.

GREENWOOD, J., Z. HERCOWITZ, AND P. KRUSELL (1997): "Long-Run Implications of Investment-Specific Technological Change," *American Economic Review*, 87, 342–362.

HALL, R. E. (1988): "The Relation between Price and Marginal. Cost in U.S. Industry," *Journal of Political Economy*, 96(5), 921–947.

——— (1990): "Invariance Properties of Solows Productivity Residual," in *Growth/ Productivity/Unemployment: Essays to Celebrate Bob Solows Birthday*, ed. by P. Diamond, pp. 71–112. The MIT Press.

HANNAN, E., AND M. DIESTLER (1988): *The Statistical Theory of Linear Systems*. Wiley.

HANSEN, G. (1993): "The Cyclical and Secular Behaviour of the Labour Input: Comparing Efficiency Units and Hours Worked," *Journal of Applied Econometrics*, 8(1), 71–80.

JORGENSON, D. W. (1966): "The Embodiment Hypothesis," *Journal of Political Economy*, 74(1), 1–17.

JORGENSON, D. W., F. M. GOLLOP, AND B. M. FRAUMENI (1987): *Productivity and US Economic Growth*. Cambridge: Harvard University Press.

JORGENSON, D. W., AND Z. GRILICHES (1967): "The Explanation of Productivity Change," *Review of Economic Studies*, 34(3), 249–283.

KATZ, L. F., AND K. MURPHY (1992): "Changes in Relative Wages: Supply and Demand Factors," *Quarterly Journal of Economics*, 107, 35–78.

KOMUNJER, I., AND S. NG (2011): "Dynamic Identification of DSGE Models," *Econometrica*, forthcoming.

KRUSELL, P., L. E. OHANIAN, J.-V. RÍOS-RULL, AND G. L. VIOLANTE (2000): "Capital-Skill Complementarity and Inequality: A Macroeconomic Analysis," *Econometrica*, 68, 1029–1053.

KYDLAND, F. E., AND E. C. PRESCOTT (1982): "Time to Build and Aggregate Fluctuations," *Econometrica*, 50, 1345–1370.

LONG, J. B. J., AND C. I. PLOSSER (1983): "Real Business Cycles," *Journal of Political Economy*, 91(1), 36–69.

NADIRI, M. I., AND I. PRUCHA (2001): "Dynamic Factor Demand Models and Productivity Analysis," in *New Developments in Productivity Analysis*, ed. by C. R. Hulten, E. R. Dean, and M. J. Harper. University of Chicago Press.

SCHORFHEIDE, F., K. SILL, AND M. KRYSHKO (2010): "DSGE Model-Based Forecasting of Non-modelled Variables," *International Journal of Forecasting*, 26(2), 348–373.

SHEA, J. (1998): "What Do Technology Shocks Do?," in *NBER Macroeconomics Annual*, ed. by B. S. Bernanke, and J. Rotemberg, vol. 13. The MIT Press.

SMETS, F. R., AND R. WOUTERS (2007): "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach," *American Economic Review*, 97(3), 586–606.

SOLOW, R. M. (1957): "Technical Change and the Aggregate Production Function," *The Review of Economics and Statistics*, 39(3), 312–320.

STOCK, J. H., AND M. W. WATSON (2012): "Disentangling the Channeles of the 2007-2009 Recession," Working Paper 18094, NBER.

UHLIG, H. (2004): "Do Technology Shocks Lead to a Fall in Total Hours Worked?," *Journal of the European Economic Association*, 2(2-3), 361–371.

——— (2005): "What are the Effects of Monetary Policy on Output? Results from an Agnostic Identification Procedure," *Journal of Monetary Economics*, 52(2), 381–419.

UZAWA, H. (1961): "Neutral Inventions and the Stability of Growth Equilibrium," *Review of Economic Studies*, 28(2), 117–124.

APPENDICES

I Proofs and Derivations

I.1 Proof of Lemma 1

Consider a variable $X_t = \sum_{i=1}^{M} X_{it}$, with long-run response g_X for variable X and g_{X_i} for the components $X_i \neq 0$. By the definition of the impulse response

$$IR_t^X(x) = (e^{g_X x} - 1)E_0[X_t] = \sum_{h=1}^{H} IR_t^{X_i}(x) = \sum_{h=1}^{H}(e^{g_{X_h} x} - 1)E_0[X_{h,t}]$$

and therefore after canceling terms

$$e^{g_X x}E_t[X_t] = \sum_{h=1}^{H} e^{g_{X_h} x}E_0[X_{h,t}]$$

Taking the $l'th$ derivative w.r.t. x yields

$$g_X^l e^{g_X x}E_0[X_t] = \sum_{h=1}^{H} g_{X_h}^l e^{g_{X_h} x}E_0[X_{h,t}]$$

and dividing by g_X^l,

$$e^{g_X x}E_0[X_t] = \sum_{h=1}^{H} \left(\frac{g_{X_h}}{g_X}\right)^l e^{g_{X_h} x}E_0[X_{h,t}]$$

