MINISTRY OF HIGHER SECONDARY AND SPECIAL EDUCATION OF REPUBLIC OF UZBEKISTAN
NAVOI STATE MINING INSTITUTE
“AUTOMATION AND CONTROL” DEPARTMENT

THE EDUCATIONAL-METHODOLOGICAL COMPLEX

ON DISCIPLINE

ALGORITHMIZATION OF COMPUTING METHODS

Field of knowledge	300 000 – It is industrial - technical sphere
Sphere of education	310 000 – Engineering
Formation direction	5 311 000 – Automation and control of technological processes and industries (on branches)

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ALGORITHMIZATION OF COMPUTING METHODS

The educational-methodical complex is developed on the basis of the approved program in the MHSSE RUz (Order No. 355 of "25" august 2016)

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In an education-methodical complex on disciplines «Algorithmization of computing methods» lecture materials, methodical instructions for practical and laboratory researches, the self study materials, the glossary, the typical curriculum, the working curriculum, lecture slides, the lecture abstract, educational technology and a technological card practical and laboratory researches, educational technology and a technological card, variants of questions by control kinds, questions of tests, the general questions on disciplines and a glossary is resulted.

The present educational-methodical complex is intended for bachelors a direction on 5311000 - «Automation and control of technological processes and industries».

Educational-methodical complex can be useful for PhD students and free researchers, and also to scientific employees studying this course.

Educational-methodical complex is discussed in department "Automation and control" (A&C) (report №1 from *25* August) on faculty meeting and it is recommended for council of Power-mechanical faculty.

The head of chair: ________ Dr., Assoc.prof. Jumayev O.A.

The educational-methodical complex is discussed on session of Faculty of Electrical and Machinery Engineering (report №1 from «*26*» August) and handed over Educational-methodical Council of institute is recommended for statements.

The chairman of the council of faculty: ____________ prof. Bazarova S.J.

The educational-methodical complex is recommended for introductions in educational process by educational-methodical Council of the Navoi State Mining Institute (report №1 from «*30*» august)

The secretary of educational-methodical Council ________ Normatova M. J.

In coordination:
The chief of educational-methodical department: _________ Karimov I.A.

THE MAINTENANCE

EDUCATION MATERIALS

THE THEMES OF ABSTRACT
(The plan, keywords and word-combinations)

Lecture №1.

Introduction. The cores concept about algorithmization of computing methods.

The purpose: Formation of knowledge, skills on studying of bases of algorithmization, the basic properties of algorithm and classification of computing methods.

The plan:

1. Classification of computing methods.
2. Preparation of problems for the personal computer decision.
3. Properties of algorithm.
4. Classification of algorithms.

Algorithmization basis. The basic properties of algorithm

Process of preparation and the decision of problems on the personal computer is while difficult enough and labour-consuming, demanding performance of variety of stages. Such stages are:

1) problem statement;
2) the mathematical formulation of a problem;
3) a choice of a numerical method of the decision;
4) working out of algorithm of the decision of problems;
5) a program writing;
6) input of the program and the initial data;
7) program debugging;
8) the problem decision on the personal computer;

The given sequence is characteristic for the decision of each problem. However in the course of problem preparation each stage can have more and less expressed character. Performance of stages in the course of problem preparation has character of consecutive approach as problem specification at the subsequent stage leads to necessity of return to the previous and repeated performance of the subsequent stages.

Let's consider more in detail performance of works at each stage in the course of preparation of a problem for the decision.

Technology OREG.

- **O –** state the **opinion.**
- **R –** produce one **reason** of the opinion.
- **E –** give an **example** for the explanatory of the reason.
- **G – generalise** the opinion.

Question for OREG: **what properties algorithms should possess?**

Algorithm with structure of the enclosed cycles – the algorithm containing a cycle in which are placed one or other several cycles. There are many ways of record of the algorithms different from each other by presentation, compactness, degree of formalisation and other indicators.

The greatest distribution was received by a graphic way and a so-called algorithmic language of record of the algorithms, focused on the person (pseudo-codes).

Algorithmic design of branching.

Branching - operating structure, организующая performance only one of two specified actions depending on justice of some condition.

Condition - a question having two variants of the answer: yes or not. Branching record is carried out in two forms: full and incomplete.

Cycle performance "while" begins with condition check, therefore such version of cycles names cycles with a precondition. Transition to action performance is carried out only in the event that the condition is carried out, otherwise there is an exit from a cycle. It is possible to tell that a cycle condition "while" is a condition of an input in a cycle. In that specific case it can appear that action was not carried out never. The cycle condition is necessary for picking up so that actions carried out in a cycle have led to infringement of its validity, differently there will be a cycling.

Control questions

1. List stages of preparation of problems for the decision on the computer.
2. What properties of algorithm in you know?
3. The basic classification of algorithms.
4. Give definitions of algorithms of branching out and cyclic structure.

Lecture №2.

Algorithmization of the numerical decision of the algebraic and transcendental equations.

A method branch of roots and a method half divisions.

The plan:

1. **A method branch of roots**
2. **A method half divisions**

1. Methods of branch of roots

The description of a method of the decision of branch of roots

The numerical decision of the nonlinear equations of a kind

$$F(x)=0 \quad (2)$$

consists in a finding of values x, satisfying (with the set accuracy) to the given equation and consists of following basic stages:

Branch (isolation, localisation) equation roots.

Specification by means of some computing algorithm of the concrete allocated root with the set accuracy.

The purpose of the first stage is the finding of pieces from a function range of definition in which one root of the solved equation contains only. Are sometimes limited to consideration only any part of the range of definition causing for those or other reasons interest. For realisation of the given stage graphic or analytical ways are used.

Analytical way of branch of roots

The analytical way of branch of roots is based on following theorems:

The theorem 1. If function F (x), defining equation F (x) =0, on the piece ends [a; b] accepts values of different signs, i.e.

Graphic way of branch of roots

The graphic way of branch of roots is based, basically, on visual perception. The branch of roots is made graphically, considering that the valid roots of the equation (1) is there are points of intersection of the schedule of function y=F (x) with an axis of abscisses y=0, it is necessary to construct the function schedule y=F (x) and on axis 0X to note the pieces containing on one root. But it is frequent for simplification of construction of the schedule of function y=F (x) the initial equation (1) replace with the equation equivalent to it $f_1(x)=f_2(x)$. Schedules of functions $y_1=f_1(x)$ and $y_2=f_2(x)$ Further are under construction, and then on axis 0X the pieces localising abscisses of points of intersection of two schedules are marked.

The decision in system MathCad

Problem: to Solve the nonlinear equation $5\sin 2x=\sqrt{1-x}$ (1) numerical method of tangents. We will find and is investigated four roots with accuracy e = 0,000001.

The decision

Let's construct in program Mathcad the function schedule

Let's preliminary transfer all to the left part and we will lead to a kind (1) then the equation will become:

2. A method half divisions

Let's consider the equation (1):

$$F(x)=0,$$

Where function *F*(x) – is continuous and defined on some piece and $F(a)F(b)<0.$

The last means that function *F* (x) has on a piece at least [*a*, *b*] one root. We will consider a case, when a root on a piece the unique [*a*, *b*].

We halve a piece. If $F\left(\frac{a+b}{2}\right)=0$, is a $\xi=\frac{a+b}{2}$ root of the equation (1). If $F\left(\frac{a+b}{2}\right)\neq 0$, it is considered that half of piece on [*a*, *b*] which ends function *F* (x) has different signs. New, narrower piece [a_1, b_1] again we halve and it is spent on it the same consideration etc. As a result on some step we will receive or exact value of a root of the equation (1), or sequence of the pieces enclosed each other $[a_1,b_1],[a_2,b_2],\dots,[a_n,b_n],\dots,$ such that

$$F(a_n)F(b_n)<0,\quad (n=1,2,\dots) \quad \text{and} \quad b_n-a_n=\frac{b-a}{2^n}. \qquad (10)$$

http://math.semestr.ru/optim/secant_method.php **Online the decision**

Control questions

1. **A method branch of roots**
2. **A method half divisions**

Lecture № 3.
Algorithmization of the numerical decision of the algebraic and transcendental equations.
Method a chord and Newton's method.

The plan:

1. **A method the Chord**
2. **Newton's method**

1. A method the Chord (a method of proportional parts)

Again we will address to the equation (1):

$$F(x)=0,$$

Where function F (x) – is continuous and defined on some piece and $[a,b]$ $F(a)F(b)<0.$

There is faster way of a finding of the isolated root of the equation ξ (1) lying on a piece $[a,b]$. We will assume for definiteness that Instead of $F(a)<0$ и $F(b)>0.$ piece division half-and-half $[a,b]$, we will divide it in the relation It $F(a):F(b).$ gives the first approach of a rootуравнения:

$$x_1 = a - \frac{F(a)}{F(b)-F(a)}(b-a). \qquad (12)$$

Then we consider pieces $[a,x_1]$ и $[x_1,b]$. We will choose that from them on which ends function F (x) has different signs, we will receive the second approach of a root of the equation etc x_2. until then yet we will not reach inequality performance – the $\left|\frac{x_{n+1}-x_n}{x_n}\right| \le \varepsilon$, где ε set accuracy of the decision. Geometrically this method is equivalent to replacement curve $y=F$ (x) a chord spent at first through points $A[a,F(a)]$ и $B[b,F(b)]$, and then the chords spent through the ends of received pieces ($[x_1,b],[x_2,b],...,[x_n,b],...,$ fig. 2). From here the name – a *method of chords.*

2. A method of tangents (Newton)

For realisation of the given method, it is necessary to construct initial function y=F (x) and to find values of function on the end of piece F (b). Then to spend a tangent through point M_1. The absciss of a point of intersection of a tangent with axis OX it also is the approached root x_1. Further to find point M_2 (x_1; F (x_1)) to construct the following tangent and to find the second approached root x_2 etc., drawing 2.

The formula for (n+1) looks like approach:

$$x_{n+1} = x_n - \frac{F(x_n)}{F'(x_n)} \qquad (3)$$

If F (a) *F "(a)> 0, x0=a, otherwise x_0=b.
Iterative process proceeds until it will be revealed that:

$$|F(x_{n+1} \le \varepsilon)|. \quad (4)$$

Advantages of a method: simplicity, speed of convergence. Method lacks: calculation of a derivative and difficulty of a choice of initial position.

At first function analyzes the end and a piece [a; b]. If the condition $f(a)\cdot f''(a)>0$, the end and a piece $f(a)\cdot f''(a)>0$ [a; b] also will be the first approach x_1 the equation root, differently the end b a piece [a b] will be the first approach of a root of the equation;. Iterative process which proceeds until Further begins $|f(x1)|>e$. As soon as iterative $|f(x1)|\le e$ process stops, and in x1содержится a required root with necessary approach.

Control questions

1. **A method the Chord**
2. **Newton's method**

Lecture № 4.
Algorithmization of the numerical decision of the algebraic and transcendental equations.
A method of iteration and a method of secants.

The plan:

1. A method of simple iteration
2. A method of secants

1. A method idle time of iterations (a method consecutive approximation)

It is said that iterative process *converges*, if at performance of consecutive iterations values of the roots turn out, all is closer and closer coming nearer to exact value of a root. Otherwise iterative process is considered *the dispersing*.

Let's copy for convenience the equation (1) in a kind:

$$x = f(x), \qquad (3)$$

That it is possible to receive by replacement: $F(x) = x - f(x)$.

Let – x_0 zero approach, i.e. the initial approached value of a root of the equation (3). Then as the following, 1st, approach we will accept

$$x_1 = f(x_0),$$

The following, 2nd, approach will be

$$x_2 = f(x_1),$$

Etc., as *n* th approach we will accept

$$x_n = f(x_{n-1}). \qquad (4)$$

Here there is a main point: whether comes nearer to the x_n true decision of the equation (3) at unlimited increase *n*? Differently, whether iterative process (4) converges?

2. A method of secants

Secants a method - a method of calculation of zero of continuous functions. Let in [and, b] the zero a continuous function f (x) contains; x_0, x_1 - various points of this piece. Iterative formula C m.:

$$f(x_1) \neq 0 \qquad (1)$$

If the sequence $f(x_1) \neq 0$ converges, it is obligatory to function zero f (x). At presence at f a continuous derivative on [and, b] local convergence C the m. to a simple root will be superlinear. If to strengthen requirements to smoothness f, it is possible to specify an exact order (local) convergence [1]., for $f(x_1) \neq 0$ $f(x_1) \neq 0$ and a such that $f(x_1) \neq 0$,

$$f(x_1) \neq 0$$

Here $f(x_1) \neq 0$

Superlinear convergence C m. for smooth functions - very important circumstance as calculations of derivatives it is not required and on each step is calculated only one new value of function. So, for comparison, in Newton's method, the order (local) convergence k-rogo is equal 2, on each step calculation of value of function and its derivative is required that, as a rule, is not less labour-consuming, than calculation of two values of function.

As convergence C m. depends on smoothness of function and a choice initial приближений, in standard machine subroutines of calculation of zero of continuous functions this method is combined with any method possessing guaranteed convergence, e.g. a method of division of a piece half-and-half. On each step of such combined method the root an is localised in a piece $f(x_1) \neq 0$, on the ends k-rogo function changes a sign (it is supposed that this condition is executed for an initial piece [a, b]). According to a nek-eye the test the next approach gets out or under the formula (1), or under the halving formula. Thus if f(x) - smooth function iterations, since nek-rogo numbers k_0, automatically go on C m. Is possible even more difficult combination of methods, e.g. algorithm zeroin (see [2]), in k-rum, except mentioned above, is used still a method of return square-law interpolation. Sometimes C m. name a method with the iterative formula

$$f(x_1) \neq 0 \qquad (2)$$

Other name of a method (2) - a false situation method, or regula falsi. Such method converges only linearly.

At generalisation C the m. on a case of system of the equations is possible a double sight at the iterative

Control questions

1. A method of simple iteration
2. A method of secants

Lecture № 5.

Algorithmization of the numerical decision of system of the algebraic and transcendental equations. A method of Gaussa.

The plan:

1. The decision of system of the linear equations a method of Gaussa
2. A method of Gaussa with a choice of the main element
3. An error estimation at the decision of system of the linear equations

1. The decision of system of the linear equations a method of Gaussa

Problems of approximation of function, and also set of other problems of applied mathematics of m of computing physics are reduced to problems about the decision of systems of the linear equations. The most universal method of the decision of system of the linear equations is the method of a consecutive exception of the unknown persons, Gaussa named a method.

For an illustration of sense of a method of Gaussa we will consider system of the linear equations:

$$\begin{cases} 4x_1 - 9x_2 + 2x_3 = 2 \\ 2x_1 - 4x_2 + 4x_3 = 3 \\ -x_1 + 2x_2 + 2x_3 = 1 \end{cases} \qquad \textbf{(1)}$$

This system we will write down in a matrix kind:

2. A method of Gaussa with a choice of the main element

The method of Gaussa is so universal that for some systems almost "bad" results turn out, various artful ways out therefore are developed. In a case when some factors of a matrix of system are close among themselves, as it is known relative errors strongly increase at subtraction, therefore the classical method of Gaussa gives the big errors. To bypass this difficulty, try to choose in a forward stroke of Gaussa that equation at which the factor at is maximum x_1 and as basic "player" choose this equation, thereby bypassing difficulties of subtraction of close numbers (if it is possible). Further, when it is necessary to null all factors of a variable x_2, except one equation – this special equation again choose that equation at which factor at maximum x_2 etc., yet we will not receive a triangular matrix.

Reverse motion occurs the same as and in a classical method of Gaussa.

3. An error estimation at the decision of system of the linear equations

To estimate errors of calculations of the decision of system of the linear equations, we need to enter concepts of corresponding norms of matrixes.

First of all, we will recollect three most often used norms for a vector $\vec{u}$:

$$\|\bar{u}\|_1 = \sum_{i=1}^{n} |u_i| \qquad \textbf{(11)}$$

$$\|\bar{u}\|_2 = \sqrt{\sum_{i=1}^{n} |u_i|^2} \quad \text{(Euklyde norm)} \qquad \textbf{(12)}$$

$$\|\bar{u}\|_3 = \lim_{p\to\infty} \sqrt[p]{\sum_{i=1}^{n} |u_i|^p} = \max_{1\le i\le n} |u_i| \quad \text{(Chebyshev norm)} \qquad \textbf{(13)}$$

For any norm of vectors it is possible to enter corresponding norm of matrixes:

$$\|A\| = \sup_{u\neq 0} \frac{\|Au\|}{\|u\|} = \sup_{\|u\|=1} \|Au\| \qquad \textbf{(14)}$$

Which is co-ordinated with norm of vectors in the sense that

$$\|Au\| \le \|A\| \cdot \|u\| \qquad \textbf{(15)}$$

It is possible to show that for three norms of a matrix resulted above cases are set A by formulas:

http://matematikam.ru/solve-equations/sistema-gaus.php

Control questions

1. The decision of system of the linear equations a method of Gaussa
2. A method of Gaussa with a choice of the main element
3. An error estimation at the decision of system of the linear equations

Lecture № 6.

Algorithmization of the numerical decision of system of the algebraic and transcendental equations. Iterative methods of Jacoby and Seidel.

The plan:

1. Iterative methods of the decision of systems of the linear equations
2 Method of simple iteration of Jacoby
3. A method of Gaussa-Seidel

1. Iterative methods of the decision of systems of the linear equations

Let's consider system of the linear equations which badly dares methods of Gaussa. We will copy system of the equations in a kind:

$$x = Bx + c \qquad \textbf{(22)}$$

Where - the B set numerical matrix n of th order, - the $c \in R^n$ set constant vector.

2 Method of simple iteration of Jacoby

This method consists in the following: any vector ($x^0 \in R^n$ initial approach) gets out $x^0 \in R^n$ and the iterative sequence of vectors under the formula is under construction:

$$x^{(n)} = Bx^{(n-1)} + c,\ n \in N \qquad \textbf{(23)}$$

Let's result the theorem giving a sufficient condition of convergence of a method of Jacoby.

The theorem. If $\| B \| < 1$, the system of the equations (22) has the unique decision and $x = \xi$ iterations (23) converge to the decision.

It is easy to notice that this theorem is simple generalisation of the theorem of the compressed displays studied by us earlier for single-step iterative process in a general view. All estimations received earlier, are transferred and for system of the equations, a difference only in concepts of corresponding norms. Generalising a method of simple iteration of Jacoby for a case of system of the equations:

$$Ax = b \qquad \textbf{(24)}$$

We build algorithm of the decision:

We copy the equation (24) in a homogeneous kind and it is multiplied by a constant - λ which further we will find from conditions of convergence of iterative process:

$$\lambda \cdot (Ax - b) = 0 \qquad \textbf{(25)}$$

We add to x both parts (25) and it is received:

$$x = x + \lambda(Ax - b) = \varphi(x, \lambda) \qquad \textbf{(26)}$$

We build the iterative formula of Jacoby:

$$x^{(n+1)} = x^{(n)} + \lambda(Ax^{(n)} - b) \qquad \textbf{(27)}$$

3. A method of Gaussa-Seidel

The set of iterative methods is developed for the decision of linear system of the equations. As the method of simple iteration of Jacoby converges slowly. One of such methods is the method of Gaussa-Seidel.

For a method illustration we will consider a numerical **example:**

$$\begin{cases} 2x - y + z = 5 \\ x + 3y - 2z = 7 \\ x + 2y + 3z = 10 \end{cases} \qquad \textbf{(29)}$$

The equations are copied in such a manner that on the main diagonal there are maximum factors for each equation.

We begin with approach $x = y = z = 0$. Using the first equation, we find for new x value under a x_1 condition $y = z = 0$.

$$x_1 = \frac{5 + y - z}{2} = \frac{5}{2} \qquad \textbf{(30)}$$

Control questions

1. Iterative methods of the decision of systems of the linear equations
2 Method of simple iteration of Jacoby
3. A method of Gaussa-Seidel

Lecture № 7.
Algorithmization interpolation methods. Interpolation functions.
The plan:

1. Introduction
2. The first interpolation Newton's formula
3. The second interpolation Newton's formula
4. The interpolation formula of Stirlinga
5. An example

1. Introduction

Interpolation – operation of approach of the function set in separate points in some set interval. The elementary problem of interpolation consists in the following. On a piece $[a, b]$ are set $n+1$ points x_i (i = 0, 1, 2, …, n), *interpolation* named *in the knots*, and values of some functions $f(x)$ in these points. It is required $f(x_0)=y_0$, $f(x_1)=y_1$,..., $f(x_n)=y_n$. to construct the *interpolating* function accepting $F(x)$ in knots of interpolation the same values, as $f(x)$, i.e. $F(x_0)=y_0$, $F(x_1)=y_1$,..., $F(x_n)=y_n$.

2. The first interpolation formul Newton.

Let in equidistant points $x_i = x_0 + i\cdot h \ \ (i=0,1,2,\dots,n)$, where h – a *step of interpolation*, preset values for $y_i = f(x_i)$ function y=$f(x)$. It is required to pick up a degree polynom $P_n(x)$ not above n satisfying to conditions (1).

$$P_n(x) = y_0 + q\Delta y_0 + \frac{q(q-1)}{2!}\Delta^2 y_0 + \dots + \frac{q(q-1)\dots(q-n+1)}{n!}\Delta^n y_0 \,, \tag{3}$$

3. The second interpolation formule Newton

The first interpolation formule Newton is almost inconvenient for interpolation functions near to the table end. In this case usually apply the *second* interpolation formule Newton:

$$P_n(x) = y_n + q\Delta y_{n-1} + \frac{q(q+1)}{2!}\Delta^2 y_{n-2} + \frac{q(q+1)(q+2)}{3!}\Delta^3 y_{n-3} + \dots + \frac{q(q+1)\dots(q+n-1)}{n!}\Delta^n y_0 \,. \tag{4}$$

4. The interpolation formul of Stirlinga

The interpolation formule of Stirlinga looks like:

$$\begin{aligned} P_{2n}(x) = {} & y_0 + q\cdot\frac{\Delta y_{-1}+\Delta y_0}{2} + \frac{q^2}{2!}\cdot\Delta^2 y_{-1} + \frac{q(q^2-1)}{3!}\cdot\frac{\Delta^3 y_{-2}+\Delta^3 y_{-1}}{2} + \\ & + \frac{q^2(q^2-1)}{4!}\cdot\Delta^4 y_{-2} + \frac{q(q^2-1)(q^2-2^2)}{5!}\cdot\frac{\Delta^5 y_{-3}+\Delta^5 y_{-2}}{2} + \\ & + \frac{q^2(q^2-1)(q^2-2^2)}{6!}\cdot\Delta^6 y_{-3} + \dots + \frac{q(q^2-1)(q^2-2^2)(q^2-3^2)\dots[q^2-(n-1)^2]}{(2n-1)!}\times \\ & \times\frac{\Delta^{2n-1}y_{-n}+\Delta^{2n-1}y_{-(n-1)}}{2} + \frac{q^2(q^2-1)(q^2-2^2)\dots[q^2-(n-1)^2]}{(2n)!}\Delta^{2n}y_{-n}\,, \end{aligned} \tag{5}$$

5. An example

The table of values of full elliptic integral is set

$$K(\alpha) = \int_0^{\pi/2}\frac{dx}{\sqrt{1-\sin^2\alpha\cdot\sin^2 x}}\,,$$

To find K (78º 30 ’).

The decision. According to the table data it is accepted $x0$ = 78; h=1; x=78º 30 ’, from here q = 0.5. Being limited to differences of the fifth order, under the formula of Stirlinga it is had:

$$K(78^\circ 30') = 2.97857 + 0.5\frac{7601+8316}{2}\cdot 10^{-5} + 0.125\cdot 715\cdot 10^{-5} - 0.0625\frac{103+135}{2}\cdot 10^{-5} -$$
$$-0.0078\cdot 32\cdot 10^{-5} + 0.0117\frac{13+8}{2}\cdot 10^{-5} = 3.019181.$$

Control questions

1. Introduction
2. The first interpolation Newton's formula
3. The second interpolation Newton's formula
4. The interpolation formula of Stirlinga
5. An example

Lecture № 8.
The Numerical decision of the differential equations. Euler's method.
The plan:

1. Types of problems for the ordinary differential equations
2. Euler's method

1. Types of problems for the ordinary differential equations

The differential equations arise in many areas of applied mathematics, physics, mechanics, technicians etc. With their help are described almost any problems of dynamics of cars and mechanisms (sections of the dynamic analysis of hydraulic systems, drives and transmissions, control systems see, for example, our site). There is a set of methods of the decision of the differential equations through elementary or special functions. However, more often these methods either are absolutely not applicable, or lead to so difficult decisions that it is easier and more expedient to use the approached numerical methods. The differential equations contain in a large quantity of problems essential nonlinearity, and functions entering into them and factors are set in the form of tables and-or experimental data that actually completely excludes possibility of use of classical methods for their decision and the analysis.

2. Euler's method

Let's consider the differential equation

$$y' = f(x, y) \qquad (1)$$

With the entry condition

$$y(x_0) = y_0 .$$

Having substituted x_0, y_0 in the equation (1), we will receive value of a derivative in a point x_0 :

$$y' |_{x=x_0} = f(x_0, y_0).$$

At the small Δx takes place:

$$y(x_0 + \Delta x) = y(x_1) = y_0 + \Delta y = y_0 + y' |_{x=x_0} \cdot \Delta x = y_0 + f(x_0, y_0) \cdot \Delta x.$$

Having designated $f(x_0, y_0) = f_0$, we will copy last equality in a kind:

$$y_1 = y_0 + f_0 \cdot \Delta x. \qquad (2)$$

Accepting now (x_1, y_1) for a new starting point, precisely also we will receive:

$$y_2 = y_1 + f_1 \cdot \Delta x.$$

Let's have generally:

$$y_{i+1} = y_i + f_i \cdot \Delta x. \qquad (3)$$

It also is *Euler's method*. The size Δx is called *as integration step*. Using this method, we receive the approached values *y* as the derivative y' actually does not remain to a constant on an interval in length Δx. Therefore we receive an error in definition of value of function *y*, that big, than it is more Δx. Euler's method is the elementary method of numerical integration of the differential equations and systems. Its lacks – small accuracy and regular accumulation of errors.

More exact is *Euler's modified method* or *Euler's method with recalculation*. Its essence that at first under the formula (3) find so-called «rough approach»:

$$\tilde{y}_{i+1} = y_i + f_i \cdot \Delta x ,$$

And then recalculation $\tilde{f}_{i+1} = f(x_{i+1}, \tilde{y}_{i+1})$ receive too approached, but more exact value:

$$y_{i+1} = y_i + \frac{f_i + \tilde{f}_{i+1}}{2} \cdot \Delta x. \qquad (4)$$

Control questions

1. Types of problems for the ordinary differential equations
2. Euler's method

Lecture № 9.

The numerical decision of the differential equations. A method of Runge-Kutta and Adams.

The plan:

1. Methods of Runge-Kutta
2. Adams's method

1. A method of Runge-Kutta

Again we will consider the differential equation

$$y' = f(x, y) \qquad (1)$$

With the entry condition $y(x_0) = y_0$.

The classical *method of Runge-Kutta* of 4th order is described by the following system of five equalities:

$$y_{i+1} = y_m + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4), \qquad (5)$$

Where

$$k_1 = f(x_i, y_i),$$

$$k_2 = f(x_i + \frac{h}{2}, y_i + \frac{hk_1}{2}),$$

$$k_3 = f(x_i + \frac{h}{2}, y_i + \frac{hk_2}{2}),$$

$$k_4 = f(x_i + h, y_i + hk_3).$$

2. Adams's method

Adams's method is applied both to the decision of the simple differential equations, and for their systems.

Problem statement

Adams's method to find the decision of system of the equations on a piece [0; 1] with accuracy ε =10-4. $\varepsilon = 10^{-4}$

$$\begin{cases} y'(x) = cy(x) - z(x), \\ z'(x) = y(x) - dz(x), \end{cases}$$

$$y(a) = k, \quad z(b) = n$$

Where *c*, *d*, *k*, *n* – the set constants

The decision of systems of the ordinary differential equations Adams's method

In the given system of the equations will substitute values of factors and entry conditions. We will receive

$$\begin{cases} y' = 2y - z \\ z' = y - 4z \end{cases} \qquad y(0) = 3, \quad z(0) = -2$$

Adams's method we will find the decision of this system on the set piece. For this purpose we will calculate a method of Runge-Kutta some initial values of function.

Let's choose a step h and, for brevity, we will enter $x_i = x_0 + ih$ и $y_i = y(x_i)$ $(i = 0, 1, 2, \dots)$

Let's consider numbers:

$$\begin{cases} k_1^{(i)} = hf(x_i, y_i) \\ k_2^{(i)} = hf\left(x_i + \frac{h}{2}, y_i + \frac{k_1^{(i)}}{2}\right) \\ k_3^{(i)} = hf\left(x_i + \frac{h}{2}, y_i + \frac{k_3^{(i)}}{2}\right) \\ k_4^{(i)} = hf(x_i + h, y_i + k_3^{(i)}) \end{cases}$$

According to a method of Runge-Kutta consecutive values y_i are defined under the formula

$$y_{i+1} = y_i + \Delta y_i$$

Where

$$\Delta y_i = \frac{1}{6}\left(k_1^{(i)} + 2 \cdot k_2^{(i)} + 2 \cdot k_3^{(i)} + k_4^{(i)}\right) (i = 0, 1, 2, \dots). \qquad (2.1)$$

Control questions

1. Methods of Runge-Kutta
2. Adams's method

Lecture №10.

Numerical integration. Quadrature formulas of trapezes and rectangles. Simpson's formula.

The plan:

1. Classification of methods
2. A method of trapezes
3. Methods of rectangles
4. Simpson's method

1. Classification of methods

It is known that the certain integral of function *f (x)* type $\int_a^b f(x)dx$ (1) numerically represents the area of a curvilinear trapeze limited to curves x=0, y=a, y=b and y = *f (x)* (fig. 1). There are two methods of calculation of this area or certain integral — a method of trapezes (fig. 2) and a method of average rectangles (fig. 3).

2. Method of Trapezes

$$S = \frac{f_0 + f_1}{2}\cdot h + \frac{f_1 + f_2}{2}\cdot h + \frac{f_2 + f_3}{2}\cdot h + \ldots + \frac{f_{n-1} + f_n}{2}\cdot h =$$
$$= h\left(\frac{f_0}{2} + \frac{f_1}{2} + \frac{f_1}{2} + \frac{f_2}{2} + \frac{f_2}{2} + \frac{f_3}{2} + \ldots \frac{f_{n-1}}{2} + \frac{f_n}{2}\right) =$$
$$= h\left(\frac{f_0 + f_n}{2} + f_1 + f_2 + \ldots + f_{n-1}\right). \quad (2)$$

Generally the formula of trapezes becomes

$$\int_a^b f(x)dx \approx h\left(\frac{f_0 + f_n}{2} + \sum_{i=2}^{n-1} f_i\right) = \frac{b-a}{n}\left(\frac{f_0 + f_n}{2} + \sum_{i=2}^{n-1} f_i\right), \quad (3)$$

Where f_i - value of subintegral function in points of splitting of an interval (a, b) on equal sites with step *h; f_0, f_n* - values of subintegral function accordingly in points a and b.

The formula of trapezes with constant step:

$$\int_a^b f(x)dx \approx \frac{1}{2}h\sum_{i=0}^{n-1}(y_i + y_{i-1}) = \frac{1}{2}h(y_0 + y_n + 2\sum_{i=1}^{n-1} y_i) \quad (4)$$

3. A method of rectangles

$$S_b = \sum_a^b \frac{|f(x_1) + (fx_2)|}{2}\varepsilon \quad (5)$$

The formula of average rectangles with constant step:

$$\int_a^b f(x)dx \approx \frac{1}{2}h\sum_{i=0}^{n-1} f\left(x_i + \frac{h}{2}\right) \quad (6)$$

4. Simpson's (Parabolas) formula

Simpson's rule – one of widest known and applied methods of numerical integration. It is similar to a rule of trapezes as also is based on splitting of the general interval of integration into smaller pieces. However its difference that for area calculation through each three consecutive ordinates of splitting the square parabola is spent. Lowering needless details and calculations we will result a definitive kind *of the formula of Simpson* [3, 4]:

$$I \approx \frac{h}{3}(y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + \ldots + 2y_{n-2} + 4y_{n-1} + y_n). \quad (6)$$

Control questions

1. Classification of methods
2. A method of trapezes
3. Methods of rectangles
4. Simpson's method

http://tgspa.ru/info/education/faculties/ffi/ito/programm/osn_chm/chislennoe_integrirovanie3b_mathcad.htm

Lecture № 11.
Numerical integration. The formula of Gauss.
The plan:

1. The quadrature formula of Gauss
2. Examples

The methods described above use the fixed points of a piece (the ends and the middle) and have a low order of accuracy (0 – methods of the right and left rectangles, 1 – methods of average rectangles and trapezes, 3 – a method of parabolas (Simpson)). If we can choose points in which we calculate values of function *f (x)* it is possible to receive methods of higher order of accuracy at the same quantity of calculations of subintegral function. So for two (as in a method of trapezes) calculations of values of subintegral function, it is possible to receive a method any more 1st, and 3rd order of accuracy:

$$I \approx \frac{b-a}{2}\left(f\left(\frac{a+b}{2}-\frac{b-a}{2\sqrt{3}}\right)+f\left(\frac{a+b}{2}+\frac{b-a}{2\sqrt{3}}\right)\right).$$

Generally, using n points, it is possible to receive a method with accuracy order 2n-1. Values of knots of a method of Gaussa on пточкам are roots of a polynom of Lezhandra of degree n.

Values of knots of a method of Gaussa and their scales are resulted in directories of special functions. The method of Gaussa on five points is most known.

Example 1.

Let's calculate integral $\int_{0.5}^{3}\frac{2x^3}{x^4}dx$ with the method of Gauss.

The decision.

$$I \approx \frac{b-a}{2}\left(f\left(\frac{a+b}{2}-\frac{b-a}{2\sqrt{3}}\right)+f\left(\frac{a+b}{2}+\frac{b-a}{2\sqrt{3}}\right)\right)$$

$$f(x)=\frac{2x^3}{x^4}.$$

$$f1(x)=f\left(\frac{a+b}{2}-\frac{b-a}{2\sqrt{3}}\right)=f\left(\frac{0.5+3}{2}-\frac{3-0.5}{2\sqrt{3}}\right)=f(1.029)=1.94\cdot$$

$$f2(x)=f\left(\frac{a+b}{2}+\frac{b-a}{2\sqrt{3}}\right)=f\left(\frac{0.5+3}{2}+\frac{3-0.5}{2\sqrt{3}}\right)=f(2.47)=0.812$$

$$\int_{0.5}^{3}\frac{2x^3}{x^4}dx=\frac{3-0.5}{2}(1.94+0.812)\approx 3.584\cdot$$

The answer: 3.584.

Control questions

1. The quadrature formula of Gauss
2. Classification of methods
3. A method of trapezes
4. Methods of rectangles
5. Simpson's method

Lecture № 12.
Root-mean-square approach of functions. A method of the least squares
The plan:

1. Root-mean-square approach of functions
2. A method of the least squares

1. Root-mean-square approach of functions

Let dependence between variables x and y is set таблично (the skilled data is set). It is required to find function somewhat in the best way describing the data. One of ways of selection of such (approaching) function is the method of the least squares. The method consists in that the sum of squares of deviations of values of required function $\bar{y}_i = \bar{y}(x_i)$ and set таблично y_i was the least:

$$S(c) = (y_1 - \bar{y}_1)^2 + (y_2 - \bar{y}_2)^2 + \ldots + (y_n - \bar{y}_n)^2 \to \min \qquad (6.1)$$

Where c a vector –of parametres of required function.

2. A method of the least squares

To construct a method of the least squares two empirical formulas: linear and square-law.

In case of linear function $y=ax+b$ *the* problem is reduced to a finding of parametres a and b from system of the linear equations

$$\begin{cases} M_{x^2} a + M_x b = M_{xy} \\ M_x a + b = M_y \end{cases}$$, Where

$$M_{x^2} = \frac{1}{n}\sum_{i=1}^{n} x_i^2, \quad M_x = \frac{1}{n}\sum_{i=1}^{n} x_i, \quad M_{xy} = \frac{1}{n}\sum_{i=1}^{n} x_i y_i, \quad M_y = \frac{1}{n}\sum_{i=1}^{n} y_i$$

а в случае квадратичной зависимости $y = ax^2 + bx + c$ к нахождению параметров a, b и c из системы уравнений:

and in case of square-law dependence $y = ax^2 + bx + c$ to a finding of parameters a, b and c from system of the equations:

$$\begin{cases} M_{x^4} a + M_{x^3} b + M_{x^2} c = M_{x^2 y} \\ M_{x^3} a + M_{x^2} b + M_x c = M_{xy} \\ M_{x^2} a + M_x b + c = M_y \end{cases}$$, Where

$$M_{x^4} = \frac{1}{n}\sum_{i=1}^{n} x_i^4, \quad M_{x^3} = \frac{1}{n}\sum_{i=1}^{n} x_i^3, \quad M_{x^2 y} = \frac{1}{n}\sum_{i=1}^{n} x_i^2 y_i$$

To choose from two functions the most suitable. For this purpose to make the table for calculation of the sum of squares of evasion under the formula (6.1). Initial given to take from table 6.

The task 2

To make the program for a finding of approaching functions of the set type with a conclusion of values of their parametres and the sums of squares of evasion corresponding to them. To choose as approaching functions the following: $y = ax + b$, $y = ax^m$, $y = ae^{mx}$. To spend linearization. To define for what kind of function the sum of squares of evasion is the least.

Control questions

1. In what an approach essence таблично the set function on a method of the least squares?
2. Than this method differs from an interpolation method?
3. How the problem of construction of approaching functions in the form of various elementary functions to a case of linear function is reduced?
4. Whether there can be a sum of squares of evasion for any approaching functions equal to zero?
5. What elementary functions are used as approaching functions?
6. How to find parametres for linear and square-law dependence, using a method of the least squares?

Lecture № 13-14
Statement of a problem of linear programming. The basic properties the decision of a problem of linear programming.

The plan:

1. The primary goal of linear programming
2. Examples of the decision of a problem

The primary goal of linear programming in a canonical form is formulated as follows:

To find the non-negative decision of system of restrictions

$$a_{i1}x_1 + a_{i2}x_2 + \ldots + a_{ij}x_j + \ldots + a_{in}x_n + b_i = 0 \ (i = 1,2,\ldots,m);$$
$$x_j \geq 0 \ (j = 1,2,\ldots,n), \qquad \text{(II. 1)}$$

Providing a maximum (minimum) of criterion function

$$Z = c_1x_1 + c_2x_2 + \ldots + c_nx_n + Q \to \max(\min) \qquad \text{(II.2)}$$

Except a record reduced form can be used partially developed

$$Z = \sum_{j=1}^{n} c_j x_j + Q \to \max;$$

$$\sum_{j=1}^{n} a_{ij}x_j + b_i = 0 \ (i = 1,2,\ldots,m); \ x_j \geq 0 \ (j = 1,2,\ldots,n)$$

and matrix forms

$$Z = Cx + Q \to \max,$$
$$Ax + B = 0, \quad x \geq 0.$$

All further reasonings will be spent only for the primary goal in a canonical form.

Usually specific targets of linear programming have distinct from initial an appearance, therefore to solve their such problems it is necessary to lead to a canonical form

Let the problem of linear programming with variables and the mixed system from m restrictions is set:

$$Z = c_1x_1 + c_2x_2 + \ldots + c_nx_n + Q \to \max; \qquad \text{(II.3)}$$

$$a_{i1}x_1 + a_{i2}x_2 + \ldots + a_{in}x_n + b_i \leq 0 \ (i = 1,2,\ldots,r);$$
$$a_{k1}x_1 + a_{k2}x_2 + \ldots + a_{kn}x_n + b_k \geq 0 \ (k = r+1,\ldots,t);$$
$$a_{l1}x_1 + a_{l2}x_2 + \ldots + a_{ln}x_n + b_l = 0 \ (l = t+1,\ldots,m);$$
$$x_j \geq 0 \ (j = 1,2,\ldots,s \leq n). \qquad \text{(II.4)}$$

For reduction of this problem to a canonical form it is necessary to replace variables, i.e. To exclude those variables which can accept both positive, and negative values. The system of restrictions-inequalities should be replaced by equivalent system of the equations with non-negative variables.

Replacement of inequalities with the equations. Replacement of system of restrictions-inequalities in (II.4) equivalent system of the equations is carried out by introduction of artificial, non-negative variables y,

$$a_{i1}x_1 + a_{i2}x_2 + \ldots + a_{in}x_n + y_i + b_i = 0;$$
$$a_{k1}x_1 + a_{k2}x_2 + \ldots + a_{kn}x_n - y_k + b_k = 0;$$
$$y_i \geq 0 \ (i = 1,2,\ldots,r); \ y_k \geq 0 \ (k = r+1,\ldots,t). \qquad \text{(II.5)}$$

Such transformation increases number of variables, without changing a problem being.

Replacement of unlimited variables. Variables which can accept negative values, are expressed through non-negative variables $x_1, x_2, \ldots, x_s$ and $y_1, y_2, \ldots, y_r$. Replacement of variables represents the system decision, concerning a replaced variable, and can be executed with the help жордановых exceptions. For replacement of one variable one step of exceptions is required, therefore to lead problem canonical form is possible only in case a rank of system of more number of unlimited variables.

After replacement the problem dares in new variables. The optimum decision in new variables is substituted in the communication equations therefore the optimum decision in initial variables turns out.

At the decision of economic and technical problems, as a rule, variables can be only positive real numbers. If in a problem any variable by the nature can accept negative values in most cases change of the formulation of conditions allows to get rid of unlimited variables.

Minimisation of form Z. Further the problem of maximisation of form Z will be considered only. If it is necessary to solve a problem of minimisation of the linear form, criterion function factors should be increased on (-1) and to solve this new problem on a maximum. The required minimum of criterion function turns out multiplication of the found maximum value on (-1), i.e. $Z_{\min} = -\max(-Z)$

Control questions

1. The primary goal of linear programming
2. Examples of the decision of a problem

Lecture № 15-16

Geometrical interpretation of a problem of linear programming.

The plan: 1. Problem statement

2. Geometrical representation.
3. An example of the decision of a problem
4. Geometrical problem interpretation

Better and more visually to present geometrical sense of a problem of linear programming, we will address to the elementary two-dimensional case (when the model includes two variables) and then we will make generalisations at presence n variables.

In case of two variables the model of linear programming has the following appearance

$$Z = c_1 x_1 + c_2 x_2 \to \max ; \qquad \text{(II.6)}$$

$$a_i x_1 + a_i x_2 + b_i \geq 0 \quad (i = 1, \ 2, \ \ldots, \ m); \qquad \text{(II.7)}$$

$$x_1 \geq 0; \quad x_2 \geq 0.$$

Each restriction (II.7) represents a straight line (fig. II.1) which breaks all space (an initial plane) on two semiplanes one of which satisfies to restriction (this area in drawing is shaded).

The system of restrictions according to the theorem 1 represents convex set, and in a considered two-dimensional case - a convex polygon of restrictions (fig. II.2). In special cases the polygon can address in a point (then the decision is unique), a straight line or a piece. If the system of restrictions is inconsistent (несовместна) it is impossible to construct a polygon of restrictions also a problem of linear programming has no decisions. Such case is shown on fig. II.3. Really, there is no point of space which simultaneously would satisfy to restriction y_1 and to restrictions y_2 and y_3.

The polygon of restrictions can be not closed (fig. II.4). In this case, as it will be shown more low, criterion function Z is not limited from above.

In a case п variables each restriction represents (n-l) a-dimensional hyperplane which divides all space into two semispaces. The system of restrictions in this case gives a convex polyhedron of decisions - the general part of the n-dimensional space, satisfying to all restrictions.

In three-dimensional space (n=3) each restriction represents a plane in space. All restrictions, being crossed, form a convex polyhedron which in special cases can be a point, a piece, a beam, a polygon or many-sided unlimited area.

For finding-out of geometrical sense of criterion function we will give to variable Z various numerical values (Z=0, Z=l, Z=2, Z=D).

To these numerical values Z there corresponds sequence of the equations and system of parallel straight lines in space (fig. II.5).

$$c_1x_1 + c_2x_2 = 0,$$
$$c_1x_1 + c_2x_2 = 1,$$
$$\ldots\ldots\ldots\ldots\ldots\ldots$$
$$c_1x_1 + c_2x_2 = D.$$

The first straight line (Z=0) passes through the beginning of co-ordinates perpendicularly (ортогонально) to the directing vector $C = (C_1 C_2)$, the subsequent straight lines are parallel to the first and will defend from it in a direction of a vector With on size 1, 2, D. As a whole variable Z defines evasion of the points lying on a straight line $Z = c_1x_1 + c_2x_2$ from a straight line $c_1x_1 + c_2x_2 = 0$, passing through the beginning of co-ordinates. To define evasion of any point from straight line Z=0, it is enough to substitute co-ordinates of this point in the criterion function equation.

In n-dimensional space of the criterion function equal to zero ($Z = c_1x_1 + c_2x_2 + \ldots + c_jx_j + \ldots + c_nx_n = 0$), geometrically there corresponds (n-1) the-dimensional hyperplane passing through the beginning of co-ordinates.

The distance from a point with co-ordinates $x' = (x'_1 + x'_2 + \ldots + x'_n)$ to a hyperplane is equal

$$R = \frac{a_1x_1' + a_2x_2' + \ldots + a_nx_n'}{\sqrt{a_1^2 + a_2^2 + \ldots + a_n^2}} \quad \text{or} \quad R\sqrt{a_1^2 + a_2^2 + \ldots + a_n^2} = a_1x_1' + a_2x_2' + \ldots + a_nx_n'$$

From here it is visible that, if in the linear form to substitute point co-ordinates, the distance from a point x' to the corresponding hyperplane Z=0, postponed in the scale equal to norm of a vector of an orthogonal plane will turn out (or in the scale equal to norm of the directing vector).

The scaled distance **y,** equal $R\sqrt{\sum_{j=1}^{n} a_j^2}$, is called as evasion of a point from a plane. As according to the theorem 3 linear form Z reaches the extreme value in an extreme point (top) of a polyhedron of restrictions geometrically the problem of linear programming consists in search of top of a polyhedron of the admissible decisions, having the maximum evasion from a hyperplane expressed by criterion function, equal to zero (fig. II.6).

Control questions

1. Problem statement
2. Geometrical representation.
3. An example of the decision of a problem
4. Geometrical problem interpretation

Lecture №17.
Finding the decision of a problem of linear programming to simplex methods.

The plan:

1. Mathematical bases a simplex of a method of the decision
2. Put a table datas

1. Mathematical bases a simplex of a method of the decision

It is known that if the problem of linear programming has the optimum decision there is at least one optimum basic decision. Thus, by search of basic decisions it is possible to receive the required decision. The number of basic decisions makes $N = C_n^k$, where n - number of variables, and K=r(A) - number of basic variables. This number very quickly grows at increase in number of variables, therefore in rather small problems continuous search becomes impracticable even by means of the COMPUTER.

The number of touched decisions can be reduced at the expense of an exception of consideration of inadmissible basic decisions. The admissible basic decision or the basic decision represents the basic decision with positive values of basic variables. Hence, to touch only basic decisions, the algorithm of search should answer a following condition: at transition from one decision to another should remain innegativity all variables. Performance of this condition does a problem of search of more foreseeable, but as a whole procedure remains ineffective as transition to another does not guarantee its improvement against one decision. What is quality of the decision? The procedure ultimate goal - achievement of a maximum of linear form Z, therefore can serve as an indicator of quality of the decision level 2 in the given basic decision. Hence, efficiency of procedure of search can be raised sharply if each step improves quality of the decision or to provide growth of linear form Z. On the basis of these reasonings it is possible to formulate the second condition to which the algorithm of the decision of a linear problem should answer: transition from one basic decision to another should provide growth of criterion function Z.

This idea can be realised only in the event that there is some basic decision which gradually improves.

The basic method of the decision of problems of linear programming is the simplex-method in which all process of the decision shares on three stages: search of the initial basic decision, search basic and then the optimum decision.

To search of basic, basic and optimum decisions apply special procedures - ordinary and modified Jordanov's exceptions.

That in system of linear forms *y=Ax* to change in places dependent variable y_r and an independent variable x_S, it is necessary to solve r-e the equation rather x, and to substitute this decision in all other equations of system.

It is obvious that to solve r-e equation rather x, is possible only in the event that $a_{rs} \neq 0$.

Definition. Step ordinary Jordanov's an exception made over system of linear forms y=Ax with the resolving element $a_{rs}\neq 0$, with r-th in the resolving line and s-th a resolving column, name the schematised operation of recalculation of factors in linear forms at change by places dependent variable y_r and independent x_s.

For definition of operations of recalculation of elements of a matrix And in system of linear forms y=Ax at replacement y_r on x_s it is necessary to present a matrix in the form of tab. II.3 and to make corresponding algebraic actions.

Table II.3

	x_1	x_2	...	x_s	...	x_n
y_1	a_{11}	a_{12}	...	a_{1s}	...	a_{1n}
y_2	a_{21}	a_{22}	...	a_{2s}	...	a_{2n}
...	...	...	...	...	...	...
y_r	a_{r1}	a_{r2}	...	a_{rs}	...	a_{rn}
...	...	...	...	...	...	...
y_m	a_{m1}	a_{m2}	...	a_{ms}	...	a_{mn}

In the new table instead of r-th forms the new form from a basic variable x_s which turns out as a result of the decision r-th forms concerning this variable will settle down

$$x_s = -\frac{a_{r1}}{a_{rs}}x_1 - \frac{a_{r2}}{a_{rs}}x_2 + \ldots + \frac{1}{a_{rs}}y_r - \ldots - \frac{a_{rn}}{a_{rs}}x_n$$

Having analysed factors at variables x_j and y_r, may be following conclusions:

1. In the new table on a place of a resolving element a_{rs} should be written down $1/a_{rs}$.
2. Other elements resolving r-th register lines in the new table with a return sign and share on resolving element, i.e. Instead $(-a_{rj}/a_{rs})$ registered
3. In the new table on a place of a resolving column it is necessary to write down elements a_{is} instead of elements

Control questions

1. Mathematical bases a simplex of a method of the decision
2. Put a table datas

Lecture №18.
Finding the decision of a problem of linear programming. A method of artificial basis.
The plan:

1. Search of the initial basic decision
2. Non-negative, independent variables

1. Search of the initial basic decision

Let the problem of linear programming from l by variables and the mixed system from *m* restrictions is set:

$$Z = c_1x_1 + c_2x_2 + \ldots + c_nx_n + Q \to \max;$$
$$a_{i1}x_1 + a_{i2}x_2 + \ldots + a_{in}x_n + b_i \geq 0 \; (i = 1,\ 2,\ \ldots,\ r);$$
$$a_{k1}x_1 + a_{k2}x_2 + \ldots + a_{kn}x_n + b_k = 0 \; (k = r+1,\ \ldots,\ m);$$
$$x_j \geq 0 \; (j = 1,\ 2,\ \ldots,\ s < n). \qquad \text{(II.8)}$$

For problem reduction to a canonical form the system of restrictions - inequalities is led to equivalent system of the equations by introduction of artificial, non-negative variables y_i

$$a_{i1}x_1 + a_{i2}x_2 + \ldots + a_{in}x_n - y_i + b_i = 0,$$
$$y_i \geq 0 \; (i = 1,\ 2,\ \ldots,\ r) \qquad \text{(II.9)}$$

Also replacement of unlimited variables is made.

After reduction of system of restrictions to system of the linear equations it is necessary to find its common decision. It is obvious that the equations received from inequalities, easily dare concerning artificial variables y_i and the common decision of this part of system of the equations will be

$$y_i = a_{i1}x_1 + \ldots + a_{in}x_n + b_i,$$
$$y_i \geq 0 \; (i = 1,\ 2,\ \ldots,\ r). \qquad \text{(II.10)}$$

For other part of system of the equations the common decision can be received with the help Jordan's exceptions (or it is established it imcompatibility).

The system decision can be combined with replacement of variables, and for this purpose it is necessary to enter unlimited variables into basis.

After search of the common decision of system the initial basic decision turns out by equating of independent variables with zero.

Thus, reception of the initial basic decision is reduced to following operations. The initial problem is led to a kind (II.10) and registers in a simplex-table (tab. II.9).

Table II.9

	$-x_1$	$-x_2$	...	$-x_n$	1
y_1	$-a_{11}$	$-a_{12}$	...	$-a_{1n}$	b_1
...	...	...	...	...	...
y_r	$-a_{r1}$	$-a_{r2}$	...	$-a_{rn}$	b_r
0	$-a_{r+1,1}$	$-a_{r+1,2}$	...	$-a_{r+1,n}$	b_{r+1}
...	...	...	...	...	...
0	$-a_{m1}$	$-a_{m2}$	...	$-a_{mn}$	b_m
Z	$-c_1$	$-c_2$	...	$-c_n$	Q

In the lines corresponding to restrictions - to inequalities, auxiliary variables register, and in lines with the equations auxiliary variables are equal to zero - (0-variables).

Jordanov's exceptions unlimited variables $x_{s+1},\ \ldots,\ x_n$ are expressed by consecutive steps through non-negative variables and simultaneously with it 0-variables are translated on table top.

The column under translated on top of the table of a 0-variable is excluded. The equations of communication for unlimited variables are remembered, and corresponding lines do not participate in the further analysis. As a result of transformations the table containing the initial basic decision, has the following appearance (tab. II.10).

At following stages of the decision of a problem the part of the table allocated with a dashed line is analyzed only. In the received basic decision independent variables are equated to zero, and basic variables and form Z appear equal to corresponding free members, i.e.

$$x_1 = 0,\ \ldots,\ x_s = 0;\ y_1 = 0,\ \ldots,\ y_p = 0; \qquad \textbf{(II.11)}$$

Control questions

1. Search of the initial basic decision
2. Non-negative, independent variables

Lecture №1.
Introduction. The cores concept about algorithmization of computing methods.

The purpose: Formation of knowledge, skills on studying of bases of algorithmization, the basic properties of algorithm and classification of computing methods.

The plan:

1. Classification of computing methods.
2. Preparation of problems for the personal computer decision.
3. Properties of algorithm.
4. Classification of algorithms.

Given a lecture course it is written according to the program on discipline «Algorithmization computing methods», studied by students of technical colleges. The lecture course is covered by following sections of the program: on concept linear нормированного spaces; methods of the numerical decision of systems of the linear equations; methods of the numerical decision of the nonlinear equations and systems; root-mean-square approach of functions; interpolation functions; numerical differentiation and integration; the numerical decision of the ordinary differential equations; numerical methods of search of an extremum of functions of one and several variables. In each theme necessary theoretical data (the basic theorems, definitions, formulas, various computing methods etc.) are resulted And also the examples illustrating application of described methods. Besides, there are exercises for the independent decision and answers to them. Appendices contain block diagrammes of computing algorithms and texts of programs for the considered numerical methods on algorithmic languages PASCAL.

The main objective a lecture course — to help development of practical skills in students with application of numerical methods. Each theme contains: computing algorithm; theoretical substantiations of its application; conditions of the termination of computing process; the examples in full or in part executed "manually"; exercises and answers to them; the appendix, in which the considered computing algorithm is presented in the form of the block diagramme and texts of programs on four (sometimes — on five) algorithmic languages.

Authors hope that mastering by numerical methods will be promoted also by a considerable quantity of in detail solved examples, and also exercises for independent work. It is necessary to notice that often various computing algorithms are illustrated by the same examples. Besides, for many examples considered in the book analytical decisions to which it is possible to compare the found numerical decisions are known. Coincidence of the results received in the different ways, is additional, evident argument of applicability of this or that numerical method. At last, the help in practical application of numerical methods will be rendered by appendices to the given book. In them block diagrammes and texts of 95 programs (with comments) on algorithmic languages used in educational practice are resulted. The material stated in appendices can be applied not only at studying of numerical methods, but also as the ready applied programs which work is checked up in program environments of firms BORLAND and MICROSOFT for personal computers.

The present a lecture course is intended for students of the higher technical educational institutions. It can appear also to useful teachers, engineers and the science officers using in the activity computing methods.

Algorithmization basis. The basic properties of algorithm

Process of preparation and the decision of problems on the personal computer is while difficult enough and labour-consuming, demanding performance of variety of stages. Such stages are:

1) problem statement;
2) the mathematical formulation of a problem;
3) a choice of a numerical method of the decision;
4) working out of algorithm of the decision of problems;
5) a program writing;
6) input of the program and the initial data;
7) program debugging;
8) the problem decision on the personal computer;

The given sequence is characteristic for the decision of each problem. However in the course of problem preparation each stage can have more and less expressed character. Performance of stages in the course of problem preparation has character of consecutive approach as problem specification at the subsequent stage leads to necessity of return to the previous and repeated performance of the subsequent stages.

Let's consider more in detail performance of works at each stage in the course of preparation of a problem for the decision.

Problem statement defines the purpose of the decision of a problem, opening its maintenance. The problem is formulated at level of professional concepts, should be correct and clear to the executor (user). Mistake directed by a problem, found out on the subsequent stages, will lead to that work on preparations of a problem for the decision should begin from the very beginning.

At problem statement the ultimate goal is found out and the general approach to the problem decision is developed. It is found out, how many decisions the problem has and whether has them in general. The general properties of the considered physical phenomenon or object are studied, possibilities of the given programming system are analyzed.

The mathematical formulation of a problem carries out formalisation of a problem by its description by means of formulas, defines the list of the initial given and received results, entry conditions, accuracy of calculation. The mathematical model of a solved problem is in essence developed.

Choice of a numerical method of the decision. In some cases the same problem can be solved by means of various numerical methods. The method choice should be defined by many factors, basic of which accuracy of results of the decision, time of the decision for the personal computer and volume of operative memory are. In each specific case as criterion for a choice of a numerical method accept any of the specified criteria or some integrated criterion.

In simple problems the given stage can be absent, as the numerical method is certain by the mathematical formulation of a problem. For example, calculation of the area of a triangle under the formula of Gerona, roots of a quadratic, etc.

Working out of algorithm of the decision of a problem. At the given stage the necessary logic sequence of calculations taking into account the chosen numerical method of the decision and other actions with which help results will be received is established.

Algorithm – some final sequence of instructions (rules) defining process of transformation of the initial and intermediate data as a result of the decision of a problem.

The program writing is carried out on the developed algorithm by means of the programming language.

Input of the program and the initial data is carried out by means of the personal computer keyboard.

Debugging of programs represents process of detection and elimination of syntactic and logic errors.

The problem decision on микро the personal computer is usually spent with a dialogue mode. In this mode the user by means of the personal computer keyboard can carry out input of the program and its updating, program translation (transfer from the programming language on machine), correction syntactic and logic errors at debugging, reception on an exit of results and the auxiliary information necessary for management by work of the personal computer.

Technology OREG.

- **O –** state the **opinion.**
- **R –** produce one **reason** of the opinion.
- **E –** give an **example** for the explanatory of the reason.
- **G – generalise** the opinion.

Question for OREG: **what properties algorithms should possess?**

Use of computers as executors of algorithms shows a number of requirements to algorithms. Unlike people, the computer can carry out only precisely certain operations. Therefore machine algorithms should possess following properties:

1. Step-type behaviour
2. Clearness;
3. Unambiguity
4. Mass character.
5. Productivity.
6. Finiteness
7. Correctness

That the executor has managed to solve the problem set for it, using algorithm, it should be able to follow its each instructions. Differently, he should understand a management essence. That is at algorithm drawing up it is necessary to consider "game rules", i.e. system of instructions (or system of commands) which understands the computer. For example, at the decision of any problem the student used the reference to functions sin x (it is trigonometrical function) and to function of Bessel (it is cylindrical function), but the computer (as well as the reader, probably) does not understand last. It is not provided by founders of the given class of cars. Hence, (as a whole) the car will not understand algorithm. We will speak in this case about "clearness" of algorithm.

As "CLEARNESS" of algorithms understand instructions which are clear to the executor.

Being clear, the algorithm should not contain nevertheless the instructions which sense can be perceived ambiguously. These properties instructions and instructions which are made for people often do not possess. For example: in the recipe of preparation of an omelette resulted above it is told: "to Break in this mix of 3 eggs and all - it it is good to shake up a spoon". At household level to us it is clear that it is a question of three eggs (and what else! - you will tell). But eggs can be both pigeon, and duck, and even ostrich's (all sharply differ on size from each other). Ambiguity here "has obviously crept in". Or type instructions: "to salt to taste","to fill two-three spoons sugar to sand","has received an estimation 4 or 5","to fry to readiness"""dig from a fence till a dinner" cannot to meet in algorithms. It is obvious that clear in certain situations for the person of the instruction of this kind can stump the computer.

Or we will recollect a parable known for all an imperial will. The tsar has ordered subordinated to execute such decree: "to Execute it is impossible to pardon". He has forgotten to put a comma in the decree, and subordinates did not know that by it to do. "It is impossible to execute instructions, to pardon" and "to execute, it is impossible to pardon" set absolutely different actions on which human life depends.

Besides, in algorithms such situations when after performance of the next instruction of algorithm to the executor it is not clear what of them should be carried out on a following step are inadmissible.

UNAMBIGUITY of algorithms is understood as uniqueness of interpretation of rules of performance of actions and an order of their performance.

As we already know, the algorithm sets full sequence of actions which it is necessary to carry out for the problem decision. Thus, as a rule, for performance of these actions them dismember (break) in certain sequence into simple steps. There is an ordered record of set of accurately divided instructions (instructions, commands), forming прерывную (or as speak, discrete) algorithm structure. To execute actions of the following instruction it is possible only having executed actions previous.

Programming is a process of decomposition of a challenge on a number of simple actions.

As STEP-TYPE BEHAVIOUR understand possibility of splitting of algorithm on the separate elementary actions which performance by the person or car does not raise the doubts.

It is very important, that the made algorithm provided the decision not one private problem, and could carry out the decision of a wide class of problems of the given type.
For example. It is necessary to solve a concrete quadratic h4 - 4x+3=0. But after all it is possible to make algorithm of the decision of any quadratic of a kind: $ax^2 + bx +$ with =0.

Really, for a case when = $b^2 - 4ac > 0$, quadratic roots it is possible to find discriminant D under known formulas.

If D <0 the valid roots do not exist. Thus, this algorithm can be used for any square at an alignment. Such algorithm will be

As FINITENESS of algorithms understand end of work of algorithm as a whole for final number of steps.

Still it is necessary to carry **PRODUCTIVITY** to desirable properties of algorithms**, she assumes that performance of algorithms should come to the end with reception of certain results**.

Similar situations in computer science arise, when no actions can be executed. In the mathematician such situations name uncertainty. For example, division of number into a zero, extraction of a square root from a negative number, and concept of infinity vaguely. Therefore, if the algorithm sets infinite sequence of actions in this case it also is considered result uncertain. But it is possible to operate in another way. Namely: to specify the reason of uncertain result. In that case, "it is impossible to divide type explanatories into a zero", "the computer to execute such not in a condition", etc. it is possible to consider as result algorithm performance.

<u>Thus, property of productivity consists that in all "cases it is possible to specify that we understand as result of performance of algorithm.</u>

And last general property of algorithms - their **correctness.** We say that algorithm **CORRECT if its performance sings correct results of the decision of tasks in view.**

Accordingly we say that the algorithm CONTAINS ERRORS if it is possible to specify such admissible initial data or conditions at which performance of algorithm either will not come to the end in general, or will not be received any results, or the received results will appear wrong.

On used structure of management of computing process algorithms classify as follows: linear structure; branching structure; cyclic structure; with structure of the enclosed cycles; the mixed (combined) structure.

For an illustration of algorithms of any structure the simple mathematical formulations of problems accessible to the pupil of any trades are used. For the decision of such problems in many cases it can appear inexpedient use of the personal computer, however consideration of ways of their programming makes sense, as they are a component more challenges.

At the decision any more or less the challenge can take place some the various algorithms leading to reception of result. It is necessary to choose the best from all possible algorithms in sense of some criterion.

Algorithm of linear structure – algorithm in which all actions are carried out consistently one after another. Such order of performance of actions is called as natural.

Algorithm of branching out structure – algorithm in which depending on performance of some logic condition computing process should go on one or other branch.

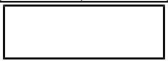
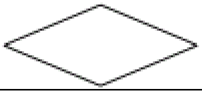
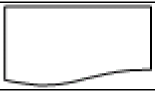
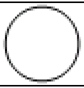
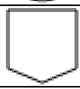
Algorithm of cyclic structure - the algorithm containing repeatedly carried out sites of computing process, named cycles.

Algorithm with structure of the enclosed cycles – the algorithm containing a cycle in which are placed one or other several cycles. There are many ways of record of the algorithms different from each other by presentation, compactness, degree of formalisation and other indicators.

The greatest distribution was received by a graphic way and a so-called algorithmic language of record of the algorithms, focused on the person (pseudo-codes).

Graphic record of algorithm should will be executed according to state standards. (гост 19.002-80"schemes of algorithms and programs. Performance rules»; гост 19.003-80"scheme of algorithms and programs. Designations conditional and graphic»).

The algorithm scheme represents sequence of the blocks ordering
Performance of certain actions, in communication between them.

The name	Symbol (drawing)	Carried out function (explanatory)
1. The block of calculations		Carries out computing action or group of actions
2. The logic block		Choice of a direction of performance of algorithm depending on a condition
3. Input/conclusion blocks		Input or output of the data without dependence from the physical carrier
		Conclusion of the data to the printer
4. The Beginning/end (input/exit)		The beginning or the program end, input or exit in the subroutine
5. The predetermined process		Calculations under the standard or user subroutine
6. The updating block		Performance of the actions changing points of algorithm
7. A connector		Communication instructions between the interrupted lines within one page
8. An interpage connector		Communication instructions between parts of the scheme located on different pages

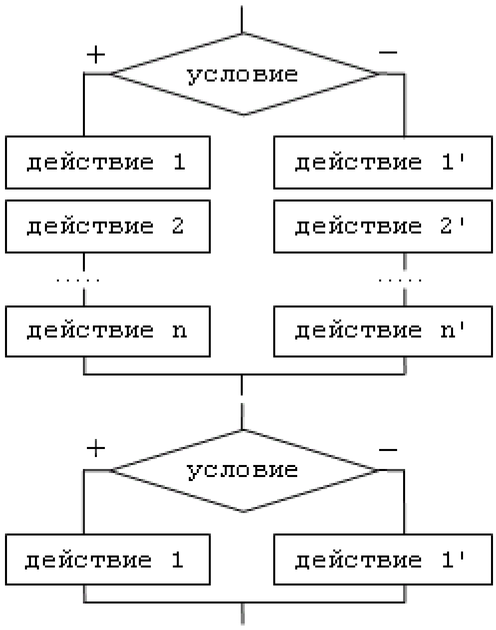
Rules of construction of block diagrammes:
1. The Block diagramme is built in one direction either from top to down, or from left to right
2. All turns of connecting lines are carried out at an angle 90 degrees

Algorithmic design of branching.

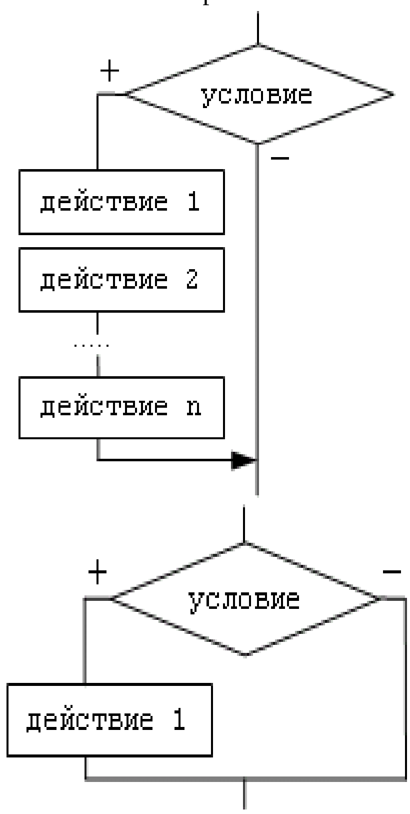
Branching - operating structure, организующая performance only one of two specified actions depending on justice of some condition.

Condition - a question having two variants of the answer: yes or not. Branching record is carried out in two forms: full and incomplete.

The full form:

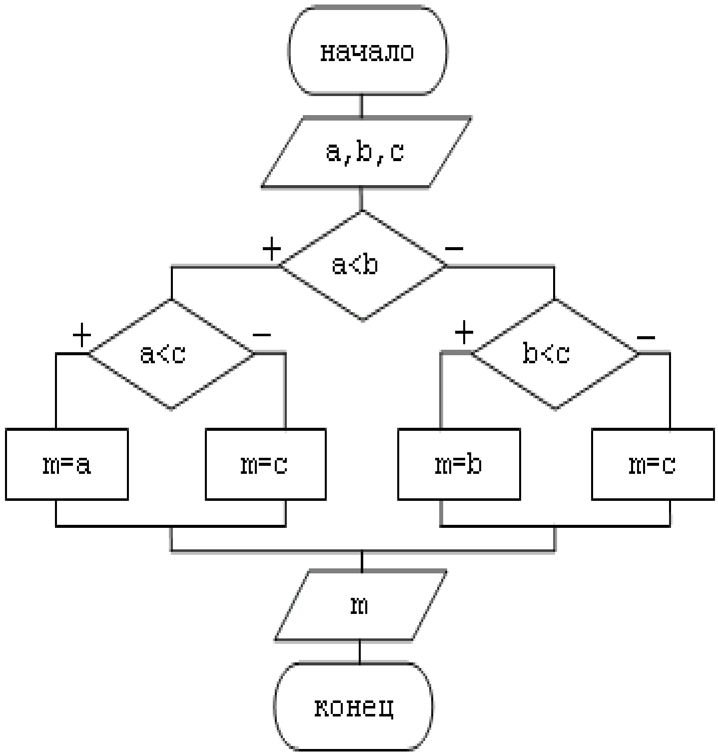


The incomplete form:

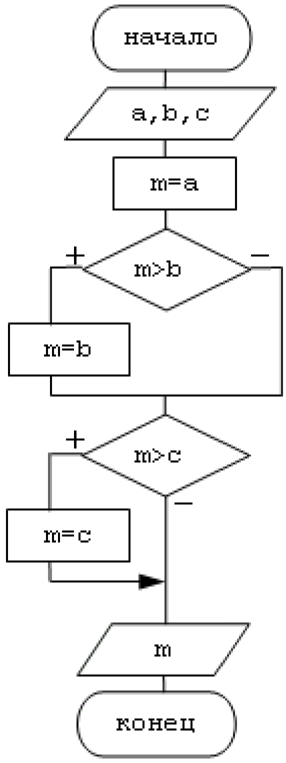


Example: to find least of three numbers.

1 variant of the decision:

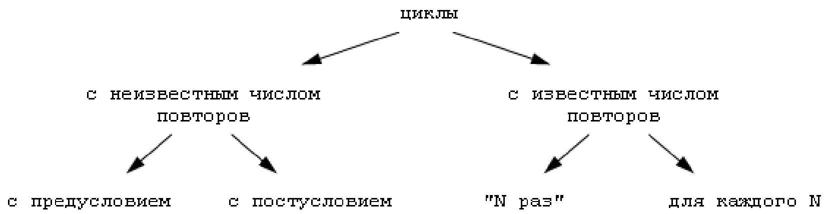


2 variant of the decision:

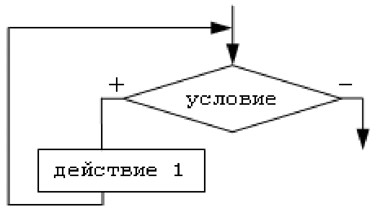


Algorithmic design of a cycle.

Cycle - operating structure, orginized repeated performance of the specified action.

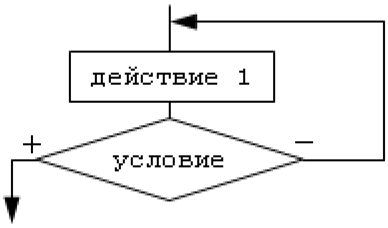


Cycle "while":



Cycle performance "while" begins with condition check, therefore such version of cycles names cycles with a precondition. Transition to action performance is carried out only in the event that the condition is carried out, otherwise there is an exit from a cycle. It is possible to tell that a cycle condition "while" is a condition of an input in a cycle. In that specific case it can appear that action was not carried out never. The cycle condition is necessary for picking up so that actions carried out in a cycle have led to infringement of its validity, differently there will be a cycling.

Cycling - infinite repetition of carried out actions. Cycle "to":



Cycle execution begins with action performance. Thus the cycle body will be realised at least once. After that there is a condition check. Therefore a cycle "to" name a cycle with a postcondition. If the condition is not carried out, there is a return to performance of actions. If the condition is true, the exit from a cycle is carried out. Thus the condition of a cycle "to" is a condition of an exit. For cycling prevention it is necessary to provide the actions leading to the validity of a condition.

Technology «the Questionnaire for a feedback»

That I have remembered on employment	**That I have understood, what have understood**	**That it was pleasant to me, has caused interest**

Control questions

1. List stages of preparation of problems for the decision on the computer.
2. What properties of algorithm in you know?
3. The basic classification of algorithms.
4. Give definitions of algorithms of branching out and cyclic structure.

