O‘ZBEKISTON RESPUBLIKASI OLIY VA O‘RTA MAXSUS TA’LIM VAZIRLIGI

ISLOM KARIMOV NOMIDAGI TOSHKENT DAVLAT TEXNIKA UNIVERSITETI

DISKRET BOSHQARISH TIZIMLARI

fanidan amaliy mashg‘ulotlarni bajarish uchun

USLUBIY KO‘RSATMALAR

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Uslubiy ko‘rsatmada «Diskret boshqarish tizimlari» fani bo’yicha amaliy mashg‘ulotlarni bajarishga doir qisqacha uslubiy ko‘rsatmalar, nazariy bilimlar hamda sodda masalalarni yechishga oid misollar keltirilgan.

Uslubiy ko‘rsatma 5330200 Informatika va axborot texnologiyalari (sanot ishlab chiqarishida) va 5312700 Intellektual muhandislik tizimlari yo‘nalishi talabalari hamda unga turdosh mutaxassislik talabalari uchun mo‘ljallangan.

Uslubiy ko‘rsatma Islom Karimov nomidagi ToshDTU ilmiy-uslubiy kengashi (25.11.20. 3 sonli kengash) qaroriga asosan chop etildi

Taqrizchilar:

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1- amaliy mashg‘ulot

Diskret boshqarish tizimlarining matematik ifodalarini o‘rganish

Ma’lumki diskret tizimning dinamikasi ayirmali tenglamalar orqali yoziladi. Diskret tizimlarni tadqiq qilish uchun ayirmali tenglamalarni yechish talab qilinadi. Lekin ayirmali tenglamaning tartibi oshib borgani sababli uni qurish murakkablashadi. Shuning uchun diskret tizimlarni tadqiq qilishda *Z* almashtirish usuli tadqiq qilinadi.

Chiziqli tizimlarda shunday misolni yechish uchun Laplas almashtirishdan foydalanar edi:

$$x(p) = \int_0^{\infty} x(t)e^{-PT}dt.$$

Z - almashtirishga o‘tish integral summa bilan almashtiriladi:

S=> $\bar{z}$, t=>nT, e^{PT}=z, u holda *Z* almashtirishi quyidagicha bo‘ladi:

$$X(Z) = \sum_{n=0}^{\infty} x[nT] \cdot Z^{-n} \ ,$$

bu yerda *X*(*Z*) – tasvir funksiya, *x*[*nT*] – uning originali.

Z almashtirishi mavjud bo‘lishi uchun *x*[*nT*] qatori yaqinlashuvchi bo‘lishi shart. *Z* almashtirishi quyidagicha belgilanadi:

$$X(Z) = Z\{x[nT]\}$$

yoki

$$x[nT] = Z^{-1}\{X(Z)\} \ .$$

<u>*Ta’rif:*</u> *Z* almashtirish deb (1) formula bilan aniqlanuvchi diskret funksiya tasviriga aytiladi.

Z almashtirishni faqatgina chiziqli impulsli tizimlarga qo‘llash mumkin.

Xossalari:

1. *Chiziqlilik xossasi.* Diskret funksiya chiziqli kombinatsiyasini tasviri ularning tasvirlarining chiziqli kombinatsiyasiga tengdir. Biror bir diskret funksiya berilgan bo‘lsin

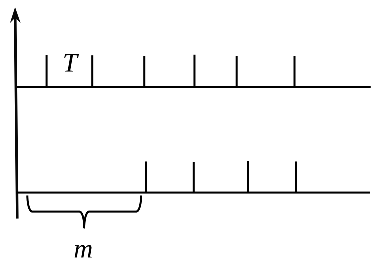
$$f[n] = \sum_{i=1}^{m} c_i \cdot f_i[n]$$

bu yerda c_i – butun son

$$F(z)=\sum_{i=1}^{n} c_i \cdot F_i(z).$$

2. *Kechikishli va ilgarilanmali xossasi.* Impulsli elimentning chiqishida impulslar *m* taktga kechikayotgan bo‘lsin:

$f(n-m)$



U holda funksiyaning tasviri quyidagicha bo‘ladi:

$$F(z)=Z\{f[n-m]\}=Z^{-m}\cdot F(z).$$

Agarda *m* taktga ilgarilab ketayotgan bo‘lsa

$$Z\{f[n-m]\}=Z^{m}\cdot F(z).$$

3. *Teorema svyortka yoki tasvirlar ko‘paytmasi.*

Agar $F_1(z)=Z\{f_1[n]\}$,

$F_2(z)=Z\{f_2[n]\}$

bo‘lsa, u holda

$$F_1(z)\cdot F_2(z)=\sum_{m=0}^{n} f_1[m]\cdot f_2[n+m]$$

bo‘ladi.

Ushbu xossalarga asoslanib ixtiyoriy signal funksiya yoki ayirmali tenglamaning *Z* tasvirini topish mumkin. Bu esa murakkab tizimning xossalarini o‘ranishda elimentar zvenolarning xossalaridan foydalanish imkonini beradi.

Misol. Ayirmali tenglama berilgan bo‘lsin:

$$x[n+2]-2x[n-1]+3x[n]=f[n]$$

Z almashtirishi topilsin.

$$Z\{x[n+2]-2x[n-1]+3x[n]\}=Z\{f[n]\}.$$

Chiziqlilik xususiyatidan foydalanib

$$Z\{x[n+2]\}-2Z\{x[n-1]\}+3Z\{x[n]\}=F[z],$$

$$Z^2x(z)-2Z^{-1}x(z)+3x(z)=F[z]$$

$$\left(Z^2-2Z^{-1}+3\right)x(z)=F[z].$$

Kirishi pog‘onali signalning Z tasvirini topamiz.

$$f(t)=1(t),\quad T=1,$$

$$f[n]=1[n],$$

$$F(z)=\sum f[n]\cdot z^{-n}=1\cdot z^0+1\cdot z^{-1}+...$$

$$S=\frac{1}{1-q},\quad q=z^{-1},$$

$$F(z)=\frac{1}{1-z^{-1}}=\frac{z}{z-1}.$$

Ixtiyoriy funksiya Z almashtirishini topish uchun mos keluvchi jadvaldan foydalanish mumkin.

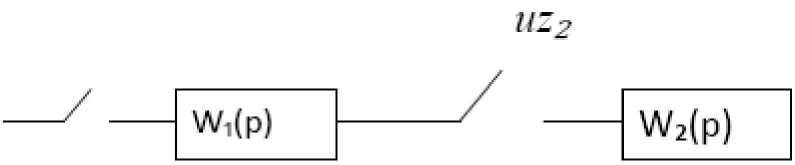
$f(t)$	$F(p)$	$f[nT]$	$F[z]$
$\delta(t)$	1	$\delta[nT]$	1
$1(t)$	$1/p$	$1[nT]$	$z/(z-1)$
t	$1/p^2$	nT	$z/(z-1)^2$
$e^{-\alpha t}$	$1/(p+\alpha)$	$e^{-\alpha nT}$	$z/(z-e^{-\alpha T})$
$1-e^{-\alpha t}$	$\alpha/(p(p+\alpha))$	$1-e^{-\alpha nT}$	$z(1-e^{-\alpha T})/(z-1)(z-e^{-\alpha T})$

Diskret tizimlar turli elementlardan tashkil topgan bo‘lib, ular bir-biri bilan turlicha ulangan bo‘lishi mumkin.

Bunday tizimlarni tadqiq qilishda uning strukturasini o‘zgartirish talab etiladi. Diskret tizimning strukturasini o‘zgartirish z almashtirish algebrasi deyiladi.

Bunda albatta impusli elementning joylashish nuqtasini hisobga olish zarur.

1.1-misol: Diskret tizim berilgan bo‘lsin:



a)

$$W_1(z) = z\{W_1(p)\}$$
$$W_2(z) = z\{W_2(p)\}$$
$$W(z) = W_1(z)W_2(z) = z\{W_1(p)\} * z\{W_2(p)\}$$

$$W(z) = z\{W_1(p)W_2(p)\}$$

b)

1.1-rasm. a va b diskret tizimning strukturaviy sxemalari

Nazorat savollari

1. Z almashtirish algebrasi deb nimaga aytiladi?
2. Diskret tizimlarni tadqiq qilishda qanaqa usullardan foydalaniladi?
3. Diskret tizimning strukturasini o‘zgartirish nima deb ataladi?

2- amaliy mashg‘ulot

Chekli ayirmali va summalarni aniqlash

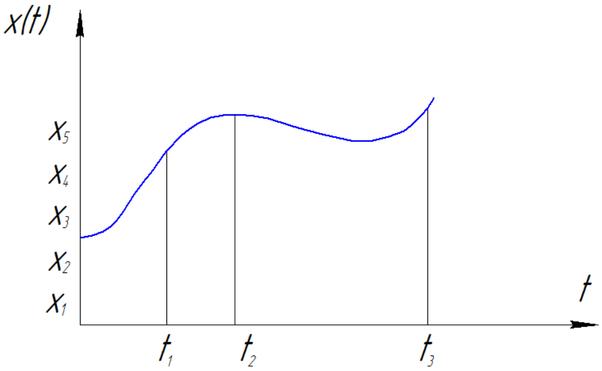
1. Diskret funksiyalar, ularning ayirmasi va yig‘indisi
2. Ayirmali tenglamalar.

Diskret tizimning dinamikasi uzluksiz tizimdan farq qiladi. Farqi shundan iboratki kalitning uzuq yoki ulangan vaqtiga qarab tizim o‘zini turlicha tutadi va signallar sakrashsimon xarakterga ega bo‘ladi. Bu esa diskret tizimlarni tekshirishda differensial tenglamalar nazariyasini qo‘llashga imkon bermaydi ya’ni Laplas almashtirishini qo‘llash mumkin emas.

Diskret funksiyalar, ularning ayirmasi va yig‘indisi

Impulsli tizimlarni hisoblashda ayirmali tenglamalardan foydalaniladi. Uzluksiz funksiyaning ayrim qiymatlarini ifodalash uchun diskret fnksiyalar qo‘llaniladi.

Diskret funksiya deb – shunday funksiyaga aytiladiki u argumentning ma’lum qiymatlaridagina mavjud, qolgan holatda nolga teng bo‘ladi. Diskret funksiyani ixtiyoriy uzluksiz funksiyadan olish mumkin.



2.1-rasm. Uzluksiz signal proyeksiyasi

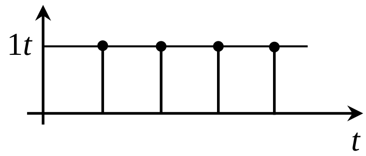
Uzluksiz funksiyadan diskret funksiyaga quyidagicha o‘tiladi: $t = nT$ bilan almashtiriladi, bu yerda T – diskret qiymatlar orasidagi vaqt, n – butun son (n=0,1,2, ...).

Demak uzluksiz funksiya berilgan bo‘lsa va diskret funksiya ko‘rinishiga o‘tilsin deyilsa, quyidagicha bo‘ladi: $x(t)$ – uzluksiz, $x(nT)$ – diskret,

$x[nT] = \{x_1, x_2, x_3....., x_n\}$ – diskret funksiyaning ko‘rinishi.

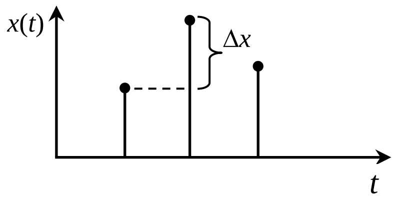
Birlik pog‘ona signal berilgan bo‘lsin

$$x[nT] = 1[nT] = \{1,1,...,1\}.$$



2.2-rasm. Birlik pog‘onalik signal

Diskret funksiyaning o‘zgarish tezligi uning birinchi tartibli ayirmasi bilan xarakterlanadi. U quyidagicha belgilanadi:



2.3-rasm. Orttirmani olish sxemasi

Δx – diskret tenglamasining o‘zgarish tezligi.

Ayirmalar 2 xil bo‘ladi: to‘g‘ri va teskari ayirma

To‘g‘ri ayirma quydagicha:

$$\Delta x[n] = x[n+1] - x[n]$$

Teskari ayirma:

$$\nabla x[n] = x[n] - x[n-1]$$

Agar ikkinchi tartibli ayirmani topcak, quyidagicha bo‘ladi:

$$\Delta^2 x[n] = \Delta[n+1] - \Delta x[n] = x[n+2] - x[n+1] - x[n+1] + x[n] = x[n+2] - 2x[n+1] + x[n]$$

,

$$\Delta^k x[n] = \sum_{v=0}^{k} (-1)^v \, C_v^k \, x[n+k-v],$$

bu yerda $C_v^k = \dfrac{k!}{(k-v)!v!}$ – Nyuton binomi birlashmasi ya’ni binominal koeffitsiyenti.

Diskret funksiyaning qiymatlaridan ixtiyoriy tartibli ayirmani topish formulasiga misol keltiramiz:

2.1-misol: $f(t) = t^2$ funksiya berilgan bo‘lsa, ikkinchi tartibli ayirmani, ya’ni Δ^2 ni aniqlang.