This implies that $g_{X_h} \leq g_X$ since otherwise the RHS converges to ∞ for $l \to \infty$. Then, since

$$exp(g_X x)E_0(X_t) = \sum_{h=1}^{H} exp(g_{X_h} x)E_0(X_{ht}),$$

$$g_X = g_{X_1} = \cdots g_{X_h} = \cdots = g_{X_H}.$$

I.2 Proof of Theorem 1

The argument has two parts. The first part is to show that a neutral technological shock has the properties stated in the theorem and the second part is to show that any other shock with these properties is a neutral shock. In order to prove the first part, note that the resource constraint implies

$$IR_t^Y(x) = IR_t^I(x) + IR_t^C(x),$$

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where I is total investment and C is total consumption of output Y. Equivalently

$$e^{g_Y x} E_0[Y_t] = e^{g_I x} E_0[I_t] + e^{g_C x} E_0[C_t].$$

Dividing by $e^{g_Y x}$ yields

$$E_0[Y_t] = e^{(g_I - g_Y)x} E_0[I_t] + e^{(g_C - g_Y)x} E_0[C_t].$$

Taking derivatives w.r.t. x:

$$0 = (g_I - g_Y)e^{(g_I - g_Y)x} E_0[I_t] + (g_C - g_Y)e^{(g_C - g_Y)x} E_0[C_t].$$

Since this holds for all x, it must be the case that $g_Y = g_I = g_C$. Capital accumulation implies that

$$IR_{t+1}^{K_j}(x) = IR_t^{K_j}(x)(1 - \delta_j) + IR_t^{I_j}(x)$$

and equivalently

$$e^{g_{K_j} x} E_0[K_{j,t+1}] = e^{g_{K_j} x} E_0[K_{j,t}] + e^{g_{I_j} x} E_0[I_{j,t}].$$

This yields

$$e^{g_{K_j} x} \{ E_0[K_{j,t+1}] - (1 - \delta_j)E_0[K_{j,t}] \} = e^{g_{K_j} x} E_0[I_{j,t}] = e^{g_{I_j} x} E_0[I_{j,t}],$$

and thus $g_{K_j} = g_{I_j}$. By Lemma 1, it must be the case that $g_{I_j} = g_I$, and thus $g_{K_j} = g_I \; \forall j$. This implies $g_Y = g_C = g_{I_j} = g_{K_j} = g$. By Assumption 1, this also means that $g_{L_j} = 0 \; \forall j$. Since the production function features constant return to scale, we hve $g = 1$. Constant returns to scale also implies that the marginal products of capital is not influenced by the shock in the long run

$$F_{K_j}(e^x K_{1,t}, \ldots, e^x K_{J,t}, e^x Z_t L_{1,t}, \ldots, e^x Z_t L_{N,t}; \theta_t) \qquad \text{(A1)}$$
$$= \; F_{K_j}(K_{1,t}, \ldots, K_{J,t}, Z_t L_{1,t}, \ldots, Z_t L_{N,t}; \theta_t),$$

and that the marginal product of labor increases by x percent,

$$F_{L_n}(e^x K_{1,t}, \ldots, e^x K_{J,t}, e^x Z_t L_{1,t}, \ldots, e^x Z_t L_{N,t}; \theta_t) \qquad \text{(A2)}$$
$$= \; e^x F_{L_n}(K_{1,t}, \ldots, K_{J,t}, Z_t L_{1,t}, \ldots, Z_t L_{N,t}; \theta_t).$$

For the impulse responses we thus get

$$IR_t^{F_{K_j}}(x) = 0$$

$$IR_t^{F_{L_j}}(x) = (e^x - 1)E_0(F_{L_j}).$$

This proves the first part of the characterization theorem.

The second part of the proof shows that any other shock with these properties is the neutral technology shock, which establishes that no other shock has these properties. To this aim, consider an innovation to a non-neutral permanent shock θ_i of x percent at time 0 and consider how this changes a variable X_t. The impulse response compares variables in two scenarios: one where the shock happens and one where it does not. Denote variables X_t with a $\tilde{}$ (i.e. $\tilde{X}_t$ is conditional on the shock) in the first scenario and without a $\tilde{}$, X_t in the second scenario. The impulse response of a variable X_t therefore equals

$$E_0(\tilde{X}_t) - E_0(X_t),$$

and for $t \geq T$:

$$\tilde{Y}_t = exp(x)Y_t, \ \tilde{K}_{jt} = exp(x)K_{jt}, \ \tilde{L}_{jt} = L_{jt}, \ \tilde{\theta}_i(t) = exp(x)\theta_i(t)$$

Using this notation we get on the one hand

$$\begin{aligned}
\tilde{Y}_t &= F(\tilde{K}_{1,t}, \ldots, \tilde{K}_{J,t}, Z_t\tilde{L}_{1,t}, \ldots, Z_t\tilde{L}_{N,t}; (\boldsymbol{\theta_1(t)}, \ldots, \tilde{\theta}_i(t), \ldots)) \qquad \text{(A3)} \\
&= F(exp(x)K_{1,t}, \ldots, exp(x)K_{J,t}, Z_tL_{1,t}, \ldots, Z_tL_{N,t}; (\boldsymbol{\theta_1(t)}, \ldots, \mathbf{exp(x)}\theta_i(t), \ldots))
\end{aligned}$$

and on the other hand that

$$Y_t = F(K_{1,t}, \ldots, K_{J,t}, Z_tL_{1,t}, \ldots, Z_0L_{N,0}; (\boldsymbol{\theta_1(t)}, \ldots, \theta_i(\mathbf{0}), \ldots)). \qquad \text{(A4)}$$

Constant returns to scale and $\tilde{Y}_t = exp(x)Y_t$ imply that

$$\begin{aligned}
\tilde{Y}_t &= exp(x)Y_t \qquad \text{(A5)} \\
&= F(exp(x)K_{1,t}, \ldots, exp(x)K_{J,t}, exp(x)Z_tL_{1,t}, \ldots, exp(x)Z_tL_{N,t}; (\boldsymbol{\theta_1(t)}, \ldots, \theta_i(t), \ldots))
\end{aligned}$$

