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# PRACTICAL DESCRIPTIVE GEOMETRY

BY

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SECOND EDITION

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# PREFACE TO THE SECOND EDITION

The first edition of this book, published in 1912, represented the author's views on the presentation of Descriptive Geometry, so as to arouse the student's interest in the subject without sacrificing rigorous training in the principles.

In three years the text has been used in many schools, and the present, second edition, aims to embody the suggestions of many leading teachers of the subject, who have, at the request of the author and publishers, expressed their views as to this development.

The main idea of the first edition, that of "practicality," has been adhered to and amplified. The most important changes are:

1. The division into more chapters.
2. A more logical sequence for certain portions of the text.
3. The notation of points and planes.
4. Graphic layouts for exercises.
5. More exercises of a practical nature, embracing more lines of engineering endeavor.
6. A number of new solutions, utilizing auxiliary planes more frequently.
7. A better system of perspective drawing.
8. A chapter on the intersections of plane solids.

Although yielding to the request of several teachers for the inclusion of graphic layouts of exercises, the author still believes that dimensioned layouts are of great value despite the amount of time expended on the layout itself. With both of these, however, in practically unlimited quantity, the instructor may decide the question for himself. The layouts of both kinds have been made so flexible, by interchanging and substituting data, that the number of variations is almost incalculable. This removes the objection, expressed by some teachers, to a definite and limited number of exercises, the solutions of which are sometimes preserved by student organizations. No such collection of solutions will benefit future classes, as the exercises can be made entirely different each succeeding year.

As to the treatment of Perspective (how and how much), the author received a large amount of constructive though conflicting advice, mainly from teachers of architecture. The writer's choice is "Conical Projection," a process that is sane, in that it is a direct application of the principles of Descriptive Geometry, and is easy and swift in operation.

It has been previously stated that much interest was manifested in the revision of this work by prominent teachers in all parts of the country, and their combined opinions moulded it into its present form. The writer wishes to convey his thanks to all who assisted in this way. For special criticisms and material he acknowledges indebtedness to Professor Thomas E. French, Ohio State University; Professor Howard Moore, Colorado College; Professor Frederick C. Biggin, Alabama Polytechnic Institute; Professor Robert H. McNeilly, Vanderbilt University; Professor Chace Newman, Michigan Agricultural College; Professor H. B. Monges, University of California; Mr. Charles R. Swineford, Armour Institute of Technology.

W. G. S.

ARMOUR INSTITUTE OF TECHNOLOGY,  
*Chicago, April, 1916.*

# PREFACE TO FIRST EDITION

In presenting a new text-book on Descriptive Geometry, the writer is fully aware of the excellence of existing treatises, and appreciates what they have contributed toward a higher standard of technical training. The authors of these treatises, pursuing their several courses, have carried the development of the subject to a high plane.

However, in spite of the indisputable excellence of many of the text-books, they seem to have failed to arouse the interest of the student, partly by ignoring the practical applications, and partly by making only a slight attempt to present the subject attractively. Some of the books are valuable only for reference and are useless in the class room; others are incomplete in the essentials; some are faddish, emphasizing certain features and treating the rest inadequately, while even the best convey to the student only a very slight idea of the practical value of the subject. This has given rise to the belief, prevailing almost universally in the student body, that Descriptive Geometry is merely a disciplinary study, having little or no relation to the life work of the individual.

The aim of the writer has been, therefore, to present the subject to the student in a simple manner, as progressively as possible, reminding him constantly of the relation which exists between Descriptive Geometry and Practical Drafting; and to avoid needless difficulty by using language and directions of the greatest possible clarity. The writer believes that a thorough knowledge of the subject is achieved not through much study of the text, but by working exercises. To this end he has provided a large number of exercises, scattered through the text, of considerable variety and capable of infinite multiplication by the clever instructor.

An examination of the subject will reveal the following features and innovations:

1. A thorough drill in fundamentals.
2. Repetitions of statements for the sake of emphasis.
3. Notation comprehensive, yet reduced to its lowest terms.
4. Analyses separated from proofs.
5. Tabulated order of the operations in each analysis and construction.

6. Exercises to the number of 860, of which about one-fourth are such as may be met in actual practice. Most of these are dimensioned for a space of standard size.

7. Notes on exceptions, checks on solutions, special cases, hints, and rules presented wherever they seem necessary or advisable.

8. Brief treatment only of such special subjects as Shades and Shadows, Perspective, etc.

9. The original seventeen "Point, Line, and Plane" problems expanded to forty-three.

10. New treatment of surfaces, less importance being attached to passing tangent planes, and more importance to plane sections, intersections, developments, and practical applications. For example, there are about thirty exercises which are based on practical illustrations of the various warped surfaces, showing actual uses for all the listed varieties.

The expansion of the problems noted in feature (9) is justified, the writer believes, in order to fully cover the ground, and to provide the necessary steps in the development of the subject. Some of them are special cases and some require little or no change in the analysis from those of preceding problems.

Students undertaking the study of Descriptive Geometry should have a fair knowledge of Mechanical Drawing, at least as much as is usually included in High School courses. The minimum prerequisite should amount to the work contained in the first eight chapters of French's "Engineering Drawing," or in Reid's "Mechanical Drawing" entire.

In preparing this text-book the writer has consulted most of the standard works on the subject, and desires to make special acknowledgment for ideas and information to the excellent books by MacCord, Church and Bartlett, Phillips and Millar, and Randall. Most of the exercises are original, but many have been adapted from such admirable books of exercises as those by Professors Hood, Marshall, and Fishleigh, as well as the problem files of Armour Institute of Technology, and the books of the American School of Correspondence.

The writer wishes to thank his colleagues, Robert V. Perry, Henry L. Nachman, John S. Reid, and Charles R. Swineford, of the Faculty of Armour Institute of Technology, for valuable suggestions and encouragement.

WILLIAM GRISWOLD SMITH.

ARMOUR INSTITUTE OF TECHNOLOGY,  
*Chicago, June 20, 1912.*

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# PRACTICAL DESCRIPTIVE GEOMETRY

## CHAPTER I

### DEFINITIONS, NOTATIONS, GEOMETRICAL THEOREMS, PRELIMINARY PRINCIPLES AND EXERCISES

**1. Definition.**—Descriptive Geometry is the Science of Drafting.

Drafting is the Art of representing objects on paper, or other flat surface, by means of projections of the object on imaginary planes.

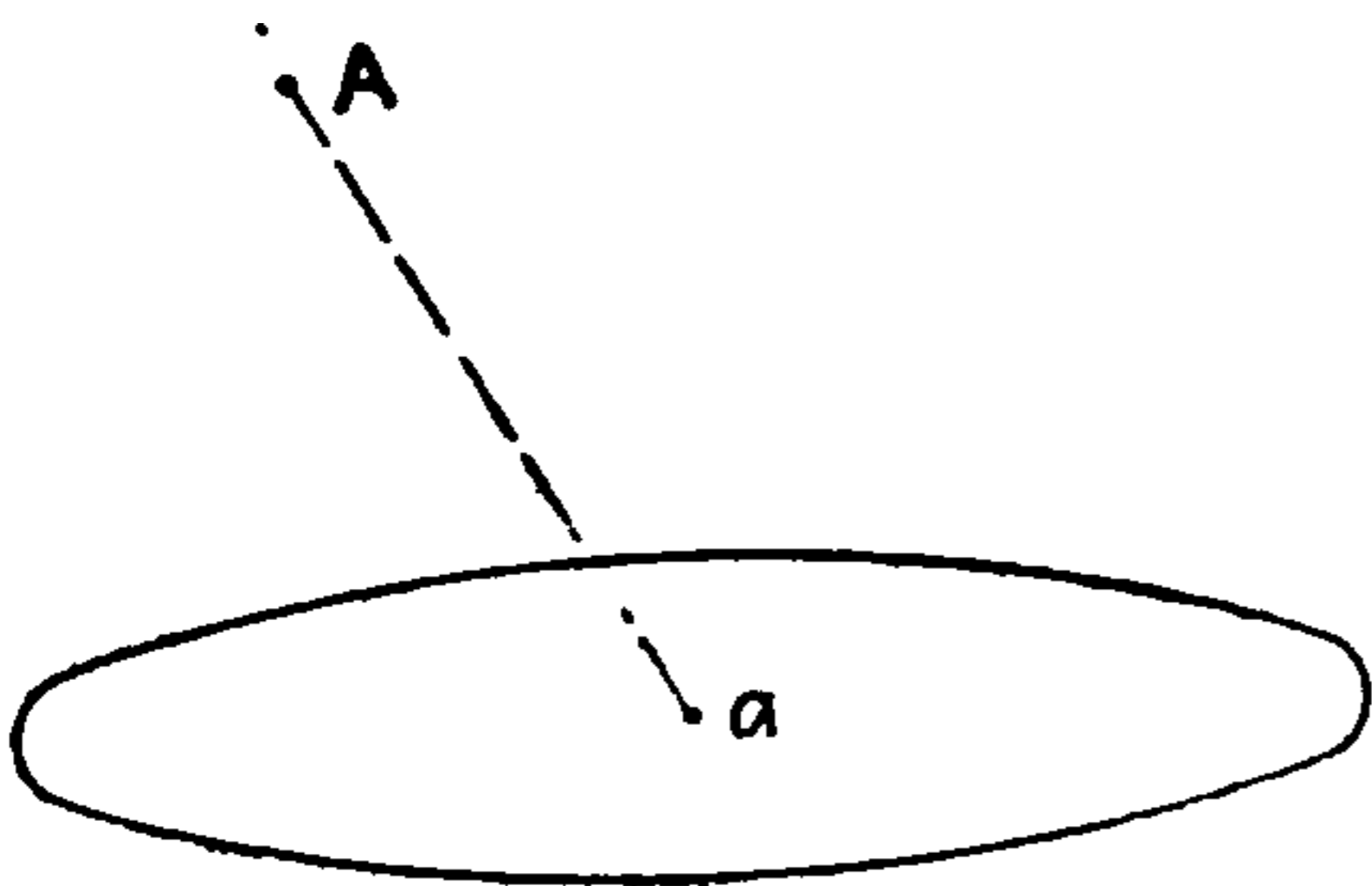


FIG. 1.—The projection of a point.

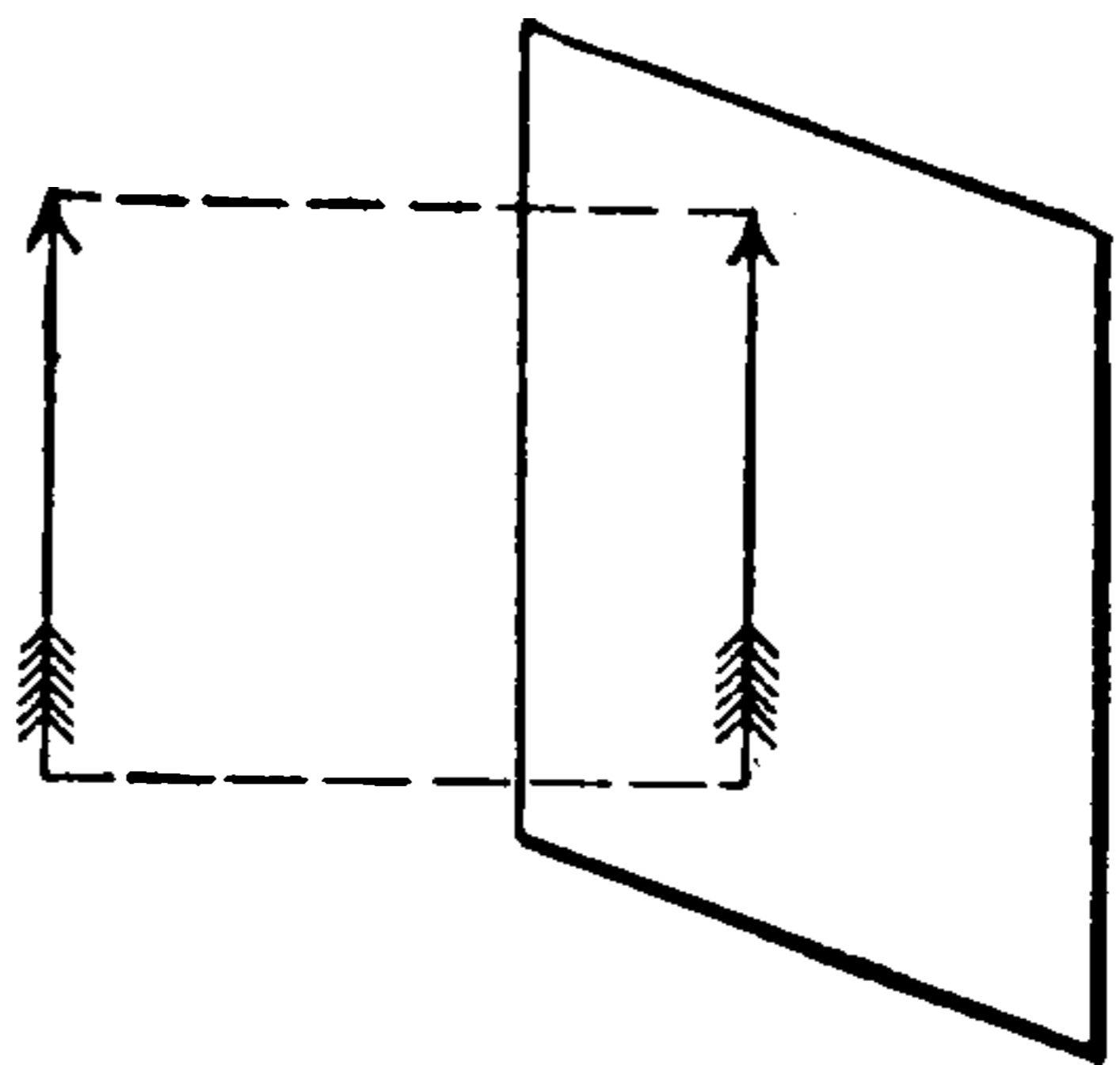


FIG. 2.—Orthographic projection.

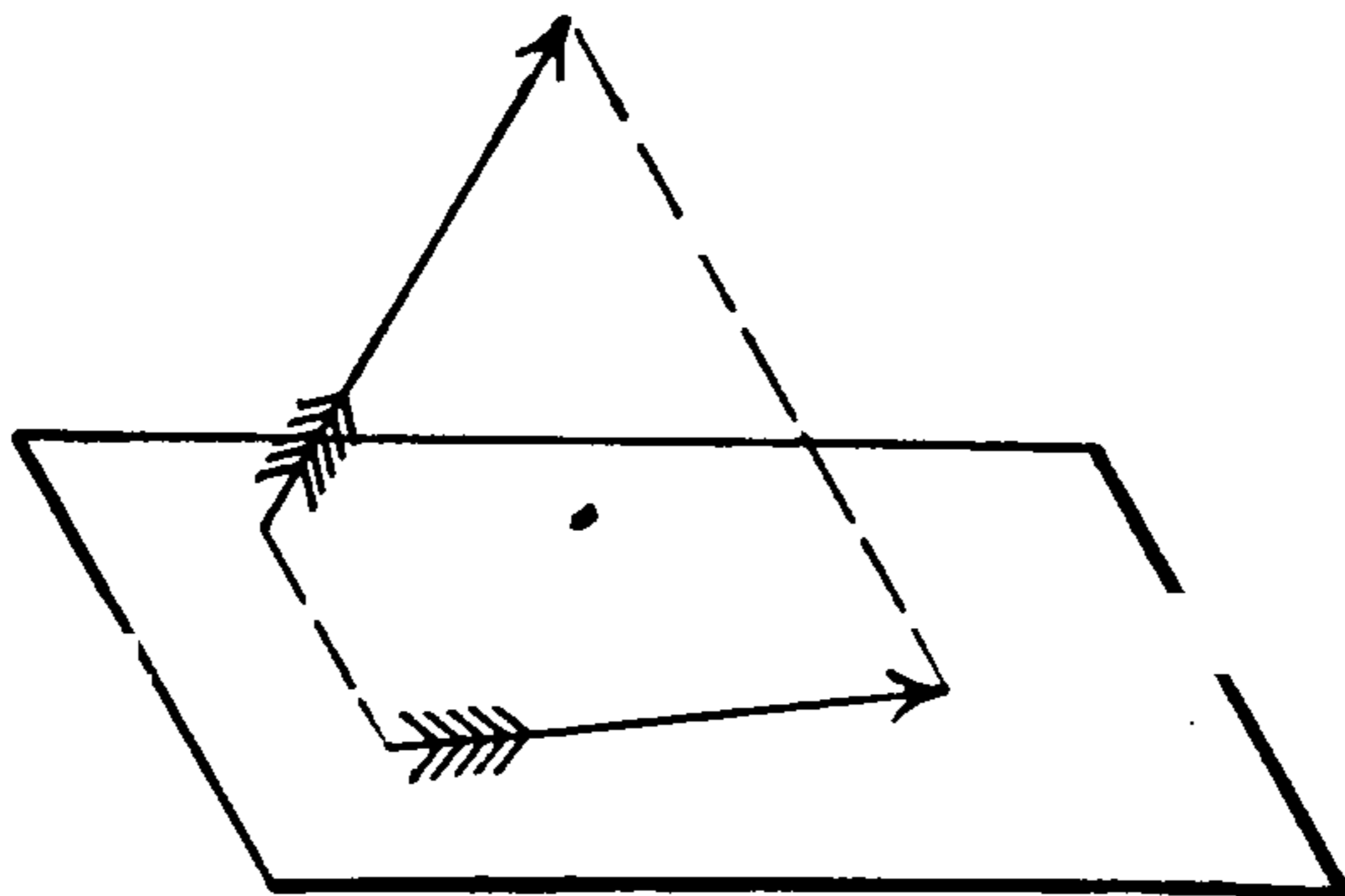


FIG. 3.—Oblique projection.

**Projections** are representations of objects made by imaginary lines, called **projectors**, running (or projected) from the important points of the object to the plane on which the image is to be made.

## CLASSIFICATION OF PROJECTIONS

**Orthographic Projection:** representation of objects projected perpendicularly.

**Perspective or Scenographic Projection:** representation of objects projected by converging lines.

**Oblique Projection:** representation of objects projected by parallel oblique lines.

**Single Plane Projections:** representation of objects in various ways for pictorial purposes. Several styles are in use and are treated in Chap. X. They are mechanical variants on perspective and are effected through various devices of orthographic and oblique projection, and by artificial foreshortening.

**2. Discussion.**—**Orthographic Projection** is almost universally used in “working drawings” and “machine sketches;” hence

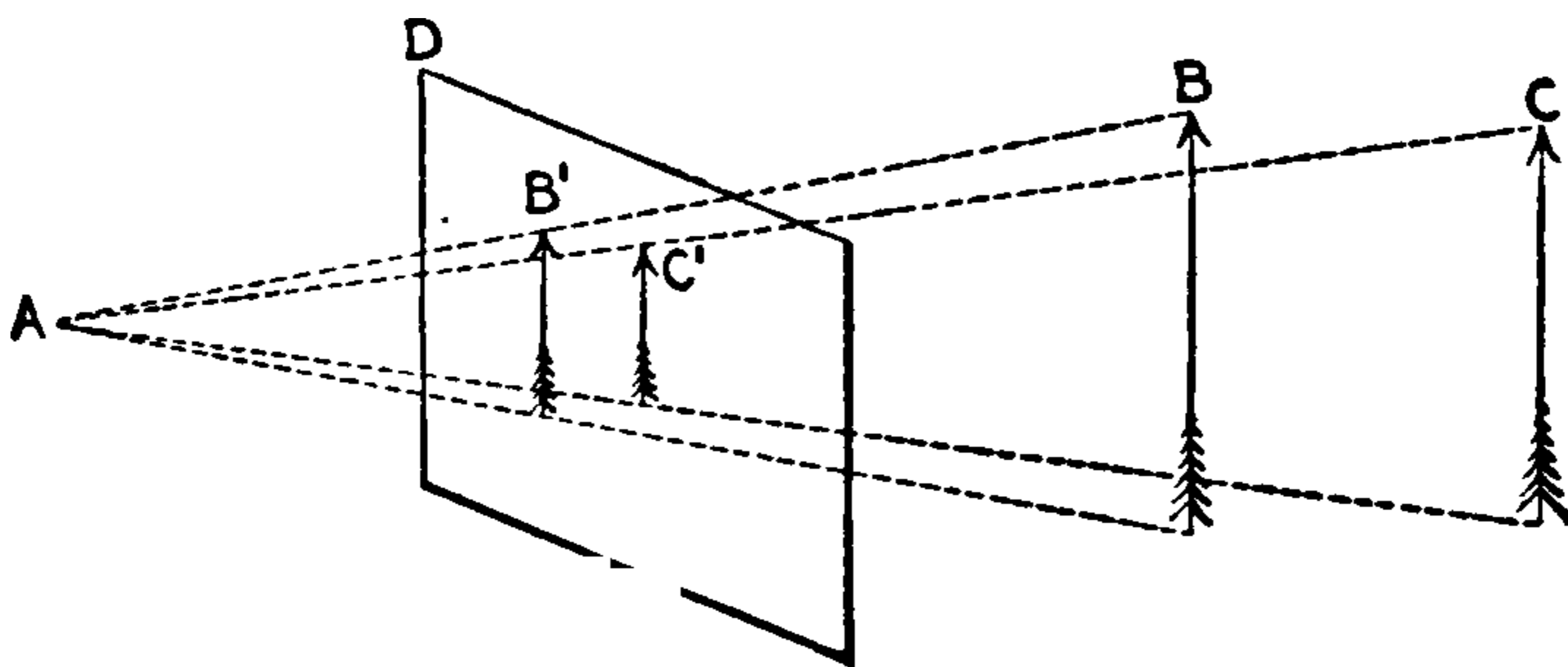


FIG. 4.—Perspective.

it is the most important class of projection. To thoroughly understand its significance, let us consider the difference between a picture and a working drawing.

A picture, or perspective, is a representation of an object, group, or scene, as it appears to the eye. Rays of light from the object reach the eye in converging lines. See Fig. 4. A is the eye of the observer; B and C are objects of equal size, but at different distances from A; D is a plane of projection between the objects and the eye. Converging rays (shown by the dotted lines) project the objects on D (piercing it at certain points). These points determine the size of the pictures of B and C, and the images B' and C' are the pictures. As B is the nearer, B' is larger than C'. Thus distant objects appear smaller than nearer ones of equal and often smaller size. By means of this fact and the knowledge of the observer of the comparative sizes of the objects, a picture shows the entire composition of a scene.

The working drawing takes no such account of distances. See Fig. 5. Here the rays are all parallel, and the projections are of equal size to the objects, be the latter near or remote. In orthographic projection, therefore, the eye must be considered to be at an **infinite** distance. If the effect of a finite viewpoint should be desired, the eye would have to be shifted (as A, Fig. 5) to the line of each ray. This, of course, is an artificial condition, and shows that a working drawing is not, and cannot be, a correct visual image of the object.

This fact would seem to be a disadvantage against the use of orthographic projection, but for manufacturing and construction work, the orthographic is much the more useful and universal

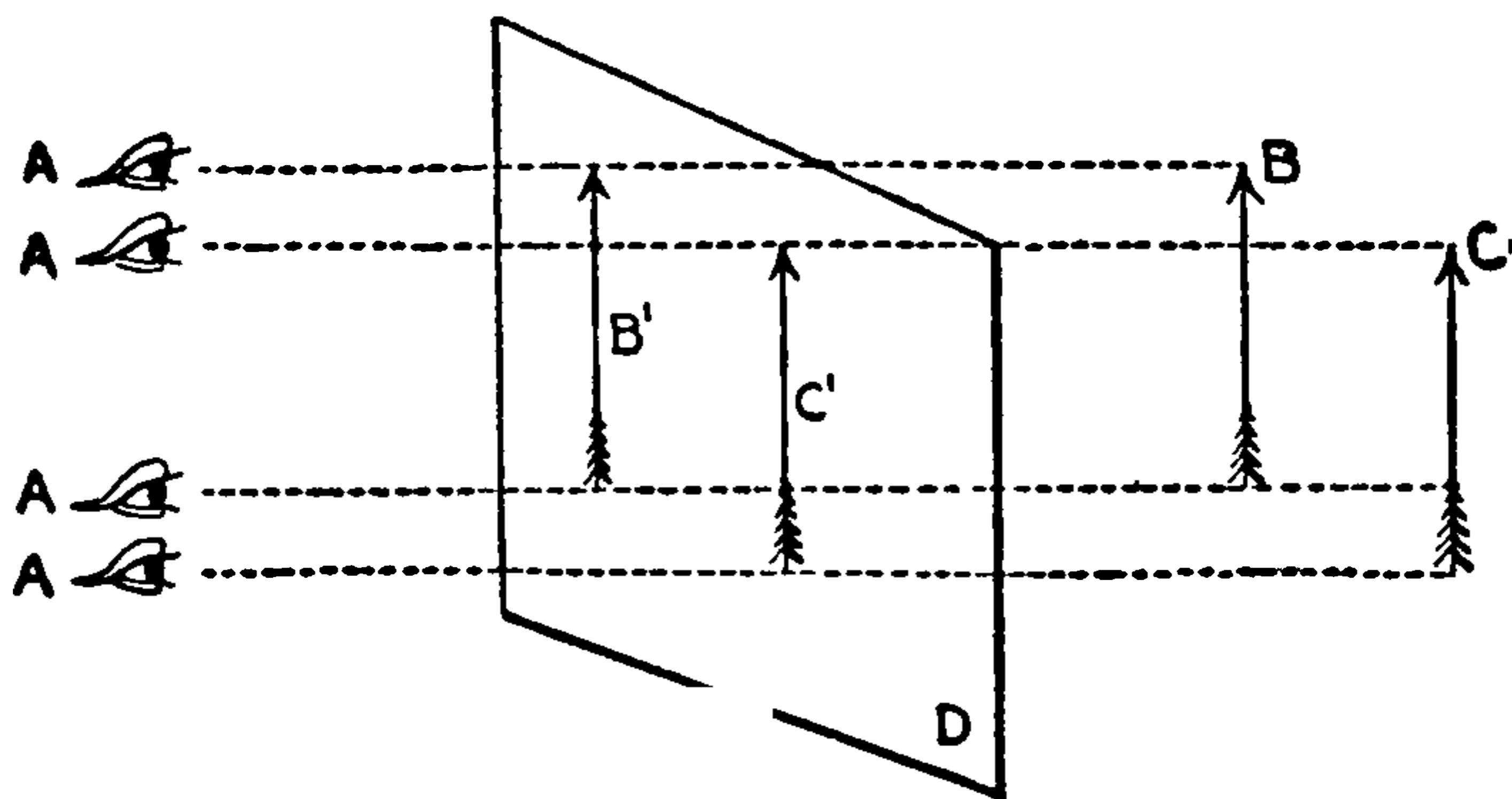


FIG. 5.

in its adoption than perspective, and much easier for the draftsman to use.

On account of the complexity of many machines and structures, it is necessary to show many dimensions in many directions, interior dimensions, thicknesses, and irregularities, which are all but impossible in a picture. This is accomplished by projecting the object orthographically on two or more imaginary planes, called the **Planes of Projection**.

### THE PLANES OF PROJECTION

**3.** The two principal planes are the **Vertical** and **Horizontal**. The **Vertical Plane** (which we shall hereafter write  $V$ ) is a plane normal to the earth's surface; that is, perpendicular to the apparent surface of the earth. The **Horizontal Plane** (hereafter written  $H$ ) is a plane perpendicular to any vertical line; that is, it is parallel to the apparent surface of the earth.  $H$  and  $V$  are perpendicular to each other. The projections on these two planes determine the relative position of any two or more points in space.

There are conditions arising often, which render a third plane desirable, and indeed necessary, in the solution of certain problems. This third plane is perpendicular to both  $H$  and  $V$ , and is called the **Profile Plane**, which will always be written  $P$ .

Any number of auxiliary planes, which may be parallel, perpendicular, or oblique to  $H$  or  $V$ , may be used, according to the

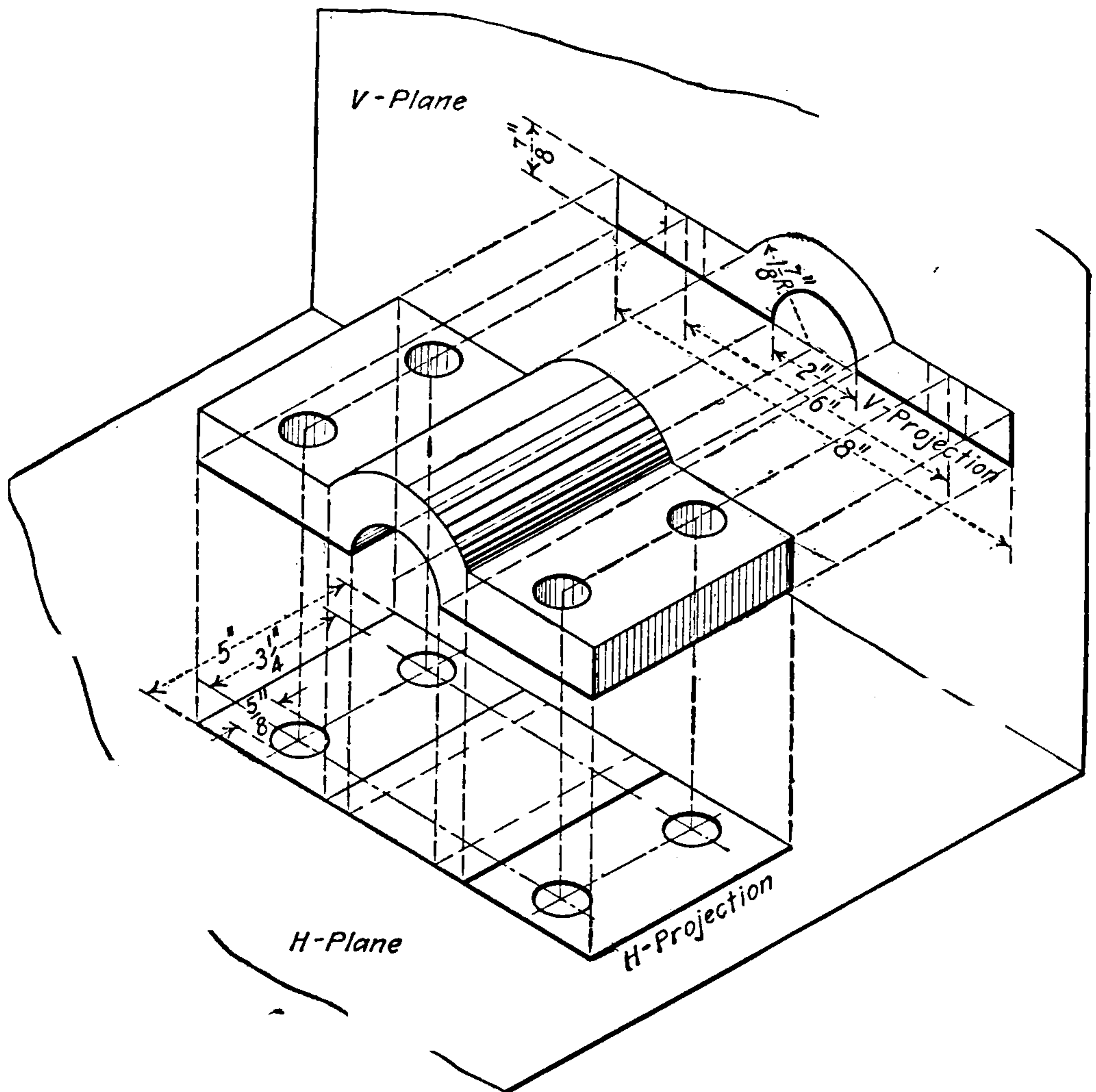


FIG. 6.—Machine part projected orthographically.

complexities of the problem. (In the architect's plans of a big building, two different vertical projections may be needed, two profiles, a horizontal projection for every floor, basement, and roof, auxiliary projections of certain oblique features, sectional interior projections, and a hundred "detail" projections of doors, windows, staircases, elevators, etc., all in orthographic projection. Besides these, it is customary to make a perspective of the projected building with colors, trees, grass, walks, etc. The average

purchaser cannot comprehend working drawings, while the picture is of little use to the workman. Perspectives are for the customers, orthographic projections for the workmen.)

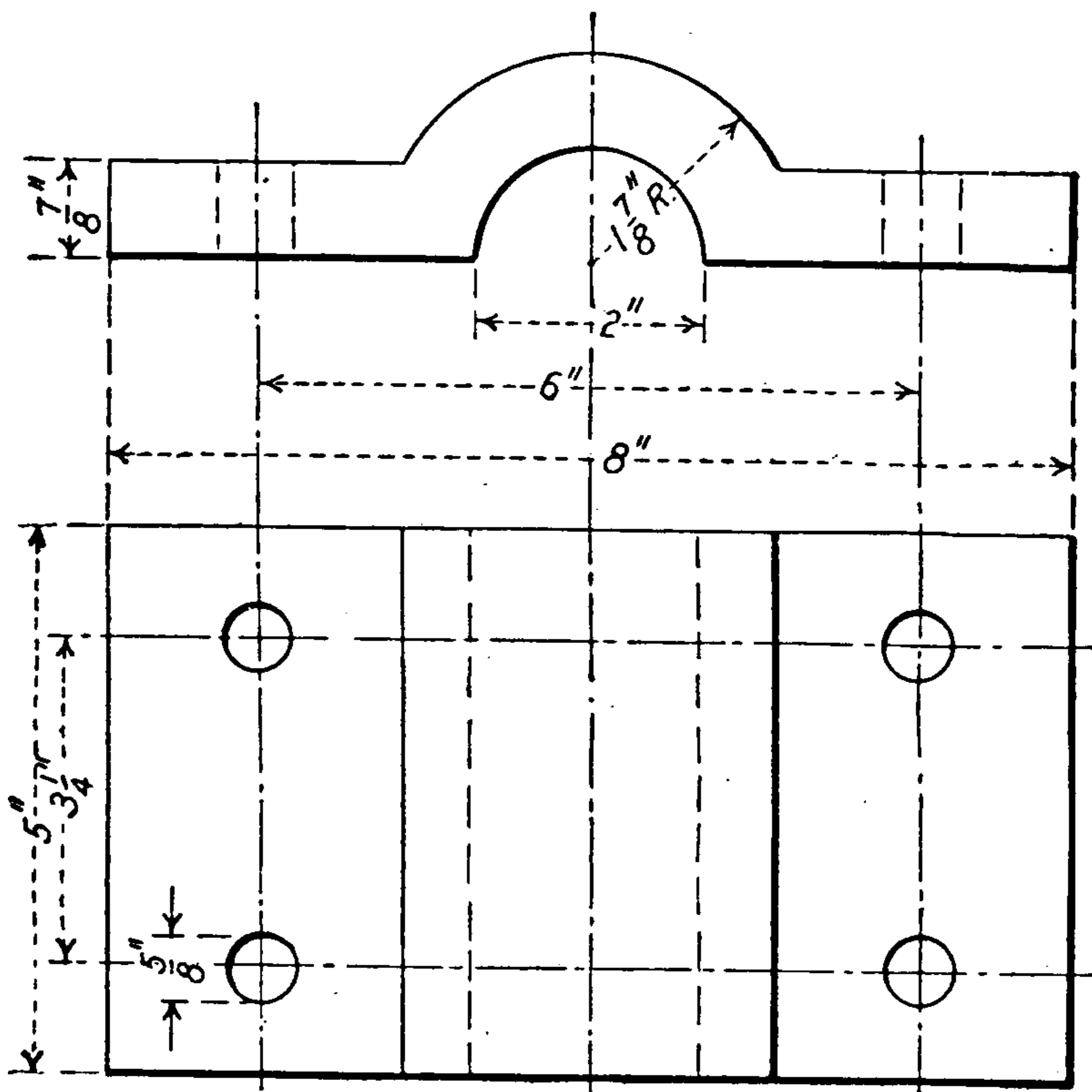


FIG. 7.—Projections of machine part in the flat.

4. It is obvious that it would be awkward and impracticable to have a drawing board made with horizontal, vertical, and profile sides, and the respective projections made on them. Not only would such an arrangement be unwieldy, but it is unnecessary. Fig. 8 shows two planes,  $H$  and  $V$ , intersecting, as they must, in a line, called the Ground Line (designated hereafter  $GL$ ).

By revolving  $V$  about  $GL$  as an axis or hinge, it will be put into coincidence with  $H$ , and the  $H$ - and  $V$ -projections can be drawn on one surface. Similarly,  $P$  can be rotated into the same surface about its intersection either with  $H$  or  $V$ . For mere solution of problems, it is immaterial whether the  $P$ -projection is revolved into  $H$  or  $V$ , but in practical work it is customary to revolve it into  $V$ ; therefore in this work the profile will always be rotated about its  $V$ -trace into  $V$ .

#### THE FOUR DIHEDRAL ANGLES

5. It will be seen from Fig. 8 that the planes of projection do not end at  $GL$ . In fact, they are unlimited in extent, and divide

space into four dihedral angles, with  $GL$  as their common intersection. In folding into the flat, therefore, no portion of the drawing board is monopolized by either  $H$  or  $V$ . There is a portion of  $H$  above  $GL$ , and a portion below  $GL$ . There is a portion of  $V$  above  $GL$ , and a portion below  $GL$ . (This circumstance will be found essential to the solution of certain problems of abstract character. In practical work the different projections, or views,

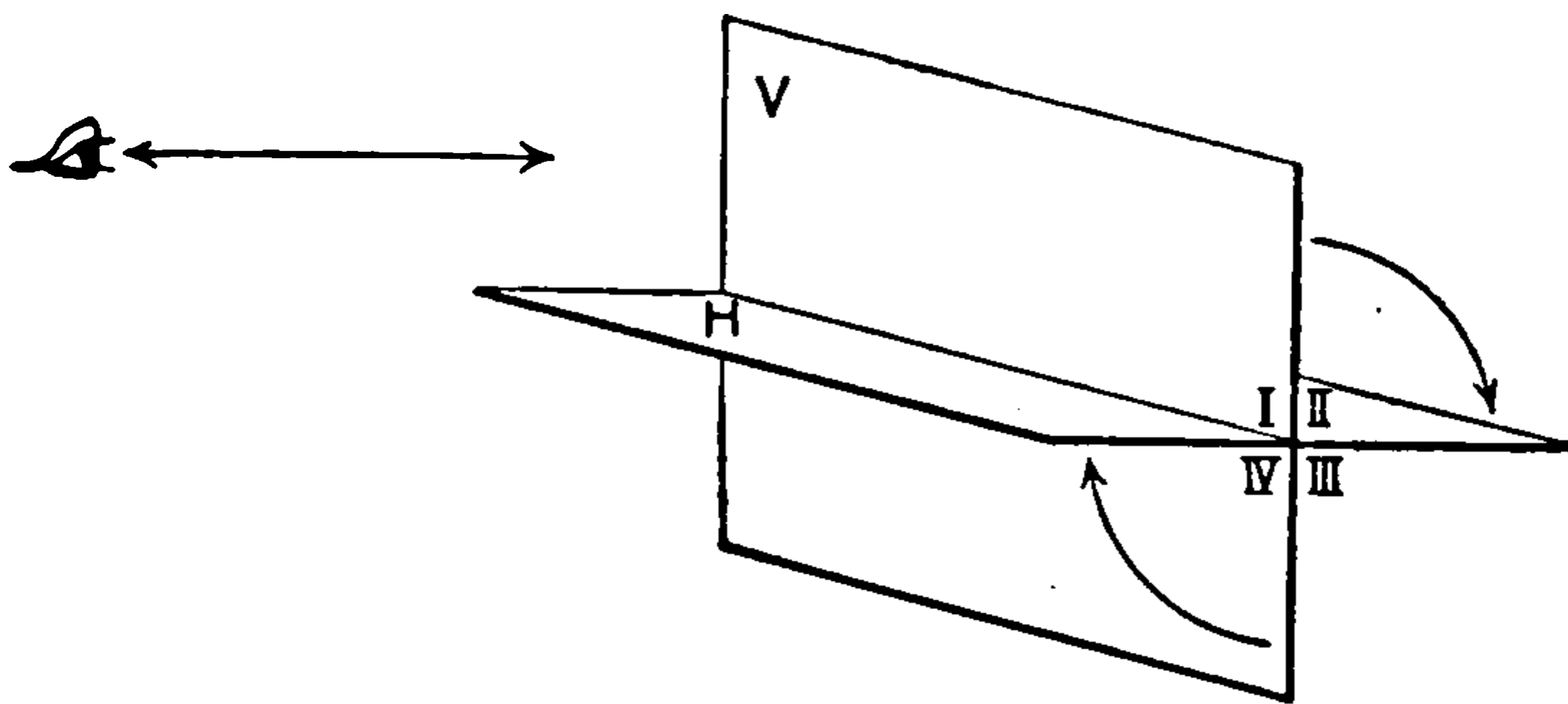


FIG. 8.

as they are often called, are entirely separated from each other, and definite spaces are reserved for  $H$ ,  $V$ , and  $P$ .)

The four dihedral angles, in any of which objects may be imagined, are numbered, as shown by the Roman numerals in Fig. 8.

**First Angle**, in front of  $V$ , above  $H$ ;

**Second Angle**, behind  $V$ , above  $H$ ;

**Third Angle**, behind  $V$ , below  $H$ ;

**Fourth Angle**, in front of  $V$ , below  $H$ .

The first angle (designated in this work as  $I$ ) is largely used in our abstract problems, on account of its accessibility and the ease of presentation by the teacher to the class, when giving a solution of a problem in space.

The third angle ( $III$ ) is commonly used in practical work, for the reason that the  $H$ -projection is usually made looking down on the object, and besides for this angle the profile comes out right-handed and detached, whereas the usual method of swinging a profile from a first-angle-projection will either bring the object out left-handed, or mixed up with the other views. Many of our problems are given in  $III$ .

## 6. Commercial Terminology of Projections.

As the terms  $V$ -projection,  $H$ -projection, etc., are not descriptive to non-professionals, certain terms are used by draftsmen.

The  $V$ -projection is often called the **Elevation**, **Front View**, **Rear View**, or **Sectional Elevation**, as the case may be.

The *H*-projection is called the **Plan, Top View, Bottom View, or Sectional Plan**, as the case may be.

The *P*-projection is called the **Profile, End View, Right or Left End View, or Profile Section**, as the case may be.

### THE REPRESENTATION OF POINTS, LINES, AND PLANES

**7. Points.**—It is obvious that the projection of a point on one plane does not determine that point. For example, let a point on the floor be the projection of some point in the room. How high is that point? It may be any distance. The point may be the projection itself, or the point may be on the ceiling above, or anywhere in between. To locate the point in space by one projection, it is necessary to add its distance, positive or negative, from the projection. A better way (the draftsman's way) is to locate the point graphically. By projecting that point on the wall, its height is determined and shown.

This definitely locates a point, for, if perpendicular lines be erected from each projection of a point, these perpendiculars will pass through the point. As there can be but one intersection of the perpendiculars, the point is absolutely located in space by its projection on two planes. The perpendicular lines from the point to the planes of projection are called **projectors**.

### THE RELATION OF PLANE AND SOLID GEOMETRY TO DESCRIPTIVE GEOMETRY

**8.** All theorems in Plane and Solid Geometry are applied to Descriptive Geometry to a greater or less extent. There are, however, a number of theorems of outstanding importance, to which reference must be made with considerable frequency. A list of these is herewith submitted. The student need not be required to prove them, but should be able to give the conclusion to every hypothesis, and to quote his references on occasion.

#### PLANE AND SOLID GEOMETRY THEOREMS

1. The opposite sides of a parallelogram are equal.
2. Triangles are equal:
  - (1) If three sides of one are equal to three sides of the other, each to each.



(2) If two sides and the included angle of one are equal to two sides and the included angle of the other.

(3) If one side and the adjacent angles of one are equal to one side and the adjacent angles of the other.

3. Right triangles are equal if any side and an acute angle of one are equal to the corresponding side and angle of the other.

4. If the two points of a line lie in a plane, the line lies entirely in the plane.

5. If a line is perpendicular to a plane, it is perpendicular to all the lines in the plane that intersect it.

6. Two straight lines perpendicular to the same plane are parallel to each other.

7. If two parallel planes are cut by a third plane, the intersections are parallel.

8. If a line external to a plane is parallel to any line in that plane it is parallel to the plane.

9. If a line is parallel to two intersecting planes, it is parallel to their intersection.

10. A line perpendicular to one of two perpendicular planes is parallel to the other.

11. A plane perpendicular to two intersecting planes is perpendicular to their intersection.

12. The dihedral angle between two planes is measured by the angle of the lines cut from them by a plane perpendicular to their intersection.

13. If a line is perpendicular to a plane, every plane containing the line is perpendicular to the plane.

14. If two planes are perpendicular, a line in one, perpendicular to their intersection, is perpendicular to the other.

15. The angle that a line makes with a plane is the angle that it makes with its projection on that plane.

16. Two lines parallel in space are projected on any plane in parallel lines.

### THEOREMS RELATING TO POINTS

9. **Theorem I.**—The *H*- and *V*-projections of a point must lie on a common perpendicular to *GL*.

**Proof.**—The *H*- and *V*-projectors of the point determine a plane that is perpendicular to both planes, and therefore to *GL*. (Why?) The intersections of this plane with *H* and *V* will then



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above **GL**. Points above **H** have their **V**-projections above **GL**. Points below **H** have their **V**-projections below **GL**.

**Theorem IV.**—Points in **H** have their **V**-projections in **GL**, and points in **V** have their **H**-projections in **GL**.

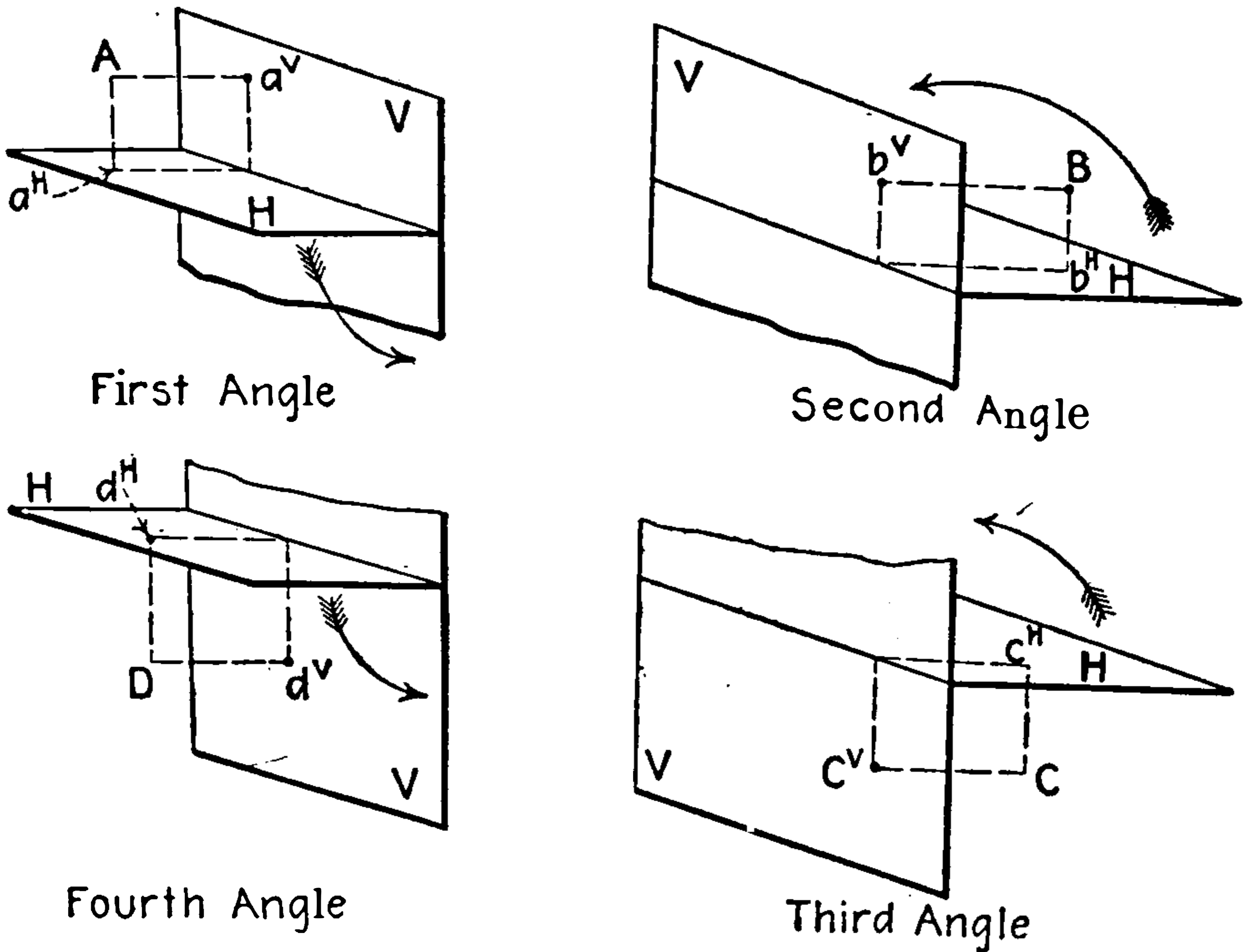


FIG. 10.—The four dihedral angles.

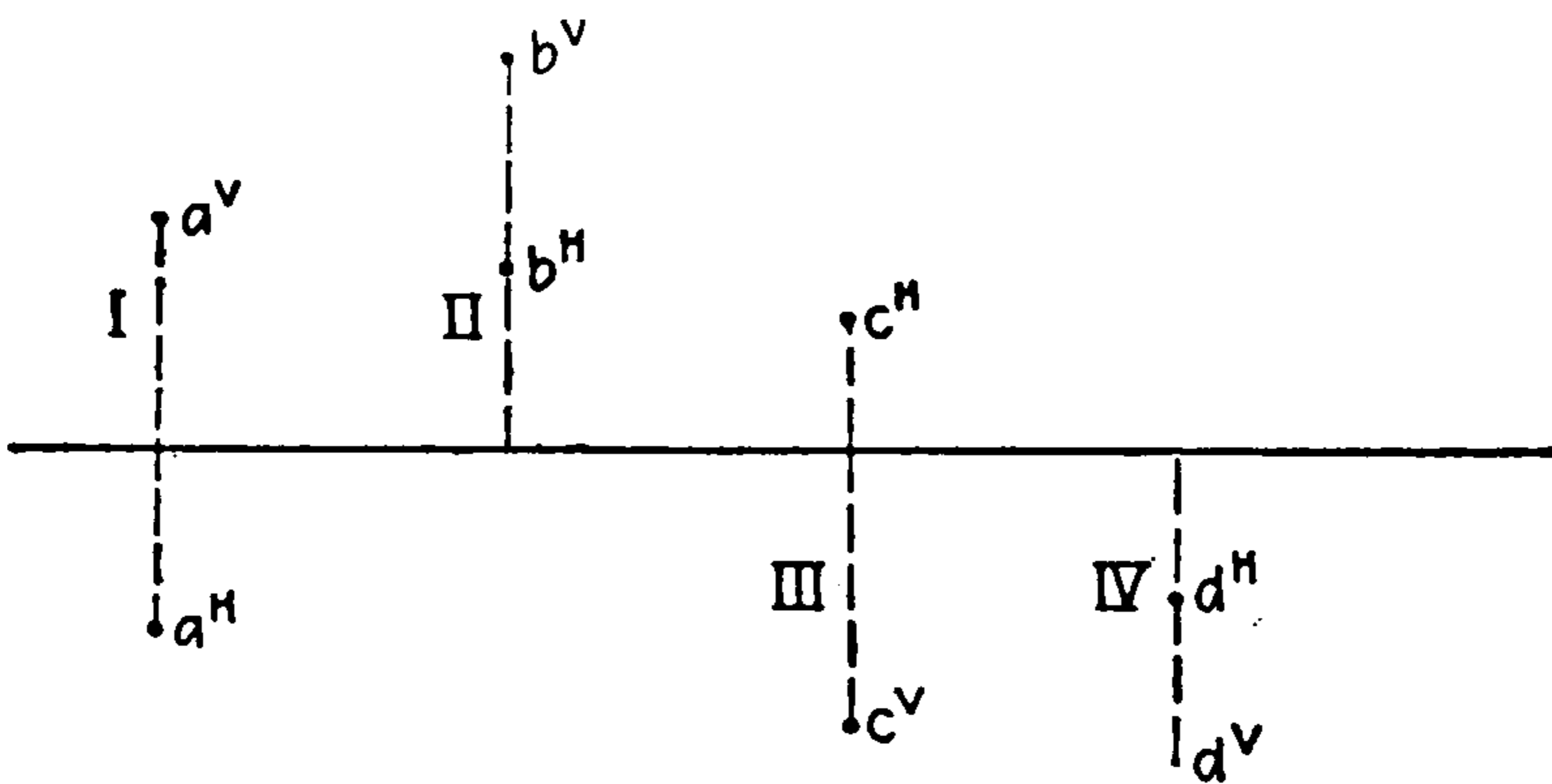


FIG. 10a.

**Proof.**—A point in **H** is zero distance from **H**, hence its **V**-projection is zero distance from **GL**. (Theorem II.)

DESIGNATION AND NOTATION OF POINTS

10. The actual point in space is designated by a capital, A, B, C, etc.

The **V**-projection is designated  $a^V$ ,  $b^V$ ,  $c^V$ , etc.

The  $H$ -projection is designated  $a^H$ ,  $b^H$ ,  $c^H$ , etc.

The  $P$ -projection is designated  $a^P$ ,  $b^P$ ,  $c^P$ , etc.

Revolved Positions.—If a point is taken from its original position, and placed in a new one, it is designated by a subscript, as  $a_1^V$ , or  $a_1^H$ . If there be a second revolution, it becomes  $a_{11}^V$ , or  $a_{11}^H$ , etc.

### NOTATION OF A POINT

There are three items in the notation of a point:

1. The distance from the margin, measured along  $GL$  to the right.

2. The distance above or below  $GL$  (+ or -) of its  $V$ -projection.

3. The distance above or below  $GL$  (+ or -) of its  $H$ -projection.

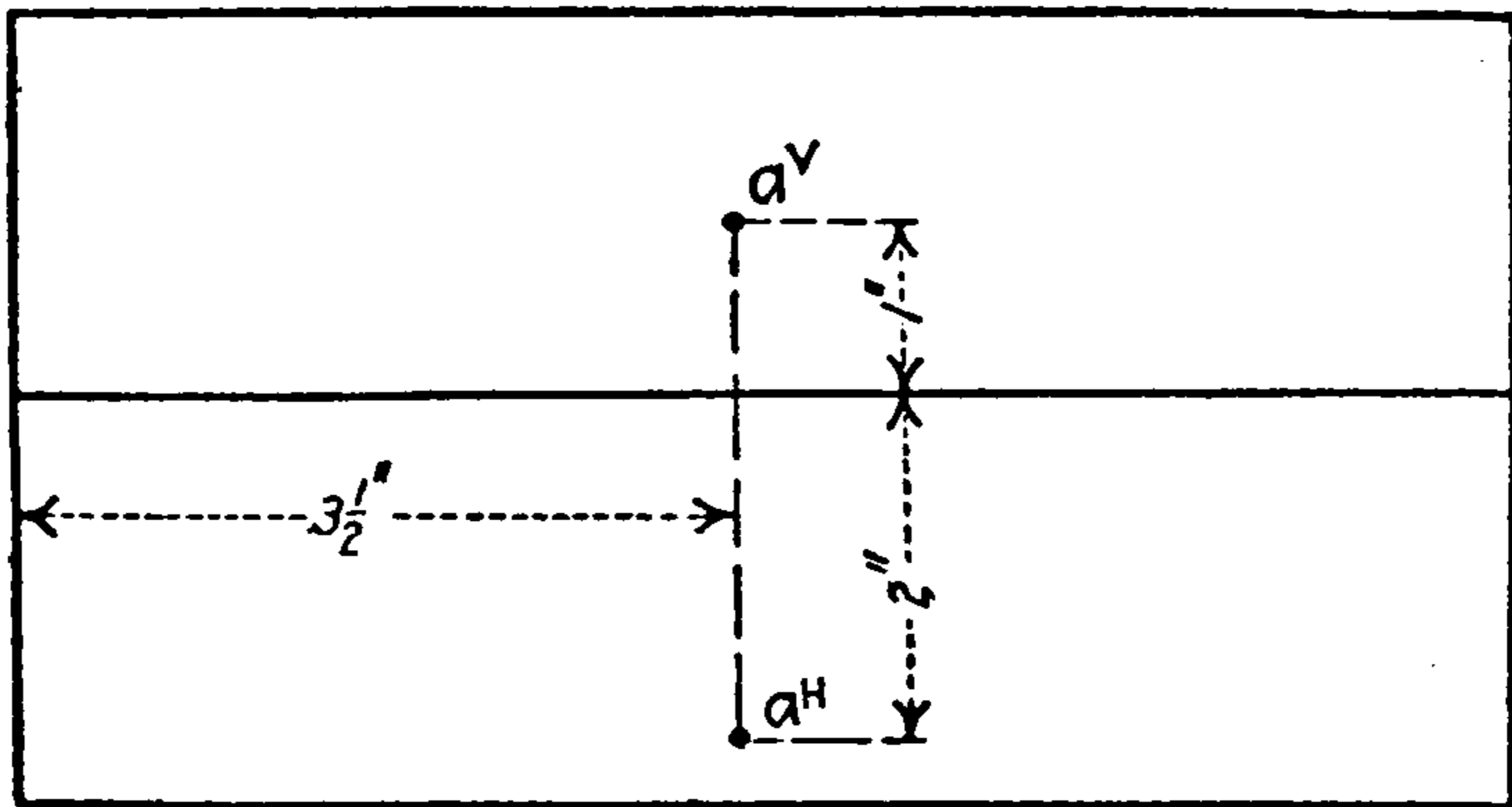


FIG. 11.

Example.—Draw on the flat the projections of  $A(3\frac{1}{2} + 1 - 2)$ .

1. Lay off a point on  $GL$ ,  $3\frac{1}{2}$  in. to the right of the margin.

2. Locate  $a^V$  1 in. above (+ 1) this point on  $GL$ .

3. Locate  $a^H$  2 in. (- 2) below this point on  $GL$ .

Notes.—1. Plus always means above  $GL$ , Minus always means below  $GL$ .

2. First locate the projections of the point on paper, and then try to imagine it in space.

3. The unit for the drawing board is 1 in., and for blackboard work 5 in. is a good scale for the exercises in this book.

### LAYOUT FOR DRAWING BOARD

11. A convenient layout is shown in Fig. 12. A sheet  $15 \times 20$  in. is given a  $1\frac{1}{2}$ -in. margin on the left for binding, and a  $\frac{1}{2}$ -in. margin on the other three sides, leaving a working space of  $14 \times$

18 in. This space is divided into six rectangles, each  $6 \times 7$  in., with *GL* drawn through the middle of each.

**Inking Plates.**—It is not advisable to ink plates, except the border lines, ground lines, and lettering. Good pencil work and neat, legible lettering should be insisted on.

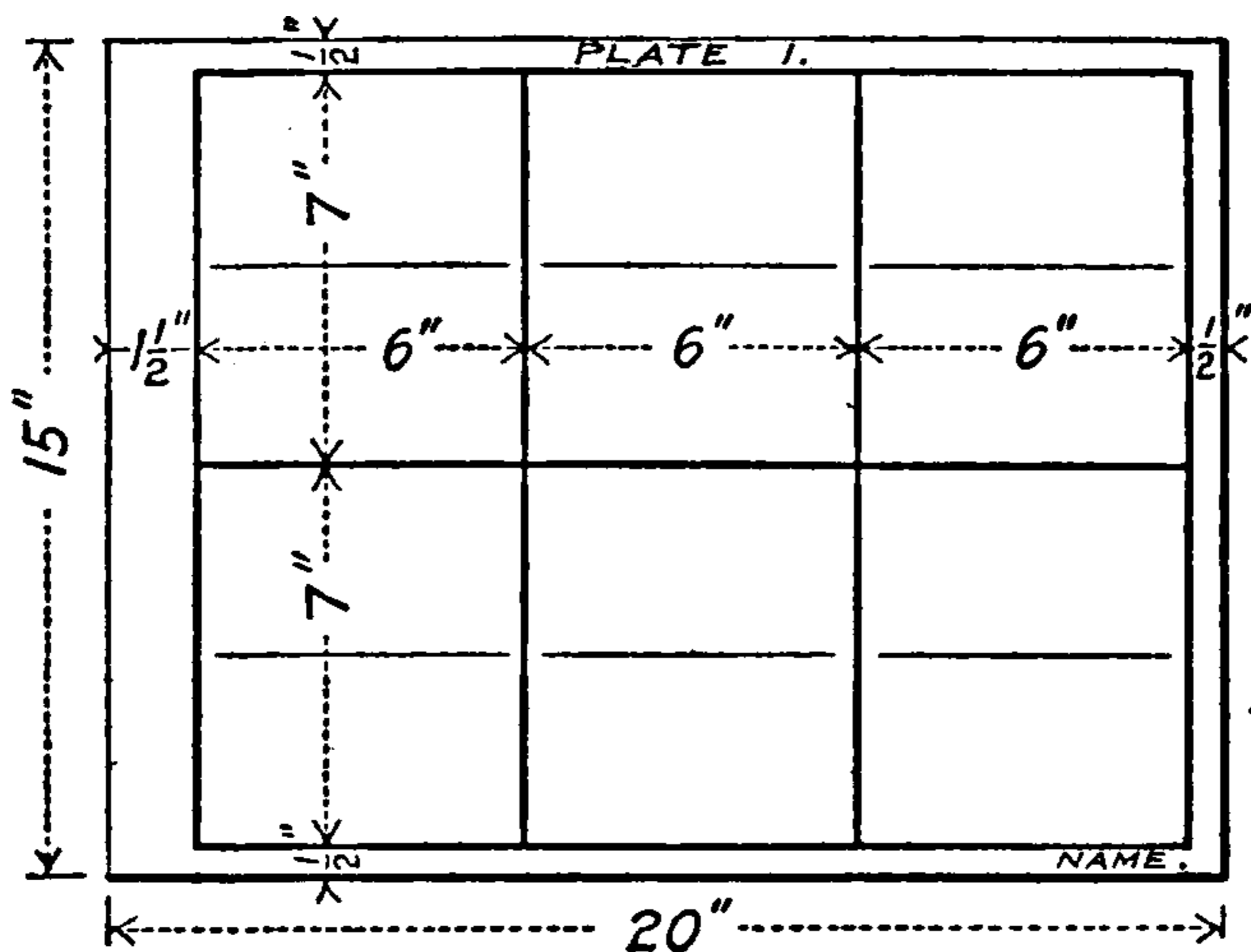


FIG. 12.

**Projectors.**—The perpendicular lines running from the projections to *GL* are called projectors and should be light dotted lines. See Figs. 10 and 11.

## 12. EXERCISES IN POINT REPRESENTATION

Locate on the flat and designate in space (see the Roman Numerals, etc., in Fig. 10a) the following points:

1.  $A(\frac{3}{4} - 1 + 2)$ ,  $B(1\frac{1}{2} + \frac{1}{2} + 2)$ ,  $C(2\frac{1}{4}, 0 + 1)$ ,  $D(3 + 1 - 1)$ ,  $E(3\frac{3}{4} - 2 - \frac{1}{2})$ ,  $F(4\frac{1}{2}, 0, 0)$ , and  $G(5\frac{1}{4} + 1\frac{1}{2}, 0)$ .
2.  $K(\frac{3}{4} + \frac{1}{2} - 2)$ ,  $L(1\frac{1}{2} - 1 - 2)$ ,  $M(2\frac{1}{4} - 1\frac{1}{2}, 0)$ ,  $N(3 - 1 + 2)$ ,  $O(3\frac{3}{4} + 2 + 1)$ ,  $P(4\frac{1}{2} + 1, 0)$ ,  $Q(5\frac{1}{4}, 0 + 1)$ .
3.  $A(\frac{3}{4}, 1$  behind  $V$ ,  $2$  below  $H)$ ,  $B(1\frac{1}{2},$  in  $H$ ,  $2$  before  $V)$ ,  $C(2\frac{1}{4}, 1$  before  $V$ ,  $1$  above  $H)$ ,  $D(3, 1$  behind  $V$ ,  $2$  above  $H)$ ,  $E(3\frac{3}{4},$  in  $GL)$ ,  $F(4\frac{1}{2}, 1$  before  $V$ ,  $2$  above  $H)$ ,  $G(5\frac{1}{4},$  in  $V$ ,  $2$  below  $H)$ .
4.  $K(\frac{3}{4},$  in  $H$ ,  $2$  behind  $V)$ ,  $L(1\frac{1}{2}, 1$  before  $V$ ,  $2$  below  $H)$ ,  $M(2\frac{1}{4}, 1$  behind  $V$ ,  $1\frac{1}{2}$  above  $H)$ ,  $N(3, 2$  before  $V$ ,  $1$  above  $H)$ ,  $O(3\frac{3}{4}$  in  $GL)$ ,  $P(4\frac{1}{2}, 1$  behind  $V$ ,  $2$  below  $H)$ ,  $Q(5\frac{1}{4}, 1\frac{1}{2}$  above  $H$ , in  $V)$ .

Locate in the flat the following points:

5.  $A$ (in  $IV$ ,  $1$  to right,  $1$  from  $V$ ,  $2$  from  $H)$ ,  
 $B$ (in  $I$ ,  $2$  to right,  $2$  from  $V$ ,  $1$  from  $H)$ ,  
 $C$ (in  $II$ ,  $3$  to right,  $1\frac{1}{2}$  from  $V$ ,  $1\frac{1}{2}$  from  $H)$ .
6.  $D$ (in  $H$ ,  $1$  to right,  $1$  behind  $V)$ ,  
 $E$ (in  $III$ ,  $2$  to right,  $2$  from  $V$ ,  $1$  from  $H)$ ,  
 $F$ (in  $I$ ,  $3$  to right,  $1$  from  $V$ ,  $1\frac{1}{2}$  from  $H)$ .
7.  $G$ (in  $II$ ,  $1$  to right,  $1\frac{1}{2}$  from  $H$ ,  $1\frac{1}{2}$  from  $V)$ .

- K(in  $V$ , between  $III$  and  $IV$ , 2 to right, 2 from  $H$ ),  
 L(in  $H$ , between  $I$  and  $IV$ , 3 to right, 1 from  $V$ ).  
 8. M(in  $IV$ , 1 to right, 2 from  $H$ , 2 from  $V$ ),  
 N(in  $III$ , 2 to right, 1 from  $V$ ,  $1\frac{1}{2}$  from  $H$ ),  
 O(in  $H$ , between  $II$  and  $III$ , 3 to right, 2 from  $V$ ).

## LINES

**13.** A straight line is the path of a point that moves constantly in one direction. Any two points therefore of a straight line are sufficient to determine it. Also, the projection of that line will contain the projections of the points. Until the subject of curved lines is taken up the term "line" will be understood to mean a straight line.

**Theorem V.**—Two projections of a line will, in general, determine the position of that line in space.

**Proof.**—Pass a plane through the  $V$ -projection of some line, and make it perpendicular to  $V$ . Also pass a plane through the  $H$ -projection of the same line and make the plane perpendicular to  $H$ . These planes will intersect in a line, and that line must be the given line.

**Theorem VI.**—If a line is parallel to  $H$ , its  $H$ -projection is parallel and equal to the line, and its  $V$ -projection is parallel to  $GL$ .

*Prove This.*—What would be the statement if the line were parallel to  $V$ ?

**Theorem VII.**—If a line is parallel to  $H$  and  $V$ , both of its projections will be parallel to  $GL$ .

**Theorem VIII.**—If a line is perpendicular to  $H$ , its  $H$ -projection will be a point, and its  $V$ -projection will be perpendicular to  $GL$ .

What would be the statement for a line perpendicular to  $V$ ?

**Theorem IX.**—If a line is oblique to both  $H$  and  $V$ , both projections will be oblique to  $GL$ , except when the line lies in a profile plane.

## 14. PROJECTIONS ON THE PROFILE PLANE

The projections of the solid shown in Fig. 13 do not completely identify the object, because the ends (bases) lie in a profile plane.

The end view (sometimes called the right section) might be one of many forms and in different positions, as the shapes given in Fig. 14. Thus the profile is sometimes necessary to complete the drawing.

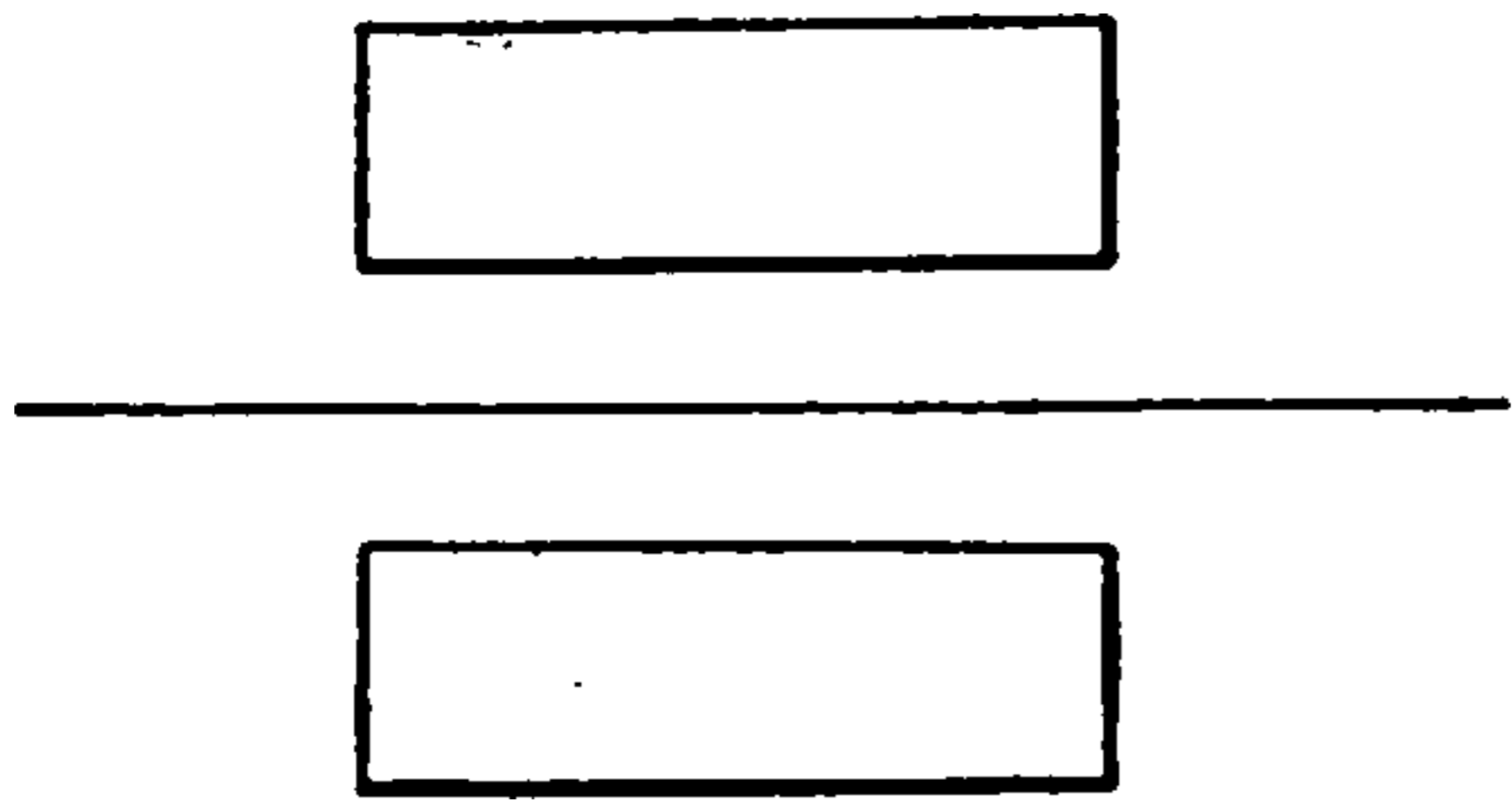


FIG. 13.

In drawing views in profile, the right end view is usually taken, although sometimes both profiles are necessary, and some cases arise where only the left view is needed.

Taking the view at the right, we see the four dihedral angles as shown in Fig. 15. Looking along  $GL$  we see  $H$  and  $V$  in their true relation (perpendicular), and the four space angles as indicated by the Roman Numerals.



FIG. 14.

Assume  $P$  at any convenient location on the right. In the regular views,  $m^v$  gives the elevation and  $m^h$  the distance in front of  $V$ . In the profile view it will be seen that the same distances obtain. If the point lies in any other angle, the pro-

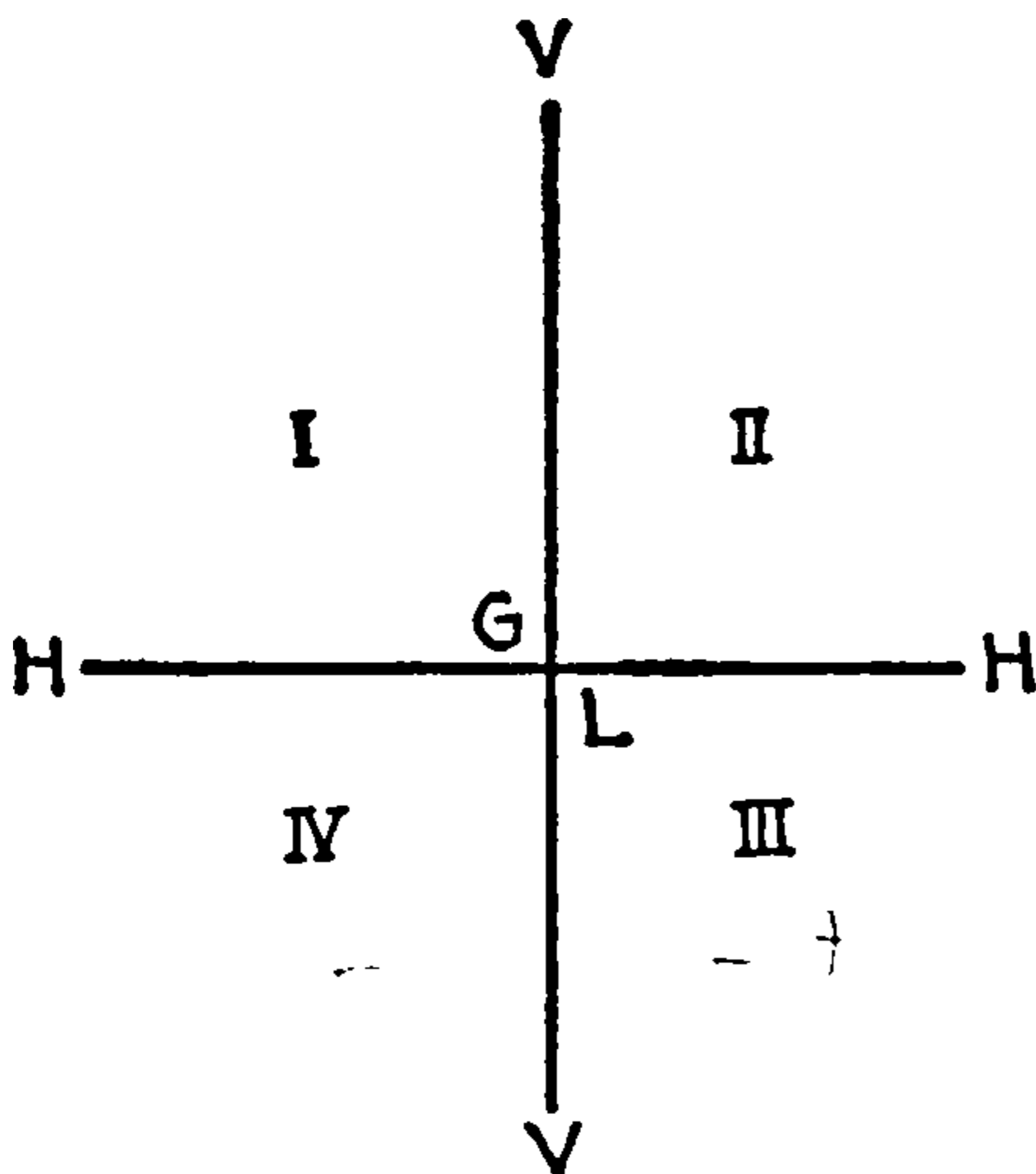


FIG. 15.

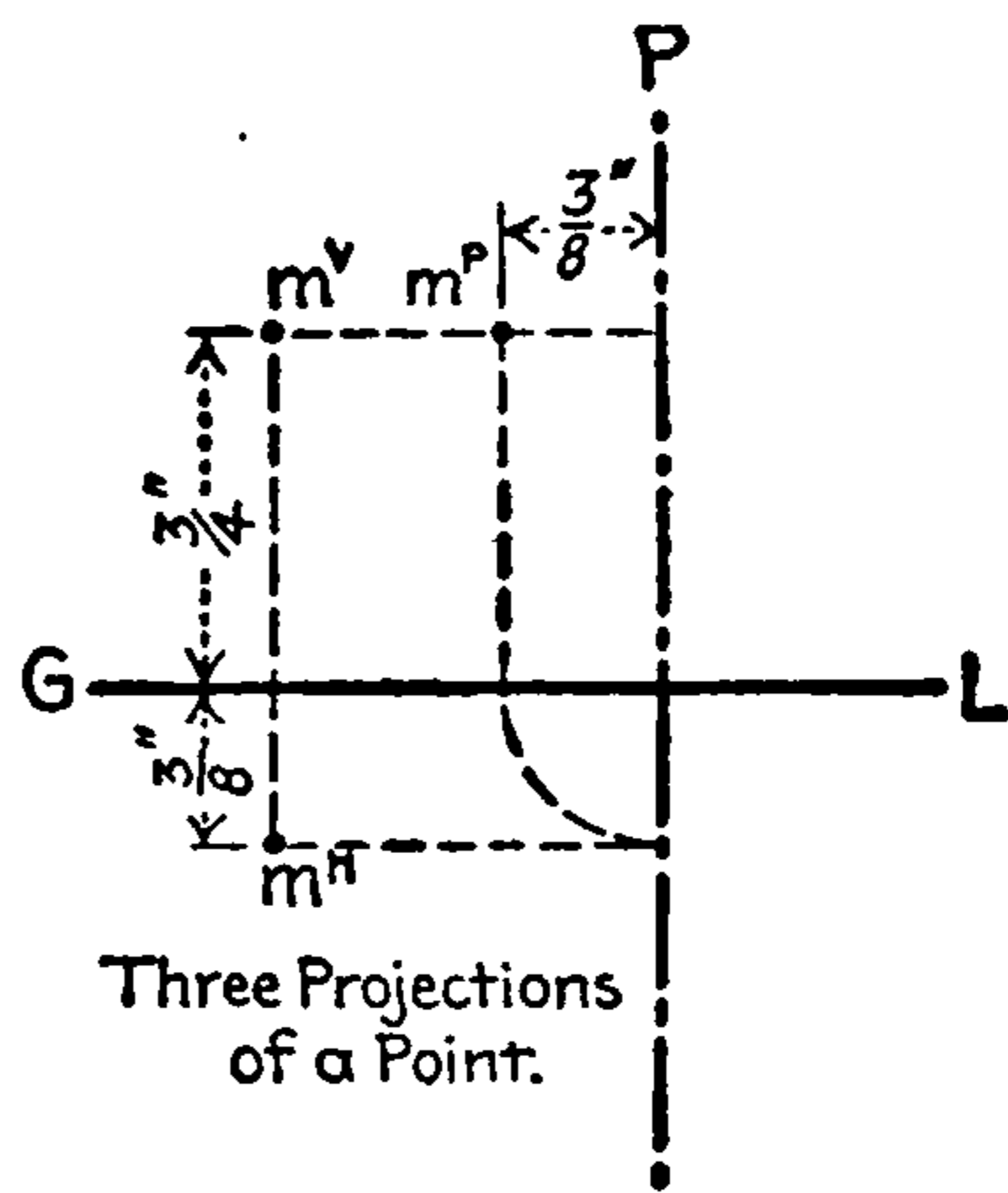


FIG. 15a.

cedure is the same. The rotation of the  $H$ -projection will determine whether the profile of the point is to lie to the right or left of  $P$ . If the  $H$ -projection is below  $GL$ , the profile will be on the left, as in Fig. 15a, and if it be above  $GL$ , the profile will be on the right. The elevation is always the same on both  $V$  and  $P$ .

**Rule.**—Project from the *V*-projection horizontally; project from the *H*-projection to *P*, and rotate to *GL*, thence up or down until the projectors intersect.

To obtain the profile of a line or plane figure, project the defining points and join them with lines. Fig. 16 gives a pictorial view of a line and its projections. Fig. 16*a* shows the same line in the flat.

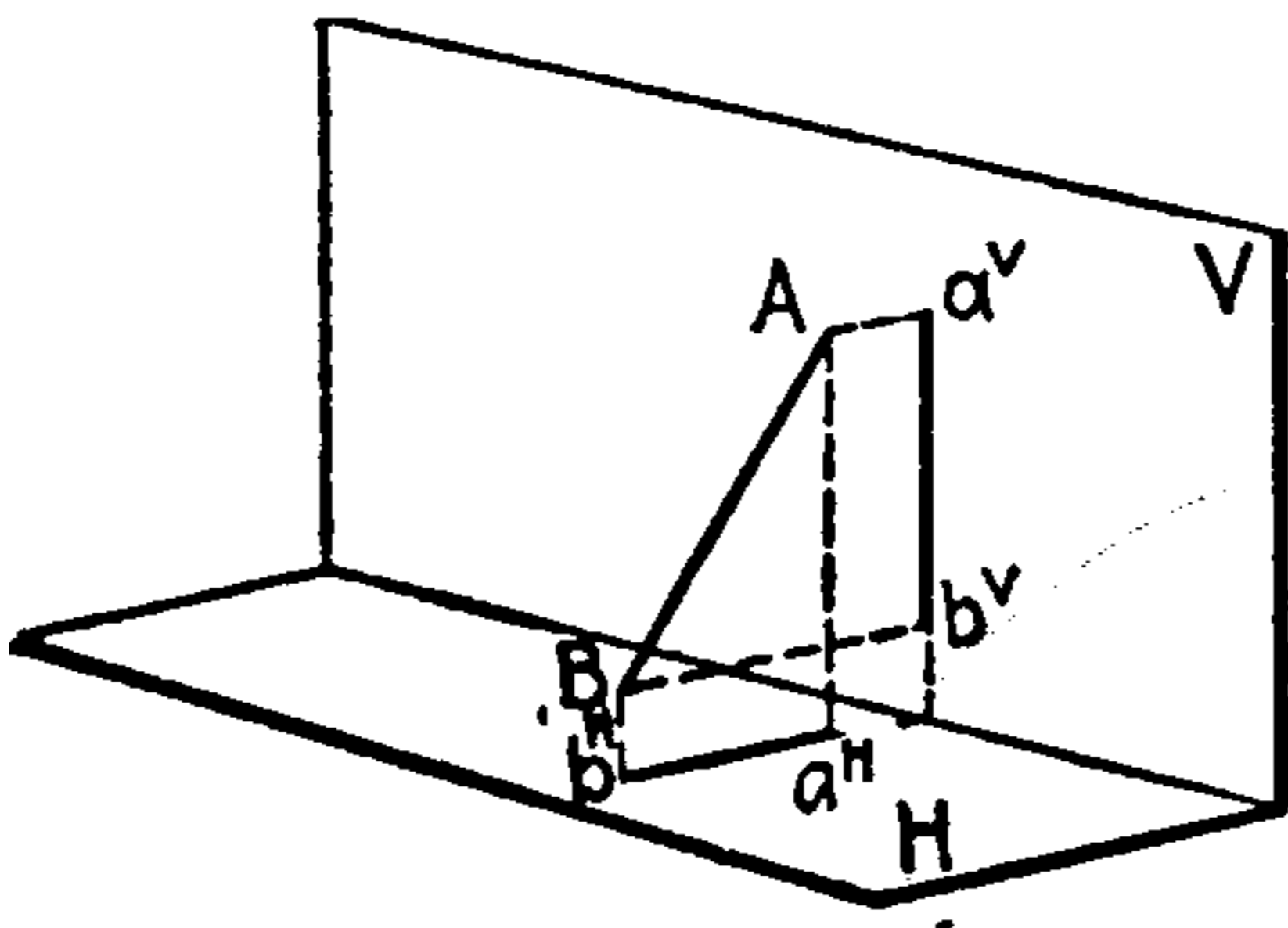
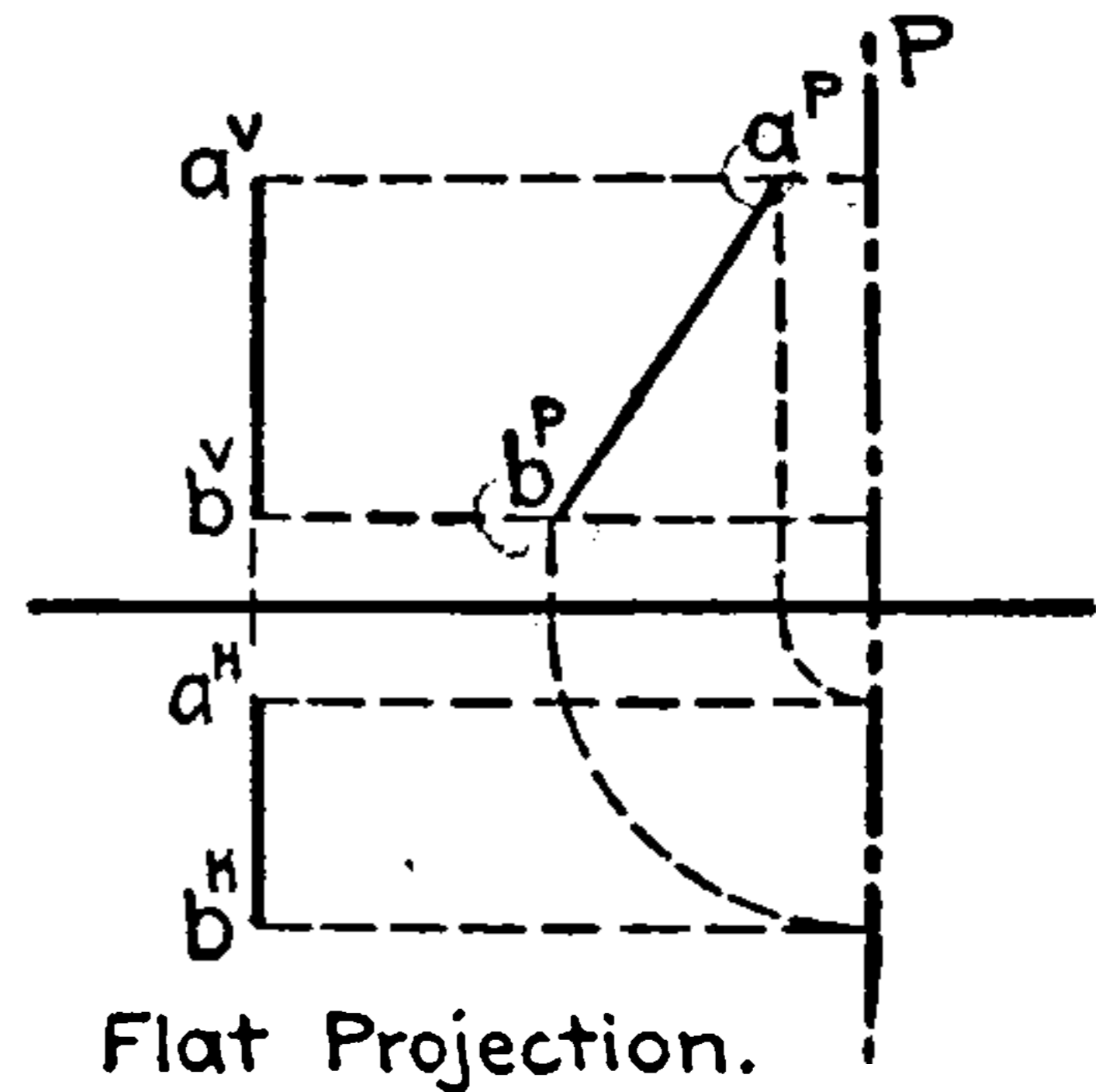


FIG. 16.


 FIG. 16*a*.

**Theorem X.**—If a line lies in *V*, its *V*-projection is the line itself, and its *H*-projection is in *GL*.

Why? What would be the statement for a line in *H*?

**Theorem XL**—If a point, line, or plane figure lies in *P*, its *V*-projection is in the *V*-trace of *P*, and its *H*-projection is in the *H*-trace of *P*.

## 15.

## EXERCISES

9. Draw the line  $A(1 + 2 - 2\frac{1}{2}) B(4 - 2 + \frac{1}{2})$ , and tell where it travels.
10. Draw the line  $C(1\frac{1}{2} - 2 + 1\frac{1}{2}) D(4\frac{1}{2} + 3 - 1)$ , and tell where it travels.
11. Draw and designate the lines:  $E(1 + 1 - 1) F(3 + 2 - 3)$ ,  $G(3\frac{1}{2} + 1 + 1\frac{1}{2}) K(4\frac{1}{2} + 2 - \frac{1}{2})$ ,  $L(5 - 1 - 2) M(5\frac{3}{4} - 3 - 1)$ .
12. Draw and designate the lines:  $A(1 - 2 + 2) B(2\frac{1}{2} - \frac{1}{2} + 1)$ ,  $C(3 + 1 + 1) D(4 + 2 + 2)$ ,  $E(4\frac{1}{2} - 1 - 3) F(5\frac{1}{2} + 1 + 1)$ .

Draw and designate the lines in the following:

13.  $K(1 + 1 - 1) L(3 + 3 - 1)$ ,  $M(2\frac{1}{2} - 2 + 2) N(4 - 2 + 1)$ , and  $O(4 + 2 - 1) P(5\frac{1}{2} + 3 + 3)$ .
14.  $A(1 + 3 + \frac{1}{2}) B(3 + 1 + \frac{1}{2})$ ,  $C(2 - 3 - 2) D(4 - 3 - 1)$ ,  $E(4\frac{1}{2} + 2 - 1) F(4\frac{1}{2} + 2 - 3)$ , and  $G(5\frac{1}{2} - 1 + 1) K(5\frac{1}{2} - 1 + 3)$ .
15.  $K(\frac{1}{2} - 1 + 1) L(2\frac{1}{2} - 3 + 1)$ ,  $M(2 - \frac{1}{2} - 1) N(4 - \frac{1}{2} - 1)$ , and  $O(3 + 2 + 1) P(5 + 2 + 3)$ .

Tell what angles these lines traverse:

16.  $A(1 + 2 - 1) B(3 - 1 - 3)$ ,  $C(2 + 3 + 1) D(4 - 2 - 2)$ , and  $E(4 - 1 + 2) F(5\frac{1}{2} + 2 - 2)$ .
17.  $G(1 - 1 - 3) K(2 + 2 + 3)$ ,  $M(2 - 3 + 1) N(3\frac{1}{2} - 1 - 1)$ , and  $O(4 + 2 - 1) P(5\frac{1}{2} + 1 + 3)$ .



18. From A(1 - 1 + 2) draw a line 2 in. long, parallel to V, and inclined  $60^\circ$  to H, downward. Does it touch H? V? Draw one  $60^\circ$  upward,  $2\frac{1}{2}$  in. long. Where does it terminate?
19. From B(2 - 2 - 1) draw a line  $\parallel$  to H, inclining  $30^\circ$  to V. How far is it on this line from B to V?
20. Draw the P-projections of the lines A(1 + 1 - 2) B(1 + 3 - 1) and C(2 - 1 + 1) D(2 - 2 + 1). Locate the profile at  $4\frac{1}{2}$  on GL.
21. Draw the P-projections of E(1 + 1 + 1) F(1 - 1 - 2) and G(2 + 2 + 1) K(2 - 1 + 1). Profile  $4\frac{1}{2}$ .
22. Draw the triangle L(2 + 1 - 1) M(2 + 2 +  $\frac{1}{2}$ ) N(2 - 1 -  $1\frac{1}{2}$ ) in its true size. Profile  $4\frac{1}{2}$ .
23. What are the sizes of the angles and sides of the triangle K(2 + 1 + 1) O(2 - 1 +  $1\frac{1}{2}$ ) P(2 -  $\frac{1}{2}$  - 2)?
24. Draw the projections of a prism parallel to H and V, of 2-in. altitude, and its base a pentagon inscribed in a  $1\frac{1}{2}$ -in. circle in P.
25. From M(1 - 2 - 1) draw a line  $\parallel$  to H, inclined  $45^\circ$  to V, 3 in. long, toward V to the right. Also from M draw a line  $\parallel$  to P,  $30^\circ$  to V, 3 in. long. Where do they terminate?

### REPRESENTATION OF PLANES

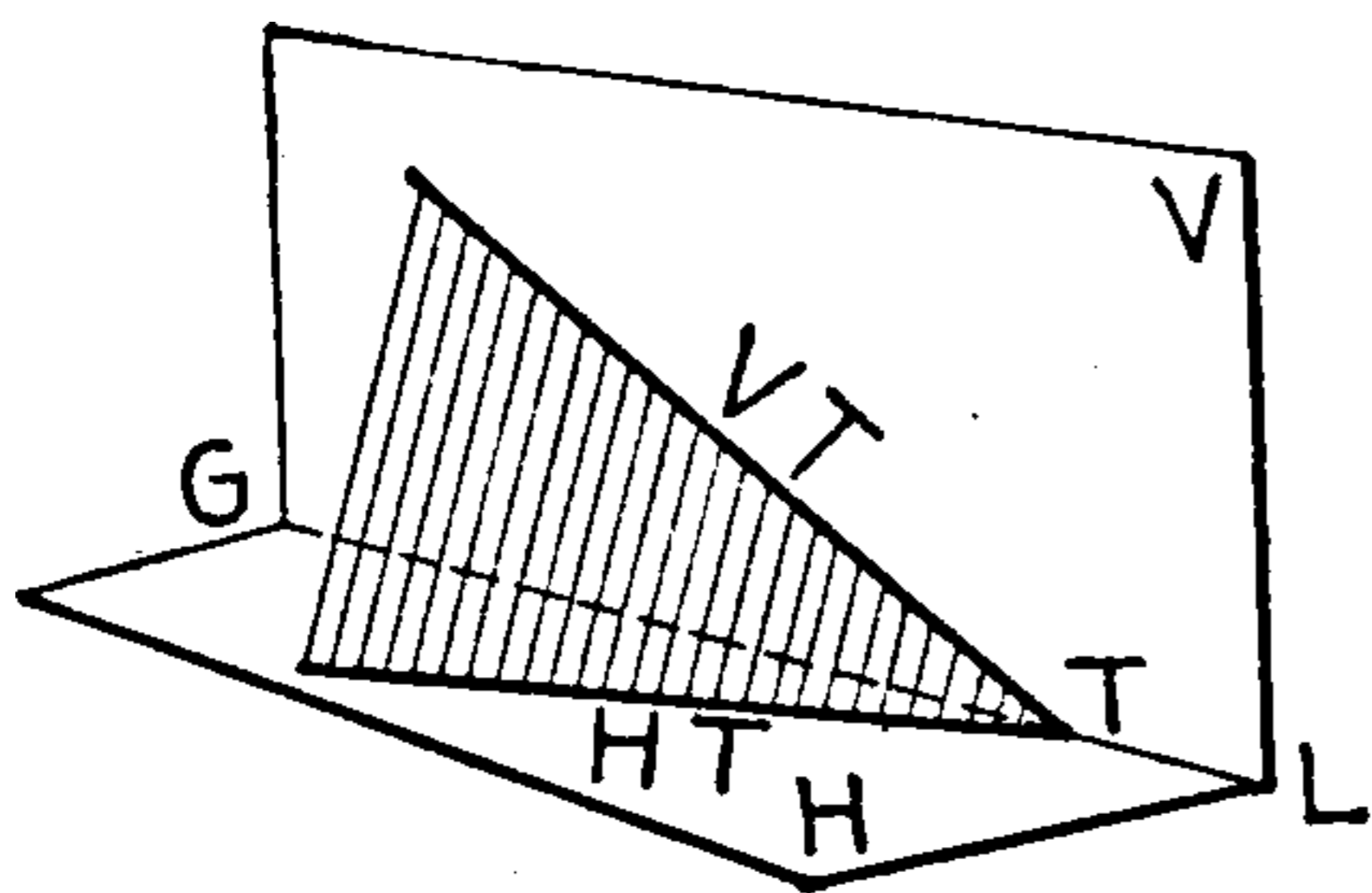


FIG. 17.

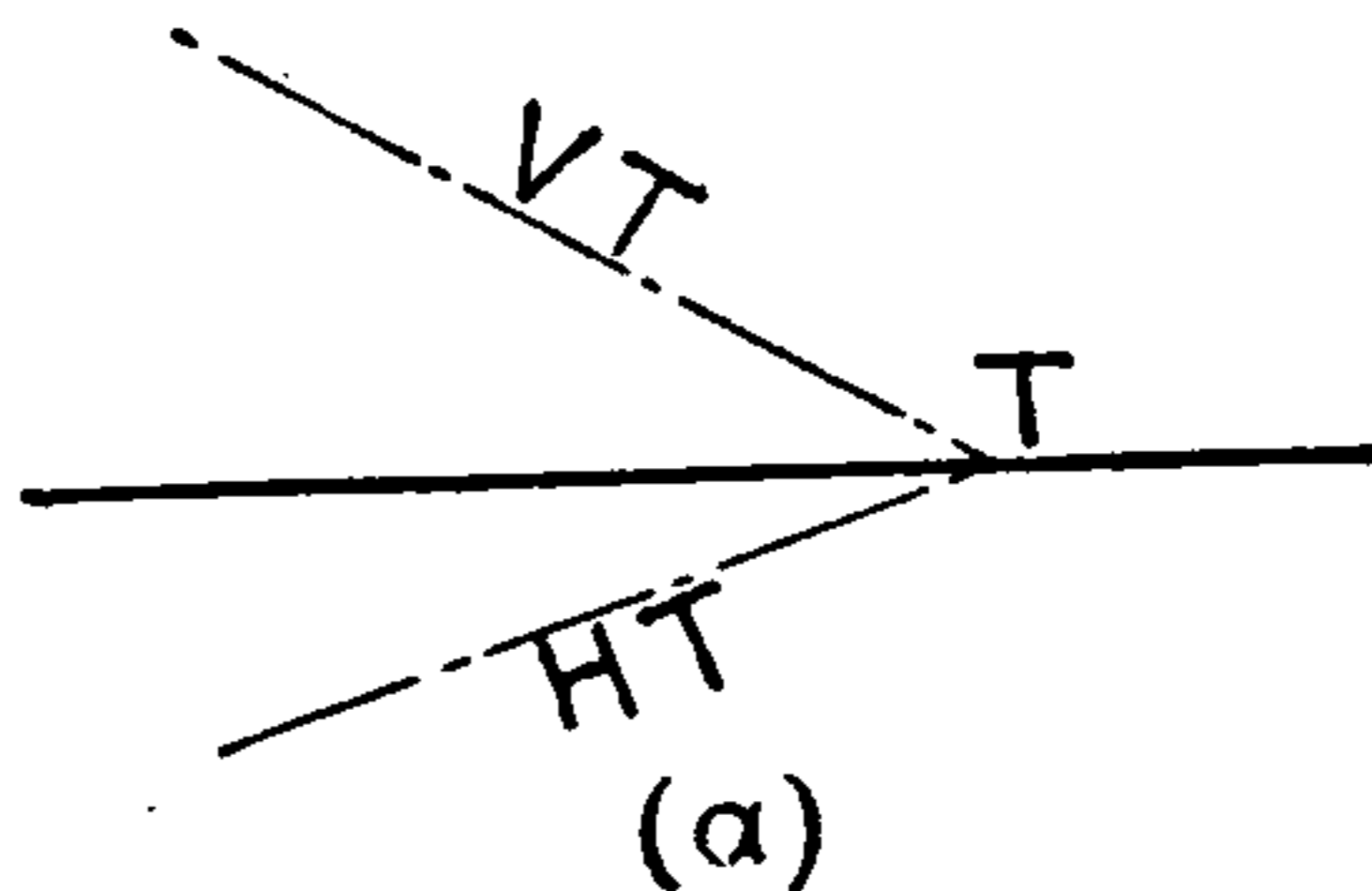


FIG. 17a.

**16.** A plane other than  $H$ ,  $V$ , or  $P$  is shown by its traces, as its intersections with the three planes are called. In most cases the  $H$ - and  $V$ -traces are sufficient, and the  $P$ -trace is chiefly used as an auxiliary to assist in the solving of problems. Fig. 17 shows an oblique plane,  $T$ . The plane  $T$  cuts  $V$  in the line  $VT$ , called its vertical trace. It cuts  $H$  in the line  $HT$ , called its horizontal trace. Fig. 17a shows the flat representation of the plane  $T$ .  $VT$  is one line of  $T$  and  $HT$  is an entirely different one.

**Theorem XII.**—The two traces of a plane completely determine the plane.

**Note.**—The student must on no account think that  $VT$  and  $HT$  are projections of the same line.  $VT$  lies in  $V$ , hence its  $H$ -projection is in  $GL$ . Therefore, if a point is assumed in the plane with its  $V$ -projection in  $VT$ , its  $H$ -projection is in  $GL$ . What is the case then with  $HT$ ?



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NOTATION

When the traces intersect in the problem limits, three points are necessary:

1. In the *V*-trace.
2. The *GL* intersection.
3. In the *H*-trace.

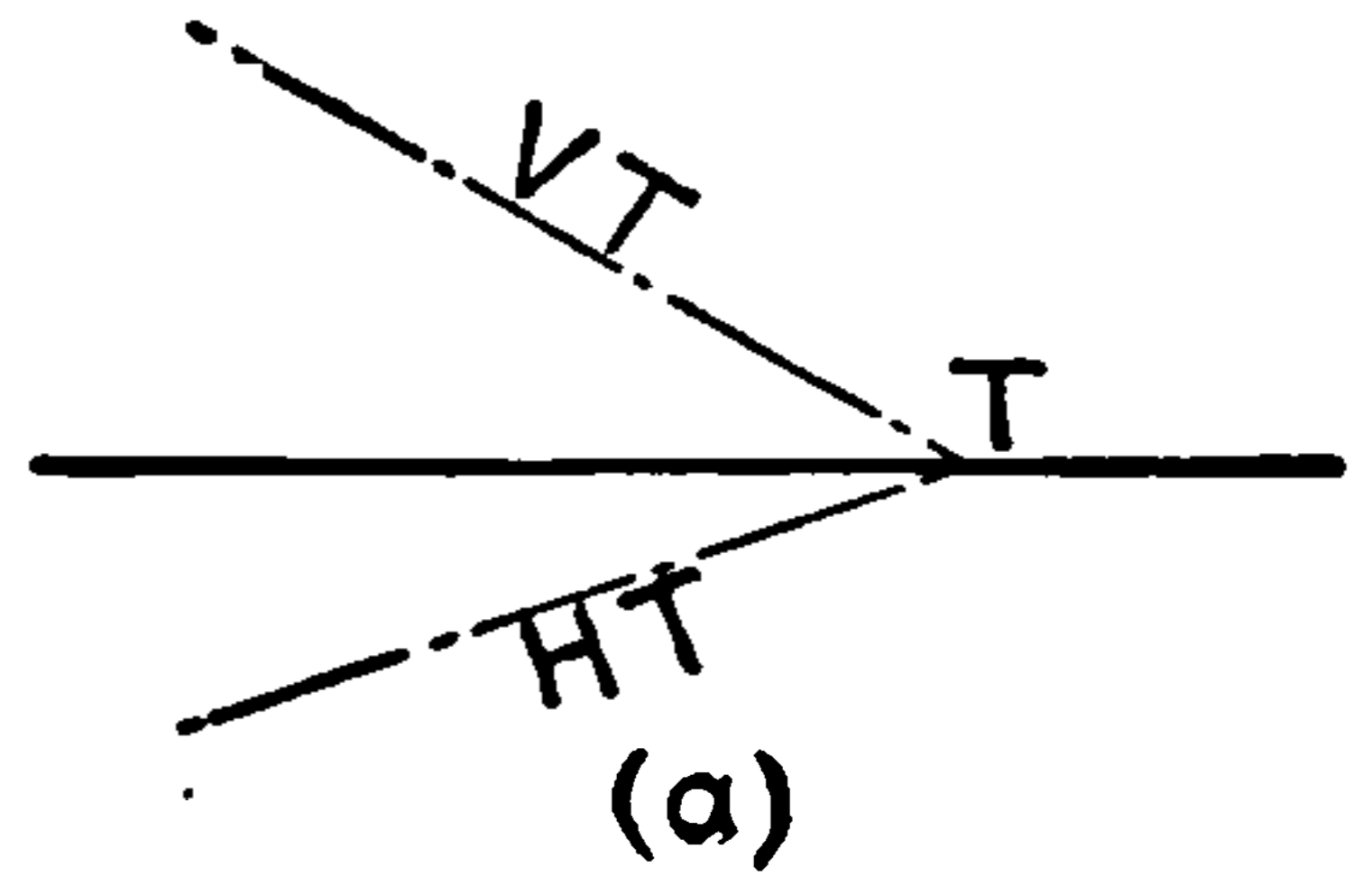


FIG. 17a.—The plane T (1 + 2) 5(1 - 1).