Lecture №2.
Algorithmization of the numerical decision of the algebraic and transcendental equations.
A method branch of roots and a method half divisions.

The purpose of educational employment: preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a lecture material to watch mastering at pupils of this knowledge, to form and develop at them skills.

The plan:

1. A method branch of roots
2. A method half divisions

1. Methods of branch of roots

The description of a method of the decision of branch of roots

The numerical decision of the nonlinear equations of a kind

$$F(x)=0 \quad (2)$$

consists in a finding of values x, satisfying (with the set accuracy) to the given equation and consists of following basic stages:

Branch (isolation, localisation) equation roots.

Specification by means of some computing algorithm of the concrete allocated root with the set accuracy.

The purpose of the first stage is the finding of pieces from a function range of definition in which one root of the solved equation contains only. Are sometimes limited to consideration only any part of the range of definition causing for those or other reasons interest. For realisation of the given stage graphic or analytical ways are used.

At end of the first stage, intervals should be defined, on each of which one root of the equation contains only.

Any iterative method consisting in construction of numerical sequence x_k usually is applied to specification of a root with demanded accuracy (k=0,1,2, …), converging to a required root x the equations.

Analytical way of branch of roots

The analytical way of branch of roots is based on following theorems:

The theorem 1. If function F (x), defining equation F (x) =0, on the piece ends [a; b] accepts values of different signs, i.e.

$$(a)*F(b)<0,$$

that on this piece contains, at least, one root of the equation.

The theorem 2. If function F (x) is strictly monotonous, a root on [a; b] the unique

$$(F'(a)*F'(b)>0).$$

For branch of roots in the analytical way the piece [A; B], drawing 1 on which there are all roots of the equation interesting the calculator. And on a piece [A; B] function F (x) should be defined, continuous and

$$(a)*F(b)<0.$$

Further there are all partial pieces [a; b], containing on one root.

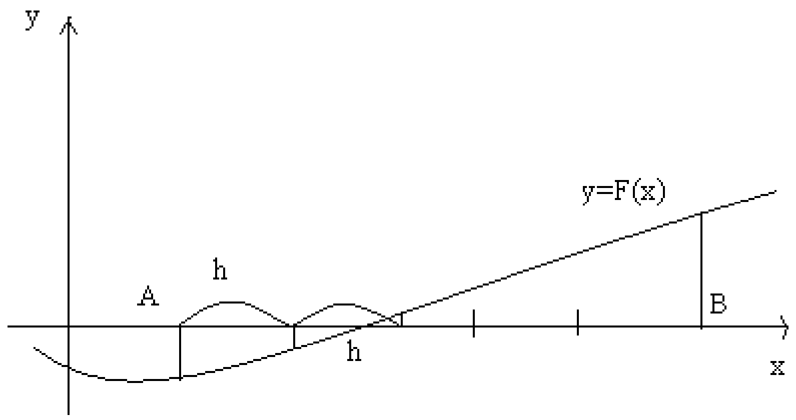
Are calculated value of function F (x), since a point x=A, moving to the right with some step h. If

$$(x)*F(x+h)<0,$$

That on a piece [x; x+h] there is a root and if function F (x) also is strictly monotonous, a root unique. If F (x_k) =0, a x_k-exact root.

Graphic way of branch of roots

The graphic way of branch of roots is based, basically, on visual perception. The branch of roots is made graphically, considering that the valid roots of the equation (1) is there are points of intersection of the schedule of function y=F (x) with an axis of abscisses y=0, it is necessary to construct the function schedule y=F (x) and on axis 0X to note the pieces containing on one root. But it is frequent for simplification of construction of the schedule of function y=F (x) the initial equation (1) replace with the equation equivalent to it $f_1(x) = f_2(x)$. Schedules of functions $y_1=f_1$ (x) and $y_2=f_2$ (x) Further are under construction, and then on axis 0X the pieces localising abscisses of points of intersection of two schedules are marked.



Drawing 1. - a piece Choice

Numerical methods of specification of roots

After the required root of equation F (x) =0 is separated, i.e. the piece [a, b] on which there is only one valid root of the equation is defined, there is an approached value of a root with the set accuracy.

Root specification can be made various methods.

The decision in system MathCad

Problem: to Solve the nonlinear equation $5\sin 2x = \sqrt{1-x}$ (1) numerical method of tangents. We will find and is investigated four roots with accuracy e = 0,000001.

The decision

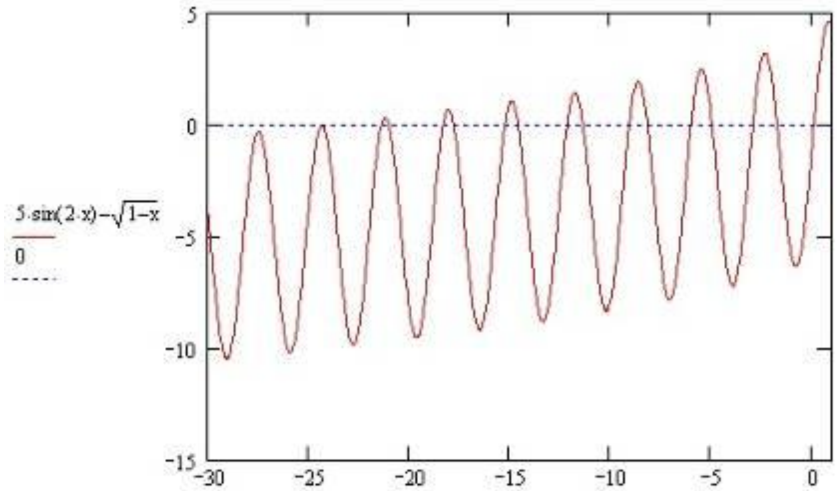
Let's construct in program Mathcad the function schedule

Let's preliminary transfer all to the left part and we will lead to a kind (1) then the equation will become:

$$f(x) := 5\cdot\sin(2\cdot x) - \sqrt{1-x}$$

,

And the schedule of function constructed in program Mathcad, will become presented on drawing 4.



Drawing 4. - the function Schedule in system Mathcad

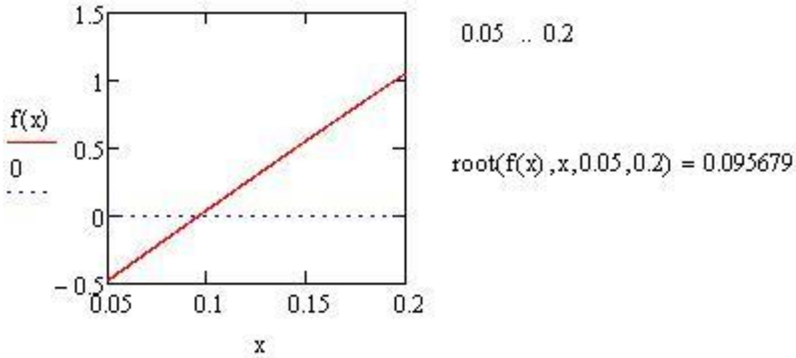
Under the schedule we define quantity and localisations of roots of the equation.

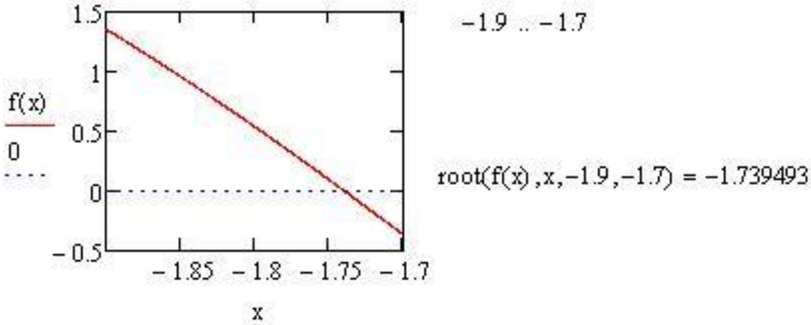
Let's find equation roots

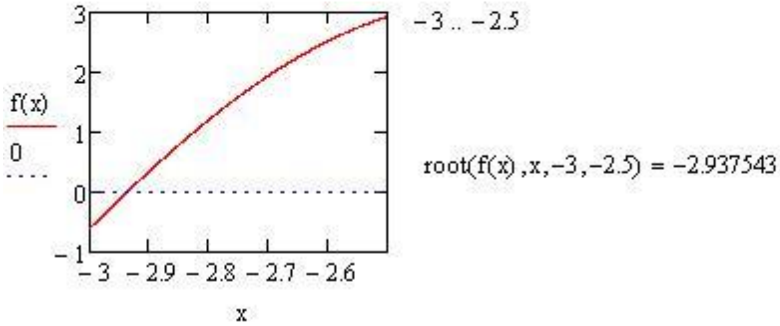
$$5\sin 2x = \sqrt{1-x}$$

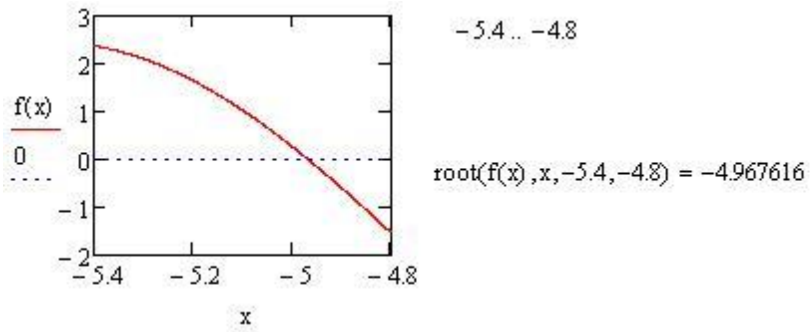
with the set accuracy e = 0,000001

$$f(x) := 5\cdot\sin(2\cdot x) - \sqrt{1-x}$$









2. A method half divisions

Let's consider the equation (1):

$$F(x)=0\,,$$

Where function F(x) – is continuous and defined on some piece and $F(a)F(b)<0.$

The last means that function F (x) has on a piece at least [a, b] one root. We will consider a case, when a root on a piece the unique [a, b].

We halve a piece. If $F\left(\frac{a+b}{2}\right)=0$, is a $\xi=\frac{a+b}{2}$ root of the equation (1). If $F\left(\frac{a+b}{2}\right)\neq 0$, it is considered that half of piece on [a, b] which ends function F (x) has different signs. New, narrower piece [a_1, b_1] again we halve and it is spent on it the same consideration etc. As a result on some step we will receive or exact value of a root of the equation (1), or sequence of the pieces enclosed each other $[a_1,b_1],[a_2,b_2],\dots,[a_n,b_n],\dots,$ such that

$$F(a_n)F(b_n)<0\,,\quad (n=1\,,2\,,\dots) \qquad (9)$$

and

$$b_n - a_n = \frac{b-a}{2^n}. \qquad (10)$$

The left ends of these pieces form the $a_1, a_2, \dots, a_n, \dots$ monotonous (not decreasing) limited sequence, and the right ends - the $b_1, b_2, \dots, b_n, \dots$ monotonous (not increasing) limited sequence. Therefore owing to equality (10) there is a general limit

$$\xi = \lim_{n\to\infty} a_n = \lim_{n\to\infty} b_n.$$

Passing in (9) to a limit at $n \to \infty$, owing to a continuity function F(x) получим: $[F(\xi)]^2 \le 0$. From here i.e $F(\xi) = 0$, . is a ξ root of the equation (1).

In practice process (10) is considered finished, if

$$b_n - a_n = \frac{b-a}{2^n} \le \varepsilon, \qquad (11)$$

Where ε – the set accuracy of the decision.

Technology «the Questionnaire for a feedback»

That I have remembered on employment	**That I have understood, what have understood**	**That it was pleasant to me, has caused interest**

Control questions

1. A method branch of roots
2. A method half divisions

http://math.semestr.ru/optim/secant_method.php **Online the decision**

Lecture № 3.
Algorithmization of the numerical decision of the algebraic and transcendental equations.
Method a chord and Newton's method.

The purpose of educational employment: preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a lecture material to watch mastering at pupils of this knowledge, to form and develop at them skills.

The plan:

1. A method the Chord
2. Newton's method

1. A method the Chord (a method of proportional parts)

Again we will address to the equation (1):

$$F(x) = 0,$$

Where function F(x) – is continuous and defined on some piece and $[a,b]$ $F(a)F(b)<0.$ There is faster way of a finding of the isolated root of the equation ξ (1) lying on a piece $[a,b]$. We will assume for definiteness that Instead of $F(a)<0$ и $F(b)>0.$ piece division half-and-half $[a,b]$, we will divide it in the relation It $F(a):F(b).$ gives the first approach of a rootуравнения:

$$x_1 = a - \frac{F(a)}{F(b)-F(a)}(b-a). \tag{12}$$

Then we consider pieces $[a,x_1]$ и $[x_1,b]$. We will choose that from them on which ends function F (x) has different signs, we will receive the second approach of a root of the equation etc x_2. until then yet we will not reach inequality performance – the $\left|\frac{x_{n+1}-x_n}{x_n}\right| \le \varepsilon$, где ε set accuracy of the decision. Geometrically this method is equivalent to replacement curve $y= F$ (x) a chord spent at first through points $A[a,F(a)]$ и $B[b,F(b)]$, and then the chords spent through the ends of received pieces ($[x_1,b],[x_2,b],...,[x_n,b],...$ fig. 2). From here the name – a *method of chords.*

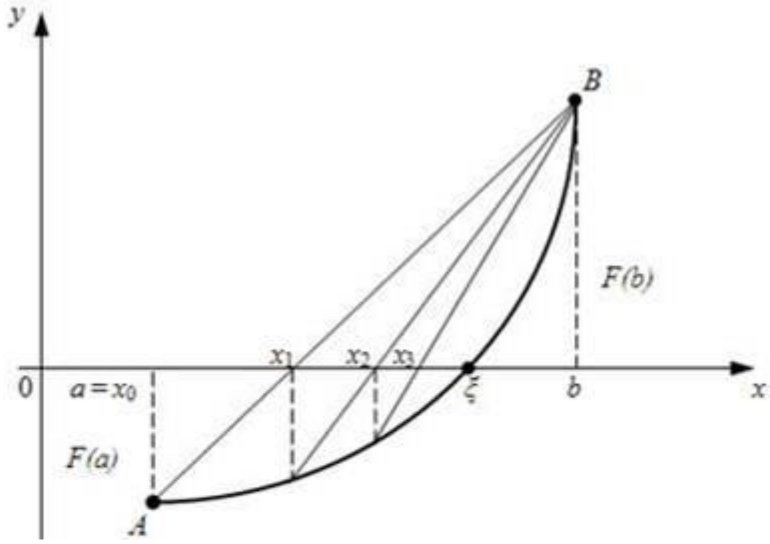
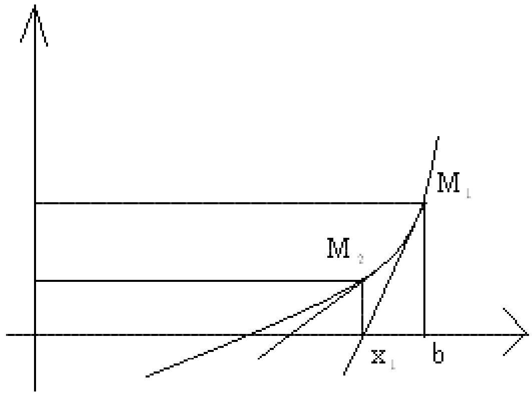


Fig. 2. Geometrical representation of a method of chords.

2. A method of tangents (Newton)

For realisation of the given method, it is necessary to construct initial function y=F (x) and to find values of function on the end of piece F (b). Then to spend a tangent through point M_1. The absciss of a point of intersection of a tangent with axis OX it also is the approached root x_1. Further to find point M_2 (x_1; F (x_1)) to construct the following tangent and to find the second approached root x_2 etc., drawing 2.



Drawing 2. - the Choice of points of a contact

The formula for (n+1) looks like approach:

$$x_{n+1} = x_n - \frac{F(x_n)}{F'(x_n)} \quad (3)$$

If F (a) *F "(a)> 0, x0=a, otherwise x_0=b.
Iterative process proceeds until it will be revealed that:

$$|F(x_{n+1} \le \varepsilon)|. \quad (4)$$

Advantages of a method: simplicity, speed of convergence. Method lacks: calculation of a derivative and difficulty of a choice of initial position.

At first function analyzes the end and a piece [a; b]. If the condition $f(a)\cdot f''(a)>0$, the end and a piece $f(a)\cdot f''(a)>0$ [a; b] also will be the first approach x_1 the equation root, differently the end b a piece [a b] will be the first approach of a root of the equation;. Iterative process which proceeds until Further begins $|f(x1)|>e$. As soon as iterative $|f(x1)|\le e$ process stops, and in x1содержится a required root with necessary approach.

Technology «the Questionnaire for a feedback»

That I have remembered on employment	That I have understood, what have understood	That it was pleasant to me, has caused interest

Control questions

1. A method the Chord
2. Newton's method

Lecture № 4.
Algorithmization of the numerical decision of the algebraic and transcendental equations.
A method of iteration and a method of secants.

The purpose of educational employment: preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a lecture material to watch mastering at pupils of this knowledge, to form and develop at them skills.

The plan:

1. **A method of simple iteration**
2. **A method of secants**

1. A method idle time of iterations (a method consecutive approximation)

It is said that iterative process *converges*, if at performance of consecutive iterations values of the roots turn out, all is closer and closer coming nearer to exact value of a root. Otherwise iterative process is considered *the dispersing*.

Let's copy for convenience the equation (1) in a kind:

$$x = f(x), \tag{3}$$

That it is possible to receive by replacement: $F(x) = x - f(x)$.

Let – x_0 zero approach, i.e. the initial approached value of a root of the equation (3). Then as the following, 1st, approach we will accept

$$x_1 = f(x_0),$$

The following, 2nd, approach will be

$$x_2 = f(x_1),$$

Etc., as n^{th} approach we will accept

$$x_n = f(x_{n-1}). \tag{4}$$

Here there is a main point: whether comes nearer to the x_n true decision of the equation (3) at unlimited increase *n*? Differently, whether iterative process (4) converges?

Conditions convergence of a method of iterations [2]: *if* at *all* values calculated x_n in the course of (4) decisions of a problem:

1) iterative $|f'(x)| < 1$ *process converges;*
2) iterative $|f'(x)| > 1$ *process disperses.*

If the derivative in $f'(x)$ some points on the x_i module is less 1, and in other points – x_j it is more 1 anything the iterative process defined about convergence it is impossible to tell. It can both to converge, and to disperse.

If iterative process disperses, the reason of it often is the unsuccessful choice of zero approach. So, on fig. 1 it is shown that the choice of zero approach essentially influences convergence of iterative process. It directly is connected with, whether there is a zero approach in x_0 area where conditions of convergence of iterative process are satisfied.

2. A method of secants

In numerical analysis, the **secant method** is a root-finding algorithm that uses a succession of roots of secant lines to better approximate a root of a function *f*. The secant method can be thought of as a finite differenceapproximation of Newton's method. However, the method was developed independently of Newton's method, and predates it by over 3,000 years.[1]

The method[edit]

The secant method is defined by the recurrence relation

As can be seen from the recurrence relation, the secant method requires two initial values, x_0 and x_1, which should ideally be chosen to lie close to the root.

Derivation of the method[edit]

Starting with initial values x_0 and x_1, we construct a line through the points $(x_0, f(x_0))$ and $(x_1, f(x_1))$, as demonstrated in the picture on the right. In slope-intercept form, this line has the equation

{\displaystyle y={\frac {f(x_{1})-f(x_{0})}{x_{1}-x_{0}}}(x-x_{1})+f(x_{1})}

We find the root of this line – the value of *x* such that $y = 0$ – by solving the following equation for *x*:

$$0={\frac {f(x_{1})-f(x_{0})}{x_{1}-x_{0}}}(x-x_{1})+f(x_{1})$$

The solution is

$$x=x_{1}-f(x_{1}){\frac {x_{1}-x_{0}}{f(x_{1})-f(x_{0})}}$$

We then use this new value of x as x_2 and repeat the process using x_1 and x_2 instead of x_0 and x_1. We continue this process, solving for x_3, x_4, etc., until we reach a sufficiently high level of precision (a sufficiently small difference between x_n and x_{n-1}).

$$x_{2}=x_{1}-f(x_{1}){\frac {x_{1}-x_{0}}{f(x_{1})-f(x_{0})}}$$

$$x_{3}=x_{2}-f(x_{2}){\frac {x_{2}-x_{1}}{f(x_{2})-f(x_{1})}}$$

…

$$x_{n}=x_{n-1}-f(x_{n-1}){\frac {x_{n-1}-x_{n-2}}{f(x_{n-1})-f(x_{n-2})}}$$

Convergence[edit]

The iterates x_{n} of the secant method converge to a root of f, if the initial values x_{0} and x_{1} are sufficiently close to the root. The order of convergence is α, where

$$\alpha ={\frac {1+{\sqrt {5}}}{2}}\approx 1.618$$

is the golden ratio. In particular, the convergence is superlinear, but not quite quadratic.

This result only holds under some technical conditions, namely that f be twice continuously differentiable and the root in question be simple (i.e., with multiplicity 1).

If the initial values are not close enough to the root, then there is no guarantee that the secant method converges. There is no general definition of "close enough", but the criterion has to do with how "wiggly" the function is on the interval $[~x_{0},~x_{1}~]$. For example, if f is differentiable on that interval and there is a point where $f^{\prime }=0$ on the interval, then the algorithm may not converge.

Comparison with other root-finding methods[edit]

The secant method does not require that the root remain bracketed like the bisection method does, and hence it does not always converge. The false position method (or *regula falsi*) uses the same formula as the secant method. However, it does not apply the formula on x_{n-1} and x_{n-2}, like the secant method, but on x_{n-1} and on the last iterate x_{k} such that $f(x_{k})$ and $f(x_{n-1})$ have a different sign. This means that the false position method always converges.

The recurrence formula of the secant method can be derived from the formula for Newton's method

$$x_{n}=x_{n-1}-{\frac {f(x_{n-1})}{f^{\prime }(x_{n-1})}}$$

by using the finite difference approximation

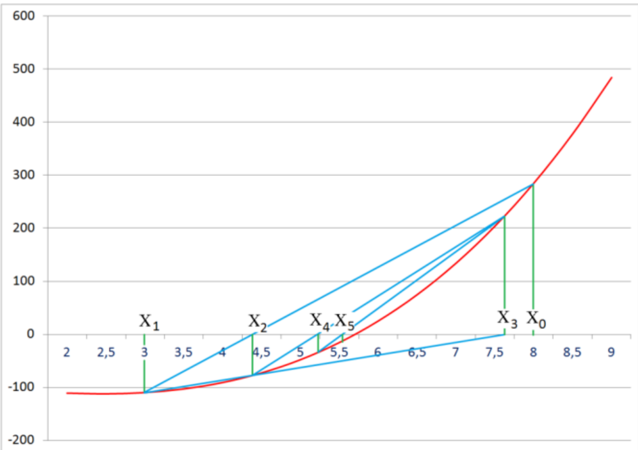
{\displaystyle f^{\prime }(x_{n-1})\approx {\frac {f(x_{n-1})-f(x_{n-2})}{x_{n-1}-x_{n-2}}}} .

The secant method can be interpreted as a method in which the derivative is replaced by an approximation and is thus a Quasi-Newton method. If we compare Newton's method with the secant method, we see that Newton's method converges faster (order 2 against α ≈ 1.6). However, Newton's method requires the evaluation of both {\displaystyle f} and its derivative {\displaystyle f^{\prime }} at every step, while the secant method only requires the evaluation of {\displaystyle f} . Therefore, the secant method may occasionally be faster in practice. For instance, if we assume that evaluating {\displaystyle f} takes as much time as evaluating its derivative and we neglect all other costs, we can do two steps of the secant method (decreasing the logarithm of the error by a factor $\alpha^2 \approx 2.6$) for the same cost as one step of Newton's method (decreasing the logarithm of the error by a factor 2), so the secant method is faster. If however we consider parallel processing for the evaluation of the derivative, Newton's method proves its worth, being faster in time, though still spending more steps.

Generalizations[edit]

Broyden's method is a generalization of the secant method to more than one dimension.

The following graph shows the function *f* in red and the last secant line in bold blue. In the graph, the *x*-intercept of the secant line seems to be a good approximation of the root of *f*.



x1=x1-(x1-x0)*f(x1)/(f(x1)-f(x0)); (Final formule)

```
#include <iostream>
#include <cstdlib>
#include <cmath>

using namespace std;

double f(double x) { return cos(x)-x+1; }  //Функция, нули которой ищем


int main() {
 double tmp,x0,x1,eps;
 int N=0;

 cout<<"eps=";
 cin>>eps;   //Точность
 cout<<"x0=";
 cin>>x0;    //Первое начальное приближение
 cout<<"x1=";
```

```
 cin>>x1;    //Второе начальное приближение

    while(fabs(x1-x0)>eps) {  //Выбран останов |x[n]-x[n+1]|<eps
     tmp=x1;
     x1=x1-(x1-x0)*f(x1)/(f(x1)-f(x0));
     x0=tmp;
     N++;
   }
     cout.setf(ios::scientific);
cout<<endl<<endl<<"x="<<x1<<endl<<"N="<<N<<endl;

cin>>N;
return 0;
}
```

Technology «the Questionnaire for a feedback»

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Control questions

1. A method of simple iteration
2. A method of secants

http://hostciti.net/calc/matematika/secant-method.html

Lecture № 5.

Algorithmization of the numerical decision of system of the algebraic and transcendental equations. A method of Gauss.

The purpose of educational employment: preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a lecture material to watch mastering at pupils of this knowledge, to form and develop at them skills.

The plan:

1. The decision of system of the linear equations a method of Gauss
2. A method of Gauss with a choice of the main element
3. An error estimation at the decision of system of the linear equations

1. The decision of system of the linear equations a method of Gaussa

Problems of approximation of function, and also set of other problems of applied mathematics of m of computing physics are reduced to problems about the decision of systems of the linear equations. The most universal method of the decision of system of the linear equations is the method of a consecutive exception of the unknown persons, Gaussa named a method.

For an illustration of sense of a method of Gaussa we will consider system of the linear equations:

$$\begin{cases} 4x_1 - 9x_2 + 2x_3 = 2 \\ 2x_1 - 4x_2 + 4x_3 = 3 \\ -x_1 + 2x_2 + 2x_3 = 1 \end{cases} \qquad \textbf{(1)}$$

This system we will write down in a matrix kind:

$$\begin{pmatrix} 4 & -9 & 2 \\ 2 & -4 & 4 \\ -1 & 2 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} \qquad \textbf{(2)}$$

As it is known, both members of equation it is possible to increase by nonzero number, and also it is possible to subtract another from one equation. Using these properties, we will try to result a matrix of system (2) in a triangular kind, i.e. to a kind, when below the main diagonal all elements – zero. This stage of the decision is called as a forward stroke.

On the forward stroke first step we will increase the first equation on 1/2 and we will subtract from the second then the variable will be excluded from the x_1 second equation. Then, we will increase the first equation on-1/4 and we will subtract from the third then the system (2) will be transformed to kind system:

$$\begin{pmatrix} 4 & -9 & 2 \\ 0 & 0.5 & 3 \\ 0 & -0.25 & 2.5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 1.5 \end{pmatrix} \qquad \textbf{(3)}$$

On the second step of a forward stroke from the third equation it is excluded x_2, i.e. from the third equation it is subtracted the second, increased, on-1/2 that results system (3) in a triangular kind (4)

$$\begin{pmatrix} 4 & -9 & 2 \\ 0 & 0.5 & 3 \\ 0 & 0 & 4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 2.5 \end{pmatrix} \qquad \textbf{(4)}$$

System (4) it is copied in a habitual kind:

$$\begin{cases} 4x_1 - 9x_2 + 2x_3 = 2 \\ 0.5x_2 + 3x_3 = 2 \\ 4x_3 = 2.5 \end{cases} \qquad \textbf{(5)}$$

Now, from system (5) we can find the decision upside-down, i.e. at first we find from the third equation $x_3 = 0.625$, further, substituting in the second equation, we find $x_2 = \frac{2 - 3x_3}{0.5} = 0.25$. Substituting and x_2 in the x_3 first equation of system (5), we find $x_1 = 0.75$. A finding of the decision from (x_1, x_2, x_3) system (5) name reverse motion.

Now, on the basis of the considered example, we will make the general algorithm of a method of Gaussa for system:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n = b_2 \\ \dots \\ a_{n1}x_1 + a_{n2}x_2 + ... + a_{nn}x_n = b_n \end{cases} \quad \textbf{(6)}$$

The method of Gaussa consists of two stages:

The forward stroke – when a matrix of system (6) is led to a triangular kind;

Reverse motion – when unknown persons upside-down, i.e. in sequence are consistently calculated: $x_n, x_{n-1}, x_{n-2}, ..., x_1$.

Forward stroke: for reduction of system (6) to a triangular kind, the equations with nonzero factors at a variable are rearranged x_1 so that they were above, than the equations with zero factors a_{i1}. Further, we subtract the first equation multiplied on a_{21}/a_{11}, from the second equation, we subtract the first equation multiplied on a_{31}/a_{11}, from the third equation etc. in general, we subtract the first equation multiplied on a_{i1}/a_{11}, from *i* - го the equations at $i = \overline{2,n}$, if $a_{i1} \neq 0$. Owing to this procedure, we have nulled all factors at a variable in x_1 each of the equations, since the second, i.e. the system (6) becomes:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n = b_1 \\ \qquad a'_{22}x_2 + ... + a'_{2n}x_n = b'_2 \\ \qquad a'_{32}x_2 + ... + a'_{3n}x_n = b'_3 \\ \qquad \dots \\ \qquad a'_{n2}x_2 + ... + a'_{nn}x_n = b'_n \end{cases} \quad \textbf{(7)}$$

Further, we apply туже the procedure, to the equations of system (7), since the second equation, i.e. the first equation is excluded from "game". Now we try to null factors at a variable x_2, since the third equation etc., yet we will not lead system to a triangular kind. If $\det A \neq 0$, the system is always led (theoretically triangular kind. It is possible to present the general algorithm of a forward stroke in a kind:

$$\begin{cases} k = \overline{1, n-1} \\ i = \overline{k+1, n} \\ l_{ik} \leftarrow \dfrac{a_{ik}}{a_{kk}} \\ b_i \leftarrow b_i - l_{ik}b_k \\ j = \overline{1,n} \\ a_{ij} \leftarrow a_{ij} - l_{ik}a_{kj} \end{cases} \quad \textbf{(8)}$$

Reverse motion: we calculate unknown persons under formulas:

$$\begin{cases} x_n \leftarrow \dfrac{b_n}{a_{nn}} \\ k = n-1, n-2, ..., 1 \\ x_k \leftarrow \dfrac{\left(b_k - \sum\limits_{j=k+1}^{n} a_{kj}x_j\right)}{a_{kk}} \end{cases} \quad \textbf{(9)}$$

The remark: for calculation of a determinant of system it is possible to use the triangular form of the received matrix then the determinant of this matrix is equal to product of diagonal elements, i.e.

$$\det A = \prod_{i=1}^{n} a_{ii} \quad \textbf{(10)}$$

2. A method of Gaussa with a choice of the main element

The method of Gaussa is so universal that for some systems almost "bad" results turn out, various artful ways out therefore are developed. In a case when some factors of a matrix of system are close among themselves, as it is known relative errors strongly increase at subtraction, therefore the classical method of Gaussa gives the big errors. To bypass this difficulty, try to choose in a forward stroke of Gaussa that equation at which the factor at is maximum x_1 and as basic "player" choose this equation, thereby bypassing difficulties of subtraction of close numbers (if it is possible). Further, when it is necessary to null all factors of a variable x_2, except one equation – this special equation again choose that equation at which factor at maximum x_2 etc., yet we will not receive a triangular matrix.

Reverse motion occurs the same as and in a classical method of Gaussa.

3. An error estimation at the decision of system of the linear equations

To estimate errors of calculations of the decision of system of the linear equations, we need to enter concepts of corresponding norms of matrixes.

First of all, we will recollect three most often used norms for a vector $\vec{u}$:

$$\|\vec{u}\|_1 = \sum_{i=1}^{n} |u_i| \qquad \textbf{(11)}$$

$$\|\vec{u}\|_2 = \sqrt{\sum_{i=1}^{n} |u_i|^2} \quad \text{(Euklyde norm)} \qquad \textbf{(12)}$$

$$\|\vec{u}\|_3 = \lim_{p\to\infty} \sqrt[p]{\sum_{i=1}^{n} |u_i|^p} = \max_{1\le i\le n} |u_i| \quad \text{(Chebyshev norm)} \qquad \textbf{(13)}$$

For any norm of vectors it is possible to enter corresponding norm of matrixes:

$$\|A\| = \sup_{u\neq 0} \frac{\|Au\|}{\|u\|} = \sup_{\|u\|=1} \|Au\| \qquad \textbf{(14)}$$

Which is co-ordinated with norm of vectors in the sense that

$$\|Au\| \le \|A\| \cdot \|u\| \qquad \textbf{(15)}$$

It is possible to show that for three norms of a matrix resulted above cases are set A by formulas:

$$\|A\|_1 = \max_{1\le k\le n} \sum_{i=1}^{n} |a_{ik}| \qquad \textbf{(16)}$$

$$\|A\|_2 = \max_{1\le i\le n} \sigma_i \qquad \textbf{(17)}$$

$$\|A\|_T = \max_{1\le i\le n} \sum_{k=1}^{n} |a_{ik}| \qquad \textbf{(18)}$$

Where - σ_i are singular matrix numbers A, i.e. these are positive values of square roots - $\sqrt{\mu_i}$ matrixes ($A^T \cdot A$ which is the is positive-defined matrix, at $\det A \neq 0$).

For material symmetric matrixes - $\sigma_i = |\lambda_i|$ where - λ_i own numbers of a matrix A.

Absolute error of the decision of system:

$$Ax + b \qquad \textbf{(19)}$$

Where - a A system matrix, - the b matrix of the right parts, is estimated by norm:

$$\Delta = \|Ax - b\| \qquad \textbf{(20)}$$

The relative error is estimated under the formula:

$$\delta = \frac{\Delta}{\|\vec{x}\|} \qquad \textbf{(21)}$$

Where $\|\vec{x}\| \neq 0$.

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Control questions

1. The decision of system of the linear equations a method of Gauss
2. A method of Gauss with a choice of the main element
3. An error estimation at the decision of system of the linear equations

http://matematikam.ru/solve-equations/sistema-gaus.php

Lecture № 6.

Algorithmization of the numerical decision of system of the algebraic and transcendental equations. Iterative methods of Jacoby and Seidel.

The purpose of educational employment: preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a lecture material to watch mastering at pupils of this knowledge, to form and develop at them skills.

The plan:

1. Iterative methods of the decision of systems of the linear equations
2 Method of simple iteration of Jacoby
3. A method of Gauss-Seidel

1. Iterative methods of the decision of systems of the linear equations

Let's consider system of the linear equations which badly dares methods of Gaussa. We will copy system of the equations in a kind:

$$x = Bx + c \qquad \textbf{(22)}$$

Where - the B set numerical matrix n of th order, - the $c \in R^n$ set constant vector.

2 Method of simple iteration of Jacoby

This method consists in the following: any vector ($x^0 \in R^n$ initial approach) gets out $x^0 \in R^n$ and the iterative sequence of vectors under the formula is under construction:

$$x^{(n)} = Bx^{(n-1)} + c, \; n \in N \qquad \textbf{(23)}$$

Let's result the theorem giving a sufficient condition of convergence of a method of Jacoby.

The theorem. If $\| B \| < 1$, the system of the equations (22) has the unique decision and $x = \xi$ iterations (23) converge to the decision.

It is easy to notice that this theorem is simple generalisation of the theorem of the compressed displays studied by us earlier for single-step iterative process in a general view. All estimations received earlier, are transferred and for system of the equations, a difference only in concepts of corresponding norms. Generalising a method of simple iteration of Jacoby for a case of system of the equations:

$$Ax = b \qquad \textbf{(24)}$$

We build algorithm of the decision:

We copy the equation (24) in a homogeneous kind and it is multiplied by a constant - λ which further we will find from conditions of convergence of iterative process:

$$\lambda \cdot (Ax - b) = 0 \qquad \textbf{(25)}$$

We add to x both parts (25) and it is received:

$$x = x + \lambda(Ax - b) = \varphi(x, \lambda) \qquad \textbf{(26)}$$

We build the iterative formula of Jacoby:

$$x^{(n+1)} = x^{(n)} + \lambda(Ax^{(n)} - b) \qquad \textbf{(27)}$$

Where a constant it is found λ from conditions of convergence of iterative process (27) which in this case looks like:

$$\left\| \varphi_x'(x^{(0)}, \lambda) \right\| < 1 \qquad \textbf{(28)}$$

Where - a $\varphi(x, \lambda)$ vector function from (26) or proceeding from the theorem of the compressed displays $\|I + \lambda A\| < 1$, where *I* - an individual matrix.

Let's consider a numerical **example:**

Let we have system of the equations:

$$\begin{cases} x_1 + 3x_2 + 4x_3 = 1 \\ 2x_1 + 3x_2 - 2x_3 = 2 \\ 3x_1 + 4x_2 + 5x_3 = 3 \end{cases}$$

We copy system in a kind:

$$\begin{cases} \lambda_1(x_1 + 3x_2 + 4x_3 - 1) + x_1 = x_1 \\ \lambda_2(2x_1 + 3x_2 - 2x_3 - 2) + x_2 = x_2 \\ \lambda_3(3x_1 + 4x_2 + 5x_3 - 3) + x_3 = x_3 \end{cases}$$

We make the iterative formula:

$$\begin{cases} x_1^{(n+1)} = x_1^{(n)} + \lambda_1(x_1^{(n)} + 3x_2^{(n)} + 4x_3^{(n)} - 1) \\ x_2^{(n+1)} = x_2^{(n)} + \lambda_2(2x_1^{(n)} + 3x_2^{(n)} - 2x_3^{(n)} - 2) \\ x_3^{(n+1)} = x_3^{(n)} + \lambda_3(3x_1^{(n)} + 4x_2^{(n)} + 5x_3^{(n)} - 3) \end{cases}$$

The factor is chosen λ_i from conditions: i.e $\|E + \lambda A\| < 1$.

$$\begin{cases} m_1 = |1 + \lambda_1| + 3|\lambda_1| + 4|\lambda_1| < 1 \\ m_2 = 2|\lambda_2| + |1 + 3\lambda_2| + 2|\lambda_2| < 1 \\ m_3 = 3|\lambda_3| + 4|\lambda_3| + |1 + 5\lambda_3| < 1 \end{cases}$$

$$\max(m_1, m_2, m_3) < 1.$$

3. A method of Gausa-Seidel

The set of iterative methods is developed for the decision of linear system of the equations. As the method of simple iteration of Jacoby converges slowly. One of such methods is the method of Gauss-Seidel.

For a method illustration we will consider a numerical **example:**

$$\begin{cases} 2x - y + z = 5 \\ x + 3y - 2z = 7 \\ x + 2y + 3z = 10 \end{cases} \quad \textbf{(29)}$$

The equations are copied in such a manner that on the main diagonal there are maximum factors for each equation.

We begin with approach $x = y = z = 0$. Using the first equation, we find for new x value under a x_1 condition $y = z = 0$.

$$x_1 = \frac{5 + y - z}{2} = \frac{5}{2} \quad \textbf{(30)}$$

Taking this value and $x = x_1 = 2{,}5$ from the $z = 0$ second equation, we find $y_1 = \frac{7 + 2z - x}{3} = \frac{3}{2}$, further from the third equation it is found. z_1 $z_1 = \frac{10 - x - 2y}{3} = \frac{3}{2}$ These three sizes give new approach and it is possible to cycle a loop from the beginning, we receive: etc $x_2 = \frac{5}{2}$ $y_2 = \frac{5}{2}$ $z_2 = \frac{5}{6}$. Iterations proceed before inequality performance $\left\| x^{(i+1)} - x^{(i)} \right\| < \varepsilon$.

The general algorithm of a method of Gaussa-Zejdelja looks like:

Let

$$Ax = b \quad \textbf{(31)}$$

Where at matrix A - all diagonal elements are distinct from zero, i.e. ($a_{ii} \neq 0$ if then $\exists\, a_{ii} = 0$ we rearrange a line so that to achieve a condition $a_{ii} \neq 0$). If i th equation of system (31) to divide on a_{ii}, and then all unknown persons except - x_i to transfer to the right part we will come to equivalent system of a kind:

$$x = Cx + D \quad \textbf{(32)}$$

where $D = (d_1, d_2, ..., d_n)$, $d_i = \frac{b_i}{a_{ii}}$, $C = (C_{ij})$

$$C_{ij} = \begin{cases} -\dfrac{a_{ij}}{a_{ii}}, \text{if } i \neq j \\ 0, \text{ if } i = j \end{cases} \quad \textbf{(33)}$$

The method of Gaussa-Zejdel consists that iterations are made under the formula:

$$x_i^{(k+1)} = \sum_{j=1}^{i-1} C_{ij} x_j^{(k+1)} + \sum_{j=i+1}^{n} C_{ij} x_j^{(k)} + d_i \quad \textbf{(34)}$$

Where - k iteration number, and $i = \overline{1, n}$.

The remark: for convergence of a method (34) enough performance at least one of conditions:

a)

$$\sum_{j=1,\, j \neq i}^{n} |a_{ij}| < |a_{ii}|, \quad i = \overline{1, n} \quad \textbf{(35)}$$

- The A symmetric and is positive-defined matrix.

Technology «the Questionnaire for a feedback»

That I have remembered on employment	That I have understood, what have understood	That it was pleasant to me, has caused interest

Control questions

1. Iterative methods of the decision of systems of the linear equations
2 Method of simple iteration of Jacoby
3. A method of Gauss-Seidel

Lecture № 7.

Algorithmization interpolation methods. Interpolation functions.

The purpose of educational employment: preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a lecture material to watch mastering at pupils of this knowledge, to form and develop at them skills.

The plan:

1. Introduction
2. The first interpolation Newton's formula
3. The second interpolation Newton's formula
4. The interpolation formula of Stirlinga
5. An example

1. Introduction

Interpolation – operation of approach of the function set in separate points in some set interval. The elementary problem of interpolation consists in the following. On a piece $[a, b]$ are set $n+1$ points x_i (i = 0, 1, 2, …, n), *interpolation* named *in the knots*, and values of some functions $f(x)$ in these points. It is required $f(x_0)=y_0\,, f(x_1)=y_1\,,..., f(x_n)=y_n\,.$ to construct the *interpolating* function accepting $F(x)$ in knots of interpolation the same values, as $f(x)$, i.e. $F(x_0)=y_0\,, F(x_1)=y_1\,,..., F(x_n)=y_n\,.$. Geometrically it means (fig. 1) that it is required to find some curve $y=F(x)$ of the certain type passing through the set set of points $(x_i\,, y_i),\ i=0,1,2,...,n$.

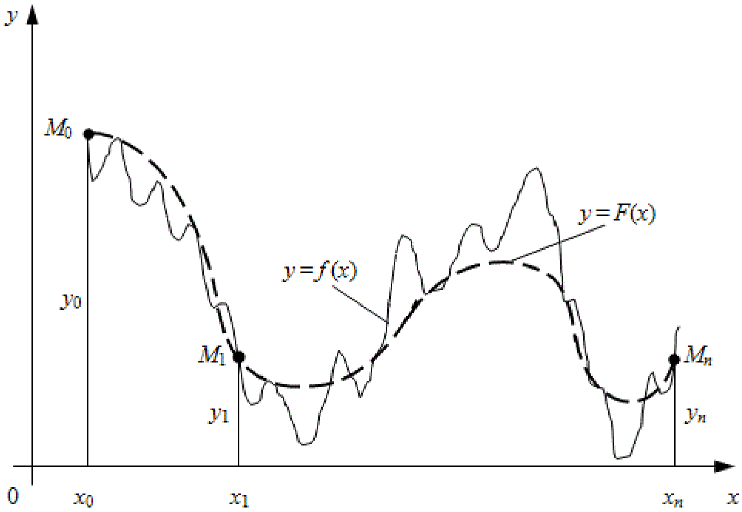


Fig. 1. Geometrical representation of interpolation of function

In such statement the interpolation problem, generally speaking, can have or uncountable set of decisions, or not have at all decisions. However a problem it becomes unequivocal разрешимой if instead of any function to search *F(x)* for a degree polynom $P_n(x)$ not above *n*, satisfying to conditions:

$$P_n(x_0)=y_0\,, P_n(x_1)=y_1\,,..., P_n(x_n)=y_n\,. \qquad (1)$$

Received интерполязионную the formula *y=F(x)* use for the approached calculation of values given функсии *f(x)* для those *x* which are distinct from interpolation knots. Such operation is called as function *interpolation f(x)*.

2. The first interpolation formul Newton.

Let in equidistant points $x_i = x_0 + i\cdot h\ \ (i=0,1,2,...,n)$, where *h* – a *step of interpolation*, preset values for $y_i = f(x_i)$ function *y=f(x)*. It is required to pick up a degree polynom $P_n(x)$ not above *n* satisfying to conditions (1). We will enter final differences for sequence of values $y_i\,,\ i=0,1,2,...,n$:

$$\begin{aligned}
&\Delta y_i = y_{i+1} - y_i\,,\\
&\Delta^2 y_i = \Delta(\Delta y_i) = \Delta y_{i+1} - \Delta y_i\,,\\
&.\ \ .\ \ .\ \ .\ \ .\ \ .\ \ .\ \ .\ \ .\ \ .\ \ .\ \ .\\
&\Delta^n y_i = \Delta(\Delta^{n-1} y_i) = \Delta^{n-1} y_{i+1} - \Delta^{n-1} y_i\,.
\end{aligned} \qquad (2)$$

Conditions (1) are equivalent to equalities:

$$\Delta^m P_n(x_0) = \Delta^m y_0$$

At $m=0,1,2,\ldots,n$

Lowering the calculations resulted in [1], we will definitively receive the *first interpolation formula Newton*:

$$P_n(x) = y_0 + q\Delta y_0 + \frac{q(q-1)}{2!}\Delta^2 y_0 + \ldots + \frac{q(q-1)\ldots(q-n+1)}{n!}\Delta^n y_0 , \qquad (3)$$

Where – $q = \dfrac{x - x_0}{h}$ number of steps of interpolation from an index point to a x_0 point x.

The formula (3) is expedient for using for function interpolation in an $y=f(x)$ index point vicinity where x_0 q on absolute size it is not enough.

In special cases it is had:

At n = 1 – the formula *of linear interpolation*:

$$P_1(x) = y_0 + q\Delta y_0 ;$$

at n = 2 – the formula *of square-law* or *parabolic interpolation*:

$$P_2(x) = y_0 + q\Delta y_0 + \frac{q(q-1)}{2}\Delta^2 y_0 .$$

3. The second interpolation formule Newton

The first interpolation formule Newton is almost inconvenient for interpolation functions near to the table end. In this case usually apply the *second* interpolation formule Newton:

$$P_n(x) = y_n + q\Delta y_{n-1} + \frac{q(q+1)}{2!}\Delta^2 y_{n-2} + \frac{q(q+1)(q+2)}{3!}\Delta^3 y_{n-3} + \ldots + \frac{q(q+1)\ldots(q+n-1)}{n!}\Delta^n y_0 . \qquad (4)$$

The detailed conclusion of the formula (4) is resulted in [1].

Let's notice that if $x<x_0$ and x it is close to x_0 it makes sense to apply the first interpolation formule Newton if $x>x_n$ and x it is close to x_n in this case is more convenient for using the second interpolation formule Newton. In other words, the first interpolation formule Newton is used usually for interpolation *forward*, and the second interpolation formule Newton – for interpolation *back*.

4. The interpolation formul of Stirlinga

The interpolation formule of Stirlinga looks like:

$$\begin{aligned} P_{2n}(x) = {} & y_0 + q\cdot\frac{\Delta y_{-1} + \Delta y_0}{2} + \frac{q^2}{2!}\cdot\Delta^2 y_{-1} + \frac{q(q^2-1)}{3!}\cdot\frac{\Delta^3 y_{-2} + \Delta^3 y_{-1}}{2} + \\ & + \frac{q^2(q^2-1)}{4!}\cdot\Delta^4 y_{-2} + \frac{q(q^2-1)(q^2-2^2)}{5!}\cdot\frac{\Delta^5 y_{-3} + \Delta^5 y_{-2}}{2} + \\ & + \frac{q^2(q^2-1)(q^2-2^2)}{6!}\cdot\Delta^6 y_{-3} + \ldots + \frac{q(q^2-1)(q^2-2^2)(q^2-3^2)\ldots[q^2-(n-1)^2]}{(2n-1)!}\times \\ & \times\frac{\Delta^{2n-1} y_{-n} + \Delta^{2n-1} y_{-(n-1)}}{2} + \frac{q^2(q^2-1)(q^2-2^2)\ldots[q^2-(n-1)^2]}{(2n)!}\Delta^{2n} y_{-n} , \end{aligned} \qquad (5)$$

where, as before $q = \dfrac{x - x_0}{h}$.

There is also a number of others interpolationally formulas: Gauss, Bessel, and so forth the Formula (5) is deduced by Lagrange with use of the first and the second interpolation formulas of Gaussa [1].

5. An example

The table of values of full elliptic integral is set

$$K(\alpha) = \int_0^{\pi/2} \frac{d\,x}{\sqrt{1 - \sin^2\alpha\cdot\sin^2 x}} ,$$

To find K (78º 30 ’).

Values of full elliptic integral K (α)

α	*K*(*α*)	*Δ K*	*Δ2K*	*Δ3K*	*Δ4K*	*Δ5K*	*Δ6K*
75º	2.76806						
		6461					

76º	2.83267		528				
		6989		84			
77º	2.90256		612		19		
		7601		103		13	
78º	2.97857		715		32		– 5
		8316		135		8	
79º	3.06173		850		40		18
		9166		175		26	
80º	3.15339		1025		66		– 1
		10191		241		25	
81º	3.25530		1266		91		43
		11457		332		68	
82º	3.36987		1598		159		
		13055		491			
83º	3.50042		2089				
		15144					
84º	3.65186						

The decision. According to the table data it is accepted $x0 = 78$; $h=1$; $x=78^{\circ}\ 30'$, from here $q = 0.5$. Being limited to differences of the fifth order, under the formula of Stirlinga it is had:

$$K(78^{\circ}30') = 2.97857 + 0.5\frac{7601+8316}{2}\cdot 10^{-5} + 0.125\cdot 715\cdot 10^{-5} - 0.0625\frac{103+135}{2}\cdot 10^{-5} -$$

$$-0.0078\cdot 32\cdot 10^{-5} + 0.0117\frac{13+8}{2}\cdot 10^{-5} = 3.019181.$$

Technology «the Questionnaire for a feedback»

That I have remembered on employment	That I have understood, what have understood	That it was pleasant to me, has caused interest

Control questions

1. Introduction
2. The first interpolation Newton's formula
3. The second interpolation Newton's formula
4. The interpolation formula of Stirlinga
5. An example

Lecture № 8.

The Numerical decision of the differential equations. Euler's method.

The purpose of educational employment: preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a lecture material to watch mastering at pupils of this knowledge, to form and develop at them skills.

The plan:

1. Types of problems for the ordinary differential equations
2. Euler's method

1. Types of problems for the ordinary differential equations

The differential equations arise in many areas of applied mathematics, physics, mechanics, technicians etc. With their help are described almost any problems of dynamics of cars and mechanisms (sections of the dynamic analysis of hydraulic systems, drives and transmissions, control systems see, for example, our site). There is a set of methods of the decision of the differential equations through elementary or special functions. However, more often these methods either are absolutely not applicable, or lead to so difficult decisions that it is easier and more expedient to use the approached numerical methods. The differential equations contain in a large quantity of problems essential nonlinearity, and functions entering into them and factors are set in the form of tables and-or experimental data that actually completely excludes possibility of use of classical methods for their decision and the analysis.

Now there is a set of various numerical methods of the decision of the ordinary differential equations (for example, Euler, Runge-Kutta, Milne, Adams, Gere, etc.) [1 – 6]. We will be limited here to consideration of methods of Euler most widely used in practice and Runge-Kutta. As to other mentioned methods they are in detail stated in the literature, see, for example: [1, 4] – Milne's method, [1, 3, 5] – Adams's method, [5, 6] – Gere's method. We also do not stop here on questions of stability of computing processes, they are in detail shined in the corresponding literature [4, 5, 7].

2. Euler's method

Let's consider the differential equation

$$y' = f(x, y) \qquad (1)$$

With the entry condition

$$y(x_0) = y_0 .$$

Having substituted x_0, y_0 in the equation (1), we will receive value of a derivative in a point x_0 :

$$y'|_{x=x_0} = f(x_0, y_0).$$

At the small Δx takes place:

$$y(x_0 + \Delta x) = y(x_1) = y_0 + \Delta y = y_0 + y'|_{x=x_0} \cdot \Delta x = y_0 + f(x_0, y_0) \cdot \Delta x.$$

Having designated $f(x_0, y_0) = f_0$, we will copy last equality in a kind:

$$y_1 = y_0 + f_0 \cdot \Delta x. \qquad (2)$$

Accepting now (x_1, y_1) for a new starting point, precisely also we will receive:

$$y_2 = y_1 + f_1 \cdot \Delta x.$$

Let's have generally:

$$y_{i+1} = y_i + f_i \cdot \Delta x. \qquad (3)$$

It also is *Euler's method*. The size Δx is called *as integration step*. Using this method, we receive the approached values *y* as the derivative y' actually does not remain to a constant on an interval in length Δx. Therefore we receive an error in definition of value of function *y*, that big, than it is more Δx. Euler's method is the elementary method of numerical integration of the differential equations and systems. Its lacks – small accuracy and regular accumulation of errors.

More exact is *Euler's modified method* or *Euler's method with recalculation*. Its essence that at first under the formula (3) find so-called «rough approach»:

$$\tilde{y}_{i+1} = y_i + f_i \cdot \Delta x ,$$

And then recalculation $\tilde{f}_{i+1} = f(x_{i+1}, \tilde{y}_{i+1})$ receive too approached, but more exact value:

$$y_{i+1} = y_i + \frac{f_i + \tilde{f}_{i+1}}{2} \cdot \Delta x. \qquad (4)$$

Фактически пересчет позволяет учесть, хот и приблизительно, изменение производной y' на шаге интегрирования Δx, так как учитываются ее значения f_i в начале и $\tilde{f}_{i+1}$ в консе шага (рис. 1), а затем берется их среднее. Метод Эйлера с пересчетом (4) является по существу методом Рунге-Кутта 2-го порядка [2], что станет очевидным из дальнейшего.

Actually recalculation allows to consider, though and approximately, derivative change y' on an integration step as Δx its values f_i in the beginning and $\tilde{f}_{i+1}$ in the end of a step (fig. 1) are considered, and then undertakes their average. Euler's method with recalculation (4) is in essence a method of Runge-Kutta of 2nd order [2] that becomes obvious of further.

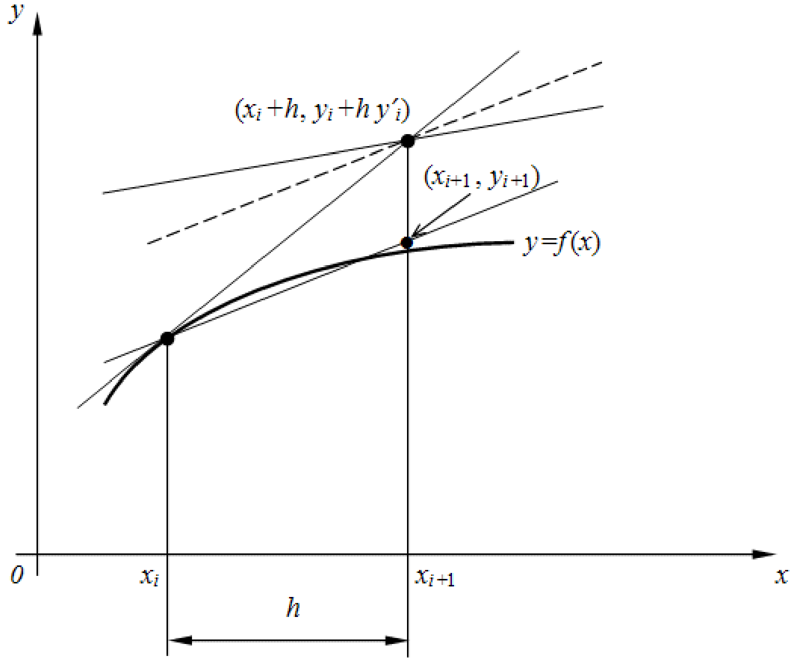


Fig. 1. Geometrical representation of a method of Euler with recalculation.

Technology «the Questionnaire for a feedback»

That I have remembered on employment	That I have understood, what have understood	That it was pleasant to me, has caused interest

Control questions

1. Types of problems for the ordinary differential equations
2. Euler's method

Lecture № 9.

The numerical decision of the differential equations. A method of Runge-Kutte and Adams.

The purpose of educational employment: preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a lecture material to watch mastering at pupils of this knowledge, to form and develop at them skills.

The plan:

1. Methods of Runge-Kutte
2. Adams's method

1. A method of Runge-Kutte

Again we will consider the differential equation

$$y' = f(x, y) \tag{1}$$

With the entry condition $y(x_0) = y_0$.

The classical *method of Runge-Kutta* of 4th order is described by the following system of five equalities:

$$y_{i+1} = y_m + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4), \tag{5}$$

Where

$$k_1 = f(x_i, y_i),$$

$$k_2 = f(x_i + \frac{h}{2}, y_i + \frac{hk_1}{2}),$$

$$k_3 = f(x_i + \frac{h}{2}, y_i + \frac{hk_2}{2}),$$

$$k_4 = f(x_i + h, y_i + hk_3).$$

Strictly speaking, there is not one, and group of methods of Runge-Kutta different from each other rather, i.e. quantity of parameters k_i. In this case we have a method of 4th order which is one of the most put into practice as provides a split-hair accuracy and at the same time differs comparative simplicity. Therefore in most cases it is mentioned in the literature simply as «a method of Runge-Kutta» without instructions of its order.

Example.

To calculate a method of Runge-Kutta integral of the differential equation $y'=x+y$ at the entry condition $y(0)=1$ on a piece [0, 0.5] with integration step $h=0.1$.

The decision. We will calculate y_1. For this purpose at first it is consistently calculated k_i:

$$k_1 = x_0 + y_0 = 0 + 1 = 1;$$

$$k_2 = x_0 + \frac{h}{2} + y_0 + \frac{hk_1}{2} = (0 + 0.05) + (1 + 0.05) = 1.1;$$

$$k_3 = x_0 + \frac{h}{2} + y_0 + \frac{hk_2}{2} = (0 + 0.05) + (1 + 0.055) = 1.105;$$

$$k_4 = x_0 + h + y_0 + hk_3 = (0 + 0.1) + (1 + 0.1105) = 1.2105.$$

Now we will receive

$$\Delta y_0 = \frac{0.1}{6}(1 + 2 \cdot 1.1 + 2 \cdot 1.105 + 1.2105) = 0.1103$$

And, hence,

$$y_1 = y_0 + \Delta y_0 = 1 + 0.1103 = 1.1103.$$

The subsequent are similarly calculated approach. Results of calculations are tabulated:

Results of numerical integration of the differential equation (1) method of Runge-Kutta of the fourth order

i	***x***	***y***	***k*** = 0.1 (*x* + *y*)	**Δ*y***
0	0	**1**	1	0.1
	0.05	1.05	1.1	0.22
	0.05	1.055	1.105	0.221
	0.1	1.1105	1.210	0.1210
				1/6 * 0.6620=**0.1103**
1	0.1	**1.1103**	1.210	0.1210
	0.15	1.1708	1.321	0.2642

	0.15	1.1763	1.326	0.2652
	0.2	1.2429	1.443	0.1443
				1/6 $\cdot$ 0.7947=**0.1324**
2	0.2	**1.2427**	1.443	0.1443
	0.25	1.3149	1.565	0.3130
	0.25	1.3209	1.571	0.3142
	0.3	1.3998	1.700	0.1700
				1/6 $\cdot$ 0.9415=**0.1569**
3	0.3	**1.3996**	1.700	0.1700
	0.35	1.4846	1.835	0.3670
	0.35	1.4904	1.840	0.3680
	0.4	1.5836	1.984	0.1984
				1/6 $\cdot$ 1.1034=**0.1840**
4	0.4	**1.5836**	1.984	0.1984
	0.45	1.6828	2.133	0.4266
	0.45	1.6902	2.140	0.4280
	0.5	1.7976	2.298	0.2298
				1/6 $\cdot$ 1.2828=**0.2138**
5	0.5	**1.7974**		

So, $y(0.5)$ =1.7974.

For comparison the exact decision of the differential equation (1):

$$y = 2e^x - x - 1,$$

Whence $y(0.5) = 2\sqrt{e} - 0.5 - 1 = 1.79744...$

Thus, exact and numerical decisions of the equation (1) have coincided to the fifth decimal sign.

The method of Runge-Kutta also is widely applied to the numerical decision of systems of the ordinary differential equations.

2. Adams's method

Adams's method is applied both to the decision of the simple differential equations, and for their systems.

Problem statement

Adams's method to find the decision of system of the equations on a piece [0; 1] with accuracy $\varepsilon = 10^{-4.}$

$$\begin{cases} y'(x) = cy(x) - z(x), \\ z'(x) = y(x) - dz(x), \end{cases}$$

$$y(a) = k, \quad z(b) = n$$

where c, d, k, n – the set constants

The decision of systems of the ordinary differential equations Adams's method

In the given system of the equations will substitute values of factors and entry conditions. We will receive

$$\begin{cases} y' = 2y - z \\ z' = y - 4z \end{cases} \quad y(0) = 3, \quad z(0) = -2$$

Adams's method we will find the decision of this system on the set piece. For this purpose we will calculate a method of Runge-Kutta some initial values of function.

Let's choose a step h and, for brevity, we will enter $x_i = x_0 + ih$ и $y_i = y(x_i)$ $(i = 0, 1, 2, \dots)$

Let's consider numbers:

$$\begin{cases} k_1^{(i)} = hf(x_i, y_i) \\ k_2^{(i)} = hf\left(x_i + \frac{h}{2}, y_i + \frac{k_1^{(i)}}{2}\right) \\ k_3^{(i)} = hf\left(x_i + \frac{h}{2}, y_i + \frac{k_3^{(i)}}{2}\right) \\ k_4^{(i)} = hf(x_i + h, y_i + k_3^{(i)}) \end{cases}$$

According to a method of Runge-Kutta consecutive values y_i are defined under the formula

$$y_{i+1} = y_i + \Delta y_i$$

Where

$$\Delta y_i = \frac{1}{6}\left(k_1^{(i)} + 2 \cdot k_2^{(i)} + 2 \cdot k_3^{(i)} + k_4^{(i)}\right)\,(i = 0, 1, 2, \dots). \quad (2.1)$$

Having substituted in these formulas initial values we will receive

$$x_0 = 0 \quad y_0 = 3 \quad z_0 = -2$$
$$x_1 = 0{,}1 \quad y_1 = 3{,}3672 \quad z_1 = -2{,}1586$$
$$x_2 = 0{,}2 \quad y_2 = 3{,}4944 \quad z_2 = -2{,}0867$$
$$x_3 = 0{,}3 \quad y_3 = 3{,}5964 \quad z_3 = -1{,}9906$$

Further calculation it is continued on Adams's method. All calculations it is written down in tables 2.1 and 2.2.

Table 2.1

k	x_k	y_k	Δy_k	p_k	Δp_k	$\Delta^2 p_k$	$\Delta^3 p_k$	z_k	Δz_k	q_k	Δq_k	$\Delta^2 q_k$	$\Delta^3 q_k$
0	0	3		0,8000	0,0893	-0,0711	0,0636	-2		1,1000	0,1002	-0,1162	0,1040
1	0,1	3,3672		0,8893	0,0183	-0,0075	0,0680	-2,1586		1,2002	-0,0160	-0,0122	-0,3354
2	0,2	3,4944		0,9076	0,0108	0,0605	0,0512	-2,0867		1,1841	-0,0282	-0,3476	0,7024
3	0,3	3,5964	0,9445	0,9183	0,0713	0,1117	-0,1448	-1,9906	1,1757	1,1559	-0,3758	0,3548	-0,6647
4	0,4	4,5409	1,0761	0,9897	0,1831	-0,0330	0,1605	-0,8149	0,3215	0,7801	-0,0210	-0,3099	0,8201
5	0,5	5,6169	1,3300	1,1727	0,1500	0,1275	-0,1562	-0,4934	1,1598	0,7590	-0,3309	0,5102	-0,9910
6	0,6	6,9469	1,3297	1,3227	0,2775	-0,0288	0,2023	0,6664	-0,1157	0,4281	0,1793	-0,4809	1,1396
7	0,7	8,2766	1,8523	1,6003	0,2488	0,1735	-0,2240	0,5507	1,2171	0,6074	-0,3016	0,6587	-1,3700
8	0,8	10,1290	1,9028	1,8490	0,4223	-0,0505		1,7678	-0,4170	0,3058	0,3571	-0,7113	
9	0,9	12,0318	2,6306	2,2713	0,3718			1,3508	1,5432	0,6629	-0,3542		
10	1	14,6623	2,7239	2,6431				2,8940	-0,6786	0,3086			

Table 2.2

k	x	y	y'	z	z'
0	0	3	8	-2	11
1	0,1	3,3672	8,893	-2,1586	12,0016
2	0,2	3,4944	9,0755	-2,0867	11,8412
3	0,3	3,5964	9,1834	-1,9906	11,5588
4	0,4	4,5409	9,8967	-0,8149	7,8005
5	0,5	5,6169	11,7272	-0,4934	7,5905
6	0,6	6,9469	13,2274	0,6664	4,2813
7	0,7	8,2766	16,0025	0,5507	6,0738
8	0,8	10,129	18,4902	1,7678	3,0578
9	0,9	12,0318	22,7128	1,3508	6,6286

(1.3) values received under the formula are necessary for specifying, having calculated them under the formula (1.4). The obtained data we will write down in the table.

Table 2.3

k	x	Δy_k	$\Delta y_k^{кор.}$	Δz_k	$\Delta z_k^{кор.}$
0	0				
1	0,1				
2	0,2				
3	0,3	0,9445	0,946075	1,1757	1,010942
4	0,4	1,0761	1,069808	0,3215	0,710767
5	0,5	1,3300	1,256483	1,1598	0,647071
6	0,6	1,3297	1,444138	-0,1157	0,441063
7	0,7	1,8523	1,733608	1,2171	0,537967
8	0,8	1,9028	2,037263	-0,4170	0,381975
9	0,9	2,6306	2,470742	1,5432	0,602158
10	1	2,7239	2,6431	-0,6786	0,3086

Technology «the Questionnaire for a feedback»

That I have remembered on employment	**That I have understood, what have understood**	**That it was pleasant to me, has caused interest**

Control questions

1. Methods of Runge-Kutte
2. Adams's method

Lecture №10.

Numerical integration. Quadrature formulas of trapezes and rectangles. Simpson's formula.

The purpose of educational employment: preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a lecture material to watch mastering at pupils of this knowledge, to form and develop at them skills.

The plan:

1. Classification of methods
2. A method of trapezes
3. Methods of rectangles
4. Simpson's method

1. Classification of methods

It is known that the certain integral of function *f (x)* type $\int_a^b f(x)dx$ (1) numerically represents the area of a curvilinear trapeze limited to curves x=0, y=a, y=b and y = *f (x)* (fig. 1). There are two methods of calculation of this area or certain integral — a method of trapezes (fig. 2) and a method of average rectangles (fig. 3).

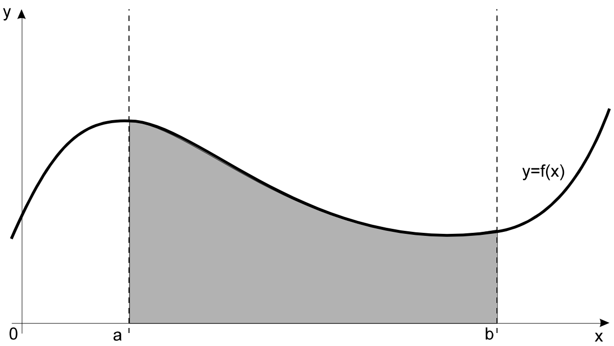


Fig. 1 curvilinear trapeze.

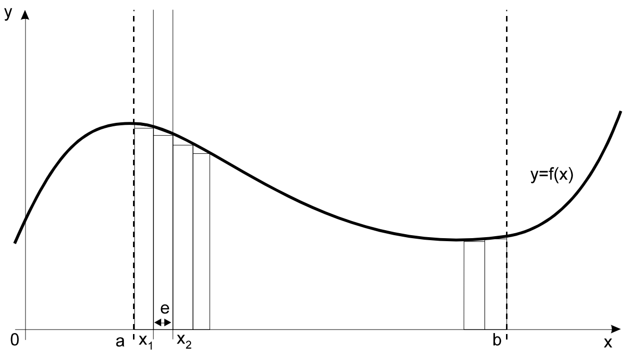


Fig. 2. Method of trapezes.

2. Method of Trapezes

The size of certain integral is numerically equal to the area of the figure formed by the schedule of function and an axis of abscisses (geometrical sense of certain integral). Hence, to find it $\int_a^b f(x)dx$ means to estimate the area of the figure limited to perpendiculars, restored to the schedule of subintegral function *f(x)* from points an and b, located on an argument axis x.

We will break an interval [a, b] on n identical sites for the problem decision. The length of each site will be equal *h = (b-a)/n* (fig. 4).

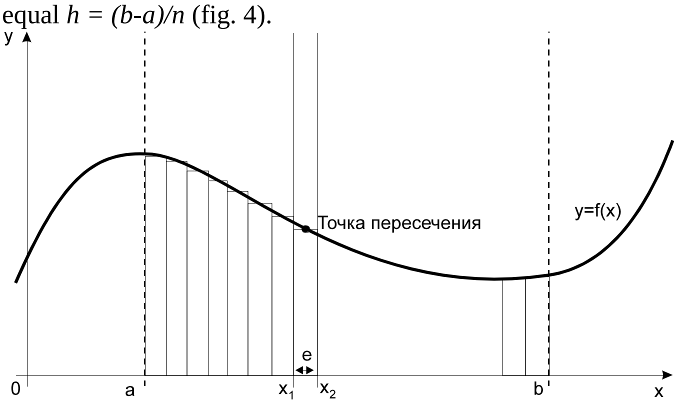


Fig. 3 Method of average rectangles.

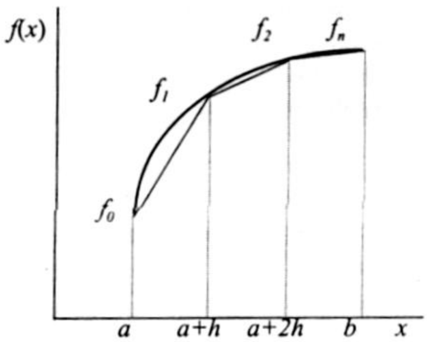


Fig. 4. Interval splitting [a, b] on n identical sites

Let's restore perpendiculars from each point before crossing with the function schedule *f (x)*. If to replace the received curvilinear fragments of the schedule of function with pieces of straight lines, then approximately the figure area and consequently also the size of certain integral is estimated as the area of all received trapezes. We will designate consistently values of subintegral functions on the ends of pieces f_0, f_1, f_2..., f_n also we will count up the area of trapezes

$$S = \frac{f_0 + f_1}{2} \cdot h + \frac{f_1 + f_2}{2} \cdot h + \frac{f_2 + f_3}{2} \cdot h + \ldots + \frac{f_{n-1} + f_n}{2} \cdot h =$$
$$= h\left(\frac{f_0}{2} + \frac{f_1}{2} + \frac{f_1}{2} + \frac{f_2}{2} + \frac{f_2}{2} + \frac{f_3}{2} + \ldots \frac{f_{n-1}}{2} + \frac{f_n}{2}\right) =$$
$$= h\left(\frac{f_0 + f_n}{2} + f_1 + f_2 + \ldots + f_{n-1}\right). \qquad (2)$$

Generally the formula of trapezes becomes

$$\int_a^b f(x)dx \approx h\left(\frac{f_0 + f_n}{2} + \sum_{i=2}^{n-1} f_i\right) = \frac{b-a}{n}\left(\frac{f_0 + f_n}{2} + \sum_{i=2}^{n-1} f_i\right), \qquad (3)$$

Where f_i - value of subintegral function in points of splitting of an interval (a, b) on equal sites with step *h*; f_0, f_n - values of subintegral function accordingly in points a and b.

The formula of trapezes with constant step:

$$\int_a^b f(x)dx \approx \frac{1}{2} h \sum_{i=0}^{n-1} (y_i + y_{i-1}) = \frac{1}{2} h (y_0 + y_n + 2 \sum_{i=1}^{n-1} y_i) \qquad (4)$$

3. A method of rectangles

The elementary methods of numerical integration are methods of rectangles. In them subintegral function is replaced with a polynom of zero degree, that is a constant. Similar replacement is ambiguous as the constant can be chosen subintegral function equal to value in any point of an interval of integration. Depending on it methods of rectangles share on: methods of the left, right and average rectangles.