Equating the last two expressions for $\tilde{Y}_t$ gives for all x and $t \geq T$ that

$$\begin{aligned}
&F(exp(x)K_{1,t}, \ldots, exp(x)K_{J,t}, Z_tL_{1,t}, \ldots, Z_tL_{N,t}; (\boldsymbol{\theta_1(t)}, \ldots, \mathbf{exp(x)}\theta_i(t), \ldots)) \qquad \text{(A6)} \\
&= F(exp(x)K_{1,t}, \ldots, exp(x)K_{J,t}, exp(x)Z_tL_{1,t}, \ldots, exp(x)Z_tL_{N,t}; (\boldsymbol{\theta_1(t)}, \ldots, \theta_i(t), \ldots)).
\end{aligned}$$

Thus the first line - the effect of a x percent shock to θ_i - is equivalent to the latter line which is the effect of a x percent shock to neutral technology ($\tilde{Z}_t = exp(x)Z_t$). Since this identity holds for all x, θ_i is a neutral technology shock.

Note that the proof at no point uses that the shock θ directly enters the production function, i.e. it applies also to non-technology shocks, e.g. preference shocks, government expenditure shocks or wage mark-up shocks.

I.3 Proof of Lemma 2

In order to check that our state space is minimal, one needs to verify the observability and controllability conditions are satisfied in our state space model. The observability matrix is given by:

$$
\mathcal{O}^n_{(n(n+k-1))\times(n+k)} = \begin{bmatrix} \mathbf{B}_{(n+k-1)\times(n+k)} \\ \mathbf{B\Phi}_{(n+k)\times(n+k)} \\ \ldots \\ \mathbf{B\Phi^n}_{(n+k)\times(n+k)} \end{bmatrix}.
$$

The observability condition is satisfied if $\mathcal{O}^n_{(n(n+k-1)\times(n+k)}$ is of full rank. First notice that $\mathbf{B}$ is of rank $n+k-1$. Now, suppose that the observability condition is violated. That would imply the existence of a $n+k$ dimensional vector $\xi \neq \mathbf{0}$ such that:[16]

$$
\mathbf{B}\xi = \mathbf{0} = \mathbf{B\Phi}\xi
$$

Given our knowledge of the $\mathbf{B}$ matrix, that would imply that the vector ξ is equal to

$$
\xi = (\chi, \underbrace{-\chi, \ldots, -\chi}_{n-1 \ elements}, 0, \ldots, 0)^{tr}
$$

for some $\chi \neq 0$. The last k elements, corresponding to the ξ vector are equal to zero since these variables are observable. As a result we have

$$
\mathbf{\Phi}\xi = \chi \left(\phi_{1,1} - \sum_{l=2}^{n} \phi_{1,l}, \ldots \phi_{j,1} - \sum_{l=2}^{n} \phi_{j,l}, \ldots, \phi_{n,1} - \sum_{l=2}^{n} \phi_{n,l}, 0, \ldots, 0 \right)^{tr},
$$

which equals using that the off-diagonal elements in the first row ($\phi_{1,j} = 0$) are zero,

---

[16]The nullspace of $\mathbf{B}$ is one-dimensional, that means it is generated by a non-zero vector $\mathbf{x}$. The nullspace of $\mathbf{B\Phi}$ is one-dimensional as well. If the observation matrix has rank $n+k-1$ then the nullspace of these of two matrices are identical and generated by the same vector $\mathbf{x}$.

$$\mathbf{\Phi}\xi \;=\; \chi\left(\phi_{1,1},\ldots,\phi_{j,1}-\sum_{l=2}^{n}\phi_{j,l},\ldots,\phi_{n,1}-\sum_{l=2}^{n}\phi_{n,l},0,\ldots,0\right)^{tr} \tag{A7}$$

Multiplying this vector with $\mathbf{B}$ maps it to zero, so that we get the set of equations:

$$-\phi_{1,1} \;=\; \phi_{j,1}-\sum_{l=2}^{n}\phi_{j,l} \quad \forall 2\leq j\leq n, \tag{A8}$$

contradicting Assumption 2 ii). Thus, by contradiction we must have that ξ is not in the nullspace of $\mathbf{B\Phi}$. Thus, the observability matrix is of full rank and the system is observable.

The controllability matrix is given by:

$$\mathcal{C}^{n}_{n+1\times(n)^2} \;=\; \left[\; \mathbf{R}_{(n+1)\times n}\,\mathbf{\Phi R}_{(n+1)\times n}\ldots\mathbf{\Phi^{n}R}_{(n+1)\times n} \;\right].$$

That the controllability matrix in our state space system is of full rank follows from Assumption 2 i).

As a result, our state space realization is observable and controllable, hence minimal.

I.4 Proof of Theorem 2

Suppose the state space is described by the matrices $(\hat{\mathbf{B}},\hat{\mathbf{\Phi}},\hat{\mathbf{R}})$ which are related to the original one as follows:

$$\begin{aligned}
\hat{\mathbf{B}} &= \mathbf{BT} \\
\hat{\mathbf{\Phi}} &= \mathbf{T}^{-1}\mathbf{\Phi T} \\
\hat{\mathbf{R}} &= \mathbf{T}^{-1}\mathbf{RU}
\end{aligned} \tag{A9}$$

We show now that $\mathbf{T}$ is the identity matrix and that $\mathbf{U}$ is as described in the theorem.