Yechish:

Diskret funksiya ko‘nishishida yozib olamiz:

$f[nT]=[nT]^2,\qquad T=1$ desak,

$f[n]=n^2$ bo‘ladi.

$$\Delta^2 f[n]=\sum_{v=0}^{2}(-1)^v C_v^k\ f[n+2-v]=(-1)^0\cdot C_0^2\cdot f[n+2-0]+(-1)^1\cdot C_1^2\cdot f[n+2-1]+$$
$$+(-1)^2\cdot C_2^2\cdot f[n+2-2]=f[n+2]-2f[n+1]+f[n]=(n+2)^2-2(n+1)^2+n^2=2.$$

bu yerda $C_0^2=\dfrac{2!}{(2-0)!\cdot 0!}=\dfrac{1\cdot 2}{1\cdot 2\cdot 1}=1;\ C_1^2=\dfrac{2!}{(2-1)!\cdot 1!}=\dfrac{1\cdot 2}{1\cdot 1}=2;\ C_2^2=\dfrac{2!}{(2-2)!\cdot 2!}=\dfrac{1\cdot 2}{1\cdot 2}=1.$

Ikkinchi tartibli teskari ayirma quyidagicha topiladi:

$\nabla x[n]=x[n]-x[n-1],$

$$\nabla^2 x[n]=\nabla x[n]-\nabla x[n-1]=x[n]-x[n-1]-x[n-1]+x[n-2]=$$
$$=x[n]-2x[n-1]+x[n-2].$$

2.2-misol: *x(t)=3t+5* $\Delta^2=?$

x(t)=f(t)

f[nT]=3nT+5 *T=1;*

f[n]=3n+5

f[n+1]=3(n+1)+5

f[n+2]=3(n+2)+5

$$\Delta^k f[n]=\sum_{j=0}^{k}(-1)^j C_j^k f[n+k-j]$$

$$\Delta^2 f[n]=\sum_{j=0}^{2}(-1)^0 C_0^2 f[n+2-0]+(-1)^1 C_1^2 f[n+2-1]+(-1)^2 C_2^2 f[n+2-2]=$$
$$f[n+2]-2f[n+1]+f[n]=3(n+2)+5-2(3(n+1)+5)+3n+5=$$
$$3n+11-6n-16+3n+5=0$$

2.3-misol: *x(t)=t*^{3} $\Delta^3=?$

x(t)=f(t)

$f[nT]=nT^3 \qquad T=1;$

$f[n]=n^3$

$f[n+1]=(n+1)^3$

$f[n+2]=(n+2)^3$

$f[n+3]=(n+3)^3$

$$\Delta^k f[n]=\sum_{j=0}^{k}(-1)^j C_j^k f[n+k-j]$$

$$\Delta^3 f[n]=\sum_{j=0}^{3}(-1)^0 C_0^3 f[n+3-0]+(-1)^1 C_1^3 f[n+3-1]+(-1)^2 C_2^3 f[n+3-2]+$$
$$(-1)^3 C_3^3 f[n+3-3]= f[n+3]-3f[n+2]+3f[n+1]-f[n]=$$
$$(n+3)^3-3\left((n+2)^3\right)+3\left((n+1)^3\right)-n^3=-6$$

Bajarish uchun topshiriqlar

1) $x(t)=t^2+3t \quad \Delta^2=?$
2) $x(t)=t^2+3t \quad \Delta^3=?$
3) $x(t)=t^3+2t \quad \Delta^2=?$
4) $x(t)=2t^2+t \quad \Delta^2=?$
5) $x(t)=3t \quad \Delta^3=?$
6) $x(t)=t^2+3(t+2) \quad \Delta^2=?$
7) $x(t)=(t+3)^2+3t \quad \Delta^2=?$
8) $x(t)=t^2+3(t+2) \quad \Delta^3=?$

11) $x(t)=t^3+(t+2)^2 \quad \Delta^2=?$

10) $x(t)=t^3+2(t+3) \quad \Delta^2=?$

11) $x(t)=t^3 \quad \Delta^2=?$

12) $x(t)=5t^2+t \quad \Delta^3=?$

13) $x(t)=t^3+3(t+5) \quad \Delta^2=?$

14) $x(t)=t^2+3(t+3) \quad \Delta^3=?$

15) $x(t)=t^3+5t \quad \Delta^2=?$

Nazorat savollari

1. Diskret tizim dinamikasining uzluksiz tizimdan farqi?
2. Diskret funksiya deb nimaga aytiladi?
3. Diskret funksiyaning o‘zgarish tezligi qanday aniqlanadi?

3-amaliy mashg‘ulot

Ayirmali tenglamalarni yechish. Ayirmali tenglamalarni bir ko‘rinishdan boshqa ko‘rinishga o‘tkazish

Ayirmali tenglamalar deb diskret funksiya va uning turli tartibli ayirmalari orasidagi o‘zaro munosabatlarni ko‘rsatuvchi tenglamaga aytiladi. Ayirmali tenglama 2 xil shaklda yozilishi mumkin:

1. Yuqori tartibli ayirmalar orqali

$$b_l \Delta^l y[n] + b_{l-1}\Delta^{l-1} y[n] + ... + b_0 y[n] = f[n]. \qquad (1)$$

2. Diskret qiymatlar orqali ifodalash

$$a_l y[n+l] + a_{l-1} y[n+l-1] + ... + a_0 y[n] = f[n], \qquad (2)$$

bu yerda $b_0, b_1, ..., b_l$ va $a_0, a_1, ..., a_l$ – o‘zgarmas koeffitsiyentlar; $y[n]$ – diskret funksiya; $f[n]$ – berilgan funksiya; l – ayirmali tenglamaning tartibi yoki darajasi.

Bu ikki tenglamadagi a va b koeffitsiyentlar bir biri bilan bog‘lanishi quyidagi munosabat orqali amalga oshiriladi:

$$a_{1-k} = \sum_{v=0}^{k} b_{l-v} (-1)^{k-v} \begin{pmatrix} l-v \\ k-v \end{pmatrix}$$

$$b_{l-k} = \sum_{v=0}^{k} a_{l-v} (-1)^{k-v} \begin{pmatrix} l-v \\ k-v \end{pmatrix}$$

3.1-misol: Quyida keltirilgan ayirmali tenglamalar orasidagi munosabatni toping:

$$b_3\Delta^3 y[n] + b_2\Delta^2 y[n] + b_1\Delta y[n] + b_0 y[n] = 0, \quad a_0, a_1, a_2, a_3 \to ?$$

Bu yerda l=3 ga teng. k=0, 1, 2, 3 ga teng bo‘lgan hollar uchun a_0, a_1, a_2, a_3 ni aniqlaymiz.

k=1 uchun hisoblaymiz:

$$a_{3-1} = \sum_{v=0}^{1} b_{3-v} (-1)^{1-v} \begin{pmatrix} 3-v \\ 1-v \end{pmatrix} = b_{3-0}(-1)^{1-0} \begin{pmatrix} 3-0 \\ 1-0 \end{pmatrix} + b_{3-1}(-1)^{1-1} \begin{pmatrix} 3-1 \\ 1-1 \end{pmatrix} = -b_3 \cdot 3 + b_2,$$

$$a_2 = -3b_3 + b_2.$$

bu yerda $C_1^3 = \dfrac{3!}{(3-1)! \cdot 1!} = 3$; $C_0^2 = \dfrac{2!}{(2-0)! \cdot 0!} = 1$.

Xudi shunday k=0, 2, 3 holatlar uchun ham hisoblanadi.

Diskret tizimning dinamikasi ayirmali tenglama orqali berilgan bo‘lsin.

$$a_m x[n+m] + a_{m-1} x[n+m-1] + \ ... \ + a_0 x[n] = f[m],$$

bu yerda $f[m]$ berilgan diskret funksiya, $a_0, a_1, a_2, ..., a_n$ o‘zgarmas koeffitsiyentlar, $x[m]$, $x[n+m-1]$, ... – izlanayotgan funksiya.

Ushbu tenglamani yechish uchun boshlang‘ich shartlar mavjud bo‘lishi kerak. Izlanayotgan funksiyaga nisbatan tenglamani yozib olamiz

$$x[n] = \left(f[n] - a_m x[n+m] + a_{m-1} x[n+m-1] + ... \right) \cdot a_0^{-1}$$

$x[n]$ funksiyaning manfiy argumentlari aniqlanmagan. Shuning uchun ayirmali tenglama iteratsiya (qadamba qadam) usulida yechiladi, ya’ni

$$x[1] = f[1] \cdot a_0^{-1}$$
$$x[2] = \left(f[2] - a_1 x[1] \right) \cdot a_0^{-1}$$
$$\vdots$$

Agarda ayirmali tenglamaning tartibi yuqori bo‘lsa, uni yechish murakkabalashadi. U holda ayirmali tenglamani yechish uchun Z almashtirishdan foydalaniladi.

3.2-misol:

$$10\Delta^3 y[n] + 3\Delta^2 y[n] + 2\Delta y[n] + 5 = 0$$

$$l = 3;\ \mathrm{b}_0 = 5;\ \mathrm{b}_1 = 2;\ \mathrm{b}_2 = 3;\ \mathrm{b}_3 = 10;\ \mathrm{a}_0,\ \mathrm{a}_1,\ \mathrm{a}_2,\ \mathrm{a}_3 = ?$$

$$a_{l-k} = \sum_{j=0}^{k} b_{l-k} (-1)^{k-j} \binom{l-j}{k-j}$$

$$a_{3-3} = b_{3-0}(-1)^{3-0}\binom{3-0}{3-0} + b_{3-1}(-1)^{3-1}\binom{3-1}{3-1} + b_{3-2}(-1)^{3-2}\binom{3-2}{3-2} +$$
$$b_{3-3}(-1)^{3-3}\binom{3-3}{3-3} = -b_3 + b_2 - b_1 + b_0 = -10 + 3 - 2 + 5 = -4$$

$$a_0 = -4$$

$$a_{3-2}=b_{3-0}(-1)^{2-0}\binom{3-0}{2-0}+b_{3-1}(-1)^{2-1}\binom{3-1}{2-1}+b_{3-2}(-1)^{2-2}\binom{3-2}{2-2}=$$

$$3b_3-2b_2+b_1=3*10-2*3+2=26$$

$$a_1=26$$

$$a_{3-1}=b_{3-0}(-1)^{1-0}\binom{3-0}{1-0}+b_{3-1}(-1)^{1-1}\binom{3-1}{1-1}=-3b_3+b_2=-3*10+3=-27$$

$$a_2=-27$$

$$a_{3-0=}b_{3-0}(-1)^{0-0}\binom{3-0}{0-0}=b_3$$

$$a_{3-0}=b_{3-0}(-1)^{0-0}\binom{3-0}{0-0}=b_3=10$$

$$a_3=10$$

$$10y[n+3]-27y[n+2]+26y[n+1]-4y[n]=f[n];$$

Bajarish uchun topshiriqlar

1) Δ^3y[n]-4Δ^2y[n]+2Δy[n]+5=0; 6) 3y[n+3]-7y[n+2]+6y[n+1]-4y[n]=f[n];

2) 7Δ^3y[n]+3Δ^2y[n]+Δy[n]+8=0; 7) y[n+3]-27y[n+2]+6y[n+1]-4y[n]=f[n];

3) Δ^3y[n]-5Δ^2y[n]+8Δy[n]-11=0; 8) 3y[n+3]-2y[n+2]+2y[n+1]-4y[n]=f[n];

4) Δ^3y[n]-4Δ^2y[n]+2Δy[n]+5=0; 9) y[n+3]-27y[n+2]+6y[n+1]-4y[n]=f[n];

5) 2Δ^3y[n]-Δ^2y[n]+8Δy[n]-3=0; 10) 7y[n+3]-7y[n+2]+6y[n+1]-y[n]=f[n].

Nazorat savollari

1. Ayirmali tenglamalar deb qanday tenglamalarga aytiladi?
2. Yuqori tartibli ayirmali tenglamalar qanday ishlanadi?
3. Ayirmali tenglama nechchi xil shaklda yozilishi mumkin?

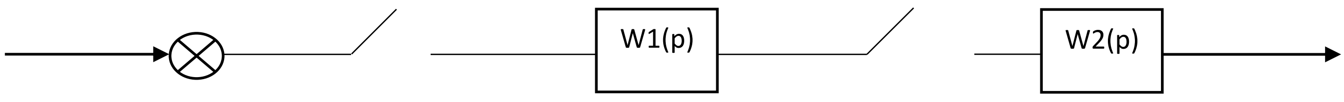
4-amaliy mashg‘ulot

Diskret tizimlarning strukturasini o‘zgartirish

Diskret tizimlar turli elementlardan tashkil topgan bo‘lib, ular bir-biri bilan turlicha ulangan bo‘lishi mumkin.

Bunday tizimlarni tadqiq qilishda uning strukturasini o‘zgartirish talab etiladi. Diskret tizimning strukturasini o‘zgartirish Z almashtirishning algebrasi deyiladi. Bunda albatta impulsli elementning joylashish nuqtasi hisobga olinishi zarur. Diskret tizim berilgan bo‘lsin.

4.1-misol



$$W(z)=\{W_1(p)\}*\{W_2(p)\}$$

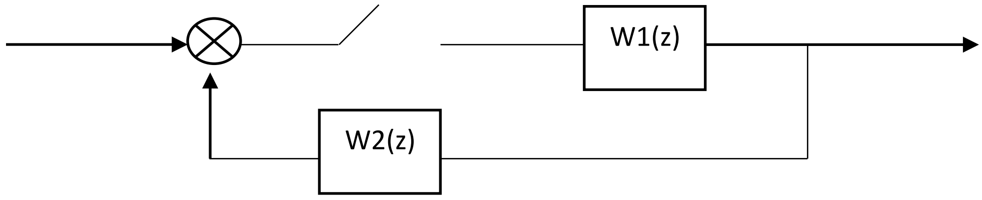
$$p\to z$$

$$W_1(z)=\frac{z}{0,5z+1};$$

$$W_2(z)=\frac{z}{1,3z+1};$$

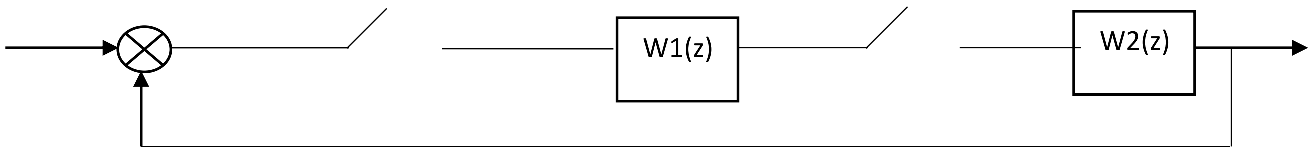
$$W(z)=\left\{\frac{z}{0,5z+1}*\frac{z}{1,3z+1}\right\}=\left\{\frac{z^2}{0,65z^2+1,8z+1}\right\};$$