All these points are in *H* or *V*, hence there are zeros in the complete notation. These zeros are understood and omitted in our notation, as being cumbersome.

1. (1 + 2) means a point in VT, 1 in. to right, 2 in. up.
2. 5 means the *GL* intersection, 5 in. to right.
3. (1 - 1) means a point in HT, 1 in. to right, 1 in. down.

Rule.—*First*.—A point in the *V*-trace. *Second*.—The *GL* intersection. *Third*.—A point in the *H*-trace.

SPECIAL CASES

Planes parallel to *GL* have no *GL* intersection and their traces are parallel to *GL*, and their notation is shown in Fig. 18.

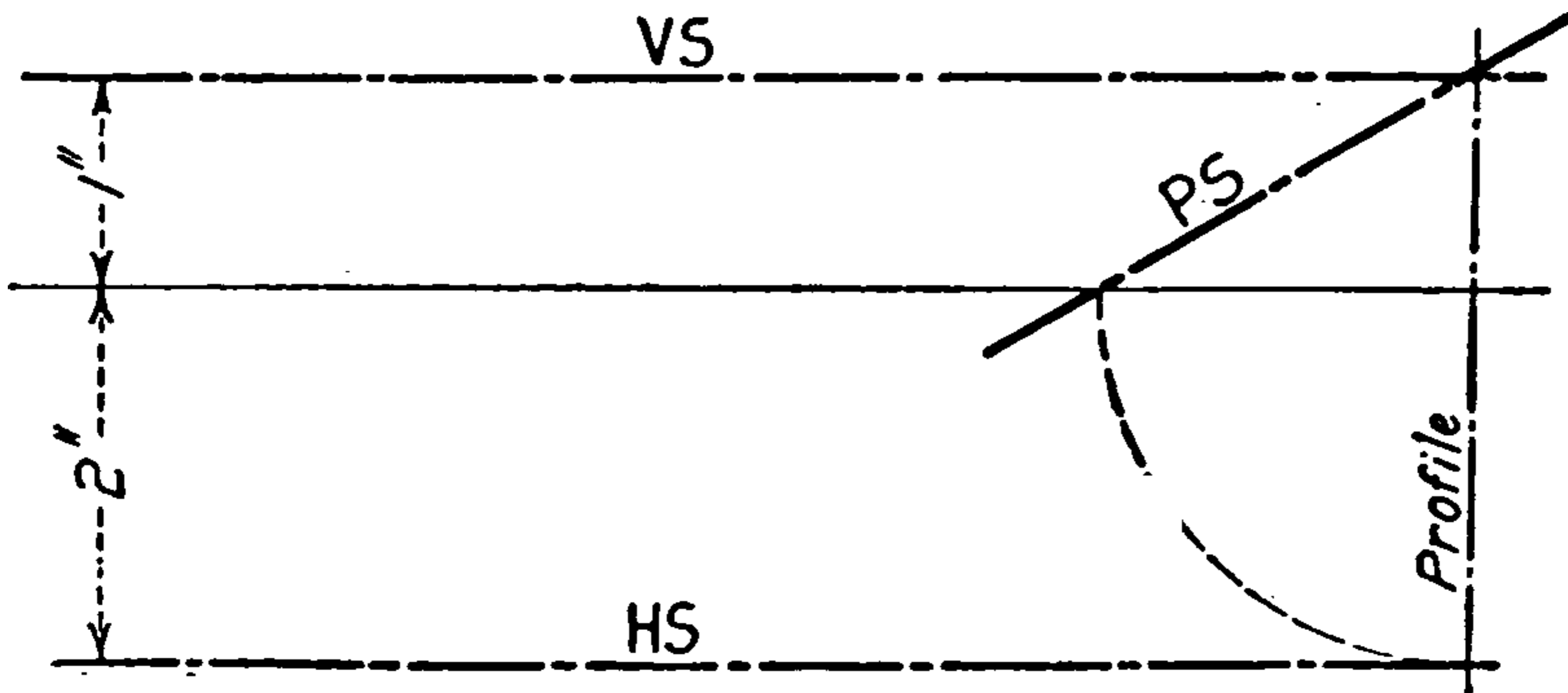


FIG. 18.—The plane S(+ 1) ∞ (- 2).

Fig. 18 shows the plane S and its profile trace, which is frequently necessary with such planes.

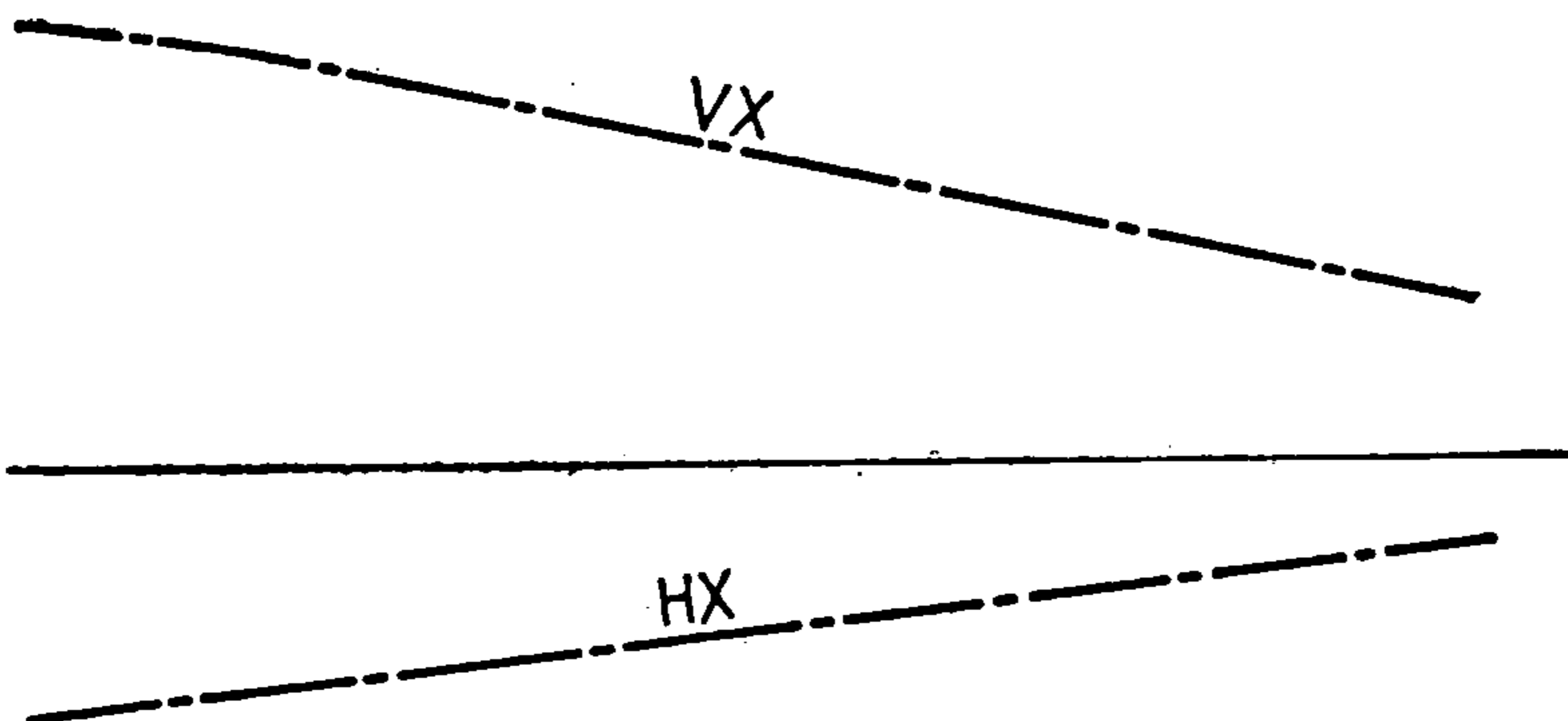


FIG. 19.—The plane X(1 + 3)(5 + 2)(1 - 1½)(5 - 1).

Planes parallel to  $H$  or  $V$  have but one trace, and that parallel to  $GL$ . They require special designation, but are seldom used except as auxiliary planes, and so it is not necessary to devise a special notation for them.

### OTHER PLANES

A plane oblique to  $GL$ , but not crossing it within the problem limits, must have four points to determine the two traces.

In this plane the first two points are in the  $V$ -trace and the second pair are in the  $H$ -trace. See Fig. 19.

A plane through  $GL$  has its  $V$ - and  $H$ -traces in  $GL$ , therefore its  $P$ -trace is its best exposition. It can be described also as a plane passing through  $GL$  making a certain angle with the horizontal, or containing some point.

### EXERCISES IN REPRESENTATION OF PLANES

18. Draw the traces of the following planes, and state their relations (parallel, perpendicular, oblique) to  $H$ ,  $V$  and  $P$ .

Draw the traces with a dot-and-dash line. In Ex. 28 and 29, place  $P$  at  $4\frac{1}{2}$ , and draw the  $P$ -traces.

26.  $T(3 + 2) 1(1 - 3)$ ,  $S(5 + 3) 3(3 - 3)$ .

27.  $X(3 + 3) 1(3 + 1)$ ,  $Y(3\frac{1}{2} + 2) 5(4 - 3)$ .

28.  $R(+ 1) \infty(+ 2)$ ,  $Q(- 1) \infty(+ 2)$ . Draw the  $P$ -traces.

29.  $S(- 2) \infty(- 1)$ ,  $T(+ 2) \infty(- 1)$ . Draw the  $P$ -traces.

30.  $X(1 + 1)(4 + 2) (1 + 1\frac{1}{2}) (4 + 3)$ ,  $U(2 - 3) 4(5\frac{1}{2} + 2\frac{1}{4})$ .

31. Draw the traces of the  $V$ -projecting plane of the line  $M(1 + 2 - \frac{1}{2}) N(2\frac{1}{2} + \frac{1}{2} - 1)$ , and of the  $H$ -projecting plane of the line  $A(3\frac{1}{2} - \frac{1}{2} - 2) B(5 + 2 + \frac{1}{2})$ .

32. Draw the traces of the  $V$ -projecting plane of  $C(1 + 3 - 1) D(3 - 1 + 3)$  and of the  $H$ -projecting plane of  $E(3\frac{1}{2} + 1 - 2) F(5 + 1 - 1)$ .

33. Draw the traces of the  $H$ -projecting plane of  $G(3 + 3 - 1) K(1 - 1 + 1)$ , and of the  $V$ -projecting plane of  $M(3\frac{1}{2} + 1 - 2) N(5\frac{1}{2} + 3 + \frac{1}{2})$ .

34. Draw the  $P$ -trace of  $S$ , which contains  $GL$  and  $O(2 + 2 - 1)$ , and of  $R$ , which contains  $GL$  and  $A(4 + 1 + 1\frac{1}{2})$ .

35. Draw the  $P$ -traces of two planes,  $T$  and  $X$ , each containing  $GL$ , and each inclined  $60^\circ$  from the horizontal,  $T$  passing through  $I$  and  $III$ ,  $X$  passing through  $II$  and  $IV$ .

**Note.**—To find the  $P$ -trace of any plane, take the plane  $P$  at any convenient place, where it will intersect the  $H$ - and  $V$ -traces. Then treat the line connecting these intersections (the  $P$ -trace) as you would any other profile line, and revolve it as in Art. 14.

### THEOREMS RELATING TO LINES AND PLANES

19. **Theorem XX.**—Two lines parallel in space have parallel projections on any plane.

**Theorem XXI.**—Two lines perpendicular in space do not ordinarily have their projections perpendicular.

**Special Case.**—If one of these lines is parallel to  $H$ , the  $H$ -projections of the two lines will be perpendicular.

Fig. 20 shows a line  $AB$ , parallel to  $H$ , and  $BC$  (any one of four different lines), whose  $H$ -projection is perpendicular to  $a^H b^H$ . Any one of the lines  $BC$  will be perpendicular to  $AB$ .

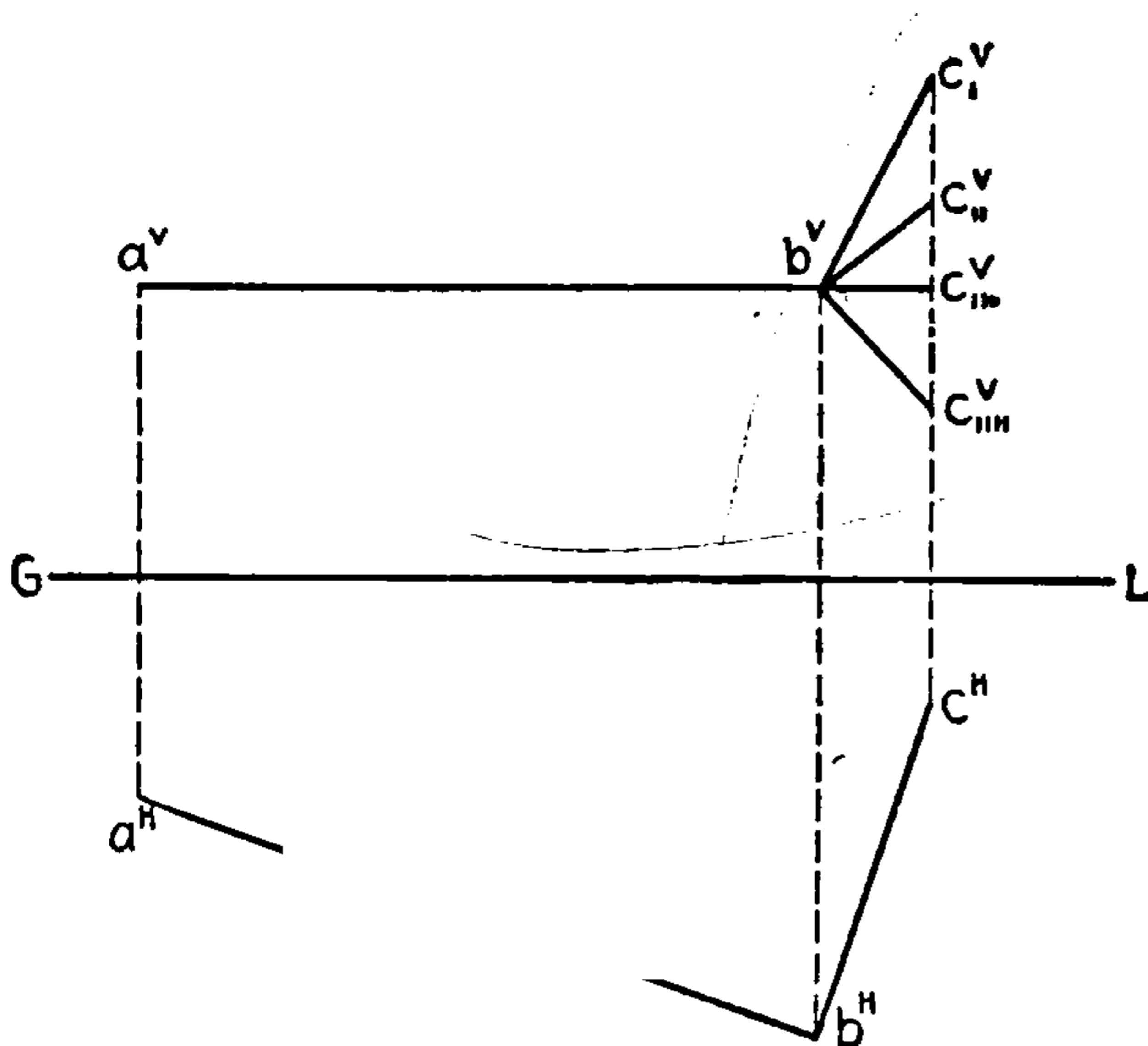


FIG. 20.

**Proof.**—If  $AB$  is parallel to  $H$ , any plane perpendicular to  $AB$  will be perpendicular to  $H$ , and will therefore be a projecting plane on  $H$  of all lines contained in it. Let the student finish the reasoning to show that all lines, whose  $H$ -projections are  $b^H c^H$ , will be perpendicular to  $AB$ .

**Theorem XXII.**—Two intersecting lines in space must have their  $H$ - and  $V$ -projections intersect in points in a line perpendicular to  $GL$ . Prove.

**Theorem XXIII.**—Two intersecting lines, both parallel to  $H$ , are projected on  $H$  in lines parallel and equal to themselves, and the included angle will be equal in projection to the angle in space. Prove.

**Theorem XXIV.**—A line, that is parallel to an oblique plane, is not, in general, projected parallel to the traces. The exception is only possible when the traces are both parallel or both perpendicular to  $GL$ .

**Theorem XXV.**—A line, that is perpendicular to a plane, always has its projections perpendicular to the respective traces of the plane.

In Fig. 21 let  $BC$  be perpendicular to  $T$ . To prove that  $bC$  is perpendicular to  $tt$ , the  $H$ -trace of  $T$ .

Let fall a perpendicular,  $Bb$ , from  $B$  to  $H$ . The lines  $BC$  and  $Bb$  determine a plane perpendicular to both  $H$  and  $T$ , and therefore to their intersection,  $tt$ . As the plane is perpendicular to  $tt$ , its intersection  $bC$  will be; that is, the  $H$ -projection is

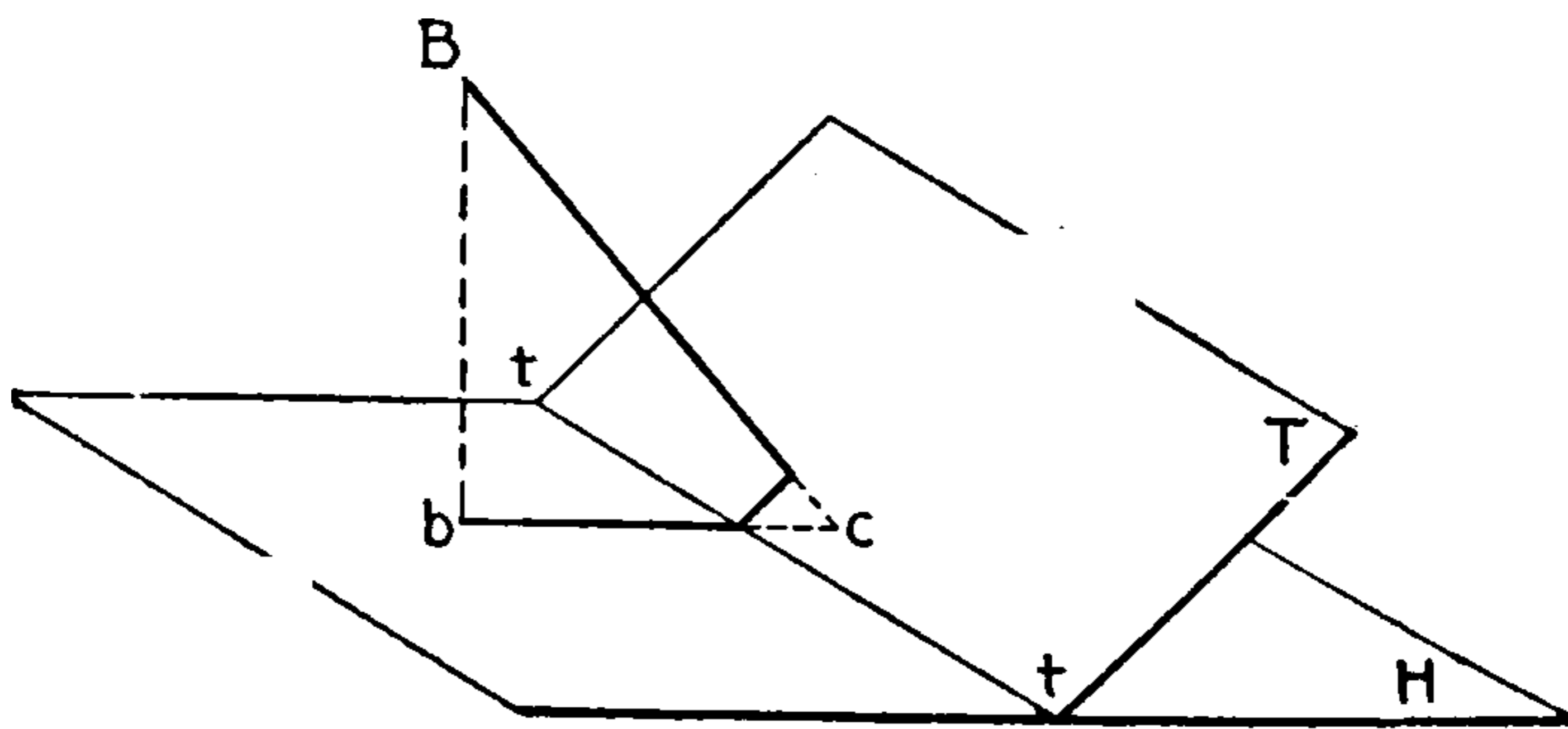


FIG. 21.

perpendicular to the  $H$ -trace. The same could be proved for the  $V$ -projection and  $V$ -trace.

**Theorem XXVI.**—Conversely, a line, whose projections are perpendicular to the respective traces of a plane, is perpendicular to the plane, with one exception.

**Exception.**—If the plane is parallel to  $GL$ , the line, whose projections are perpendicular to its traces, may not be perpendicular to the plane. Prove this by the  $P$ -trace of such a plane, and the  $P$ -projection of such a line.

**Theorem XXVII.**—A line in any plane pierces  $H$  in the  $H$ -trace of the plane, unless the line is parallel to  $H$ . Similarly, it will pierce  $V$  in the  $V$ -trace of the plane in which it lies. Prove.

**Theorem XXVIII.**—If a line is parallel to  $H$ , it is parallel to the  $H$ -trace of any plane in which it lies. Similarly for  $V$ . Prove.



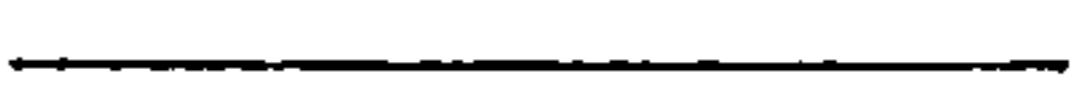

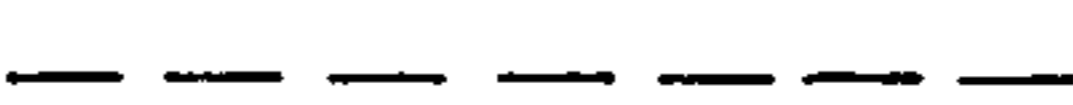



**Theorem XXIX.**—Any plane that is passed through one of two parallel lines either contains or is parallel to the other. Let the student prove this with a figure. Refer to Plane and Solid Geometry theorems.

## EXERCISES IN REVIEW

**20. Directions.**—All the exercises here given can be drawn in a space 6 in.  $\times$  7 in., with *GL* drawn horizontally across the middle.

Letter all points carefully, using small letters for all projections,  $a^V$ ,  $b^V$ , etc., for *V*-projections,  $a^H$ ,  $b^H$ , etc., for *H*-projections,  $a^P$ ,  $b^P$ , for *P*-projections.

Use the following conventions for all lines etc.

Given lines—solid, medium,		$\frac{1}{64}$ in.
Required lines—solid, heavy		$\frac{1}{32}$ in.
Auxiliary lines—solid, light		very fine.
Invisible lines—dotted, medium		$\frac{1}{64}$ in.
Projectors—dotted, light		very fine.
Paths of Revolution—the same.		
Traces of Given planes—medium		$\frac{1}{64}$ in.
Traces of Required planes—heavy		$\frac{1}{32}$ in.
Traces of Auxiliary planes—light		very fine.

All planes are considered transparent, and there will be no invisible lines until problems involving solids are given.  $\parallel$  and  $\perp$  are abbreviations for parallel and perpendicular.

36. Through  $A(5 - 1 + 2)$  draw  $AB \parallel$  to  $C(4 + 2 - 2)$   $D(2 + 1 - \frac{1}{2})$ .
  37. Through  $M(4 + 2 + 1)$  draw  $MN \parallel$  and equal to  $O(3 - 1 - 2)$   $P(1 + \frac{1}{2} - \frac{1}{2})$ .
  38. From  $A(4\frac{1}{2} - 2 + 1)$  draw two lines intersecting  $B(1 + 1 - 1)$   $C(3 + 3 - 2)$  in points other than  $B$  and  $C$ .
  39. Draw two lines  $EF$  and  $KL$  intersecting in the point  $D(3 - 1 + 1)$ . Connect these two lines with a line  $\parallel$  to  $H$ .
  40. Draw two lines  $MN$  and  $OP$  intersecting in the point  $K(2\frac{1}{2} + 3 - 1)$ . Connect these two lines with a line  $\parallel$  to  $V$ .
  41. Draw a line  $\perp$  to  $V$  intersecting  $A(1 + \frac{1}{2} - 2)$   $B(4 + 3 - 1)$  and  $C(1\frac{1}{2} + 2 - \frac{1}{2})$   $D(3\frac{1}{2} + 1 - 1)$ .
  42. Draw a line  $\perp$  to  $H$  intersecting  $E(1\frac{1}{2} - 3 + 2)$   $F(5 - 1 + 3)$  and  $G(1\frac{1}{2} - 1 + 2\frac{1}{2})$   $K(4 - 2 + 1)$ .
  43. Draw four lines at right angles to  $L(1 + 2 - 1)$   $M(3 + \frac{1}{2} - 1)$  through the point  $M$ .
  44. Through  $N(2 - 1 + 2)$  draw a line  $\perp$  to  $T(1 - 3)$   $5(3 + 3)$ .
  45. Through  $O(5 + 1 - 1)$  draw a line  $\perp$  to  $S(5 + 3)$   $3(1 - 3)$ .
  46. Through  $P(2, 0, 0)$  draw the traces of a plane  $R$  that is  $\perp$  to  $A(2 - 2 + 2)$   $B(4 - \frac{1}{2} + 1)$ .
  47. Is  $C(2 + 2 - 1)$   $D(2 + \frac{1}{2} - 2)$   $\perp$  to the plane  $Q(+ 3) \in (- 1)$ ?  
If not, draw the traces of  $T$ , that is  $\perp$  to  $CD$ .
- Draw the *P*-projections of the following points and lines, locating *P*  $4\frac{1}{2}$  in. to the right:
48.  $A(1 + 2 + 1)$ ,  $B(1\frac{1}{2} + 2 - 1)$ ,  $C(2 - 1 - 2)$ ,  $D(2\frac{1}{2} - 2 + 1)$ , and  $E(3, 0 + 1)$ .

49. The lines  $F(1 + 3 + 1)$   $G(1 - 1 + 1)$  and  $K(2 + 1 - 1)$   $G(2 - 2 - 1)$ . What relation are these lines to  $H$  and  $V$ ?
50.  $M(1 + 2 - 2)$   $N(1, 0 - 1)$ . What angles does this line make with  $H$  and  $V$ ?
51. Through  $O(2 - 3 + 1)$  draw a 3-in. line  $\parallel$  to  $P$ ,  $45^\circ$  to  $H$ . In what space angle does the line terminate?
52. Let  $A(3, 0 + 1\frac{1}{2})$   $B(4\frac{1}{2}, 0 + 1)$  be the diagonal of a square in  $H$ . With this square as the upper base of a cube, draw the projections of a cube. Draw the traces of the six planes bounding the cube.

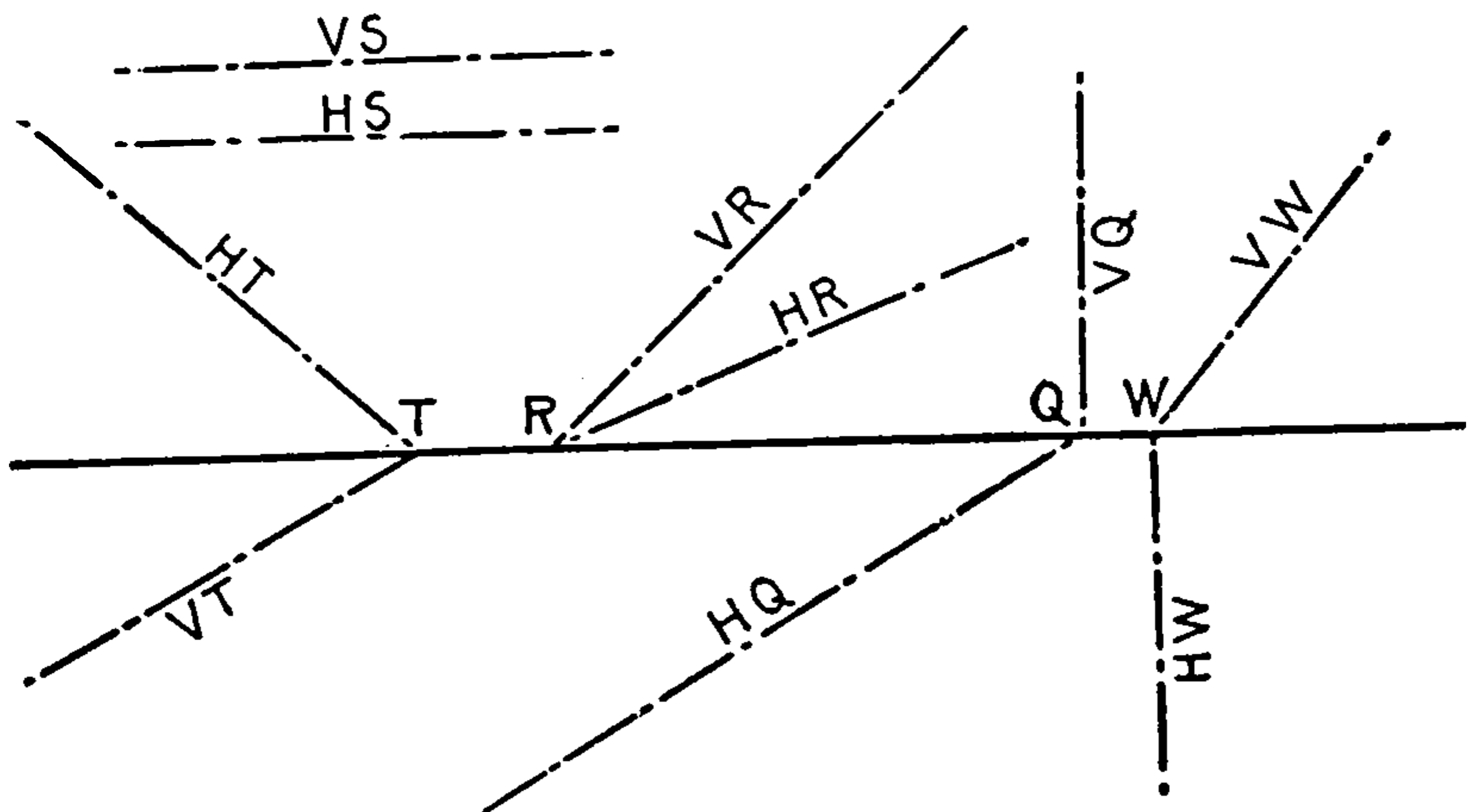


FIG. 22.

53. Draw the three projections of a hexagonal prism in  $I$ , its center line  $\parallel$  to  $H$  and  $V$ , with a  $1\frac{1}{4}$ -in. base and 2-in. altitude. Draw the  $H$ - and  $V$ -traces of the plane of one of its oblique faces.
54. Draw the prism given in Ex. 53 in  $III$ , and draw the  $H$ - and  $V$ -traces of the plane of one of its oblique faces.
55. Let  $C(2, 0 - 1)$   $D(3\frac{1}{2}, 0 - 2)$  be the base-diagonal of a hexagonal prism of 2-in. altitude, standing on  $H$ . Draw the traces of the six face planes of the prism.
56. Write the notation of the planes in Fig. 22, and describe them. Use your own dimensions.



## CHAPTER II

### PROBLEMS RELATING TO POINTS, LINES, AND PLANES

#### INTRODUCTORY

21. In the solution of problems that compose this chapter, we (1) analyze the operation in space, (2) offer proof of the correctness of the analysis, when it seems necessary, and (3) show the construction of a typical example of the problem "on the flat." The analysis should be learned, not by memorizing, but by visualizing the situation and process in space.

Descriptive Geometry, as the science of drafting, is intended to solve drafting difficulties. Drafting presents very few difficulties when objects are in their "natural position;" that is, when the lines, plane figures, and solids are parallel to  $H$  or  $V$ . It is only when the objects are in oblique positions, that we find their relations difficult. These difficulties show themselves in distortions, called "foreshortening."

The obvious general solution, therefore, is to bring the objects into their simplest relations (parallelism), and this is done by one of two methods. The first method might be called the "draftsman's method," and is equivalent to the changing of position by the observer in order to bring the objects into their desired relation. The second might be termed the "mathematician's method," and consists in rotating the objects, or in securing the results by means of a series of operations, such as passing planes, making intersections, rotating, and the like, which finally achieve the solution. Both methods have their advantages, and both are presented in many cases, but where the superiority of either is conspicuous, that method only is offered.

It has been shown that certain situations can only be represented by the use of a third plane of projection, the Profile, perpendicular to both  $H$  and  $V$ . In a like manner, there are certain solutions of this *type*, but different in situation, that can best be represented by projecting on a plane that is perpendicular to  $H$  or  $V$ , but not to both. This method, the draftsman's method, is merely a variant on profile projection, and is performed in the



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If a second removal is required:

**X** is the plane perpendicular to **Q** and oblique to **H**.

**Y** is the plane perpendicular to **R** and oblique to **V**.

The accompanying figure shows the projection of a line on **Q**; note that the **Q**-projections of the two points, **A** and **B**, are the same in elevation as their **V**-projections. This fact is the chief instrument.

### FUNDAMENTAL PROBLEM I

**24. Problem 1.**—To project an oblique line on an auxiliary plane parallel to the given line, and perpendicular to **H** or **V**.

Let **AB** (Fig. 24) be the given line.

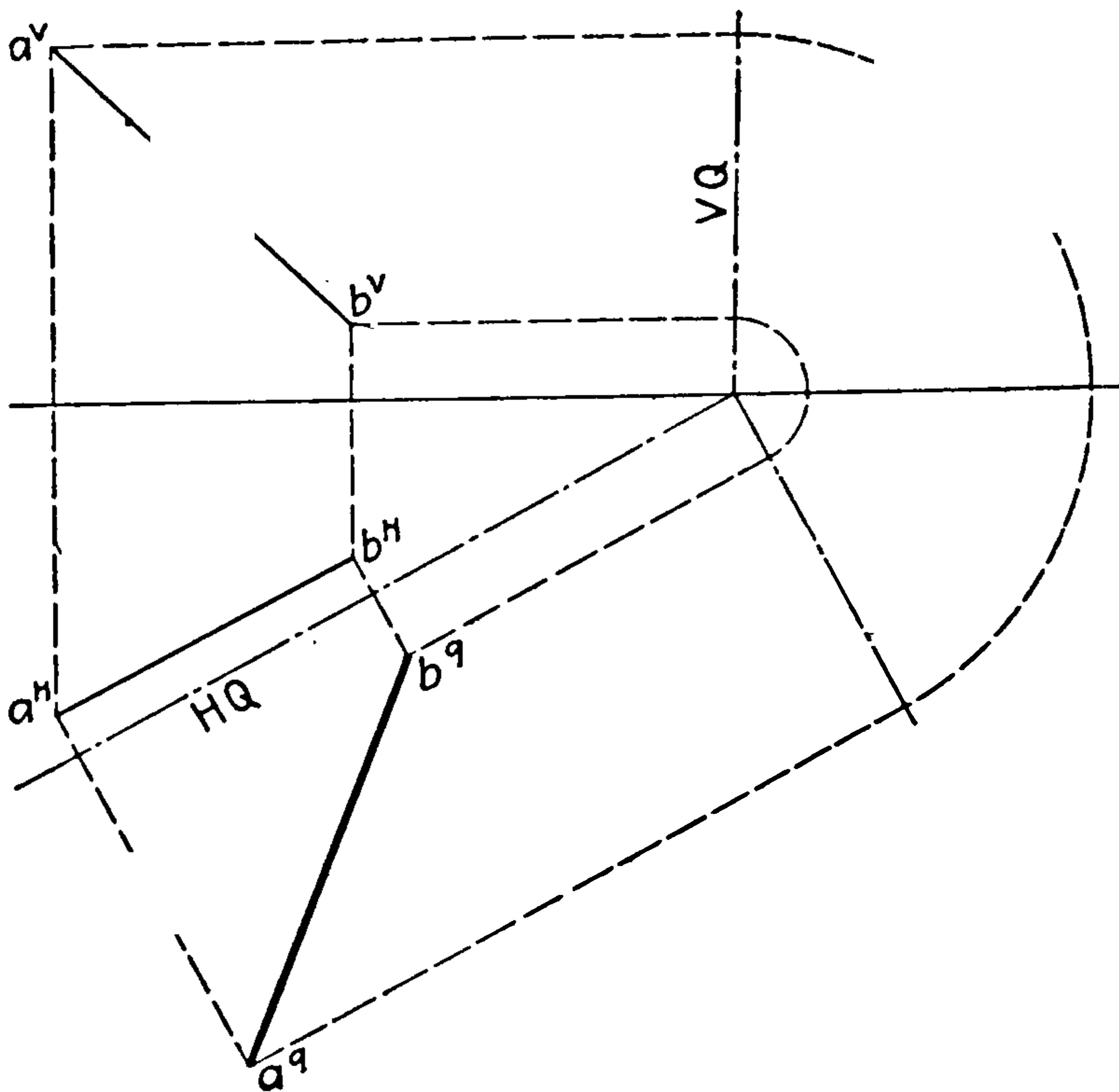


FIG. 24.

**Construction.**—1. Draw **HQ** parallel to  $a^h b^h$ . (**VQ** is drawn here, but is not necessary in this problem.)

2. Run projectors from  $a^h$  and  $b^h$  perpendicular to **HQ**.

3. Locate  $a^q$  on its projector the same distance from **HQ** that  $a^v$  is from **GL**. Locate  $b^q$  similarly.

**Conclusion.**—The line  $a^q b^q$  is the projection of **AB** on **Q**. (Let the student give the full explanation.)

**Corollary I.**—The Q-projection of AB is the true length of the line. Why?

**Corollary II.**—The angle between  $a^q b^q$  and HQ is the true angle between AB and H. Why?

Note.—When the other trace of Q or R is not needed, it will be omitted from figures hereafter.

**25. Supplementary Problem.**—To lay off a given distance on a given line from a given point.

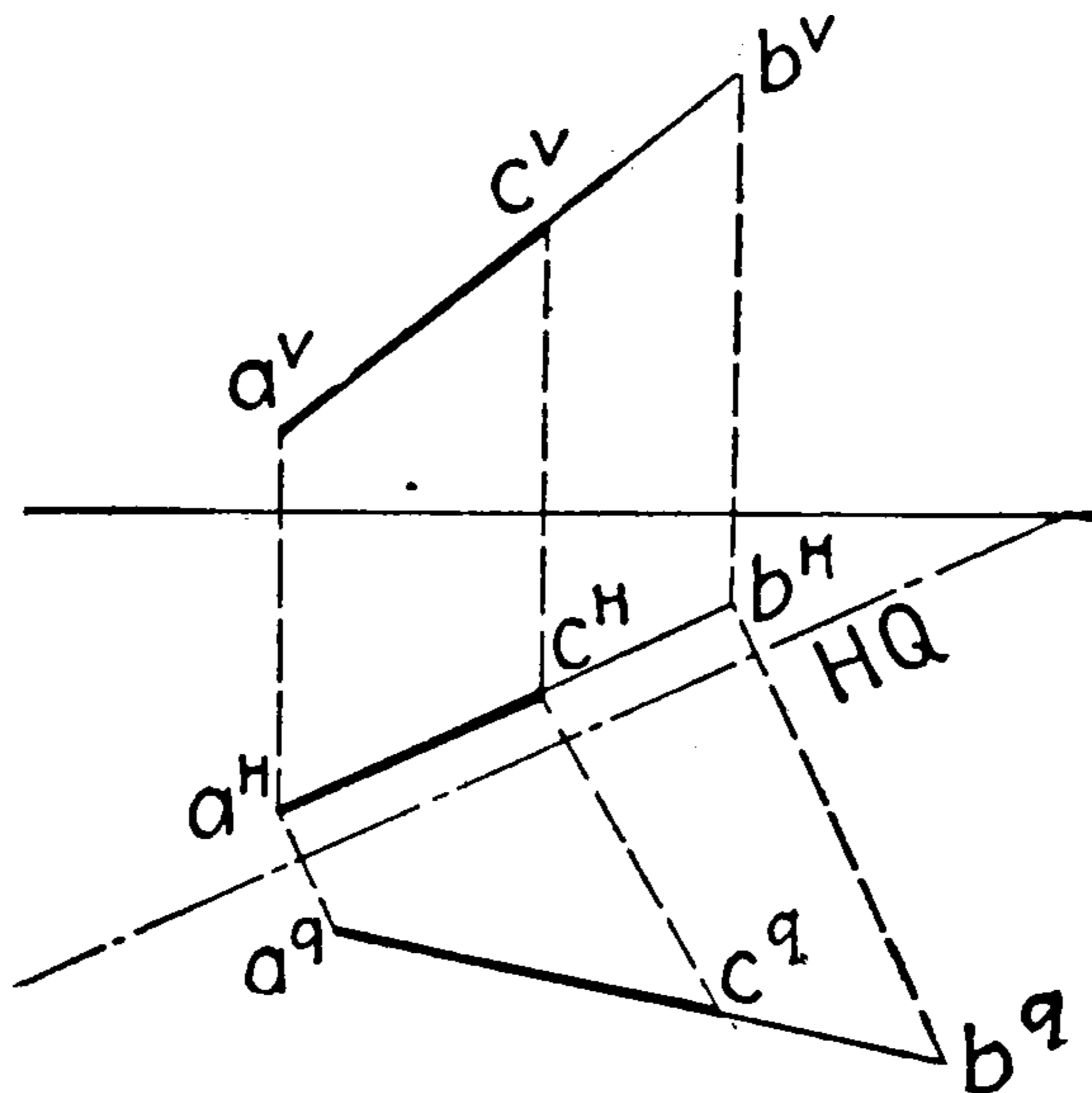


FIG. 25.

**Required.**—To lay off 1 in. on AB (Fig. 25) from A toward B.

**Constructions.**—1. Obtain the Q-projection of AB.

2. Lay off  $a^q c^q$  equal to 1 in.

3. Project  $c^q$  back to  $c^h$ , thence to  $c^v$ .

**Conclusion.**—AC is 1 in. long.

### EXERCISES

**26. Note.**—Exercises throughout this book are presented in two ways, with dimensioned coördinates, and graphically. The various sets are not exact duplicates. The graphic “layouts” are not always given dimensions, that being left to the student or instructor.

Project on Q the following lines and measure their true length and their angle with the horizontal.

57. A(2 + 2 - 1) B(4 - ½ - 2).

58. C(1 + 1 + 1) D(2 + 3 + 3).

59. E(5 + 1 + 2) F(3 - 3 + ½).

60. G(2 - 1 - 2) K(4 - 2 - 1).

61. L(1 - 3 + 1) M(3 - 1½ + 3)

62. O(4 + 2 - 3) P(2 + 1 + 2).

Project on R the following lines, and measure their true lengths and inclination from the vertical.

63. A(4 + 2 + 1) B(2 + 1 - 2).

64. C(1 - 1 - 2) D(3 - 2½ - 1).

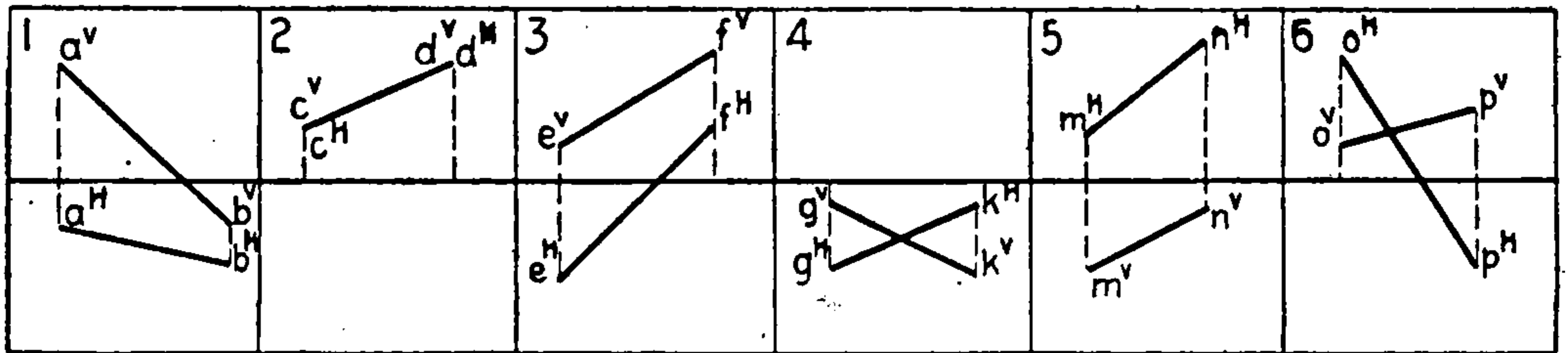
65. E(2 - 1 + 2) F(3 - 2½ + 1).

66. G(3 - 3 - 3) K(5 - 1 - 1).

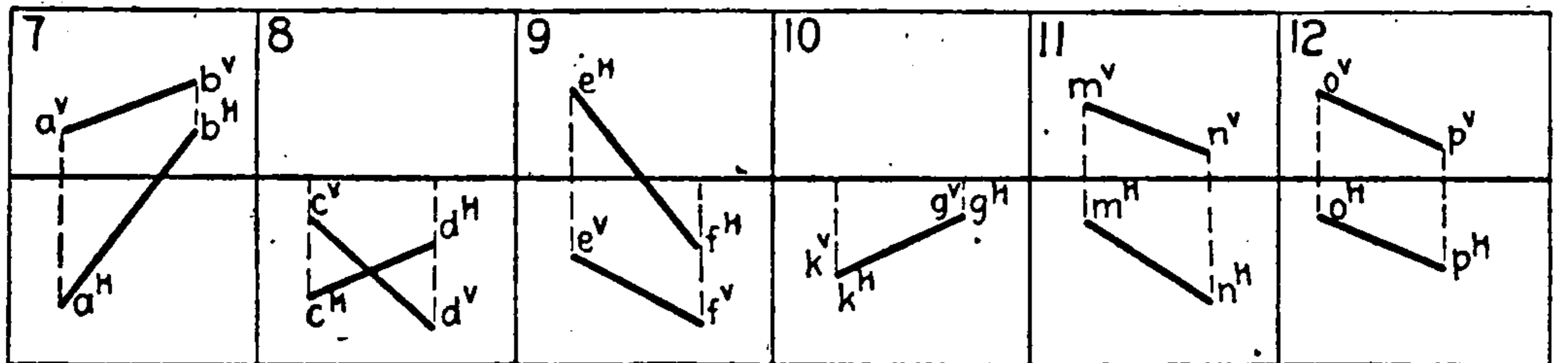
67. L(2 + 2 - ½) M(3½ + 1 - 3).

68. N(2 + 1 - 1) O(4 - 1 - 3).

Lay off on the above lines a distance of — in. (amount specified by instructor) from the first point toward the second.



Graphic Layout No. 1.—Make Q-projections of these lines. Dimensions to suit.



Graphic Layout No. 2.—(a) Make R-projections of these lines. (b) Lay off on the above lines a distance of — in., from the first point toward the second.

Note.—The requirements for these layouts may be transposed, if desired.

## FUNDAMENTAL PROBLEM II

27. Problem 2.—To find the Q-trace or R-trace of a plane oblique to H and V.

Required, to find the Q-trace of the plane T in Fig. 26.

Construction.—1. Draw HQ perpendicular to HT, and VQ perpendicular to GL.

2. Construct a right triangle on HQ as a base, making the altitude equal to that part of VQ included between GL and VT.

Note.—This is equivalent to revolving into H the right triangle cut by Q from H, V and T.

Conclusion.—The hypotenuse of this triangle is the Q-trace of T, hereafter written QT.

Note.—Thus T is a projecting plane on Q, and all points in T are projected on Q in QT. This makes it a simple instrument for all problems relating to points in T.

28. **Supplementary Problems.**—To find the dihedral angle between an oblique plane and a plane of projection.

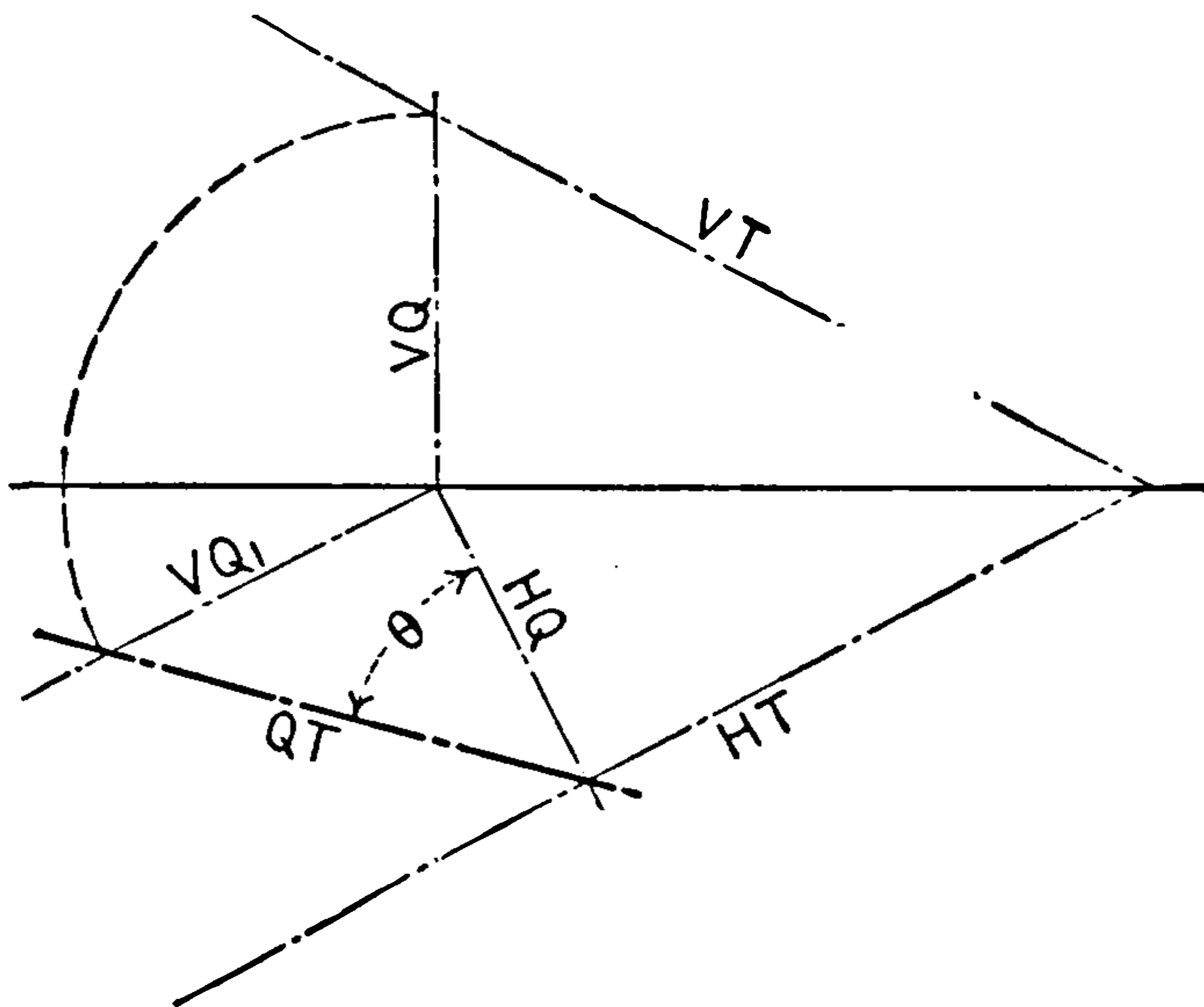


FIG. 26.

**Conclusion.**—The angle  $\theta$  in Fig. 26 is the true measure of the angle between T and H. If the R-trace of T had been found, it would have yielded the angle between T and V.

**Proof.**—The plane Q is perpendicular to the intersection of T and H, therefore the angle between HQ and QT is the measure of the dihedral angle. R does the same for T and V.

29. EXERCISES

1 	2 	3 	4 	5 
	7 	8 	9 	10 

Graphic Layout No. 3.—Find the Q- and R-traces of plane —. Also measure the dihedral angles between the planes and H and V, in degrees.

Find the Q-trace and R-trace of the various planes given here. Also measure the dihedral angle in degrees that each of these planes makes with H and V.

69. S(1 + 3) 4(1 - 2).

70. T(5 - 1) 2(5 + 2).

71.  $U(2 + 2) 3\frac{1}{2}(5 - 2)$ .  
 72.  $W(3 - 1) 5(3 - 3)$ .  
 73.  $Z(4 + 3) 1\frac{1}{2}(4 + 1\frac{1}{2})$ .  
 74.  $S(1 + 3) (5 + 2) (1 - 2\frac{1}{4}) (5 - 1\frac{1}{2})$ .  
 75.  $T(5 - 3) (1 - 2) (5 + 1\frac{1}{2}) (1 + 1)$ .  
 76.  $U(1 + 3) 2\frac{1}{2}(5 - 1)$ .  
 77.  $W(1 - 1) 3(5 + 2\frac{1}{2})$ .

### FUNDAMENTAL PROBLEM III

**30. Problem 3.**—To find the points in which a line pierces  $H$  and  $V$ .

**Analysis.**—1. Extend the  $V$ -projection of the line until it intersects  $GL$ , and project this point to the  $H$ -projection. The point thus found will be the  $H$ -piercing point.

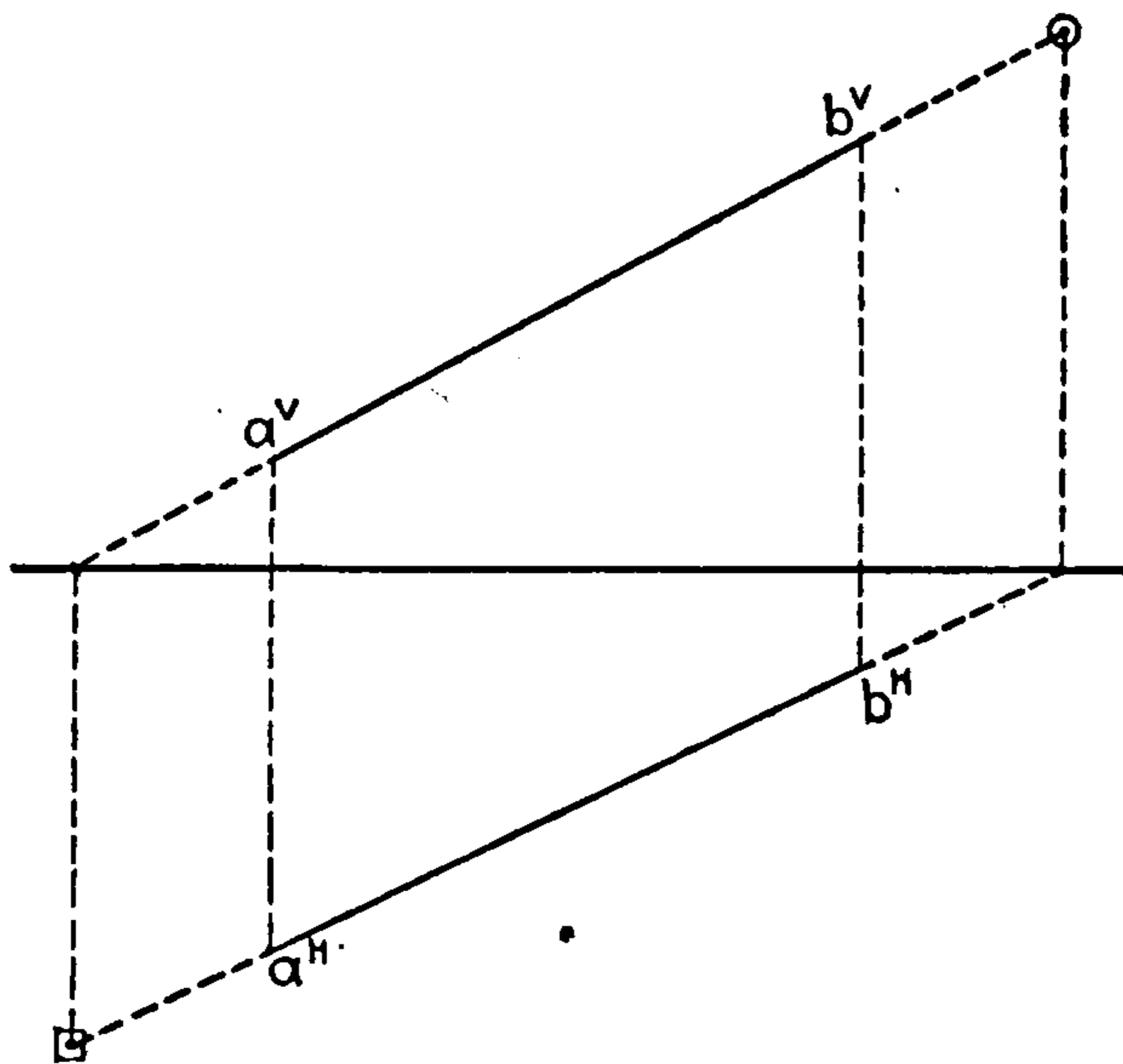


FIG. 27.

Interchanging  $V$  and  $H$  in the analysis will yield the  $V$ -piercing point.

**Proof.**—All points in  $H$  have their  $V$ -projections in  $GL$ , hence this point is in  $H$ , and therefore where the line pierces  $H$ .

**Construction.**—Let  $AB$  (Fig. 27) be any oblique line.

1. Extend  $a^v b^v$  to  $GL$ .
2. Project the intersection to  $\square$  on  $a^h b^h$ , or its extension. ( $\square$  is the symbol hereafter used for the  $H$ -piercing point.)
3. Extend  $a^h b^h$  to  $GL$ .
4. Project the intersection to  $\circ$  on  $a^v b^v$ , or its extension. ( $\circ$  is the symbol for  $V$ -piercing point.)

**Axioms.**—A line parallel to  $V$  has no  $\odot$ , hence the  $H$ -projection of such a line is parallel to  $GL$ .

A line parallel to  $H$  has no  $\square$ , hence the  $V$ -projection of such a line is parallel to  $GL$ .

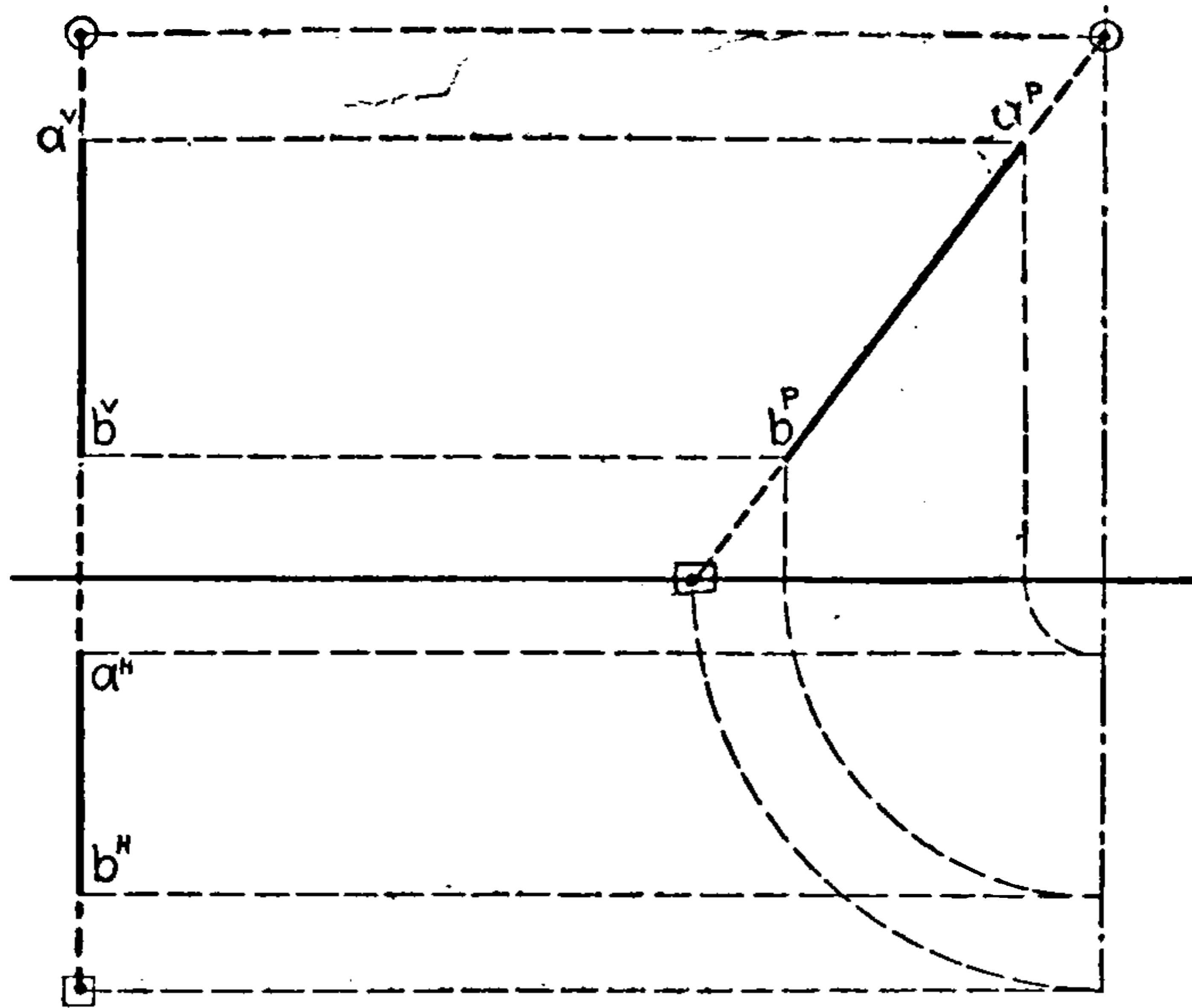


FIG. 28.— $H$ - and  $V$ -piercing points of lines located in  $P$ -planes.

**Construction.**—An examination of Figs. 28 and 28a will be sufficient to show the operation in this case.

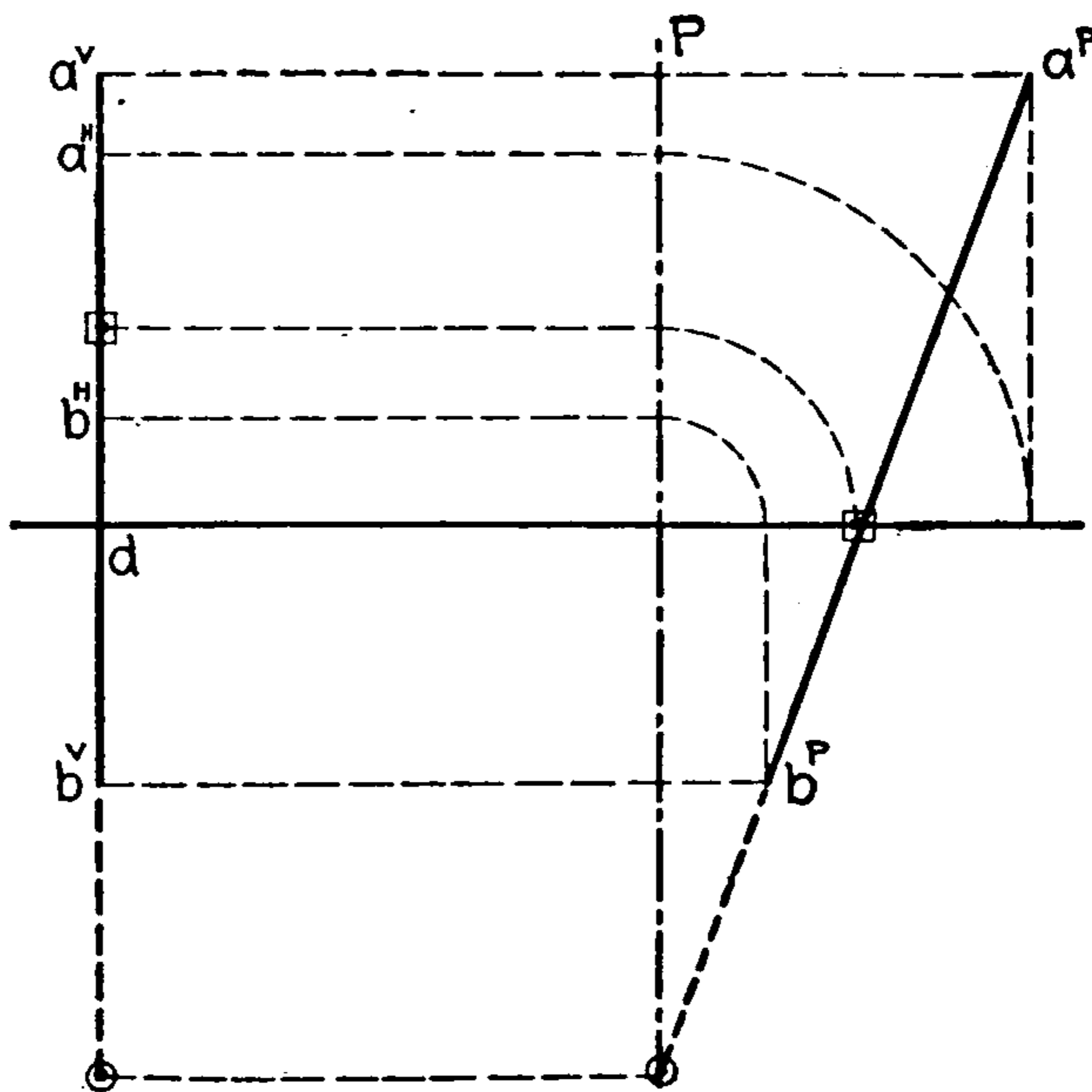


FIG. 28a.

**31. Supplementary Problem.**—To find the point where a line pierces a profile plane, and show it in  $P$ -projection.



**Construction.**—Let  $AB$  (Fig. 28b) be the given line. Required its  $P$ -piercing point. Locate  $P$  at any suitable point.

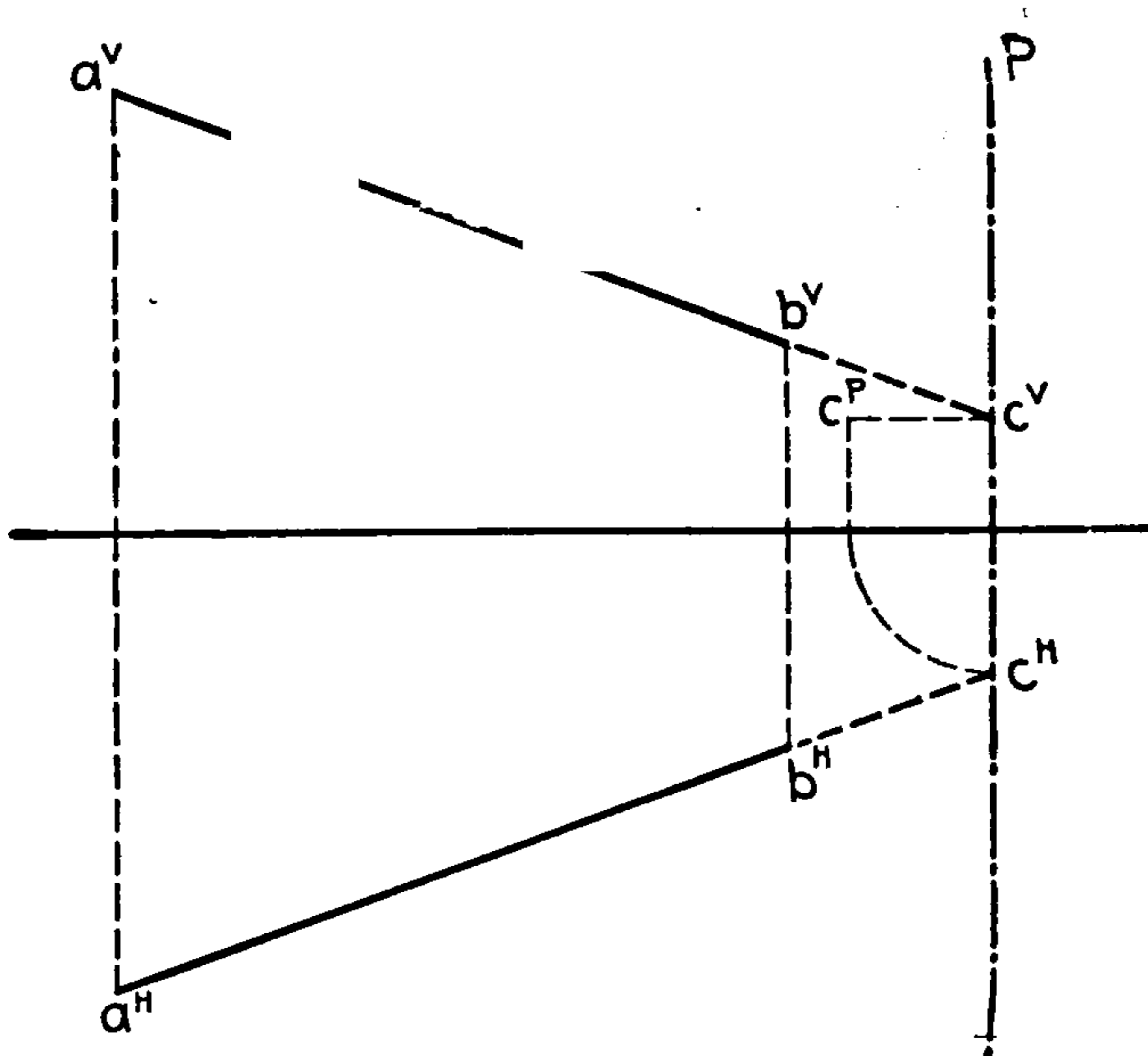


FIG. 28b.

1. Extend  $a^v b^v$  to the  $P$ -trace at  $c^v$ .
2. Extend  $a^h b^h$  to the  $P$ -trace at  $c^h$ .  $C$  is the piercing point.
3. Revolve  $P$  into  $V$ , locating  $c^p$ .

### EXERCISES

**32.** Locate the  $H$ - and  $V$ -piercing points of the following lines, and tell what angles they traverse. If  $P$  is required, place it at  $4\frac{1}{2}$  in. from the margin.

78.  $A(1 + 2 - \frac{1}{2})$   $B(2\frac{1}{2} + \frac{1}{2} - 2\frac{1}{2})$ ,  $C(3\frac{1}{2} + \frac{1}{2} + 2)$   $D(5 + 1\frac{1}{2} + \frac{1}{4})$ .
79.  $E(1 - \frac{1}{2} - 1\frac{3}{4})$   $F(3 - 2 - \frac{1}{4})$ ,  $G(3\frac{1}{2} + 2 + 1)$   $K(5 - 1 - 2)$ .
80.  $M(1 + 2 + 2)$   $N(2\frac{1}{2} - 1 - 1)$ ,  $O(3 - 1 + 2)$   $P(5 - 2 + \frac{1}{2})$ .
81.  $A(1 + 2 + 1)$   $B(3 + \frac{1}{2} - 2)$ ,  $C(3 + 2 + 1)$   $D(4 + 1 + \frac{1}{2})$ .
82.  $E(2 - \frac{1}{2} - 1\frac{1}{4})$   $F(3 - 1\frac{1}{2} - 2)$ ,  $G(3 - 1 + 2)$   $K(5 - 3 - 1)$ .
83.  $M(1 + 2 - 1\frac{1}{2})$   $N(1 + 1 - \frac{1}{2})$ ,  $O(2 - 2 - 2\frac{1}{2})$   $P(2 - 1 - \frac{1}{2})$ .
84.  $A(1 + 2 + \frac{1}{2})$   $B(1 + \frac{3}{4} + 1\frac{1}{2})$ ,  $C(2 - 2 + \frac{1}{2})$   $D(2 - 1 - \frac{1}{2})$ .
85.  $E(1 - 1 - 2)$   $F(1 + 2\frac{1}{2} + \frac{1}{2})$ ,  $G(2 - 2\frac{1}{2} - \frac{1}{2})$   $K(2 + \frac{1}{2} + 1)$ .
86.  $M(1 + 2 - 2)$   $N(1 - 1 - \frac{1}{2})$ ,  $O(2 + 2 + 1\frac{1}{4})$   $P(2 - 1\frac{1}{2} + \frac{1}{2})$ .
87.  $A(1 + 2 - \frac{1}{2})$   $B(1 - \frac{1}{2} - 1)$ ,  $C(2 + \frac{1}{2} - 2)$   $D(2 - 2 + \frac{1}{2})$ .
88.  $E(1 + 2 - 1)$   $F(1 - 1 + 1\frac{1}{2})$ ,  $G(2 - 2 - 1)$   $K(2 + 1 + 1)$ .

Find the  $P$ -piercing points of the following lines, locating  $P$   $4\frac{1}{2}$  in. to the right.

89.  $A(2 + \frac{1}{2} + 2)$   $B(3\frac{1}{2} - 1 + \frac{1}{2})$ .
90.  $C(2 + 1\frac{1}{4} - 1)$   $D(3\frac{1}{2} + 2\frac{1}{2} - \frac{1}{4})$ .
91.  $E(1\frac{1}{2} + 1 + 1\frac{1}{2})$   $F(3\frac{1}{2} + 1 + 1\frac{1}{2})$ .
92.  $G(2 + \frac{1}{2} - 1)$   $K(3\frac{1}{2} + 1\frac{1}{2} - 1)$ .



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**Construction.**—Let  $T$  (Fig. 29) be the plane.

1. Assume  $a^V$  in  $VT$ ;  $a^H$  will be in  $GL$ .
2. Assume  $b^H$  in  $HT$ ;  $b^V$  will be in  $GL$ .
3. Draw  $a^V b^V$  and  $a^H b^H$ .  $AB$  will be entirely in  $T$ .

**Special Case.**—To assume a line parallel to  $H$  or  $V$ , lying in an oblique plane.

If parallel to  $H$ , draw the  $V$ -projection parallel to  $GL$  from an assumed point in the  $V$ -trace. From the  $H$ -projection of this point draw the  $H$ -projection parallel to the  $H$ -trace.

In Fig. 29a, the line  $CD$  is a parallel line to  $H$  lying in  $S$ , and the line  $EF$  is parallel to  $V$  lying in  $R$ .

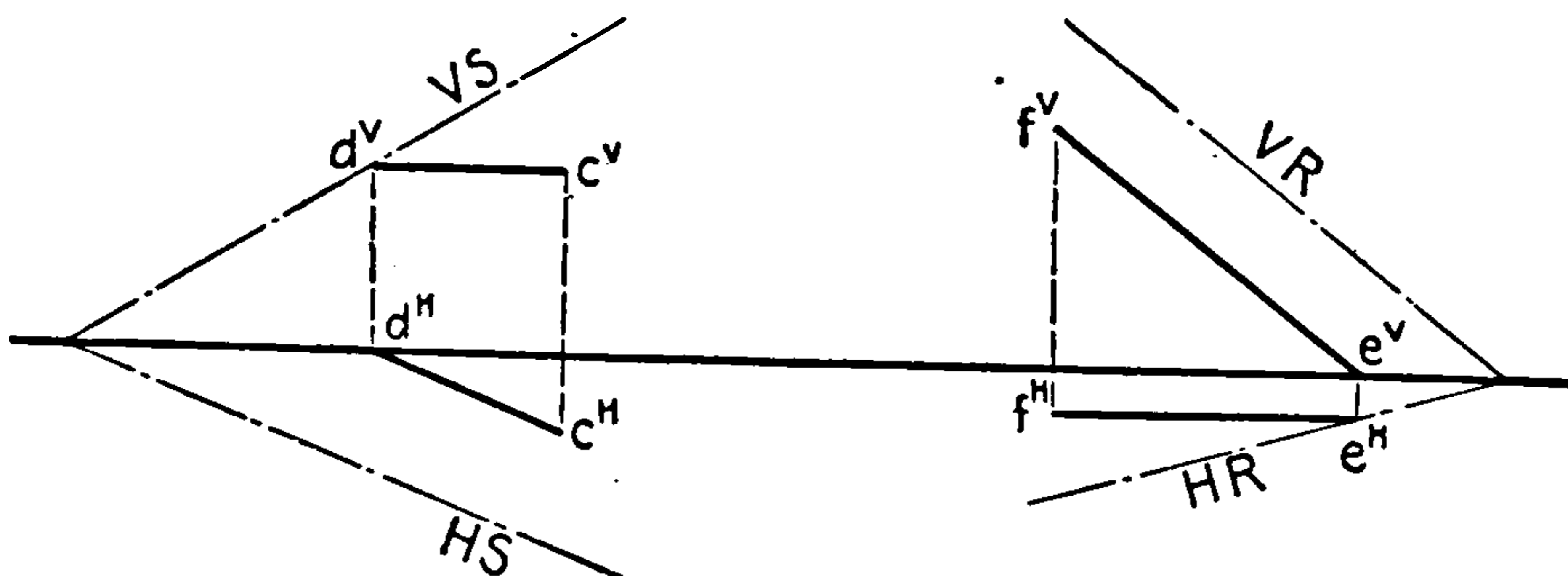


FIG. 29a.

**34. Problem 5.**—Given one projection of a point in a plane, to locate the other projection.

**Construction.**—Through the given projection draw the projection of a line and assume the line to be in the plane, thus locating the other projection. The required projection of the point will be on the second projection of the assumed line.

**Hint.**—The convenient line for most cases of the foregoing is a line parallel to  $H$  or  $V$ .

**Note.**—If the given projection of the point lies in the projecting trace of a projecting plane, the point is indeterminate, and can be one of an infinite number of points lying in the plane.

**Problem 6.**—To find the traces of a plane determined by three given points.

- Analysis.**—1. Join the points, two and two, by lines.  
 2. Find the piercing points of the lines.  
 3. Draw the  $H$ -trace through the  $H$ -piercing points, and the  $V$ -trace through the  $V$ -piercing points.

**Proof.**—The lines have their piercing points in the respective traces of the plane, therefore the lines (and all their points) lie in the plane thus determined.

**Construction.**—Let  $A$ ,  $B$ , and  $C$  be the given points (Fig. 30).

1. Join the projections of  $AB$  and  $AC$ .

2. Find the four piercing points of  $AB$  and  $AC$ .

3. Through the  $H$ -piercing points draw  $HT$ , and through the  $V$ -piercing points draw  $VT$ .

**Check.**—1. The line  $BC$  should have its piercing points in the respective traces of  $T$ , and

2. The traces should meet in a point on  $GL$ , or be parallel to  $GL$ .

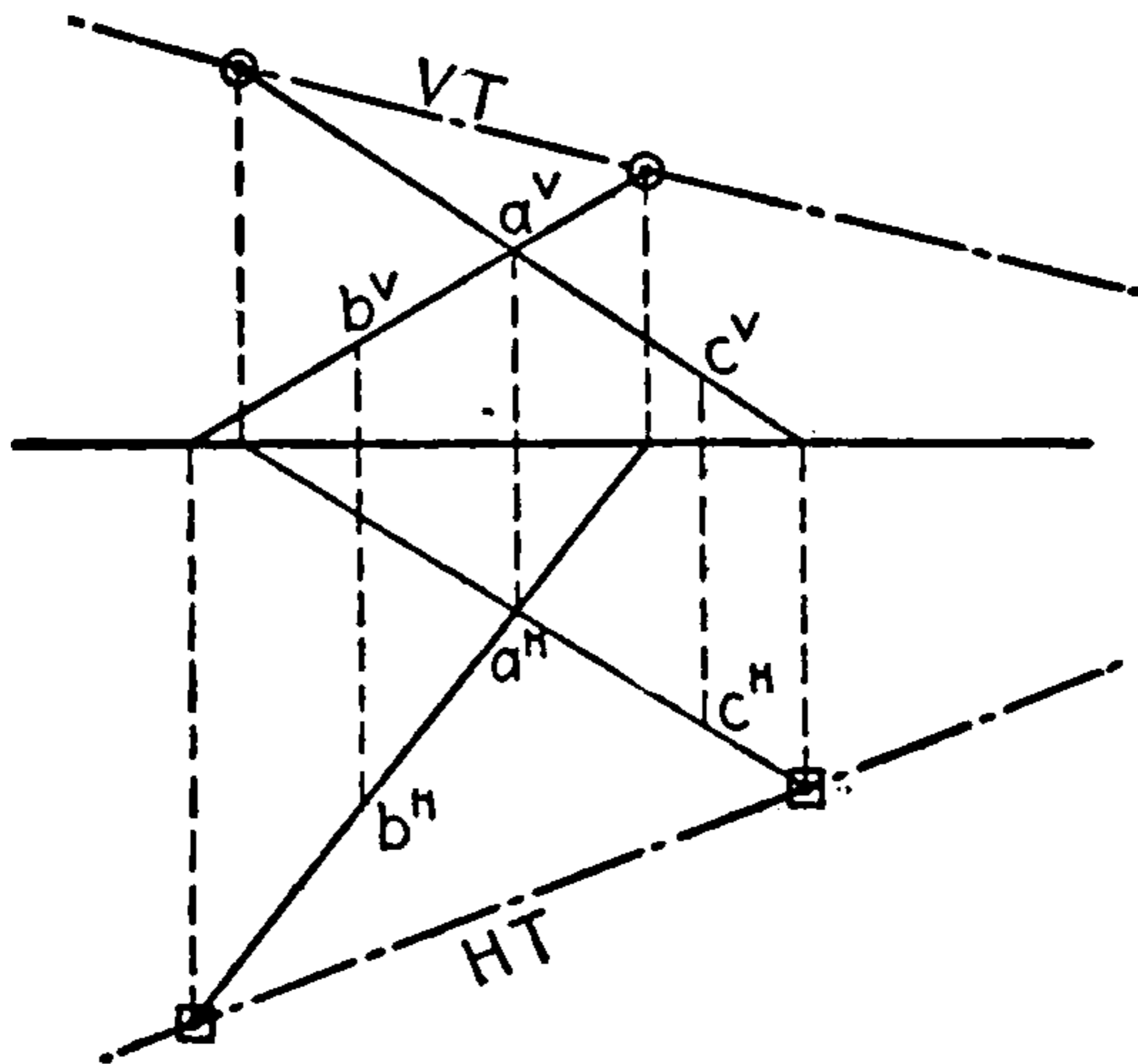


FIG. 30.

**Hints.**—1. Sometimes the points may be so placed that the piercing points are not all to be found within the limits of the problem. In such a case, an auxiliary line may be drawn from any of the three points to any point on the line joining the other two. For instance, if such a proceeding were necessary, a line could be drawn from  $B$  to any point,  $M$ , on the line  $AC$ .  $BM$  would lie in the required plane, and its piercing points would help to determine the traces.

2. Ordinarily, only three piercing points are necessary to be found, because when the  $H$ -trace is found, its intersection with  $GL$  may be joined to one of the  $V$ -piercing points.

**If two intersecting lines are given.**—This makes no change in the problem.

**If two parallel lines are given.**—Find their piercing points and draw the traces, as in the previous case.

**If a point and a line are given.**—The point may be connected with any point in the line, thus making two intersecting lines, or through the point a line may be drawn parallel to the given line, thus making two parallel lines.

## EXERCISES

35. Locate the following lines in the planes given. The points to be determined are lettered as unknowns,  $x$ ,  $y$ ,  $z$ , etc., without  $+$  or  $-$  signs.

96. AB, any oblique line in  $T(1 + 3) 1(4 - 2)$ .

97. CD, any oblique line in  $S(5 + 2) 2(2 - 3)$ .

98. EF, any oblique line in any plane oblique to  $GL$ .

99. Draw a line in the third angle that lies in  $T(1 + 2) 3(1 - 3)$ .

100. Locate  $C(2 + 1, x) D(3, + 2, y)$  in  $S(1 + 2) 4(1 - 2\frac{1}{2})$ . State values of  $x$  and  $y$ .

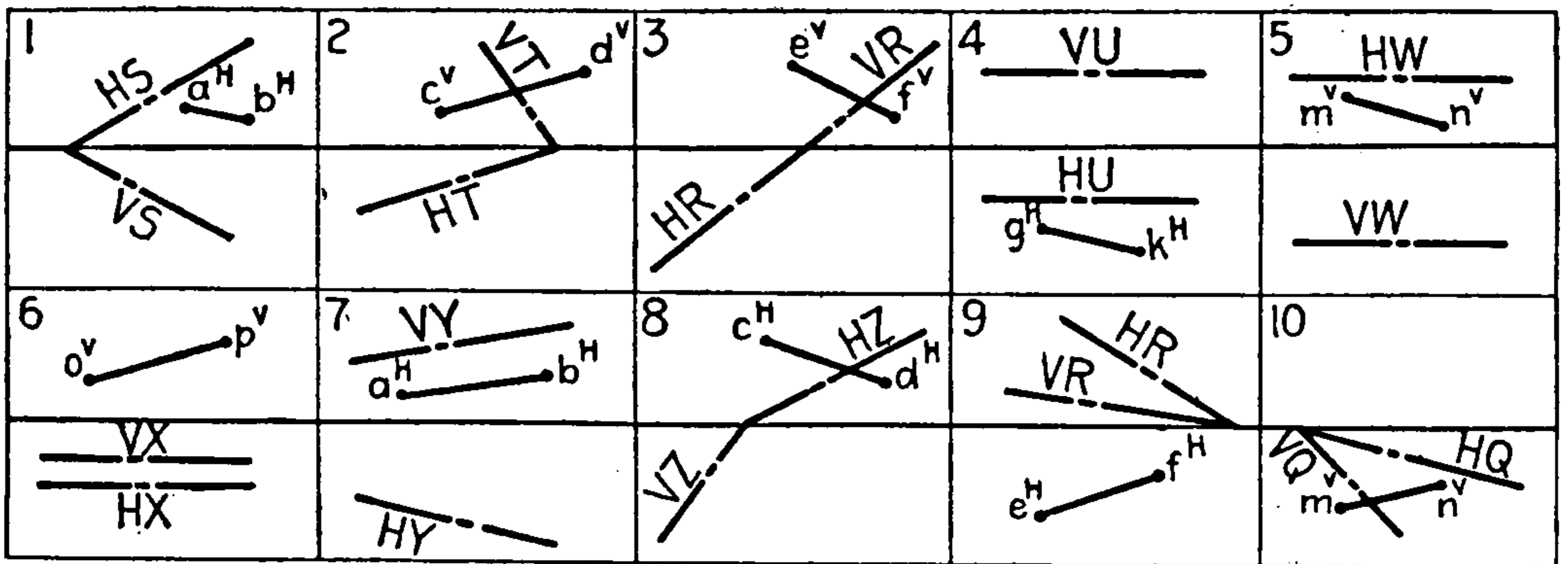
101. Locate  $E(4 +, x) F(5 + 2, y)$  in  $W(1 + 3) 3(5 - 1)$ .

102. Draw a line in  $S(1 + 2) 4(1 - 2\frac{1}{2})$  that is  $\parallel$  to  $H$ , 1 in. above  $H$ .

103. Draw a line in  $W(1 + 3) 3(5 - 1)$  that is  $\parallel$  to  $V$ ,  $\frac{1}{2}$  in. behind  $V$ .

104. Locate  $M(5 + 1, x) N(3, 0, y)$  in  $W(1 + 3) 3(5 - 1)$ .

105. Locate  $O(3 + 2, x) P(4 + \frac{1}{2}, y)$  in  $T(1 + 3) 3(5 - 3)$ .



Graphic Layout No. 6.—Locate the other projections of the line in the given plane—

106. Find the locus of all points in  $S(5 + 2) 2(5 - 3)$ :

(a) 1 in. above  $H$ .

(b) 1 in. below  $H$ .

(c) 2 in. behind  $V$ .

(d) 2 in. before  $V$ .

107. Locate in  $R(+ 2) \in (- 1\frac{1}{2})$  the following lines:

$A(1 + 1, x) B(2 + 1\frac{1}{2}, y)$ .

$C(2, x, - 1) D(4, y - 2\frac{1}{2})$ .

EF, the locus of all points 1 in. before  $V$ .

108. Locate the following lines in  $X(+ 1\frac{1}{2}) \in (- 1)$ :

$F(1 - 1, x) G(2 - 2, y)$ .

$K(2\frac{1}{2} + 2\frac{1}{2}, x) L(4 +, y)$ .

MN, the locus of all points  $\frac{3}{4}$  in. above  $H$ .

109. Locate  $O(2 + 2, x) P(4 + 1, y)$  in  $T(1 + 3) (5 + 1\frac{1}{2}) (1 - 2) (5 - 1)$ .

Locate the following points in the planes given:

110.  $A(1, x - 1), B(3 + 3, y)$  in  $S(1 + 3) 5(1 - 1\frac{1}{2})$ .

111.  $C(1\frac{1}{2} - 2, x), D(4, y + 2)$  in  $U(5 - 1) 1(3 + 3)$ .

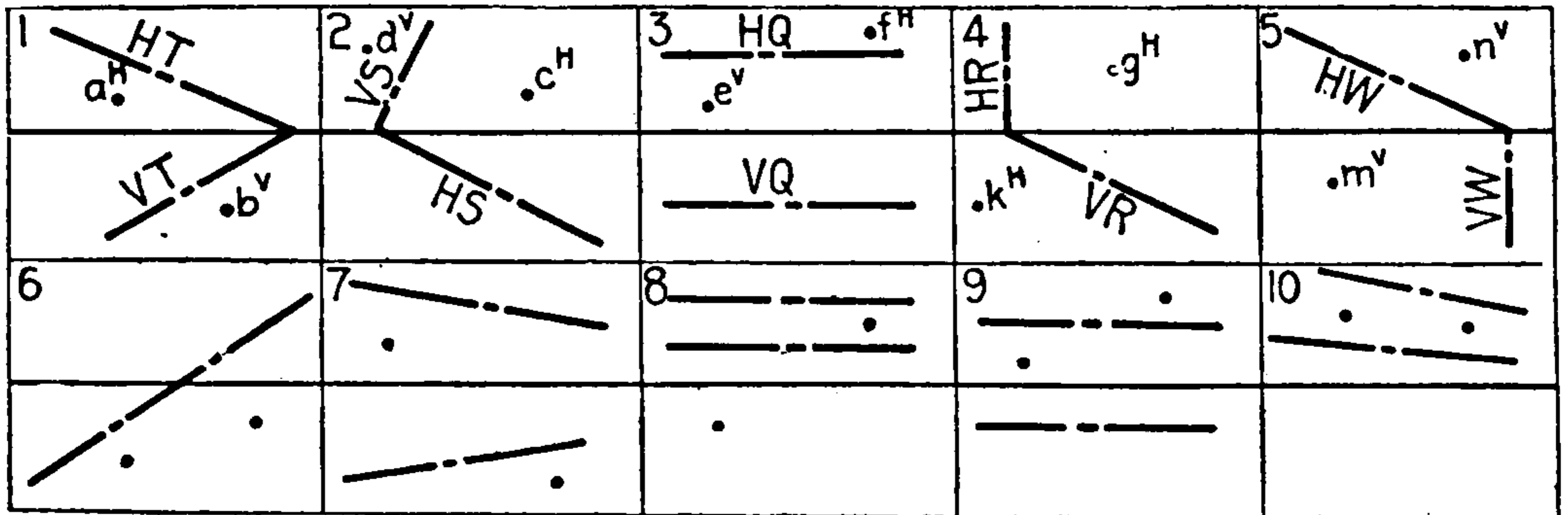
112.  $E(2 + 1, x), F(3, y - 3)$  in  $W(+ 2) \in (- 1)$ .

113.  $G(2 + 1\frac{1}{2}, x), K(3 + 3, y)$  in  $Q(+ 2) \in (+ 1)$ .

114.  $L(1 + 1, x)$ ,  $M(4\frac{1}{2}, y + 1)$  in  $R(1 + 2) 3(5 - 2)$ .

115.  $N(1, x + \frac{1}{2})$ ,  $O(3 + 2, y)$  in  $S(1 + 3) 5(1 + 1)$ .

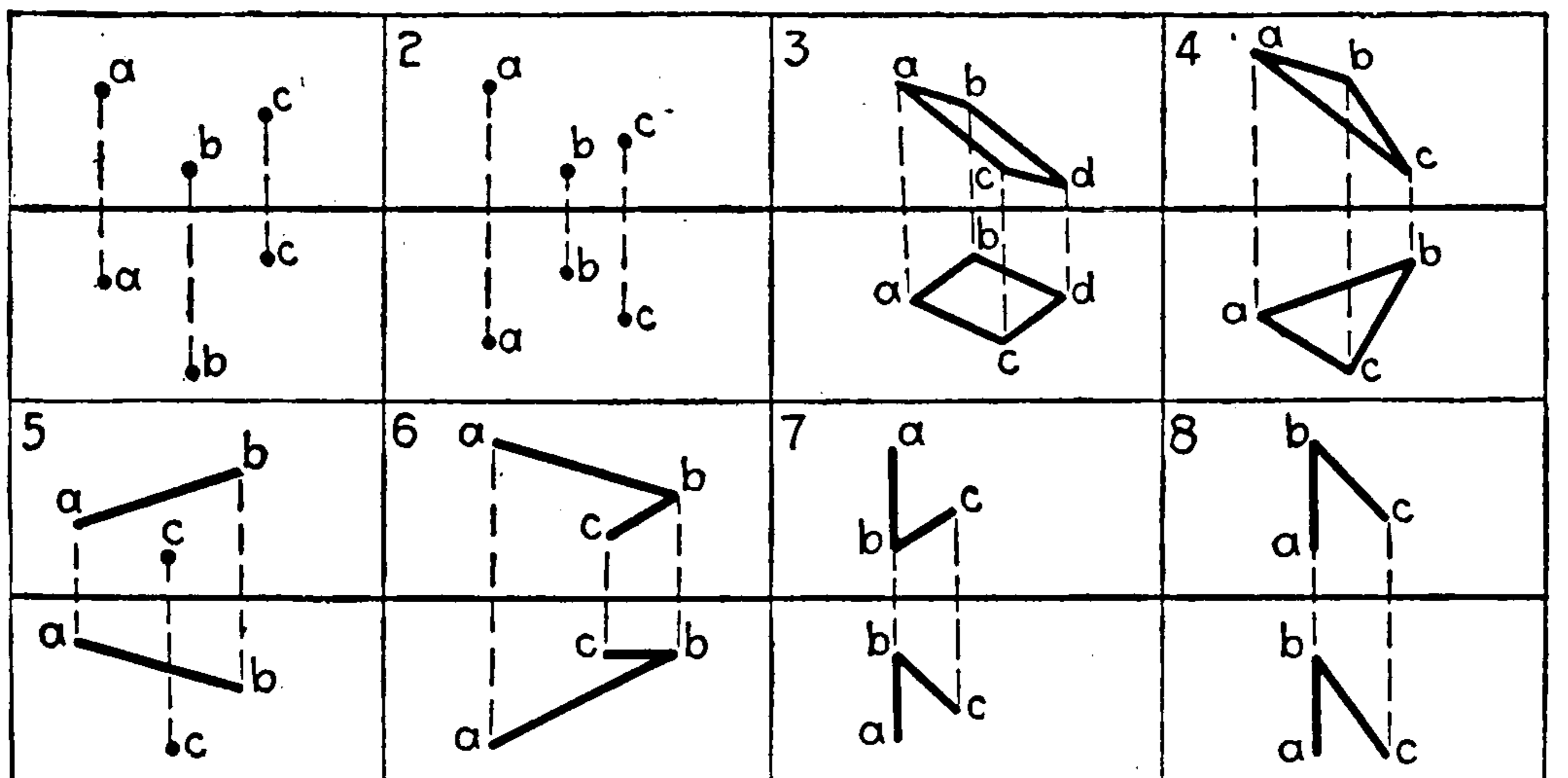
116.  $P(1 + 1\frac{1}{2}, x)$ ,  $A(4, y - \frac{1}{2})$ ,  $B(4\frac{1}{2} + 2, z)$  in  $T(1 + 3)(5 + 1\frac{1}{2})$   
 $(1 - 2)(5 - 1)$ .



Graphic Layout No. 7.—Given one projection of points in planes, to locate the other.

Draw the traces of the planes containing the following points, lines, and plane figures:

117.  $A(2 + 2 - 1)$ ,  $B(3 + \frac{1}{2} - 3)$ , and  $C(3\frac{1}{2} + 1\frac{1}{4} - 1\frac{1}{4})$ .  
 118.  $D(2 + 1 - \frac{1}{2})$ ,  $E(2\frac{1}{2} - \frac{1}{2} + \frac{1}{2})$  and  $F(4 + 1\frac{1}{2} + \frac{3}{4})$ .  
 119.  $G(2 - 2 + 1)$ ,  $K(3 - \frac{1}{2} + 3)$ , and  $L(3\frac{1}{2} - 1\frac{1}{4} + 1\frac{1}{4})$ .  
 120.  $M(2\frac{1}{2} + \frac{1}{2} - \frac{1}{2})$   $N(3\frac{1}{2} + 1\frac{1}{4} - \frac{3}{8})$ , and  $O(4\frac{1}{2} + \frac{1}{4} - 1\frac{1}{2})$ .  
 121.  $A(1 - 1 + 2\frac{1}{2})$   $B(4 - \frac{3}{4} + 1)$ , and  $C(3 - 2 + \frac{1}{2})$ .  
 122. The triangle  $D(1\frac{1}{2} + 1 - 1\frac{1}{2})$   $E(3 + 1 + \frac{1}{2})$   $F(4 - 1 + 2)$ .  
 123. The parallelogram  $M(1\frac{1}{2} - 1 + \frac{5}{8})$   $N(2 - 1\frac{5}{8} + \frac{3}{8})$   $O(2\frac{5}{8} - \frac{3}{8} + 1\frac{5}{8})$   $P(3\frac{1}{8} - 1 + 1\frac{3}{8})$ .  
 124.  $HT$  is a line through  $A(4, 0, 0)$  and  $B(2, 0 - 2\frac{1}{4})$ .  $T$  contains  $D(1 + 1 - 1\frac{1}{2})$ . Draw  $VT$ .



Graphic Layout No. 8.—Locate these points in either  $I$  or  $III$ , and draw the traces of the planes containing them.

125. The points  $D(2\frac{1}{2} + 1\frac{1}{2} - 1)$ ,  $E(3\frac{1}{4} + \frac{3}{4} - 1)$ ,  $F(3\frac{1}{4} + 2\frac{1}{4} - 1)$ , and  $G(4 + 1\frac{1}{2} - 1)$  are the corners of a vertical base of a cube. Draw the projections of the cube and the traces of its six bounding planes.
126.  $L(3, 0 - 1)$   $M(4, 0 - 2\frac{1}{2})$  is the base diagonal of a square pyramid of 2-in. altitude, standing on  $H$ . Draw the traces of the four oblique planes forming its faces.
127.  $O(3, 0 - 1\frac{1}{2})$  is the center of the base of a 2-in. hexagonal oblique pyramid, whose apex is  $P(2 + 2 - 1)$ . Hexagon base is in  $H$ ; draw the traces of the six oblique bounding planes.

**36. Problem 7.**—To pass a plane through a given point, parallel to two given lines.

**Analysis.**—1. Draw through the given point two lines, parallel respectively to the given lines.

2. Pass a plane through the lines thus drawn.

**Conclusion.**—This plane contains the given point and is parallel to both lines, from the geometrical theorem that a line, parallel to both lines, from the geometrical theorem that a line,

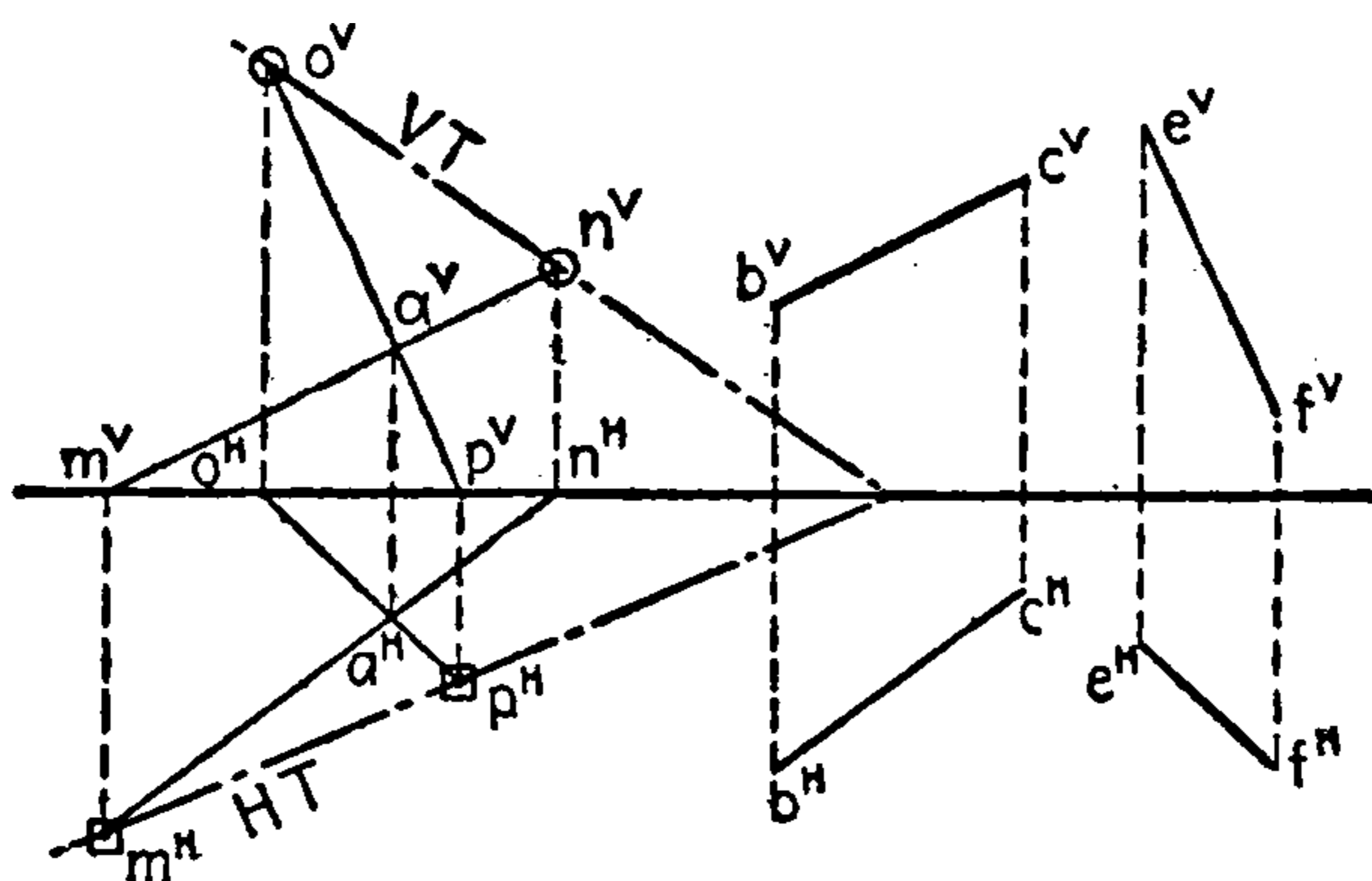


FIG. 31.

external to a plane, that is parallel to any line in that plane, is parallel to that plane.