Let $\chi_i = (0,\ldots,\underbrace{1}_{i},\ldots,0)'$ be the unit vector with the $i'th$ entry equal to 1 and other entries equal to zero. Consider the long-run effect of χ_1, that is the long-run effect of a neutral technology shock, which equals

$$\mathbf{BT}^{-1}(\mathbf{I}-\mathbf{\Phi})^{-1}\mathbf{RU}\chi_1 = \mathbf{B}(\mathbf{I}-\mathbf{\Phi})^{-1}\mathbf{RU}\chi_1$$

since $\mathbf{BT^{-1}} = \mathbf{B}$. Let

$$\mathbf{U}\chi_1 = \sum_{i=1}^{n+1} u_{i1}\chi_i,$$

where u_{i1} is the $(i, 1)$ entry of $\mathbf{U}$. Then the long-run effect of χ_1 equals

$$\sum_{i=1}^{n+1} u_{i1}v_i,$$

where v_i is the true long-run effect (i.e. for the state space described by the true matrices $(\mathbf{B}, \mathbf{\Phi}, \mathbf{R})$ of χ_i:

$$v_i = \mathbf{B}(\mathbf{I} - \mathbf{\Phi})^{-1}\mathbf{R}\chi_i.$$

We impose the balanced growth restriction which states that the long-run effect of χ_1 equals v_1, so that

$$v_1 = \sum_{i=1}^{n+1} u_{i1}v_i,$$

The RHS is the long-run response to the shock $\mathbf{U}\chi_1$ which equals the long-run response of neutral technology (χ_1) on the LHS (v_1). Theorem 1 implies that only neutral technology has this property so that $\mathbf{U}\chi_1 = \chi_1$, i.e. first column of U is the vector $(1, 0, \ldots, 0)'$. Since $\mathbf{UU'} = I$ this implies that the first row of $\mathbf{U}$ equals $(1, 0, \ldots, 0)$. Finally we use that the first row of

$$\mathbf{T^{-1}RU}$$

is $(1, 0, \ldots, 0)$. Using the properties of $\mathbf{U}$, we also know that the first row of

$$\mathbf{T^{-1}R}$$

is $(1, 0, \ldots, 0)$. Since $\mathbf{\tilde{R}}$ is invertible, we have that $\kappa_2 = \kappa_3 = \ldots = \kappa_n = 0$. Furthermore since $r_{zz} = 1$ we also have $\kappa_1 = 0$, so that $\mathbf{\hat{R}} = \mathbf{RU}$, what completes the proof since $\mathbf{\hat{R}\hat{R}'} = \mathbf{RUU'R'} = \mathbf{RR'}$.

II Standard Approaches to Controlling for Input Heterogeneity

II.1 Jorgenson's Correction

The fact that inputs heterogeneity complicates the measurement of technology is a well known problem in the growth accounting literature. Here we discuss the most widely accepted pro-

cedure that was developed by Jorgenson (1966). An alternative but closely related procedure due to Hansen (1993) is discussed in Appendix II.2. Central to these approaches is the approximation of the growth rate of L_t^e in terms of a weighted sum of the hours worked by different groups of individuals:

$$\Delta \log(L_t^e) \approx \sum_{j=1}^{J} a_{j,t} \Delta \log(L_{j,t}). \tag{A10}$$

The procedures differ in the way the weights $\{a_{j,t}\}$ are computed. Jorgenson uses the following Tornqvist aggregator:

$$a_{j,t} = \frac{\nu_{j,t} + \nu_{j,t-1}}{2}, \quad \text{where } \nu_{j,t} = \frac{w_{i,t} L_{i,t}}{\sum_j w_{j,t} L_{j,t}}. \tag{A11}$$

As shown in Diewert (1976), this would be the right correction to make in the case that L_t^e is a deterministic homogeneous translog function of the J groups considered, $\log(L_t^e) = f(\log(\mathbf{L_t}))$, where $\mathbf{L_t}$ is the vector of hours worked by the J groups.[17] Using the properties of quadratic function (e.g., translog as defined in footnote 17), one obtains:

$$
\begin{aligned}
\Delta \log(L_t^e) &= f(\log(\mathbf{L_t})) - f(\log(\mathbf{L_{t-1}})) \tag{A13} \\
&= \frac{1}{2} \left[\nabla f(\log(\mathbf{L_t})) + \nabla f(\log(\mathbf{L_{t-1}})) \right]' (\log(\mathbf{L_t}) - \log(\mathbf{L_{t-1}})),
\end{aligned}
$$

where the matrix $\nabla f(\log(\mathbf{L_t}))$ collects the partial derivatives of $f(.)$. Under the additional assumption that prices equal marginal products at all points in time, the Jacobian $\nabla f(\log(\mathbf{L_t}))$ is equal to $\frac{w_{i,t} L_{i,t}}{\sum_j w_{j,t} L_{j,t}}$. Thus, equation A10 is exact for a homogeneous translog aggregator when the weights are Tornquivst indexes of labor shares of different groups. All other functional forms, e.g., CES aggregator, will generate a bias.