4.2-misol



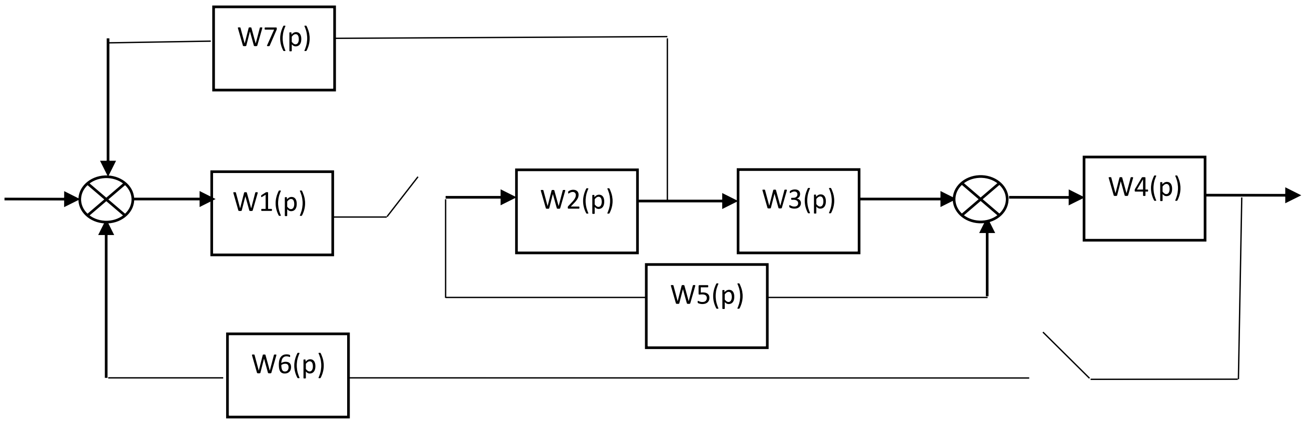
$$W(z)=\frac{w_1(z)}{1+w_1(z)*w_2(z)}$$

4.3-misol



$$W(z)=\frac{w_1(z)*w_2(z)}{1+w_1(z)*w_2(z)}$$

4.4-misol



$$\overline{W}_1(z)=\frac{z\{w_2(p)\}}{1+z\{w_7(p)*w_2(p)\}};$$

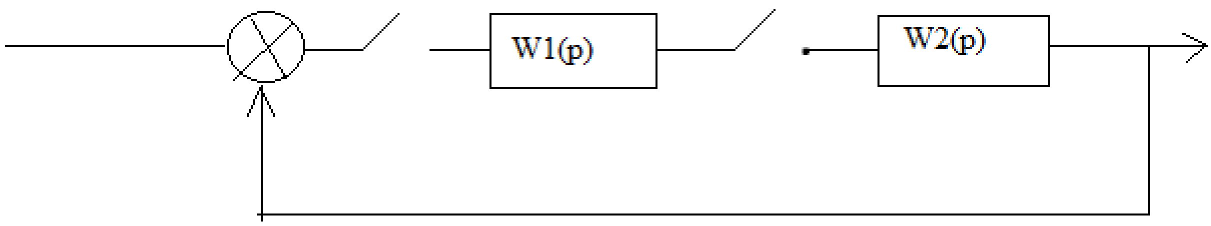
$$\overline{W}_2(z)=\frac{z\{w_3(p)\}*\overline{W_1}(z)}{1+z\{w_3(p)*w_5(p)\}*\overline{W_1}(z)};$$

$$\overline{W}_3(z)=\overline{W}_1(z)*\overline{W}_2(z)*z\{w_4(p)\};$$

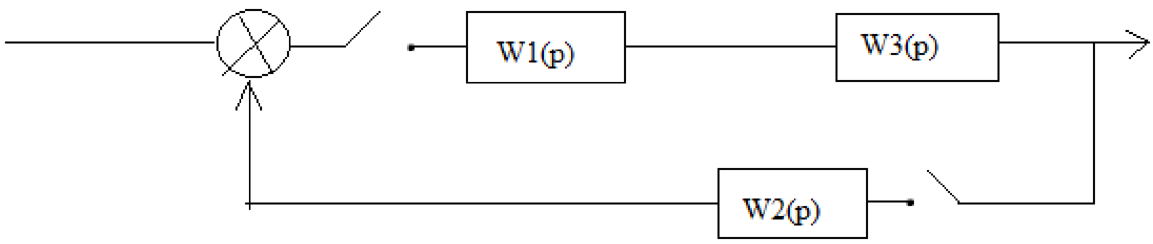
$$\overline{W}_4(z)=\frac{\overline{W_3}(z)}{1+z\{w_1(p)*w_6(p)\}*\overline{W_3}(z)}$$

Bajarish uchun topshiriqlar.

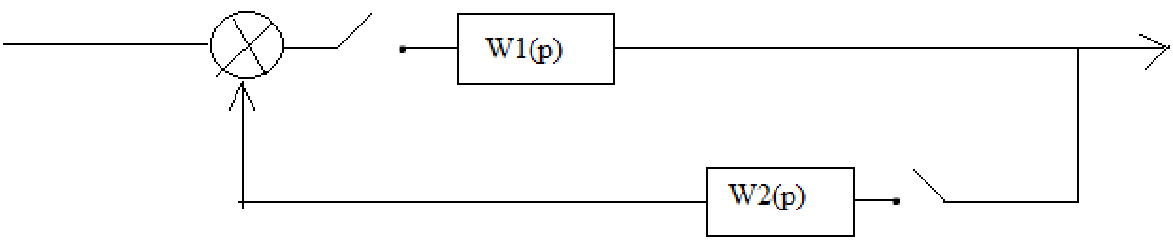
1)



2)



3)



5-amaliy mashg‘ulot

Diskret boshqarish tizimlarning analitik usulda turg‘unligini tahlil qilish

Tizimning turg‘unligini aniqlashda avvalo tahlil qilinayotgan tizimning umumiy uzatish funksiyasini aniqlash lozim bo‘ladi. Agar Tizim strukturasi uncha murakkab bo‘lmasa, u holda bu masala qiyinchiliklarsiz hal qilinishi mumkin, aks holda ushbu masalani yechishda z o‘zgarish algebrasidan va diskret tizimlarning strukturaviy sxemalarining shakl almashtirish usulidan foydalanish kerak bo‘ladi. Quyida diskret tizimlar struktura sxemalarining shakl almashtirishning ba’zi bir, ko‘p qo‘llaniladigan usullari keltirilgan.

Diskret tizim turg‘un bo‘lishligi uchun uning xarakteristik tenglamasini ildizlari radiusi 1 ga teng bo‘lgan aylana ichida yotishi zaruriy va yetarlidir. Diskret tizimning xarakteristik tenglamasi darajasi oshib borgan sari uning ildizlarini toppish murakkablashadi. Bunday hollarda turli mezonlardan foydalanamiz. Diskret tizimni xarakteristik tenglamasi transendent ko‘rinishiga egadir. Shuning uchun Gurvits mezonini diskret tizimlarga tadbiq qilinadigan quyidagi Bi chiziqli almashtirish amalga oshiriladi.

$$z \to \frac{1+\omega}{1-\omega}$$

Gurvis mezoni bo‘yicha tizimni turg‘unligini aniqlash.

5.1-misol. $5z^2 + 4z + 4 = 0$

$z \to \dfrac{1+\omega}{1-\omega};$

$5*\left(\dfrac{1+\omega}{1-\omega}\right)^2 + 4*\left(\dfrac{1+\omega}{1-\omega}\right) + 4 = 0$

$5*(1+\omega)^2 + 4*(1+\omega)*(1-\omega) + 4*(1-\omega) = 0$

$5*(1+2\omega+\omega^2) + 4*(1-\omega^2) + 4*(1-\omega) = 0$

$5+10\omega+5\omega^2 + 4-4\omega^2 + 4-4\omega = 0$

$\omega^2 + 6\omega + 13 = 0$

$a_0 = 1;\ a_1 = 6;\ a_2 = 13;$

$\Delta_2 = \begin{bmatrix} a_1 & a_3 \\ a_0 & a_2 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 1 & 13 \end{bmatrix} = 78 > 0$ tizim turg‘un.

5.2-misol. $z^3 + 2z^2 + z = 0$

$z \to \dfrac{1+\omega}{1-\omega};$

$$\left(\frac{1+\omega}{1-\omega}\right)^3+2*\left(\frac{1+\omega}{1-\omega}\right)^2+\left(\frac{1+\omega}{1-\omega}\right)=0$$

$$(1+\omega)^3+2(1+\omega)(1-\omega)^2+3(1+\omega)(1-\omega)=0$$

$$1+3\omega+3\omega^2+\omega^3+2\omega^3-2\omega^2-2\omega+2+\omega^3-\omega^2-\omega+1=4\omega^3-4=0$$

$a_0=-4;\ a_1=4;$ Yetarli va zaruriy shart bajarilmadi tizim noturg‘un.

5.3-misol. $4z^2-2z+3=0$

$$z\to\frac{1+\omega}{1-\omega};$$

$$4\left(\frac{1+\omega}{1-\omega}\right)^2-2\left(\frac{1+\omega}{1-\omega}\right)+3=0$$

$$4(1+\omega)^2-2(1+\omega)(1-\omega)+3(1-\omega)^2=0$$

$$4\left(1+2\omega+\omega^2\right)-2\left(1-\omega^2\right)+3\left(1-2\omega+\omega^2\right)=0$$

$$4+8\omega+4\omega^2-2+2\omega^2+3-6\omega+3\omega^2=0$$

$$9\omega^2+2\omega+5=0$$

$a_0=9;\ a_1=2;\ a_2=5;$

$\Delta_2=\begin{bmatrix}a_1 & a_3\\ a_0 & a_2\end{bmatrix}=\begin{bmatrix}2 & 0\\ 9 & 5\end{bmatrix}=10>0$ tizim turg‘un.

Bajarish uchun topshiriqlar

1) $7z^2-4z+3=0$
2) $5z^3+7z-4=0$
3) $5z^3+4z^2+4z=0$
4) $z^2-4z+4=0$
5) $9z^2-3z+8=0$
6) $5z^3-4z^2-z=0$
7) $5z^3+4z^2+z=0$
8) $z^3-2z^2+z=0$
9) $3z^3+4z^2+z=0$
10) $7z^2-4z+4=0$

Nazorat savollari.

1. Diskret tizim turg‘un bo‘lish sharti.

2. Gurvis mezoni nima?

3. Impulsli tizimlar turg‘unligining zaruriy va yetarli sharti

4. Impulsli tizimlarda strukturaviy shakl almashtirish usullari

6-amaliy mashg‘ulot

Diskret boshqarish tizimlarining chastotali usulda turg‘unligini tahlil qilish

Chastotaviy xarakteristikalardan foydalanib DSlarning dinamik xossalarini o‘rganish mumkin.Ulardan eng asosiysi tizimning turg‘unligini aniqlashda Mixaylov va Naykvist mezonlaridan foydalanish mumkin. Mixaylov mezoni diskret tizimlarga tatbiq qilishda uning xarakteristik tenglamalaridan foydalaniladi.

$$a_m z^m + a_{m-1} z^{m-1} + a_0 = 0$$

Bunda:

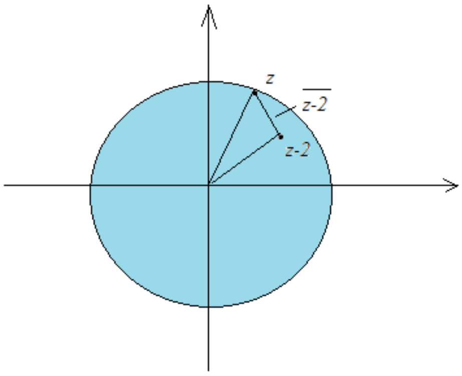
a_0, a_m lar o‘zgarmas koeffitsiyentlar bo‘lib, tizimning xossalaridan topiladi. Xarakteristik tenglamadagi o‘zgaruvchi z ni trigonometrik ko‘rinishga o‘tkazib olamiz. Ma’lumki implusli tizim turg‘un bo‘lishi uchun uning ildizlari birlik doirada yotishi shart. Shundan kelib chiqqan holda xarakteristik tenglamani quyidagicha yozishimiz mumkin:

$$z = \cos\overline{\omega} + j\sin\overline{\omega}$$

$$|z_i| < 1$$

$$a_m (z - z_1)(z - z_2)...(z - z_m) = 0$$

Quydagi shart bajarilsin:



6.1-rasm. Birlik doira

Chastota $0 \to 2\pi$ gacha o‘zgartirilganda $z - z_k$ vektorning burilish burchagi $\varphi = 2\pi$ ga teng bo‘larkan. Agarda barcha ildizlar birlik doirani ichida yotsa u holda xarakteristik tenglama vektorining burilish burchagi 2π ga teng bo‘ladi. Diskret tizim turg‘un bo‘lishi uchun chastota $\overline{\omega} = 0 \to 2\pi$ gacha o‘zgartirilganda uning xarakteristik tenglamasining burilish burchagi 2π m ga teng bo‘lishi zaruriy va yetarlidir.

Bu yerda:

m-xarakteristik tenglamaning tartibi yoki darajasi. Turg‘unlikni topishda Mixaylov mezoni qo‘llanilganda dastlab quyidagi munosabat tekshiriladi.

$$\overline{\omega} = 0$$

$$G(0) = a_m + a_{m-1} + ... + a_0$$

$\overline{\omega} = \pi$ qo‘yamiz, ya’ni 180^0

$$G(\pi) = (-1)^m a_m + (-1)^{m-1} a_{m-1} + ... + a_0$$

m ning juft yoki toqligi tekshiriladi. Agarda m juft bo‘lsa, u holda $G(0), G(\pi) > 0$ m toq bo‘lsa, $G(\pi) < 0$. Chastotaviy xarakteristikalar qurilayotganda xarakteristik tenglama 2 qismga ajratib olinadi. $G(jn)$ da dastlab quyidagi munosabat tekshiriladi.

$$G\overline{\omega} = x\overline{\omega} + jy\overline{\omega}$$

$$x\overline{\omega} = a_m \sin(m\overline{\omega}) + a_{m-1}\sin((m-1)\overline{\omega}) + ...a_0.$$

$$y\overline{\omega} = a_m \sin(m\overline{\omega}) + a_{m-1}\sin((m-1)\overline{\omega}) + ...a_1 \sin\overline{\omega}.$$

Mixaylov mezoni bo‘yicha turg‘unlikni aniqlash.

6.1-misol. $5z^2 + 4z + 4 = 0$

$m = 2;$

$G(0) = 5 + 4 + 4 = 13 > 0;$

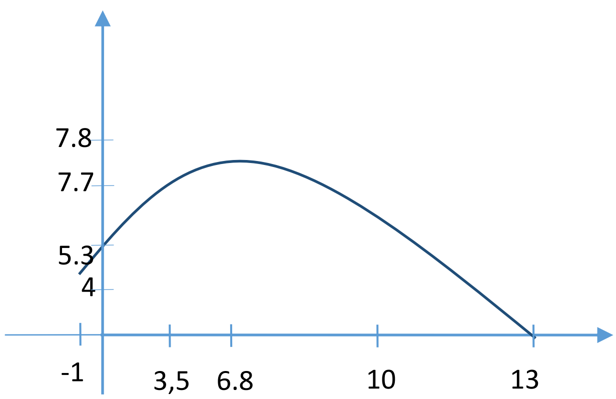
$G(\pi) = 5*(-1)^2 + 4*(-1)^1 + 4*(-1)^0 = 5 > 0;$

$G(\overline{\omega}) = x(\overline{\omega}) + jy(\overline{\omega});$

$x(\overline{\omega}) = 5\cos(2\overline{\omega}) + 4\cos(\overline{\omega}) + 8;$

$y(\overline{\omega}) = 5\sin(2\overline{\omega}) + 4\sin(\overline{\omega});$

Ω	0°	30°	45°	60°	90°
x(ω)	13	10	6.8	3.5	-1
y(ω)	0	5.3	7.8	7.7	4



Tizim turg‘un.