**Construction.**—Let  $A$  (Fig. 31) be the given point, and  $BC$  and  $EF$  the given lines.

1. Through  $A$  draw  $MN \parallel$  to  $BC$ , and  $OP \parallel$  to  $EF$ .
2. Find the  $H$ - and  $V$ -piercing points of  $MN$  and  $OP$ .
3. Through these piercing points draw the traces of the plane.

**Note.**—If the given lines are parallel, the required plane is indefinite, and an infinite number of planes may be passed.

**37. Problem 8.**—To pass a plane through a given line parallel to another given line.

**Analysis.**—1. Through any point of the given line, draw a line parallel to the second line.

2. Pass the required plane through the two intersecting lines.

Let the student make the construction.

Note.—If a plane be passed through the second line parallel to the first, the two planes will be parallel.

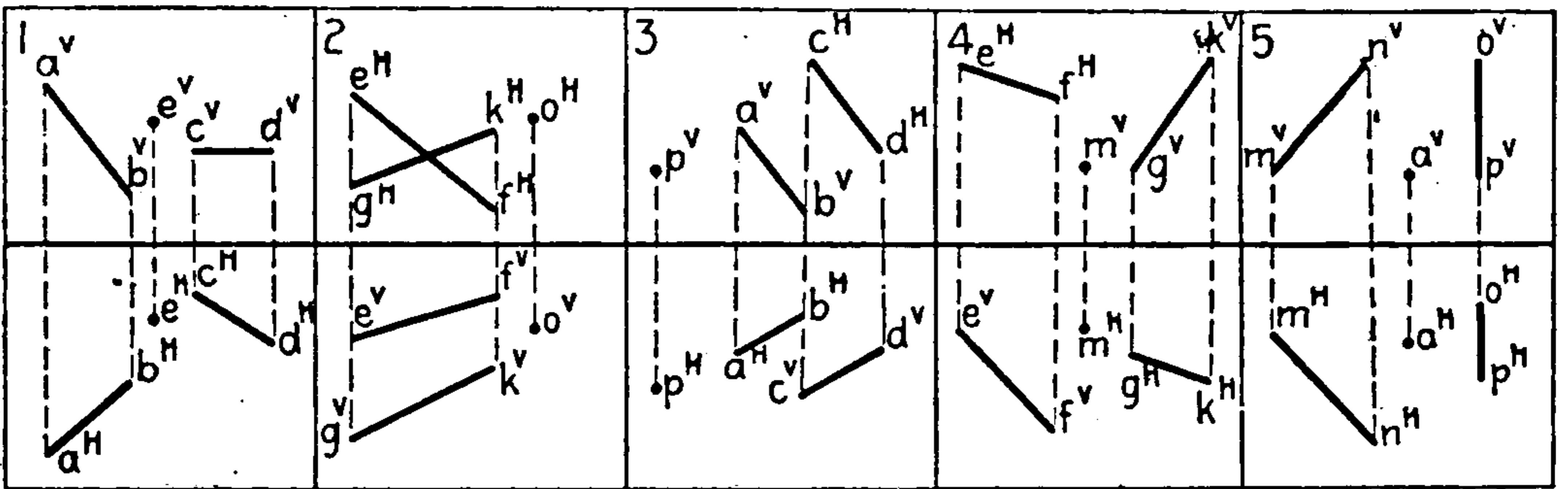
EXERCISES

38. Pass planes through the following points parallel to the lines given:

128. Through  $O(3 - 1 + 1) \parallel$  to  $A(1 - 1\frac{1}{2} + 1)$   $B(2 - \frac{1}{2} + 2)$  and  $C(4 - 1 + 1)$   $D(5 - 1\frac{1}{2} + 2)$ .

129. Through  $A(4 + 1 - 1) \parallel$  to  $E(1 + 2 - \frac{1}{2})$   $F(1 + 1 - 1)$  and  $G(2 + \frac{1}{2} - 1)$   $K(3 + 2 - 2)$ .

130. Through  $M(4 + 1 - 1\frac{1}{2}) \parallel$  to  $L(1\frac{1}{2} + 1 - 1)$   $N(2\frac{1}{2}, 0 - \frac{1}{2})$  and  $O(3 + 1 + \frac{1}{2})$   $P(4 + 2 + 1\frac{1}{2})$ .

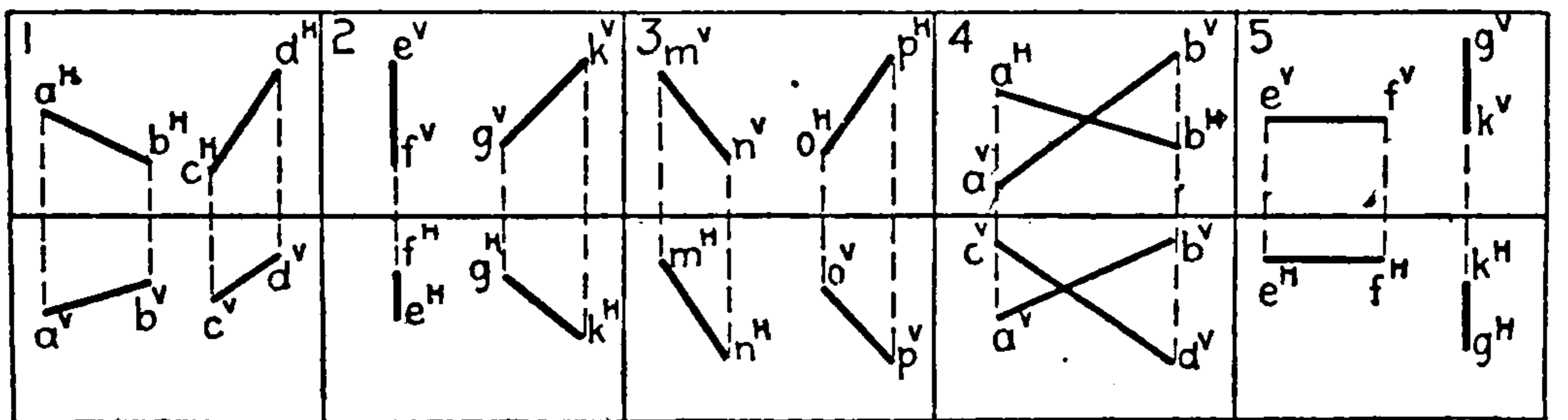


Graphic Layout No. 9.—Through the given point, pass a plane parallel to the two given lines.

131. Through  $A(1 + 3 - 2)$   $B(2 + 1 + 1)$  pass a plane  $\parallel$  to  $C(3 + 1 - 1)$   $D(4 + 1\frac{1}{2} - 2)$ .

132. Through  $E(4 - 1 + 2)$   $F(3 - 2 + 1)$  pass a plane  $\parallel$  to  $G(1 + 1\frac{1}{2} - 2)$   $K(2 + 1 - 1)$ .

133. Through  $M(3 - 2 + 1)$   $N(4 - \frac{1}{2} - 2)$  pass a plane  $\parallel$  to  $O(1 + 2 + 1)$   $P(2 + 3 - 1)$ .



Graphic Layout No. 10.—Through the line —, Ex. —, pass a plane parallel to line —.

REVOLUTION OF POINTS

39. Definition.—The revolution of a point about a line (called its axis) consists in moving the point in a plane perpendicular to the axis, and keeping it at a constant distance from the axis.

The Path of Revolution is a circle.



## FOURTH FUNDAMENTAL PROBLEM

**40. Problem 9.**—To revolve a point through any given angle about an axis perpendicular to  $H$  or  $V$ .

**Construction.**—Let  $O$  (Fig. 32) be the point,  $AB$  the axis perpendicular to  $H$ , and  $\theta$  the given angle.

1. With  $a^H b^H$  as a center draw a circle through  $o^H$ .

2. Through  $o^V$  draw a line parallel to  $GL$ . (These will be the  $H$ - and  $V$ -projections of the path of revolution.)

3. Move  $o^H$  through the given angle to  $o_1^H$ .

4. Project to  $o_1^V$ .

**Conclusion.**—This gives the new position of the point. If the axis were perpendicular to  $V$ , the circle would be projected on  $V$  in its true size and on  $H$  as a line parallel to  $GL$ .

**41. Problem 10.**—To revolve a line about a vertical or horizontal axis through any given angle.

**Analysis.**—1. As the line is the sum of all points in it, take any two points of it and revolve them about the axis, as in Problem 9.

2. Join these revolved points, and the resulting projections will be projections of the line.

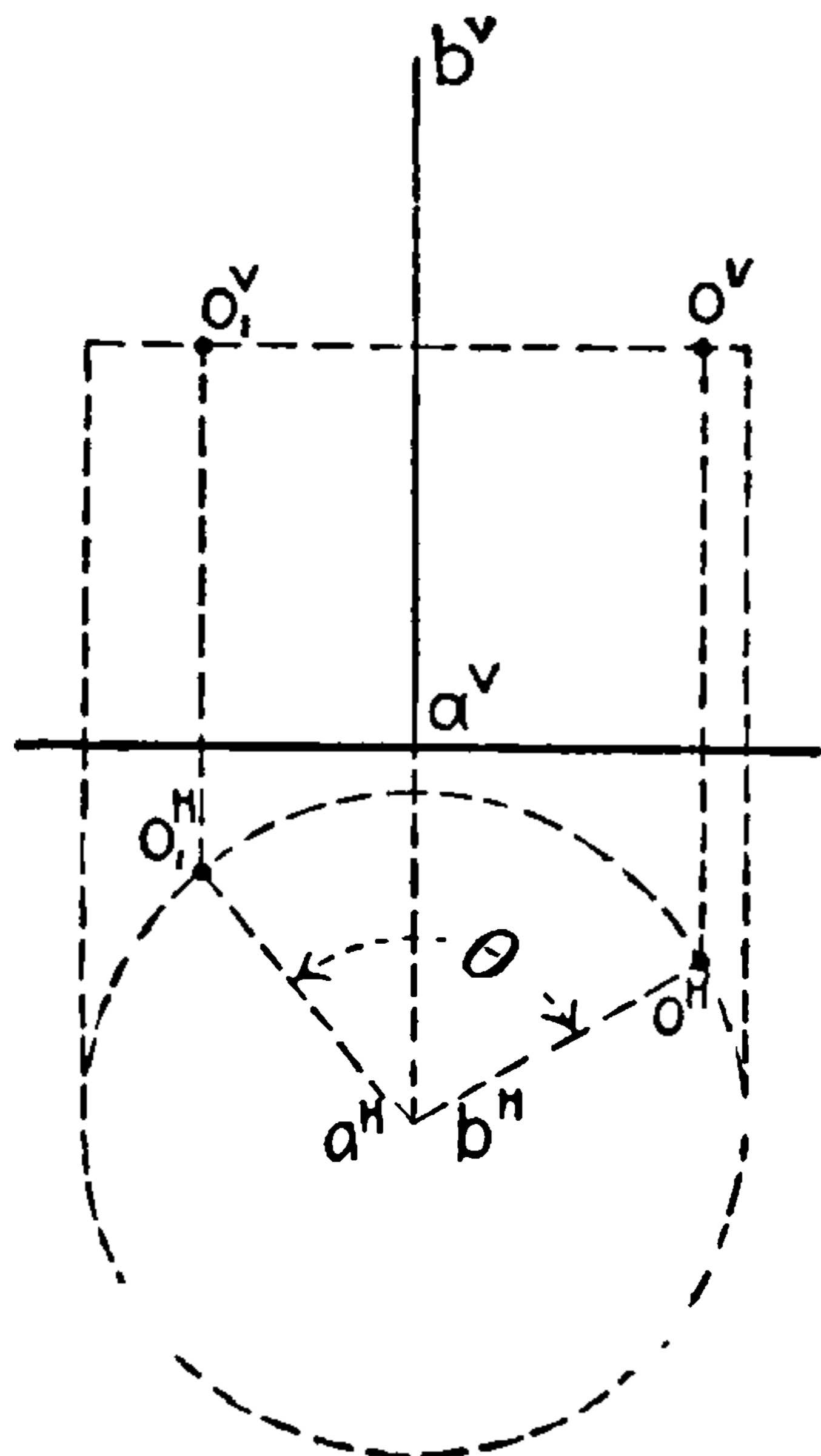


FIG. 32.

### 42.

### EXERCISES

134. Revolve  $A(4 + 1 - 1)$  about  $B(3, 0 - 2)$   $C(3 + 3 - 2)$  counter-clockwise  $120^\circ$ .
135. Revolve  $D(4 + 2 - 1)$  about  $E(3 + \frac{1}{2}, 0)$   $F(3 + \frac{1}{2} - 2)$  clockwise  $240^\circ$ .
136. Revolve  $G(1 + 1 - 1)$  about  $K(2 - 1 + 1)$   $L(2 - 1 + 2)$  clockwise  $135^\circ$ , and  $M(5 - 1 + 1)$  about  $N(4, 0 + 2\frac{1}{2})$   $P(4 - 2 + 2\frac{1}{2})$  until it reaches  $H$ .
137. Revolve  $A(3 - 1 - \frac{1}{2})$  clockwise  $90^\circ$  about  $B(4, 0 - 1)$   $C(4 - 2 - 1)$ . Continue the revolution until it returns to the fourth angle.
- >138. Revolve  $D(5 - \frac{1}{2} - 1)$  counter-clockwise  $90^\circ$  about  $E(4 + \frac{1}{2}, 0)$   $F(4 + \frac{1}{2} - 2)$ . Continue the revolution until it reaches  $H$  again.
139. Revolve  $G(4 + \frac{1}{2} - 1)$   $K(3 + 2 - 2)$  about  $P(3 + 1, 0)$   $O(3 + 1 - 3)$  clockwise  $135^\circ$ .
140. Revolve  $L(2 + 2 - 1)$   $M(3\frac{1}{2} + 1 - 2\frac{1}{2})$  about  $K(1\frac{1}{2}, 0 - 2)$   $O(1\frac{1}{2} + 3 - 2)$  into a position parallel to  $V$ .



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in a series of parallel circles. This means that every point remains in the same relation to the axis as every other point throughout the revolution, and therefore all points in the line remain in the same relation to each other, and the line is unchanged.

Note.—In revolving parallel to  $V$ , the  $H$ -projection is not changed in length, but the  $V$ -projection is.

Construction.—Fully shown in Fig. 33.

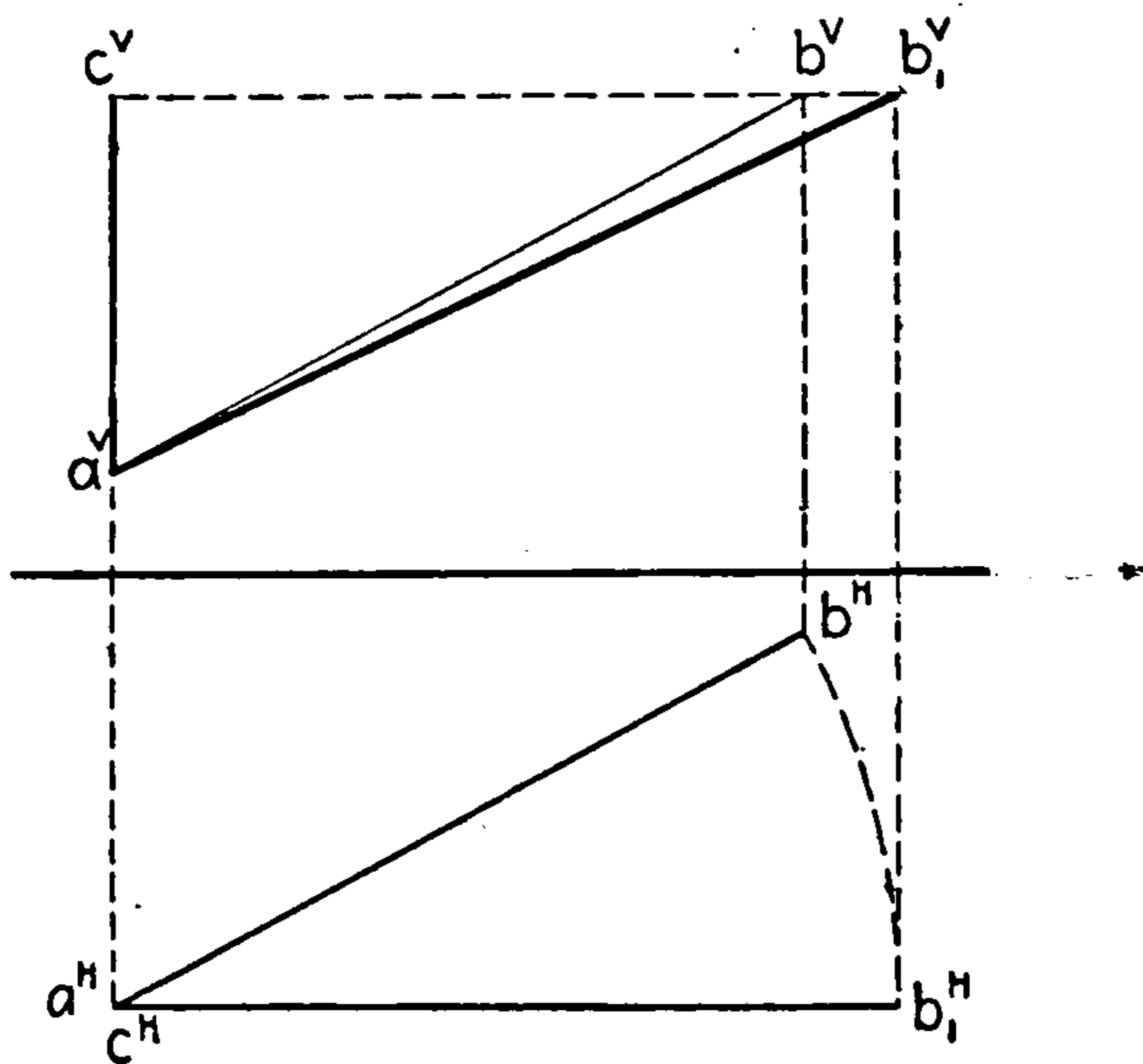


FIG. 33.

**44. The true length of a line in a profile plane.**  
The  $P$ -projection shows its true length.

45.

### EXERCISES

143. Measure A(1 + 1 - 1) B(2 + 3 - 2) by revolving  $\parallel$  to  $H$ , (2)  $\parallel$  to  $V$ . Check result by Problem 1, Corollary 1.
144. Measure C(5 - 1 + 3) D(3 - 3 + 2), (1) by revolving  $\parallel$  to  $V$ , (2)  $\parallel$  to  $H$ . Check.
145. Measure E(2 + 1 + 1) F(4 + 2½ + 2½), (1) by revolving  $\parallel$  to  $V$ , (2)  $\parallel$  to  $H$ . Check.
146. Measure G(3 - 3 - 3) K(1 + 1 - 1), (1) by revolving  $\parallel$  to  $H$ , (2)  $\parallel$  to  $V$ . Check.
147. Measure L(2 + 1 - 2) M(3 - 1 + 1), (1) by revolving  $\parallel$  to  $V$ , (2)  $\parallel$  to  $V$ . Check.
148. Measure M(4 - 2 + 1) N(5 + 1 + 1½), (1) by revolving  $\parallel$  to  $H$ , (2)  $\parallel$  to  $H$ . Check.

Measure the following lines. Place  $P$  at 4½.

149. A(1 - 2 + ½) B(1 - ½ + 1) and C(2 - 1 - 2) D(2 - 2 - ½).
150. E(1 + 3 - 1) F(1 + 1 - 2) and G(2 + ½ + 1¼) K(2 + 3 + ½).
151. M(1 + 2 - 1) N(1 - ½ - 2) and O(2 - 1 + 1) P(2 + 2 + ½).

152.  $A(1 - 2 + 1)$   $B(1 + 2 - 2)$  and  $C(2 + 1 + 2)$   $D(2 - 2 - \frac{1}{2})$ .  
 153.  $E(1 + 2 + 1)$   $F(1 + 1 - 1\frac{1}{2})$  and  $G(2 - \frac{1}{2} - 1)$   $K(2 - 2 + \frac{1}{2})$ .  
 154. Lay off — in. (amount specified by instructor) on any of the lines in Problems 143–153 from the first point toward the second.  
 155. Through  $M(1 + 1 - 3)$  draw a line — in. long, parallel to  $O(3 + 1 - 2)$   $P(4 + 3 - 1)$ .  
 156. Through  $A(5 - 2 + 3)$  draw a line — in. long, parallel to  $B(1 - 1 + 1)$   $C(2 - 1\frac{1}{2} + 2)$ .  
 157.  $D(2 - 3 + 1)$   $E(3\frac{1}{2} - 3 + 2\frac{1}{4})$  is the base diagonal of a square pyramid,  $2\frac{1}{2}$ -in. altitude, in III. Draw its projections.  
 (a) Measure one of its slanting edges.  
 (b) Measure the line running from a point  $1\frac{3}{4}$  in. up one edge to a point  $\frac{1}{2}$  in. up the adjacent edge.  
 158. Scale 100 ft. = 1 in. A rope conveyor is to be constructed from  $A(75 \text{ ft.}, 0, -180 \text{ ft.})$  to a loading point on a hill 225 ft. high, 200 ft. north,  $60^\circ$  east, of the dumping point. How much cable must be purchased for a duplex belt, allowing 30 ft. for pulleys, slack, etc.?

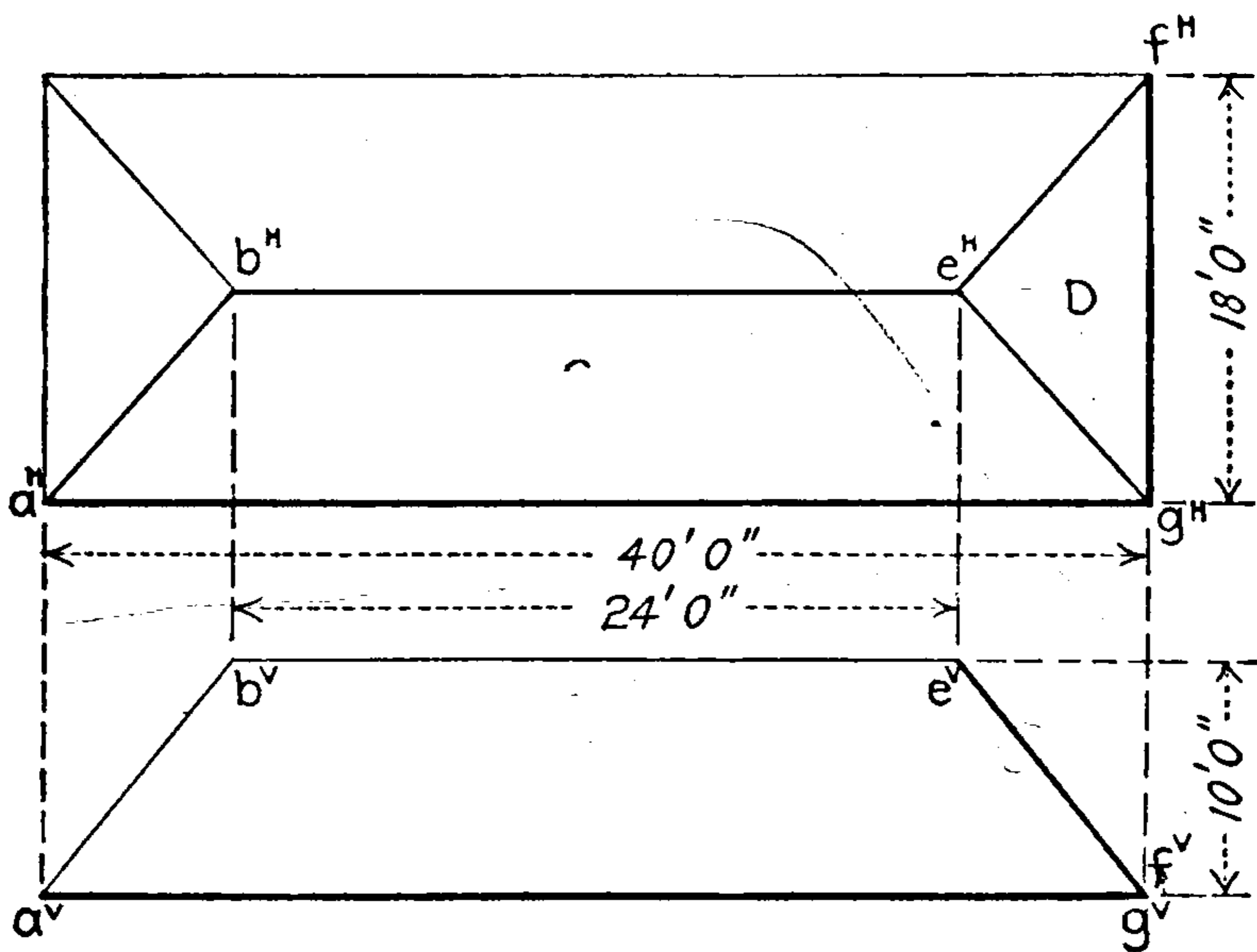


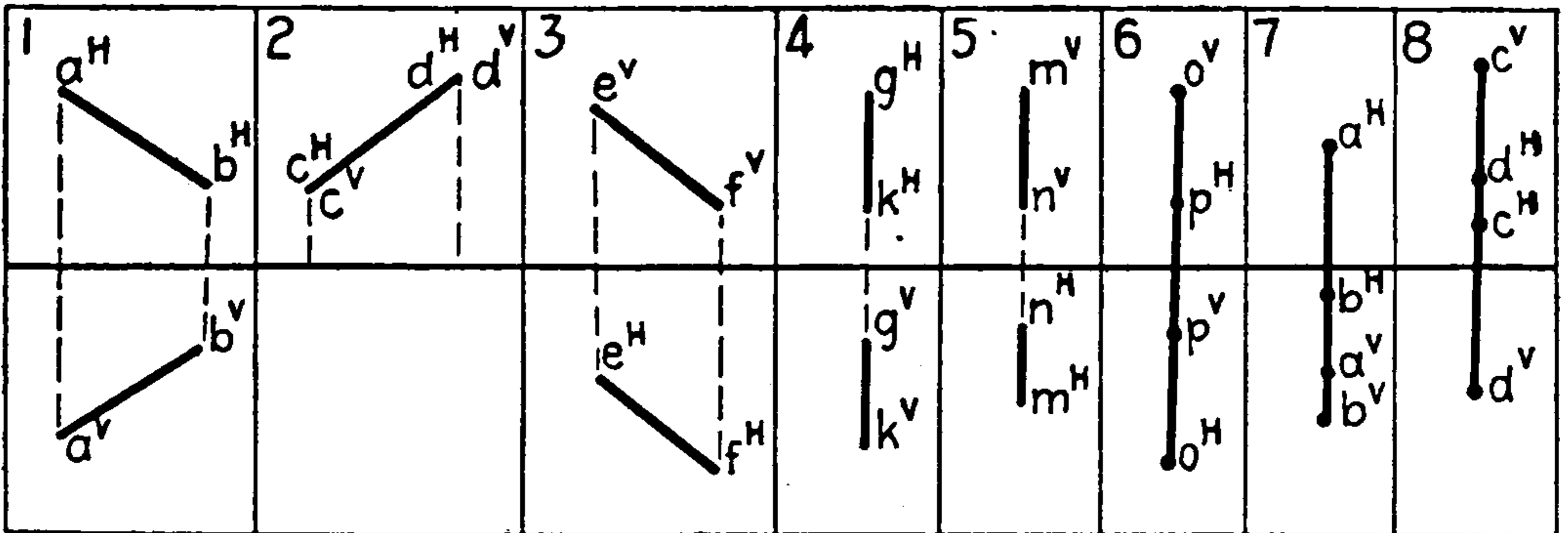
FIG. 34.

159. A pipe runs through a factory building from a point  $M(5 \text{ ft.} + 10 \text{ ft.} - 2 \text{ ft.})$  to  $N(30 \text{ ft.} - 2 \text{ ft.} 6 \text{ in.} - 14 \text{ ft.})$  in the floor below.  
 (a) What is its length?  
 (b) At what point in the floor must the hole for it be made?  
 (c) At a point 6 ft. from the floor hole, measured along the pipe, is a support from the floor. How far from the wall (V) and the floor (H) is this point of support? Scale  $\frac{1}{4}$  in. = 1 ft.  
 160. Fig. 34 shows the plan and elevation of a plain hip roof. Draw it to any convenient scale.  
 (a) What angle is it drawn in? Draw its *P*-projection.  
 (b) Locate the traces of the roof planes, C and D.  
 (c) What is the length of the hip rafter AB.

(d) Locate a point on the roof C, 3 ft. below the ridge and 10 ft. to the right of the center.

(e) Locate a point on the rafter, 3 ft. from F.

161. Scale 1 in. = 20 ft. A 60-ft. stack, 3 ft. in diameter, rises from A(60 ft., 0 - 35 ft.), on the top of a flat roof (H). Five guy-wires are attached to the stack at 40 ft. above the roof, and are anchored at points on the roof, respectively, B(30 ft., 0 - 10 ft.), C(85 ft., 0 - 7 ft. 6 in.), D(90 ft., 0 - 47 ft. 6 in.), E(70 ft., 0 - 67 ft. 6 in.), and F(25 ft., 0 - 52 ft. 6 in.). Draw the stack and guys, and measure the true length of the guys.



Graphic Layout No. 13.—(a) Measure the true length of given line ——. (b) Lay off from one point of given line — a point — in. distant on the line toward the second point.

### FIFTH FUNDAMENTAL PROBLEM

46. Problem 12.—To revolve a point in space into H or V about an axis lying in that plane.

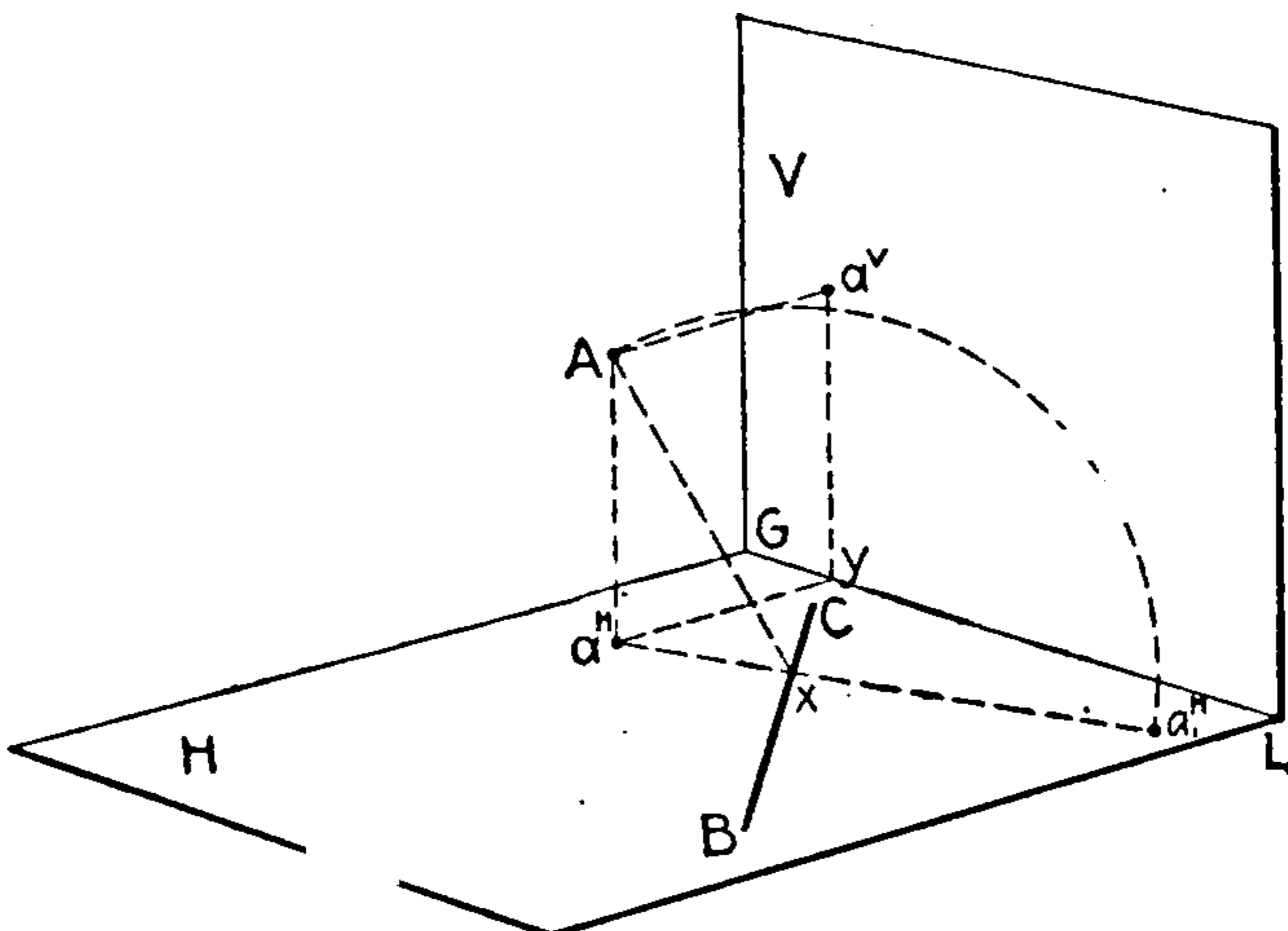


FIG. 35.

Process.—Fig. 35 shows a point A in space and a line BC in H. To revolve A about BC into H, it is necessary to draw the radius

of the circle through which  $A$  is to revolve. As will be seen in the picture,  $Ax$  is the radius, and it is the hypotenuse of the right triangle,  $Aa^H - a^Hx$ . By laying off this hypotenuse in either direction perpendicular to  $BC$  at  $x$ , the revolved position is obtained.

**Analysis.**—To revolve a point into  $H$  about an axis in  $H$ .

1. Draw the trace of the plane of revolution through the  $H$ -projection of the point, perpendicular to the axis.

2. Construct the right triangle as follows: (1) the base = the perpendicular distance from the  $H$ -projection to the line; (2) the altitude = the  $V$ -projection to  $GL$ .

3. Lay off the hypotenuse of this triangle from the axis on the trace of the plane of revolution.

**Construction.**—Let  $A$  (Fig. 36) be the given point and  $BC$  the given line in  $H$ .

1. Through  $a^H$  draw  $xa_1^H$  perpendicular to  $b^Hc^H$ .

2. Construct the triangle  $a^Hx - a^Hz (= a^Vy)$ .

3. Lay off  $xz$  on  $xa_1^H$ , locating  $a_1^H$ .

**Conclusion.**— $a_1^H$  is the revolved position of  $A$  about  $BC$ .

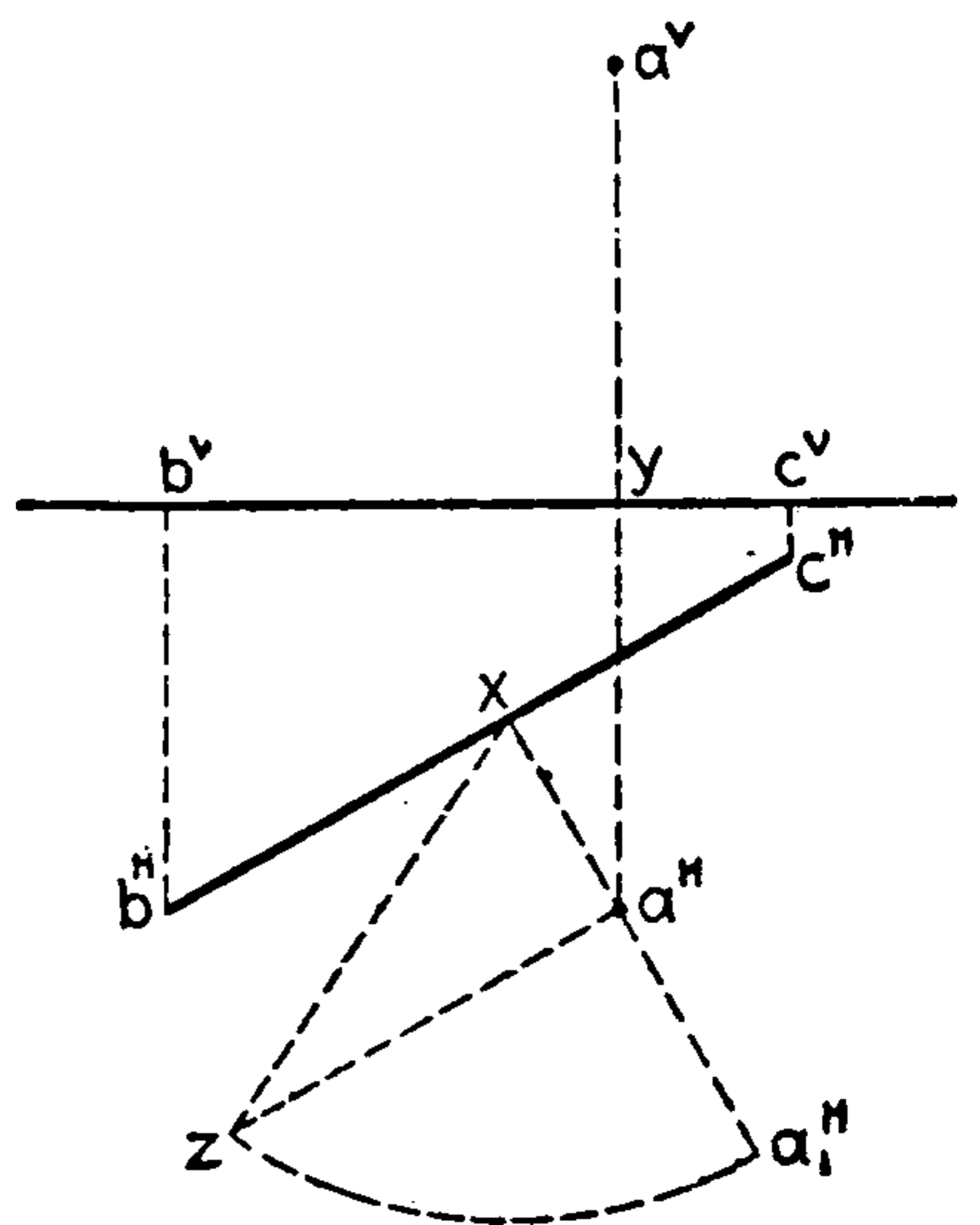


FIG. 36.

**Notes.**—To revolve a point into  $V$  about an axis in  $V$ , make an interchange of  $H$  and  $V$  in the foregoing.

2. If the point is directly over the axis, the base of the triangle becomes zero, and the hypotenuse will be equal to the altitude.

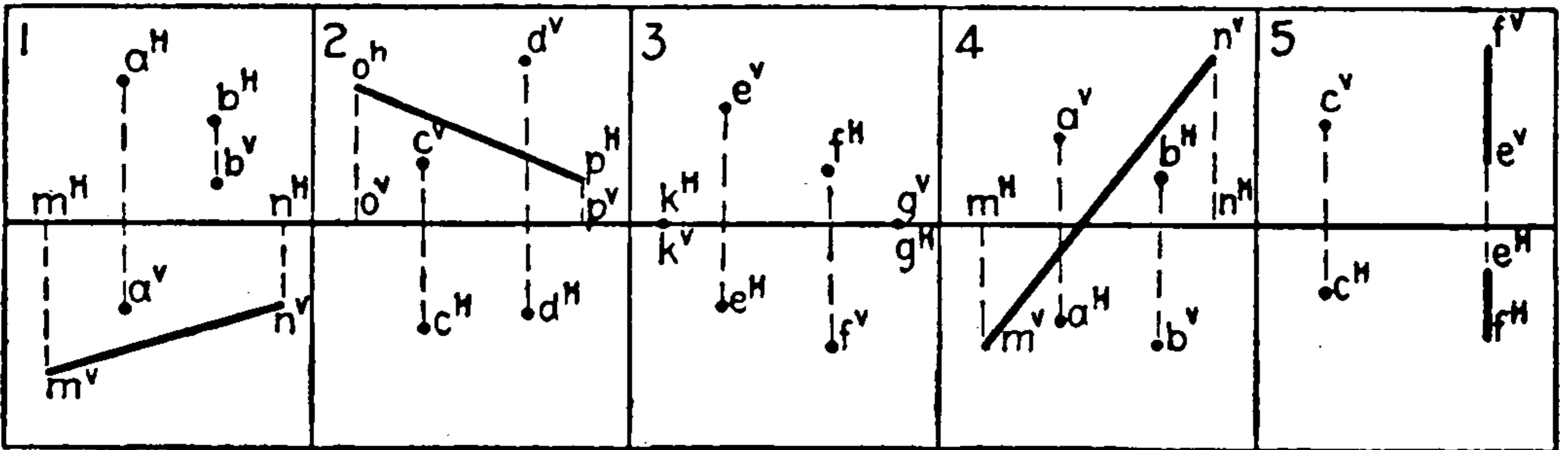
3. The “trace of the plane of revolution” and “the projection of the path of revolution” will be abbreviated hereafter to “the path of revolution.”

47.

EXERCISES

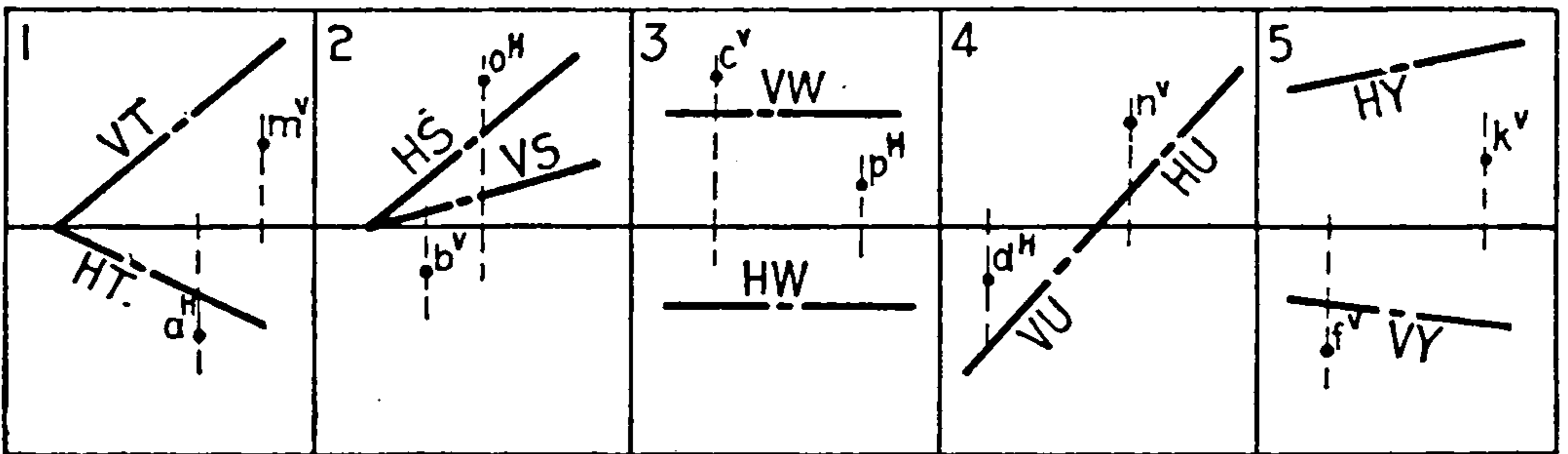
- 162. Revolve  $A(2 - 1 + 3)$  and  $B(4 + 2 - 2\frac{1}{2})$  into  $H$  about  $C(1, 0 - 2)$   $D(5, 0 - \frac{1}{2})$ .
- 163. Revolve  $E(1 + 1 - 2)$  and  $F(4 - 1 + 2)$  into  $V$  about  $G(2 + 3, 0)$   $K(5 + \frac{1}{2}, 0)$ .
- 164. Revolve  $M(1 - 2 - 1)$ ,  $N(2 + 1 - 2)$ ,  $O(3 + 1 + 1)$ , and  $P(4 - 1 + 1\frac{1}{2})$  about  $GL$ , (1) into  $V$ ; (2) into  $H$ .
- 165. Revolve  $K(2 + 1\frac{1}{2} - 1)$  about  $L(1 + 3, 0)$   $N(5, 0, 0)$  into  $V$ . Revolve  $M)3 - 2 + 1)$  about  $P(2, 0, 0)$   $O(5, 0 - 1)$  into  $H$ .
- 166. Locate  $A(1 + 1, x)$  and  $B(2, y - 1)$  in  $T(1 + 3)$   $5(1 - 2)$ , and revolve into  $V$  about  $VT$ , and into  $H$  about  $HT$ .

167. Locate  $C(4 + 2, x)$  and  $D(2, y - 1)$  in  $S(5 + 3) 1(5 - 1)$ , and revolve into  $V$  about  $VS$ , and into  $H$  about  $HS$ .
168. Locate  $E(2, x - 2)$  and  $F(4 + 1, y)$  in  $U(+ 2) \infty(- 1)$ , and revolve them about  $VU$  and  $HU$  into  $V$  and  $H$ .



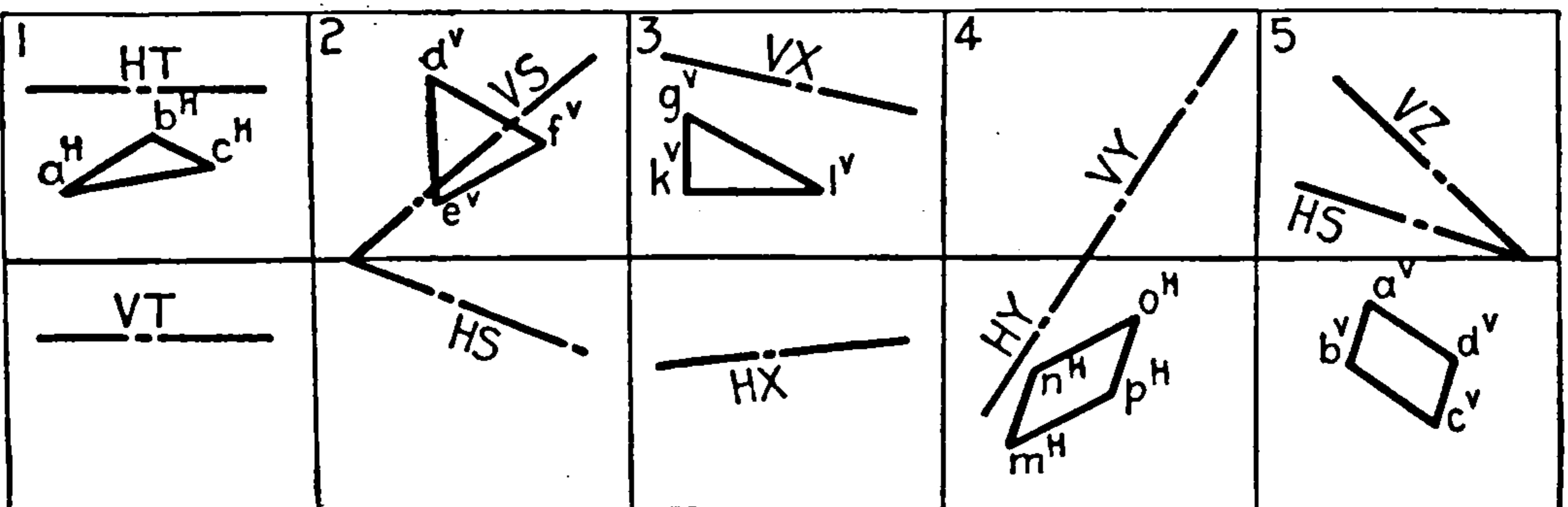
Graphic Layout No. 14.—Revolve the points in Ex. — about the given line into the plane of the latter.

169. Locate  $K(2 - 2, x)$  and  $L(4, y + 1)$  in  $W(- 1) \infty(+ 3)$ , and revolve them about  $VW$  and  $HW$  into  $V$  and  $H$ .



Graphic Layout No. 15.—Locate the points in plane — and revolve them into — about the — trace.

170. Locate  $O(+ 2)$  and  $P(+ 1)$  in the  $V$ -trace of  $T(1 + 3) 5(1 - 2)$  and revolve them into  $H$  about the  $H$ -trace.



Graphic Layout No. 16.—Locate the plane figure—in its plane, and revolve about one of the traces to show its true size.

171. Locate  $A(2 - 1, z)$  and  $B(4, 0 + 2)$  in  $S(- 1) \infty(+ 2)$ , and revolve  $A$  into  $H$  about  $HS$ , and  $B$  into  $V$  about  $VS$ .

172. Revolve the triangle  $M(1\frac{1}{2} + 1\frac{1}{2} - 1)$   $N(2\frac{1}{2} + 1 - 2\frac{1}{4})$   $O(3\frac{1}{2} + 1\frac{1}{4} - \frac{1}{2})$  into  $V$  about the  $V$ -trace of its plane. Will this give its true size, and the true angles at the corners?
173. Revolve the parallelogram  $A(3 - 1 + 1)$   $B(3\frac{1}{2} - \frac{3}{4} + 2)$   $C(5 - 2\frac{1}{4} + 1\frac{1}{2})$   $D(4\frac{1}{2} - 2\frac{1}{2} + \frac{1}{2})$  into  $H$  about the  $H$ -trace of its plane. Will this give its true size?

MEASUREMENT OF ANGLES

48. Problem 13.—To measure the true angle between any two intersecting lines.

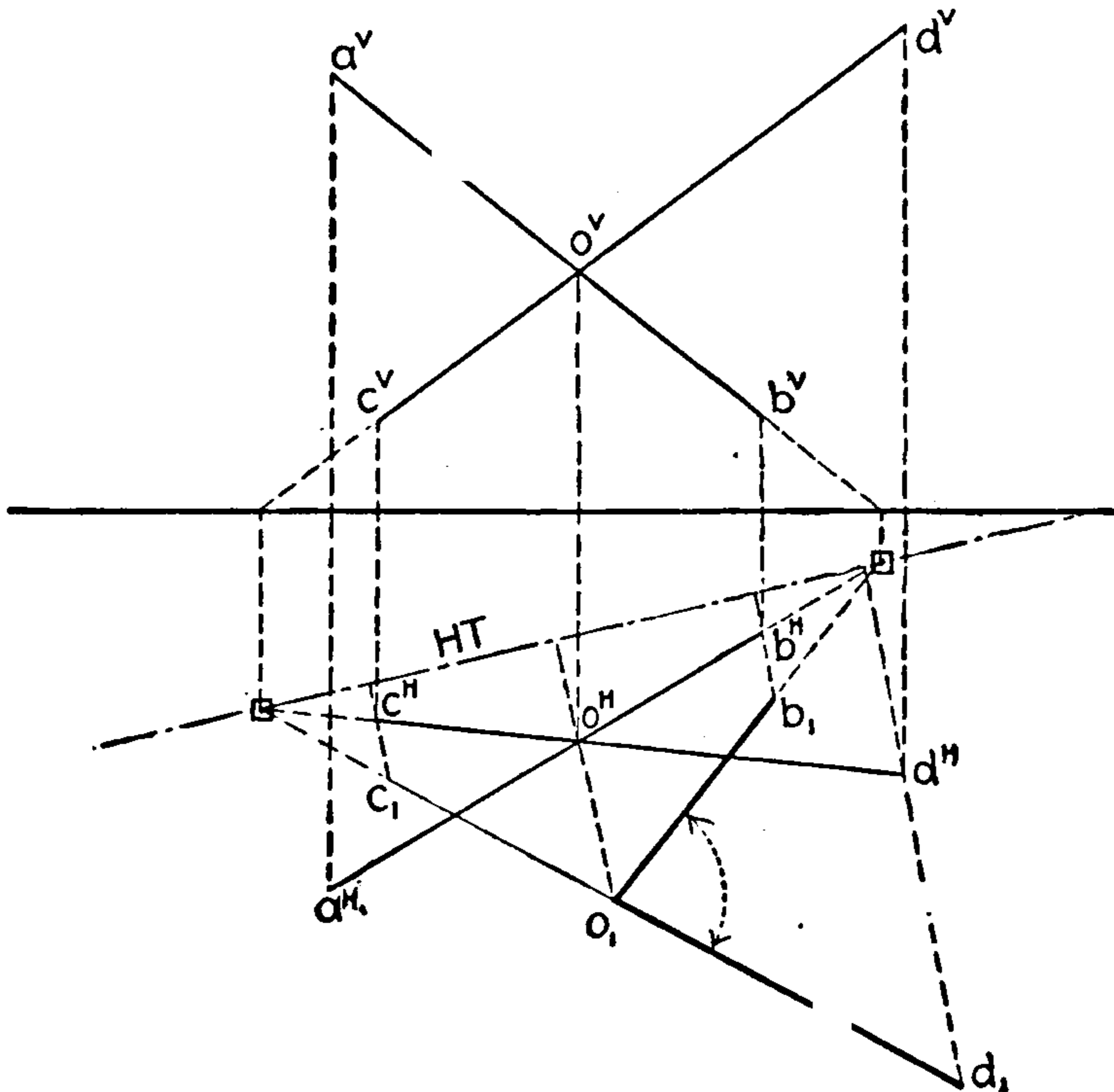


FIG. 37.

Two intersecting lines lie in one plane, and, if that plane be folded so as to coincide with  $H$  or  $V$ , the revolved projections will show the true relation of the lines.

Analysis.—1. Pass a plane through the two lines.

Note.—One trace only need be obtained.

2. Revolve the plane about its  $H$ -trace into  $H$ , or about its  $V$ -trace into  $V$ .

Note.—Revolving the various points in a plane is equivalent to revolving the plane.

Conclusion.—The angle between the lines in revolved position is the measure of the actual angle.

Construction.—Let  $AB$  and  $BC$  (Fig. 37) be the two intersecting lines whose angle is to be measured.



1. Pass the plane through the two lines; HT is the necessary trace.

2. Revolve AB and BC about the trace.

Conclusion.—The required angle is  $b_1o_1d_1$ .

Note 1.—It is usual to consider the acute angle as the measure, although the obtuse angle is not incorrect.

Note 2.—In case any of the necessary piercing points are inaccessible, it is proper to draw as many auxiliary lines from one line to the other as may be needed to obtain the trace.

### SPECIAL CASES

49. Case I.—When one side is parallel to H.

Analysis.—1. Revolve the oblique line about the horizontal line as an axis until it becomes parallel to H.

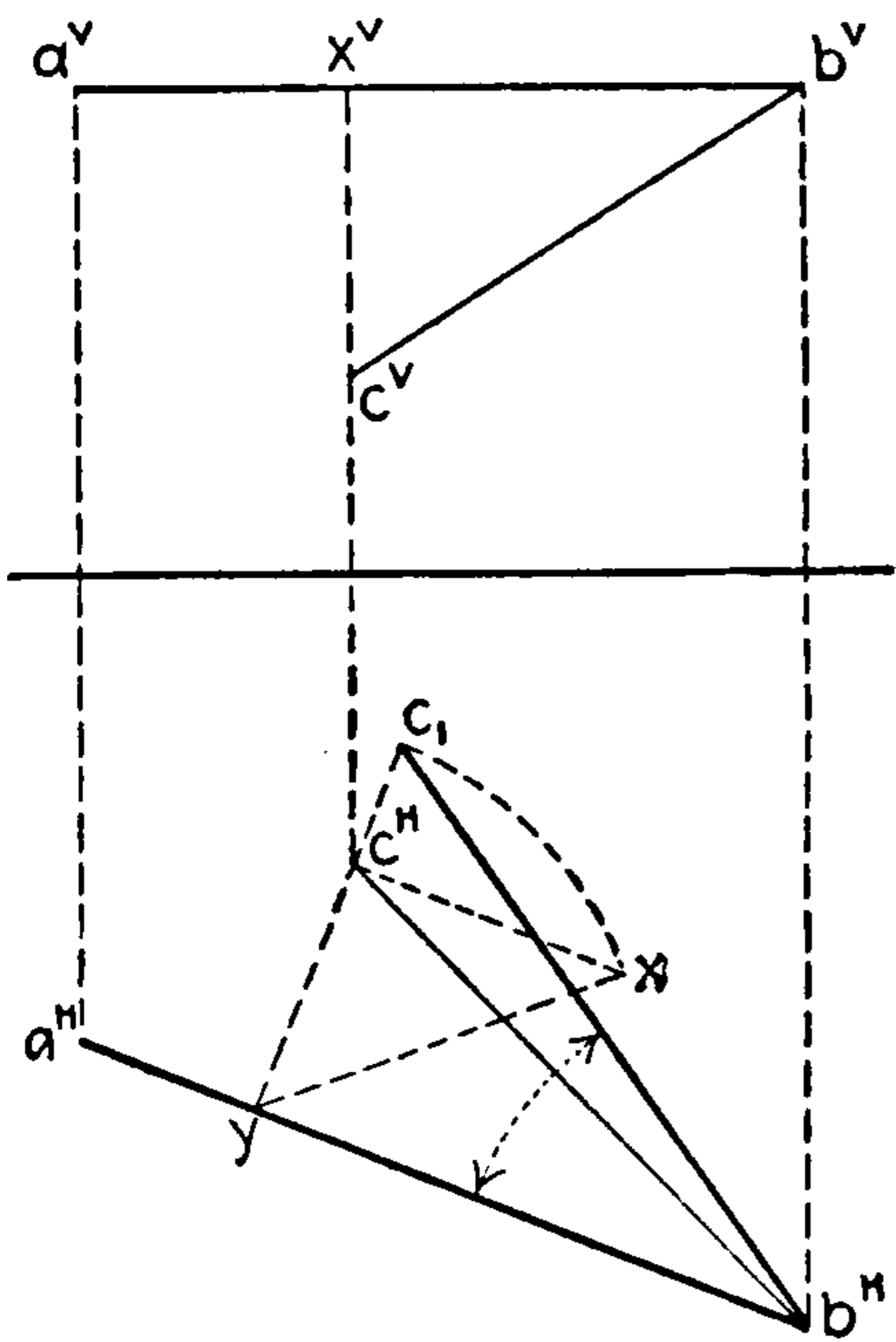


FIG. 37a.

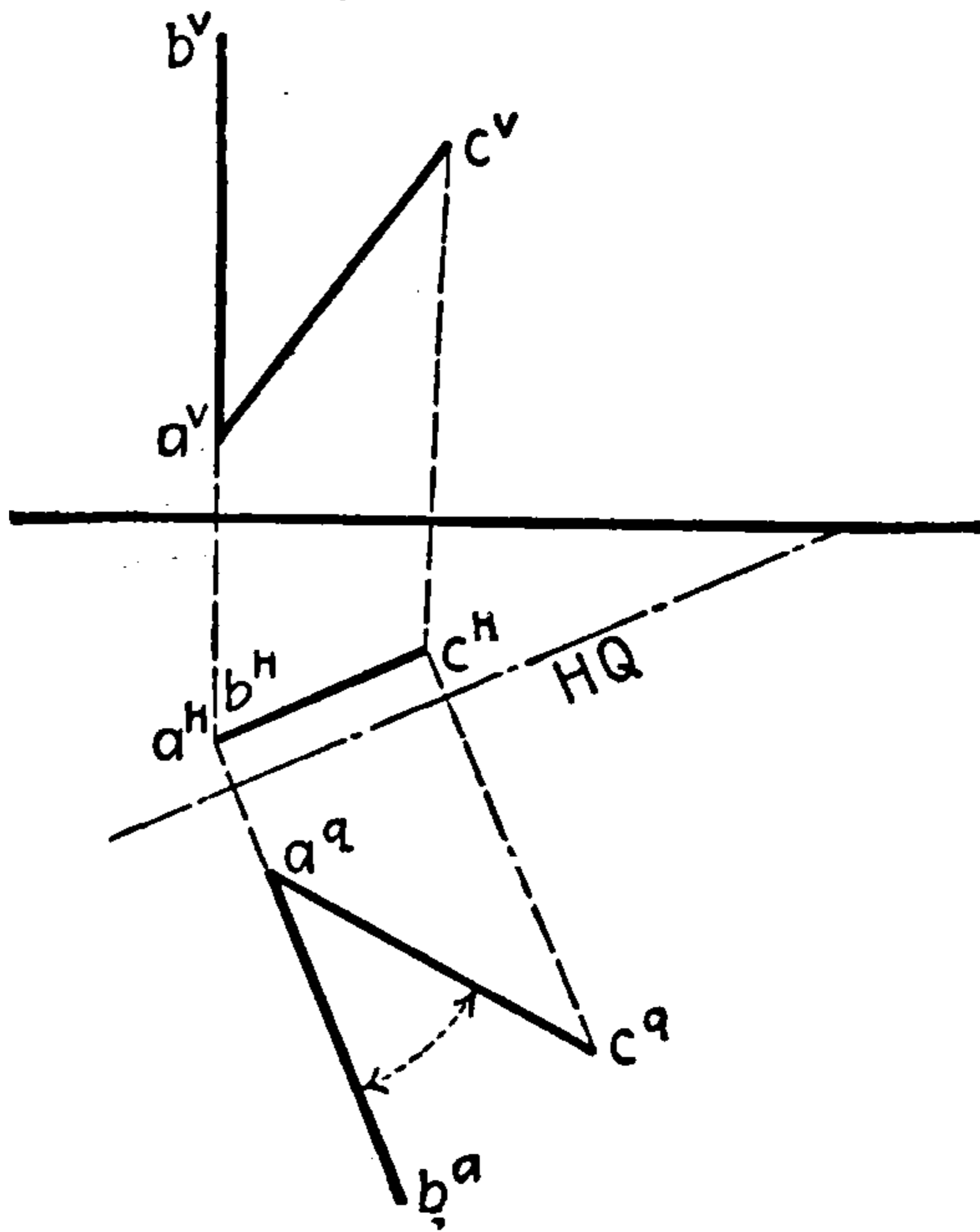


FIG. 37b.

Conclusion.—The projected angle is now equal to the true angle, since both are parallel to H, and their relations are unchanged.

Construction.—Required to measure the angle ABC (Fig. 37a), AB being parallel to H.

1. Revolve C about AB as an axis into the horizontal plane of AB.

Note 1.—This can be easiest done by transferring  $GL$  to  $a^vb^v$ , and applying Problem 12.

Note 2.—The same process can be performed when one of the lines is parallel to V or P.

Note 3.—If preferred, the regular processes can be used instead of this special one.



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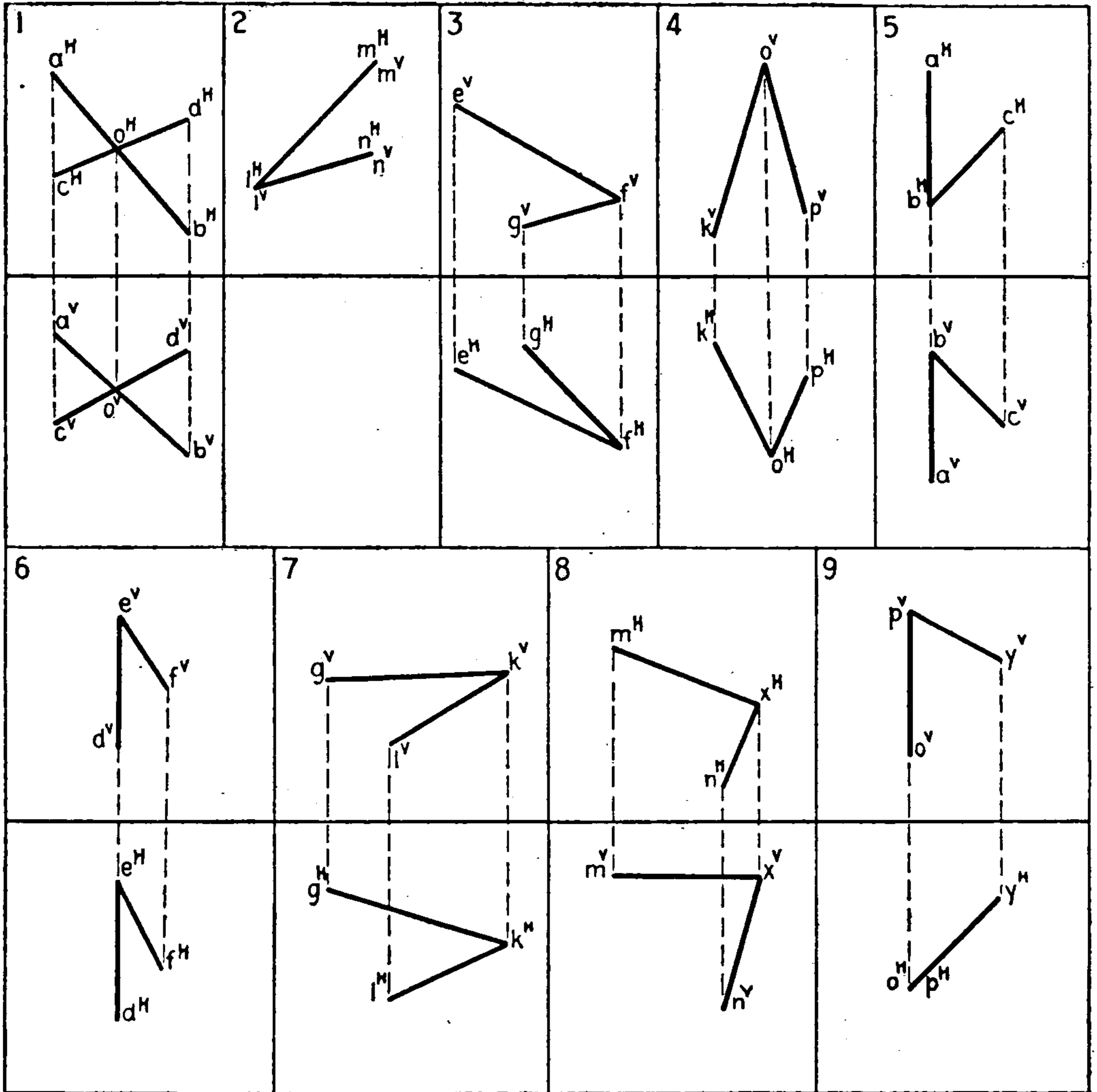
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Note.—If the angle can be measured by any of the special cases, the bisector may be found by the same process.

53.

## EXERCISES



Graphic Layout No. 17.—(a) Measure angles between given lines in Ex. ——. (b) Draw the projections of the bisector of the angle in Ex. ——.

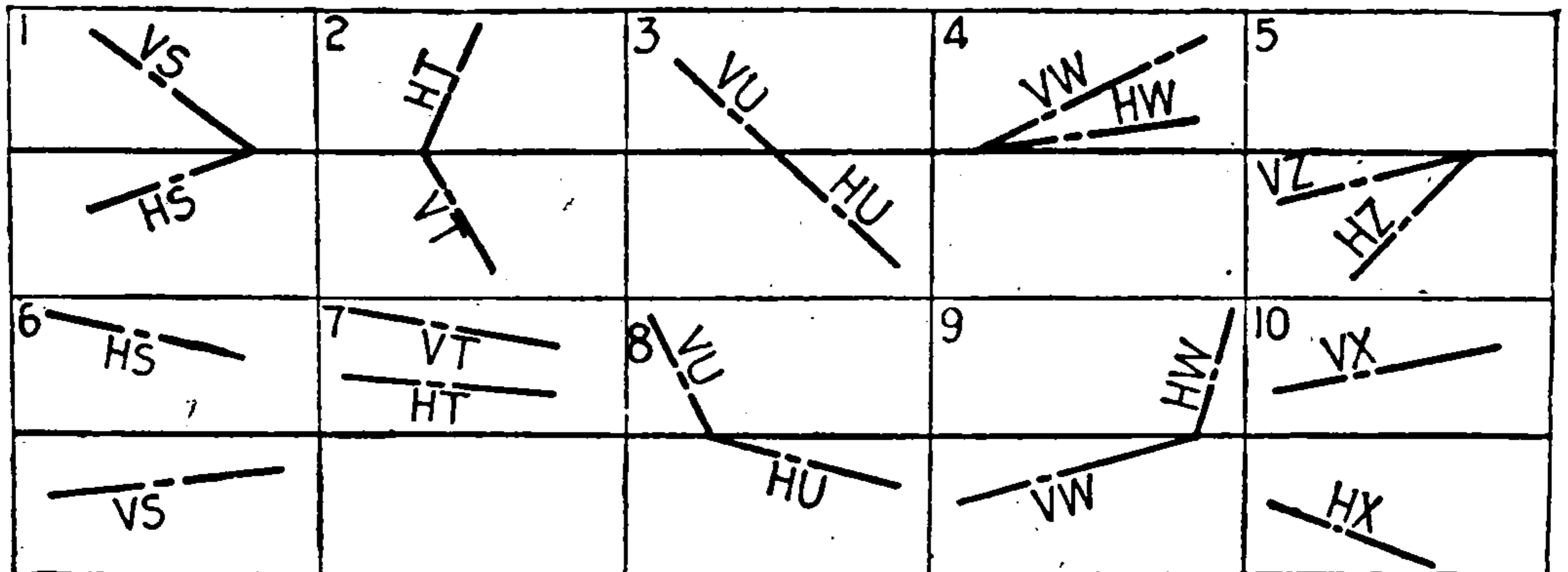
Measure the angles included between the following lines:

174.  $A(2 + 2 - 2)$   $B(3\frac{1}{2} + 1 - \frac{1}{2})$  and  $BC(4\frac{1}{2} + 3 - 1\frac{1}{2})$ .  
 175.  $D(1 - 2 + 3)$   $E(4 - 1\frac{1}{2} + 1\frac{1}{2})$  and  $EF(2 - 1 + 1)$ .  
 176.  $G(1 + 2\frac{1}{2} - 1)$   $K(3\frac{1}{2} + 1\frac{1}{2} - 2)$  and  $KL(2 + 1 - 2\frac{1}{2})$ .  
 177.  $M(1\frac{1}{2} + 3 - 2)$   $O(2 + 1 - \frac{1}{2})$  and  $OP(5 + \frac{1}{2} - 1\frac{1}{2})$ .  
 178.  $A(2 + 3 - 2)$   $B(2 + 1 - 1)$  and  $BC(3\frac{1}{2} + 2 - 3)$ .  
 179.  $D(4 + 1 - 1)$   $E(4 + 2\frac{1}{2} - 2)$  and  $EF(3 + 1 - 1)$ .  
 180.  $G(3 + 1 - 3)$   $K(3 + 3 - 1)$  and  $KL(2 + 3 - 1)$ .  
 181.  $M(1 + 2 - 1)$   $N(1 + 2 - 3)$  and  $NO(2\frac{1}{2} + \frac{1}{2} - 1)$ .  
 182.  $A(1 + 1\frac{1}{2} - 2)$   $B(4 + 1\frac{1}{2} - 2)$  and  $BC(2 + 1 - 1)$ .

- 183.  $D(3 + 3 - 1\frac{1}{2}) E(1 + 1 - \frac{1}{2})$  and  $EF(3 + 1 - 3)$  by revolving into the profile.
- 184.  $G(2 + 1\frac{1}{2} - 3) K(4 + 1\frac{1}{2} - 1)$  and  $KL(3 + 3 - 1\frac{1}{2})$ .
- 185.  $M(1 - 2\frac{1}{2} + 1) N(4 - \frac{1}{2} + 1)$  and  $NO(2 - 1 + 2)$ .
- 186.  $A(2 + 2 + 2) B(4 + \frac{1}{2} + \frac{1}{2})$  and  $BC(3 - 1 - 1)$ .
- 187.  $D(4 - 2 - 2) E(2 + \frac{1}{2} + \frac{1}{2})$  and  $EF(3 + 2 - 1)$ .
- 188.  $G(3 + 1 + 1) K(3 - 1\frac{1}{2} - 1\frac{1}{2})$  and  $KL(1 - 1 - 1)$
- 189.  $M(4, 0, 0) N(2 - 1 + 1\frac{1}{2})$  with  $GL$ .

Measure the angle between the traces of the following planes:

- 190.  $T(1 + 1) 4(1 - 2\frac{1}{2})$ .
- 191.  $S(1 + 3) 4(1 + 2)$ .
- 192.  $R(4 - 1\frac{1}{2}) 1(4 + 2)$ .
- 193.  $Q(1 + 2) 3(5 - 2)$ .
- 194.  $U(4 + 2) 3(2 - 2)$ .
- 195.  $W(5 + 3) 3\frac{1}{2}(1 - 1)$ .
- 196.  $T(1 + 2) 1(3 - 3)$  and  $S(2 + 1\frac{1}{2}) 5(5 - 3)$
- 197.  $R(1 + 3)(5 + 1\frac{1}{2})(1 - 2)(5 - 1)$ .
- 198.  $W(1 + 3)(5 + 1\frac{1}{2})(1 + 2)(5 + 1)$ .



Graphic Layout No. 18.—(a) Measure the angle between the  $H$ - and  $V$ -traces of plane ——. (b) Draw the bisector of the angle between the traces of plane ——.

Measure the angles in the following triangles:

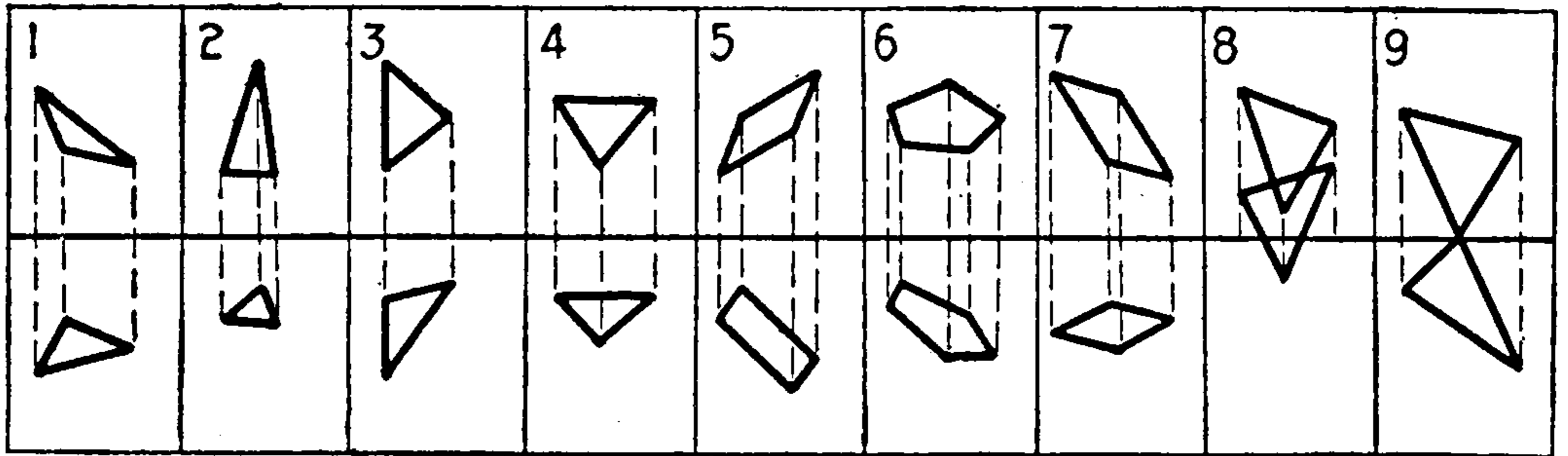
- 199.  $A(2 - 1 + 3) B(3 - 2 + 1\frac{1}{2}) C(5 - 1\frac{1}{2} + 2)$ .
- 200.  $D(2\frac{1}{2} + 1 - 1) E(4 + 3 - \frac{1}{2}) F(4 + 2 - 2)$ .
- 201.  $G(2 + 2 - 2) K(2 + 1 - 1\frac{1}{2}) L(3 + 2 - 2)$ .
- 202.  $M(2\frac{1}{2} - 1\frac{1}{2} + 2) O(3 + \frac{3}{4} - 1) P(4 - \frac{3}{4} + 1)$ .
- 203.  $A(2 + 2 + 2) B(2\frac{1}{2} + 1 + 1) C(4 + 1\frac{1}{2} + 1\frac{1}{2})$ .
- 204.  $D(5 - 1 - 1) E(4 - 2 - 2) F(2\frac{1}{2} + 1 - \frac{1}{2})$ .
- 205.  $G(2 + 1\frac{1}{2}, 0) K(3\frac{1}{2}, 0, 0) L(3, 0 - 2)$ .
- 206. Measure the angles and true size of the parallelogram  $M(3 + \frac{1}{2} - 1\frac{1}{2}) N(3 + 1 - \frac{1}{2}) O(4 + 2 - 1) P(4 + 1\frac{1}{2} - 2)$ .
- 207. Measure the angles of the pentagon  $A(3\frac{1}{2} + \frac{1}{2}, x) B(4 + 1\frac{1}{4}, y) C(4\frac{1}{2} + \frac{1}{8}, z) D(5\frac{1}{2} + \frac{3}{4}, m) E(5 + 1\frac{1}{4}, n)$  that lies in the plane  $T(6 + 3) 1(5 - 1\frac{1}{2})$ .

208. Prove graphically that  $A(2 + \frac{3}{4} - 1)$   $B(3 + 2 - 2)$   $C(4\frac{1}{2} + 2 - \frac{1}{2})$  is a right angle. Write an explanation.
209. Draw a line from  $D(2 + 2 - 1)$  intersecting  $E(2 + 1 - 2)$   $F(5 + 2 - \frac{1}{2})$  at  $\text{---}^\circ(30, 45, 60, 75)$ .
210. Draw a line from  $G(3 + 1 - 1)$  intersecting  $K(1 + 2 - 3)$   $L(4 + 2 - 3)$  at  $\text{---}^\circ(30, 45, 60, 75)$ .
211. Draw a line from  $M(2 + \frac{1}{2} - 2)$  intersecting  $N(1 + 2 - 1)$   $O(3 + 1 - 1)$  at  $\text{---}^\circ(30, 45, 60, 75)$ .
212. Draw a line from  $P(2 + 2 - 1)$  intersecting  $A(3 + 1 - 2)$   $B(3 + 2 - \frac{1}{2})$  at  $\text{---}^\circ(30, 45, 60, 75)$ .
213. Draw a perpendicular from  $C(2 + 2 - \frac{1}{2})$  to  $D(2 + \frac{1}{2} - \frac{1}{2})$   $E(2 + 2 - 1\frac{1}{2})$ .
214. Measure the angle between two adjacent edges and the angle between an edge and a base line of the pyramid given in Ex. 157, Art. 45.
215. Measure the angle that  $E(2 + \frac{1}{2} - 2\frac{1}{2})$   $F(4 + 2 - 1)$  makes (1) with its own  $V$ -projection, (2) with its  $H$ -projection.
216. Draw the roof in Fig. 34 to a definite scale, and measure the following angles:
- Hip rafter  $EF$  with hip rafter  $EG$ .
  - Hip rafter  $EF$  with eave  $FG$ .
  - Hip rafter  $AB$  with eave  $AG$ .
  - Hip rafter  $AB$  with ridge  $BE$ .
217. Draw the roof in Fig. 34 to a definite scale, and measure the true size of the roof planes  $C$  and  $D$ .  
 Compute their areas from the scale.  
 What would it cost to roof it at \$1.25 per square yard?
218. The  $V$ -trace of  $T(1 + 3)$   $4(x, y)$  makes  $60^\circ$  with its  $H$ -trace. Draw the  $H$ -trace.
219. The  $H$ -trace of  $S(x, y)$   $3(5 - 1\frac{1}{2})$  makes  $120^\circ$  with its  $V$ -trace. Draw the  $V$ -trace.
220. A ray of light passes through  $A(1 + 1 - 2)$  and is reflected from  $H$  to  $B(4 + 1\frac{1}{2} - 1)$ . At what point and angle does it strike  $H$ ?
221. A ray of light passes through  $C(1 + 2 - 1)$  and, after reflecting from  $V$  and  $H$ , passes through  $D(5 + \frac{1}{2} - 2)$ . At what points and angles does it strike  $H$  and  $V$ ?
222. Draw a line from  $E(4, x - 1)$  in  $T(5 + 3)$   $2(5 - 2)$  making  $60^\circ$  with the  $H$ -trace.
223. Draw a line from  $K(2 + 1 - 1\frac{1}{2})$  in the triangle  $KL(2 + 2 - 2)$   $N(3 + 2 - 2)$  making  $60^\circ$  with the opposite side.
224. Find the center of the triangle  $A(2 + 1\frac{1}{2} - 1)$   $B(3 + 1 - \frac{1}{2})$   $C(3\frac{1}{2} + 3 - 2)$ .
225. Find the center of the triangle  $D(1\frac{1}{2} - 2 + 1)$   $E(2\frac{1}{2} - 1 + \frac{1}{2})$   $F(3\frac{1}{2} - 1\frac{1}{2} + 2)$ .

Draw the projections of the bisectors of the following angles (Ex. 226-247):

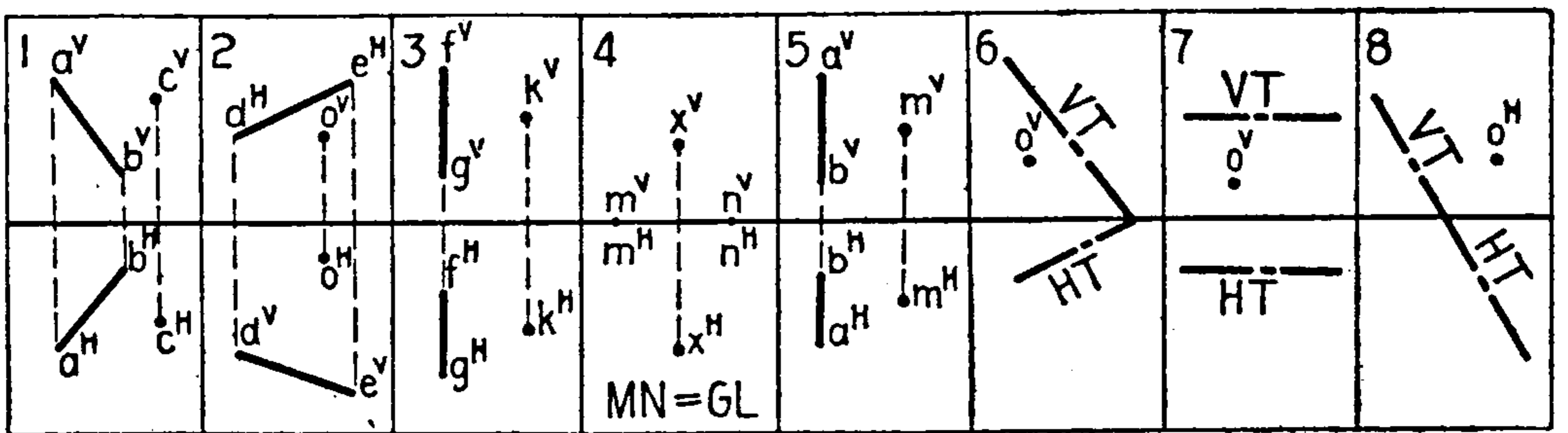
226.  $A(4 + 2\frac{1}{2} - 1)$   $B(2 + 1 - 1\frac{1}{2})$   $C(4 + 1\frac{3}{4} - \frac{1}{4})$ .
227.  $D(2 - 1 + 3)$   $E(2\frac{3}{4} - 3 + 1\frac{1}{2})$   $F(4 - 1\frac{1}{4} + 1)$ .
228.  $G(2 + 2 - 1)$   $K(3\frac{1}{2} + 3 - 3)$   $L(2 + \frac{1}{2} - 1\frac{3}{4})$ .

229.  $M(4 - \frac{1}{2} + 2)$   $N(2 - 3 + 2\frac{1}{2})$   $O(4 - 1\frac{1}{2} + \frac{1}{2})$ .  
 230.  $A(2 + 1\frac{1}{2} - 3)$   $B(2 + \frac{1}{2} - \frac{3}{4})$   $C(2 + 3 - 2)$ .  
 231.  $D(2 - 1 + \frac{1}{2})$   $E(2 - \frac{1}{2} + 1\frac{1}{2})$   $F(2 - 3 + \frac{1}{2})$   
 232.  $G(1 + \frac{1}{2} - 3)$   $K(2 + 1 - 1)$   $L(4 + 2 - \frac{1}{2})$ .  
 233.  $M(1 + 2\frac{1}{2} - 2)$   $N(1 + 1 - 1)$   $O(3 + 2 - 2\frac{1}{2})$ .  
 234.  $A(2\frac{1}{2} + 2\frac{1}{2} - \frac{1}{2})$   $B(2\frac{1}{2} + \frac{1}{2} - 1\frac{3}{4})$   $C(1 + 2 - 1)$ .  
 235.  $D(2 + 1\frac{1}{2} - 1)$   $E(5, 0, 0)$   $F(2, 0, 0)$ .



Graphic Layout No. 19.—(a) Measure the angle of the polygon ——. (b) Locate the center of triangle — (1, 2, 3, 4, 8, 9).

236.  $G(1 - 1 + \frac{1}{2})$   $K(1 - 2\frac{1}{2} + 2)$   $L(1 - 1\frac{1}{2} + 1\frac{1}{2})$ .  
 237. Between the  $H$ - and  $V$ -traces of  $T(1 + 3)$   $5(1 - 1\frac{1}{2})$ .  
 238. Between the  $H$ - and  $V$ -traces of  $S(1 + 3)$   $2(5 - 1)$ .  
 239. Between the  $H$ - and  $V$ -traces of  $R(5 + 3)$   $1(5 + 2)$ .  
 240. Between the  $V$ - and  $P$ -traces of  $R(5 + 3)$   $1(5 + 2)$   
 241. Between the  $V$ - and  $P$ -traces of  $Q(+ 1)$   $\infty(- 2)$ .



Graphic Layout No. 20.—(a) Draw a perpendicular from point — to line — in Ex. —. In Ex. 6–8 locate the point in the plane, and draw the perpendicular to the — trace. (b) From the given point in Ex. — draw a line making —° (15, 30, 45, 60, 75) with the given line or trace.

242. Between the  $H$ - and  $V$ -traces of  $U(5 + 3)(1 - 1)(5 + 1\frac{1}{2})(1 - \frac{1}{2})$ .  
 243. Between the  $H$ - and  $P$ -traces of  $U$ (Ex. 242).  
 244. Between the  $H$ - and  $V$ -traces of  $W(2 + 3)$   $3(4 - 3)$ .  
 245. Between the  $H$ - and  $P$ -traces of  $W(2 + 3)$   $3(4 - 3)$ .  
 246.  $M(1 + 2 - 3)$   $N(3 + 1 - \frac{1}{2})$  and its  $V$ -projection.  
 247.  $O(4 - 1 + 3)$   $P(2 - 3 + 1\frac{1}{2})$  and its  $H$ -projection.

**54. Problem 15.**—To find the shortest distance from a point to a line.

### TWO METHODS

**First Method.**—Analysis.—1. Pass a plane through the given point and line.

2. Revolve the point and line about the trace of the auxiliary plane, as in the preceding problems.

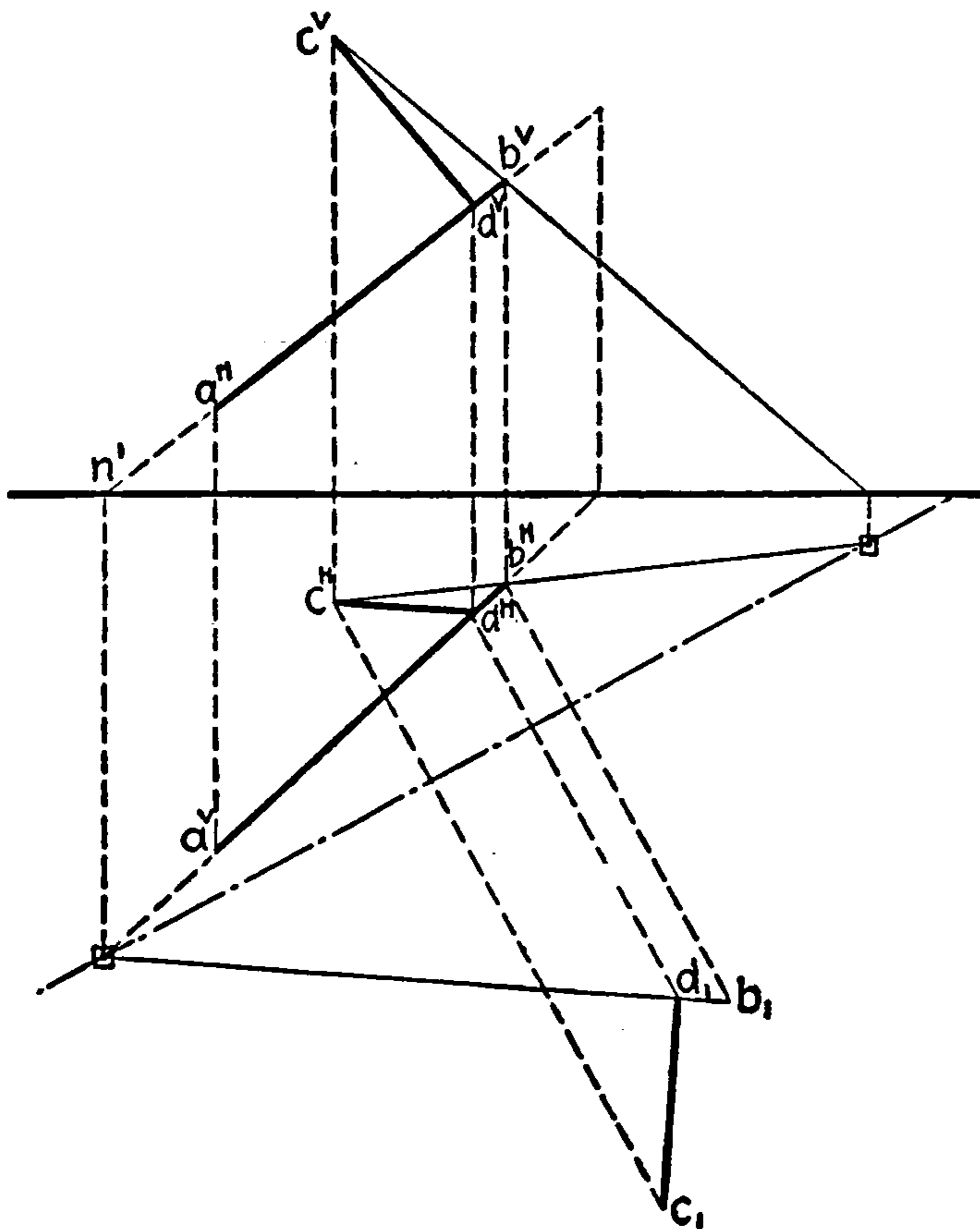


FIG. 39.

3. Erect the perpendicular from the point to the line.

**Conclusion.**—This perpendicular is the true shortest distance, as the point and line are in their true relation.

**Construction.**—Let C (Fig. 39) be the given point and AB the given line.

1. Pass the plane T through AB and C, obtaining HT only.
2. Revolve AB and C about HT to a, b, and c.
3. Erect the perpendicular  $c_1d_1$  from  $c_1$  to  $a_1b_1$ . This is the shortest distance.

**Note.**—If the shortest line is desired,  $c_1d_1$  may be revolved back to the original projections of the line as shown in the figure.

**55. Second Method** by auxiliary planes.

Let  $AB$  (Fig. 40) be the given line and  $C$  the given point.

**Construction.**—1. Project  $AB$  and  $C$  on  $Q$ , which is taken parallel to  $AB$  (and perpendicular to  $H$ , of course).

2. Draw  $QX$  perpendicular to  $a^q b^q$ , and project  $AB$  and  $C$  on  $Q$ .

**Conclusion.**—The distance from  $c^x$  to  $a^x b^x$  (a point in projection) is the true distance. Projecting back,  $CD$  is the shortest line.

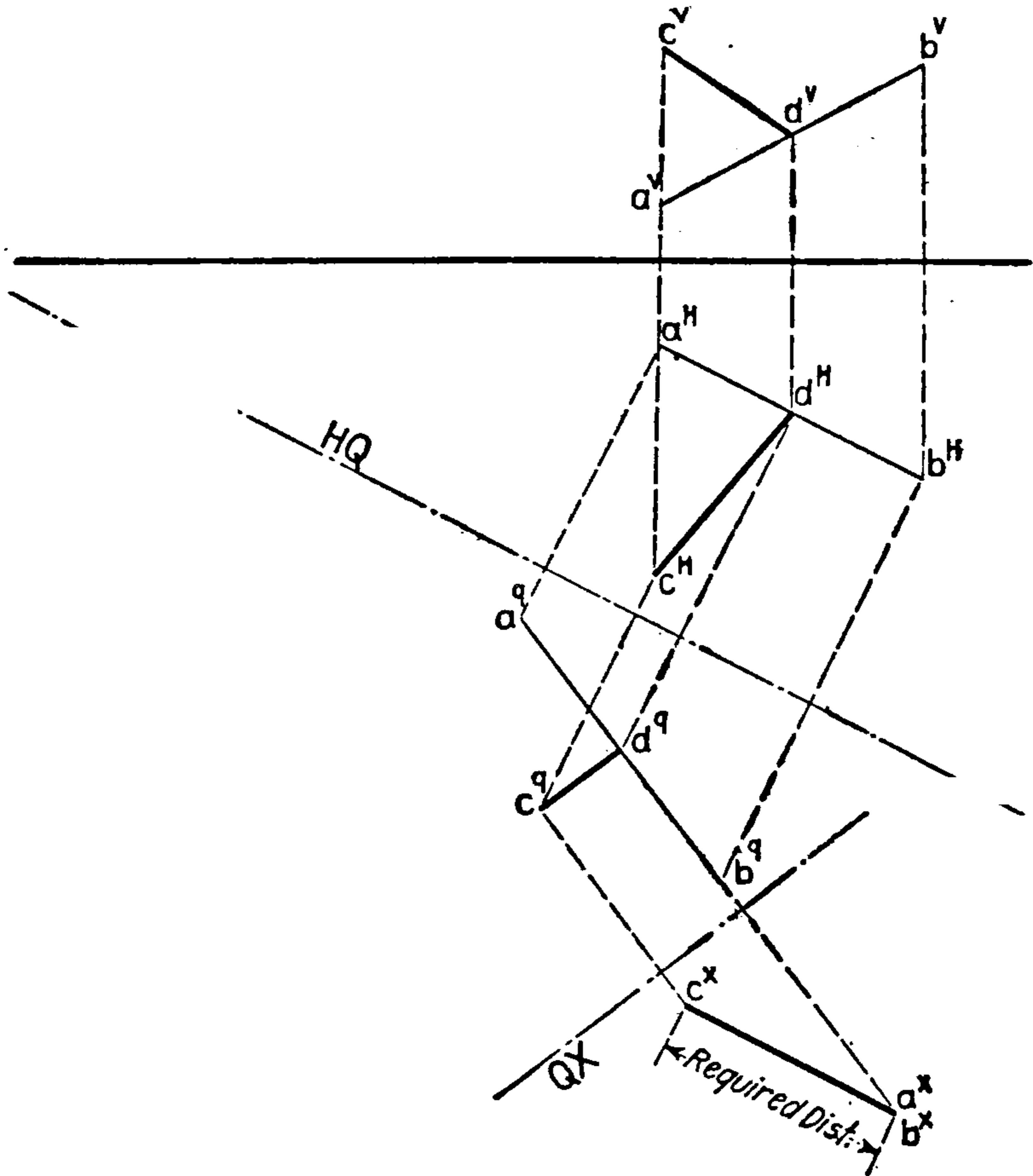


FIG. 40.

**56.**

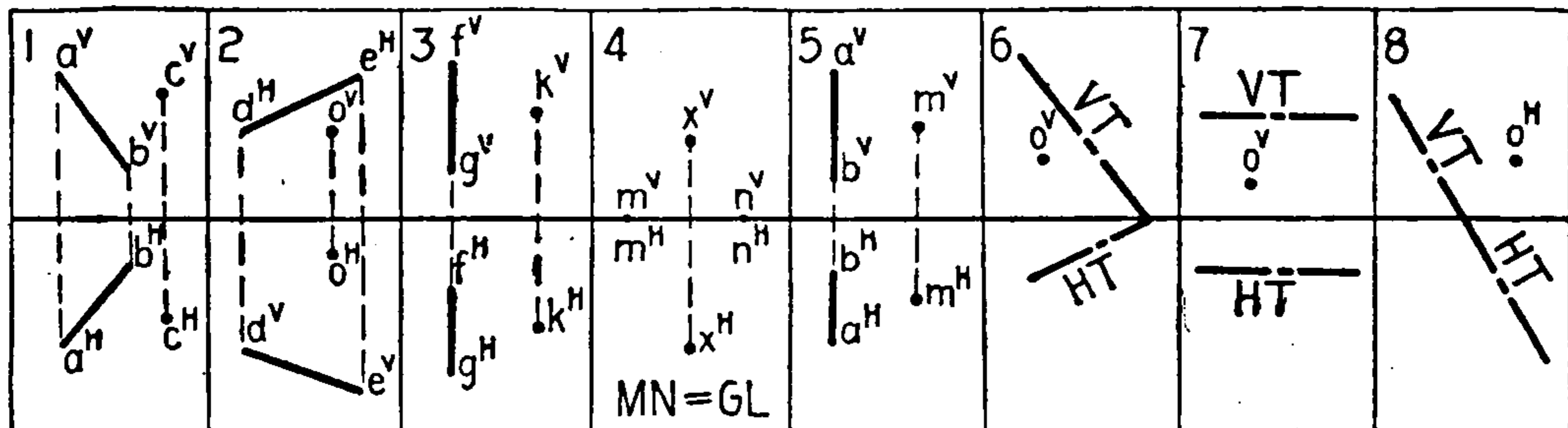
**EXERCISES**

Find the shortest distance from the following points to the following lines:

- 248.  $A(4 + 3 - 1)$  to  $B(2 + 2\frac{1}{2} - \frac{1}{2})$   $C(4 + 1 - 3)$ .
- 249.  $E(1 - 3 + 2\frac{1}{2})$  to  $D(1\frac{1}{2} - \frac{1}{2} + 1)$   $F(4\frac{1}{2} - 3 + 2)$
- 250.  $G(3 + 2 - 3)$  to  $K(1 - 2 + 3)$   $L(4 - 1 + 1)$ .
- 251.  $M(3 + 1 - 3)$  to  $N(2 + 2, 0)$   $O(4 + 1, 0)$ .\*
- 252.  $P(3 + 2 - 1\frac{1}{2})$  to  $GL$ .
- 253.  $A(2 + 1 + 1)$  to  $B(1 - 3 - 3)$   $C(3 - 1 - 1)$ .
- 254.  $D(3 - 1 - 1)$  to  $E(2 + 2 + 2)$   $F(4 + \frac{1}{2} + \frac{1}{2})$ .



255.  $G(1 + 3, 0)$  on the  $V$ -trace of  $T(1 + 3) 4(1 + 2)$  to  $HT$ . Also  $K(2\frac{1}{2}, 0 - 1)$  on  $HT$  to  $VT$ .
256. A 2-in. steam pipe runs through a basement from a point in the floor,  $M(10 \text{ ft.}, 0, -18 \text{ ft. } 6 \text{ in.})$  to a point on one wall,  $N(50 \text{ ft. } + 13 \text{ ft.}, 0)$ . At a point  $O(22 \text{ ft. } + 18 \text{ ft. } - 2 \text{ ft.})$  in the ceiling it is desired to make the shortest possible connection, using  $1\frac{1}{2}$ -in. pipe. Where will the joint be made, and how long will the branch pipe be, allowing 3 in. for connection?



**Graphic Layout No. 21.**—(a) Measure the distance from the given point in Ex. — to the given line. (In Ex. 6–8 locate the point in the plane given, and measure the distance to the — ( $H$ - or  $V$ -trace).) (b) Draw the projections of the shortest possible line.

### FIGURES IN OBLIQUE PLANES

**57. Problem 16.**—To draw the projections of any plane figure in any oblique plane.

#### METHODS

There are two important processes of achieving this result. The student, by mastering both, will have complete command over any situation that will arise. The difference in the method is mainly in the manner of counter-revolving the points.

**Analysis.**—Assuming that a given point in a given oblique plane is to be the center of some regular polygon lying in the plane.

1. Revolve the given point about the trace of the given plane, as in Problem 12.

2. Construct the required polygon in its true size about the revolved position of the center.

3. Counter-revolve the points (corners) of the polygon into the original position of the plane.

**Note.**—The above is the analysis for both methods.

#### 58. METHOD BY ORDINARY REVOLUTION

**Construction.**—Let  $O$  (Fig. 41) be a point in  $T$ . Let it be required to draw the projections of a 1-in. square in  $T$ , about  $O$  as a center, having two of its sides parallel to  $H$ .



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2. Obtain the Q-trace of T and the Q-projection of O.
3. Revolve O about HT to  $o_1$  in the manner shown in Fig. 42.
4. Construct the required hexagon about  $o_1$  as a center.
5. Counter-revolve the corners of the hexagon: (1) to QT; (2) to the *H*-projection; (3) to the *V*-projection.

Note 1.—The arrows, in Fig. 42, show the direction of this operation in the case of point A.

Note 2.—The *V*-projections of the various points are the same distance from *GL* as the Q-projections are from HQ.