A fundamental problem of this strategy arises when hours in efficiency units is not a deterministic aggregator of hours worked. An implicit assumption in this procedure is that the parameters of the aggregator have to be constant, making it for example difficult to explain movements in the skill premium. Thus even if the aggregator satisfies the functional form

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[17] Defined by

$$ln f(x) = \alpha_0 + \sum_{k=1}^{K} \alpha_k ln x_n + \frac{1}{2} \sum_{m=1}^{K} \sum_{l=1}^{K} \gamma_{ml} ln x_m ln x_l, \tag{A12}$$

where $\sum_{k=1}^{K} \alpha_k = 1$, $\gamma_{ml} = \gamma_{lm}$ and $\sum_{l=1}^{K} \gamma_{ml} = 0$ for $j = 1, 2, \ldots, K$.

requirements at every point of time but parameters are changing over time, technology is measured with a bias. In order to make this point explicit, suppose that $\log(L_t^e) = f(\log(\mathbf{L_t}), \boldsymbol{\Theta_t})$, where $\boldsymbol{\Theta_t}$ is a vector of time varying observable or unobservable factors and parameters. In this environment, one immediately verifies that equation A14 is an incorrect expansion for L_t^e as it neglects changes in $\boldsymbol{\Theta_t}$.

II.2 Hansens' Correction

Hansen (1993) measures the efficiency units of labor as

$$\sum_i \alpha_i L_{i,t}, \tag{A14}$$

where α_i is the constant weight of group i. The weights α_i are the average hourly earnings

$$\alpha_i = \frac{HE_i}{HE}, \tag{A15}$$

where HE_i is average hourly earnings for group i and HE is average hourly earnings.

We first compute a log-linear approximation of $\log(\sum_i \alpha_i L_{i,t})$ with respect to $\log(L_{i,t})$:

$$\log(\sum_i \alpha_i L_{i,t}) \approx \sum_i \frac{\alpha_i \overline{L_i}}{\sum_j \alpha_j \overline{L_j}} \log(L_{i,t}), \tag{A16}$$

where $\overline{L_i}$ is the average labor supply of group i. In addition to this approximation, a second difference between Hansen and Jorgensen is that they use different coefficients. Jorgensens uses $\nu_{j,t}$, an average of two adjacent periods whereas Hansen uses

$$\frac{\alpha_i \overline{L_i}}{\sum_j \alpha_j \overline{L_j}}, \tag{A17}$$

a time average for the full sample. This means the second bias in the measurement due to differences in computing averages of wages equals

$$\nu_{j,t} - \frac{\alpha_i \overline{L_i}}{\sum_j \alpha_j \overline{L_j}}. \tag{A18}$$

After these approximations, Hansen measurement is equal to Jorgenson and thus is unbiased if and only if the aggregator is a homogeneous translog function (with constant coefficients).

II.3   Estimation of Solow Residual in Practice

The current the state-of-the-art measurement of Solow residual in the data is based on IV-regression methods described in Basu, Fernald, and Kimball (2006). As their methodology differs from the Solow residual construction we used in the main text, a few details should be mentioned. First, it is well-known that if there might be increasing returns to scale, time-varying factor utilization, or if factors are not paid their marginal products, tfp measured as Solow residual will be biased. To overcome this limitation, Basu, Fernald, and Kimball (2006), following the insight in Hall (1988, 1990), treat Equation (2) as a regression. As input choices are likely endogenous to innovations in the technology estimated as the residual, the regression is estimated using instrumented variables. The instruments are required to affect the input choice but to be uncorrelated with innovation in technology. The authors use oil prices, growth in real government defense spending, and "monetary shocks" from a non-structural VAR. Their estimates are based on the data described in Jorgenson, Gollop, and Fraumeni (1987) that controls for changes in labor composition using the Jorgenson's correction.

III   Calibration of the Simple RBC Model

The vector of structural parameters of our model is given by:

$$\theta = [\underbrace{\beta, \delta, \alpha, h_s^*, h_u^*, u, \phi^*, \overline{\nu}, \gamma}_{\theta_1}, \underbrace{\rho_\phi, \sigma_\phi, x_l, \rho_z, \sigma_z, \sigma_a, \rho_q, \sigma_q, \rho_g, \sigma_g}_{\theta_2}].$$

Model period is one quarter. We use quarterly post-84 data on the US economy in order to calibrate the vector θ. The parameters in θ_1 are pinned down using long run average for selected time series. In particular, the parameters β, α and δ are chosen so that, in a deterministic steady state of the model, the real interest rate, the depreciation rate of capital and a labor income share are respectively 1%, 2.5% and 66%, values that are common in the business cycle literature. The growth rate of neutral technology shocks, γ, is chosen so to match an average growth rate of GDP per capita equal to 2%. The parameters h_u^*, h_s^*, u and ϕ^* are chosen so that the model matches a fraction of 0.29 hours worked by low-skilled individual, 0.36 by high-skilled individuals, a fraction of low-skilled individuals over total population of 0.64 and a skill premium equal to 1.7. These numbers are calculated using CPS quarterly

data (1979-2006) on wages and hours worked by education level.[18] Finally, we fix the average Frisch elasticity $\bar{\nu}$ to 1.