On a method of average rectangles the integral is equal to the sum of the areas of rectangles where the rectangle basis any small size (accuracy), and the height is defined on a point of intersection of the top basis of a rectangle which the function schedule should cross in the middle. Accordingly we receive the formula of the areas for a method of average rectangles:

$$S_b = \sum_a^b \frac{|f(x_1) + (fx_2)|}{2} \varepsilon \qquad (5)$$

The formula of average rectangles with constant step:

$$\int_a^b f(x)dx \approx \frac{1}{2} h \sum_{i=0}^{n-1} f\left(x_i + \frac{h}{2}\right) \qquad (6)$$

4. Simpson's (Parabolas) formula

Simpson's rule – one of widest known and applied methods of numerical integration. It is similar to a rule of trapezes as also is based on splitting of the general interval of integration into smaller pieces. However its difference that for area calculation through each three consecutive ordinates of splitting the square parabola is spent. Lowering needless details and calculations we will result a definitive kind *of the formula of Simpson* [3, 4]:

$$I \approx \frac{h}{3}(y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + \ldots + 2y_{n-2} + 4y_{n-1} + y_n). \qquad (6)$$

Here n - even number. This formula is much more exact than the formula of trapezes. So, at integration of multinomials of degree not above Simpson's third method gives exact values of integral.

Examples

Let's consider probability integral:

$$I = \int_{-2}^{2} e^{-\frac{x^2}{2}} dx.$$

Exact value of integral of probability to the fifth significant figure equally 2.3925.

Example 1. To calculate integral of probability a method of trapezes with step h = 1.0, 0.5, 0.25.

The decision. Results of calculations are tabulated:

Integration step h	Value of integral	The received error: The absolute	The relative, %
1.0	2.3484	– 0.0441	1.843
0.5	2.3813	– 0.0112	0.468
0.25	2.3898	– 0.0027	0.113

Example 2. To calculate integral of probability Simpson's method with step h = 1.0, 0.5, 0.25.
The decision. Results of calculations are tabulated:

Integration step h	Value of integral	The received error: The absolute	The relative, %
1.0	2.3743	– 0.0182	0.760
0.5	2.3923	– 0.0002	0.008
0.25	2.3926	+ 0.0001	0.004

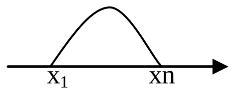
The resulted examples show, how much Simpson's method is more exact than the formula of trapezes.

Example 3.

Application of the formula of average rectangles for the decision of problems of numerical integration (on a calculation example $\int_1^2 (x^2+1)\sin(x-0.5)dx$).

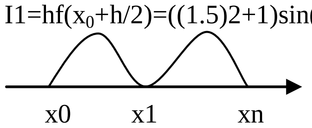
The decision.

$$\int_1^2 (x^2+1)\sin(x-0.5)dx = h\sum_{i=0}^{n-1} f\left(x_i+\frac{h}{2}\right)$$



Let's calculate integral I1 under the formula of a method of average rectangles (6):
$h_1=1$

$I1=hf(x_0+h/2)=((1.5)2+1)\sin(1.5-0.5)=2.734$

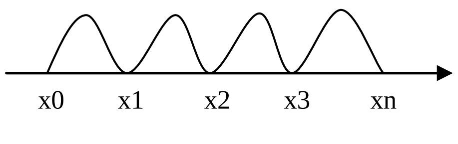


Let's reduce a step twice and we will calculate integral I2 under the formula of a method of average rectangles (6):

$h_2=1/2$
I2= h(f(x0+h/2)+f(x1+h/2))=1/2 ((1.25)2+1)sin(1.25-0.5)+ ((1.75)2+1)sin(1.75-0.5))=2.8005
Let's calculate criterion for integrals I1 and I2, as I2≥1 the criterion is calculated under the formula:

$|(I2-I1)/I2|=0.023746>\varepsilon$

The received criterion is not carried out, we calculate integral I3, reducing a step twice:



h2=1/4
I3=h(f(x0+h/2)+f(x1+h/2)+f(x2+h/2)+f(x3+h/2))=1/4((1.125)2+1)sin(1.125-0.5)+(1.375)2+1)sin(1.375-0.5)+(1.625)2+1)sin(1.625-0.5)+ (1.875)2+1)sin(1.875-0.5))=2.814

Let's calculate criterion for integrals I2 and I3, as I3≥1 the criterion is calculated under the formula:
$|(I3-I2)/I3|=0.004797<\varepsilon$

The received criterion is carried out, hence, we have calculated the set integral with demanded accuracy.

The answer: $\int_1^2 (x^2+1)\sin(x-0.5)dx =$ 2.814 with accuracy0.01.

Technology «the Questionnaire for a feedback»

That I have remembered on employment	That I have understood, what have understood	That it was pleasant to me, has caused interest

Control questions

1. Classification of methods
2. A method of trapezes
3. Methods of rectangles
4. Simpson's method

http://tgspa.ru/info/education/faculties/ffi/ito/programm/osn_chm/chislennoe_integrirovanie3b_mathcad.htm

Lecture № 11.
Numerical integration. The formula of Gauss.

The purpose of educational employment: preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a lecture material to watch mastering at pupils of this knowledge, to form and develop at them skills.

The plan:

1. The quadrature formula of Gauss
2. Solve the equations

The methods described above use the fixed points of a piece (the ends and the middle) and have a low order of accuracy (0 – methods of the right and left rectangles, 1 – methods of average rectangles and trapezes, 3 – a method of parabolas (Simpson)). If we can choose points in which we calculate values of function *f (x)* it is possible to receive methods of higher order of accuracy at the same quantity of calculations of subintegral function. So for two (as in a method of trapezes) calculations of values of subintegral function, it is possible to receive a method any more 1st, and 3rd order of accuracy:

$$I \approx \frac{b-a}{2}\left(f\left(\frac{a+b}{2}-\frac{b-a}{2\sqrt{3}}\right)+f\left(\frac{a+b}{2}+\frac{b-a}{2\sqrt{3}}\right)\right).$$

Generally, using n points, it is possible to receive a method with accuracy order 2n-1. Values of knots of a method of Gaussa on пточкам are roots of a polynom of Lezhandra of degree n.

Values of knots of a method of Gaussa and their scales are resulted in directories of special functions. The method of Gaussa on five points is most known.

Example 1.

Let's calculate integral $\int\limits_{0.5}^{3}\frac{2x^3}{x^4}dx$ with the method of Gauss.

The decision.

$$I \approx \frac{b-a}{2}\left(f\left(\frac{a+b}{2}-\frac{b-a}{2\sqrt{3}}\right)+f\left(\frac{a+b}{2}+\frac{b-a}{2\sqrt{3}}\right)\right)$$

$$f(x)=\frac{2x^3}{x^4}.$$

$$f1(x)=f\left(\frac{a+b}{2}-\frac{b-a}{2\sqrt{3}}\right)=f\left(\frac{0.5+3}{2}-\frac{3-0.5}{2\sqrt{3}}\right)=f(1.029)=1.94\cdot$$

$$f2(x)=f\left(\frac{a+b}{2}+\frac{b-a}{2\sqrt{3}}\right)=f\left(\frac{0.5+3}{2}+\frac{3-0.5}{2\sqrt{3}}\right)=f(2.47)=0.812$$

$$\int\limits_{0.5}^{3}\frac{2x^3}{x^4}dx=\frac{3-0.5}{2}(1.94+0.812)\approx 3.584\cdot$$

The answer: 3.584.

Example 2.

Let's calculate integral a method $\int\limits_{0.5}^{2.3}\pi\cdot\sin(\pi x)dx$ of Gauss.

The decision.

$$f(x)=\pi\cdot\sin(\pi x).$$

$$f1(x)=f\left(\frac{a+b}{2}-\frac{b-a}{2\sqrt{3}}\right)=f\left(\frac{0.5+2.3}{2}-\frac{2.3-0.5}{2\sqrt{3}}\right)=f(0.88)=-1.156\cdot$$

$$f2(x)=f\left(\frac{a+b}{2}+\frac{b-a}{2\sqrt{3}}\right)=f\left(\frac{0.5+2.3}{2}+\frac{2.3-0.5}{2\sqrt{3}}\right)=f(1.92)=0.781$$

$$\int_{0.5}^{2.3} \pi \cdot \sin(\pi x) dx = \frac{2.3 - 0.5}{2} \left(-1.156 + 0.781\right) \approx -0.588.$$

The answer: - 0.588.

Technology «the Questionnaire for a feedback»

That I have remembered on employment	That I have understood, what have understood	That it was pleasant to me, has caused interest

Control questions

1. The quadrature formula of Gauss
2. Solve the equations

Lecture № 12.

Root-mean-square approach of functions. A method of the least squares

The purpose of educational employment: preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a lecture material to watch mastering at pupils of this knowledge, to form and develop at them skills.

The plan:

1. Root-mean-square approach of functions
2. A method of the least squares

1. Root-mean-square approach of functions

Let dependence between variables *x* and *y* is set table (the skilled data is set). It is required to find function somewhat in the best way describing the data. One of ways of selection of such (approaching) function is the method of the least squares. The method consists in that the sum of squares of deviations of values of required function $\bar{y}_i = \bar{y}(x_i)$ and set tabel y_i was the least:

$$S(c) = (y_1 - \bar{y}_1)^2 + (y_2 - \bar{y}_2)^2 + \ldots + (y_n - \bar{y}_n)^2 \to \min \qquad (6.1)$$

Where *c a* vector –of parametres of required function.

2. A method of the least squares

To construct a method of the least squares two empirical formulas: linear and square-law.

In case of linear function *y=ax+b the* problem is reduced to a finding of parametres *a* and *b* from system of the linear equations

$$\begin{cases} M_{x^2} a + M_x b = M_{xy} \\ M_x a + b = M_y \end{cases}, \text{ Where}$$

$$M_{x^2} = \frac{1}{n}\sum_{i=1}^{n} x_i^2, \quad M_x = \frac{1}{n}\sum_{i=1}^{n} x_i, \quad M_{xy} = \frac{1}{n}\sum_{i=1}^{n} x_i y_i, \quad M_y = \frac{1}{n}\sum_{i=1}^{n} y_i$$

а в случае квадратичной зависимости $y = ax^2 + bx + c$ к нахождению параметров a, b и c из системы уравнений:

and in case of square-law dependence $y = ax^2 + bx + c$ to a finding of parameters *a*, *b* and *c* from system of the equations:

$$\begin{cases} M_{x^4} a + M_{x^3} b + M_{x^2} c = M_{x^2 y} \\ M_{x^3} a + M_{x^2} b + M_x c = M_{xy} \\ M_{x^2} a + M_x b + c = M_y \end{cases}, \text{ Where}$$

$$M_{x^4} = \frac{1}{n}\sum_{i=1}^{n} x_i^4, \quad M_{x^3} = \frac{1}{n}\sum_{i=1}^{n} x_i^3, \quad M_{x^2 y} = \frac{1}{n}\sum_{i=1}^{n} x_i^2 y_i$$

To choose from two functions the most suitable. For this purpose to make the table for calculation of the sum of squares of evasion under the formula (6.1). Initial given to take from table 6.

The task 2

To make the program for a finding of approaching functions of the set type with a conclusion of values of their parametres and the sums of squares of evasion corresponding to them. To choose as approaching functions the following: $y = ax + b$, $y = ax^m$, $y = ae^{mx}$. To spend linearization. To define for what kind of function the sum of squares of evasion is the least.

Initial data is placed in table 6.

Approximate fragment of performance of laboratory work

(George E. Forsyth and Michael A. Malcolm and Cleve B. Moler. Computer Methods for Mathematical Computations. Prentice-Hall, Inc., 1977.)

$i := 1..10$

$x_1 := 0.5$ $\quad x_2 := 0.1$ $\quad x_3 := 0.4$ $\quad x_4 := 0.2$ $\quad x_5 := 0.6$

$y_1 := 1.8$ $\quad y_2 := 1.1$ $\quad y_3 := 1.8$ $\quad y_4 := 1.4$ $\quad y_5 := 2.1$

$x_6 := 0.3$ $\quad x_7 := 0.4$ $\quad x_8 := 0.7$ $\quad x_9 := 0.3$ $\quad x_{10} := 0.8$

$y_6 := 1.8$ $\quad y_7 := 1.6$ $\quad y_8 := 2.2$ $\quad y_9 := 1.5$ $\quad y_{10} := 2.3$

$$mx2 := 1\cdot\frac{\left[\sum_{i=1}^{10} \left(|x_i|\right)^2\right]}{10} \qquad mx := 1\cdot\frac{\left(\sum_{i=1}^{10} x_i\right)}{10} \qquad mxy := 1\cdot\frac{\left(\sum_{i=1}^{10} x_i\cdot y_i\right)}{10} \qquad my := 1\cdot\frac{\left(\sum_{i=1}^{10}\right)}{10}$$

$mx2 = 0.229$ $\qquad mx = 0.43$ $\qquad mxy = 0.828$ $\qquad my = 1.76$

Given

$mx2a + mx{\cdot}b = mxy$

$mx{\cdot}a + b = my$

$\mathrm{Find}(a,b) \rightarrow$

Table 6

№ \ i		1	2	3	4	5	6	7	8	9	10
1	*x*	0.5	0.1	0.4	0.2	0.6	0.3	0.4	0.7	0.3	0.8
	y	1.8	1.1	1.8	1.4	2.1	1.8	1.6	2.2	1.5	2.3
2	*x*	1.7	1.5	3.7	1.1	6.2	0.3	6.5	3.6	3.8	5.9
	y	1.5	1.4	1.6	1.3	2.1	1.1	2.2	1.8	1.7	2.3
3	*x*	1.7	1.1	1.6	1.2	1.9	1.5	1.8	1.4	1.3	1.0
	y	6.7	5.6	6.7	6.1	7.4	6.9	7.9	5.9	5.6	5.3
4	*x*	1.3	1.2	1.5	1.4	1.9	1.1	2.0	1.6	1.7	1.8
	y	5.5	5.9	6.3	5.8	7.4	5.4	7.6	6.9	6.6	7.5
5	*x*	2.3	1.4	1.0	1.9	1.5	1.8	2.1	1.6	1.7	1.3
	y	5.3	3.9	2.9	5.0	4.0	4.9	5.1	4.5	4.1	3.7
6	*x*	1.8	2.6	2.3	1.3	2.0	2.1	1.1	1.9	1.6	1.5
	y	4.4	6.4	5.3	3.7	4.9	5.6	3.0	5.0	4.3	3.7
7	*x*	1.9	2.1	2.0	2.9	3.0	2.6	2.5	2.7	2.2	2.8
	y	6.6	7.6	6.7	9.2	9.4	7.8	8.4	8.0	7.9	8.7
8	*x*	2.0	1.4	1.0	1.7	1.3	1.6	1.9	1.5	1.2	2.1
	y	7.5	6.1	4.8	7.4	5.7	7.0	7.1	6.8	6.0	8.9
9	*x*	2.0	1.2	1.8	1.9	1.1	1.7	1.6	1.4	1.5	1.3
	y	7.5	5.9	7.0	8.0	5.0	7.4	6.4	6.6	6.3	5.7
10	*x*	1.9	1.1	1.4	2.3	1.7	2.1	1.6	1.5	1.0	1.2
	y	4.7	3.4	3.8	5.2	4.6	5.5	3.9	3.9	3.2	3.5

Technology «the Questionnaire for a feedback»

That I have remembered on employment	That I have understood, what have understood	That it was pleasant to me, has caused interest

Control questions

1. In what an approach essence таблично the set function on a method of the least squares?
2. Than this method differs from an interpolation method?
3. How the problem of construction of approaching functions in the form of various elementary functions to a case of linear function is reduced?
4. Whether there can be a sum of squares of evasion for any approaching functions equal to zero?
5. What elementary functions are used as approaching functions?
6. How to find parametres for linear and square-law dependence, using a method of the least squares?

Lecture № 13-14

Statement of a problem of linear programming. The basic properties the decision of a problem of linear programming.

The purpose of educational employment: preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a lecture material to watch mastering at pupils of this knowledge, to form and develop at them skills.

The plan:

1. The primary goal of linear programming
2. Examples of the decision of a problem

The primary goal of linear programming in a canonical form is formulated as follows:

To find the non-negative decision of system of restrictions

$$a_{i1}x_1 + a_{i2}x_2 + \ldots + a_{ij}x_j + \ldots + a_{in}x_n + b_i = 0 \ (i = 1,2,\ldots,m);$$
$$x_j \geq 0 \ (j = 1,2,\ldots,n), \qquad \text{(II. 1)}$$

Providing a maximum (minimum) of criterion function

$$Z = c_1x_1 + c_2x_2 + \ldots + c_nx_n + Q \to \max(\min) \qquad \text{(II.2)}$$

Except a record reduced form can be used partially developed

$$Z = \sum_{j=1}^{n} c_j x_j + Q \to \max;$$

$$\sum_{j=1}^{n} a_{ij} x_j + b_i = 0 \ (i = 1,2,\ldots,m); \ x_j \geq 0 \ (j = 1,2,\ldots,n)$$

and matrix forms

$$Z = \dot{C}x + Q \to \dot{m}\dot{a}x,$$
$$Ax + B = 0, \quad x \geq 0.$$

All further reasonings will be spent only for the primary goal in a canonical form.

Usually specific targets of linear programming have distinct from initial an appearance, therefore to solve their such problems it is necessary to lead to a canonical form

Let the problem of linear programming with variables and the mixed system from m restrictions is set:

$$Z = c_1x_1 + c_2x_2 + \ldots + c_nx_n + Q \to \max; \qquad \text{(II.3)}$$

$$a_{i1}x_1 + a_{i2}x_2 + \ldots + a_{in}x_n + b_i \leq 0 \ (i = 1,2,\ldots,r);$$
$$a_{k1}x_1 + a_{k2}x_2 + \ldots + a_{kn}x_n + b_k \geq 0 \ (k = r+1,\ldots,t);$$
$$a_{l1}x_1 + a_{l2}x_2 + \ldots + a_{ln}x_n + b_l = 0 \ (l = t+1,\ldots,m);$$
$$x_j \geq 0 \quad (j = 1,2,\ldots,s \leq n). \qquad \text{(II.4)}$$

For reduction of this problem to a canonical form it is necessary to replace variables, i.e. To exclude those variables which can accept both positive, and negative values. The system of restrictions-inequalities should be replaced by equivalent system of the equations with non-negative variables.

Replacement of inequalities with the equations. Replacement of system of restrictions-inequalities in (II.4) equivalent system of the equations is carried out by introduction of artificial, non-negative variables y,

$$a_{i1}x_1 + a_{i2}x_2 + \ldots + a_{in}x_n + y_i + b_i = 0 \ ;$$
$$a_{k1}x_1 + a_{k2}x_2 + \ldots + a_{kn}x_n - y_k + b_k = 0 \ ;$$
$$y_i \geq 0 \ (i = 1,2,\ldots,r); \ y_k \geq 0 \ (k = r+1,\ldots,t). \qquad \text{(II.5)}$$

Such transformation increases number of variables, without changing a problem being.

Replacement of unlimited variables. Variables which can accept negative values, are expressed through non-negative variables $x_1, x_2, \ldots, x_s$ and $y_1, y_2, \ldots, y_r$. Replacement of variables represents the system decision, concerning a replaced variable, and can be executed with the help жордановых exceptions. For replacement of one variable one step of exceptions is required, therefore to lead problem canonical form is possible only in case a rank of system of more number of unlimited variables.

After replacement the problem dares in new variables. The optimum decision in new variables is substituted in the communication equations therefore the optimum decision in initial variables turns out.

At the decision of economic and technical problems, as a rule, variables can be only positive real numbers. If in a problem any variable by the nature can accept negative values in most cases change of the formulation of conditions allows to get rid of unlimited variables.

Minimisation of form Z. Further the problem of maximisation of form Z will be considered only. If it is necessary to solve a problem of minimisation of the linear form, criterion function factors should be increased on (-1) and to solve this new problem on a maximum. The required minimum of criterion function turns out multiplication of the found maximum value on (-1), i.e. $Z_{min} = -\max(-Z)$

Таблиsa II. 1

Example II.1. THE Colliery works in a complex with concentrating factory. Daily average extraction of mine makes D=3300 t and planned instantaneous ash content coal A = 19,2 %. All coal of mine is transferred to enrichment, therefore every day tasks on quality of coal is corrected for constant maintenance instantaneous ash content processed raw materials. As a result of receipt of party of coal with high instantaneous ash content the concentrating factory demands to lower next days instantaneous ash content extracted coal to 18 %. In this connection it is required to correct daily tasks mining to mine sites so that extraction decrease as a whole, but to mine was minimum. Indicators of work of sites of mine are resulted in III.1.

II. 1

Site number	Daily loading according to plan D_{ni}, t	instantaneous ash content extracted coal A_i, %	The greatest possible loading on a site D_i^{max}, t
1	900	20	1000
2	850	23	920
3	850	18	950
4	700	15	800

Loadings can be increased by a site at the expense of redistribution of empty trolleys and manpower resources. Coal from sites 1 and 2 is transported on the conveyor line having daily productivity no more P_1 = 1850 t and from sites 3 and 4 on line with daily productivity no more P_2=1700 t.

The task in view can be shown to a problem of linear programming with non-negative variables if as variables to accept loadings on faces, and with unlimited variables if for variables corrective amendments of daily tasks of sites are accepted. For more evident illustration of all stages of the decision of problems of linear programming two variants of statement of a problem here will be considered.

Variant 1. If as variables x_i to accept loadings on clearing sites, but a main objective of the decision of a problem - maintenance of the maximum extraction can be described the following criterion function

$$D = x_1 + x_2 + x_3 + x_4 \rightarrow \max.$$

Thus following restrictions should be carried out: on quality of coal

$$A_1x_1 + A_2x_2 + A_3x_3 + A_4x_4 = A_{n}(x_1 + x_2 + x_3 + x_4)$$

On extraction of sites

$$x_1 \le D_1^{max}, \qquad x_2 \le D_2^{max},$$

$$x_3 \le D_3^{max}, \qquad x_4 \le D_4^{max},$$

But throughput of transport communications

$$x_1 + x_2 \le \Pi_1, \qquad x_3 + x_4 \le \Pi_2$$

On the physical essence loading on a face - size positive, therefore $x_i \ge 0 \; (i = 1, 2, 3, 4)$

After substitution of the initial data and reduction of similar members the problem becomes

$$D = x_1 + x_2 + x_3 + x_4 \rightarrow \max$$

$$\begin{aligned} 2x_1 + 5x_2 - 3x_4 &= 0, \\ x_1 - 1000 &\le 0, \\ x_2 - 920 &\le 0. \\ x_3 - 950 &\le 0 \\ x_4 - 800 &\le 0 \\ x_1 + x_2 - 1850 &\le 0 \\ x_3 + x_4 - 1700 &\le 0 \end{aligned}$$

$$x_i \ge 0 \; (i = 1, 2, 3, 4)$$

For problem reduction to a canonical form it is necessary to replace inequalities with equivalent restrictions-equalities by introduction of auxiliary non-negative variables y_i.

$$D = x_1 + x_2 + x_3 + x_4 \rightarrow \max$$

$$\begin{aligned} 2x_1 + 5x_2 - 3x_4 &= 0, \\ -x_1 - y_1 + 1000 &= 0, \\ -x_2 - y_2 + 920 &= 0, \\ -x_3 - y_3 + 950 &= 0, \end{aligned}$$

$$
\begin{aligned}
&&&-x_4 &&-y_4 &&+800=0\\
&-x_1 &-x_2 &&&-y_5 &&+1850=0\\
&&-x_3 &-x_4 &&-y_6 &&+1700=0
\end{aligned}
$$

$$x_i \geq 0 \quad (i=1,\dots,4)$$

$$y_j \geq 0 \quad (j=1,\dots,6)$$

Variant 2. We will designate variable updatings of daily tasks on sites through Δ_i then the problem purpose - extraction maximisation - will be reached at

$$\Delta = \Delta_1 + \Delta_2 + \Delta_3 + \Delta_4 \to \max$$

Thus restrictions should be carried out: on quality

$$\frac{(D_{n1}+D_1)A_1 + (D_{n2}+D_2)A_2 + (D_{n3}+D_3)A_3 + (D_{n4}+D_4)A_4}{(D_{n1}+D_1)+(D_{n2}+D_2)+(D_{n3}+D_3)+(D_{n4}+D_4)} = A_{nn}$$

On extraction of sites

$$0 \leq D_{n1} + \Delta_1 \leq D_1^{\max},$$

$$0 \leq D_{n2} + \Delta_2 \leq D_2^{\max},$$

$$0 \leq D_{n3} + \Delta_3 \leq D_3^{\max},$$

$$0 \leq D_{n4} + \Delta_4 \leq D_4^{\max},$$

On throughput of transport communications

$$(D_{n1}+\Delta_1)+(D_{n2}+\Delta_2) \leq \Pi_1,$$

$$(D_{n3}+\Delta_3)+(D_{n4}+\Delta_4) \leq \Pi_2.$$

After substitution of the initial data, replacement of bilaterial restrictions unilateral and transition to equivalent system of the equations, a problem can lead the kind

$$D = \Delta_1 + \Delta_2 + \Delta_3 + \Delta_4 \to \max$$

$$D = \Delta_1 + \Delta_2 + \Delta_3 + \Delta_4 \to \max$$

$$
\begin{aligned}
&0{,}02\Delta_1 &+0{,}05\Delta_2 &&-0{,}03\Delta_4 &&+39{,}5=0,\\
&-\Delta_1 &&&&-u_1 &+100=0,\\
&&-\Delta_2 &&&-u_2 &+70=0,\\
&&&-\Delta_3 &&-u_3 &+100=0,\\
&&&&-\Delta_4 &-u_4 &+100=0,\\
&\Delta_1 &&&&-u_5 &+900=0,\\
&&\Delta_2 &&&-u_6 &+850=0,\\
&&&\Delta_3 &&-u_7 &+850=0,\\
&&&&\Delta_4 &-u_8 &+700=0,\\
&-\Delta_1 &-\Delta_2 &&&-u_9 &+100=0,\\
&&&-\Delta_3 &-\Delta_4 &-u_{10} &+150=0,
\end{aligned}
$$

$$u_j \geq 0 \quad (j=1, \ \dots, \ 10)$$

Variables Δ can accept both positive, and negative values, therefore for reduction of this problem to a canonical form it is necessary to express them through non-negative variables u_j. This operation will be carried out more low at the further decision of a problem.

The optimum decision of a problem of the linear programming led to a canonical form, the non-negative decision of system of restrictions (II.1), providing a criterion function maximum (II.2) is.

At the decision of system of restrictions there can be three cases:

1. System of restrictions not compatibility and the optimum decision is impossible. Not compatibility systems of restrictions it is caused by the economic and technological reasons. More often not compatibility speaks insufficient quantity of resources because of what restrictions on planned amounts of works cannot be executed. Besides, in planning problems mining works not compatibility systems of restrictions it is often caused by impossibility of performance of requirements to quality of a mineral at the developed industrial situation (the certain maintenance of useful and harmful components in blocks or faces). Restriction revealing because of which all system not compatibility, allows to specify problem statement.

2. The system of restrictions has the unique decision $x_1 = \beta_1 \geq 0;\ x_2 = \beta_2 \geq 0;\ x_n = \beta_n \geq 0$. In this case the problem of linear programming is reduced to the decision of system the linear equation and substitution of this unique decision in criterion function, i.e.

$$Z_{\max} = c_1\beta_1 + c_2\beta_2 + \dots + c_n\beta_n + Q$$

3. The system of restrictions has uncountable set of decisions. From the point of view of maximisation of form Z this case represents the greatest interest and will be considered more low.

Computing procedure of search of the optimum decision of a problem of linear programming is based on following theorems.

The theorem 1. The set of admissible decisions of the primary goal of linear programming is convex.

The theorem 2. The non-negative basic decision of system of linear restrictions (II.1) is a point of set of decisions of the primary goal of linear programming.

The point $\bar{x}$ belonging to set **X,** is called as extreme if it cannot be presented as a convex combination of other points.

As number of the variable equations in system (II.1**)** $\mathbf{x_j \geq 0}$ (j=1, 2..., n) there is more than number of restrictions I (i=l, 2..., n) *THE* system has set of decisions. One of possible decisions of system can be found, if (n-m) any variables to equate to zero. Then the received system from T THE equations with n unknown persons is easy for solving (if a determinant made of factors at unknown persons, does not address in zero, i.e. When lines and columns of a matrix of factors are linearly independent). The decision received thus is called as basic, and making it T variables also are called as basic. The others (n-m) variables are called as not basic or free. In each concrete system of the equations (II.1**)** usually there are some basic decisions with various basic variables.

The theorem 3. The linear form of a problem of linear programming reaches the unique maximum value in an extreme point of set of decisions.

From theorems 2 and 3 it is possible to draw the important conclusion - it is necessary to search for the optimum decision of the primary goal of linear programming among set of admissible basic decisions of system of restrictions.

Technology «the Questionnaire for a feedback»

That I have remembered on employment	That I have understood, what have understood	That it was pleasant to me, has caused interest

Control questions

1. The primary goal of linear programming
2. Examples of the decision of a problem

Lecture № 15-16

Geometrical interpretation of a problem of linear programming.

The purpose of educational employment: preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a lecture material to watch mastering at pupils of this knowledge, to form and develop at them skills.

The plan:

1. Problem statement
2. Geometrical representation.
3. An example of the decision of a problem
4. Geometrical problem interpretation

Better and more visually to present geometrical sense of a problem of linear programming, we will address to the elementary two-dimensional case (when the model includes two variables) and then we will make generalisations at presence n variables.

In case of two variables the model of linear programming has the following appearance

$$Z = c_1 x_1 + c_2 x_2 \to \max ; \qquad \text{(II.6)}$$

$$a_i x_1 + a_i x_2 + b_i \geq 0 \quad (i = 1, \ 2, \ \ldots, \ m); \qquad \text{(II.7)}$$

$$x_1 \geq 0; \quad x_2 \geq 0.$$

Each restriction (II.7) represents a straight line (fig. II.1) which breaks all space (an initial plane) on two semiplanes one of which satisfies to restriction (this area in drawing is shaded).

The system of restrictions according to the theorem 1 represents convex set, and in a considered two-dimensional case - a convex polygon of restrictions (fig. II.2). In special cases the polygon can address in a point (then the decision is unique), a straight line or a piece. If the system of restrictions is inconsistent (несовместна) it is impossible to construct a polygon of restrictions also a problem of linear programming has no decisions. Such case is shown on fig. II.3. Really, there is no point of space which simultaneously would satisfy to restriction y_1 and to restrictions y_2 and y_3.

The polygon of restrictions can be not closed (fig. II.4). In this case, as it will be shown more low, criterion function Z is not limited from above.

In a case п variables each restriction represents (n-l) a-dimensional hyperplane which divides all space into two semispaces. The system of restrictions in this case gives a convex polyhedron of decisions - the general part of the n-dimensional space, satisfying to all restrictions.

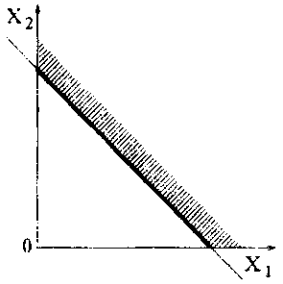


Fig. II.1. Geometrical sense of restriction

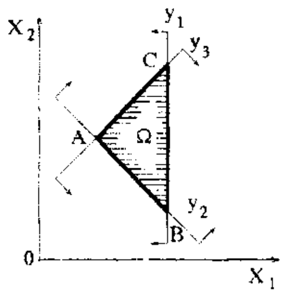


Fig. II.2. Geometrical interpretation of system of restrictions

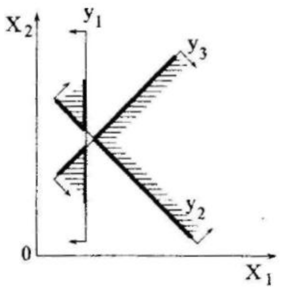


Fig. II.3. Imcompatibility systems of restrictions

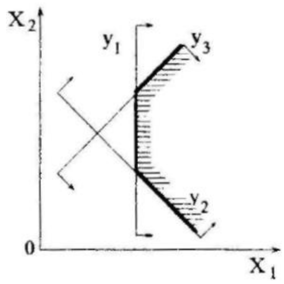


Fig. II.4. Limitlessness of criterion function

In three-dimensional space (n=3) each restriction represents a plane in space. All restrictions, being crossed, form a convex polyhedron which in special cases can be a point, a piece, a beam, a polygon or many-sided unlimited area.

For finding-out of geometrical sense of criterion function we will give to variable Z various numerical values (Z=0, Z=l, Z=2, Z=D).

To these numerical values Z there corresponds sequence of the equations and system of parallel straight lines in space (fig. II.5).

$$c_1x_1 + c_2x_2 = 0,$$
$$c_1x_1 + c_2x_2 = 1,$$
$$\ldots\ldots\ldots\ldots\ldots$$
$$c_1x_1 + c_2x_2 = D.$$

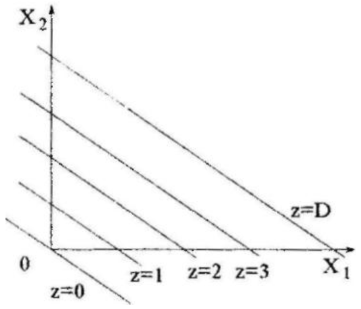


Fig. II.5. Geometrical interpretation of criterion function

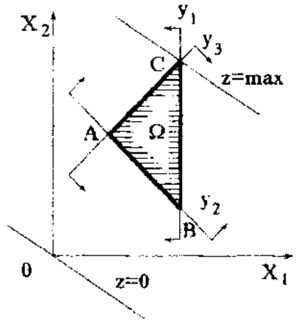


Fig. II.6. Geometrical sense of the optimum decision of a problem of linear programming

The first straight line (Z=0) passes through the beginning of co-ordinates perpendicularly (ортогонально) to the directing vector $C = (C_1C_2)$, the subsequent straight lines are parallel to the first and will defend from it in a direction of a vector With on size 1, 2, D. As a whole variable Z defines evasion of the points lying on a straight line $Z = c_1x_1 + c_2x_2$ from a straight line $c_1x_1 + c_2x_2 = 0$, passing through the beginning of co-ordinates. To define evasion of any point from straight line Z=0, it is enough to substitute co-ordinates of this point in the criterion function equation.

In n-dimensional space of the criterion function equal to zero ($Z = c_1x_1 + c_2x_2 + \ldots + c_jx_j + \ldots + c_nx_n = 0$), geometrically there corresponds (n-1) the-dimensional hyperplane passing through the beginning of co-ordinates.

The distance from a point with co-ordinates $x' = (x'_1 + x'_2 + \ldots + x'_n)$ to a hyperplane is equal

$$R = \frac{a_1x_1' + a_2x_2' + \ldots + a_nx_n'}{\sqrt{a_1^2 + a_2^2 + \ldots + a_n^2}}$$

or

$$R\sqrt{a_1^2 + a_2^2 + \ldots + a_n^2} = a_1x_1' + a_2x_2' + \ldots + a_nx_n'$$

From here it is visible that, if in the linear form to substitute point co-ordinates, the distance from a point x' to the corresponding hyperplane Z=0, postponed in the scale equal to norm of a vector of an orthogonal plane will turn out (or in the scale equal to norm of the directing vector).

The scaled distance **y,** equal $R\sqrt{\sum_{j=1}^{n} a_j^2}$, is called as evasion of a point from a plane. As according to the theorem 3 linear form Z reaches the extreme value in an extreme point (top) of a polyhedron of restrictions geometrically the problem of linear programming consists in search of top of a polyhedron of the admissible decisions, having the maximum evasion from a hyperplane expressed by criterion function, equal to zero (fig. II.6).

If a polyhedron of restrictions will not close (fig. II.4 see) evasion is equal to infinity as a straight line parallel to criterion function, it is possible to move as much as necessary upwards, without leaving for area of admissible values of variables.

The graphic method of their decision is based on geometrical interpretation of linear problems. This method can be used effectively at the decision of problems with two (sometimes with three) variable and reduced to them as it is impossible to represent graphically spaces большей dimensions. For the graphic decision of a problem of linear programming it is necessary in accepted system of co-ordinates to construct the equation of all restrictions which set will give a polyhedron of restrictions. Then build the equation of criterion function equal to zero, i.e. Passing through the beginning of co-ordinates, Z=0. After that, moving the direct (plane) corresponding to criterion function, in parallel to itself, find a point of a contact of this direct (plane) with a polygon (polyhedron) of restrictions - the top of a polygon having the maximum evasion from direct (plane), Z=0.

Let's show use graphic мегода on a concrete example.

Example II.2. It is required to define annual volumes of extraction of ore on three enterprises (tab. II.2).

Table II.2

Indicators	The enterprise		
	1	2	3
The maximum annual extraction Q_i^{max}, million ***t***.	16,7	17,4	15,9
Annual production rate of structure q_i, million ***t***.	2,5	2,35	2,5

The metal maintenance α_i, %	5,9	6,4	7,7
Has arrived from extraction and processings 1 million t. ores p_i, billion roubl.	9,24	9,6	12,12

In total in work there are 13 structures. The ore extracted on three mines, is processed at one concentrating factory, and the average maintenance of metal in ore should be within 6,4 - 6,6 %.

For operated variables we accept number of the structures, allocated to mines for ore transportation on concentrating factory $-x_1, x_2$ и x_3, and for criterion of an optimality - total profit on extraction and ore processing.

Then criterion function of a problem will become

$$\sum_{i=1}^{3} p_i q_i x_i = 9{,}24 \cdot 2{,}5x_1 + 9{,}6 \cdot 2{,}35x_2 + 12{,}12 \cdot 2{,}5x_3 =$$

$$= 23{,}1x_1 + 22{,}56x_2 + 30{,}3x_3 \to \max.$$

At the decision it is necessary to observe following restrictions:

On extraction of mines

$$q_i x_i \le Q_i^{\max};$$

$$2{,}5x_1 \le 16{,}7; \quad 2{,}35x_2 \le 17{,}4; \quad 2{,}5x_3 \le 15{,}9;$$

on number of structures

$$x_1 + x_2 + x_3 = 13;$$

on quality

$$6{,}4 \le \frac{\sum_{i=1}^{n} q_i x_i \alpha_i}{\sum_{i=1}^{n} q_i x_i} \le 6{,}6$$

On positivity of variables $x_1 \ge 0; \quad x_2 \ge 0; \quad x_3 \ge 0.$

Having substituted values q_i and α in restrictions on quality and having executed necessary transformations, we will receive

$$-1{,}75x_1 - 0{,}47x_2 + 2{,}75x_3 \le 0;$$

$$-1{,}25x_1 + 3{,}25x_3 \ge 0.$$

Using restriction – equality $x_1 + x_2 + x_3 = 13,$ we will express in criterion function $\mathbf{x_2}$ through $\mathbf{x_1}$ and $\mathbf{x_3}$. As a result we will receive the following economic-mathematical model with two variables:

$$\begin{aligned}
&23{,}1x_1 + 22{,}6(13 - x_1 - x_3) + 30{,}3x_3 \to \max,\\
&2{,}5x_1 \le 16{,}7,\\
&2{,}35(13 - x_1 - x_3) \le 17{,}4,\\
&2{,}5x_3 \le 15{,}9,\\
&-1{,}75x_1 - 0{,}47(13 - x_1 - x_3) + 2{,}75x_3 \le 0,\\
&-1{,}25x_1 + 3{,}25x_3 \ge 0,\\
&x_1 \ge 0, \quad x_3 \ge 0.
\end{aligned}$$

After transformation it is had

$$\begin{aligned}
&0{,}5x_1 + 7{,}7x_3 \to \max,\\
&2{,}5x_1 \le 16{,}7,\\
&2{,}35x_1 + 2{,}35x_3 \ge 13{,}2,\\
&2{,}5x_3 \le 15{,}9,\\
&-1{,}28x_1 + 3{,}22x_3 \le 6{,}1,\\
&-1{,}25x_1 + 3{,}25x_3 \ge 0,\\
&x_1 \ge 0, \quad x_3 \ge 0.
\end{aligned}$$

Geometrical interpretation of a problem is resulted on fig. II.7, where $x_1 = 0, \ x_3 = 0$ - axes of co-ordinates. Besides, five more restrictions are constructed, and by short shading and arrows admissible semiplanes are shown. The system of restrictions forms area of admissible decisions - convex polygon ABCD.

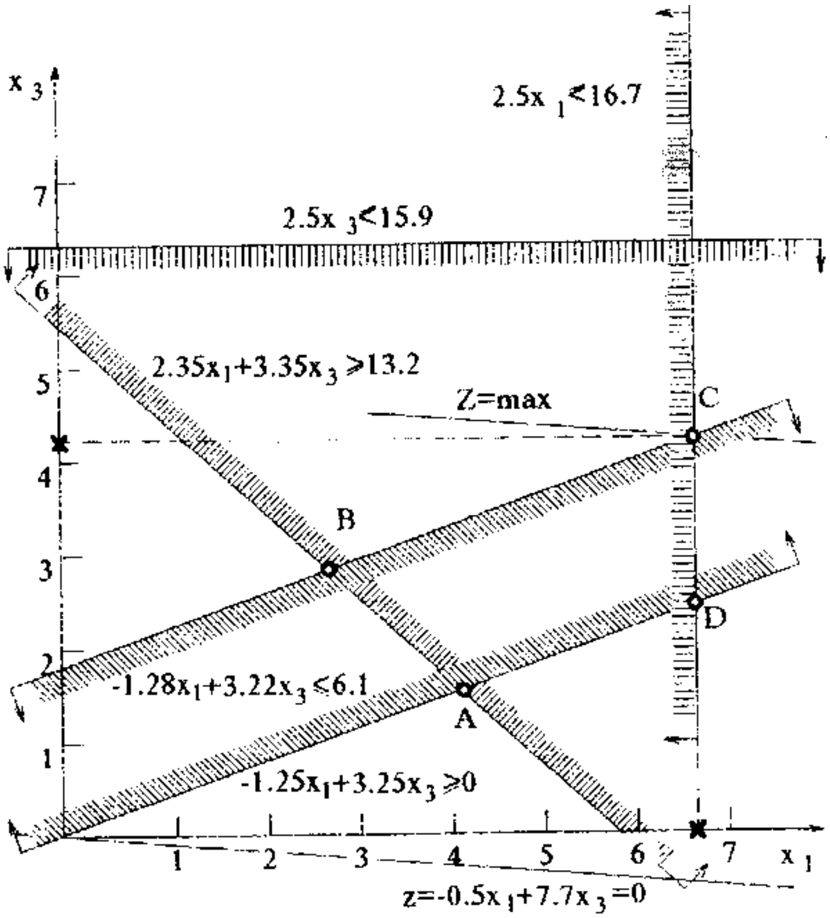


Fig. II.7. Geometrical interpretation of a problem

Through the beginning of co-ordinates there passes a straight line corresponding to the equation of criterion function, equal to zero ($Z = 0,5x_1 + 7,7x_3 = 0$). We move this straight line in parallel to themselves until it will not concern tops of a polygon of the restrictions, having the maximum removal from an initial straight line (Z=0). The top With gives to us x_1, and x_3, turning criterion function in a maximum. Lowering from a point C perpendiculars on co-ordinate axes, we will receive x_1=6,68 and x_3=4,55. Then $x_2 = 13 - x_1 - x_3 = 1,77$.

So the maximum value of criterion function is reached at x_1=6,68, **x_2=1,77** and x_3=4,55. Divisibility of number of structures speaks about necessity of their distribution and movement management on an open cycle.

Technology «the Questionnaire for a feedback»

That I have remembered on employment	That I have understood, what have understood	That it was pleasant to me, has caused interest

Control questions

1. Problem statement
2. Geometrical representation.
3. An example of the decision of a problem
4. Geometrical problem interpretation

Lecture №17.

Finding the decision of a problem of linear programming to simplex methods.

The purpose of educational employment: preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a lecture material to watch mastering at pupils of this knowledge, to form and develop at them skills.

The plan:

1. Mathematical bases a simplex of a method of the decision
2. Fill the table

1. Mathematical bases a simplex of a method of the decision

It is known that if the problem of linear programming has the optimum decision there is at least one optimum basic decision. Thus, by search of basic decisions it is possible to receive the required decision. The number of basic decisions makes $N = C_n^k$, where n - number of variables, and K=r(A) - number of basic variables. This number very quickly grows at increase in number of variables, therefore in rather small problems continuous search becomes impracticable even by means of the COMPUTER.

The number of touched decisions can be reduced at the expense of an exception of consideration of inadmissible basic decisions. The admissible basic decision or the basic decision represents the basic decision with positive values of basic variables. Hence, to touch only basic decisions, the algorithm of search should answer a following condition: at transition from one decision to another should remain innegativity all variables. Performance of this condition does a problem of search of more foreseeable, but as a whole procedure remains ineffective as transition to another does not guarantee its improvement against one decision. What is quality of the decision? The procedure ultimate goal - achievement of a maximum of linear form Z, therefore can serve as an indicator of quality of the decision level 2 in the given basic decision. Hence, efficiency of procedure of search can be raised sharply if each step improves quality of the decision or to provide growth of linear form Z. On the basis of these reasonings it is possible to formulate the second condition to which the algorithm of the decision of a linear problem should answer: transition from one basic decision to another should provide growth of criterion function Z.

This idea can be realised only in the event that there is some basic decision which gradually improves.

The basic method of the decision of problems of linear programming is the simplex-method in which all process of the decision shares on three stages: search of the initial basic decision, search basic and then the optimum decision.

To search of basic, basic and optimum decisions apply special procedures - ordinary and modified Jordanov's exceptions.

That in system of linear forms $y=Ax$ to change in places dependent variable y_r and an independent variable x_S, it is necessary to solve r-e the equation rather x, and to substitute this decision in all other equations of system.

It is obvious that to solve r-e equation rather x, is possible only in the event that $a_{rs} \neq 0$.

Definition. Step ordinary Jordanov's an exception made over system of linear forms y=Ax with the resolving element $a_{rs}\neq 0$, with r-th in the resolving line and s-th a resolving column, name the schematised operation of recalculation of factors in linear forms at change by places dependent variable y_r and independent x_s.

For definition of operations of recalculation of elements of a matrix And in system of linear forms y=Ax at replacement y_r on x_s it is necessary to present a matrix in the form of tab. II.3 and to make corresponding algebraic actions.

Table II.3

	x_1	x_2	...	x_s	...	x_n
y_1	a_{11}	a_{12}	...	a_{1s}	...	a_{1n}
y_2	a_{21}	a_{22}	...	a_{2s}	...	a_{2n}
...	...	...	...	...	...	...
y_r	a_{r1}	a_{r2}	...	a_{rs}	...	a_{rn}
...	...	...	...	...	...	...
y_m	a_{m1}	a_{m2}	...	a_{ms}	...	a_{mn}

In the new table instead of r-th forms the new form from a basic variable x_s which turns out as a result of the decision r-th forms concerning this variable will settle down

$$x_s = -\frac{a_{r1}}{a_{rs}}x_1 - \frac{a_{r2}}{a_{rs}}x_2 + \ldots + \frac{1}{a_{rs}}y_r - \ldots - \frac{a_{rn}}{a_{rs}}x_n$$

Having analysed factors at variables x_j and y_r, may be following conclusions:

1. In the new table on a place of a resolving element a_{rs} should be written down $1/a_{rs}$.

2. Other elements resolving r-th register lines in the new table with a return sign and share on resolving element, i.e. Instead $(-a_{rj}/a_{rs})$ registered

3. In the new table on a place of a resolving column it is necessary to write down elements a_{is} instead of elements

4. Instead of the elements a_{ij} which are not belonging to the resolving line and a column, in the new table elements register **bij** $= (a_{ij}a_{rs} - a_{is}a_{rj})/a_{rs}$

Thus, for performance of one step Jordanov's exceptions with a resolving element a_{rs} it is necessary to carry out four operations by the rules formulated here and as a result the new system of forms in the form of tab. II-4 will be received.

Table II.4

	x_1	x_2	...	y_r	...	x_n
y_1	b_{11}	b_{12}	...	$\frac{a_{1s}}{a_{rs}}$	...	b_{1n}
...	...	...	...	...	...	...
x_s	$\frac{-a_{r1}}{a_{rs}}$	$\frac{-a_{r2}}{a_{rs}}$	...	$\frac{1}{a_{rs}}$	...	$\frac{-a_{rn}}{a_{rs}}$
...	...	...	...	...	...	...
y_m	b_{m1}	b_{m2}	...	$\frac{a_{ms}}{a_{rs}}$	...	b_{mn}

Modified Jordanovs exceptions. If system of linear forms y=Ax to present in a kind y = (-1)-A(-1)x and in this system to make replacement of a dependent variable y_r on independent x_s, with the help Jordanovs exceptions such procedure is called as modified Jordanov's exceptions.

Procedure modified Jordanov's exceptions is deduced similarly and consists in the following.

1. The system y=Ax is represented in a kind y=(-1)-A(-1)x and is brought in tab. II.5

Table II.5

	$-x_1$	$-x_2$	...	$-x_s$	...	$-x_n$
y_1	α_{11}	α_{12}	...	α_{1s}	...	α_{1n}
...	...	...	...	...	...	...
y_r	α_{r1}	α_{r2}	...	α_{rs}	...	α_{rn}
...	...	...	...	...	...	...
y_m	α_{m1}	α_{m2}	...	α_{ms}	...	α_{mn}

The note. $\alpha_{ij} = -a_{ij} \quad (i = 1,2,\ldots,m); \ (j = 1,2,\ldots,n)$

2. Resolving element α_{rs} replace with unit.
3. Other elements of a resolving line remain without changes.
4. A sign at other elements of a resolving column change for the opposite.
5. All elements α_{ij} which are not belonging to the resolving column and a line, replace with elements

$$\beta_{ij} = \alpha_{ij}\alpha_{rs} - \alpha_{is}\alpha_{ri}$$

6. All elements of the new table divide into resolving element α_{rs}

As a result of one step modified Jordanov's exceptions with a resolving element α_{rs} new tab. II.6 turns out.

Table II.6

	$-x_1$	$-x_2$	...	$-y_r$	...	$-x_n$
y_1	$\frac{\beta_{11}}{\alpha_{rs}}$	$\frac{\beta_{12}}{\alpha_{rs}}$	...	$\frac{-\alpha_{1s}}{\alpha_{rs}}$	...	$\frac{\beta_{1n}}{\alpha_{rs}}$
...	...	...	...	...	...	...
x_s	$\frac{\alpha_{r1}}{\alpha_{rs}}$	$\frac{\alpha_{r2}}{\alpha_{rs}}$	...	$\frac{1}{\alpha_{rs}}$	...	$\frac{\alpha_{rn}}{\alpha_{rs}}$
...	...	...	...	...	...	...
y_m	$\frac{\beta_{m1}}{\alpha_{rs}}$	$\frac{\beta_{m2}}{\alpha_{rs}}$	...	$\frac{-\alpha_{ms}}{\alpha_{rs}}$	...	$\frac{\beta_{mn}}{\alpha_{rs}}$

For preservation of monotony of calculations at the decision of various problems only procedure modified Jordanov's exceptions further will be used. Unlike ordinary in modified Jordanov's exceptions the sign varies on opposite at a resolving column, instead of at a line.

Technology «the Questionnaire for a feedback»

That I have remembered on employment	That I have understood, what have understood	That it was pleasant to me, has caused interest

Control questions

1. Mathematical bases a simplex of a method of the decision
2. Fill the table

Lecture №18.

Finding the decision of a problem of linear programming. A method of artificial basis.

The purpose of educational employment: preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a lecture material to watch mastering at pupils of this knowledge, to form and develop at them skills.

The plan:

1. Search of the initial basic decision
2. Non-negative, independent variables

1. Search of the initial basic decision

Let the problem of linear programming from l by variables and the mixed system from *m* restrictions is set:

$$Z = c_1x_1 + c_2x_2 + \ldots + c_nx_n + Q \to \max;$$
$$a_{i1}x_1 + a_{i2}x_2 + \ldots + a_{in}x_n + b_i \geq 0 \ (i = 1,\ 2,\ \ldots,\ r);$$
$$a_{k1}x_1 + a_{k2}x_2 + \ldots + a_{kn}x_n + b_k = 0 \ (k = r+1,\ \ldots,\ m);$$
$$x_j \geq 0 \ (j = 1,\ 2,\ \ldots,\ s < n). \qquad \text{(II.8)}$$

For problem reduction to a canonical form the system of restrictions - inequalities is led to equivalent system of the equations by introduction of artificial, non-negative variables y_i

$$a_{i1}x_1 + a_{i2}x_2 + \ldots + a_{in}x_n - y_i + b_i = 0,$$
$$y_i \geq 0 \ (i = 1,\ 2,\ \ldots,\ r) \qquad \text{(II.9)}$$

Also replacement of unlimited variables is made.

After reduction of system of restrictions to system of the linear equations it is necessary to find its common decision. It is obvious that the equations received from inequalities, easily dare concerning artificial variables y_i and the common decision of this part of system of the equations will be

$$y_i = a_{i1}x_1 + \ldots + a_{in}x_n + b_i,$$
$$y_i \geq 0 \ (i = 1,\ 2,\ \ldots,\ r). \qquad \text{(II.10)}$$

For other part of system of the equations the common decision can be received with the help Jordan's exceptions (or it is established it imcompatibility).

The system decision can be combined with replacement of variables, and for this purpose it is necessary to enter unlimited variables into basis.

After search of the common decision of system the initial basic decision turns out by equating of independent variables with zero.

Thus, reception of the initial basic decision is reduced to following operations. The initial problem is led to a kind (II.10) and registers in a simplex-table (tab. II.9).

Table II.9

	$-x_1$	$-x_2$	...	$-x_n$	1
y_1	$-a_{11}$	$-a_{12}$	...	$-a_{1n}$	b_1
...	...	...	...	...	...
y_r	$-a_{r1}$	$-a_{r2}$	...	$-a_{rn}$	b_r
0	$-a_{r+1,1}$	$-a_{r+1,2}$	...	$-a_{r+1,n}$	b_{r+1}
...	...	...	...	...	...
0	$-a_{m1}$	$-a_{m2}$	...	$-a_{mn}$	b_m
Z	$-c_1$	$-c_2$	...	$-c_n$	Q

In the lines corresponding to restrictions - to inequalities, auxiliary variables register, and in lines with the equations auxiliary variables are equal to zero - (0-variables).

Jordanov's exceptions unlimited variables $x_{s+1},\ \ldots,\ x_n$ are expressed by consecutive steps through non-negative variables and simultaneously with it 0-variables are translated on table top.

The column under translated on top of the table of a 0-variable is excluded. The equations of communication for unlimited variables are remembered, and corresponding lines do not participate in the further analysis. As a result of transformations the table containing the initial basic decision, has the following appearance (tab. II.10).

At following stages of the decision of a problem the part of the table allocated with a dashed line is analyzed only. In the received basic decision independent variables are equated to zero, and basic variables and form Z appear equal to corresponding free members, i.e.

$$x_1 = 0,\ \ldots,\ x_s = 0;\ y_1 = 0,\ \ldots,\ y_p = 0; \qquad \textbf{(II.11)}$$

$$\begin{bmatrix} y_{p+1} \\ \cdot \\ \cdot \\ \cdot \\ y_p \end{bmatrix} = \overline{\beta_1};\ Z = q.$$

Table II.10

<table>
<tr><td rowspan="2"></td><td rowspan="2"></td><td colspan="3">Non-negative, independent variables</td></tr>
<tr><td>0,…,0</td><td>$-x_1, \ldots, -x_s, -y_1, \ldots, -y_n$</td><td>1</td></tr>
<tr><td></td><td>0
.
.
.
0</td><td rowspan="3">A_0</td><td>‖0‖</td><td>$\overline{\beta_0}$</td></tr>
<tr><td>Unlimited variables</td><td>x_{s+1}
.
.
.
x_n</td><td>A_2</td><td>$\overline{\beta_2}$</td></tr>
<tr><td>Non-negative basic variables</td><td>y_{p+1}
.
.
.
y_p</td><td>A_1</td><td>$\overline{\beta_1}$</td></tr>
<tr><td></td><td>Z</td><td></td><td>$\overline{\gamma_1}$</td><td>q</td></tr>
</table>

If at least one unlimited variable cannot be expressed through non-negative variables because of occurrence of zero in a column of the simplex-table corresponding to it such problem is not led to a canonical form and cannot be solved a simplex-method.

If in a line corresponding to a 0-variable, all elements except a free member are equal to zero, system of restrictions imcompatible.

Technology «the Questionnaire for a feedback»

That I have remembered on employment	That I have understood, what have understood	That it was pleasant to me, has caused interest

Control questions

1. Search of the initial basic decision
2. Non-negative, independent variables

Practical work №1

The numerical decision of the algebraic and transcendental equations iterative methods.

(4 hours)

The purpose of educational employment: preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a practical material to watch mastering at pupils of this knowledge, to form and develop at them skills.

The plan:

1. Iteration method
2. Method of Chords
3. Method half divisions

Let's consider the equation

$$f(x) = 0 \tag{1.1}$$

Where $f(x)$ defined and continuous on some final or infinite interval $a < x < b$.

Any value x^* turning function $f(x)$ in zero $f(x^*) \equiv 0$, is called as a root of the equation (1.1), and the way of a finding of this value x^* and is the decision of the equation (1.1).

To find roots of the equation of a kind (1.1) precisely it is possible only in rare instances. Besides, often the equation contains the factors known only approximately and therefore, the problem about exact definition of roots of the equation loses meaning. Methods of the numerical decision of the equations of the kind (1.1) are developed, allowing to find the approached values of roots of this equation.

Thus it is necessary to solve two problems:

1) branch of roots, i.e. Search enough small areas, in each of which are concluded only one root of the equation;

2) calculation of roots with the set accuracy.

Let's take advantage of known result of the mathematical analysis: if continuous function accepts on the ends of some interval of value of different signs an interval contains at least one root of the equation.

For allocation of the areas containing one root, it is possible to use, for example, graphic in the way, or moving along a range of definition with some step, to check on the ends of intervals a condition of change of a sign on function.

For the decision of the second problem exists numerous methods from which we will consider four: a method of iterations, a method half divisions, a method of chords, a method of tangents.

The task 1

To make branch of roots: graphically and under the program (accuracy $\varepsilon = 10^{-1}$). Individual tasks are resulted in table 1.

The task 2

1. To spend specification of roots by a method half divisions.

As initial approach we will choose $c = (a+b)/2$, then we investigate function on the ends of pieces $[a, c]$ and $[c, b]$. That piece at which value of function on the ends has opposite signs gets out. Process proceeds until the condition $|b - a| < \varepsilon$ will be satisfied. Accuracy ε to accept the equal 10^{-3}.

2. To make specification of roots by a method of simple iteration.

Let roots are separated and $[a, b]$ contains a unique root. The equation (1.1) we will lead to an iterative kind:

$$x = \varphi(x) \tag{1.2}$$

where function $\varphi(x)$ is differentiated on $[a, b]$ and for any. $x \in [a,b] \, | \varphi'(x) | < 1$. Function $\varphi(x)$ can be picked up in a kind

$$\varphi(x) = x + kf(x), \tag{1.3}$$

Where k is from a condition $|\varphi'(k,x)| = |1 + kf'(x)| < 1$, for $\forall x \in [a,b]$.

Last condition guarantees convergence of iterative sequence $x_1, x_2, \cdots x_{n-1}, x_n \cdots$ to a root ζ. As a condition of the termination of the account we will consider inequality performance

$$|x_n - x_{n-1}| < \frac{\varepsilon(1-q)}{q}; \quad q = max|\varphi'(x)| \tag{1.4}$$

3. To make specification of roots by a method of chords or tangents (X, K in table 1) with the set accuracy $\varepsilon = 10^{-4}$.

The settlement formula for a method of chords:

$$x_{n+1} = \frac{x_0 f(x_n) - x_n f(x_0)}{(f(x_n) - f(x_0))},$$

For a method of tangents:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)},$$

Value x_0 for a method of chords and an index point for a method of tangents gets out of a condition of performance of an inequality $f(x_0) f''(x_0) > 0$.

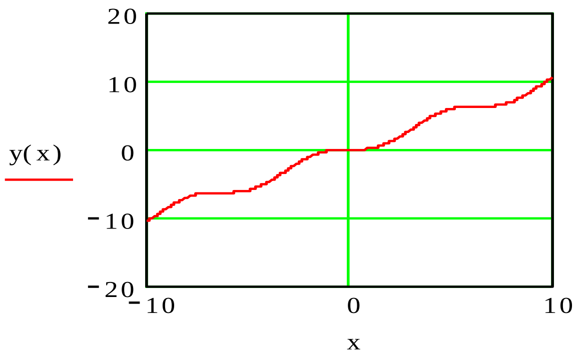
As a result of calculations under these formulas the sequence of the approached values of a root can be received $x_1, x_2, \cdots x_{n-1}, x_n \cdots$. Process of calculations comes to an end at condition performance $|x_n - x_{n-1}| < \varepsilon$ ($\varepsilon = 10^{-5}$). In each case to print quantity of the iterations necessary for achievement of set accuracy.

APPROXIMATE VARIANT OF PERFORMANCE OF WORK ON MATHCAD

1. Definition, construction of tables of values and schedules of functions and branch of roots of the equation *y=x-sinx-0,25*.

We separate roots graphically.
We calculate values of argument and function.

$$y(x) := x - \sin(x) - 0.25$$



$i := 0..10$ $x_i := -5 + i$ $F_i := y(x_i)$

We type i, x_i F_i. More low, x= and nearby we click the mouse, we type F=, also nearby we click the mouse.

x =

	0
0	-5
1	-4
2	-3
3	-2
4	-1
5	0
6	1
7	2
8	3
9	4
10	5

F =

	0
0	-6.209
1	-5.007
2	-3.109
3	-1.341
4	-0.409
5	-0.25
6	-0.091
7	0.841
8	2.609
9	4.507
10	5.709

Given

$$x - \sin(x) - 0.25 = 0$$

$$\text{Find}(x) \rightarrow 1.1712296525016659 9$$

2. The decision with use of operators ***given, find***.

3. The symbolical decision.

$$x - \sin(x) - 0.25 \text{ solve}, x \rightarrow 1.17122965250166599$$

4. At the left the decision a method of iterations, in the middle a method of tangents, on the right a method of chords.

$i := 0..10$ $\quad$ $i := 0..10$ $\quad$ $i := 0..10$

$x_0 := 1$ $\quad$ $x_0 := 1$ $\quad$ $x_0 := 1$

$$x_{i+1} := \sin(x_i) + 0.25 \qquad x_{i+1} := x_i - \frac{\left[x_i - (\sin(x_i) + 0.25)\right]}{1 + \cos(x_i)} \qquad x_{i+1} := \frac{\left[x_0 \cdot (x_i - \sin(x_i) - 0.25) - x_i \cdot (x_0 - \sin(x_0) - 0.25)\right]}{(x_i - \sin(x_i) - 0.25) - (x_0 - \sin(x_0) - 0.25)}$$

x =

	0
0	1
1	1.091471
2	1.137306
3	1.157505
4	1.165804
5	1.169105
6	1.170401
7	1.170907
8	1.171104
9	1.171181
10	1.171211
11	1.171222

x =

	0
0	1
1	1.059385
2	1.101462
3	1.129285
4	1.146676
5	1.157108
6	1.163197
7	1.16669
8	1.168674
9	1.169794
10	1.170424
11	1.170778

x =

	0
0	1
1	0
2	1.576998
3	1.126117
4	1.177917
5	1.170273
6	1.171367
7	1.17121
8	1.171232
9	1.171229
10	1.17123
11	1.17123

Table 1

N	The equation
1	$x + x\ln(x+0.5) - 0.5 = 0$
2	$x2^x - 1 = 0$
3	$x^3 - 2x^2 + x - 3 = 0$
4	$x^3 + 12x - 2 = 0$
5	$5x - 8\ln(x) - 8 = 0$
6	$x^4 + 0.5x^3 - 4x^2 - 3x - 0.5 = 0$
7	$x - \cos(x) - 0.27 = 0$
8	$x^3 - 6x^2 + 20 = 0$
9	$5x^3 + 10x^2 + 5x - 1 = 0$
10	$0.1x^2 - x\ln(x) = 0$

Solve the equation with Newton's method

1. $x^3 + 2x^2 + 2 = 0$
2. $x^3 - 2x + 2 = 0$
3. $x^3 + x - 3 = 0$
4. $x^3 - 0{,}2x^2 + 0{,}4x - 1{,}4 = 0$

14. $x^3 - 3x^2 + 9x - 10 = 0$
15. $x^3 + 3x - 1 = 0$
16. $x^3 + 0{,}4x^2 + 0{,}6x - 1{,}6 = 0$
17. $x^3 - 0{,}1x^2 + 0{,}4x - 1{,}4 = 0$

5.	$x^3+3x^2+12x+3=0$	18.	$x^3-0{,}2x^2+0{,}5x-1=0$
6.	$x^3-0{,}1x^2++0{,}4x+1{,}2=0$	19.	$x^3-3x^2+6x-5=0$
7.	$x^3-0{,}2x^2+0{,}5x-1{,}4=0$	20.	$x^3+2x+4=0$
8.	$x^3-3x^2+12x-12=0$	21.	$x^3+0{,}2x^2+0{,}5x+0{,}8=0$
9.	$x^3+4x-6=0$	22.	$x^3+0{,}1x^2+0{,}4x-1{,}2=0$
10.	$x^3+3x^2+6x-1=0$	23.	$x^3-0{,}1x^2+0{,}4x-1{,}5=0$
11.	$x^3-3x^2+6x-2=0$	24.	$x^3-0{,}2x^2+0{,}3x-1{,}2=0$
12.	$x^3-3x^2+12x-9=0$	25.	$x^3+0{,}2x^2+0{,}5x-2=0$
13.	$x^3+3x+1=0$	26.	$x^3+0{,}2x^2+0{,}5x-1{,}2=0$

Technology «the Questionnaire for a feedback»

That I have remembered on employment	That I have understood, what have understood	That it was pleasant to me, has caused interest

Control questions

1. Stages of the decision of the equation from one unknown person.
2. Ways of branch of roots.
3. How the graphic branch of roots is specified by means of calculations?
4. To give the verbal description of algorithm of a method половинного divisions.
5. Necessary conditions of convergence of a method половинного divisions.
6. Condition of the termination of the account of a method of simple iteration. A method error.
7. The verbal description of algorithm of a method of chords. Graphic representation of a method. Error calculation.
8. The verbal description of algorithm of a method of tangents (Newton). Graphic representation of a method. A condition of a choice of an index point.

Practical work № 2.
Newton's interpolation polynom and Lagrange
(4 hours)

The purpose of educational employment: preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a practical material to watch mastering at pupils of this knowledge, to form and develop at them skills.

The plan:

1. Interpolation polynom of Newton
2. Interpolation polynom of Lagrange

Let function *f(x)* is set as table, or its calculation demands bulky calculations. We will replace approximately function *f(x)* with any function *F(x)* so that the deviation *f(x)* from *F(x)* was in the set area somewhat minimum. Similar replacement is called as function approximation *f(x)*, and function *F(x)* – approximating (approaching) function.

The classical approach to the decision of a problem of construction of approaching function is based on the requirement of strict coincidence of values *f (x)* and *F (x)* in points x_i ($i=0,1,2, \dots n$), i.e.

$$F(x_0)=y_0,\, F(x_1)=y_1, \dots, F(x_n)=y_n. \tag{3.1}$$

In this case a finding of the approached function name interpolation (or interpolation), points $x_0, x_1, \cdots x_n$ – interpolation knots.

Often интерполирование it is conducted for the functions set by tables with equidistant values of argument *x*. In this case the table step $h=x_{i+1}-x_i$ $(i=0,1,2,\dots)$ is constant size. For such tables construction интерполяsионных formulas (as, however, and calculation under these formulas) considerably becomes simpler.

По заданной таблисе значений функsии составит формулу интерполяsионного многочлена **Лагранжа** (3.2) и построит график $L_2(x)$.. Исходные данные берутся из таблиsы 3.1.

The task 1

Under the set table of values of function to make the formula interpolation a multinomial **of Lagrange** (3.2) and to construct the schedule $L_2(x)$. The initial data undertakes from table 3.1.

$$L_2(x)=y_0\frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}+y_1\frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}+y_2\frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} \tag{3.2}$$

Table 3.1.

№	x_0	x_1	x_2	y_0	y_1	y_2
1	2	3	5	4	1	7
2	4	2	3	5	2	8
3	0	2	3	-1	-4	2
4	7	9	13	2	-2	3
5	-3	-1	3	7	-1	4
6	1	2	4	-3	-7	2
7	-2	-1	2	4	9	1
8	2	4	5	9	-3	6
9	-4	-2	0	2	8	5
10	-1	1.5	3	4	-7	1
11	2	4	7	-1	-6	3
12	-9	-7	-4	3	-3	4
13	0	1	4	7	-1	8
14	8	5	0	9	2	4
15	-7	-5	-4	4	-4	5

The task 2

To calculate one value of the set function for intermediate value of argument (*a*) with the help interpolation a multinomial **of Lagrange** (3.3) and to estimate an interpolation error. For task performance the initial data undertakes from table 3.2, 3.3 or 3.4.

$$L_n(x) = \sum_{i=0}^{n} y_i \frac{(x-x_0)\ldots(x-x_{i-1})(x-x_{i+1})\ldots(x-x_n)}{(x_i-x_0)\ldots(x_i-x_{i-1})(x_i-x_{i+1})\ldots(x_i-x_n)} \qquad (3.3)$$

For an error $R_n(x)$ inequality is carried out

$$|R_n(x)| \le \frac{M_{n+1}}{(n+1)!}|\Pi_{n+1}(x)|, \qquad x \in [x_o, x_n] \qquad (3.4)$$

where $M_{n+1} = \max|f^{(n+1)}(x)|$.

The table 3.2

№ Variant	Valuea	№ tables
1	-2	3.3
2	3.77	3.4
3	0.55	3.3
4	4.83	3.4
5	3.5	3.3
6	5.1	3.4
7	1.75	3.3
8	4.2	3.4
9	-1.55	3.3
10	6.76	3.4

The table 3.3

x	-3.2	-0.8	0.4	2.8	4.0	6.4	7.6
$f(x) = 2.1\sin(0.37x)$	-1.94	-0.61	0.31	1.81	2.09	1.47	0.68

The table 3.4

x	1.3	2.1	3.7	4.5	6.1	7.7	8.5
$f(x) = \lg(x)/x + x^2$	1.777	4.563	13.84	20.39	37.34	59.41	72.4

The table 3.5

x	0.10	0.15	0.20	0.25	0.30	0.35	0.40
$f(x) = \cos(x)$	0.995	0.988	0.980	0.969	0.955	0.939	0.921

The table 3.6

x	0.65	0.70	0.75	0.80	0.85	0.90	0.95
$f(x) = \sin(x)$	0.605	0.644	0.681	0.71	0.75	0.783	0.813

The task 3.

To condense a part of the table set on a piece *[a, b]* functions, using interpolation **Newton's** multinomial (3.5) and to estimate an error of interpolation *D* (the formula (3.6)). Table 3.7 of final differences to count manually on a piece *[a, b]* with step *h*. For task performance the initial data undertakes from tables 3.8, 3.5 and 3.6.

$$P_2(x) = y_0 + t\Delta y_0 + \frac{t(t-1)}{2!}\Delta^2 y_0 + \frac{t(t-1)(t-2)}{3!}\Delta^3 y_0, \qquad (3.5)$$

where $t = \dfrac{x - x_0}{h}$.

$$D \approx \frac{t(t-1)(t-2)}{3!} f'''(\xi) \qquad , \qquad (3.6)$$

where ξ – Some internal point of the least interval containing all knots $x_i\,(i = \overline{0,n})$ and x.

The formula (3.5) is called as the first interpolation **Newton's** formula**.** If calculated value of a variable is closer to the piece end *[a, b]*, apply Newton's second formula – interpolation back (the formula (3.6)).

$$P_n(x) = y_n + t\Delta y_{n-1} + \frac{t(t+1)}{2!}\Delta^2 y_{n-2} + \frac{t(t-1)(t-2)}{3!}\Delta^3 y_{n-3} \quad (3.6)$$

где $t = \dfrac{x - x_n}{h}$ и $D = \dfrac{t(t+1)(t+2)}{3!} f'''(\xi)$.