6.2-rasm. Mixaylov godografi

6.2-misol. $10z^5 + 5z^4 + 3z^3 + 2z^2 + z + 8 = 0$

$m = 5;$

$$G(0) = 10 + 4 + 3 + 2 + 1 + 8 = 28 > 0;$$

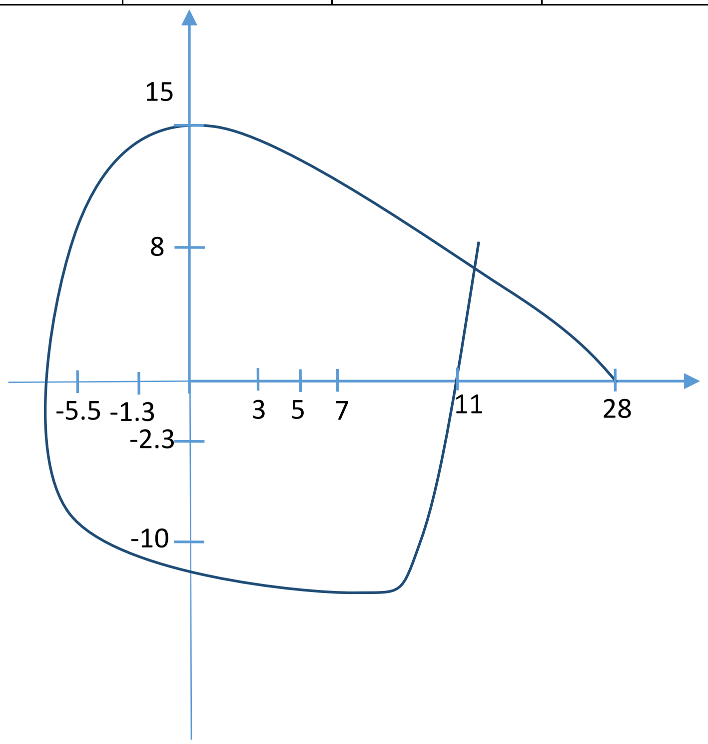
$$G(\pi) = 10*(-1)^5 + 4*(-1)^4 + 3*(-1)^3 + 2*(-1)^2 + 1*(-1)^1 + 8 = 0;$$

$$G(\overline{\omega}) = x(\overline{\omega}) + jy(\overline{\omega});$$

$$x(\overline{\omega}) = 10\cos(5\overline{\omega}) + 5\cos(4\overline{\omega}) + 3\cos(3\overline{\omega}) + 2\cos(2\overline{\omega}) + \cos(\overline{\omega}) + 8;$$

$$y(\overline{\omega}) = 10\sin(5\overline{\omega}) + 5\sin(4\overline{\omega}) + 3\sin(3\overline{\omega}) + 2\sin(2\overline{\omega}) + \sin(\overline{\omega});$$

Ω	0^0	30^0	45^0	60^0	90^0
x(ω)	28	-1.3	-5.5	7	11
y(ω)	0	15	-2.3	-10	8



n=4 bo‘lsa, tizim turg‘un

Bajarish uchun topshiriqlar

1) $7z^2 - 4z + 3 = 0$; $7z^5 + z^4 + z^3 + z + 5 = 0$;

2) $5z^3+7z-4=0$; $2z^5+3z^4+2z^2+4z+7=0$;

3) $5z^3+4z^2+4z=0$; $10z^5+3z^3+2z^2+z+8=0$;

4) $z^2-4z+4=0$; $6z^5+2z^4+2z^3+z+3=0$;

5) $9z^2-3z+8=0$; $z^5+3z^4+3z^3+4z^2+z+7=0$;

6) $5z^3-4z^2-z=0$; $8z^5+2z^4+3z^3+2z^2+5z=0$;

7) $5z^3+4z^2+z=0$; $5z^4+3z^3+2z^2+z+2=0$;

8) $z^3-2z^2+z=0$; $10z^5-6z^4+3z^3+2z^2+z-5=0$;

9) $3z^3+4z^2+z=0$; $10z^5-5z^4+3z^3+2z^2+z-8=0$;

10) $7z^2-4z+4=0$; $10z^5+5z^4-3z^3-2z^2+z+8=0$;

Nazorat savollari

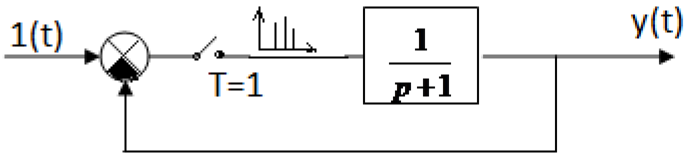
1. Mixaylov mezoni bo‘yicha turg‘unlik qanday aniqlanadi?
2. DSlarning dinamik xossalarini qanday o‘rganish mumkin?
3. Mixaylov mezoni qo‘llanilganda dastlab qanday munosabat tekshiriladi?

7-amaliy mashg‘ulot

Diskret tizimlarda o‘tkinchi jarayonni hisoblash

Boshqarish tizimning sifat ko‘rsatkichlarini yaxshilash uchun uning o‘tkinchi jarayonini qurish kerak bo‘ladi. Bunda uning tarkibida kechiktiruvchi zveno bor yo‘qligiga e’tibor beriladi. O‘tkinchi jarayonni qurishdagi asosiy matematik apparat “*z*” almashtirishdir. Uning yordamida jarayonni qurish masalasini ko‘rib chiqamiz.

1. Shakllantiruvchi zveno yo‘q tizim berilgan bo‘lsin:



7.1-rasm. Berk tizimning struktura sxemasi

Jarayonni qurish algoritmi quyidagidan iborat:

1) Ochiq tizimning uzatish funksiyasi topiladi.

$$W_o(Z) = Z\left\{\frac{1}{p+1}\right\} = \frac{Z}{Z-e^{-1}} = \frac{Z}{Z-0.37}$$

2) Berk tizimning uzatish funksiyasi topiladi.

$$W_b(Z) = \frac{W_o(Z)}{1+W_o(Z)} = \frac{\dfrac{Z}{Z-0.37}}{1+\dfrac{Z}{Z-0.37}} = \frac{Z}{2Z-0.37};$$

3) Kirish signalida Z almashtirishi topiladi.

$$X(t) = Z\{1(t)\} = \frac{Z}{Z-1}$$

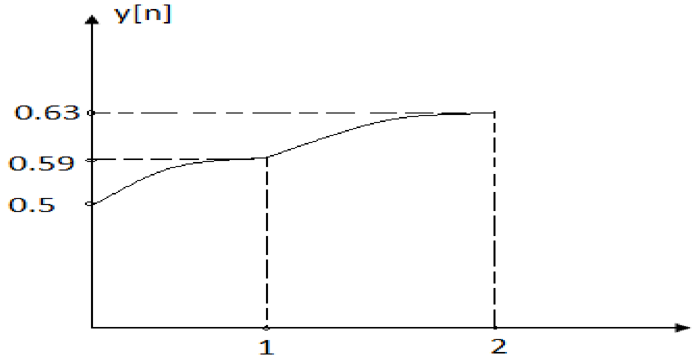
4) Chiqish signalining Z almashtirishini topamiz.

$$y(Z) = W_b(Z) * x(Z) = \frac{Z}{2Z-0.37} * \frac{Z}{Z-1} = \frac{Z^2}{2Z^2-2.37Z+0.37};$$

5) Suratdagi polinomni maxrajdagi polinomga bo‘lib, Z-Loran qatoriga yoyiladi.

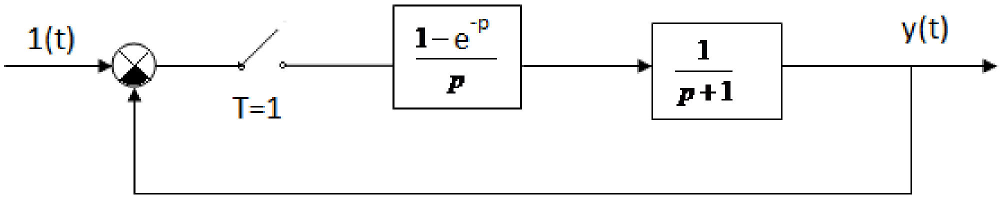
Z^2	$2Z^2-2.37Z+0.37$
$Z^2-1.18Z+0.18$	$0.5+0.59Z^{-1}+0.63Z^{-2}$
$1.18Z-0.18$	
$1.18Z-1.39+0.22Z^{-1}$	
$1.26-0.22Z^{-1}$	
$1.26-1.5Z^{-1}+0,23$	
$1.28Z^{-1}-0,23$	

6) Hosil bo‘lgan qatordan foydalanib o‘tkinchi jarayon grafigi quriladi. Bunda absissa o‘qiga takt soni ordinata o‘qiga chiqish signalining diskret qiymatlari qo‘yiladi



7.2-rasm. O‘tish jarayoni grafigi.

2. Shakllantiruvchi zveno mavjud holatda



7.3-rasm. Shakllantiruvchi zveno diskret signal struktura sxemasi

Hisoblash algoritmi quyidagidan iborat:

1) Ochiq tizimning uzatish funksiyasi topiladi.

$$W_o(Z) = Z\left\{\frac{1-e^{-p}}{p} * \frac{1}{p+1}\right\} = Z\left\{(1-e^{-p}) * \frac{1}{p(p+1)}\right\} = Z\{1-e^{-p}\} * Z\left\{\frac{1}{p(p+1)}\right\} =$$

$$\frac{Z-1}{Z} * \frac{Z(1-e^{-1})}{(Z-1)(Z-e^{-1})} = \frac{1-0.37}{Z-0.37} = \frac{0.63}{Z-0.37};$$

2) Berk tizimning uzatish funksiyasini topish

$$W_b(Z) = \frac{W_o(Z)}{1+W_o(Z)} = \frac{\frac{0.63}{Z-0.37}}{1+\frac{0.63}{Z-0.37}} = \frac{0.63}{Z+0.26};$$

3) Kirish signalining Z tasvirini topamiz.

$x(Z) = Z\{1(t)\} = \frac{Z}{Z-1}$ birlik pog‘onali signal bo‘lganligi uchun

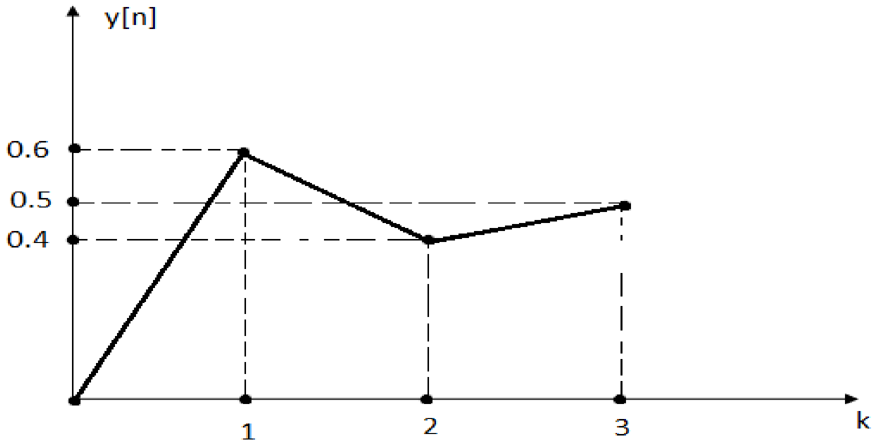
4) Chiqish signalining Z tasvirini topamiz.

$$y(Z)=W_b(Z)*x(Z)=\frac{0.63}{Z+0.26}*\frac{Z}{Z-1}=\frac{0.63*Z}{Z^2-0.74Z-0,26}$$

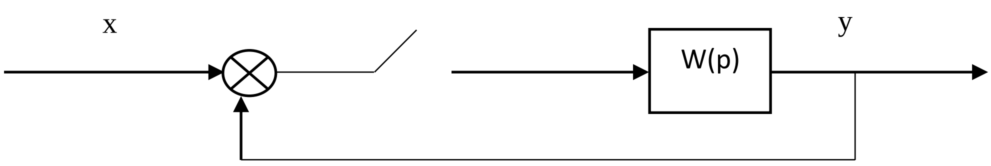
5) Polinomlarni bir biriga bo‘lamiz

$0.63Z$	$Z^2-0.74Z-026$
$0.63Z-0.46-0.88Z^{-1}$	$0.63Z^{-1}+0.43Z^{-2}+0.47Z^{-3}$
$0.43+0.18Z^{-1}$	
$0.43-0.79Z^{-1}-0.13Z^{-2}$	
$0.47Z^{-1}+0.13Z^{-2}$	

6) O‘tkinchi jarayon grafigini quramiz.



7.4-rasm. O‘tkinchi jarayon grafigi



$$w(p)=\frac{10}{2p+1};$$

1) Ochiq tizimning diskret uzatish funksiyasi topiladi.

$$W_0(z) = z\left\{\frac{10}{2p+1}\right\} = \frac{10z}{2z-0,37};$$

2) Berk tizimning diskret uzatish funksiyasi topiladi.

$$W_0(z) = z\left\{\frac{W_0}{1+W_0(z)}\right\} = \frac{10z}{\cancel{2z-0,37}} \cdot \frac{\cancel{2z-0,37}}{12z-0,37} = \frac{10z}{12z-0,37};$$

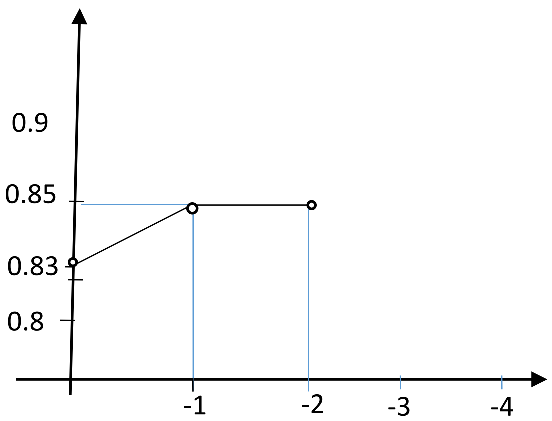
3)Kirish signalining z tasviri topiladi.

$$X(z) = z\{1(t)\} = \frac{z}{z-1};$$

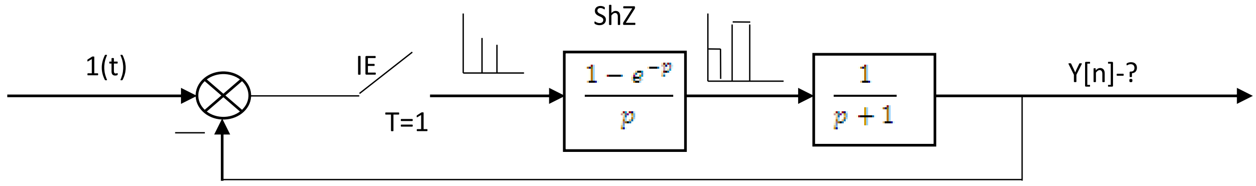
4)Chiqish signalining z tasviri topiladi.

$$Y(z) = W_b(z) \cdot X(z) = \frac{10z}{12z-0,37} \cdot \frac{z}{z-1} = \frac{10z^2}{12z^2-12,37z+0,37};$$

$$\begin{array}{l|l} 10Z & 12z^2-12,37z+0,37 \\ \cline{2-2} 10Z-10.26+0.3Z^{-1} & 0.83Z^{-1}+0.85Z^{-2}+0.85Z^{-3} \\ \cline{1-1} \quad 10.26-0.3Z^{-1} & \\ \quad 10.26\text{-}10.51Z^{-1}-0.31Z^{-2} & \\ \cline{1-1} \qquad 10.21Z^{-1}+0.31Z^{-2} & \end{array}$$



Shakllantiruvchi zveno qo‘shib tizimning o‘tkinchi funksiyasini qurish.