The remaining parameters in θ_2 are calibrated via a Simulated Method of Moments (SMM) algorithm. In particular, let $\mathbf{m_T}$ be a vector of sample moments for selected time series of length $\mathbf{T}$ computed using US data. We denote by $\mathbf{m_T}(\theta)$ their model counterpart when the vector of structural parameter is θ. θ is chosen to minimize a weighted distance between model and data moments:

$$\min_{\theta_2} \quad [\mathbf{m_T} - \hat{\mathbf{m}}(\theta)]'\mathbf{W_T}[\mathbf{m_T} - \hat{\mathbf{m}}(\theta)],$$

where $\mathbf{W_T}$ is a diagonal matrix whose nonzero elements are the inverse of the variance of the corresponding moment. The empirical moments included in the vector $\mathbf{m_T}$ are standard measures of cyclical variation and comovement for post 1984 quarterly US data. The time series used are the growth rate in GDP, private non-durable consumption, private nonresidential investment, total hours worked in the business sector, total hours of low and high skilled individuals in the business sector, nominal wages for these two demographic groups, labor productivity. For each of these time series, we compute the sample standard deviation, the first order autocorrelation and the cross-correlation with GDP growth. We collect these sample moments in the vector $\mathbf{m_T}$. The associated model's moments are calculated via a Monte Carlo procedure. In particular, for each θ, we solve for the policy functions using first order perturbation. We next simulate a realization of length $\mathbf{T}$ for the model's counterparts of the above time series and calculate the vector $\hat{\mathbf{m}}_{\mathbf{T}}(\theta)$. We repeat this procedure $M = 300$ times, each time changing the seed used in the simulation. We then take the (component wise) median of $\hat{\mathbf{m}}(\theta)$ across the Monte Carlo replications.

Table A-1 summarizes the procedure used for the calibration of our model and reports numerical values for the structural parameters. Table A-2 reports the fit our model in terms of the calibration targets. We can verify that the calibrated model is consistent along many dimensions with the behavior of aggregate time series.

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[18]We define high-skilled individuals as those possessing college education and low-skilled individuals as those with no college education. See Appendix V.

Table A-1: **Calibrated Parameter Values: RBC Model**

Parameter	Value	Source
α	0.33	Labor Income Share
δ	0.025	Depreciation of Capital Stock
β	0.99	Real Interest Rate
γ	1.004	Average GDP growth per capita
h_s^*	0.36	Weekly Hours per Individual (College)
h_u^*	0.29	Weekly Hours per Individual (no College)
u	0.64	% of Individuals without College
μ_ϕ	0.39	Skill Premium
$\overline{\nu}$	1.00	Fixed
x_s	0.85	Calibrated
ρ_ϕ	0.74	Calibrated
ρ_a	1.00	Calibrated
ρ_z	0.26	Calibrated
ρ_g	0.97	Calibrated
ρ_q	0.99	Calibrated
$\sigma_a \times 100$	1.14	Calibrated
$\sigma_\phi \times 100$	1.32	Calibrated
$\sigma_z \times 100$	0.74	Calibrated
$\sigma_g \times 100$	0.18	Calibrated
$\sigma_q \times 100$	0.12	Calibrated

Table A-2: **RBC Model Calibration Targets: Data and Model**

Moment	Data	Model	Moment	Data	Model	Moment	Data	Model
St Dev(ΔY_t)	0.88	0.63	Acorr(ΔY_t)	0.15	0.47	Corr$(\Delta Y_t, \Delta C_t)$	0.17	0.55
St Dev(ΔC_t)	0.47	0.56	Acorr(ΔC_t)	0.14	0.42	Corr$(\Delta Y_t, \Delta I_t)$	0.13	0.33
St Dev(ΔI_t)	2.15	3.25	Acorr(ΔI_t)	0.30	0.36	Corr$(\Delta Y_t, \Delta I_t)$	0.04	0.44
St Dev(ΔH_t)	0.77	0.75	Acorr(ΔH_t)	0.02	0.53	Corr$\left(\Delta Y_t, \Delta \frac{Y_t}{H_t}\right)$	0.18	0.11
St Dev$\left(\Delta \frac{Y_t}{H_t}\right)$	0.56	0.53	Acorr$\left(\Delta \frac{Y_t}{H_t}\right)$	-0.11	-0.20	Corr$(\Delta Y_t, \Delta W_{s,t})$	0.30	0.06
St Dev($\Delta W_{s,t}$)	0.98	1.53	Acorr($\Delta W_{s,t}$)	-0.07	0.12	Corr$(\Delta Y_t, \Delta W_{u,t})$	0.05	0.06
St Dev($\Delta W_{u,t}$)	0.78	1.13	Acorr($\Delta W_{u,t}$)	-0.01	0.06	$\frac{\text{St Dev}(\Delta H_{s,t})}{\text{St Dev}(H_{u,t})}$	1.20	1.15

IV   A New-Keynesian Model with Heterogeneous Labor

In this section we describe the New-Keynesian model which we use in the main text. The model is identical to Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007), except that we have two different type of labor (u)nskilled and (s)killed labor. There is a mass one of workers (on the unit interval), unskilled workers on the interval $[0, u]$ and skilled workers on $[1 - u, 1]$. We also have a richer specification of uncertainty. The sources of uncertainty in the model are shocks to TFP, investment, the disutility of labor, discount factor, the wage markup, the price markup, the skill premium, government spending and monetary policy.

IV.1   Final-Good Firms

The final consumption good Y is a composite made of intermediate goods Y_j and is sold in a perfectly competitive market at price P_t and equals

$$Y_t = \left[\int_0^1 Y_{jt}^{\frac{1}{1+\lambda_{f,t}}} dj \right]^{1+\lambda_{f,t}}, \tag{A19}$$

where $\lambda_{f,t}$ is an exogenous shock whose law of motion will be specified later, and Y_{jt} is intermediate good i. The inflation rate $\pi_t = P_t/P_{t-1}$. Bonds pay a return $e^b R$, where e^b is a risk-premium shock on the nominal return R.