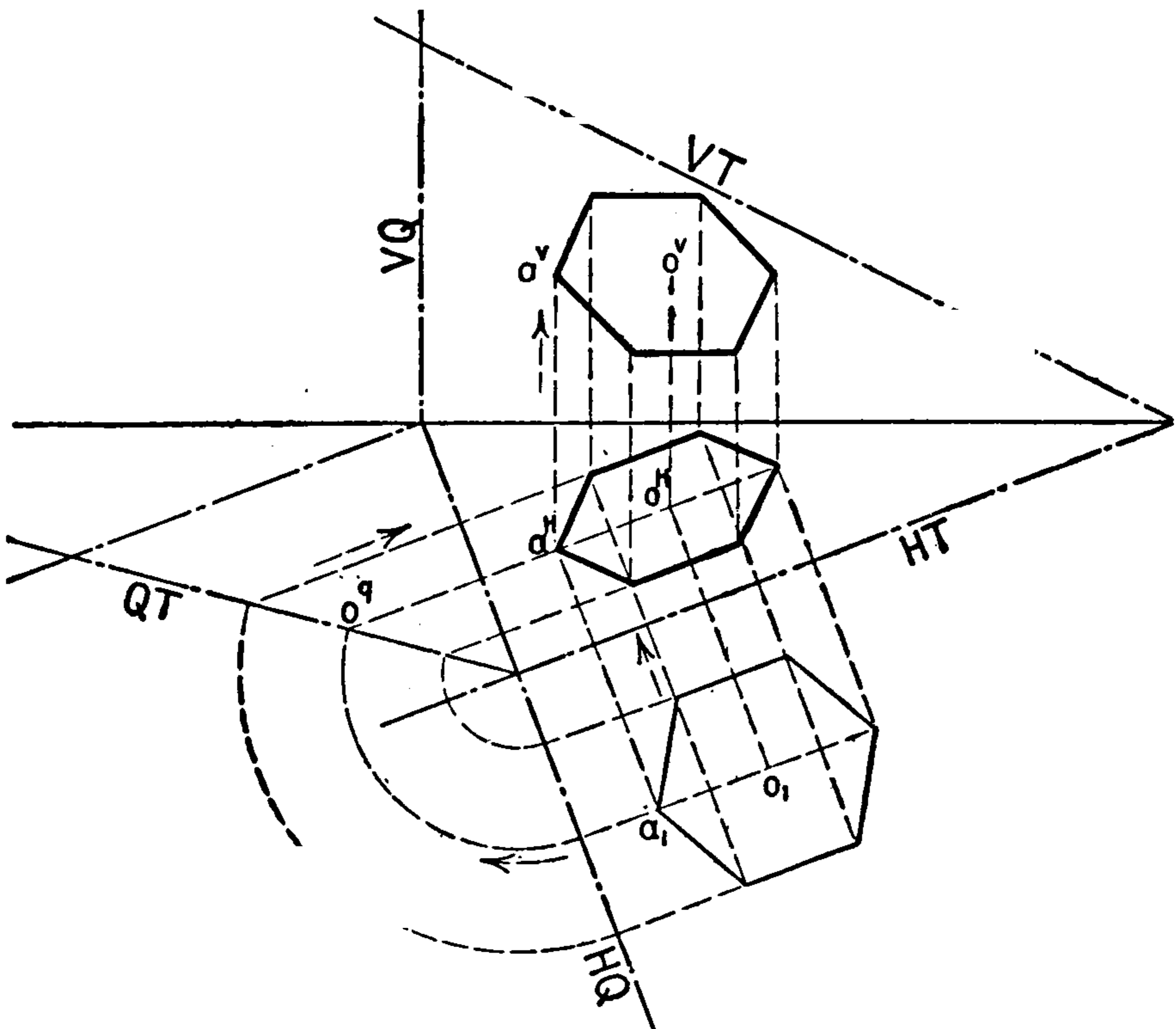


FIG. 42.—A hexagon drawn in an oblique plane by the “auxilliary plane” method.

**60. Supplementary Problem.**—To draw the projections of a circle in an oblique plane.

**Construction.**—Let O (Fig. 43) be the center of a  $1\frac{1}{2}$ -in. circle lying in T.

1. Revolve O about HT to  $o_1$ .
2. Draw the circle full size about  $o_1$  as a center.
3. Draw the diameters parallel and perpendicular to HT,  $a_1b_1$  and  $c_1d_1$  respectively.
4. Counter-revolve the points  $a_1$ ,  $b_1$ ,  $c_1$ , and  $d_1$ . They will be the four vertices of the ellipse, which is the *H*-projection of the circle. The major axis will be  $a^Hb^H$  and the minor axis  $c^Hd^H$ .

The ellipse may be drawn on these axes in any desired manner, preferably the "trammel method." (See Art. 147.)

For the *V*-projection, revolve *O* about *VT* and counter-revolve the points *e*<sub>1</sub>, *f*<sub>1</sub>, *g*<sub>1</sub>, and *k*<sub>1</sub> in the same way; *e*<sup>v</sup>*f*<sup>v</sup> will then be the major axis, and *g*<sup>v</sup>*k*<sup>v</sup> will be the minor axis.

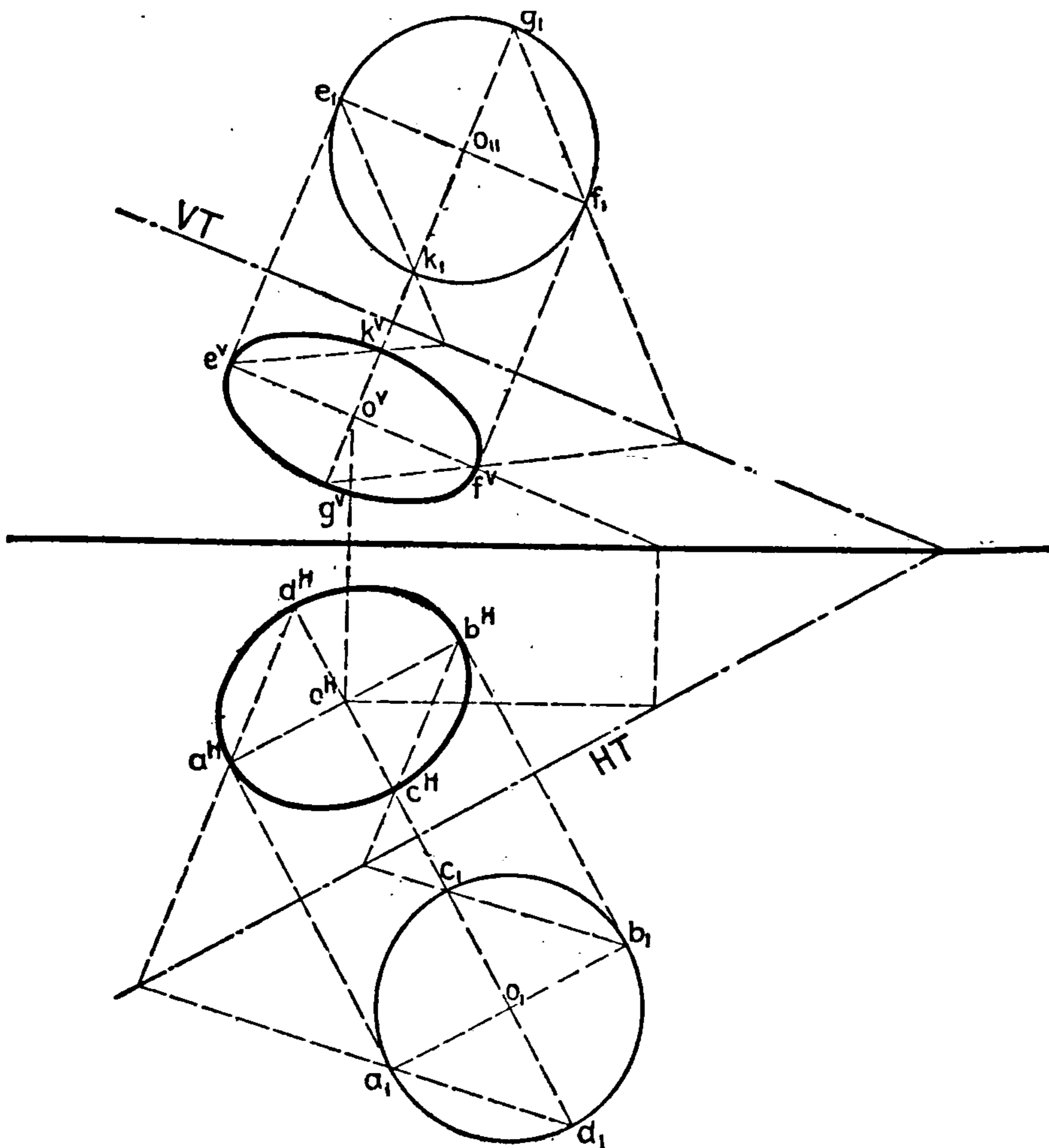


FIG. 43.—The projections of a circle.

61.

EXERCISES

Draw the projections of the following plane figures in the planes given according to the stated conditions:

257. Equilateral triangle,  $1\frac{1}{2}$ -in. sides.  $T(2 + 3) 2(5 - 1\frac{1}{2})$ . Center at  $A(3\frac{1}{2} + 1, x)$  in *T*.

258. Triangle Ex. 257.  $U(2 + 2) 4(4 - 3)$ . One side  $\parallel$  to *H*, 1 in. from *HU*.

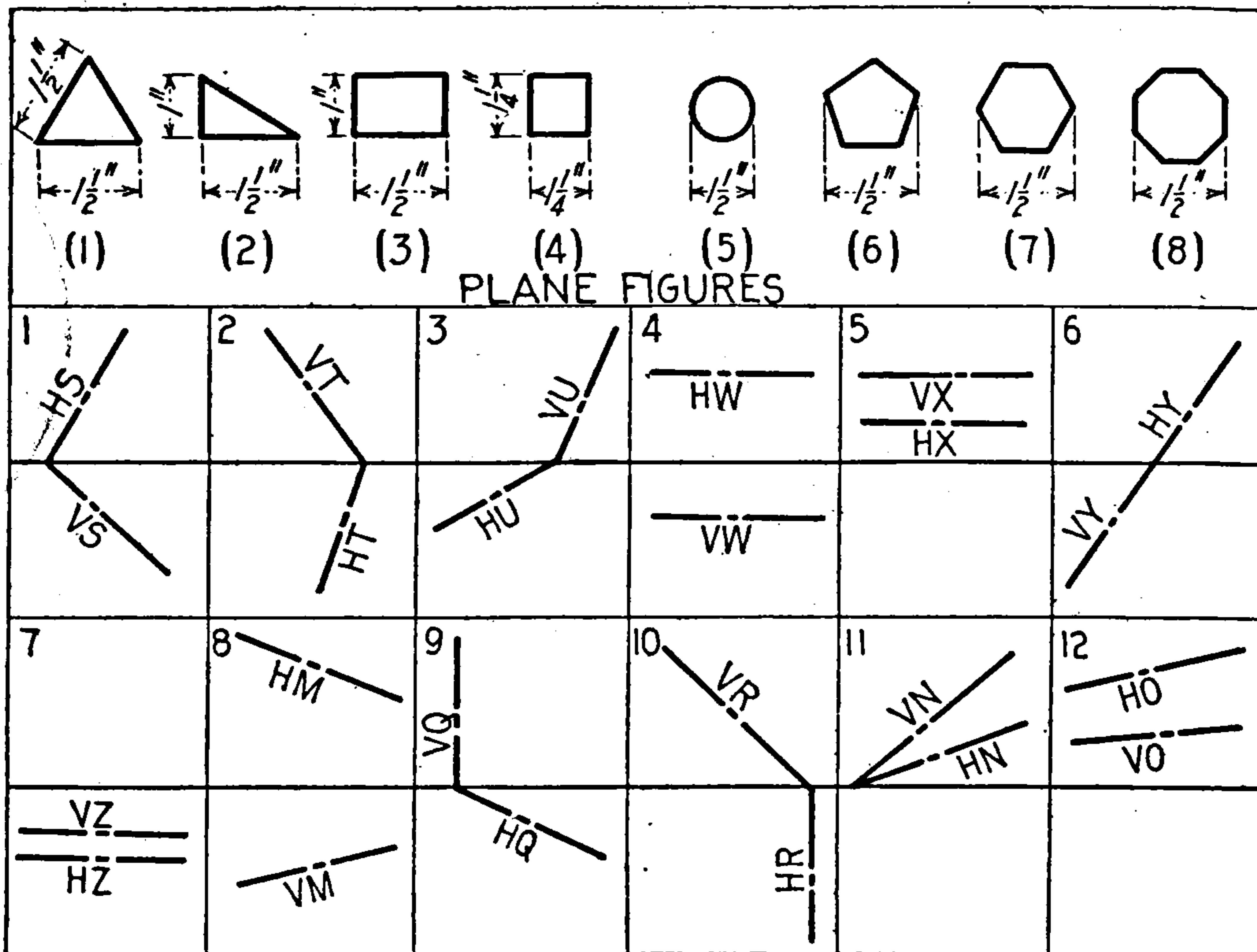
259. Triangle Ex. 257.  $S(5 + 2) 2(5 - 1\frac{1}{2})$ . Center at  $O(4 + 1, x)$  in *S*.

260. Triangle Ex. 257.  $T(2 - 3) 4(2 + 2)$ . One side  $\parallel$  to *V*, 1 in. from *VT*.

261. Triangle Ex. 257.  $U(+2) \infty (-1)$ . Center at  $M(3, y - \frac{1}{2})$ .

262. Triangle Ex. 257.  $W(-3) \infty (+2)$ . One side  $\parallel$  to *H*, 1 in. from *VW*.

263. Triangle Ex. 257.  $X(+\frac{1}{2}) \infty (-2)$ . Center at  $N(3 + 1\frac{1}{2}, y)$ .  
 264. Triangle Ex. 257.  $Q(1 + 3) 3(5 - 1)$ . Center at  $K(4 + 1, x)$ .  
 265. Triangle Ex. 257.  $R(1 + 1) (5 + 2) (1 - \frac{1}{2}) (5 - 1)$ . Center at  $F(3 + 1, x)$ .  
 266. Triangle Ex. 257.  $S(5 + 2) 1(5 + 1)$ . Center at  $D(3, x - 1)$ .



Graphic Layout No. 22.—Draw the projections of plane figure — in given plane —, under one of the following conditions, as selected by the instructor:

- (a) Assume a point in the given plane, as the center of the given plane figure.  
 (b) Assume a line in the given plane as one of the sides of the plane figure.  
 (c) Assume a line in the given plane as the diagonal or diameter of the given plane figure.  
 (d) Draw one corner of the given figure in  $H$ , and one side inclined  $\text{---}^\circ$  (10, 15, 30, 40, 45) to the  $H$ -trace.

Note.— $V$  may be substituted for  $H$  in condition (d).

- (e) Circumscribe a circle about the plane figure — in plane —.  
 (f) Inscribe a circle in the plane figure — in plane —.

267. Triangle Ex. 257.  $Y(1 + 2\frac{1}{2}) 3(5 - 2\frac{1}{2})$ . Center at  $A(2\frac{1}{2}, x - 1)$ .  
 268. Triangle Ex. 257.  $P(4 + 3) 4(4 - 3)$ . One side  $\parallel$  to  $H$ , 1 in. below  $H$  in  $III$ .  
 269. Substitute a right triangle 1 in.  $\times$   $1\frac{1}{2}$  in. in any of the planes in Ex. 257–268, under the same conditions.  
 270. Inscribe a circle in any of the triangles in Ex. 257–269.

271. Substitute one of the following polygons for the triangle in Ex. 257–268:
- $1\frac{1}{4}$  in. square.
  - 1-in.  $\times$   $1\frac{1}{2}$ -in. rectangle.
  - $1\frac{1}{2}$ -in. regular pentagon.
  - $1\frac{1}{2}$ -in. regular hexagon.
  - $1\frac{1}{2}$ -in. circle.
  - $1\frac{1}{2}$ -in. octagon.
272. Draw the projections of polygon ——— in Ex. 271 in any of the planes given in Ex. 257–268 with one corner in  $H$  and another corner  $\frac{1}{2}$  in. from the  $H$ -trace.
273. Triangle in Ex. 269 in any of the planes in Ex. 257–268, with its hypotenuse  $\parallel$  to  $V$ , 1 in. from the  $V$ -trace.
274. Square, Ex. 271 (a) in any of the planes in Ex. 257–268, with its diagonal  $\parallel$  to  $H$ ,  $\frac{3}{4}$  in. above or below  $H$ .
275. Hexagon,  $1\frac{1}{4}$  in. short diameter, with a  $\frac{3}{4}$ -in. circle in the center, drawn in any of the planes in Ex. 257–268, with its center at any chosen point in the given plane.
276. Circumscribe a circle about any of the foregoing polygons.
277. Square, Ex. 271 (a) in any of the planes in Ex. 257–268, with one corner in  $V$ , and a side inclined  $30^\circ$  to the  $V$ -trace.
278. The line  $M(3 + 1, x)$   $N(4 + \frac{1}{2}, y)$  in  $R(1 + 1)$   $(5 + 2)$   $(1 - \frac{1}{2})$   $(5 - 1)$  is one of the diameters of a circle in  $R$ . Draw the projections of the circle.
279. An ellipse, major axis  $A(2, 0 - 1)$   $B(3\frac{1}{2}, 0 - 1)$ , minor axis  $C(2\frac{3}{4}, 0 - \frac{1}{2})$   $D(2\frac{3}{4}, 0 - 1\frac{1}{2})$  is the projection of a circle lying in a plane, whose  $H$ -trace is  $E(1, 0 - 2)$   $F(4, 0 - 2)$ . Draw the  $V$ -trace of the plane and the  $V$ -projection of the circle.
280. A  $1\frac{1}{4}$ -in. circle whose center in  $M(3, 0 - 1)$  is the  $H$ -projection of an ellipse lying in any of the planes in Ex. 258, 259, 261, 263, 264, 265, 266, 267. Draw the  $V$ -projection and the true size of the ellipse.
281. The roof of a tower is a hexagonal pyramid, base 14 ft., and altitude 18 ft. In each of the three front faces are rectangular windows, 2 ft. 9 in.  $\times$  6 ft. 3 in., each 3 ft. 3 in. from the tower base. Draw the projections. Scale  $\frac{1}{8}$  in. = 1 ft.

### SIXTH FUNDAMENTAL PROBLEM

**62. Problem 17.—To find the line of intersection of two planes.**

**Axioms.—1.** The intersection of two planes is a straight line.

**2.** Two points determine a straight line.

**Analysis.—1.** Find both projections of the point where the  $V$ -traces of the given planes intersect.

**2.** Find both projections of the point where the  $H$ -traces intersect.

**3.** Join the respective projections of these two points, which are common to both planes.

**Conclusion.—**The line thus drawn is common to both planes, and therefore their intersection.

Note.—The  $H$ -projection of any  $V$ -trace is in  $GL$ , and therefore the  $H$ -projection of any point in the  $V$ -trace is in  $GL$ .

Proof.—The  $V$ -traces, being each a line in one of the given planes, intersect in a point common to both planes, and therefore in the line of intersection. Similarly for the intersection of the  $H$ -traces. Having found these two points, the line of intersection is determined.

Construction.—Left to the student. Refer to Fig. 44.

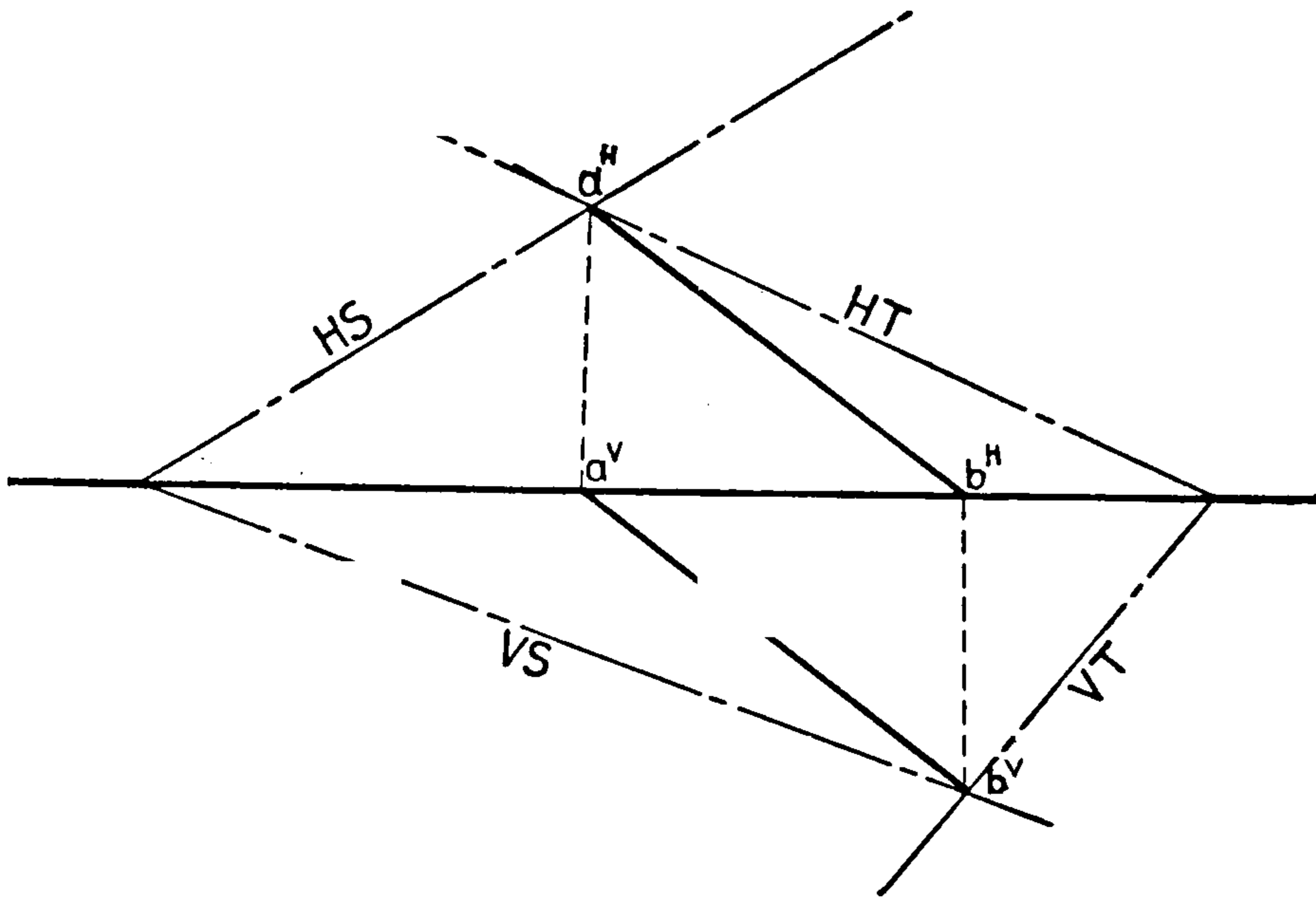


FIG. 44.

**63. Special Cases.**—When the traces do not intersect within the limits of the problem, or not at all.

Analysis.—1. If one pair of traces intersect, locate their point of intersection.

2. Draw an auxiliary plane that will intersect both given planes. The lines of intersection of the auxiliary and given planes will intersect in a point on the required line of intersection.

3. If neither pair of traces intersect, draw a second auxiliary plane which will determine another point on the required line, thus determining the line of intersection.

Construction.—For most oblique planes, such as the two shown in Fig. 45, it is quickest and most convenient to use planes parallel to  $H$  or  $V$  as auxiliaries. Such planes have one trace only, and that is parallel to  $GL$ . Such planes intersect oblique planes in lines parallel to their traces; that is, a plane parallel

to  $V$  will intersect an oblique plane in a line parallel to its  $V$ -trace, etc. Let the student give the reason from Solid Geometry.

1. Draw  $VQ$  ( $Q \parallel$  to  $H$ ).  $Q$  will cut  $CM$  and  $DM$  from  $T$  and  $S$  respectively.

2. Locate  $m^V$  and  $m^H$ , the intersection of  $CM$  and  $DM$ .

3. Draw  $HR$  ( $R \parallel$  to  $V$ ).  $R$  will cut  $AO$  and  $BO$  from  $T$  and  $S$  respectively.

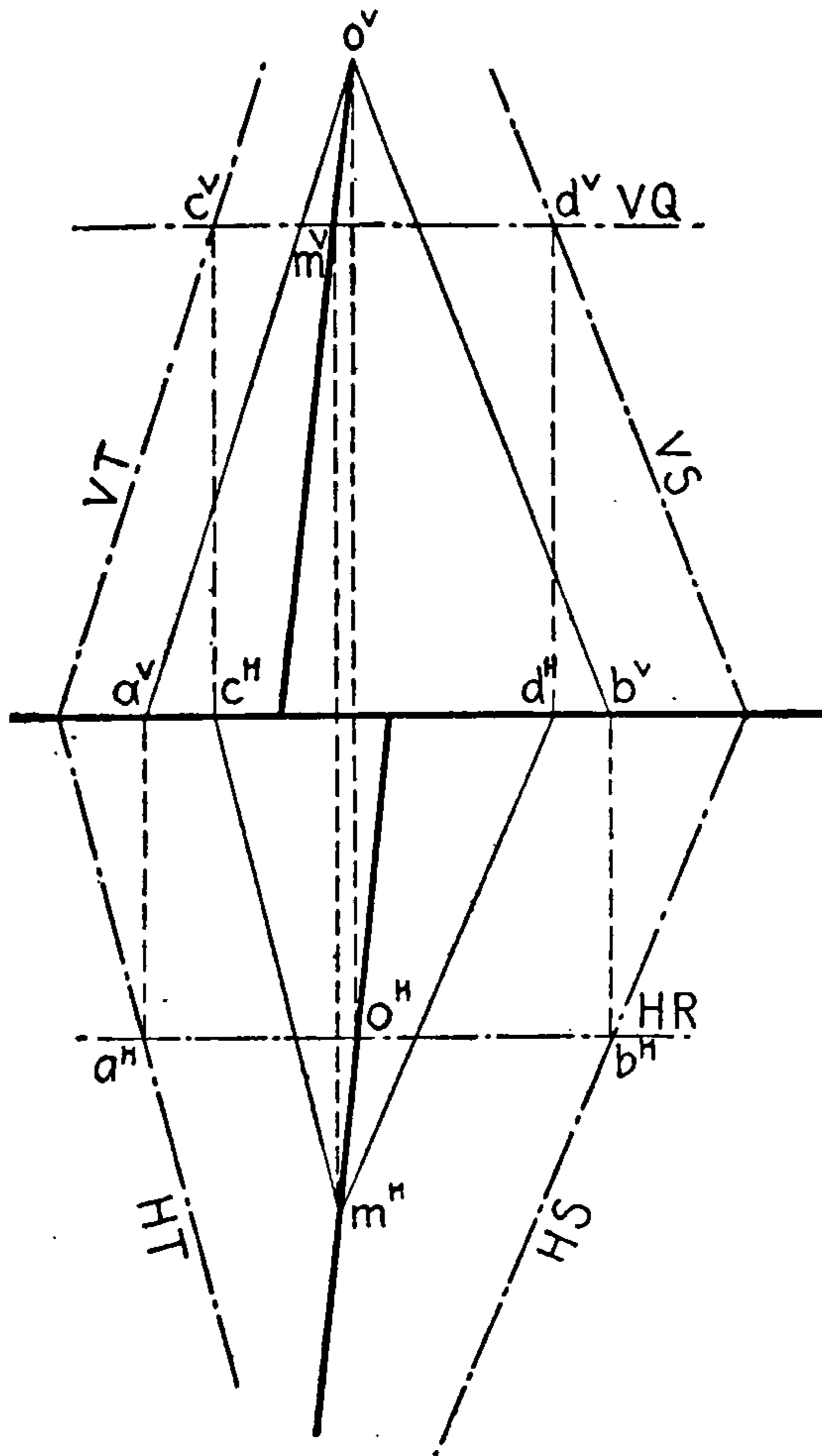


FIG. 45.

4. Locate  $o^V$  and  $o^H$ , the intersection of  $AO$  and  $BO$ .

5. Draw  $o^V m^V$  and  $o^H m^H$ , the projections of the required intersection.

**64. Case II.**—When it is convenient to use an auxiliary plane parallel to one of the given planes.

Fig. 46 shows a plane  $Q$ , parallel to  $S$ , being used to determine the direction of the line of intersection of  $T$  and  $S$ .

**Construction.**—1. Pass a plane  $Q$  parallel to  $S$ , so that its intersection  $XY$  with  $T$  will come inside the problem limits.

2. Draw  $XY$ , the intersection of  $T$  and  $Q$ .



3. Find  $M$  (one point on the intersection of  $T$  and  $S$ ).

4. Draw  $MN$  parallel to  $XY$ .

**Conclusion.**— $MN$  is the required intersection of  $T$  and  $S$ .

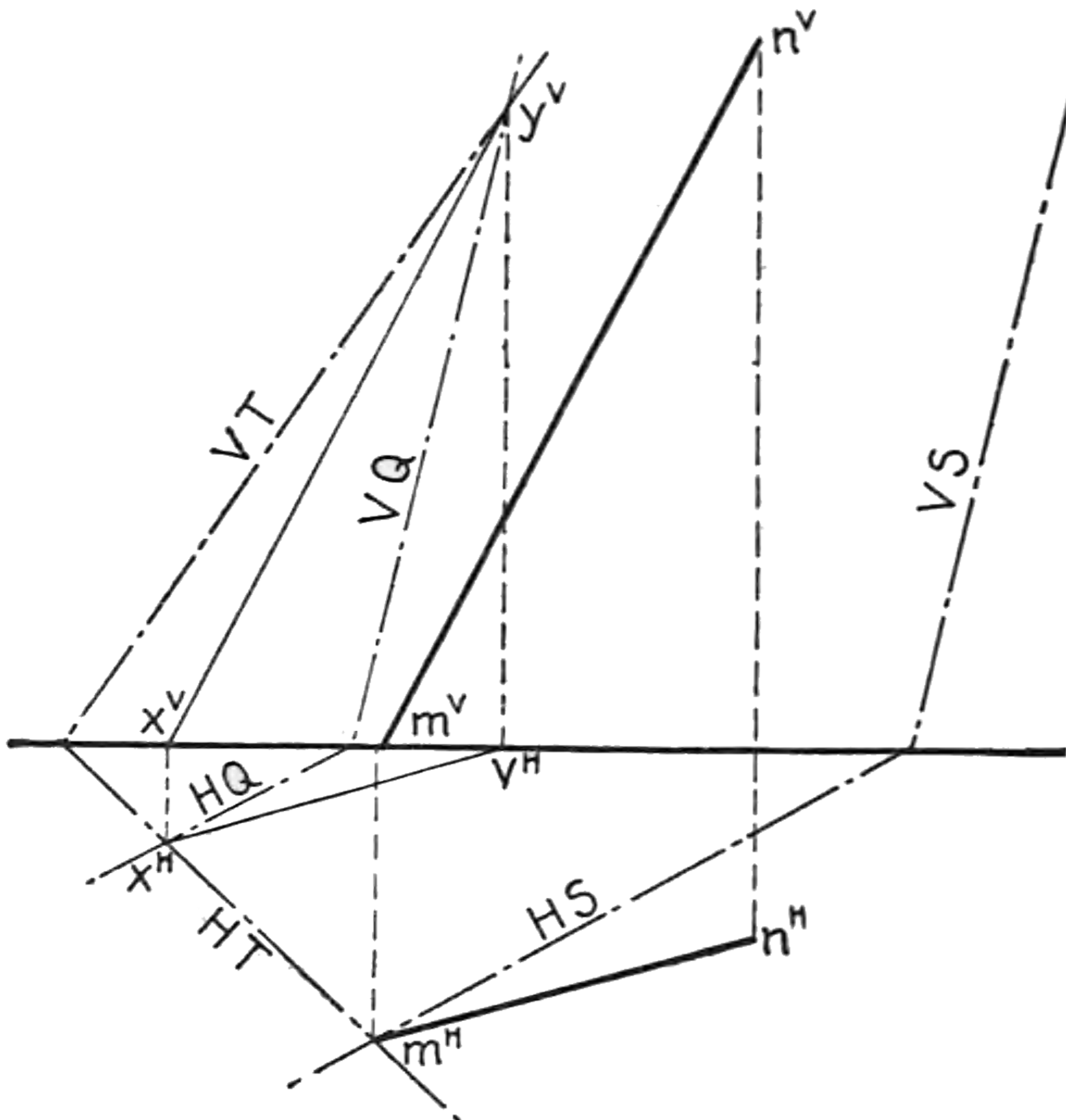


FIG. 46.

**65. Case III.**—When both planes are parallel to  $GL$ .

Although the planes intersect, their traces are all parallel. Inasmuch as both planes are parallel to  $GL$ , their intersection

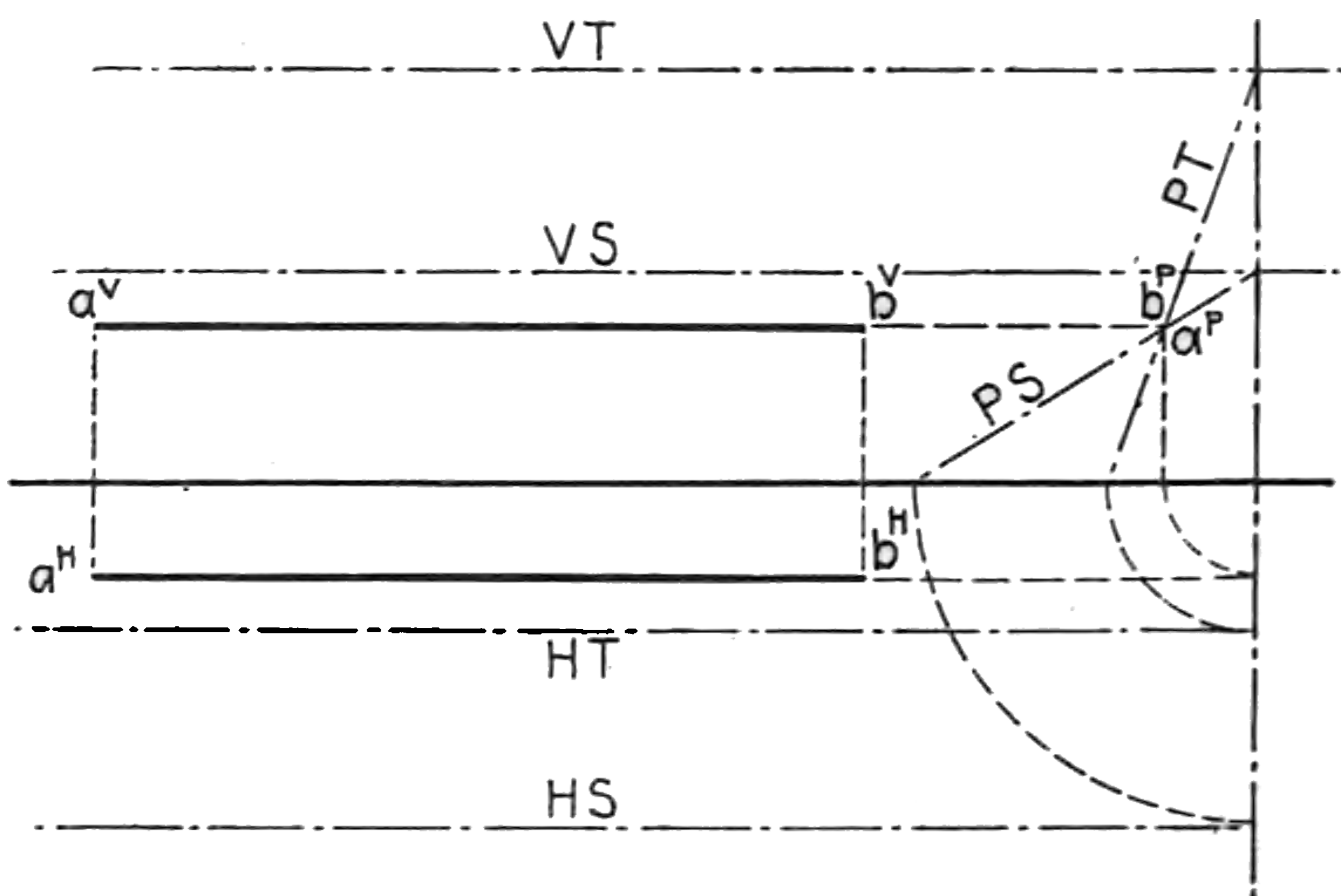


FIG. 47.

will also be parallel to  $GL$ . (See Solid Geometry problems). Hence only one point on the intersection is required.

Figs. 47 and 48 show two different auxiliary planes that may



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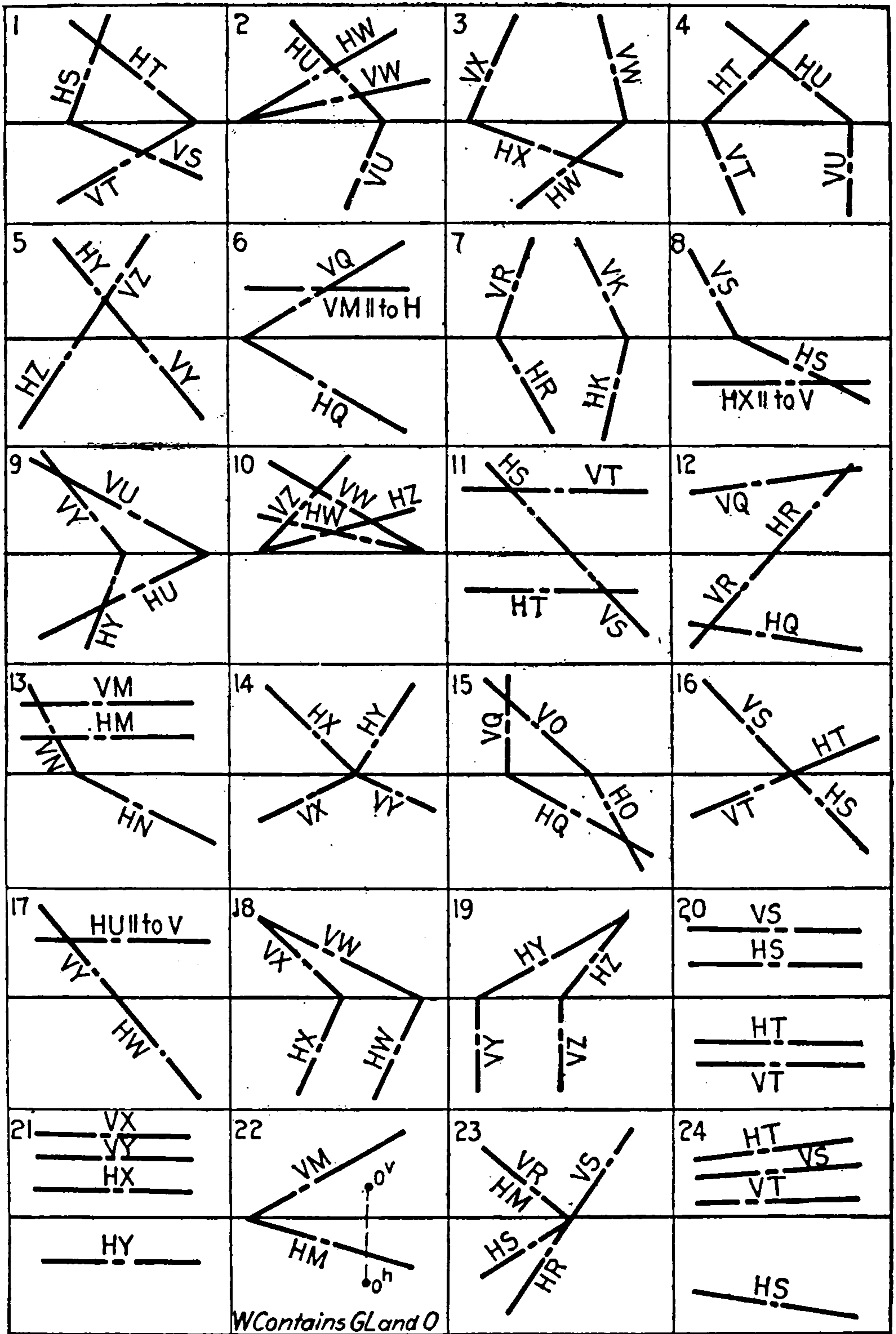
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296.  $S(+1\frac{1}{2})$ , parallel to  $H$  and  $X(5+2)3(1-2)$ .
297.  $U(+1)$ , parallel to  $V$  and  $W(5+2)3\frac{1}{2}(1-3)$ .
298.  $X(1+2)3(1-1)$  and  $Y(5+1\frac{1}{2})3(5-3)$ .
299.  $R(1+3)3(1-2)$  and  $S(5+2)3(5+1)$ .
300.  $T(1+2)3(5-2)$  and  $W(2-3)3(4+3)$ .
301.  $U(4+2)3(1-1)$  and  $X(5+2)4(5-2)$ .
302.  $Y(1+3)3(5-2)$  and  $Z(2+3)4(1-1)$ .
303.  $R(1+3)5(3-3)$  and  $Q(5+2)3\frac{1}{2}(1\frac{1}{2}-3)$ .
304.  $S(1+3)2(1-3)$  and  $U(5+2)3(5-3)$ .
305.  $T(1+3)2(5-1)$  and  $X(5+2)4(1-2)$ .
306.  $W(1-3)2\frac{1}{2}(5+2)$  and  $Y(4\frac{1}{2}-3)4(1+1)$ .
307.  $Q(3+3)1(2-3)$  and  $Z(4-3)5(3-3)$ .
308.  $R(1+\frac{3}{4})(5+2\frac{1}{4})(1-\frac{1}{2})(5-1\frac{1}{2})$  and  $S(1+3)3(5-3)$ .
309.  $T(2+3)2(5-2)$  and  $U(5+2)3(1-3)$ .
310.  $W(5-3)1(1-2)$  and  $X(1-2\frac{1}{2})3(5+1\frac{1}{4})$ .
311.  $Y(3+3)1\frac{1}{2}(4-2)$  and  $Z(5+2)2(4\frac{1}{2}-2)$ .
312.  $Q(1+2)(5+3)(1-1)(5-1\frac{1}{2})$  and  $R(1+1)(5+1\frac{1}{2})(1-2)(5-3)$ .
313.  $S(1+1)(5+3)(1-1\frac{1}{2})(3-2)$  and  $T(1-1\frac{1}{2})(3-2)(1-1)(5-3)$ .
314.  $U(1+2)5(1-3)$  and a  $30^\circ$  plane which contains  $GL$ .
314.  $W(5-1)1(4+3)$  and a plane containing  $GL$  and  $A(3+1-2)$ .
315.  $X(1+1)(5+3)(1-\frac{1}{2})(5+1\frac{1}{2})$  and a  $45^\circ$  plane through  $GL$ .
316. Draw a line through  $M(4+1-1)$  parallel to  $T(1+3)4(1-1)$  and  $S(5+3)1(3-3)$ .
317. Draw a line 2 in. long through  $O$ ,  $(4+1-2)$  parallel to  $W(1+3)2(5-1)$  and  $X(5+3)3(1-1\frac{1}{2})$ .
318. Draw a line  $2\frac{1}{2}$  in. long through  $A(2+3-3)$  parallel to  $X(1+3)3(5-3)$  and  $Y(3+3)2(1-3)$ .
319. Find the point of intersection of the three planes,  $R(1+3)5(1-2)$ ,  $S(-2)\infty(-\frac{1}{2})$ , and  $T(5+2)1(3-3)$ .
320. Draw the projections of the tetrahedron bounded by  $H$  and the three planes in Ex. 319.
321. Develop the surface of the tetrahedron in Ex. 320.
322. Draw the projections of the frustum of the tetrahedron in Ex. 320 made by  $W(+\frac{1}{2})\infty(-2)$ .
323. Develop the surface of the frustum of the tetrahedron in Ex. 322.
324. Draw the projections of the tetrahedron formed by the intersection of the planes  $S(3+2)\frac{1}{2}(5+1\frac{1}{2})$ ,  $T(3-3)2(4+2)$ ,  $R(3-3)4\frac{1}{2}(2+2\frac{3}{4})$  and the plane  $Q$ , 2 in. below and parallel to  $H$ .
325. Develop the surface of tetrahedron in Ex. 324.
326. Draw the projections of the frustum of the tetrahedron in Ex. 324, made by the plane  $Y(-1\frac{3}{4})\infty(+3)$ .
327. Develop the surface of the frustum in Ex. 326.

### THE PIERCING POINT OF A LINE AND PLANE

**67. Problem 18.**—To find the point where an oblique plane is pierced by a line.



Graphic Layout No. 23.—Draw the projections of the line of intersection of the planes given in Ex. —.

## FIRST METHOD

**Construction.**—Let  $T$  (Fig. 49) be the given plane and  $AB$  the given line.

1. Obtain  $QT$ .
2. Obtain  $a^q b^q$ .
3. Locate  $O^q$  (the piercing point) where  $a^q b^q$  intersects  $QT$ , and project it back to  $O^H O^V$ .

**Conclusion.**—The point  $O$  is the required piercing point.

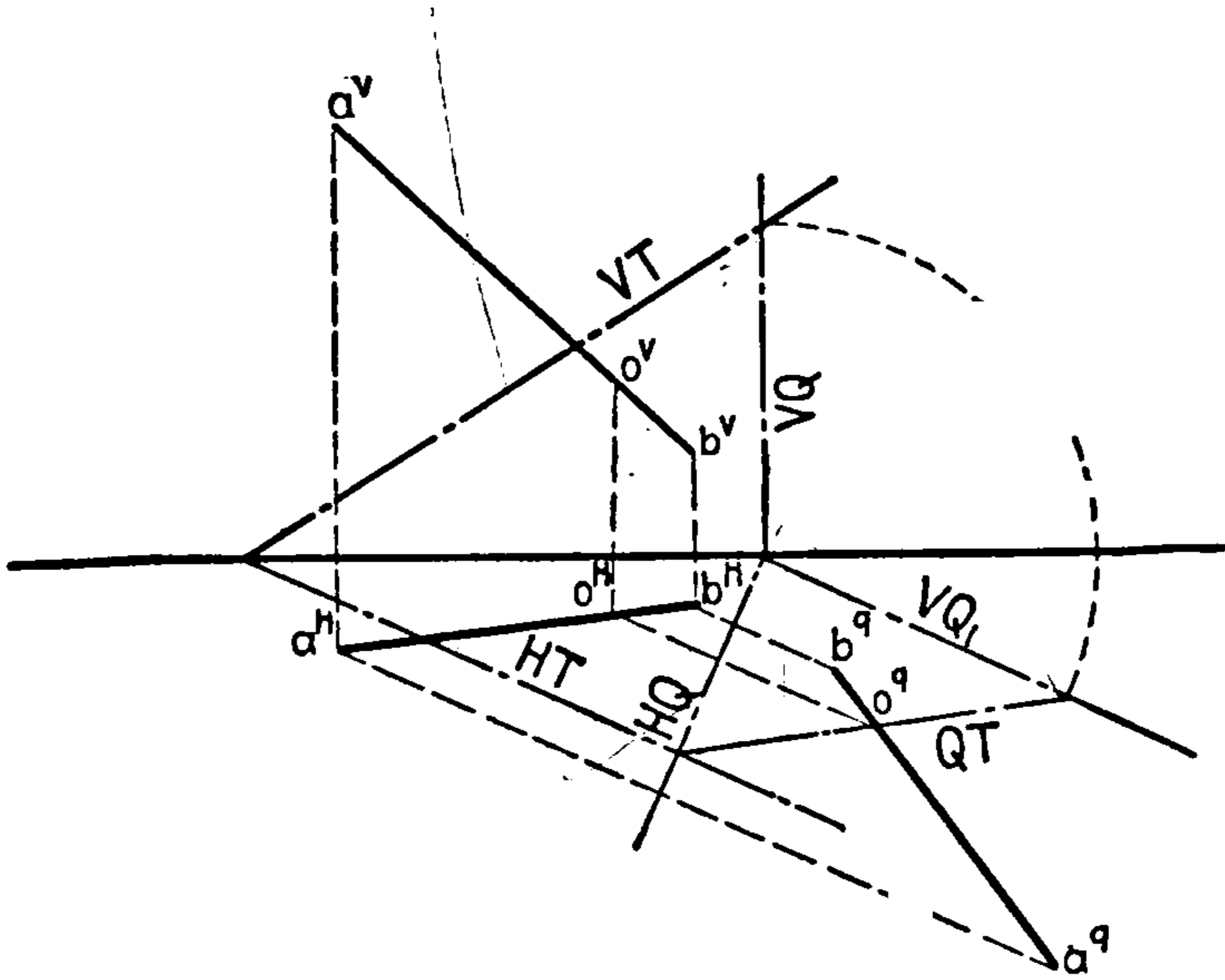


FIG. 49.

## SECOND METHOD

- Analysis.**—1. Pass a plane through the given line.  
 2. Find the line of intersection of the auxiliary and given planes.

**Conclusion.**—Where this intersection crosses the given line is the piercing point.

**Proof.**—Since the point thus found is not only on the given line, but in the given plane, it must be the common point of the two, *i.e.*, the “piercing point.”

**Construction.**—Let  $AB$  (Fig. 50) be the given line and  $T$  the given plane.

1. Pass a plane ( $Q$ ) through  $AB$ .

**Note.**—While any of the infinite number of planes containing  $AB$  may be used, it is simpler and preferable to use the  $H$ - or  $V$ -projecting plane. In this construction the  $H$ -projecting plane ( $Q$ ) is used.

2. Obtain  $MN$ , the intersection of  $Q$  and  $T$ .

3. Locate C, where MN crosses AB.

Conclusion.—C is the point in which AB pierces T.

Note.—This problem figures in most of the succeeding problems, and should be thoroughly mastered.

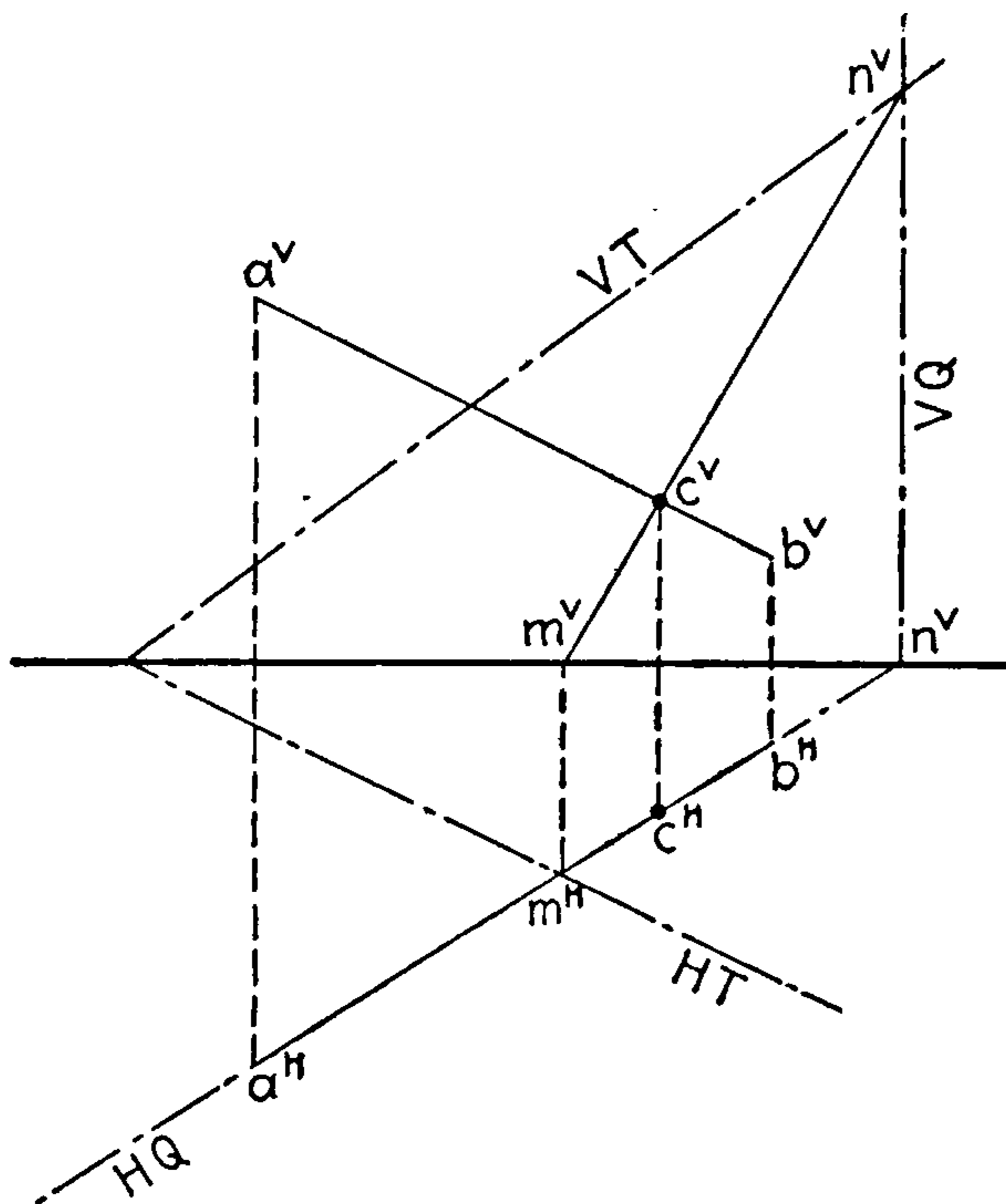


FIG. 50.

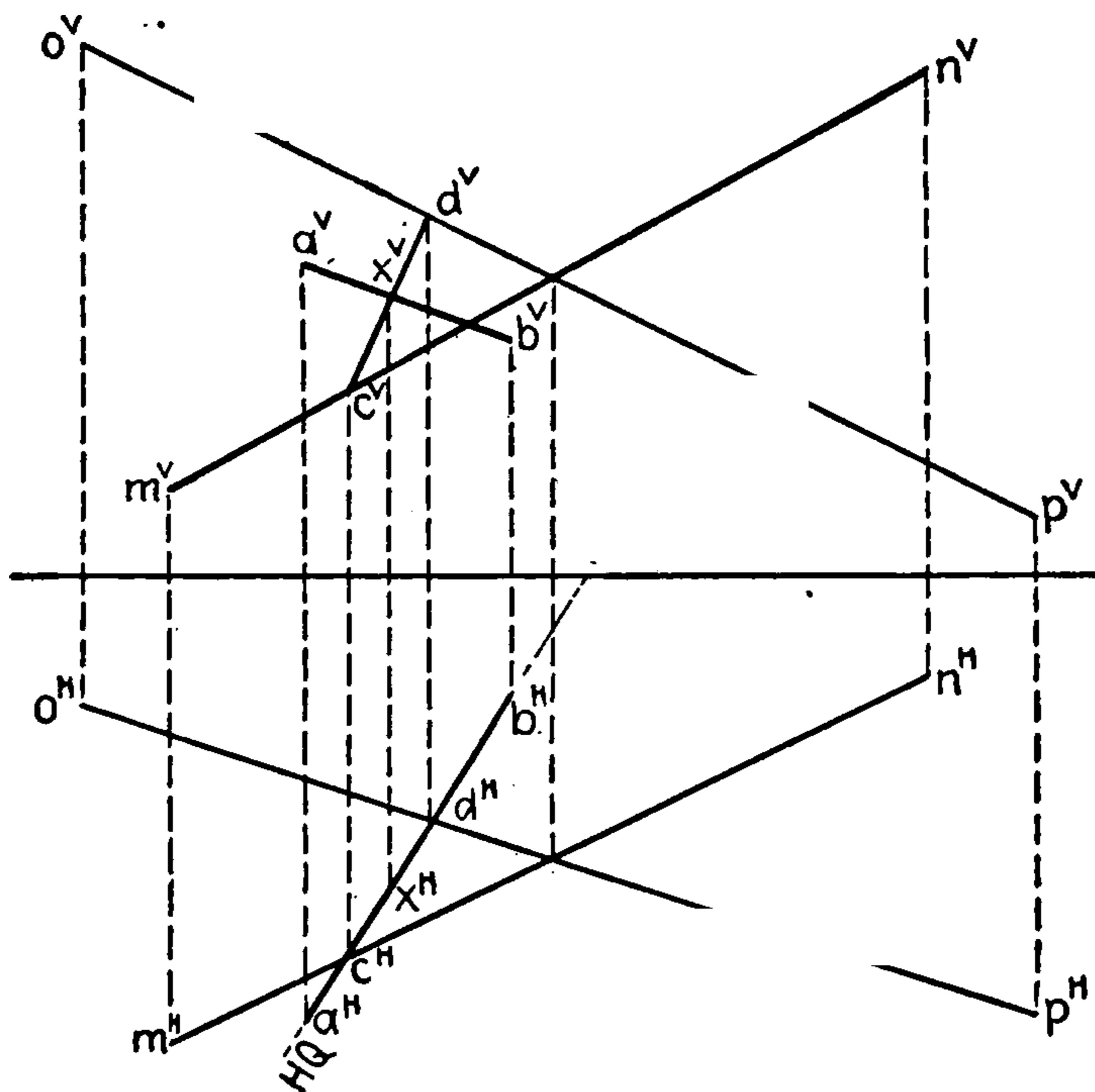


FIG. 51.

**68. Problem 19.**—To solve **Problem 18** without the traces of the given plane.

**Construction.**—Let the two intersecting lines MN and OP determine the given plane, and AB be the given line (Fig. 51).

1. Pass the projecting plane Q through AB.

(Note.—HQ is sufficient.)

2. Project the Q-piercing points of MN and OP,  $c^H$  and  $d^H$ , to their V-projections,  $c^V$  and  $d^V$ .

3. Draw  $c^Vd^V$ , the intersection of Q and the plane MNOP.

4. Where  $c^Vd^V$  crosses  $a^Vb^V$ , at  $x^V$ , the piercing point is located. Project to  $x^H$  on  $a^Hb^H$ .

Note.—This problem is of great value in surface intersections.

### EXERCISES

**69.** Find the piercing points of the following lines and planes. Familiarize yourself with both methods by doing the exercises both ways.

328. A(2 + 3 - 2) B(4 + 1 - 1) and S(5 + 3) 1(5 - 1½).

329. C(1 + 1½ - ½) D(4 + 1½ - ½) and S in Ex. 328.

330. D(3 + ½ - 1¼) E(5 + 2 - 2) and S in Ex. 328.

331. F(3 + 2, 0) G(3 + 2 - 3) and S in Ex. 328.

332. K(3 + ½ + ½) L(4 + 3 + 3) and S in Ex. 328.

333. M(3 - 1 + 1) N(4 - 2 + 3) and T(2 - 2) 5(3 + 3).

334. O(1 - 1 + 2) P(5 - 1 + 2) and T in Ex. 333.

335. A(1 + ½ + 1½) B(5 + ½ + 1½) and T in Ex. 333.

336. C(3½ - 2 + 3) D(3½ - ½ + ½) and T in Ex. 333.

337. E(3½ + 1 + 3¼) F(5 + 2 + 1) and T in Ex. 333.

338. G(2 + 2 - 1) K(4 + 1 - 3) and X(5 + 3) 3(1 - 3).

339. M(1 + 2 - 1) N(4 + 2 - 1) and X, Ex. 338.

340. O(4 + 3 - 1) P(2 - 3 + 1) and X, Ex. 338.

341. A(3 - 3 + 2) B(1 - 1 + 1) and Y(- 1) ∞(+ 2).

342. C(1 - 2 + 3) D(3 - ½ + ½) and Y, Ex. 341.

343. E(3 + 2 - 2) F(3 + ½ - 1½) and U(+ 1½) ∞(- 1).

344. G(1 + 2 - 2) K(3½ + 1 - ¾) and U(Ex. 343).

345. M(2 + 2 - 1) N(3 + 1 - 1½) and R(5 + 2) 1(5 + 1).

346. O(3 + 3 - ½) P(3 + 1 - 2) and R(5 + 2) 1(5 + 1).

347. A(1 + 1 - 1) B(5 + 1 - 1) and R (Ex. 346).

348. C(3 - 1 + 2) D(5, 0 + 3) and R(Ex. 346).

349. E(2 + 3 - 3) F(2 + ½ - 1½) and T(5 + 2) (1 + 1) (5 - 3) (1 - 1½).

350. G(2 + 2 - ½) K(3 + ½ - 1½) and T, Ex. 349.

351. M(1 + 2 - ½) N(5 + 2 - ½) and T, Ex. 349.

352. A(4 + 2 - 1) B(3 + 1 - 3) and S(1 + 3) 2(5 - 2).

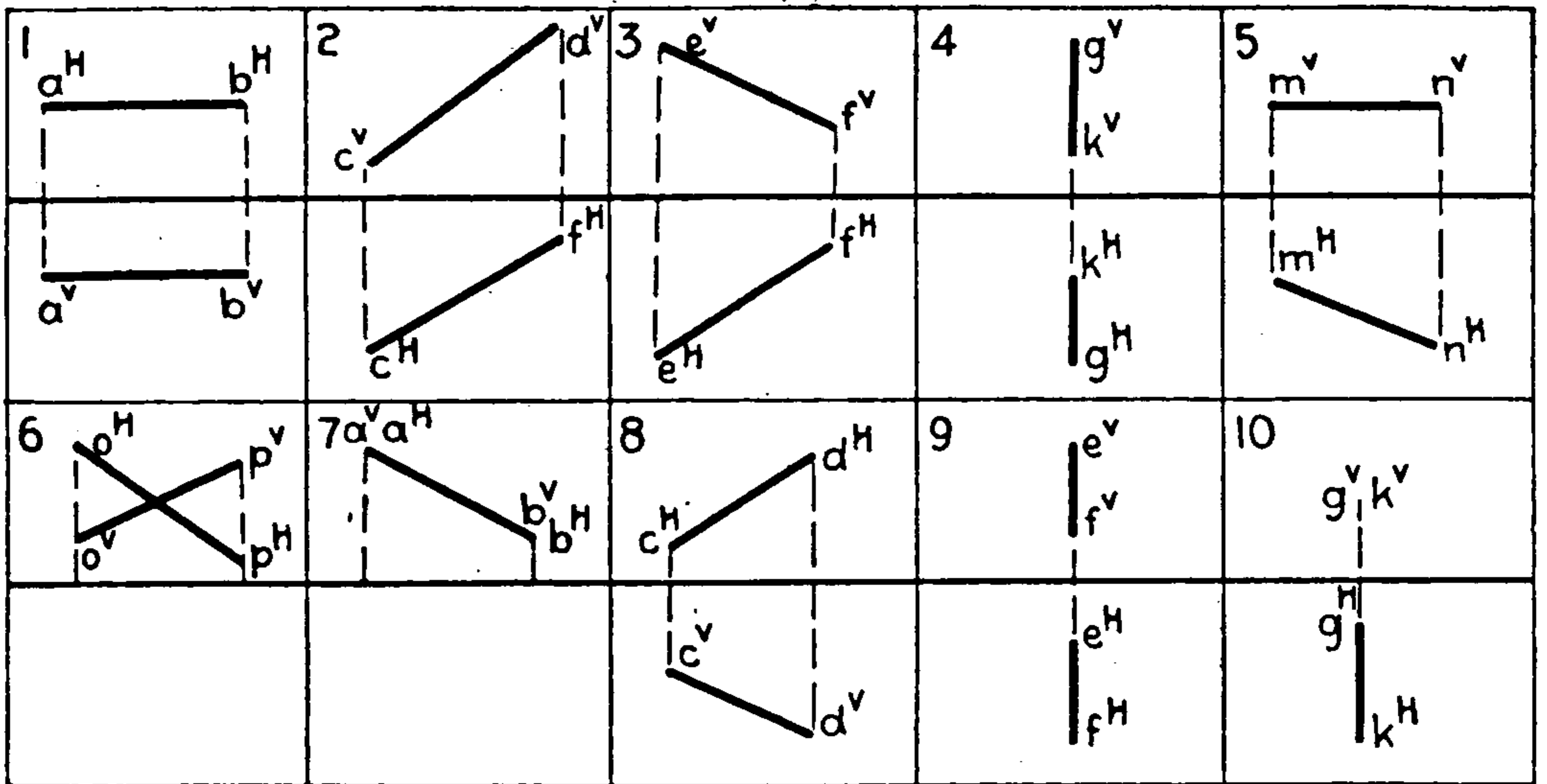
353. C(1 + 1 - 2) D(4 + 1 - 2) and S, Ex. 352.

354. E(2 + 2 - ½) F(2 + 1 - 2) and S, Ex. 352.

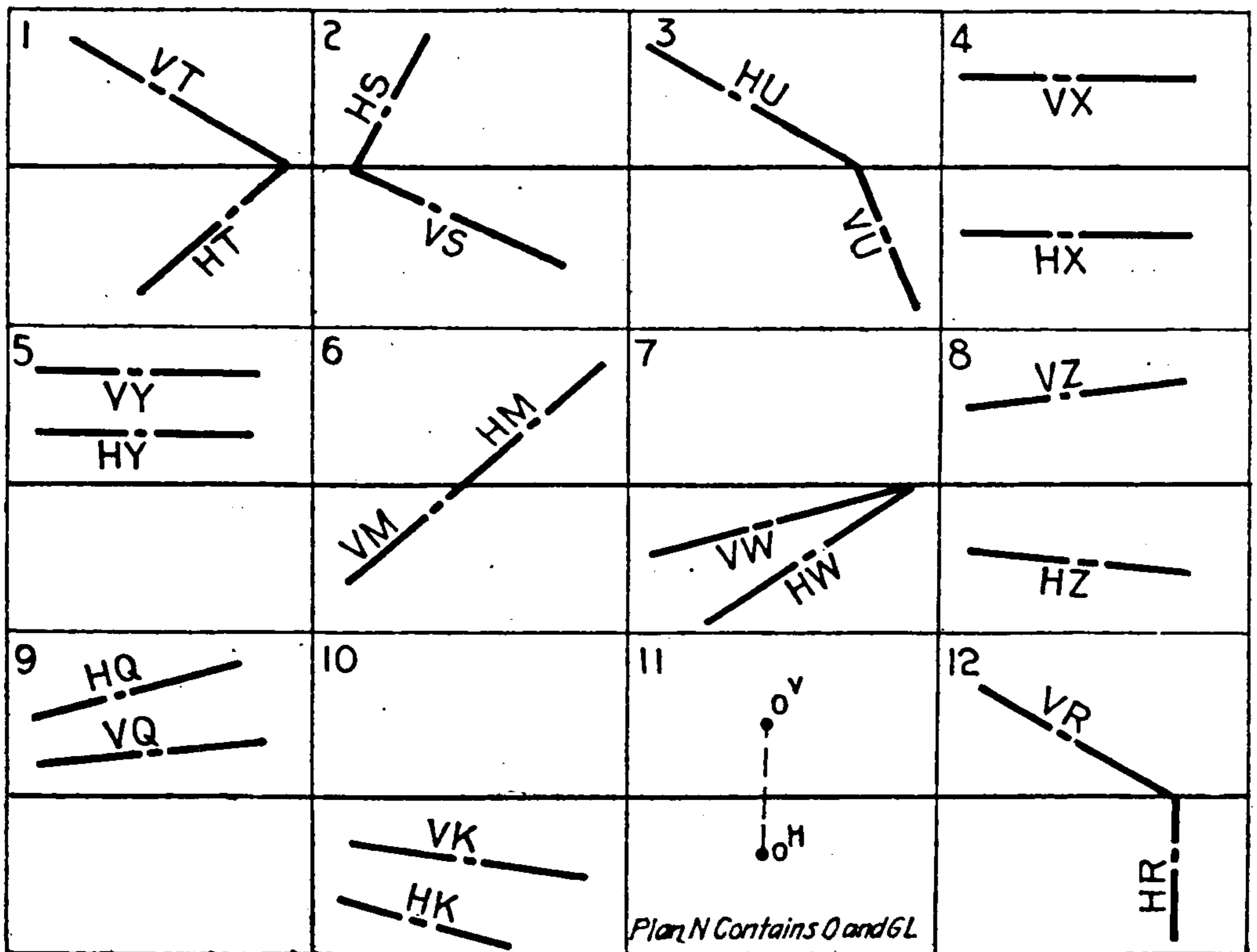
355. G(3 + 2 - 1½) K(3¾, 0 - 2) and S, Ex. 352.

Locate the piercing points of the following lines with the planes of the various given lines, without drawing the traces of the planes.

356.  $M(2 + 2 - 1)$   $N(3\frac{1}{2} + 1 - 2\frac{1}{2})$  and the plane of  $O(5 + 3 - 1)$   
 $P(1 + 1 - 1\frac{1}{2})$   $K(4 + \frac{1}{2} - 2)$ .



Given Lines



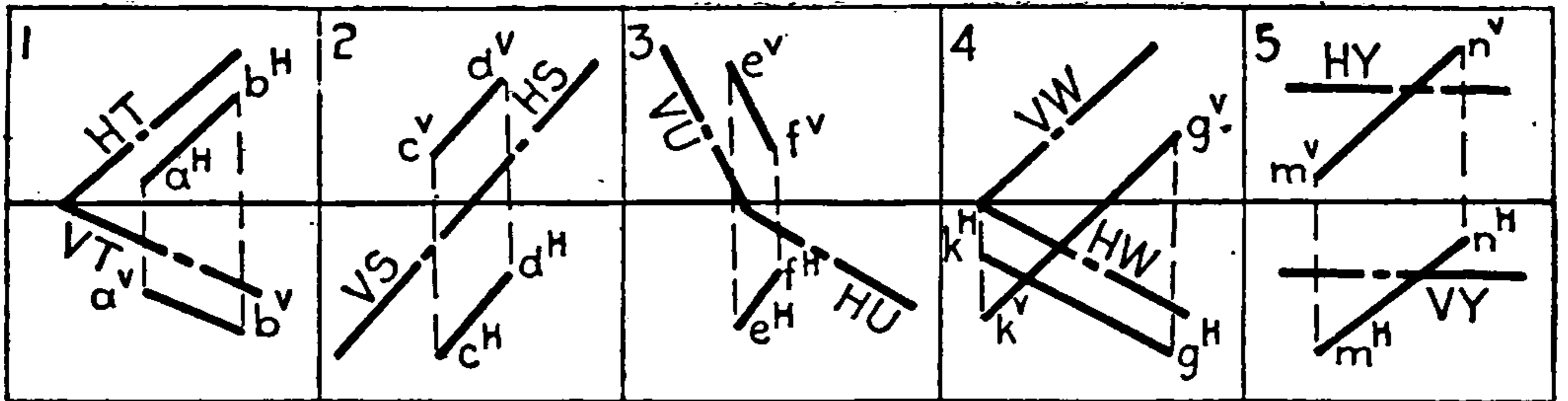
Given Planes

Graphic Layout No. 24.—Locate the point in which the line — pierces the plane —. Note.—Draw the line so that it is likely to pierce the plane. The H and V projections and traces may be transposed, if desired.

357.  $A(5 - \frac{1}{2} + 3)$   $B(2 - 2\frac{1}{2} + 1)$  and the plane of  $C(5 - 2 + 2)$   
 $D(1 - 1 + 1\frac{1}{2})$   $E(5, 0 + 1)$ .

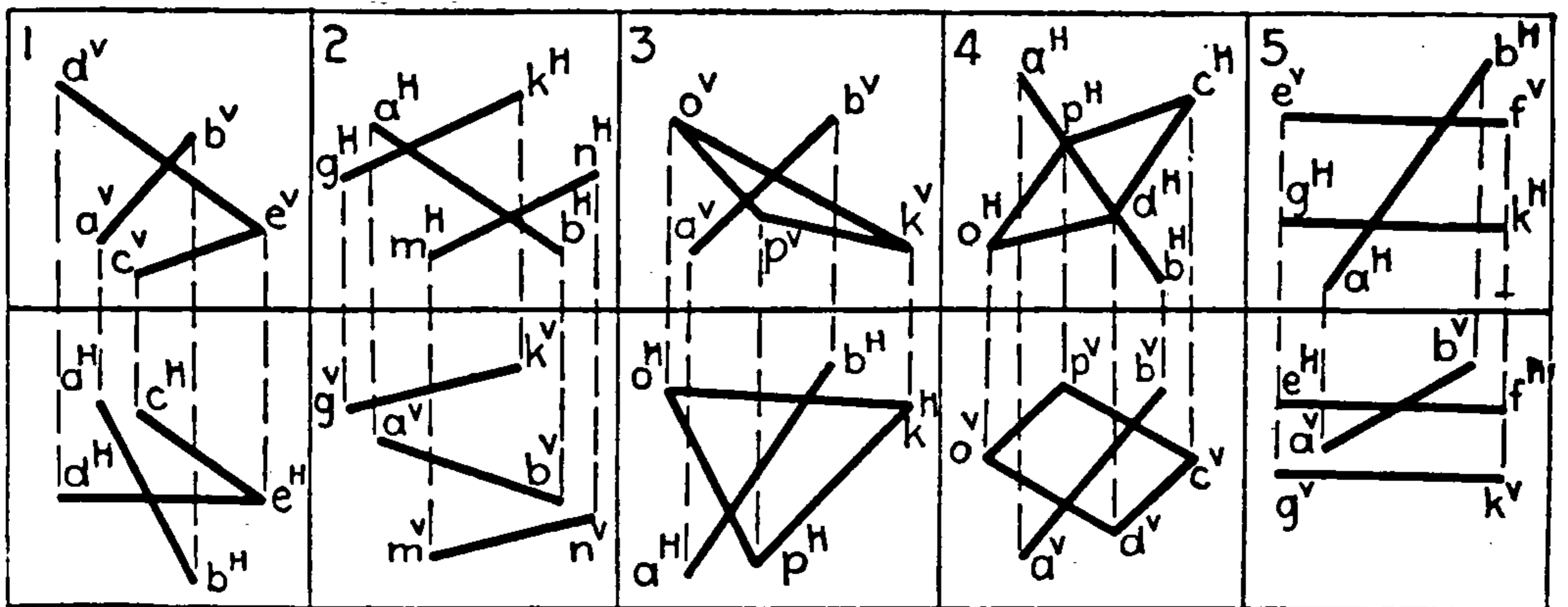


358.  $P(3 + 2 - 3)$   $O(3 + 1 - \frac{1}{2})$  and the plane of  $G(3 + 2 - 2)$   
 $K(1 + 1 - 1\frac{1}{2})$   $L(5 + 1, 0)$ .  
 359.  $M(2 - 3 + 2)$   $N(2 - \frac{1}{2} + 1)$  and the plane of  $A(1 - 2 + 2\frac{1}{2})$   
 $B(4 - 1 + 1)$   $C(1 - \frac{1}{2} + 1\frac{1}{2})$ .  
 360.  $D(1 + 3 - 1)$   $E(3\frac{1}{2} + 1 - 2)$  and the plane of  $F(2 + 1\frac{1}{2} - 3)$   
 $G(4 + 2\frac{1}{2} - 1)$  and  $K(2\frac{1}{2} + \frac{1}{2} - 2)$   $L(4\frac{1}{2} + 1\frac{1}{2}, 0)$ .  
 361.  $M(3 + 2 - \frac{1}{2})$   $N(3 + \frac{1}{2} - 1\frac{1}{2})$  and the plane of  $FG - KL$ , Ex.  
 360.



Graphic Layout No. 25.—Find where the given line pierces the given plane in Ex. —.

362.  $O(4 - 2 + \frac{1}{2})$   $P(2\frac{1}{2} - \frac{1}{2} + 2\frac{1}{2})$  and the plane of  $A(2 - 1\frac{1}{2}, 0)$   
 $B(4 - \frac{1}{2} + 2)$  and  $C(2 - 2\frac{1}{2} + 1)$   $D(4 - 1\frac{1}{2} + 3)$ .  
 363. From the middle point  $A$  of the line  $B(1 + 2 - 1)$   $C(1 + 1 - 3)$   
 draw a line which shall intersect  $M(2 + \frac{1}{2} - 1)$   $N(4 + 3 - 3)$  and  
 $O(3\frac{1}{2} + 1\frac{1}{2} - 1)$   $P(4 + \frac{1}{2} - 1\frac{1}{2})$ .



Graphic Layout No. 26.—Find where the given line  $AB$  pierces the plane of the intersecting or parallel lines in Ex. —, without drawing the plane-traces.

364. Draw a right hexagonal prism, its center line  $E(1 + \frac{3}{4} - 1)$   $F(3\frac{1}{2} + \frac{3}{4} - 2)$ , with one of its faces in  $H$ . Find the points in which it is pierced by  $G(1\frac{1}{2} + 2 - \frac{1}{2})$   $K(3, 0 - 2\frac{3}{4})$ .  
 365. Draw the projections of the shortest line on the surface of the hexagonal prism (Ex. 364) between the piercing points of  $GK$ .  
 366. Hexagonal prism in Ex. 364. Find the piercing points of the three parallel lines  $A(1, 0 - 2)$   $B(3 + 2 - 2)$ ,  $C(1\frac{1}{4}, 0 - 1)$   $D(3\frac{1}{4} + 2 - 1)$ , and  $E(2, 0 - 1\frac{1}{2})$   $F(4 + 2 - 1\frac{1}{2})$ .



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376. Develop the surface of the pyramid in Ex. 375 between  $H$  and  $S$ . Make a paper model of it.
377. Prism in Ex. 364. Find its intersection with the plane  $U(2 + 3) 2-(3 - 3)$ . Crosshatch the section and show it in its true size.
378. Develop the surface of the prism in Ex. 377 between one of its bases and  $U$ . Make a paper model of it.
379. Prism in Ex. 364. Find its intersection, and the true size of the intersection, with a profile plane through  $A(2\frac{1}{2}, 0, 0)$ .
380. Develop the surface of the prism in Ex. 379 between one base and the profile plane. Make a paper model.
381. Prism in Ex. 364. Draw its intersection by the plane  $S(2 + 3) 4\frac{1}{2}-(1 - 3)$ . Crosshatch the intersection and show its true size.
382. Develop the prism in Ex. 381 between one of its bases and  $S$ . Make a paper model.

### PERPENDICULARS TO PLANES

**70. Problem 20.**—To draw a line perpendicular to a given plane, through a given point.

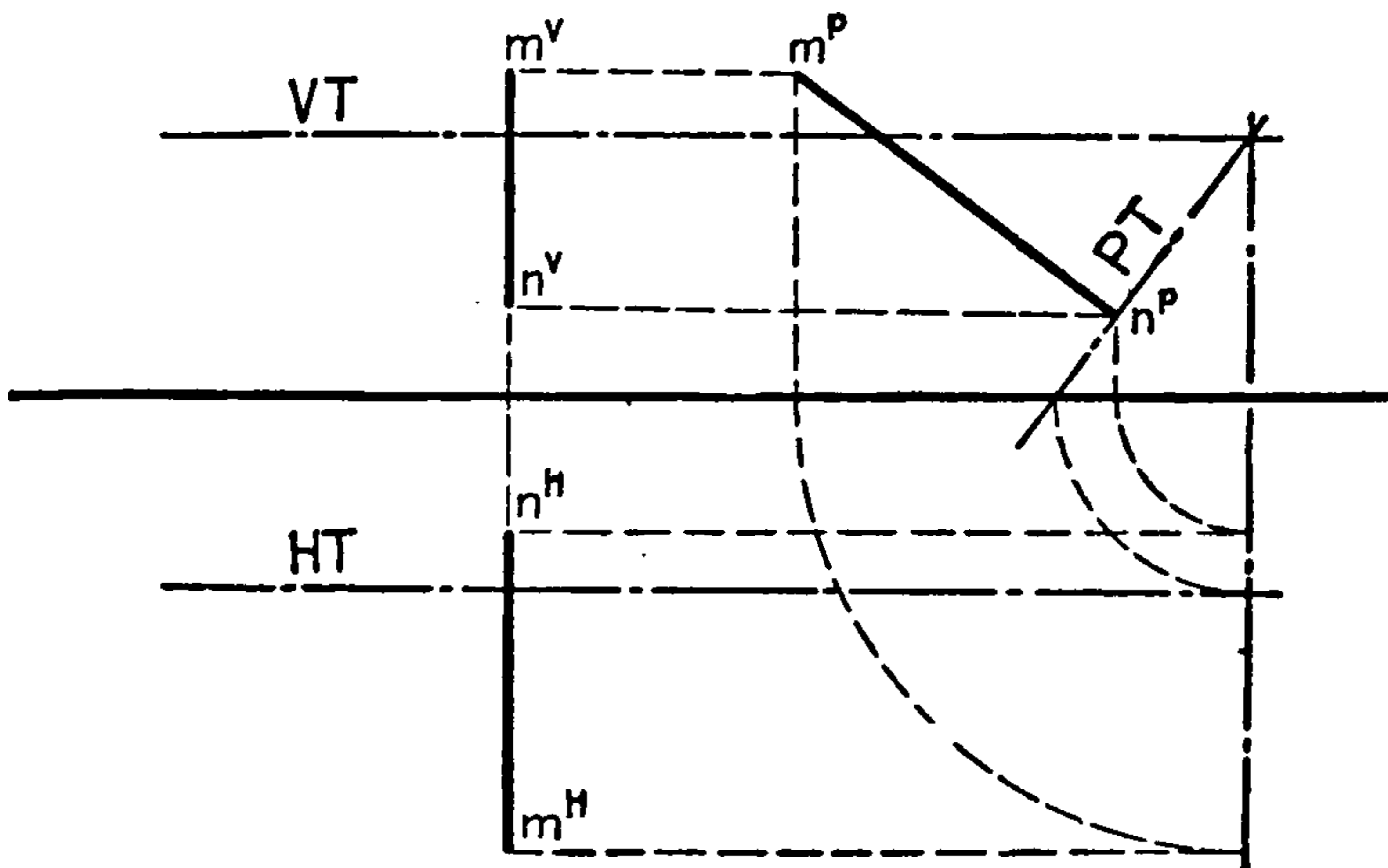


FIG. 52.

**Analysis.**—1. Through the  $V$ -projection of the given point draw the  $V$ -projection of the required line perpendicular to the  $V$ -trace of the given plane.

2. Through the  $H$ -projection of the given point draw the  $H$ -projection of the required line perpendicular to the  $H$ -trace of the given plane.

**Construction.**—Let the student make the construction for the ordinary case.

**Special Case.**—When the given plane is parallel to  $GL$ .

**Construction.**—Let  $M$  (Fig. 52) and  $T$  be the given point and plane.

1. Obtain the  $P$ -projection of  $M$ .
2. Obtain the  $P$ -trace of  $T$ .
3. Draw the perpendicular  $m^P n^P$  to  $PT$ .
4. Project  $n^P$  to  $n^V$  and  $n^H$  on a line through  $M$  perpendicular to the traces of  $T$ .

Conclusion.— $MN$  is perpendicular to  $T$ .

Note.— $N$  need not be a point in the plane, but any point in the line.

71.

EXERCISES

1 	2 	3 	4 
5 	6 	7 	8 
9 	10 	11 	12 

Graphic Layout No. 28.—From  $A(3 + 2 - 2)$  (or assume the point, if preferred) draw a perpendicular to plane —. Find where the perpendicular pierces the given plane. Find the true length of the perpendicular from the point to the plane.

383. From  $A(2 + 2 - 2\frac{1}{2})$  draw a perpendicular to  $T(5 + 2) 1(4 - 3)$ , and find where it pierces it.
384. From  $B(1 - 2 + 1)$  draw a perpendicular to  $S(5 - 3) 5(2 + 3)$ , and find where it pierces it.
385. From  $C(4 - 1 - 2)$  draw a perpendicular to  $R(3 - 3) 1(4 - 3)$ , and find where it pierces it.
386. From  $D(2 + 1 - \frac{1}{2})$  draw a perpendicular to  $X(+ 2) \infty(- 2)$ , and find where it pierces it.

387. From  $E(2 - 1 + \frac{1}{2})$  draw a perpendicular to  $U(5 - 2) (1 - 1) (5 + 3) (1 - 1\frac{1}{2})$ , and find where it pierces it.
388. From  $F(3 + 2 - 2)$  draw a perpendicular to  $W(1 + 2) 3(5 - 2)$ , and find its  $W$ -piercing point.
389. From  $G(3, 0, 0)$  draw a perpendicular to  $Y(1 + 2) 4(1 - 3)$ , and find where it pierces it.
390. From  $K(3, 0, 0)$  draw a perpendicular to  $T(-2) \infty(-1)$ .
391. Scale  $\frac{1}{8}$  in. = 1 ft. The roof of a building contains two points:  $A(16 \text{ ft.}, 0, -8 \text{ ft.})$ , and  $B(22 \text{ ft.}, +14 \text{ ft. } 6 \text{ in.}, -2 \text{ ft.})$ . The ridge and lower edge run 24 ft. horizontally to the southeast. At  $C(28 \text{ ft.}, 0 - 6 \text{ ft.})$  it is desired to erect a rafter brace perpendicular to the roof. At what point will it support the roof, and how long will it be?
392. Scale  $\frac{1}{8}$  in. = 1 ft. A tower, a hexagonal pyramid, 18-ft. base, 16-ft. altitude, base center at  $O(24 \text{ ft.}, 0 - 8 \text{ ft.})$ . From  $B(42 \text{ ft.} + 20 \text{ ft.} - 18 \text{ ft.})$  it is desired to run a telephone wire perpendicularly to the nearest face. Where is the hole drilled, and what is the length of the wire?

**72. Problem 21.**—To erect a perpendicular of any given length from a point in a plane.

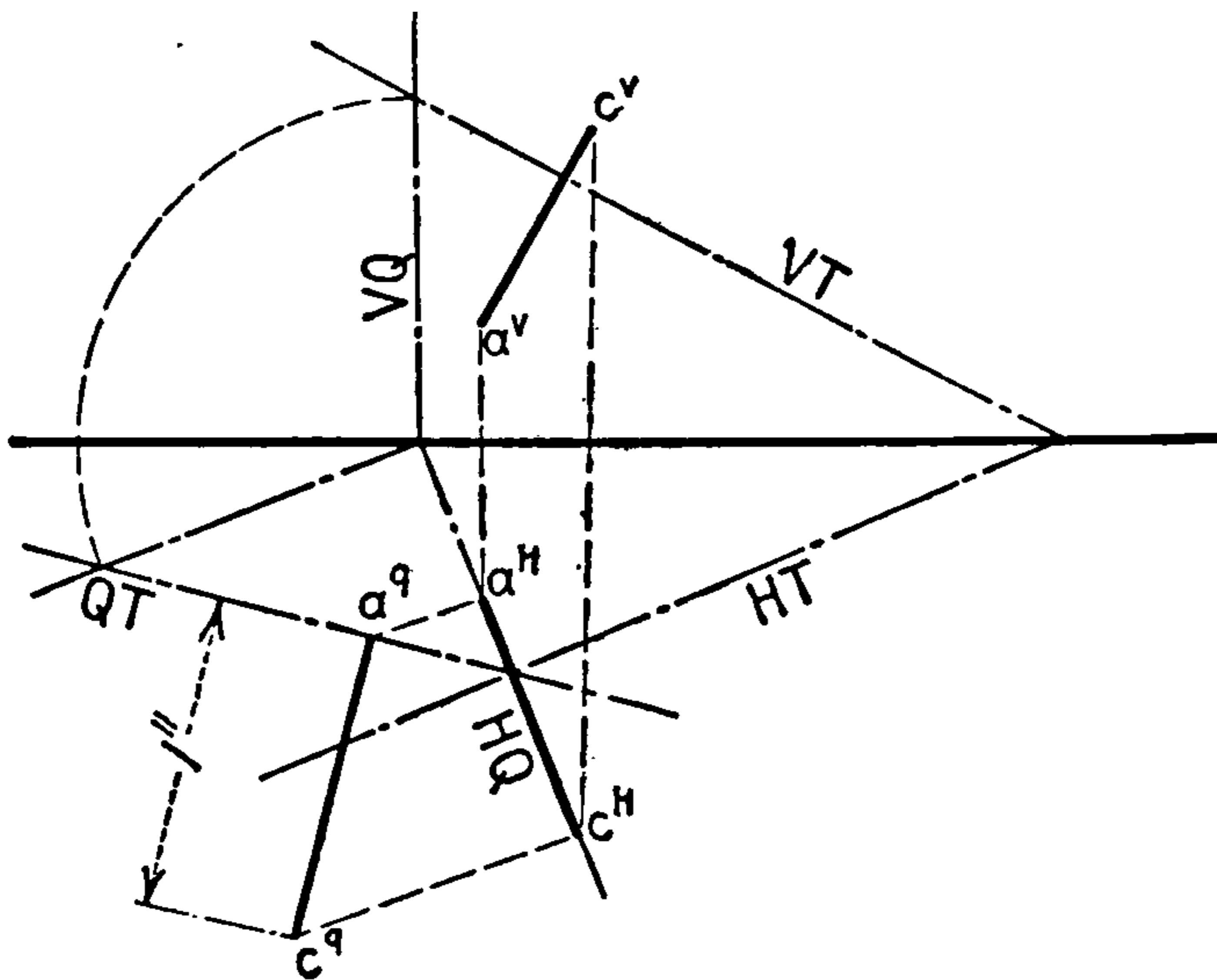


FIG. 53.

**Construction.**—Let  $A$  (Fig. 53) be the given point, and  $T$  the given plane. Required to erect a perpendicular 1 in. long.

1. Draw a perpendicular to  $T$  of indefinite length through  $A$ .
2. Obtain  $QT$ .
3. Obtain  $a^q$ .
4. Erect the perpendicular  $a^q c^q$ , 1 in. long.
5. Project  $c^q$  back to  $c^h$ , thence to  $c^v$ .

**Conclusion.**— $AC$  is perpendicular to  $T$ , and 1 in. long.

SECOND METHOD

- Analysis.—1. Erect a perpendicular of indefinite length from the given point, according to Problem 20.  
 2. Assume a point on the perpendicular.  
 3. Revolve the definite line, thus drawn, so as to show its true length.  
 4. Lay off the required distance on the true length of the line from the given point.  
 5. Return the limiting point to its proper position in space.

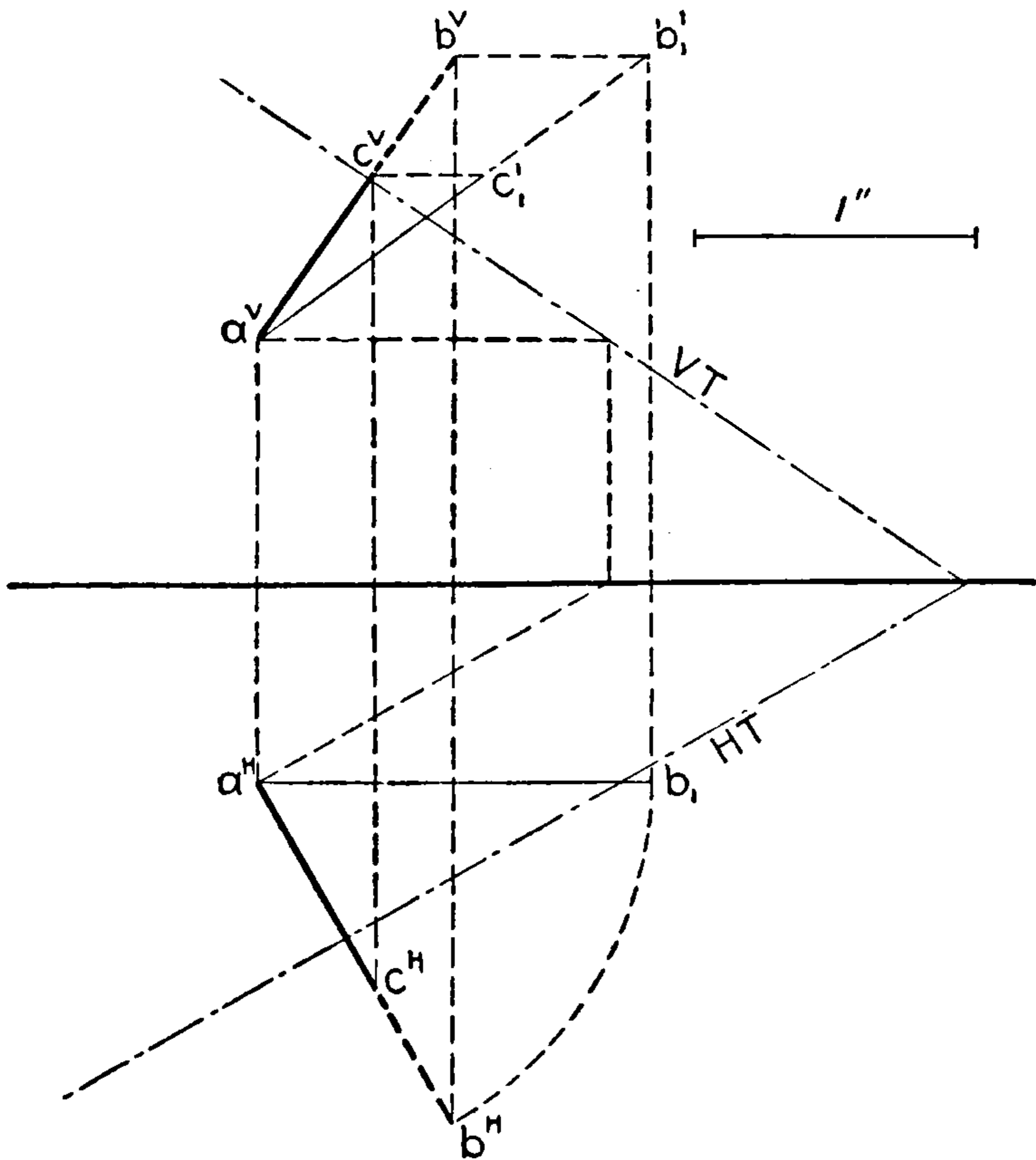


FIG. 54.

Construction.—To erect a perpendicular 1 in. long at A (Fig. 54) in the plane T.

1. Draw  $a^H b^H$  and  $a^V b^V$ , respectively perpendicular to HT and VT.
2. Having assumed B on the line, revolve it parallel to V, to show its true length.

3. Lay off 1 in. on this revolved line  $a^V c_1^V$ .

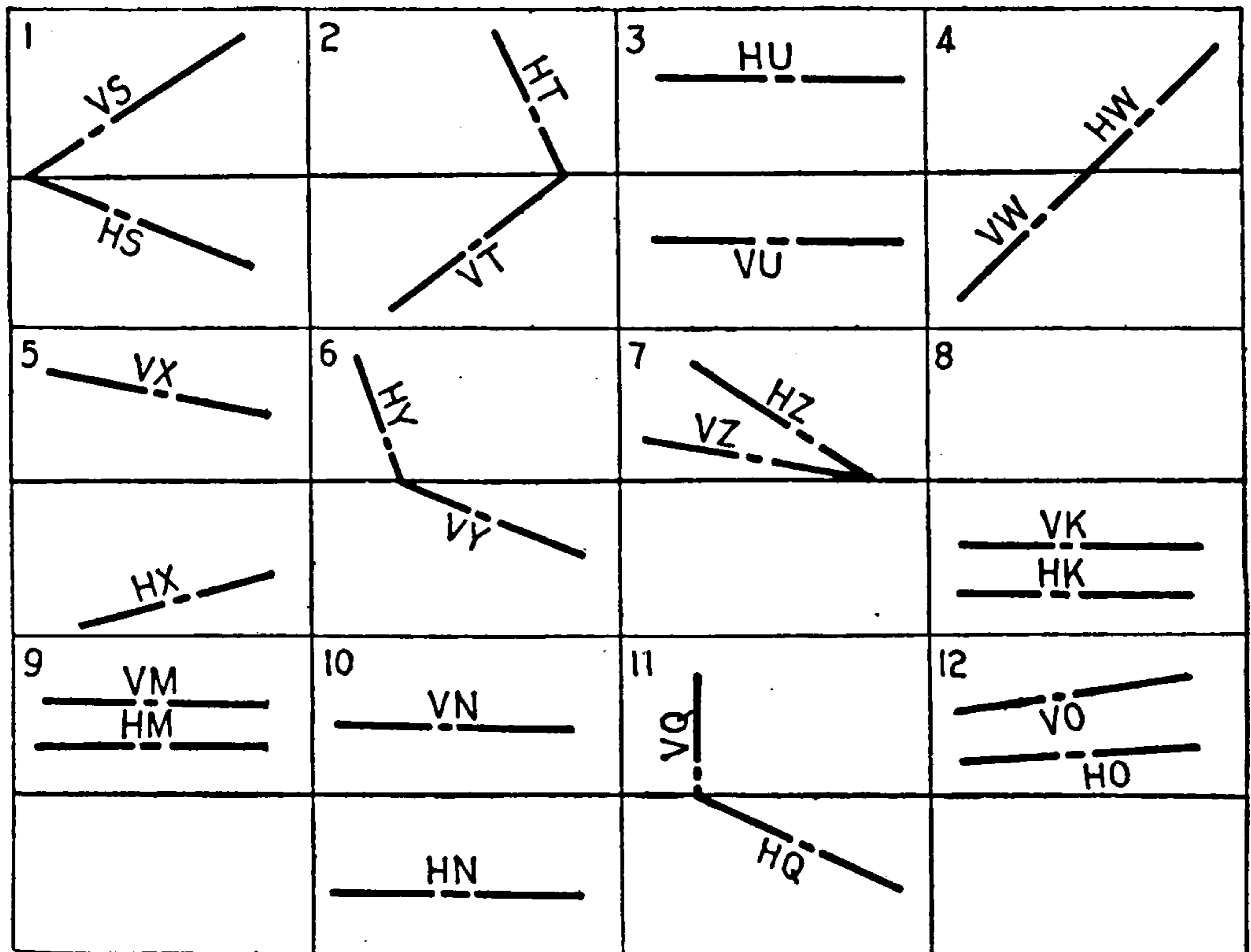
4. Return  $c_1^V$  to  $c^V c^H$  on AB.

Conclusion.—AC is perpendicular to T, and 1 in. long.

74.

## EXERCISES

393. From  $A(2 + 1, x)$  in  $T(1 + 3) 4(1 - 2)$  erect a perpendicular 2 in. long.
394. From  $B(1, y - 1)$  in  $S(+ 2) \infty(- 1)$  erect a perpendicular  $1\frac{1}{2}$  in. long.
395. From  $C(2, z - 1)$  in  $R(1 + 2\frac{1}{2}) 5(1 + 1)$  erect a perpendicular 2 in. long.
396. From  $D(3 - 1, y)$  in  $Q(1 - 3) (5 - 1) (1 + 1\frac{1}{2}) (5 + \frac{1}{2})$  erect a perpendicular  $1\frac{1}{2}$  in. long.



Graphic Layout No. 29.—Assume a point in plane —, and erect a  $1\frac{1}{2}$ -in. perpendicular to the plane from the assumed point.

397. From  $E(2 - 1\frac{1}{2}, x)$  in  $U(2 - 3) 3\frac{1}{2}(5 + 3)$  erect a perpendicular 1 in. long.
398. Draw the projections of a right prism,  $1\frac{3}{4}$ -in. altitude, having  $M(1 + 2 - 1\frac{1}{4}) O(1\frac{3}{4} + 1 - 2) P(2\frac{1}{2} + 1\frac{1}{2} - 1)$  as its base.
399. Assume the projections of any plane figure in space and erect a right prism of 2-in. altitude on it as a base.

## THE PROJECTIONS OF OBLIQUE SOLIDS

75. Problem 22.—To draw the projections of a solid having its base in an oblique plane.

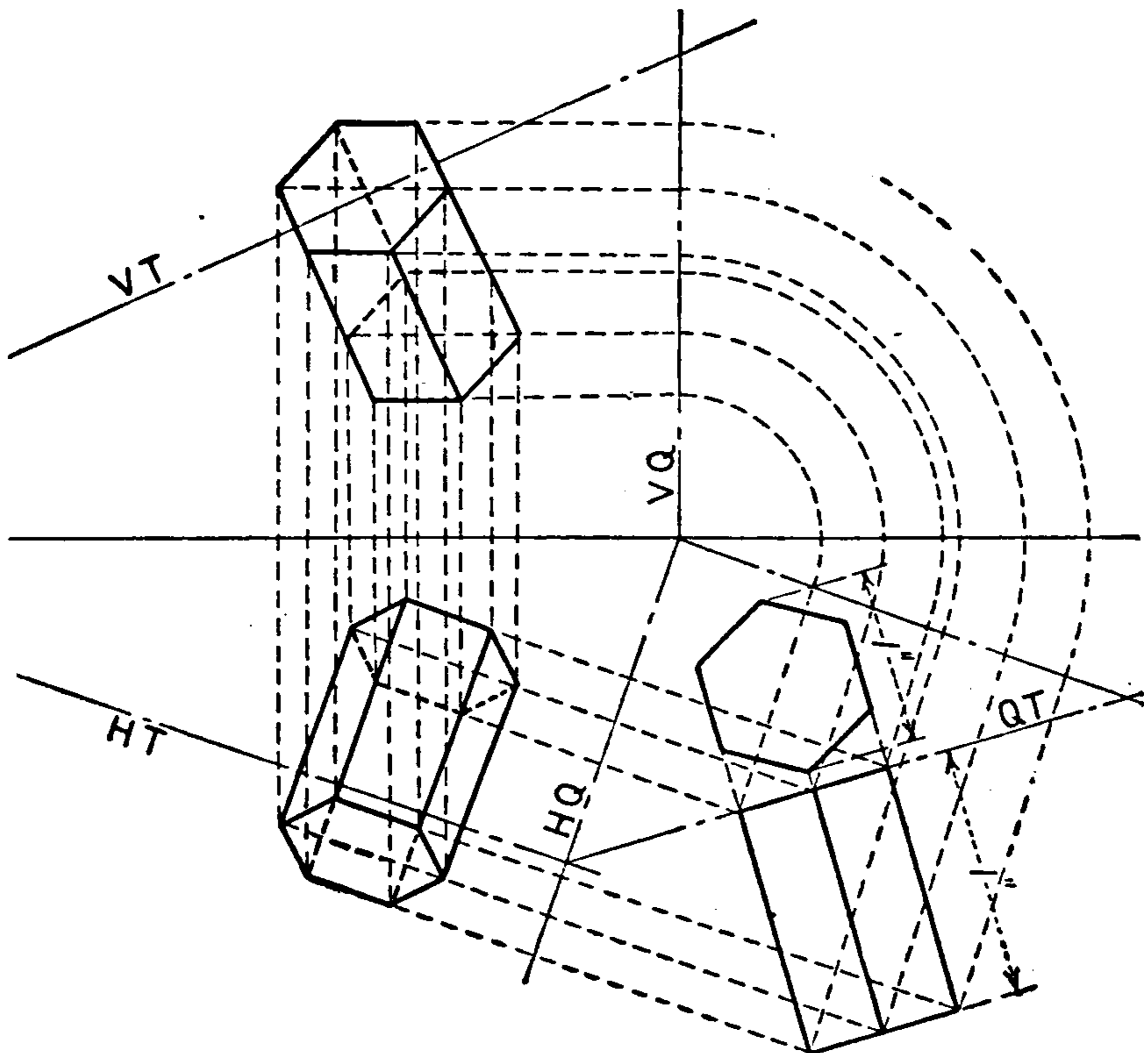


FIG. 55.

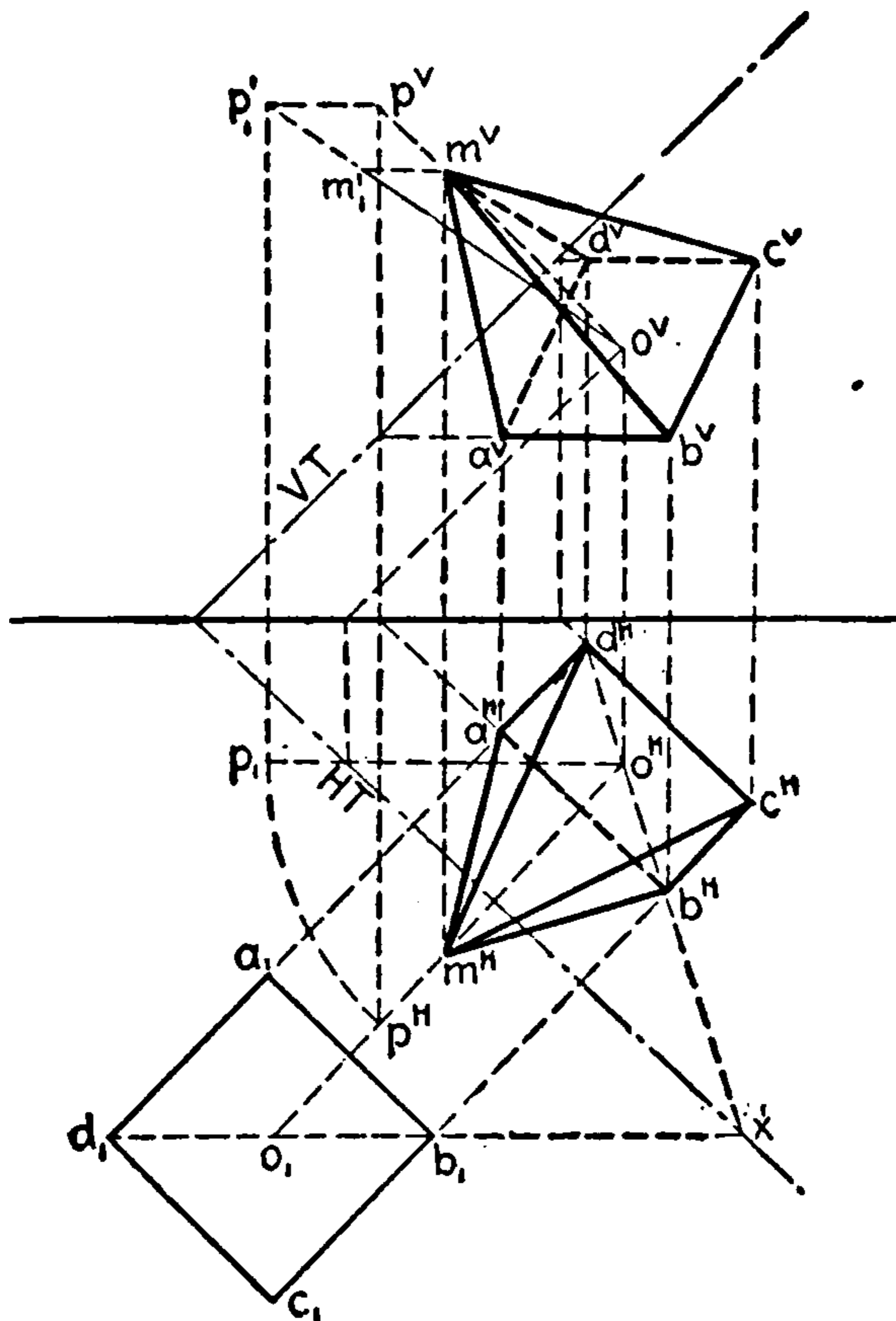


FIG. 56.