The table3.7

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
x_0	y_0	$\Delta y_0 = y_1 - y_0$	$\Delta^2 y_0 = \Delta y_1 - \Delta y_0$	$\Delta^3 y_0 = \Delta^2 y_1 - \Delta^2 y_0$
$x_1 = x_0 + h$	y_1	$\Delta y_1 = y_2 - y_1$	$\Delta^2 y_1 = \Delta y_2 - \Delta y_1$	
$x_2 = x_1 + h$	y_2	$\Delta y_2 = y_3 - y_2$		
$x_3 = x_2 + h$	y_3			

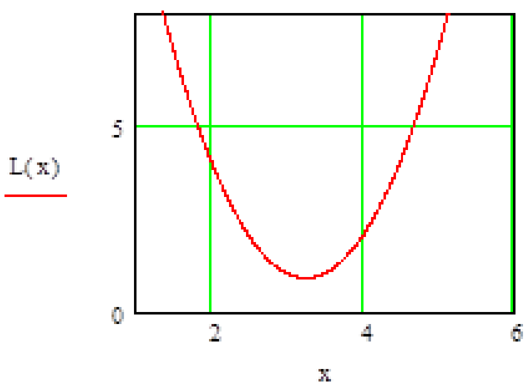
Table 3.8

№	a	b	h_0	h	№ tables
1	0.65	0.80	0.05	0.01	3.6
2	0.25	0.40	0.05	0.025	3.5
3	0.75	0.90	0.05	0.01	3.6
4	0.70	0.85	0.05	0.025	3.6
5	0.80	0.95	0.05	0.025	3.6
6	0.1	0.25	0.05	0.025	3.5
7	0.15	0.3	0.05	0.025	3.5
8	0.7	0.85	0.05	0.025	3.6
9	0.2	0.35	0.05	0.01	3.5
10	0.80	0.95	0.05	0.01	3.6

Approximate fragment of performance of work in MathCAD

$x0 := 2 \quad x1 := 3 \quad x2 := 5 \quad y0 := 4 \quad y1 := 1 \quad y2 := 7$

$$L(x) := \left[\frac{y0\cdot(x - x1)\cdot((x - x2))}{(x0 - x1)\cdot(x0 - x2)} + \frac{y1\cdot(x - x0)\cdot((x - x2))}{(x1 - x0)\cdot(x1 - x2)} + \frac{y2\cdot(x - x0)\cdot((x - x1))}{(x2 - x0)\cdot(x2 - x1)}\right]$$



$x0 := 2 \quad x1 := 3 \quad x2 := 5 \quad y0 := 4 \quad y1 := 1 \quad y2 := 7$

$$L(x) := \left[\frac{4\cdot(x-3)\cdot((x-5))}{(2-3)\cdot(2-5)} + \frac{1\cdot(x-2)\cdot((x-5))}{(3-2)\cdot(3-5)} + \frac{7\cdot(x-2)\cdot((x-3))}{(5-2)\cdot(5-3)}\right]$$

$$2\cdot x^2 - 13\cdot x + 22$$

Technology «the Questionnaire for a feedback»

That I have remembered on employment	That I have understood, what have understood	That it was pleasant to me, has caused interest

Control questions:

1. In what feature of approach таблично the set function by a method of interpolation?
2. How existence and uniqueness interpolation a multinomial is proved?
3. How interpolation a multinomial degree is connected with quantity of knots of interpolation?
4. How are under construction interpolation multinomials of Lagrange and Newton?
5. In what feature of these two ways of interpolation?
6. How the estimation of an error of a method of interpolation is made by a multinomial of Lagrange?
7. How the method interpolation for specification of tables of functions is used?
8. In what difference between the first and the second interpolation Newton's formulas?

Practical work № 3

Calculation of integrals by the approached methods

(4 hours)

The purpose of educational employment: preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a practical material to watch mastering at pupils of this knowledge, to form and develop at them skills.

The plan:

1. Method of trapezes and Simpson
2. Methods of rectangles
3. The quadrature formula of Gauss

1. A method of trapezes and Simpson

Формулы, используемые для приближенного вычисления однократных интегралов, называют квадратурными формулами. Простой прием построения квадратурных формул состоит в том, что подынтегральная функция $f(x)$ заменяется на отрезке [a,b] интерполяsионным многочленом, например, многочленом Лагранжа $L_n(x)$; для интеграла имеем приближенное равенство (4.1). Предполагается, что отрезок [a,b] разбит на n частей точками (узлами) x_i, наличие которых подразумевается при построении многочлена $L_n(x)$. Для равноотстоящих узлов

The formulas used for approached calculation of unitary integrals, name квадратурными formulas. Simple reception of construction quadrature formulas consists that subintegral function $f(x)$ is replaced on a piece [a,b] interpolation with a multinomial, for example, a multinomial of Lagrange $L_n(x)$; for integral it is had the approached equality (4.1). It is supposed that the piece [a,b] is broken on n parts by points (knots) x_i which presence is meant at construction of multinomial $L_n(x)$. For equidistant knots $x_i = x_0 + ih,\ h = \frac{b-a}{n},\ x_0 = a,\ x_n = b.$

$$\int_a^b f(x)dx \approx \int_a^b L_n(x)\,dx \qquad (4.1)$$

At certain assumptions we receive the formula of trapezes

$$\int_a^b f(x)dx \approx h\left(\frac{y_0+y_n}{2} + y_1 + y_2 + \ldots + y_{n-1}\right), \qquad (4.2)$$

Where y_i values –of function in interpolation knots.

We have the following estimation of an error of a method of integration under the formula of trapezes (4.2):

$$|R_n| \le M\frac{|b-a|\cdot h^2}{12},\ \text{где}\ M = \max\left|f^{(2)}(x)\right|,\ x \in [a,b]. \qquad (4.3)$$

In many cases of more exact there is Simpson's formula (the formula of parabolas):

$$\int_a^b f(x)dx \approx \frac{2h}{3}\left(\frac{y_0+y_{2m}}{2} + 2y_1 + y_2 + \ldots + 2y_{2m-1}\right). \qquad (4.4)$$

For Simpson's formula it is had the following estimation of an error:

$$|R_n| \le M\frac{|b-a|\cdot h^4}{180},\ \text{где}\ M = \max\left|f^{(4)}(x)\right|,\ x \in [a,b].$$

The task 1

To make the program of calculation of integral from the set function on a piece $[a,b]$ under the formula of trapezes with step h=0.1 and h=0.05. To compare results. To estimate accuracy under the formula (4.3). To compare results. The initial data for task performance undertakes from table 4.

The task 2

To make the program of calculation of integral from the set function on a piece $[a,b]$ under Simpson's formula a method of the repeated account with accuracy $\varepsilon = 10^{-6}$. The initial data for task performance undertakes from table 4.

To calculate integral in MathCAD from the set function on a piece [a, b] under the formula of trapezes and direct way.

$a := 0 \qquad b := 1 \qquad n := 10 \qquad h := \frac{(b-a)}{n}$

$i := 0 .. 10 \qquad x_0 := a \qquad x_i := x_0 + i\cdot h$

$y := 0.37\cdot e^{\sin(x)}$

$$s := h \cdot \left(\sum_{i=1}^{n-1} y_i + \frac{y_0 + y_n}{2} \right)$$

$$s = 0.604$$

$$\int_0^1 0.37 \, e^{\sin(x)} \, dx = 0.604$$

Таблиѕа 4

N	Функсия	a	b
1	$0.37e^{\sin x}$	0	1
2	$0.5x + x\ln x$	1	2
3	$(x+1.9)\sin(x/3)$	1	2
4	$\frac{1}{x}\ln(x+2)$	2	3
5	$\frac{3\cos x}{2x+1.7}$	0	1
6	$(2x+0.6)\cos(x/2)$	1	2
7	$2.6x^2 \ln x$	1.2	2.2
8	$(x^2+1)\sin(x-0.5)$	1	2
9	$x^2\cos(x/4)$	2	3
10	$\frac{\sin(0.2x-3)}{x^2+1}$	3	4

3. A method of rectangles

The elementary methods of numerical integration are methods of rectangles. In them subintegral function is replaced with a polynom of zero degree, that is a constant. Similar replacement is ambiguous as the constant can be chosen subintegral function equal to value in any point of an interval of integration. Depending on it methods of rectangles share on: methods of the left, right and average rectangles.

On a method of average rectangles the integral is equal to the sum of the areas of rectangles where the rectangle basis any small size (accuracy), and the height is defined on a point of intersection of the top basis of a rectangle which the function schedule should cross in the middle. Accordingly we receive the formula of the areas for a method of average rectangles:

$$S_b = \sum_a^b \frac{|f(x_1) + (fx_2)|}{2} \varepsilon \quad (5)$$

The formula of average rectangles with constant step:

$$\int_a^b f(x)dx \approx \frac{1}{2} h \sum_{i=0}^{n-1} f\left(x_i + \frac{h}{2}\right) \quad (6)$$

5. The quadrature formula of Gaussa

The methods described above use the fixed points of a piece (the ends and the middle) and have a low order of accuracy (0 – methods of the right and left rectangles, 1 – methods of average rectangles and trapezes, 3 – a method of parabolas (Simpson)). If we can choose points in which we calculate values of function *f (x)* it is possible to receive methods of higher order of accuracy at the same quantity of calculations of subintegral function. So for two (as in a method of trapezes) calculations of values of subintegral function, it is possible to receive a method any more 1st, and 3rd order of accuracy:

$$I \approx \frac{b-a}{2}\left(f\left(\frac{a+b}{2} - \frac{b-a}{2\sqrt{3}}\right) + f\left(\frac{a+b}{2} + \frac{b-a}{2\sqrt{3}}\right)\right).$$

Generally, using n points, it is possible to receive a method with accuracy order 2n-1. Values of knots of a method of Gaussa on n points are roots of a polynom of Lezhandra of degree n.

Values of knots of a method of Gaussa and their scales are resulted in directories of special functions. The method of Gaussa on five points is most known.

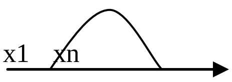
<u>Examples</u>

Example 1.

Application of the formula of average rectangles for the decision of problems of numerical integration (on calculation example $\int_1^2 (x^2+1)\sin(x-0.5)dx$).

The decision.

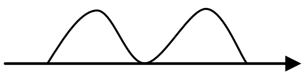
$$\int_1^2 (x^2+1)\sin(x-0.5)dx = h\sum_{i=0}^{n-1} f\left(x_i+\frac{h}{2}\right)$$



Let's calculate integral I1 under the formula of a method of average rectangles (6):

h1=1

I1=hf(x0+h/2)=((1.5)2+1)sin(1.5-0.5)=2.734



x0 x1 xn

Let's reduce a step twice and we will calculate integral I2 under the formula of a method of average rectangles (6):

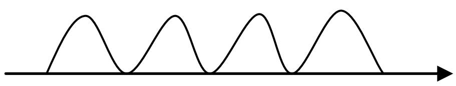
h2=1/2

I2= h(f(x0+h/2)+f(x1+ h/2))= (1/2) ((1.25)2+1)sin(1.25-0.5)+ ((1.75)2+1)sin(1.75-0.5))=2.8005

Let's calculate criterion for integrals I1 and I2, as I2≥1 the criterion is calculated under the formula:

$|(I_2-I_1)/I_2|=0.023746>\varepsilon$

The received criterion is not carried out, we calculate integral I3, reducing a step twice:

x0  x1 x2 x3 xn

h2=1/4

I3=h(f(x0+h/2)+f(x1+h/2)+f(x2+h/2)+f(x3+h/2))=(1/4)((1.125)2+1)sin(1.125-0.5)+(1.375)2+1)sin(1.375--0.5)+(1.625)2+1)sin(1.625-0.5)+ (1.875)2+1)sin(1.875-0.5))=2.814

Let's calculate criterion for integrals I2 and I3, as I3≥1 the criterion is calculated under the formula:

$|(I_3-I_2)/I_3|=0.004797<\varepsilon$

The received criterion is carried out, hence, we have calculated the set integral with demanded accuracy.

The answer: $\int_1^2 (x^2+1)\sin(x-0.5)dx =$ 2.814 with accuracy0.01.

Example 2. We Will calculate integral $\int_{0.5}^{3} \frac{2x^3}{x^4}dx$ method of Gaussa.

The decision.

$$I \approx \frac{b-a}{2}\left(f\left(\frac{a+b}{2}-\frac{b-a}{2\sqrt{3}}\right)+f\left(\frac{a+b}{2}+\frac{b-a}{2\sqrt{3}}\right)\right)$$

$$f(x)=\frac{2x^3}{x^4}.$$

$$f1(x)=f\left(\frac{a+b}{2}-\frac{b-a}{2\sqrt{3}}\right)=f\left(\frac{0.5+3}{2}-\frac{3-0.5}{2\sqrt{3}}\right)=f(1.029)=1.94.$$

$$f2(x)=f\left(\frac{a+b}{2}+\frac{b-a}{2\sqrt{3}}\right)=f\left(\frac{0.5+3}{2}+\frac{3-0.5}{2\sqrt{3}}\right)=f(2.47)=0.812$$

$$\int_{0.5}^{3}\frac{2x^3}{x^4}dx=\frac{3-0.5}{2}(1.94+0.812)\approx 3.584.$$

The answer: 3.584.

Example 3. We Will calculate integral $\int_{0.5}^{2.3}\pi\cdot\sin(\pi x)dx$ method of Gaussa.

The decision.

$$f(x)=\pi\cdot\sin(\pi x).$$

$$f1(x)=f\left(\frac{a+b}{2}-\frac{b-a}{2\sqrt{3}}\right)=f\left(\frac{0.5+2.3}{2}-\frac{2.3-0.5}{2\sqrt{3}}\right)=f(0.88)=-1.156.$$

$$f2(x)=f\left(\frac{a+b}{2}+\frac{b-a}{2\sqrt{3}}\right)=f\left(\frac{0.5+2.3}{2}+\frac{2.3-0.5}{2\sqrt{3}}\right)=f(1.92)=0.781$$

$$\int_{0.5}^{2.3}\pi\cdot\sin(\pi x)dx=\frac{2.3-0.5}{2}(-1.156+0.781)\approx -0.588.$$

The answer: - 0.588.

Exercise

Calculate the set integrals under formulas of rectangles, a trapeze and Simpson if the integration piece is broken on n=2 and n=4 equal parts. To estimate an error of result and сравныт the approached values of integral with the exact.

1\. $\int_0^1\frac{dx}{1+x^2}$ $\left(\text{З}=\frac{\pi}{4}\approx 0{,}785\right)$. 2. $\int_0^1\frac{dx}{1+x}$ $(\text{З}=\ln 2\approx 0{,}693)$.

3\. $\int_0^{\pi/4}\sin 4x\,dx$ $(\text{З}=0{,}5)$. 4. $\int_0^1\frac{dx}{\sqrt{1+x^2}}$ $(\text{З}=\ln(1+\sqrt{2})\approx 0{,}881)$.

5\. $\int_1^e\ln x\,dx$ $(\text{З}=1)$. 6. $\int_0^1\ln(x+1)dx$ $(\text{З}=2\ln 2-1\approx 0{,}386)$.

7\. $\int_0^{\pi/2}x\cos x\,dx$ $\left(\text{З}=\frac{\pi}{2}-1\approx 0{,}571\right)$. 8. $\int_0^1\frac{e^x dx}{1+e^{2x}}$ $\left(\text{З}=\operatorname{arctg} e-\frac{\pi}{4}\approx 0{,}433\right)$.

9\. $\int_0^{\pi}\cos^3 x\,dx$ $(\text{З}=0)$. 10. $\int_0^{\pi/4}\frac{dx}{\cos x}$ $(\text{З}=\ln(1+\sqrt{2})\approx 0{,}881)$.

11\. $\int_0^{\pi/2}\frac{dx}{1+\sin x}$ $(\text{З}=1)$. 12. $\int_0^1\operatorname{arctg}x\,dx$ $\left(\text{З}=\frac{1}{4}(\pi-2\ln 2)\approx 0{,}438\right)$.

13\. $\int_0^1\frac{dx}{1+e^x}$ $(\text{З}\approx 0{,}38)$. 14. $\int_0^{\sqrt{2}/2}\arcsin x\,dx$ $\left(\text{З}=\frac{\sqrt{2}}{8}(\pi+4)-1\approx 0{,}26\right)$.

15\. $\int_0^{\pi/4}\operatorname{tg}x\,dx$ $\left(\text{З}=\frac{1}{2}\ln 2\approx 0{,}346\right)$. 16. $\int_{\pi/4}^{\pi/2}\operatorname{ctg}x\,dx$ $\left(\text{З}=\frac{1}{2}\ln 2\approx 0{,}346\right)$.

17\. $\int_0^1 x\,e^x\,dx$ $(\text{З}=1)$. 18. $\int_0^1\sqrt{1+x}\,dx$ $\left(\text{З}=\frac{2}{3}(2\sqrt{2}-1)\approx 1{,}22\right)$.

Technology «the Questionnaire for a feedback»

That I have remembered on employment	That I have understood, what have understood	That it was pleasant to me, has caused interest

Control questions

1\. What advantages of the formula of parabolas in comparison with the formula of trapezes and a consequence of that are these advantages?
2\. Whether formulas (4.2) are true, (4.4) for is unequal straining knots?
3\. In what cases the approached formulas of trapezes and parabolas appear exact?
4\. How the step size influences accuracy of numerical integration?
5\. In what way it is possible to predict approximate size of a step for achievement of the set accuracy of integration?
6\. Whether it is possible to achieve unlimited reduction of an error of integration by consecutive reduction of a step?

Practical work № 4.
Approximation results of experiment with a method of the least square.
Creation non-linear empirical connection
(4 hours)

The purpose of educational employment: preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a practical material to watch mastering at pupils of this knowledge, to form and develop at them skills.

The plan:

1. Root-mean-square approach of functions
2. A method of the least squares

1. Root-mean-square approach of functions

Let dependence between variables *x* and *y* is set таблично (the skilled data is set). It is required to find function somewhat in the best way describing the data. One of ways of selection of such (approaching) function is the method of the least squares. The method consists in that the sum of squares of deviations of values of required function $\bar{y}_i = \bar{y}(x_i)$ and set таблично y_i was the least:

$$S(c) = (y_1 - \bar{y}_1)^2 + (y_2 - \bar{y}_2)^2 + \ldots + (y_n - \bar{y}_n)^2 \rightarrow \min \qquad (6.1)$$

Where *c a* vector –of parametres of required function.

2. A method of the least squares

To construct a method of the least squares two empirical formulas: linear and square-law.

In case of linear function *y=ax+b the* problem is reduced to a finding of parametres *a* and *b* from system of the linear equations

$$\begin{cases} M_{x^2} a + M_x b = M_{xy} \\ M_x a + b = M_y \end{cases}$$, Where

$$M_{x^2} = \frac{1}{n}\sum_{i=1}^{n} x_i^{\,2}, \quad M_x = \frac{1}{n}\sum_{i=1}^{n} x_i, \quad M_{xy} = \frac{1}{n}\sum_{i=1}^{n} x_i y_i, \quad M_y = \frac{1}{n}\sum_{i=1}^{n} y_i$$

and in case of square-law dependence $y = ax^2 + bx + c$ to a finding of parameters *a*, *b* and *c* from system of the equations:

$$\begin{cases} M_{x^4} a + M_{x^3} b + M_{x^2} c = M_{x^2 y} \\ M_{x^3} a + M_{x^2} b + M_x c = M_{xy} \\ M_{x^2} a + M_x b + c = M_y \end{cases}$$, Where

$$M_{x^4} = \frac{1}{n}\sum_{i=1}^{n} x_i^{\,4}, \quad M_{x^3} = \frac{1}{n}\sum_{i=1}^{n} x_i^{\,3}, \quad M_{x^2 y} = \frac{1}{n}\sum_{i=1}^{n} x_i^{\,2} y_i$$

To choose from two functions the most suitable. For this purpose to make the table for calculation of the sum of squares of evasion under the formula (6.1). Initial given to take from table 6.

The task 2

To make the program for a finding of approaching functions of the set type with a conclusion of values of their parametres and the sums of squares of evasion corresponding to them. To choose as approaching functions the following: $y = ax + b$, $y = ax^m$, $y = ae^{mx}$. To spend linearization. To define for what kind of function the sum of squares of evasion is the least.

Initial data is placed in table 6.

Approximate fragment of performance of laboratory work

(George E. Forsyth and Michael A. Malcolm and Cleve B. Moler. Computer Methods for Mathematical Computations. Prentice-Hall, Inc., 1977.)

$i := 1..10$

$x_1 := 0.5$ $\quad y_1 := 1.8$

$x_2 := 0.1$ $\quad y_2 := 1.1$

$x_3 := 0.4$ $\quad y_3 := 1.8$

$x_4 := 0.2$ $\quad y_4 := 1.4$

$x_5 := 0.6$ $\quad y_5 := 2.1$

$x_6 := 0.3$ $\quad y_6 := 1.8$

$x_7 := 0.4$ $\quad y_7 := 1.6$

$x_8 := 0.7$ $\quad y_8 := 2.2$

$x_9 := 0.3$ $\quad y_9 := 1.5$

$x_{10} := 0.8$ $\quad y_{10} := 2.3$

$$mx2 := 1\frac{\left[\sum_{i=1}^{10} \left(\left|x_i\right|\right)^2\right]}{10} \qquad mx := 1\cdot\frac{\left(\sum_{i=1}^{10} x_i\right)}{10} \qquad mxy := 1\frac{\left(\sum_{i=1}^{10} x_i\cdot y_i\right)}{10} \qquad my := 1\frac{\left(\sum_{i=1}^{10} y_i\right)}{10}$$

$mx2 = 0.229$ $\quad mx = 0.43$ $\quad mxy = 0.828$ $\quad my = 1.76$

Given

$mx2a + mx\cdot b = mxy$

$mx\cdot a + b = my$

Find(a,b)$\rightarrow$

T

№ \ *i*		1	2	3	4	5	6	7	8	9	10
1	*x*	0.5	0.1	0.4	0.2	0.6	0.3	0.4	0.7	0.3	0.8
	y	1.8	1.1	1.8	1.4	2.1	1.8	1.6	2.2	1.5	2.3
2	*x*	1.7	1.5	3.7	1.1	6.2	0.3	6.5	3.6	3.8	5.9
	y	1.5	1.4	1.6	1.3	2.1	1.1	2.2	1.8	1.7	2.3
3	*x*	1.7	1.1	1.6	1.2	1.9	1.5	1.8	1.4	1.3	1.0
	y	6.7	5.6	6.7	6.1	7.4	6.9	7.9	5.9	5.6	5.3
4	*x*	1.3	1.2	1.5	1.4	1.9	1.1	2.0	1.6	1.7	1.8
	y	5.5	5.9	6.3	5.8	7.4	5.4	7.6	6.9	6.6	7.5
5	*x*	2.3	1.4	1.0	1.9	1.5	1.8	2.1	1.6	1.7	1.3
	y	5.3	3.9	2.9	5.0	4.0	4.9	5.1	4.5	4.1	3.7
6	*x*	1.8	2.6	2.3	1.3	2.0	2.1	1.1	1.9	1.6	1.5
	y	4.4	6.4	5.3	3.7	4.9	5.6	3.0	5.0	4.3	3.7
7	*x*	1.9	2.1	2.0	2.9	3.0	2.6	2.5	2.7	2.2	2.8
	y	6.6	7.6	6.7	9.2	9.4	7.8	8.4	8.0	7.9	8.7
8	*x*	2.0	1.4	1.0	1.7	1.3	1.6	1.9	1.5	1.2	2.1
	y	7.5	6.1	4.8	7.4	5.7	7.0	7.1	6.8	6.0	8.9
9	*x*	2.0	1.2	1.8	1.9	1.1	1.7	1.6	1.4	1.5	1.3
	y	7.5	5.9	7.0	8.0	5.0	7.4	6.4	6.6	6.3	5.7
10	*x*	1.9	1.1	1.4	2.3	1.7	2.1	1.6	1.5	1.0	1.2
	y	4.7	3.4	3.8	5.2	4.6	5.5	3.9	3.9	3.2	3.5

Technology «the Questionnaire for a feedback»

That I have remembered on employment	That I have understood, what have understood	That it was pleasant to me, has caused interest

Control questions:

1. In what an approach essence таблично the set function on a method of the least squares?
2. Than this method differs from an interpolation method?
3. How the problem of construction of approaching functions in the form of various elementary functions to a case of linear function is reduced?
4. Whether there can be a sum of squares of evasion for any approaching functions equal to zero?
5. What elementary functions are used as approaching functions?
6. How to find parametres for linear and square-law dependence, using a method of the least squares?

Practical work № 5.
The geometrical decision of a problem of linear programming

(2 hours)

The purpose of educational employment: preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a practical material to watch mastering at pupils of this knowledge, to form and develop at them skills.

The plan:

1. Geometrical interpretation of a problem of linear programming
2. Using geometrical interpretation, find decisions of problems

1. Geometrical interpretation of a problem of linear programming

1.29. To find a maximum and a minimum of function $F=x_1+x_2$ under conditions

$$\begin{cases} 2x_1+4x_2\leqslant 16, \\ -4x_1+2x_2\leqslant 8, \\ x_1+3x_2\geqslant 9, \end{cases}$$
$$x_1,\ x_2\geqslant 0.$$

T h e d e c i s i o n . We will construct a polygon of decisions. For this purpose in inequalities of system of restrictions and conditions nonnegativity variables signs on inequalities we will replace with signs on exact equalities:

$$\begin{cases} 2x_1+4x_2=16, & \text{(I)} \\ 4x_1+2x_2=\ 8, & \text{(II)} \\ x_1+3x_2=\ 9, & \text{(III)} \\ x_1=0, & \text{(IV)} \\ x_2=0. & \text{(V)} \end{cases}$$

Having constructed the received straight lines, will find corresponding semiplanes and their crossing (fig. 1.6).

Apparently from fig. 1.6, a polygon of decisions of a problem is triangle ABC. Co-ordinates of points of this triangle satisfy to a condition nonnegativity and to inequalities of system of restrictions of a problem. Hence, the problem will be solved, if among points of triangle ABC to find such in which function $F=x_1+x_2$ accepts the maximum and minimum values. For a finding of these points we will construct a straight line $x_1+x_2=4$ (number 4 is taken any) and a vector C = (1 ; 1) .

Moving the given straight line in parallel to itself in a direction of a vector With, we see that its last general point with a polygon of decisions of a problem is point C. Hence, in this point function F accepts the maximum value. As with - a point of intersection of straight lines I and II its co-ordinates satisfy to the equations of these straight lines:

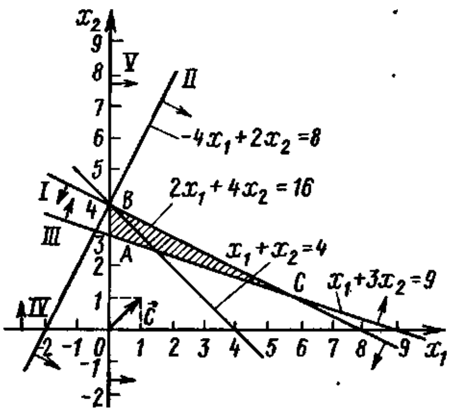


Fig. 1.6

$$\begin{cases} 2x_1+4x_2=16, \\ x_1+3x_2=\ 9. \end{cases}$$

Having solved this system of the equations, we will receive $x_1^*=6,\ x_2^*=1$. Thus, the maximum value of function $F_{max}=7$.

For a finding of the minimum value of criterion function of a problem it is moved a straight line $x_1+x_2=4$ in a direction opposite to a direction of vector C = (1 ; 1) . In this case, apparently from fig. 1.6, last general point of a straight line with a polygon of decisions of a problem is A.Sledovatelno's point, in this point function F accepts the minimum value. For definition of co-ordinates of a point And we solve system of the equations

$$\begin{cases} x_1+3x_2=9, \\ x_1=0, \end{cases}$$

whence $x_1^*=0,\ x_2^*=3$. Substituting the found values of variables in criterion function, we will receive $F_{min} = 3$.

1.30. To find the maximum value of function $F = -16x_1 - x_2 + x_3 + 5x_4 + 5x_5$ under conditions

$$\begin{cases} 2x_1+\ x_2+x_3=10, \\ -2x_1+3x_2+x_4=\ 6, \\ 2x_1+4x_2-x_5=\ 8, \end{cases}$$

$$x_1,\ x_2,\ x_3,\ x_4,\ x_5 \geqslant 0.$$

T h e d e c i s i o n . Unlike considered above problems in an initial problem of restriction are set in the form of the equations. Thus number of unknown persons equally five. Therefore the given problem should be reduced to a problem in which the number of unknown persons would be equal to two. In the case under consideration it can be made by transition from the initial problem which have been written down in the form of basic, to the Problem which has been written down in the form of standard.

It has been above shown that the initial problem is written down, in the form of the basic for a problem consisting in a finding of the maximum value of function $F = 2x_1 + 3x_2$ under conditions

$$\begin{cases} 2x_1+\ x_2\leqslant 10, \\ -2x_1+3x_2\leqslant\ 6, \\ 2x_1+4x_2\geqslant\ 8, \end{cases}$$

$$x_1,\ x_2,\ x_3,\ x_4,\ x_5 \geqslant 0.$$

From criterion function of an initial problem variables x_3, x_4, x_5 are excluded by means of substitution of their values from the corresponding equations of system of restrictions.

Let's construct a polygon of decisions of the received problem (fig. 1.7). Apparently from fig. 1.7, the maximum value problem criterion function accepts in a point from crossing of straight lines I and II. Along each of boundary straight lines value of one of the variables, excluded at transition to corresponding inequality, is equal to zero. Therefore in each of tops of the received polygon of decisions of last problem at least two variables of an initial problem accept zero values. So, in

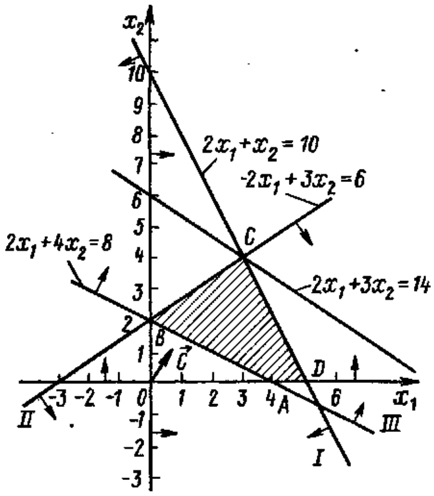


Рис. 1.7

To point C it is had $x_3=0$ and $x_4=0$. Substituting these values in the first and second equations of system of restrictions of an initial problem, we receive system of two equations

$$\begin{cases} 2x_1+\ x_2=10, \\ -2x_1+3x_2=\ 6, \end{cases}$$

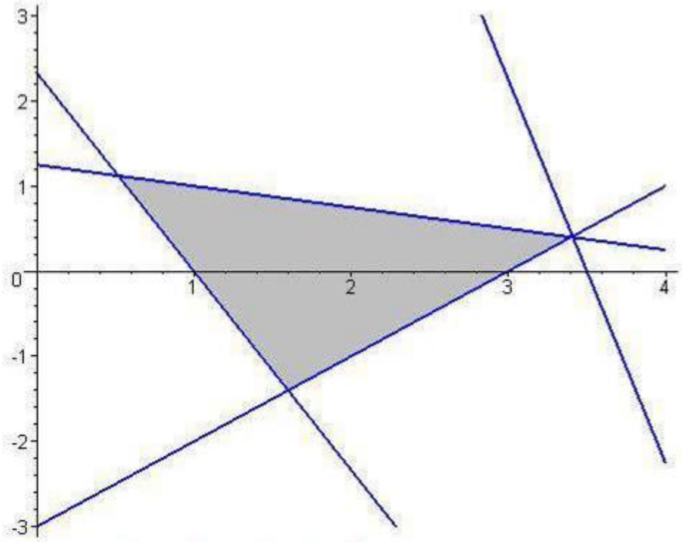
Solving which it is found $x_1^*=3,\ x_2^*=4$.

Substituting the found values x_1 and x_2 in the third equation of system of restrictions of an initial problem, we define value of a variable x_5, equal 14.

Hence, the optimum plan of a considered problem is $X^* = (3; 4; 0; 0; 14)$. At this plan value of criterion function is $F_{max} = 18$.

Решение задачи в Maple

```
restart;
plots[inequal]({x1+4*x2<=5,x1-x2<=3,7*x1+3*x2>=7, 9*x1+2*x2=31.5}, x1=0..4, x2=-3..3,
    optionsfeasible=(color=grey),
    optionsopen=(color=blue,thickness=2),
    optionsclosed=(color=blue, thickness=2),
    optionsexcluded=(color=white) );
```




```
simplex[maximize](9*x1+2*x2,{x1+4*x2<=5,x1-x2<=3,7*x1+3*x2>=7});
```

$$\left\{x1 = \frac{17}{5}, x2 = \frac{2}{5}\right\}$$

2. Using geometrical interpretation, find decisions of problems

1. **1.32.** $F = x_1 + x_2 \to \max;$

$$\begin{cases} x_1 + 2x_2 \leqslant 14, \\ -5x_1 + 3x_2 \leqslant 15, \\ 4x_1 + 6x_2 \geqslant 24, \end{cases}$$
$x_1,\ x_2 \geqslant 0.$

2. **1.33.** $F = x_1 + 2x_2 \to \max;$

$$\begin{cases} 4x_1 - 2x_2 \leqslant 12, \\ -x_1 + 3x_2 \leqslant 6, \\ 2x_1 + 4x_2 \geqslant 16, \end{cases}$$
$x_1,\ x_2 \geqslant 0.$

3. **1.34.** $F = -2x_1 + x_2 \to \min;$

$$\begin{cases} 3x_1 - 2x_2 \leqslant 12, \\ -x_1 + 2x_2 \leqslant 8, \\ 2x_1 + 3x_2 \geqslant 6, \end{cases}$$
$x_1,\ x_2 \geqslant 0.$

4. **1.35.** $F = -x_1 + 4x_2 + 2x_4 - x_5 \to \max;$

$$\begin{cases} x_1 - 5x_2 + x_3 = 5, \\ -x_1 + x_2 + x_4 = 4, \\ x_1 + x_2 + x_5 = 8. \end{cases}$$
$x_1,\ x_2,\ \ldots,\ x_5 \geqslant 0.$

5. **1.36.** $F = -5x_1 + x_2 - x_3 \to \max;$

$$\begin{cases} 3x_1 - x_2 - x_3 = 4, \\ x_1 - x_2 + x_3 - x_4 = 1, \\ 2x_1 + x_2 + 2x_3 + x_5 = 7. \end{cases}$$
$x_1,\ x_2,\ \ldots,\ x_5 \geqslant 0.$

6. For manufacture of tables and cases the furniture factory uses necessary resources. Norms of expenses of resources on one product of the given kind, profit on realisation of one product and total of available resources of each kind are resulted in the following table:

Resources	Norms of expenditures of resources on one product		The General an amount of resources
	Table	The Case	
Wood (m^3): I aspect	0,2	0,1	40
II aspect	0,1	0,3	60
Labour input (person-hour)	1,2	1,5	371,4
Profit on realisation of one product (rouble.)	6	8	

To define, how many tables and cases the factory should produce, that the profit on their realisation was maximum.

7. For manufacture of two kinds of products A and B the turning, milling and grinding equipment is used. Norms of expenses of time for each of equipment types on one product of the given kind are resulted in the table. In it the general fund of working hours of each of equipment types, and also profit on realisation of one product is specified.

Type the equipment	Expenditures of time (machine tool-hour) on working off of one product		The General fund of useful working hours of the equipment (hour)
	A	B	
The milling	10	8	168
The turning	5	10	180
The grinding	6	12	144
Profit on realisation of one product (roub.)	14	18	

To find the plan of release of products A and B, providing the maximum profit on their realisation.

8. At furniture factory it is necessary to cut out preparations of three kinds from plywood standard sheets in the quantities accordingly equal of 24, 31 and 18 pieces Each sheet of plywood can be cut on for
Cooking by two ways. The quantity of received preparations at the given way раскроя is resulted in the table. In it the size of a waste which turn out at the given way раскроя one sheet of plywood is specified.

Aspect preform	Amount of preforms (piece) in open on a mode	
	1	2
I	2	6
II	5	4
III	2	3
Magnitude of a waste (sm^3)	12	16

To define, how many sheets of plywood and on what way follow раскроит so that has been received not less the necessary quantity of preparations at the minimum waste.

9-10. On a fur farm silver foxes and polar foxes can be grown up. For maintenance of normal conditions of their cultivation it is used three kinds of forages. The quantity of a forage of each kind which foxes and polar foxes should receive daily, is resulted in the table. In it are specified total of a forage of each kind which can be used a fur farm, and profit on realisation of one skin of a fox and a polar fox.

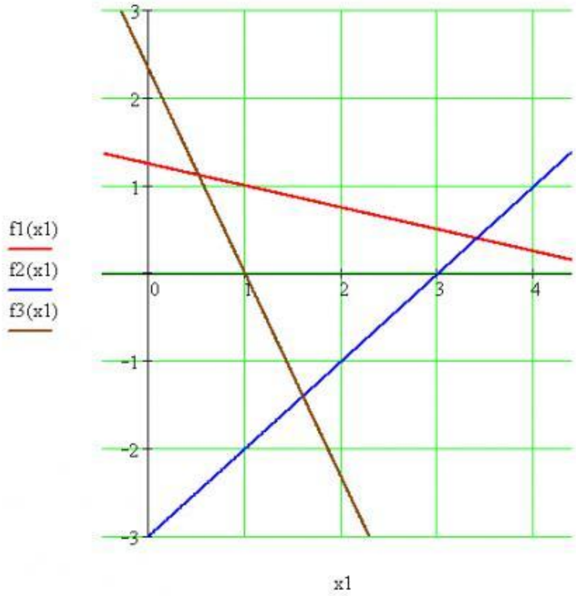
Feed kind	Quantity of units of a feed which daily should receive		Feed Total
	fox	A polar fox	
I	2	3	180
II	4	1	240
III	6	7	426
Profit on realisation of one glass-paper (roub.)	16	12	

To define, how many foxes and polar foxes should be grown up on a fur farm that the profit on realisation of their skins was maximum.

http://www.cyberforum.ru/mathcad/thread361265.html

$f1(x1) := x1 + 4x2 = 5 \text{ solve}, x2 \rightarrow \frac{-1}{4} \cdot x1 + \frac{5}{4}$ $\quad f2(x1) := x1 - x2 = 3 \text{ solve}, x2 \rightarrow x1 - 3$ $\quad f3(x1) := 7x1 + 3x2 = 7 \text{ solve}, x2 \rightarrow \frac{-7}{3} \cdot x1 + \frac{7}{3}$

$L(x1, x2) := 9x1 + 2x2$ $\quad c1 := \frac{d}{dx1} L(x1, x2) \rightarrow 9$ $\quad c2 := \frac{d}{dx2} L(x1, x2) \rightarrow 2$ $\quad Grad(x1) := \frac{c2}{c1} \cdot x1$



Given

$x2 = f1(x1)$

$x2 = f3(x1)$

$$\begin{pmatrix} x11 \\ x21 \end{pmatrix} := Find(x1, x2) \rightarrow \begin{pmatrix} \frac{13}{25} \\ \frac{28}{25} \end{pmatrix}$$

Given

$x2 = f1(x1)$

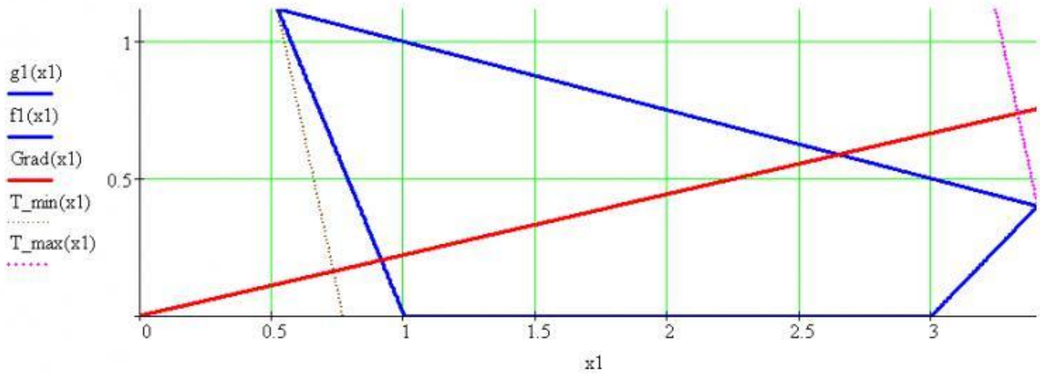
$x2 = f2(x1)$

$$\begin{pmatrix} x12 \\ x22 \end{pmatrix} := Find(x1, x2) \rightarrow \begin{pmatrix} \frac{17}{5} \\ \frac{2}{5} \end{pmatrix}$$

$g1(x1) := \max(f3(x1), 0, f2(x1))$

$$T\_min(x1) := \begin{pmatrix} c1 \\ c2 \end{pmatrix} \cdot \begin{pmatrix} x1 - x11 \\ x2 - x21 \end{pmatrix} \text{ solve}, x2 \rightarrow \frac{-9}{2} \cdot x1 + \frac{173}{50}$$

$$T\_max(x1) := \begin{pmatrix} c1 \\ c2 \end{pmatrix} \cdot \begin{pmatrix} x1 - x12 \\ x2 - x22 \end{pmatrix} \text{ solve}, x2 \rightarrow \frac{-9}{2} \cdot x1 + \frac{157}{10}$$



$x1\_min := x11 \rightarrow \frac{13}{25}$ $\quad x2\_min := x21 \rightarrow \frac{28}{25}$ $\quad L\_min := L(x1\_min, x2\_min) \rightarrow \frac{173}{25}$

$x1\_max := x12 \rightarrow \frac{17}{5}$ $\quad x2\_max := x22 \rightarrow \frac{2}{5}$ $\quad L\_max := L(x1\_max, x2\_max) \rightarrow \frac{157}{5}$

Technology «the Questionnaire for a feedback»

That I have remembered on employment	That I have understood, what have understood	That it was pleasant to me, has caused interest

Control questions:

1. Geometrical interpretation of a problem of linear programming
2. Using geometrical interpretation, find decisions of problems

LABORATORY MATERIALS

Laboratory work №1
The numerical decision of the algebraic and transcendental equations by iterative methods Chord and Newton
(4 hours)

The purpose of educational employment: preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a laboratory material to watch mastering at pupils of this knowledge, to form and develop at them skills.

Plan:

1. Iteration method
2. Method of Chords
3. Method half divisions

Let's consider the equation

$$f(x) = 0 \tag{1.1}$$

Where $f(x)$ defined and continuous on some final or infinite interval $a < x < b$.

Any value x^* turning function $f(x)$ in zero $f(x^*) \equiv 0$, is called as a root of the equation (1.1), and the way of a finding of this value x^* and is the decision of the equation (1.1).

To find roots of the equation of a kind (1.1) precisely it is possible only in rare instances. Besides, often the equation contains the factors known only approximately and therefore, the problem about exact definition of roots of the equation loses meaning. Methods of the numerical decision of the equations of the kind (1.1) are developed, allowing to find the approached values of roots of this equation.

Thus it is necessary to solve two problems:

1) branch of roots, i.e. Search enough small areas, in each of which are concluded only one root of the equation;

2) calculation of roots with the set accuracy.

Let's take advantage of known result of the mathematical analysis: if continuous function accepts on the ends of some interval of value of different signs an interval contains at least one root of the equation.

For allocation of the areas containing one root, it is possible to use, for example, graphic in the way, or moving along a range of definition with some step, to check on the ends of intervals a condition of change of a sign on function.

For the decision of the second problem exists numerous methods from which we will consider four: a method **of iterations**, a method half **divisions**, a method **of chords**, a method **of tangents**.

The task 1

To make branch of roots: graphically and under the program (accuracy $\varepsilon = 10^{-1}$). Individual tasks are resulted in table 1.

The task 2

1. To spend specification of roots by a method half divisions.

As initial approach we will choose $c = (a+b)/2$, then we investigate function on the ends of pieces $[a,c]$ and $[c,b]$. That piece at which value of function on the ends has opposite signs gets out. Process proceeds until the condition $|b-a| < \varepsilon$ will be satisfied. Accuracy ε to accept the equal 10^{-3}.

2. To make specification of roots by a method of simple iteration.

Let roots are separated and $[a,b]$ contains a unique root. The equation (1.1) we will lead to an iterative kind:

$$x = \varphi(x) \tag{1.2}$$

where function $\varphi(x)$ is differentiated on $[a,b]$ and for any. $x \in [a,b] \, |\varphi'(x)| < 1$. Function $\varphi(x)$ can be picked up in a kind

$$\varphi(x) = x + kf(x), \tag{1.3}$$

Where k is from a condition $|\varphi'(k,x)| = |1 + kf'(x)| < 1$, for $\forall x \in [a,b]$.

Last condition guarantees convergence of iterative sequence $x_1, x_2, \cdots x_{n-1}, x_n \cdots$ to a root ζ. As a condition of the termination of the account we will consider inequality performance

$$|x_n - x_{n-1}| < \frac{\varepsilon(1-q)}{q}; \; q = max|\varphi^{/}(x)| \qquad (1.4)$$

3. To make specification of roots by a method of chords or tangents (X, K in table 1) with the set accuracy $\varepsilon = 10^{-4}$.

The settlement formula for a method of chords:

$$x_{n+1} = \frac{x_0 f(x_n) - x_n f(x_0)}{(f(x_n) - f(x_0))},$$

For a method of tangents:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)},$$

Value x_0 for a method of chords and an index point for a method of tangents gets out of a condition of performance of an inequality $f(x_0) f^{//}(x_0) > 0$.

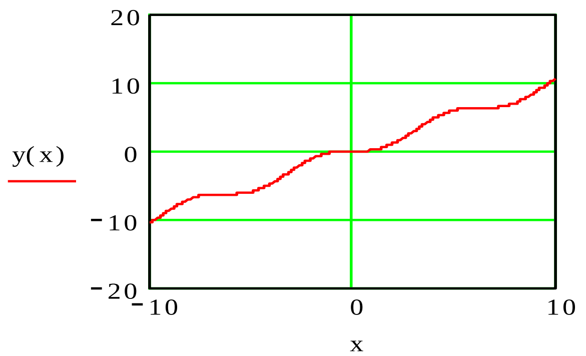
As a result of calculations under these formulas the sequence of the approached values of a root can be received $x_1, x_2, \cdots x_{n-1}, x_n \cdots$. Process of calculations comes to an end at condition performance $|x_n - x_{n-1}| < \varepsilon$ ($\varepsilon = 10^{-5}$). In each case to print quantity of the iterations necessary for achievement of set accuracy.

APPROXIMATE VARIANT OF PERFORMANCE OF WORK ON MATHCAD

1. Definition, construction of tables of values and schedules of functions and branch of roots of the equation *y=x-sinx-0,25*.

We separate roots graphically.
We calculate values of argument and function.

$$y(x) := x - \sin(x) - 0.25$$



$i := 0..10$ $\qquad$ $x_i := -5 + i$ $\qquad$ $F_i := y(x_i)$

We type i, x_i F_i. More low, x= and nearby we click the mouse, we type F=, also nearby we click the mouse.

x =

	0
0	-5
1	-4
2	-3
3	-2
4	-1
5	0
6	1
7	2
8	3
9	4
10	5

F =

	0
0	-6.209
1	-5.007
2	-3.109
3	-1.341
4	-0.409
5	-0.25
6	-0.091
7	0.841
8	2.609
9	4.507
10	5.709

Given

$$x - \sin(x) - 0.25 = 0$$

$$\text{Find}(x) \rightarrow 1.17122965250166599$$

2. The decision with use of operators ***given, find***.
3. The symbolical decision.

$$x - \sin(x) - 0.25 \text{ solve}, x \rightarrow 1.17122965250166599$$

4. At the left the decision a method of iterations, in the middle a method of tangents, on the right a method of chords.

$i := 0..10$ $\quad$ $i := 0..10$ $\quad$ $i := 0..10$

$x_0 := 1$ $\quad$ $x_0 := 1$ $\quad$ $x_0 := 1$

$$x_{i+1} := \sin(x_i) + 0.25$$

$$x_{i+1} := x_i - \frac{\left[x_i - (\sin(x_i) + 0.25)\right]}{1 + \cos(x_i)}$$

$$x_{i+1} := \frac{\left[x_0 \cdot (x_i - \sin(x_i) - 0.25) - x_i \cdot (x_0 - \sin(x_0) - 0.25)\right]}{(x_i - \sin(x_i) - 0.25) - (x_0 - \sin(x_0) - 0.25)}$$

x =

	0
0	1
1	1.091471
2	1.137306
3	1.157505
4	1.165804
5	1.169105
6	1.170401
7	1.170907
8	1.171104
9	1.171181
10	1.171211
11	1.171222

x =

	0
0	1
1	1.059385
2	1.101462
3	1.129285
4	1.146676
5	1.157108
6	1.163197
7	1.16669
8	1.168674
9	1.169794
10	1.170424
11	1.170778

x =

	0
0	1
1	0
2	1.576998
3	1.126117
4	1.177917
5	1.170273
6	1.171367
7	1.17121
8	1.171232
9	1.171229
10	1.17123
11	1.17123

Table 1

N	Method	The equation
1	K	$x + x\ln(x+0.5) - 0.5 = 0$
2	K	$x2^x - 1 = 0$
3	X	$x^3 - 2x^2 + x - 3 = 0$
4	K	$x^3 + 12x - 2 = 0$
5	X	$5x - 8\ln(x) - 8 = 0$
6	K	$x^4 + 0.5x^3 - 4x^2 - 3x - 0.5 = 0$
7	X	$x - \sin(x) - 0.25 = 0$
8	K	$x^3 - 6x^2 + 20 = 0$
9	X	$5x^3 + 10x^2 + 5x - 1 = 0$
10	K	$0.1x^2 - x\ln(x) = 0$

Newton's methods

Example 1. To solve the cubic equation $x^3 + x - 10 = 0$ with relative accuracy ε =0,001 **method of tangents** of Newton-Rafsona.

The decision. In this case $F(x) = x^3 + x - 10$. Hence, $F'(x) = 3x^2 + 1$. As zero approach we will accept x_0=3

(Exact value of a root ξ=2). Then under the formula($x_{n+1} = x_n - \frac{F(x_n)}{F'(x_n)}$) received:

$$x_1 = 3 - \frac{20}{28} = 2.285714,$$

$$x_2 = 2.285714 - \frac{4.227400}{16.673465} = 2.032173,$$

Let's check up, whether the set relative accuracy is reached ε:

$$\left|\frac{x_2 - x_1}{x_1}\right| = \left|\frac{2.032173 - 2.285714}{2.285714}\right| \approx 0.110924 > \varepsilon = 0.001$$

Continue iterations:

$$x_3 = 2.032173 - \frac{0.424493}{13.389181} = 2.000469$$

Again we will check up, whether the set relative accuracy is reached ε:

$$\left|\frac{x_3 - x_2}{x_2}\right| = \left|\frac{2.000469 - 2.032173}{2.032173}\right| \approx 0.015601 > \varepsilon = 0.001$$

The following iteration to within 6 decimal signs gives almost exact value of a root:

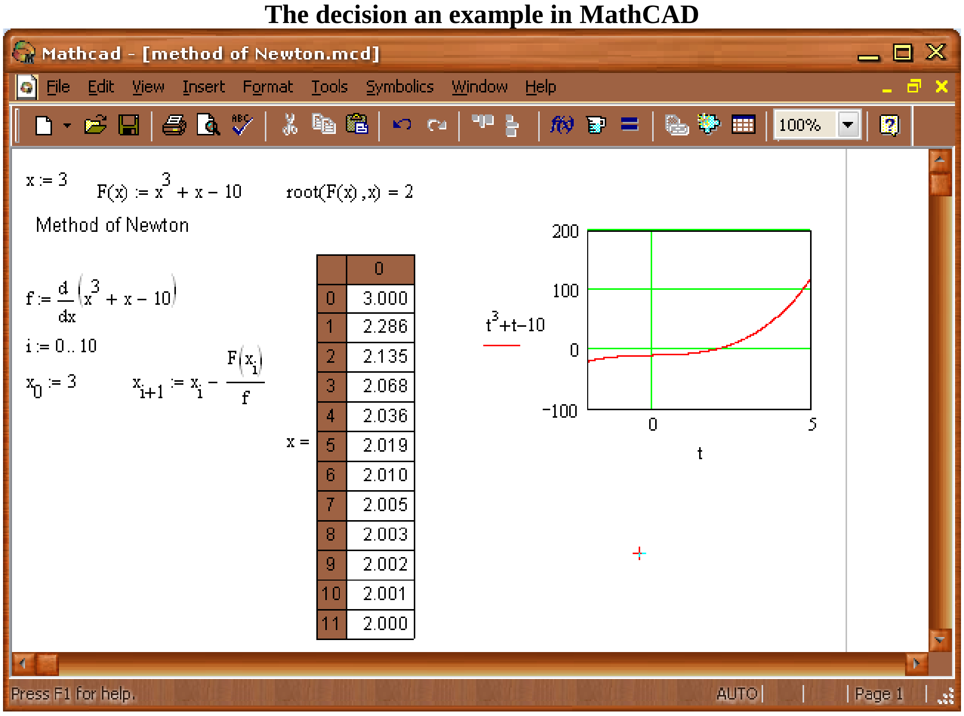
$$x_4 = 2.000469 - \frac{0.006098}{13.005629} = 2.0000001$$

However, here again it is necessary to check up, whether the set relative accuracy is reached ε:

$$\left|\frac{x_4 - x_3}{x_3}\right| = \left|\frac{2.0000001 - 2.000469}{2.000469}\right| \approx 0.000234 < \varepsilon = 0.001$$

The found root of the equation is equal 2.0000001. Thus, computing process has converged for 4 iterations, and we have received a required root with the set relative accuracy ε.

The decision an example in MathCAD



<u>Variants for laboratory works 1,2</u>

Solve the following the equation with accuracy 0,001

1) $x^3 - 9x^2 + 31x + 37 = 0$
2) $\ln x + x + 13 = 0$
3) 1.5cos(x-0.6)+x-2.047=0, [0;π/2]
4) 2x-1.3^x = 0, [0;10];
5) $\sqrt{\frac{\pi}{2}}e^{0,6x} + \frac{x}{0,36 + x^3} = 0,[-1;1]$
6) x^2+4sinx-1.6280819=0, [0;1];
7) x+lgx=0,5;
8) x^3+0,4x^2+0,6x-1,6=0;
9) $x^3 - 9x^2 + 31x + 30 = 0$
10) $\ln x + x - 13 = 0$
11) 1.5cos(x-0.6)+x+2.047=0, [0;π/2]
12) 3x-1.3^x = 0, [0;10];
13) $x + \frac{x}{0,36 + x^3} = 0,[-1;1]$
14) x^2+4cosx-1.628=0, [0;1];
15) x+lnx=0,5;
16) x^3+0,4x^2+0,6x-1,6=0;

Solve the equation with Newton's method

1. $x^3 + 2x^2 + 2 = 0$
2. $x^3 - 2x + 2 = 0$
3. $x^3 + x - 3 = 0$

14. $x^3 - 3x^2 + 9x - 10 = 0$
15. $x^3 + 3x - 1 = 0$
16. $x^3 + 0,4x^2 + 0,6x - 1,6 = 0$

4. $x^3 - 0{,}2x^2 + 0{,}4x - 1{,}4 = 0$
5. $x^3 + 3x^2 + 12x + 3 = 0$
6. $x^3 - 0{,}1x^2 + +0{,}4x + 1{,}2 = 0$
7. $x^3 - 0{,}2x^2 + 0{,}5x - 1{,}4 = 0$
8. $x^3 - 3x^2 + 12x - 12 = 0$
9. $x^3 + 4x - 6 = 0$
10. $x^3 + 3x^2 + 6x - 1 = 0$
11. $x^3 - 3x^2 + 6x - 2 = 0$
12. $x^3 - 3x^2 + 12x - 9 = 0$
13. $x^3 + 3x + 1 = 0$
17. $x^3 - 0{,}1x^2 + 0{,}4x - 1{,}4 = 0$
18. $x^3 - 0{,}2x^2 + 0{,}5x - 1 = 0$
19. $x^3 - 3x^2 + 6x - 5 = 0$
20. $x^3 + 2x + 4 = 0$
21. $x^3 + 0{,}2x^2 + 0{,}5x + 0{,}8 = 0$
22. $x^3 + 0{,}1x^2 + 0{,}4x - 1{,}2 = 0$
23. $x^3 - 0{,}1x^2 + 0{,}4x - 1{,}5 = 0$
24. $x^3 - 0{,}2x^2 + 0{,}3x - 1{,}2 = 0$
25. $x^3 + 0{,}2x^2 + 0{,}5x - 2 = 0$
26. $x^3 + 0{,}2x^2 + 0{,}5x - 1{,}2 = 0$

Technology «the Questionnaire for a feedback»

That I have remembered on employment	That I have understood, what have understood	That it was pleasant to me, has caused interest

The literature

1. Демидович Б.П., Марон И.А. Основы вычислительной математики. – М.: Наука, 1970. – 664 с.
2. Мак-Кракен Д., Дорн У. Численные методы и программирование на ФОРТРАНе. – М.: Мир, 1977. – 584 с.

Control questions:

1. Stages of the decision of the equation from one unknown person.
2. Ways of branch of roots.
3. How the graphic branch of roots is specified by means of calculations?
4. To give the verbal description of algorithm of a method половинного divisions.
5. Necessary conditions of convergence of a method половинного divisions.
6. Condition of the termination of the account of a method of simple iteration. A method error.
7. The verbal description of algorithm of a method of chords. Graphic representation of a method. Error calculation.
8. The verbal description of algorithm of a method of tangents (Newton). Graphic representation of a method. A condition of a choice of an index point.

Laboratory work № 2
The numerical decision of system of the linear algebraic equations methods of Gauss, simple iteration and Seidel.
(4 hours)

The purpose of educational employment: preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a laboratory material to watch mastering at pupils of this knowledge, to form and develop at them skills.

The plan:

1. Methods of Gauss
2. Methods of simple iteration.
3. The decision of system of the linear algebraic equations a method of Seidel

1. Methods of Gauss

Problems of approximation of function, and also set of other problems of applied mathematics of m of computing physics are reduced to problems about the decision of systems of the linear equations. The most universal method of the decision of system of the linear equations is the method of a consecutive exception of the unknown persons, Gaussa named a method.

For an illustration of sense of a method of Gaussa we will consider system of the linear equations:

$$\begin{cases} 4x_1 - 9x_2 + 2x_3 = 2 \\ 2x_1 - 4x_2 + 4x_3 = 3 \\ -x_1 + 2x_2 + 2x_3 = 1 \end{cases} \qquad \textbf{(1)}$$

This system we will write down in a matrix kind:

$$\begin{pmatrix} 4 & -9 & 2 \\ 2 & -4 & 4 \\ -1 & 2 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} \qquad \textbf{(2)}$$

As it is known, both members of equation it is possible to increase by nonzero number, and also it is possible to subtract another from one equation. Using these properties, we will try to result a matrix of system (2) in a triangular kind, i.e. to a kind, when below the main diagonal all elements – zero. This stage of the decision is called as a forward stroke.

On the forward stroke first step we will increase the first equation on 1/2 and we will subtract from the second then x_1 the variable will be excluded from the second equation. Then, we will increase the first equation on-1/4 and we will subtract from the third then the system (2) will be transformed to kind system:

$$\begin{pmatrix} 4 & -9 & 2 \\ 0 & 0.5 & 3 \\ 0 & -0.25 & 2.5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 1.5 \end{pmatrix} \qquad \textbf{(3)}$$

On the second step of a forward stroke from the third equation it is excluded x_2, i.e. from the third equation it is subtracted the second, increased, on-1/2 that results system (3) in a triangular kind (4)

$$\begin{pmatrix} 4 & -9 & 2 \\ 0 & 0.5 & 3 \\ 0 & 0 & 4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 2.5 \end{pmatrix} \qquad \textbf{(4)}$$

System (4) it is copied in a habitual kind:

$$\begin{cases} 4x_1 - 9x_2 + 2x_3 = 2 \\ 0.5x_2 + 3x_3 = 2 \\ 4x_3 = 2.5 \end{cases} \qquad \textbf{(5)}$$

Now, from system (5) can find the decision upside-down, i.e. at first we find from the third equation x_3=0.625, further, substituting in the second equation, we find x_2=(2-3x_3)/0.5. Substituting x_2 and x_3 in the first equation of system (5), we find x_1=0.75. A decision finding (x_1, x_2, x_3) from system (5) name reverse motion.

Example:

Solve the equation with a method of Gaussa.

$$\begin{cases} x_1 + x_2 - x_3 - x_4 = 0 \\ \quad x_2 + 2x_3 - x_4 = 2 \\ x_1 - x_2 - x_4 = -1 \\ -x_1 + 3x_2 - 2x_3 = 0 \end{cases}$$

The decision:

$$\begin{cases} x_1 + x_2 - x_3 - x_4 = 0 \\ x_2 + 2x_3 - x_4 = 2 \\ 2x_2 - x_3 = 1 \\ 4x_2 - 3x_3 - x_4 = 0 \end{cases} \Rightarrow \begin{cases} x_1 + x_2 - x_3 - x_4 = 0 \\ x_2 + 2x_3 - x_4 = 2 \\ 5x_3 - 2x_4 = 2 \\ 11x_3 - 3x_4 = 8 \end{cases} \Rightarrow \begin{cases} x_1 + x_2 - x_3 - x_4 = 0 \\ x_2 + 2x_3 - x_4 = 2 \\ 5x_3 - 2x_4 = 2 \\ -7x_4 = -7 \end{cases}$$

$$\Rightarrow \begin{cases} x_1 + x_2 - x_3 - x_4 = 0 \\ x_2 + 2x_3 - x_4 = 2 \\ 5x_3 - 2x_4 = 2 \\ x_4 = 1 \end{cases} \Rightarrow \begin{cases} x_1 + x_2 - x_3 - x_4 = 0 \\ x_2 + 2x_3 - x_4 = 2 \\ x_3 = 1 \\ x_4 = 1 \end{cases} \Rightarrow \begin{cases} x_1 + x_2 - x_3 - x_4 = 0 \\ x_2 = 1 \\ x_3 = 1 \\ x_4 = 1 \end{cases} \Rightarrow \begin{cases} x_1 = 1 \\ x_2 = 1 \\ x_3 = 1 \\ x_4 = 1 \end{cases}$$

Example. Solve following systems the equation a method of Gaussa with accuracy 0,001.

$$\begin{cases} 0,68x_1 + 0,05x_2 - 0,11x_3 + 0,08x_4 = 2,15 \\ 0,21x_1 - 0,13x_2 + 0,27x_3 - 0,8x_4 = 0,44 \\ -0,11x_1 - 0,84x_2 + 0,28x_3 + 0,06x_4 = -0,83 \\ -0,08x_1 + 0,15x_2 - 0,5x_3 - 0,12x_4 = 1,16 \end{cases}$$

The decision of systems the equation in MathCAD

Comments. Function ***augment (A, b)*** forms the expanded matrix of system addition to a system matrix on the right a column of the right parts. Function ***rref*** leads the expanded matrix of system to a step kind, carrying out direct and return courses гауссова exceptions. Last column contains the system decision.

$$A := \begin{bmatrix} 1 & 2 & 3 \\ 1 & -3 & 2 \\ 1 & 1 & 1 \end{bmatrix} \qquad b := \begin{bmatrix} 7 \\ 5 \\ 3 \end{bmatrix} \qquad \mathrm{rref(augment}(A, b)) = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

<u>**The task I** **II.:** Solve systems the equation with a method of Gaussa.</u>

№ 1
$$\begin{cases} 4,4x_1 - 2,5x_2 + 19,2x_3 - 10,8x_4 = 4,3 \\ 5,5x_1 - 9,3x_2 - 14,2x_3 + 13,2x_4 = 6,8 \\ 7,1x_1 - 11,5x_2 + 5,3x_3 - 6,7x_4 = -1,8 \\ 14,2x_1 + 23,4x_2 - 8,8x_3 + 5,3x_4 = 7,2 \end{cases}$$

№ 2
$$\begin{cases} 8,2x_1 - 3,2x_2 + 14,2x_3 + 14,8x_4 = -8,4 \\ 5,6x_1 - 12x_2 + 15x_3 - 6,4x_4 = 4,5 \\ 5,7x_1 + 3,6x_2 - 12,4x_3 - 2,3x_4 = 3,3 \\ 6,8x_1 + 13,2x_2 - 6,3x_3 - 8,7x_4 = 14,3 \end{cases}$$

№ 3
$$\begin{cases} 5,7x_1 - 7,8x_2 - 5,6x_3 - 8,3x_4 = 2,7 \\ 6,6x_1 + 13,1x_2 - 6,3x_3 + 4,3x_4 = -5,5 \\ 14,7x_1 - 2,8x_2 + 5,6x_3 - 12,1x_4 = 8,6 \\ 8,5x_1 + 12,7x_2 - 23,7x_3 + 5,7x_4 = 14,7 \end{cases}$$

№ 4
$$\begin{cases} 3,8x_1 + 14,2x_2 + 6,3x_3 - 15,5x_4 = 2,8 \\ 8,3x_1 - 6,6x_2 + 5,8x_3 + 12,2x_4 = -4,7 \\ 6,4x_1 - 8,5x_2 - 4,3x_3 + 8,8x_4 = 7,7 \\ 17,1x_1 - 8,3x_2 + 14,4x_3 - 7,2x_4 = 13,5 \end{cases}$$

№ 5
$$\begin{cases} 15,7x_1 + 6,6x_2 - 5,7x_3 + 11,5x_4 = -2,4 \\ 8,8x_1 - 6,7x_2 + 5,5x_3 - 4,5x_4 = 5,6 \\ 6,3x_1 - 5,7x_2 - 23,4x_3 + 6,6x_4 = 7,7 \\ 14,3x_1 + 8,7x_2 - 15,7x_3 - 5,8x_4 = 23,4 \end{cases}$$

№ 6
$$\begin{cases} 4,3x_1 - 12,1x_2 + 23,2x_3 - 14,1x_4 = 15,5 \\ 2,4x_1 - 4,4x_2 + 3,5x_3 + 5,5x_4 = 2,5 \\ 5,4x_1 + 8,3x_2 - 7,4x_3 - 12,7x_4 = 8,6 \\ 6,3x_1 - 7,6x_2 + 1,34x_3 + 3,7x_4 = 12,1 \end{cases}$$

2. Methods of simple iteration.

Methods of the decision of systems of the linear equations

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n = b_2 \\ ...... \\ a_{m1}x_1 + a_{m2}x_2 + ... + a_{mn}x_n = b_n \end{cases} \qquad (2.1)$$

Or in a vector kind

$$Ax = b \qquad (2.2)$$

It is possible to divide on two basic groups: direct methods and iterative. Direct methods give the exact decision for final number of operations; Kramer's methods and Gaussa concern them, for example. Iterative methods give the decision of system of the equations as a limit consecutive приближений. For iterative methods performance of conditions of convergence and additional transformations of system in equivalent to it is necessary.

The task 1

1. To solve system of the linear equations a **method of Gauss**. Tasks are resulted in table 2.

The comment. The control of carried out calculations is the important element of the decision of any computing problem. For the forward stroke control use the control sums which represent the sums of factors at unknown persons and a free member for each equation of the set system.

For the control of calculations in the basic part of the scheme of unique division (columns of factors at unknown and free members) over the control sums carry out the same actions, as over other elements of the same line. In the absence of computing errors the control sum for every line in limits influences of errors of a rounding off and their accumulation should coincide with the lower case sum - the second column of the control. The lower case sums represent the sums of all elements from the basic part of this line.

The task 2

To solve system (2.1) method of simple iteration. It is supposed further that a matrix *And* square and невырожденная.

Let's preliminary result system (2.2) in an iterative kind:

$$x = Cx + f \qquad (2.3)$$

For any initial vector x_0 iterative process

$$x^{n+1} = Cx^n + f$$

Converges, if one of conditions is executed

а) $$\sum_{j=1}^{n} |c_{i,j}| = \alpha < 1, \qquad 1 \le i \le n, \qquad (2.4)$$

б) $$\sum_{i=1}^{n} |c_{i,j}| = \alpha < 1, \qquad 1 \le j \le n, \qquad (2.5)$$

в) $$\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}^2} = \alpha < 1. \qquad (2.6)$$

Process of calculations is finished at condition performance

$$\rho_i(x^{k-1}, x^k) \le \varepsilon(1-\alpha)/\alpha \qquad (2.7)$$

where ρ_i ($i=1,2,3$)– one of the metrics, defined by the left part (2.4) - (2.6) on which convergence, ε the set – accuracy (ε $=10^{-4}$ – has been established).

The task 3

To solve system (2.1) **method of Seidel.**

The method of Zejdel differs from a method of simple iteration by that having found any value for components, we on a following step use it for search following components. Calculations are conducted under the formula

$$x_i^{(k+1)} = -\sum_{j=1}^{i-1} \frac{a_{ij}}{a_{ii}} x_j^{(k+1)} - \sum_{j=i+1}^{n} \frac{a_{ij}}{a_{ii}} x_j^{(k)} + \frac{b_i}{a_{ii}}. \qquad (2.8)$$

Each of conditions (2.4) - (2.6) is sufficient for convergence of iterative process on a **method of Zejdel.** Practically more conveniently following transformation of system (2.2). Домножая both parts (2.2) on A^T, we will receive system equivalent to it

$$CX = d\,,$$

where $C=A^TA$ and $d=A^Tb$. Further, having divided each equation on c_{ij}, we will lead system to a kind (2.8). Similar transformation also guarantees convergence of iterative process.

APPROXIMATE variant of performance of laboratory work

Example. Solve system of the equations

$X_1+2X_2+3X_3=7$,
$X_1-3X_2+2X_3=5$,

$X_1+X_2+X_3=3$.

1. The symbolical decision of systems of the equations

The fragment of a brief with corresponding calculations is resulted more low. Here = - logic equality.

Given

$$x1+2\cdot x2+3\cdot x3=7$$

$$x1-3\cdot x2+2\cdot x3=5$$

$$x1+x2+x3=3$$

$$\mathrm{Find}(x1,x2,x3)\rightarrow\begin{pmatrix}1\\0\\2\end{pmatrix}$$

2. The decision of system of the linear algebraic equations as matrix equation Ax=b

Order of performance of the task.

1. Establish a mode of automatic calculations.
2. Enter a matrix of system and a matrix-column of the right parts.
3. Calculate the system decision under the formula $x=A^{-1}b$.
4. Check up correctness of the decision multiplication of a matrix of system to a decision vector-column.
5. Find the decision of system by means of function lsolve and compare results.

$$A:=\begin{pmatrix}1&2&3\\1&-3&2\\1&1&1\end{pmatrix}\qquad b:=\begin{pmatrix}7\\5\\3\end{pmatrix}$$

$$x:=A^{-1}\cdot b\qquad x=\begin{pmatrix}1\\0\\2\end{pmatrix}\qquad A\cdot x-b=\begin{pmatrix}0\\0\\0\end{pmatrix}$$

check

Let's solve system by means of function ***lsolve*** **and** will compare result to the decision $x=A^{-1}b$.

$$x:=\mathrm{lsolve}(A,b)\qquad x=\begin{pmatrix}1\\0\\2\end{pmatrix}$$

3. The decision of linear system a method of Gaussa

Comments. Function ***augment (A, b)*** forms the expanded matrix of system addition to a system matrix on the right a column of the right parts. Function ***rref*** leads the expanded matrix of system to a step kind, carrying out direct and return courses гауссова exceptions. Last column contains the system decision.

$$\mathrm{rref}(\mathrm{augment}(A,b))=\begin{pmatrix}1&0&0&1\\0&1&0&0\\0&0&1&2\end{pmatrix}$$

4. The decision of system Kramer's method

Order of performance of work.

1. We calculate D a determinant of matrix A.
2. Let's set matrix DX1, replacement of the first column of matrix A, a matrix b. We calculate a determinant of matrix DX1.
3. Let's set matrix DX2, replacement of the second column of matrix A, a matrix b. We calculate a determinant of matrix DX2.
4. Let's set matrix DX3, replacement of the third column of matrix A, a matrix b. We calculate a determinant of matrix DX3.

5. We define the decision of system of the linear equations x_1, x_2, x_3.

$D := |A| \qquad D = 9$

$DX1 := \begin{pmatrix} 7 & 2 & 3 \\ 5 & -3 & 2 \\ 3 & 1 & 1 \end{pmatrix} \qquad DX1 := |DX1| \qquad DX1 = 9$

$DX2 := \begin{pmatrix} 1 & 7 & 3 \\ 1 & 5 & 2 \\ 1 & 3 & 1 \end{pmatrix} \qquad DX2 := |DX2| \qquad DX2 = 0$

$DX3 := \begin{pmatrix} 1 & 2 & 7 \\ 1 & -3 & 5 \\ 1 & 1 & 3 \end{pmatrix} \qquad DX3 := |DX3| \qquad DX3 = 18$

$x1 := \frac{DX1}{D} \quad x1 = 1 \qquad x2 := \frac{DX2}{D} \quad x2 = 0 \qquad x3 := \frac{DX3}{D} \quad x3 = 2$

5. The decision of system linear algebraic the equation a method of simple iterations

Order of performance of the task

1. Enter matrixes C and d.
2. Transform initial system Cx=d to a kind x=b+Ax.
3. Define zero approach of the decision.
4. Set quantity of iterations.
5. Calculate consecutive approach.