IV.2   Intermediate-Goods Firms

A monopolist produces intermediate good $j \in [0, 1]$ using the following technology:

$$Y_{jt} = \begin{cases} K_{jt}^\alpha [e^{Z_t} L_{jt}^e]^{1-\alpha} - Z_t F & \text{if} \geq F \\ 0 & \text{otherwise} \end{cases}, \tag{A20}$$

where $0 < \alpha < 1$,

$$L_{jt}^e = L_{s,jt}^\phi L_{u,jt}^{1-\phi}$$

and $L_{s,t} = sh_{s,t}$ is total hours worked by skilled individuals, and $L_t = L_{s,t} + L_{u,t}$ is total hours worked. Here, L_{jt}^e and k_{jt} denote the time t labor and capital services used to produce the jth intermediate good. The fixed cost of production are denoted $F > 0$. Intermediate firms rent

capital and labor in perfectly competitive factor markets. Profits are distributed to households at the end of each time period. Let R_t^k and W_t denote the nominal rental rate on capital t services and the wage rate, respectively. A firm's real marginal cost is $s_t = \delta S_t(Y)/\delta Y$, where

$$S_t(Y) = \min_{K, L_s, L_u} r_t^k K + w_t^u L_u + w_t^s L_s \tag{A21}$$

$$Y \text{ given by (A20)} \tag{A22}$$

where $r_t^k = R_t^k/P_t$, $w_t^s = W_t^s/P_t$ and $w_t^u = W_t^u/P_t$. Given our functional forms, we have

$$s_t = \left(\frac{1}{\alpha}\right)^\alpha \left[\left(\frac{1}{\phi(1-\alpha)}\right)^\phi \left(\frac{1}{(1-\phi)(1-\alpha)}\right)^{1-\phi}\right]^{1-\alpha} (r_t^k)^\alpha \left[(w_t^s)^\phi (w_t^u)^{1-\phi}\right]^{1-\alpha} e^{Z_t(\alpha-1)} \tag{A23}$$

Price setting by firms is as in Calvo (1983) with a constant probability, $1 - \theta_p$, of being able to reoptimize its nominal price.

IV.3 Households

There is a continuum of households, indexed by $j \in [0, 1]$. As in Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007) all households - skilled and unskilled - are homogeneous with respect to consumption and asset holdings but are heterogeneous with respect to the wage rate they earn and the hours they work. The utility function of the jth household of type $T \in \{u, s\}$

$$E_{t-1}^j \sum_{l=0}^\infty \beta^{l-t} \left[u(c_{t+l} - hc_{t+l-1}) - \frac{e^{A_{t+l}}}{1+\nu_T} h_{j,t+l}^{1+\nu_T}\right]. \tag{A24}$$

Here, E_{t-1}^j is the expectation operator, conditional on aggregate and household $j's$ idiosyncratic information up to, and including, time $t-1$; c_t denotes time t consumption; h_{jt} denotes time t hours worked. The household's stock of physical capital, $\bar{k}_t$, evolves according to

$$\bar{k}_{t+1} = (1-\delta)\bar{k}_t + e^{q_t}\left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t \tag{A25}$$

The physical rate of depreciation is denoted δ, I_t denotes time t investment, and S is the adjustment cost function, with the following properties: $S(e^\gamma) = 0$, $S'(e^\gamma) = 0$ and $S''(e^\gamma) = \kappa$, where γ is mean growth rate of Z_t.

Capital services, k_t, are related to the physical stock of capital by $k_t = u_t\bar{k}_t$. Here, u_t denotes the utilization rate of capital, which at cost $a(u_t)\bar{k}_t$ (in consumption goods) is set by the household. We assume that $u_* = 1$ in steady state, that $a(1) = 0$ and we define $\gamma_u = a''(1)$.

IV.4   The Wage Decision

Households are monopoly suppliers of a differentiated labor service, $h_{u,jt}$ for unskilled and $h_{s,jt}$ for skilled workers. They sell this service to a representative, competitive firm for skilled/unskilled workers that transforms it into an aggregate labor input, $L_{s,t}$ and $L_{u,t}$ respectively, using the following technologies:

$$L_{T,t} = \left[\int_0^1 h_{T,jt}^{\frac{1}{1+\lambda_{w,t}}} dj \right]^{1+\lambda_{w,t}}, \tag{A26}$$

for $T \in \{s, u\}$. In each period, a household faces a constant probability, $1 - \theta_w$, of being able to reoptimize its nominal wage.

IV.5   Monetary Policy

We assume that monetary policy is described by an interest rate rue given by

$$\frac{R_t}{R_t^*} = \left(\frac{R_{t-1}}{R^*} \right)^{\rho_R} \left[\left(\frac{\pi_t}{\pi^*} \right)^{r_\pi} \left(\frac{Y_t}{Y_t^*} \right)^{r_y} \right]^{1-\rho_R} exp(\epsilon_t^r), \tag{A27}$$

where R^* is the steady state nominal gross interest rate, π^* is steady state inflation rate, and Y_t^* is the natural level of output, i.e. the output level in the flexible price and wage economy.

IV.6   The aggregate resource constraint

The aggregate resource constraint is

$$c_t + g_t + u_t + a(u_t) \leq Y_t, \tag{A28}$$

where g_t is government expenditure.