## FIRST METHOD

Construction.—Let  $T$  (Fig. 55) be the given plane, and let it be required to draw a 1-in.  $\times$  1½-in. hexagonal prism.

1. Obtain  $QT'$ .
2. With  $QT$  as a ground line draw the solid in its natural position.
3. Project it back on  $H$  and  $V$ , following the dotted projectors, as shown in the figure.

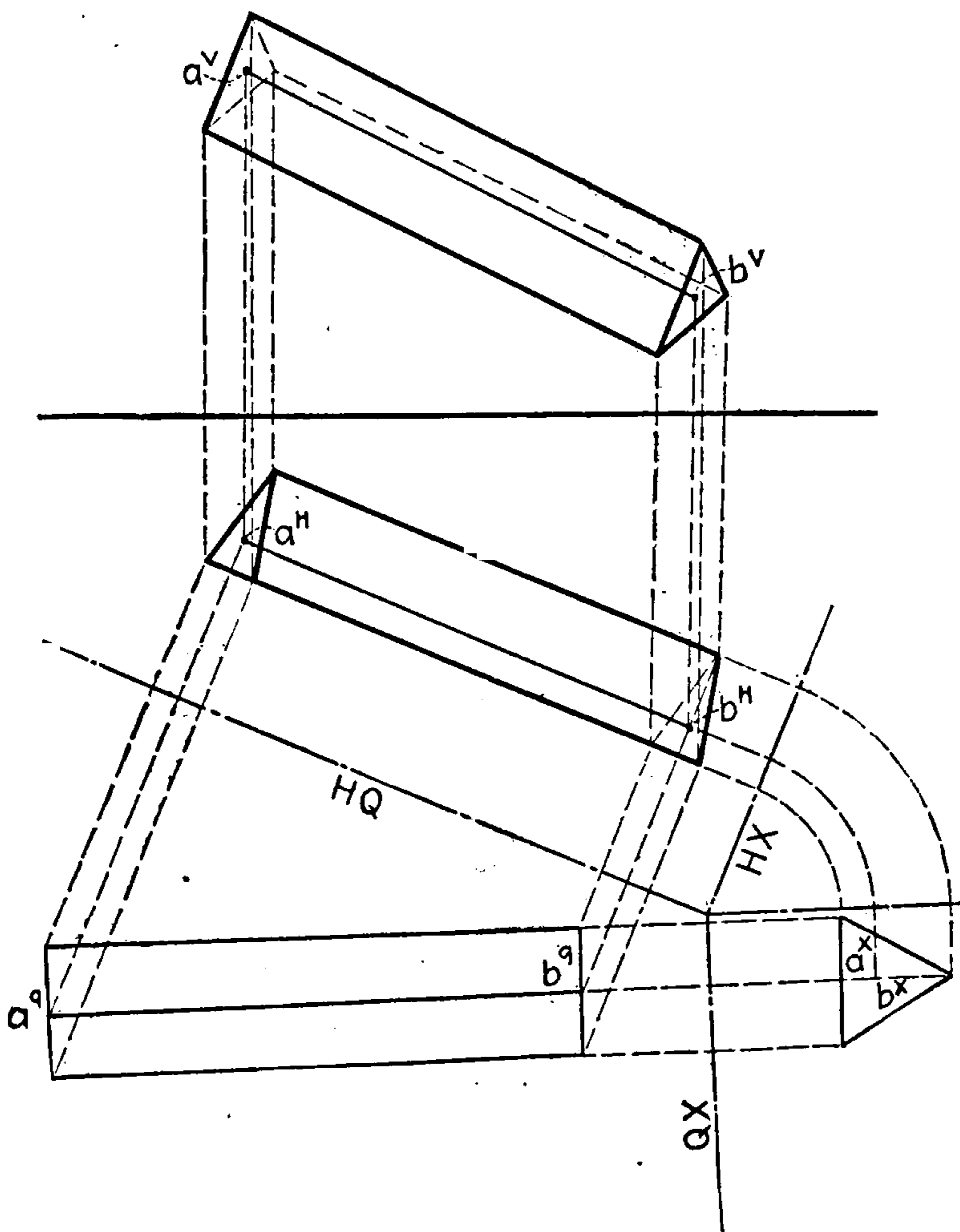


FIG. 57.

76.

## SECOND METHOD

Construction.—Let  $O$  (Fig. 56) be the center of a 1-in. square in the plane  $T$ . Let it be required to draw a pyramid of 2-in. altitude on this square as a base.

1. Revolve  $O$  about  $HT$  into  $H$ .



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401. Solid —,  $R(1 - 3) 1(5 + 2)$ ,  $A(4 - 1, y)$ .  
 402. Solid —,  $S(4 + 2) 2(4 - 1)$ ,  $B(4, x - \frac{1}{2})$ .  
 403. Solid —,  $T(1 - 3) 3(1 + 2)$ ,  $C(1\frac{1}{2} - 1, x)$ .  
 404. Solid —,  $U(1 + 2) 2\frac{1}{2}(4 - 2)$ ,  $D(2\frac{1}{2}, y - 1)$ .  
 405. Solid —,  $W(5\frac{1}{2} - 2) 4(2\frac{1}{2} + 2)$ ,  $E(3\frac{1}{2} - \frac{3}{4}, x)$ .  
 406. Solid —,  $X(4 + 3) 1(5 + 1)$ ,  $F(3 + y - \frac{1}{2})$ .  
 407. Solid —,  $Y(1 + 1\frac{1}{2}) (5 + \frac{3}{4}) (1 - 1) (5 - \frac{1}{2})$ ,  $G(2 + 1, x)$ .  
 408. Solid —,  $Z(5 - 3) (1 - 2) (5 + 2\frac{1}{4}) (1 + 1\frac{1}{2})$ ,  $K(3 - 1\frac{3}{4}, y)$ .  
 409. Solid —,  $S(1 + 3) 2(5 - 1\frac{1}{2})$ ,  $L(4 + 2, x)$ .  
 410. Solid —,  $T(5 - 2\frac{1}{2}) 4(1 + 1)$ ,  $M(2, y + 1\frac{1}{2})$ .  
 411. Solid —,  $U(+ 2) \infty(- 1)$ ,  $N(3 + 1, x)$ .  
 412. Solid —,  $W(- 3) \infty(+ 1)$ ,  $O(3 - 1, y)$ .  
 413. Solid —,  $X(+ 1) \infty(- \frac{1}{2})$ ,  $P(2, x - 2)$ .

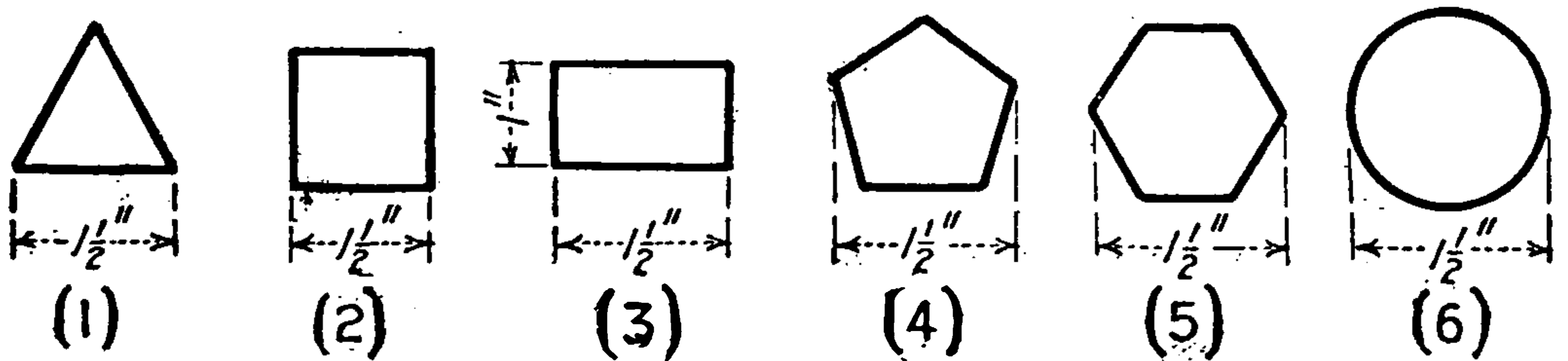
Erect one of the foregoing solids on one of the following center lines. Draw one side of the base  $\parallel$  to  $H$  or  $V$ , according to assignment.

414. Solid —, Center line  $A(1 + 2 - 1) B(3 + 2 - 1)$ .  
 415. Solid —, Center line  $C(2\frac{1}{2} - 1 + 1\frac{1}{2}) D(1 - 1 + 1\frac{1}{2})$ .  
 416. Solid —, Center line  $E(1 - 1 + 1\frac{1}{2}) F(3 - 2 + 1\frac{1}{2})$ .  
 417. Solid —, Center line  $G(5 - 1 + 2) K(3 - 1 + 1)$ .  
 418. Solid —, Center line  $M(1 + 2 - \frac{1}{2}) N(2 + \frac{1}{2} - 2\frac{1}{2})$ .  
 419. Solid —, Center line  $O(4 - 3 + 1) P(5 - 1 + 2\frac{1}{2})$ .  
 420. Solid —, Center line  $A(1 + 3 - 3) B(2 + 1 + \frac{1}{2})$ .  
 421. Solid —, Center line  $C(4 - 1 + 1) D(5 - 2\frac{1}{2} + 3)$ .  
 422. Solid —, Center line  $E(2 + 2 - \frac{1}{2}) F(2 + 1 - 2)$ .  
 423. Solid —, Center line  $G(2 + 2\frac{1}{2} - 2\frac{1}{2}) K(2 + 1 - 1)$ .  
 424. Solid —, Center line  $M(2 - 2 + \frac{3}{4}) N(2 - \frac{3}{4} + 2)$ .  
 425. Solid —, Center line  $O(2 - 1 + \frac{3}{4}) P(2 - \frac{3}{4} + 2\frac{1}{2})$ .

Erect solid  $b$ ,  $d$ ,  $f$ ,  $k$ ,  $m$ , or  $o$ , according to assignment, with its apex in the point given, and its base in the plane given in the following exercises. Disregard the altitudes given, as they may not fit.

426. Solid —, Apex  $A(5 + 1\frac{1}{2} - \frac{1}{2})$ , Base  $T(1 + 3) 1(4 - 2)$ .  
 427. Solid —, Apex  $B(3 + 2 - 2)$ , Base  $S(1 + 2) 2\frac{1}{2}(2\frac{1}{2} - 3)$ .  
 428. Solid —, Apex  $C(1 + 3 - 2)$ , Base  $U(2 + 2) 5(2 - 1)$ .  
 429. Solid —, Apex  $D(3 - 2\frac{1}{2} + 3)$ , Base  $W(5 - 2) 3(5 + 3)$ .  
 430. Solid —, Apex  $E(1 + 2 - \frac{1}{2})$ , Base  $X(5 + 2) 4(1 - 1)$ .  
 431. Solid —, Apex  $F(3 - 1 + 2)$ , Base  $Y(1 - 3) 2(5 + 1)$ .  
 432. Solid —, Apex  $G(3 + 1 - 2\frac{1}{2})$ , Base  $Z(5 + 3) 1(5 + 1)$ .  
 433. Solid —, Apex  $K(3, 0, 0)$ , Base  $Q(1 + 3) (5 + 2) (1 - 2\frac{1}{4}) (5 - 1\frac{1}{2})$ .  
 434. Solid —, Apex  $M(4 - 3 + 2\frac{1}{2})$ , Base  $R(1 - 1\frac{1}{2})(5 - \frac{3}{4}) (1 - 2) (5 - 1)$ .  
 435. Solid —, Apex  $N(5 + 2 - \frac{1}{2})$ , Base  $S(1 + 3) 3(5 - 3)$ .  
 436. Solid —, Apex  $O(1 - 3 + 1)$ , Base  $T(5 - 2) 3\frac{1}{2}(2 + 2)$ .  
 437. Solid —, Apex  $P(3 + 3 - 3)$ , Base  $U(+ 2) \infty(- 1)$ .  
 438. Solid —, Apex  $A(2 - 2 + 2)$ , Base  $W(- 2\frac{1}{2}) \infty(+ 1)$ .  
 439. Solid —, Apex  $B(2 - 2 + 1)$ , Base  $X(+ \frac{1}{2}) \infty(+ 1\frac{1}{4})$ .  
 440. Erect a 1-in. cube on  $T(1 + 2) 4(1 - 1)$ , with its base center at  $O(2 + 1, x)$ , diagonal  $\parallel$  to  $H$ .

441. Draw a 1-in. cube on  $S(+2) \infty(-1)$  with one corner at  $A(3 + \frac{1}{4}, x)$  and another touching  $H$ .
442. Draw a cube, one of whose edges is the line  $A(4 + 1 - 2) B(3 + 1 - 1)$  and another touches  $H$ .
443. The center line of a cube is  $C(2 + 2 - 2) D(3 + 1 - 1\frac{1}{2})$ . Draw the cube, (a) with an edge  $\parallel$  to  $H$ ; (b) with its base diagonal  $\parallel$  to  $H$ .
444. The line  $E(2 + 1\frac{1}{2}, x) F(3 + \frac{3}{4}, y)$  in  $S(+2) \infty(-1\frac{1}{2})$  is one edge of a cube, base in  $S$ . Draw the projections of the cube.



1 HS VS	2 VT HT	3 VU HU	4 VW HW	5 HX VX	6 VY HY
7 VQ HQ	8 VR HR	9 HZ VZ	10 VM HM	11 VO HO	12 VK HK

Graphic Layout No. 30.—Let the polygons be bases of 2-in. prisms or pyramids (or cones or cylinders for No. 6).

(a) Erect prism (or cylinder), base — on plane —, assuming a center in the plane.

(b) Erect pyramid (or cone), base — on plane —, assuming a center in the plane.

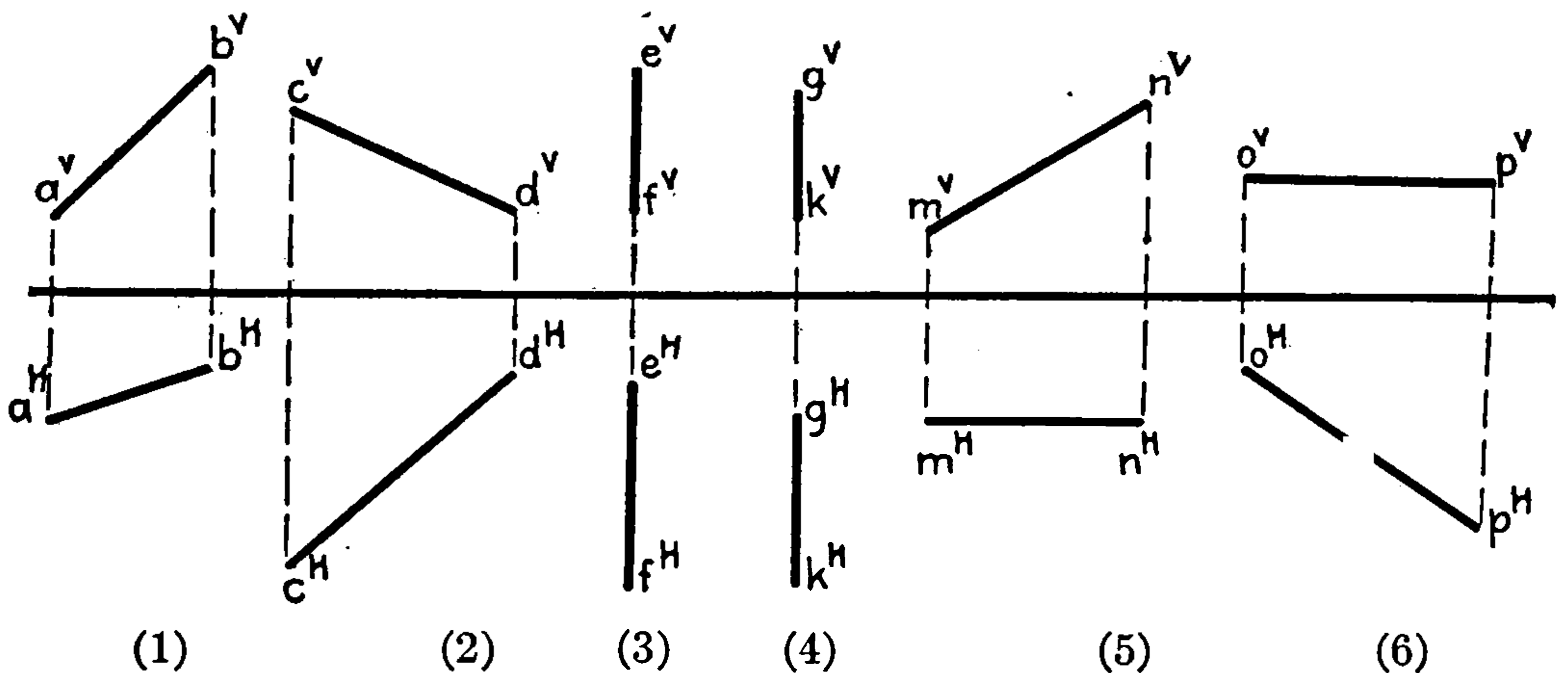
(c) With the base — on plane — under any of the conditions given in Graphic Layout No. 22, erect a 2-in. pyramid or prism.

(d) Assume a convenient point in space (e.g.,  $A(3 + 2 - 2)$  or  $B(3 - 2 + 2)$ ) as the apex of pyramid —, or cone —, to be drawn with its base in plane —.

(e) Draw prism, base —, with its center line parallel to — ( $H, V,$  or  $P$ ), one of its faces touching plane —.

(f) Draw hollow prism or cylinder, with  $\frac{1}{4}$ -in. walls, under the conditions (a) or (e).

445. The lines  $G(2 + 1 - 1)$   $K(1 + \frac{1}{2} - 1\frac{1}{4})$  and  $KL(2\frac{1}{2} + 1\frac{1}{4} - 1\frac{1}{2})$  form two sides of a parallelogram, which is the base of a prism of  $1\frac{1}{2}$ -in altitude. Draw the prism.
446. The line  $M(1 + 1 - 1)$   $N(2 + \frac{1}{4} - \frac{3}{4})$  is one side of a rectangle which forms the base of a prism  $1\frac{1}{2}$ -in. altitude. The opposite side passes through  $D(2 + 1 - 1)$ . Draw the prism.
447. Draw a line from  $P(2\frac{1}{2} + 2 - \frac{1}{2})$  that shall intersect  $M(1 + 2 - 1\frac{1}{2})$   $N(3 + \frac{1}{4} - 1)$  and be at a distance of 2 in. from  $P(3\frac{1}{2} + 1 - 2\frac{1}{2})$ .



Center lines for solids.

Graphic Layout No. 31.—(a) With line — as center line, draw 2-in. prism (or cylinder) with base —. (b) With line — as center line, draw a 2-in. pyramid (or cone) with base —.

Note.—The signs on these lines may be transposed.

**79. Problem 23.**—To pass a plane through a given point perpendicular to a given line.

**Principle.**—The traces of a plane perpendicular to any line are respectively perpendicular to the projections of the line; hence we know the direction of the traces.

**Analysis.**—Apply the principle given.

**Construction.**—Let  $O$  (Fig. 58) be the given point, and  $AB$  the given line.

1. Draw  $o^h p^h$  perpendicular to  $a^h b^h$ .
2. Draw  $o^v p^v$  parallel to  $GL$ .
3. Obtain  $\odot (P)$  of  $OP$ .
4. Through  $\odot$  draw  $VT$  perpendicular to  $a^v b^v$ .
5. Where  $VT$  intersects  $GL$  draw  $HT$ , perpendicular to  $a^h b^h$ .

**Conclusion.**— $T$  contains  $O$ , and is perpendicular to  $AB$ .

**Notes.**—1. If the first trace does not intersect  $GL$  within the problem limits, draw a second auxiliary line parallel to the second required trace.



459. The center line of a stick  $1\frac{1}{2}$  in. square is the line  $O(2 + 3 - 2) P(3 + 1 - 1)$ . Two  $\frac{1}{2}$ -in. holes are drilled straight through the stick,  $\frac{3}{4}$  in. from the ends. Draw the projections of the stick and holes, dotting invisible lines.

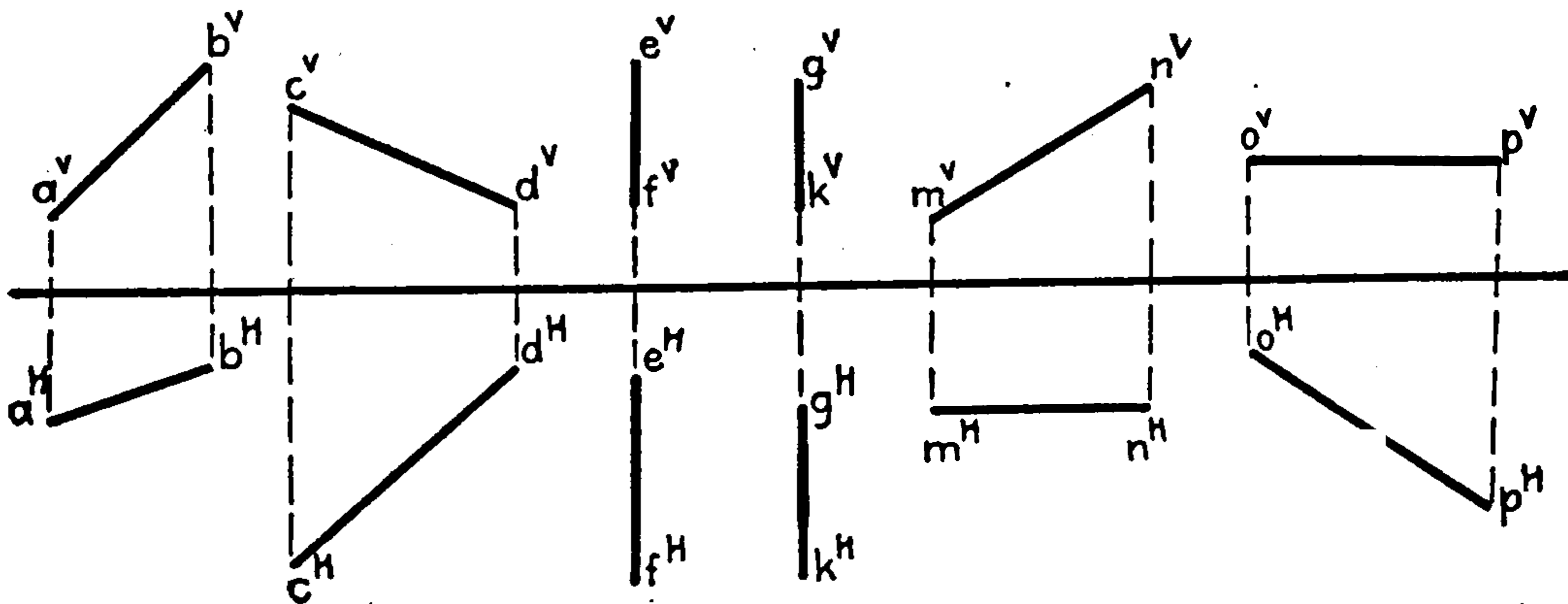
Note.—This exercise can be performed by the method of the Supplementary Problem, Problem 22. Conversely, Ex. 414–425 can also be constructed by the use of Problem 23.

460. The upper surface of a vein of coal is found to be perpendicular to a shaft bored in the direction of  $C(3 - 1 + \frac{1}{2}) D(1 - 3 + 2)$ . The ore is located at a point  $O(2, x, y)$  on  $CD$ . Draw the traces of the plane of the ore.

461. Scale 1 in. = 10 ft. A 4-ft. stack on a building has its center line  $A(15 \text{ ft.} - 30 \text{ ft.} + 15 \text{ ft.}) B(15 \text{ ft., } 0 + 15 \text{ ft.})$ , a guy wire runs from a point 15 ft. up the stack to a point  $C(23 \text{ ft.} - 22 \text{ ft.} + 7 \text{ ft. } 6 \text{ in.})$  in a roof perpendicular to the wire. Find the traces of the roof plane, and draw the projections of the roof, a rectangle, 40 ft. long by 20 ft. slant height, with  $C$  as its center.

462. Scale 1 in. = 6 ft. A 3-in. shaft carrying a 54-in. pulley (8-in. face) mounted 7 ft. from the upper end, runs from  $D(18 \text{ ft. } 0 + 12 \text{ ft.})$  in the ceiling of a factory building to  $E(18 \text{ ft., } 18 \text{ ft., } 0)$  in a wall. Draw the projections of the shaft and pulley (without spokes).

463. Scale 1 in. = 6 ft. A guide pulley, 54-in.  $\times$  12-in. face, is running on an oblique 2-in. shaft, whose center line is  $F(12 \text{ ft.} - 6 \text{ ft.} + 8 \text{ ft. } 6 \text{ in.}) G(24 \text{ ft.} - 12 \text{ ft.} + 19 \text{ ft. } 6 \text{ in.})$  Draw the projections of the pulley at the center of  $FG$ .



Graphic Layout No. 32.—(a) Assume a point in space, and pass a plane through it perpendicular to line —.

(b) Pass a plane  $\perp$  to line — at its middle point.

(c) Let line — be one edge of a cube, with one corner (or edge) touching — ( $H$  or  $V$ ). Draw the projections of the cube.

(d) Let line — be the center line of a  $1\frac{1}{2}$ -in. square stick, with a  $\frac{1}{2}$ -in. hole drilled lengthwise through the center. Draw its projections.

(e) Let line — be the center line of a  $1\frac{1}{2}$ -in. square stick with a  $\frac{1}{2}$ -in. hole drilled crosswise through the center. Draw its projections.

(f) Let line — be the center line of a 2-in. shaft, carrying a 24-in.  $\times$  8-in. pulley. Draw the projections of the pulley to any convenient scale.

DISTANCE OF A POINT TO A PLANE

81. Problem 24.—To find the distance from a point to a plane.

Construction.—Let  $O$  (Fig. 59) and  $T$  be the given point and plane.

1. Obtain  $QT$  and  $O^q$ .
2. Draw a perpendicular from  $O^q$  to  $QT$ .

Conclusion.—This perpendicular will be the distance required.

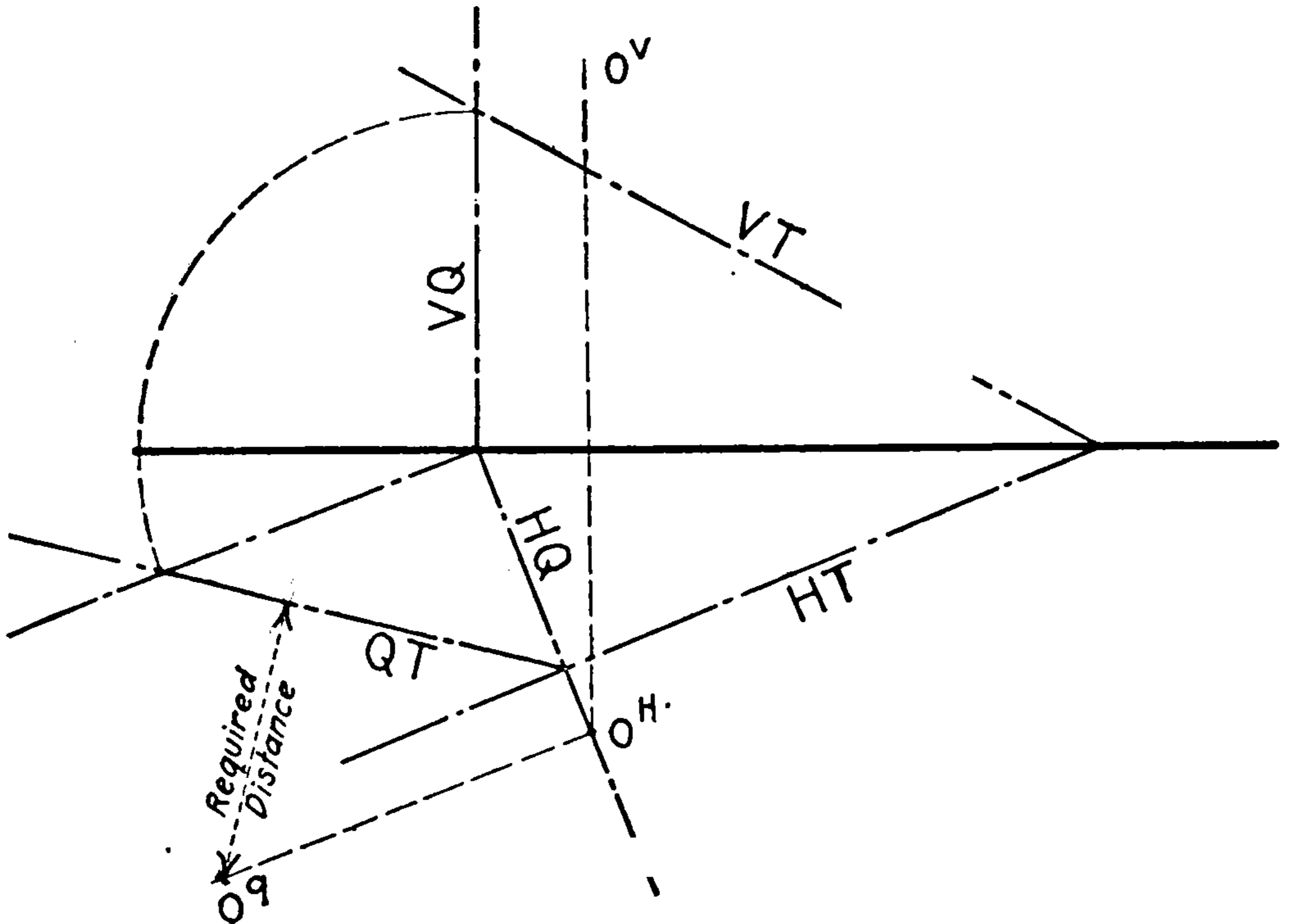


FIG. 59.

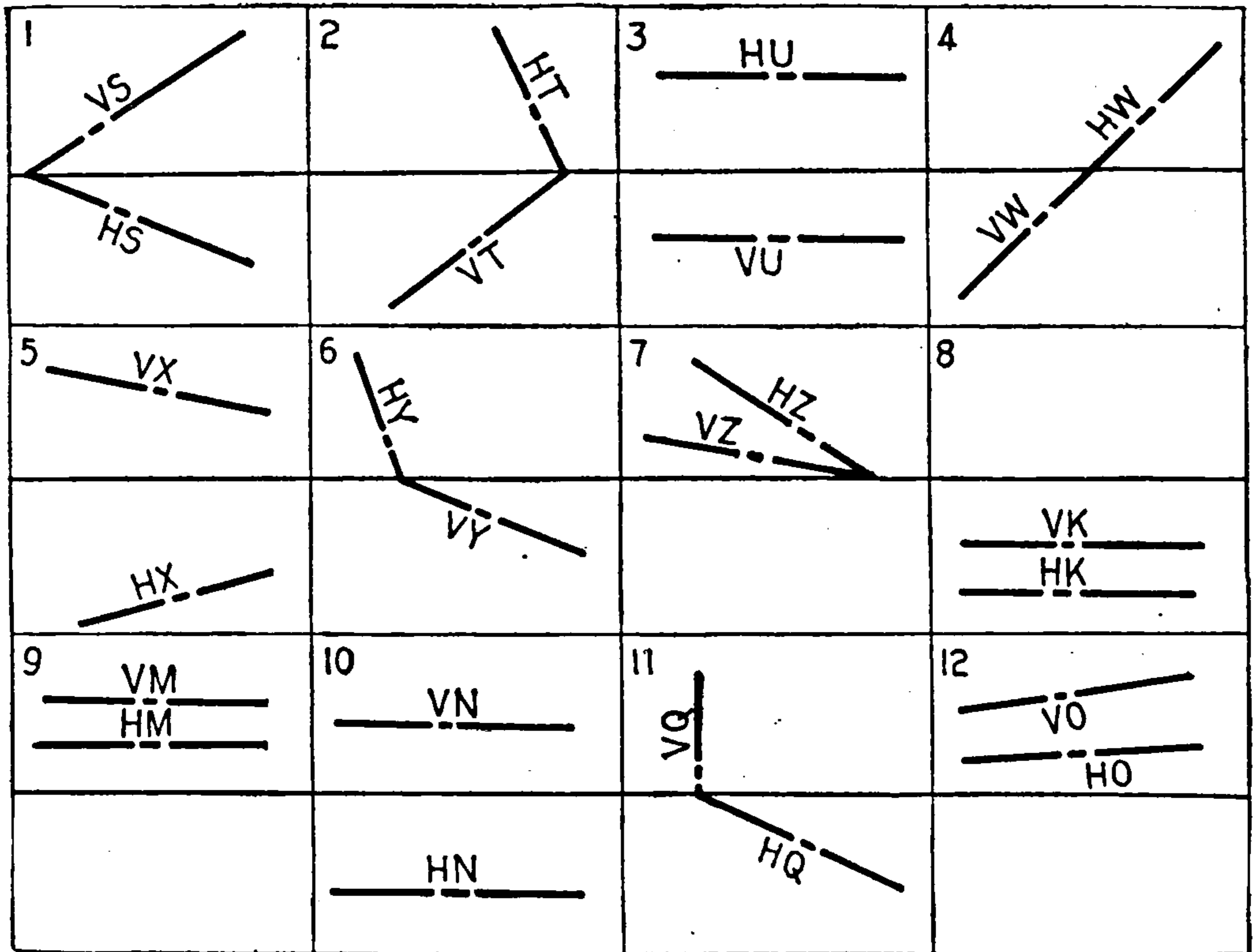
82.

EXERCISES

464. Shortest distance  $A(1 + \frac{1}{2} - 2)$  to  $T(1 + 2) 4(4 - 3)$ .
465. Shortest distance  $B(1 - 3 + 2)$  to  $S(4 - 2) 1(1 + 3)$ .
466. Shortest distance  $C(2 + 1 - 3)$  to  $R(5 + 2) 2(5 - 2)$ .
467. Shortest distance  $D(2 + \frac{1}{2} - \frac{1}{2})$  to  $Q(1 + 3) (5 + 2) (1 - 2\frac{1}{4}) (5 - 1\frac{1}{2})$ .
468. Shortest distance  $E(1 + \frac{1}{2} - 3)$  to  $U(1 + 2) 3(5 - 2)$ .
469. Shortest distance  $F(4 - 1 + 1)$  to  $W(5 + 3) 2(5 - 1)$ .
470. Shortest distance  $M(4 - 2 + 2)$  to  $X(1 + 3) 2(5 - 1)$ .
471. Shortest distance  $N(2 + 3 - 2)$  to  $Y(+ 1\frac{1}{2}) \infty(+ 1)$ .
472. Shortest distance  $O(3 - 1 + 1)$  to  $Z(+ 1 \infty(- 2))$ .
473. Shortest distance upper right-hand rear corner of a 3-in. cube with faces in  $H$  and  $V$ , to the plane of the three adjacent corners.
474. Scale  $1\frac{1}{2}$ -in. = 1 ft. An 8-in. circular flue runs from a point  $A(1$  ft.,  $0 - 8$  in.) in a floor perpendicular to a plane whose traces intersect



at a point 4 ft. to the right, the  $V$ -trace being inclined  $67\frac{1}{2}$  to  $GL$ , and the  $H$ -trace  $45^\circ$ . Draw the projections of the roof and floor openings for the flue.



Graphic Layout No. 33.—Assume a point in space, and measure its distance from plane —.

### PROJECTION OF LINES ON OBLIQUE PLANES

83. Problem 25.—To project a line on an oblique plane.

Construction.—Let  $AB$  (Fig. 60) be projected on  $T$ .

1. Let fall perpendiculars from  $A$  and  $B$  to  $T$ .
2. Obtain  $QT$  and  $a^q b^q$ .
3. Draw the projectors  $a^q m^q$  and  $b^q n^q$ , perpendicular to  $QT$ .
4. Project the points  $m^q$  and  $n^q$  back to the  $H$ - and  $V$ -projections of the perpendiculars drawn in operation (1).

Conclusion.— $MN$  is the projection of  $AB$  on  $T$ .

Check.—If  $AB$  intersects  $MN$  at the point in which  $AB$  pierces  $T$ , the operation is probably correct. Why?

84.

### EXERCISES

475. Project  $A(1\frac{1}{2} + 2 - 1)$   $B(3 + 1 - 2)$  on  $Q(5 + 3)$   $5(3 - 2)$ .

476. Project  $C(1 - 2 + 1)$   $D(2 - \frac{1}{2} + 2)$  on  $R(3 - 3)$   $4(4 + 3)$

477. Project  $E(2 + 1 - 1)$   $F(4 + 1 - 1)$  on  $S(1 + 3)$   $4(1 - 2)$ .



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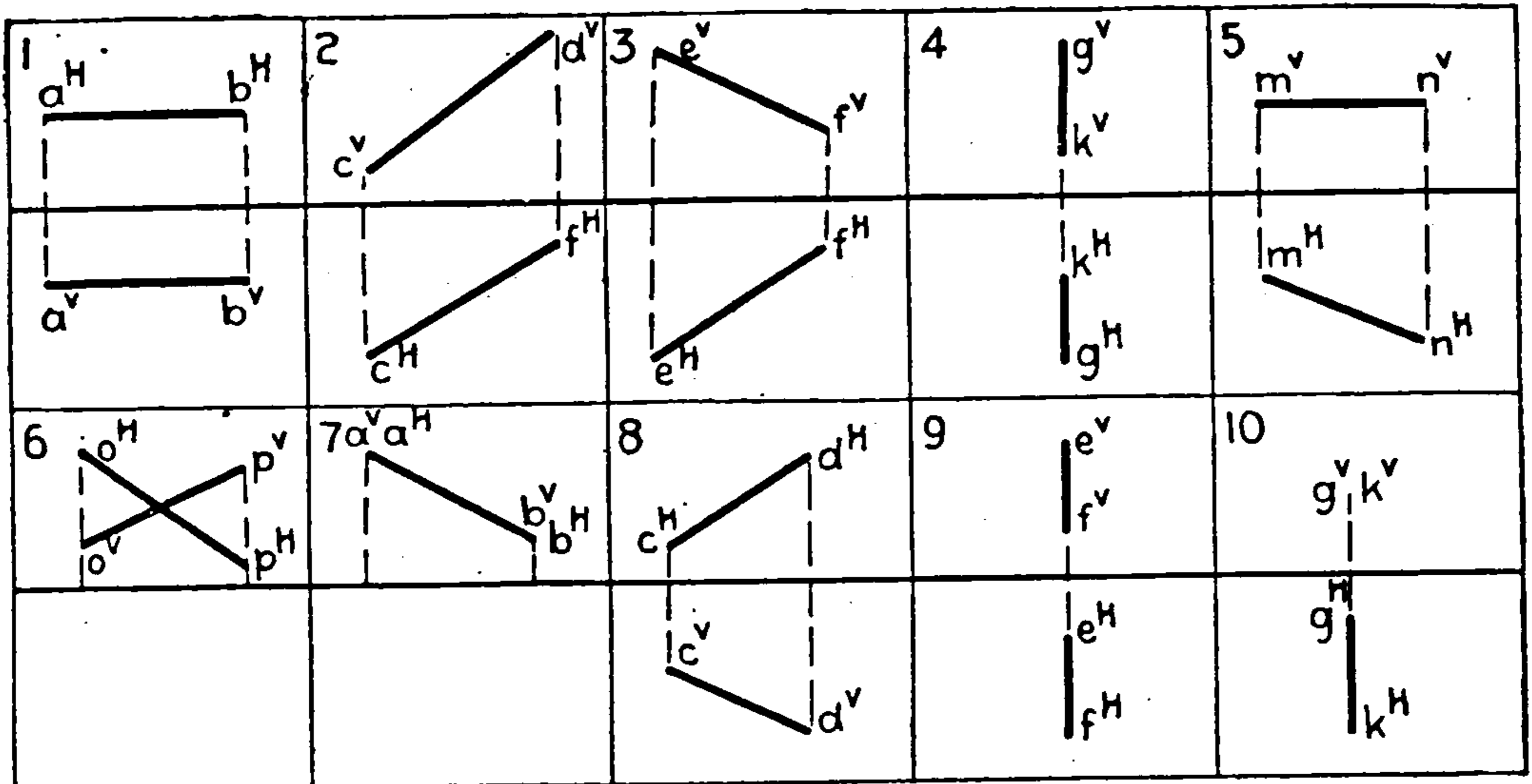
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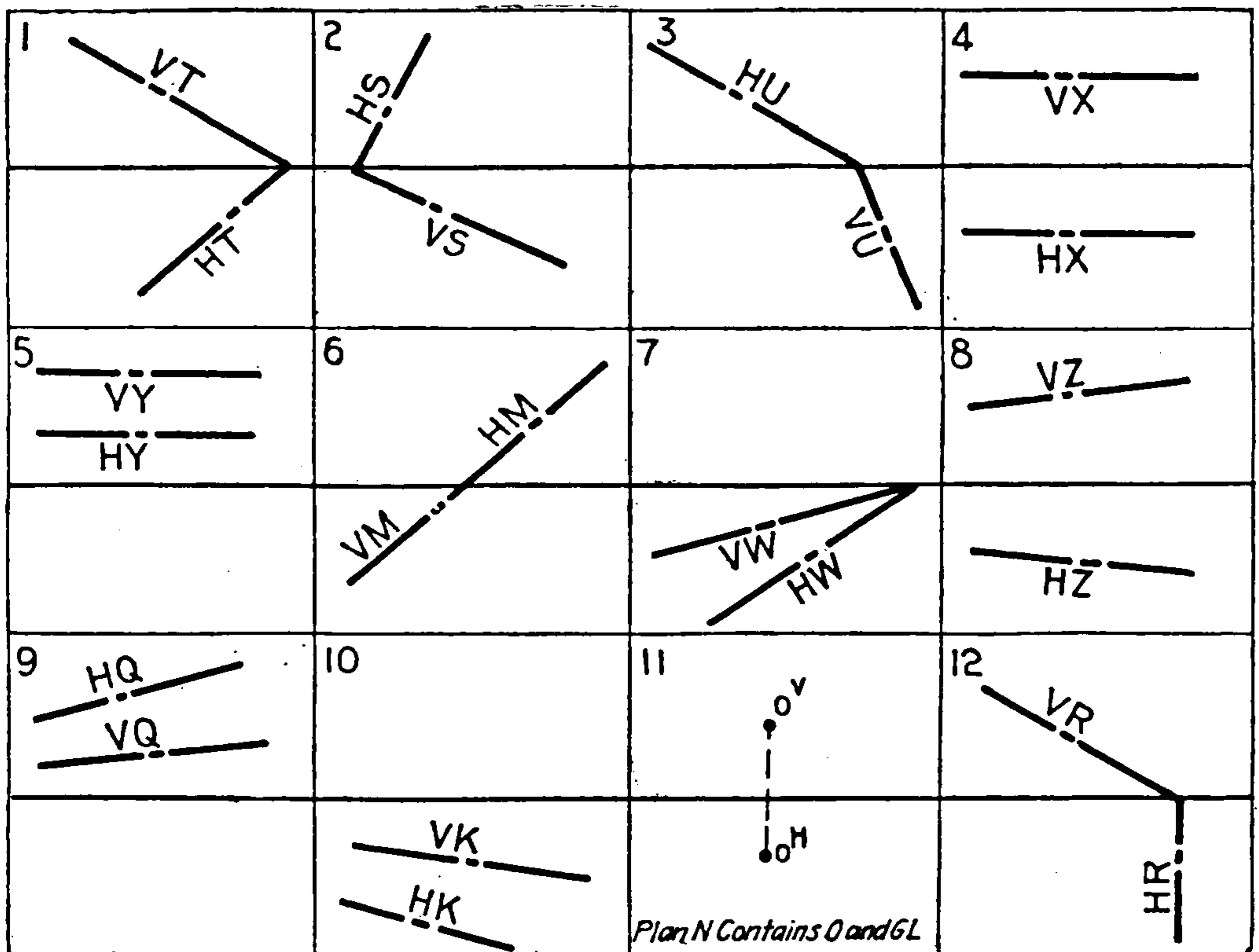
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Given Lines



Given Planes

Graphic Layout No. 35.—Project given line — on plane —.

ANGLES BETWEEN LINES AND PLANES

**85. Problem 26.**—To measure the angle between any line and any plane.

**First Method.**—The measure of the angle between a line and a plane is the angle between the line and its projection on that plane. Therefore one analysis suggests itself; as follows:

**Analysis.**—1. Project the line on the plane by Problem 25.

2. Measure the angle between the given line and its projection thus found, by Problem 13.

Let the student make the construction.

**85. Second Method.**—This method, while not so obvious is much quicker and better.

**Analysis.**—1. From any point in the given line erect a perpendicular to the given plane.

2. Measure the angle between the given line and this perpendicular.

3. Subtract this angle from  $90^\circ$ , which gives the required angle between the line and plane.

**Proof.**—By letting fall a perpendicular to the plane, a right-angled triangle is formed between the given line, its projection, and the perpendicular. The angles between the line and its projection, and between the line and the perpendicular are complementary, and add up to  $90^\circ$ . Hence this method might be called the “complementary method.”

**Construction.**—Let it be required to measure the angle between AB (Fig. 61) and the plane T.

1. From B draw the line  $b^v p^v - b^h p^h$  perpendicular to T.

2. Through the *H*-piercing points of AB and BP ( $o^h$  and  $p^h$ ) draw HQ.

**Note.**—If the *V*-piercing points are more convenient, they may be used in preference.

3. Revolve B into *H* about HQ to  $b_1$ .

4. Lay off a perpendicular  $b_1 m_1$  to  $p^h b_1$ .

**Conclusion.**—The required angle will be  $\theta$ , the complement of the angle between the line and the projector.

## 86.

## EXERCISES

Measure the angles that the following lines make with the planes given.

485.  $A(1 + 2 - 3)$   $B(3 + 1 - 1)$  and  $T(2 + 3)$   $2(5 - 2)$ .  
 486.  $C(2 + 3 - \frac{1}{2})$   $D(4 + 1 - 1)$  and  $S(5 + 3)$   $5(1 - 2)$ .  
 487.  $E(2 + 1 - 2)$   $F(4 + 1 - 2)$  and  $R(1 + 3)$   $4(1 - 2)$ .  
 488.  $G(2 + 1 - \frac{1}{2})$   $K(2 + 2 - 3)$  and  $R$ (Ex. 487).  
 489.  $L(4 + 3 - \frac{1}{2})$   $M(1 + 1 - 2)$  and  $R$ (Ex. 487).  
 490.  $N(2 - 1\frac{1}{2} + 1)$   $P(4 - 1\frac{1}{2} + 1)$  and  $Q(5 - 2)$   $2(5 - 1\frac{1}{2})$ .  
 491.  $O(1 + 2 - 1)$   $K(3 + 1 - 1)$  and  $Q$ (Ex. 490).  
 492.  $A(2 + 2 - 1)$   $B(1 + \frac{1}{2} - 3)$  and  $U(+ 2)$   $\infty(- 1)$ .  
 493.  $C(2 + 3 - 2)$   $D(2 + 1, 0)$  and  $U$ (Ex. 492).

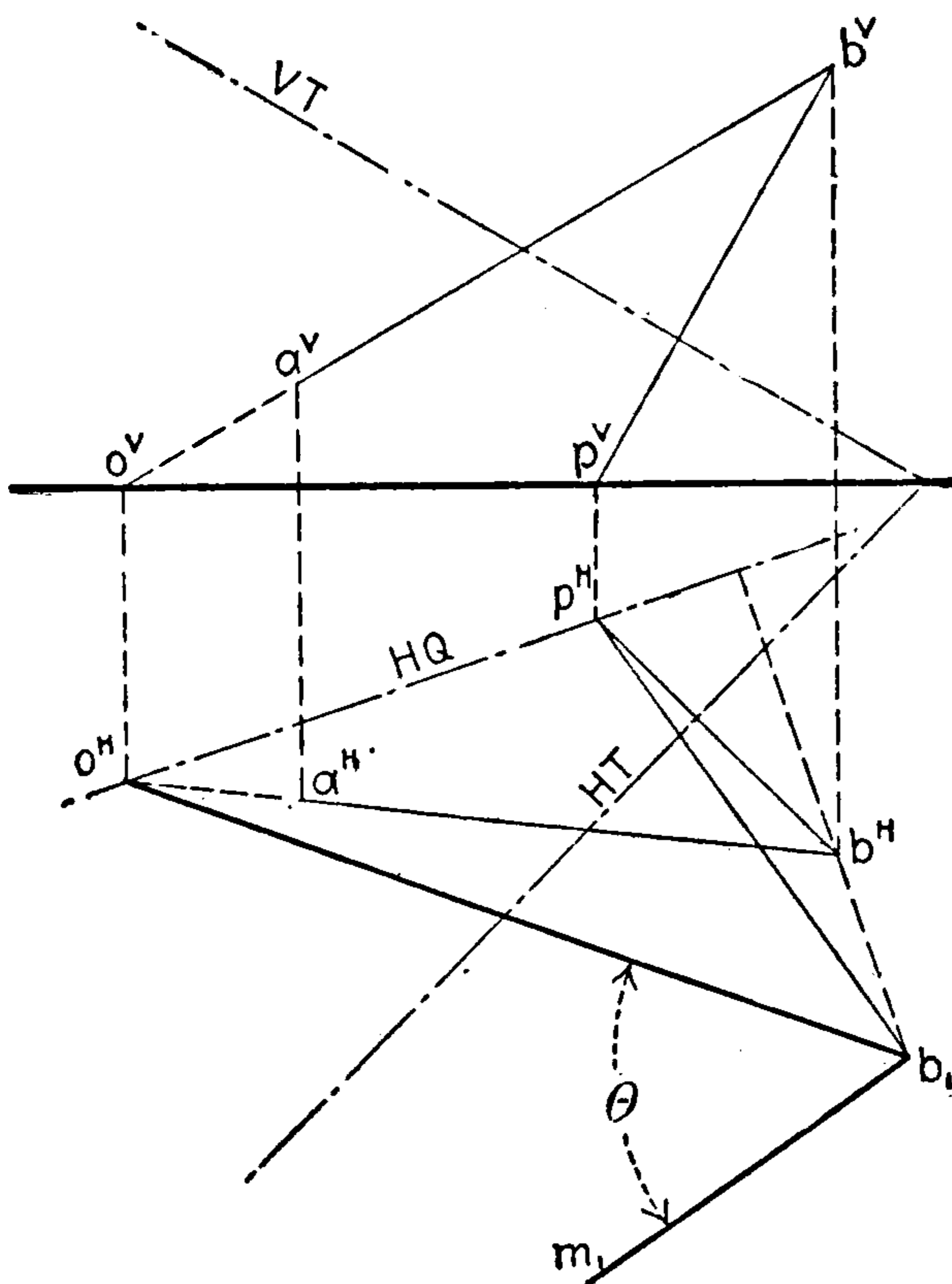


FIG. 61.

494.  $E(1 - 3 + \frac{1}{2})$   $F(2 - 2 + 2)$  and  $W(+ 1)$   $\infty(- 2)$ .  
 495.  $G(3 + 1 - 1)$   $K(4 + \frac{1}{2} - 1\frac{1}{2})$  and  $X(1 + 1)$   $3(5 - 1)$ .  
 496.  $L(1 + 2 - \frac{1}{2})$   $E(4 + 2 - \frac{1}{2})$  and  $X$ (Ex. 495).  
 497.  $M(2 - 1 - 2)$   $N(3 - 2 - \frac{1}{2})$  and  $Y(1 + 3)$   $5(1 + 1)$ .  
 498.  $O(2 + 2 - \frac{1}{2})$   $P(2 + \frac{1}{2} - 3)$  and  $Y$ (Ex. 497).  
 499.  $A(1 + \frac{1}{2} - \frac{1}{2})$   $B(2 + 3 - 3)$  and  $Z(1 + 3)$   $(5 + 2)$   $(1 - 2\frac{1}{4})$   
 $(1 - 1\frac{1}{2})$ .  
 500.  $C(2 + 2 - 2)$   $D(2 + 1 - \frac{1}{2})$  and  $Z$ (Ex. 499).  
 501.  $E(2 + 1 - 3)$   $F(3 + 3 - 2)$  and  $Q(1 + 3)$   $2(5 - 1)$ .  
 502.  $A(1 + 2 - 2\frac{1}{2})$   $B(2 + 1 - 1)$  with  $H$ ,  $V$ , and  $P$ .  
 503. Any of the foregoing lines with  $H$ ,  $V$ , and  $P$ .



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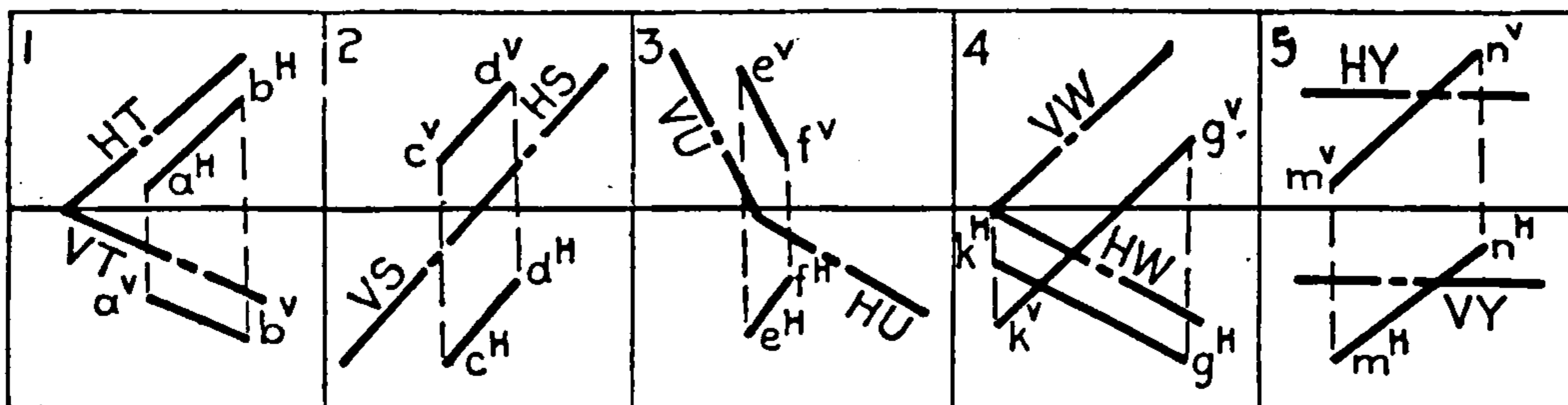
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509. Draw a line from  $M(4 + 3 - 3)$  making an angle of  $60^\circ$  with  $T(1 + 2)$   $5(1 - 3)$  intersecting the line  $A(3 + 2, 0)$   $B(1, 0 - 3)$ . What angle does it make with  $AB$ ?
510. Draw the projections of the locus of all lines from  $O(3 + 2 - 3)$  making  $67\frac{1}{2}^\circ$  with  $S(5 + 2)$   $2(5 - 1\frac{1}{2})$ .
511. Draw a line from  $N(2 + 2 - 2)$  making  $45^\circ$  with  $R(+ 2) \infty (- 1)$  intersecting  $C(2 + 2, 0)$   $D(4, 0 - 1)$ .
512. Draw a line from  $P(2 + 3 - 2)$  at  $60^\circ$  from  $U(1 + 1\frac{1}{2})$   $3(1 - 1\frac{1}{2})$ , intersecting  $E(1 + 1\frac{1}{2}, 0)$   $F(1, 0 - 1\frac{1}{2})$ .



Graphic Layout No. 37.—Measure the angle between the line and plane in Ex. —.

513. Draw the projections of the locus of all lines through  $M(3 + 2 - 1)$  making  $60^\circ$  with  $W(- 1) \infty (+ 2)$ .
514. Draw a line from  $P(3, 0, 0)$  making  $45^\circ$  with  $T(1 + 2)$   $(5 + 1)$   $(1 - 3)$   $(5 - 1\frac{1}{2})$ , intersecting  $Q(2 + 1\frac{3}{4}, 0)$   $K(3, 0 - 2\frac{1}{4})$ .
515. Draw the locus of all lines through  $N(3 + 2, 0)$  making  $67\frac{1}{2}^\circ$  with the plane through  $GL$  and  $F(2 + 1 - 2)$ .

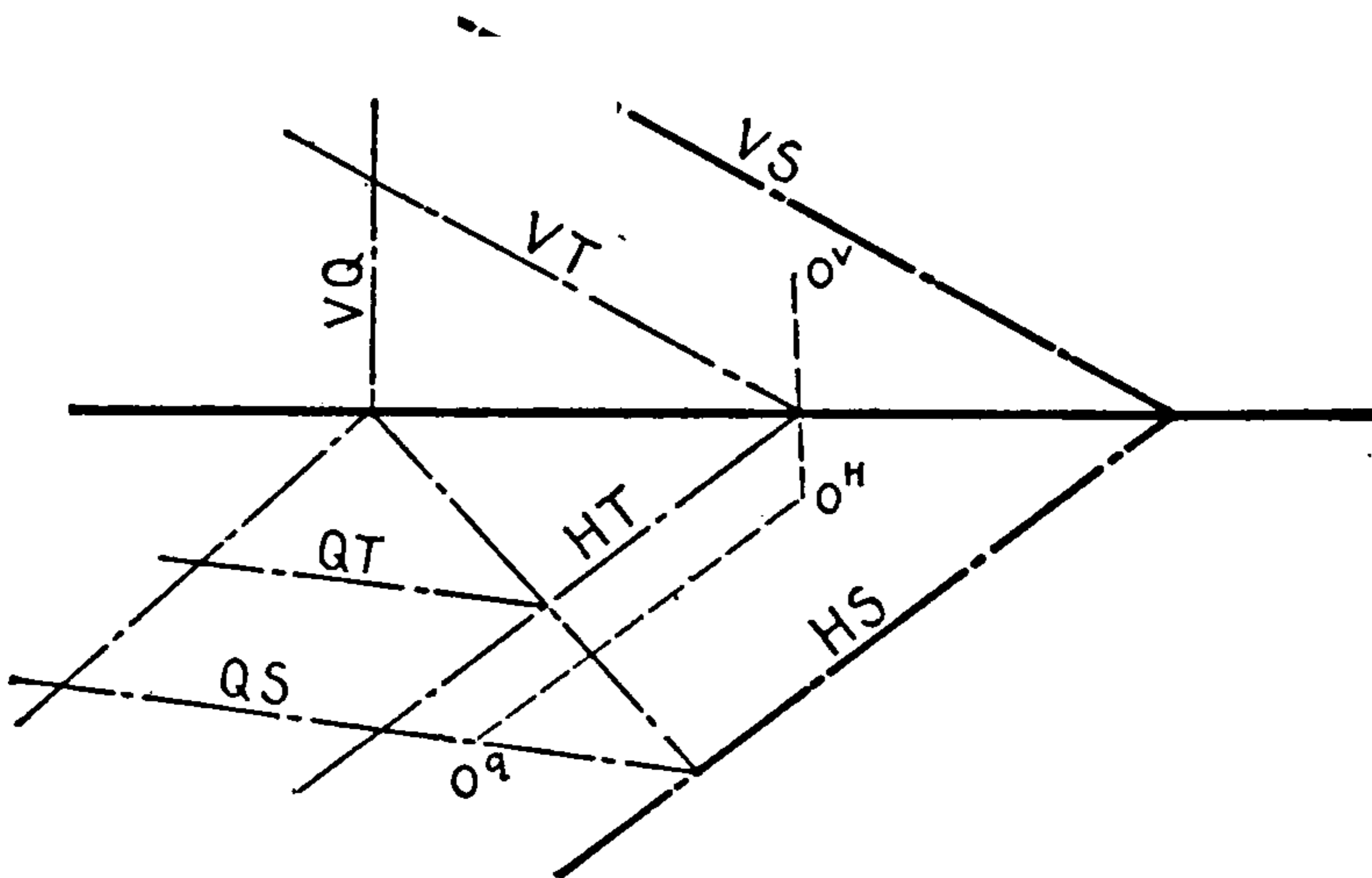


FIG. 62.

PARALLEL PLANE PROBLEMS

87. Problem 27.—To pass a plane through a given point parallel to a given plane.

Principle.—Corresponding traces of parallel planes are parallel.

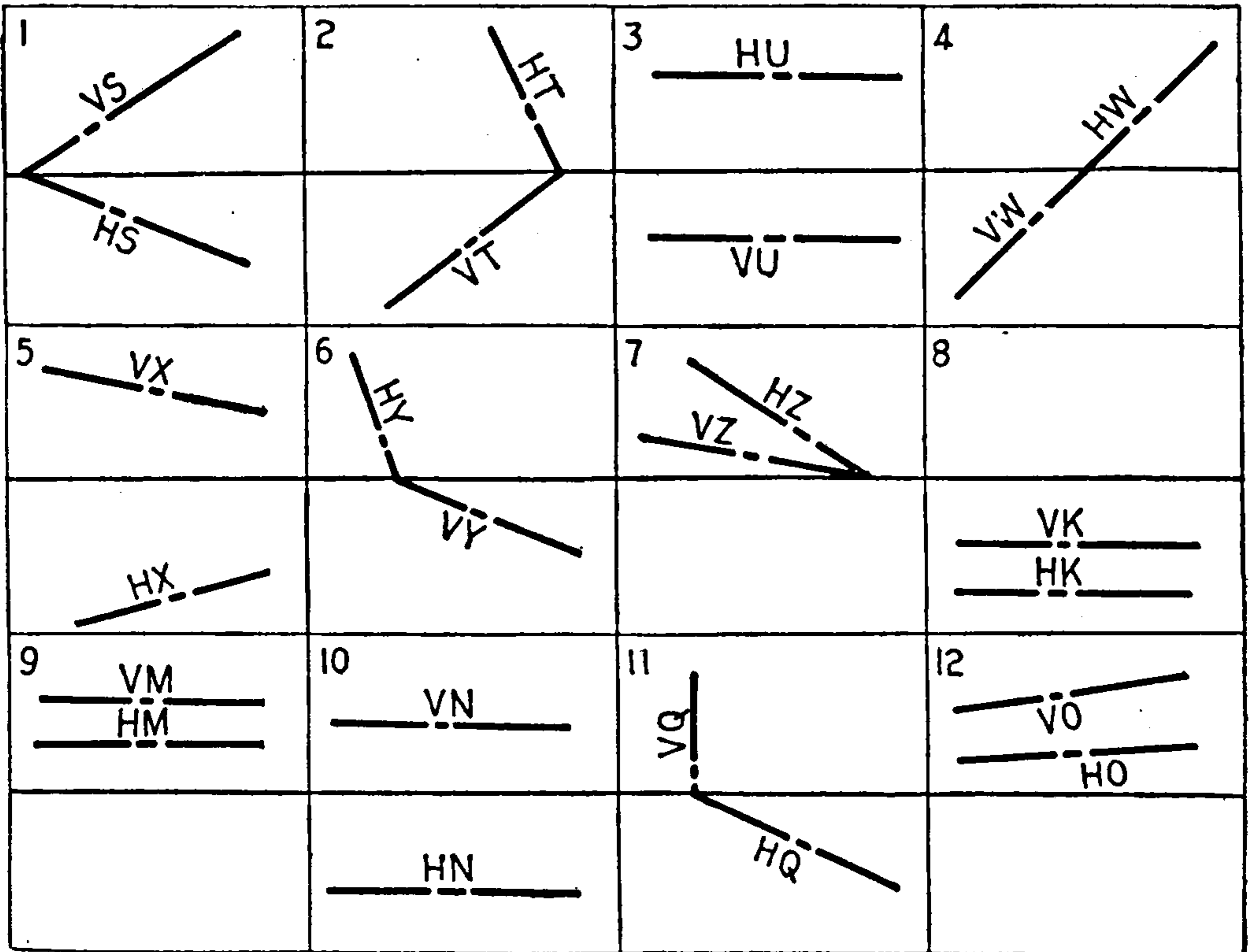
Construction.—Let  $O$  (Fig. 62) be the given point. Required to pass a plane through  $O$  parallel to  $T$ .

1. Locate  $QT$ .
2. Locate  $o^q$ .
3. Draw  $QS$  through  $o^q$  parallel to  $QT$ .
4. Where  $QS$  intersects  $HQ$ , draw  $HS$  parallel to  $HT$ . Then draw  $VS$  parallel to  $VT$ .

Conclusions.—The plane  $S$  contains  $O$  and is parallel to  $T$ .

88.

EXERCISES



Graphic Layout No. 38.—Assume a point in space, and pass a plane through it  $\parallel$  to plane ——. What is the distance between the planes?

516. Through  $A(1 + 2 - \frac{1}{2})$  pass a plane  $\parallel$  to  $R(3 + 3) 3(5 - 3)$ . What is the distance between planes?
517. Through  $B(3 + 2 - 2)$  pass a plane  $\parallel$  to  $Q(1 + 1) 3(1 - 3)$ . What is the distance between planes?
518. Through  $C(1 - 1 + 1\frac{1}{2})$  pass a plane  $\parallel$  to  $S(5 + 2) 1(5 - 1\frac{1}{2})$ . What is the distance between planes?
519. Through  $D(2 + 1 - \frac{1}{2})$  pass a plane  $\parallel$  to  $T(+ 3) \infty(- 2)$ . What is the distance between planes?
520. Through  $E(1 + 3 - 1)$  pass a plane  $\parallel$  to the plane containing  $GL$  and  $P(3 + 1 - 2)$ . What is the distance between planes?







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of intersection, and therefore to both of the given planes. Two methods of finding this angle are given herewith.

**First Method.—Analysis.**—1. Find the intersection of the given planes.

2. At any assumed point on the line of intersection pass a plane perpendicular to it.

**Note.**—Only one trace of this auxiliary plane is usually necessary.

3. Measure the angle between the intersections of the auxiliary plane with the two given planes.

**Construction.**—Let S and T (Fig. 64), be the two planes.

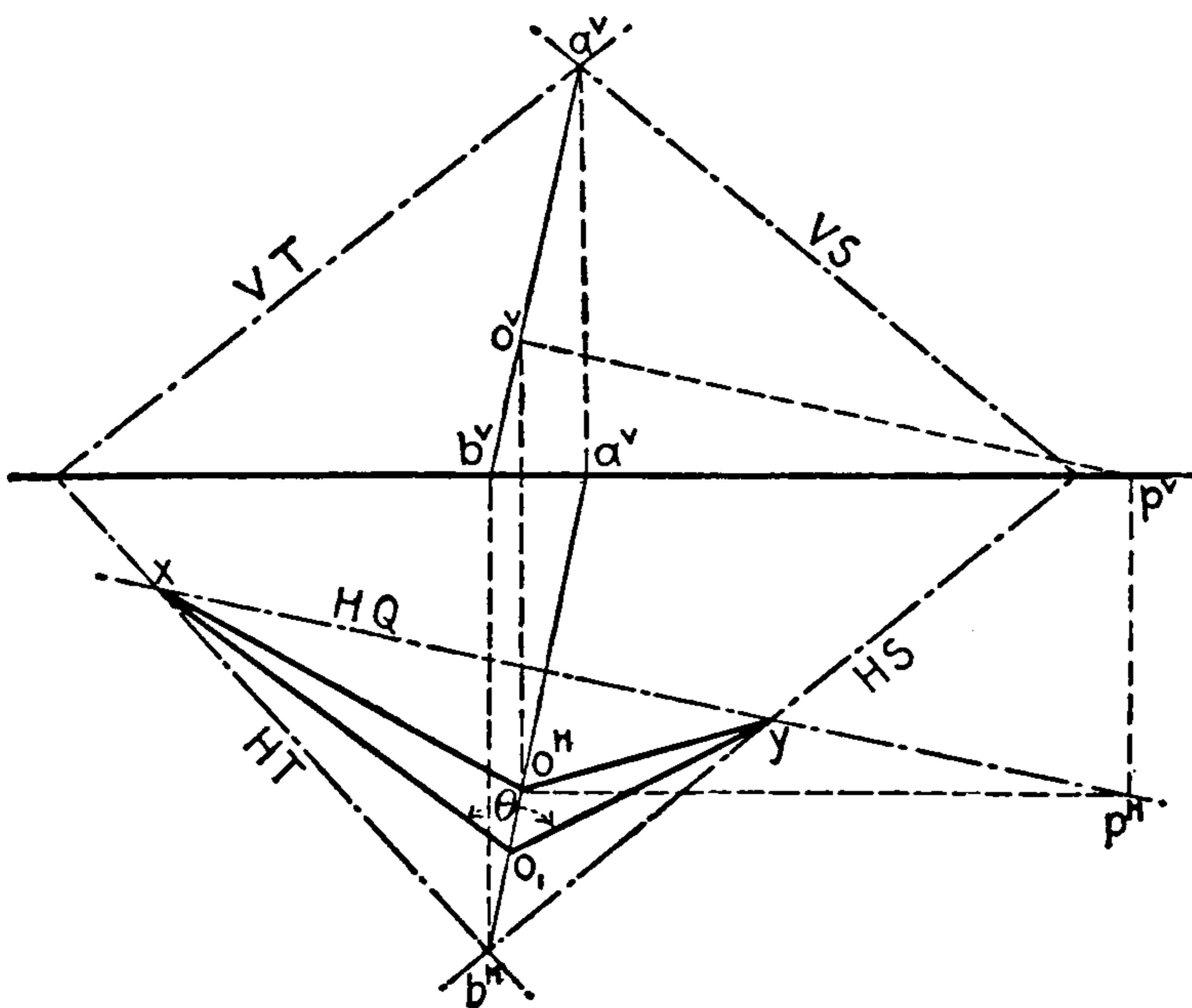


FIG. 64.

1. Obtain their line of intersection, AB.
2. Through O, assumed on AB, draw OP, perpendicular to AB and parallel to V.
3. Obtain P, the *H*-piercing point of OP.
4. Draw HQ through  $p^H$ , perpendicular to  $a^H b^H$ .
5. Connect  $o^H$  with x and y (intersections of HQ with HT and HS).
6. Revolve O to  $o_1$ .

**Conclusion.**— $XO_1Y(\theta)$  is the required angle.

**Second Method.—Analysis.**—1. From any point let fall two perpendiculars, one to each of the two given planes.

2. Measure the angle included between these perpendiculars.

This will be the correct measure of the required dihedral angle.

**Proof.**—The two perpendiculars from the point to the planes determine a plane perpendicular to both planes. Fig. 65 shows a view of two intersecting planes S and T. O is taken anywhere, and perpendiculars OM and ON are let fall to T and S respectively. The plane of MON cuts lines MP and NP from T and S, and the four lines, OM, ON, MP and NP form a quadrilateral, the sum of whose angles is  $360^\circ$ . Prove this. The angles at M and N are right angles by construction, and therefore add up to  $180^\circ$ . This leaves  $180^\circ$  as the sum of angles  $\phi$  and  $\theta$ , at O and P respectively. The angle  $\phi$  is therefore the supplement of  $\theta$ , which is the angle between T and S. But the measure of any angle, plane or dihedral, may be either the acute angle or its supplement, so the angle between the planes S and T may be either  $\theta$  or  $\phi$ ,

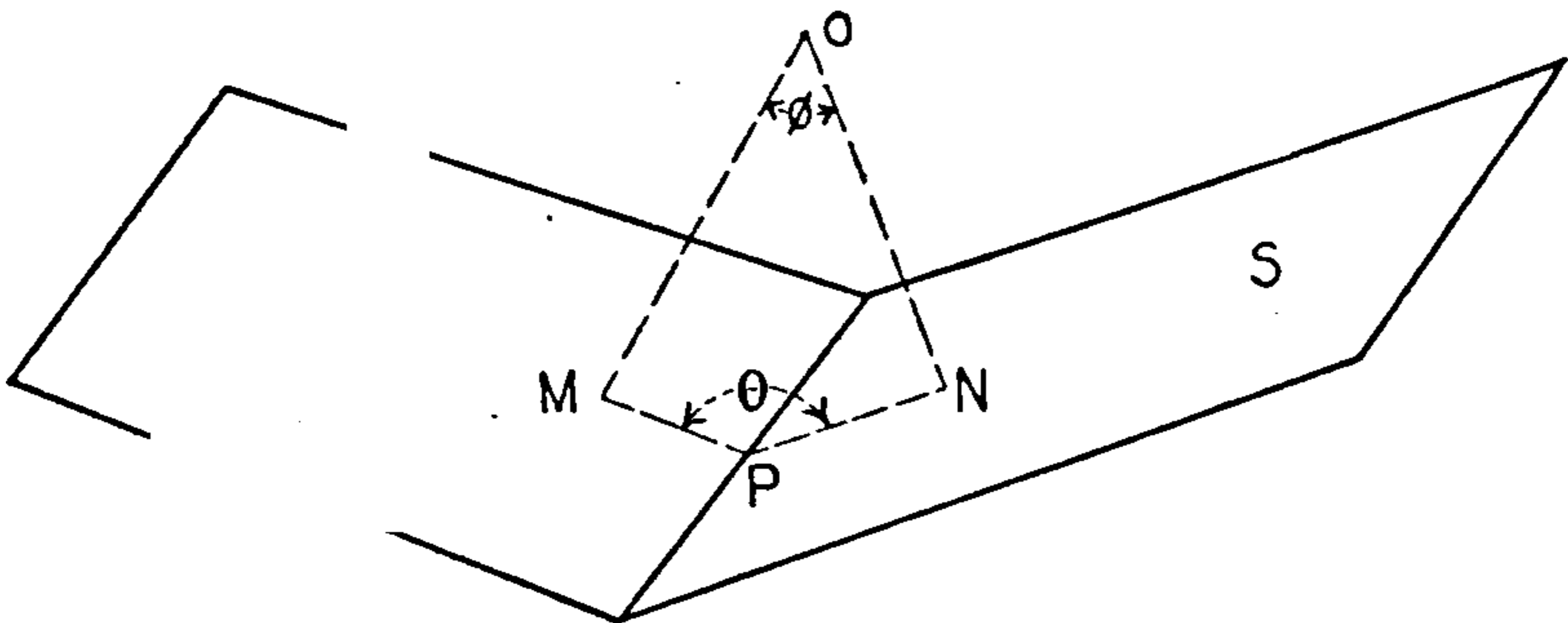


FIG. 65.

and as  $\phi$  is the angle between the perpendiculars, it is also the measure of the given dihedral angle.

**Note.**—Although the angle between two lines, or two planes, may be either the acute or the obtuse angle, as stated in the foregoing, the acute angle is usually taken as the true measure.

92.

EXERCISES

Dimensions of planes for following exercises:

S(5 + 2) 1(5 - 3).

T(+ 2)  $\infty$  (- 1).

R(- 1)  $\infty$  (- 1½).

Q(5 + 3) 5(5 - 2).

X(1 + 2) 3(5 - 2).

W(5 - 2) 4(1 - 1).

Y(1 - 3) 4(1 - 1).

Z(5 - 3) (1 - 2) (5 + 1½) (1 + 1).

P(3 + 3) 3(3 - 3).

M(1 - 3) (5 - 1½) (1 - 2) (5 - 1).

U containing A(2 + 1 - 1½) and GL.

537. Measure the dihedral angle of the planes — and —.

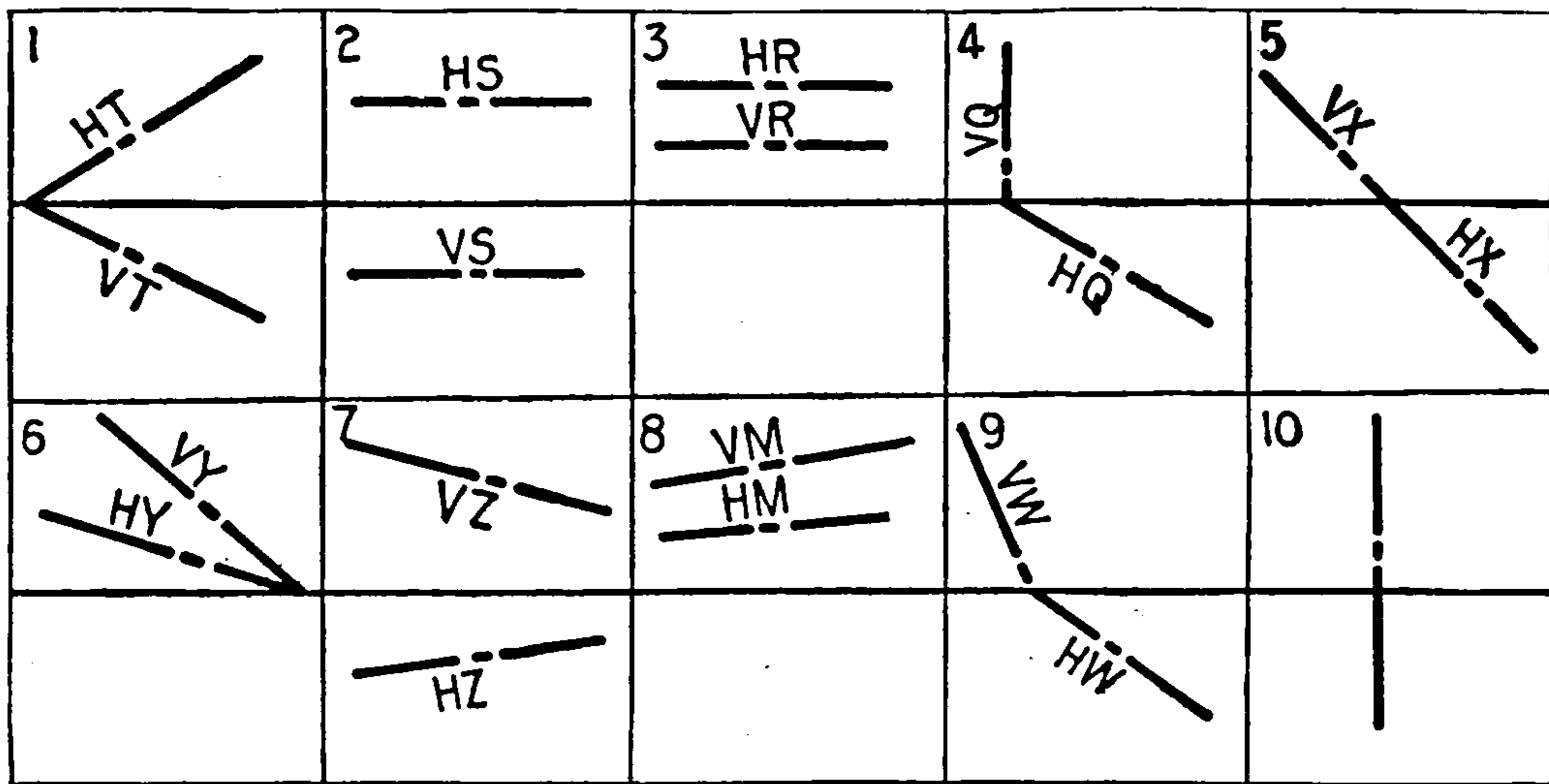
538. Measure the angles that the plane — makes with  $H$ ,  $V$ , and  $P$ .

539. Measure the hip roof angle between the roofs  $C$  and  $D$  in Fig. 34, Art. 45.

540. A room is 18 ft. long, 10 ft. wide, and 10 ft. high. The center line of a chute runs from a point in the ceiling 2 ft. from the right end wall and  $1\frac{1}{2}$  ft. from the rear, to a point in the floor, 5 ft. from the left end wall and 2 ft. from the front. The chute is 2 ft. square, with the diagonal parallel to the floor.

(a) Find the floor opening for the chute.

(b) Find the angles of its faces with the floor. Scale 1 in. = 3 ft.



Graphic Layout No. 40.—Exercises 537 and 538 apply to either graphic or dimensioned layout.

541. A hexagonal roof tower, 12-ft. base diameter, 16-ft. altitude, has specially rolled angle irons for its frame. What is the angle at which they must be rolled?

**93. Problem 30.**—To find the traces of a plane that shall make a given angle with a given plane and contain a given line of that plane.

Analysis.—Assume a point in the given line, and pass a plane through the point perpendicular to the line.

2. Revolve the line cut by this auxiliary plane from the given plane about the trace of the auxiliary plane into  $H$  or  $V$ , as the case may require.

3. From the revolved position of the assumed point on the given line lay off a line, making the given angle with the revolved line.

4. Where this new line crosses the trace of the auxiliary plane will be a point on the trace of the required plane.

5. Since the required plane must contain the given line, its



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**Conclusion.**—This plane will be perpendicular to the given plane. Why?

Let the student make the construction.

**Note.**—If the given line is perpendicular to the given plane, the number of solutions is infinite.

### EXERCISES

1 	2 	3 	4 	5 
6 	7 	8 	9 	10 
11 	12 	13 	14 	15 

**Graphic Layout No. 41.**—(a) Plane ———. Draw traces of a plane under the conditions of Ex. 542.

(b) Plane ———. Draw traces of a plane under the conditions of Ex. 543.

(c) Plane ———. Draw traces of a plane under the conditions of Ex. 544.

**94.** Use the following planes in Ex. 542–544, and 546.

$$M(1 + 2) 5(1 - 3).$$

$$N(5 - 1) 2(5 + 2\frac{1}{2}).$$

$$O(1 + 3) (5 + 1\frac{1}{2}) (1 + 2) (5 + 1).$$

$$P(3 + 3) 3(3 - 3).$$

$$Q(2 - 3) 2(5 + 2).$$

$$R(1 + 1) 4\frac{1}{2}(4\frac{1}{2} + 3).$$

$$S(5 + 2) 3(1 - 2).$$

$$T(+ 2) \infty(- 1).$$

$$U(+ 1\frac{1}{2}) \infty(+ 2).$$

$$W(+ 2) \infty(+ 1).$$

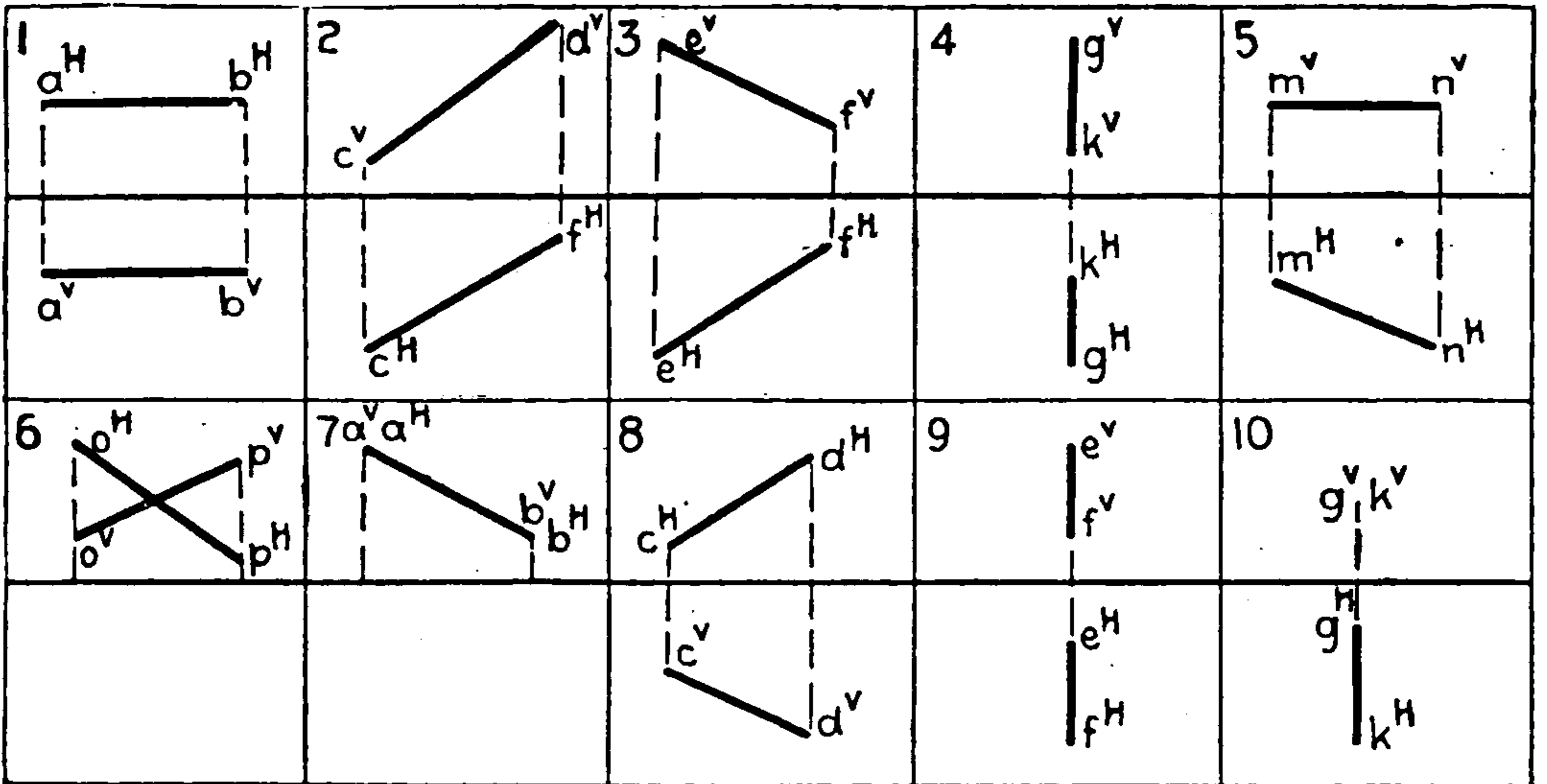
$$X(+ 1\frac{1}{2}) \infty(- 2\frac{1}{2}).$$

$$Y(1 + 2) (5 + 1) (1 - 2\frac{1}{2}) (5 - 1\frac{1}{4}).$$

$Z(5 + 1) 1(5 + 2\frac{1}{2}).$

$K(1 + 3) 2(5 - 1).$

542. Assume a line 1 in. from and parallel to  $\overset{M}{\text{---}}$  ( $H$  or  $V$ ) in the plane  $\overset{N}{\text{---}}$ . Draw the traces of plane that shall contain this line and make a dihedral angle of  $\text{---}^\circ$  with the given plane.
543. Plane  $\text{---}$ . Assume a line running obliquely to  $H$ ,  $V$ , and  $P$  in the plane, and draw the traces of a plane, which contains the line, and makes  $\text{---}^\circ$  with the given plane.
544. Plane  $\text{---}$ . Assume a profile line in the plane, and draw the traces of a plane, which contains the line, and makes  $\text{---}^\circ$  with the given plane.



Graphic Layout No. 42.—Draw the traces of two planes containing line  $\text{---}$ , and making a dihedral angle of  $\text{---}^\circ$  (15, 30, 45, 60, 75, 90).

545. The dihedral angle between two adjacent faces of a right pyramid, base a 2-in. equilateral triangle, is  $\text{---}^\circ$  (95, 100, 105, 110, 120, 135, 150). Draw the projections of the pyramid.
546. Assume a line in space, and pass a plane through the line, perpendicular to given plane  $\text{---}$ . (Use either dimensioned or graphic layout).

PROBLEMS UTILIZING THE RIGHT CIRCULAR CONE

95. Properties of the Right Circular Cone.

Three properties of the cone are particularly useful in the solution of certain problems.

1. The elements of the cone all make the same angle with the plane of the base, and they are all of the same length, because one way of generating a cone is to revolve a right-angled triangle about one of its perpendicular sides.

2. All planes tangent to a cone make the same angle with the plane of the base, because a tangent plane is perpendicular to the generating triangle of the cone, and contains its hypotenuse.



3. Every element of the cone and every plane tangent to the cone pass through the apex.

With these properties understood, it is a simple matter to solve the six following problems:

**96. Problem 31.**—To draw a line through a point so as to make given angles with  $H$  and  $V$ .

**Limits.**—The sum of these angles cannot be more than  $90^\circ$ . If a line be parallel to  $GL$ , the sum is  $0^\circ$ , and if it lie in a profile

plane, the sum is  $90^\circ$ . For any other position, the sum will be less than  $90^\circ$ .

**Construction.**—Let it be required to draw a line through  $A$ , Fig. 66, that shall make  $45^\circ$  with  $H$ , and  $30^\circ$  with  $V$ .

1. Make  $A$  the apex of a right circular cone, whose base is in  $H$ , and whose elements make  $45^\circ$  with  $H$ .

All the elements make  $45^\circ$  with  $H$ , and all pass through  $A$ . Therefore, it remains to be found which are the four possible elements that make  $30^\circ$  with  $V$ .

2. From  $A$  draw a line

$a^v b^v$ ,  $a^h b^h$  parallel to  $H$ , making  $30^\circ$  with  $V$ , and equal in length to the elements of the cone.

3. Let this line be an element of a second cone with its apex at  $A$ , and base parallel to  $V$ . Draw the projections of the cone (whole or in part).

4. Where the bases of the cones intersect ( $B$  or  $E$ ), the common element can be drawn.

**Conclusion.**—The line  $AB$  (as well as the lines  $AC$ ,  $AD$ , and  $AE$ ) is inclined  $45^\circ$  to  $H$ , and  $30^\circ$  to  $V$ .

Let the student give the reason for this and supply a proof and a means of checking its correctness.

97.

### EXERCISES

547. Through  $M(3 + 1\frac{1}{2} - 2)$  draw four lines, each making  $60^\circ$  with  $H$  and  $15^\circ$  with  $V$ .

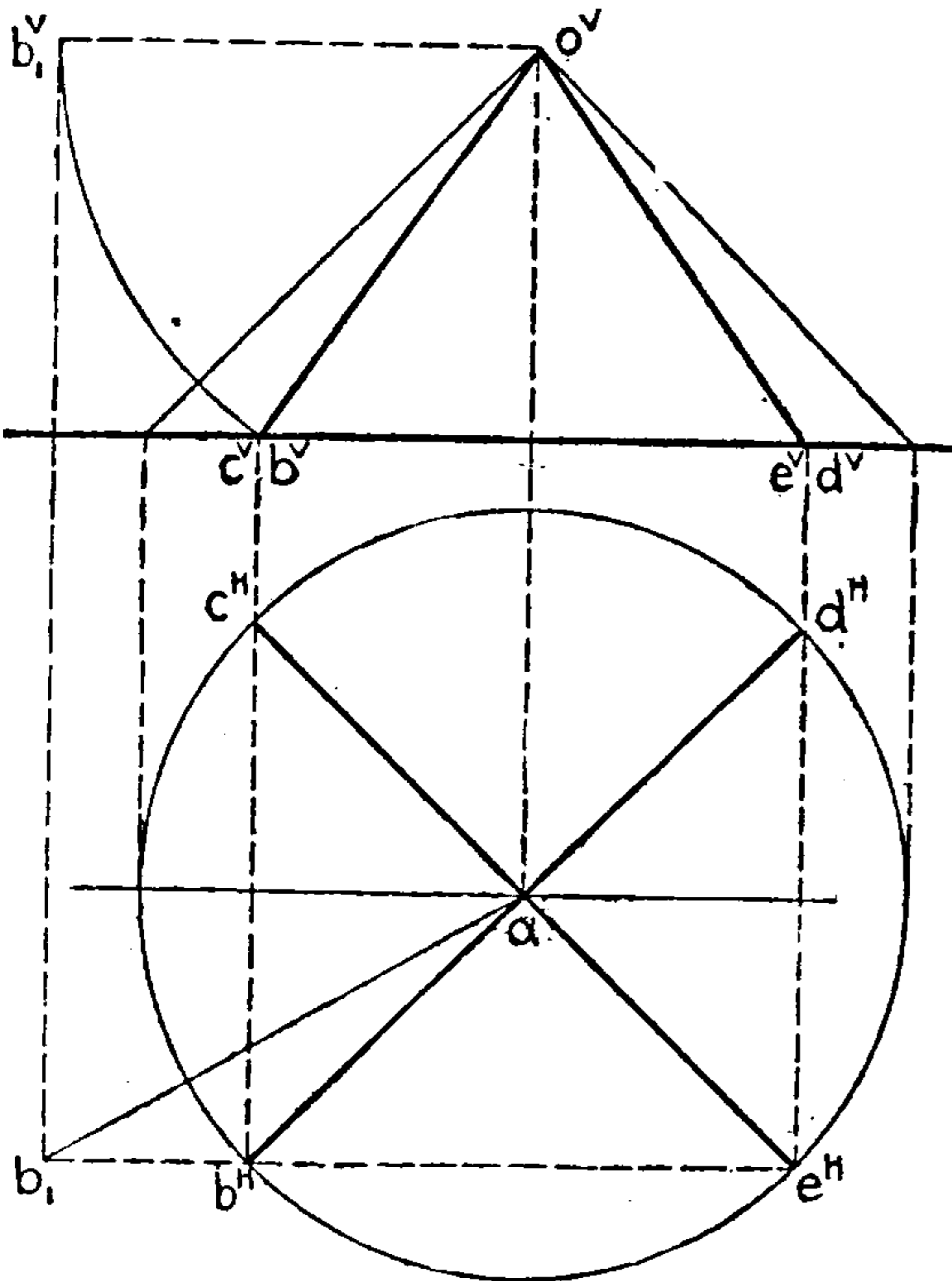


FIG. 66.



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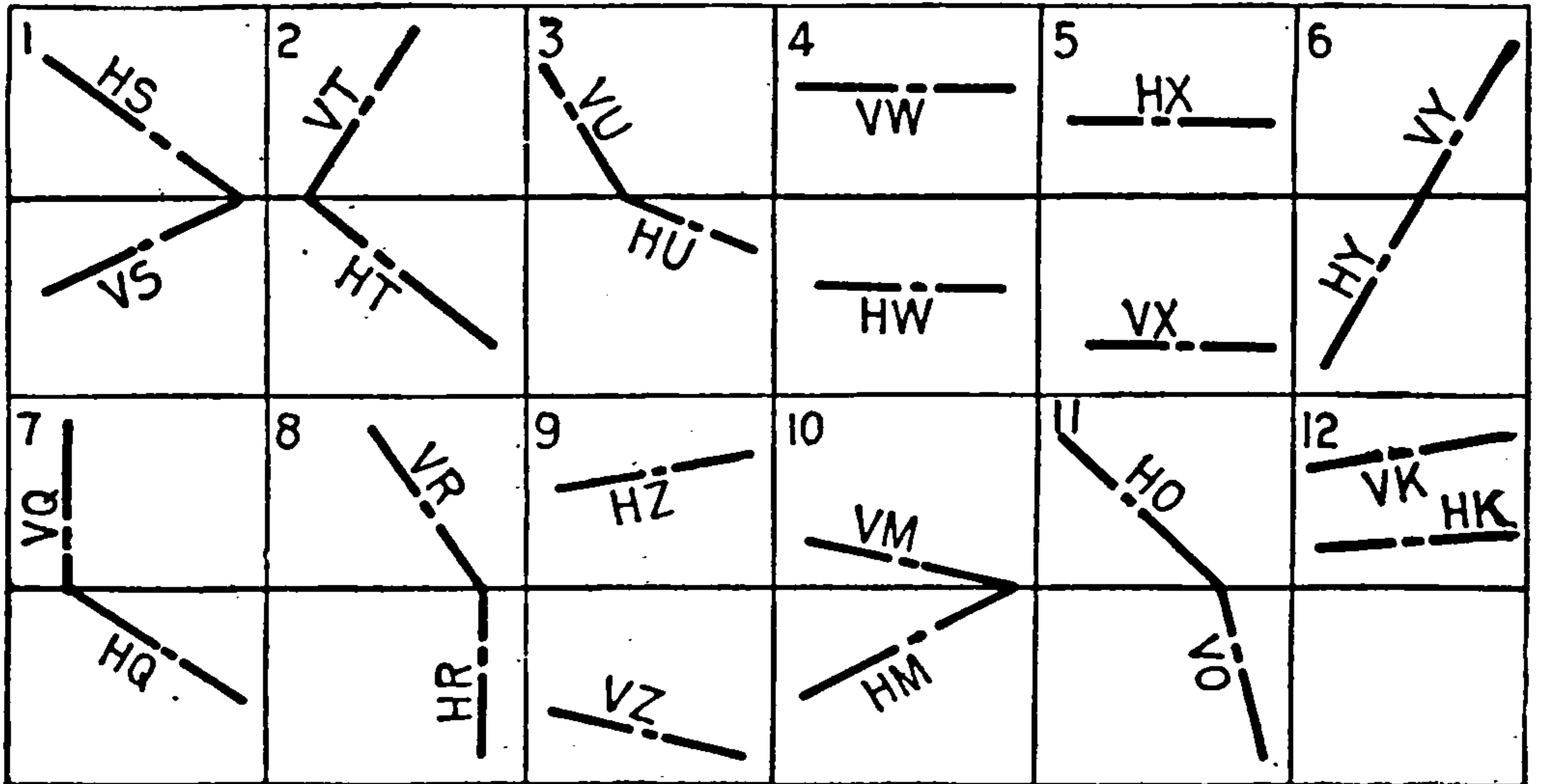
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562.  $K(1 + 3) 2(5 - 1)$ .

563.  $N$  through  $GL$  and  $E(2 + 2 - \frac{1}{2})$ .

564. A tetrahedron is limited by the points  $A(2, 0 - 1)$ ,  $B(3\frac{1}{2}, 0 - 1\frac{1}{2})$ ,  $C(4, 0 - 3)$ , and  $D(3 + 1\frac{1}{2} - 2\frac{1}{2})$ . Measure the dihedral angle between face — and face —.

565. Tetrahedron (Ex. 564). Measure the angles of the three oblique faces with  $H$  and  $V$ .



Graphic Layout No. 42.—Measure the angles between given plane — and  $H$  and  $V$ .

**100. Supplementary Problem 33(a).**—Given one trace of a plane and the angle between the plane and the plane of projection, to find the other trace.

Analysis.—1. Construct a cone, whose elements make the given angle with its base, tangent to the given trace, with the center of its base in  $GL$ .

2. Draw the other trace through the apex of the cone and the intersection of the given trace with  $GL$ .

Notes.—1. If the given trace does not intersect  $GL$  within bounds, two cones can be constructed, and the trace drawn through the two vertices.

2. If the given trace is parallel to  $GL$ , the other trace will also be parallel, and the quickest solution is effected through the profile plane.

3. By inverting the cone, another plane could be found that would fulfil the conditions.

**101. Supplementary Problem 33(b).**—Given the  $H$ -trace of a plane and its angle with  $V$ , to find the  $V$ -trace.

Analysis.—1. From any point in the given trace as an apex, construct a cone of the required elemental angle, having its base in  $V$ .

2. From the intersection of the given trace with  $GL$  draw the  $V$ -trace tangent to the base of the auxiliary cone.

Note.—See notes following 33(a).

102.

EXERCISES

1 	2 	3 	4 
5 	6 	7 	8 

Graphic Layout No. 43.— $H$ - or  $V$ -traces of planes to be used with Ex. 566–579.

566. Given  $H$ -traces:  $S_5(1 - 2)$ ,  $T_4(2 - 3)$ ,  $U_\infty(-1)$ ,  $W(1 - 3) (5 - 2)$ ,  $X_5(1 + 1\frac{1}{2})$ ,  $Y_\infty(+1\frac{1}{2})$ ,  $Z(1 + 2) (5 + 1)$ .

Draw the  $V$ -trace of plane —, when the plane makes —° with  $H$ .

567. Draw the  $V$ -trace of plane — (Ex. 566), when the plane makes —° with  $V$ .

568. The  $H$ -trace — (Ex. 566) is the  $H$ -trace of two planes, each inclined to  $H$  at —°. Draw their  $V$ -traces. Measure the angle between the two planes.

569. Draw the traces of the bisecting plane of the dihedral angle formed by the two required planes in Ex. 568.

570. Given trace — (Ex. 566). This is the  $H$ -trace of two planes making —° with  $V$ . Draw their  $V$ -traces.

571. Measure the dihedral angle between the two required planes in Ex. 570.

572. Draw the traces of the bisecting plane of the dihedral angle between the required planes in Ex. 570.

573. Given  $V$ -traces:  $X(3 + 3)_2$ ,  $Y(5 + 1)_1$ ,  $Z(+2)_\infty$ ,  $M(5 + 2)(1 + 1)$ ,  $S(1 - 1\frac{1}{2})_5$ ,  $T(2 - 3)_3$ ,  $U(-1)_\infty$ ,  $W(1 - 2\frac{1}{2}) (5 - 1)$ . Draw the  $H$ -trace of —, when the plane makes —° with  $V$ .

574. Draw the  $H$ -trace of — (Ex. 573), when the plane makes —° with  $H$ . What is its angle with  $V$ ?

575. The  $V$ -trace — (Ex. 573) is the trace of the two planes making —° with  $V$ . Draw their  $H$ -traces.

576. The  $V$ -trace — (Ex. 573) is the trace of a plane making —° with  $H$  and another plane making —°. Draw their  $H$ -traces.

577. Measure the dihedral angle between the two planes required in Ex. 575.
578. Measure the dihedral angle between the two planes required in Ex. 576.
579. Draw the traces of the plane bisecting the dihedral angle between the two planes required in Ex. 575 or Ex. 576.
580. Through a point  $A(3, 0 - 1)$  in a vein of ore a horizontal line on the surface runs north,  $37^\circ$  west. The "dip" (inclination of the ore to the horizontal) is  $\text{---}^\circ$ . Find the  $V$ -trace of the plane of the ore.
581. The lower edge of a rectangular roof is 30 ft. long and inclined  $40^\circ$  to the  $V$ -plane. The roof is inclined  $60^\circ$  to the horizontal and the ridge is 8 ft. higher than the lower edge. Draw the  $H$ - and  $V$ -projections of the roof, and insert three windows, each 2 ft. 6 in.  $\times$  6 ft. 3 in., symmetrically placed. Draw to a scale  $\frac{1}{8}$  in. = 1 ft.
582. Given the line  $\text{---}$  as the  $\text{---}$  trace of a plane inclined  $\text{---}^\circ$  to  $\text{---}$ . Draw the other trace. What angle does it make with  $\text{---}$ ?
583. The line  $\text{---}$  is the  $\text{---}$  trace of two planes making  $\text{---}^\circ$  with  $\text{---}$ . Draw their traces.
584. The line  $\text{---}$  is the  $\text{---}$  trace of two planes, one making  $\text{---}^\circ$  with  $\text{---}$ , and the other  $\text{---}^\circ$  with  $\text{---}$ . Draw their traces.
585. Measure the dihedral angle between the required planes in Ex. 583 or 584.
586. Draw the traces of the bisector of the dihedral angle of the required planes in Ex. 583 or 584.