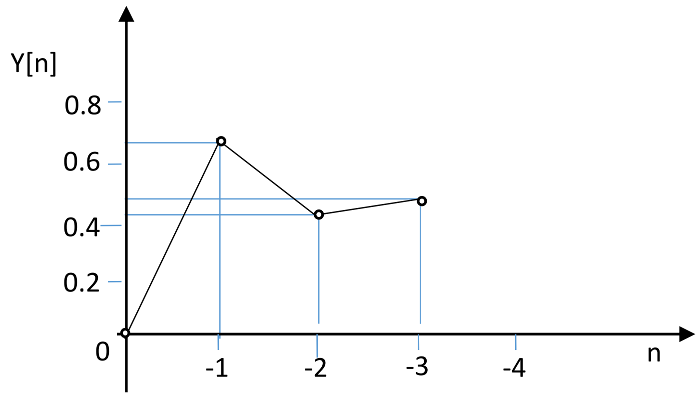
1) $W_0(z) = z\left\{\frac{1-e^{-p}}{p} \cdot \frac{1}{p+1}\right\} = z\{1-e^{-p}\} \cdot z\left\{\frac{1}{p(p+1)}\right\} = \frac{z-1}{z} \cdot \frac{(1-e^{-1})z}{(z-1)(z-e^{-1})} = \frac{0,63}{z-0,37};$

2) $W_b(z) = \frac{0,63}{z-0,37} / 1 + \frac{0,63}{z-0,37} = \frac{0,63}{2z+0,26};$

3) $X(z) = z\{1(t)\} = \frac{z}{z-1};$

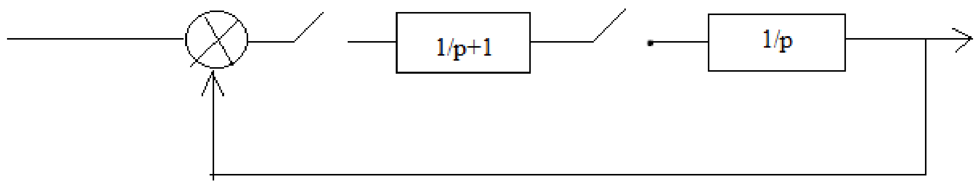
4) $Y(z) = W_b(z) \cdot X(z) = \frac{0,63}{z+0,26} \cdot \frac{z}{z-1} = \frac{0,63z}{z^2-0,74z-0,26};$

$$
\begin{array}{l|l}
0,63Z & z^2-0,74z-0,26 \\
\hline
0,63Z-0,46-0,16Z^{-1} & 0,63Z^{-1}+0,46Z^{-2}+0,5Z^{-3} \\
\hline
\quad 0,46+0.16Z^{-1} & \\
\quad 0,46\text{-}0,34Z^{-1}-0,11Z^{-2} & \\
\hline
\qquad 0,5Z^{-1}+0,11Z^{-2} & \\
\qquad 10.21Z^{-1}+0.31Z^{-2} &
\end{array}
$$

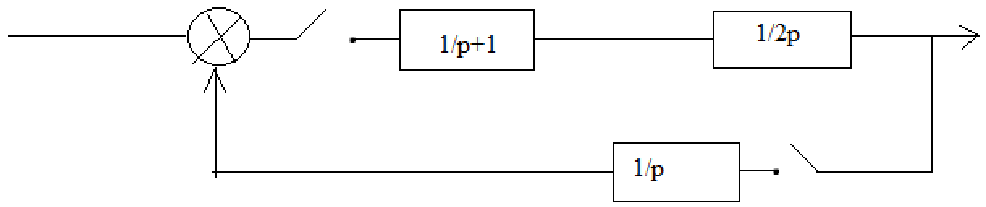


Bajarish uchun topshiriqlar

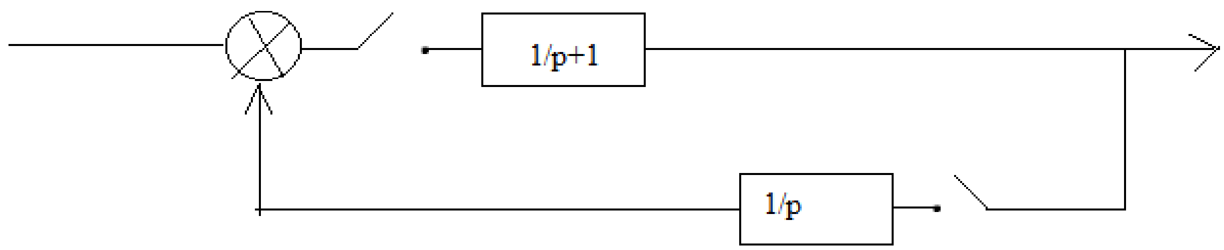
1)



2)



3)



1. Impulsli tizimning chastotaviy xarakteristikasi godografini quring va Naykvist mezonidan foydalanib yopiq tizimning chegaraviy kuchaytirish koeffitsiyentini aniqlang 2.Yopiq tizimning o‘tish jarayoni chizig‘ini quring. 3. Tizimdagi o‘tish jarayonini impulsli tizimning bitta vaqtda tugaydigan optimal kuchaytirish koeffitsiyentini toping.

Nazorat savollari

1. Impulsli tizim deb qanday tizimga aytiladi ? Misollar keltiring.
2. Qanday qilib impulsli tizim xisoblash shakliga keltiriladi?
3. Impulsli tizimning tung‘unligini aniqlash usullari. Kuchaytirish koeffisentlari , chegaraviy va optimal kuchchaytirish koeffitsiyentlari?
4. Impulsli tizimda o‘tkinchi jarayonni ko‘rish usullari?

8-amaliy mashg‘ulot

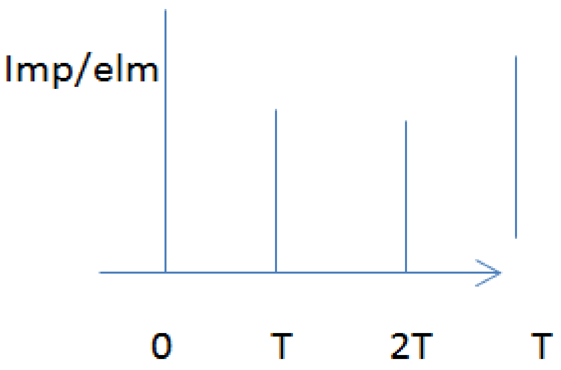
Strukturaviy metod yordamida chiziqli diskret tizimlardagi o‘tkinchi jarayonlarni hisoblash

1.DSlarning strukturaviy modeli.

2.Fiksatorsiz DSlardagi o‘tkinchi jarayonlarni hisoblash.

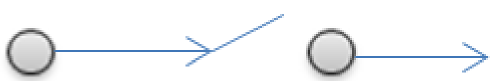
3.Fiksatorli Diskret tizimlardagi o‘tkinchi jarayonni hisoblash.

Diskret tizimlardagi o‘tkinchi jarayonni hisoblashda strukturaviy usullar keng qo‘llaniladi. Bu usullar graflar nazariyasiga asoslangan bo‘lib, implusli elementning ishlash rejimini hamda obyektning boshlang`ich holati 0 bo‘lmagandagi xususiyatini hisobga olish imkonini beradi. Ideal implusli element ishlash rejimining strukturaviy modeli quyidagicha bo‘ladi.



Implusli elementning holati 3 ta bo‘ladi.

1.Kalit ulangunga qadar.

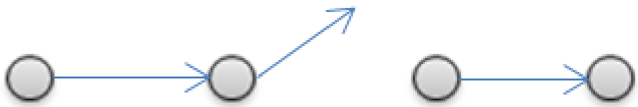


$$y(jt),\ j=0,1,2$$

2.Kalit ulangan vaqtda.

$$y\left(jt^{+}\right)$$

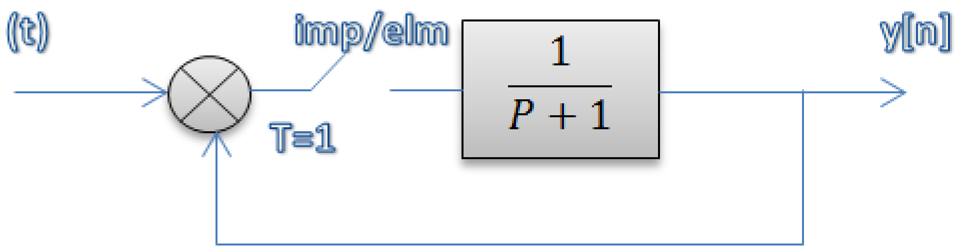
3.Kalit uzilgandan so‘ng.

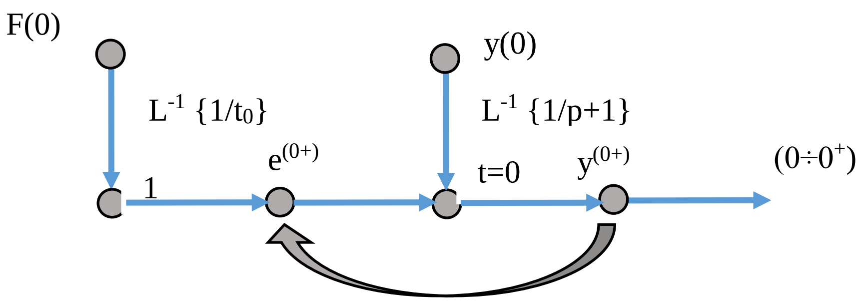


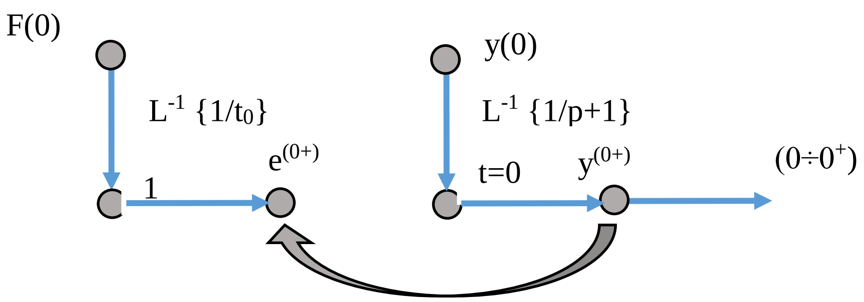
$$y=\left[\left(j+1\right)T\right]$$

Strukturaviy usul yordamida o‘tkinchi jarayon hisoblanganda DSning strukturaviy sxemasidan hamda implusli elementning ishlash rejimidan foydalangan holda tizimning strukturaviy modeli quriladi.

Fiksatorsiz diskret tizim berilgan bo‘lsin:



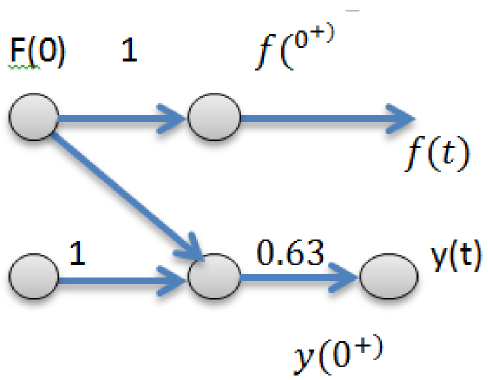




$$L^{-1}\left\{\frac{1}{P_{t=0}}\right\}=1(t)_{t=0}=1$$

$$L^{-1}\left\{\frac{1}{p+1}\right\}=e^{-t}\big/t=e^{-0}=1=i^{-1}=\frac{0}{63}.$$

O‘zgaruvchilarning o‘zaro bog‘liqligini topish uchun ikkiyoqlama grafga o‘tiladi.



$$W_{f_1 y}=\frac{P}{1-L}$$

$$\frac{1}{1+1}=0.5$$

$y(0)=0$ boshlang‘ich shart

$$f^{(0^+)}=1*f(0)=1$$

$$y^{0^+}=1*0,5+1\text{y}(0)=0,5$$

$$f(T)=1*f^{0^+}=1*1=1$$

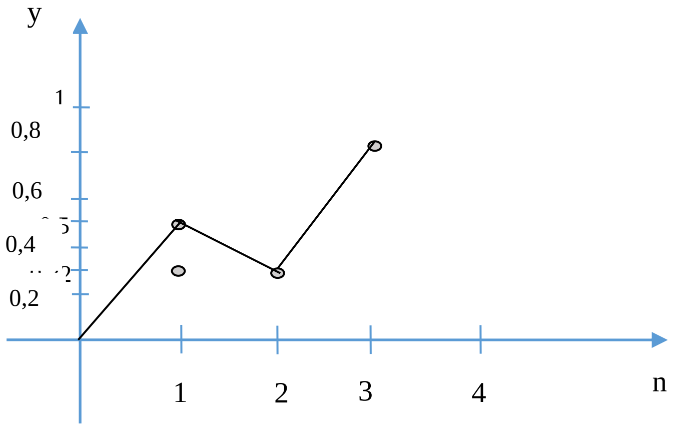
$$y(T)=0,63*y\left(0^+\right)=0,63*0,5=0,32$$

$$f\left(T^{+}\right)=1*f\left(T\right)=1*1=1$$

$$y\left(T^{+}\right)=0,5*f\left(T\right)+1y\left(T\right)=0,5*1+1*0,32=0,82$$

$$f\left(2T\right)=1*f\left(T^{+}\right)=1$$

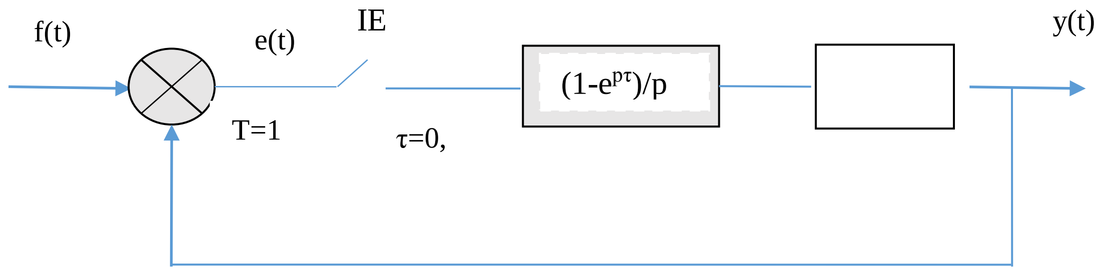
$$y\left(2T\right)=1*0,5+1y\left(T^{+}\right)=0,5+1*0,82=1,32$$