IV.7   Stochastic Structure

In addition to monetary policy there are eight additional sources of uncertainty. The law of motion for these shocks are given by:

$$z_t - z_{t-1} = \gamma + \rho_z(z_{t-1} - z_{t-2}) + \varepsilon_{z,t} \tag{A29}$$

$$\epsilon_t^r = \varepsilon_{r,t} \tag{A30}$$

$$A_t = \rho_a A_{t-1} + \varepsilon_{a,t} \tag{A31}$$

$$\phi_t = \rho_\phi \phi_{t-1} + \varepsilon_{\phi,t} \tag{A32}$$

$$g_t = \rho_g g_{t-1} + \varepsilon_{g,t} \tag{A33}$$

$$q_t = \rho_q q_{t-1} + \varepsilon_{q,t} \tag{A34}$$

$$b_t = \rho_b b_{t-1} + \varepsilon_{b,t} \tag{A35}$$

$$\lambda_{wt} = \rho_w \lambda_{wt-1} + \varepsilon_{w,t} \tag{A36}$$

$$\lambda_{ft} = \rho_w \lambda_{ft-1} + \varepsilon_{f,t} \tag{A37}$$

The innovations follow a standard normal random vector.

Table A-3: **Calibrated Parameter Values: New Keynesian Model**

Parameter	Value	Source	Parameter	Value	Source
α	0.20	SSK	x_s	0.85	SMM
h	0.66	SSK	ρ_ϕ	0.08	SMM
κ	2.5	SSK	ρ_a	0.99	SMM
γ_u	0.30	SSK	ρ_z	0.05	SMM
θ_p	0.66	SSK	ρ_g	0.90	SMM
θ_w	0.25	SSK	ρ_q	0.38	SMM
λ_p	0.15	SSK	ρ_w	0.10	SMM
λ_w	0.15	SSK	ρ_p	0.10	SMM
ρ_r	0.86	SSK	ρ_b	0.50	SMM
ψ_π	3.05	SSK	$\sigma_a \times 100$	1.71	SMM
ψ_g	0.06	SSK	$\sigma_\phi \times 100$	1.01	SMM
π^*	2.94	SSK	$\sigma_z \times 100$	0.60	SMM
g^*	0.28	SSK	$\sigma_g \times 100$	0.01	SMM
δ	0.025	Depreciation of Capital Stock	$\sigma_q \times 100$	0.82	SMM
γ	1.004	Average GDP growth per capita	σ_w	0.41	SMM
h_s^*	0.36	Weekly Hours per Individual (College)	$\sigma_p \times 100$	0.15	SMM
h_u^*	0.29	Weekly Hours per Individual (no College)	$\sigma_b \times 100$	0.36	SMM
u	0.64	% of Individuals without College	$\sigma_r \times 100$	0.02	SMM
μ_ϕ	0.39	Skill Premium			
$\overline{\nu}$	1.00	Fixed			

Table A-4: **New Keynesian Model Estimation Targets: Data and Model**

Moment	Data	Model	Moment	Data	Model	Moment	Data	Model
St Dev(ΔY_t)	0.75	0.63	Acorr(ΔY_t)	0.48	0.47	Corr$(\Delta Y_t, \Delta C_t)$	0.82	0.55
St Dev(ΔC_t)	0.64	0.56	Acorr(ΔC_t)	0.42	0.42	Corr$(\Delta Y_t, \Delta I_t)$	0.70	0.36
St Dev(ΔI_t)	2.38	3.25	Acorr(ΔI_t)	0.39	0.36	Corr$(\Delta Y_t, \Delta \pi_t)$	-0.14	-0.05
St Dev(π_t)	1.58	1.03	Acorr(π_t)	0.40	0.57	Corr$(\Delta Y_t, \Delta H_t)$	0.73	0.54
St Dev(ΔH_t)	1.03	0.75	Acorr(ΔH_t)	0.27	0.53	Corr$\left(\Delta Y_t, \Delta \frac{Y_t}{H_t}\right)$	-0.02	0.03
St Dev$\left(\Delta \frac{Y_t}{H_t}\right)$	0.70	0.53	Acorr$\left(\Delta \frac{Y_t}{H_t}\right)$	-0.11	-0.18	Corr$(\Delta Y_t, \Delta W_{s,t})$	0.30	0.06
St Dev$(\Delta W_{s,t})$	0.83	1.53	Acorr$(\Delta W_{s,t})$	-0.07	0.07	Corr$(\Delta Y_t, \Delta W_{u,t})$	0.05	0.06
St Dev$(\Delta W_{u,t})$	1.25	1.13	Acorr$(\Delta W_{u,t})$	-0.01	-0.23	Corr$(\Delta Y_t, R_t)$	-0.18	0.18
St Dev(R_t)	1.24	2.53	Acorr(R_t)	0.96	0.98			

V   Data Construction

GDP growth: Log difference of gross domestic product in per capita term (chained dollars). Data are quarterly, 1979:Q1-2012:Q4. The source is *Bureau of Economic Analysis*, National Income and Product Accounts Tables, Table 7.1.

Consumption growth: Log difference of personal consumption expenditures in per capita term (chained dollars). Data are quarterly, 1979:Q1-2012:Q4. The source is *Bureau of Economic Analysis*, National Income and Product Accounts Tables, Table 7.1.

Investment growth: Log difference of nonresidential gross private domestic investment (chained dollars). Data are quarterly, 1979:Q1-2012:Q4. The source is *Bureau of Economic Analysis*, National Income and Product Accounts Tables, Table 1.1.6.

Inflation: Log difference of GDP deflator. Data are quarterly, 1979:Q1-2012:Q4. The series is downloaded from the FRED database of the Federal Reserve Bank of St. Louis (GDPDEF).

Federal Funds Rate: Quarterly averages of monthly effective Federal Funds Rate. Data are quarterly, 1979:Q1-2012:Q4. The series is downloaded from the FRED database of the Federal Reserve Bank of St. Louis (FEDFUNDS).