$ORIGIN := 1$

$C := \begin{bmatrix} 100 & 6 & -2 \\ 6 & 200 & -10 \\ 1 & 2 & 100 \end{bmatrix} \qquad d := \begin{bmatrix} 200 \\ 600 \\ 500 \end{bmatrix}$

$i := 1..3 \qquad j := 1..3$

$b_i := \frac{d_i}{C_{i,i}} \qquad A_{i,j} := \frac{-C_{i,j}}{C_{i,i}} \qquad A_{i,i} := 0$

$A = \begin{bmatrix} 0 & -0.06 & 0.02 \\ -0.03 & 0 & 0.05 \\ -0.01 & -0.02 & 0 \end{bmatrix} \qquad b = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$

$x^{\langle 1 \rangle} := b \qquad k := 2..10 \qquad x^{\langle k \rangle} := b + A \cdot x^{\langle k-1 \rangle}$

x =

	1	2	3	4	5	6	7	8	9	10
1	2	1.92	1.907	1.907	1.907	1.907	1.907	1.907	1.907	1.907
2	3	3.19	3.188	3.189	3.189	3.189	3.189	3.189	3.189	3.189
3	5	4.92	4.917	4.917	4.917	4.917	4.917	4.917	4.917	4.917

$X := x^{\langle 10 \rangle} \qquad X = \begin{bmatrix} 1.907 \\ 3.189 \\ 4.917 \end{bmatrix}$

6. The decision of system of the linear algebraic equations a method of Seidel

Order of performance of the task

1. Enter matrixes C and d.
2. Transform system Cx=d to a kind x=b+A1x+A2x.
3. Define zero approach of the decision.
4. Set quantity of iterations.
5. Calculate consecutive approach.

$ORIGIN := 1$

$$C := \begin{bmatrix} 100 & 6 & -2 \\ 6 & 200 & -10 \\ 1 & 2 & 100 \end{bmatrix} \qquad d := \begin{bmatrix} 200 \\ 600 \\ 500 \end{bmatrix}$$

$$i := 1..3 \qquad b_i := \frac{d_i}{C_{i,i}} \qquad i := 2..3 \qquad j := 1..2$$

$$A1_{i,j} := \frac{-C_{i,j}}{C_{i,i}} \qquad A2_{j,i} := \frac{-C_{j,i}}{C_{j,j}}$$

$$A1_{i,i} := 0 \quad A1_{j,i} := 0 \quad A2_{i,i} := 0 \quad A2_{i,j} := 0 \quad A := A1 + A2$$

$$A1 = \begin{bmatrix} 0 & 0 & 0 \\ -0.03 & 0 & 0 \\ -0.01 & -0.02 & 0 \end{bmatrix} \quad A2 = \begin{bmatrix} 0 & -0.06 & 0.02 \\ 0 & 0 & 0.05 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & -0.06 & 0.02 \\ -0.03 & 0 & 0.05 \\ -0.01 & -0.02 & 0 \end{bmatrix} \qquad b = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$$

$$x^{\langle 1 \rangle} := b \qquad y^{\langle 1 \rangle} := b \qquad k := 2..10$$

$$x^{\langle k \rangle} := b + A2 \cdot x^{\langle k-1 \rangle} \qquad x^{\langle k \rangle} := x^{\langle k \rangle} + A1 \cdot x^{\langle k-1 \rangle}$$

x =

	1	2	3	4	5	6	7	8	9	10
1	2	1.92	1.905	1.905	1.905	1.905	1.905	1.905	1.905	1.905
2	3	3.19	3.192	3.193	3.193	3.193	3.193	3.193	3.193	3.193
3	5	4.92	4.917	4.917	4.917	4.917	4.917	4.917	4.917	4.917

table 2

№ вар.	a_{1i}	a_{2i}	a_{3i}	b_{1i}
1	0.35 0.12 - 0.13	0.12 0.71 0.15	- 0.13 0.15 0.63	0.10 0.26 0.38
2	0.71 0.10 - 0.10	0.10 0.34 0.64	0.12 - 0.04 0.56	0.29 0.32 - 0.10
3	0.34 - 0.04 0.06	- 0.04 0.44 0.56	0.10 - 0.12 0.39	0.33 - 0.05 0.28

4	0.10 - 0.04 - 0.43	- 0.04 0.34 0.05	- 0.63 0.05 0.13	- 0.15 0.31 0.37
5	0.63 0.05 0.15	0.05 0.34 0.10	0.15 0.10 0.71	0.34 0.32 0.42
6	1.20 - 0.50 - 0.30	- 0.20 1.70 0.10	0.30 - 1.60 - 1.50	- 0.60 0.30 0.40
7	0.30 - 0.10 - 1.50	1.20 - 0.20 - 0.30	- 0.20 1.60 0.10	- 0.60 0.30 0.70
8	0.20 0.58 0.05	0.44 - 0.29 0.34	0.91 0.05 0.10	0.74 0.02 0.32
9	6.36 7.42 1.77	1.75 19.03 0.42	1.0 1.75 6.36	41.70 49.49 27.67
10	3.11 - 1.65 0.60	- 1.66 3.15 0.78	- 0.60 - 0.78 - 2.97	- 0.92 2.57 1.65

Exercises

Method of Gauss of system of the linear algebraic equations of Ah=b. To compare to the exact decision ξ.

1. $A = \begin{bmatrix} 5 & 0 & 1 \\ 1 & 3 & -1 \\ -3 & 2 & 10 \end{bmatrix}, \quad b = \begin{bmatrix} 11 \\ 4 \\ 6 \end{bmatrix}, \quad \xi = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}.$

2. $A = \begin{bmatrix} 2 & 0 & -1 \\ -1 & 3 & 1 \\ 1 & -1 & 4 \end{bmatrix}, \quad b = \begin{bmatrix} -3 \\ 2 \\ 3 \end{bmatrix}, \quad \xi = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}.$

3. $A = \begin{bmatrix} 2 & 0 & -1 \\ 1 & -3 & 1 \\ 1 & 1 & 3 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}, \quad \xi = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}.$

4. $A = \begin{bmatrix} 5 & 1 & -1 \\ -1 & 3 & 1 \\ 1 & -2 & 4 \end{bmatrix}, \quad b = \begin{bmatrix} -5 \\ 5 \\ 1 \end{bmatrix}, \quad \xi = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}.$

5. $A = \begin{bmatrix} 3 & 1 & -1 \\ -2 & 4 & 1 \\ 1 & 1 & 3 \end{bmatrix}, \quad b = \begin{bmatrix} -1 \\ 5 \\ -3 \end{bmatrix}, \quad \xi = \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}.$

6. $A = \begin{bmatrix} 3 & 1 & -1 \\ 2 & 4 & 1 \\ 1 & -1 & 3 \end{bmatrix}, \quad b = \begin{bmatrix} 6 \\ 9 \\ 4 \end{bmatrix}, \quad \xi = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}.$

7. $A = \begin{bmatrix} 2 & -1 & 0 \\ 2 & 5 & -2 \\ 1 & -1 & 3 \end{bmatrix}, \quad b = \begin{bmatrix} -2 \\ -4 \\ 2 \end{bmatrix}, \quad \xi = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}.$

8. $A = \begin{bmatrix} 3 & -1 & 1 \\ 0 & 2 & -1 \\ -1 & 1 & 5 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 3 \\ -5 \end{bmatrix}, \quad \xi = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}.$

9. $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & 0 \\ 1 & -1 & 5 \end{bmatrix}, \quad b = \begin{bmatrix} 7 \\ 7 \\ 11 \end{bmatrix}, \quad \xi = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}.$

Technology «the Questionnaire for a feedback»

That I have remembered on employment	That I have understood, what have understood	That it was pleasant to me, has caused interest

Control questions:

1. The method of Gaussa concerns what type - direct or iterative?
2. In what consists a straight line and reverse motion in the scheme of unique division?
3. How it will be organised, the control over calculations in direct and reverse motion?
4. How the iterative sequence for a finding of the decision of system of the linear equations is under construction?
5. How it is formulated sufficient conditions of convergence of iterative process?
6. How these conditions are connected with a choice of the metrics of space?
7. In what difference of iterative process of a method of Zejdel from similar process of a method of simple iteration?

Laboratory work № 3
Problems of Cochy for the ordinary differential equations.
Euler's methods, Runge-Kutte and Adams

(4 hours)

The purpose of educational employment: preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a laboratory material to watch mastering at pupils of this knowledge, to form and develop at them skills.

The plan:

1. Euler's method
2. Method of Runge-Kutte.
3. Adam's method

Let the differential equation of the first order is given

$$y' = f(x, y). \qquad (5.1)$$

It is required to find on a piece $[a, b]$ the decision $y(x)$, satisfying to the entry condition

$$y(a) = y_0 \qquad (5.2)$$

Let's assume that conditions of the theorem of existence and uniqueness are executed. For the decision we use **Euler's** method (a method of the first order of accuracy, settlement formulas (5.3)) and a method **of Runge-Kutta** (a method of the fourth order of accuracy, settlement formulas (5.4)) with step h and $2h$. We will notice that results can strongly differ, whereas Euler's method, having only the first order of accuracy, is used, as a rule, for estimated calculations. A rough estimation of an error of a method of Runge-Kutta to calculate ε under the formula (5.5 [2].

$$y_{i+1} = y_i + hf(x_i, y_i), \quad \text{where } h \text{ – a splitting step.} \qquad (5.3)$$

$$y_{i+1} = y_i + \frac{k_1 + 2k_2 + 2k_3 + k_4}{6}, \text{ where} \qquad (5.4)$$

$$k_1 = hf(x_i, y_i),\ k_2 = hf(x_i + \frac{h}{2}, y_i + \frac{k_1}{2}),\ k_3 = hf(x_i + \frac{h}{2}, y_i + \frac{k_2}{2}),$$

$$k_4 = hf(x_i + h, y_i + k_3).$$

$$\varepsilon = \frac{|y_{2h} - y_h|}{15} \qquad (5.5)$$

APPROXIMATE fragment of performance of work

1. To solve the differential equation y ' =f (x, y) **Euler's method** on a piece [a, b] with step h with the entry condition y (a) =y_0, f (x, y) = (3x-y) / (x^2+y), a=2, b=3, h=0.1, y_0=1.

2. To solve the differential equation y' =f (x, y) a method of Runge-Kutta on a piece [a, b] with step h with the entry condition y (a) =y_0.

Euler's method

$a := 2 \qquad b := 3 \qquad x_0 := a$

$i := 0..10 \qquad h := 0.1 \qquad x_{i+1} := x_0 + i \cdot h \qquad y_0 := 1$

$$y_{i+1} := y_i + h \cdot \frac{3 \cdot x_i - y_i}{(x_i)^2 + y_i}$$

x =

	0
0	2
1	2
2	2.1
3	2.2
4	2.3
5	2.4
6	2.5
7	2.6
8	2.7
9	2.8
10	2.9
11	3

y =

	0
0	1
1	1.1
2	1.196
3	1.287
4	1.374
5	1.457
6	1.536
7	1.613
8	1.687
9	1.758
10	1.827
11	1.895

Method of Runge-Kutta

$a := 2 \qquad b := 3 \qquad x_0 := a$

$i := 0..10 \qquad h := 0.1 \qquad x_{i+1} := x_0 + i \cdot h \qquad y_0 := 1$

$$f(x, y) := \frac{3 \cdot x - y}{x^2 + y} \qquad y_{i+1} := y_i + h \cdot f(x_i, y_i)$$

$$k_1 := h \cdot f(x_i, y_i) \qquad k_2 := h \cdot f\left[\left(x_i + \frac{h}{2}\right), y_i + \frac{k_1}{2}\right]$$

$$k_3 := h \cdot f\left[\left(x_i + \frac{h}{2}\right), y_i + \frac{k_2}{2}\right] \qquad k_4 := h \cdot f\left[\left(x_i + h\right), y_i + k_3\right]$$

$$y_{i+1} := y_i + \frac{k_1 + 2 \cdot k_2 + 2 \cdot k_3 + k_4}{6}$$

x =

	0
0	2
1	2
2	2.1
3	2.2
4	2.3
5	2.4
6	2.5
7	2.6
8	2.7
9	2.8
10	2.9
11	3

y =

	0
0	1
1	1.066
2	1.132
3	1.199
4	1.265
5	1.331
6	1.397
7	1.463
8	1.529
9	1.596
10	1.662
11	1.728

3. Adams's method

The decision of systems of the ordinary differential equations Adams's method

In the given system of the equations we will substitute values of factors and entry conditions. We will receive

$$\begin{cases} y' = 2y - z \\ z' = y - 4z \end{cases} \qquad y(0) = 3, \quad z(0) = -2$$

Adams's method we will find the decision of this system on the set piece. For this purpose we will calculate a method of Runge-Kutta some initial values of function.

Let's choose a step h and, for brevity, will enter $x_i = x_0 + ih$ и $y_i = y(x_i)$ $(i = 0, 1, 2, \dots)$

Let's consider numbers:

$$\begin{cases} k_1^{(i)} = hf(x_i, y_i) \\ k_2^{(i)} = hf\left(x_i + \frac{h}{2}, y_i + \frac{k_1^{(i)}}{2}\right) \\ k_3^{(i)} = hf\left(x_i + \frac{h}{2}, y_i + \frac{k_3^{(i)}}{2}\right) \\ k_4^{(i)} = hf(x_i + h, y_i + k_3^{(i)}) \end{cases}$$

According to a method of Runge-Kutta consecutive values y_i are defined under the formula

$$y_{i+1} = y_i + \Delta y_i$$

where

$$\Delta y_i = \frac{1}{6}\left(k_1^{(i)} + 2 \cdot k_2^{(i)} + 2 \cdot k_3^{(i)} + k_4^{(i)}\right) (i = 0, 1, 2, \dots\ ). \quad (2.1)$$

Having substituted in these formulas initial values we will receive

$$x_0 = 0 \qquad y_0 = 3 \qquad z_0 = -2$$
$$x_1 = 0{,}1 \quad y_1 = 3{,}3672 \quad z_1 = -2{,}1586$$
$$x_2 = 0{,}2 \quad y_2 = 3{,}4944 \quad z_2 = -2{,}0867$$
$$x_3 = 0{,}3 \quad y_3 = 3{,}5964 \quad z_3 = -1{,}9906$$

Further calculation it is continued on Adams's method. All calculations it is written down in tables 2.1 and 2.2.

Table 2.1

k	x_k	y_k	Δy_k	p_k	Δp_k	$\Delta^2 p_k$	$\Delta^3 p_k$	z_k	Δz_k	q_k	Δq_k	$\Delta^2 q_k$	$\Delta^3 q_k$
0	0	3		0,8000	0,0893	-0,0711	0,0636	-2		1,1000	0,1002	-0,1162	0,1040
1	0,1	3,3672		0,8893	0,0183	-0,0075	0,0680	-2,1586		1,2002	-0,0160	-0,0122	-0,3354
2	0,2	3,4944		0,9076	0,0108	0,0605	0,0512	-2,0867		1,1841	-0,0282	-0,3476	0,7024
3	0,3	3,5964	0,9445	0,9183	0,0713	0,1117	-0,1448	-1,9906	1,1757	1,1559	-0,3758	0,3548	-0,6647
4	0,4	4,5409	1,0761	0,9897	0,1831	-0,0330	0,1605	-0,8149	0,3215	0,7801	-0,0210	-0,3099	0,8201
5	0,5	5,6169	1,3300	1,1727	0,1500	0,1275	-0,1562	-0,4934	1,1598	0,7590	-0,3309	0,5102	-0,9910
6	0,6	6,9469	1,3297	1,3227	0,2775	-0,0288	0,2023	0,6664	-0,1157	0,4281	0,1793	-0,4809	1,1396
7	0,7	8,2766	1,8523	1,6003	0,2488	0,1735	-0,2240	0,5507	1,2171	0,6074	-0,3016	0,6587	-1,3700
8	0,8	10,1290	1,9028	1,8490	0,4223	-0,0505		1,7678	-0,4170	0,3058	0,3571	-0,7113	
9	0,9	12,0318	2,6306	2,2713	0,3718			1,3508	1,5432	0,6629	-0,3542		
10	1	14,6623	2,7239	2,6431				2,8940	-0,6786	0,3086			

Table 2.2

k	x	y	y'	z	z'
0	0	3	8	-2	11
1	0,1	3,3672	8,893	-2,1586	12,0016
2	0,2	3,4944	9,0755	-2,0867	11,8412
3	0,3	3,5964	9,1834	-1,9906	11,5588
4	0,4	4,5409	9,8967	-0,8149	7,8005
5	0,5	5,6169	11,7272	-0,4934	7,5905
6	0,6	6,9469	13,2274	0,6664	4,2813
7	0,7	8,2766	16,0025	0,5507	6,0738
8	0,8	10,129	18,4902	1,7678	3,0578
9	0,9	12,0318	22,7128	1,3508	6,6286

(1.3) values received under the formula are necessary for specifying, having calculated them under the formula (1.4). The obtained data we will write down in the table.

Table 2.3

k	x	Δy_k	$\Delta y_k^{кор.}$	Δz_k	$\Delta z_k^{кор.}$
0	0				
1	0,1				
2	0,2				
3	0,3	0,9445	0,946075	1,1757	1,010942
4	0,4	1,0761	1,069808	0,3215	0,710767
5	0,5	1,3300	1,256483	1,1598	0,647071
6	0,6	1,3297	1,444138	-0,1157	0,441063
7	0,7	1,8523	1,733608	1,2171	0,537967
8	0,8	1,9028	2,037263	-0,4170	0,381975
9	0,9	2,6306	2,470742	1,5432	0,602158
10	1	2,7239	2,6431	-0,6786	0,3086

The task 1

To write the program of the decision of the differential equation *y ' =f (x, y)* Euler's method on a piece [a,b] with step h and *2h* and the entry condition $y(a) = y_0$. The Initial data for task performance undertakes from table 5. To compare results.

The task 2

To write the program of the decision of the differential equation *y ' =f (x, y) a* method of Runge-Kutta on a piece [a,b] with step h and *2h* and the entry condition $y(a) = y_0$. To estimate an error under the formula (5.5). The initial data for task performance undertakes from table 5.

Table 5

N	*Функsия*	a	b	y_0	h
1	$\frac{3x-y}{x^2+y}$	2	3	1	0.1
2	$\frac{2x+y+4}{2y+x}$	3	4	1	0.1
3	$\frac{x^2-y}{2x+y+1}$	0	1	2	0.1
4	$\frac{x^2-y+2}{xy+3x}$	2	3	1	0.1
5	$\frac{3-x-y^2}{2-xy^2}$	1	2	1	0.1
6	$\frac{2-x-y^2x}{3x+y}$	0	1	1	0.1
7	$\frac{1+3xy}{5-x+y^2}$	0	1	2	0.1
8	$\frac{x^2y+2}{2x-y}$	0	1	1	0.1
9	$\frac{x^2+y+2}{2x-y}$	2	3	2	0.1
10	$\frac{xy+4}{2y-xy+1}$	0	1	3	0.1

Technology «the Questionnaire for a feedback»

That I have remembered on employment	That I have understood, what have understood	That it was pleasant to me, has caused interest

Control questions:

1. To check up for the differential equation of a condition of the theorem of existence and uniqueness.
2. The approached methods of the decision of the differential equations are subdivided into what basic groups?
3. In what form it is possible to receive the decision of the differential equation on Euler's method?
4. What geometrical sense of the decision of the differential equation Euler's method?
5. In what form it is possible to receive the decision of the differential equation on a method of Runge-Kutta?
6. What way of an estimation of accuracy is used at the approached integration of the differential equations by Euler's methods and Runge-Kutta?
7. How to calculate an error under the set formula, using a method of double recalculation?

Laboratory work № 4
The numerical decision of system of the nonlinear algebraic equations to methods of simple iteration.
(4 hours)

The purpose of educational employment: preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a laboratory material to watch mastering at pupils of this knowledge, to form and develop at them skills.

The plan:

1. The decision of systems of the nonlinear equations
2. Method of simple iterations
3. Example for decision of systems of the nonlinear equations
4. The decision of systems nonlinear the equation in MathCAD.

The decision of systems of the nonlinear equations

The system of the nonlinear equations is given

$$\begin{cases} f_1(x_1, x_2, x_3, \ldots, x_n) = 0, \\ f_2(x_1, x_2, x_3, \ldots, x_n) = 0, \\ \ldots \\ f_n(x_1, x_2, x_3, \ldots, x_n) = 0, \end{cases} \qquad (1)$$

or

$$f_i(x_1, x_2, x_3, \ldots, x_n) = 0, i = \overline{1 \ldots n}.$$

It is necessary to solve this system, i.e. to find a vector $\bar{X} = [x_1, x_2, x_3, \ldots, x_n]$, Satisfying to system *(1)* with accuracy ε.

The vector $\bar{X}$ defines a point in n-dimensional Evklidovom space, i.e. $\bar{X} \in$ to this space and satisfies to all equations of system *(1).*

Unlike systems of the linear equations for *systems of the nonlinear equations* direct methods of the decision are unknown. At the decision *of systems of the nonlinear equations* iterative methods are used. Efficiency of all iterative methods depends on a choice of initial approach (index point), i.e. a vector $\overline{X^0} = [x_1^0, x_2^0, \ldots, x_n^0]$.

The area in which initial approach $\overline{X^0}$ converges to the required decision, is called as area of convergence G. If initial approach $\overline{X^0}$ lies outside of G it is not possible to receive the system decision.

The index point $\overline{X^0}$ choice is in many respects defined by intuition and experience of the expert. [5]

Method of simple iterations

For application of this method the initial system *(1)* should be transformed to a kind

$$\begin{cases} x_1 = \varphi_1(x_1, x_2, x_3, \ldots, x_n) = 0, \\ x_2 = \varphi_2(x_1, x_2, x_3, \ldots, x_n) = 0, \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ x_n = \varphi_n(x_1, x_2, x_3, \ldots, x_n) = 0, \end{cases} \qquad (2)$$

or

$$x_i = \varphi_i(x_1, x_2, x_3, \ldots, x_n), i = \overline{1, n}.$$

Further, having chosen initial approach $\overline{X^0} = [x_1^0, x_2^0, \ldots, x_n^0]$ and using system *(2)*, we build iterative process of search in the scheme:

$$x_i^k =_i (x_1^{k-1}, x_2^{k-1}, x_3^{k-1}, \ldots, x_n^{k-1}),$$

i.e. on each k-th step of search the vector of variables $\overline{X}$ is found, using values of the variables received on a step *(k-1)*.

Iterative process of search stops, as soon as the condition will be satisfied

$$\left|x_j^k - x_j^{k-1}\right| \le \varepsilon, j = \overline{1, n}. \qquad (3)$$

Thus the condition *(3)* should be carried out simultaneously on all variables.

The method of simple iterations is used for the decision of such systems of the nonlinear equations in which the condition of convergence of iterative process of search, namely (3) is satisfied, i.e. the sum of absolute sizes of the private derivative all transformed equations of system *(2)* on j-th variable is less than unit.

$$\sum_{i=1}^{n} \left| \frac{\partial \varphi_i}{\partial x_j} \right| < 1, j = \overline{1, n}.$$

For two system equitions will be represent system (*) in a kind:

$$\begin{cases} x_1 = g_1(x_1, x_2), \\ x_2 = g_2(x_1, x_2). \end{cases} \qquad \textbf{(5)}$$

It is represented the right members of equation in a kind:

$$\begin{aligned} g_1(x_1,x_2) &= x_1 + \lambda_{11} f_1(x_1,x_2) + \lambda_{12} f_2(x_1,x_2), \\ g_2(x_1,x_2) &= x_2 + \lambda_{21} f_1(x_1,x_2) + \lambda_{22} f_2(x_1,x_2). \end{aligned} \qquad (6)$$

For a method of simple iteration

$$\begin{aligned} g_1(x_1,x_2) &= x_1 + \lambda_{11} f_1(x_1,x_2) + \lambda_{12} f_2(x_1,x_2), \\ g_2(x_1,x_2) &= x_2 + \lambda_{21} f_1(g_1,x_2) + \lambda_{22} f_2(x_1,x_2). \end{aligned} \qquad (7)$$

For a method of Zejdel

For search of factors λ_{ii} solve system

$$\left\{ \begin{array}{l} \left\{ \begin{array}{l} 1 + \lambda_{11} \left.\dfrac{\partial f_1}{\partial x_1}\right|_{x^{(0)}} + \lambda_{12} \left.\dfrac{\partial f_2}{\partial x_1}\right|_{x^{(0)}} = 0, \\ \lambda_{11} \left.\dfrac{\partial f_1}{\partial x_2}\right|_{x^{(0)}} + \lambda_{12} \left.\dfrac{\partial f_2}{\partial x_2}\right|_{x^{(0)}} = 0; \end{array} \right. \\ \left\{ \begin{array}{l} \lambda_{21} \left.\dfrac{\partial f_1}{\partial x_1}\right|_{x^{(0)}} + \lambda_{22} \left.\dfrac{\partial f_2}{\partial x_1}\right|_{x^{(0)}} = 0, \\ 1 + \lambda_{21} \left.\dfrac{\partial f_1}{\partial x_1}\right|_{x^{(0)}} + \lambda_{22} \left.\dfrac{\partial f_2}{\partial x_1}\right|_{x^{(0)}} = 0. \end{array} \right. \end{array} \right. \qquad (8)$$

Let's use further system *(7)* for search of roots of the equation. In the program **$g_1(x_1,x_2)$=y1** and **$g_2(x_1,x_2)$=y2**.

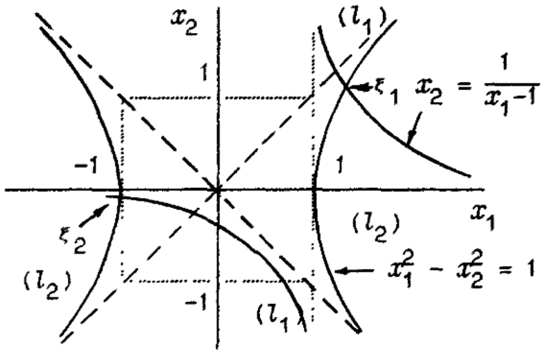
Example. The system of the equations is given

$$\begin{cases} x_2(x_1 - 1) - 1 = 0, (l_1) \\ x_1^2 - x_2^2 - 1 = 0, (l_2) \end{cases} \qquad (1)$$

to find with accuracy $\varepsilon=10^{-3}$ its decision located in the first quarter of a plane $0x_1x_2$.

The decision. The curves defined by the equations (1), are represented on fig. 1. These curves are crossed in two points ξ_1 and ξ_2. Let's result system (1) in a kind convenient for iterations, and we will find the decision system (1) ξ_1 with the set accuracy.

We take as initial value (the graphic decision) $x^{(0)} = \begin{pmatrix} 1,5 \\ 1,5 \end{pmatrix}$ and for definition of factors λ_{ij} olve system of the equations.



- A curve (l_1) – a hyperbole (two branches) - A curve (l_2) – a hyperbole (two branches)

Fig. 1.

Let's calculate private derivatives of functions.

$$f_1(x_1,x_2) = x_2(x_1 - 1) - 1, \quad f_2(x_1,x_2) = x_1^2 - x_2^2 - 1$$

In a point $x^{(0)}$:

$$\left.\frac{\partial f_1}{\partial x_1}\right|_{\mathbf{x}^{(0)}} = \left.x_2\right|_{\mathbf{x}^{(0)}} = 1,5; \qquad \left.\frac{\partial f_2}{\partial x_1}\right|_{\mathbf{x}^{(0)}} = \left.2x_1\right|_{\mathbf{x}^{(0)}} = 3;$$

$$\left.\frac{\partial f_1}{\partial x_2}\right|_{\mathbf{x}^{(0)}} = \left.(x_1 - 1)\right|_{\mathbf{x}^{(0)}} = 0,5; \qquad \left.\frac{\partial f_2}{\partial x_2}\right|_{\mathbf{x}^{(0)}} = \left.-2x_2\right|_{\mathbf{x}^{(0)}} = -3.$$

Having solved system

$$\begin{cases} \begin{cases} 1 + 1,5\,\lambda_{11} + 3\,\lambda_{12} = 0, \\ 0,5\,\lambda_{11} - 3\,\lambda_{12} = 0; \end{cases} \\ \begin{cases} 1,5\,\lambda_{21} + 3\,\lambda_{22} = 0, \\ 1 + 0,5\,\lambda_{21} - 3\,\lambda_{22} = 0, \end{cases} \end{cases}$$

let's find $\lambda_{11} = -\frac{1}{2}, \quad \lambda_{12} = -\frac{1}{12}, \quad \lambda_{21} = -\frac{1}{2}, \quad \lambda_{22} = \frac{1}{4}.$

Condition $\lambda_{11}\lambda_{22} - \lambda_{12}\lambda_{21} \neq 0$ is executed. The resulted system has the following appearance:

$$\begin{cases} x_1 = g_1(x_1, x_2) = x_1 - \frac{1}{2}\left[x_2(x_1 - 1) - 1\right] - \frac{1}{12}\left[x_1^2 - x_2^2 - 1\right], \\ x_2 = g_2(x_1, x_2) = x_2 - \frac{1}{2}\left[x_2(x_1 - 1) - 1\right] + \frac{1}{4}\left[x_1^2 - x_2^2 - 1\right]. \end{cases} \quad (2)$$

Using the received representations (2) for functions $g_1(x_1, x_2)$ and $g_2(x_1, x_2)$, will find vectors consecutive approximation. An estimation of an error of each approach we will define in distance between vectors of two consecutive iterations on *m-norm*.

$$d_k = \|\mathbf{x}^{(k)} - \mathbf{x}^{(k-1)}\| = \max\{|x_1^{(k)} - x_1^{(k-1)}|, |x_2^{(k)} - x_2^{(k-1)}|\}.$$

Then will receive

$$\mathbf{x}^{(1)} = \begin{pmatrix} x_1^{(1)} \\ x_2^{(1)} \end{pmatrix} = \begin{pmatrix} g_1(x_1^{(0)}, x_2^{(0)}) \\ g_2(x_1^{(0)}, x_2^{(0)}) \end{pmatrix} = \begin{pmatrix} g_1(1,5;\ 1,5) \\ g_2(1,5;\ 1,5) \end{pmatrix} = \begin{pmatrix} 1,70833 \\ 1,37500 \end{pmatrix},$$

$d_1 = 0,20833;$

$$\mathbf{x}^{(2)} = \begin{pmatrix} x_1^{(2)} \\ x_2^{(2)} \end{pmatrix} = \begin{pmatrix} g_1(x_1^{(1)}, x_2^{(1)}) \\ g_2(x_1^{(1)}, x_2^{(1)}) \end{pmatrix} = \begin{pmatrix} g_1(1,70833; 1,37500) \\ g_2(1,70833; 1,37500) \end{pmatrix} = \begin{pmatrix} 1,71904 \\ 1,39497 \end{pmatrix},$$

$d_2 = 0,01997;$

$$\mathbf{x}^{(3)} = \begin{pmatrix} x_1^{(2)} \\ x_2^{(2)} \end{pmatrix} = \begin{pmatrix} g_1(x_1^{(2)}, x_2^{(2)}) \\ g_2(x_1^{(2)}, x_2^{(2)}) \end{pmatrix} = \begin{pmatrix} g_1(1,71904; 1,39497) \\ g_2(1,71904; 1,39497) \end{pmatrix} = \begin{pmatrix} 1,71676 \\ 1,39574 \end{pmatrix},$$

$d_3 = 0,00281;$

$$\mathbf{x}^{(4)} = \begin{pmatrix} x_1^{(3)} \\ x_2^{(3)} \end{pmatrix} = \begin{pmatrix} g_1(x_1^{(3)}, x_2^{(3)}) \\ g_2(x_1^{(3)}, x_2^{(3)}) \end{pmatrix} = \begin{pmatrix} g_1(1,71676; 1,39574) \\ g_2(1,71676; 1,39574) \end{pmatrix} = \begin{pmatrix} 1,71662 \\ 1,39533 \end{pmatrix},$$

$d_4 = 0,00041;$

$$\mathbf{x}^{(5)} = \begin{pmatrix} x_1^{(4)} \\ x_2^{(4)} \end{pmatrix} = \begin{pmatrix} g_1(x_1^{(4)}, x_2^{(4)}) \\ g_2(x_1^{(4)}, x_2^{(4)}) \end{pmatrix} = \begin{pmatrix} g_1(1,71662; 1,39533) \\ g_2(1,71662; 1,39533) \end{pmatrix} = \begin{pmatrix} 1,71667 \\ 1,39533 \end{pmatrix}.$$

took $\|x^{(5)} - x^{(4)}\| = 0,00005$. As on a condition $\varepsilon = 10^{-3}$, that according to an estimation (25b) it is possible to take 5 approach as the decision $\xi = x^{(5)} = \begin{pmatrix} 1,7167 \\ 1,3953 \end{pmatrix}$.

The decision of systems nonlinear the equation in MathCAD.

MathCAD gives the chance to find the decision of system of the equations numerical methods, thus the maximum number of the equations in MathCAD2001i is finished to 200.

For the decision of system of the equations it is necessary to execute following stages.

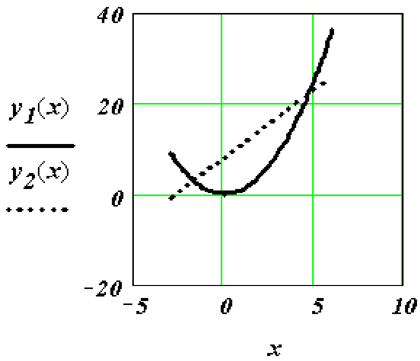
The task of initial approach for all unknown persons entering into system of the equations. At a small number of unknown persons this stage can be executed graphically, as shown in an example.

Example. The system of the equations is given:

$$y = x^2;$$
$$y = 8 + 3x.$$

To define initial approach for decisions of this system.

$$y_1(x) := x^2 \qquad x := -3, -2.95 .. 6$$
$$y_2(x) := 8 + 3 \cdot x$$



It is visible that the system has two decisions: for the first decision as initial approach the point (-2, 2), and for the second decision – a point (5, 20) can be accepted. ¨

Calculation of the decision of system of the equations with the set accuracy. Already known computing block *Given* is for this purpose used.

Function *Find* calculates the decision of system of the equations with the set accuracy, and the call of this function looks like *Find (x)*, where *x* – the list of variables on which the decision is searched. Initial values to these variables are set in the block <Entry conditions>. The number of arguments of function should be equal to number of unknown persons.

Following expressions are inadmissible in the decision block:

- Restrictions with a sign [1];
- Discrete variable or the expressions containing a discrete variable in any form;
- Blocks of the decision of the equations cannot be enclosed each other, each block can have only one keyword *Given* and a name of function *Find* (or *Minerr*).

Example. Using block *Given*, to calculate all decisions of system of the previous example. To execute check of the found decisions.

x:=-2 *y:=2* Initial approach for the first decision

Given

$y=8+3\cdot x$

$y=x^2$

$S_A:=Find(x,y)$

$$S_A = \begin{pmatrix} -1.702 \\ 2.895 \end{pmatrix}$$ *Projections of the first decision*

x:=5 *y:=20* Initial approach for the second decision

Given

$y=8+3\cdot x$

$y=x^2$

$x>0$ Restriction on positivity of a projection x the second decision

$S_B:=Find(x,y)$

$$S_B = \begin{pmatrix} 4.702 \\ 22.105 \end{pmatrix}$$ *Projections of the second decision*

Example. Using function ***Minerr***, calculate the decision of system of the equations

$$x + y = 0.95;$$
$$(x^2 + 1)^2 + (y^2 + 1)^2 = 5.5.$$

Exercises

Method of iterations to solve systems of the equations with accuracy $\varepsilon=10^{-2}$.

1. $\begin{cases} x_1^{2/3} + x_2^{2/3} = 4, \\ x_1^2 - 2x_2 = 0 \end{cases}$ $(x_1 > 0)$.
2. $\begin{cases} x_1^{2/3} + x_2^{2/3} = 4, \\ x_1^2 - 2x_2 = 0 \end{cases}$ $(x_1 < 0)$.
3. $\begin{cases} x_2 - \sqrt{x_1+1} = 0, \\ x_1^2 + x_2^2 - 2x_2 = 0 \end{cases}$ $(x_1 > 0)$.
4. $\begin{cases} x_1 \cos x_1 - x_2 = 0, \\ x_1^2 + x_2^2 - 1 = 0 \end{cases}$ $(x_1 > 0)$.
5. $\begin{cases} x_1 \cos x_1 - x_2 = 0, \\ x_1^2 + x_2^2 - 1 = 0 \end{cases}$ $(x_1 < 0)$.
6. $\begin{cases} 2x_1^2 + x_2^2 = 1, \\ x_2 - x_1^{2/3} = 0 \end{cases}$ $(x_2 > 0)$.
7. $\begin{cases} 2x_1^2 + x_2^2 = 1, \\ x_2 + x_1^{2/3} = 0 \end{cases}$ $(x_2 < 0)$.
8. $\begin{cases} x_2 - \sin x_1 = 0, \\ x_1^2 + x_2^2 - 1 = 0 \end{cases}$ $(x_1 > 0)$.
9. $\begin{cases} x_2 - \sin x_1 = 0, \\ x_1^2 + x_2^2 - 1 = 0 \end{cases}$ $(x_1 < 0)$.
10. $\begin{cases} x_1^2 + x_2^2 - 2x_2 = 0, \\ x_2 - e^{-x_1} = 0 \end{cases}$ $(x_1 < 0)$.
11. $\begin{cases} x_1^2 + x_2^2 - 2x_2 = 0, \\ x_2 - e^{-x_1} = 0 \quad (x_1 > 0). \end{cases}$
12. $\begin{cases} x_1^2 - 2x_1 + x_2^2 - 2x_2 + 1 = 0, \\ x_2 - \sqrt{x_1+1} = 0 \quad (x_1 > 0). \end{cases}$
13. $\begin{cases} x_1^2 + x_2^2 - 2x_1 = 0, \\ x_2 - \ln x_1 = 0 \end{cases}$ $(x_2 > 0)$.
14. $\begin{cases} x_1^2 + 2x_1 + x_2^2 + 2x_2 + 1 = 0, \\ x_2 - \sqrt{x_1+1} = -1. \end{cases}$
15. $\begin{cases} x_1^{2/3} + x_2^{2/3} = 1, \\ x_1^2 + x_2^2 - 2x_1 = 0 \end{cases}$ $(x_2 > 0)$.
16. $\begin{cases} x_1^{2/3} + x_2^{2/3} = 1, \\ x_2^2 + x_1^2 - 2x_1 = 0 \end{cases}$ $(x_2 < 0)$.
17. $\begin{cases} x_1 \sin x_1 - x_2 = 0, \\ x_1^2 + x_2^2 - 1 = 0 \quad (x_1 > 0). \end{cases}$
18. $\begin{cases} x_1 \sin x_1 - x_2 = 0, \\ x_1^2 + x_2^2 - 1 = 0 \quad (x_1 < 0). \end{cases}$

The note. For the curve image $(x_1^2 + x_2^2)^2 = 2(x_1^2 - x_2^2)$ (lemniscate Bernulli) to take advantage of polar co-ordinates.

Technology «the Questionnaire for a feedback»

That I have remembered on employment	That I have understood, what have understood	That it was pleasant to me, has caused interest

Control questions:

1. The decision of systems of the nonlinear equations
2. Method of simple iterations
3. Example for decision of systems of the nonlinear equations
4. The decision of systems nonlinear the equation in MathCAD.

http://pers.narod.ru/study/mathcad/07.html#start

Laboratory work № 5
Finding the decision of a problem of linear programming to Simplex methods
(2 hours)

The purpose of educational employment: preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a laboratory material to watch mastering at pupils of this knowledge, to form and develop at them skills.

The plan:

1. A simplex method of the decision of a problem of linear programming
2. Examples of the decision of a problem of linear programming with a simplex method

1. A simplex method of the decision of a problem of linear programming

Decisions of any problem of linear programming can be found either a simplex method, or a method of artificial basis. Before to apply one of the specified methods, it is necessary to write down an initial problem in the form of the primary goal of linear programming if it has no such form of record.

2. Examples of the decision of a problem of linear programming with a simplex method

1.41. For manufacturing of various products A, B and C before acceptance uses three various kinds of raw materials. Norms of the expense of raw materials on manufacture of one product of each kind, the price of one product A, B and C, and also total of raw materials of each kind which can be used the enterprise, are resulted in tab. 1.5.

Table 1.5

Raw materials kind	Norms of expenses of raw materials (kg) on one product			Total of raw materials (kg)
	A	B	C	
I	18	15	12	360
II	6	4	8	192
II	5	3	3	180
The price of one product (rbl.)	9	10	16	

Products A, B and C can be made in any parities (sale is provided), but manufacture is limited by the raw materials of each kind allocated to the enterprise.

To make the plan of manufacture of products at which the total cost of all production made by the enterprise is maximum.

The decision. We will make mathematical model of a problem. Required release of products A we will designate through x, products B - through x_2, products C - through x_3. As there are restrictions on the fund of raw materials of each kind allocated to the enterprise, variables x_1, x_2, x_3 should be satisfy to the following system of inequalities:

$$\begin{cases} 18x_1+15x_2+12x_3\leqslant 360, \\ 6x_1+4x_2+8x_3\leqslant 192, \\ 5x_1+3x_2+3x_3\leqslant 180. \end{cases} \quad (29)$$

The total cost of production made by the enterprise under condition of release x_t products A, x_2 products B and x_3 products C makes

$$F=9x_1+10x_2+16x_3. \quad (30)$$

Under the economic maintenance variables x_1, X_2 and X_3 can accept only non-negative values:

$$x_1,\ x_2,\ x_3\geqslant 0. \quad (31)$$

Thus, we come to the following mathematical problem: among all non-negative decisions of system of inequalities (29) it is required to find such at which function (30) accepts the maximum value.

Let's write down this problem in the form of the primary goal of linear programming. For this purpose we will pass from restrictions-inequalities to restrictions-equalities. We will enter three additional variables therefore restrictions will register in the form of system of the equations

$$\begin{cases} 18x_1+15x_2+12x_3+x_4=360, \\ 6x_1+4x_2+8x_3+x_5=192, \\ 5x_1+3x_2+3x_3+x_6=180. \end{cases}$$

These additional variables on economic sense mean not used at the given plan of manufacture quantity of raw materials of this or that kind. For example, x_4 is not used quantity of raw materials of I kind.

The transformed system of the equations we will write down in the vector form:

$$x_1P_1+x_2P_2+x_3P_3+x_4P_4+x_5P_5+x_6P_6=P_0,$$

where

$$P_1=\begin{pmatrix}18\\6\\5\end{pmatrix};\quad P_2=\begin{pmatrix}15\\4\\3\end{pmatrix};\quad P_3=\begin{pmatrix}12\\8\\3\end{pmatrix};\quad P_4=\begin{pmatrix}1\\0\\0\end{pmatrix};$$

$$P_5=\begin{pmatrix}0\\1\\0\end{pmatrix};\quad P_6=\begin{pmatrix}0\\0\\1\end{pmatrix};\quad P_0=\begin{pmatrix}360\\192\\180\end{pmatrix}.$$

As among vectors $P_1, P_2, P_3, P_4, P_5, P_6$ are three individual vectors, for the given problem it is possible to write down the basic plan directly. That is plan X = (0; 0; 0; 360; 192; 180), defined by system of three-dimensional individual vectors P_4, P_5, P_6, which form basis of three-dimensional vector space.

We make the simplex table for I iteration (tab. 1.6), we count up values F_0, $z_i - c_i$ and the initial basic plan for an optimality is checked:

$$F_0=(C, P_0)=0;\ z_1=(C, P_1)=0;\ z_2=(C, P_2)=0;\ z_3=(C, P_3)=0;$$

$$z_1-c_1=0-9=-9;\ z_2-c_2=0-10=-10;\ z_3-c_3=-16.$$

For basis vectors $z_i-c_i=0$.

The table 1.6

i	Базис	$C_б$	P_0	9	10	16	0	0	0
				P_1	P_2	P_3	P_4	P_5	P_6
1	P_4	0	360	18	15	12	1	0	0
2	P_5	0	192	6	4	8	0	1	0
3	P_6	0	180	5	3	3	0	0	1
4			0	−9	−10	−16	0	0	0

Apparently from tab. 1.6, values of all basic variables x_1, x_2, x_3 are equal to zero, and additional variables accept the values according to problem restrictions. These values of variables answer such "plan" at which it is made nothing, the raw materials are not used also value of criterion function to equally zero (i.e. Cost of made production is absent). This plan, of course, is not optimum.

It is visible and from 4th line of tab. 1.6 as in it is available three negative number: $z_1-c_1=-9$, $z_2-c_2=-10$ и $z_3-c_3=$-16. Negative numbers not only testify to possibility of increase in a total cost of made production, but also show, on this sum how many will increase at introduction in the plan of unit of this or that kind of production.

So, number-9 means that at inclusion in the plan of manufacture of one product And the release increase is provided - production for 9 rouble. If to include in the manufacture plan on one product In and With the total cost of produced production will increase accordingly for 10 and 16 rouble. Therefore from the economic point of view the most expedient inclusion in the plan of manufacture of products an is it is necessary to make and on the basis of a formal sign of a simplex method as the maximum negative number on absolute size Δ_i costs in 4th line of a column of vector P_3. Hence, into basis we will enter vector P_3. We define a vector which is subject to an exception of basis. It is for this purpose found θ_0=min (b_i/a_{i3}) for $a_{i3}>0$, i.e. θ_0= min (360/12; 192/8; 180/3) = 192/8.

Найдя число 192/8 = 24, мы тем самым с экономической точки зрения определили, какое количество изделий С предприятие может изготовлят с учетом норм расхода и имеющихся объемов сырья каждого вида. Так как сырья данного вида соответственно имеется 360, 192 и 180 кг, а на одно изделие С требуется затратит сырья каждого вида соответственно 12, 8 и 3 кг, то максимальное число изделий С, которое может быт изготовлено предприятием, равно min(360/12; 192/8; 180/3)=192/8=24, т. е. ограничивающим фактором для производства изделий С является имеющийся объем сырья II вида. С учетом его наличия предприятие может изготовит 24 изделия С. При этом сырье II вида будет полиостью использовано.

Having found number 192/8 = 24, we thereby from the economic point of view have defined, what quantity of products About the enterprise can produce taking into account norms of the expense and available volumes of raw materials of each kind. As raw materials of the given kind accordingly there are 360, 192 and 180 kg, and With it is required to spend for one product of raw materials of each kind accordingly 12, 8 and 3 kg the maximum number of products With which can be made the enterprise, is equal min (360/12; 192/8; 180/3) =192/8=24, i.e. Limiting factor for manufacture of products C is the available volume of raw materials of II kind. Taking into account its presence the enterprise can make 24 products C. At these raw materials of II kind полиостью it will be used.

Hence, vector P_5 is subject to an exception of basis. The column of vector P_3 and 2nd line are directing. We make the table for II iteration (tab. 1.7).

Table 1.7

i	Базис	$C_б$	P_0	9	10	16	0	0	0
				P_1	P_2	P_3	P_4	P_5	P_6
1	P_4	0	72	9	9	0	1	−3/2	0
2	P_3	16	24	3/4	1/2	1	0	1/8	0
3	P_6	0	108	11/4	3/2	0	0	−3/8	1
4			384	3	−2	0	0	2	0

At first we fill a line of the vector again entered into basis, i.e. A line, which number coincides with number of a directing line. Here 2nd line is directing. Elements of this line of tab. 1.7 turn out from corresponding elements of tab. 1.6 their division into a resolving element. Thus in column C_6 we write down factor C_3=16 standing in a column of vector P_3 entered into basis. Then we fill elements of columns for the vectors entering into new basis. In these columns on crossing of lines and columns of the vectors with the same name we put down units, and all other elements it is believed equal to zero.

To definition of other elements of tab. 1.7 it is applied a triangle rule. These elements can be calculated and is direct under recurrent formulas.

Let's calculate the elements of tab. 1.7 standing In a column of vector P_0. The first of them is in 1st line of this column. For its calculation it is found three numbers:

1) the number standing in tab. 1.6 on crossing of a column of vector P_0 and 1st line (360);
2) the number standing in tab. 1.6 on crossing of a column of a vector of Rz and 1st line (12);
3) the number standing in tab. 1.7 on crossing of a column of vector P_0 and 2nd line (24).

Subtracting from the first product of two others, we find a required element: 360-12*24=72; we write down it in 1st line of a column of vector P_0 tab. 1.7.

The second element of a column of vector P_0 tab. 1.7 has been already calculated earlier. For calculation of the third element of a column of vector P_0 also it is found three numbers. The first of them (180) is on crossing of 3rd line and a column of vector P_0 tab. 1.6, the second (3) - on crossing of 3rd line and a column of vector P_3 tab. 1.6, the third (24) - on crossing of 2nd line and a column of vector P_0 tab. 1.8. So, the specified element is 180-24·3=108. Number 108 it is written down in 3rd line of a column of a vector of P_0 of tab. 1.7.

Value F_0 can be found in 4th line of a column of the same vector in two ways:

1) under the formula $F_0=(C, P_0)$, i.e. F_0=0*72+16*24+0*108=384;
2) by a triangle rule; in this case the triangle is formed by numbers 0,-16, 24. This way results besides to result: 0 (-16) *24 = 384.

At definition by a rule of a triangle of elements of a column of a vector of P_0 the third standing in the bottom top of a triangle, all time remains invariable and first two numbers varied only. We will consider it at a finding of elements of a column of vector P_1 tab. 1.7. For calculation of the specified elements first two numbers we take from columns of vectors P_1 and P_3 tab. 1.6, and the third - from tab. 1.7. This number costs on crossing of 2nd line and a column of vector P_1 of last table. As a result we receive values of required elements: 18-12·3/4 =9; 5-3·(3/4)=11/4.

Число z_1-c_1 в 4-й строке столбsa вектора P_1 табл. 1.7 можно найти двумя способами:

The number z_1-c_1 can be found tab. 1.7 in 4th line of a column of vector P_1 in two ways:

1) under the formula $z_1-c_1=(C, P_1)-c_1$ had 0*9+16*3/4 + +0*11/4-9=3;
2) by a triangle rule we will receive -9-(-16)*(3/4)=3. Similarly we find elements of a column of vector P_2. Elements of a column of a vector of R it is calculated by a triangle rule. However constructed for definition of these elements triangles look differently.

At calculation of an element of 1st line of the specified column the triangle formed by numbers 0,12 and 1/8 turns out. Hence, the required element is equal 0-12 * (1/8) =-3/2. The element standing in 3rd line of the given column, is equal 0-3*(1/8)=-3/8.

Upon termination of calculation of all elements of tab. 1.7 in it the new basic plan and factors of decomposition of vectors $P\ (j=\overline{1,6})$ through basic vectors P_4, P_3, P_6 and values are received Δ_j' и F_0'.

Apparently from this table, the new basic plan of a problem is plan X = (0; 0; 24; 72; 0; 108). At the given plan of manufacture 24 products are produced With and remains not used 72 kg of raw materials of I kind and 108 kg of raw materials of III kind. Cost of all production made at this plan is equal 384 rbl. the Specified numbers tab. 1.7 are written down in a column of vector P_0. Apparently, the data of this column still represents parametres of a considered problem though they have undergone considerable changes. The data on other columns has changed, and their economic maintenance became more difficult. So, for example, we take the data of a column of vector P_2. Number 1/2 in 2nd line of this column shows, on how many it is necessary to reduce manufacturing of products With if to plan release of one product of Century Numbers 9 and 3/2 in 1st and 3rd lines of vector P_2 show accordingly, how many it is required raw materials I and II kind at inclusion in the plan of manufacture of one product In, and number-2 in 4th line shows that if release of one product will be planned In, it will provide output increase in cost expression for 2 rbl. Differently if to include in the production plan one product In it will demand reduction of release of a product With on 1/2 units and will demand additional expenses of raw materials of I kind of 9 kg and 3/2 kg of raw materials of III kind, and the total cost of produced production according to the new optimum plan will increase for 2 rbl. Thus, numbers 9 and 3/2 act as though as new "norms" of expenses of raw materials I and III kind on manufacturing of one product In (apparently from tab. 1.6, earlier they were equal 15 and 3) that speaks reduction of release of products With.

The same economic sense the data of a column of vector P_1 has also tab. 1.7. The numbers which have been written down in a column of vector P5 have A bit different economic maintenance. Number 1/8 in 2nd line of this column, shows that the increase in volumes of raw materials of II kind at 1 kg would allow to increase release of products C by 1/8 units Simultaneously 3/2 kg of raw materials of I kind and 3/8 kg of raw materials of III kind would be required in addition. The increase in release of products C at 1/8 units will lead to output growth иа 2 rbl.

From stated above the economic maintenance of given tab. 1.7 follows that the plan of a problem found on II iteration is not optimum. It is visible and from 4th line of tab. 1.7 as in a column of vector P_2 of this line there is a negative number-2. Means, it is necessary to enter vector P_0 into basis, i.e. In the new plan it is necessary to provide release of products of B. For definition of possible number of manufacturing of products In it is necessary to consider available quantity of raw materials of each kind, namely: possible release of products In is defined min (b_i'/a_{i2}') for $a_{i2}'>0$, i.e. find

$$0_0 = \min\left(\frac{72}{9}; \quad \frac{24\cdot 2}{1}; \quad \frac{108\cdot 2}{3}\right) = \frac{72}{9} = 8.$$

Hence, vector P_4, otherwise, release of products B is subject to an exception of basis is limited available the enterprises by raw materials of I kind. Taking into account available volumes of these raw materials the enterprise should make 8 products B. Number 9 is a resolving element, and the column of vector P_2 and 1st line of tab. 1.7 are directing. We make the table for III iteration (tab. 1.8).

Table 1.8

i	Базис	$C_б$	P_0	9	10	16	0	0	0
				P_1	P_2	P_3	P_4	P_5	P_6
1	P_2	10	8	1	1	0	1/9	−1/6	0
2	P_3	16	20	1/4	0	1	−1/18	5/24	0
3	P_6	0	96	5/4	0	0	−1/6	−1/8	1
4			400	5	0	0	2/9	5/3	0

In tab. 1.8 at first we fill elements of 1st line which represents a line of vector P_2 again entered into basis. Elements of this line it is received from elements of 1st line of tab. 1.7 by division of the last into a resolving element (i.e. On 9). Thus in column C_6 of the given line it is written down C_2=10.

Then we fill elements of columns of vectors of basis and by a triangle rule we calculate elements of other columns. As a result in tab. 1.8 new basic plan X = (0 is received; 8; 20; 0; 0; 96) and factors of decomposition of vectors P_j $(j=\overline{1,6})$ through basic vectors P_2, P_3, P_6 and corresponding values Δ_j'' и F_0''.

We check, whether the given basic plan is optimum or not. For this purpose we will consider 4th line of tab. 1.8. This line among numbers Δ_j'' are no negative. It means that the found basic plan is optimum and F_{max}=400.

Hence, the output plan including manufacturing of 8 products In and 20 product C, is optimum. At the given plan of release of products the raw materials I and II kinds completely are used and remain not used 96 kg of raw materials of III kind, and cost of made production is equal 400 rbl.

The optimum plan of production does not provide manufacturing of products A. Introduction in an output plan of products of kind A would lead to reduction of the specified total cost. It is visible from 4th line of a column of vector P_1 where number 5 shows that at the given plan inclusion in it of release of unit of a product And leads only to reduction of a combined value of cost by 5 rbl.

The decision of the given example a simplex method could be spent, using only one table (tab. 1.9). In this * to the table all three iterations of computing process are written consistently down one for another.

Table 1.9

i	Базис	$C_б$	P_0	9	10	16	0	0	0
				P_1	P_2	P_3	P_4	P_5	P_6
1	P_4	0	360	18	15	12	1	0	0
2	P_5	0	192	6	4	8	0	1	0
3	P_6	0	180	5	3	3	0	0	1
4			0	−9	−10	−16	0	0	0
1	P_4	0	72	9	9	0	1	−3/2	0
2	P_3	16	24	3/4	1/2	1	0	1/8	0
3	P_6	0	108	11/4	3/2	0	0	−3/8	1
4			384	3	−2	0	0	2	0
1	P_2	10	8	1	1	0	1/9	−1/6	0
2	P_3	16	20	1/4	0	1	−1/18	5/24	0
3	P_6	0	96	5/4	0	0	−1/6	−1/8	1
4			400	5	0	0	2/9	5/3	0

1.42. To find a function $F=2x_1-6x_2+5x_5$ maximum under conditions

$$\begin{cases} -2x_1+x_2+x_3+x_5=20, \\ -x_1-2x_2+x_4+3x_5=24, \\ 3x_1-x_2-12x_5+x_6=18, \end{cases}$$

$$x_j\geq 0 \quad (j=\overline{1,6}).$$

The decision. System of the equations of a problem we will write down in the vector form:

$$x_1P_1 + x_2P_2 + x_3P_3 + x_4P_4 + x_5P_5 + x_6P_6 = P_0,$$

where

$$P_1 = \begin{pmatrix} -2 \\ -1 \\ 3 \end{pmatrix}; \quad P_2 = \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}; \quad P_3 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}; \quad P_4 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix};$$

$$P_5 = \begin{pmatrix} 1 \\ 3 \\ -12 \end{pmatrix}; \quad P_6 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}; \quad P_0 = \begin{pmatrix} 20 \\ 24 \\ 18 \end{pmatrix}.$$

As among vectors P_1, P_2, P_3, P_4, P_5, P_6 is available three individual vectors for the given problem it is possible to find the basic plan directly. That is plan X = (0; 0; 20; 24; 0; 18). We make the simplex table (tab. 1.10) and it is checked, whether the given basic plan is optimum.

Table 1.10

i	Базис	$C_б$	P_0	2	−6	0	0	5	0
				P_1	P_2	P_3	P_4	P_5	P_6
1	P_3	0	20	−2	1	1	0	1	0
2	P_4	0	24	−1	−2	0	1	3	0
3	P_6	0	18	3	−1	0	0	−12	1
4			0	−2	6	0	0	−5	0

Apparently from tab. 1.10, the initial basic plan is not optimum. Therefore we pass to the new basic plan. It can be made, as in columns of vectors P_1 and P_5 which 4th line contains negative numbers, there are positive elements. For transition to the new basic plan we will enter into basis vector P_5 and we will exclude from basis vector P_4. We make table of II iteration.

Table 1.11

i	Базис	$C_б$	P_0	2	−6	0	0	5	0
				P_1	P_2	P_3	P_4	P_5	P_6
1	P_3	0	12	−5/3	5/3	1	−1/3	0	0
2	P_5	5	8	−1/3	−2/3	0	1/3	1	0
3	P_6	0	114	−1	−9	0	4	0	1
			40	−11/3	8/3	0	5/3	0	0

Apparently from tab. 1.11, the new basic plan of a problem is not optimum as in 4th line of a column of vector P_1 there is a negative number-11/3. As in a column of this vector there are no the positive elements, the given problem has no optimum plan.

Decisions of problem in MathCAD

Criterion function is set **$L=9x_1+2x_2 \rightarrow max$**

Condition:

$x_1+4x_2 \le 5$

$x1-x2 \le 3$

$7x_1+3x_2 \ge 7$

$x_1,x_2 \ge 0$

L(x1,x2):=9·x1+2·x2

x1:=10 x2:=10

Given

x1+4·x2≤5 x1≥0

x1-x2≤3 x2≥0

7·x1+3·x2≥7

$$\begin{pmatrix} x1 \\ x2 \end{pmatrix} := Maximize(L, x1, x2) = \begin{pmatrix} 3.4 \\ 0.4 \end{pmatrix} \quad L(x1, x2) = 31.4$$

3. Using the considered method, find the decision following problems

1.49. $F=3x_1+2x_3-6x_6\to\max;$

$$\begin{cases} 2x_1+x_2-3x_3+6x_6=18, \\ -3x_1+2x_3+x_4-2x_6=24, \\ x_1+3x_3+x_5-4x_6=36, \end{cases}$$

$$x_j\geqslant 0\ (j=\overline{1,6}).$$

1.50. $F=2x_1+3x_2-x_4\to\max;$

$$\begin{cases} 2x_1-x_2-2x_4+x_5=16, \\ 3x_1+2x_2+x_3-3x_4=18, \\ -x_1+3x_2+4x_4+x_6=24, \end{cases}$$

$$x_j\geqslant 0\ (j=\overline{1,6}).$$

1.51. $F=8x_2+7x_4+x_6\to\max;$

$$\begin{cases} x_1-2x_2-3x_4-2x_6=12, \\ 4x_2+x_3-4x_4-3x_6=12, \\ 5x_2+5x_4+x_5+x_6=25, \end{cases}$$

$$x_j\geqslant 0\ (j=\overline{1,6}).$$

1.52. $F=x_1+3x_2-5x_4\to\max;$

$$\begin{cases} 2x_1+4x_2+x_3+2x_4=28, \\ -3x_1+5x_2-3x_4+x_5=30, \\ 4x_1-2x_2+8x_4+x_6=32, \end{cases}$$

$$x_j\geqslant 0\ (j=\overline{1,6}).$$

1.53. $F=3x_1+2x_5-5x_6\to\max;$

$$\begin{cases} 2x_1+x_2-3x_5+5x_6=34, \\ 4x_1+x_3+2x_5-4x_6=28, \\ -3x_1+x_4-3x_5+6x_6=24, \end{cases}$$

$$x_j\geqslant 0\ (j=\overline{1,6}).$$

1.54. $F=x_1+2x_2-x_3\to\max;$

$$\begin{cases} -x_1+4x_2-2x_3\leqslant 6, \\ x_1+x_2+2x_3\geqslant 6, \\ 2x_1-x_2+2x_3=4, \end{cases}$$

$$x_1,\ x_2,\ x_3\geqslant 0.$$

1.55. $F=8x_1-3x_2+x_3+6x_4-5x_5\to\max;$

$$\begin{cases} 2x_1+4x_2+x_3+x_4-2x_5=28, \\ x_1-2x_2+x_4+x_5=31, \\ -x_1+3x_2+5x_3+4x_4-8x_5=118, \end{cases}$$

$$x_1,\ x_2,\ x_3,\ x_4,\ x_5\geqslant 0.$$

1.56. $F=2x_1-3x_2+4x_3+5x_4-x_5+8x_6\to\max;$

$$\begin{cases} x_1+5x_2-3x_3-4x_4+2x_5+x_6=120, \\ 2x_1+9x_2-5x_3-7x_4+4x_5+2x_6=320, \end{cases}$$

$$x_j\geqslant 0\ (j=\overline{1,6}).$$

1.57. $F=-3x_1+5x_2-3x_3+x_4+x_5+8x_6\to\max;$

$$\begin{cases} x_1-3x_2+4x_3+5x_4-6x_5+x_6=60, \\ 7x_1-17x_2+26x_3+31x_4-35x_5+6x_6=420, \end{cases}$$

$$x_j\geqslant 0\ (j=\overline{1,6}).$$

1.58. $F=5x_1-x_2+8x_3+10x_4-5x_5+x_6\to\max;$

$$\begin{cases} 2x_1-x_2+3x_4+x_5-x_6=36, \\ -x_1+2x_2+x_3+2x_4+2x_6=20, \\ 3x_2-x_2+2x_3-x_4+3x_5+x_6=30, \end{cases}$$

$$x_j\geqslant 0\ (j=\overline{1,6}).$$

Technology «the Questionnaire for a feedback»

That I have remembered on employment	That I have understood, what have understood	That it was pleasant to me, has caused interest

Control questions:

1. A simplex method of the decision of a problem of linear programming
2. Examples of the decision of a problem of linear programming with a simplex method.

SELF STUDY MATERIALS

THEMES OF ABSTRACTS (SELF STUDY WORKS)

1. Integer linear programming and its application at the decision of problems of planning of mining manufacture

- Features of integer linear problems and methods of their decision
- Use boolean variables at construction of models of integer problems of planning
- Model of planning of placing coal of the concentrating factories
- Model of operational planning of arrangement of the self-propelled equipment on clearing blocks of mine
- Problem model about cutting
- The decision of integer problems a method from sections
- The decision of integer problems a method of branches and borders
- Partial search in problems with boolean variables

2. Nonlinear programming and its use in planning and management of mining manufacture

- General characteristic, the basic types and features of problems of nonlinear programming
- Methods of the decision of problems of unconditional optimisation
- Direct methods of the decision of problems of conditional optimisation
- Methods of transformation for the decision of problems of conditional optimisation
- The approached methods of decisions of nonlinear problems

3. Dynamic optimising models of planning and management of mining manufacture

- The general statement and geometrical interpretation of dynamic problems of optimisation
- Optimality principle, the basic functional equation and order of the decision of problems a method of dynamic programming
- Problem about definition of an optimum trajectory of moving of system and its decision a method of dynamic programming
- Use of dynamic programming for the decision of static problems of distribution of resources
- Dynamic problem of distribution of resources
- Problem of search of the shortest distances on a network
- Use of dynamic programming by optimisation of alternative counts

4. Network planning and management of realisation of programs

- The basic definitions and stages of network planning and management
- Network representation of programs (network model)
- Calculation of time parametres of network model
- Construction of the planned schedule of realisation of programs
- Planned schedule optimisation on time at the limited resources
- Planned schedule optimisation on expenses
- Management of process of realisation of programs

5. Analytical models of systems of mass service

- Systems of mass service
- Order of the decision of problems of mass service
- Modelling of systems of mass service with refusals
- The opened systems of mass service with expectation
- The closed systems of mass service

6. Statistical modelling of productions

- Problems of modelling of processes and classification of types of interaction of cars and mechanisms
- Modelling of direct interaction of cars and mechanisms
- Interaction modelling through a warehouse
- Statistical modelling of systems of mass service

7. Decision-making in the conditions of uncertainty

- Elements of the theory of statistical decisions
- Choice of criterion of decision-making and definition of rational accuracy of the initial information
- The basic concepts of the theory of games
- Methods of the decision of pair games

COURSE THE PROJECT (WORK)

The academic year project (work) is not intended

THE COLLECTION OF PROBLEMS OF EXERCISES

Solve the equation with Newton's method

1. $x^3+2x^2+2=0$
2. $x^3-2x+2=0$
3. $x^3+x-3=0$
4. $x^3-0{,}2x^2+0{,}4x-1{,}4=0$
5. $x^3+3x^2+12x+3=0$
6. $x^3-0{,}1x^2++0{,}4x+1{,}2=0$
7. $x^3-0{,}2x^2+0{,}5x-1{,}4=0$
8. $x^3-3x^2+12x-12=0$
9. $x^3+4x-6=0$
10. $x^3+3x^2+6x-1=0$
11. $x^3-3x^2+6x-2=0$
12. $x^3-3x^2+12x-9=0$
13. $x^3+3x+1=0$
14. $x^3-3x^2+9x-10=0$
15. $x^3+3x-1=0$
16. $x^3+0{,}4x^2+0{,}6x-1{,}6=0$
17. $x^3-0{,}1x^2+0{,}4x-1{,}4=0$
18. $x^3-0{,}2x^2+0{,}5x-1=0$
19. $x^3-3x^2+6x-5=0$
20. $x^3+2x+4=0$
21. $x^3+0{,}2x^2+0{,}5x+0{,}8=0$
22. $x^3+0{,}1x^2+0{,}4x-1{,}2=0$
23. $x^3-0{,}1x^2+0{,}4x-1{,}5=0$
24. $x^3-0{,}2x^2+0{,}3x-1{,}2=0$
25. $x^3+0{,}2x^2+0{,}5x-2=0$
26. $x^3+0{,}2x^2+0{,}5x-1{,}2=0$

<u>Variants for laboratory works 1,2</u>

Solve the following the equation with accuracy 0,001

17) $x^3-9x^2+31x+37=0$
18) $\ln x+x+13=0$
19) 1.5cos(x-0.6)+x-2.047=0, [0;π/2]
20) $2x-1.3^x=0$, [0;10];
21) $\sqrt{\frac{\pi}{2}}e^{0,6x}+\frac{x}{0,36+x^3}=0,[-1;1]$
22) $x^2+4\sin x-1.6280819=0$, [0;1];
23) **x+lgx=0,5;**
24) **$x^3+0{,}4x^2+0{,}6x-1{,}6=0$;**
25) $x^3-9x^2+31x+30=0$
26) $\ln x+x-13=0$
27) 1.5cos(x-0.6)+x+2.047=0, [0;π/2]
28) $3x-1.3^x=0$, [0;10];
29) $x+\frac{x}{0,36+x^3}=0,[-1;1]$
30) $x^2+4\cos x-1.628=0$, [0;1];
31) **x+lnx=0,5;**
32) **$x^3+0{,}4x^2+0{,}6x-1{,}6=0$;**

Decide following equations with a method of iterations, in the middle a method of tangents, on the right a method of chords.

Table 1

N	Method	The equation			
1	K	$x+x\ln(x+0.5)-0.5=0$	6	K	$x^4+0.5x^3-4x^2-3x-0.5=0$
2	K	$x2^x-1=0$	7	X	$x-\sin(x)-0.25=0$
3	X	$x^3-2x^2+x-3=0$	8	K	$x^3-6x^2+20=0$
4	K	$x^3+12x-2=0$	9	X	$5x^3+10x^2+5x-1=0$
5	X	$5x-8\ln(x)-8=0$	10	K	$0.1x^2-x\ln(x)=0$

<u>The tasks.: Solve systems the equation with a method of Gauss.</u>

№ 1
$$\begin{cases} 4{,}4x_1 - 2{,}5x_2 + 19{,}2x_3 - 10{,}8x_4 = 4{,}3 \\ 5{,}5x_1 - 9{,}3x_2 - 14{,}2x_3 + 13{,}2x_4 = 6{,}8 \\ 7{,}1x_1 - 11{,}5x_2 + 5{,}3x_3 - 6{,}7x_4 = -1{,}8 \\ 14{,}2x_1 + 23{,}4x_2 - 8{,}8x_3 + 5{,}3x_4 = 7{,}2 \end{cases}$$

№ 2
$$\begin{cases} 8{,}2x_1 - 3{,}2x_2 + 14{,}2x_3 + 14{,}8x_4 = -8{,}4 \\ 5{,}6x_1 - 12x_2 + 15x_3 - 6{,}4x_4 = 4{,}5 \\ 5{,}7x_1 + 3{,}6x_2 - 12{,}4x_3 - 2{,}3x_4 = 3{,}3 \\ 6{,}8x_1 + 13{,}2x_2 - 6{,}3x_3 - 8{,}7x_4 = 14{,}3 \end{cases}$$

№ 3
$$\begin{cases} 5{,}7x_1 - 7{,}8x_2 - 5{,}6x_3 - 8{,}3x_4 = 2{,}7 \\ 6{,}6x_1 + 13{,}1x_2 - 6{,}3x_3 + 4{,}3x_4 = -5{,}5 \\ 14{,}7x_1 - 2{,}8x_2 + 5{,}6x_3 - 12{,}1x_4 = 8{,}6 \\ 8{,}5x_1 + 12{,}7x_2 - 23{,}7x_3 + 5{,}7x_4 = 14{,}7 \end{cases}$$

№ 4
$$\begin{cases} 3{,}8x_1 + 14{,}2x_2 + 6{,}3x_3 - 15{,}5x_4 = 2{,}8 \\ 8{,}3x_1 - 6{,}6x_2 + 5{,}8x_3 + 12{,}2x_4 = -4{,}7 \\ 6{,}4x_1 - 8{,}5x_2 - 4{,}3x_3 + 8{,}8x_4 = 7{,}7 \\ 17{,}1x_1 - 8{,}3x_2 + 14{,}4x_3 - 7{,}2x_4 = 13{,}5 \end{cases}$$

№ 5
$$\begin{cases} 15{,}7x_1 + 6{,}6x_2 - 5{,}7x_3 + 11{,}5x_4 = -2{,}4 \\ 8{,}8x_1 - 6{,}7x_2 + 5{,}5x_3 - 4{,}5x_4 = 5{,}6 \\ 6{,}3x_1 - 5{,}7x_2 - 23{,}4x_3 + 6{,}6x_4 = 7{,}7 \\ 14{,}3x_1 + 8{,}7x_2 - 15{,}7x_3 - 5{,}8x_4 = 23{,}4 \end{cases}$$

№ 6
$$\begin{cases} 4{,}3x_1 - 12{,}1x_2 + 23{,}2x_3 - 14{,}1x_4 = 15{,}5 \\ 2{,}4x_1 - 4{,}4x_2 + 3{,}5x_3 + 5{,}5x_4 = 2{,}5 \\ 5{,}4x_1 + 8{,}3x_2 - 7{,}4x_3 - 12{,}7x_4 = 8{,}6 \\ 6{,}3x_1 - 7{,}6x_2 + 1{,}34x_3 + 3{,}7x_4 = 12{,}1 \end{cases}$$

Solve system of the linear algebraic equations of Ah=b with a method of Gauss. To compare to the exact decision ξ.

1. $A = \begin{bmatrix} 5 & 0 & 1 \\ 1 & 3 & -1 \\ -3 & 2 & 10 \end{bmatrix}$, $b = \begin{bmatrix} 11 \\ 4 \\ 6 \end{bmatrix}$, $\xi = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$.

2. $A = \begin{bmatrix} 2 & 0 & -1 \\ -1 & 3 & 1 \\ 1 & -1 & 4 \end{bmatrix}$, $b = \begin{bmatrix} -3 \\ 2 \\ 3 \end{bmatrix}$, $\xi = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$.

3. $A = \begin{bmatrix} 2 & 0 & -1 \\ 1 & -3 & 1 \\ 1 & 1 & 3 \end{bmatrix}$, $b = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$, $\xi = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$.