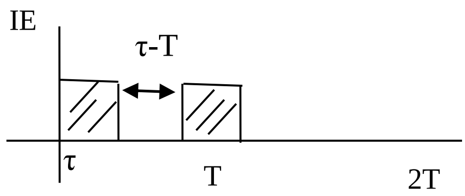


y_0, $y(0^{+})$, $y(T)$, $y(T^{+})$, $y(2T)$, $y(2T^{+})$

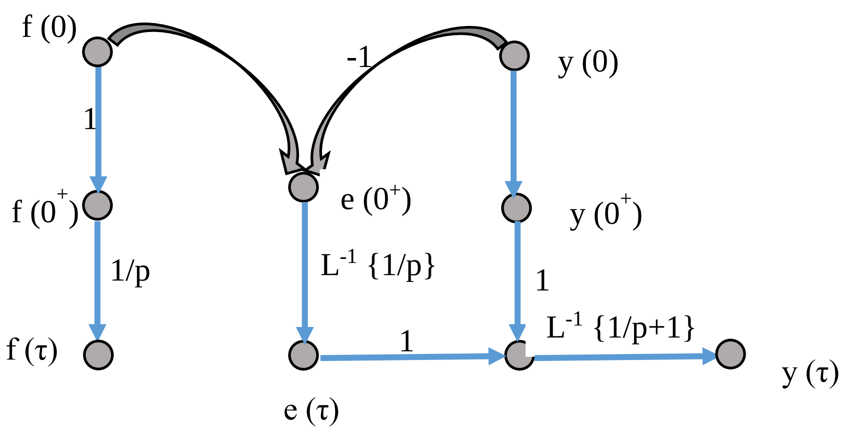
Fiksatorli diskret tizimlarni hisoblash

Diskret tizim berilgan bo‘lsin:



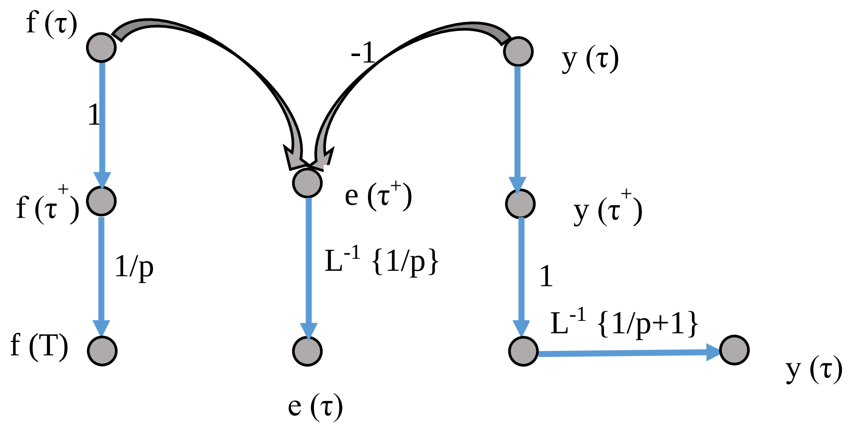


Impulsli signalning ishlashi:



$0^+ \div \tau$

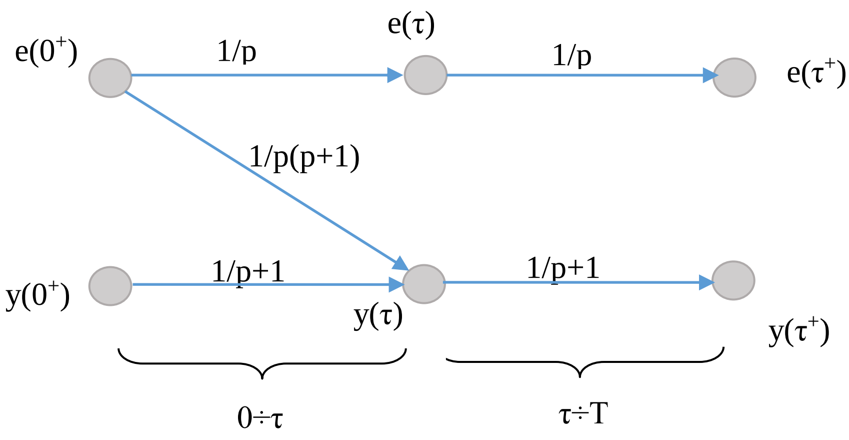
$$e(0^+) = f(0) - y(0)$$



τ÷T

1. O‘zgaruvchilar orasidagi munosabat 2 yoqlamali graf shakliga keltirib olinadi;

Bunda kalit ulangan momentlar τ vaqtga davr qiymat hisobga olinadi.



Har bir o‘zgaruvchi qiymatining matematik ifodasini yozamiz:

$$y(\tau) = L^{-1}\left\{\frac{1}{p+1}\right\} y(0^+) + L^{-1}\left\{\frac{1}{p(p+1)}\right\} e(0^+)$$

$$y(\tau) = e^{-\tau} y(0^+) + (1 - e^{-\tau}) e(0^+) = e^{-0,5} y(0^+) + (1 - e^{-\tau}) e(0^+) =$$
$$0,6 y(0^+) + 0,4 e(0^+);$$

Endi τ÷T hisoblaymiz.

$$y(\mathrm{T}) = L^{-}\left\{\frac{1}{p+1}\right\} \cdot y(\tau) = e^{-(\mathrm{T}-\tau)} y(\tau) = e^{-0,5} y(\tau) = 0,6 y(\tau);$$

$$y(0) = 0$$

$$f(0) = 1$$

$$e(0^{+}) = f(0) - y(0) = 1 - 0 = 1$$

$$y(\tau) = 0,6 \cdot 0 + 0,4 \cdot 1 = 0,4$$

$$y(\mathrm{T}) = 0,6 \cdot 0,4 = 0,24$$

$$e(\mathrm{T}^{+}) = 1 - 0,24 = 0,76$$

Har doim tekshirish kerak

$$y(\mathrm{T}+\tau) = 0,6 \cdot 0,24 + 0,4 \cdot 0.76 = 0,44$$

$$y(2\mathrm{T}) = 0.6 \cdot 0,44 = 0,26.$$

Nazorat savollari

1. Implusli elementning holati nechta bo‘ladi?
2. O‘zgaruvchilarning o‘zaro bog‘liqligini topish uchun qanaqa grafga o‘tiladi?

9-amaliy mashg‘ulot

Diskret tizimlarning holat tenglamasi

1.Holat tenglamasining ko‘rinishlari.

2.Modellashtirish tizimlarini qurish usullari.

3.Bevosita rejalashtirish usuli.

Diskret tizimlarni tadqiq qilishda holat parametrlarining fazasi usulida qo‘llaniladi.

Diskret tizimlarda holat tenglamasi shu turishda impulsli elementning ulash rejimi hisobiga olinishi shart, bunda kalitning ulanishi hamda ulanishlar orasidagi jarayon uchun alohida holat tenglamasi tuzib olinadi.

Kalitlar ulangan vaqtda holat tenglamasi quyidagicha yoziladi:

$$V(2T) = BV(jT)$$

V-holat o‘zgaruvchilarining qiymati; B-kalitlar matritsasi;

Kalitlar orasidagi ulanishlar orasidagi tenglama:

$$(j-1)T < \tau < jT$$

Holat tenglamasi oddiy differensial tenglama bilan yoziladi:

$$\frac{dv(t)}{d(t)} = AV(t)$$

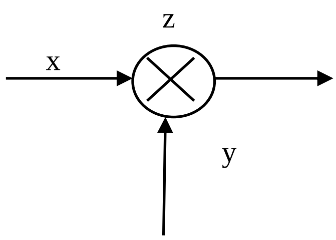
Holat tenglamasining modellashtirish sxemasini qurishning 3 xil usuli mavjud:

1.Bevosita rivojlantirish;

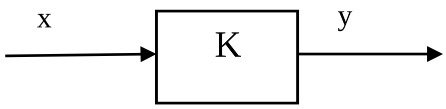
2.Parallel rivojlantirish;

3.Ketma-ket rejalashtirish;

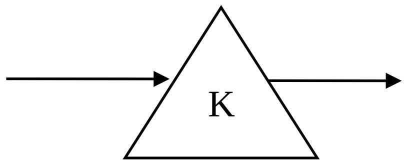
Modellashtirish sxemasini qurishda 3 xil element ishtirok etadi:



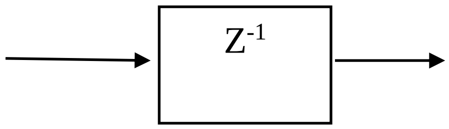
1.Summatorlar $z = x + y$;



2.Kuchaytiruvchi element x y $y = K - x$



3.Kechikuvchi zveno x y



Faraz qilaylik, diskret tizimning dinamikasi quyidagi ko‘rinishda bo‘lsin:

$$y(K+n)+a_1\, y(K+n-1)+a_2\, y(K+n-2)+...+a_{n-1}\, y(K+1)+a_n$$
$$y(K)=bn(K) \quad (1)$$

y(K)=chiqish signali;

u(K)=kirish signali;

K=taktlar soni;

n=ayirmali tenglamaning darajasi;

Umumiy hollarda holat tenglamasini quyidagicha yozish mumkin:

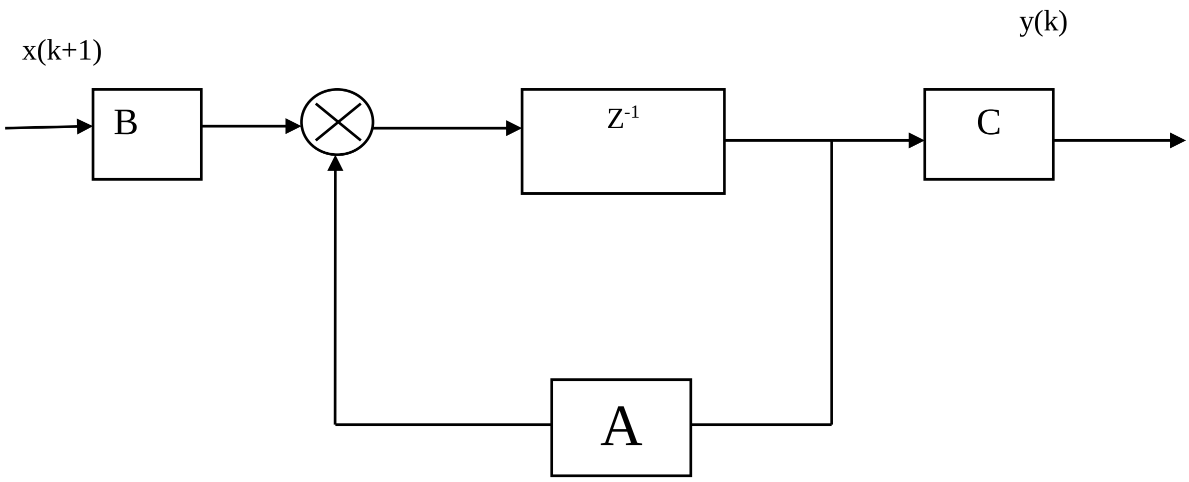
$$x(k+1)=AX(K)+Bn(K)$$

$$y(k+1)=CX(K)$$

Bunda modellashtirish sxemasini tasvirlab olishimiz mumkin:

$$x(k+1)=z*x(k)$$

$$x(k)=Z^{-1}\ x(k+1)$$



Chiziqli diskret tizimlarni holatlar makonida matematik ifodalash

Nazorat savollari

1. Holat o‘zgaruvchilari tenglamalarini qanday yozish mumkin?
2. Umumiy hollarda holat tenglamasi qanday ifodalanadi?
3. Holat tenglamasini modellashtirish sxemasini nechchi xil usulda qurish mumkin?

10-amaliy mashg‘ulot

Diskret tizimlarning holat tenglamalari va modellash sxemalari

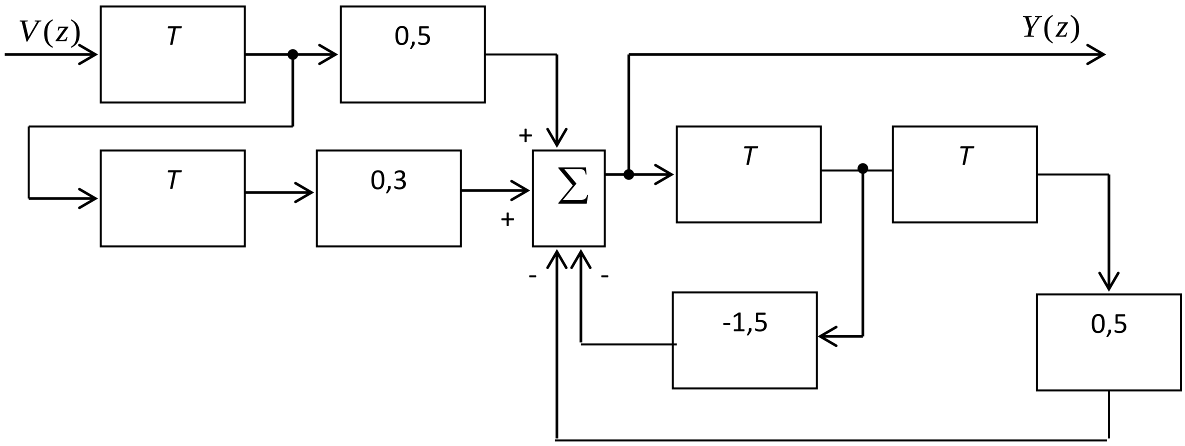
Diskret tizim quyidagi ayirmali tenglama bilan ifodalangan bo‘lsin:

$$y(k) = 0,5\upsilon(k-1) + 0,3\upsilon(k-2) + 1,5y(k-1) - 0,5y(k-2), \qquad (10.1)$$

bunda $\upsilon(k)-$ tizimning kirish signali; $y(k)-$tizimning chiqish koordinatasi. Bu tenglamani $z-$o‘zgartirib, diskret uzatish funksiyasini hosil qilamiz:

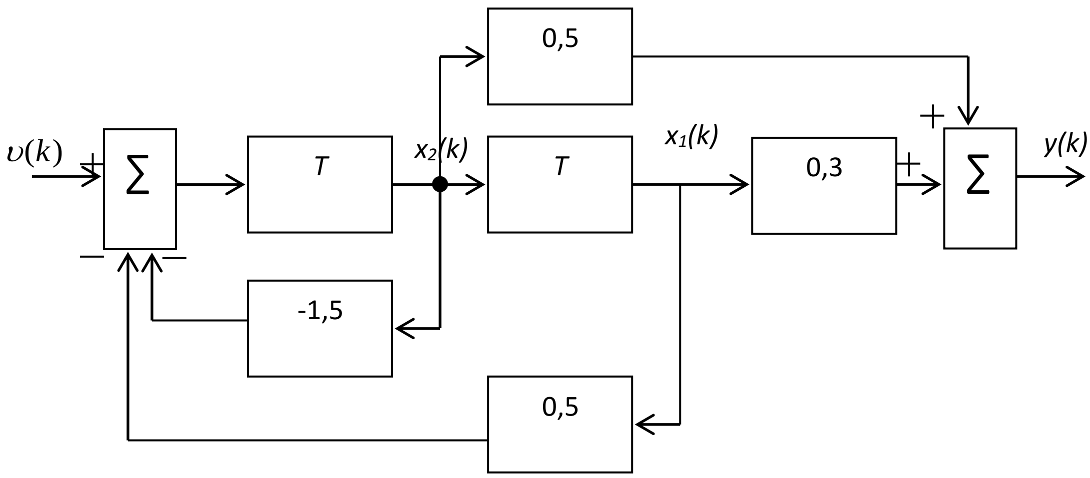
$$K(z) = \frac{Y(z)}{V(z)} = \frac{0,5z^{-1} + 0,3z^{-2}}{1 - 1,5z^{-1} + 0,5z^{-2}} = \frac{0,5z + 0,3}{z^2 - 1,5z + 0,5}. \qquad (10.2)$$

Keltirilgan ikki formula (10.1) va (10.2)ga modellashning turli sxemalari mos kelishi mumkin. Shulardan biri 10.1 – rasmda berilgan.