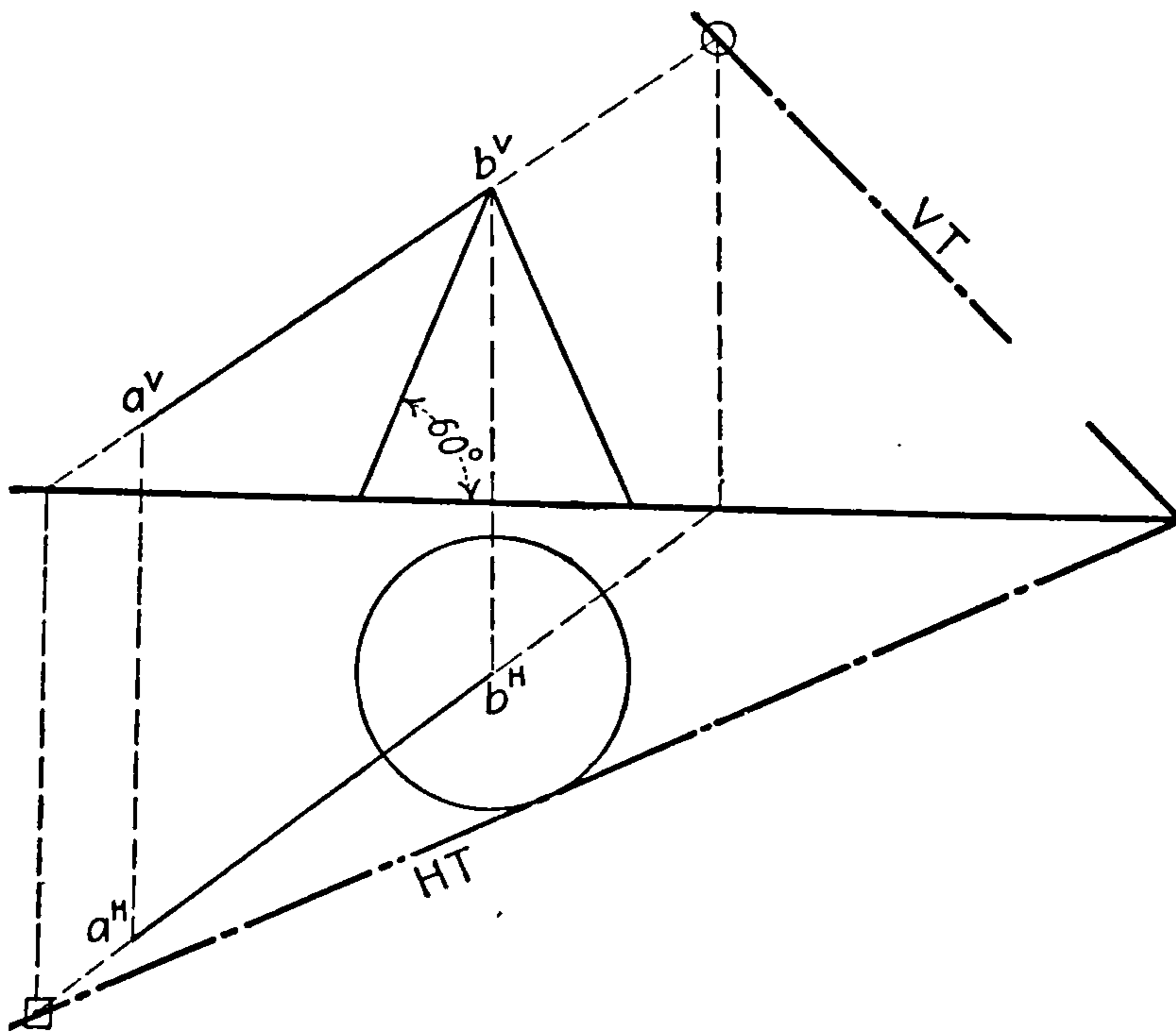


FIG. 68.

**103. Problem 34.**—To pass a plane through a given line, so as to make any required angle with a plane of projection.



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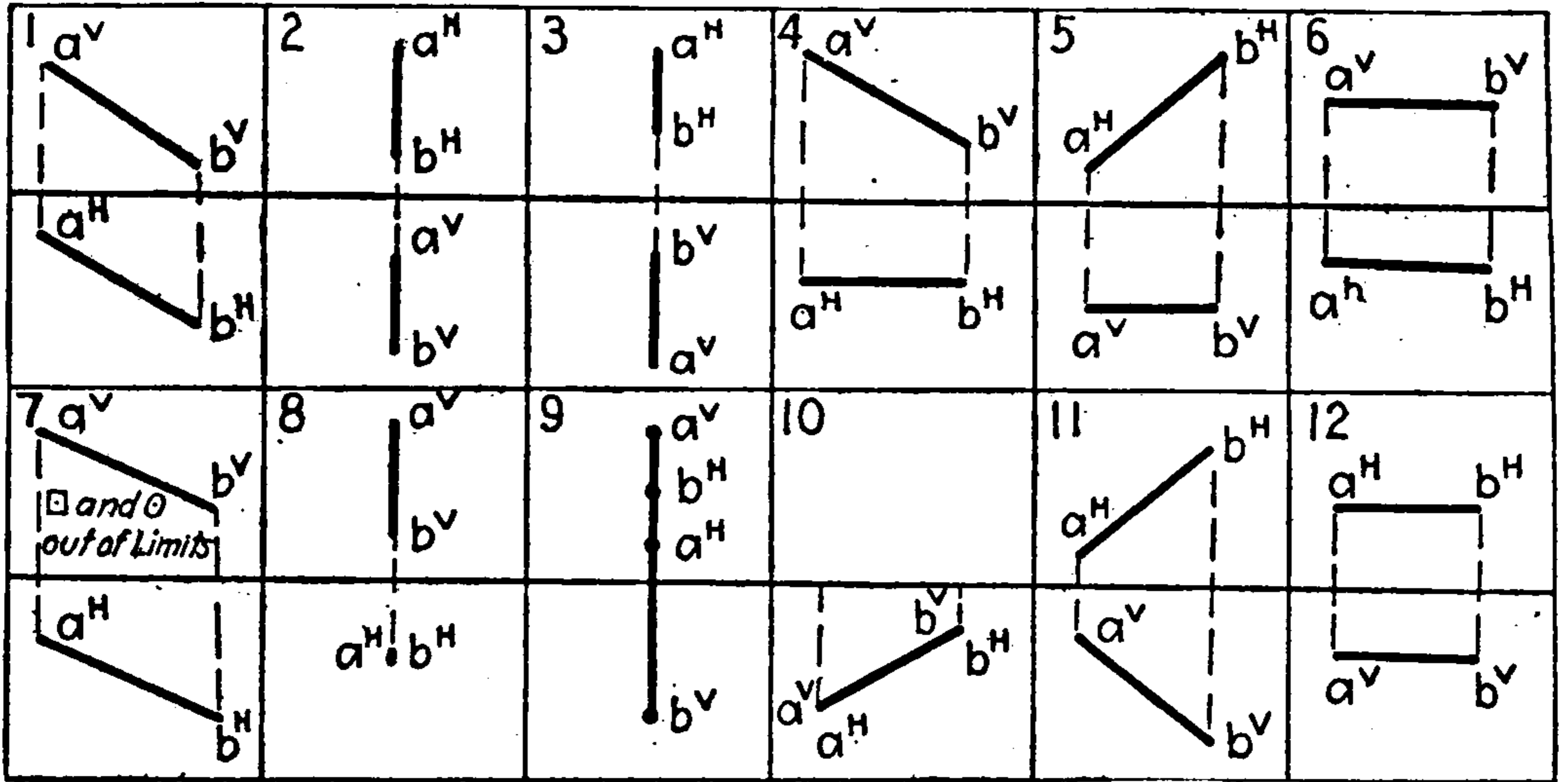
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595. Measure the dihedral angle between the two planes required in Ex. 593 or Ex. 594.
596. Use one of the given lines as a roof ridge or rafter and draw the roof as directed by the instructor, triangular, rectangular, etc., at any inclination given, and with or without windows.



Graphic Layout No. 44.—To be used with Ex. 587–596.

**105. Problem 35.**—To find the traces of a plane which shall make any required angles with  $H$  and  $V$ .

**Limitations.**—The sum of these angles must be between the limits of  $90^\circ$  and  $180^\circ$ . A plane parallel to  $H$  or  $V$  is perpendicular to the other. A profile plane is  $90^\circ$  to each. Oblique planes are between these limits.

**Construction.**—Let it be required to draw the traces of a plane inclined  $45^\circ$  to  $V$  and  $60^\circ$  to  $H$ .

1. Draw the projections of a sphere of any diameter with its center in  $GL$ . (See Fig. 69.)

2. Draw a  $45^\circ$  cone tangent to the sphere, having its base on  $V$  and apex in  $H$ .

3. Draw a  $60^\circ$  cone tangent to the sphere, having its base on  $H$  and apex in  $V$ .

4. Draw  $VT$  through the apex in  $V$ , tangent to the base of the other cone, and  $HT$  through the apex in  $H$ , tangent to the base that is in  $H$ .

**Conclusion.**—This plane tangent to both cones thus fulfils the conditions of the problem

**Notes.**—1. Eight planes fulfilling these conditions can be drawn, all tangent to the sphere.

2. Any plane parallel to  $GL$  has the sum of its angles with  $H$  and  $V$  equal to  $90^\circ$ , hence, if the sum of the given angles is equal to  $90^\circ$ , the problem can be easiest solved by the use of the profile plane.

3. If it be required to pass such a plane through a given point, an auxiliary plane may be drawn anywhere, as in the foregoing, filling the conditions, and then a plane may be passed through the point parallel to the auxiliary plane. The plane  $S$ , Fig. 69, is passed through the point  $O$ , parallel to  $T$  already found. The plane  $S$  makes the same angles with  $H$  and  $V$  that  $T$  does.

**Second Method.**—(From Church and Bartlett.)—“If a line is perpendicular to a plane, the angles between the line and the planes of projection will be complements of the angles between the plane and the planes of projection. Hence if we construct

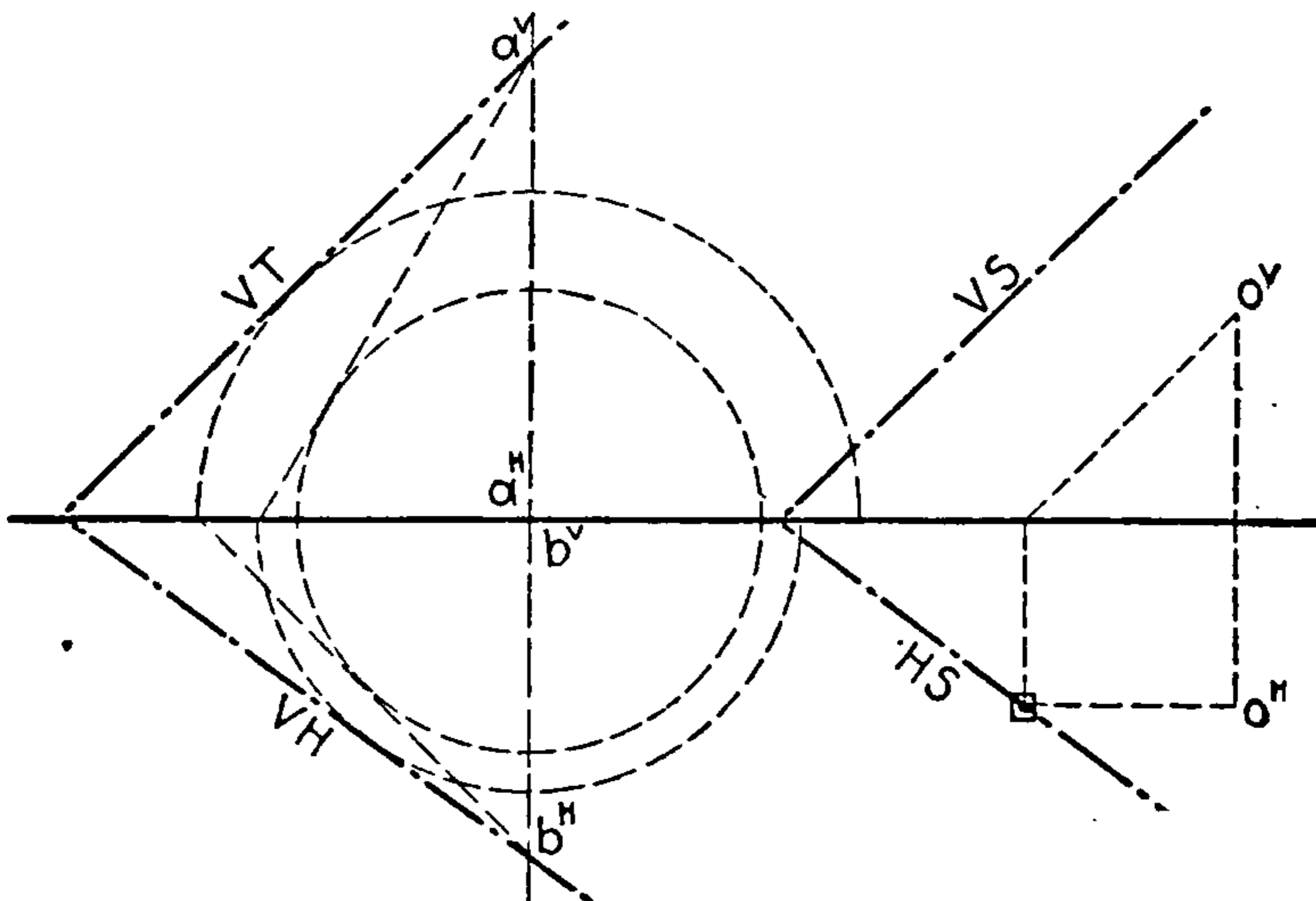


FIG. 69.

a line, making with  $H$  and  $V$  the complements of the required angles, a perpendicular plane through this point will make the required angles with  $H$  and  $V$ .”

106.

EXERCISES

- 597. Draw the traces of a plane inclined  $20^\circ$  to  $V$  and  $10^\circ$  to  $H$  through the point  $A(3 + 1 - 2)$ .
- 598. (a) Draw the traces of a plane inclined  $\text{---}^\circ$  to  $V$  and  $\text{---}^\circ$  to  $H$  through an assumed point in  $III$ . (b) Assume the point in  $II$ . (c) Assume the point in  $IV$ .
- 599. Pass a plane under the conditions in Ex. 597 or 598, using the second method.
- 600. A line shaft making  $\text{---}^\circ$  with  $H$  and  $\text{---}^\circ$  with  $V$  carries a 12-in. pulley with 6-in. face, with its center at  $M(3 \text{ ft. } 6 \text{ in.}, +2 \text{ ft. } - 1 \text{ ft. } 8 \text{ in.})$ . Draw the projections of the pulley to a scale 1 in. = 1 ft.



601. A pulley 24 in.  $\times$  8 in., whose center plane is inclined  $\text{---}^\circ$  to  $H$  and  $\text{---}^\circ$  to  $V$ , runs on a shaft 8 ft. long extending from a point in the wall ( $V$ ) to a point in the floor ( $H$ ). The pulley is in the center of the shaft. Draw its projections.
602. A man is reading a newspaper 20 in.  $\times$  30 in. and his eye at  $D$  (10 ft. + 4 ft. - 6 ft.) is 2 ft. distant from the paper. His line of sight is inclined  $\text{---}^\circ$  to  $H$  and  $\text{---}^\circ$  to  $V$ , and travels to the geometrical center of the sheet. Draw the projections of the paper.
608. Assume a point in the  $\text{---}$  angle, and draw the traces of all the planes that can contain the point and be inclined  $\text{---}^\circ$  to  $H$  and  $\text{---}^\circ$  to  $V$ .

## SPECIAL PROBLEMS

**107. Problem 36.**—To find the shortest distance between two lines not in the same plane.

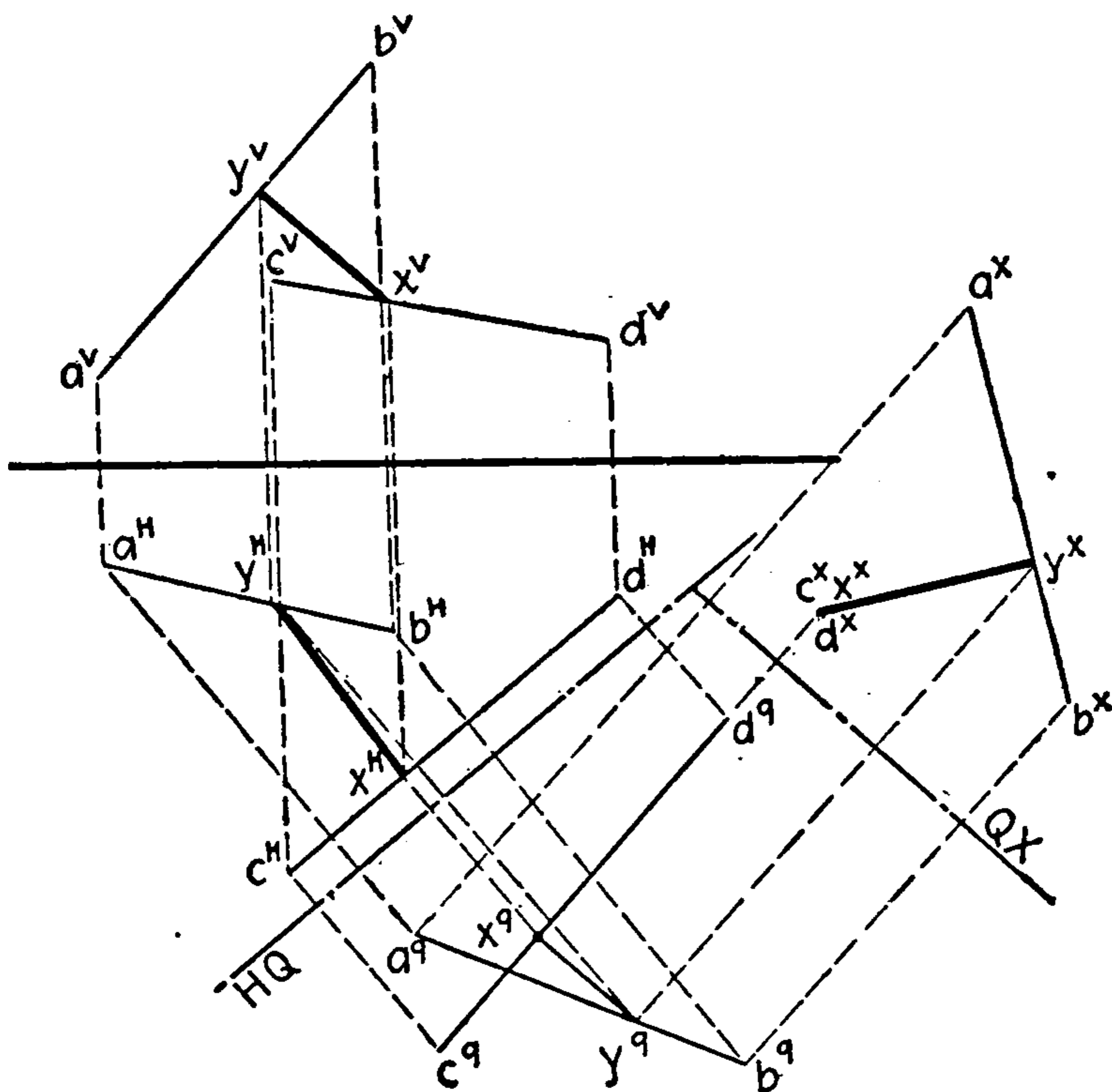


FIG. 70.

**Construction.**—Required the shortest distance between  $AB$  and  $CD$  (Fig. 70).

Referring to Fig. 71, we find that, when a line is perpendicular to  $H$  or  $V$  (as  $AB$  is to  $H$ ), the common perpendicular is immediately drawn, without further operations on  $AB$  and  $CD$ . Why? Therefore, this problem is easiest solved by bringing one of the lines into this relation.

Return to Fig. 70. Draw  $Q$  parallel to  $CD$ .



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3. At the intersection of this projection with the given line, erect a perpendicular from the plane to the outside line. This line is the common perpendicular.

**Construction.**—Let AB and CD (Fig. 72) be the given lines.

1. Draw EF parallel to AB intersecting CD.
2. Pass the plane Q through EF and CD.
3. Project AB to ab on Q.
4. Where ab crosses CD at o, erect the common perpendicular oO.

This method, the standard for many years, is still preferred by many. Let the student make the construction in orthographic projection.

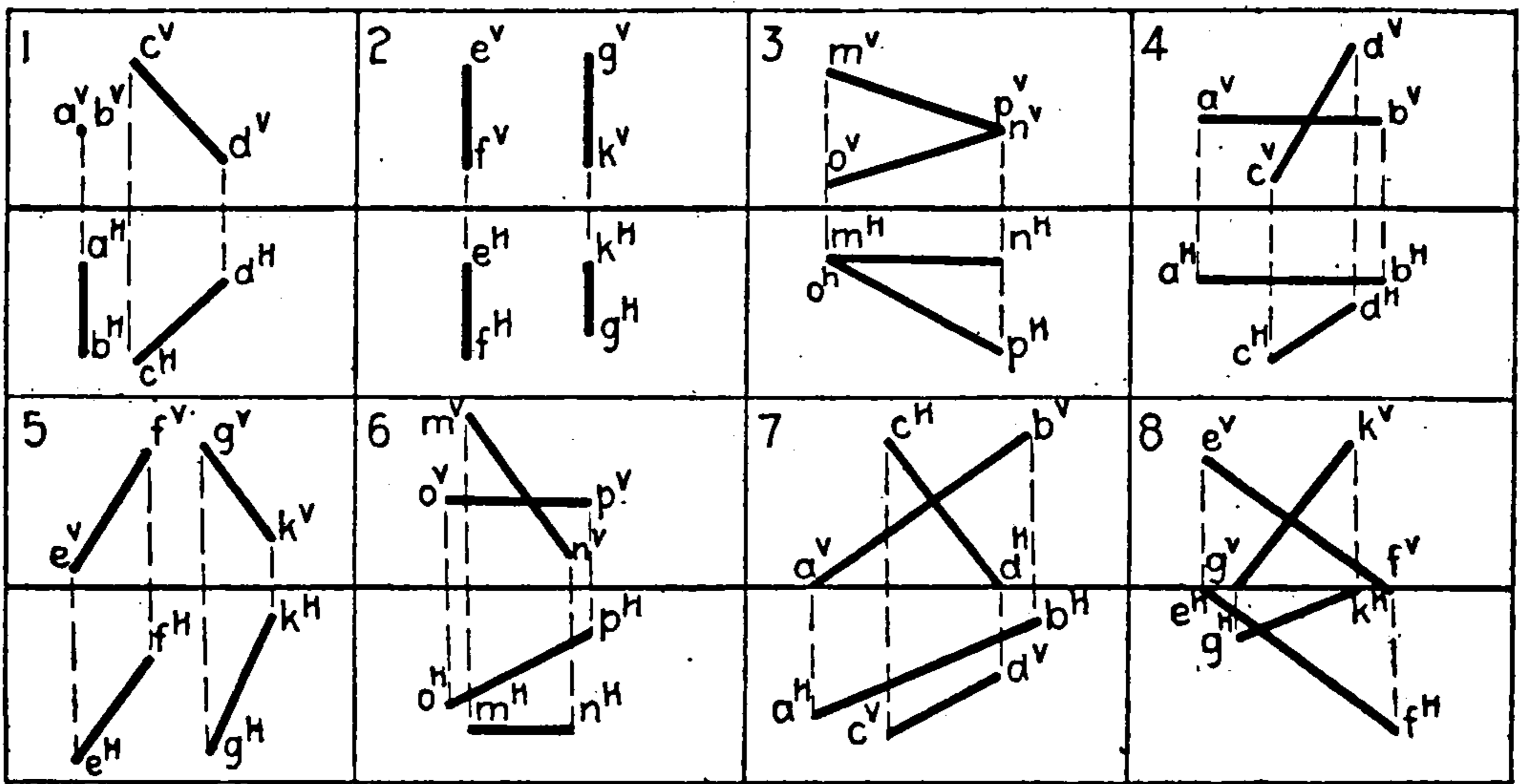
110.

## EXERCISES

Draw the projections of the common perpendicular to the following lines:

604. A(1 + 2 - 2) B(4, 0 - 1) and C(3, 0 - 3) D( $4\frac{1}{2}$  + 2 - 1).
605. E(4 + 1 - 1) F(4 + 1 - 3) and G(1 - 1 - 1) K(3 - 3 - 3).
606. M(1 + 2 - 1) N(3 + 2 - 1) and O(2 + 3 - 3) P(3 + 1 - 2).
607. A(2 - 2 + 1) B( $3\frac{1}{2}$ , 0 + 3) and C(2 -  $1\frac{1}{2}$ , 0) D(4 - 3 + 1).
608. E(2 + 3 -  $\frac{1}{2}$ ) F(2 + 1 -  $2\frac{1}{2}$ ) and G(3 + 3 - 3) K(C, 0, 0).
609. M(2 + 3, 0) N(2, 0 - 2) and O( $2\frac{1}{2}$ , 0 - 3) P(4, 0 +  $2\frac{1}{2}$ ).
610. X(2 + 3, 0) Y( $3\frac{1}{2}$ , 0 - 2) and GL.
611. A(1 + 1 - 1) B(4 + 1 - 3) and C(2 + 3 - 1) D(4, 0 - 1).
612. E(1 + 3 - 1) F(3, 0 - 1) and G(3 +  $2\frac{1}{2}$  - 2) K(4, 0, 0).
613. M(1 + 2 -  $\frac{1}{2}$ ) N(2, 0 - 2) and O(3 + 3 - 3) P( $3\frac{1}{2}$  + 1 -  $\frac{1}{2}$ ).
614. Draw the shortest possible line parallel to — (H, V, or P) intersecting both lines in Ex. — (604–613).
615. Given a 2-in. cube. Draw the shortest line between the non-intersecting diagonals of any two adjacent faces.
616. Two wires run through a building, A (12 ft. + 23 ft., 0) B(28 ft., 0 - 6 ft.) and C(16 ft. + 12 ft., 0) D(24 ft., 0 - 16 ft.). Draw the shortest connecting wire. Scale  $\frac{1}{8}$  in. = 1 ft.
617. A  $2\frac{1}{2}$ -in. pipe is run from a hole in a wall at E(13 ft. + 12 ft., 0) at  $45^\circ$  to H and  $45^\circ$  to V. At F(8 ft. + 6 ft., 0) a 1-in. pipe runs straight out from the wall, then bends perpendicularly and runs downward to the right at  $40^\circ$  to H. Make the shortest possible connection. How long must it be, allowing 2 in. at each end for connections? Scale  $\frac{1}{4}$  in. = 1 ft.
618. Draw the shortest possible horizontal pipe for Ex. 617.
619. Draw the shortest possible vertical pipe for Ex. 617.
620. Scale  $\frac{1}{4}$  in. = 1 ft. In a mill there is a 6-ft. cubical bin on the floor with one edge at A(15 ft., 0, 0) B(21 ft., 0, 0). A 24-in. square chute runs its center line from C(3 ft. + 9 ft., 0) in the wall to D(16 ft., 0 - 10 ft.) in the floor. It is desired to run a 9-in. cylindrical pipe from a hole at E(18 ft. + 10 ft., 0) to F(8 ft., 0 - 13 ft.). Can the pipe pass both

the bin and the chute? If so, how much will there be to spare in each case? If not, locate the nearest point to transfer F.



Graphic Layout No. 45.—(Note.—Symbols may be transposed at option of instructor.)

- 621. Draw the shortest line between the two lines in Ex. — of Layout 45.
- 622. Draw the shortest horizontal between the two lines in Ex. — of Layout 45.
- 623. Draw the shortest vertical between the two lines in Ex. — of Layout 45.
- 624. Draw the shortest line between  $GL$  and any oblique line in Layout 45.

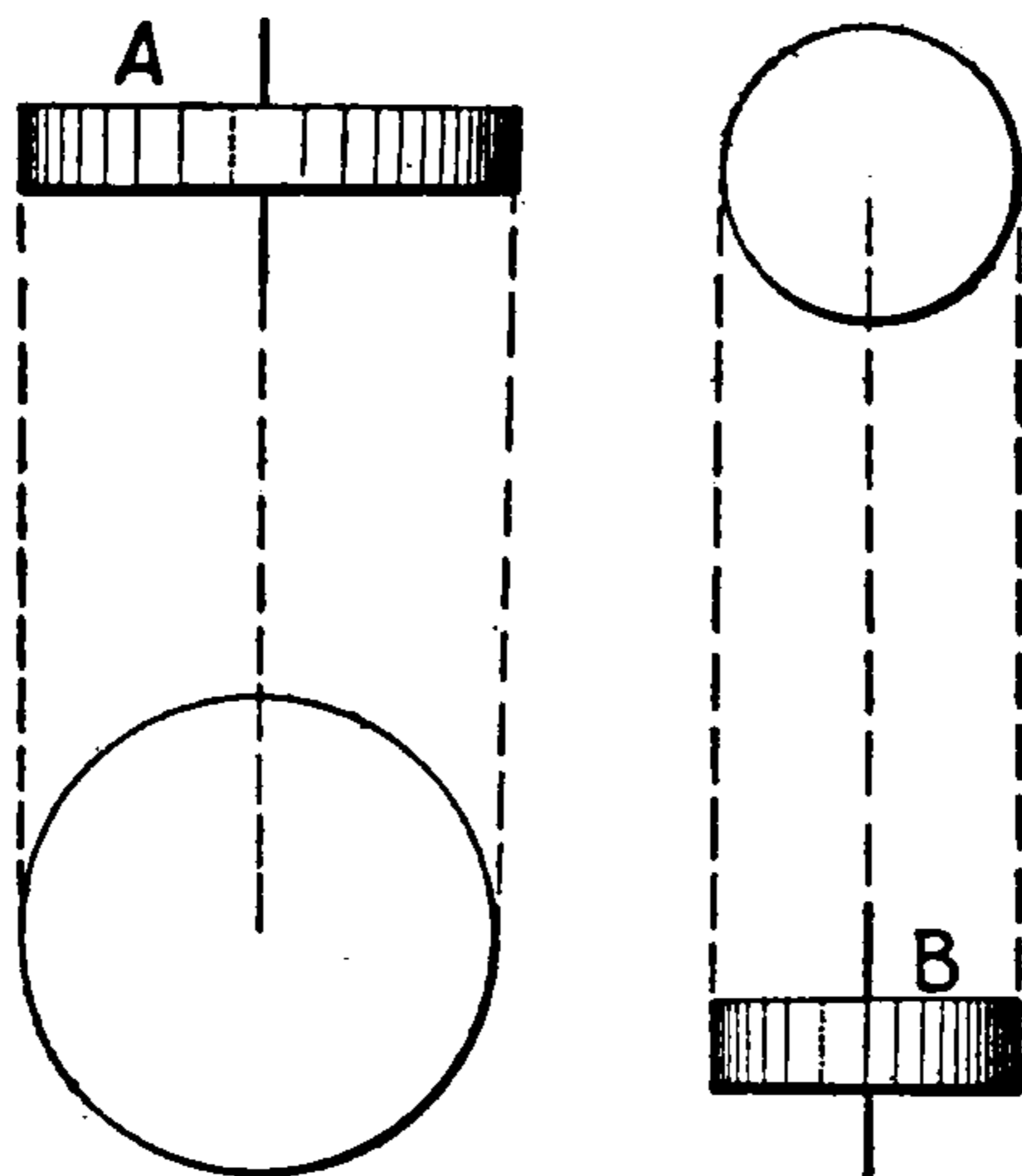


FIG. 73.—Pulleys on non-intersecting shafts.

**111. The Problem of the Guide Pulley.**

It is a well-known fact in machine design that a belt could be run over the pulleys A and B, shown in Fig. 73, so as to drive in one direction or the other, but it would be impossible to reverse

the direction if attached to run in one. It is possible to construct an "idler," or guide pulley, with its shaft at such an angle that the belt will not run off either pulley, running in either direction.

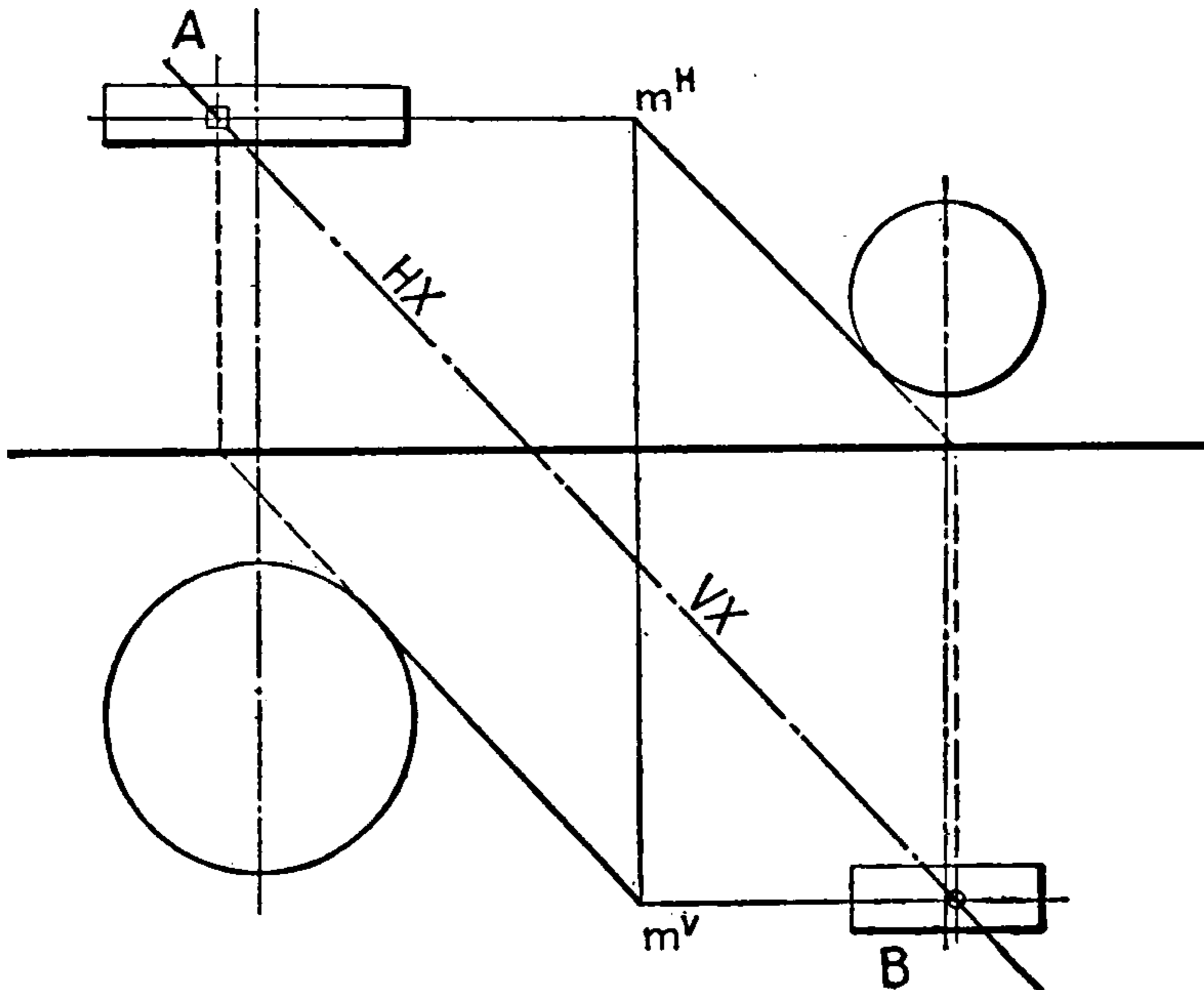


FIG. 74.—Locating the plane of the guide pulleys.

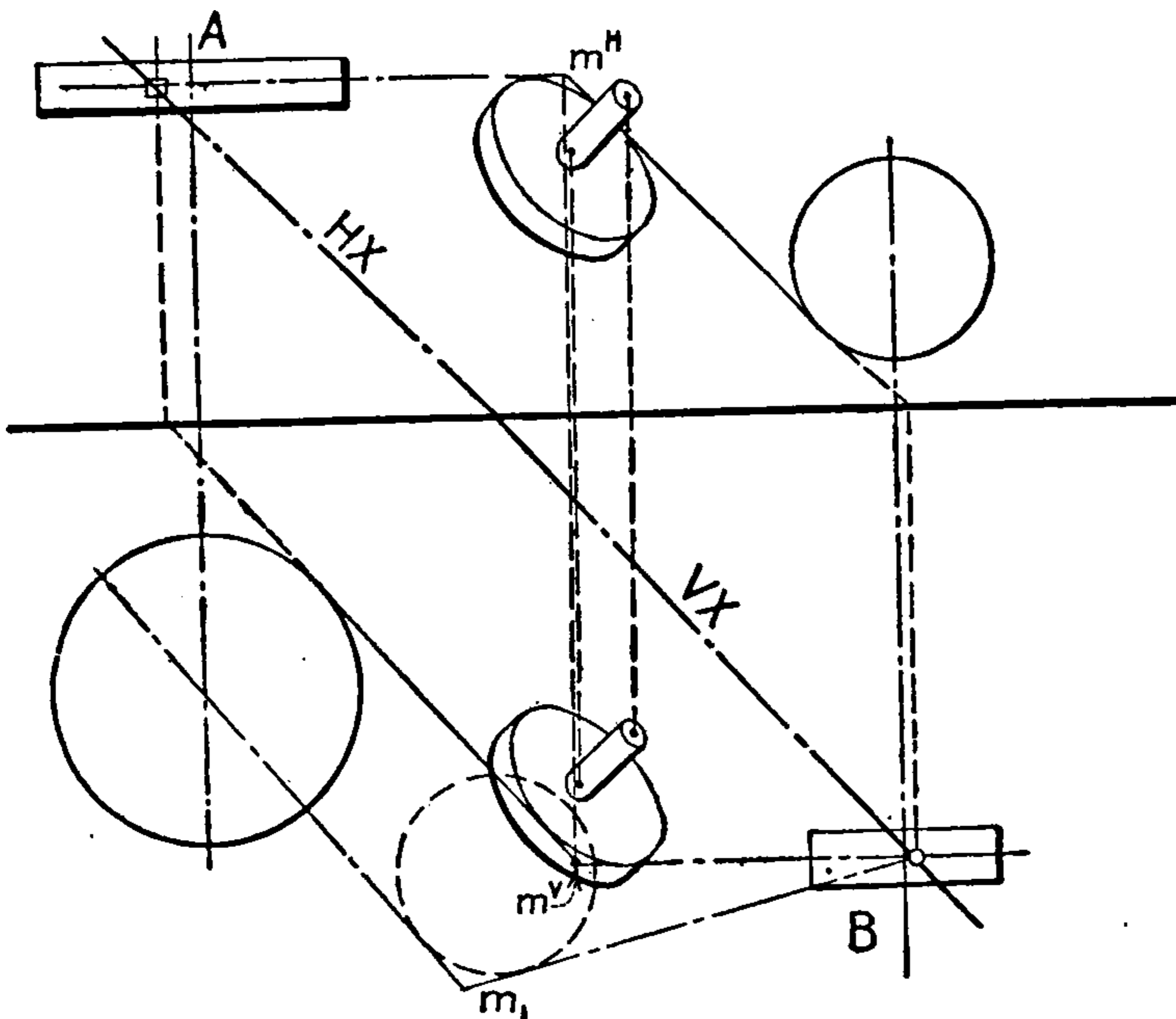


FIG. 75.—Showing revolution and counter-revolution of the guide pulley.

**Construction.**—1. Select a point,  $M$  (Fig. 74), between the pulleys, preferably about midway, common to the center planes of the pulleys.

2. Run a tangent from  $M$  to each of the pulleys in their center plane.



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114. PRACTICAL EXERCISES

- 633. Find the length of all the oblique rafters in Fig. 78. Scale  $\frac{1}{8}$  in. = 1 ft.
- 634. Find the true angles between the various oblique rafters and the horizontal ridges that intersect them (Fig. 78).
- 635. Measure all the valley angles between the planes A and C, A and D, etc., in the roof (Fig. 78).

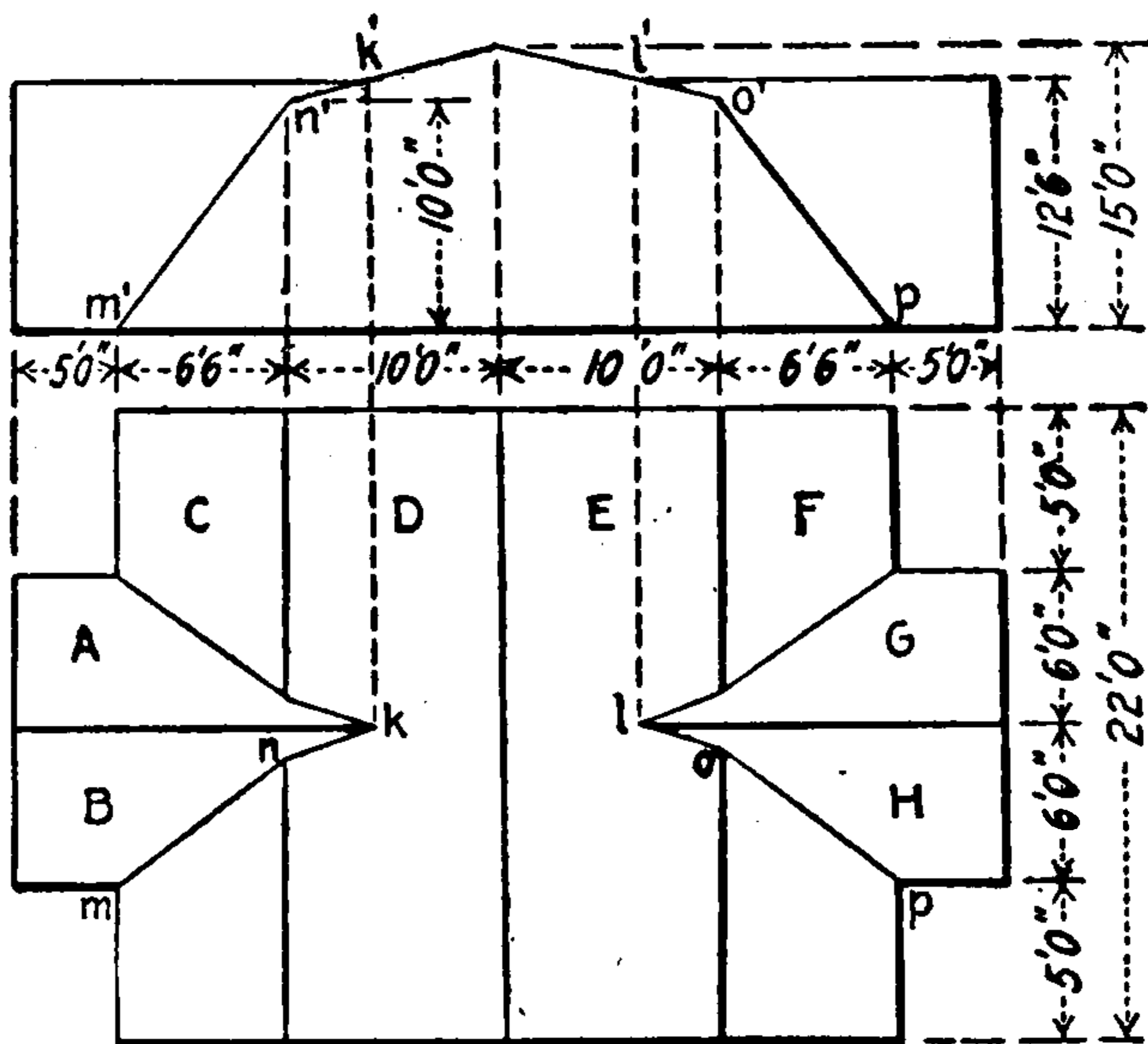
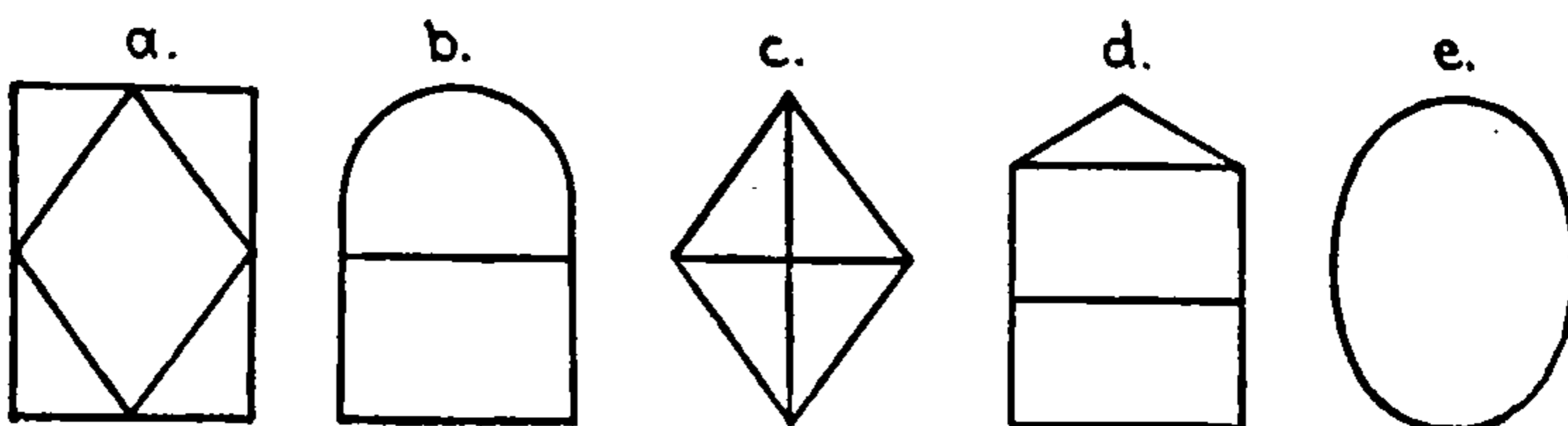


FIG. 78.

- 636. A rectangular chimney 3 ft. 6 in.  $\times$  5 ft. 4 in. is to be put through roof C in Fig. 78. Draw its *H*- and *V*-projections, and the true size of the opening in the roof.



Windows 2'0"  $\times$  3'6".

FIG. 79.

- 637. Roof in Fig. 78. Place window — (Fig. 79) in the center of roof —.
- 638. Cut an opening 4 ft. square in roof —, Fig. 78.
- 639. An octagonal stone tower, 18 ft. in diameter, and 40 ft. high, is surmounted by a pyramidal cap, 12 ft. high. Draw its projections, and draw in the center of each of the four visible faces window — (Fig. 79). Draw only half of the plan and the elevation of the cap. Scale  $\frac{1}{4}$  in. = 1 ft.
- 640. A 14-in. steam feed pipe, whose center line is A(1 ft. + 3 ft. 3 in. - 3 ft.) B(4 ft., 0 - 1 ft.). The plane of the roof shed is T(5 ft. + 2 ft.) 6 in. (2 ft. - 3 ft.). Draw the projections of the opening in the roof, and its true size. Determine the dip of the roof. Scale 1 in. = 1 ft.



641. A trapeze bar, 3 ft. long, is suspended by ropes 3 ft. long. Find how high it is raised by twisting through  $\text{---}^\circ$  ( $60^\circ$ ,  $75^\circ$ ,  $90^\circ$ ,  $105^\circ$ ,  $120^\circ$ ).
642. What angles does the pipe in Ex. 640 make with  $H$ ,  $V$ , and  $T$ . If it keeps its direction, what is the size of the hole in the floor?
643. A conical tower is 15 ft. in diameter and 14 ft. high. Draw two lines in the surface from the apex to the base, making  $\text{---}^\circ$  ( $30^\circ$ ,  $45^\circ$ ,  $60^\circ$ ) with each other.
644. Draw a cube of  $1\frac{1}{2}$ -in. sides on  $H$ . Draw the traces of a plane that shall cut a regular hexagon from the cube. Revolve the hexagon to prove that it is regular, and calculate the angle that  $Q$  makes with  $H$ .
645. A building 28 ft. 6 in.  $\times$  41 ft. 3-in. ground plan is covered by a hip roof having a central ridge 18 ft. long, commencing 10 ft. from the front of the house. The front corner rafters are inclined  $45^\circ$  from the horizontal. Draw the plan and elevation of the roof. What is its height? Measure the hip angles. What are the angles made with the horizontal by the rear corner rafters?

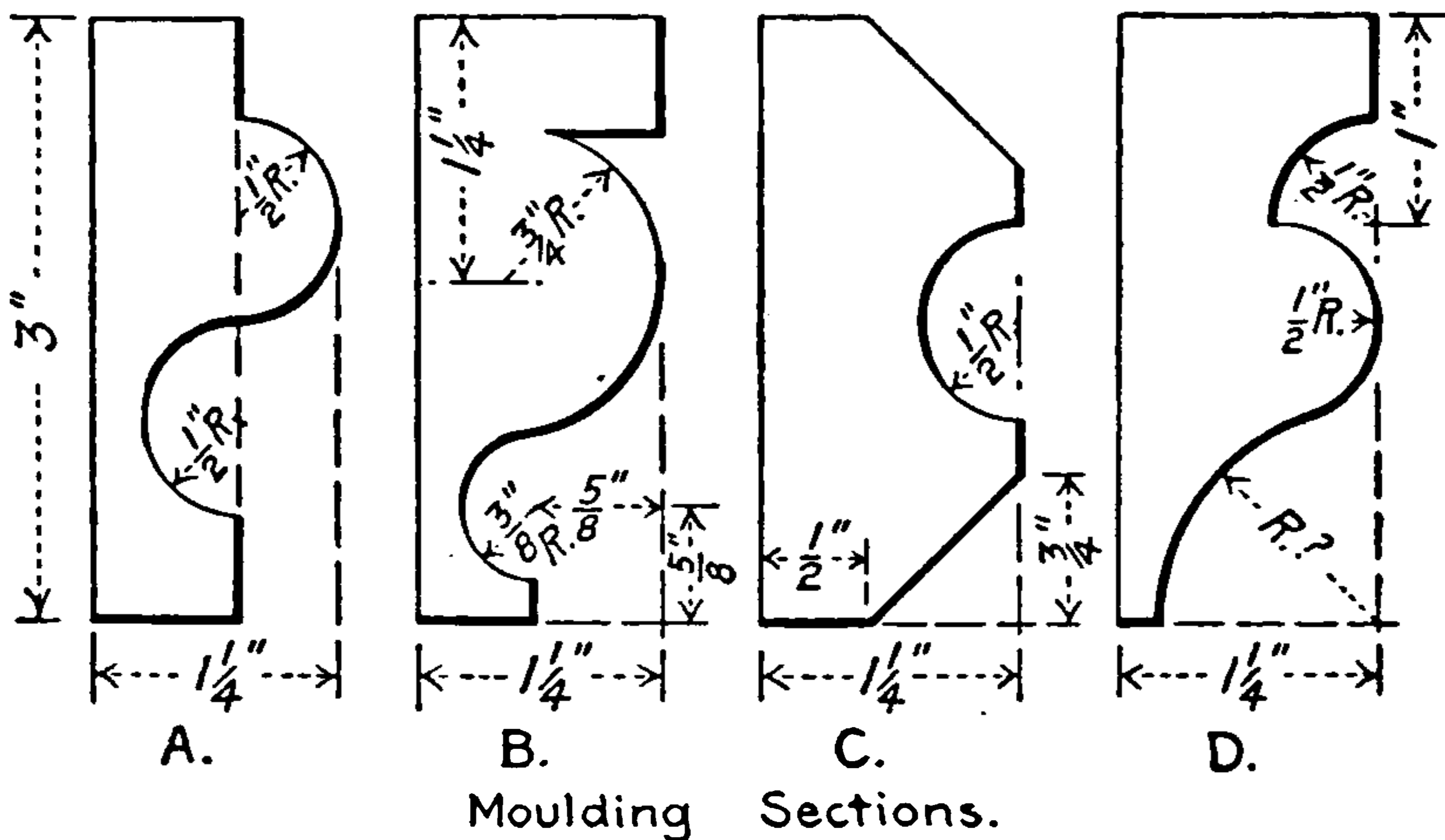


FIG. 80.

646. Two pulleys 24 in.  $\times$  8 in. are located on parallel horizontal shafts. The center line of one is  $A(4 \text{ ft.} + 6 \text{ ft. } 6 \text{ in.} - 18 \text{ in.})$   $B(6 \text{ ft.} + 6 \text{ ft. } 6 \text{ in.} - 6 \text{ in.})$ , and the other passes through  $C(6 \text{ ft.} + 2 \text{ ft.} - 5 \text{ ft.})$ . The first pulley is midway between  $A$  and  $B$ . The belt passes through a wall whose plane is  $S(10 \text{ ft.} + 4 \text{ ft.})$   $6 \text{ ft.}$  ( $2 \text{ ft.} - 4 \text{ ft.}$ ). Draw the projections and true size of the opening in the wall to accommodate the belt. Allow 2-in. clearance at all points. Scale  $\frac{1}{2}$  in. = 1 ft.
647. Inscribe a sphere in the tetrahedron  $A(2, 0 - 1)$   $B(4, 0 - 1)$   $C(3\frac{1}{2}, 0 - 3)$   $D(3\frac{1}{4} + 2 - 1\frac{3}{4})$ .
648. Circumscribe a sphere about the tetrahedron in Ex. 647.
649. Inscribe a sphere in the tetrahedron  $M(2 + 1\frac{1}{2} - 1)$   $N(3 + 2 - \frac{1}{2})$   $O(3\frac{1}{4} + 1\frac{1}{2} - 1\frac{1}{2})$   $P(3 + 1 - 2)$ .
650. Circumscribe a sphere about the tetrahedron in Ex. 649.
651. Pass a plane equidistant from  $A(2 + 3 - 2)$   $B(3 + 1 - 1)$  and  $C(5 + 3 - \frac{1}{2})$   $D(3\frac{1}{2} + 1 - 3)$ .
652. Draw the  $H$ - and  $V$ -projections of a  $1\frac{1}{2}$ -in. hex nut, drilled and chamfered, on the center line  $M(2 + 1 - 3)$   $N(4 + 2 - 1)$ . Full size.



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665. A tunnel through a hill runs from A(425 ft. + 275 ft. - 100 ft.) to B(2100 ft. + 150 ft. - 1300 ft.). Find its length and grade.
666. A plane T passes through  $GL$  and the point  $K(3 + 1 - 2)$ . A plane S parallel to it cuts  $V$   $1\frac{1}{4}$  in. above  $GL$ . Find the  $H$ -trace of S, the distance between T and S, and their angles with  $H$  and  $V$ .
667. Pass a plane through  $L(1 + 2 - 1)$  equally distant from  $M(4 + 1 - \frac{3}{4})$ ,  $N(3 + 2 - \frac{1}{2})$ , and  $O(2 + 2 - \frac{1}{4})$ . Find a point in the plane thus found which is equally distant from M, N and O.
668. Erect a cube on  $H$  with its base diagonal  $A(3, 0 - \frac{1}{2})$   $B(4, 0 - 1\frac{1}{4})$ . Show its picture as seen from  $C(3\frac{1}{2} + 1\frac{3}{4} - 3)$ . Show the picture in its true size.

Note.—Pass the picture plane perpendicular to the line from the point of sight to the center of the cube.

669. From a hole,  $A(18$  ft. + 10 ft., 0), in a mill wall, run a pipe that will make a straight connection with two pipes, whose center lines are  $B(5$  ft. + 10 ft. - 3 ft. 6 in.)  $C(19$  ft., 0 - 10 ft.) and  $D(3$  ft., 0 - 8 ft.)  $E(10$  ft. + 10 ft. - 8 ft.). What are the lengths of the two portions of the pipe? Scale  $\frac{1}{4}$  in. = 1 ft.

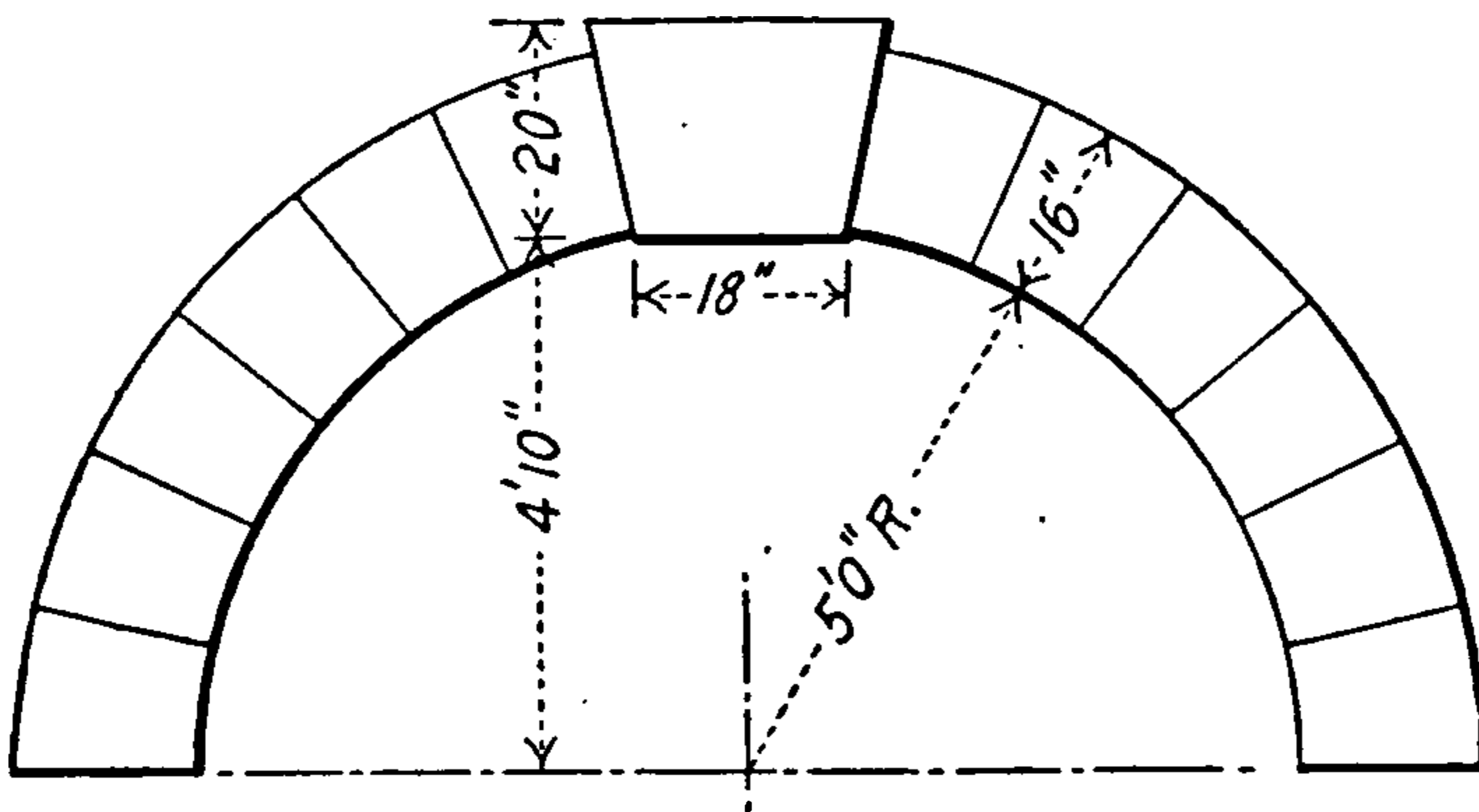
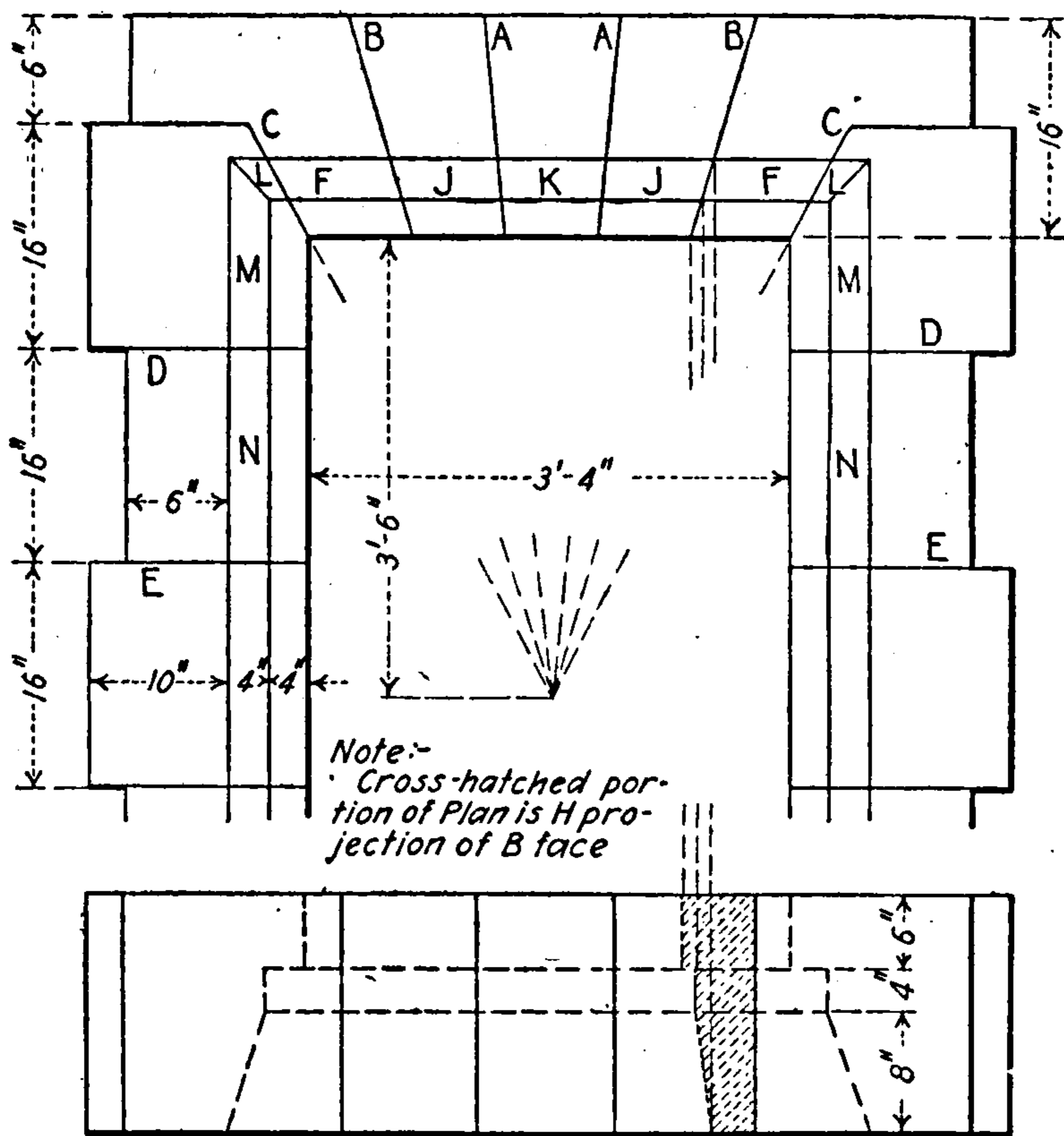


FIG. 81.—Stone arch.

670. Draw the projections of the circle passed through  $A(2 + 2 - \frac{1}{4})$ ,  $B(2\frac{1}{2} + 1 - 1)$ , and  $C(3\frac{1}{2} + 2 - 1\frac{1}{2})$ .
671. The hinge of a door is  $M(4 + 3, 0)$   $N(4, 0, 0)$ . When shut, it is to the left of  $MN$ . A taut wire runs from  $O(3 + 2, 0)$  to  $P(5, 0 - 1)$ . Find the outline of a slot that will allow the door to open freely through  $90^\circ$ .
672. Let  $H$  be a mirror. A ray of light passes through  $A(1 + 1\frac{1}{2} - 2\frac{1}{2})$  strikes  $H$  and passes through  $B(3\frac{1}{2} + 1 - \frac{1}{2})$ . Where does it reflect on  $H$ , and where does it strike  $V$ ?
673. Find the path of a ray of light that passes through  $O(5 + 2 - 1\frac{1}{4})$  and is reflected from  $H$  and  $V$  through  $P(2 + 1 - 2)$ .
674. The top of a large bin in a mill has for its corners the points  $A(4$  ft. + 7 ft. 6 in., 0),  $B(20$  ft. + 7 ft. 6 in., 0),  $C(4$  ft. + 2 ft. 3 in. - 8 ft. 9 in.), and  $D(20$  ft. + 2 ft. 3 in. - 8 ft. 9 in.). From  $M(16$  ft. + 12 ft., 0) the center line of a 24-in. square chute runs downward to the left at  $45^\circ$  to  $H$  and  $30^\circ$  to  $V$ . Draw the projections of the hole in the bin top made by the chute, and find its true size. Scale  $\frac{1}{4}$  in. = 1 ft.

- 675. Draw the projections of four spheres,  $1\frac{1}{4}$  in.,  $1\frac{1}{2}$  in.,  $1\frac{3}{4}$  in., and 2 in. in diameter, each one tangent to the other three.
- 676. Develop the surface of the keystone of the arch shown in Fig. 81. The masonry is 14 in. thick. Use any convenient scale.
- 677. M(8 in. +  $7\frac{1}{2}$  in. - 2 in.) N(17 in. + 4 in. - 8 in.) is the center line of a hollow tile, 4 in. X 12 in., of square section with rounded corners,  $\frac{1}{2}$ -in. walls. The rounded corners are  $1\frac{1}{4}$ -in. radius. Draw its projections, scale 3 in. = 1 ft.
- 677. A borehole cuts a 6-ft. core from a vein, whose strike is north  $60^\circ$  east, and whose dip is  $45^\circ$  to the northwest. The borehole bears south  $75^\circ$  west and dips  $30^\circ$ . What is the real thickness of the vein?

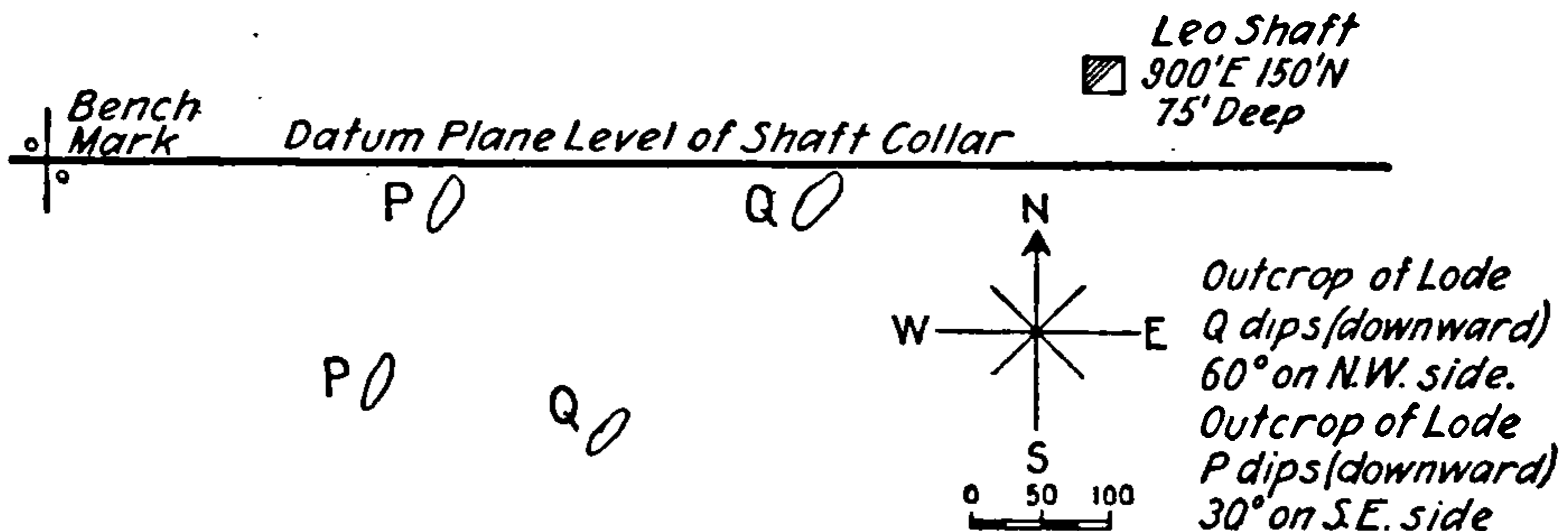


Ex. 681.—Plate band arch.

- 678. At A, where the elevation is 9800 ft., a vertical borehole strikes a vein at 300 ft. B bears north,  $30^\circ$  east, 1500 ft. from A, and has an elevation of 9600 ft. An inclined hole is drilled, bearing north  $75^\circ$  east, dipping  $75^\circ$ , striking the vein at 525 ft. At C, whose elevation is 9900 ft., 1200 ft. from A, south  $60^\circ$  east, a vertical borehole cuts the ore at 375 ft. Required the dip and strike of the vein. Let H be at 9500 ft. level.
- 679. At A(1 + 3 - 3) a borehole is drilled, bearing north  $30^\circ$  east, and dipping  $75^\circ$ . It intersects a vein, whose strike is due north, and cuts an 8-ft. core. The true thickness of the vein is 5 ft. What is the dip of the vein?
- 680. Vertical holes on a hill side (considered a plane) are bored 150 ft. at

A(100 ft. + 400 ft. - 300 ft.), 200 ft. at B(350 ft. + 250 ft. - 325 ft.), and 125 ft. at C(600 ft. + 325 ft. - 100 ft.), striking a vein of ore. What is the line of outcrop, and what are the dip and strike of the vein?

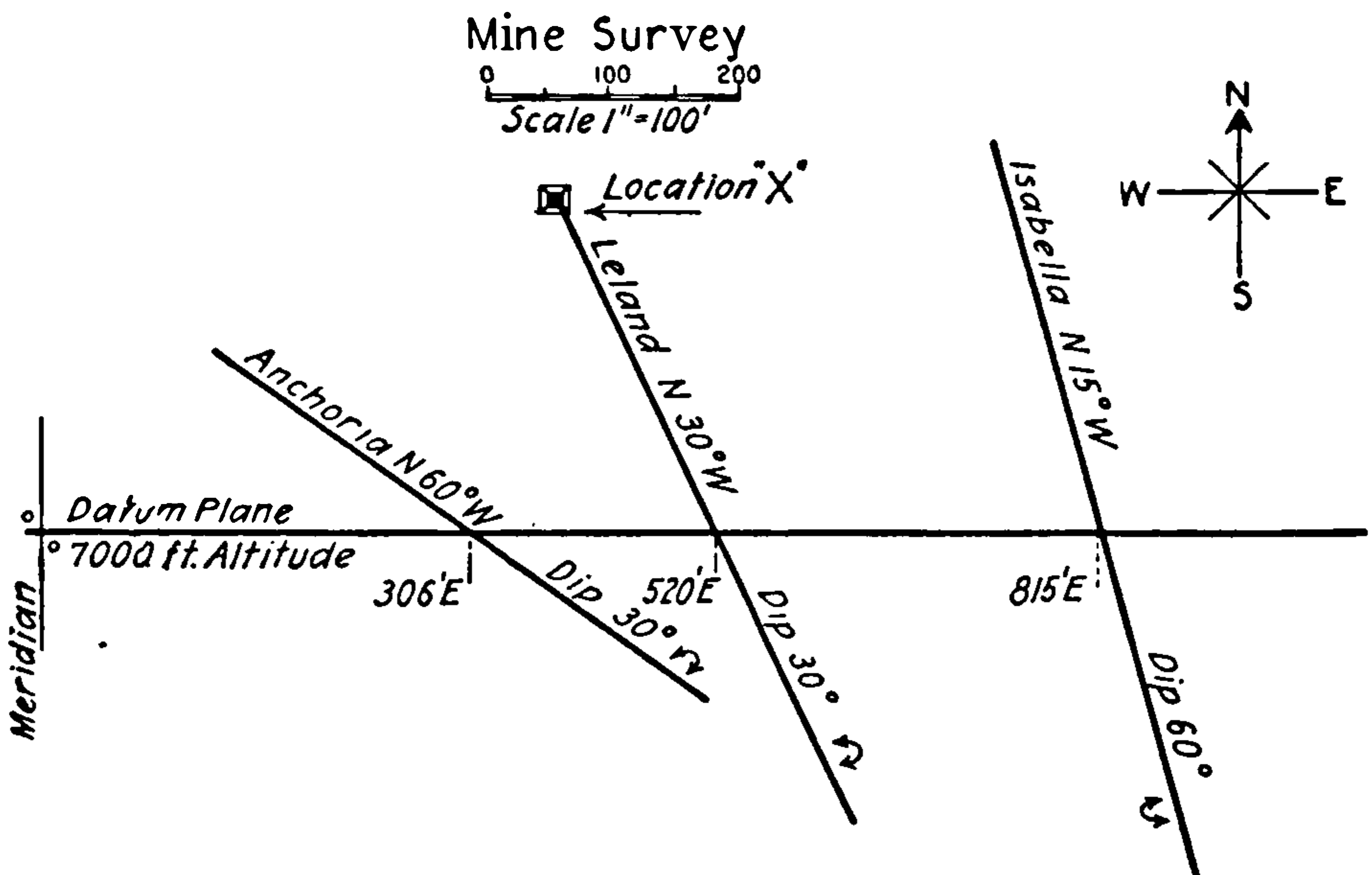
- 681. Draw the projections of the Plate Band Arch. Lay out the patterns for joints A, B, C, D, E. Scale  $\frac{1}{2}$  in. = 1 ft.
- 682. Plate Band Arch. Lay out the patterns for the slanting faces, F, J, K, L, M, and N. Scale  $\frac{1}{2}$  in. = 1 ft.
- 683. Plate Band Arch. Make working drawings in three views of Block — (A, B, C, N, M). Scale 1 in. = 1 ft



Ex. 684.—Mine survey of Prince and Queen veins.

- 684. Given the outcrops of Prince and Queen veins, and the shaft Leo. Required the length, bearing, and inclination of the shortest cross-cut from the present bottom point of the shaft to the intersection of P and Q. Also the length, bearing, and level of the drift from the lowest point of this line to the line of the shaft.

Note.—Draw compass mark and scale.



Ex. 685.—Mine survey No. 2.

- 685. Mine Survey of Anchoria, Leland, and Isabella lodes. Required: (1) Location and depth of a shaft to the intersection of A - L - I; (2)



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## CHAPTER III

### INTERSECTIONS AND DEVELOPMENTS OF PLANE SOLIDS

115. Problem 39.—To find a plane section of a pyramid or prism.

Analysis.—1. Obtain the piercing points of the various oblique edges with the given plane.

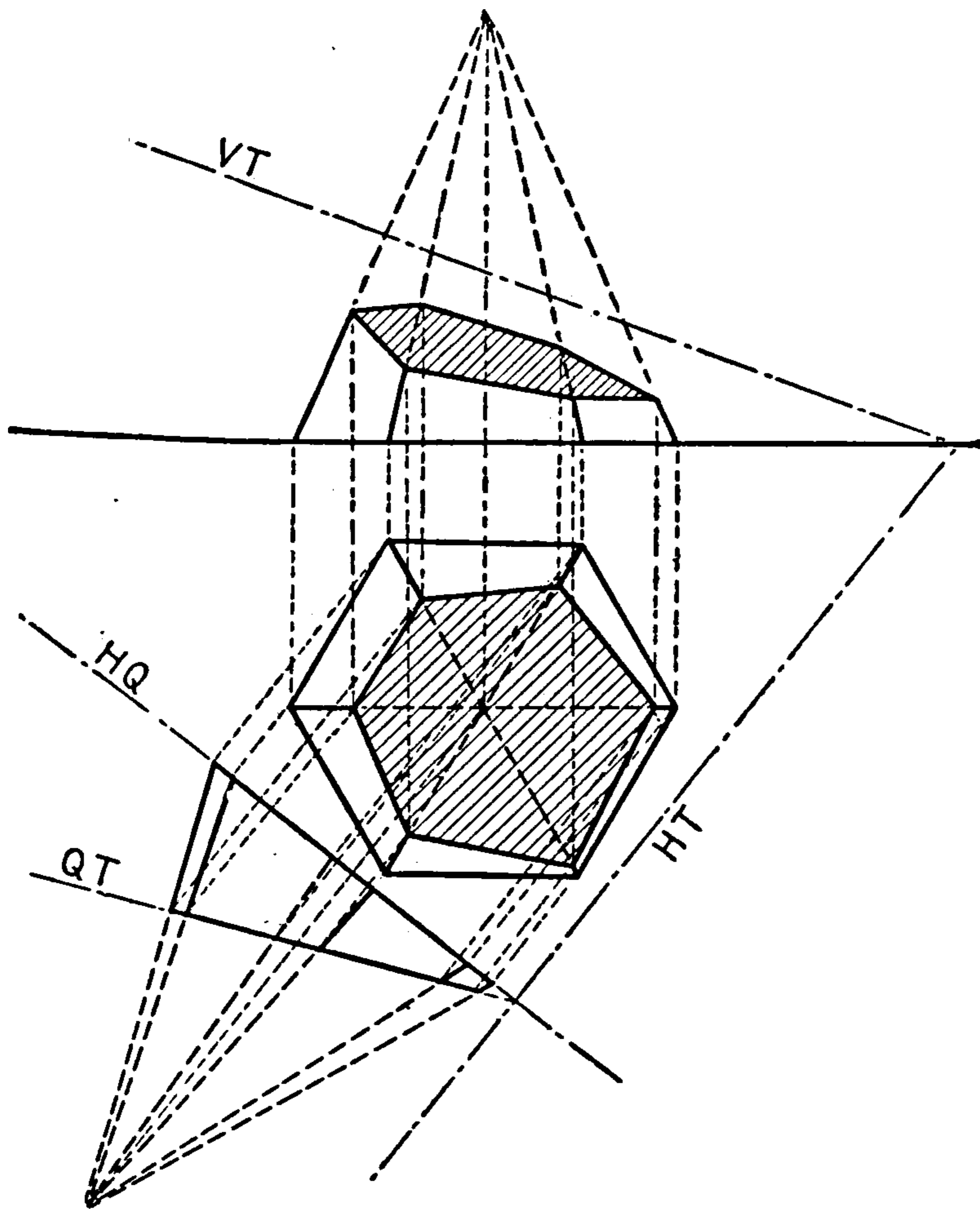


FIG. 82.

2. Connect the points two and two by straight lines in order.

Construction.—Let it be required to draw the section line of the hexagonal pyramid made by the plane T (Fig. 82).

1. Draw the Q-projection of the pyramid.

2. Draw the Q-trace of T.

3. Project the various piercing points to *H* and *V*. (The Q-projections of these points are all located on QT.)

4. Join the points in order, forming the irregular hexagon enclosing the crosshatched space.

## DEVELOPMENT

**116. Definition.**—To develop the surface of a solid is to bring its various faces and bases into one plane, all in their true sizes and relative positions, so that the plane figure thus outlined can be cut out and folded into the form of the solid.

**Note.**—In sheet metal drafting this is called “laying out the pattern.”

**Problem 40.**—To develop the surface of a truncated prism.

**Construction.**—Let it be required to develop the square prism (Fig. 83), between its *H*-base and the T-section.

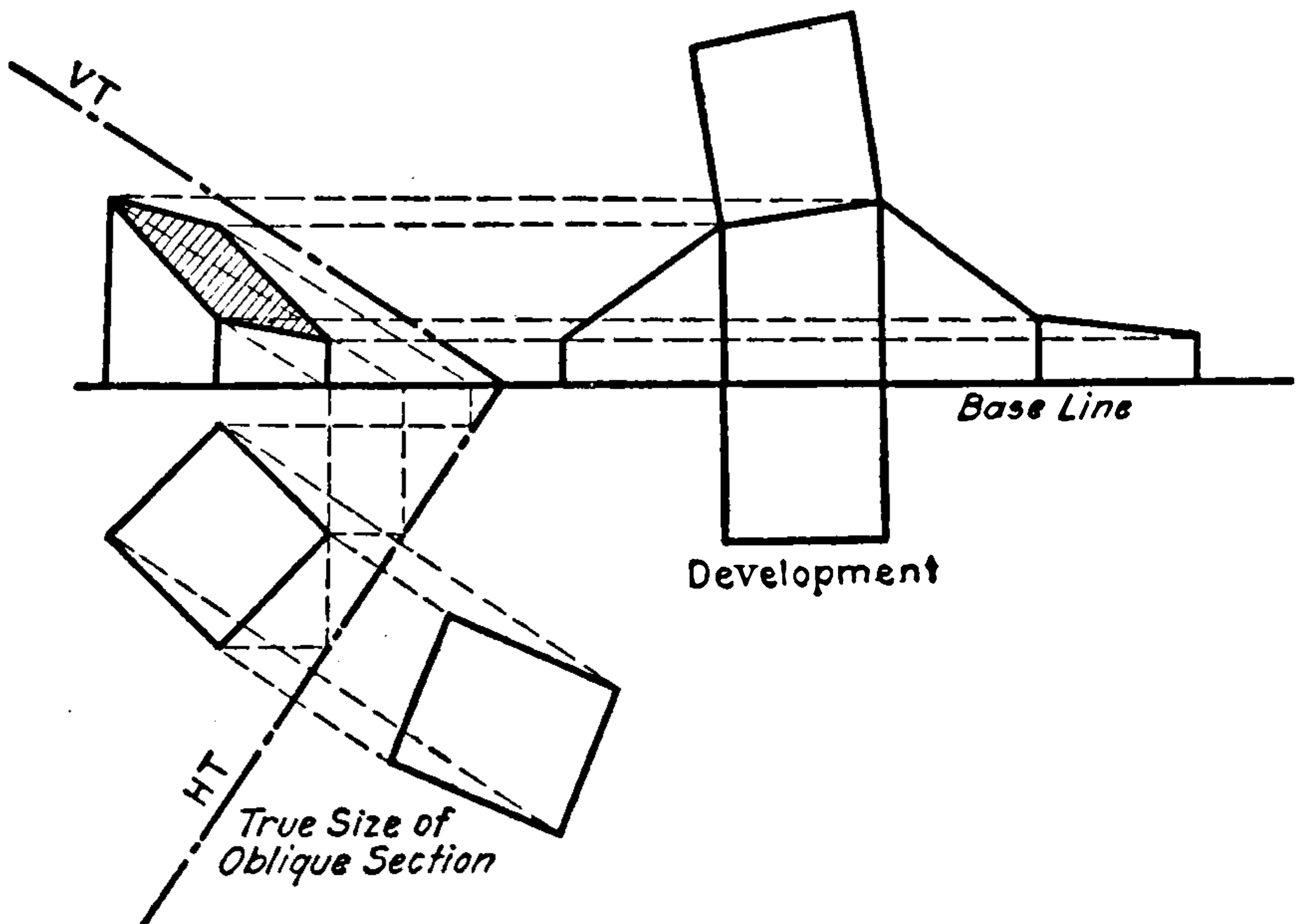


FIG. 83.

1. Lay out the “base line” on a right section of the prism. (In this case use the *H*-base.)

2. Lay off the true length of the base edges on this line.

3. Erect perpendiculars at these points, making them equal in length to the segments of the longitudinal edges, in order.

4. Connect the ends of these perpendiculars two and two by straight lines.



5. Attach the top and bottom bases, A and B, to their proper edges in their true sizes.

**117. Problem 41.**—To develop a truncated pyramid.

**Construction.**—Let it be required to develop the pyramid shown in Fig. 82 between the *H*-base and the *T*-section.

1. Obtain the true lengths of the edges, and of the segments between bases.

2. Lay out the triangles in true size and order, with a common vertex.

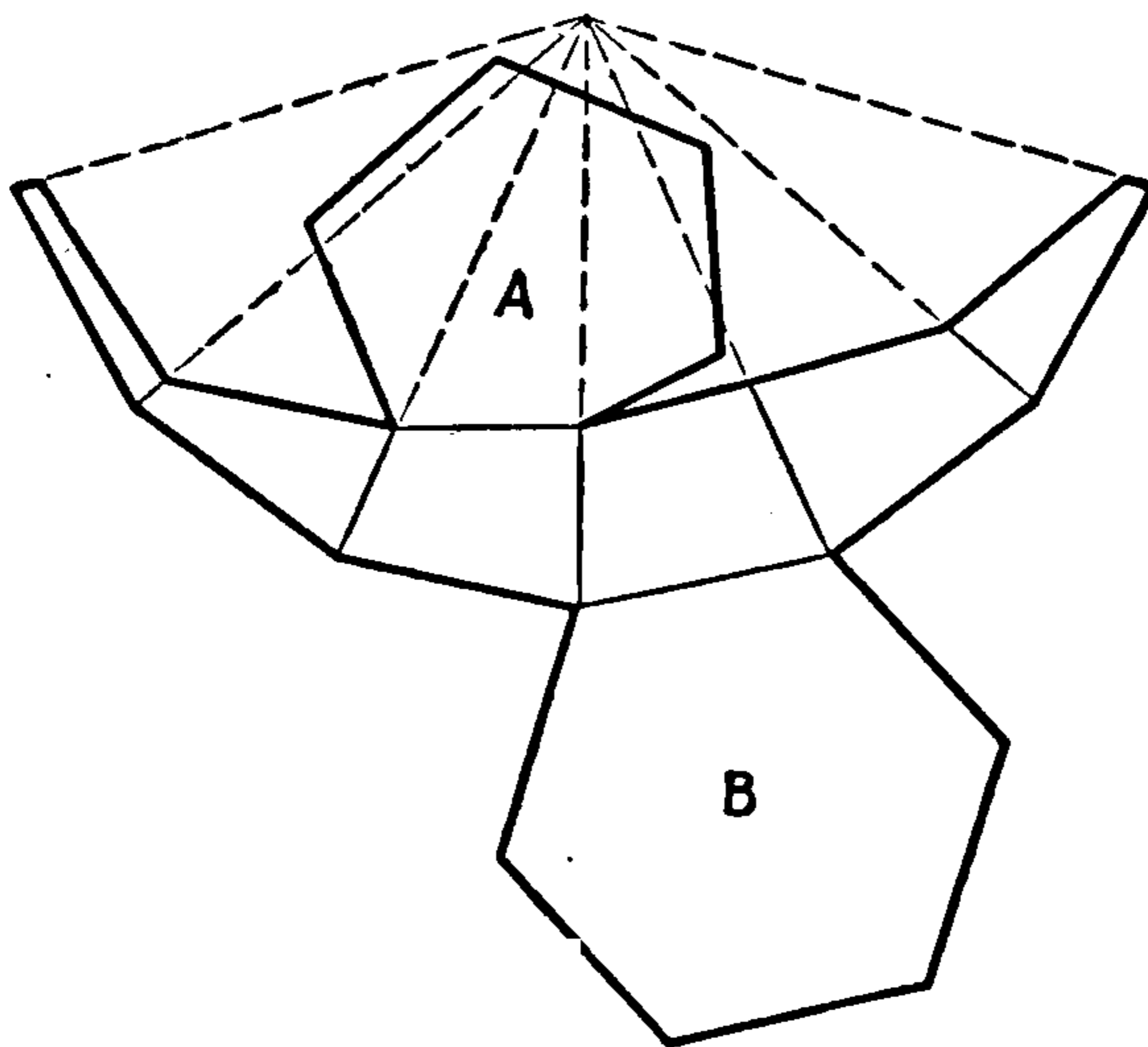


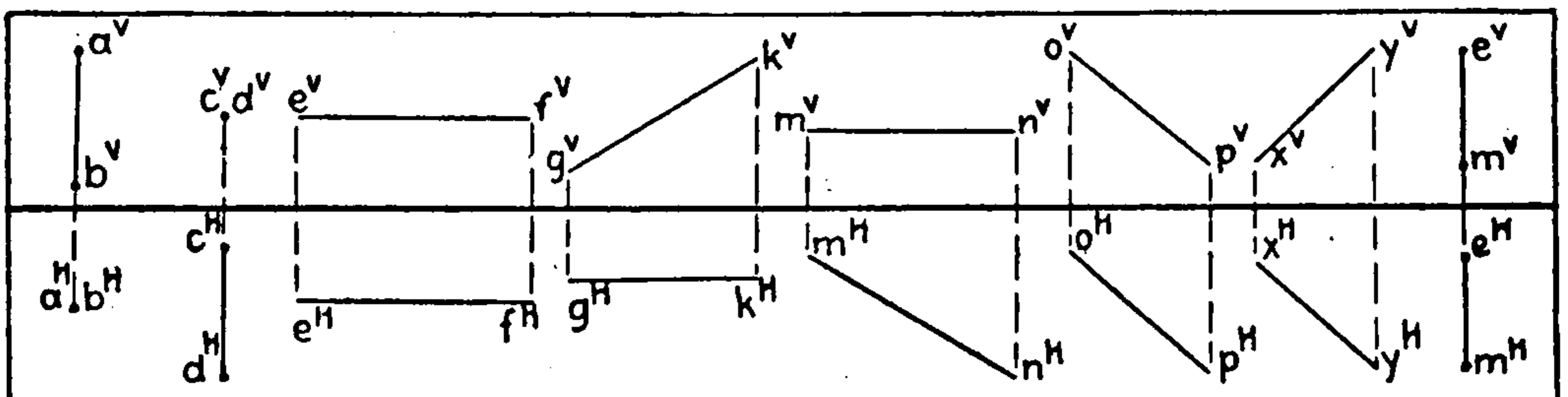
FIG. 84.—Development of truncated pyramid.

3. Lay off on the edges in this figure the true lengths of the various segments, and connect these points by straight lines, two and two.

4. Attach the bases A and B in their true sizes to the proper edges.

118.

### EXERCISES



Graphic Layout No. 46.—Center lines for regular solids.



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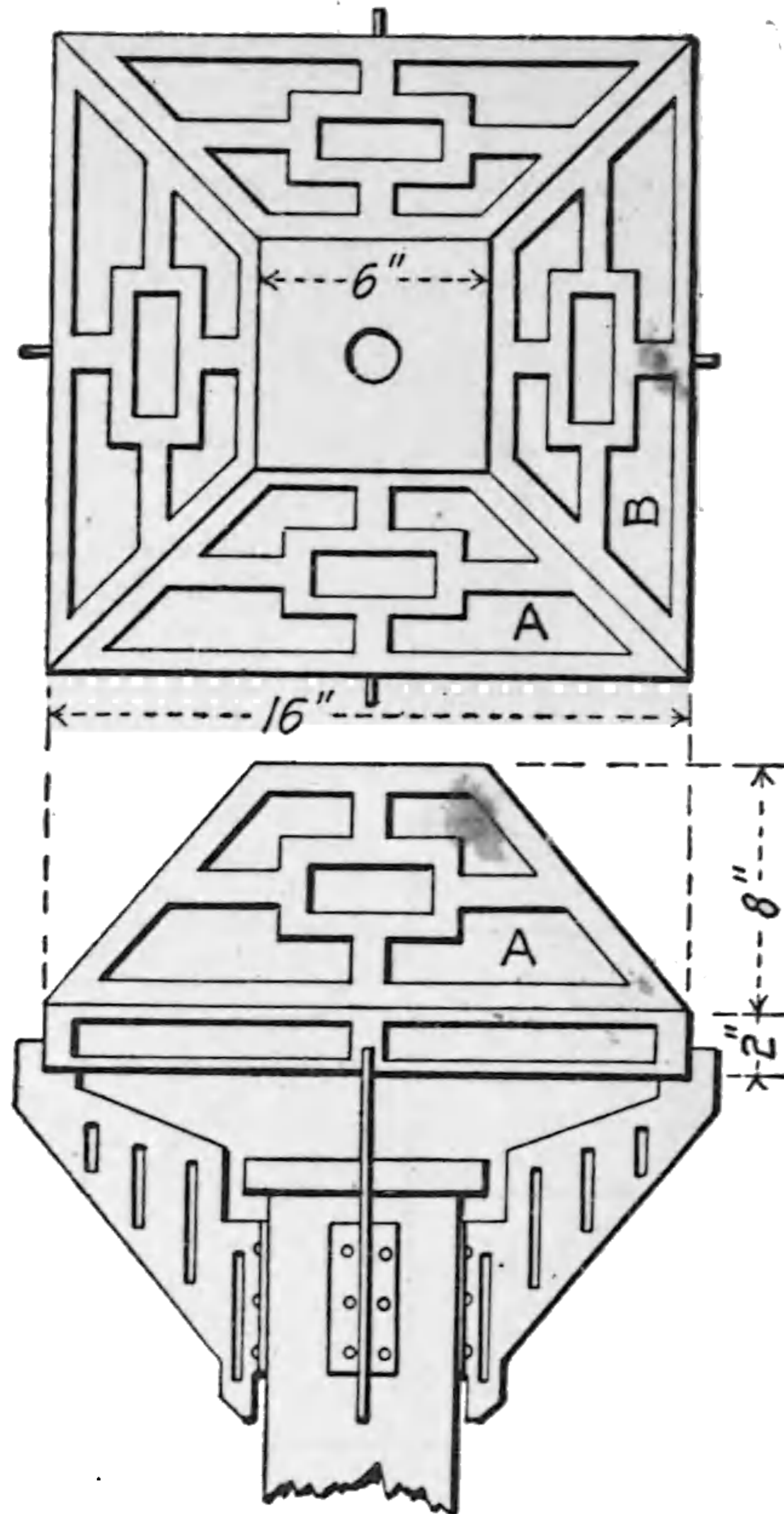


FIG. 85.

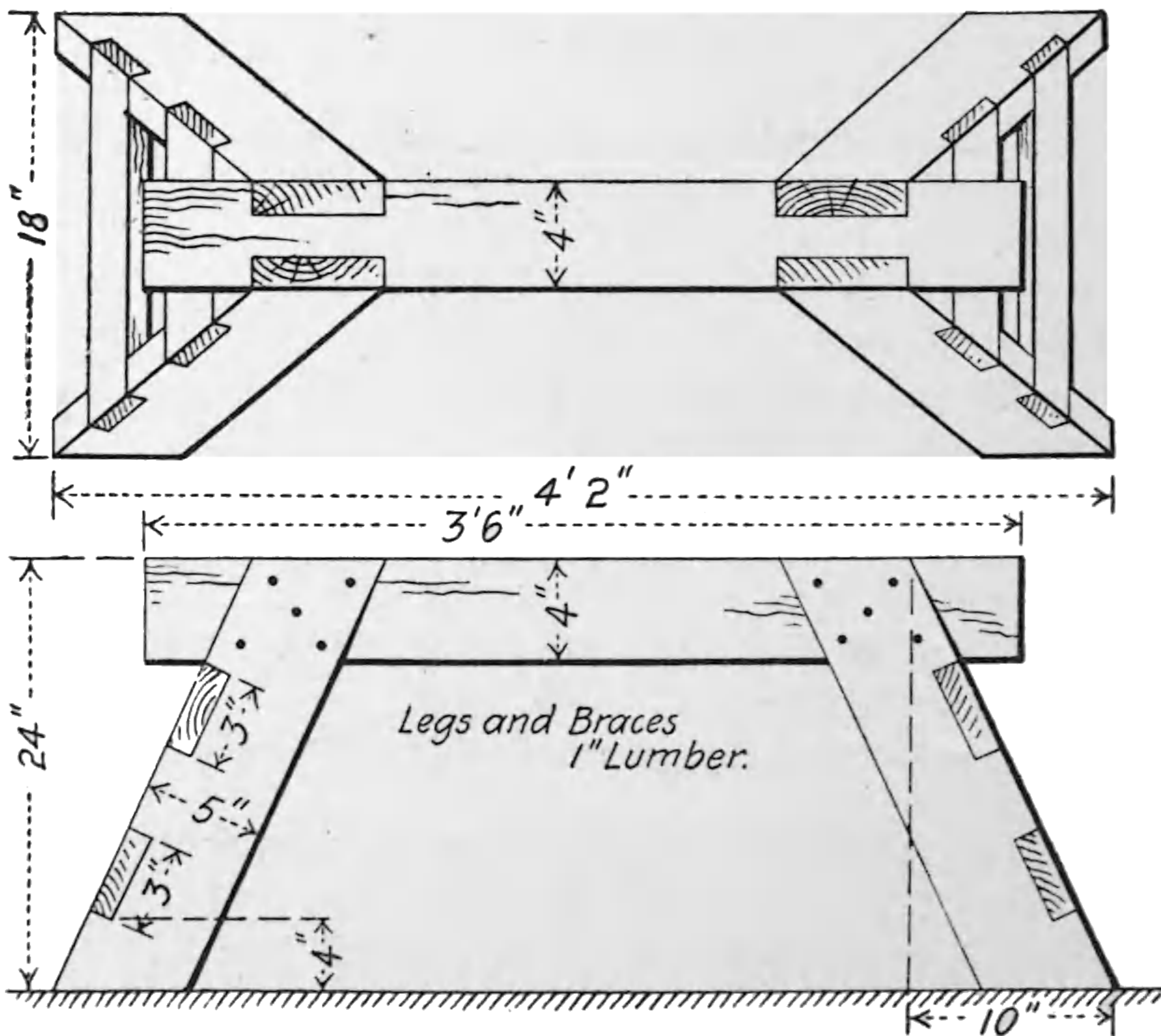


FIG. 86.—Saw horse.

691. Make a detail of the backbone of the saw horse shown in Fig. 86, showing the angles of the cuts to be made. Draw to any convenient scale.

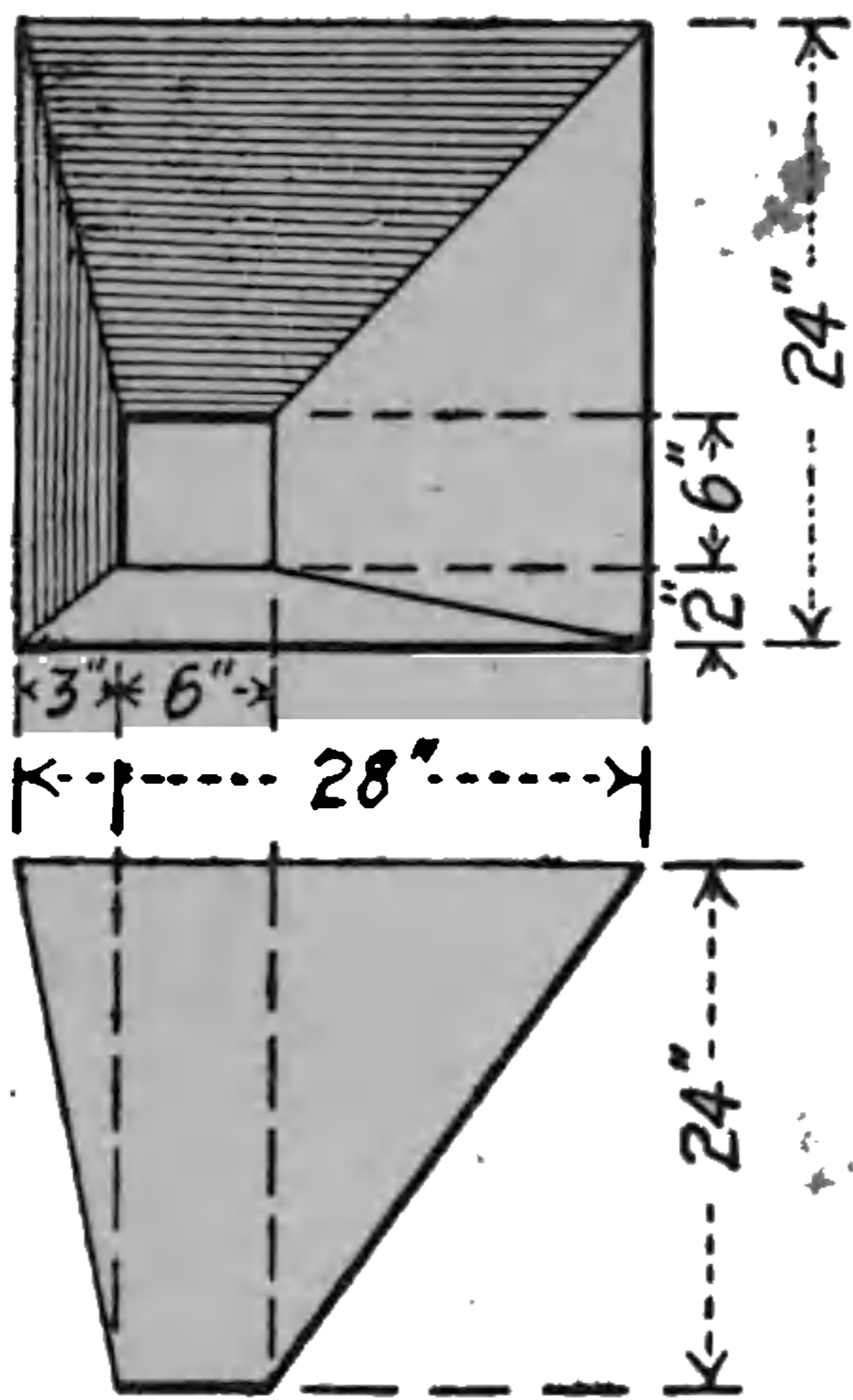


FIG. 87.—Grain hopper.

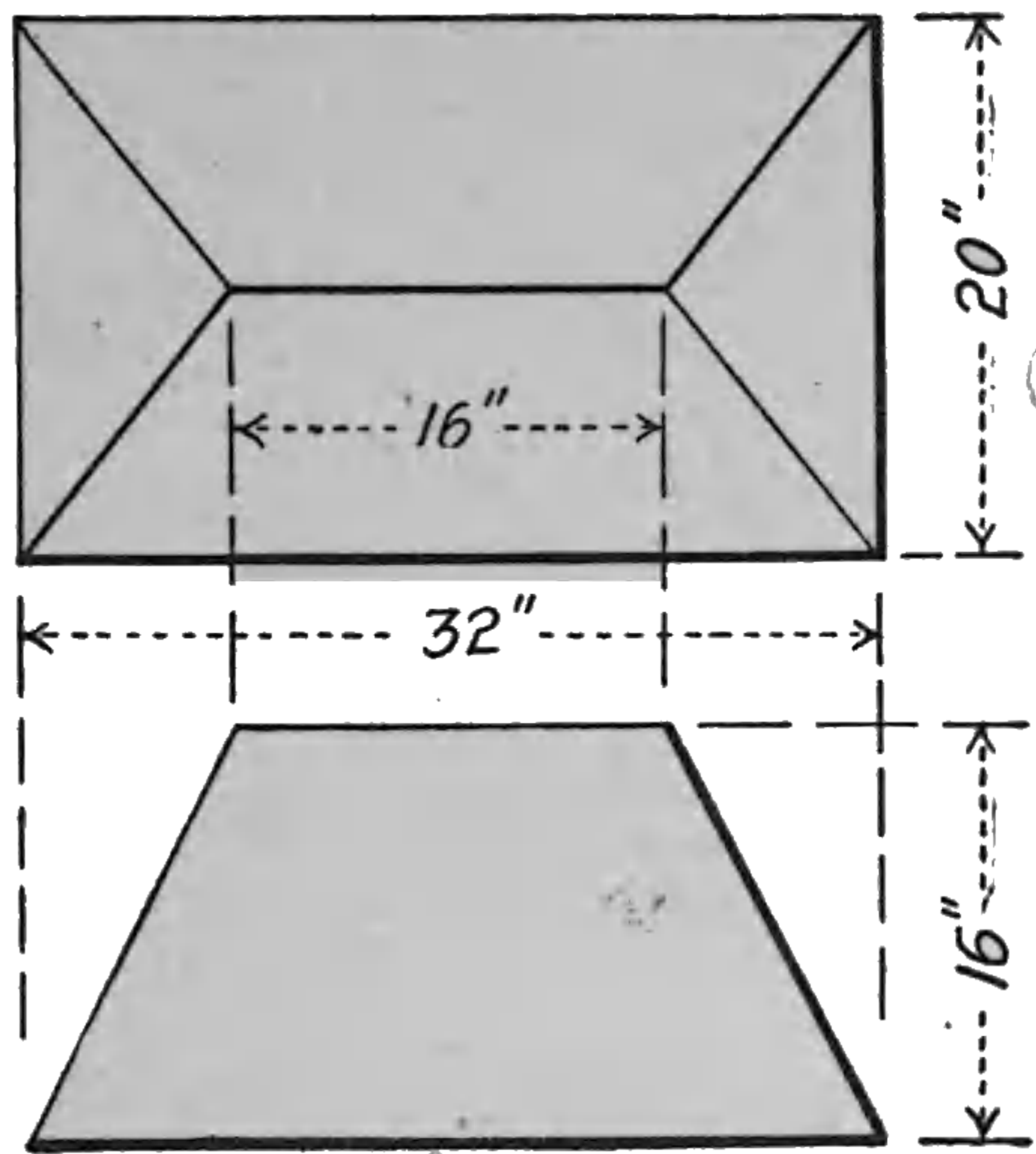


FIG. 88.—Ventilator cap.