4. $A = \begin{bmatrix} 5 & 1 & -1 \\ -1 & 3 & 1 \\ 1 & -2 & 4 \end{bmatrix}$, $b = \begin{bmatrix} -5 \\ 5 \\ 1 \end{bmatrix}$, $\xi = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$.

5. $A = \begin{bmatrix} 3 & 1 & -1 \\ -2 & 4 & 1 \\ 1 & 1 & 3 \end{bmatrix}$, $b = \begin{bmatrix} -1 \\ 5 \\ -3 \end{bmatrix}$, $\xi = \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$.

6. $A = \begin{bmatrix} 3 & 1 & -1 \\ 2 & 4 & 1 \\ 1 & -1 & 3 \end{bmatrix}$, $b = \begin{bmatrix} 6 \\ 9 \\ 4 \end{bmatrix}$, $\xi = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$.

7. $A = \begin{bmatrix} 2 & -1 & 0 \\ 2 & 5 & -2 \\ 1 & -1 & 3 \end{bmatrix}$, $b = \begin{bmatrix} -2 \\ -4 \\ 2 \end{bmatrix}$, $\xi = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$.

8. $A = \begin{bmatrix} 3 & -1 & 1 \\ 0 & 2 & -1 \\ -1 & 1 & 5 \end{bmatrix}$, $b = \begin{bmatrix} 1 \\ 3 \\ -5 \end{bmatrix}$, $\xi = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$.

9. $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & 0 \\ 1 & -1 & 5 \end{bmatrix}$, $b = \begin{bmatrix} 7 \\ 7 \\ 11 \end{bmatrix}$, $\xi = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$.

The decision of system of the linear algebraic equations a method of Seidel

№ вар.	a_{1i}	a_{2i}	a_{3i}	b_{1i}
1	0.35 0.12 - 0.13	0.12 0.71 0.15	- 0.13 0.15 0.63	0.10 0.26 0.38
2	0.71 0.10 - 0.10	0.10 0.34 0.64	0.12 - 0.04 0.56	0.29 0.32 - 0.10
3	0.34 - 0.04 0.06	- 0.04 0.44 0.56	0.10 - 0.12 0.39	0.33 - 0.05 0.28
4	0.10 - 0.04 - 0.43	- 0.04 0.34 0.05	- 0.63 0.05 0.13	- 0.15 0.31 0.37
5	0.63 0.05 0.15	0.05 0.34 0.10	0.15 0.10 0.71	0.34 0.32 0.42
6	1.20 - 0.50 - 0.30	- 0.20 1.70 0.10	0.30 - 1.60 - 1.50	- 0.60 0.30 0.40
7	0.30 - 0.10 - 1.50	1.20 - 0.20 - 0.30	- 0.20 1.60 0.10	- 0.60 0.30 0.70
	0.20	0.44	0.91	0.74

8	0.58 0.05	- 0.29 0.34	0.05 0.10	0.02 0.32
9	6.36 7.42 1.77	1.75 19.03 0.42	1.0 1.75 6.36	41.70 49.49 27.67
10	3.11 - 1.65 0.60	- 1.66 3.15 0.78	- 0.60 - 0.78 - 2.97	- 0.92 2.57 1.65

Method of iterations to solve systems of the equations with accuracy $\varepsilon=10^{-2}$.

1. $\begin{cases} x_1^{2/3} + x_2^{2/3} = 4, \\ x_1^2 - 2x_2 = 0 \end{cases} (x_1 > 0).$ 2. $\begin{cases} x_1^{2/3} + x_2^{2/3} = 4, \\ x_1^2 - 2x_2 = 0 \end{cases} (x_1 < 0).$

3. $\begin{cases} x_2 - \sqrt{x_1+1} = 0, \\ x_1^2 + x_2^2 - 2x_2 = 0 \end{cases} (x_1 > 0).$ 4. $\begin{cases} x_1 \cos x_1 - x_2 = 0, \\ x_1^2 + x_2^2 - 1 = 0 \end{cases} (x_1 > 0).$

5. $\begin{cases} x_1 \cos x_1 - x_2 = 0, \\ x_1^2 + x_2^2 - 1 = 0 \end{cases} (x_1 < 0).$ 6. $\begin{cases} 2x_1^2 + x_2^2 = 1, \\ x_2 - x_1^{2/3} = 0 \end{cases} (x_2 > 0).$

7. $\begin{cases} 2x_1^2 + x_2^2 = 1, \\ x_2 + x_1^{2/3} = 0 \end{cases} (x_2 < 0).$ 8. $\begin{cases} x_2 - \sin x_1 = 0, \\ x_1^2 + x_2^2 - 1 = 0 \end{cases} (x_1 > 0).$

9. $\begin{cases} x_2 - \sin x_1 = 0, \\ x_1^2 + x_2^2 - 1 = 0 \end{cases} (x_1 < 0).$ 10. $\begin{cases} x_1^2 + x_2^2 - 2x_2 = 0, \\ x_2 - e^{-x_1} = 0 \end{cases} (x_1 < 0).$

11. $\begin{cases} x_1^2 + x_2^2 - 2x_2 = 0, \\ x_2 - e^{-x_1} = 0 \quad (x_1 > 0). \end{cases}$ 12. $\begin{cases} x_1^2 - 2x_1 + x_2^2 - 2x_2 + 1 = 0, \\ x_2 - \sqrt{x_1+1} = 0 \quad (x_1 > 0). \end{cases}$

13. $\begin{cases} x_1^2 + x_2^2 - 2x_1 = 0, \\ x_2 - \ln x_1 = 0 \end{cases} (x_2 > 0).$ 14. $\begin{cases} x_1^2 + 2x_1 + x_2^2 + 2x_2 + 1 = 0, \\ x_2 - \sqrt{x_1+1} = -1. \end{cases}$

15. $\begin{cases} x_1^{2/3} + x_2^{2/3} = 1, \\ x_1^2 + x_2^2 - 2x_1 = 0 \end{cases} (x_2 > 0).$ 16. $\begin{cases} x_1^{2/3} + x_2^{2/3} = 1, \\ x_2^2 + x_1^2 - 2x_1 = 0 \end{cases} (x_2 < 0).$

17. $\begin{cases} x_1 \sin x_1 - x_2 = 0, \\ x_1^2 + x_2^2 - 1 = 0 \quad (x_1 > 0). \end{cases}$ 18. $\begin{cases} x_1 \sin x_1 - x_2 = 0, \\ x_1^2 + x_2^2 - 1 = 0 \quad (x_1 < 0). \end{cases}$

The note. For the curve image $(x_1^2 + x_2^2)^2 = 2(x_1^2 - x_2^2)$ (lemniscate Bernulli) to take advantage of polar co-ordinates.

The task

To write the program of the decision of the differential equation *y ' =f (x, y)* Euler's method on a piece [a,b] with step h and *2h* and the entry condition *y(a) =y_0*. The Initial data for task performance undertakes from table 5. To compare results.

The task

To write the program of the decision of the differential equation *y ' =f (x, y) a* method of Runge-Kutta on a piece [a,b] with step h and *2h* and the entry condition *y (a) =y_0*. To estimate an error under the formula (5.5). The initial data for task performance undertakes from table 5.

Table 5

N	*Function*	*a*	*b*	y_0	*h*
1	$\frac{3x-y}{x^2+y}$	2	3	1	0.1
2	$\frac{2x+y+4}{2y+x}$	3	4	1	0.1
3	$\frac{x^2-y}{2x+y+1}$	0	1	2	0.1
4	$\frac{x^2-y+2}{xy+3x}$	2	3	1	0.1
5	$\frac{3-x-y^2}{2-xy^2}$	1	2	1	0.1
6	$\frac{2-x-y^2x}{3x+y}$	0	1	1	0.1
7	$\frac{1+3xy}{5-x+y^2}$	0	1	2	0.1
8	$\frac{x^2y+2}{2x-y}$	0	1	1	0.1
9	$\frac{x^2+y+2}{2x-y}$	2	3	2	0.1
10	$\frac{xy+4}{2y-xy+1}$	0	1	3	0.1

Under the set table of values of function to make the formula interpolation a multinomial of Lagrange (3.2) and to construct the schedule $L_2(x)$. The initial data undertakes from table 3.1.

$$L_2(x) = y_0 \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} + y_1 \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} + y_2 \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} \qquad (3.2)$$

Table 3.1.

№	x_0	x_1	x_2	y_0	y_1	y_2
1	2	3	5	4	1	7
2	4	2	3	5	2	8
3	0	2	3	-1	-4	2
4	7	9	13	2	-2	3
5	-3	-1	3	7	-1	4
6	1	2	4	-3	-7	2
7	-2	-1	2	4	9	1
8	2	4	5	9	-3	6
9	-4	-2	0	2	8	5
10	-1	1.5	3	4	-7	1
11	2	4	7	-1	-6	3
12	-9	-7	-4	3	-3	4
13	0	1	4	7	-1	8
14	8	5	0	9	2	4
15	-7	-5	-4	4	-4	5

Calculate the set integrals under formulas of rectangles, a trapeze and Simpson if the integration piece is broken on n=2 and n=4 equal parts. To estimate an error of result and сравныть the approached values of integral with the exact.

1. $\int_0^1 \frac{dx}{1+x^2} \left(\mathfrak{I} = \frac{\pi}{4} \approx 0{,}785 \right)$. 2. $\int_0^1 \frac{dx}{1+x}$ $(\mathfrak{I} = \ln 2 \approx 0{,}693)$.

3. $\int_0^{\pi/4} \sin 4x \, dx$ $(\mathfrak{I} = 0{,}5)$. 4. $\int_0^1 \frac{dx}{\sqrt{1+x^2}}$ $(\mathfrak{I} = \ln(1+\sqrt{2}) \approx 0{,}881)$.

5. $\int_1^e \ln x \, dx$ $(\mathfrak{I} = 1)$. 6. $\int_0^1 \ln(x+1) dx$ $(\mathfrak{I} = 2\ln 2 - 1 \approx 0{,}386)$.

7. $\int_0^{\pi/2} x \cos x \, dx$ $(\mathfrak{I} = \frac{\pi}{2} - 1 \approx 0{,}571)$. 8. $\int_0^1 \frac{e^x dx}{1+e^{2x}}$ $(\mathfrak{I} = \mathrm{arctg}\, e - \frac{\pi}{4} \approx 0{,}433)$.

9. $\int_0^{\pi} \cos^3 x \, dx$ $(\mathfrak{I} = 0)$. 10. $\int_0^{\pi/4} \frac{dx}{\cos x}$ $(\mathfrak{I} = \ln(1+\sqrt{2}) \approx 0{,}881)$.

11. $\int_0^{\pi/2} \frac{dx}{1+\sin x}$ $(\mathfrak{I} = 1)$. 12. $\int_0^1 \mathrm{arctg}\, x \, dx$ $(\mathfrak{I} = \frac{1}{4}(\pi - 2\ln 2) \approx 0{,}438)$.

13. $\int_0^1 \frac{dx}{1+e^x}$ $(\mathfrak{I} \approx 0{,}38)$. 14. $\int_0^{\sqrt{2}/2} \arcsin x \, dx$ $(\mathfrak{I} = \frac{\sqrt{2}}{8}(\pi+4) - 1 \approx 0{,}26)$.

15. $\int_0^{\pi/4} \mathrm{tg}\, x \, dx$ $(\mathfrak{I} = \frac{1}{2}\ln 2 \approx 0{,}346)$. 16. $\int_{\pi/4}^{\pi/2} \mathrm{ctg}\, x \, dx$ $(\mathfrak{I} = \frac{1}{2}\ln 2 \approx 0{,}346)$.

17. $\int_0^1 x e^x \, dx$ $(\mathfrak{I} = 1)$. 18. $\int_0^1 \sqrt{1+x} \, dx$ $(\mathfrak{I} = \frac{2}{3}(2\sqrt{2} - 1) \approx 1{,}22)$.

To make the program for a finding of approaching functions of the set type with a conclusion of values of their parametres and the sums of squares of evasion corresponding to them. To choose as approaching functions the following: $y = ax + b$, $y = ax^m$, $y = ae^{mx}$. To spend linearization. To define for what kind of function the sum of squares of evasion is the least.

Initial data is placed in table 6.

Approximate fragment of performance of laboratory work

T

№ \ *i*		1	2	3	4	5	6	7	8	9	10
1	*x*	0.5	0.1	0.4	0.2	0.6	0.3	0.4	0.7	0.3	0.8
	y	1.8	1.1	1.8	1.4	2.1	1.8	1.6	2.2	1.5	2.3
2	*x*	1.7	1.5	3.7	1.1	6.2	0.3	6.5	3.6	3.8	5.9
	y	1.5	1.4	1.6	1.3	2.1	1.1	2.2	1.8	1.7	2.3

3	x	1.7	1.1	1.6	1.2	1.9	1.5	1.8	1.4	1.3	1.0
	y	6.7	5.6	6.7	6.1	7.4	6.9	7.9	5.9	5.6	5.3
4	x	1.3	1.2	1.5	1.4	1.9	1.1	2.0	1.6	1.7	1.8
	y	5.5	5.9	6.3	5.8	7.4	5.4	7.6	6.9	6.6	7.5
5	x	2.3	1.4	1.0	1.9	1.5	1.8	2.1	1.6	1.7	1.3
	y	5.3	3.9	2.9	5.0	4.0	4.9	5.1	4.5	4.1	3.7
6	x	1.8	2.6	2.3	1.3	2.0	2.1	1.1	1.9	1.6	1.5
	y	4.4	6.4	5.3	3.7	4.9	5.6	3.0	5.0	4.3	3.7
7	x	1.9	2.1	2.0	2.9	3.0	2.6	2.5	2.7	2.2	2.8
	y	6.6	7.6	6.7	9.2	9.4	7.8	8.4	8.0	7.9	8.7
8	x	2.0	1.4	1.0	1.7	1.3	1.6	1.9	1.5	1.2	2.1
	y	7.5	6.1	4.8	7.4	5.7	7.0	7.1	6.8	6.0	8.9
9	x	2.0	1.2	1.8	1.9	1.1	1.7	1.6	1.4	1.5	1.3
	y	7.5	5.9	7.0	8.0	5.0	7.4	6.4	6.6	6.3	5.7
10	x	1.9	1.1	1.4	2.3	1.7	2.1	1.6	1.5	1.0	1.2
	y	4.7	3.4	3.8	5.2	4.6	5.5	3.9	3.9	3.2	3.5

2. Using geometrical interpretation, find decisions of linear programming tasks

1.32. $F = x_1 + x_2 \to \max;$

$$\begin{cases} x_1 + 2x_2 \leqslant 14, \\ -5x_1 + 3x_2 \leqslant 15, \\ 4x_1 + 6x_2 \geqslant 24, \end{cases}$$
$$x_1,\ x_2 \geqslant 0.$$

1.33. $F = x_1 + 2x_2 \to \max;$

$$\begin{cases} 4x_1 - 2x_2 \leqslant 12, \\ -x_1 + 3x_2 \leqslant 6, \\ 2x_1 + 4x_2 \geqslant 16, \end{cases}$$
$$x_1,\ x_2 \geqslant 0.$$

1.34. $F = -2x_1 + x_2 \to \min;$

$$\begin{cases} 3x_1 - 2x_2 \leqslant 12, \\ -x_1 + 2x_2 \leqslant 8, \\ 2x_1 + 3x_2 \geqslant 6, \end{cases}$$
$$x_1,\ x_2 \geqslant 0.$$

1.35. $F = -x_1 + 4x_2 + 2x_4 - x_5 \to \max;$

$$\begin{cases} x_1 - 5x_2 + x_3 = 5, \\ -x_1 + x_2 + x_4 = 4, \\ x_1 + x_2 + x_5 = 8. \end{cases}$$
$$x_1,\ x_2,\ \ldots,\ x_5 \geqslant 0.$$

1.36. $F = -5x_1 + x_2 - x_3 \to \max;$

$$\begin{cases} 3x_1 - x_2 - x_3 = 4, \\ x_1 - x_2 + x_3 - x_4 = 1, \\ 2x_1 + x_2 + 2x_3 + x_5 = 7. \end{cases}$$
$$x_1,\ x_2,\ \ldots,\ x_5 \geqslant 0.$$

3. Using the Simplex method, find the decision following linear programming problems

1.49. $F = 3x_1 + 2x_3 - 6x_6 \to \max;$

$$\begin{cases} 2x_1 + x_2 - 3x_3 + 6x_6 = 18, \\ -3x_1 + 2x_3 + x_4 - 2x_6 = 24, \\ x_1 + 3x_3 + x_5 - 4x_6 = 36, \end{cases}$$
$$x_j \geqslant 0\ (j = \overline{1,6}).$$

1.50. $F = 2x_1 + 3x_2 - x_4 \to \max;$

$$\begin{cases} 2x_1 - x_2 - 2x_4 + x_5 = 16, \\ 3x_1 + 2x_2 + x_3 - 3x_4 = 18, \\ -x_1 + 3x_2 + 4x_4 + x_6 = 24, \end{cases}$$
$$x_j \geqslant 0\ (j = \overline{1,6}).$$

1.51. $F = 8x_2 + 7x_4 + x_6 \to \max;$

$$\begin{cases} x_1 - 2x_2 - 3x_4 - 2x_6 = 12, \\ 4x_2 + x_3 - 4x_4 - 3x_6 = 12, \\ 5x_2 + 5x_4 + x_5 + x_6 = 25, \end{cases}$$
$$x_j \geqslant 0\ (j = \overline{1,6}).$$

1.52. $F = x_1 + 3x_2 - 5x_4 \to \max;$

$$\begin{cases} 2x_1 + 4x_2 + x_3 + 2x_4 = 28, \\ -3x_1 + 5x_2 - 3x_4 + x_5 = 30, \\ 4x_1 - 2x_2 + 8x_4 + x_6 = 32, \end{cases}$$
$$x_i \geqslant 0\ (i = \overline{1,6}).$$

1.53. $F = 3x_1 + 2x_5 - 5x_6 \to \max;$

$$\begin{cases} 2x_1 + x_2 - 3x_5 + 5x_6 = 34, \\ 4x_1 + x_3 + 2x_5 - 4x_6 = 28, \\ -3x_1 + x_4 - 3x_5 + 6x_6 = 24, \end{cases}$$
$$x_j \geqslant 0\ (j = \overline{1,6}).$$

1.54. $F = x_1 + 2x_2 - x_3 \to \max;$

$$\begin{cases} -x_1 + 4x_2 - 2x_3 \leqslant 6, \\ x_1 + x_2 + 2x_3 \geqslant 6, \\ 2x_1 - x_2 + 2x_3 = 4, \end{cases}$$
$$x_1,\ x_2,\ x_3 \geqslant 0.$$

1.55. $F = 8x_1 - 3x_2 + x_3 + 6x_4 - 5x_5 \to \max;$

$$\begin{cases} 2x_1 + 4x_2 + x_3 + x_4 - 2x_5 = 28, \\ x_1 - 2x_2 + x_4 + x_5 = 31, \\ -x_1 + 3x_2 + 5x_3 + 4x_4 - 8x_5 = 118, \end{cases}$$
$$x_1,\ x_2,\ x_3,\ x_4,\ x_5 \geqslant 0.$$

1.56. $F = 2x_1 - 3x_2 + 4x_3 + 5x_4 - x_5 + 8x_6 \to \max;$

$$\begin{cases} x_1 + 5x_2 - 3x_3 - 4x_4 + 2x_5 + x_6 = 120, \\ 2x_1 + 9x_2 - 5x_3 - 7x_4 + 4x_5 + 2x_6 = 320, \end{cases}$$
$$x_j \geqslant 0\ (j = \overline{1,6}).$$

1.57. $F = -3x_1 + 5x_2 - 3x_3 + x_4 + x_5 + 8x_6 \to \max;$

$$\begin{cases} x_1 - 3x_2 + 4x_3 + 5x_4 - 6x_5 + x_6 = 60, \\ 7x_1 - 17x_2 + 26x_3 + 31x_4 - 35x_5 + 6x_6 = 420, \end{cases}$$
$$x_j \geqslant 0\ (j = \overline{1,6}).$$

1.58. $F = 5x_1 - x_2 + 8x_3 + 10x_4 - 5x_5 + x_6 \to \max;$

$$\begin{cases} 2x_1 - x_2 + 3x_4 + x_5 - x_6 = 36, \\ -x_1 + 2x_2 + x_3 + 2x_4 + 2x_6 = 20, \\ 3x_2 - x_2 + 2x_3 - x_4 + 3x_5 + x_6 = 30, \end{cases}$$
$$x_j \geqslant 0\ (j = \overline{1,6}).$$

1.37. For manufacture of tables and cases the furniture factory uses necessary resources. Norms of expenses of resources on one product of the given kind, profit on realisation of one product and total of available resources of each kind are resulted in the following table:

Resources	Norms of expenditures of resources on one product		The General an amount of resources
	Table	The Case	
Wood (м3): I aspect	0,2	0,1	40
II aspect	0,1	0,3	60
Labour input (person-hour)	1,2	1,5	371,4
Profit on realisation of one product (rouble.)	6	8	

To define, how many tables and cases the factory should produce, that the profit on their realisation was maximum.

1.38. For manufacture of two kinds of products A and B the turning, milling and grinding equipment is used. Norms of expenses of time for each of equipment types on one product of the given kind are resulted in the table. In it the general fund of working hours of each of equipment types, and also profit on realisation of one product is specified.

Type the equipment	Expenditures of time (machine tool-hour) on working off of one product		The General fund of useful working hours of the equipment (hour)
	A	B	
The milling	10	8	168
The turning	5	10	180
The grinding	6	12	144
Profit on realisation of one product (roub.)	14	18	

To find the plan of release of products A and B, providing the maximum profit on their realisation.

1.39. At furniture factory it is necessary to cut out preparations of three kinds from plywood standard sheets in the quantities accordingly equal of 24, 31 and 18 pieces Each sheet of plywood can be cut on for
Cooking by two ways. The quantity of received preparations at the given way раскроя is resulted in the table. In it the size of a waste which turn out at the given way раскроя one sheet of plywood is specified.

Aspect preform	Amount of preforms (piece) in open on a mode	
	1	2
I	2	6
II	5	4
III	2	3
Magnitude of a waste (sm^3)	12	16

To define, how many sheets of plywood and on what way follow раскроить so that has been received not less the necessary quantity of preparations at the minimum waste.

1.40. On a fur farm silver foxes and polar foxes can be grown up. For maintenance of normal conditions of their cultivation it is used three kinds of forages. The quantity of a forage of each kind which foxes and polar foxes should receive daily, is resulted in the table. In it are specified total of a forage of each kind which can be used a fur farm, and profit on realisation of one skin of a fox and a polar fox.

Feed kind	Quantity of units of a feed which daily should receive		Feed Total
	fox	A polar fox	
I	2	3	180
II	4	1	240
III	6	7	426
Profit on realisation of one glass-paper (roub.)	16	12	

To define, how many foxes and polar foxes should be grown up on a fur farm that the profit on realisation of their skins was maximum.

GLOSSARY

Absolute error-The size equal to a difference between true value of number and its approached value, received as a result of calculation or measurement

Absolute deviation-The deviation equal to the maximum value of absolute size of a difference between approximating and initial functions on the given piece

Adaptive (adapting) algorithms-The algorithms capable automatically to adapt to character of change of function

Adequacy of mathematical model-The basic requirement shown to mathematical model of the considered phenomenon, consisting that the model should precisely enough (within the limits of admissible errors) to reflect characteristic features of the phenomenon

Approximation-Function approach at which *f (x)* it is required to replace the given function approximately with some function *φ (υ)* that, That a deviation (somewhat) *φ(x)* from *f(x)* in the set area was the least

Approximating function-Function with which initial function at approximation is replaced

Global interpolation-Interpolation, which interpolating function *φ(x)* is under construction at once for all considered interval of change *x*

$$\delta(a) = \frac{|A-a|}{|a|}$$

The problem put correctly-Problem in which for any values of the initial data from some class its decision exists, it is unique and it is steady under the initial data

Significant figures-All figures of the given number, since the first nonzero figure

Integrated (or continuous) approximation-Approximation at approach construction on continuous set of points

Интерполирование-Type of dot approximation, at which *interpolating function* $\varphi(x)$, Initial function *f (x)* accepts in the set points *xi the* same values *yi*, as

Iteration-Repeated repetition of process consecutive approximate

Square-law (parabolic) interpolation-Interpolation at which in quality интерполяционной functions on a piece [*x*i-1, *x*i+1] the square trinomial is accepted

Correct numerical algorithm (method)-The numerical algorithm (method) having the unique numerical decision at any values of the initial data, and also in case of stability of this decision concerning errors of the initial data

Piece (local) interpolation-Interpolation, at which interpolating function *φ(x)* Is under construction separately for different parts of a considered interval of change*x*

Kusochno-linear (or linear) interpolation-The elementary and often used kind of local interpolation at which the set points incorporate rectilinear pieces, and function comes nearer a broken line with tops in the given points

Method of splines-One of methods of numerical integration, especially effective at strictly limited number of knots

Ineradicable errors-Errors, которыене can be reduced the calculator neither prior to the beginning of the problem decision, nor in the course of its decision

Certain integral from function *f (x)* on a piece *φ (x)*-Limit of the integrated sum at such unlimited increase in number of points of splitting at which the length of greatest of elementary pieces aspires to zero

Relative error-The relation of an absolute error to the approached value of number

Parabolic (square-law) interpolation-Interpolation, at which in quality интерполяционной functions on a piece [*x*i-1, *x*i+1] the square trinomial is accepted

Error of approximation of a derivative-The size characterising a deviation of approached value derivative of its true value

Error of restriction of the function received by means of a number-Error arising because of the account of only limited number of members of a number

Error of roundings off-The error connected with limitation of a digit grid of the computer

Adapting (adaptive) algorithms-The algorithms capable automatically to adapt to character of change of function

Function derivative y = *f(x)*-Limit of the relation of an increment of function Δy to an argument increment Δx at aspiration Δx to zero

Spline-function-In special way constructed multinomial of the third degree

Convergence of a numerical method-Aspiration of values of the decision of discrete model of a problem to corresponding values of the decision of an initial problem at aspiration to zero of parametre of digitization

Dot approximation-Approximation at which approach is under construction on the set discrete set of points $\{xi\}$

Steady problem (on initial parameter *x*)-Problem, in which small increment of initial size Δx leads to a small increment of required size Δy

Function odd concerning a point x_0-Function for which $f(x\text{-}x0) = -f(x0\text{-}x)$

Function even concerning a point x_0-Function for which $f(x\text{-}x0) = f(x0\text{-}x)$

Numerical methods-Methods of the decision of the difficult mathematical problems, allowing to reduce the problem decision to performance of final number of arithmetic actions over numbers; thus results turn out in the form of numerical values

Step-Difference between the next values of argument

Extrapolation-интерполирование, applied to the approached function evaluation out of a considered piece ($x<x0$, $x>xn$)

The software interface is a set of tools that interact with devices and software in a computer system.

Stopwatch computers (DHM) are computational computers that operate both digital and analogous; They have the advantages of UHM and SRC.

Informatics is an area of human activity connected with updating information with the help of computers and their application environment.

Workstations are powerful microscopes that are used by a person specializing in certain types of work (graphics, engineering, publishing, etc.).

Cybernetics is science, technology, biological, social and other systems.

Keyboard - a device designed to enter text and managed information manually on a PC.

Shell is a program that is split between any application and user or other software.

The operating system (OS) is a set of software that manages the resources of the computer, displays applications and interacts with external devices and other software, and also interacts with the user's computer.

Digital computers (RHM) are a discrete computer machine that operates in discrete, accurate digital information.

Servers are powerful microEUs that many use to split requests from all stations.

Telecommunications is a remote data transmission based on computer networks and modern technical communications.

The user interface is software and hardware that interacts with user software or exposure.

GLOSSARIY

Matematik model xatoligi – real jarayonning matematik tavsiflanishi naoniqligidan kelib chiqadigan xatolik.

Boshlang‘ich ma’lumotlar xatoligi – boshlang‘ich ma’lumotlarning noaniqligi tufayli yuzaga keladigan xatolik;

Uslub xatoligi – masalani yechishda qo‘llanilayotgan uslublarning noaniqligidan kelib chiqadigan xatolik;

Hisoblash xatoligi- hisoblashlarda vujudga keladigan xatoliklar;

Yaxlitlash xatoligi – yaxlitlash natijasida hosil bo‘ladigan xatolik.

Xatolik - hisoblashlarda qatnashayotgan taqribiy a son bilan shu sonning aniq qiymati A orasidagi farq (A – a) aytiladi. Agar A>*a* bo‘lsa, xatolik musbat va A<*a* bo‘lsa, xatolik manfiy bo‘ladi. Xatoliklarni baholash to‘g‘ri bo‘lishi uchun absolyut xatolik tushunchasi kiritiladi.

Xatolikning moduliga a taqribiy sonning **absolyut xato**si deyiladi va Δa kabi belgilanadi, ya’ni

$$\Delta a = |A - a|$$

Taqribiy a soni absolyut xatoligining shu son moduliga nisbati a taqribiy sonning **nisbiy xato**ligi deyiladi va $\delta(a)$ kabi belgilanadi, ya’ni

$$\delta(a) = \frac{|A-a|}{|a|}$$

Aniq A son noma’lum bo‘lganligi sababli absolyut va nisbiy xatoliklar ham noma’lum bo‘ladi, shuning uchun xatolikning chegarasi ko‘rsatiladi.

$|A - a| \le h$ tengsizlikni qanoatlantiruvchi h kattalik **absolyut xatolikning chegarasi** deyiladi.

$\frac{|A-a|}{|a|} \le \varepsilon$ tengsizlikni qanoatlantiruvchi ε soni **nisbiy xatolikning chegarasi** deyiladi.

Maple sistemasi - kompyuterda turli yo’nalishdagi: iqtisodiyot, mexanika, matematika, fizika, muhandislik masalalarining analitik va sonli yechimlarini aniq, tez, samarali hal etish uchun mo’ljallangan sistemadir

MathCAD paketi muxandislik hisob ishlarini bajarish uchun dasturiy vosita bo`lib, u professional matematiklar uchun mo`ljallangan. Uning yordamida o`zgaruvchi va o`zgarmas parametrli algebraik va differentsial tenglamalarni yechish, funksiyalarni tahlil qilish va ularning ekstremumini izlash, topilgan yechimlarni tahlil qilish uchun jadvallar va grafiklar qurish mumkin.

MATLAB sistemasi - kompyuterda turli yo’nalishdagi: mexanika, matematika, fizika, muxandislik va boshqaruv masalalarini yechish, turli xil mexaniq, energetik va dinamik sistemalarni modellashtirish, loyihalash, tavsiflash va tahlil qilish masalalarining aniq, tez, samarali hal etish uchun mo’ljallangan sistema va turli xil sohali foydalanuvchilarga muljallangan dasturlash tilidir.

Massiv – bir turdagi ma’lumotlarning raqamlangan va tartiblangan to’plamidir.

Amaliy dasturlar paketi (ADP) – bu muayyan (funksional tizimosti, biznes- ilova) sinf vazifalarini hal etish uchun mo’ljallangan dasturlar majmui.

Axborot jamiyati – ko’pchilik ishlovchilarning axborot, ayniqsa uning oliy shakli bo’lmish bilimlarni ishlab chiqarish, saqlash, qayta ishlash va amalga oshirish bilan band bo’lgan jamiyatidir.

Axborot zahiralari – aloxida hujjat va alohida hujjat to’plami, axborot tizimlari (kutubxona, arxiv, fond, ma'lumotlar banklari, boshqa axborot tizimlari) dagi hujjatlar va hujjatlar to’plamidir.

Axborot texnologiyasi (AT) – ob’yekt, jarayon yoki hodisaning holati haqida yangi sifat axboroti olish uchun ma'lumotlar yig’ish, qayta ishlash va uzatish (boshlang’ich axborot) vosita va uslublari jamlanmasidan foydalanadigan jarayondir.

Loyihalash – bu ob’yektning birlamchi bayoni va (yoki) uni mavjud qiladigan algoritm asosida berilgan sharoitda ham mavjud bo‘lmagan ob’yektni yaratish uchun zarur bo‘lgan bayonini tuzish jarayonidir. Loyihalash berilgan talablarga javob beradigan, yangi buyumni yaratish yoki yangi jarayonni amalga oshirish uchun zarur va yetarli bo‘lgan loyihalanadigan predmet bayonini olish maqsadidagi izlanish, tadqiqot, hisob va konstruktsiyalash bo‘yicha ishlar majmuidir.

Loyihalashni avtomatlashtirish – loyihani ishlab chiqish jarayonini bajarishning shunday usuli tushuniladiki, bunda loyihalash protseduralari va operatsiyalari loyihalovchining EHM bilan chambarchas muloqotida amalga oshad.

Avtomatlashtirilgan loyihalash tizimi (ALT) – avtomatlashtirilgan loyihalashni bajaruvchi loyihalovchi tashkilot yoki mutaxassislar jamoasi bilan bog‘langan avtomatlashtirilgan loyihalash vositalarining majmuidir.

Ma’lumotlar bazasini boshqarish tizimi (MBBT) – ma’lumotlar strukturasi ko‘rinishida tashkil qilingan informatsion baza bilan ishlashni ta’minlaydigan dasturaviy-metodik kompleksdir.

Mashina grafikasining dasturaviy-metodik komplekslari (DMK) foydalanuvchining EHM bilan muloqotida grafik informatsiya almashinuvini, geometrik masalalarni yechishni, tasvirlarni shakllantirishni va grafik informatsiyani avtomatik ravishda tayyorlashni ta’minlaydi.

ALTning dasturiy ta’minoti avtomatlashtirilgan loyihalashni bajarish uchun zarur bo‘lgan hamma dasturlar va ekspluatatsion hujjatlaridan iborat. Dasturaviy ta’minot umumtizimiy va maxsus (amaliy)larga bo‘linadi.

ALT matematik ta’minoti asosini algoritmlar tashkil qiladi; bu algoritmlar bo‘yicha ALTning dasturaviy ta’minoti ishlab chiqiladi. ALTda matematik ta’minotning elementlari har xil bo‘ladi. Ular ichida invariant elementlar – funksional modellarni tuzish printsiplari, algebraik va differentsial tenglamalarning sonli yechimi metodlari, ekstremal masalalarni qo‘yish, ekstrimumni qidirishlar mavjud.

ALTning texnikaviy ta’minoti – avtomatlashtirilgan loyihalashni bajarish uchun mo‘ljallangan o‘zaro bog‘langan va o‘zaro ta’sir qiluvchi texnikaviy vositalar majmuidir.

ALT lingvistik ta’minoti asosini maxsus til vositalari (loyihalash tillari) tashkil qiladi; ular avtomatlashtirilgan loyihalash protseduralarini va loyihaviy yechimlarni bayon qilish uchun mo‘ljallangan. Lingvistik ta’minotning asosiy qismi – insonning EHM bilan muloqot qilish tillari. Loyihalashning muammoli-yo‘nalgan tillari (MYT) loyihalashning algoritmik tillariga (Visual Basic, Visual C++, Delphi, Java, Visual Fox Pro va sh.k.) o‘xshash.

Videomonitor (displey)-ShK ga kiritiladigan va undan chiqadigan axborotni aks ettiruvchi moslamadir

Dasturiy interfeys – hisoblash tizimi doirasida qurilma va dasturlar o’zaro ta'sirini ta'minlovchi vositalar yig’indisi.

Duragay hisoblash mashinalari (DHM) – kombinatsiyalashgan holda amal qiluvchi hisoblash mashinalari bo’lib, hamda raqamli ham o’xshashli shaklda taqdim etilgan axborot bilan ishlaydi; ular UXM va RXMning afzalliklarini o’zida jo etgan.

Informatika – kompyuterlar yordami va ularni qo’llash muhiti vositasida axborotni yangilash jarayonlari bilan bog’liq inson faoliyati sohasidir.

Ishchi stantsiyalar – muayyan turdagi (grafik, muxandislik, nashriyot va boshqalar) ishlarni bajarishga ixtisoslashtirilgan bir kishi foydalanadigan qudratli mikro- EHM lardir.

Kibernetika – texnik , biologik, ijtimoiy va boshqa turli tizimlarda boshqaruvning umumiy tamoyillari haqidagi fandir.

Klaviatura – son bilan ko’rsatiladigan, matnli va boshqariluvchi axborotni ShKga qo’l yordamida kirishi uchun mo’ljallangan moslama.

Qobiq – biror bir dastur va foydalanuvchi o’rtasidagi katlam yoki boshqa dastur ustida uskurtma bo’lgan dastur.

Operatsion tizim (OT) – bu EHM zahiralarini boshqarish, amaliy dasturlarni chiqarish va ularning tashqi qurilmalar, boshqa dasturlar bilan o’zaro aloqasini amalga oshiruvchi, shuningdek, foydalanuvchining kompyuter bilan muloqotini ta'minlovchi dasturiy vositalar yig’indisidir.

Raqamli hisoblash mashinalari (RHM) – diskretli ishlaydigan hisoblash mashinalari bo’lib, diskret, aniqrogi raqamli shaklda taqdim etilgan axborot bilan ishlaydi.

Serverlar – barcha stansiya tarmoqlaridan olingan so’rovlarni qayta ishlash uchun ajratilgan ko’p kishi foydalanadigan qudratli mikroEHMlardir.

Telekommunikatsiya – kompyuter tarmoqlari va zamonaviy texnik aloqa vositalari negizida ma'lumotlarni masofadan uzatishdir.

Foydalanuvchi interfeysi – foydalanuvchining dasturiy yoki EHM bilan o’zaro ta'siridagi dasturiy va apparat vositalaridir.

ГЛОССАРИЙ

Ошибка математической модели - математическое описание ошибки реального процесса из-за путаницы.

Исходная ошибка данных - ошибка, возникающая из-за неопределенности исходных данных;

Ошибки стиля - ошибки, возникающие из-за неопределенности методов, используемых для решения проблемы;

Ошибка расчета - ошибки в расчетах;

Ошибка округления - это ошибка, вызванная округлением.

Ошибка - это разница (а - а) между приблизительной цифрой а и точным значением этого числа. Если А> а, ошибка положительна и А <а, ошибка отрицательна. Чтобы правильно исправить ошибки, включена концепция абсолютной ошибки.

Модуль ошибки называется абсолютной ошибкой приблизительного числа и определяется как Δа

□а = □А – а□

Отношение уравнения а к абсолютной ошибке называется относительной ошибкой приблизительного числа и обозначается как □ (а), т. Е.

$$\delta(a) = \frac{|A-a|}{|a|}$$

Поскольку абсолютное значение А неизвестно, абсолютные и относительные ошибки также неизвестны, поэтому отображается предел ошибки.

□А–а□ □ h Абсолютным пределом является абсолютный предел ошибки.

$\frac{|A-a|}{|a|} \leq \varepsilon$ Число неравенств удовлетворяет □ - граница относительной ошибки.

Система Maple - это система для компьютера, для быстрого, эффективного решения аналитических и численных решений различных фаз: экономика, механика, математика, физика, инженерия

MathCAD - программный инструмент для инженерных расчетов, предназначенный для профессиональных математиков. Его можно использовать для построения таблиц и графиков для решения алгебраических и дифференциальных уравнений с переменными и переменными параметрами, для анализа функций и поиска их крайностей, для анализа полученных результатов.

Система MATLAB представляет собой систему, предназначенную для решения задач механики, математики, физики, инженерии и управления, моделирования, проектирования, описания и анализа различных механических, энергетических и динамических систем в компьютерной системе и язык программирования для различных пользователей отрасли.

Massive - это нумерованный и упорядоченный набор типов данных.

Практический программный пакет (ADP) - это набор приложений, предназначенных для решения конкретных (функциональных, систематических, бизнес-задач) класса.

Информационное общество - это сообщество, в котором большинство сотрудников занимается производством, хранением, обработкой и реализацией информации, в частности формой информации.

Информационные ресурсы представляют собой набор документов и документов в отдельных документах и отдельной документации, информационных системах (библиотеках, архивах, запасах, банках данных и других информационных системах).

Информационные технологии (ИТ) - это процесс, который использует набор инструментов, инструментов и методов для сбора, обработки и передачи данных (исходной информации) для получения новой качественной информации о статусе предмета, процесса или события.

Разработка - это процесс создания описания, которое необходимо для создания объекта, который не существует даже в контексте первичного объекта и / или его существующего алгоритма. Дизайн представляет собой набор исследований, дизайна, дизайна и строительных конструкций, предназначенных для удовлетворения конкретных потребностей проектируемого объекта, который необходим и достаточен для создания нового продукта или для осуществления нового процесса.

Автоматизация проектирования - реализуется метод реализации проекта, в котором проектирование и операции проектирования выполняются в тесном диалоге с проектором.

Автоматизированная система проектирования (ALT) - это набор автоматизированных средств проектирования, которые связаны с автоматизированной проектной организацией или группой экспертов.

Система управления базами данных (MBMS) - это программно-методический комплекс, обеспечивающий доступ к информационной базе, созданной в виде структур данных.

Программирование и методические комплексы машинной графики (DMK) обеспечивают графический обмен информацией пользователя с компьютером, решение геометрических задач, автоматическое создание изображений и графической информации.

Программное обеспечение ALT состоит из всего программного обеспечения и эксплойтов, необходимых для автоматизированного проектирования. Программное обеспечение разделено на общие и специальные приложения.

Алгоритмы являются основой математической поддержки ALT; Будут разработаны программные алгоритмы. ALT имеет разные математические средства управления. Они включают в себя принципы создания инвариантных элементов - функциональные модели, методы численного решения алгебраических и дифференциальных уравнений, экстремальные проблемы, поиск экстремальных.

Техническая поддержка ALT - это набор взаимосвязанных и взаимосвязанных технических средств для автоматизированного проектирования.

Основой лингвистической поддержки ALT является создание специальных языковых инструментов (языков проекта); Они предназначены для описания автоматизированных процедур проектирования и проектных решений. Основная часть лингвистического питания - это язык общения с людьми. Программно-ориентированные языки программирования подобны алгоритмическим языкам программирования (Visual Basic, Visual C ++, Delphi, Java, Visual Fox Pro и т. Д.).

Videomonitor - это устройство, которое отображает информацию, которая вставляется в SCS и выходит из нее

Программный интерфейс представляет собой набор инструментов, которые взаимодействуют с устройствами и программным обеспечением в вычислительной системе.

Вычислительные машины секундомера (DHM) - вычислительные вычислительные машины, которые работают как цифровыми, так и аналогичными; Они имеют преимущества UHM и SRC.

Информатика - это область человеческой деятельности, связанная с обновлением информации с помощью компьютеров и их прикладной среды.

Рабочие станции - это мощные микроскопы, которые используются человеком, специализирующимся на определенных видах работ (графика, инжиниринг, публикация и т. Д.).

Кибернетика - наука, технология, биологические, социальные и другие системы.

Клавиатура - устройство, предназначенное для ввода текста и управляемой информации вручную на ПК.

Shell - это программа, которая разбивается между любым приложением и пользователем или другим программным обеспечением.

Операционная система (ОС) представляет собой набор программного обеспечения, которое управляет ресурсами компьютера, выводит приложения и взаимодействует с внешними устройствами и другим программным обеспечением, а также взаимодействует с компьютером пользователя.

Цифровые вычислительные машины (RHM) - это дискретная компьютерная машина, которая работает в дискретной, точной цифровой информации.

Серверы - это мощные микроЭУ, которые многие используют для разделения запросов от всех станций.

Телекоммуникация - это дистанционная передача данных на основе компьютерных сетей и современных технических коммуникаций.

Пользовательский интерфейс - это программное и аппаратное обеспечение, которое взаимодействует с программным обеспечением пользователя или экспозицией.

ANNEXES

THE TYPICAL PROGRAM ON DISCIPLINE

ЎЗБЕКИСТОН РЕСПУБЛИКАСИ ОЛИЙ ВА ЎРТА МАХСУС ТАЪЛИМ ВАЗИРЛИГИ

Рўйхатга олинди	Олий ва ўрта махсус таълим вазирлиги
№ БД – 5311000 – 2.06	
2016 йил «8» 08	2016 йил «25» 01

ҲИСОБЛАШ УСУЛЛАРИНИ АЛГОРИТМЛАШ

ФАН ДАСТУРИ

Билим соҳалари:	300 000 – Ишлаб чиқариш техник соҳа 100 000 – Гуманитар соҳа
Таълим соҳалари:	310 000 – Муҳандислик иши 110 000 – Педагогика
Таълим йўналишлари:	5311000 – Технологик жараёнлар ва ишлаб чиқаришни автоматлаштириш ва бошқариш (кимё, нефть-кимё ва озиқ-овқат саноати); 5111000 – Касб таълими (Технологик жараёнлар ва ишлаб чиқаришни автоматлаштириш ва бошқариш)

Тошкент – 2016

Фаннинг ўқув дастури Олий ва ўрта махсус, касб-ҳунар таълими йўналишлари бўйича Ўқув-услубий бирлашмалар фаолиятини Мувофиқлаштирувчи Кенгашнинг 2016 йил «8» 08 даги «3» -сонли мажлис баёни билан маъқулланган ҳамда вазирликнинг 2016 йил «25» 08 даги «355» -сонли буйруғининг 2 -иловаси билан фан дастури рўйхати тасдиқланган.

Фан дастури Тошкент давлат техника университетида ишлаб чиқилди.

Тузувчилар:

Игамбердиев Х.З. - Тошкент давлат техника университети «Бошқаришда ахборот технологиялари» кафедраси профессори, т.ф.д.;

Зарипов О.О. - Тошкент давлат техника университети «Электроника ва автоматика» факультети декани, т.ф.д.

Тақризчилар:

Азимов Б.М. - Тошкент ахборот технологиялари университети ҳузуридаги «Дастурий маҳсулотлар ва аппарат-дастурий мажмуалар яратиш» марказининг лаборатория мудири, т.ф.д., профессор;

Абдукадиров А.А. Тошкент давлат техника университети «Ишлаб чиқариш жараёнларини автоматлаштириш» кафедраси профессори, т.ф.д.

Фан дастури Тошкент давлат техника университети Кенгашида кўриб чиқилган ва тавсия қилинган (2016 йил «6» 07 даги 12 -сонли баённома).

КИРИШ

5311000 – «Технологик жараёнлар ва ишлаб чиқаришни автоматлаштириш ва бошқариш» (тармоқлар бўйича) йўналиши бўйича бакалаврларни тайёрлаш ўқув режасида «Ҳисоблаш усулларини алгоритмлаш» ўқув фани умумкасбий фанлар туркумига киритилган.

«Ҳисоблаш усулларини алгоритмлаш» фанидан ҳар хил синфдаги математик масалаларнинг тақрибий ечимларининг алгоритмларини назарий асослаш, қуриш ва амалда қўллаш масалалари ўрганилади.

Ўқув фанининг мақсади ва вазифалари

Ўқув фанининг мақсади – тажриба йўли билан тўпланган натижаларни қайта ишлаш, алгебраик, дифференциал ва интеграл тенгламаларни тақрибий ечимини топишда алгоритмларни тузиш учун мантиқий фикрлаш қобилиятини талабаларда шакллантиришдан иборат.

Ўқув фанининг вазифаси – талабаларни тажриба орқали олинган натижаларни қайта ишлаш, алгебраик, дифференциал ва интеграл тенгламаларни тақрибий ечимини топишда алгоритмларни тузиш учун маъқул вариантларни танлашга ўргатишдан иборат.

Фан бўйича талабаларнинг билим, кўникма ва малакасига қўйиладиган талаблар

«Ҳисоблаш усулларини алгоритмлаш» ўқув фанини ўзлаштириш жараёнида амалга ошириладиган масалалар доирасида бакалавр:

– алгебра, дифференциал ва интеграл тенгламаларини ечимини топишда тақрибий ечим усуллари ***ҳақида тасаввурга эга бўлиши***;

– матрица ва детерминант, дифференциал ва интеграл тенгламаларнинг хусусий ечимларини олиш усулларини ***билиши;***

– мустақил равишда тақрибий ечимлар алгоритмларини туза олиш ***кўникмаларига эга бўлиши керак***.

Қўйилган вазифалар ўқиш жараёнида талабаларни маъруза, лаборатория ва амалий машғулотларда фаол иштирок этиши, адабиётлар билан ишлаши билан амалга оширилади.

Фаннинг ўқув режадаги бошқа фанлар билан ўзаро боғлиқлиги ва услубий жиҳатдан узвий кетма-кетлиги

«Ҳисоблаш усулларини алгоритмлаш» фани мутахассислик фани ҳисобланиб, 3-семестрда ўқитилади. Дастурни амалга ошириш ўқув режасида режалаштирилган «Информатика ва ахборот технологиялари» ва «Олий математика» фанларидан етарли билим ва кўникмаларга эга бўлиш талаб этилади.

Фаннинг ишлаб чиқаришдаги ўрни

Кимё саноати корхоналарида ва илмий текшириш институтларида турли ҳисоб ишларини амалга оширишда ҳисоблаш усулларини алгоритмлашдан фойдаланиб, ишлаб чиқариш унумдорлиги ва марадорлигини ошириш бўйича олиб борилаётган ишлар умумий ҳажмнинг анчагина қисмини ташкил қилади.

Шунинг учун ҳам ҳисоблаш усулларини алгоритмлашни ўрганишга алоҳида талаблар қўйилади. Айниқса мураккаб системалар фаолиятини таҳлил қилишда ҳисоблаш усулларини алгоритмлашдан кенг фойдаланилмоқда. Шунинг учун ушбу фан асосий ихтисослик фани ҳисобланиб, технологик жараёнларнинг ажралмас бўғини сифатида қаралади.

Фанни ўқитишда замонавий ахборот ва педогогик технологиялар

Талабаларнинг ҳисоблаш усулларини алгоритмлаш фанини ўзлаштиришлари учун ўқитишнинг илғор ва замонавий усулларидан фойдаланиш, янги информацион-педагогик технологияларни тадбиқ қилиш муҳим аҳамиятга эгадир. Фанни ўзлаштиришда дарслик, ўқув ва услубий қўлланмалар, маъруза матнлари, тарқатма материаллар, электрон материаллар, виртуал стендлар ҳамда намуналар ва макетлардан фойдаланилади. Маъруза, амалий ва лаборатория дарсларида мос равишдаги илғор педагогик технологиялардан фойдаланилади.

Асосий қисм
Фаннинг назарий машғулотлари мазмуни

Илмий ишларнинг самарадорлигини оширишда математик усулларни ва математик моделлаштиришни қўллаш.

Математик тавсиф тенгламаларининг ечиш усуллари:

Алгебраик ва транцендент тенгламаларни тўғри ва итерация усуллари билан ечиш усулларини алгоритмларини тузиш. (Алгебраик ва транцендент тенгламаларни илдизларини ажратиш. Тенг ярмига бўлиш усули. Ватарлар усули. Ньютон усули. Қўшма усул. Итерация усули).

Алгебраик ва транцендент тенгламалар системаларини тўғри ва итерация усуллари билан ечиш усулларини алгоритмлаш (Гаусс усули. Итерацион (Якоби ва Зайдел) усуллари). Итерация усулларининг яқинлашиш жараёни шартларини ўрганиш. Стационар итерацион усуллларининг яқинлашиш жараёнини етарли ва зарурий шартлари.

Интерполяция усулларини алгоритмлаш. Алгебраик кўп ҳадлар билан интерполяциялаш. Яқинлашиш жараёни шартларини ўрганиш.

Дифференциал тенгламаларни тақрибий ечимларини аниқлаш. Эйлер усули.

Интеграл тенгламаларнинг тақрибий ечимлари. Тўртбурчак ва трапеция усуллари. Симпсон формуласи.

Тажриба натижаларини қайта ишлаш. Энг кичик квадратлар усули.

Ночизиқли тенгламаларни тақрибий ечимлари.

Амалий машғулотлар мазмуни, уларни ташкил этиш бўйича кўрсатма ва тавсиялар

Амалий машғулотларда талабалар маърузаларда ўрганилган назарий билимларини бойитадилар ва мустаҳкамлайдилар. Амалий машғулотларни қуйидаги мавзуларда олиб бориш тавсия этилади:

Алгебраик ва транцендент тенгламаларни ечимини тўғри ва итерацион усуллар билан олиш.

Алгебраик ва транцендент тенгламалар системасини Гаусс усулида ечиш.

Интеграл тенгламаларни Симпсон усулида ечиш.

Тажриба натижаларини Ньютон ва Логранж усули билан интерполяциялаш.

Тажриба натижаларини энг кичик квадратлар усули билан аппроксимациялаш.

Ночизиқли эмпирик боғлиқликларни тузиш.

Амалий машғулотларни ташкил этиш бўйича кафедра профессор-ўқитувчилари томонидан кўрсатма ва тавсиялар ишлаб чиқилади. Унда талабалар асосий маъруза мавзулари бўйича олган билим ва кўникмаларини амалий масалалар ечиш орқали янада бойитадилар. Шунингдек, дарслик ва ўқув қўлланмалар асосида талабалар билимларини мустаҳкамлашга эришиш, тарқатма материаллардан фойдаланиш, илмий мақолалар ва тезисларни чоп этиш орқали билимини ошириш, масалалар ечиш, мавзулар бўйича кўргазмали қуроллар тайёрлаш ва бошқалар тавсия этилади.

Лаборатория ишлари мазмуни, уларни ташкил этиш бўйича кўрсатмалар

Лаборатория ишлари талабаларда ҳисоблаш усулларини алгоритмлашнинг қуллаш ва уларнинг атрофлича таҳлил қилиш бўйича амалий кўникма ва малака ҳосил қилади.

Лаборатория ишларининг тавсия этиладиган мавзулари:

1. Алгебраик ва трансендент тенгламаларни оддий итерация ҳамда ватарлар усули билан ечиш.
2. Оддий итерация усули билан чизиқли бўлмаган тенгламалар системасини ечиш
3. Ньютон усули билан алгебраик ва трансцендент тенгламаларни тақрибий ечиш.
4. Чизиқли алгебраик тенгламалар системасини оддий итерация усули билан ечиш.
5. Чизиқли алгебраик тенгламалар системасини Зейдель усули билан ечиш

Мустақил ишни ташкил этишнинг шакли ва мазмуни

Талаба мустақил ишни тайёрлашда муайян фаннинг хусусиятларини ҳисобга олган ҳолда қуйидаги шакллардан фойдаланиши тавсия этилади:

- дарслик ва ўқув қўлланмалар бўйича фанларнинг боблари ва мавзуларини ўрганиш;
- тарқатма материаллар бўйича маърузалар қисмини ўзлаштириш;
- автоматлаштирилган ўргатувчи ва назорат қилувчи тизимлар билан ишлаш;
- махсус адабиётлар бўйича фанлар бўлимлари ёки мавзулари устида ишлаш;
- янги техникаларни, аппаратураларни, жараён ва технологияларни ўрганиш;
- талабаларнинг ўқув – илмий - тадқиқот ишларини бажариш билан боғлиқ бўлган фанлар бўлимлари ва мавзуларни чуқур ўрганиш;
- фаол ва муаммоли ўқитиш услубидан фойдаланиладиган ўқув машғулотлари;
- масофавий (дистанцион) таълим.

Тавсия этилаётган мустақил ишларнинг мавзулари:

Яхлитлаш хатоликларининг тўпланиши.

Алгебраик тенгламалар системасини ечишда Гаусс усулини қўллаш шартлари.

Дифференциал тенгламаларни Адамс усули билан ечиш.

Биринчи тартибли дифференциал тенгламаларни тақрибий интеграллаш усули билан ечиш.

Майдон ва ҳажмларни каррали интеграл ёрдамида ҳисоблаш.

Интерполяция хатоликлари.

Аппроксимация усуллари ва мезонлари.

Дастурнинг информацион-услубий таъминоти

Мазкур фанни ўқитиш жараёнида таълимнинг замонавий методлари, педогогик ва ахборот-коммуникация технологиялари қўлланилиши назарда тутилган:

- ҳисоблаш усулларини алгоритмлашнинг назарий асослари бўлимига тегишли маъруза дарсларида замонавий компютер технологиялари ёрдамида презентацион ва электрон-дидактик технологиялари;

- ҳисоблаш усулларини алгоритмлашнинг бўйича ўтказиладиган амалий машғулотларда ақлий хужум, гуруҳли фикрлаш педагогик технологияларини қўллаш назарда тутилади.

- ҳисоблаш усулларини алгоритмлашнинг махсус бўлимларига тегишли бўлган тажриба машғулотларида кичик гуруҳлар мусобақалари, гуруҳли фикрлаш педогогик технологияларини қўллаш назарда тутилади.

Фойдаланилаётган асосий дарсликлар ва ўқув қўлланмалар рўйхати

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THE MINISTRY OF THE HIGHER AND AVERAGE
THE VOCATIONAL EDUCATION
REPUBLIC OF UZBEKISTAN
NAVOI MINING-METALLURGICAL COMPLEX
NAVOI STATE MINING INSTITUTE

FACULTY ELECTRICAL AND MECHANICAL ENGINEERING

«AUTOMATION AND CONTROL» DEPARTMENT

Registered:
No 1 – AY
"28" 08 2017 y/

«CONFIRMATION»
Vice rector on study
N.A.Abduazizov
«28» 08 2017 y

THE WORKING CURRICULUM
ON DISCIPLINE
«ALGORITHMIZATION OF COMPUTING METHODS»

Field of knowledge	300 000 – industrial - technical sphere
Sphere of education	310 000 – Engineering
Formation direction	5 311 000 – Automation and control of technological processes and industries (on branches)

Semestre	3
The general auditoria hours	72
Including:	
Lecture	36
Practical employment	18
Laboratory employment	18
Independent work	52
In total	124

Navoi – 2017

Composer:

Urinov Sh.R. – senior lecturer of department of the "Automation and Control"

The working program is discussed and confirmed at session №1 from 08/25/2017 of chair «Automation and control»

The head of chair: ________ Jumaev O.A.

The working curriculum is confirmed on council of session Power-mechanical faculty by report №1 from August, 26th, 2017.

The chairman of the council of faculty: ___________ Bazarova S.J.

In coordination:

The chief of educational-methodical department: _______ **Karimov I.A.**

INTRODUCTION

Formation is not simply process of reception of the sum of necessary knowledge, but also process of formation of spiritual essence of the person. To the full it concerns and higher education. For this reason, education is inseparable from training process.

The modern period of a life of a human society is characterized by unknown growth of information streams; therefore, the important place in preparation of modern experts is played by information, mathematical and natural science disciplines.

The program is made on the basis of requirements of the state educational standards of professional higher education to an obligatory minimum of the maintenance and level of preparation for students of all specialties.

The course is discipline in which are put modelling, numerical methods, computer modelling of system management in the industrial enterprises their designing and operation in national economy branch.

The purposes and problems of studying of discipline

One of the purposes of teaching of information, mathematical and natural-science disciplines in technical college is increase of the general level of information and mathematical culture of the future experts. Thus the problem of education of high culture of the creative reference with a science also dares.

The subject purpose is – students educated logic abilities of mind for algorithm construction on definition approximate the decision the algebraic, differential both integrated equation and processing of results of experimental data.

Subject problems is – to study students a choice corresponding variants for algorithm construction on definition approximate the decision the algebraic, differential both integrated equation and processing of results of experimental data. As a part of the primary goal of computer science today it is possible to allocate following directions for practical appendices:

• architecture of computing systems (receptions and methods of construction of the systems intended for automatic data processing);

• interfaces of computing systems (receptions and management methods to equipment rooms and the software);

• programming (receptions, methods and means of working out of computer programs);

• transformation of the data (receptions and methods of transformation of structures of the data);

• information protection (generalization of receptions, working out of methods and protection frames of the data);

• automation (functioning of hardware-software means without participation of the person);

• standardization (compatibility maintenance between equipment rooms and software, and also between formats of the data presentation, computing systems concerning various types).

Requirements to level of development of the maintenance of discipline

The student should master:

- Knows about methods approximate search at the decision algebraic, differential and integrated the equation;
- To know reception methods private the decision matrix and determined, differential and integrated the equation;
- It is required to have independently skill to make the approximate solved algorithms.
- Tasks in view in process study the student actively to take part in lecture, laboratory and practical employment, to work with references.

The student should be able:

- The nobility higher mathematics and computer science;
- To count characteristics different methods the problem decision;
- Structure of construction algorithms and applied programming.

Communication of the subject with other disciplines

The subject is a part to subjects on a speciality, and will be spent in 3rd semestre.

The discipline is connected with subjects «Computer science and an information technology», «Applied programming», "Higher mathematics", etc. and the student should know knowledge and skills in these subjects.

Communication of the subject with manufacture

In the presents time in all industrial enterprises computer technologies are widely used. Therefore special requirements algorithmization of computing methods is shown. With помошью new applied programs it is effectively solved different technical problems in manufacture. Systems at first in computers will be developed by experts and are then realised in practice. In bases computer modelling operatively and very much a split-hair accuracy difficult productions cope and regulated.

Algorithmization of computing methods provides accuracy and economic efficiency. On it, the subject is the basic a special subject and a manufacture part.

Means of maintenance of development of discipline

Original properties the direction allows program development by interactive methods. Thus the basic the attention goes on аудиторные employment and самотоятельные preparations, to theoretical employment and also on formation of opinions on technological process of object of manufacture.

Development of program materials*:*

- On problem themes;
- On difficultly independently mastered to knowledge;

- By parts causing especially interest;
- New means, experience of the foreign states ("Siemens", "Metran", "Honeywell");

On interactive methods of training:

- Reception independent formations and work, colloquiums and in process discussion on mastered knowledge to spend employment.

In the course of independent preparation, the student should show ability to use the technical literature, Internet materials, instructions, standard documents, during time аудиторных employment to show abilities correctly to perceive received data.

The program is carried out on the basis of rating estimations of new principles of educational process used at the organization.

THE MAINTENANCE OF DISCIPLINE OF THEORETICAL EMPLOYMENT

LECTURE EMPLOYMENT (36 hour)

Introduction. Subject problems – 2 hour.

Introduction. Base concept about algorithmization of computing methods. (2 hour)

Classification of computing methods; Preparation of problems for the personal computer decision; Properties of algorithm; Classification of algorithms.

Methods the decision the equation mathematical characteristics - 22 hour.

Algorithmization of the numerical decision of the algebraic and transcendental equations. A method branch of roots and a method half divisions. (2 hour)

Method branch of roots; the Method of half divisions

Algorithmization of the numerical decision of the algebraic and transcendental equations. A method a chord and Newton's method. (2 hour)

Method the Chord; Newton's Method

Algorithmization of the numerical decision of the algebraic and transcendental equations. A method of iteration and a method of secants. (2 hour)

Method of simple iteration; the Method of secants

Algorithmization of the numerical decision of system of the algebraic and transcendental equations. A method of Gauss. (2 hour)

The decision of system of the linear equations a method of Gaussa; the Method of Gaussa with a choice of the main element; an error Estimation at the decision of system of the linear equations

Algorithmization of the numerical decision of system of the algebraic and transcendental equations. Iterative methods of Jacoby and Seidel. (2 hour)

Iterative methods of the decision of systems of the linear equations; the Method of simple iteration of Jacobi; the Method of Gaussa-Seidel

Algorithmization interpolation methods. Interpolation of functions. (2 hour)

Introduction; the First interpolation Newton's formula; the Second interpolation Newton's formula; the interpolation formula of Stirling; the Example

The numerical decision of the differential equations. Euler's method. (2 hour)

Types of problems for the ordinary differential equations; Euler's Method

The numerical decision of the differential equations. A method of Runge-Kutte and Adams. (2 hour)

Methods of Runge-Kutte; Adams's Method

Numerical integration. Quadrature formulas of trapezes and rectangles. Simpson's formula. (2 hour)

Classification of methods; the Method of trapezes; Methods of rectangles; Simpson's Method.

Numerical integration. The formula of Gauss. (2 hour)

The quadrature formula of Gauss

Root-mean-square approach of functions. A method of the least squares. (2 hour)

Root-mean-square approach of functions; the Method of the least squares

Algorithmizations methods linear programming - 12 hour

Statement of a problem of linear programming. The basic properties the decision of a problem of linear programming. (4 hour)

The primary goal of linear programming; Examples of the decision of a problem

Geometrical interpretation of a problem of linear programming. (4 hour)

Problem statement; Geometrical representation; the Example of the decision of a problem; Geometrical problem interpretation

Finding the decision of a problem of linear programming to Simplex methods. (2 hour)

Mathematical bases a simplex of a method of the decision

Finding the decision of a problem of linear programming. A method of artificial basis. (2 hour)

Search of the initial basic decision

THE THEMATIC PLAN OF THE PRACTICAL TRAINING (18 HOUR)

The numerical decision of the algebraic and transcendental equations iterative methods. (4 hour)
Calculation of integrals by the approached methods (4 hour)
Newton's interpolation polynom and Lagrange (4 hour)
Approximation results of experiment with a method of the least square. Construction nonlinear empirical connection. (4 hour)
The geometrical decision of a problem of linear programming. (2 hour)

THE THEMATIC PLAN OF LABORATORY RESEARCHES (18 HOUR)

The numerical decision of the algebraic and transcendental equations iterative methods the Chord and Newton. (4 hour)
The numerical decision of system of the linear algebraic equations methods of Gauss, simple iteration and Seidel. (4 hour)
The numerical decision of system of the nonlinear algebraic equations to methods of simple iteration. (4 hour)
Problems of Cochy for the ordinary differential equations. Euler's methods, Runge-Kutte and Adams. (4 hour)
Finding the decision of a problem of linear programming to Simplex methods. (2 hour)

THE TERM PAPER

Ability to use the computer in the work begins the usual requirement to engineering and scientific, to workers. Therefore in all technical colleges of the country as obligatory the subjects training the future experts to art of programming (as though these subjects were not called) are entered. As a rule, these subjects are included in the curriculum at an initial stage of training of students that allows to raise essentially efficiency of teaching of disciplines on following courses, using the computer in educational-pedagogical process in performance of various homeworks.

Term paper in discipline it is not intended.

SELF-STUFY WORKS

The purpose of independent formation consists that students should develop the mental abilities both the received knowledge, and skills in lecture, laboratory and a practical training with use of multimedia means and news of an information technology, and also on the basis of the basic and additional literature.

The student during time preparation independent work with calculate properties of a subject offered to use following forms:

- Studying paragraphs and to those subjects under textbooks and manuals;
- Progress distributing materials on lecture parts;
- To work with training and supervised by the automated systems;
- To work as parts and themes to subjects on special literatures;
- Studying new technicians, equipments, processes and technologies;
- Deeply to study to a part and those subjects which are connected on performance educational-research works of the student;
- Educational employment using with active and problem training methods;
- Remote formation.

Themes of self-study works

Integer linear programming and its application at the decision of problems of planning of mountain manufacture
Nonlinear programming and its use in planning and management of mountain manufacture
Dynamic optimising models of planning and management of mountain manufacture
Network planning and management of realisation of programs
Analytical models of systems of mass service
Statistical modelling of productions
Decision-making in the conditions of uncertainty
Meeting complete errors.
Condition application of a method of Gaussa at the decision algebraic systems the equation
The decision differential equation with Adams's method.
The decision differential the equation of the first order with methods confidants integration.
Calculation of fields and volumes a method multiple integrals.
Interpolation errors
Methods and criteria of approximation.
Integer linear programming and its application at the decision of problems of planning of mountain manufacture
Nonlinear programming and its use in planning and management of mountain manufacture
Dynamic optimising models of planning and management of mountain manufacture
Network planning and management of realisation of programs
Analytical models of systems of mass service
Statistical modelling of productions
Decision-making in the conditions of uncertainty

CRITERIA OF THE ESTIMATION

The implementation of a national training program to a qualitatively new stage of higher education institutions to assess students' knowledge of the country and aims to introduce a system to control the quality of education is to prepare highly qualified specialists competitive. The level of knowledge of students in higher educational institutions of the system is evaluated. Evaluation system based on the student's knowledge of all the students to increase their knowledge during the regular reading and creative activities aimed at stimulating improved performance.

The evaluation criteria of the Ministry of Higher and Secondary Special Education of the Republic of Uzbekistan No. 333 dated August 25, 2013, included in the amendments and supplements to the order of the Ministry of Justice of the Republic of Uzbekistan on August 26, 2013, 1981-2 with the state registration number of the "Higher Education Institutions Regulations on the monitoring and assessment of students' knowledge system" in accordance with the requirements of the Ministry of Higher and Secondary Special Education of the Republic of Uzbekistan August 14, 2009, "the organization of independent work of students" on the order of 286 applications, and the Ministry of Higher and Secondary Special Education of the router 332 as of August 15, 2012/1 dated approved. Curriculum and training program has been developed on the basis of science.

The evaluation criteria of the subjects is recommended to be used to assess students' knowledge, but also the academic performance of students in this process will allow us to have an idea about how you can collect points.

Control charts, control type, shape, and number of a maximum score in control, as well as information on current and interim control points in the first session, students will be announced on science.

1. Monitoring and evaluation procedure

Subject 5311000 - Automation and Control of technological processes and industries" on the undergraduate curriculum plan for 2 course to 4 semestre. The level of students' level of knowledge and the development of the State to ensure compliance with the standards of education refers to the following types of control:

- current control - to determine the student's level of knowledge and practical skills on the subject and method of evaluation. Based on current monitoring on the subject, request oral and practical work experience, studies carried out by checking homework and conversation;

- control during the semester, mandatory training program (which includes a number of topics of science) department to determine a student's level of knowledge and practical skills, and after the completion of the evaluation method. Space control are held twice a semester, held its shape in the form of written work shall be allocated on the basis of the total number of hours of education;

- the ultimate control - a fan at the end of the semester by the students theoretical knowledge and practical skills for the development of a method to assess the level. The final control is based on the concept and phrase "written" form.

To control the level of knowledge and skills of students on the basis of the rating system is characterized by the development of science degree student scores.

Each semester student in the development of science assessed by the number of 100-point system.

These types of controls on current and interim control 100 points - 70 points and finished with 30 points distributed control.

2. Chart of subject

Day	Semestre	Quantity of the week	Common hour for semestr (rating points)	Lecture	Laboratory	Practice	Seminariom	Hour for self-study	**Ap**-auditorium points **SSe**-self-study points	Type of control (estimation)												Course work (lproject) exist subketcs
										Common points in %	FE	FE – 1	FE – 2	IE	IE – 1	IE – 2	∑FE+IE	Saralash balli	FE	Form of final control	Maximum points	
3	4	18	120	36	18	18	-	48	Ap	60	35	12	12	35	12	12	70	39	30	writ ing	100	
									SSe	40		5	6		5	6						

3. RATING TABLES AND CRITERIA ESTIMATION

3.1. Rating design

№	Control kinds	Quantity	Quan. and a point	Common score
I. The CURRENT CONTROL (35 % = 35 score)				
1.1	Performance of practical tasks	5	3,5x5	17,5
1.2	Performance of laboratory tasks	5	3,5x5	17,5
1.3	Self-study work - (it is given everyone practical and laboratory employment, scores is delivered with addition practical and laboratory research)			
II. The INTERMEDIATE CONTROL (35 % = 35 score)				
2.1	1 – the intermediate control, written work (2 questions)	1	6x2	12
2.2	2 – the intermediate control, written work (2 questions)	1	6x2	12
2.3	Independent work – (it is set as tretiy a question to the intermediate control)	2	5+6	11
The general points – ABOUT: (CC+IC)				**70**
III. The TOTAL CONTROL (30 % = 30 ball)				
3.1.	The total control (3 questions)	1	10x3	30
IN TOTAL				**100**

3.2. Evaluation criteria

1.1. (Experience) student work tasks performed by a full 3 - 3.5 points, 2.5 depending on the level of quality to answer the questions but did not given a full 3 percentage points, depending on the degree of fulfillment 1.9 - 2.5 points.

Practical topics are as follows:

1. *The Numerical decision of the algebraic and transcendental equations iterative methods*
2. *Calculation of integrals by the approached methods*
3. *Interpolation Newton's polynom and Lagrange*
4. *Approximation results of experiment with a method of the least square. Construction nonlinear empirical connection*
5. *The Geometrical decision of a problem of linear programming*

Laboratory topics are as follows:

1. *The Numerical decision of the algebraic and transcendental equations iterative methods the Chord and Newton*
2. *The Numerical decision of system of the linear algebraic equations methods of Gauss, simple iteration and Seidel*
3. *The Numerical decision of system of the nonlinear algebraic equations to methods of simple iteration*
4. *Problems of Coshy for the ordinary differential equations. Euler's methods, Runge-Kutte and Adams*
5. *The Finding the decision of a problem of linear programming to Simplex methods*

1.2.* This control on the answers given on behalf of the student's independent work into practice with the operation concluded notebook notebook:

• task is fully opened, and the direct result of creative ideas-4,3-5 (5,2-6) points
• disclosed only conclusion the task 3,6-4,3 (4,3-5,2)
• illuminated the nature of the task, but there are some disadvantages 2,8-3,5 (3,3-4,2).
• know the answers to the questions had a partial response or independent business-0-2,8 (0-3,3).