10.1– rasm. Diskret uzatish funksiyasini modellash sxemasi.

Modelni (10.1) tenglamada ifodalangan tizimning holat o‘zgaruvchilarida tasvirlash (aks ettirish) uchun holat o‘zgaruvchisi sifatida tutib qoluvchi har bir elementning chiqish signali – *T*ni qabul qilamiz. Biz ko‘rayotgan holatda tizim ikkinchi tartibli, shuning uchun holat o‘zgaruvchilari ikkita: $x_1(k)$ va $x_2(k)$. Shunda modellash sxemasi quyidagi ko‘rinishda bo‘ladi (10.2 – rasm).



10.2– rasm. Ayirmali tenglamani holat o‘zgaruvchilarida modellash sxemasi.

Bu sxemaga muvofiq va tutib qoluvchi elementlarning kirish joyi $x_1(k+1)$ va $x_2(k+1)$ ko‘rinishida ifodalanishini e’tiborga olib, holat tenglamalarini quyidagicha yozish mumkin:

$$\begin{cases} x_1(k+1), \\ x_2(k+1) = -0,5x_1(k) + 1,5x_2(k) + \upsilon(k), \\ y(k) = 0,3x_1(k) + 0,5x_2(k). \end{cases} \tag{10.3}$$

Vektor – matritsali ko‘rinishda hosil qilamiz:

$$\begin{cases} \begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -0,5 & 1,5 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \upsilon(k), \\ y(k) = \begin{bmatrix} 0,3 & 0,5 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}. \end{cases} \tag{10.4}$$

Uzil – kesil ixcham ko‘rinishda:

$$\begin{cases} X(k+1) = AX(k) + B\upsilon(k), \\ y(k) = CX(k). \end{cases} \tag{10.5}$$

Bu mulohazalarni eng umumiy holat uchun yoyib, holat o‘zgaruvchilari tenglamalarini yozamiz:

$$\begin{cases} X(k+1) = AX(k) + BV(k), \\ Y(k) = CX(k) + DV(k), \end{cases} \tag{10.6}$$

bunda A – asosiy, V – kirish, S – chiqish, D – bog‘lanish matritsalari bo‘lib, ularning o‘lchami, uzluksiz tizimlardagi kabi, tegishli ravishda, $n\times n, n\times m, p\times n, p\times m.$ real tizimlarda bog‘lanish matritsasi, odatda, nolga teng, shuning uchun uni hisobga olmaymiz.

Holat tenglamalarini yechish

(10.6) tizimning birinchi holat tenglamasi matritsasini ko‘rib chiqamiz:

$$X(k+1) = AX(k) + BV(k). \tag{10.7}$$

Uni iteratsiya usuli (2.4 bo‘limga q) bilan ham, z – o‘zgartirish usuli bilan ham yechish mumkin.

Birinchi usul bilan yechamiz. Bunda $k-$ning hamma qiymatlari uchun $X(0)$ va $V(0)$ ni bilishi kerak:

$$k=0: \quad X(1)=AX(0)+BV(0);$$
$$k=1: \quad X(2)=AX(1)+BV(1)=A[AX(0)+BV(0)]+BV(1)=A^2X(0)+ABV(0)+BV(1);$$
$$k=2: \quad X(3)=AX(2)+BV(2)=A[A^2X(0)+ABV(0)+BV(1)]+BV(2)=$$
$$=A^3X(0)+A^2BV(0)+ABV(1)+BV(2);$$
$$\vdots$$
$$k=n-1: \quad X(n)=A^nX(0)+A^{n-1}BV(0)+A^{n-2}BV(1)+\ldots+ABV(n-2)+BV(n-1).$$

(10.7) tenglamaning yechimi, umumiy ko‘rinishda:

Ikkinchi usul bilan yechamiz. Buning uchun (10.7) tenglamani yoyib yozamiz:

$$\begin{cases} x_1(k+1)=a_{11}x_1(k)+\ldots+a_{1n}x_n(k)+b_{11}\upsilon_1(k)+\ldots+b_{1m}\upsilon_m(k), \\ x_n(k+1)=a_{n1}x_1(k)+\ldots+a_{nn}x_n(k)+b_{n1}\upsilon_1(k)+\ldots+b_{nm}\upsilon_m(k). \end{cases}$$

Bu tenglamalarga $z-$o‘zgarishni qo‘llaymiz:

$$\begin{cases} z[X_1(z)-x_1(0)]=a_{11}X_1(z)+\ldots+a_{1n}X_n(z)+b_{11}V_1(z)+\ldots+b_{1m}V_m(z), \\ z[X_n(z)-x_n(0)]=a_{n1}X_1(z)+\ldots+a_{nn}X_n(z)+b_{n1}V_1(z)+\ldots+b_{nm}V_m(z). \end{cases}$$

Keyingi tenglamalar vektor – matritsali ko‘rinishda:

$$z[X(z)-X(0)]=AX(z)+BV(z),$$

bundan kelib chiqadi:

$$X(z)=z\left[zE-A\right]^{-1}X(0)+\left[zE-A\right]^{-1}BV(z), \qquad (10.9)$$

bunda E– “bir”li diagonal matritsa $diag[11\cdots1]$. (10.9)ni teskari $z-$o‘zgartirib, quyidagi ko‘rinishli yechimni hosil qilamiz:

$$X(n)=\Phi(n)X(0)+\sum_{k=0}^{n-1}\Phi(n-1-k)BV(k). \qquad (10.10)$$

(10.10) va (10.8)ni bir – biri bilan taqqoslashdan kelib chiqadi:

$$\Phi(n) = Z^{-1}\left\{z\left[zE - A\right]^{-1}\right\} = A^n,$$

$$\Phi(n-1-k) = Z^{-1}\left\{\left[zE - A\right]^{-1}\right\} = A^{n-1-k}. \qquad (10.11)$$

Bittadan kirish va chiqish joyi bo‘lgan diskret tizimni ko‘rib chiqib, tizimning uzatish funksiyasini matritsalar bo‘yicha hosil qilish mumkin.

Agar dastlabki shartlar nol bo‘lsa, (ya’ni $X(0)=0$), (10.9)dan kelib chiqadi:

$$X(z) = \left[zE - A\right]^{-1} BV(z).$$

Bu ifodani ikkinchi tenglama (10.5)ning z o‘zgarishiga qo‘yib, hosil qilamiz:

$$Y(z) = CX(z) = C\left[zE - A\right]^{-1} BV(z),$$

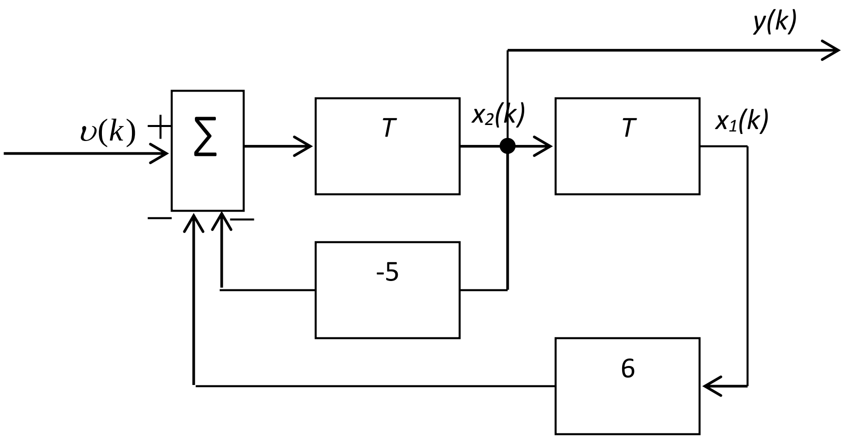
bundan:

$$K(z) = \frac{Y(z)}{V(z)} = C\left[zE - A\right]^{-1} B. \qquad (10.12)$$

10.1-misol. Diskret tizimning uzatish funksiyasi quyidagicha bo‘lsin:

$$K(z) = \frac{Y(z)}{V(z)} = \frac{z}{z^2 - 5z + 6}.$$

Modelni holatlar makonida tasvirlaymiz (10.3 – rasm) .



10.3– rasm. Modelning sxemasi

Holat tenglamalarining yoyilgan holatda ko‘rinishi:

$$\begin{cases} x_1(k+1) = x_2(k), \\ x_2(k+1) = -6x_1(k) + 5x_2(k) + \upsilon(k), \\ y(k) = x_2(k); \end{cases}$$

Vektor – matritsali ko‘rinishi:

$$\begin{cases} X(k+1) = \begin{bmatrix} 0 & 1 \\ -6 & 5 \end{bmatrix} X(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \upsilon(k), \\ y(k) = \begin{bmatrix} 0 & 1 \end{bmatrix} X(k). \end{cases}$$

Tenglamani iteratsiya usuli bilan yechamiz. Buning uchun $k = 0,1,2,\ldots;$ va $y(0)$ da $X(0); \upsilon(k) = 1$ deb faraz qilamiz.

$$X(1) = AX(0) + B\upsilon(0) = \begin{bmatrix} 0 & 1 \\ -6 & 5 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} 1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad y(1) = \begin{bmatrix} 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = 1;$$

$$X(2) = AX(1) + B\upsilon(1) = \begin{bmatrix} 0 & 1 \\ -6 & 5 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} 1 = \begin{bmatrix} 1 \\ 6 \end{bmatrix}, \quad y(2) = \begin{bmatrix} 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 6 \end{bmatrix} = 6;$$

$$X(3) = AX(2) + B\upsilon(2) = \begin{bmatrix} 0 & 1 \\ -6 & 5 \end{bmatrix}\begin{bmatrix} 1 \\ 6 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} 1 = \begin{bmatrix} 6 \\ 25 \end{bmatrix}, \quad y(3) = \begin{bmatrix} 0 & 1 \end{bmatrix}\begin{bmatrix} 6 \\ 25 \end{bmatrix} = 25;$$

Tenglamalarni z o‘zgartirish yordamida yechamiz. $[zE - A] = \begin{bmatrix} z & -1 \\ 6 & z-5 \end{bmatrix}; \; |zE - A| = z^2 - 5z + 6; \; z^2 - 5z + 6 = 0$ tenglamaning ildizlari: $z_1 = 2, \; z_2 = 3;$ shunda $[zE - A]^{-1} = \dfrac{1}{|zE - A|}[zE - A]_{np} = \dfrac{1}{z^2 - 5z + 6}\begin{bmatrix} z-5 & 1 \\ -6 & z \end{bmatrix}$ bu yerda $[zE - A]_{np}$ – bu, $[zE - A]$ matritsasiga nisbatan biriktirilgan matritsa.

$X(z) = [zE - A]^{-1} BV(z).$ $V(z) = \dfrac{z}{z-1}$ bo‘lgani uchun:

$$X(z)=\frac{1}{z^2-5z+6}\begin{bmatrix} z-5 & 1 \\ -6 & z \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}\frac{z}{z-1}=\frac{1}{z^2-5z+6}\begin{bmatrix} 1 \\ z \end{bmatrix}\frac{z}{z-1}=\begin{vmatrix} \dfrac{z}{(z-1)(z^2-5z+6)} \\ \dfrac{z^2}{(z-1)(z^2-5z+6)} \end{vmatrix},$$

$$Y(z)=CX(z)=\begin{bmatrix} 0 & 1 \end{bmatrix}\begin{bmatrix} \dfrac{z}{(z-1)(z^2-5z+6)} \\ \dfrac{z^2}{(z-1)(z^2-5z+6)} \end{bmatrix}=\frac{z^2}{(z-1)(z-2)(z-3)}.$$

$\dfrac{Y(z)}{z}=\dfrac{z}{(z-1)(z-2)(z-3)}$ ifodani oddiy kasrlarga yoyib chiqamiz.

$$\frac{z}{(z-1)(z-2)(z-3)}=\frac{\beta_1}{z-1}+\frac{\beta_2}{z-2}+\frac{\beta_3}{z-3},$$ bu yerda

$$\beta_1=\left.\frac{(z-1)z}{(z-1)(z-2)(z-3)}\right|_{z=1}=\frac{1}{2}; \qquad \beta_2=\left.\frac{(z-2)z}{(z-1)(z-2)(z-3)}\right|_{z=2}=-2;$$

$$\beta_3=\left.\frac{(z-3)z}{(z-1)(z-2)(z-3)}\right|_{z=3}=1{,}5.$$

Shunda $Y(z)=\dfrac{0{,}5z}{z-1}-\dfrac{2z}{z-2}+\dfrac{1{,}5z}{z-3}$ bo‘yicha va tasvirlar bilan originallarning muvofiqlik jadvalidan $y(k)=0{,}5-2(2)^k+1{,}5(3)^k$ ekanini topamiz; bundan kelib chiqadiki, $y(0)=0;\ y(1)=1;\ y(2)=6;\ y(3)=25.$ Ikkala usulning natijalari bir xil chiqdi.

Nihoyat, (10.12) formula bo‘yicha diskret uzatish funksiyasini keltirib chiqaramiz.

Yozamiz: $[zE-A]^{-1}=\dfrac{1}{z^2-5z+6}\begin{bmatrix} z-5 & 1 \\ -6 & z \end{bmatrix}=\begin{bmatrix} \dfrac{z-5}{\Delta} & \dfrac{1}{\Delta} \\ -\dfrac{6}{\Delta} & \dfrac{z}{\Delta} \end{bmatrix},$ bu yerda $\Delta=z^2-5z+6.$

$$K(z)=C[ZE-A]^{-1}B=[0 \quad 1]\begin{bmatrix}\frac{z-5}{\Delta} & \frac{1}{\Delta}\\ -\frac{6}{\Delta} & \frac{z}{\Delta}\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}=\begin{bmatrix}-\frac{6}{\Delta} & \frac{z}{\Delta}\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}=\frac{z}{\Delta}=\frac{z}{z^2-5z+6},$$ bular

dastlabki uzatish funksiyasiga mos.

Nazorat savollari

1. Holat tenglamalarini yechishning qanday usullarini bilasiz?
2. Normal shakldan kanonik shaklga qanday o‘tish mumkin?
3. Holat tenglamalaridagi *A*, *B*, *C*, *D* matritsalar mos ravishda qanday ma’noni bildiradi?

11-amaliy mashg‘ulot

Impulsli tizimlar holat tenglamalarining asosiy shakllari

Umumiy holda IABS dinamikasi (10.6) tenglamalar bilan ifodalanadi:

$$X(k+1)=AX(k)+BV(k),$$
$$Y(k)=CX(k).$$

Agar *A* matrisa Frobenius shaklida ifodalangan bo‘lsa:

$$A=\begin{bmatrix}0 & 1 & 0 & \dots & 0\\ \dots & \dots & \dots & \dots & \dots\\ 0 & 0 & 0 & \dots & 1\\ -a_n & -a_{n-1} & -a_{n-2} & \dots & -a_1\end{bmatrix}, \qquad (11.1)$$

normal shakldagi holat tenglamalariga ega bo‘lamiz.