692. Make detail drawings of the legs and braces of the saw horse, shown in Fig. 86, showing the angles of the various cuts. Draw to any convenient scale.

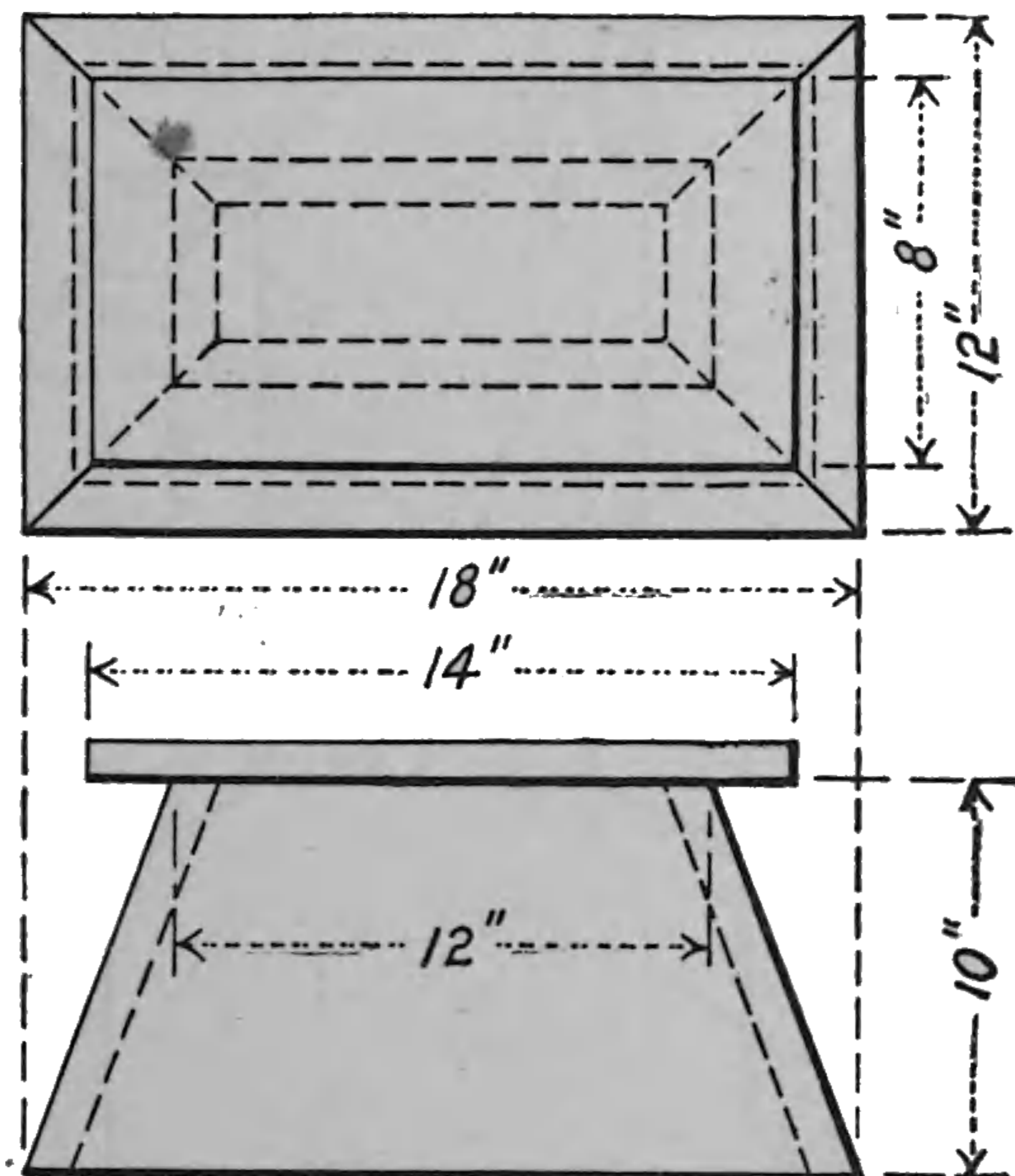


FIG. 89.—Stand.

693. Make a pattern to any convenient scale of the grain hopper, shown in Fig. 87. Use two spaces for the projections and pattern. Make a paper model of the hopper, putting on a flap for holding together.

694. Make a pattern to any convenient scale, of the ventilator cap, shown in Fig. 88.

695. Fig. 89 shows a stool, or plant stand, made of 1-in. lumber. The edges of the four inclined supporting pieces are chamfered to fit. Make a working drawing with full details to scale, showing the angles

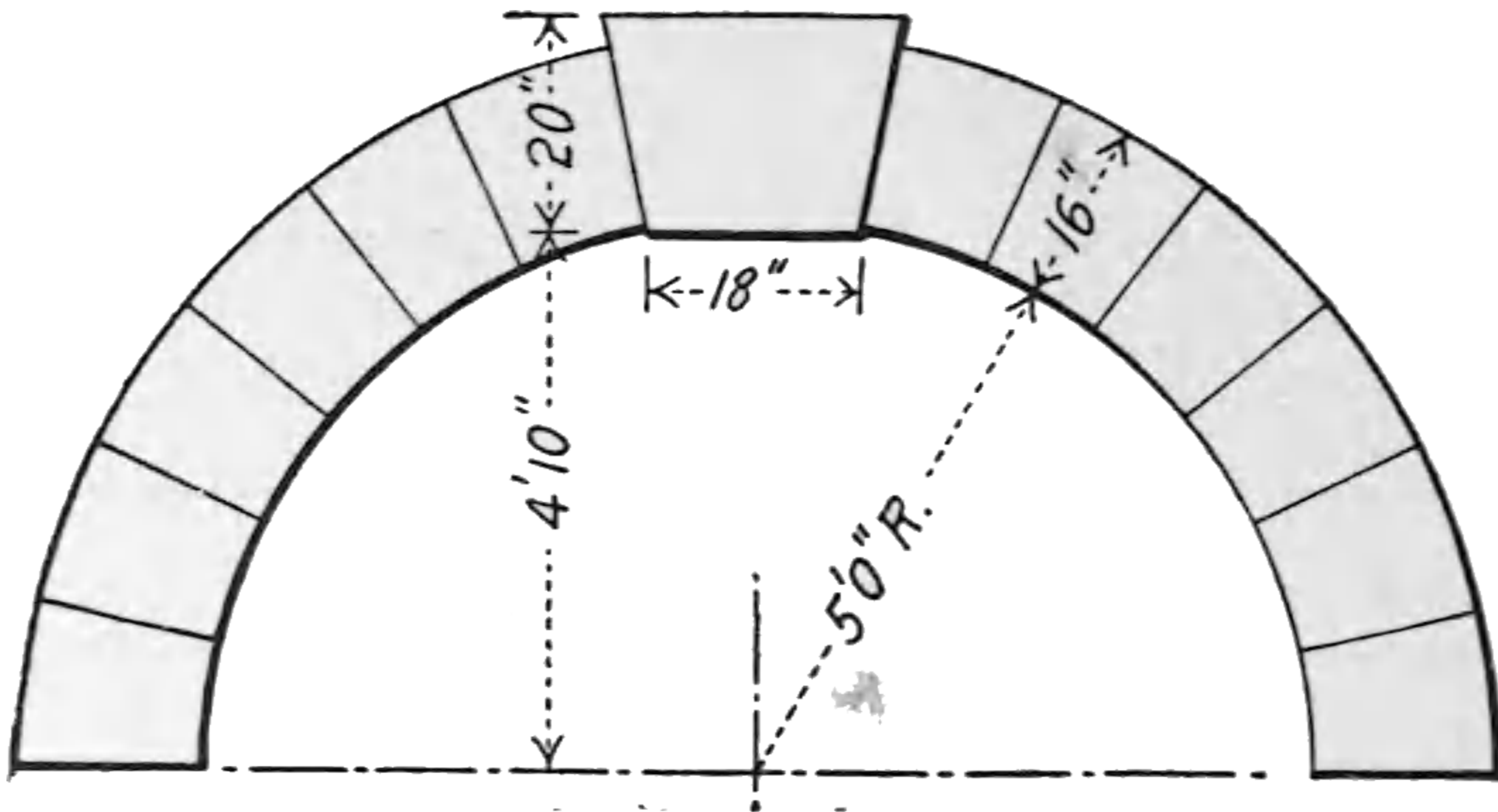


FIG. 90.—Stone arch.

of the chamfered pieces, without duplicating any of the parts. Take two spaces for the work.

696. Develop the surface of the keystone of the arch shown in Fig. 90. The masonry is 12 in. thick. Use any convenient scale.

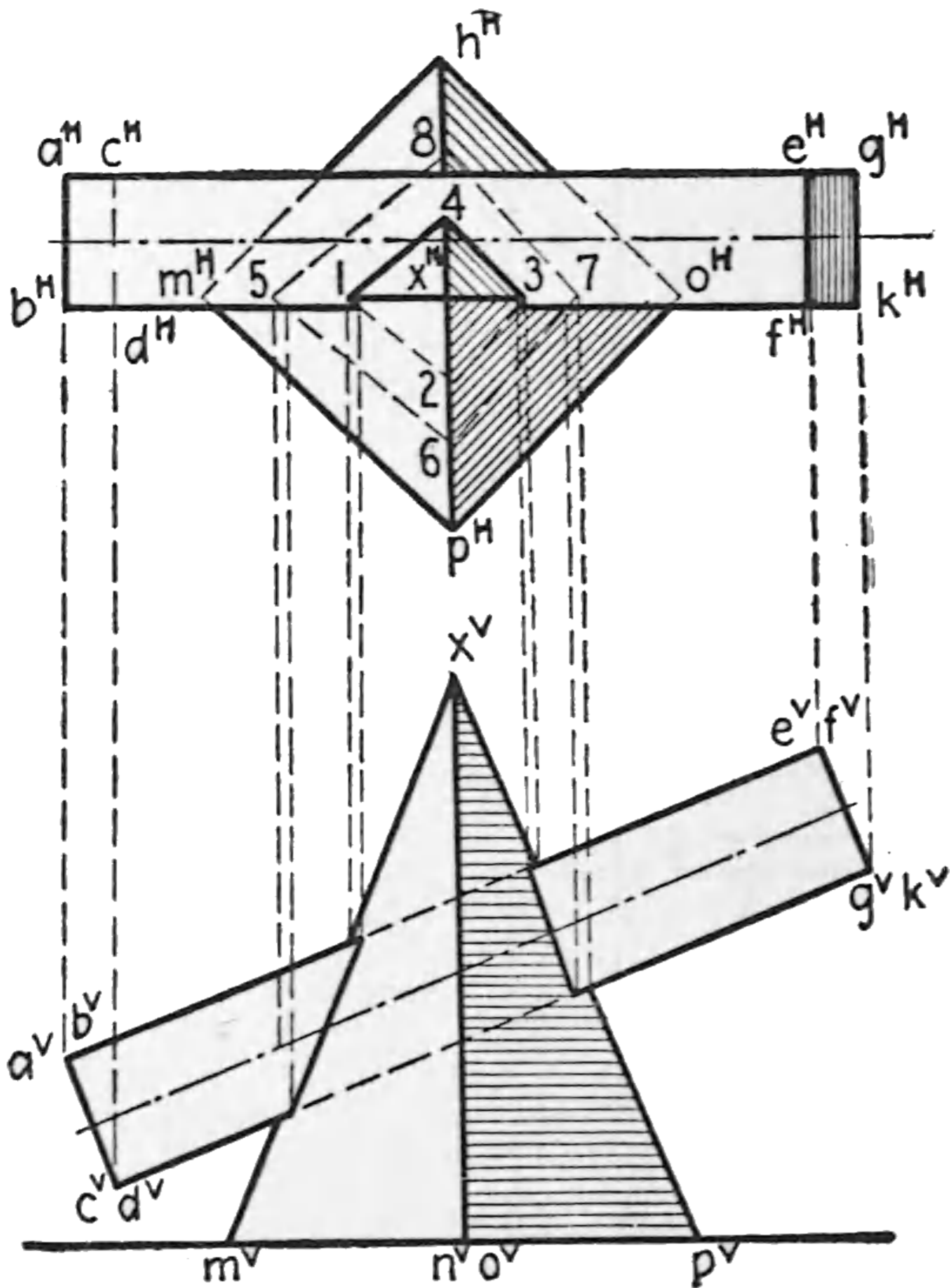


FIG. 91.

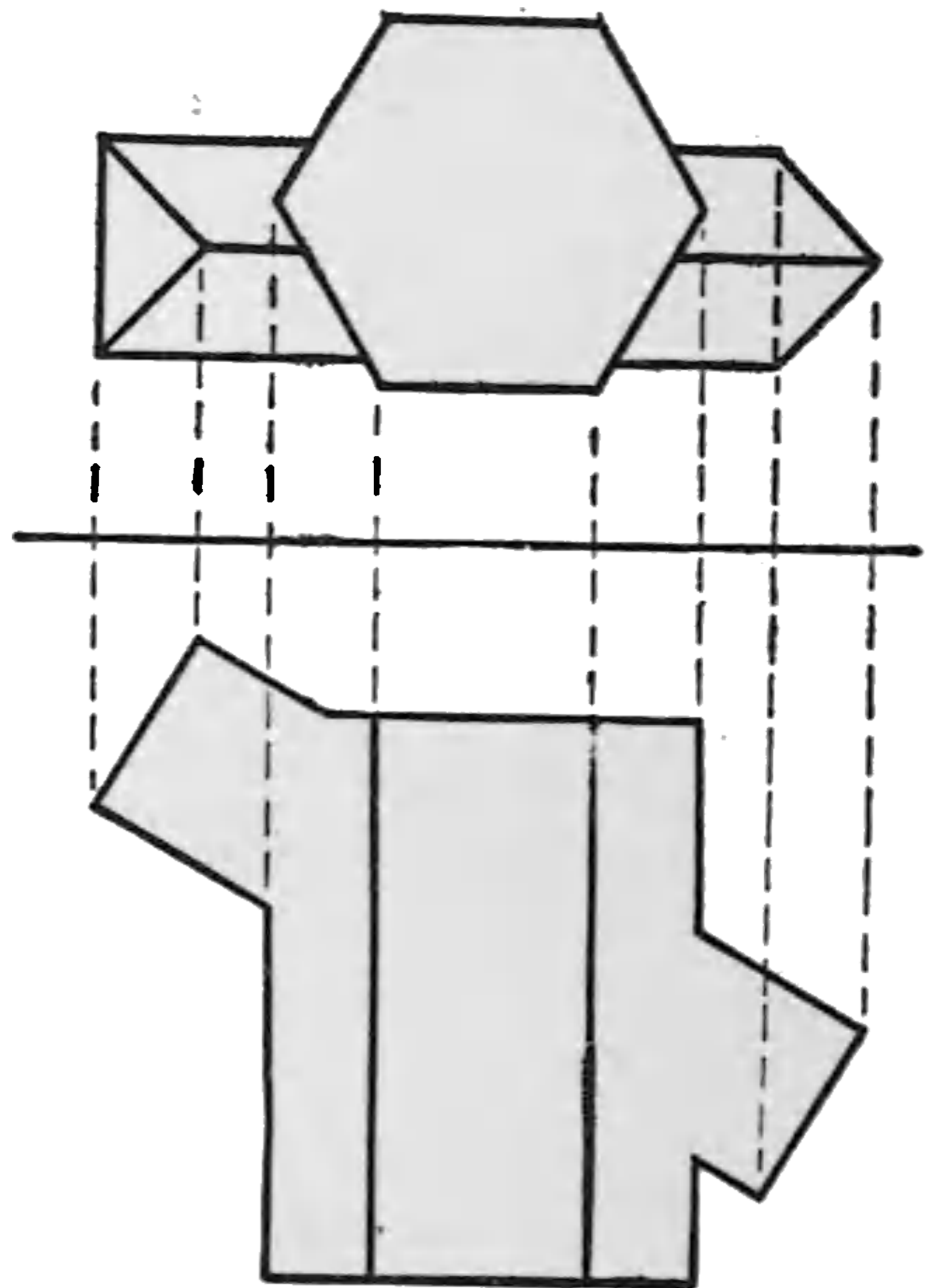


FIG. 92.

**119. Problem 42.—To find the line of intersection between two plane solids.**

**Construction.**—To find the line of intersection between the square pyramid and square prism, shown in Fig. 91.



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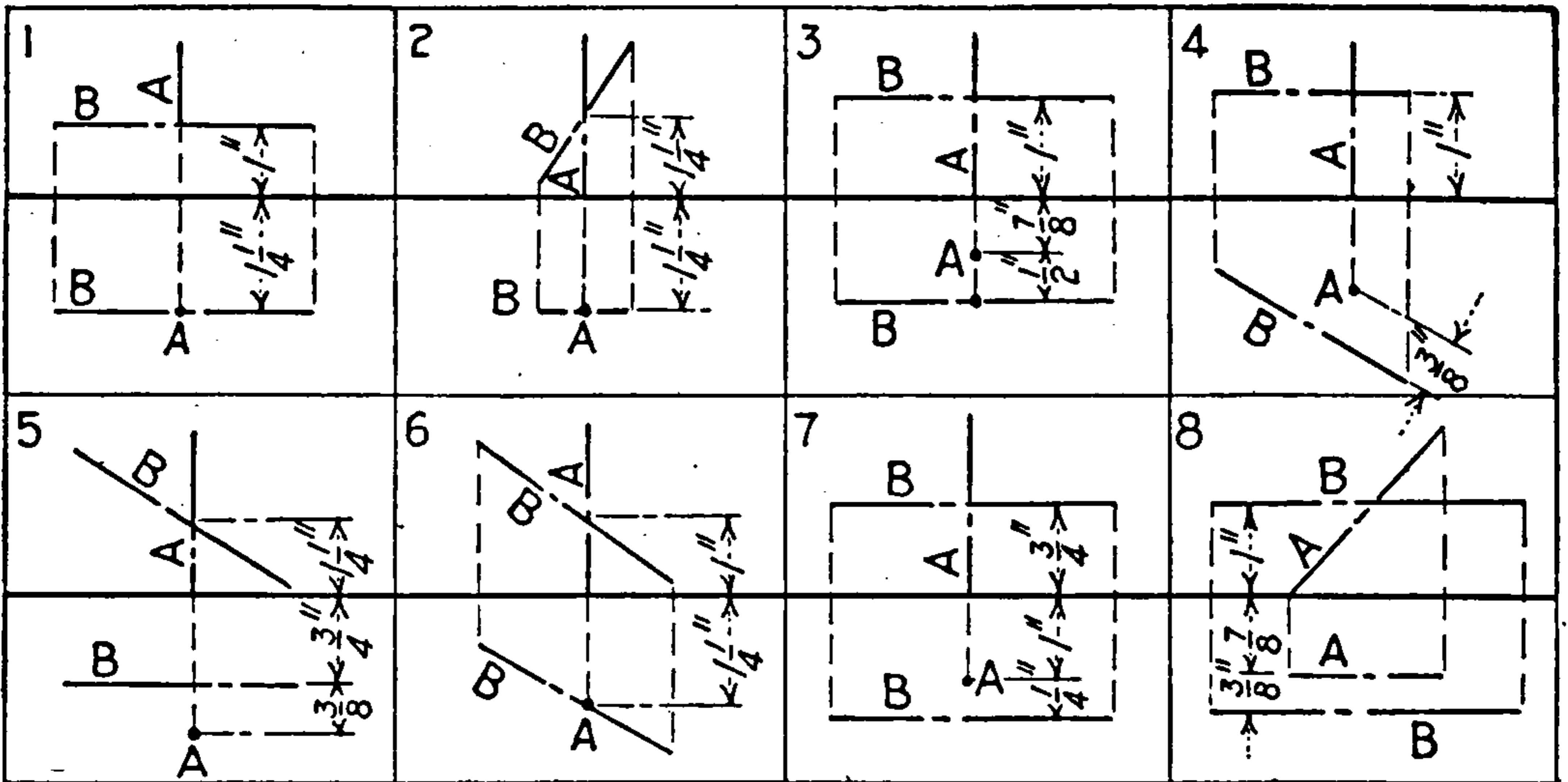
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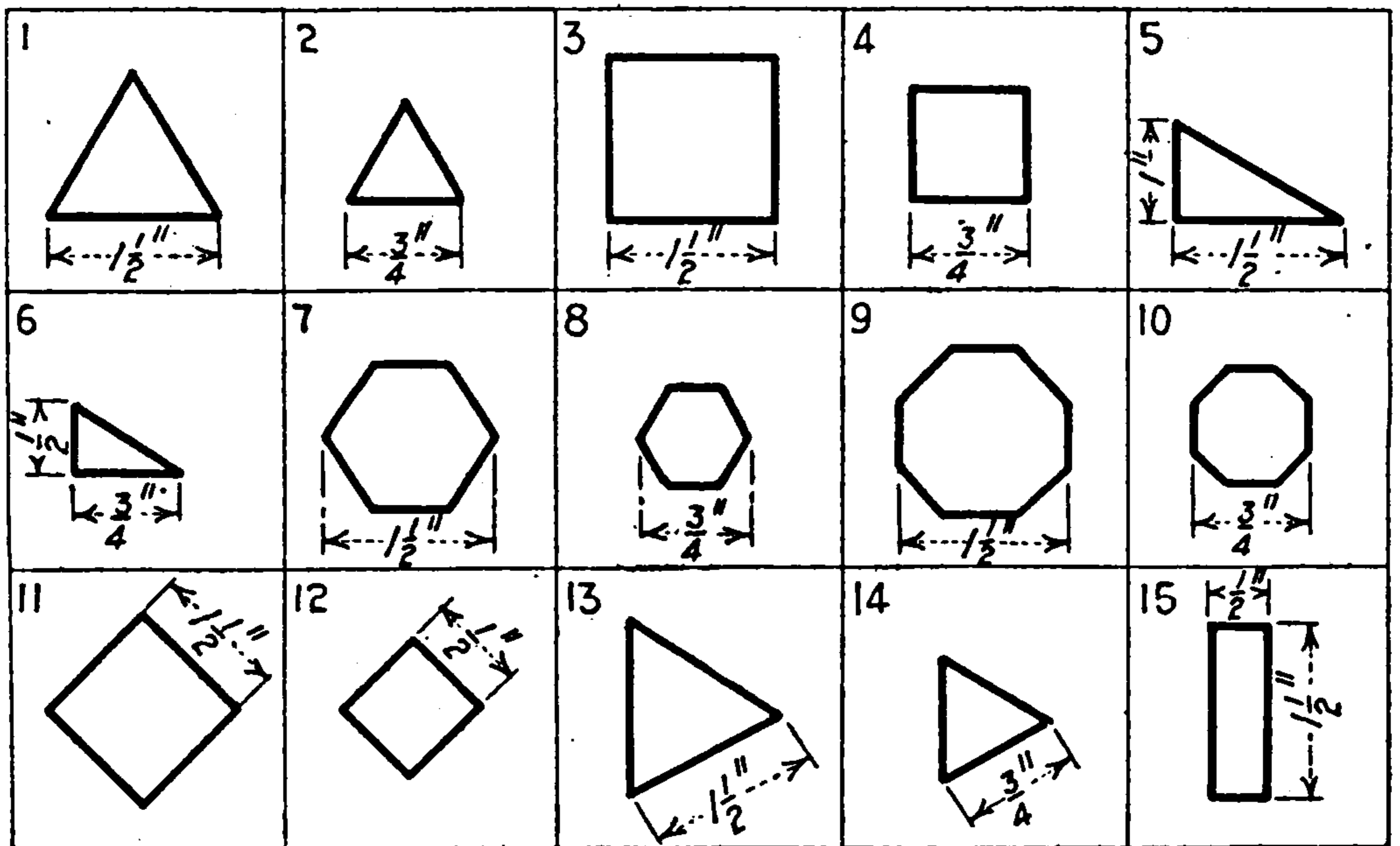
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121.

EXERCISES



Graphic Layout No. 48.—Center lines for intersecting solids. Letter to suit, locating in *I* or *III*. Another pair (9) can be made like (8) with axes intersecting.



Graphic Layout No. 49.—Bases for solids.

698. Let A — be the center line of a right pyramid, base —, altitude — in. (2,  $2\frac{1}{2}$ , 3), and B — be the center line of the right pyramid, base —, altitude — in. (2,  $2\frac{1}{2}$ , 3).

- (a) Draw the line of intersection.
- (b) Develop the surface of pyramid A.
- (c) Develop the surface of pyramid B.

699. Let A — be the center line of a right pyramid, base —, altitude — in. (2,  $2\frac{1}{2}$ , 3), and B — be the center line of the right prism, base —, altitude 3 in.

- (a) Draw the line of intersection.
  - (b) Develop the surface of pyramid A.
  - (c) Develop the surface of prism B.
700. Let A — be the center line of a right prism, base —, altitude 2 in. and B — be the center line of a right pyramid, base —, altitude — in. (2, 2½, 3).
- (a) Draw the line of intersection.
  - (b) Develop the surface of prism A.
  - (c) Develop the surface of pyramid B.
701. Let A — be the center line of a right prism, base —, altitude 2 in., and B — be the center line of a right prism, base —, altitude 3 in.
- (a) Draw the line of intersection.
  - (b) Develop the surface of prism A.
  - (c) Develop the surface of prism B.
702. Make a paper model of the two given solids.

CONCRETE FORM PROBLEMS

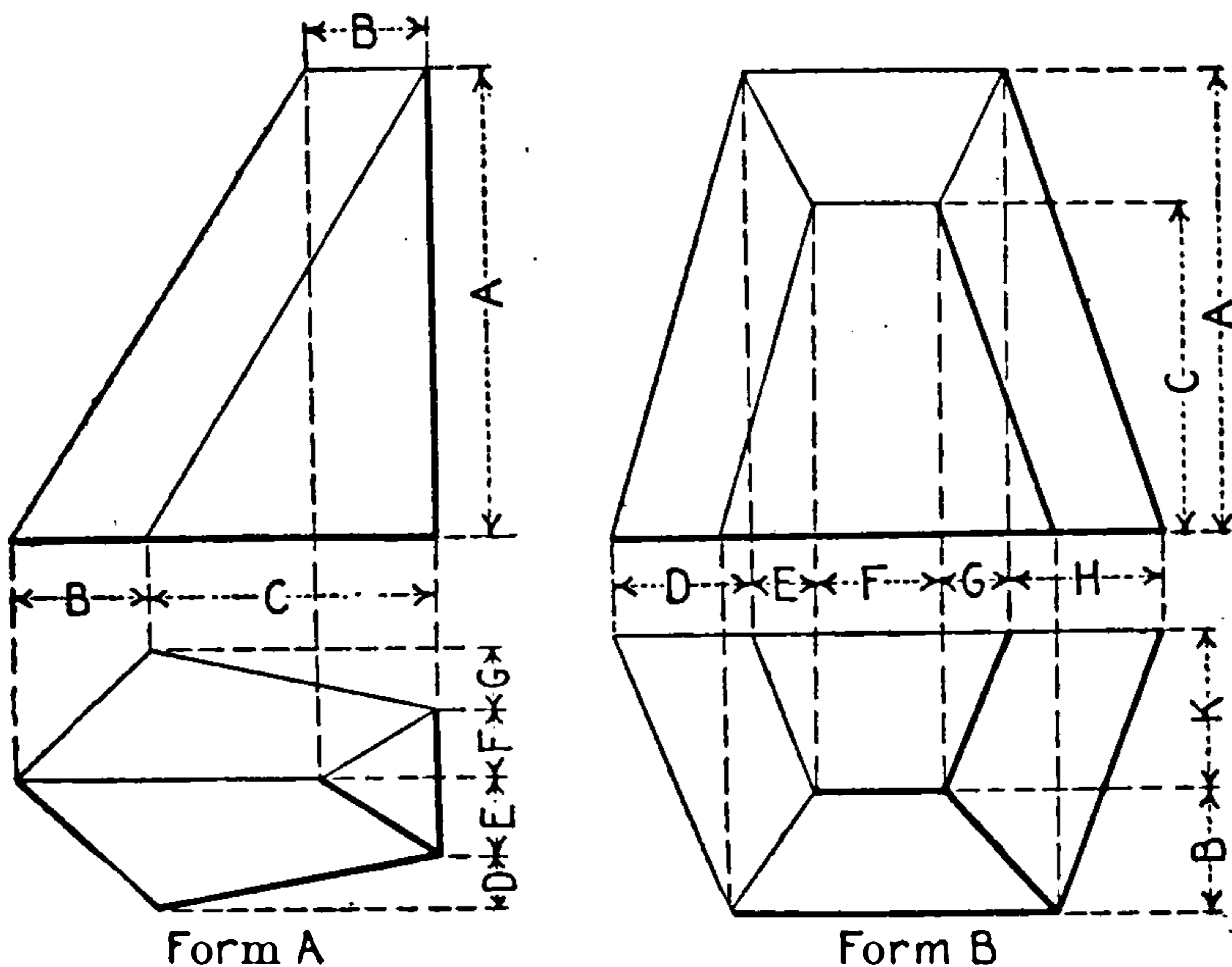


FIG. 95.

A	B	C	D	E	F	G	H	K
ft. 20	ft. 8	ft. 16	ft. in. 3	ft. in. 4	ft. in. 4 6	ft. in. 2 6	ft. in. 3	ft. 8
18	7	15	2 9	3	5	3 6	2 9	6
16	6	13	2 6	2 6	3	2 9	2 6	5
12	5	9	2	2	3	2	2 3	4

703. Develop the patterns for the concrete form — (A or B), dimensions — (1, 2, 3, or 4), constructed by the Santa Fe Railroad (dimensions altered).



704. In laying out the timbering for the gable, shown in Fig. 94, use sizes selected from the following list:

4 in. × 6 in.	6 in. × 6 in.	8 in. × 8 in.	10 in. × 10 in.
4 in. × 8 in.	6 in. × 8 in.	8 in. × 10 in.	10 in. × 12 in.
4 in. × 10 in.	6 in. × 10 in.	8 in. × 12 in.	10 in. × 14 in.
4 in. × 12 in.	6 in. × 12 in.	8 in. × 14 in.	10 in. × 16 in.

Dimensions of the gable may be altered, if desired. Detail each piece, laying off the angles of the trim, with full details of measurements. Scale to suit.

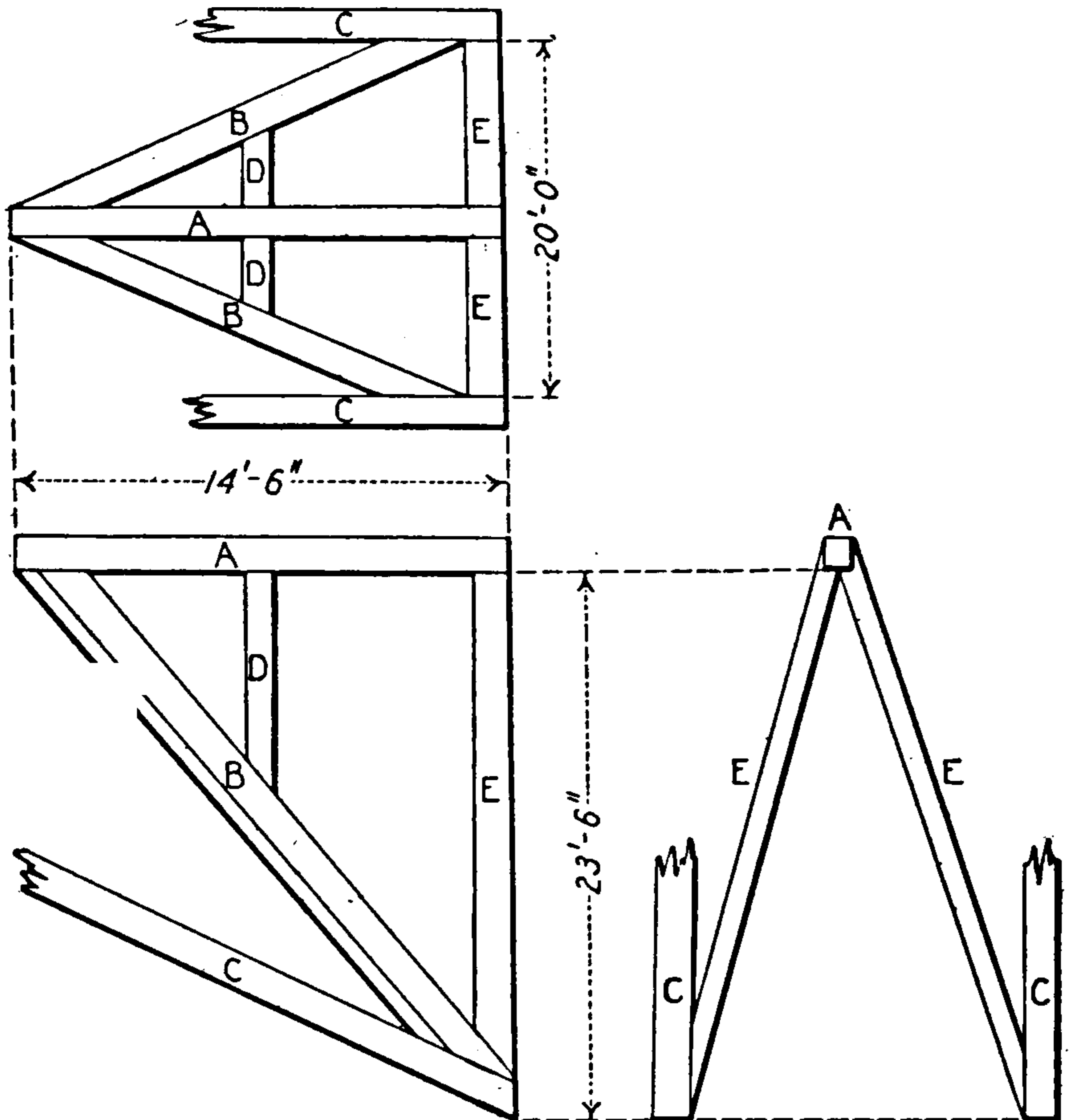


FIG. 94.—Timbering of a gable in Main Building, Armour Institute of Technology.

705. Two timbers, whose center lines are A(4 ft. + 10 ft. - 2 ft.) B(18 ft., 0 - 5 ft. 6 in.) and C(4 ft., 0 - 8 ft.) D(18 ft. + 10 ft. - 8 ft.), are connected by the shortest possible timber brace, notched to fit. Select the timber sizes from the foregoing list, and draw the projections of the three pieces in assembly and detail, with full dimensions of notches and trims. AB and CD are trimmed to fit horizontal timbers. Scale  $\frac{1}{4}$  in. = 1 ft.
706. Using the same data as Ex. 705, make the crossbrace the shortest possible horizontal timber.



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710. Develop the patterns for the hopper elbow —, in — (2, 3, 4) pieces (Fig. 98). Any convenient scale.
711. A rectangular air pipe, 10 in. × 12 in., follows the center line A(3 ft. + 6 ft., 0) B(3 ft. + 6 ft. - 2 ft.) C(7 ft. 6 in. + 2 ft. - 2 ft.) D(7 ft. 6 in., 0 - 2 ft.), coming through the wall horizontally, running down-

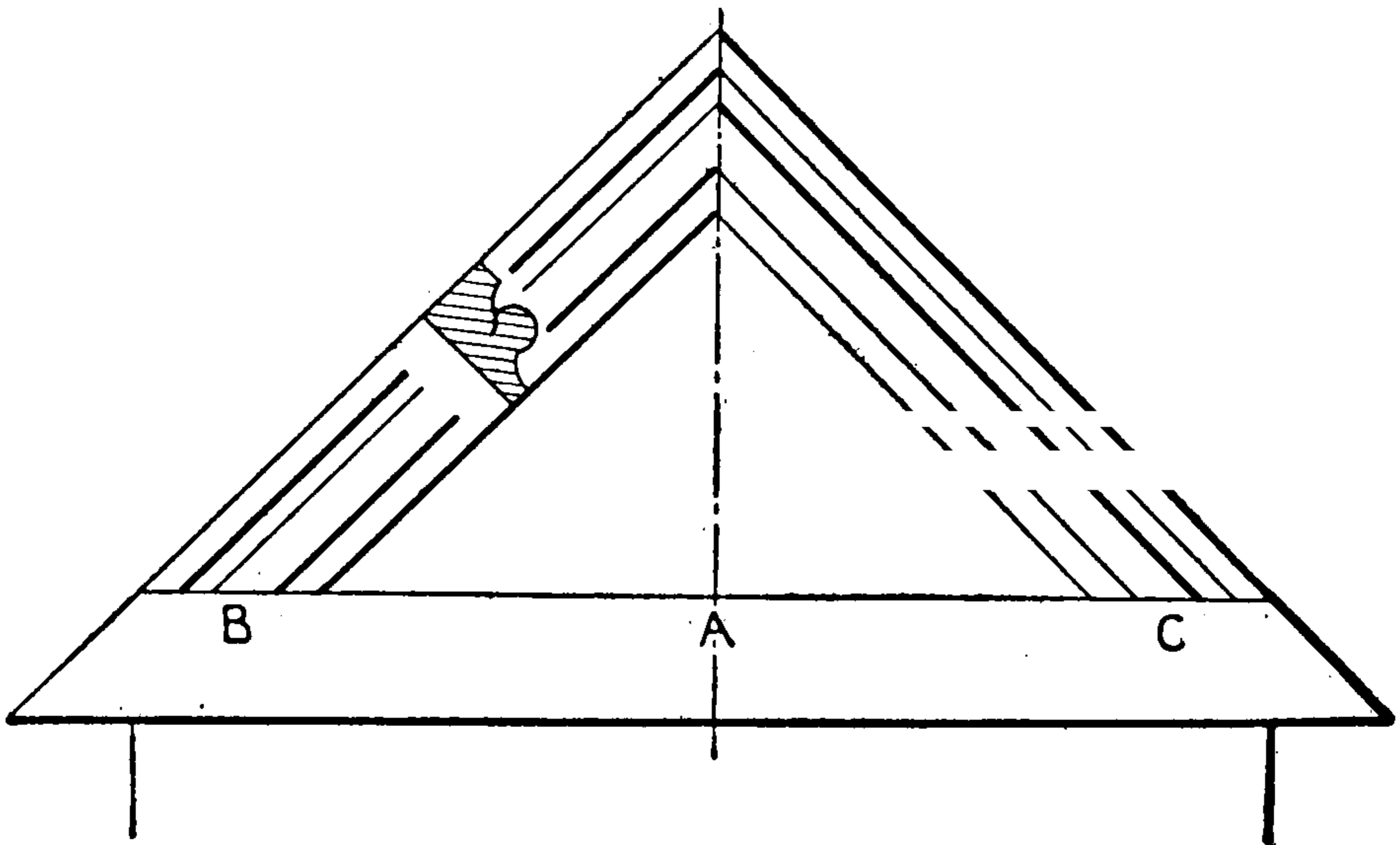
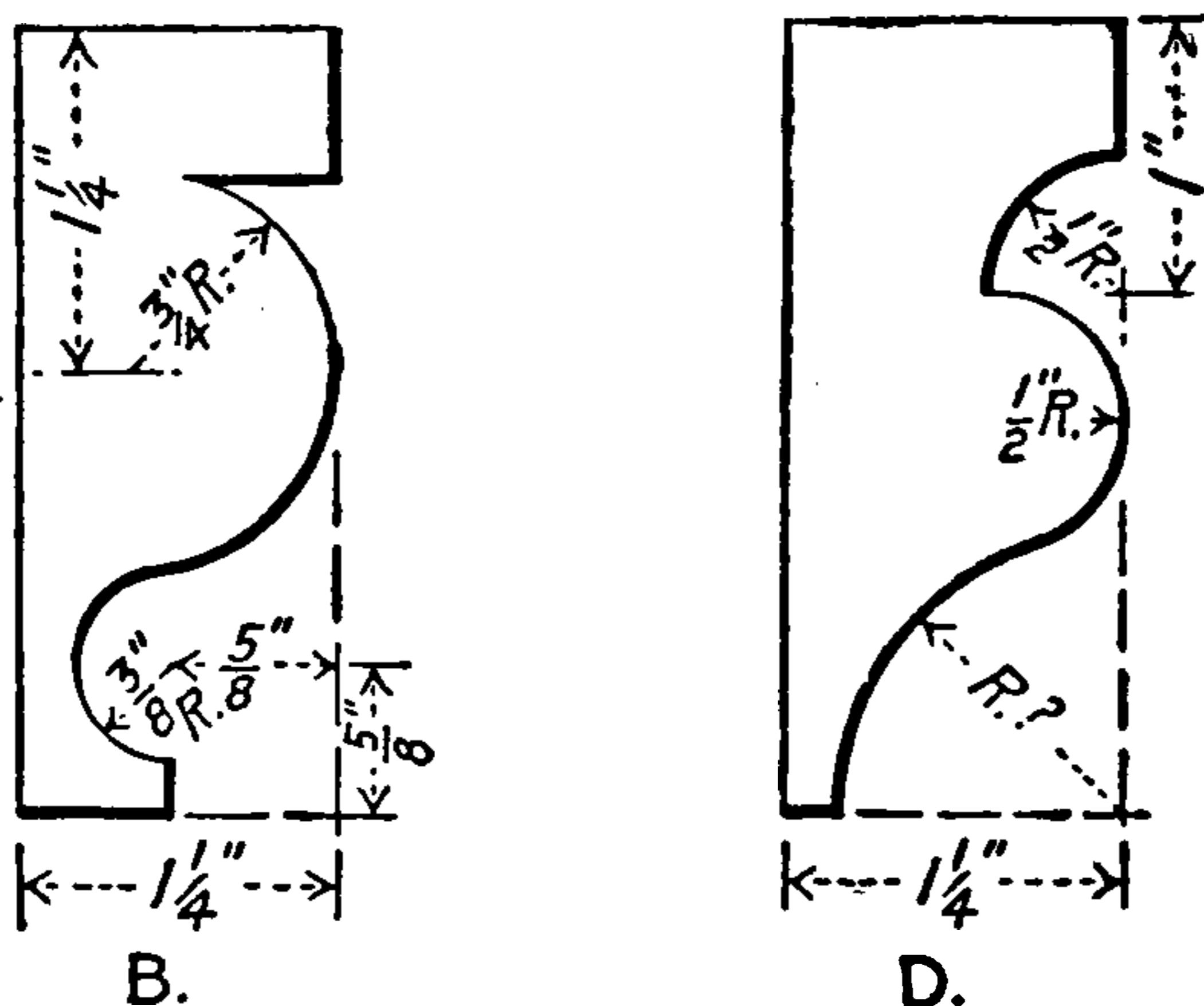


FIG. 99.

ward obliquely, and then down through the floor. Keep the cross-section uniform except at the turns. Lay out the joints in projection and lay out the patterns for the piping. Scale  $\frac{1}{2}$  in. = 1 ft.

712. Fig. 99 shows a  $45^\circ$  gable, with "Water Table" A inclined at  $30^\circ$ , using moulding "D," Fig. 100, multiplying each dimension by 3. Required



Moulding Sections

FIG. 100.

the line of intersection of the moulding with the water table at points B and C. Also, lay out the pattern for this moulding in sheet-metal work. (Note.—Develop as a cylinder, and show the shape of each end in development).

713. Fig. 101 shows a 45° gable, using moulding "B," Fig. 100, multiplying each dimension by 4, with half-round pilasters as shown at C and D. Draw the line of intersection in regions E and F. Lay out the patterns for this moulding in sheet metal. Show each end in the development. Lay out also the patterns for the pilaster in sheet metal. Any convenient scale.

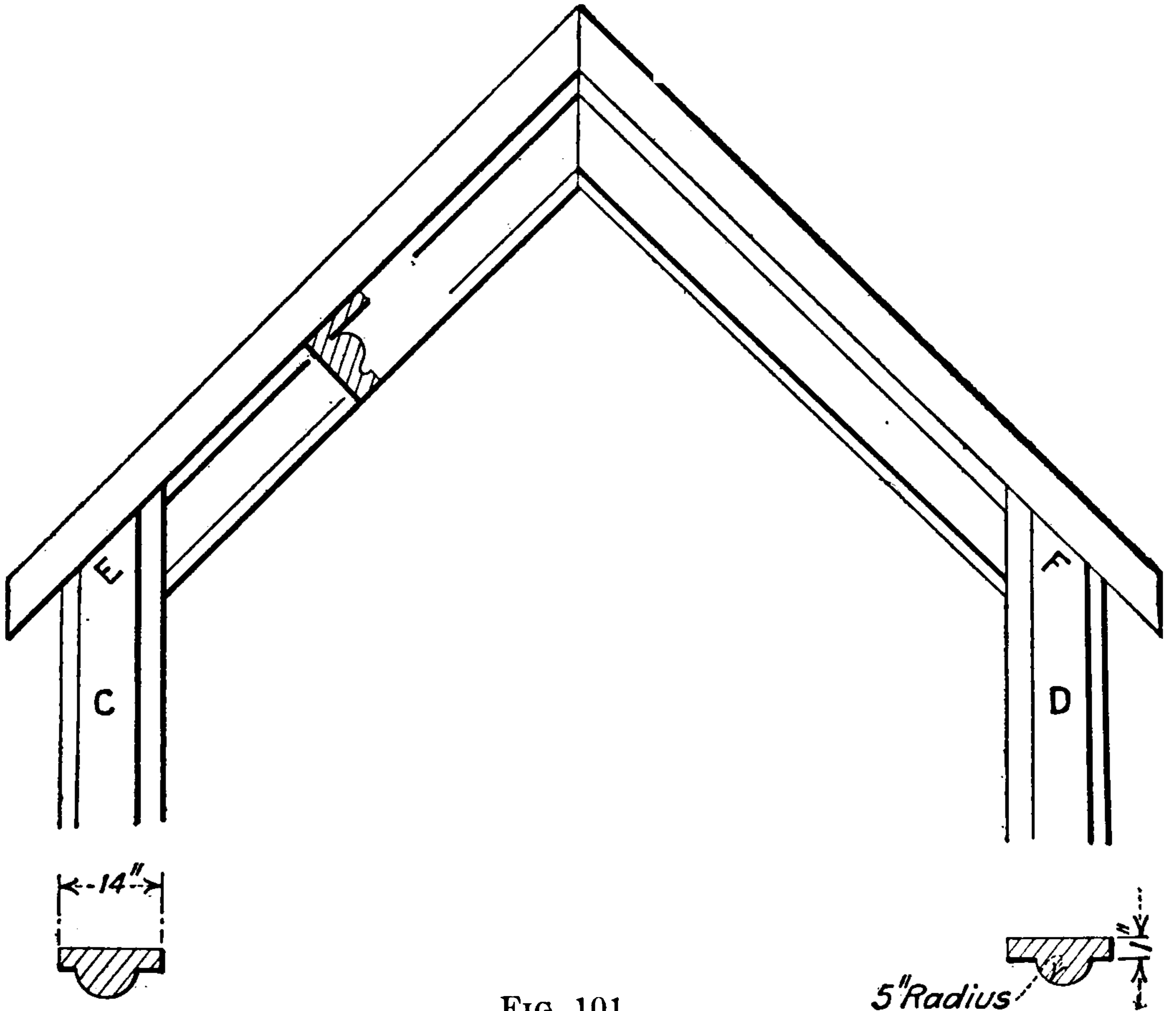


FIG. 101.

5" Radius

## CHAPTER IV.

### SHADES AND SHADOWS

**122.** One of the interesting applications of Descriptive Geometry is the determination of the shades and shadows of objects. Catalogue cuts and architect's drawings employ it most extensively, but on many occasions it will be found useful in shop drawings.

#### **123. Definitions.**

**Shade** is that part of an object not exposed to the direct rays of light.

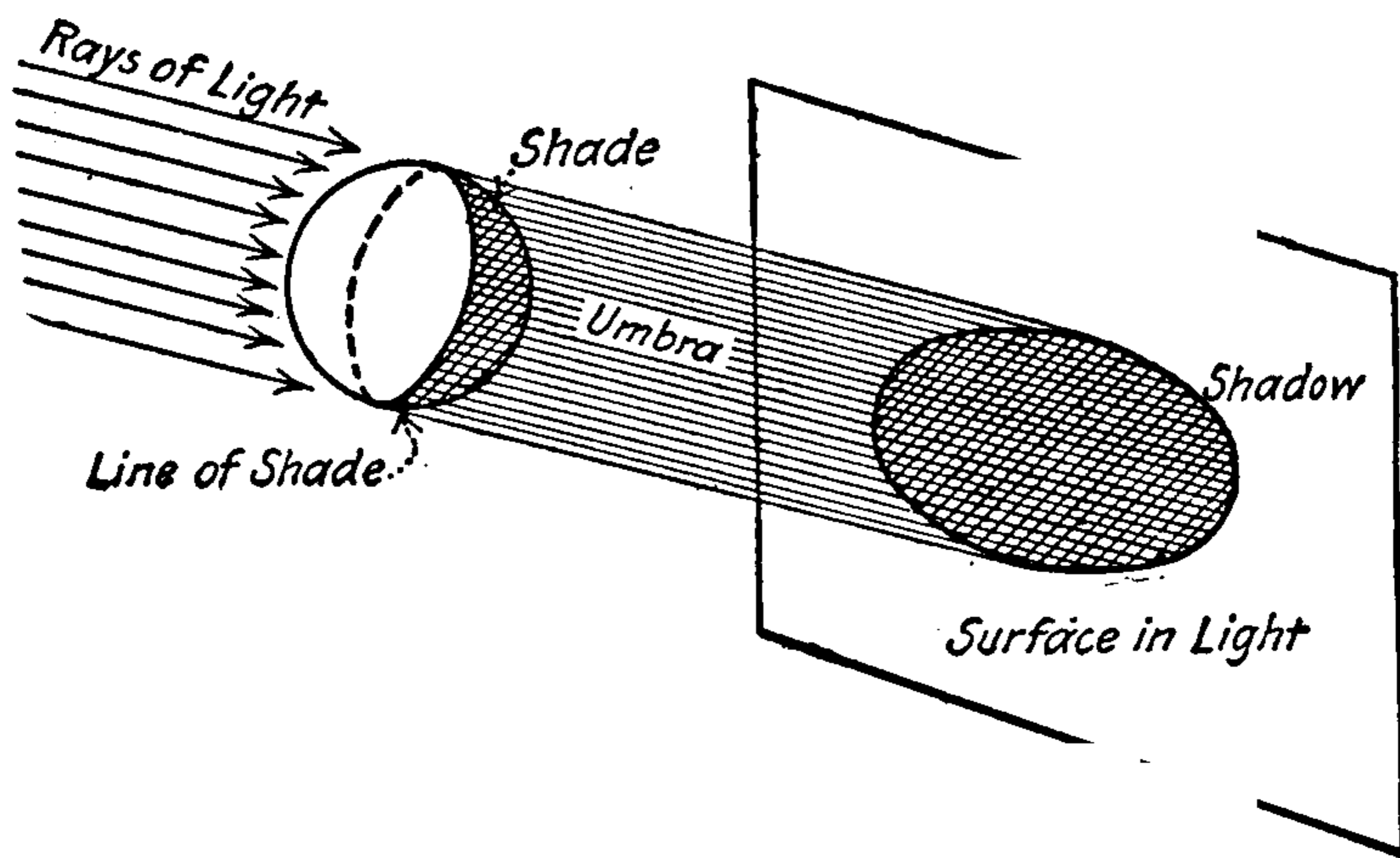


FIG. 102.

**Shadow** is that part of a surface in light from which rays of light are excluded.

**Umbra** is the unlighted space behind an object in light.

Note.—Shadow is the intersection of the umbra with a surface in light; that is, it is the interrupted portion of an umbra by an interposed surface.

The **Line of Shade** is the imaginary line between the light and shaded portions of an object.

**124.** The sun is our source of light, and its rays are assumed to be straight lines, all parallel. This assumption is very nearly true, and the deviation of the rays is so very slight that no account need be taken of the error. For convenience in conventional



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2. Find the piercing point of the ray with  $H$  or  $V$ , the one it meets first.

**Conclusion.**— $O$  casts its shadow on  $V$  at  $P$ .

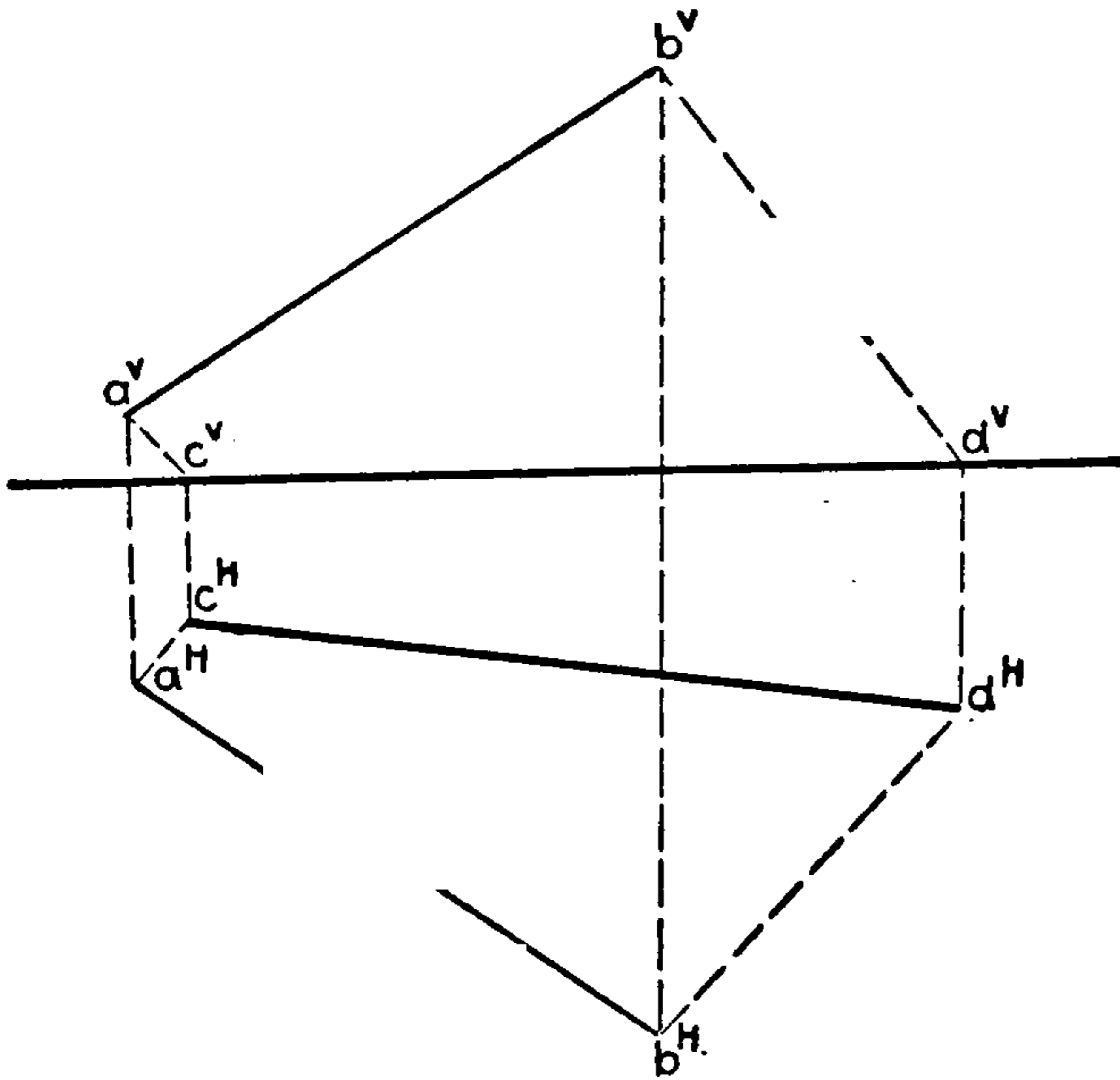


FIG. 104.—The shadow of a line.

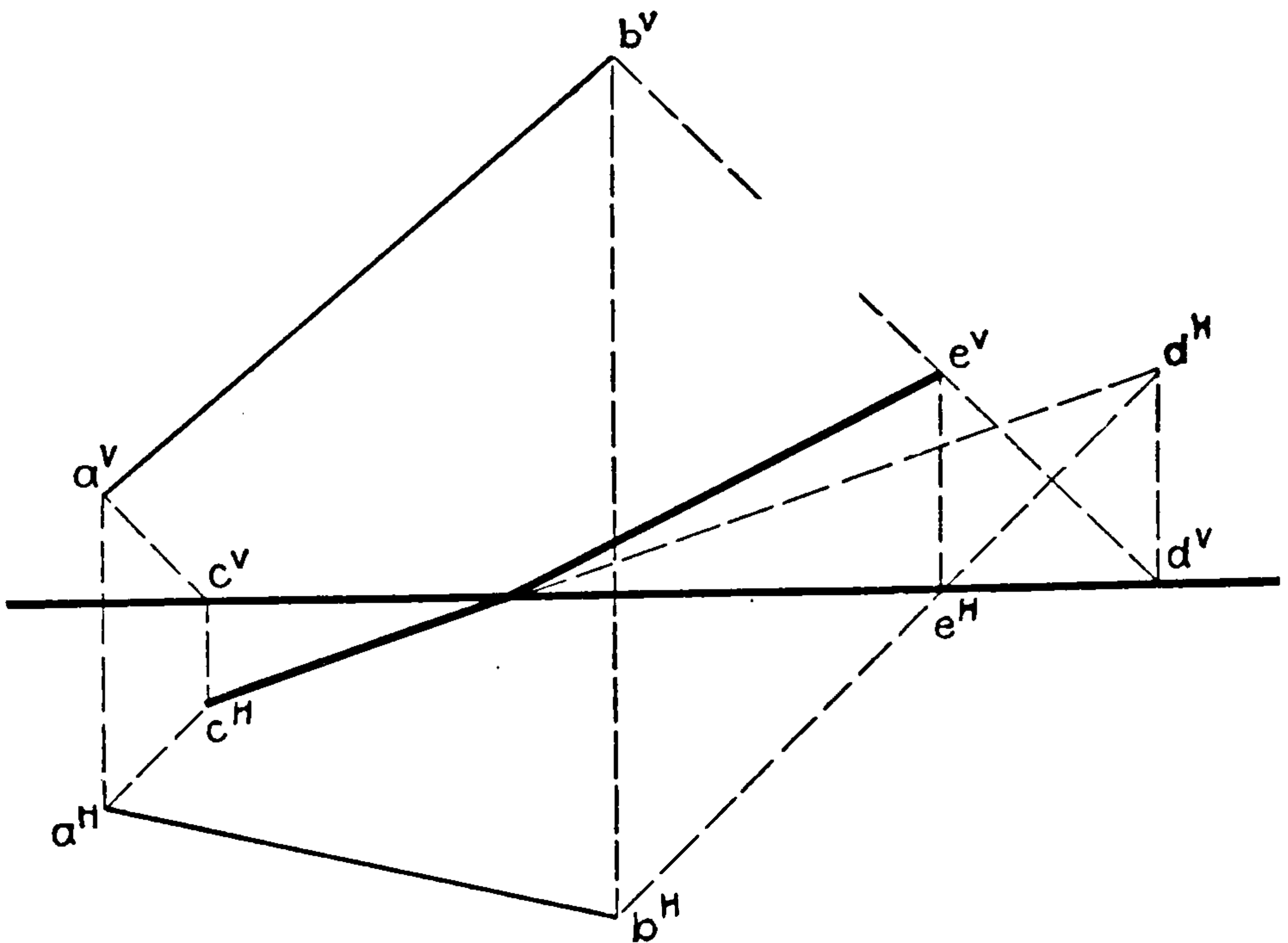


FIG. 105.—Shadow of a line on  $H$  and  $V$ .

**128. Problem 44.**—To find the shadow of a line.

**Construction.**—Let  $AB$ , Fig. 104, be the given line.

1. Find the piercing points,  $C$  and  $D$ , of  $A$  and  $B$ .

2. Draw  $CD$ ; it is the shadow on  $H$ .

**Special Case.**—When a line casts its shadow on more than one plane.

**Construction.**—Let  $AB$ , Fig. 105, be the given line.

1. Find its  $H$ -piercing points,  $C$  and  $D$ , and the  $V$ -piercing point,  $E$ , of  $B$ .

2. Draw  $c^H d^H$  until it meets  $GL$ , and connect it at this point with  $e^V$ .

**Note.**—The shadow line is the trace of the umbra plane, hence this shadow is the  $H$ -trace on  $H$ , and the  $V$ -trace on  $V$  of the plane  $ABCD$ .

**129. Problem 45.**—To find the shadow of a plane figure on a plane.

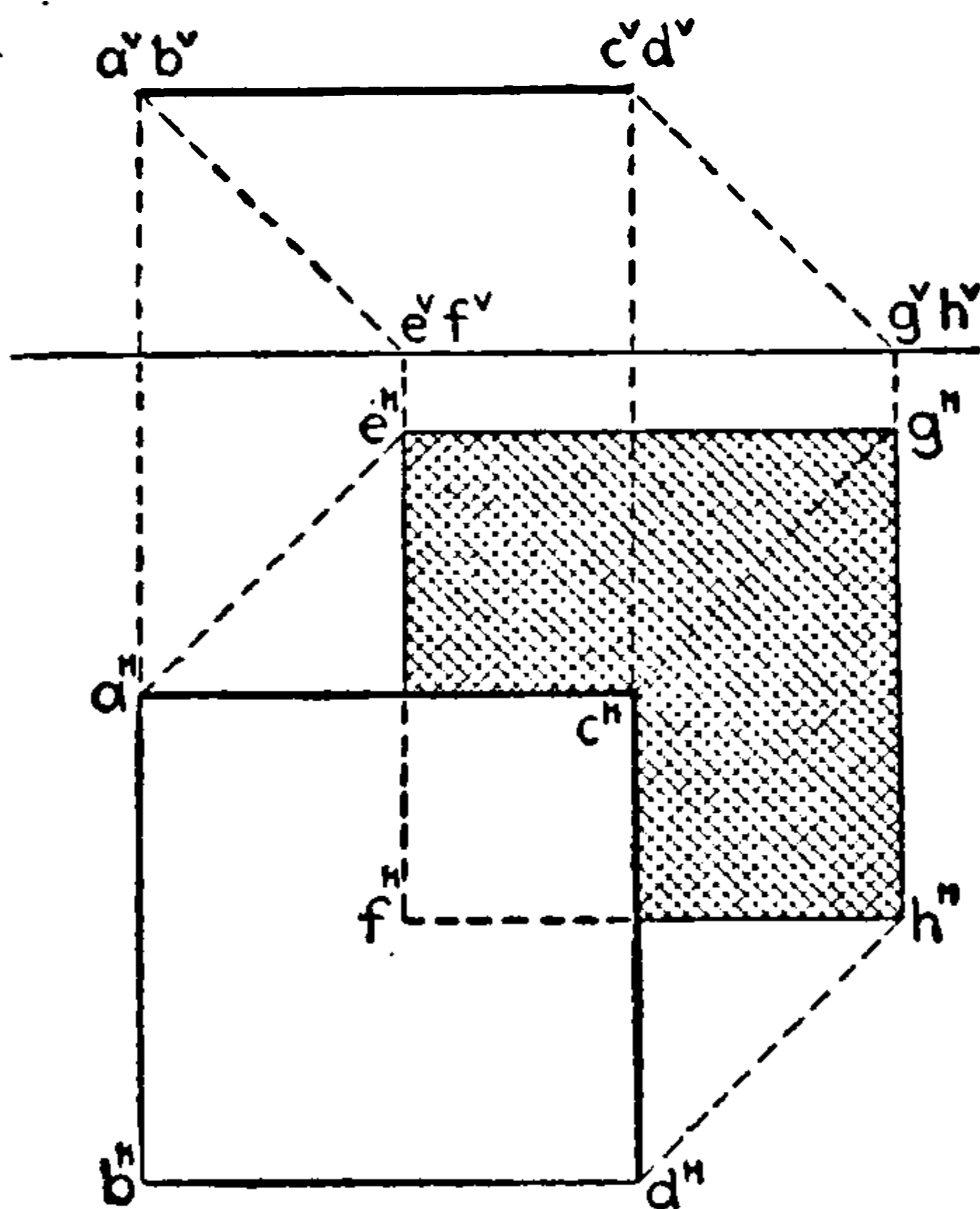


FIG. 106.—Shadow of a square.

**Construction.**—Let the square  $ABCD$  (Fig. 106) be the figure; to find its shadow.

1. Obtain the  $H$ -piercing points of the rays from the four corners,  $A$ ,  $B$ ,  $C$ , and  $D$ , at  $e^H$ ,  $f^H$ ,  $g^H$ , and  $h^H$ .

2. Join them in order, and fill in the enclosed space, except where it is covered (hidden) by the opaque object.

**Note.**—If the plane figure is parallel to the surface in light, its shadow will be an equal plane figure. Why?

**130. Problem 46.**—To draw the shadow of a solid on a plane.

**Construction.**—Let the cube shown in Fig. 107 be the given solid.

1. Draw rays from all the corners.

2. Disregard the ray through  $A$ , as it will be inside the umbra.



3. Obtain the  $H$ -piercing points of the rays from B, C, and D, at  $e^H$ ,  $f^H$ , and  $g^H$ .

4. Connect these points with the broken line, and fill in back to the  $H$ -projection of the cube.

Note.—The shadows of BM and DN are the lines  $m^H e^H$  and  $n^H g^H$ .

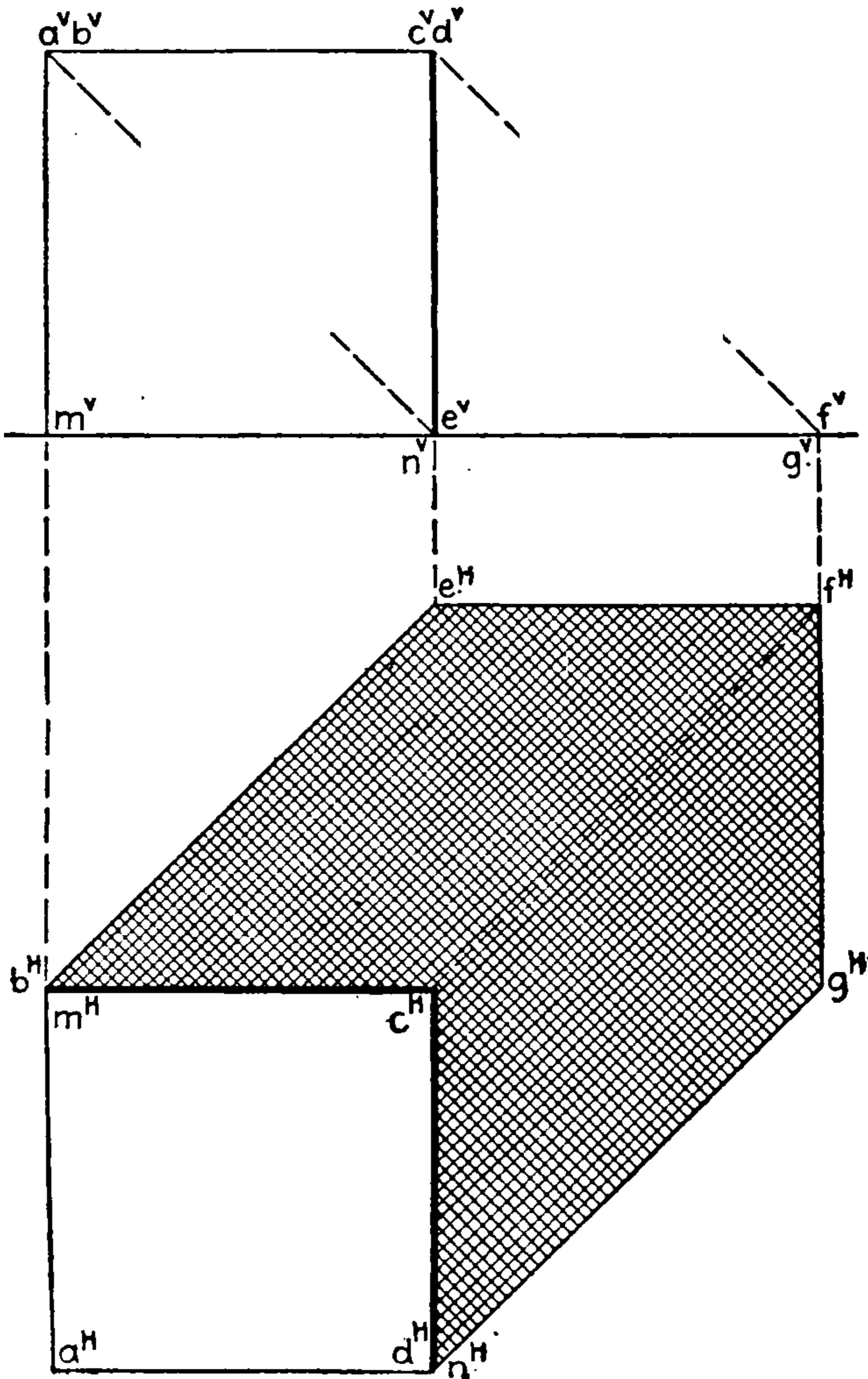


FIG. 107.—Shadow of a cube.

**Special Case.**—When the shadow falls on more than one plane.

**Construction.**—Let the hexagonal prism, Fig. 108, be the object. To find its shadow on  $H$  and  $V$ .

1. From all the exposed points draw the rays and obtain their piercing points. In this case it will be found that the point X is the only one to cast its shadow on  $H$ .

2. Obtain the shadow of MX on  $H$  and  $V$ ,  $y^H - n^V$ , and of AB on  $H$  and  $V$ ,  $a^H - c^H - c^V$  (Problem 44, Special Case).



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719. Hexagonal prism resting on  $H$ , center line  $G(1\frac{1}{2} + \frac{3}{4} - 1) K(3\frac{1}{2} + \frac{3}{4} - 2)$ .
720. Raise the prism in Ex. 719  $\frac{1}{2}$  in. off  $H$ .
721. Cylinder, 2-in. diameter, axis  $M(2, 0 - 1\frac{1}{2}) N(2 + 3 - 1\frac{1}{2})$ :
722. Surmount the cylinder in Ex. 721 with a — (square or hexagonal) plinth, 3-in. diagonal and  $\frac{1}{2}$  in. thick.
723. Cylinder tangent to  $H$ , axis  $O(1 + \frac{3}{4} - 1) P(3 + \frac{3}{4} - 2)$ .
724. Prism — (square or hexagonal), base diagonal 2 in., center line  $A(2, 0 - 2) B(2 + 2 - 2)$ . Shadow on  $H$  and  $T(5 + 2) 2(5 - 2)$ .
725. Prism — in Ex. 724 cutting through the plane  $S(+2) \infty(-3)$ . Shadow on  $S$ .
726. Square prism, 1-in. base, center line  $A(2, 0 - 2) B(2 + 2 - 2)$ , surmounted by a square plinth,  $1\frac{1}{2}$ -in. base,  $\frac{1}{2}$  in. thick.

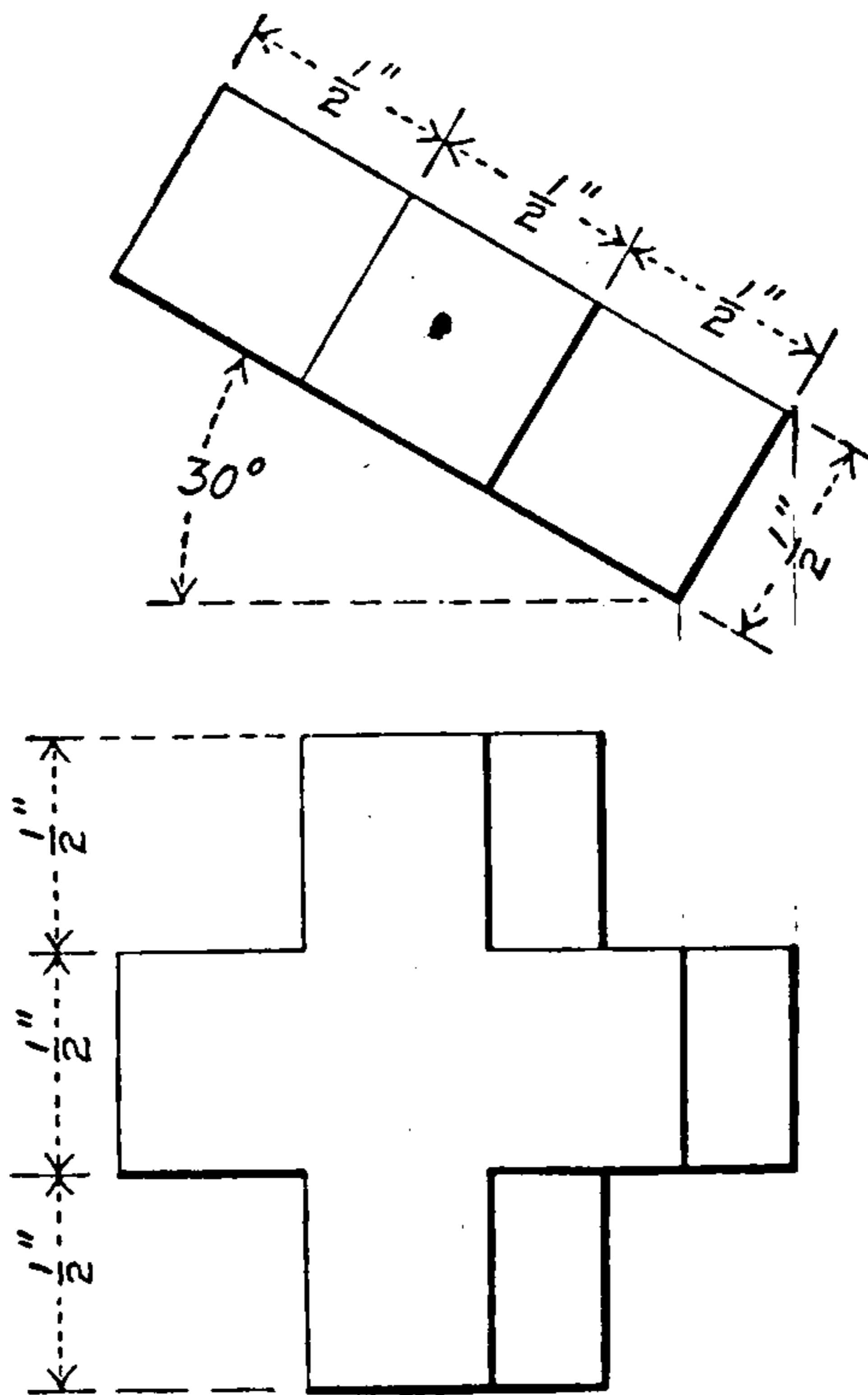


FIG. 109.

727. Prism and plinth in Ex. 726. Shadow on  $H$ ,  $V$ , and  $T(5 + 2) 2(5 - 2)$ .
728. Prism and plinth in Ex. 726, emerging through  $S(+2) \infty(-3)$ . Shadow on  $S$ .
729. Hexagonal prism,  $1\frac{1}{2}$ -in. base diagonal, center line  $A(2, 0 - 2) B(2 + 2 - 2)$ , surmounted by a hexagonal plinth, 2-in. base diagonal,  $\frac{1}{2}$  in. thick.
730. Prism and plinth, Ex. 729. Shadow on  $H$ ,  $V$ , and  $T(5 + 2) 2(5 - 2)$ .
731. Prism and plinth, Ex. 729, cutting through plane  $S(+2) \infty(-3)$ . Shadow on  $S$ .
732. Cylinder 1-in. diameter, surmounted by — (square, round, or hexagonal) plinth, base diagonal (or diameter) 2 in.,  $\frac{1}{2}$  in. thick.
733. Cylinder and plinth, Ex. 732. Shadow on  $H$ ,  $V$ , and  $T(5 + 2) 2(5 - 2)$ .

734. Cylinder and plinth, Ex. 732, cutting through  $S(+2) \infty(-3)$ .  
Shadow on  $S$ .

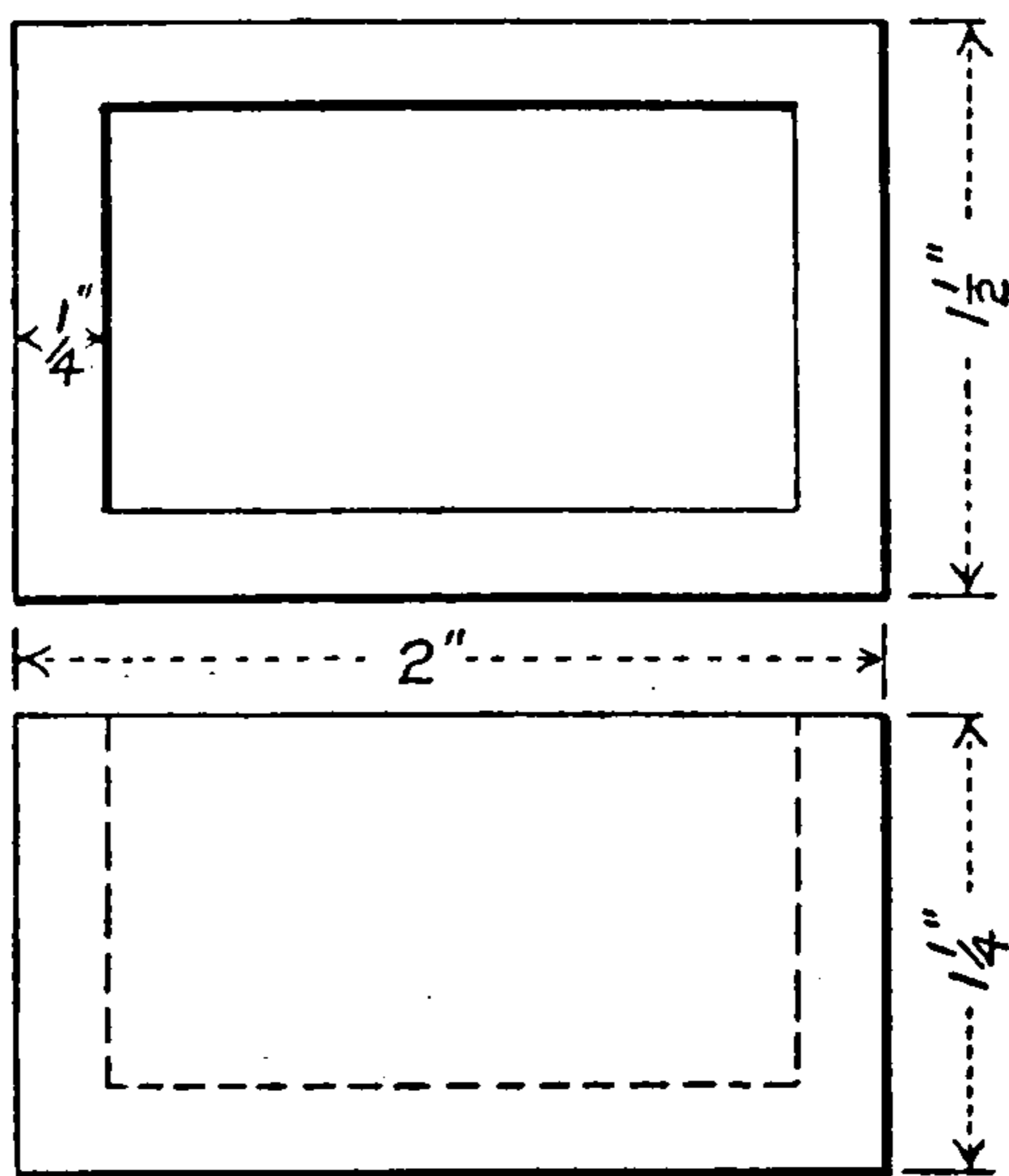


FIG. 110.

*(1 1/2, 3, 9)*

735. Cross, Fig. 109, base center at  $A(2, 0 - 1)$ .

736. Twist the cross, Ex. 735,  $\text{---}^\circ$  ( $10^\circ, 20^\circ, 30^\circ, 45^\circ, 60^\circ, 75^\circ$ ).

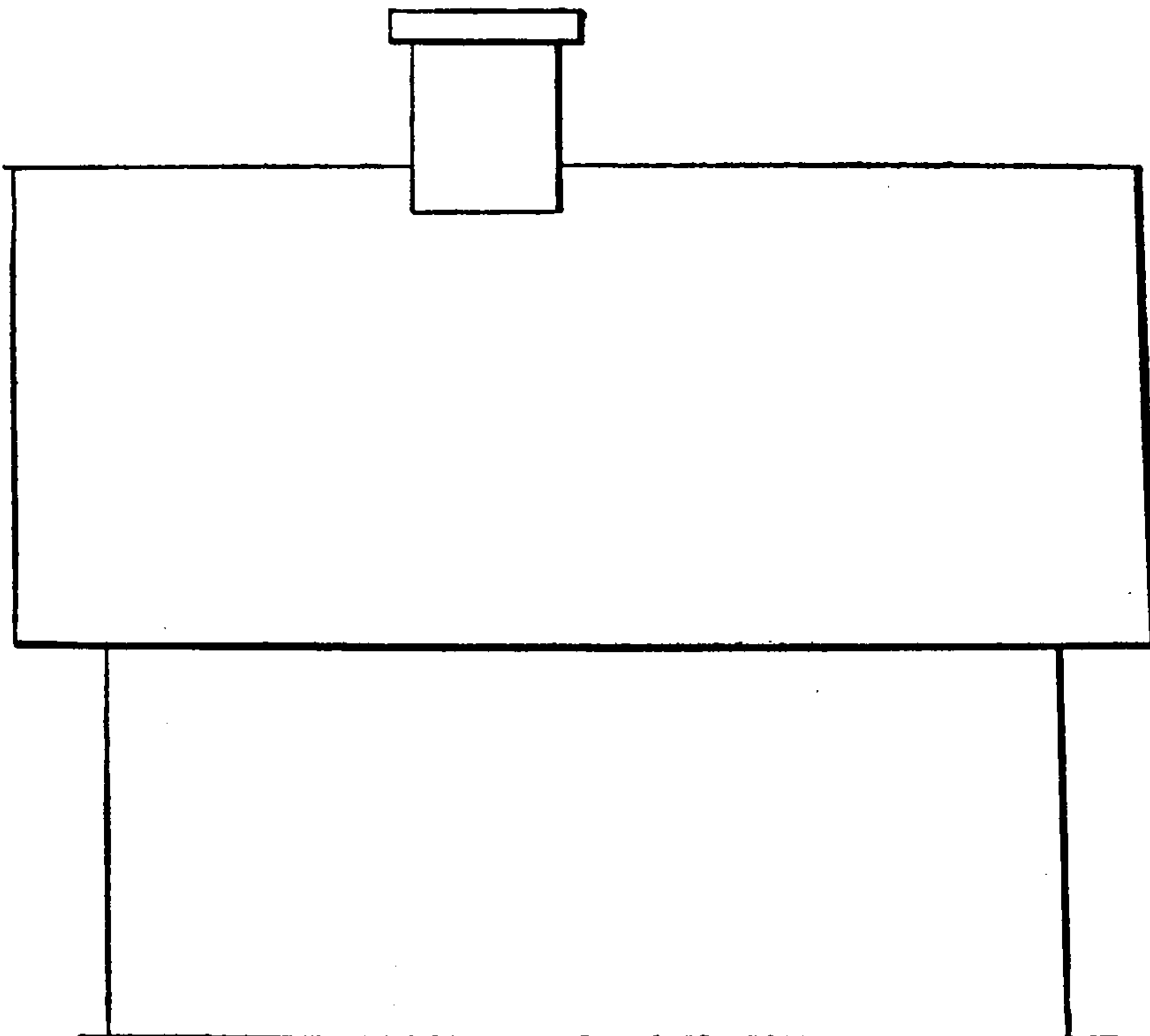


FIG. 111.

737. Cross, Ex. 736, center at  $B(2, 0 - 2)$ . Shadow on  $H, V,$  and  $T(5 + 2)$   
 $2(5 - 2)$ .

738. Place the cross, Ex. 736, on a stepped pyramid made up of four  $\frac{1}{4}$  in. thick plinths, respectively  $1\frac{3}{4}$  in.,  $1\frac{1}{2}$  in.,  $1\frac{1}{4}$  in., 1 in. square.
739. Draw the box, Fig. 110,  $30^\circ$  from V. Shades and shadows, inside and outside, and on *H* and V.
740. Draw the box, Fig. 110, on  $S(1 + 1) 3(3 - 3)$ . Shadows on *H*, V and S.
741. Draw the box, Fig. 110, on end, open front. Shadows on *H*, V, and itself.
742. Twist the box, Fig. 741, —  $^\circ$  ( $10^\circ$ ,  $20^\circ$ ,  $30^\circ$ ). Shadows on *H*, V, and itself.
743. Side elevation of a house with  $45^\circ$  gable roof. Draw plan and end view. Required shadow of chimney on roof, roof edges on walls, and entire house on the ground. Dimensions and scale whatever convenient.

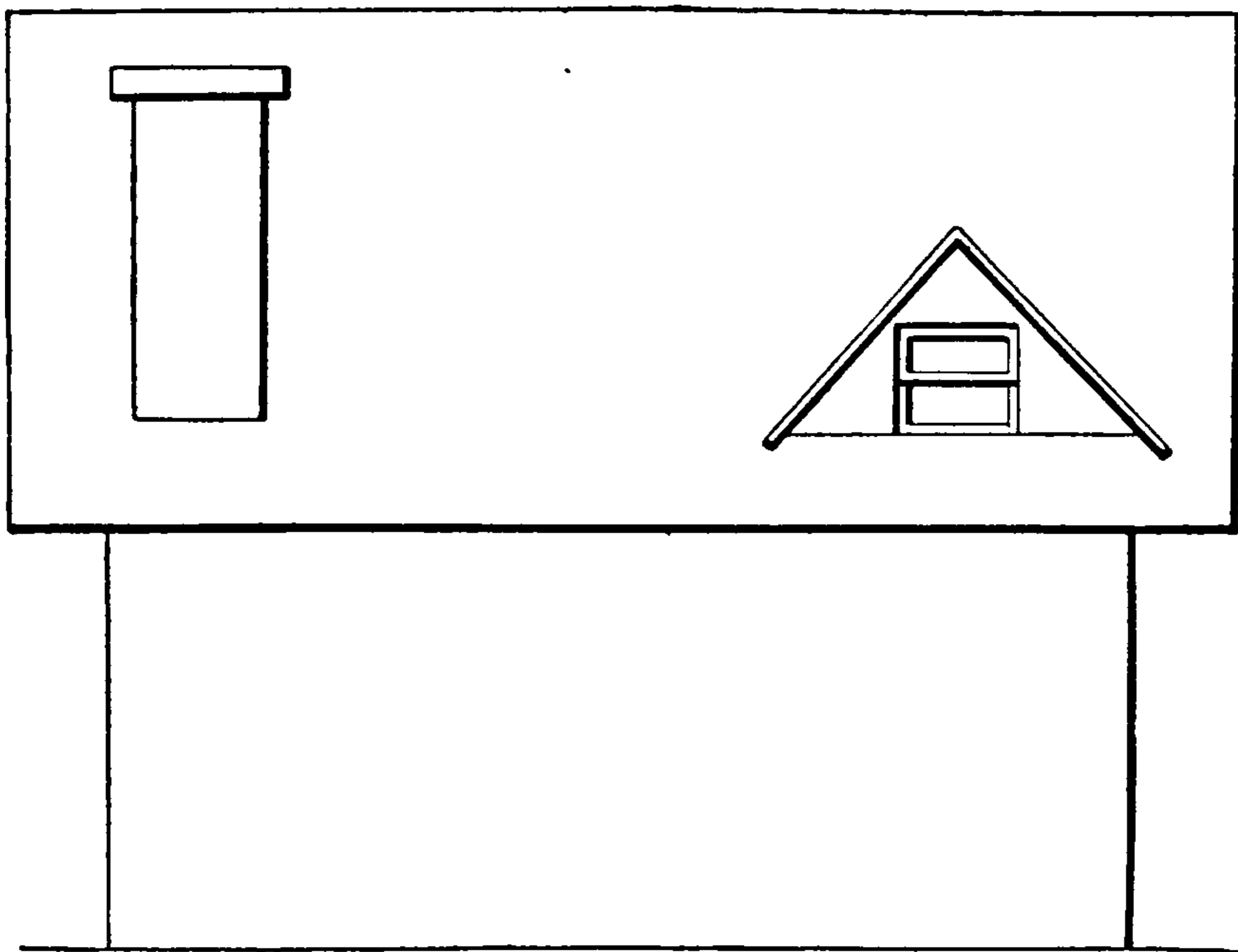


FIG. 112.

744. Side elevation of a house with  $30^\circ$  gable roof. Draw the plan and end view. Draw the shadow of the chimney and dormer on the roof, also the roof edges on the walls, and the shadow of the house on the ground.



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### 135. Projections of Curves.

A curve is projected on  $H$ ,  $V$  and  $P$  in the same manner as a straight line; that is, all the points in the curve are projected on the plane of projection in lines perpendicular to the plane, and the sum of these projections of points makes up the projection of the line. As these projectors are all parallel, the surface which they compose is called the **Projecting Cylinder** of the curve. (Compare with the projecting plane of a straight line.)

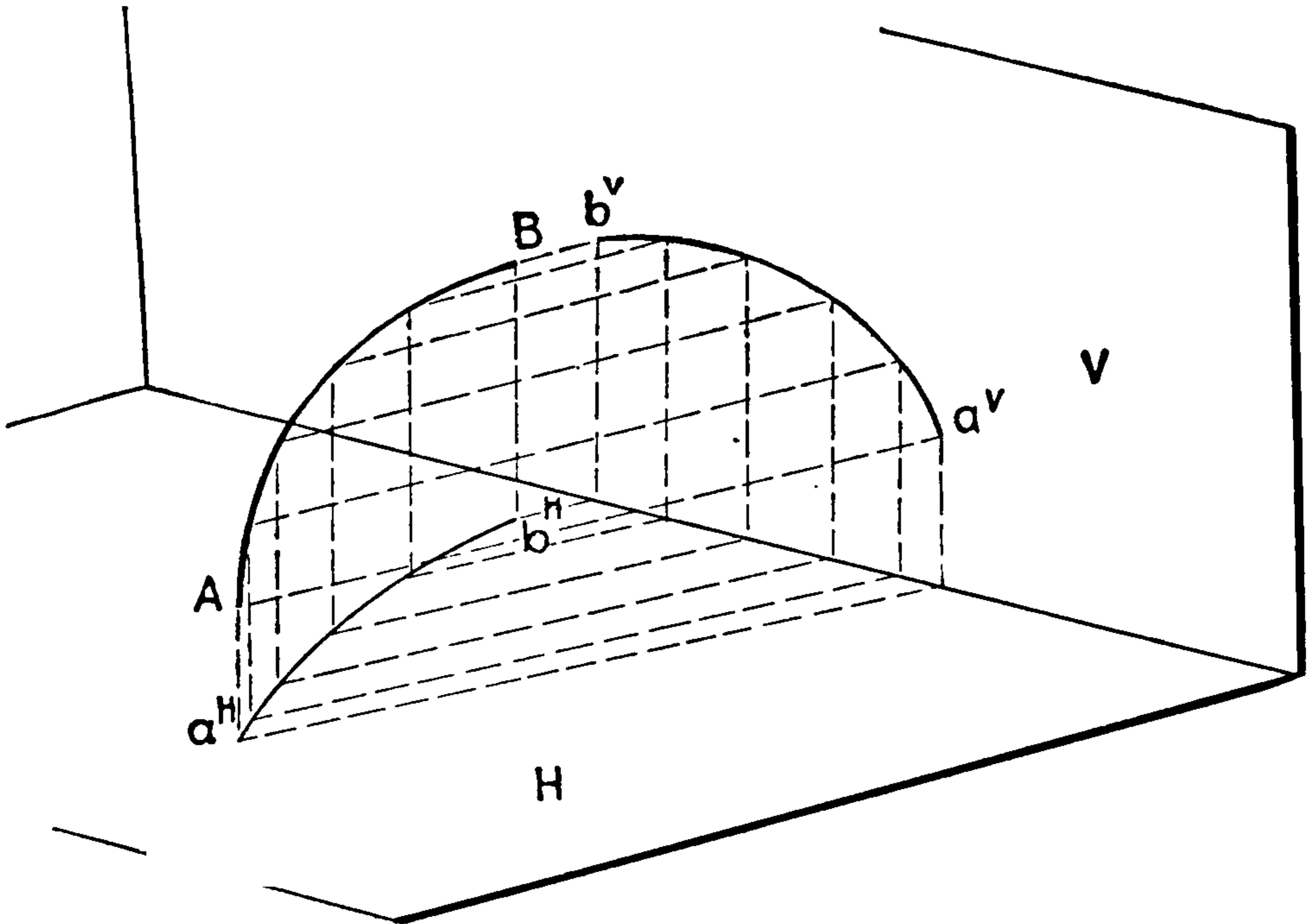


FIG. 113.—Projections of a curve.

Usually a small number (six or eight) points in space will very accurately determine a curve. Its projections are drawn by running a smooth line through the projections of the points in order, on both  $H$  and  $V$ , and on  $P$ , if necessary. (See Fig. 113.)

Note.—Plane curves, in certain positions, will project on one plane as a straight line, and the projecting cylinder is in that case a projecting plane. However, most curves do not lie so conveniently, and space curves never project as a straight line. Any curve, that is parallel or oblique to the plane of projection, will project as a curve on that plane. Any plane curve, lying in a plane perpendicular to the plane of projection, will project as a straight line on that plane.

### 136. Plane Curves Lying in a Profile Plane.

No curve, lying in a profile plane, can be shown in its true character by the  $H$ - and  $V$ -projections, because they will both be

straight lines. The  $P$ -projection will show not only the true character of the curve, but its true dimensions. The  $H$ - and  $V$ -projections of a number of points in a curve will determine it, but will not show it. (See Fig. 114.)

In a similar way, any curve, lying in a plane parallel to  $H$ , is projected in its true size on  $H$ , and as a straight line on  $V$ .

If the curve lie in a plane parallel to  $V$ , its  $V$ -projection gives its true size, and its  $H$ -projection will be a straight line.

If a curve lie in a plane perpendicular to  $H$  and oblique to  $V$ , its  $H$ -projection will be a straight line, and the  $V$ -projection will be curved, and will show the character of the curve. The curve, however, may be entirely changed by this foreshortening; *e.g.*, the projection of an oblique circle is an ellipse.

The  $H$ - and  $V$ -piercing points of a curve are found by the same rule that obtains for straight lines; that is, where the  $V$ -projection crosses  $GL$ , project to the  $H$ -projection to find the  $H$ -piercing point, etc.

### 137. Tangents and Normals to Curves.

**Definition.**—A tangent to a curve is a line containing two consecutive points of the curve.

The tangent may be straight or curved, but, when curved, it is always especially designated. When spoken of as a tangent, the understanding is that it is a straight line tangent.

The tangent (from its definition) must lie in the plane of the curve at the point of tangency. Also (from its definition) it must have the same direction as the curve at the point of tangency. It is especially important to remember that the tangent must lie in the plane of the curve, as many errors occur from disregarding this fact.

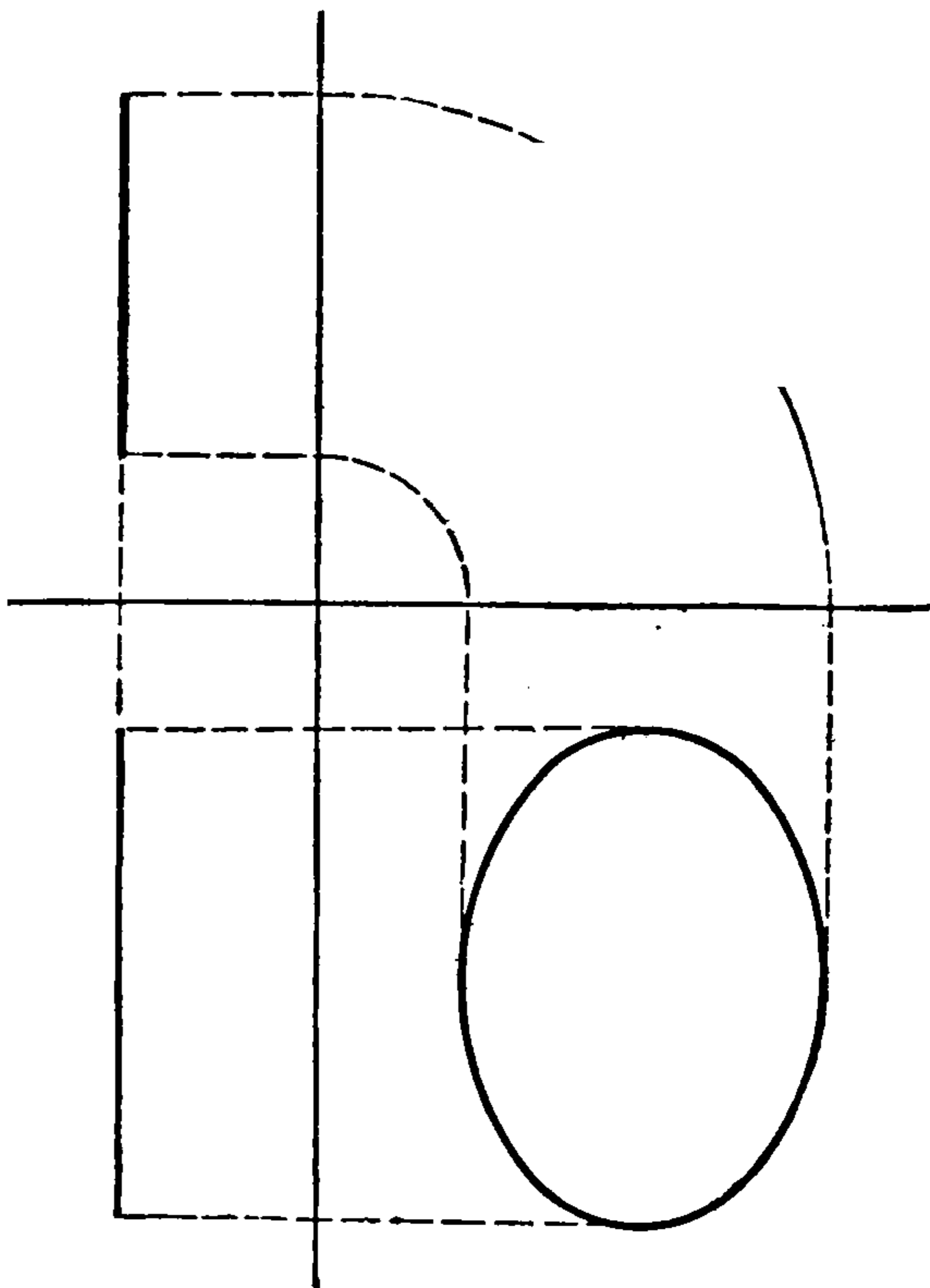


FIG. 114.—Curve in a profile plane.



Note.—The apparent discrepancy between the foregoing definition of the tangent and that given in most Plane Geometries amounts to nothing. They define a tangent as a line touching a curve at one point. Their definition answers sufficiently for beginners in Geometry, and for a section of the subject that confines its operations to a single plane, but the student can easily see how such a definition would be utterly inadequate in Space Geometry.

As the tangent is a short straight line in the curve (two consecutive points), the curve may be considered as being composed of an infinite number of infinitesimal tangents, intersecting consecutively. Fig. 115 shows a series of straight lines, A, B, C, D, E, F, G, etc., intersecting at short finite intervals. They compose so

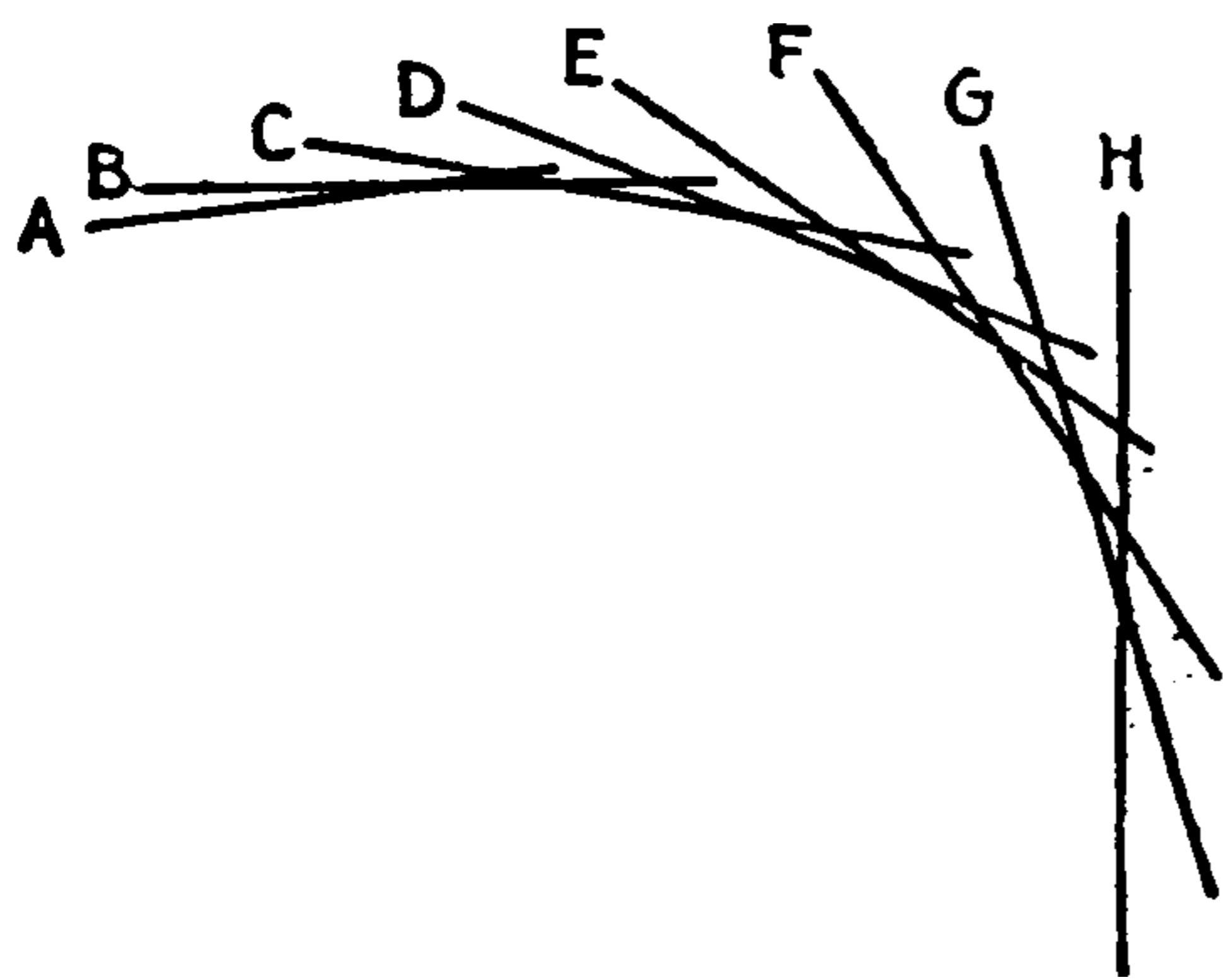


FIG. 115.

nearly a curve that it is easy to imagine that minute divisions would compose an actual curve. From the figure it can easily be seen how the tangent at any point determines the direction of the curve at that point.