Self-study work to check the current topics are as follows:

1. *The Numerical decision of the algebraic and transcendental equations iterative methods*
2. *Calculation of integrals by the approached methods*
3. *Interpolation Newton's polynom and Lagrange*
4. *Approximation results of experiment with a method of the least square. Construction nonlinear empirical connection*
5. *The Geometrical decision of a problem of linear programming*
6. *The Numerical decision of the algebraic and transcendental equations iterative methods the Chord and Newton*
7. *The Numerical decision of system of the linear algebraic equations methods of Gauss, simple iteration and Seidel*
8. *The Numerical decision of system of the nonlinear algebraic equations to methods of simple iteration*
9. *Problems of Koshi for the ordinary differential equations. Euler's methods, Runge-Kutta and Adams*
10. *The Finding the decision of a problem of linear programming to Simplex methods*

2.1. 1 and 2 - written evaluation procedure, which will be asked to answer 2 questions. Each question is worth 6 points.

- If you are the essence of open questions, the answers are complete and accurate, and creative ideas - 10.3 - 12 points
- answering general questions, but if some of the facts reported in full - 8.5 - 10.3 points
- confusion if you are trying to answer the questions - - 6.6 - 8.5 points.
- If you did not answer questions or questions confusion - 0 - 6.6 points.

1 - Intermediate control questions

1. *Classification of computing methods.*
2. *Preparation of problems for the personal computer decision.*
3. *Properties of algorithm.*
4. *Classification of algorithms.*
5. *The Method branch of roots*
6. *The Method half divisions*
7. *The Method the Chord*
8. *Newton's Method*
9. *The Method of simple iteration*
10. *The Method of secants*
11. *The Decision of system of the linear equations a method of Gauss*
12. *The Method of Gauss with a choice of the main element*
13. *An error Estimation at the decision of system of the linear equations*
14. *Iterative methods of the decision of systems of the linear equations*
15. *The Method of simple iteration of Jacoby*
16. *The Method of Gaussa-Seidel*
17. *The First interpolation Newton's formula*
18. *The Second interpolation Newton's formula*
19. *Interpolation the formula of Stirlinga*

2 - Intermediate control questions

1. *Types of problems for the ordinary differential equations*
2. *Euler's Method*
3. *Methods of Runge-Kutte*
4. *Adams's Method*
5. *The Method of trapezes*
6. *Methods of rectangles*
7. *Simpson's method*
8. *Quadrature the formula of Gauss*
9. *Root-mean-square approach of functions*
10. *The Method of the least squares*
11. *The Primary goal of linear programming*
12. *Geometrical representation LP.*
13. *Geometrical interpretation of problem LP*
14. *Mathematical bases a simplex of a method of the decision*
15. *Search of the initial basic decision*
16. *Features of a transport problem*
17. *Constructions of basic decision TT*
18. *Conditions and a method of construction of the optimum decision of a transport problem*
19. *Algorithm of the decision of a transport problem on a network*

2.2.* Intermediate interim control of the independent work of the students on the topics included in the check written to the third question:

- • When fully opened, and the direct result of creative ideas-4,3-5 (5,2-6) points
- • question has disclosed only conclusion 3,6-4,3 (4,3-5,2)
- • reflected the essence of the question, but there are some disadvantages 2,8-3,5 (3,3-4,2).
- • know the answers to the questions had a partial response or independent business-0-2,8 (0-3,3).

Self-study question for intermediate control to the following:

1. Integer linear programming and its application at the decision of problems of planning of mountain manufacture
2. Nonlinear programming and its use in planning and management of mountain manufacture
3. Dynamic optimizing models of planning and management of mountain manufacture

4. Network planning and management of realization of programs
5. Analytical models of systems of mass service
6. Statistical modelling of productions
7. Decision-making in the conditions of uncertainty

3.1. The final evaluation of the student to give a written answer to question 3.

- issued a written question by 10 points.
- If you opened the essence of the questions, explain the basic facts correct - 26 - 30 points
- correct answers, but there are some disadvantages - 21 - 26 points
- If the answers to common questions and problems and more - 16 - 21 points
- correct answers to the questions would not be complete without at least a lot of shortcomings, and - 0 - 16

Questions of final control subjects

1. *Classification of computing methods.*
2. *Preparation of problems for the personal computer decision.*
3. *Properties of algorithm.*
4. *Classification of algorithms.*
5. *The Method branch of roots*
6. *The Method half divisions*
7. *The Method the Chord*
8. *Newton's Method*
9. *The Method of simple iteration*
10. *The Method of secants*
11. *The Decision of system of the linear equations a method of Gauss*
12. *The Method of Gauss with a choice of the main element*
13. *An error Estimation at the decision of system of the linear equations*
14. *Iterative methods of the decision of systems of the linear equations*
15. *The Method of simple iteration of Jacoby*
16. *The Method of Gauss-Seidel*
17. *The First interpolation Newton's formula*
18. *The Second interpolation Newton's formula*
19. *interpolation the formula of Stirlinga*
20. *Types of problems for the ordinary differential equations*
21. *Euler's Method*
22. *Methods of Runge-Kutte*
23. *Adams's Method*
24. *The Method of trapezes*
25. *Methods of rectangles*
26. *Simpson's Method*
27. *Quadrature the formula of Gauss*
28. *Root-mean-square approach of functions*
29. *The Method of the least squares*
30. *The Primary goal of linear programming*
31. *Geometrical representation LP.*
32. *Geometrical interpretation of problem LP*
33. *Mathematical bases a simplex of a method of the decision*
34. *Search of the initial basic decision*
35. *Features of a transport problem*
36. *Constructions of basic decision TT*
37. *Conditions and a method of construction of the optimum decision of a transport problem*
38. *Algorithm of the decision of a transport problem on a network*

Final evaluation of written work procedures

The method of evaluation system of students' knowledge of student’s written work on the development of critical thinking and writing skills to express their opinion.

The final control in the form of written works. Written questions and options at the beginning of each academic year in the faculty of the department of the newly established department will be discussed and approved at the meeting.

Written questions put to work version of the content, and the level of coverage significantly tested the head of the department, endorsed with his signature. According to the written work was scheduled for the last two weeks of the semester, it is on this subject during training sessions per week. Written statements based on 3 questions. Writing assessment criteria for the evaluation of the final tests, the maximum 30 points allocated for each question a maximum of 10 points. Within two days after the written work of students and teachers, to test and evaluate attention. The written work of students of the imagination must be sufficient to assess the knowledge and practical skills.

The order of ranking results

Through evaluation of the student's knowledge of the types of items to the end of each semester, students are teachers, and all the points recorded in his notebook.

INFORMATION-METHODICAL MAINTENANCE

The basic literature:

Asosiy

1. Yusupbekov N.R., Muxiddinov D.P. Texnologik jarayonlarni modellashtirish va optimallashtish asoslari. – T.: Fan va texnologiya, 2015. – 438 b.
2. Yusupbekov N.R., Muxiddinov D.P., Bazarov M.B. Elektron hisoblash mashinalari kimyo texnologiyasida qo’llash. – T.: Fan, 2010. – 2010. – 492 b.
3. Абзалимов Р.Р. Численные методы решения уравнений на ЭВМ. – Т.: ТГТУ, 2008. – 2008. – 168 с.
4. Юсупбеков Н.Р., Мухитдинов Д.П., Базаров М.Б., Халилов А.Ж. Бошқариш системаларини компьютерли моделлаштириш асослари. Олий ўқув юртлари учун ўқув қўлланма. –Н.: Навоий-Голд-Сервес, 2009.
5. Granville S. Computational Methods of Linear Algebra. Singapure: World Scientific Publishing, 2014. – 328 p.

The additional literature:

1. Кнут Д.Э. Искусство программирования: Том 2: Получисленные алгоритмы: Пер. с англ./ Под общ. ред. Ю.В. Козаченко. -Москва: Издат. дом "Вильямс", 2004. – 832 с.
2. Грибанов В.П., Калмыкова О.В., Сорока Р.И. Основы алгоритмизации и программирование. - М.: Моск. гос. ун-т экономики, статистики и информатики, 2001. - 129 с.
3. Заковряшин А.И. Алгоритмизация и программирование вычислительных задач. - М: Science Press, 2002. – 380 с.
4. Генри мл. Уоррен. Алгоритмические трюки для программистов / Пер. с англ. - М.: Издательский дом "Вильяме", 2004. – 382 с.
5. Голицына О.Л., Попов И.И. Основы алгоритмизации и программирования. – М.: Форум, 2008. – 432 с.

Electronic resources

1. www.ziyonet.uz
2. www.twirpx.com
3. www.boorfi.org
4. www.bumlib.com
5. www.ozon.ru
6. www.exponenta.ru
7. www.edu.uz
8. www.ndki.uz
9. www.megasoft.uz
10. www.utube.uz

THE PLANNED SCHEDULE ON DISCIPLINE

CALENDAR - THEMATIC PLAN

On discipline: Algorithmization of computing methods

Lecturer: docent. Urinov Sh.R. **Faculty:** PMF

Consultations and a practical training conducts: ______________

Laboratory researches conducts: _Urinov Sh.R_ **The course** II**, Group ______**

№	Type of lecture	Theme and the summary	Otve-deno	The information about executed works		The teacher's signature
				Number	O'clock	
1	Lecture	Introduction. The cores concept about algorithmization of computing methods.	2			
2	Lecture	Algorithmization of the numerical decision of the algebraic and transcendental equations. A method branch of roots and a method half divisions.	2			
3	Lecture	Algorithmization of the numerical decision of the algebraic and transcendental equations. A method a chord and Newton's method.	2			
4	Lecture	Algorithmization of the numerical decision of the algebraic and transcendental equations. A method of iteration and a method of secants.	2			
5	Lecture	Algorithmization of the numerical decision of system of the algebraic and transcendental equations. A method of Gauss.	2			
6	Lecture	Algorithmization of the numerical decision of system of the algebraic and transcendental equations. Iterative methods of Jacoby and Seidel.	2			
7	Lecture	Algorithmization interpolation methods. Interpolation functions.	2			
8	Lecture	The numerical decision of the differential equations. Euler's method.	2			
9	Lecture	The numerical decision of the differential equations. A method of Runge-Kutta and Adams.	2			
10	Lecture	Numerical integration. Quadrature formulas of trapezes and rectangles. Simpson's formula.	2			
11	Lecture	Numerical integration. The formula of Gauss.	2			
12	Lecture	Root-mean-square approach of functions. Method of the least squares.	2			
13-14	Lecture	Statement of a problem of linear programming. The basic properties the decision of a problem of linear programming.	4			
15-16	Lecture	Geometrical interpretation of a problem of linear programming.	4			
17	Lecture	Finding the decision of a problem of linear programming to Simplex methods.	2			
18	Lecture	Finding the decision of a problem of linear programming. A method of artificial basis.	2			
		IN TOTAL	**36**			

The head of chair:

The teacher:

CALENDAR - THEMATIC PLAN

On discipline: Algorithmization of computing methods

Lecturer: docent. Urinov Sh.R. **Faculty:** PMF

Consultations and a practical training conducts: ______________

Laboratory researches conducts: _ Urinov Sh.R _ **The course** II, **Group** ______

№	**Type of lesson**	**Theme and the summary**	**Otve-deno**	**The information about executed works**		**The teacher's signature**
				Number	O'clock	
1	**2**	**3**	**4**	**5**	**6**	**7**
1-2	Practical work	The numerical decision of the algebraic and transcendental equations iterative methods.	4			
3-4	Practical work	Newton's interpolation polynom and Lagrange	4			
5-6	Practical work	Calculation of integrals by the approached methods	4			
7-8	Practical work	Approximation results of experiment with a method of the least square. Creation non-linear empirical connection	4			
9	Practical work	The geometrical decision of a problem of linear programming.	2			
TOTAL:			**18**			
1-2	Laboratory	The numerical decision of the algebraic and transcendental equations iterative methods Chord and Newton.	4			
3-4	Laboratory	The numerical decision of system of the linear algebraic equations methods of Gauss, simple iteration and Seidel	4			
5-6	Laboratory	Problems of Cochy for the ordinary differential equations. Euler's methods, Runge-Kutte and Adams.	4			
7-8	Laboratory	The numerical decision of system of the nonlinear algebraic equations to methods of simple iteration.	4			
9	Laboratory	Finding the decision of a problem of linear programming to Simplex methods.	2			
Resulto:			**18**			

The head of chair:

The teacher:

TECHNOLOGY TRAINING AND THE TECHNOLOGICAL CARD

TECHNOLOGY TRAINING AND THE TECHNOLOGICAL CARD ON LECTURE

Theme №1	**Introduction. The cores concept about algorithmization of computing methods**

TECHNOLOGY OF TRAINING
(Carrying out) of lecture employment

Employment time - 2 hours (80 minutes)	Quantity of students: 40-50
Mode of study	Introduction-thematic lecture
The lecture plan	– Classification of computing methods. – Preparation of problems for the personal computer decision. – Properties of algorithm. – Classification of algorithms.

The purpose of educational employment: **preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a lecture material to watch mastering at pupils of this knowledge, to form and develop at them skills.**

***Problems of the teacher*:**	***Results of educational activity:***
• To acquaint students with a lecture material, the basic directions of activity on a considered material and the curriculum; • Logically consistently, it is given reason and clearly to state thoughts, correctly to build oral and written speech; • Studying of the basic concepts and the terms applied within the limits of a lecture material; • To make training more clear and accessible, to interest students, and the same to motivate them for the further training and self-education; • Maintenance of perception, judgement and primary storing of communications and relations in object of studying; • Establishment of correctness and sensibleness of mastering of a new teaching material, revealing of incorrect representations and their correction; • Maintenance of mastering of new knowledge and ways of actions; • Formation of complete representation of knowledge on a theme; • To show a distributing material, to give talks and to give practical tasks.	• Mastering of new knowledge and ways of actions; • Primary check of understanding; • Fastening of knowledge and ways of actions; • Generalisation and ordering of knowledge. • Careful studying and the all-round analysis of a lecture material and increase of an educational level which would provide the decision of the put problem; • Increase of a degree of quality of knowledge through introduction of innovative technologies; • Level monitoring обученности pupils on steps, classes, subjects, it is concrete on each student, for the purpose of revealing of the real reasons influencing progress, dynamics of conformity of level of teaching to educational standards. • Monitoring of professional skill of teachers. • To continue activity on the organisation of interaction of participants of educational space; • To create conditions, to generalise an advanced experience and to motivate students; • Increase scientific информативности in a field of knowledge of a subject matter and related subjects.
Training methods	**Lecture – visualisation, conversation**
Technics of training	**Blitz – the interrogation, focusing questions**
Modes of study	**Collective, face-to-face**
Tutorials	**Projector, supply with information, visual materials, educational - methodical grants**
Training conditions	**The audience provided with tutorials**
Monitoring and estimation of knowledge	**The oral control: a question-answer**

TECHNOLOGICAL CARD
Carrying out of lecture employment on a theme:
«Introduction. The cores concept about algorithmization of computing methods»

Stages, time	*The activity maintenance*	
	The teacher	*Students*
1 stage. Introduction ***(15 minutes)***	1.1. Informs a theme, the purpose, planned results of educational employment and the plan of its carrying out. 1.2. For the purpose of actualisation of knowledge of students asks focusing questions.	1.1. Listen. 1.2. Answer questions.
2 stage. Basic (information) **(55 minutes)**	2.1. Consistently states a material of lecture on plan questions. – Classification of computing methods. – Preparation of problems for the personal computer decision. – Properties of algorithm. – Classification of algorithms.	2.1. Listen, discuss the maintenance of schemes and tables, visual materials, specify, ask questions. Write down the main thing.
3 stage. The final ***(10 minutes)***	3.1. Spends a blitz – interrogation on a theme of lecture employment. Does the total conclusion. 3.2. Gives the task for independent work.	3.1. Answer questions. 3.2. Listen, write down.

Theme №2	Algorithmization of the numerical decision of the algebraic and transcendental equations. A method branch of roots and a method half divisions.

TECHNOLOGY OF TRAINING
(Carrying out) of lecture employment

Employment time - 2 hours (80 minutes)	Quantity of students: 40-50
Mode of study	Introduction-thematic lecture
The lecture plan	– A method branch of roots – A method half divisions

The purpose of educational employment: **preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a lecture material to watch mastering at pupils of this knowledge, to form and develop at them skills.**

***Problems of the teacher*:**	***Results of educational activity:***
• To acquaint students with a lecture material, the basic directions of activity on a considered material and the curriculum; • Logically consistently, it is given reason and clearly to state thoughts, correctly to build oral and written speech; • Studying of the basic concepts and the terms applied within the limits of a lecture material; • To make training more clear and accessible, to interest students, and the same to motivate them for the further training and self-education; • Maintenance of perception, judgement and primary storing of communications and relations in object of studying; • Establishment of correctness and sensibleness of mastering of a new teaching material, revealing of incorrect representations and their correction; • Maintenance of mastering of new knowledge and ways of actions; • Formation of complete representation of knowledge on a theme; • To show a distributing material, to give talks and to give practical tasks.	• Mastering of new knowledge and ways of actions; • Primary check of understanding; • Fastening of knowledge and ways of actions; • Generalisation and ordering of knowledge. • Careful studying and the all-round analysis of a lecture material and increase of an educational level which would provide the decision of the put problem; • Increase of a degree of quality of knowledge through introduction of innovative technologies; • Level monitoring обученности pupils on steps, classes, subjects, it is concrete on each student, for the purpose of revealing of the real reasons influencing progress, dynamics of conformity of level of teaching to educational standards. • Monitoring of professional skill of teachers. • To continue activity on the organisation of interaction of participants of educational space; • To create conditions, to generalise an advanced experience and to motivate students; • Increase scientific информативности in a field of knowledge of a subject matter and related subjects.
Training methods	**Lecture – visualisation, conversation**
Technics of training	**Blitz – the interrogation, focusing questions**
Modes of study	**Collective, face-to-face**
Tutorials	**Projector, supply with information, visual materials, educational - methodical grants**
Training conditions	**The audience provided with tutorials**
Monitoring and estimation of knowledge	**The oral control: a question-answer**

TECHNOLOGICAL CARD
Carrying out of lecture employment on a theme:
Algorithmization of the numerical decision of the algebraic and transcendental equations. A method branch of roots and a method half divisions.

Stages, time	*The activity maintenance*	
	The teacher	*Students*
1 stage. Introduction ***(15 minutes)***	1.1. Informs a theme, the purpose, planned results of educational employment and the plan of its carrying out. 1.2. For the purpose of actualisation of knowledge of students asks focusing questions.	1.1. Listen. 1.2. Answer questions.
2 stage. Basic (information) **(55 minutes)**	2.1. Consistently states a material of lecture on plan questions. – A method branch of roots – A method half divisions	2.1. Listen, discuss the maintenance of schemes and tables, visual materials, specify, ask questions. Write down the main thing.
3 stage. The final ***(10 minutes)***	3.1. Spends a blitz – interrogation on a theme of lecture employment. Does the total conclusion. 3.2. Gives the task for independent work.	3.1. Answer questions. 3.2. Listen, write down.

Theme №3	Algorithmization of the numerical decision of the algebraic and transcendental equations. Method a chord and Newton's method

TECHNOLOGY OF TRAINING
(Carrying out) of lecture employment

Employment time - 2 hours (80 minutes)	Quantity of students: 40-50
Mode of study	Introduction-thematic lecture
The lecture plan	– A method the Chord – Newton's method –
The purpose of educational employment: **preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a lecture material to watch mastering at pupils of this knowledge, to form and develop at them skills.**	

Problems of the teacher:	***Results of educational activity:***
• To acquaint students with a lecture material, the basic directions of activity on a considered material and the curriculum; • Logically consistently, it is given reason and clearly to state thoughts, correctly to build oral and written speech; • Studying of the basic concepts and the terms applied within the limits of a lecture material; • To make training more clear and accessible, to interest students, and the same to motivate them for the further training and self-education; • Maintenance of perception, judgement and primary storing of communications and relations in object of studying; • Establishment of correctness and sensibleness of mastering of a new teaching material, revealing of incorrect representations and their correction; • Maintenance of mastering of new knowledge and ways of actions; • Formation of complete representation of knowledge on a theme; • To show a distributing material, to give talks and to give practical tasks.	• Mastering of new knowledge and ways of actions; • Primary check of understanding; • Fastening of knowledge and ways of actions; • Generalisation and ordering of knowledge. • Careful studying and the all-round analysis of a lecture material and increase of an educational level which would provide the decision of the put problem; • Increase of a degree of quality of knowledge through introduction of innovative technologies; • Level monitoring обученности pupils on steps, classes, subjects, it is concrete on each student, for the purpose of revealing of the real reasons influencing progress, dynamics of conformity of level of teaching to educational standards. • Monitoring of professional skill of teachers. • To continue activity on the organisation of interaction of participants of educational space; • To create conditions, to generalise an advanced experience and to motivate students; • Increase scientific информативности in a field of knowledge of a subject matter and related subjects.
Training methods	**Lecture – visualisation, conversation**
Technics of training	**Blitz – the interrogation, focusing questions**
Modes of study	**Collective, face-to-face**
Tutorials	**Projector, supply with information, visual materials, educational - methodical grants**
Training conditions	**The audience provided with tutorials**
Monitoring and estimation of knowledge	**The oral control: a question-answer**

TECHNOLOGICAL CARD
Carrying out of lecture employment on a theme:
Algorithmization of the numerical decision of the algebraic and transcendental equations. Method a chord and Newton's method

Stages, time	*The activity maintenance*	
	The teacher	*Students*
1 stage. Introduction ***(15 minutes)***	1.1. Informs a theme, the purpose, planned results of educational employment and the plan of its carrying out. 1.2. For the purpose of actualisation of knowledge of students asks focusing questions.	1.1. Listen. 1.2. Answer questions.
2 stage. Basic (information) **(55 minutes)**	2.1. Consistently states a material of lecture on plan questions. – A method the Chord – Newton's method	2.1. Listen, discuss the maintenance of schemes and tables, visual materials, specify, ask questions. Write down the main thing.
3 stage. The final ***(10 minutes)***	3.1. Spends a blitz – interrogation on a theme of lecture employment. Does the total conclusion. 3.2. Gives the task for independent work.	3.1. Answer questions. 3.2. Listen, write down.

Theme №4	**Algorithmization of the numerical decision of the algebraic and transcendental equations. A method of iteration and a method of secants**

TECHNOLOGY OF TRAINING
(Carrying out) of lecture employment

Employment time - 2 hours (80 minutes)	Quantity of students: 40-50
Mode of study	Introduction-thematic lecture
The lecture plan	– A method of simple iteration – A method of secants

The purpose of educational employment: **preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a lecture material to watch mastering at pupils of this knowledge, to form and develop at them skills.**	
***Problems of the teacher*:**	***Results of educational activity:***
• To acquaint students with a lecture material, the basic directions of activity on a considered material and the curriculum; • Logically consistently, it is given reason and clearly to state thoughts, correctly to build oral and written speech; • Studying of the basic concepts and the terms applied within the limits of a lecture material; • To make training more clear and accessible, to interest students, and the same to motivate them for the further training and self-education; • Maintenance of perception, judgement and primary storing of communications and relations in object of studying; • Establishment of correctness and sensibleness of mastering of a new teaching material, revealing of incorrect representations and their correction; • Maintenance of mastering of new knowledge and ways of actions; • Formation of complete representation of knowledge on a theme; • To show a distributing material, to give talks and to give practical tasks.	• Mastering of new knowledge and ways of actions; • Primary check of understanding; • Fastening of knowledge and ways of actions; • Generalisation and ordering of knowledge. • Careful studying and the all-round analysis of a lecture material and increase of an educational level which would provide the decision of the put problem; • Increase of a degree of quality of knowledge through introduction of innovative technologies; • Level monitoring обученности pupils on steps, classes, subjects, it is concrete on each student, for the purpose of revealing of the real reasons influencing progress, dynamics of conformity of level of teaching to educational standards. • Monitoring of professional skill of teachers. • To continue activity on the organisation of interaction of participants of educational space; • To create conditions, to generalise an advanced experience and to motivate students; • Increase scientific информативности in a field of knowledge of a subject matter and related subjects.
Training methods	**Lecture – visualisation, conversation**
Technics of training	**Blitz – the interrogation, focusing questions**
Modes of study	**Collective, face-to-face**
Tutorials	**Projector, supply with information, visual materials, educational - methodical grants**
Training conditions	**The audience provided with tutorials**
Monitoring and estimation of knowledge	**The oral control: a question-answer**

TECHNOLOGICAL CARD
Carrying out of lecture employment on a theme:
Algorithmization of the numerical decision of the algebraic and transcendental equations. A method of iteration and a method of secants

Stages, time	*The activity maintenance*	
	The teacher	*Students*
1 stage. Introduction ***(15 minutes)***	1.1. Informs a theme, the purpose, planned results of educational employment and the plan of its carrying out. 1.2. For the purpose of actualisation of knowledge of students asks focusing questions.	1.1. Listen. 1.2. Answer questions.
2 stage. Basic (information) **(55 minutes)**	2.1. Consistently states a material of lecture on plan questions. – A method of simple iteration – A method of secants	2.1. Listen, discuss the maintenance of schemes and tables, visual materials, specify, ask questions. Write down the main thing.
3 stage. The final ***(10 minutes)***	3.1. Spends a blitz – interrogation on a theme of lecture employment. Does the total conclusion. 3.2. Gives the task for independent work.	3.1. Answer questions. 3.2. Listen, write down.

Theme №5	**Algorithmization of the numerical decision of system of the algebraic and transcendental equations. Method of Gauss**

TECHNOLOGY OF TRAINING
(Carrying out) of lecture employment

Employment time - 2 hours (80 minutes)	Quantity of students: 40-50
Mode of study	Introduction-thematic lecture
The lecture plan	– The decision of system of the linear equations a method of Gauss – A method of Gauss with a choice of the main element – An error estimation at the decision of system of the linear equations

The purpose of educational employment: **preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a lecture material to watch mastering at pupils of this knowledge, to form and develop at them skills.**

Problems of the teacher:	***Results of educational activity:***
• To acquaint students with a lecture material, the basic directions of activity on a considered material and the curriculum; • Logically consistently, it is given reason and clearly to state thoughts, correctly to build oral and written speech; • Studying of the basic concepts and the terms applied within the limits of a lecture material; • To make training more clear and accessible, to interest students, and the same to motivate them for the further training and self-education; • Maintenance of perception, judgement and primary storing of communications and relations in object of studying; • Establishment of correctness and sensibleness of mastering of a new teaching material, revealing of incorrect representations and their correction; • Maintenance of mastering of new knowledge and ways of actions; • Formation of complete representation of knowledge on a theme; • To show a distributing material, to give talks and to give practical tasks.	• Mastering of new knowledge and ways of actions; • Primary check of understanding; • Fastening of knowledge and ways of actions; • Generalisation and ordering of knowledge. • Careful studying and the all-round analysis of a lecture material and increase of an educational level which would provide the decision of the put problem; • Increase of a degree of quality of knowledge through introduction of innovative technologies; • Level monitoring обученности pupils on steps, classes, subjects, it is concrete on each student, for the purpose of revealing of the real reasons influencing progress, dynamics of conformity of level of teaching to educational standards. • Monitoring of professional skill of teachers. • To continue activity on the organisation of interaction of participants of educational space; • To create conditions, to generalise an advanced experience and to motivate students; • Increase scientific информативности in a field of knowledge of a subject matter and related subjects.
Training methods	**Lecture – visualisation, conversation**
Technics of training	**Blitz – the interrogation, focusing questions**
Modes of study	**Collective, face-to-face**
Tutorials	**Projector, supply with information, visual materials, educational - methodical grants**
Training conditions	**The audience provided with tutorials**
Monitoring and estimation of knowledge	**The oral control: a question-answer**

TECHNOLOGICAL CARD
Carrying out of lecture employment on a theme:
Algorithmization of the numerical decision of system of the algebraic and transcendental equations. Method of Gaussa

Stages, time	*The activity maintenance*	
	The teacher	*Students*
1 stage. Introduction ***(15 minutes)***	1.1. Informs a theme, the purpose, planned results of educational employment and the plan of its carrying out. 1.2. For the purpose of actualisation of knowledge of students asks focusing questions.	1.1. Listen. 1.2. Answer questions.
2 stage. Basic (information) **(55 minutes)**	2.1. Consistently states a material of lecture on plan questions. – The decision of system of the linear equations a method of Gaussa – A method of Gaussa with a choice of the main element – An error estimation at the decision of system of the linear equations	2.1. Listen, discuss the maintenance of schemes and tables, visual materials, specify, ask questions. Write down the main thing.
3 stage. The final ***(10 minutes)***	3.1.Spends a blitz – interrogation on a theme of lecture employment. Does the total conclusion. 3.2. Gives the task for independent work.	3.1. Answer questions. 3.2. Listen, write down.

Theme №6	Algorithmization of the numerical decision of system of the algebraic and transcendental equations. Iterative methods of Jacoby and Seidel

TECHNOLOGY OF TRAINING
(Carrying out) of lecture employment

Employment time - 2 hours (80 minutes)	Quantity of students: 40-50
Mode of study	Introduction-thematic lecture
The lecture plan	– Iterative methods of the decision of systems of the linear equations – Method of simple iteration of Jacoby – A method of Gauss-Seidel

The purpose of educational employment: **preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a lecture material to watch mastering at pupils of this knowledge, to form and develop at them skills.**

Problems of the teacher:	***Results of educational activity:***
• To acquaint students with a lecture material, the basic directions of activity on a considered material and the curriculum; • Logically consistently, it is given reason and clearly to state thoughts, correctly to build oral and written speech; • Studying of the basic concepts and the terms applied within the limits of a lecture material; • To make training more clear and accessible, to interest students, and the same to motivate them for the further training and self-education; • Maintenance of perception, judgement and primary storing of communications and relations in object of studying; • Establishment of correctness and sensibleness of mastering of a new teaching material, revealing of incorrect representations and their correction; • Maintenance of mastering of new knowledge and ways of actions; • Formation of complete representation of knowledge on a theme; • To show a distributing material, to give talks and to give practical tasks.	• Mastering of new knowledge and ways of actions; • Primary check of understanding; • Fastening of knowledge and ways of actions; • Generalisation and ordering of knowledge. • Careful studying and the all-round analysis of a lecture material and increase of an educational level which would provide the decision of the put problem; • Increase of a degree of quality of knowledge through introduction of innovative technologies; • Level monitoring обученности pupils on steps, classes, subjects, it is concrete on each student, for the purpose of revealing of the real reasons influencing progress, dynamics of conformity of level of teaching to educational standards. • Monitoring of professional skill of teachers. • To continue activity on the organisation of interaction of participants of educational space; • To create conditions, to generalise an advanced experience and to motivate students; • Increase scientific информативности in a field of knowledge of a subject matter and related subjects.
Training methods	**Lecture – visualisation, conversation**
Technics of training	**Blitz – the interrogation, focusing questions**
Modes of study	**Collective, face-to-face**
Tutorials	**Projector, supply with information, visual materials, educational - methodical grants**
Training conditions	**The audience provided with tutorials**
Monitoring and estimation of knowledge	**The oral control: a question-answer**

TECHNOLOGICAL CARD
Carrying out of lecture employment on a theme:
Algorithmization of the numerical decision of system of the algebraic and transcendental equations. Iterative methods of Jacoby and Zejdel

Stages, time	*The activity maintenance*	
	The teacher	*Students*
1 stage. Introduction ***(15 minutes)***	1.1. Informs a theme, the purpose, planned results of educational employment and the plan of its carrying out. 1.2. For the purpose of actualisation of knowledge of students asks focusing questions.	1.1. Listen. 1.2. Answer questions.
2 stage. Basic (information) **(55 minutes)**	2.1. Consistently states a material of lecture on plan questions. – Iterative methods of the decision of systems of the linear equations – Method of simple iteration of Jacoby – A method of Gaussa-Zejdel	2.1. Listen, discuss the maintenance of schemes and tables, visual materials, specify, ask questions. Write down the main thing.
3 stage. The final ***(10 minutes)***	3.1. Spends a blitz – interrogation on a theme of lecture employment. Does the total conclusion. 3.2. Gives the task for independent work.	3.1. Answer questions. 3.2. Listen, write down.

Theme №7	Algorithmization interpolation methods. Interpolation functions

TECHNOLOGY OF TRAINING
(Carrying out) of lecture employment

Employment time - 2 hours (80 minutes)	Quantity of students: 40-50
Mode of study	Introduction-thematic lecture
The lecture plan	– Introduction – The first interpolationNewton's formula – The second interpolation Newton's formula – The interpolation formula of Stirlinga – An example

The purpose of educational employment: **preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a lecture material to watch mastering at pupils of this knowledge, to form and develop at them skills.**

Problems of the teacher:	***Results of educational activity:***
• To acquaint students with a lecture material, the basic directions of activity on a considered material and the curriculum; • Logically consistently, it is given reason and clearly to state thoughts, correctly to build oral and written speech; • Studying of the basic concepts and the terms applied within the limits of a lecture material; • To make training more clear and accessible, to interest students, and the same to motivate them for the further training and self-education; • Maintenance of perception, judgement and primary storing of communications and relations in object of studying; • Establishment of correctness and sensibleness of mastering of a new teaching material, revealing of incorrect representations and their correction; • Maintenance of mastering of new knowledge and ways of actions; • Formation of complete representation of knowledge on a theme; • To show a distributing material, to give talks and to give practical tasks.	• Mastering of new knowledge and ways of actions; • Primary check of understanding; • Fastening of knowledge and ways of actions; • Generalisation and ordering of knowledge. • Careful studying and the all-round analysis of a lecture material and increase of an educational level which would provide the decision of the put problem; • Increase of a degree of quality of knowledge through introduction of innovative technologies; • Level monitoring обученности pupils on steps, classes, subjects, it is concrete on each student, for the purpose of revealing of the real reasons influencing progress, dynamics of conformity of level of teaching to educational standards. • Monitoring of professional skill of teachers. • To continue activity on the organisation of interaction of participants of educational space; • To create conditions, to generalise an advanced experience and to motivate students; • Increase scientific информативности in a field of knowledge of a subject matter and related subjects.
Training methods	**Lecture – visualisation, conversation**
Technics of training	**Blitz – the interrogation, focusing questions**
Modes of study	**Collective, face-to-face**
Tutorials	**Projector, supply with information, visual materials, educational - methodical grants**
Training conditions	**The audience provided with tutorials**
Monitoring and estimation of knowledge	**The oral control: a question-answer**

TECHNOLOGICAL CARD
Carrying out of lecture employment on a theme:
Algorithmization interpolation methods. Interpolation functions

Stages, time	*The activity maintenance*	
	The teacher	*Students*
1 stage. Introduction ***(15 minutes)***	1.1. Informs a theme, the purpose, planned results of educational employment and the plan of its carrying out. 1.2. For the purpose of actualisation of knowledge of students asks focusing questions.	1.1. Listen. 1.2. Answer questions.
2 stage. Basic (information) **(55 minutes)**	2.1. Consistently states a material of lecture on plan questions. – Introduction – The first interpolationNewton's formula – The second interpolation Newton's formula – The interpolation formula of Stirlinga – An example	2.1. Listen, discuss the maintenance of schemes and tables, visual materials, specify, ask questions. Write down the main thing.
3 stage. The final ***(10 minutes)***	3.1. Spends a blitz – interrogation on a theme of lecture employment. Does the total conclusion. 3.2. Gives the task for independent work.	3.1. Answer questions. 3.2. Listen, write down.

Theme №8	The numerical decision of the differential equations. Euler's method

TECHNOLOGY OF TRAINING

(Carrying out) of lecture employment

Employment time - 2 hours (80 minutes)	Quantity of students: 40-50
Mode of study	Introduction-thematic lecture
The lecture plan	– Types of problems for the ordinary differential equations – Euler's method
The purpose of educational employment: **preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a lecture material to watch mastering at pupils of this knowledge, to form and develop at them skills.**	
***Problems of the teacher*:** • To acquaint students with a lecture material, the basic directions of activity on a considered material and the curriculum; • Logically consistently, it is given reason and clearly to state thoughts, correctly to build oral and written speech; • Studying of the basic concepts and the terms applied within the limits of a lecture material; • To make training more clear and accessible, to interest students, and the same to motivate them for the further training and self-education; • Maintenance of perception, judgement and primary storing of communications and relations in object of studying; • Establishment of correctness and sensibleness of mastering of a new teaching material, revealing of incorrect representations and their correction; • Maintenance of mastering of new knowledge and ways of actions; • Formation of complete representation of knowledge on a theme; • To show a distributing material, to give talks and to give practical tasks.	***Results of educational activity:*** • Mastering of new knowledge and ways of actions; • Primary check of understanding; • Fastening of knowledge and ways of actions; • Generalisation and ordering of knowledge. • Careful studying and the all-round analysis of a lecture material and increase of an educational level which would provide the decision of the put problem; • Increase of a degree of quality of knowledge through introduction of innovative technologies; • Level monitoring обученности pupils on steps, classes, subjects, it is concrete on each student, for the purpose of revealing of the real reasons influencing progress, dynamics of conformity of level of teaching to educational standards. • Monitoring of professional skill of teachers. • To continue activity on the organisation of interaction of participants of educational space; • To create conditions, to generalise an advanced experience and to motivate students; • Increase scientific информативности in a field of knowledge of a subject matter and related subjects.
Training methods	**Lecture – visualisation, conversation**
Technics of training	**Blitz – the interrogation, focusing questions**
Modes of study	**Collective, face-to-face**
Tutorials	**Projector, supply with information, visual materials, educational - methodical grants**
Training conditions	**The audience provided with tutorials**
Monitoring and estimation of knowledge	**The oral control: a question-answer**

TECHNOLOGICAL CARD

Carrying out of lecture employment on a theme:

The numerical decision of the differential equations. Euler's method

Stages, time	*The activity maintenance*	
	The teacher	*Students*
1 stage. Introduction ***(15 minutes)***	1.1. Informs a theme, the purpose, planned results of educational employment and the plan of its carrying out. 1.2. For the purpose of actualisation of knowledge of students asks focusing questions.	1.1. Listen. 1.2. Answer questions.
2 stage. Basic (information) **(55 minutes)**	2.1. Consistently states a material of lecture on plan questions. – Types of problems for the ordinary differential equations – Euler's method	2.1. Listen, discuss the maintenance of schemes and tables, visual materials, specify, ask questions. Write down the main thing.
3 stage. The final ***(10 minutes)***	3.1. Spends a blitz – interrogation on a theme of lecture employment. Does the total conclusion. 3.2. Gives the task for independent work.	3.1. Answer questions. 3.2. Listen, write down.

Theme №9	The numerical decision of the differential equations. A method of Runge-Kutte and Adams

TECHNOLOGY OF TRAINING
(Carrying out) of lecture employment

Employment time - 2 hours (80 minutes)	Quantity of students: 40-50
Mode of study	Introduction-thematic lecture
The lecture plan	– Methods of Runge-Kutte – Adams's method

The purpose of educational employment: **preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a lecture material to watch mastering at pupils of this knowledge, to form and develop at them skills.**

***Problems of the teacher*:**	***Results of educational activity:***
• To acquaint students with a lecture material, the basic directions of activity on a considered material and the curriculum; • Logically consistently, it is given reason and clearly to state thoughts, correctly to build oral and written speech; • Studying of the basic concepts and the terms applied within the limits of a lecture material; • To make training more clear and accessible, to interest students, and the same to motivate them for the further training and self-education; • Maintenance of perception, judgement and primary storing of communications and relations in object of studying; • Establishment of correctness and sensibleness of mastering of a new teaching material, revealing of incorrect representations and their correction; • Maintenance of mastering of new knowledge and ways of actions; • Formation of complete representation of knowledge on a theme; • To show a distributing material, to give talks and to give practical tasks.	• Mastering of new knowledge and ways of actions; • Primary check of understanding; • Fastening of knowledge and ways of actions; • Generalisation and ordering of knowledge. • Careful studying and the all-round analysis of a lecture material and increase of an educational level which would provide the decision of the put problem; • Increase of a degree of quality of knowledge through introduction of innovative technologies; • Level monitoring обученности pupils on steps, classes, subjects, it is concrete on each student, for the purpose of revealing of the real reasons influencing progress, dynamics of conformity of level of teaching to educational standards. • Monitoring of professional skill of teachers. • To continue activity on the organisation of interaction of participants of educational space; • To create conditions, to generalise an advanced experience and to motivate students; • Increase scientific информативности in a field of knowledge of a subject matter and related subjects.
Training methods	**Lecture – visualisation, conversation**
Technics of training	**Blitz – the interrogation, focusing questions**
Modes of study	**Collective, face-to-face**
Tutorials	**Projector, supply with information, visual materials, educational - methodical grants**
Training conditions	**The audience provided with tutorials**
Monitoring and estimation of knowledge	**The oral control: a question-answer**

TECHNOLOGICAL CARD

Carrying out of lecture employment on a theme:

The numerical decision of the differential equations. A method of Runge-Kutta and Adams

Stages, time	*The activity maintenance*	
	The teacher	*Students*
1 stage. Introduction ***(15 minutes)***	1.1. Informs a theme, the purpose, planned results of educational employment and the plan of its carrying out. 1.2. For the purpose of actualisation of knowledge of students asks focusing questions.	1.1. Listen. 1.2. Answer questions.
2 stage. Basic (information) **(55 minutes)**	2.1. Consistently states a material of lecture on plan questions. – Methods of Runge-Kutta – Adams's method	2.1. Listen, discuss the maintenance of schemes and tables, visual materials, specify, ask questions. Write down the main thing.
3 stage. The final ***(10 minutes)***	3.1. Spends a blitz – interrogation on a theme of lecture employment. Does the total conclusion. 3.2. Gives the task for independent work.	3.1. Answer questions. 3.2. Listen, write down.

Theme №10	Numerical integration. Quadrature formulas of trapezes and rectangles. Simpson's formula

TECHNOLOGY OF TRAINING

(Carrying out) of lecture employment

Employment time - 2 hours (80 minutes)	Quantity of students: 40-50
Mode of study	Introduction-thematic lecture
The lecture plan	– Classification of methods – A method of trapezes – Methods of rectangles – Simpson's method
The purpose of educational employment: **preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a lecture material to watch mastering at pupils of this knowledge, to form and develop at them skills.**	

Problems of the teacher:	***Results of educational activity:***
• To acquaint students with a lecture material, the basic directions of activity on a considered material and the curriculum; • Logically consistently, it is given reason and clearly to state thoughts, correctly to build oral and written speech; • Studying of the basic concepts and the terms applied within the limits of a lecture material; • To make training more clear and accessible, to interest students, and the same to motivate them for the further training and self-education; • Maintenance of perception, judgement and primary storing of communications and relations in object of studying; • Establishment of correctness and sensibleness of mastering of a new teaching material, revealing of incorrect representations and their correction; • Maintenance of mastering of new knowledge and ways of actions; • Formation of complete representation of knowledge on a theme; • To show a distributing material, to give talks and to give practical tasks.	• Mastering of new knowledge and ways of actions; • Primary check of understanding; • Fastening of knowledge and ways of actions; • Generalisation and ordering of knowledge. • Careful studying and the all-round analysis of a lecture material and increase of an educational level which would provide the decision of the put problem; • Increase of a degree of quality of knowledge through introduction of innovative technologies; • Level monitoring обученности pupils on steps, classes, subjects, it is concrete on each student, for the purpose of revealing of the real reasons influencing progress, dynamics of conformity of level of teaching to educational standards. • Monitoring of professional skill of teachers. • To continue activity on the organisation of interaction of participants of educational space; • To create conditions, to generalise an advanced experience and to motivate students; • Increase scientific информативности in a field of knowledge of a subject matter and related subjects.
Training methods	**Lecture – visualisation, conversation**
Technics of training	**Blitz – the interrogation, focusing questions**
Modes of study	**Collective, face-to-face**
Tutorials	**Projector, supply with information, visual materials, educational - methodical grants**
Training conditions	**The audience provided with tutorials**
Monitoring and estimation of knowledge	**The oral control: a question-answer**

TECHNOLOGICAL CARD

Carrying out of lecture employment on a theme:

Numerical integration. Quadrature formulas of trapezes and rectangles. Simpson's formula

Stages, time	*The activity maintenance*	
	The teacher	*Students*
1 stage. Introduction ***(15 minutes)***	1.1. Informs a theme, the purpose, planned results of educational employment and the plan of its carrying out. 1.2. For the purpose of actualisation of knowledge of students asks focusing questions.	1.1. Listen. 1.2. Answer questions.
2 stage. Basic (information) **(55 minutes)**	2.1. Consistently states a material of lecture on plan questions. – Classification of methods – A method of trapezes – Methods of rectangles – Simpson's method	2.1. Listen, discuss the maintenance of schemes and tables, visual materials, specify, ask questions. Write down the main thing.
3 stage. The final ***(10 minutes)***	3.1. Spends a blitz – interrogation on a theme of lecture employment. Does the total conclusion. 3.2. Gives the task for independent work.	3.1. Answer questions. 3.2. Listen, write down.

Theme №11	Numerical integration. The formula of Gauss

TECHNOLOGY OF TRAINING
(Carrying out) of lecture employment

Employment time - 2 hours (80 minutes)	Quantity of students: 40-50
Mode of study	Introduction-thematic lecture
The lecture plan	– The quadrature formula of Gauss – Solve the equations

The purpose of educational employment: **preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a lecture material to watch mastering at pupils of this knowledge, to form and develop at them skills.**

***Problems of the teacher*:**	***Results of educational activity:***
• To acquaint students with a lecture material, the basic directions of activity on a considered material and the curriculum; • Logically consistently, it is given reason and clearly to state thoughts, correctly to build oral and written speech; • Studying of the basic concepts and the terms applied within the limits of a lecture material; • To make training more clear and accessible, to interest students, and the same to motivate them for the further training and self-education; • Maintenance of perception, judgement and primary storing of communications and relations in object of studying; • Establishment of correctness and sensibleness of mastering of a new teaching material, revealing of incorrect representations and their correction; • Maintenance of mastering of new knowledge and ways of actions; • Formation of complete representation of knowledge on a theme; • To show a distributing material, to give talks and to give practical tasks.	• Mastering of new knowledge and ways of actions; • Primary check of understanding; • Fastening of knowledge and ways of actions; • Generalisation and ordering of knowledge. • Careful studying and the all-round analysis of a lecture material and increase of an educational level which would provide the decision of the put problem; • Increase of a degree of quality of knowledge through introduction of innovative technologies; • Level monitoring обученности pupils on steps, classes, subjects, it is concrete on each student, for the purpose of revealing of the real reasons influencing progress, dynamics of conformity of level of teaching to educational standards. • Monitoring of professional skill of teachers. • To continue activity on the organisation of interaction of participants of educational space; • To create conditions, to generalise an advanced experience and to motivate students; • Increase scientific информативности in a field of knowledge of a subject matter and related subjects.
Training methods	**Lecture – visualisation, conversation**
Technics of training	**Blitz – the interrogation, focusing questions**
Modes of study	**Collective, face-to-face**
Tutorials	**Projector, supply with information, visual materials, educational - methodical grants**
Training conditions	**The audience provided with tutorials**
Monitoring and estimation of knowledge	**The oral control: a question-answer**

TECHNOLOGICAL CARD
Carrying out of lecture employment on a theme:
Numerical integration. The formula of Gaussa

Stages, time	*The activity maintenance*	
	The teacher	*Students*
1 stage. Introduction ***(15 minutes)***	1.1. Informs a theme, the purpose, planned results of educational employment and the plan of its carrying out. 1.2. For the purpose of actualisation of knowledge of students asks focusing questions.	1.1. Listen. 1.2. Answer questions.
2 stage. Basic (information) **(55 minutes)**	2.1. Consistently states a material of lecture on plan questions. – The quadrature formula of Gaussa – Solve the equations	2.1. Listen, discuss the maintenance of schemes and tables, visual materials, specify, ask questions. Write down the main thing.
3 stage. The final ***(10 minutes)***	3.1. Spends a blitz – interrogation on a theme of lecture employment. Does the total conclusion. 3.2. Gives the task for independent work.	3.1. Answer questions. 3.2. Listen, write down.

Theme №12	Root-mean-square approach of functions. A method of the least squares

TECHNOLOGY OF TRAINING
(Carrying out) of lecture employment

Employment time - 2 hours (80 minutes)	Quantity of students: 40-50
Mode of study	Introduction-thematic lecture
The lecture plan	– Root-mean-square approach of functions – A method of the least squares

The purpose of educational employment: **preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a lecture material to watch mastering at pupils of this knowledge, to form and develop at them skills.**

***Problems of the teacher*:**	***Results of educational activity:***
• To acquaint students with a lecture material, the basic directions of activity on a considered material and the curriculum; • Logically consistently, it is given reason and clearly to state thoughts, correctly to build oral and written speech; • Studying of the basic concepts and the terms applied within the limits of a lecture material; • To make training more clear and accessible, to interest students, and the same to motivate them for the further training and self-education; • Maintenance of perception, judgement and primary storing of communications and relations in object of studying; • Establishment of correctness and sensibleness of mastering of a new teaching material, revealing of incorrect representations and their correction; • Maintenance of mastering of new knowledge and ways of actions; • Formation of complete representation of knowledge on a theme; • To show a distributing material, to give talks and to give practical tasks.	• Mastering of new knowledge and ways of actions; • Primary check of understanding; • Fastening of knowledge and ways of actions; • Generalisation and ordering of knowledge. • Careful studying and the all-round analysis of a lecture material and increase of an educational level which would provide the decision of the put problem; • Increase of a degree of quality of knowledge through introduction of innovative technologies; • Level monitoring обученности pupils on steps, classes, subjects, it is concrete on each student, for the purpose of revealing of the real reasons influencing progress, dynamics of conformity of level of teaching to educational standards. • Monitoring of professional skill of teachers. • To continue activity on the organisation of interaction of participants of educational space; • To create conditions, to generalise an advanced experience and to motivate students; • Increase scientific информативности in a field of knowledge of a subject matter and related subjects.
Training methods	**Lecture – visualisation, conversation**
Technics of training	**Blitz – the interrogation, focusing questions**
Modes of study	**Collective, face-to-face**
Tutorials	**Projector, supply with information, visual materials, educational - methodical grants**
Training conditions	**The audience provided with tutorials**
Monitoring and estimation of knowledge	**The oral control: a question-answer**

TECHNOLOGICAL CARD
Carrying out of lecture employment on a theme:
Root-mean-square approach of functions. A method of the least squares

Stages, time	*The activity maintenance*	
	The teacher	*Students*
1 stage. Introduction ***(15 minutes)***	1.1. Informs a theme, the purpose, planned results of educational employment and the plan of its carrying out. 1.2. For the purpose of actualisation of knowledge of students asks focusing questions.	1.1. Listen. 1.2. Answer questions.
2 stage. Basic (information) **(55 minutes)**	2.1. Consistently states a material of lecture on plan questions. – Root-mean-square approach of functions – A method of the least squares	2.1. Listen, discuss the maintenance of schemes and tables, visual materials, specify, ask questions. Write down the main thing.
3 stage. The final ***(10 minutes)***	3.1. Spends a blitz – interrogation on a theme of lecture employment. Does the total conclusion. 3.2. Gives the task for independent work.	3.1. Answer questions. 3.2. Listen, write down.

Theme №13-14	Statement of a problem of linear programming. The basic properties the decision of a problem of linear programming

TECHNOLOGY OF TRAINING
(Carrying out) of lecture employment

Employment time - 4 hours (160 minutes)	Quantity of students: 40-50
Mode of study	Introduction-thematic lecture
The lecture plan	– The primary goal of linear programming – Examples of the decision of a problem
The purpose of educational employment: **preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a lecture material to watch mastering at pupils of this knowledge, to form and develop at them skills.**	

***Problems of the teacher*:**	***Results of educational activity:***
• To acquaint students with a lecture material, the basic directions of activity on a considered material and the curriculum; • Logically consistently, it is given reason and clearly to state thoughts, correctly to build oral and written speech; • Studying of the basic concepts and the terms applied within the limits of a lecture material; • To make training more clear and accessible, to interest students, and the same to motivate them for the further training and self-education; • Maintenance of perception, judgement and primary storing of communications and relations in object of studying; • Establishment of correctness and sensibleness of mastering of a new teaching material, revealing of incorrect representations and their correction; • Maintenance of mastering of new knowledge and ways of actions; • Formation of complete representation of knowledge on a theme; • To show a distributing material, to give talks and to give practical tasks.	• Mastering of new knowledge and ways of actions; • Primary check of understanding; • Fastening of knowledge and ways of actions; • Generalisation and ordering of knowledge. • Careful studying and the all-round analysis of a lecture material and increase of an educational level which would provide the decision of the put problem; • Increase of a degree of quality of knowledge through introduction of innovative technologies; • Level monitoring обученности pupils on steps, classes, subjects, it is concrete on each student, for the purpose of revealing of the real reasons influencing progress, dynamics of conformity of level of teaching to educational standards. • Monitoring of professional skill of teachers. • To continue activity on the organisation of interaction of participants of educational space; • To create conditions, to generalise an advanced experience and to motivate students; • Increase scientific информативности in a field of knowledge of a subject matter and related subjects.
Training methods	**Lecture – visualisation, conversation**
Technics of training	**Blitz – the interrogation, focusing questions**
Modes of study	**Collective, face-to-face**
Tutorials	**Projector, supply with information, visual materials, educational - methodical grants**
Training conditions	**The audience provided with tutorials**
Monitoring and estimation of knowledge	**The oral control: a question-answer**

TECHNOLOGICAL CARD

Carrying out of lecture employment on a theme:

Statement of a problem of linear programming. The basic properties the decision of a problem of linear programming

Stages, time	*The activity maintenance*	
	The teacher	*Students*
1 stage. Introduction ***(15 minutes)***	1.1. Informs a theme, the purpose, planned results of educational employment and the plan of its carrying out. 1.2. For the purpose of actualisation of knowledge of students asks focusing questions.	1.1. Listen. 1.2. Answer questions.
2 stage. Basic (information) **(135 minutes)**	2.1. Consistently states a material of lecture on plan questions. – The primary goal of linear programming – Examples of the decision of a problem	2.1. Listen, discuss the maintenance of schemes and tables, visual materials, specify, ask questions. Write down the main thing.
3 stage. The final ***(10 minutes)***	3.1. Spends a blitz – interrogation on a theme of lecture employment. Does the total conclusion. 3.2. Gives the task for independent work.	3.1. Answer questions. 3.2. Listen, write down.

Theme №15-16	**Geometrical interpretation of a problem of linear programming**

TECHNOLOGY OF TRAINING
(Carrying out) of lecture employment

Employment time - 4 hours (160 minutes)	Quantity of students: 40-50
Mode of study	Introduction-thematic lecture
The lecture plan	– Problem statement – Geometrical representation. – An example of the decision of a problem – Geometrical problem interpretation

The purpose of educational employment: **preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a lecture material to watch mastering at pupils of this knowledge, to form and develop at them skills.**

Problems of the teacher:	***Results of educational activity:***
• To acquaint students with a lecture material, the basic directions of activity on a considered material and the curriculum; • Logically consistently, it is given reason and clearly to state thoughts, correctly to build oral and written speech; • Studying of the basic concepts and the terms applied within the limits of a lecture material; • To make training more clear and accessible, to interest students, and the same to motivate them for the further training and self-education; • Maintenance of perception, judgement and primary storing of communications and relations in object of studying; • Establishment of correctness and sensibleness of mastering of a new teaching material, revealing of incorrect representations and their correction; • Maintenance of mastering of new knowledge and ways of actions; • Formation of complete representation of knowledge on a theme; • To show a distributing material, to give talks and to give practical tasks.	• Mastering of new knowledge and ways of actions; • Primary check of understanding; • Fastening of knowledge and ways of actions; • Generalisation and ordering of knowledge. • Careful studying and the all-round analysis of a lecture material and increase of an educational level which would provide the decision of the put problem; • Increase of a degree of quality of knowledge through introduction of innovative technologies; • Level monitoring обученности pupils on steps, classes, subjects, it is concrete on each student, for the purpose of revealing of the real reasons influencing progress, dynamics of conformity of level of teaching to educational standards. • Monitoring of professional skill of teachers. • To continue activity on the organisation of interaction of participants of educational space; • To create conditions, to generalise an advanced experience and to motivate students; • Increase scientific информативности in a field of knowledge of a subject matter and related subjects.
Training methods	**Lecture – visualisation, conversation**
Technics of training	**Blitz – the interrogation, focusing questions**
Modes of study	**Collective, face-to-face**
Tutorials	**Projector, supply with information, visual materials, educational - methodical grants**
Training conditions	**The audience provided with tutorials**
Monitoring and estimation of knowledge	**The oral control: a question-answer**

TECHNOLOGICAL CARD
Carrying out of lecture employment on a theme:
Geometrical interpretation of a problem of linear programming

Stages, time	*The activity maintenance*	
	The teacher	*Students*
1 stage. Introduction ***(15 minutes)***	1.1. Informs a theme, the purpose, planned results of educational employment and the plan of its carrying out. 1.2. For the purpose of actualisation of knowledge of students asks focusing questions.	1.1. Listen. 1.2. Answer questions.
2 stage. Basic (information) **(135 minutes)**	2.1. Consistently states a material of lecture on plan questions. – Problem statement – Geometrical representation. – An example of the decision of a problem – Geometrical problem interpretation	2.1. Listen, discuss the maintenance of schemes and tables, visual materials, specify, ask questions. Write down the main thing.
3 stage. The final ***(10 minutes)***	3.1. Spends a blitz – interrogation on a theme of lecture employment. Does the total conclusion. 3.2. Gives the task for independent work.	3.1. Answer questions. 3.2. Listen, write down.

Theme №17	Finding the decision of a problem of linear programming to simplex methods

TECHNOLOGY OF TRAINING
(Carrying out) of lecture employment

Employment time - 2 hours (80 minutes)	Quantity of students: 40-50
Mode of study	Introduction-thematic lecture
The lecture plan	– Mathematical bases a simplex of a method of the decision – Fill the table

The purpose of educational employment: **preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a lecture material to watch mastering at pupils of this knowledge, to form and develop at them skills.**	
***Problems of the teacher*:**	***Results of educational activity:***
• To acquaint students with a lecture material, the basic directions of activity on a considered material and the curriculum; • Logically consistently, it is given reason and clearly to state thoughts, correctly to build oral and written speech; • Studying of the basic concepts and the terms applied within the limits of a lecture material; • To make training more clear and accessible, to interest students, and the same to motivate them for the further training and self-education; • Maintenance of perception, judgement and primary storing of communications and relations in object of studying; • Establishment of correctness and sensibleness of mastering of a new teaching material, revealing of incorrect representations and their correction; • Maintenance of mastering of new knowledge and ways of actions; • Formation of complete representation of knowledge on a theme; • To show a distributing material, to give talks and to give practical tasks.	• Mastering of new knowledge and ways of actions; • Primary check of understanding; • Fastening of knowledge and ways of actions; • Generalisation and ordering of knowledge. • Careful studying and the all-round analysis of a lecture material and increase of an educational level which would provide the decision of the put problem; • Increase of a degree of quality of knowledge through introduction of innovative technologies; • Level monitoring обученности pupils on steps, classes, subjects, it is concrete on each student, for the purpose of revealing of the real reasons influencing progress, dynamics of conformity of level of teaching to educational standards. • Monitoring of professional skill of teachers. • To continue activity on the organisation of interaction of participants of educational space; • To create conditions, to generalise an advanced experience and to motivate students; • Increase scientific информативности in a field of knowledge of a subject matter and related subjects.
Training methods	**Lecture – visualisation, conversation**
Technics of training	**Blitz – the interrogation, focusing questions**
Modes of study	**Collective, face-to-face**
Tutorials	**Projector, supply with information, visual materials, educational - methodical grants**
Training conditions	**The audience provided with tutorials**
Monitoring and estimation of knowledge	**The oral control: a question-answer**

TECHNOLOGICAL CARD
Carrying out of lecture employment on a theme:
Finding the decision of a problem of linear programming to simplex methods

Stages, time	*The activity maintenance*	
	The teacher	*Students*
1 stage. Introduction ***(15 minutes)***	1.1. Informs a theme, the purpose, planned results of educational employment and the plan of its carrying out. 1.2. For the purpose of actualisation of knowledge of students asks focusing questions.	1.1. Listen. 1.2. Answer questions.
2 stage. Basic (information) **(55 minutes)**	2.1. Consistently states a material of lecture on plan questions. – Mathematical bases a simplex of a method of the decision – Fill the table	2.1. Listen, discuss the maintenance of schemes and tables, visual materials, specify, ask questions. Write down the main thing.
3 stage. The final ***(10 minutes)***	3.1. Spends a blitz – interrogation on a theme of lecture employment. Does the total conclusion. 3.2. Gives the task for independent work.	3.1. Answer questions. 3.2. Listen, write down.

Theme №18	Finding the decision of a problem of linear programming. A method of artificial basis

TECHNOLOGY OF TRAINING
(Carrying out) of lecture employment

Employment time - 2 hours (80 minutes)	Quantity of students: 40-50
Mode of study	Introduction-thematic lecture
The lecture plan	– Search of the initial basic decision – Non-negative, independent variables

The purpose of educational employment: **preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a lecture material to watch mastering at pupils of this knowledge, to form and develop at them skills.**	
***Problems of the teacher*:** • To acquaint students with a lecture material, the basic directions of activity on a considered material and the curriculum; • Logically consistently, it is given reason and clearly to state thoughts, correctly to build oral and written speech; • Studying of the basic concepts and the terms applied within the limits of a lecture material; • To make training more clear and accessible, to interest students, and the same to motivate them for the further training and self-education; • Maintenance of perception, judgement and primary storing of communications and relations in object of studying; • Establishment of correctness and sensibleness of mastering of a new teaching material, revealing of incorrect representations and their correction; • Maintenance of mastering of new knowledge and ways of actions; • Formation of complete representation of knowledge on a theme; • To show a distributing material, to give talks and to give practical tasks.	***Results of educational activity:*** • Mastering of new knowledge and ways of actions; • Primary check of understanding; • Fastening of knowledge and ways of actions; • Generalisation and ordering of knowledge. • Careful studying and the all-round analysis of a lecture material and increase of an educational level which would provide the decision of the put problem; • Increase of a degree of quality of knowledge through introduction of innovative technologies; • Level monitoring обученности pupils on steps, classes, subjects, it is concrete on each student, for the purpose of revealing of the real reasons influencing progress, dynamics of conformity of level of teaching to educational standards. • Monitoring of professional skill of teachers. • To continue activity on the organisation of interaction of participants of educational space; • To create conditions, to generalise an advanced experience and to motivate students; • Increase scientific информативности in a field of knowledge of a subject matter and related subjects.
Training methods	**Lecture – visualisation, conversation**
Technics of training	**Blitz – the interrogation, focusing questions**
Modes of study	**Collective, face-to-face**
Tutorials	**Projector, supply with information, visual materials, educational - methodical grants**
Training conditions	**The audience provided with tutorials**
Monitoring and estimation of knowledge	**The oral control: a question-answer**

TECHNOLOGICAL CARD
Carrying out of lecture employment on a theme:
Finding the decision of a problem of linear programming. A method of artificial basis

Stages, time	*The activity maintenance*	
	The teacher	*Students*
1 stage. Introduction ***(15 minutes)***	1.1. Informs a theme, the purpose, planned results of educational employment and the plan of its carrying out. 1.2. For the purpose of actualisation of knowledge of students asks focusing questions.	1.1. Listen. 1.2. Answer questions.
2 stage. Basic (information) **(55 minutes)**	2.1. Consistently states a material of lecture on plan questions. – Search of the initial basic decision – Non-negative, independent variables	2.1. Listen, discuss the maintenance of schemes and tables, visual materials, specify, ask questions. Write down the main thing.
3 stage. The final ***(10 minutes)***	3.1. Spends a blitz – interrogation on a theme of lecture employment. Does the total conclusion. 3.2. Gives the task for independent work.	3.1. Answer questions. 3.2. Listen, write down.

TECHNOLOGY TRAINING AND THE TECHNOLOGICAL CARD ON PRACTICE

Theme № 1-2	**The numerical decision of the algebraic and transcendental equations iterative methods.**

TECHNOLOGY OF TRAINING
(Carrying out) of practical employment

Employment time - 4 hours (160 minutes)	Quantity of students: 15-20
Mode of study	Introduction - thematic lecture
The plan practical employment	– Iteration method – Method of Chords – Method half divisions
The purpose of educational employment: preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a practical material to watch mastering at pupils of this knowledge, to form and develop at them skills.	
Problems of the teacher:	***Results of educational activity:***
• To acquaint students with a practical material, the basic directions of activity on a considered material and the curriculum; • Logically consistently, it is given reason and clearly to state thoughts, correctly to build oral and written speech; • Studying of the basic concepts and the terms applied in frameworks практичкеского of a material; • To make training more clear and accessible, to interest students, and the same to motivate them for the further training and self-education; • Maintenance of perception, judgement and primary storing of communications and relations in object of studying; • Establishment of correctness and sensibleness of mastering of a new teaching material, revealing of incorrect representations and their correction; • Maintenance of mastering of new knowledge and ways of actions; • Formation of complete representation of knowledge on a theme; • To show a distributing material, to give talks and to give practical tasks.	• Mastering of new knowledge and ways of actions; • Primary check of understanding; • Fastening of knowledge and ways of actions; • Generalisation and ordering of knowledge. • Careful studying and the all-round analysis of a lecture material and increase of an educational level which would provide the decision of the put problem; • Increase of a degree of quality of knowledge through introduction of innovative technologies; • Level monitoring обученности pupils on steps, classes, subjects, it is concrete on each student, for the purpose of revealing of the real reasons influencing progress, dynamics of conformity of level of teaching to educational standards. • Monitoring of professional skill of teachers. • To continue activity on the organisation of interaction of participants of educational space; • To create conditions, to generalise an advanced experience and to motivate students; • Increase scientific информативности in a field of knowledge of a subject matter and related subjects.
Training methods	Practice – visualisation, conversation
Technics of training	Blitz – the interrogation, focusing questions
Modes of study	Collective, face-to-face
Tutorials	Projector, supply with information, visual materials, educational - methodical grants
Training conditions	The audience provided with tutorials
Monitoring and estimation of knowledge	The oral control: a question-answer

TECHNOLOGICAL CARD
Carrying out of practical employment on a theme:
The numerical decision of the algebraic and transcendental equations iterative methods.

Stages, time	*The activity maintenance*	
	The teacher	*Students*
1 stage. Introduction ***(15 minutes)***	1.1. Informs a theme, the purpose, planned results of educational employment and the plan of its carrying out. 1.2. For the purpose of actualisation of knowledge of students asks focusing questions.	1.1. Listen. 1.2. Answer questions.
2 stage. Basic (information) **(135 minutes)**	2.1. Consistently states a material of practice concerning the plan. – Iteration method – Method of Chords – Method half divisions	2.1. Listen, discuss the maintenance of schemes and tables, visual materials, specify, ask questions. Write down the main thing.
3 stage. The final ***(10 minutes)***	3.1. Spends a blitz – interrogation on a theme of practical employment. Does the total conclusion. 3.2. Gives the task for independent work.	3.1. Answer questions. 3.2. Listen, write down.

Theme № 3-4	Interpolation polynom Newton and Lagrange

TECHNOLOGY OF TRAINING
(Carrying out) of practical employment

Employment time - 4 hours (160 minutes)	Quantity of students: 15-20
Mode of study	Introduction - thematic lecture
The plan practical employment	– Interpolation polynom of Newton – Interpolation polynom of Lagrange
The purpose of educational employment: **preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a practical material to watch mastering at pupils of this knowledge, to form and develop at them skills.**	
***Problems of the teacher*:**	***Results of educational activity:***
• To acquaint students with a practical material, the basic directions of activity on a considered material and the curriculum; • Logically consistently, it is given reason and clearly to state thoughts, correctly to build oral and written speech; • Studying of the basic concepts and the terms applied in frameworks практичкеского of a material; • To make training more clear and accessible, to interest students, and the same to motivate them for the further training and self-education; • Maintenance of perception, judgement and primary storing of communications and relations in object of studying; • Establishment of correctness and sensibleness of mastering of a new teaching material, revealing of incorrect representations and their correction; • Maintenance of mastering of new knowledge and ways of actions; • Formation of complete representation of knowledge on a theme; • To show a distributing material, to give talks and to give practical tasks.	• Mastering of new knowledge and ways of actions; • Primary check of understanding; • Fastening of knowledge and ways of actions; • Generalisation and ordering of knowledge. • Careful studying and the all-round analysis of a lecture material and increase of an educational level which would provide the decision of the put problem; • Increase of a degree of quality of knowledge through introduction of innovative technologies; • Level monitoring обученности pupils on steps, classes, subjects, it is concrete on each student, for the purpose of revealing of the real reasons influencing progress, dynamics of conformity of level of teaching to educational standards. • Monitoring of professional skill of teachers. • To continue activity on the organisation of interaction of participants of educational space; • To create conditions, to generalise an advanced experience and to motivate students; • Increase scientific информативности in a field of knowledge of a subject matter and related subjects.
Training methods	Practice – visualisation, conversation
Technics of training	Blitz – the interrogation, focusing questions
Modes of study	Collective, face-to-face
Tutorials	Projector, supply with information, visual materials, educational - methodical grants
Training conditions	The audience provided with tutorials
Monitoring and estimation of knowledge	The oral control: a question-answer

TECHNOLOGICAL CARD
Carrying out of practical employment on a theme:
Interpolation polynom Newton and Lagrange

Stages, time	*The activity maintenance*	
	The teacher	*Students*
1 stage. Introduction ***(15 minutes)***	1.1. Informs a theme, the purpose, planned results of educational employment and the plan of its carrying out. 1.2. For the purpose of actualisation of knowledge of students asks focusing questions.	1.1. Listen. 1.2. Answer questions.
2 stage. Basic (information) **(135 minutes)**	2.1. Consistently states a material of practice concerning the plan. – Interpolation polynom of Newton – Interpolation polynom of Lagrange	2.1. Listen, discuss the maintenance of schemes and tables, visual materials, specify, ask questions. Write down the main thing.
3 stage. The final ***(10 minutes)***	3.1. Spends a blitz – interrogation on a theme of practical employment. Does the total conclusion. 3.2. Gives the task for independent work.	3.1. Answer questions. 3.2. Listen, write down.

Theme № 5-6	**Calculation of integrals by the approached methods**

TECHNOLOGY OF TRAINING
(Carrying out) of practical employment

Employment time - 4 hours (160 minutes)	Quantity of students: 15-20
Mode of study	Introduction - thematic lecture
The plan practical employment	– Method of trapezes and Simpson – Methods of rectangles – The quadrature formula of Gauss

The purpose of educational employment: **preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a practical material to watch mastering at pupils of this knowledge, to form and develop at them skills.**

***Problems of the teacher*:**	***Results of educational activity:***
• To acquaint students with a practical material, the basic directions of activity on a considered material and the curriculum; • Logically consistently, it is given reason and clearly to state thoughts, correctly to build oral and written speech; • Studying of the basic concepts and the terms applied in frameworks практичкеского of a material; • To make training more clear and accessible, to interest students, and the same to motivate them for the further training and self-education; • Maintenance of perception, judgement and primary storing of communications and relations in object of studying; • Establishment of correctness and sensibleness of mastering of a new teaching material, revealing of incorrect representations and their correction; • Maintenance of mastering of new knowledge and ways of actions; • Formation of complete representation of knowledge on a theme; • To show a distributing material, to give talks and to give practical tasks.	• Mastering of new knowledge and ways of actions; • Primary check of understanding; • Fastening of knowledge and ways of actions; • Generalisation and ordering of knowledge. • Careful studying and the all-round analysis of a lecture material and increase of an educational level which would provide the decision of the put problem; • Increase of a degree of quality of knowledge through introduction of innovative technologies; • Level monitoring обученности pupils on steps, classes, subjects, it is concrete on each student, for the purpose of revealing of the real reasons influencing progress, dynamics of conformity of level of teaching to educational standards. • Monitoring of professional skill of teachers. • To continue activity on the organisation of interaction of participants of educational space; • To create conditions, to generalise an advanced experience and to motivate students; • Increase scientific информативности in a field of knowledge of a subject matter and related subjects.
Training methods	Practice – visualisation, conversation
Technics of training	Blitz – the interrogation, focusing questions
Modes of study	Collective, face-to-face
Tutorials	Projector, supply with information, visual materials, educational - methodical grants
Training conditions	The audience provided with tutorials
Monitoring and estimation of knowledge	The oral control: a question-answer

TECHNOLOGICAL CARD
Carrying out of practical employment

Stages, time	*The activity maintenance*	
	The teacher	*Students*
1 stage. Introduction ***(15 minutes)***	1.1. Informs a theme, the purpose, planned results of educational employment and the plan of its carrying out. 1.2. For the purpose of actualisation of knowledge of students asks focusing questions.	1.1. Listen. 1.2. Answer questions.
2 stage. Basic (information) **(135 minutes)**	2.1. Consistently states a material of practice concerning the plan. – Method of trapezes and Simpson – Methods of rectangles – The quadrature formula of Gauss	2.1. Listen, discuss the maintenance of schemes and tables, visual materials, specify, ask questions. Write down the main thing.
3 stage. The final ***(10 minutes)***	3.1. Spends a blitz – interrogation on a theme of practical employment. Does the total conclusion. 3.2. Gives the task for independent work.	3.1. Answer questions. 3.2. Listen, write down.