Holat tenglamalarining boshqa shaklini hosil qilamiz. Yopiq IABS ning diskret uzatish funksiyasi quyidagi ko‘rinishga ega bo‘lsin:

$$K_3(z)=\frac{Y(z)}{V(z)}=\frac{b_0z^m+b_1z^{m-1}+\dots+b_m}{a_0z^n+a_1z^{n-1}+\dots+a_n}, \qquad (11.2)$$

bunda $z_1, z_2, \dots, z_n$ – xarakteristik tenglama ildizlari va $m<n$.

Agar ildizlar oddiy bo‘lsa, $K_Ç(z)$ ni oddiy kasrlarga yoyish mumkin:

$$K_3(z) = \frac{\beta_1}{z - z_1} + \frac{\beta_2}{z - z_2} + \ldots + \frac{\beta_n}{z - z_n}, \qquad (11.3)$$

bunda $\beta_i = (z - z_i)K_3(z)\Big|_{z=z_i}$. Shunda

$$Y(z) = K_3(z)V(z) = \sum_{i=1}^{n} \frac{\beta_i}{z - z_i} V(z). \qquad (11.4)$$

Ifodalaymiz: $\frac{\beta_i}{z - z_i} V(z) = X_i(z),$ shunda $Y(z) = \sum_{i=1}^{n} X_i(z)$ bo‘ladi.

Teskari $z -$o‘zgartirishdan foydalanib va $x_i(k+1) = Z^{-1}\{zX_i(z)\},$ a $\upsilon_i(k) = Z^{-1}\{V_i(z)\}$ ekanini hisobga olib, originallarga o‘tamiz:

$$\begin{cases} x_1(k+1) = z_1 x(k) + \beta_1 \upsilon(k), \\ \qquad \ldots \\ x_n(k+1) = z_n x_n(k) + \beta_n \upsilon(k), \\ y(k) = x_1(k) + x_2(k) + \ldots + x_n(k). \end{cases} \qquad (11.5)$$

yoki matritsa ko‘rinishida:

$$\begin{cases} X(k+1) = \begin{bmatrix} z_1 & 0 & \ldots & 0 \\ 0 & z_2 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & & z_m \end{bmatrix} X(k) + \begin{bmatrix} \beta_1 \\ \ldots \\ \beta_n \end{bmatrix} V(k), \\ Y(k) = [1, \ldots, 1]\, X(k)0 \end{cases} \qquad (11.6)$$

Uzil – kesil ixcham ko‘rinishda

$$\begin{cases} X(k+1) = ZX(k) + BV(k), \\ Y(k) = CX(k). \end{cases} \qquad (11.7)$$

Holat tenglamalarining (11.7) shakli qoida (qonun) bo‘lib qolgan. Uning asosiy matritsasi diagonalli: $Z = diag[z_1\ z_2 \ldots z_n]$. Agar xarakteristik tenglamaning ildizlari orasida karrali qiymatlar bo‘lsa, asosiy matritsa Jordan shakliga ega bo‘ladi.

11.1-misol. Yopiq IABSning uzatish funksiyasi quyidagi ko‘rinishda bo‘lsin:

$$K_3(z) = \frac{0{,}4z}{z^2 - 0{,}7z + 0{,}1}.$$

Xarakteristik tenglama $z^2 - 0{,}7z + 0{,}1 = 0$ ning ildizlari $z_1 = 0{,}2$ va $z_2 = 0{,}5$ bo‘ladi.

β_i ni aniqlaymiz:

$$\beta_1 = \frac{0{,}4z(z - z_1)}{(z - z_1)(z - z_2)}\Big|_{z=0{,}2} = \frac{0{,}4 \cdot 0{,}2}{0{,}2 - 0{,}5} = -\frac{4}{15}, \quad \beta_2 = \frac{0{,}4z(z - z_2)}{(z - z_1)(z - z_2)}\Big|_{z=0{,}5} = \frac{0{,}4 \cdot 0{,}5}{0{,}5 - 0{,}2} = \frac{2}{3}.$$

Tenglamani (11.6) ga ko‘ra, holat makoni uchun yozamiz:

$$\begin{cases} X(k+1) = \begin{bmatrix} 0{,}2 & 0 \\ 0 & 0{,}5 \end{bmatrix} X(k) + \begin{bmatrix} -\dfrac{4}{15} \\ \dfrac{2}{3} \end{bmatrix} V(k), \\ Y(k) = \begin{bmatrix} 1 & 1 \end{bmatrix} X(k). \end{cases}$$

Holat tenglamalarini o‘zgartirish

Normal shakldan qoidaga aylanib qolgan shaklga o‘tish uchun modal matritsa $- M$ dan foydalaniladi. Xususan, agar matritsa Frobenius matritsasi bo‘lsa, va turli xususiy raqamlar $- z_1, z_2, \ldots, z_n$ ga ega bo‘lsa, modal matritsa, uzluksiz ABSlardagi kabi, quyidagi ko‘rinishga ega bo‘ladi:

$$M = \begin{bmatrix} 1 & \ldots & 1 \\ z_1 & \ldots & z_n \\ \vdots & & \vdots \\ z_1^{n-1} & \ldots & z_n^{n-1} \end{bmatrix},$$

bunda $n -$ impulsli tizimning xarakteristik tenglamasi tartibi; $z_i -$ tenglama ildizlari.

$X(k) = MQ(k)$ ifodadan, ya'ngi holat o'zgaruvchisi $-Q(k)$ni kiritib, dastlabki tenglamalar (10.6) va (10.13)ni, uzluksiz tizimlardagiga o'xshatib, quyidagicha yozamiz: $\begin{cases} Q(k+1) = ZQ(k) + M^{-1}BV(k), \\ Y(k) = CMQ(k). \end{cases}$

11.2-misol. IABS quyidagi normal shakldagi tenglamalar bilan ifodalansin:

$$\begin{cases} X(k+1) = \begin{bmatrix} 0 & 1 \\ -20 & -4 \end{bmatrix} X(k) + \begin{bmatrix} 1 \\ 2 \end{bmatrix} V(k), \\ Y(k) =\geq \begin{bmatrix} 1 & 4 \\ 1 & 5 \end{bmatrix} X(k). \end{cases}$$

A matritsa kuzatuvchi (yordamchi) bo'lgani uchun tizimning xarakteristik tenglamasi quyidagicha bo'ladi: $\det[A - zE] = (0-z)(-4-z) - (-20)\cdot 1 = 0$ bundan kelib chiqadiki, $z^2 + 4z + 20 = 0$ va ildizlari $z_{1,2} = -2 \pm 4j$. Shunda modal matritsa quyidagi ko'rinish oladi:

$$M = \begin{bmatrix} 1 & 1 \\ -2+4j & -2-4j \end{bmatrix}.$$

$X(k)$ni $MQ(k)$ga tenglab, dastlabki tenglamani quyidagi ko'rinishga keltiramiz:

$$\begin{cases} Q(k+1) = M^{-1} \begin{bmatrix} 0 & 1 \\ -20 & -4 \end{bmatrix} MQ(k) + M^{-1} \begin{bmatrix} 1 \\ 2 \end{bmatrix} V(k), \\ Y(k) = \begin{bmatrix} 1 & 4 \\ 1 & 5 \end{bmatrix} MQ(k), \end{cases}$$

bu yerda

$$M^{-1} = \frac{1}{|M|} M_{i\partial} = \frac{1}{-8j} \begin{bmatrix} -2-4j & -1 \\ 2-4j & 1 \end{bmatrix} = 0{,}5\frac{j}{4} \begin{bmatrix} -2-4j & -1 \\ 2-4j & 2 \end{bmatrix},$$

$$M^{-1} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 0{,}5\frac{j}{4} \begin{bmatrix} -2-4j & -1 \\ 2-4j & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 0{,}5\frac{j}{4} \begin{bmatrix} -4-4j \\ 4-4j \end{bmatrix} = 0{,}5 \begin{bmatrix} -j & 1 \\ j & 1 \end{bmatrix}.$$

Uzil – kesil hosil qilamiz:

$$\begin{cases} Q(k+1) = \begin{bmatrix} -2+4j & 0 \\ 0 & -2-4j \end{bmatrix} Q(k) + 0{,}5 \begin{bmatrix} -j & 1 \\ j & 1 \end{bmatrix} V(k), \\ Y(k) = \begin{bmatrix} 1 & 4 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -2+4j & -2-4j \end{bmatrix} Q(k) = \begin{bmatrix} -7+16j & -7-16j \\ -9+20j & -9-20j \end{bmatrix} Q(k). \end{cases}$$

Dastlabki holat tenglamalaridagi asosiy matritsa ixtiyoriy bo‘lgan umumiy holatlarda qoida bo‘lib qolgan shaklga o‘tish uchun $X = MQ$ $z -$o‘zgartiriladi. Bunda modal matritsa, uzluksiz tizimlardagi kabi vektor – ustun $-x^i$ lardan tashkil topadi. Vektor – ustunlar esa, quyidagi tenglamalarni yechib hosil qilinadi:

$$\left[z_i E - A\right] x^i = 0. \qquad (11.8)$$

Modal matritsa quyidagi ko‘rinishda bo‘ladi:

$$M = \begin{bmatrix} x_1^1 & x_1^2 & \cdots & x_1^n \\ x_2^1 & x_2^2 & \cdots & x_2^n \\ \cdots & \cdots & \cdots & \cdots \\ x_n^1 & x_n^2 & \cdots & x_n^n \end{bmatrix}. \qquad (11.9)$$

Nazorat savollari

1. Agar xarakteristik tenglamaning ildizlari orasida karrali qiymatlar bo‘lsa, asosiy matritsa qanday shaklga ega bo‘ladi?
2. Modal matritsa qanday ko‘rinishda bo‘ladi?
3. Holat tenglamalarining qanday shakli kanonik deb nomlanadi?

12-amaliy mashg‘ulot

Diskret tizimlarning boshqarilishi va kuzatilishi

Diskret tizimlarning boshqarilishi va kuzatilishi tushunchalari uzluksiz tizimlardagi kabi ma’noga ega.

(10.5) tenglamalar bilan ifodalangan diskret tizim quyidagi shartlarda to‘liq bajariladigan tizim deb ataladi: $t(0)$, $t(l)$ vaqt onlari va $x(t_0)$, $x(t_i)$ holatlar uchun boshqarish $-\upsilon(k)$ mavjud; bunda $0 \le k \le l$; boshqarish dastlabki holat $-x(t_0)$ni so‘nggi holat $-x(t_i)$ ga o‘tkazadi.

Boshqarilish mezoni boshqarilish matritsasi $K_y = \left[B \vdots AB \vdots \cdots \vdots A^{n-1}B\right]$ning buzilmagan (aytilmagan) ligiga bog‘liq. Bittadan kirish va chiqish joyiga ega tizim uchun boshqarilish mezoni quyidagi shartga keltiriladi:

$$\det K_y \neq 0. \qquad (12.1)$$

Buni isbotlash uchun (10.8) formuladan foydalanish mumkin. Vaqt onini $l = n$ deya qabul qilib, topamiz:

$$x(n) = A^n x(0) + A^{n-1} B\upsilon(0) + \ldots + B\upsilon(n-1). \qquad (12.2)$$

Bu ifodani quyidagi ko‘rinishga keltirish mumkin:

$$x(n) = A^n x(0) + \left[B \vdots AB \vdots \cdots \vdots A^{n-1}B\right] \begin{bmatrix} \upsilon(n-1) \\ \upsilon(n-2) \\ \cdots \\ \upsilon(0) \end{bmatrix}. \qquad (12.3)$$

(12.3)dan hosil qilamiz:

$$\begin{bmatrix} \upsilon(n-1) \\ \upsilon(n-2) \\ \ldots\ldots\ldots\ldots \\ \upsilon(0) \end{bmatrix} = \left[B \vdots AB \vdots \cdots \vdots A^{n-1}B\right]^{-1} \left[x(n) - A^n x(0)\right] = K_y^{-1}\left[x(n) - A^n x(0)\right], \qquad (12.4)$$

Bunga erishish uchun $\det K_y \neq 0$ bo‘lishi kerak, chunki $K_y^{-1} = \dfrac{1}{\det K_y} K_{óïð}$.

Agar diskret tizimni kuzatish oni $t = t(l)$da, o‘lchash ma’lumotlari $-y(t_i)$ va ma’lum qiymatlar $-\upsilon(t_i)$ bo‘yicha, $k = 0$ onda, holat vektorini tiklash mumkin bo‘lsa, bunday diskret tizim “to‘la kuzatiladigan” deyiladi. Kuzatish, kirish joyidagi o‘zgaruvchanga bog‘liq bo‘lmagani uchun tizimni avtonom deb qarash mumkin, ya’ni (10.8)ni quyidagicha yozish mumkin:

$$X(n) = A^n X(0). \qquad (12.5)$$

Kuzatilish mezoni kuzatilish matritsasi $K_f = \left[C^T \vdots A^T C^T \vdots \cdots \vdots (A^T)^{n-1} C^T\right]$ning aynimasligiga bog‘liq. Bir o‘lchamli tizim uchun bu mezonning ko‘rinishi:

$$\det K_H \neq 0. \qquad (12.6)$$

$l = n-1$ va (12.5) ni hisobga olib, $y(0),\ldots,y(n-1)$ qiymatlarni topamiz:

$$y_0 = C^T x(0), y(1) = C^T A x(0), \ldots, y(n-1) = C^T A^{n-1} x(0),$$

yoki ixcham ko‘rinishda yozamiz:

$$\begin{bmatrix} y(0) \\ y(1) \\ \cdots\cdots \\ y(n-1) \end{bmatrix} = \left[C^T \vdots C^T A^T \vdots \cdots \vdots C^T (A^T)^{n-1} \right] x(0) = K_H x(0). \qquad (12.7)$$

Agar matritsa $-K_H$ ortga qaytuvchi ($\det K_H \neq 0$) bo'lsa, quyidagini topish mumkin:

$$x(0) = K_H^{-1} \begin{bmatrix} y(0) \\ y(1) \\ \cdots\cdots\cdot \\ y(n-1) \end{bmatrix}. \qquad (12.8)$$

Nazorat savollari

1. Diskret tizimlarning boshqarilishi va kuzatilishi tushunchalari qanday ma'nolarga ega?
2. Bittadan kirish va chiqish joyiga ega tizim uchun boshqarilish mezoni qanday shartga keltiriladi?
3. Kuzatilish mezoni kuzatilish matritsasi qanday bog'liq?

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Mundarija

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DISKRET BOSHQARISH TIZIMLARI

fanidan amaliy mashg‘ulotlarni bajarish uchun

USLUBIY KO‘RSATMALAR

Muharrir: Sidikova K.A.