Theme № 7-8	Approximation results of experiment with a method of the least square. Creation non-linear empirical connection

TECHNOLOGY OF TRAINING
(Carrying out) of practical employment

Employment time - 4 hours (160 minutes)	Quantity of students: 15-20
Mode of study	Introduction - thematic lecture
The plan practical employment	– Root-mean-square approach of functions – A method of the least squares
The purpose of educational employment: **preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a practical material to watch mastering at pupils of this knowledge, to form and develop at them skills.**	
***Problems of the teacher*:**	***Results of educational activity:***
• To acquaint students with a practical material, the basic directions of activity on a considered material and the curriculum; • Logically consistently, it is given reason and clearly to state thoughts, correctly to build oral and written speech; • Studying of the basic concepts and the terms applied in frameworks практичкеского of a material; • To make training more clear and accessible, to interest students, and the same to motivate them for the further training and self-education; • Maintenance of perception, judgement and primary storing of communications and relations in object of studying; • Establishment of correctness and sensibleness of mastering of a new teaching material, revealing of incorrect representations and their correction; • Maintenance of mastering of new knowledge and ways of actions; • Formation of complete representation of knowledge on a theme; • To show a distributing material, to give talks and to give practical tasks.	• Mastering of new knowledge and ways of actions; • Primary check of understanding; • Fastening of knowledge and ways of actions; • Generalisation and ordering of knowledge. • Careful studying and the all-round analysis of a lecture material and increase of an educational level which would provide the decision of the put problem; • Increase of a degree of quality of knowledge through introduction of innovative technologies; • Level monitoring обученности pupils on steps, classes, subjects, it is concrete on each student, for the purpose of revealing of the real reasons influencing progress, dynamics of conformity of level of teaching to educational standards. • Monitoring of professional skill of teachers. • To continue activity on the organisation of interaction of participants of educational space; • To create conditions, to generalise an advanced experience and to motivate students; • Increase scientific информативности in a field of knowledge of a subject matter and related subjects.
Training methods	Practice – visualisation, conversation
Technics of training	Blitz – the interrogation, focusing questions
Modes of study	Collective, face-to-face
Tutorials	Projector, supply with information, visual materials, educational - methodical grants
Training conditions	The audience provided with tutorials
Monitoring and estimation of knowledge	The oral control: a question-answer

TECHNOLOGICAL CARD
Carrying out of practical employment

Stages, time	*The activity maintenance*	
	The teacher	*Students*
1 stage. Introduction ***(15 minutes)***	1.1. Informs a theme, the purpose, planned results of educational employment and the plan of its carrying out. 1.2. For the purpose of actualisation of knowledge of students asks focusing questions.	1.1. Listen. 1.2. Answer questions.
2 stage. Basic (information) **(135 minutes)**	2.1. Consistently states a material of practice concerning the plan. – Root-mean-square approach of functions – A method of the least squares	2.1. Listen, discuss the maintenance of schemes and tables, visual materials, specify, ask questions. Write down the main thing.
3 stage. The final ***(10 minutes)***	3.1. Spends a blitz – interrogation on a theme of practical employment. Does the total conclusion. 3.2. Gives the task for independent work.	3.1. Answer questions. 3.2. Listen, write down.

Theme № 9	The geometrical decision of a problem of linear programming

TECHNOLOGY OF TRAINING
(Carrying out) of practical employment

Employment time - 2 hours (80 minutes)	Quantity of students: 15-20
Mode of study	Introduction - thematic lecture
The plan practical employment	– Geometrical interpretation of a problem of linear programming – Using geometrical interpretation, find decisions of problems
***The purpose of educational employment:* preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a practical material to watch mastering at pupils of this knowledge, to form and develop at them skills.**	
***Problems of the teacher*:** • To acquaint students with a practical material, the basic directions of activity on a considered material and the curriculum; • Logically consistently, it is given reason and clearly to state thoughts, correctly to build oral and written speech; • Studying of the basic concepts and the terms applied in frameworks практичкеского of a material; • To make training more clear and accessible, to interest students, and the same to motivate them for the further training and self-education; • Maintenance of perception, judgement and primary storing of communications and relations in object of studying; • Establishment of correctness and sensibleness of mastering of a new teaching material, revealing of incorrect representations and their correction; • Maintenance of mastering of new knowledge and ways of actions; • Formation of complete representation of knowledge on a theme; • To show a distributing material, to give talks and to give practical tasks.	***Results of educational activity:*** • Mastering of new knowledge and ways of actions; • Primary check of understanding; • Fastening of knowledge and ways of actions; • Generalisation and ordering of knowledge. • Careful studying and the all-round analysis of a lecture material and increase of an educational level which would provide the decision of the put problem; • Increase of a degree of quality of knowledge through introduction of innovative technologies; • Level monitoring обученности pupils on steps, classes, subjects, it is concrete on each student, for the purpose of revealing of the real reasons influencing progress, dynamics of conformity of level of teaching to educational standards. • Monitoring of professional skill of teachers. • To continue activity on the organisation of interaction of participants of educational space; • To create conditions, to generalise an advanced experience and to motivate students; • Increase scientific информативности in a field of knowledge of a subject matter and related subjects.
Training methods	Practice – visualisation, conversation
Technics of training	Blitz – the interrogation, focusing questions
Modes of study	Collective, face-to-face
Tutorials	Projector, supply with information, visual materials, educational - methodical grants
Training conditions	The audience provided with tutorials
Monitoring and estimation of knowledge	The oral control: a question-answer

TECHNOLOGICAL CARD
Carrying out of practical employment on a theme:
Finding the decision of a problem of linear programming to Simplex methods

Stages, time	*The activity maintenance*	
	The teacher	*Students*
1 stage. Introduction ***(15 minutes)***	1.1. Informs a theme, the purpose, planned results of educational employment and the plan of its carrying out. 1.2. For the purpose of actualisation of knowledge of students asks focusing questions.	1.1. Listen. 1.2. Answer questions.
2 stage. Basic (information) **(55 minutes)**	2.1. Consistently states a material of practice concerning the plan. – Geometrical interpretation of a problem of linear programming – Using geometrical interpretation, find decisions of problems	2.1. Listen, discuss the maintenance of schemes and tables, visual materials, specify, ask questions. Write down the main thing.
3 stage. The final ***(10 minutes)***	3.1. Spends a blitz – interrogation on a theme of practical employment. Does the total conclusion. 3.2. Gives the task for independent work.	3.1. Answer questions. 3.2. Listen, write down.

Theme № 1-2	The numerical decision of the algebraic and transcendental equations by iterative methods Chord and Newton

TECHNOLOGY OF TRAINING
(Carrying out) laboratory work

Employment time - 4 hours (160 minutes)	Quantity of students: 15-20
Mode of study	Introduction - thematic lecture
The plan of laboratory works	– Iteration method – Method of Chords – Method half divisions
The purpose of educational employment: **preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on laboratory work to watch mastering at pupils of this knowledge, to form and develop at them skills.**	
***Problems of the teacher*:** • To acquaint students with a laboratory material and the device, the basic directions of activity on a considered material and the curriculum; • Logically consistently, it is given reason and clearly to state thoughts, correctly to build oral and written speech; • Studying of the basic concepts and the terms applied in frameworks практичкеского of a material; • To make training more clear and accessible, to interest students, and the same to motivate them for the further training and self-education; • Maintenance of perception, judgement and primary storing of communications and relations in object of studying; • Establishment of correctness and sensibleness of mastering of a new teaching material, revealing of incorrect representations and their correction; • Maintenance of mastering of new knowledge and ways of actions; • Formation of complete representation of knowledge on a theme; • To show a distributing material, to give talks and to give laboratory works.	***Results of educational activity:*** • Mastering of new knowledge and ways of actions; • Primary check of understanding; • Fastening of knowledge and ways of actions; • Generalisation and ordering of knowledge. • Careful studying and the all-round analysis of a laboratory material and increase of an educational level which would provide the decision of the put problem; • Increase of a degree of quality of knowledge through introduction of innovative technologies; • Level monitoring обученности pupils on steps, classes, subjects, it is concrete on each student, for the purpose of revealing of the real reasons influencing progress, dynamics of conformity of level of teaching to educational standards. • Monitoring of professional skill of teachers. • To continue activity on the organisation of interaction of participants of educational space; • To create conditions, to generalise an advanced experience and to motivate students; • Increase scientific информативности in a field of knowledge of a subject matter and related subjects.
Training methods	**Laboratory – visualisation, conversation**
Technics of training	**Blitz – the interrogation, focusing questions**
Modes of study	**Collective, face-to-face**
Tutorials	**Visual materials, educational-methodical grants, laboratory installations and devices**
Training conditions	**The audience provided with tutorials**
Monitoring and estimation of knowledge	**The oral control: a question-answer**

TECHNOLOGICAL CARD
Carrying out laboratory work

Stages, time	*The activity maintenance*	
	The teacher	*Students*
1 stage. Introduction ***(15 minutes)***	1.1. Laboratory work and the plan of its carrying out informs a theme, the purpose, planned results. 1.2. For the purpose of actualisation of knowledge of students asks focusing questions.	1.1. Listen. 1.2. Answer questions.
2 stage. Basic (information) **(135 minutes)**	2.1. Consistently states a material laboratory concerning the plan. – Iteration method – Method of Chords – Method half divisions	2.1. Listen, discuss the maintenance of schemes and tables, specify, ask questions, carry out laboratory work. Write down the main thing results.
3 stage. The final ***(10 minutes)***	3.1. Spends a blitz – interrogation on a theme laboratory work. Does the total conclusion. 3.2. Gives the task for independent work.	3.1. Answer questions. 3.2. Listen, write down.

Theme № 3-4	The numerical decision of system of the linear algebraic equations methods of Gauss, simple iteration and Seidel.

TECHNOLOGY OF TRAINING
(Carrying out) laboratory work

Employment time - 4 hours (160 minutes)	Quantity of students: 15-20
Mode of study	Introduction - thematic lecture
The plan of laboratory works	– Methods of Gauss – Methods of simple iteration. – The decision of system of the linear algebraic equations a method of Seidel
The purpose of educational employment: **preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on laboratory work to watch mastering at pupils of this knowledge, to form and develop at them skills.**	

***Problems of the teacher*:**	***Results of educational activity:***
• To acquaint students with a laboratory material and the device, the basic directions of activity on a considered material and the curriculum; • Logically consistently, it is given reason and clearly to state thoughts, correctly to build oral and written speech; • Studying of the basic concepts and the terms applied in frameworks практичкеского of a material; • To make training more clear and accessible, to interest students, and the same to motivate them for the further training and self-education; • Maintenance of perception, judgement and primary storing of communications and relations in object of studying; • Establishment of correctness and sensibleness of mastering of a new teaching material, revealing of incorrect representations and their correction; • Maintenance of mastering of new knowledge and ways of actions; • Formation of complete representation of knowledge on a theme; • To show a distributing material, to give talks and to give laboratory works.	• Mastering of new knowledge and ways of actions; • Primary check of understanding; • Fastening of knowledge and ways of actions; • Generalisation and ordering of knowledge. • Careful studying and the all-round analysis of a laboratory material and increase of an educational level which would provide the decision of the put problem; • Increase of a degree of quality of knowledge through introduction of innovative technologies; • Level monitoring обученности pupils on steps, classes, subjects, it is concrete on each student, for the purpose of revealing of the real reasons influencing progress, dynamics of conformity of level of teaching to educational standards. • Monitoring of professional skill of teachers. • To continue activity on the organisation of interaction of participants of educational space; • To create conditions, to generalise an advanced experience and to motivate students; • Increase scientific информативности in a field of knowledge of a subject matter and related subjects.
Training methods	**Laboratory – visualisation, conversation**
Technics of training	**Blitz – the interrogation, focusing questions**
Modes of study	**Collective, face-to-face**
Tutorials	**Visual materials, educational-methodical grants, laboratory installations and devices**
Training conditions	**The audience provided with tutorials**
Monitoring and estimation of knowledge	**The oral control: a question-answer**

TECHNOLOGICAL CARD
Carrying out laboratory work

Stages, time	*The activity maintenance*	
	The teacher	*Students*
1 stage. Introduction ***(15 minutes)***	1.1. Laboratory work and the plan of its carrying out informs a theme, the purpose, planned results. 1.2. For the purpose of actualisation of knowledge of students asks focusing questions.	1.1. Listen. 1.2. Answer questions.
2 stage. Basic (information) **(135 minutes)**	2.1. Consistently states a material laboratory concerning the plan. – Methods of Gauss – Methods of simple iteration. – The decision of system of the linear algebraic equations a method of Seidel	2.1. Listen, discuss the maintenance of schemes and tables, specify, ask questions, carry out laboratory work. Write down the main thing results.
3 stage. The final ***(10 minutes)***	3.1. Spends a blitz – interrogation on a theme laboratory work. Does the total conclusion. 3.2. Gives the task for independent work.	3.1. Answer questions. 3.2. Listen, write down.

Theme № 5-6	**Problems of Cochy for the ordinary differential equations. Euler's methods, Runge-Kutte and Adams**

TECHNOLOGY OF TRAINING
(Carrying out) laboratory work

<table>
<tr><td>Employment time -
4 hours (160 minutes)</td><td>Quantity of students: 15-20</td></tr>
<tr><td>Mode of study</td><td>Introduction - thematic lecture</td></tr>
<tr><td>The plan of laboratory works</td><td>– Euler's method
– Method of Runge-Kutte.
– Adam's method</td></tr>
<tr><td colspan="2">The purpose of educational employment: preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on laboratory work to watch mastering at pupils of this knowledge, to form and develop at them skills.</td></tr>
<tr><td>Problems of the teacher:
• To acquaint students with a laboratory material and the device, the basic directions of activity on a considered material and the curriculum;
• Logically consistently, it is given reason and clearly to state thoughts, correctly to build oral and written speech;
• Studying of the basic concepts and the terms applied in frameworks практичкеского of a material;
• To make training more clear and accessible, to interest students, and the same to motivate them for the further training and self-education;
• Maintenance of perception, judgement and primary storing of communications and relations in object of studying;
• Establishment of correctness and sensibleness of mastering of a new teaching material, revealing of incorrect representations and their correction;
• Maintenance of mastering of new knowledge and ways of actions;
• Formation of complete representation of knowledge on a theme;
• To show a distributing material, to give talks and to give laboratory works.</td><td>Results of educational activity:
• Mastering of new knowledge and ways of actions;
• Primary check of understanding;
• Fastening of knowledge and ways of actions;
• Generalisation and ordering of knowledge.
• Careful studying and the all-round analysis of a laboratory material and increase of an educational level which would provide the decision of the put problem;
• Increase of a degree of quality of knowledge through introduction of innovative technologies;
• Level monitoring обученности pupils on steps, classes, subjects, it is concrete on each student, for the purpose of revealing of the real reasons influencing progress, dynamics of conformity of level of teaching to educational standards.
• Monitoring of professional skill of teachers.
• To continue activity on the organisation of interaction of participants of educational space;
• To create conditions, to generalise an advanced experience and to motivate students;
• Increase scientific информативности in a field of knowledge of a subject matter and related subjects.</td></tr>
<tr><td>Training methods</td><td>Laboratory – visualisation, conversation</td></tr>
<tr><td>Technics of training</td><td>Blitz – the interrogation, focusing questions</td></tr>
<tr><td>Modes of study</td><td>Collective, face-to-face</td></tr>
<tr><td>Tutorials</td><td>Visual materials, educational-methodical grants, laboratory installations and devices</td></tr>
<tr><td>Training conditions</td><td>The audience provided with tutorials</td></tr>
<tr><td>Monitoring and estimation of knowledge</td><td>The oral control: a question-answer</td></tr>
</table>

TECHNOLOGICAL CARD
Carrying out laboratory work

<table>
<tr><td rowspan="2">Stages, time</td><td colspan="2">The activity maintenance</td></tr>
<tr><td>The teacher</td><td>Students</td></tr>
<tr><td>1 stage.
Introduction
(15 minutes)</td><td>1.1. Laboratory work and the plan of its carrying out informs a theme, the purpose, planned results.
1.2. For the purpose of actualisation of knowledge of students asks focusing questions.</td><td>1.1. Listen.

1.2. Answer questions.</td></tr>
<tr><td>2 stage. Basic (information)
(135 minutes)</td><td>2.1. Consistently states a material laboratory concerning the plan.
– Euler's method
– Method of Runge-Kutte.
– Adam's method</td><td>2.1. Listen, discuss the maintenance of schemes and tables, specify, ask questions, carry out laboratory work.
Write down the main thing results.</td></tr>
<tr><td>3 stage.
The final
(10 minutes)</td><td>3.1. Spends a blitz – interrogation on a theme laboratory work. Does the total conclusion.
3.2. Gives the task for independent work.</td><td>3.1. Answer questions.
3.2. Listen, write down.</td></tr>
</table>

Theme №7-8	**The numerical decision of system of the nonlinear algebraic equations to methods of simple iteration.**

TECHNOLOGY OF TRAINING
(Carrying out) laboratory work

Employment time - 4 hours (160 minutes)	Quantity of students: 15-20
Mode of study	Introduction - thematic lecture
The plan of laboratory works	– The decision of systems of the nonlinear equations – Method of simple iterations – Example for decision of systems of the nonlinear equations – The decision of systems nonlinear the equation in MathCAD.

The purpose of educational employment: **preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on laboratory work to watch mastering at pupils of this knowledge, to form and develop at them skills.**

Problems of the teacher:	***Results of educational activity:***
• To acquaint students with a laboratory material and the device, the basic directions of activity on a considered material and the curriculum; • Logically consistently, it is given reason and clearly to state thoughts, correctly to build oral and written speech; • Studying of the basic concepts and the terms applied in frameworks практичкеского of a material; • To make training more clear and accessible, to interest students, and the same to motivate them for the further training and self-education; • Maintenance of perception, judgement and primary storing of communications and relations in object of studying; • Establishment of correctness and sensibleness of mastering of a new teaching material, revealing of incorrect representations and their correction; • Maintenance of mastering of new knowledge and ways of actions; • Formation of complete representation of knowledge on a theme; • To show a distributing material, to give talks and to give laboratory works.	• Mastering of new knowledge and ways of actions; • Primary check of understanding; • Fastening of knowledge and ways of actions; • Generalisation and ordering of knowledge. • Careful studying and the all-round analysis of a laboratory material and increase of an educational level which would provide the decision of the put problem; • Increase of a degree of quality of knowledge through introduction of innovative technologies; • Level monitoring обученности pupils on steps, classes, subjects, it is concrete on each student, for the purpose of revealing of the real reasons influencing progress, dynamics of conformity of level of teaching to educational standards. • Monitoring of professional skill of teachers. • To continue activity on the organisation of interaction of participants of educational space; • To create conditions, to generalise an advanced experience and to motivate students; • Increase scientific информативности in a field of knowledge of a subject matter and related subjects.
Training methods	**Laboratory – visualisation, conversation**
Technics of training	**Blitz – the interrogation, focusing questions**
Modes of study	**Collective, face-to-face**
Tutorials	**Visual materials, educational-methodical grants, laboratory installations and devices**
Training conditions	**The audience provided with tutorials**
Monitoring and estimation of knowledge	**The oral control: a question-answer**

TECHNOLOGICAL CARD
Carrying out laboratory work

Stages, time	*The activity maintenance*	
	The teacher	*Students*
1 stage. Introduction ***(15 minutes)***	1.1. Laboratory work and the plan of its carrying out informs a theme, the purpose, planned results. 1.2. For the purpose of actualisation of knowledge of students asks focusing questions.	1.1. Listen. 1.2. Answer questions.
2 stage. Basic (information) **(135 minutes)**	2.1. Consistently states a material laboratory concerning the plan. – The decision of systems of the nonlinear equations – Method of simple iterations – Example for decision of systems of the nonlinear equations – The decision of systems nonlinear the equation in MathCAD.	2.1. Listen, discuss the maintenance of schemes and tables, specify, ask questions, carry out laboratory work. Write down the main thing results.
3 stage. The final ***(10 minutes)***	3.1. Spends a blitz – interrogation on a theme laboratory work. Does the total conclusion. 3.2. Gives the task for independent work.	3.1. Answer questions. 3.2. Listen, write down.

Theme № 9	Finding the decision of a problem of linear programming to Simplex methods

TECHNOLOGY OF TRAINING
(Carrying out) laboratory work

Employment time - 2 hours (80 minutes)	Quantity of students: 15-20
Mode of study	Introduction - thematic lecture
The plan of laboratory works	– Simplex method of the decision of a problem of linear programming – Examples of the decision of a problem of linear programming with a simplex method
The purpose of educational employment: **preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on laboratory work to watch mastering at pupils of this knowledge, to form and develop at them skills.**	
***Problems of the teacher*:** • To acquaint students with a laboratory material and the device, the basic directions of activity on a considered material and the curriculum; • Logically consistently, it is given reason and clearly to state thoughts, correctly to build oral and written speech; • Studying of the basic concepts and the terms applied in frameworks практичкеского of a material; • To make training more clear and accessible, to interest students, and the same to motivate them for the further training and self-education; • Maintenance of perception, judgement and primary storing of communications and relations in object of studying; • Establishment of correctness and sensibleness of mastering of a new teaching material, revealing of incorrect representations and their correction; • Maintenance of mastering of new knowledge and ways of actions; • Formation of complete representation of knowledge on a theme; • To show a distributing material, to give talks and to give laboratory works.	***Results of educational activity:*** • Mastering of new knowledge and ways of actions; • Primary check of understanding; • Fastening of knowledge and ways of actions; • Generalisation and ordering of knowledge. • Careful studying and the all-round analysis of a laboratory material and increase of an educational level which would provide the decision of the put problem; • Increase of a degree of quality of knowledge through introduction of innovative technologies; • Level monitoring обученности pupils on steps, classes, subjects, it is concrete on each student, for the purpose of revealing of the real reasons influencing progress, dynamics of conformity of level of teaching to educational standards. • Monitoring of professional skill of teachers. • To continue activity on the organisation of interaction of participants of educational space; • To create conditions, to generalise an advanced experience and to motivate students; • Increase scientific информативности in a field of knowledge of a subject matter and related subjects.
Training methods	**Laboratory – visualisation, conversation**
Technics of training	**Blitz – the interrogation, focusing questions**
Modes of study	**Collective, face-to-face**
Tutorials	**Visual materials, educational-methodical grants, laboratory installations and devices**
Training conditions	**The audience provided with tutorials**
Monitoring and estimation of knowledge	**The oral control: a question-answer**

TECHNOLOGICAL CARD
Carrying out laboratory work on a theme:
Finding the decision of a problem of linear programming to Simplex methods

Stages, time	*The activity maintenance*	
	The teacher	*Students*
1 stage. Introduction ***(15 minutes)***	1.1. Laboratory work and the plan of its carrying out informs a theme, the purpose, planned results. 1.2. For the purpose of actualisation of knowledge of students asks focusing questions.	1.1. Listen. 1.2. Answer questions.
2 stage. Basic (information) **(55 minutes)**	2.1. Consistently states a material laboratory concerning the plan. – Simplex method of the decision of a problem of linear programming – Examples of the decision of a problem of linear programming with a simplex method	2.1. Listen, discuss the maintenance of schemes and tables, specify, ask questions, carry out laboratory work. Write down the main thing results.
3 stage. The final ***(10 minutes)***	3.1. Spends a blitz – interrogation on a theme laboratory work. Does the total conclusion. 3.2. Gives the task for independent work.	3.1. Answer questions. 3.2. Listen, write down.

TESTS ON DISCIPLINE

1. In what method consecutive approach are calculated under the formula $x_{n+1} := x_n - \frac{f(x_n)\cdot(a - x_n)}{f(a) - f(x_n)}$

Method of chords
Method of tangents
Method of division of a piece half-and-half
Method a trapeze

2. The condition of monotonous convergence consecutive приближений in a method of chords is:

Preservation of a sign on the second derivative initial function
Preservation of a sign on the first derivative initial function
Coincidence of signs on the first and second derivatives of initial function
Coincidence of signs by the first

3. What speed of convergence of a method of tangents?

The square-law
The linear
The cubic
The face-to-face

4. What speed of convergence of a method of chords?

The linear
The square-law
The cubic
The face-to-face

5. Criterion of convergence of an iterative method:

Own numbers of a matrix of transition on the module there is less than unit
Own numbers of a matrix of transition on the module there is more than unit
The system matrix - is a matrix with diagonal prevalence
Matrix

6. The formula of Zejdel is an initial formula of a method:

Зейделя
Simple iterations
Relaxations
The reflective

7. The relaxation method converges, if:

The relaxation parametre w lies on an interval (0,2)
The relaxation parametre w on the module is less 2
The relaxation parametre w is not negative
The reflective

8. The number of conditionality of a matrix of system influences on:

Sensitivity decisions to an error of the initial data
Speed of convergence of iterative process
Choice of initial approach
The adaptive

9. By means of a sedate method is:
Maximum on the module own number
The maximum own number
Minimum on the module own number
The extreme

10. By means of a method of rotations:
The matrix is led to a diagonal kind
The matrix is led to a triangular kind
The matrix is transposed
Vector

11. What speed of convergence of a method of tangents?
The square-law
The linear
The cubic
The face-to-face

12. What speed of convergence of a method of chords?
The linear
The square-law
The cubic
The face-to-face

13. Criterion of convergence of an iterative method:
Own numbers of a matrix of transition on the module there is less than unit
Own numbers of a matrix of transition on the module there is more than unit
The system matrix - is a matrix with diagonal prevalence
Matrix

14. The formula of Zejdel is an initial formula of a method:
Seidel
Simple iterations
Relaxations
The reflective

15. The relaxation method converges, if:
The relaxation parametre w lies on an interval (0,2)
The relaxation parametre w on the module is less 2
The relaxation parametre w is not negative
The reflective

QUESTIONS FOR FLOWING, INTERMEDIATE AND TOTAL EXAMINATION

1. Classification of computing methods.
2. Preparation of problems for the personal computer decision.
3. Properties of algorithm.
4. Classification of algorithms.
5. Method branch of roots
6. Method half divisions
7. Method the Chord
8. Newton's method
9. Method of simple iteration
10. Method of secants
11. The decision of system of the linear equations a method of Gaussa
12. Method of Gaussa with a choice of the main element
13. Error estimation at the decision of system of the linear equations
14. Iterative methods of the decision of systems of the linear equations
15. Method of simple iteration of Jacoby
16. Method of Gauss-Seidel
17. The first interpolation Newton's formula
18. The second interpolation Newton's formula
19. The interpolation formula of Stirlinga
20. Types of problems for the ordinary differential equations
21. Euler's method
22. Methods of Runge-Kutta
23. Adams's method
24. Method of trapezes
25. Methods of rectangles
26. Simpson's method
27. The quadrature formula of Gaussa
28. Root-mean-square approach of functions
29. Method of the least squares
30. The primary goal of linear programming
31. Geometrical representation LP.
32. Geometrical interpretation of problem LP
33. Mathematical bases a simplex of a method of the decision
34. Search of the initial basic decision
35. Features of a transport problem
36. Constructions of basic decision TP
37. Conditions and a method of construction of the optimum decision of a transport problem
38. Algorithm of the decision of a transport problem on a network

1st Intermediate control work notebook from subject "Algorithmization of computing methods"

Var-1

1. A method branch of roots
2. The decision of system of the linear equations a method of Gauss
3. The first interpolation Newton's formula

Var-2

1. A method half divisions
2. Iterative methods of the decision of systems of the linear equations
3. Euler's method

2nd Intermediate control work notebook from subject "Algorithmization of computing methods"

Var-1

1. Euler's method
2. The quadrature formula of Gaussa
3. Mathematical bases a simplex of a method of the decision

Var-2

1. Methods of Runge-Kutta
2. Root-mean-square approach of functions
3. Search of the initial basic decision

Var-3

1. Adams's method
2. Method of the least squares
3. Features of a transport problem

Var-4

1. Method of trapezes
2. The primary goal of linear programming
3. Constructions of basic decision TP

Var-5

1. Methods of rectangles
2. Geometrical representation LP.
3. Conditions and a method of construction of the optimum decision of a transport problem

Var-6

1. Simpson's method
2. Geometrical interpretation of problem LP
3. Algorithm of the decision of a transport problem on a network

VARIANTS TOTAL EXAMINATION

Variant № 1

1. Classification of computing methods.
2. Iterative methods of the decision of systems of the linear equations
3. Simpson's method

Variant № 2

1. Preparation of problems for the personal computer decision.
2. Method of simple iteration of Jacoby
3. The quadrature formula of Gauss

Variant № 3

1. Root-mean-square approach of functions
2. Method of Gaussa-Seidel
3. Properties of algorithm

Variant № 4

1. Classification of algorithms.
2. The first interpolation Newton's formula
3. Method of the least squares

Variant № 5

1. The primary goal of linear programming
2. The second interpolation Newton's formula
3. Method branch of roots

Variant № 6

1. Method half divisions
2. The interpolation formula of Stirling
3. Geometrical representation linear programming.

Variant № 7

1. Geometrical interpretation of problem linear programming
2. Types of problems for the ordinary differential equations
3. Method the Chord

Variant № 8

1. Newton's method
2. Euler's method
3. Mathematical bases a simplex of a method of the decision

Variant № 9

1. Search of the initial basic decision
2. Methods of Runge-Kutta
3. Method of simple iteration

Variant № 10

1. Method of secants
2. Adams's method
3. Features of a transport problem

Variant № 11

1. Constructions of basic decision transport task
2. Method of trapezes
3. The decision of system of the linear equations a method of Gauss

Variant № 12

1. Method of Gauss with a choice of the main element
2. Methods of rectangles
3. Conditions and a method of construction of the optimum decision of a transport problem

Variant № 13

1. Algorithm of the decision of a transport problem on a network
2. Error estimation at the decision of system of the linear equations
3. Classification of computing methods.

Variant № 14

1. Iterative methods of the decision of systems of the linear equations
2. The quadrature formula of Gaussa
3. Algorithm of the decision of a transport problem on a network

Variant № 15

1. Conditions and a method of construction of the optimum decision of a transport problem
2. Simpson's method
3. Error estimation at the decision of system of the linear equations

Variant № 16

1. Method of Gaussa with a choice of the main element
2. Methods of rectangles
3. Constructions of basic decision transport tasks

Variant № 17

1. Features of a transport problem
2. Method of trapezes
3. The decision of system of the linear equations a method of Gaussa

Variant № 18

1. Classification of computing methods.
2. Iterative methods of the decision of systems of the linear equations
3. Simpson's method

Variant № 19

1. Preparation of problems for the personal computer decision.
2. Method of simple iteration of Jakoby
3. The quadrature formula of Gauss

Variant № 20

1. Root-mean-square approach of functions
2. Method of Gaussa-Zeidel
3. Properties of algorithm.

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GOOD ADVICE

5 IMPORTANT INSTRUCTIONS FOR USE THE COMPUTER FOR THE BEGINNER

Many neglect with what that was instructions for use of the computer and complain that something is impossible to them. And such people also have the most unusual problems. Sowing for the computer of such person, at times you do not know from what to begin - the computer happens it is finished to such degree of a brothel that it would not be desirable it even to touch. And all it it is possible to avoid if to adhere to several simple rules. Them the set is final, but I have allocated 5 most important in my opinion instructions for use with the computer, observing which beginner can facilitate a life and and the master in case of a computer output out of operation. Rules are simple also them easily to remember, and if it is impossible it is possible to write on a leaflet and to hang up near to the monitor.

1 rule. Correct deenergizing of the computer.

For correct deenergizing of the computer it is necessary to press button "Start-up" - "Deenergizing" - "Deenergizing" and to wait while Windows will finish work. Only after that to disconnect from the socket. Computer deenergizing выдергиванием plugs from the socket is inadmissible. If you have got used to switch off thus an iron or a teapot with the computer do not think so to arrive at all.

And why is not present? On the included computer the set of system processes and programs which before deenergizing needs to be finished and stopped works. If to chop off a computer food you risk that next time, the computer will not join at all and, probably, it is necessary to reinstall system and all necessary programs.

Once again: for computer deenergizing press start-up - deenergizing

Then press "deenergizing" and wait while the computer will not be switched off.

2 rule. It is impossible to store the data on "desktop"
And in a folder "My documents".

Remember that the desktop which appears at computer inclusion, and a folder "My documents" is not a reliable place for storage of the valuable information. Yes, I understand conveniently at once to drag on it films or something else with флешки, etc. Why it is impossible? Because, if your computer will be infected with viruses and it will cease to be started, you probably will lose that was stored there. Besides sometimes for not clear reasons Windows gives failure and at the next inclusion of the computer the desktop appears empty. What it is possible to store on a desktop? The answer - labels of programs (but not programs), time files (утере which you will not be afflicted).

3 rule. The antivirus should be established.

Because of flooding of the Internet by viruses there was necessary a presence of skills of struggle against them at usual users. As much do not have time for studying of the similar information it is necessary to have at least an antivirus on the computer which will remove from you a part of cares of safety from viruses. I repeat will remove a part of cares because if you will indefatigably climb on suspicious sites here any antivirus will not be in time for you. I recommend to use "Yandex the version of an antivirus of Kaspersky", it works half a year free of charge.

4 rule. Correct installation of programs.

Learn to instal correctly programs and games. That is to establish them in a proper place. Developers Windows have given vent to users to instal applications where it will like, but there is an order observing which your computer can longer serve and is better. Programs establish on disk C: in a folder "Program Files". For each program create a folder with a program name. The same concerns also games, but them establish on disk D: in a folder of "Game" or "Games". The second is more preferable. For each game create a folder with her name not to mix files and not to spoil game. And for removal of programs and games use a file Uninstall.exe which usually is in a program folder. If you simply remove a folder with the program that in the system register there will be a set of superfluous records which will brake in due course computer work.

5 rule. A file order.

Accustom itself to order conducting on the computer. Be not lazy to create folders. Store all valuable on disk D: or the friend distinct from With: a disk. As usually disk C: it is used as system, in case of reinstallation it is recommended to format it and on it all leaves. If you store films on the computer, photos and books create corresponding folders. If at you, for example, the big collection of films, sort them on genres or years and place in separate folders. It in the first will increase speed of access to them to the computer, secondly will facilitate to you search of the necessary film. The same concerns photos, books, music, clips and everything.

If to adhere to these rules, to you no failure of the computer will be terrible, and the master come to you to repair the computer will praise you for accuracy.

INSTRUCTIONS FOR USE THE COMPUTER

It is forbidden to assort independently the computer and all its accessories. At occurrence of malfunctions it is necessary to address

All cables connecting the system block with other devices, it is necessary to insert and take out only at the switched off computer. The exception is made by USB-devices: they can be connected to the included computer;

It is forbidden to establish, delete independently, деактивироват and to change the software and network options on the computer.

It is forbidden to subject the computer and remote terminal units to physical, thermal and chemical influences. (It is impossible to sit on the computer, to spill on it tea, coffee, to spill out sunflower seeds, to put at the battery and other heating devices);

At power cutoff a source of an uninterrupted food (UPS) allows the computer to remain in working order from 5 till 20 minutes. At power cutoff indoors the user should spend correct deenergizing of the computer in an immediate order.

IT IS FORBIDDEN

To open computers, the network and peripheral equipment; to connect to the computer the additional equipment without the coordination with the system administrator, to change options BIOS, and also to make loading of workstations from diskettes;

Autocratically to connect the computer to a network, and also to change the IP-address of the computer which has been given out by the system administrator. Data transmission in a network with use of others IP addresses as the address of the sender is distribution of a false information and creates threat of safety of the information on other computers;

To use a network for mass distribution advertising (spam), commercial announcements, the pornographic information, appeals to violence, kindling national or religious hostility, insults, threats, etc.

Work with e-mail

1. E-mail is given to employees of the organisation only for performance of the direct official duties. Its use to suit the own ends is forbidden. Mail box creation is spent by the system administrator.

2. All electronic letters created and хранимые on computers of the organisation, are the property of the organisation and are not considered as the personal;

3. Users should not allow to someone to send the letter from another's name. It concerns their chiefs, secretaries, assistants or other colleagues;

4. As clients of e-mail the confirmed post programs can be used only;

5. To carry out mass dispatch of not co-ordinated preliminary electronic letters. It is meant mass dispatch both dispatch to set of addressees, and plural dispatch to one addressee (spam).

6. To employees of the organisation, using Internet, it is forbidden to transfer or load a material which is obscene on the computer, pornographic or breaks the current legislation of the Russian Federation;

7. All programs used for access to a network the Internet, should be confirmed the network manager and on them necessary levels of safety should be adjusted.

Instructions for use the personal computer.

Windows - a good advice and secrets

Keep the information on a hard disk in good order (including on logic With, D, E)

Supervise empty seat presence (not less than 200 Mb) on a system disk (disk).

Folder «My Documents» move from a system disk (for example on disk D). Documents copy and store in this folder, instead of on a desktop.

Once in 3-6 months do archival copies of documents on CD or флэшку.

Regularly clear a basket, folders of time files, кеш. Use for this purpose the special software.

Work rules on the personal computer on the Internet.

Use different logins and passwords in different systems, try to use difficult passwords.

Look where you enter (in an address line look on урл) or to whom send the registration data.

Use some mail boxes for the various purposes (personal, for work, for registration at forums and social networks).

Do not download doubtful programs from unchecked sources and from letters from strangers.

Use the reliable and actual (updated) antivirus. Do not use more than one antivirus simultaneously.

Instructions for use the laptop.

Hold away from the laptop of a cup from coffee, tea and other vessels with liquids.

Do not accept food behind the laptop.

Do not use the laptop dirty hands.

Do not apply excessive efforts by pressing of buttons of the keyboard.

Regularly wipe the laptop special means and vacuum.

Standard programs Windows XP.

By means of a standard set of programs of operating system Windows XP it is possible to carry out the whole spectrum of various problems, such as: drawing, typing, electronic letters, creation and assembling of films, music listening.

To find standard programs it is possible through button "Start-up" in the menu «All programs».

In a folder Automatic loading there are programs for automatic start at operating system start.

You will find simple toys in a folder of Game.

To lose music or films it is possible with program Windows Media Player, and to create cinema by means of Windows Movie Maker.

Use the Internet through browser Internet Explorer, accept and send your correspondence in Outlook Express.

In a folder “Standard programs” programs: Paint for drawing. WordPad for editing of texts. A directory – for the account of contacts. A notebook. The calculator. A command line (console). The master of compatibility of programs. Connection to a remote desktop. A conductor. Synchronisation.

In "the Entertainment" folder. Programs loudness and a sound recording.

Folder "Office". Masters of copying and clearing. The program “Data on system”.

“The table of symbols” the safety Center

Folder “Special possibilities”. Programs for the limited people.

The computer for children: the friend or the enemy?

Instructions for use the computer

Whether the computer is necessary to children? What for dangers «the clever car» for small почемучки conceals? Whether there can be a computer for the child the friend and the assistant?

The computer for children in the modern world is not a wonder, not a miracle overseas. Almost in each family the personal computer is, and sometimes at all one. As a rule, it is intended for adults: for work, entertainments, search of the necessary information. However, as soon as the kid starts to creep, and then and to go, the computer gets and to sphere of its interests.

First the child is interested in the computer only because behind it all time something is done by adults. Then he starts to understand that the computer is a fascinating electronic toy, which 10 times most "abruptly" available "usual" toys. And if hobby of the child for the computer not to check, soon it in general will replace to the child not only toys and entertainments, but also a family, friends, real dialogue.

Certainly, such it is impossible to suppose. The computer for children can become the enemy, but at the correct approach it can become and the good friend. Simply instructions for use the computer for children need to be established as soon as possible and never to recede from them.

From the very first days of using the computer for children it is necessary to establish time frameworks. So, for the child of 3-4th years optimum time makes 25 minutes, 5-6-летнему the child cannot to allow use the computer longer 35 minutes in day.

The senior children to "tear off" from the computer it is more difficult, but to do it it is necessary. It is impossible to allow for children of 8-12 years to spend behind the computer more than 1 hour per day, to teenagers of 16 years – more than 2 hours. Thus necessarily it is necessary to learn to carry out children visual gymnastics each half an hour are will allow to keep good sight.

It is impossible to allow for children to sit at the computer in twilight or to be behind it in any "curve" pose – all it conducts to infringements of children's health. The long finding in an atypical pose for an organism can become the reason of curvatures of a backbone, and sitting in the dark and driving by "nose" on the monitor screen can lead to visual acuity decrease.

The computer for children should be established and located so that it was convenient to child to work with it. For these purposes the specialised computer chair with adjustable height and a support for feet will approach. The monitor should be established so that its centre was hardly below level of eyes of the child. Thus the monitor should not «бликовать» or deform colours.

The computer is dangerous to children not only and not so much infringements of physical health, how many infringements of psychological character, it is no secret, that modern computer games and films, Internet resources do not promote good psychological education of children.

To check in what game the 6-year-old kid simply plays the computer – it after all does it under parental supervision. And independently to establish other games or programs on the computer the child cannot. But how to be with children-schoolboys and the senior children? How them to protect from severe criminal blockbusters, games-streljalok, completely not children's sites?

Here to the aid of parents «computer nurses» come special. These programs supervise access of the child to the computer and to resources of a network the Internet. It is possible to establish both the complex program, and separate filters for sites, time counters etc.

Thus always it is necessary to suppose possibility that the senior child is quite capable to "deceive or"crack"the program – often children much"grounded"in these questions. Then it is necessary to agree only with the child and to build mutual relations on trust.

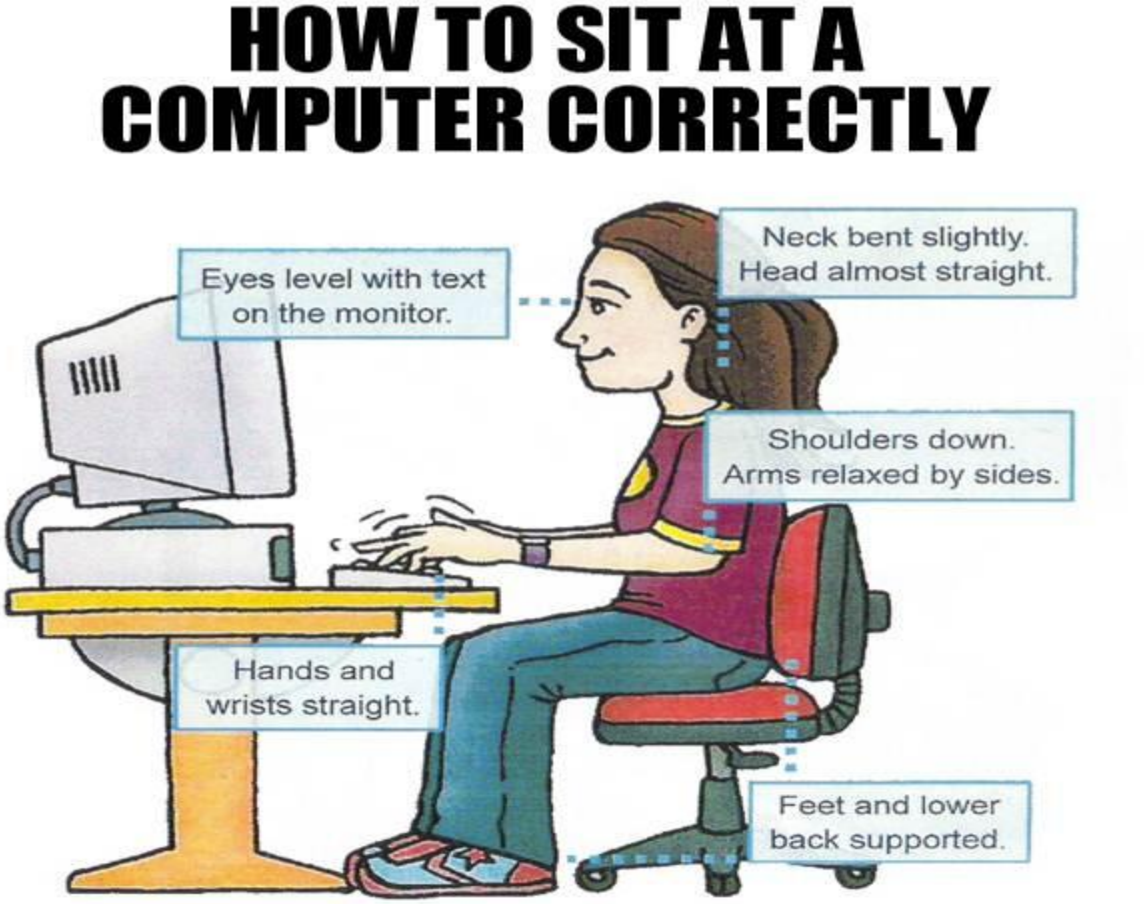
The computer for children it becomes frequent unique means of entertainment simply because they plainly do not know, how it is possible to have a good time in another way: the ball and a skipping rope are forgotten, the children do not rush any more noisy crowd playing "Cossacks-robbers"...

So give we, adults, we will show to children that the computer is only the car, let even very clever, but not capable to replace the present human pleasure and warmth of live dialogue!

TEN RULES FOR PROPER SITTING AT THE COMPUTER

1) Adjust the chair to your body's proportion, so you can sit properly, so that you can use the back of the chair.
2) To avoid stress on your shoulders, neck and lower back (see pictures), raise or lower the chair so that when your elbows are at a 90-degree angle, your palms will rest comfortably on your desk.
3) Place your keyboard and screen directly in front of you. When using the alphabet, place that part of the keyboard in front of you. When working with the numbers on the right side of the keyboard, place that part in front of your working hand. Your body should be 20 centimeters (about 8 inches) from the keyboard.
4) Your elbows should be bent and should rest on the arms of your chair at a comfortable height or on your desk.
5) Position the mouse so that it will allow you to work with a bent elbow that will rest on the arms of your chair or on the table. Your wrist should extend in a direct line from your middle finger. You can place a small pillow or silicon pad under your wrist to raise it to the proper height for the mouse and keyboard.
6) Arrange your work space so that objects you use often are within reach when your elbow remains bent. Remove everything you don't use often from your work space.
7) Spread your feet a bit and place them on a sturdy, comfortable platform - the floor or a foot rest -- not bent under the chair.
8) Hold your hand directly in front of you. If you cannot touch the top end of your screen, move the screen nearer or farther until you can.
9) The top part of the screen should be at eye level. If you wear reading glasses or multifocal glasses, tilt the top part of the screen away from you so its angle is similar to the one you use when reading a book.
10) Get up from your chair every hour for a few minutes, stretch and perform muscle release exercises.

This Health Tip courtesy of Ms. Leah Migdal - Chief of Ergonomics, the Department of Physiotherapy.



5 Essential Characteristics of Cloud Computing

Ref: The NIST Definition of Cloud Computing
http://csrc.nist.gov/publications/nistpubs/800-145/SP800-145.pdf

On-demand self-service | Ubiquitous network access | Location transparent resource pooling | Rapid elasticity | Measured service with pay per use

Source: http://aka.ms/532


Computer Lab Rules
Enter and
Exit Quietly
No Food or Drink
Don't Spread Germs
Organize Before
Leaving Your Area
Print Only With
Permission
Exit Inappropriate
Sites or Images
You
Matter
CAUTION
No Running
in the Lab
Respect Your Teacher
and Classmates
Never Give Out
Personal Info
© oakdome.com

Computer Lab Rules

C **Come into the lab quietly and go to your assigned computer.** Do not touch other keyboards or mice on the way to your computer. Read board and begin assignment if one exists or wait for instructions before you do anything.

O **Only visit approved internet sites and only when you have permission to do so.** Never give out personal information. Do not share your passwords with anyone other than your parents or teacher, if school related. If you see anything that makes you uncomfortable, turn off your monitor and let your teacher know immediately.

M **Make sure you leave your work area neat and organized.** Exit out of all programs. Hang up your headphones. Straighten your keyboard and mouse. Push in your chair. Throw away any trash.

P **Print only if you have permission.** Only press the Print button once! Ask your neighbors for help before you raise your hand to ask your teacher. Keep your fingers out of your nose and mouth.

U **Use only your assigned computer.** Do not move, change, or delete any of the icons on the desktop. Do not edit files that do not belong to you. Help others with your mouth and not their mouse.

T **Treat your classmates, your teacher, and all equipment with respect.** Do not talk while the teacher is talking. Come to the computer lab with clean hands. No banging on the mouse or keyboard. Do not twist the monitors for your neighbors to see.

E **Eat and drink OUTSIDE of the lab only.** No food or drink allowed in the lab. Wash your hands with soap before returning from the bathroom. Do not get out of your seat unless you have permission.

R **Raise your hand if you need help or if you need to go to the bathroom.** Read the monitor screen BEFORE asking questions. No RUNNING in the lab.

STANDARD DOCUMENTS

The Republic Uzbekistan law "About information"

(From December, 11th, 2003)

Article 1. The purpose of the present Law

The purpose of the present Law is regulation of relations in the field of information, uses of information resources and information systems.

Article 2. The legislation on information

The legislation on information consists of the present Law and other certificates of the legislation.

If the international contract of Republic Uzbekistan establishes other rules, than what are provided by the legislation of Republic Uzbekistan on information rules of the international contract are applied.

Article 3. The basic concepts

In the present Law following basic concepts are applied:

- Information - organizational social and economic and scientific and technical process of creation of conditions for satisfaction of requirements legal and physical persons in the information with use of information resources, an information technology and information systems;
- Information resource - the information, a databank, a database as a part of information system;
- The proprietor of information resources or information systems - legal or the physical person who is carrying out possession, using and the order information resources or information systems;
- The owner of information resources or information systems - legal or the physical person who is carrying out possession, using and the order information resources or information systems within the rights established by the law or the proprietor of information resources, information systems;
- Information technology - set of methods, devices, ways and the processes used for gathering, storage, search, processing and information distribution;
- The information system - organizational ordered set of information resources, an information technology and a communication facility, allowing to carry out gathering, storage, search, processing and using the information.

Article 4. The State policy in the field of information

The state policy in the field of information is directed on creation of national information system taking into account modern world tendencies of development and perfection of information resources, an information technology and information systems.

The basic directions of a state policy in the field of information are:

- Realisation of constitutional laws of everyone on free reception and information distribution, providing of access to information resources;
- Creation of a uniform information field of Republic Uzbekistan on the basis of information systems of state structures, branch and territorial information systems, and also information systems legal and physical persons;
- Creation of conditions for access to the international information networks and the world information network the Internet;
- Formation of the state information resources, creation and development of information systems, maintenance of their compatibility and interaction;
- The organisation of manufacture of modern means of an information technology;
- Assistance to formation of the market of information resources, services and information technology;
- Stimulation of development of manufacture of software products;
- Support and business stimulation, creation of favorable conditions for attraction of investments;
- Preparation and improvement of professional skill of shots, stimulation of scientific researches.

Article 5. State regulation in the field of information

State regulation in the field of information is carried out by the Cabinet of Republic Uzbekistan and the special representative it body.

Article 6. The Special representative body

The special representative body:

- Organises and co-ordinates work on formation of the state information resources;
- Develops government programs of information and development of an information technology;
- Promotes creation of information systems of state structures, branch and territorial information systems;
- Develops standards, norms and rules in the field of information;
- Organises work on certification of means and services of information systems and an information technology;
- Co-ordinates activity legal and physical persons on maintenance of protection of their information resources and information systems;
- Promotes development of the market of information resources, services and an information technology;
- Organises marketing researches and monitoring in the field of information;
- Carries out measures on protection of the rights and legitimate interests of users of information resources;

- Provides information safety and priority use of information systems in interests of defensibility and safety of Republic Uzbekistan;
- Carries out other powers according to the legislation.

Article 7. The Legal regime of information resources and information systems

The legal regime of information resources and information systems is defined by the norms establishing:

Order of documenting of the information, formation of information resources and creation of information systems;

- The property right to information resources and information systems;
- Categories of information resources on access level to them;
- Order of protection of information resources and information systems;
- Order of gateway connections of information systems.

Article 8. Information documenting

Information documenting is an indispensable condition of inclusion of the information in information resources. The order of documenting of the information is established by the special representative body.

The information, хранимая and processed in the information resources, confirmed with the electronic digital signature, is the electronic document and has an identical validity with the document on the paper carrier.

The relations connected with formation and use of the electronic document and the electronic digital signature, are regulated by the law.

Article 9. The Property right to information resources and information systems

Information resources and information systems can be in Republic Uzbekistan in public and a private property.

The bases of occurrence of the property right to information resources and information systems are:

- Creation of information resources and information systems at the expense of means of the state budget, own means legal and physical persons or other sources which have been not forbidden by the legislation;
- The contract of purchase and sale or other transaction containing conditions of transition of the property right to information resources and information systems to other person;
- Inheritance.

The law can be provided and other bases of occurrence of the property right to information resources and information systems.

Article 10. The state information resources

The state information resources are formed from:

- Information resources of state structures;
- Information resources legal and the physical persons created at the expense of means of the state budget;
- Information resources legal and the physical persons containing the state secrets and the confidential information;
- The documentary information legal and the physical persons, given when due hereunder.

Legal and physical persons are obliged to give when due hereunder the documentary information in corresponding state structures for formation of the state information resources.

Order of obligatory granting of the documentary information legal and physical persons, the list of the state structures responsible for formation and use of the state information resources, are established by the Republic Uzbekistan Cabinet.

The order of granting of the information carried to the state secrets and the confidential information, is established by the legislation.

Article 11. Access categories to information resources

Information resources on access categories are divided into popular information resources and information resources with the limited access.

Popular information resources are the information resources intended for an unlimited circle of users.

The information resources containing the information on the state secrets and the confidential information or the information access to which is limited by proprietors of information resources concern information resources of the limited access.

Proprietors and owners of information resources should provide access equal in rights legal and physical persons to popular information resources.

Reference of information resources to access categories is defined by the proprietor of information resources in an order established by the legislation.

Article 12. Using information resources

Users of information resources possess the equal rights to access to information resources, except for information resources with the limited access.

For using information resources the payment when due hereunder can be raised.

Lists of the information and services in a supply with information, to information resources proprietors and owners of information resources give data on an order and access conditions to users free of charge.

The information received on the lawful bases from information resources legal and physical persons, can be used them for creation of the derivative information with an obligatory reference to the source of the information.

The order of reception of the information from information resources is defined by the proprietor or the owner of information resources with observance of the requirements established by the legislation.

Article 13. The information resources containing the information about personal given physical persons

The order of formation and use of the information resources containing the information about personal given physical persons, is established by the legislation.

The information on the personal given physical persons concerns a category confidential.

Article 14. Access legal and physical persons to the information resources containing data on them

Legal and physical persons have the right to an easy approach to the information resources containing data on them, on specification of these data with a view of maintenance of their completeness and reliability.

Access legal and physical persons to the information resources containing data on them, can be limited by the law.

Refusal of the proprietor or the owner of information resources legal and to physical persons in access to the information resources containing data on them, can be appealed against in court.

Article 15. National information system

The national information system includes information systems of state structures, branch and territorial information systems, and also information systems legal and physical persons.

The national information system is created at the expense of means of the state budget, and also own means legal and physical persons and other sources which have been not forbidden by the legislation.

The national information system is created taking into account compatibility of information systems entering into its structure with the international information systems. The information exchange with use of national information system is made on a contractual basis, except for the cases provided by the legislation.

Article 16. Use of information systems for fulfilment of transactions

Use of information systems for fulfilment of transactions between legal and physical persons is regulated by the legislation.

Article 17. Certification of means of information systems

The means making information systems, are subject to certification in an order established by the legislation.

Means of information systems of state structures, branch and territorial information systems, information systems legal and the physical persons, the information intended for processing, containing the state secrets or the confidential information, and also protection frames of these systems are subject to obligatory certification.

Article 18. Gateway connections of information systems

Gateway connections of information systems are carried out for information interchange between various information systems. Proprietors, owners of information systems provide possibility of gateway connection among themselves according to the established norms and rules.

Gateway connections of various information systems are carried out on a contractual basis between proprietors, owners of information systems.

Order and conditions of realisation of gateway connections and interaction of various information systems are established by the special representative body.

Article 19. Protection of information resources and information systems

Protection of information resources and information systems is carried out with a view of:

- Maintenance of information safety of the person, society and the state;
- Prevention of leak, plunder, loss, distortion, blocking, fake of information resources and other unapproved access to them;
- Prevention of unapproved actions on destruction, blocking, copying, distortion of the information and other forms of intervention in information resources and information systems;
- Preservations of the state secrets and the confidential information containing in information resources.

Article 20. The organisation of protection of information resources and information systems

Information resources and information systems the wrongful reference with which can cause a damage to their proprietors, owners or others legal and to physical persons are subject to protection.

State structures, legal and physical persons are obliged to provide protection of information resources and the information systems containing the information on the state secrets and the confidential information.

The order of the organisation of protection of information resources and information systems is established by their proprietors, owners independently.

The order of the organisation of protection of information resources and the information systems containing the information on the state secrets and the confidential information, is defined by the Republic Uzbekistan Cabinet.

Article 21. Inclusion in the international information networks

State structures, legal and physical persons can include the information systems in the international information networks and in the world information network the Internet in the order established by the legislation.

Inclusion of the information systems containing information resources of limited access, to the international information networks and in the world information network the Internet is carried out only after acceptance of necessary protective measures.

Article 22. The Resolution of disputes

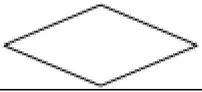
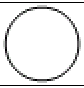
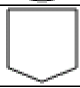
Disputes in the field of information are resolved in an order established by the legislation.

Article 23. The Liability of infringement of the legislation on information

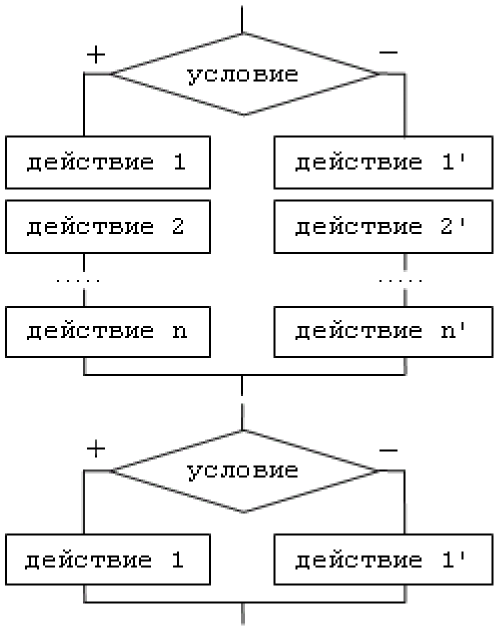
The persons guilty of infringement of the legislation on information, bear responsibility when due hereunder.

The president
Republics Uzbekistan
I.KARIMOV

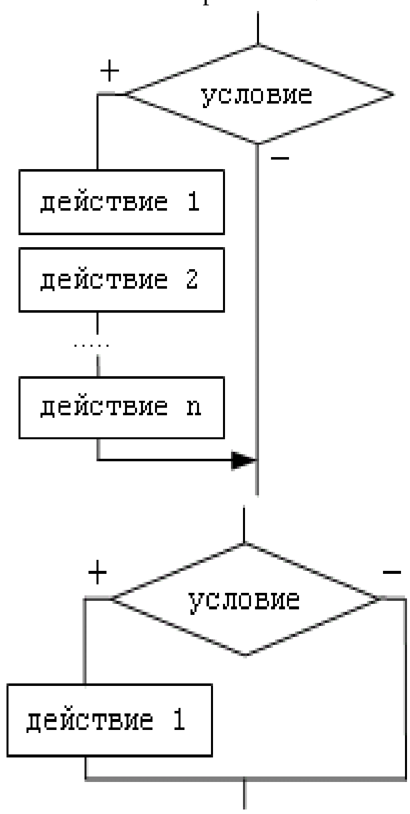
DISTRIBUTING MATERIALS

The name	Symbol (drawing)	Carried out function (explanatory)
1. The block of calculations		Carries out computing action or group of actions
2. The logic block		Choice of a direction of performance of algorithm depending on a condition
3. Input/conclusion blocks		Input or output of the data without dependence from the physical carrier
		Conclusion of the data to the printer
4. The Beginning/end (input/exit)		The beginning or the program end, input or exit in the subroutine
5. The predetermined process		Calculations under the standard or user subroutine
6. The updating block		Performance of the actions changing points of algorithm
7. A connector		Communication instructions between the interrupted lines within one page
8. An interpage connector		Communication instructions between parts of the scheme located on different pages

The full form:



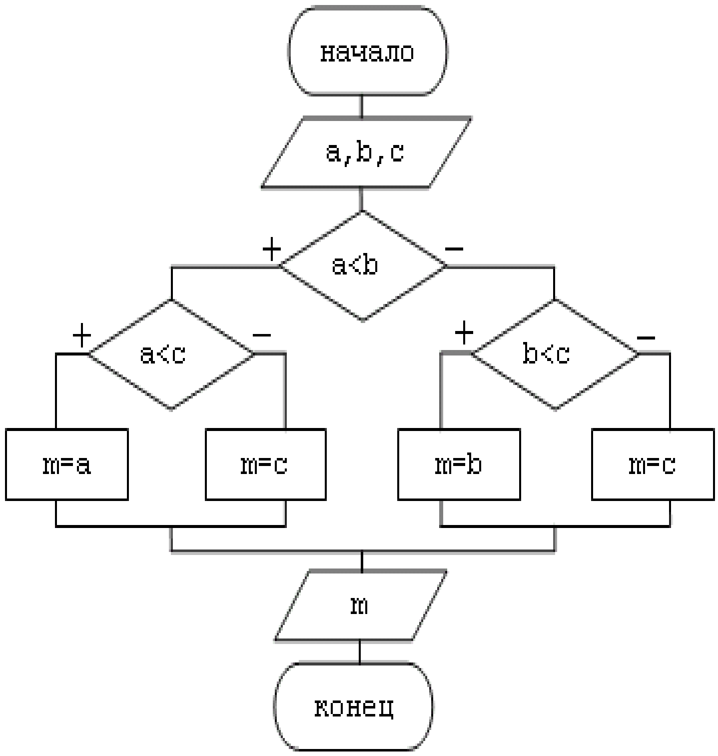
The incomplete form:

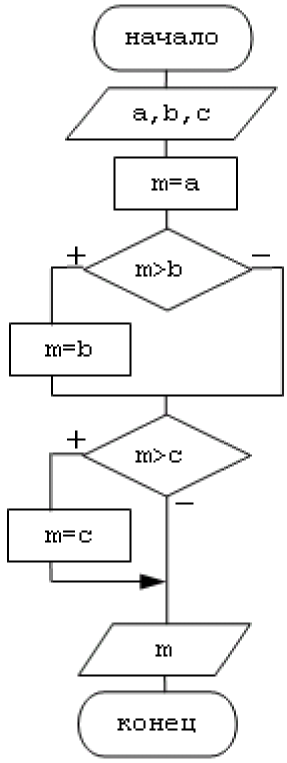


Example: to find least of three numbers.

1 Decision variant:

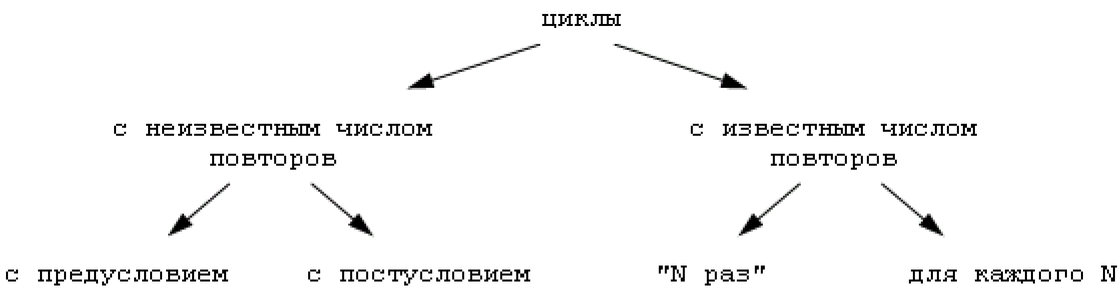
2 variant of the decision:



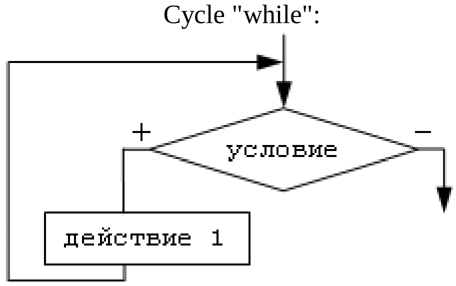


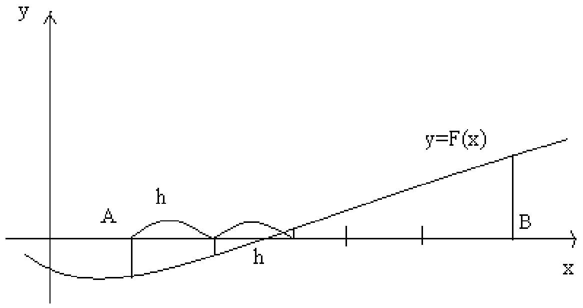
Algorithmic design of a cycle.

Cycle - operating structure, organized repeated performance of the specified action.



Cycle "while":





Rice 1. A piece choice

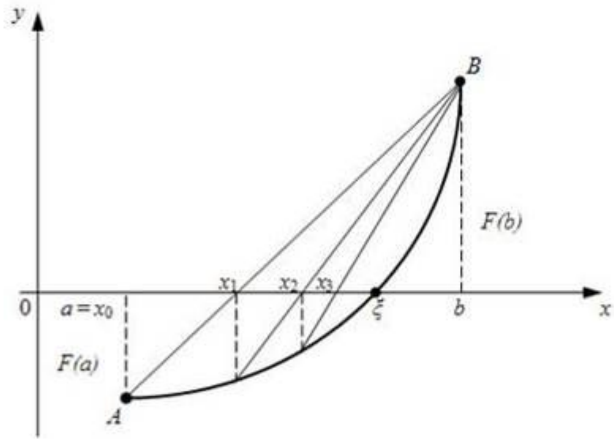


Fig. 2. Geometrical representation of a method of chords.

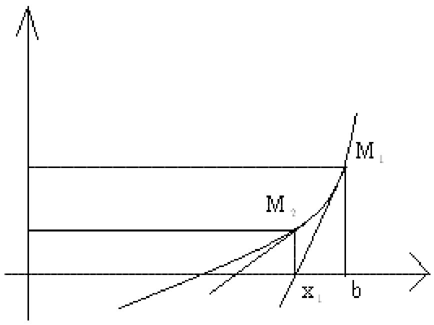


Fig. 2. A choice of points of a contact

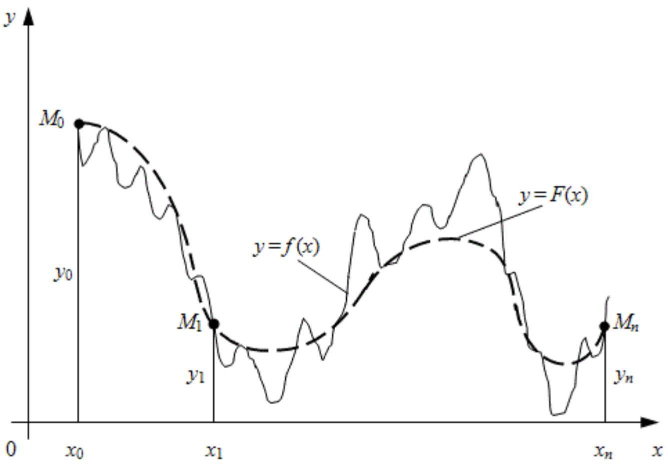


Fig. 1. Geometrical representation of interpolation of function

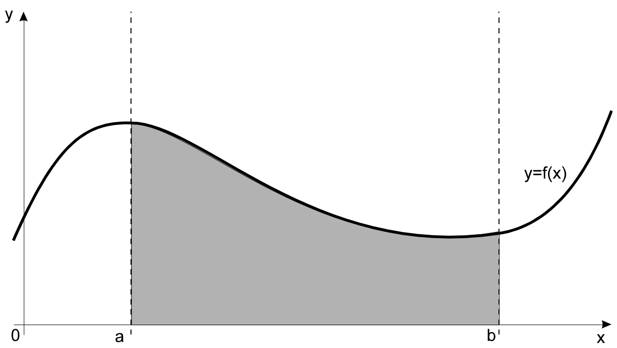


Fig. 1 Curvilinear trapeze.

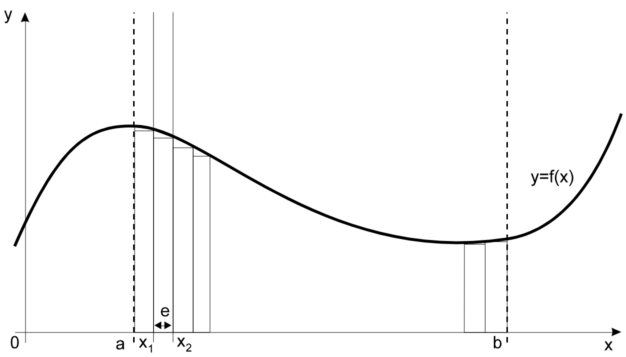


Fig. 2 Method of trapezes.

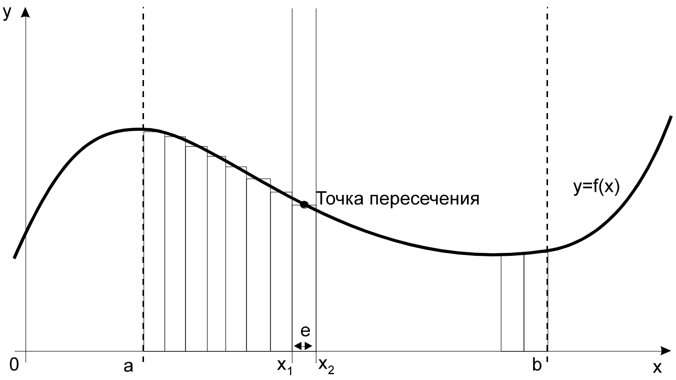


Fig. 3 Method of average rectangles.

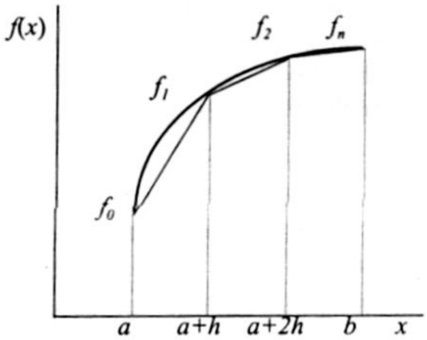


Fig. 4. Interval splitting [a, b] on n identical sites

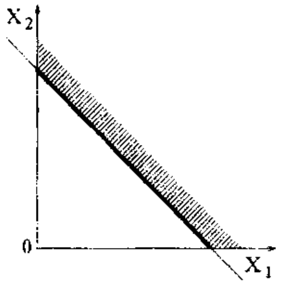


Fig. II.1. Geometrical sense of restriction

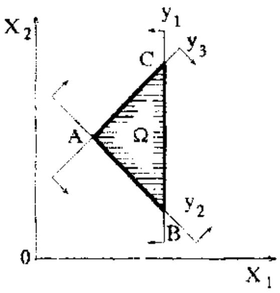


Fig. II.2. Geometrical interpretation of system of restrictions

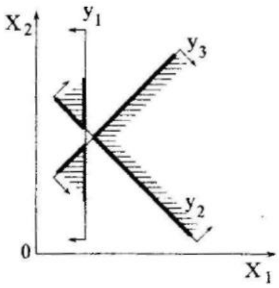


Fig. II.3. Incompatibility systems of restrictions

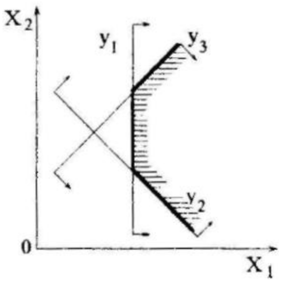


Fig. II.4. Limitlessness of criterion function

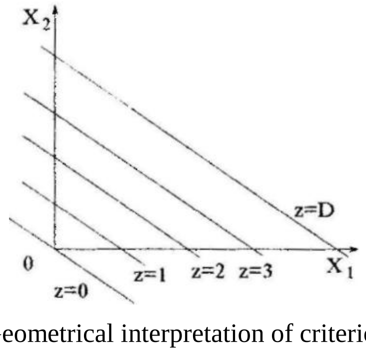


Fig. II.5. Geometrical interpretation of criterion function

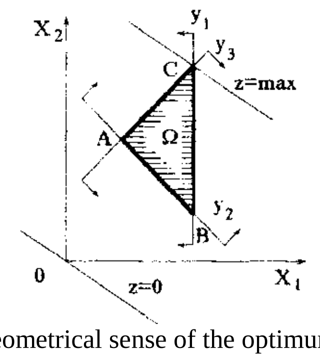


Fig. II.6. Geometrical sense of the optimum decision of a problem of linear programming

THE SUMMARY

In an educational-methodical complex lecture, practical materials, the test questions flowing, intermediate, total control questions in a subject «Algorithmization of computing methods» are resulted. An educational methodical complex are intended for students of a direction a bachelory 5311000 - «Automation and Control technological processes and industries».

The educational methodical complex is intended as the textbook for pupils of technical colleges and colleges.

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