### 138. Projections of the Tangent.

The projections of the tangent to a curve will be tangent to the projections of the curve on the respective planes. The converse is not necessarily true, because a line lying in a different plane from that of the curve may have its projection tangent to that of the curve on one plane, but not on any other.

#### To Draw a Tangent to a Curve.

There are exact and approximate methods. In the case of the circle, the tangent is always perpendicular to the radius at the point of contact. In the case of other conic sections, the exact method will be taken up in connection with the study of those curves. For ordinary drafting work a sufficiently accurate tangent may be drawn by placing a straight edge against the curve, and laying off the tangent with the eye. A simple approximation, that can be used in nearly all cases of curves and is exact for the circle, is this: Let it be required to draw the tangent at A (Fig. 116) on the curve shown. Set the dividers at some small arbitrary span, and lay off B and C on

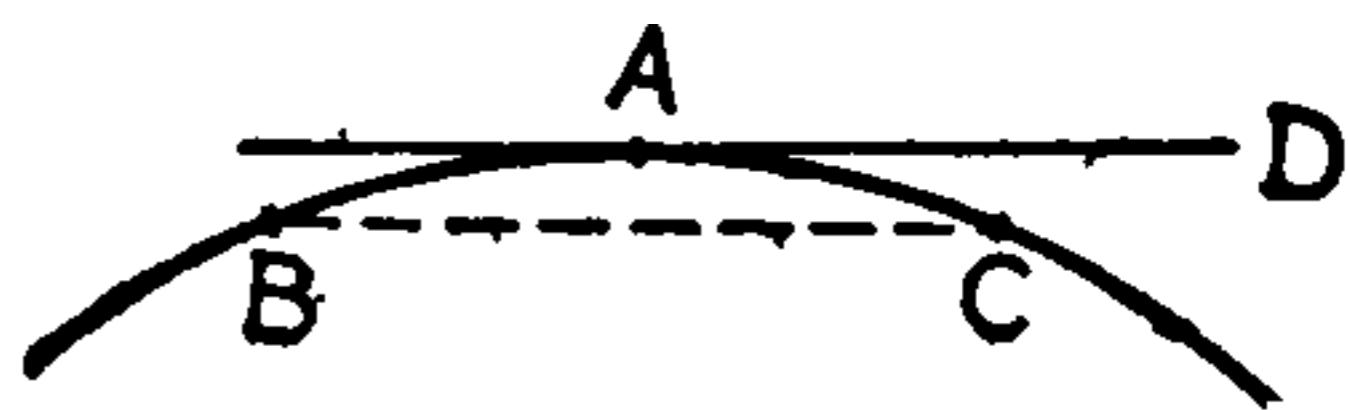


FIG. 116.



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*Fourth.*—To rectify a quadrant of a circle.

Let AB (Fig. 119) be the given quadrant. Draw the tangent at B, and from A draw a line AC, which makes  $60^\circ$  with CB. BC is then a very close approximation to AB. This is undoubtedly the closest approximation for such an arc. Let the

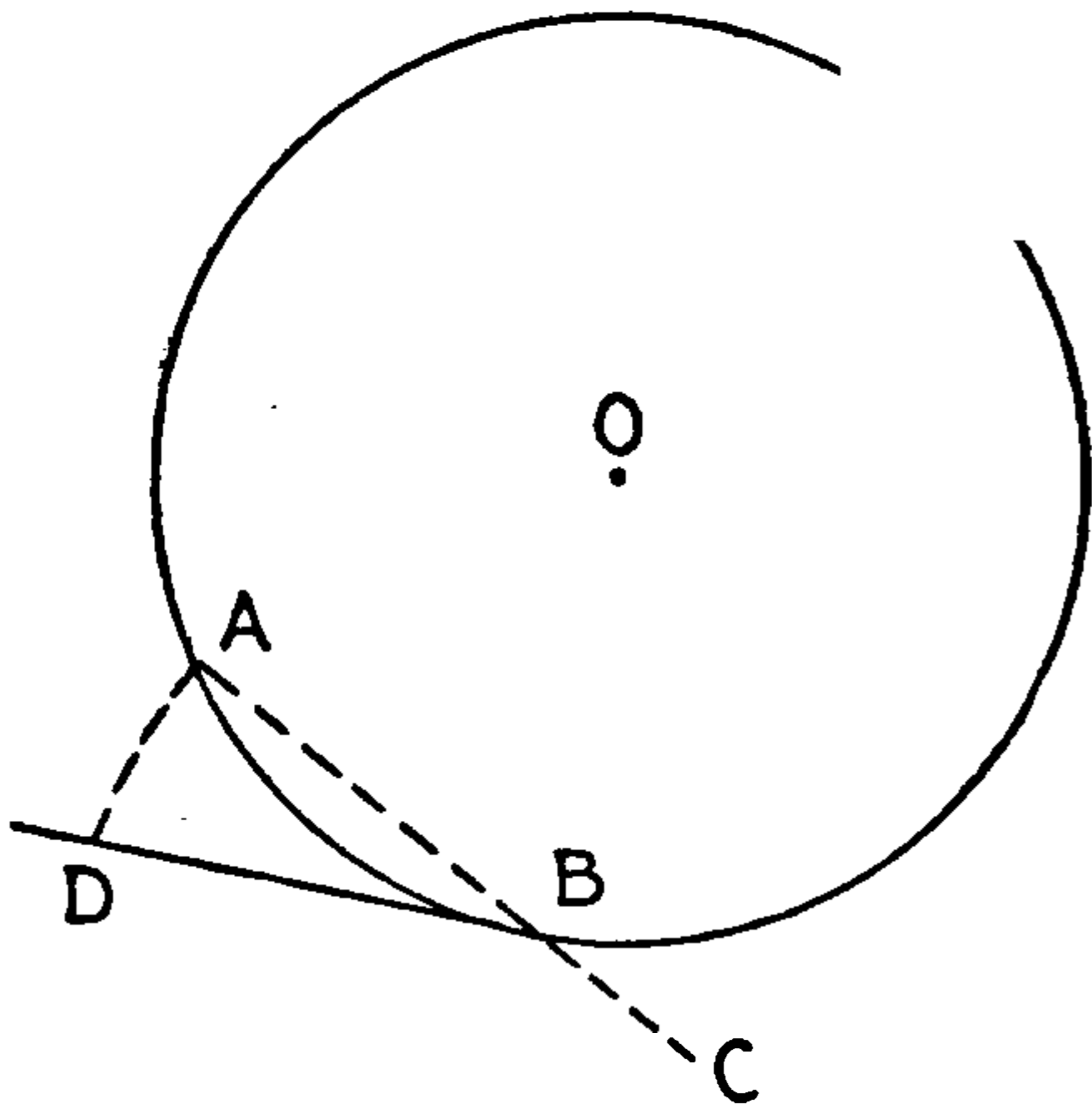


FIG. 118.—The rectification of a circular arc.

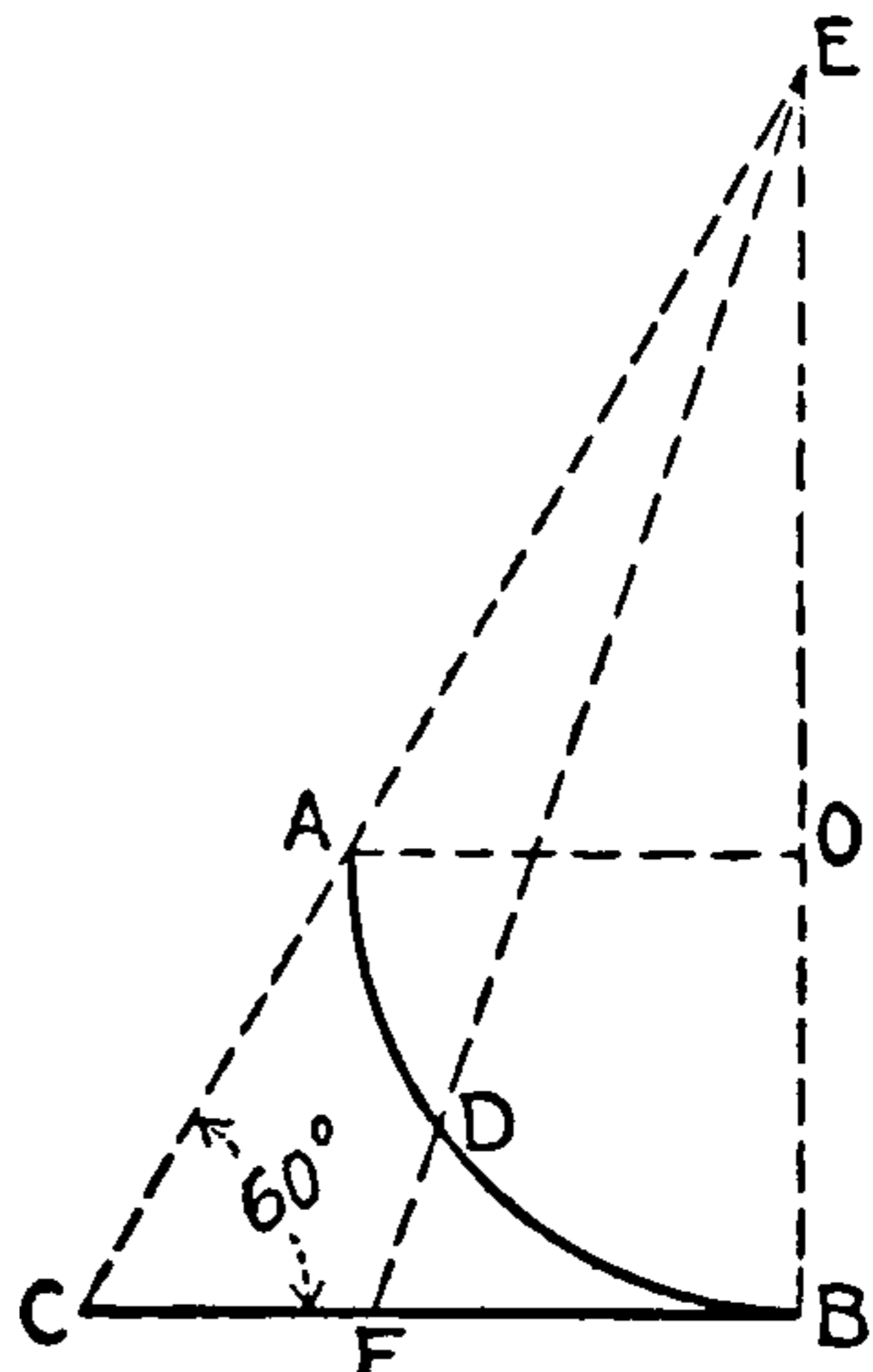


FIG. 119.—The rectification of a quadrant.

student prove by trigonometry that it is numerically equal to  $\frac{\pi}{2}R$ . Although there is no geometrical proof for this, your calculation will show that it is closer than the accuracy of your instruments.

A convenient method of rectifying a small arc comes with this method. To rectify BD, Fig. 119, draw the line ED through to the tangent at F. The line BF equals BD.

*Fifth.*—To rectify an arc of a circle when its angle is known.

Here the numerical method is best. The arc AB (angle =  $\theta$ )

$$= 2\pi R \frac{\theta^\circ}{360^\circ} = .01745R\theta^\circ.$$

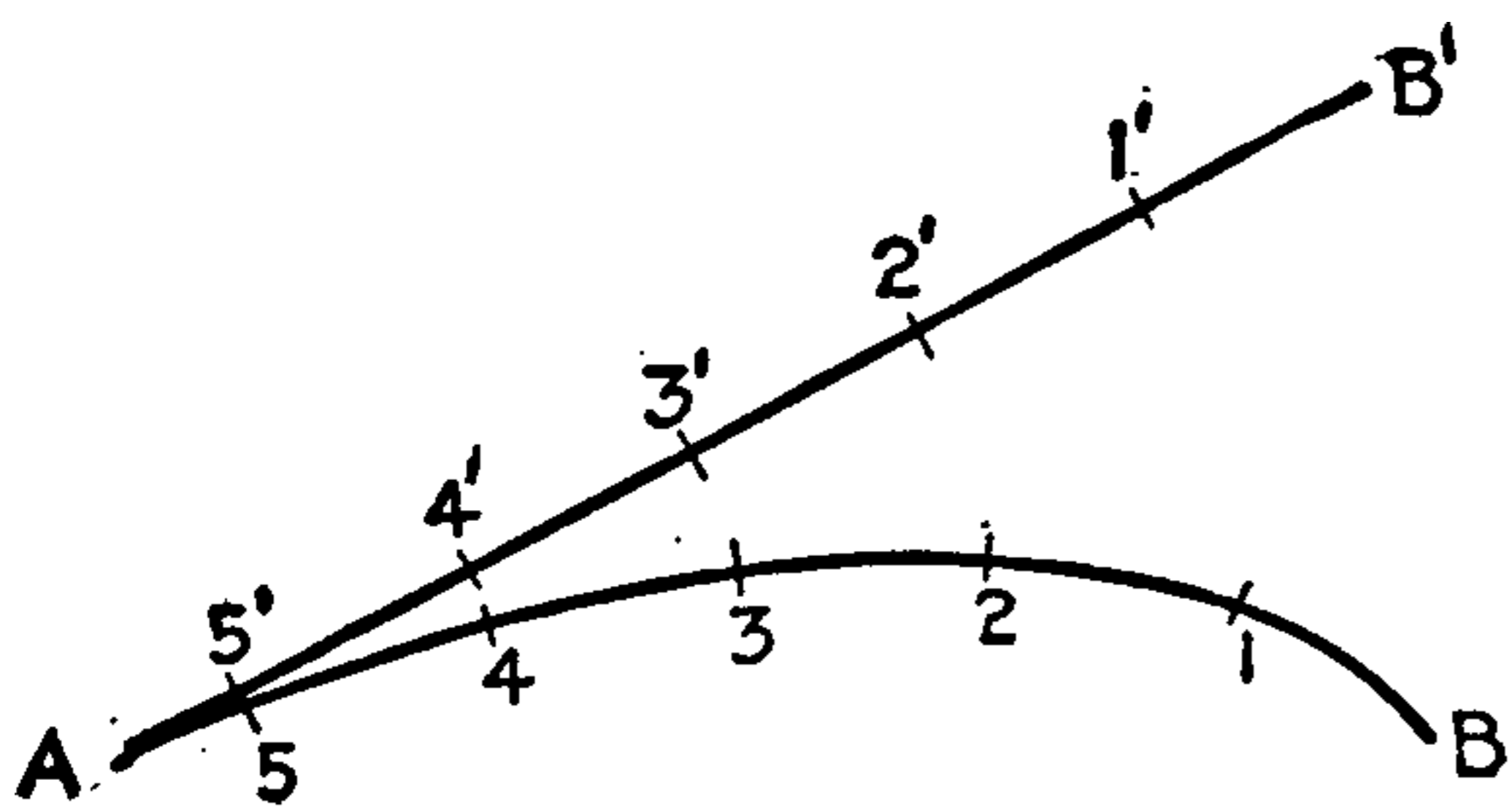


FIG. 117.

**141. To Lay Off Any Length of Line, Straight or Curved, on Any Curve.**

Regard the line AB' (Fig. 117) as a curved line. Lay off AB on the curve in exactly the same manner that it was laid off on the straight line.

This is often a necessary operation in practical drafting; example, in drawing cycloids for gear outlines.

## 142. Conic Sections.

### Definitions.

**Circle.**—A circle is a plane curve generated by a point moving so as to be at a constant distance from another point.

**Ellipse.**—An ellipse is a plane curve generated by a point moving so that the **sum** of its distances from two fixed points in its plane is constant.

**Hyperbola.**—A hyperbola is a plane curve generated by a point moving so that the **difference** between its distances from two fixed points in its plane is constant.

**Parabola.**—A parabola is a plane curve generated by a point moving so that its distances from a line **and** a point in the plane are always equal.

The four foregoing curves are called “Conic Sections” from the fact that they are the possible curves that can be cut from a right circular cone by a plane. A plane can also cut a straight line from a cone. The necessary conditions for cutting these lines from a cone are:

**Straight Line.**—Plane through the apex.

**Circle.**—Plane perpendicular to the axis.

**Ellipse.**—Plane oblique to the axis, cutting all the elements on one side of the apex.

**Hyperbola.**—Plane, parallel to the axis, or a plane oblique to the axis at a smaller angle than that made by the elements of the cone.

**Parabola.**—Plane parallel to one element of the cone.

## 143. Gearing Curves.

**Cycloid.**—A cycloid is a plane curve generated by a point on the circumference of a circle, as the circle rolls on a straight line.

**Epicycloid.**—An epicycloid is a plane curve generated by a point on the circumference of a circle, as it rolls on another circle.

**Hypocycloid.**—A hypocycloid is a plane curve generated by a point on the circumference of a circle as it rolls on the inside of another circle.

**Involute.**—An involute is a plane curve generated by a point on a tangent, as the tangent rolls on a plane curve or polygon. The involute of a circle is the only one of any considerable importance, and is the curve used in gearing.

The foregoing are called "gearing curves," because they are mainly used as the outlines of teeth in gear wheels. The outlines of bevel gears are (theoretically) spherical cycloids, but their consideration is negligible in this work, and practically so in actual drafting work.

### SPACE CURVES

**144. Helix.**—A helix is a curve generated by a point moving about a straight line at a constant distance, and in the direction of the line at a constant velocity.

**Note.**—This line is the line of the screw thread, twist drill, screw conveyor, worm gear, coiled spring, and many other important practical devices. It is a curve on the surface of a cylinder.

There is also a conical helix that sometimes appears in practical work. It is usually drawn so that it makes one complete revolution about the axis in traveling from the base to the apex, ascending the surface at a uniform rate of motion. The plan of the conical helix is the 'Spiral of Archimedes. By drawing the plan first and projecting the points of the curve as it intersects the various elements of the cone, the elevation is easily drawn. It is rather commonly used in sheet metal work and in spiral springs.

There is no need of defining or considering any other of the space curves.

### GENERAL DEFINITIONS

**145. Axis.**—The line about which a point revolves, as in the case of the helix. It is also a center line for some curves, as the ellipse, parabola, and hyperbola.

**Minor Axis and Major Axis.**—In the ellipse there are two dimensions, sometimes called the long and short diameters, but more generally the major and minor axes. They are respectively the longest and shortest possible straight lines running through the center from one point in the curve to another. The constant sum, spoken of in the definition, is equal to the length of the major axis.

**Focus.**—The two fixed points, mentioned in the definitions of the ellipse and hyperbola, and the single one, mentioned in the definition of the parabola.

**Vertex.**—The nearest point in the curve to the focus. It has the smallest radius of curvature of any point in the curve, and



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F and F' as centers. The intersections, C and D, of these arcs are the ends of the minor axis.

4. Set the compasses at arbitrary radii, A1, A2, etc., and strike arcs with them from F and F' as centers.

5. Set the compasses at complementary radii, B1, B2, etc., and strike arcs from F and F' as centers.

6. Through A, B, C, D, and the intersections of complementary radii, draw a smooth line. This will be the required ellipse.

**Proof.**—The sum of the distances of any of the points, thus found, to the foci is always equal to AB.

**147. Approximate and Mechanical Methods of Drawing the Ellipse.**

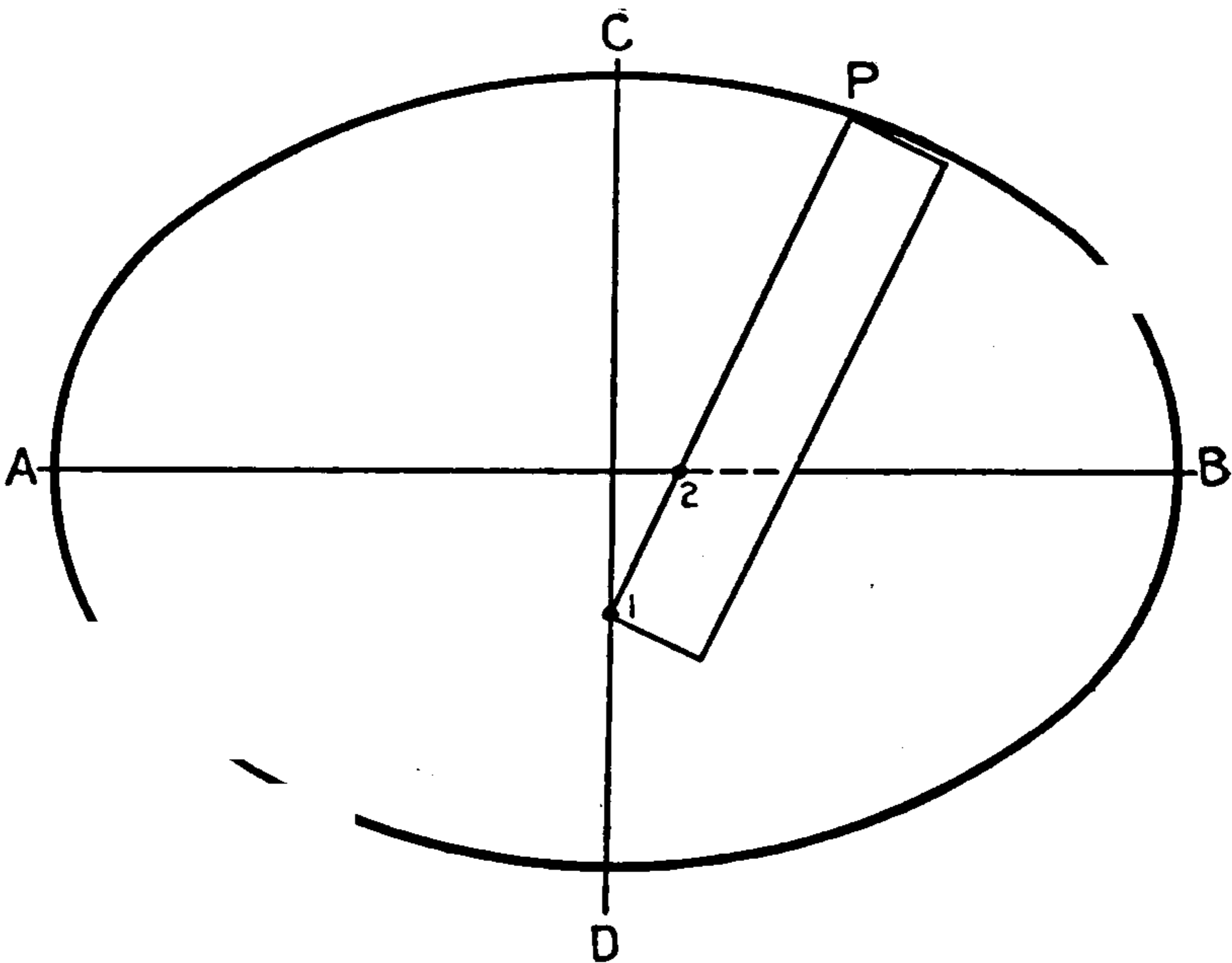


FIG. 121.—Drawing the ellipse with a trammel.

There are numerous ways of drawing an ellipse, exact and approximate. These are found in most text-books on Mechanical Drawing. The best of these is the "Trammel Method."

**The Trammel Method.**—1. Lay off the Major and Minor Axes, of any desired dimensions, AB and CD (Fig. 121).

2. Take a card, or any straight edge, and lay off P1 and P2, respectively equal to the semi-major and semi-minor axes, BO and CO.

3. Place the straight edge so that 2 lies on AB and 1 lies on CD, and mark the point P.

4. Do this in as many positions as desired, marking a number of points on the curve.

5. Draw a smooth curve through the points thus obtained.

**Uses for the Ellipse.**—In ornamental work the ellipse is extensively used, in its entirety and in portions. The ellipse is the projection of any circle in an oblique plane, and its major axis is always equal to the diameter of the circle. In machine design, the ellipse is used in certain “quick return” motions. In sheet metal work and boiler-making it is often necessary to cut elliptical holes in the plates. In designing “focusing reflectors” the ellipse is necessary, because all rays from one focus of an ellipse reflect from the curve to the other focus.

**Limits.**—When the distance between foci is zero, the ellipse is a circle. When it is equal to the “constant sum,” the ellipse is a straight line.

**148. Problem 48.**—To draw the tangent to an Ellipse at a given point on the curve.

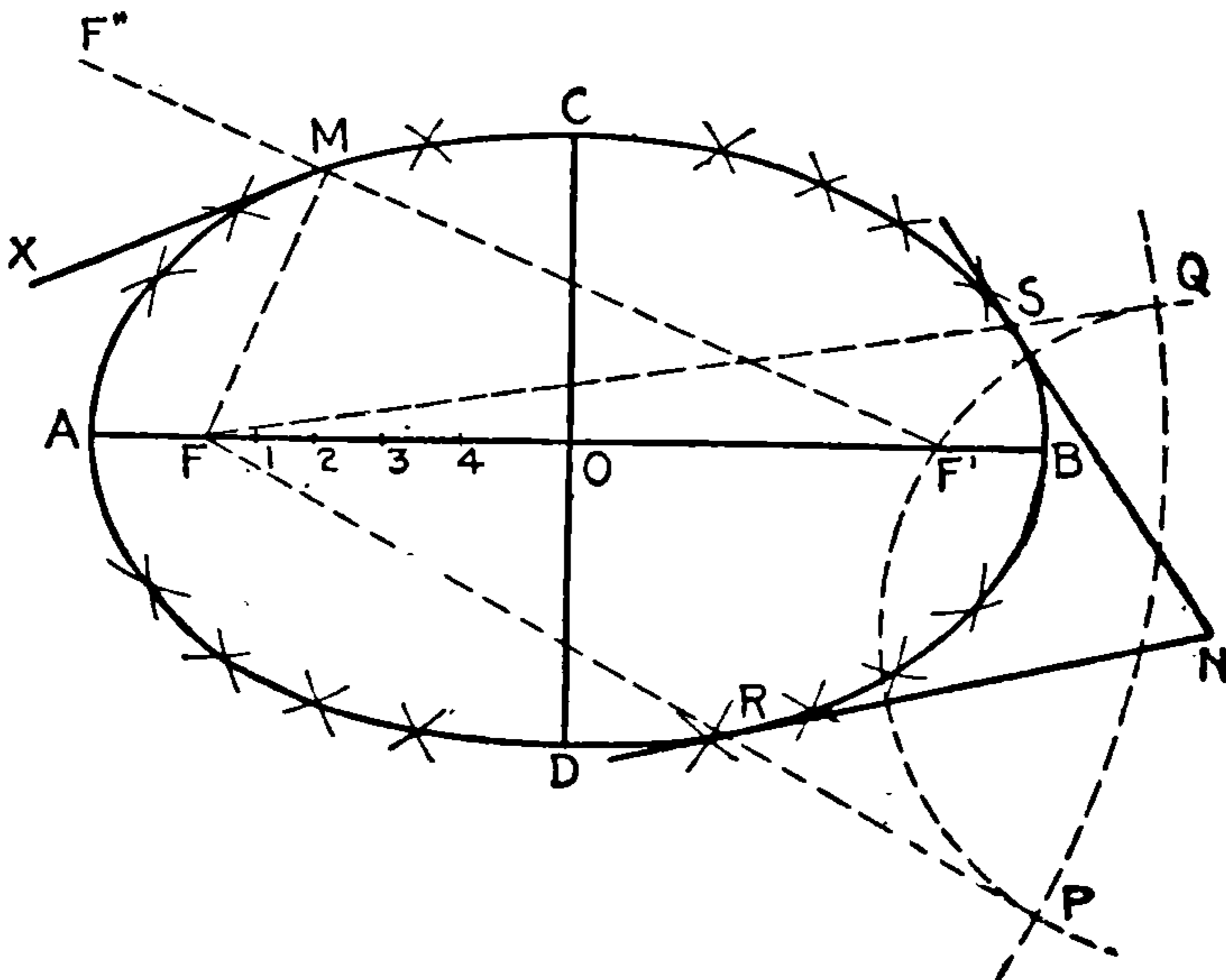


FIG. 120.—The ellipse.

**Construction.**—Let M (Fig. 120) be the point.

1. Draw MF and MF', the focal lines.
2. Extend MF' beyond M to F''.
3. Bisect the exterior focal angle FMF''.

The bisector MX will be the tangent at M.

**149. Problem 49.**—To draw the tangent to an Ellipse from a point outside.

Let N (Fig. 120) be the point.

1. Draw a circle with N as center and NF' as radius.



2. Intersect the circle in points P and Q by an arc whose radius is equal to AB, and whose center is at F.
3. Connect either P or Q with F.
4. Where PF or QF intersect the ellipse at R or S will be the points to which tangents can be drawn from N.

If the axes of an ellipse are given, the foci may be determined by striking an arc from C or D as center, with a radius equal to AO. Where this arc intersects AB will be the foci.

**150. Problem 50.**—To draw a **Hyperbola**, having given its foci and the “constant difference.”

Let F and F' (Fig. 122) be the foci, and the “constant difference” be equal to the distance VV'.

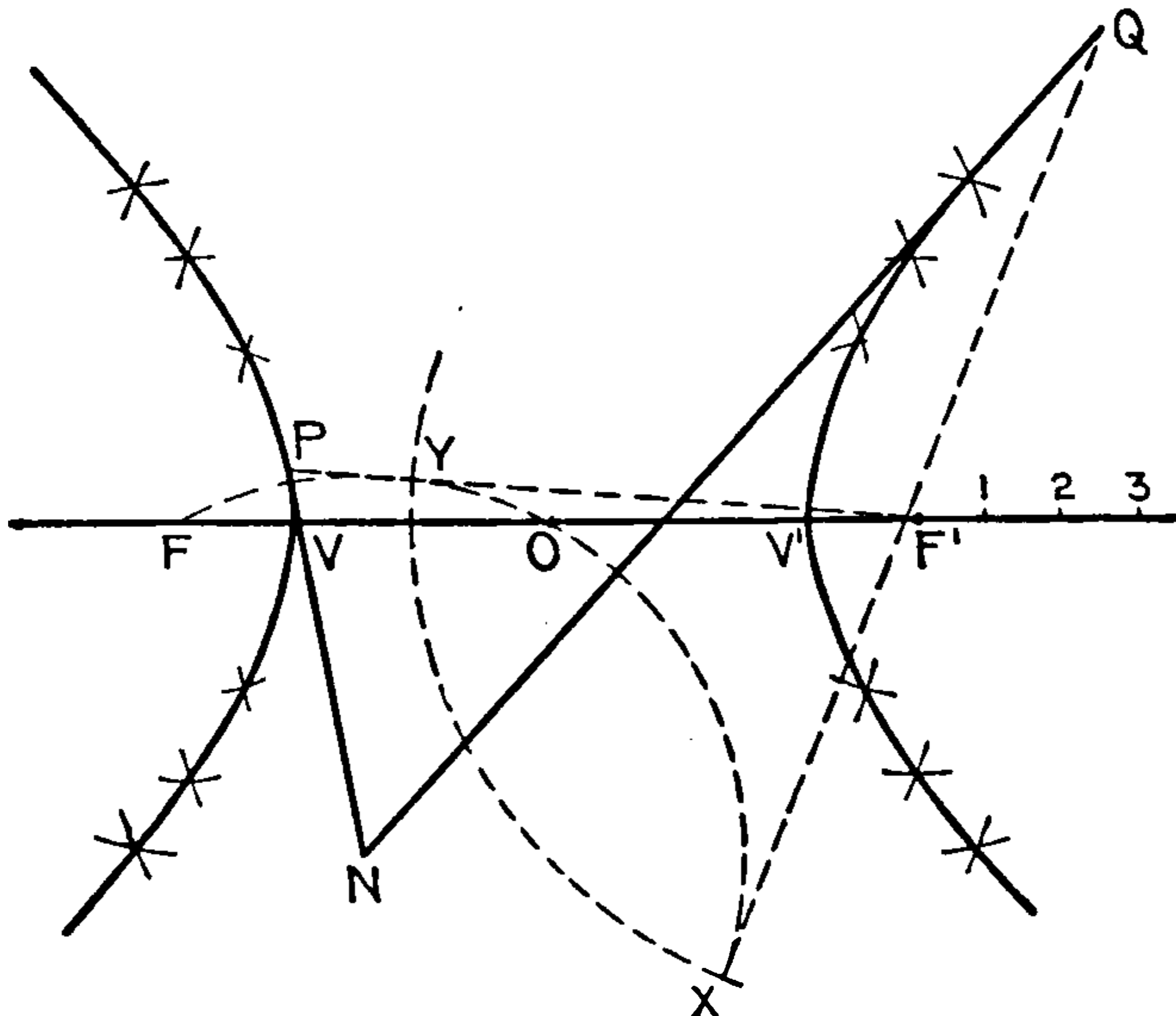


FIG. 122.—The hyperbola.

**Construction.**—1. Lay off F and F' and V and V' on the same line, at their proper distances, all symmetrically placed about a center, O. (V and V' will be the vertices.)

2. Spread the compass at any arbitrary spans, as V1, V2, etc., and strike arcs with these radii with F and F' as centers.

3. Using the complementary radii, V'1, V'2, etc., strike arcs with F and F' as centers, intersecting those already drawn.

4. Draw a smooth curve through the proper intersections and V and V'. This will be the required hyperbola.

**Note.**—The hyperbola is in reality two curves, each having a vertex, and each unlimited in extent. The hyperbola varies in accordance with the ratio between FF' and VV'. The most important use of the hyperbola is that it is the generating line of the pitch surface of “skew” gears.



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1. Draw a line CG through F perpendicular to XD. This is the axis of the curve.

2. Bisect the distance from F to XD at E. E will be the vertex.

3. Draw a number of lines at points, 1, 2, 3, etc., parallel to XD, at arbitrary distances.

4. Locate two points on each of the lines, equally distant from F and XD.

5. Draw a smooth line through the points thus found, and it will be the required parabola.

**154. Problem 54.**—To draw the Tangent to a Parabola at any point on the curve.

**Construction.**—Let M (Fig. 123) be the given point.

1. Draw MX, perpendicular to XD.

2. Draw the focal line MF.

3. Bisect the angle FMX. The bisector is the tangent.

**Note.**—Extend the line XM in the direction MY, and draw the normal MO. Notice that MY is parallel to the axis, and that MO is the bisector of the angle FMY, making OMF equal to OMY. This fact is taken advantage of in designing the reflector for the searchlight. M is a random point on the parabola, and shows that any ray from F striking the parabola at M (any point) is reflected in a line parallel to the axis, as MY. This concentrates all the reflected rays into a cylinder of light, preventing diffusion.

**155. Problem 55.**—To draw a Tangent to a Parabola from a point outside.

**Construction.**—Let N (Fig. 123) be the given point.

1. With N as a center, draw a circle through F.

2. From its intersections, C and D, with DX, draw the perpendiculars, CE and DK.

3. E and K, the intersections with the curve, are the points to which tangents can be drawn.

**156. Problem 56.**—To draw a Cycloid, having given the directrix and generating circle.

**Construction.**—Let AB be the directrix and O be the center of the generating circle, which is to be drawn tangent to the directrix, of a radius R.

1. Lay off on AB, a distance equal to  $2\pi R$ , and divide the circle and AB into any convenient number of equal parts, 12, 16 or 24.

2. Draw the path of the center of the circle, parallel to AB, at a distance equal to R.

3. Erect perpendiculars to the line of centers at each of the divisions of AB. The intersections will be successive positions of the center of the generating circle.

4. From the divisions, 1, 2, 3, etc., of the generating circle, run projectors parallel to AB.

5. From each of the successive positions of the center, 1', 2', 3', etc., describe arcs intersecting the corresponding projectors.

6. Through these intersections draw a smooth curve, which will be the required Cycloid.

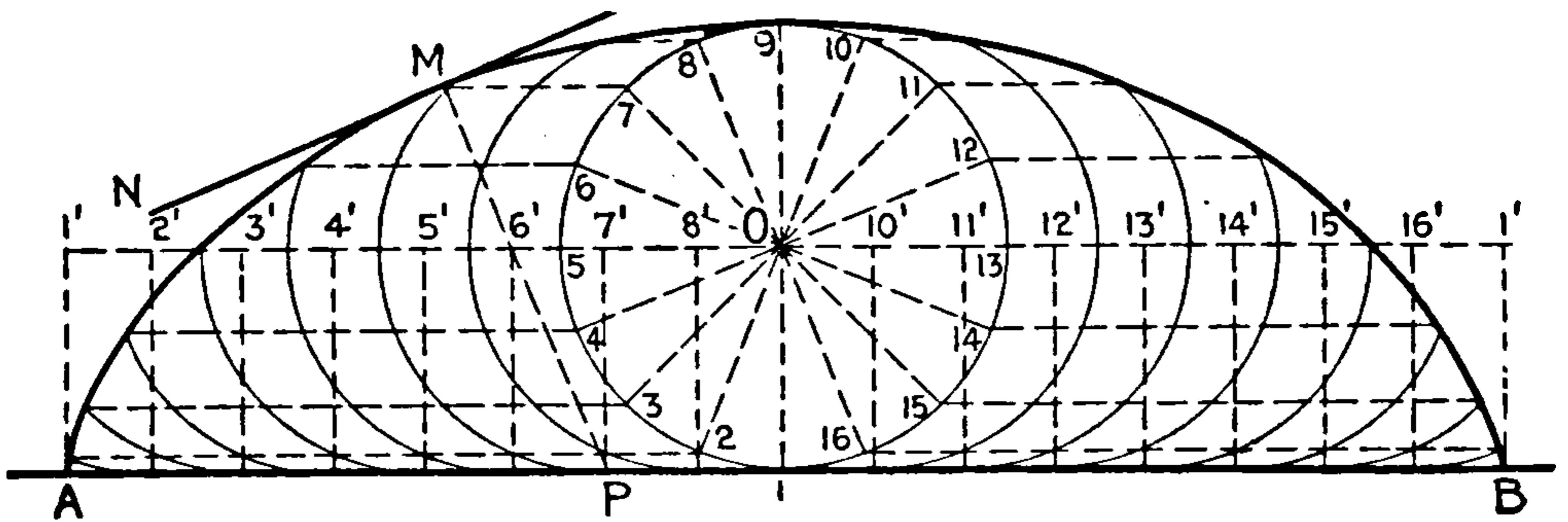


FIG. 124.—The cycloid.

**157. Problem 57.**—To draw the tangent to the Cycloid at any point on the curve.

**Construction.**—Let M (Fig. 124) be the given point, and P be the point of tangency with AB of the generating circle, when M is its intersection with the curve.

1. Draw MP. This line is the normal at M.
2. Draw the tangent perpendicular to MP.

**Note.**—It is owing to the fact that the normal to the cycloid at any point passes through the point P, where the generating circle and directrix are tangent, that the cycloid is a correct curve for use in gear outlines. It is this property that makes it possible to maintain a constant angular velocity ratio in gears, as will be seen when the study of Kinematics, or “Mechanism,” is taken up.

**158. Problem 58.**—To draw an Epicycloid and a Hypocycloid.

The only difference between drawing these curves and the cycloid, is that the directrix now is a circle instead of a straight line. Fig. 125 shows both curves. AB is an arc of the pitch circle, as the directrix is usually called in Machine Design, and the center of the generating circle of the Epicycloid is at C, and its path is the arc CD (center at O). The center of the Hypocycloidal circle is at E, and its path is the arc EF, center at O.

**Construction.**—1. Divide the generating circles into any convenient number of equal parts.

2. Step off these arcs on AB (Art. 141).

3. From the points 1, 2, 3, etc., draw arcs with their centers at O.

4. Through the divisions on AB draw radial lines intersecting CD and EF in the points occupied by the centers as the circles roll on AB.

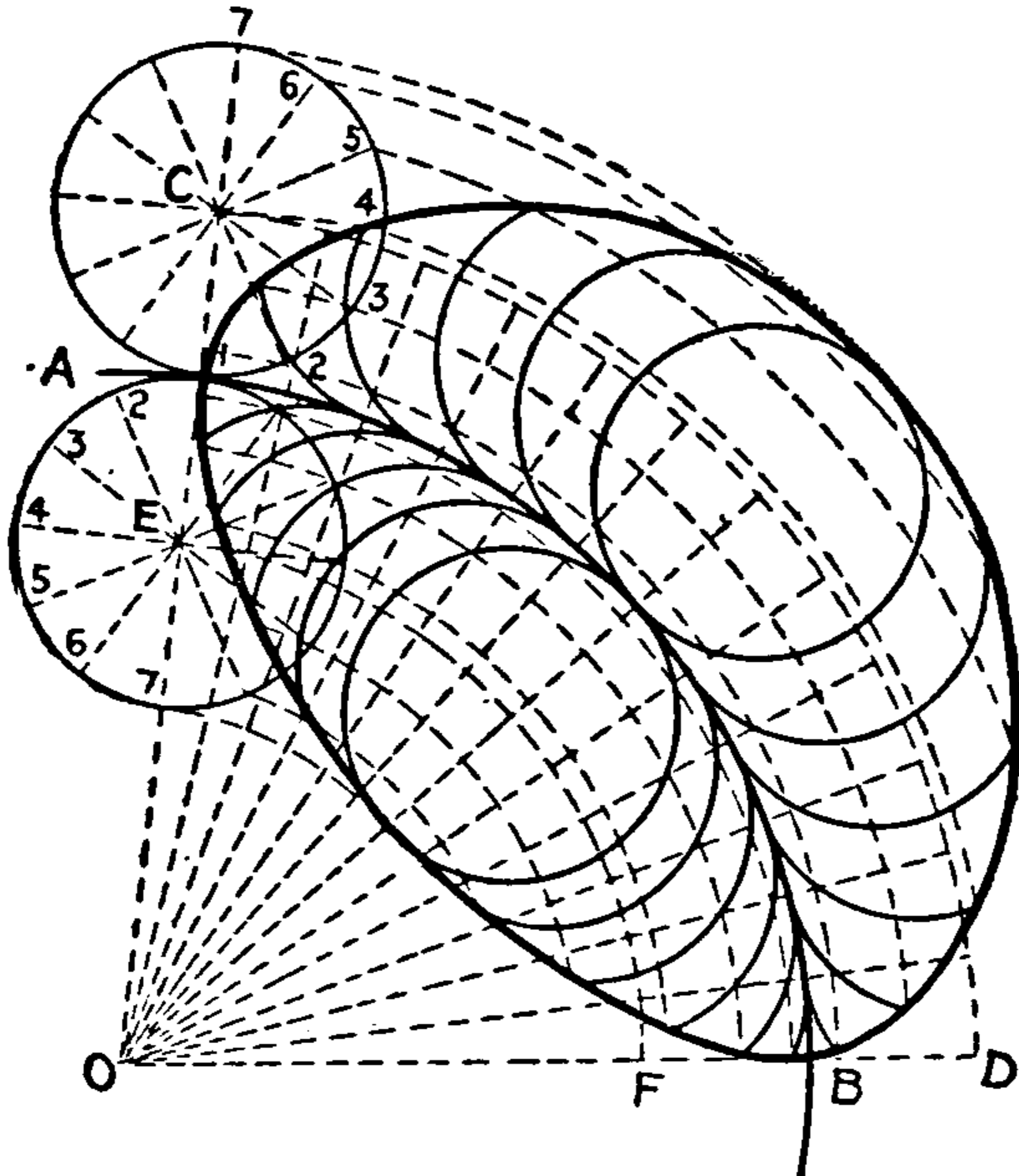


FIG. 125.—The epicycloid and hypocycloid.

5. From these center points strike arcs with radii equal to those of the generating circles, intersecting the arcs of operation (3).

6. Through the respective intersections draw the curves.

The tangents are drawn precisely as the tangents to the cycloid are drawn; that is, the normal at any point passes through the point of tangency of the generating circle and the pitch circle.

**Note.**—The hypocycloid generated by a circle half the size of the pitch circle is a straight line and is called a radial hypocycloid.

**159. Problem 59.**—To draw an Involute to a Circle.

Let AB (Fig. 126) be the directing circle, or Evolute, with its center at O.

**Construction.**—1. Divide any arc of AB, as, for example, the semicircle, into any number of equal parts.

2. At the points 1, 2, 3, etc., draw tangents in the direction of the origin, A.



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**162. Problem 62.**—To draw the projections of a Helix, having given its Pitch and Diameter.

Let  $OP$  (Fig. 128) be the axis,  $O^v12$  the pitch, and  $D$  the diameter.

**Construction.**—1. Draw the circle and divide it into any number of equal parts.

2. Divide the pitch into the same number of equal parts.

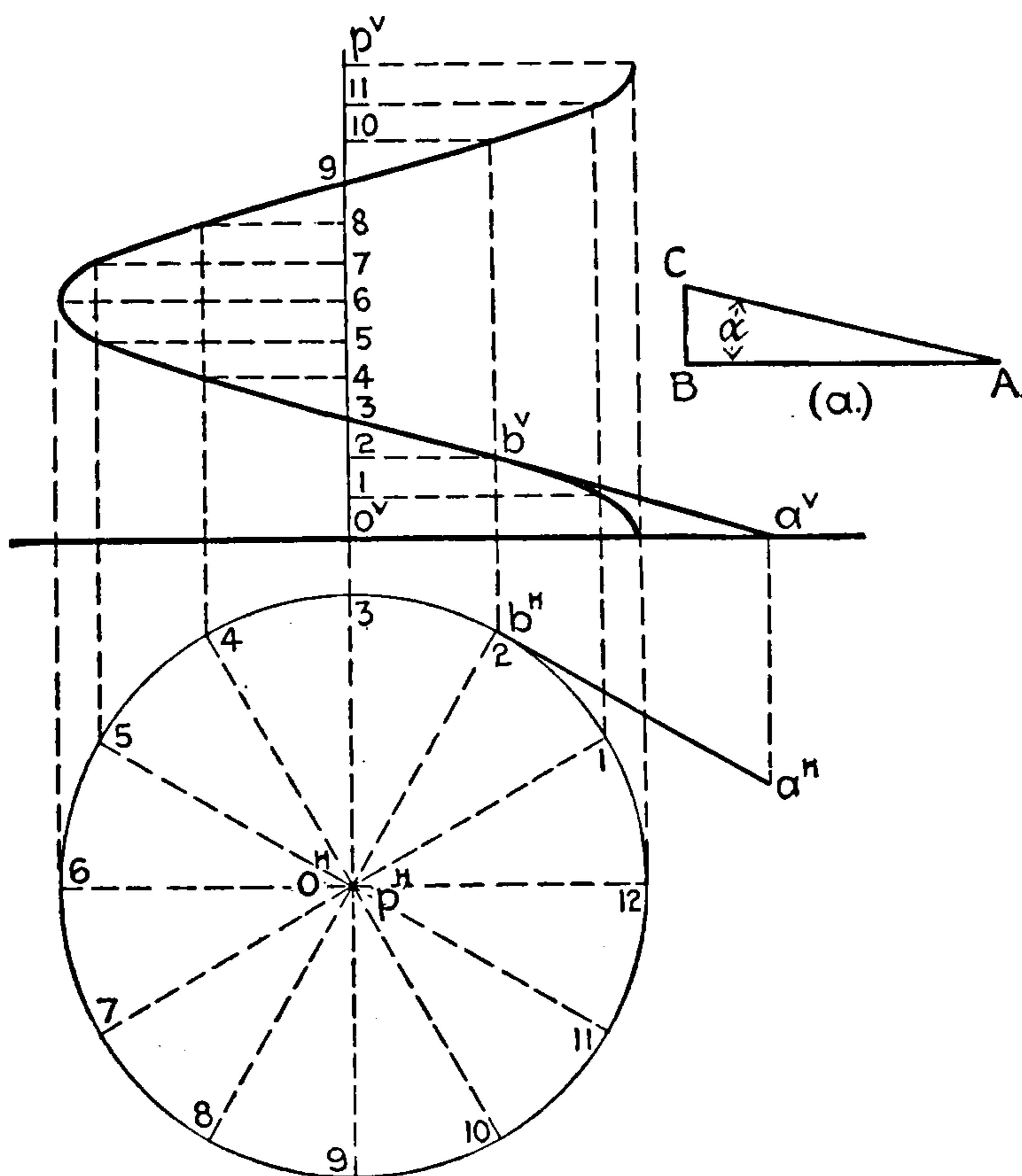


FIG. 128.

3. Draw perpendiculars to the axis from each of the divisions made in operation (2).

4. Draw projectors from each of the divisions of the circle, intersecting the corresponding perpendiculars from the axis.

5. Draw the curve through these intersections.

### 163. The Angle of the Helix.

From the definition (Art. 144), we see that the generating point on the helix ascends a given distance in traversing a given arc, and that the rate of ascent is constant. This means that any element of the curve makes a certain angle with the horizontal (the axis of the helix being taken in all these problems

as vertical), and that all the elements in any one helix make the same angle with the horizontal. Thus, in Fig. 128(a), if AB is the length of an elemental arc of the circle, and BC is the length of an elemental division of the pitch, the hypotenuse of the right triangle ABC will yield the inclination and the length of an element of the helix. Therefore, if a right triangle, having its base angle equal to  $\alpha$ , be wrapped about a cylinder, the hypotenuse will become the helix. From this it is evident that

$$\tan \alpha = \frac{\text{pitch}}{\pi \cdot \text{diam.}}$$

#### 164. The Tangent to the Helix.

If the right triangle, just mentioned, be unwrapped from the cylinder, its hypotenuse becomes the tangent to the helix at the point where the unwrapping ceases.

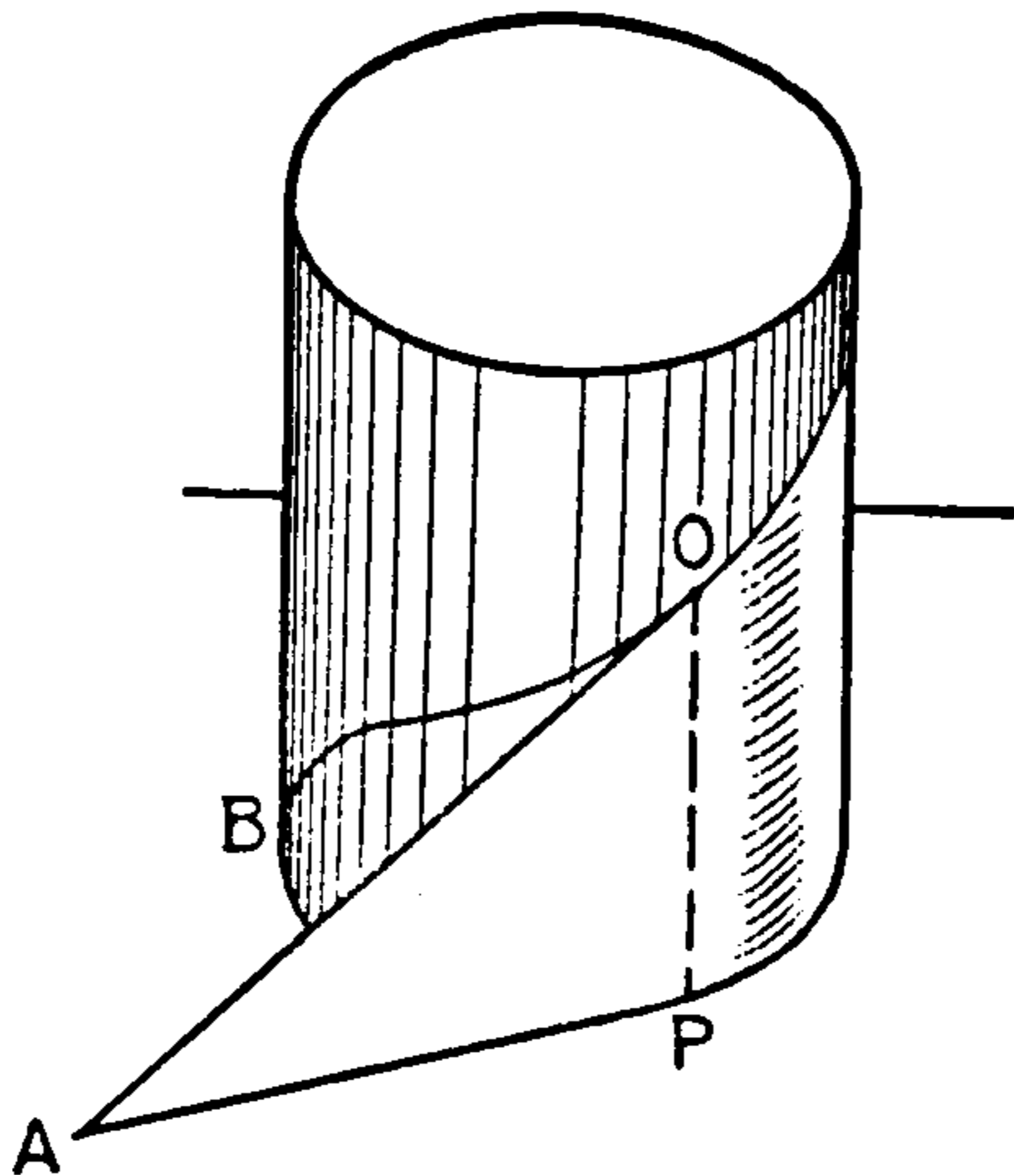


FIG. 129.—The generation of a helix.

This is so because the tangent must make the same angle with the horizontal that the helix does, because it contains two consecutive points (one element) of the curve.

Therefore, the tangent becomes the helix if it is rolled on the cylinder, and the length of the helix from its origin to the point of tangency is equal to the length of the tangent from the plane of the origin to the point of tangency. Referring to the picture, Fig. 129, will show that  $AO = BO$ , and further that  $AP = BP$ . AP is the *H*-projection of the tangent, and BP is the *H*-projection of the helix. From this the rule is derived: **The H-projection of the Helix is of the same length as the H-projection of the Tangent from the plane of the origin to the point of tangency.**



**165. Problem 63.**—To draw the Tangent to the Helix at any point on the curve.

**Construction.**—Let B (Fig. 128) be the given point. 1. Draw the tangent to the circle at  $b^H$ .

2. Rectify the arc  $b^H12$ , and lay it off on the tangent  $a^Hb^H$ .

3. Project  $a^H$  to  $a^V$  on  $GL$ .

4.  $a^Vb^V$  will be the  $V$ -projection, and  $a^Hb^H$  the  $H$ -projection of the tangent required.

166.

## EXERCISES IN CURVED LINES

**Note.**—No ground line is necessary in drawing any of the curves but the helix.

745. Draw an ellipse,  $AB = 5$  in.,  $FF' = 3$  in. Draw a tangent at a point on the curve 2 in. from  $F$ . Draw two tangents from a point 4 in. from  $F$  and 2 in. from  $F'$ .

746. Draw an ellipse,  $AB = 4\frac{1}{2}$  in., and  $CD = 3$  in. Locate  $F$  and  $F'$ , and draw a tangent at a point on the curve 1 in. from  $A$ . Draw two tangents from  $E$ , which is 3 in. from  $F'$  and 2 in. from  $F$ .

747.  $A$  and  $B$  are two tangent ellipses of equal size. The foci of  $A$  are  $C(1, 0 - 2)$   $D(3\frac{1}{2}, 0 - 2)$ , and the more distant focus of  $B$  is at  $E(5, 0 + 1)$ . Draw the two ellipses.

748. Draw a hyperbola.  $FF' = 3$  in. and the "constant difference" = 2 in. Draw the tangent at a point on the curve 2 in. from  $F$ . Draw a tangent from a point 2 in. from  $F'$  and  $2\frac{1}{2}$  in. from  $F$ .

749. Draw a parabola with the distance between focus and directrix 1 in. Draw tangents from assumed points on and outside the curve.

750. Draw a parabola whose distance from focus to vertex is  $\frac{3}{4}$  in. Draw tangents from assumed points on and outside the curve.

751. Draw a cycloid with a  $1\frac{1}{2}$ -in. generating circle. Draw a tangent at the point on the curve at  $\frac{5}{16}$  of the revolution.

752. Draw an epicycloid and a hypocycloid on a pitch circle of 6-in. radius, with  $1\frac{1}{4}$ -in. generating circles. Draw the normals to each curve at  $\frac{5}{16}$  of a revolution from the origin.

753. Draw an involute of a 1-in. circle.

754. Draw an involute of a  $\frac{1}{2}$ -in. square.

755. Draw an involute of a hexagon of  $\frac{1}{2}$ -in. sides.

756. Draw an involute of a  $\frac{3}{4}$ -in. equilateral triangle.

757. Draw a helix of 2-in. diameter, 3-in. pitch. Draw its tangent at  $60^\circ$  from the origin. Also one at  $90^\circ$ .

758. Design a focusing reflector of diameter — in. (4, 5, 6, 7), in which the rays from a light, placed — in. (2, 3, 4, 5) from the vertex are concentrated at a point — in. (8, 9, 10, 12, 14, 16) from the light.



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They are sometimes convex, sometimes concave, and sometimes both. The annular torus has both in continuous surface.

Warped Surfaces are numerous in their variety, but the most important are the helicoid and hyperboloid of revolution of one nappe.

169. The following table will give a convenient summary of the surfaces.

Single-Curved	<ul style="list-style-type: none"> <li>Cone—all elements intersecting</li> <li>Cylinder—all elements parallel</li> <li>Convolute—consecutive elements intersecting.</li> </ul>	} Ruled Surfaces.		
Warped			<ul style="list-style-type: none"> <li>Helicoid</li> <li>Hyperboloid of Revolution</li> <li>Hyperbolic Paraboloid</li> </ul>	
			<ul style="list-style-type: none"> <li>Conoid</li> <li>Cylindroid</li> <li>Warped Cone</li> <li>Cow's Horn.</li> </ul>	
	Double-Curved		<ul style="list-style-type: none"> <li>Sphere</li> <li>Oblate Spheroid</li> <li>Prolate Spheroid</li> <li>Paraboloid (both varieties)</li> <li>Hyperboloid of Révolution of Two Nappes</li> <li>Torus.</li> </ul>	
			Surfaces of Revolution	<ul style="list-style-type: none"> <li>Right Circular Cone, Right Circular Cylinder</li> <li>Hyperboloid of Revolution of One Nappe</li> <li>Sphere, Spheroids, Ovoids, etc.</li> </ul>

### 170. Tangent Planes and Developments.

The most important problems in surfaces are to be found in drawing tangent planes to surfaces, the intersections of surfaces, and developing the surfaces.

A **Tangent Plane** to a surface at any point is a plane composed of all the tangent lines to all the lines of a surface passing through that point. As two intersecting lines determine a plane, the general rule for tangent planes is this: Any two tangent lines at any point on any surface determine the tangent plane at that point.

Any plane, passed through the point of tangency so as to intersect a surface and its tangent plane, will cut from them a line and its tangent.

In ruled surfaces, one and sometimes two straight lines of the surface can be drawn through each point in the surface, therefore this line (element, as it always is) can be used for one

of the tangents in the foregoing rule. This is true, because a straight line is its own tangent. On this account, the rule will read: "In ruled surfaces, one tangent and the element at any point will determine the tangent plane at that point."

**Normal.**—The normal line at any point of a curved surface is the perpendicular to the tangent plane at that point. The normal plane is any plane containing the normal line.

**Development.**—By Development is meant the rolling out of a surface into a plane figure, which can be rolled, or folded, back to make the original surface. Only single-curved surfaces are strictly developable, because they only have consecutive elements

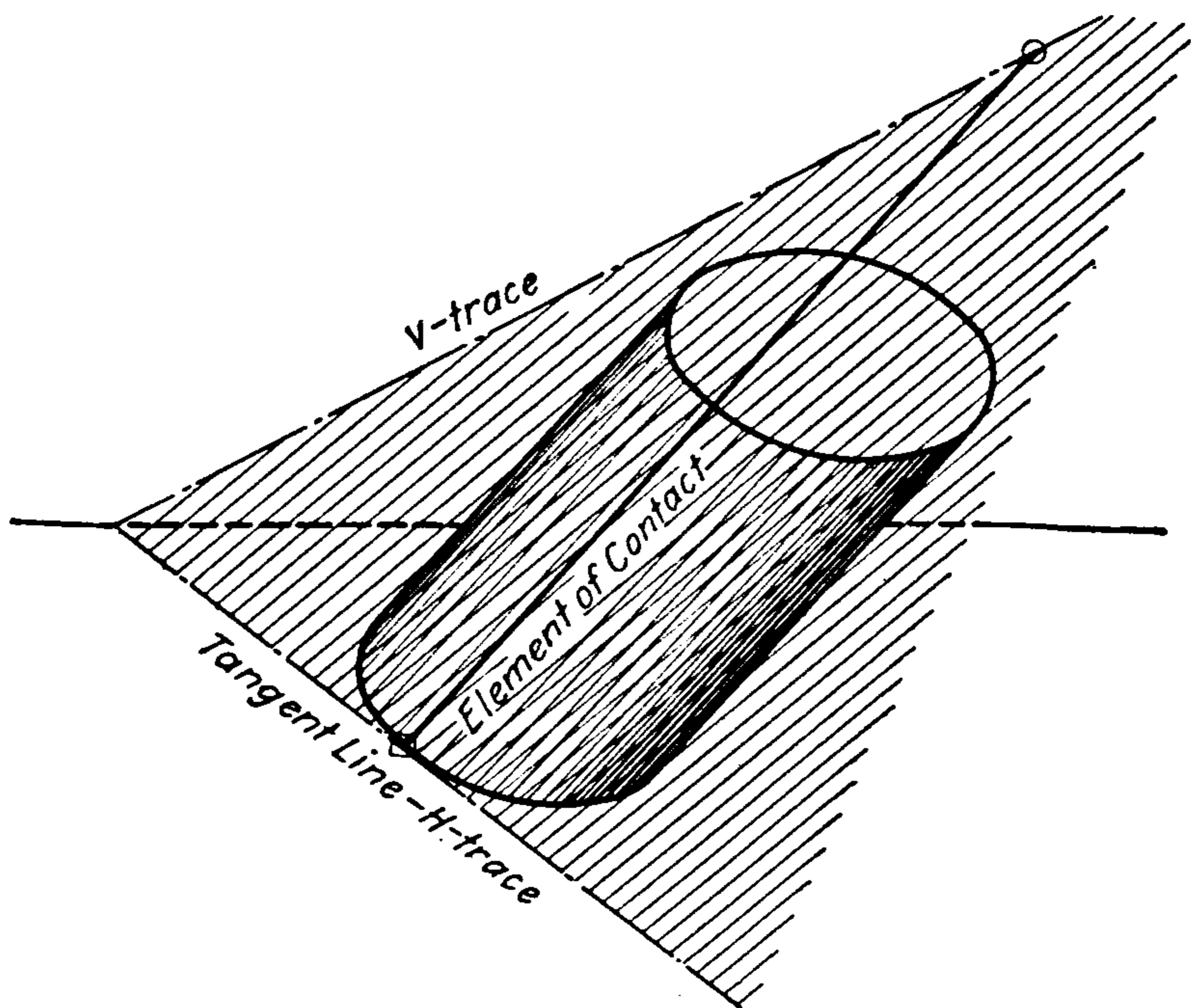


FIG. 130.—Tangent plane to a cylinder determined by two tangent lines.

lying in planes. Approximate developments can be made of other surfaces, as will appear when the surfaces are taken up separately.

## THE CONE

**171. Definitions.**—A **Cone** is generated by a straight line moving so as to touch constantly a curved line directrix, while one point in the generatrix remains stationary.

**Apex or Vertex**, the stationary point on the generatrix.

**Base**, a limiting plane that cuts all the elements.

**Axis**, a straight line through the apex and the center of the base.

A **Frustum** is a portion of the surface between two parallel planes on the same side of the apex, which cut all the elements.

**Discussion.**—The surface of the cone that lies between the base and apex is usually all that is considered. Theoretically, however, the surface is unlimited, and extends beyond both limits to infinity. This theoretical extension creates two divisions of the surfaces, one on each side of the apex, called respectively the **Upper Nappe** and **Lower Nappe**. In practical work this condition almost never exists, and is usually ignored.

**Varieties of Cones.**—The different varieties of cones are distinguished by their bases and axes. The bases may be circular, elliptical, hyperbolic, etc., or combinations of curves, and the axes are either perpendicular (**right**) or oblique to the bases. Thus we designate a cone as a **right circular cone**, an **oblique elliptical cone**, etc. The base and apex, or the base and axis, are sufficient to determine the cone.

**Limits.**—A cone, whose apex is at infinity, becomes a **cylinder**, and one, whose apex is in the plane of the base, becomes a **plane**.

A **Tangent Plane** to a cone at any point on the surface is tangent all along the element through that point. This element is called the **Element of Contact**. As all elements pass through the apex, every tangent plane must contain the apex. This fact is of great importance in passing tangent planes. It means that **any line through the apex**, except lines inside the cone, will lie in one or two tangent planes. From this fact, and from Art. 155, we derive the special rule for passing tangent planes to cones.

**Rule.**—**One tangent line and the apex are sufficient to determine a tangent plane to any cone.**

**Axiom.**—If the base of a cone is in  $H$ , the  $H$ -trace of any tangent plane will be tangent to the base.

**172. Development of Cones.**—When the surface of a cone is rolled out, we find that there is one point common to all elements, the apex, and this point will remain a point in the development. From this point all the elements radiate in fan shape, making up an integral of all the minute triangles that form the surface of the cone. In the case of a **right circular cone**, the elements are all of equal length, and the base rolls out into an arc of a circle, whose radius is the length of the elements (sometimes called "**slant height**"). The length of the arc thus derived is



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2. Obtain the elements of the cone cut by this plane.

3. Where these elements intersect the given line will be located its piercing points with the cone.

Let the student make the construction.

Note.—There are occasions when a plane intersecting a number of elements can be used to better advantage than a plane through the apex. Such a plane will cut a curve from the cone surface, and the piercing points will be at the intersections of the given line with the curve.

**176. Problem 66.**—To pass a plane Tangent to a Cone at a point on the surface.

**Analysis.**—Read the rule given in Art. 171.

**Construction.**—Refer to Fig. 133. Let  $O$  be the given point.

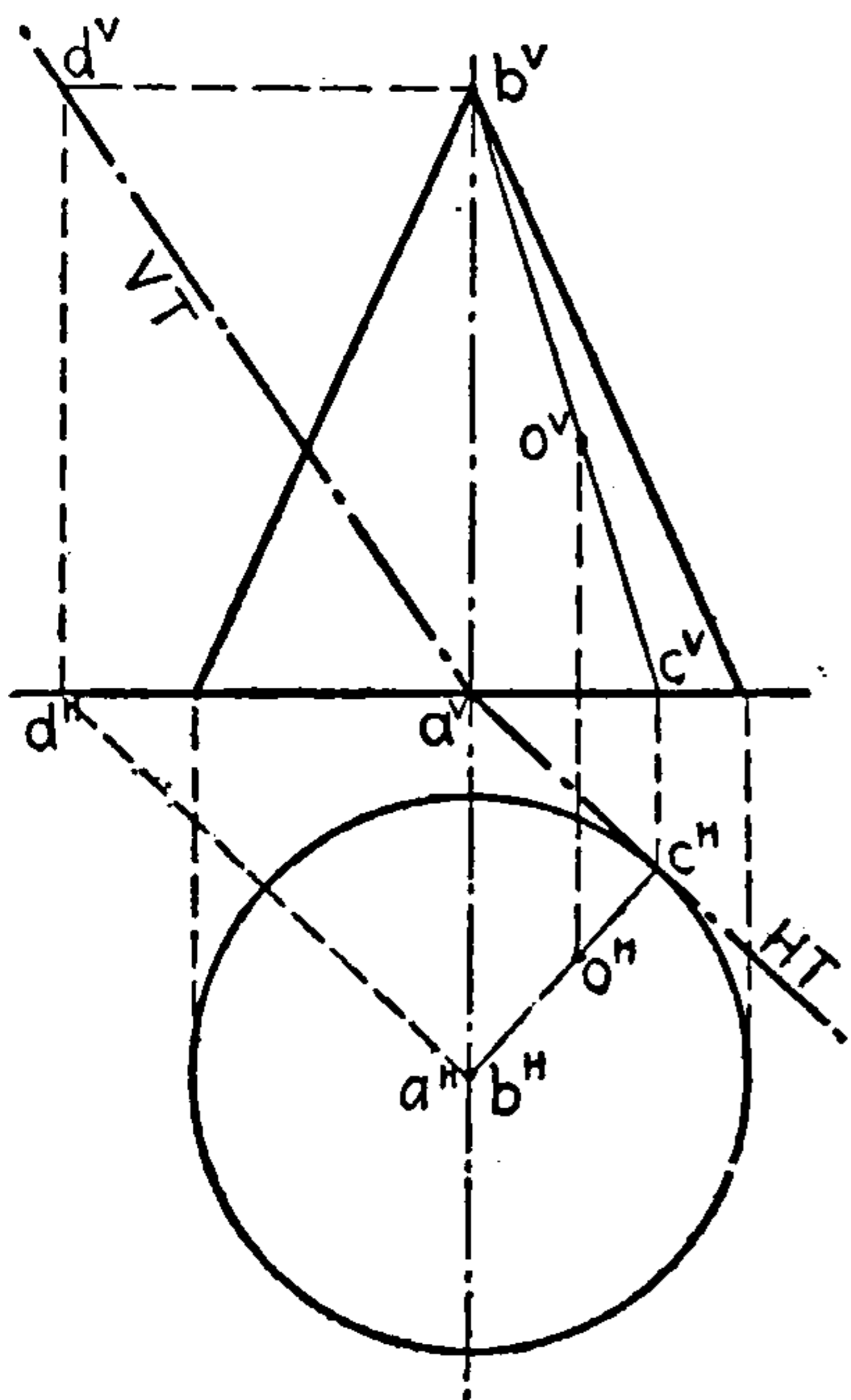


FIG. 133.

1. Draw the element through  $O$ . This is the **Element of Contact**, and one line of the plane.

2. Where the element of contact intersects the base, draw the tangent to the base.

This is the *H*-trace of the required plane, and the *V*-trace may be obtained in several ways known to the student.

**177. Problem 67.**—To pass a plane Tangent to a Cone through a Point outside the surface.

**Analysis.**—1. Draw a line through the apex and the given point. (See Art. 171.)

2. Find the piercing point of this line with the plane of the base.

3. From this point draw one of the two possible tangents to the base.

4. Pass the required plane through the tangent line and the apex.

**Construction.**—Let the student make it.

**Note 1.**—If the line through the given point and the apex is parallel to the base, the tangent line in the plane of the base will be parallel to said line, and may be drawn on either side of the base.

**Note 2.**—If the line through the apex is too nearly parallel to the plane of the base to obtain the piercing point within the limits of the drawing, a plane may be passed parallel to the base, nearer the apex, and a tangent





3. If the base be in  $H$ , and the axis oblique, different angles, but not all, may be assigned to the problem.

4. If the base be in  $H$ , and a different curve from the circle, even though the axis be perpendicular, there will be some variety to the angles that may be required.

Each problem will fix its own limitations.

**Construction.**—Let it be required to pass a plane tangent to the cone  $AB$  (Fig. 134) making  $60^\circ$  with  $H$ .

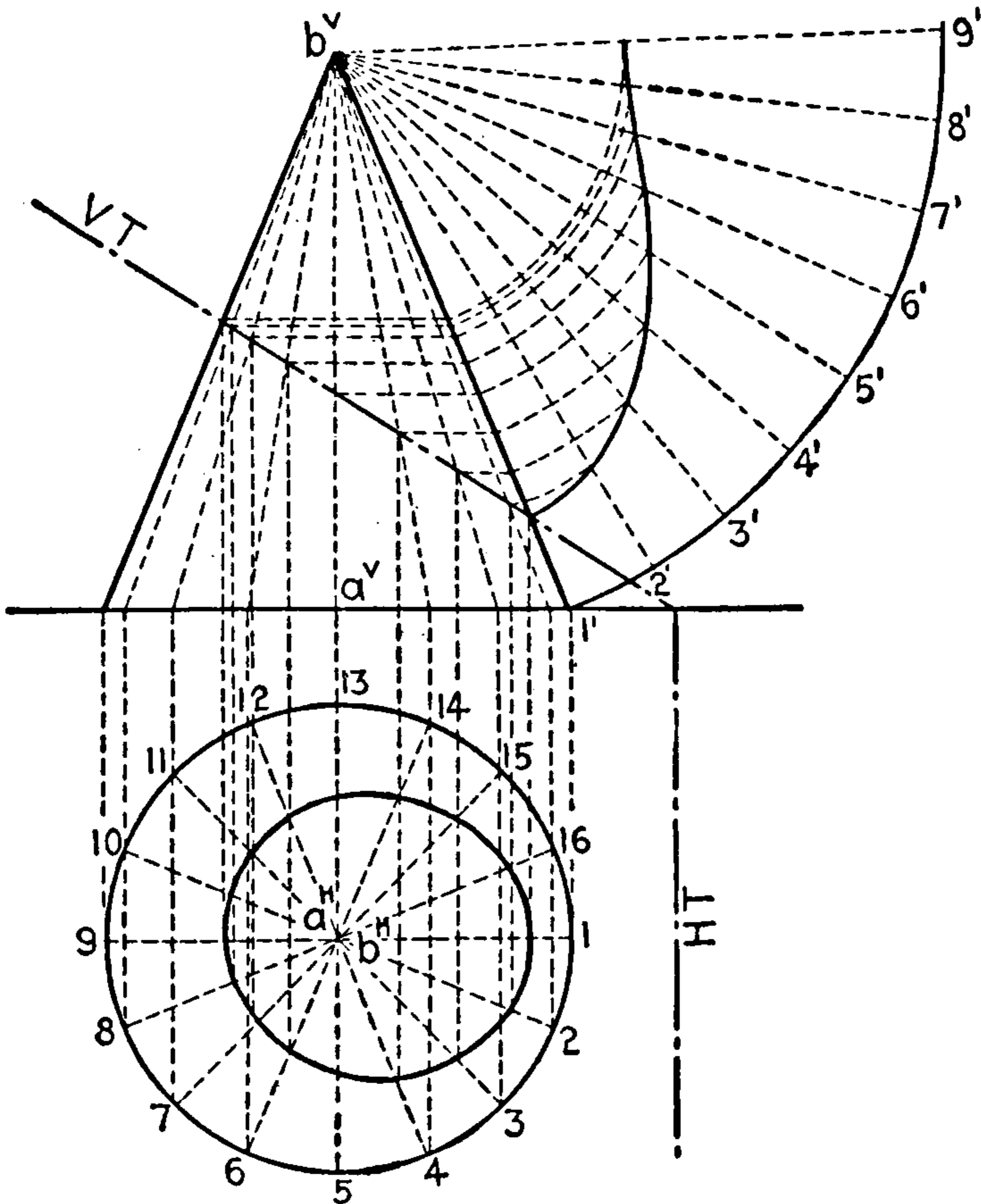


FIG. 135.

1. With  $B$  as its apex, draw the projections of a  $60^\circ$  cone, axis  $BC$ .

2. Draw the  $H$ -trace of  $T$ , tangent to the bases of the two cones. This may be drawn on either side of the cones.

3. Locate the  $V$ -trace of  $T$  by any lines through  $B$ , lying in  $T$ .

**Note.**—Under certain conditions four planes can be passed making the same angle with  $H$ , all tangent to the cone.

**180. Problem 70.**—To develop the surface of a right circular cone.



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**Construction.**—1. If it is desired to have a conical elbow of two pieces to turn through an axial angle of  $2\theta$ , pass a plane through the cone at an angle of  $\theta$  with the base.

2. Turn the truncated portion on the lower part, and the required elbow is made.

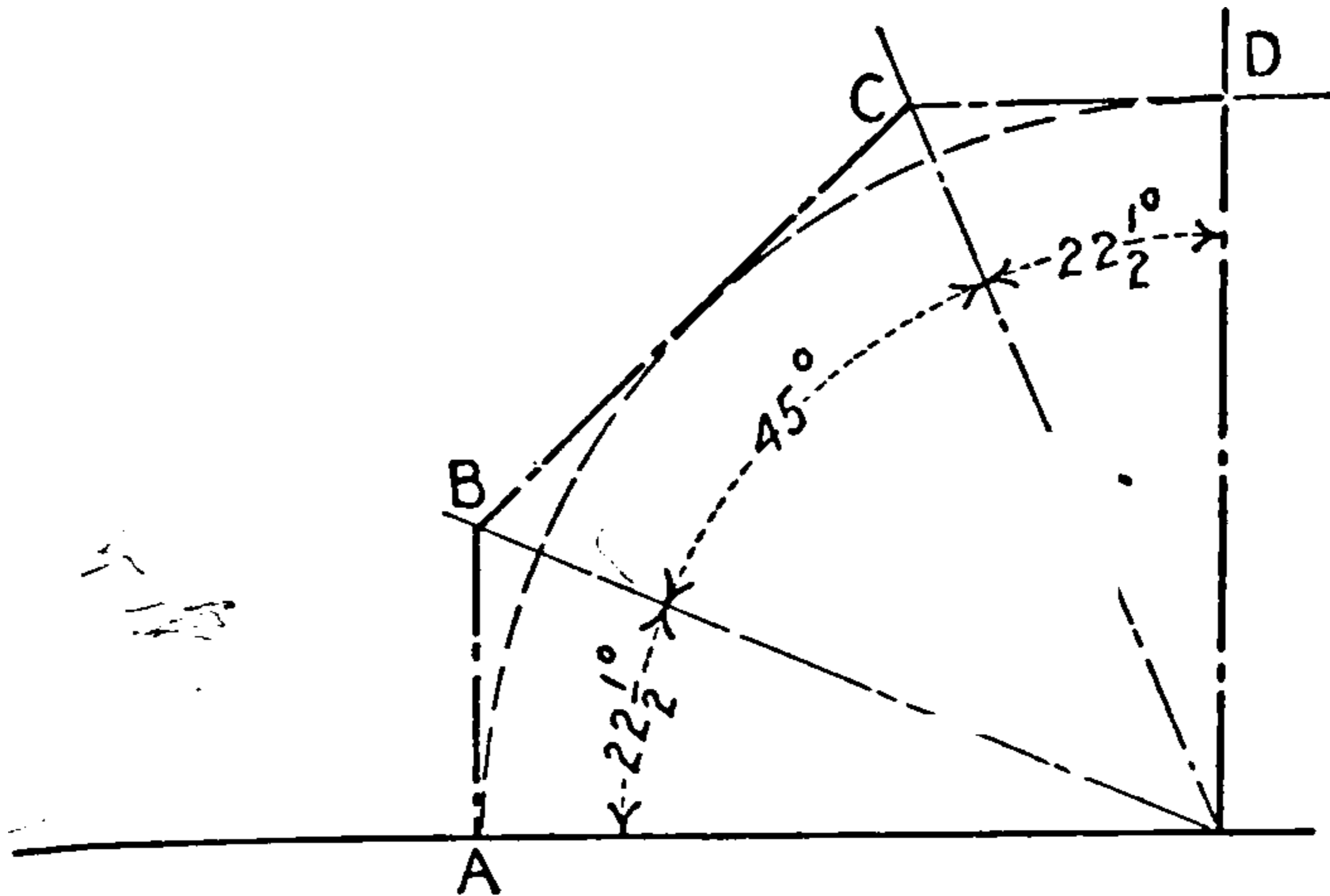


FIG. 138.—Center line layout.

### MULTI-JOINT ELBOWS

**183.** The construction of this is essentially that of the single-joint elbow.

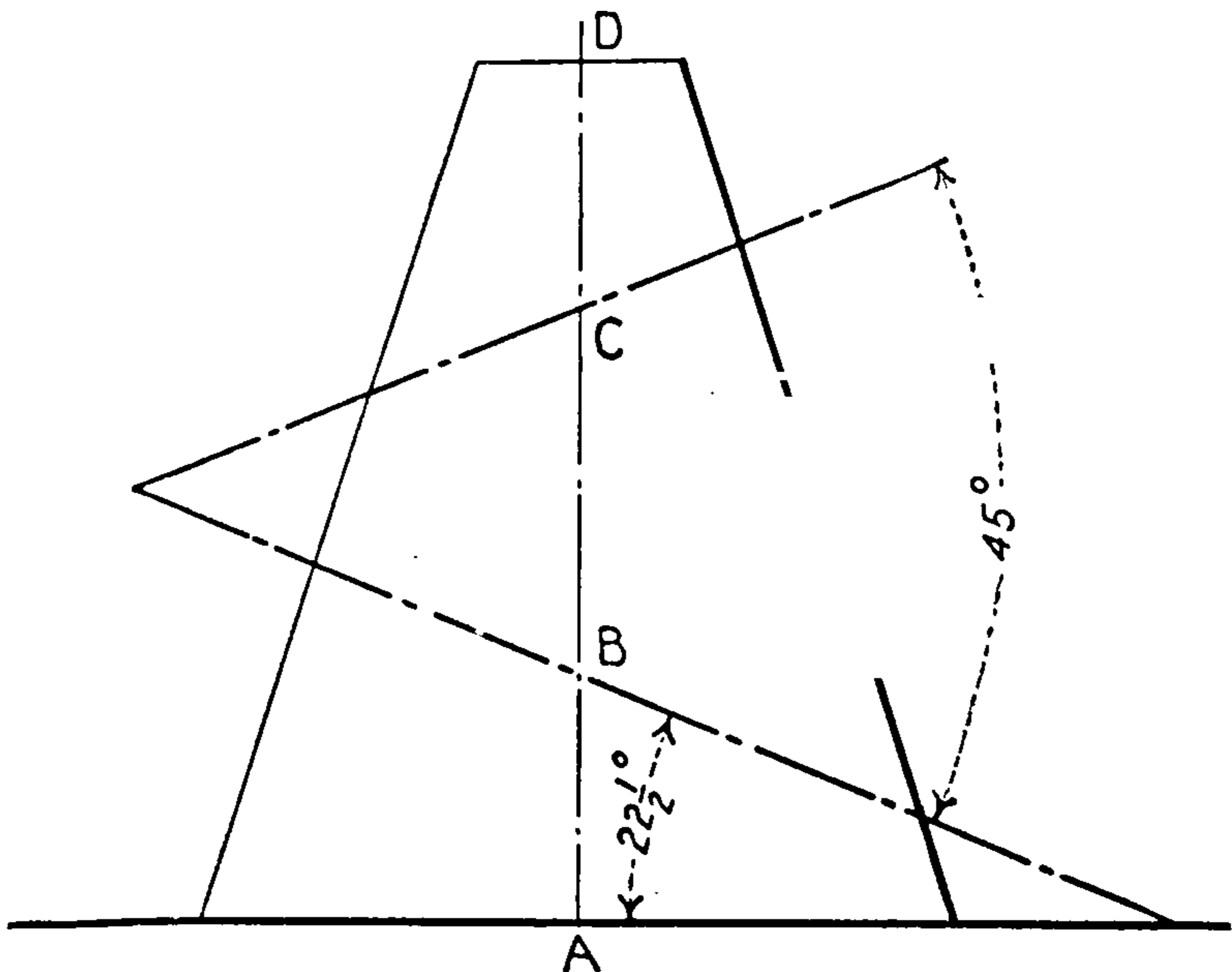


FIG. 139.—Laying out the seams.

**Example.**—Required to reduce a 6-in. pipe to a 2-in. pipe, through a  $90^\circ$  bend, in three pieces.

1. Layout the center lines according to Fig. 138.

2. Lay off the total length of the center lines as the altitude of the frustum of a cone of 6-in. base and 2-in. top.

3. Cut the frustum by planes through B and C at the angles shown in Fig. 139.

4. Reverse these truncated portions as shown in Fig. 140.

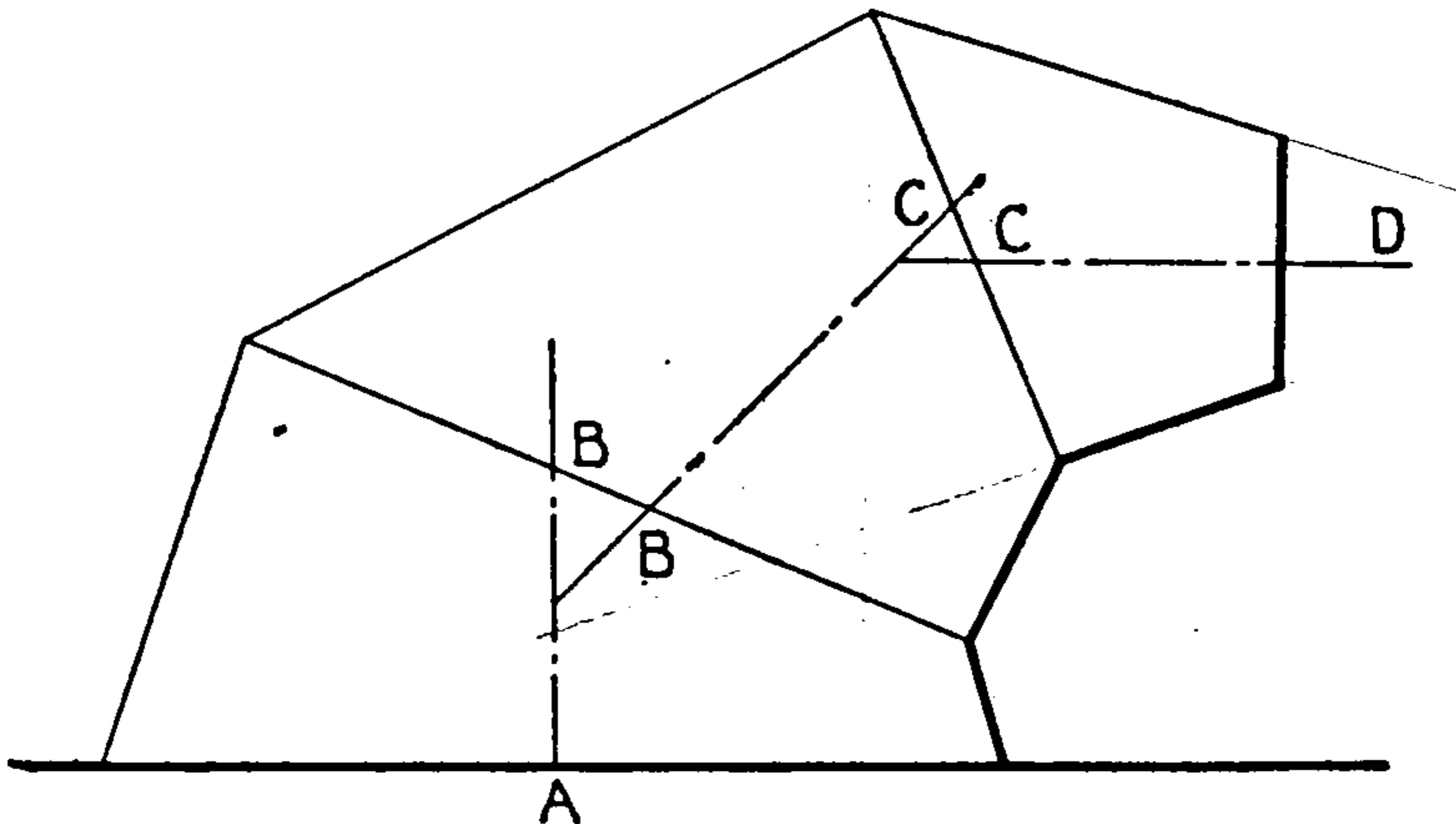


FIG. 140.—Three-piece elbow.

184.

EXERCISES

1		2		3		4		5	
6	<i>Cone Tangent to H</i> 	7	<i>Cone Tangent to H</i> 	8		9		10	
									<i>Cone Tangent to V</i> 

Graphic Layout No. 50.—Center lines of right circular cones.

Base — in. (1, 1½, 1¾, 2, 2½).

Altitude — in. (1½, 2, 2½, 3, 3½).

Note.—All these axes are in *I*. The signs may be reversed, and the lines drawn in *III* at the option of the instructor.

Dimensioned Layout.—Center lines. Let A or B be the center of the base.

(1)  $A(3 + 3 - 1\frac{1}{2}) B(3, 0 - 1\frac{1}{2})$ .

(2)  $A(1 + 1 - 1\frac{1}{2}) B(3\frac{1}{2} + 1 - 1\frac{1}{2})$ .



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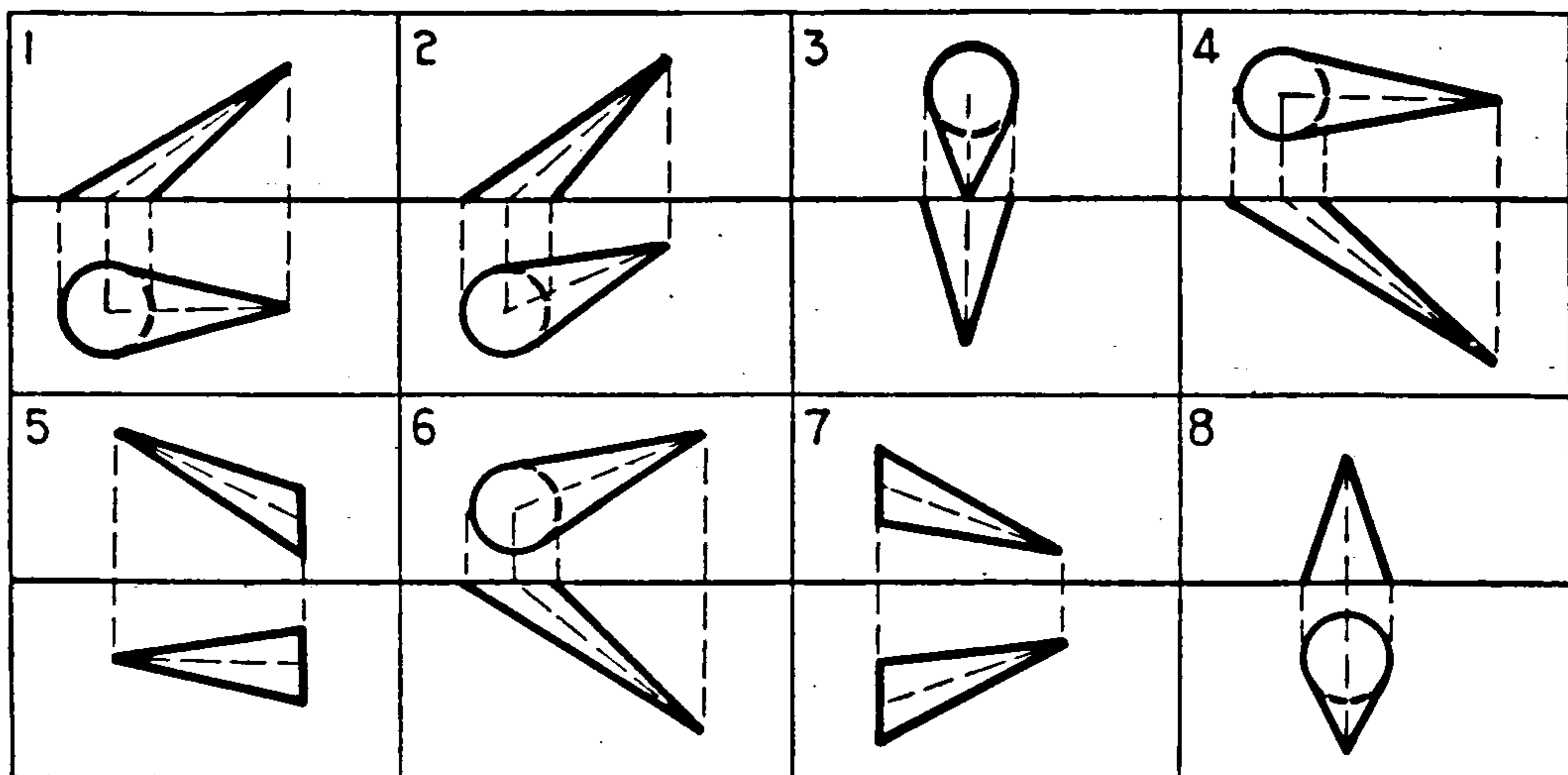
- (5)  $W(-1) \infty (-\frac{1}{2})$ .  
 (6)  $X(1+3) 3(5-3)$ .  
 (7)  $Y(1+1) 5(1+2\frac{1}{2})$ .  
 (8)  $Z(2+3) 3(2-2)$ .  
 (9)  $P(3\frac{1}{2}+3) 3\frac{1}{2}(3\frac{1}{2}-3)$ .

**Piercing Lines.**—Signs may be reversed, if desired.

- (1)  $A(2, 0 - 1\frac{1}{4}) B(4 + 2 - 2)$ .  
 (2)  $C(2, 0 - 2) B(4 + 2 - \frac{1}{2})$ .  
 (3)  $E(3 + 2 - \frac{1}{2}) F(3 + 1 - 2\frac{1}{2})$ .  
 (4)  $G(3 + 2 - 2) K(3 + \frac{1}{2} - 1)$ .  
 (5)  $M(1 + 1 - 1\frac{1}{2}) N(4 + 1 - 1\frac{1}{2})$ .

759. Locate a point on the — (front or rear) surface of cone —, — in. away from  $H$ .

760. Cut the surface of cone — by plane —. Find the true size of the section.



**Graphic Layout No. 52.**—Oblique cones with circular bases, diameters  $1\frac{1}{2}$  in.,  $1\frac{3}{4}$  in., 2 in.,  $2\frac{1}{2}$  in., and axes  $1\frac{1}{2}$  in., 2 in.,  $2\frac{1}{2}$  in., 3 in. long.

Oblique cones with elliptical bases, 1 in.  $\times$   $1\frac{1}{2}$  in., 1 in.  $\times$  2 in.,  $1\frac{1}{2}$  in.  $\times$  2 in.,  $1\frac{1}{2}$  in.  $\times$   $2\frac{1}{2}$  in., 2 in.  $\times$   $2\frac{1}{2}$  in., and axes  $1\frac{1}{2}$ , 2,  $2\frac{1}{2}$ , 3 in. long.

Use this Layout in Ex. 770–781.

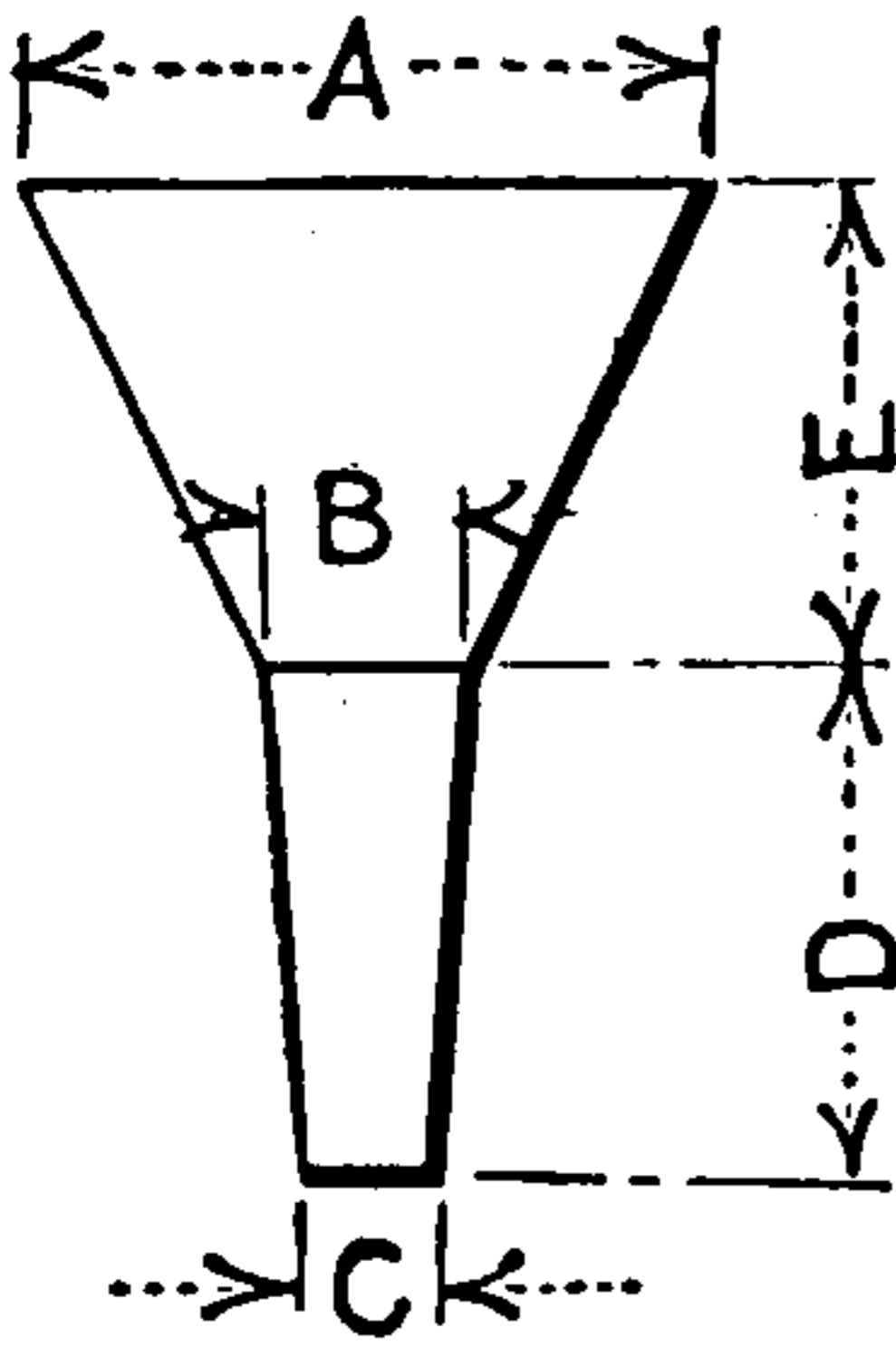
761. Develop the surface between the base and cutting plane, in Ex. 760.  
 762. Develop the surface between the apex and cutting plane, in Ex. 760.  
 763. Find the points in which the surface of cone — is pierced by line —.  
 764. Find the shortest line on the surface of cone — between the piercing points of line —.  
 765. Assume a point on the surface of cone —, and draw the traces of a plane tangent at that point.  
 766. Assume a point in space outside the cone, and draw the traces of a plane (or two planes, if desired) tangent to the cone —.  
 767. Draw the traces of one of the tangent planes to cone —, inclined —° to — ( $H$ ,  $V$ , or  $P$ ).

768. Draw the traces of the tangent plane (or planes) to the cone —, parallel to line — (one of the given piercing lines).
769. What are the limitations of the specifications of Ex. 766 and 767, if any?

Dimensioned Layout.—Let the axes be the following lines, with the circular or elliptical base — at A. Use in Ex. 770–781.

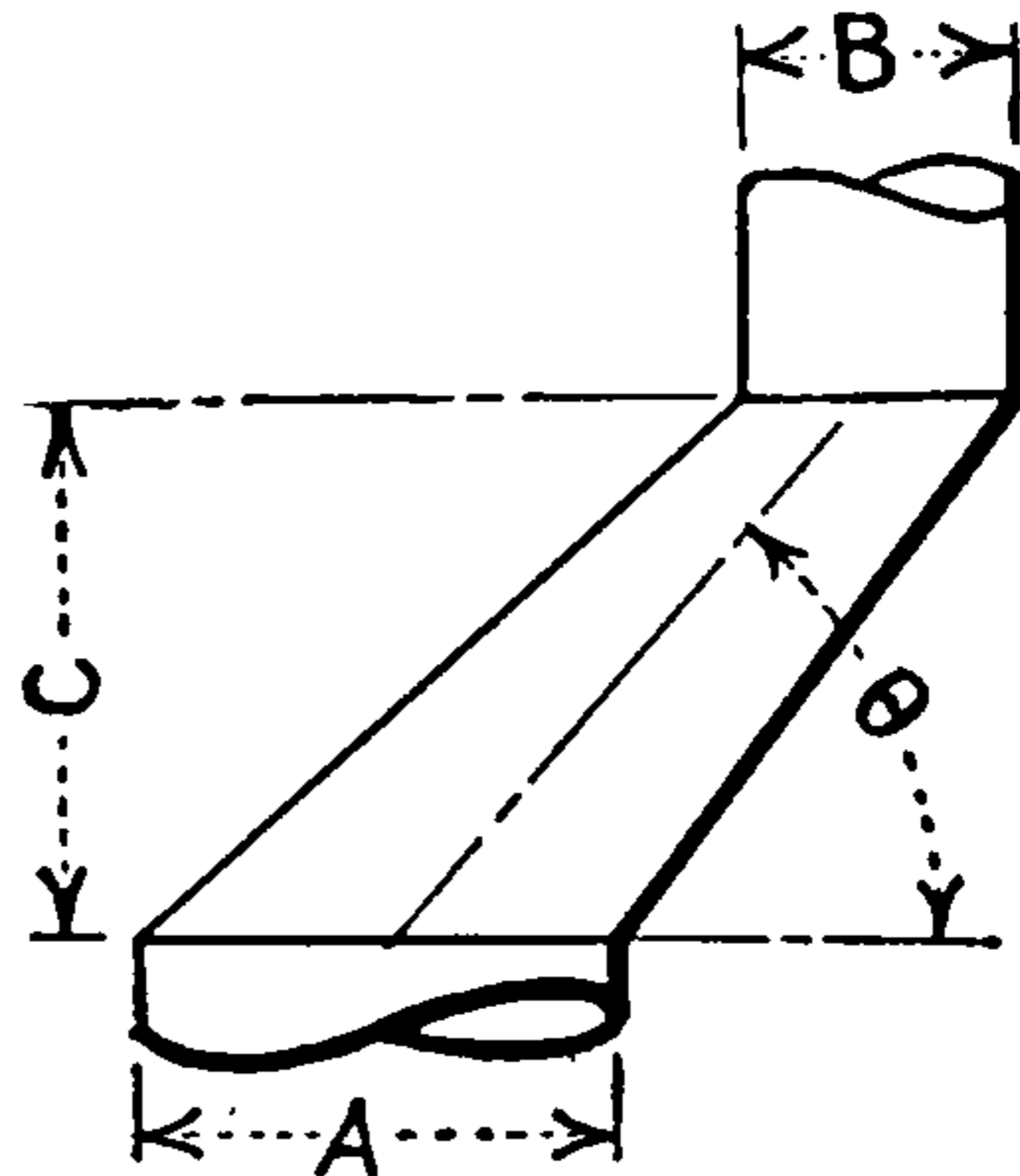
- (1) A (2, 0 - 1½) B(4 + 2 - 1½).
  - (2) A(2, 0 - 1) B(4 + 2 - 2).
  - (3) A(3 + 1, 0) B(3 + 2½ - 1½).
  - (4) A(2 + 1½, 0) B(3½ + 1½ - 2).
  - (5) A(2 + 1, 0) B(3½ + 2 - 2).
  - (6) A(4 + 1 - 1½) B(2 + 2 - 1½).
  - (7) A(4 + 2 - 2) B(2 + 1 - ½).
  - (8) A(3, 0 - 1) B(3 + 2 - 2).
770. Cone —, base —, axis —; locate a point on the — (front or rear) surface, — in. away from *H*.
771. Cone —, base —, axis —; draw the intersection, and its true size, made by plane —.
772. Develop the surface of the base frustum in Ex. 770.
773. Develop the surface of the apex frustum in Ex. 770.
774. Cone —, base —, axis —. Find the points in which it is pierced by line —.
775. Draw the shortest line on the surface of the cone between the piercing points in Ex. 774.
776. Cone —, base —, axis —; assume a point on the surface, and draw the traces of a plane tangent at that point.
777. Assume a point in space, outside of cone —, base —, axis —; draw the traces of a plane (or both planes, if desired) tangent to the cone through the assumed point.
778. Cone —, base —, axis —; draw the traces of a plane tangent, and inclined —° to — (*H*, *V*, or *P*).
779. Cone —, base —, axis —; draw the traces of a plane tangent to the cone and parallel to line —.
780. What are the limitations of the specifications of Ex. 778 and 779, if any?
781. Cone —, base —, axis —; draw the projections and true size of its right section at the middle point of its axis.
782. Lay out the patterns for Funnel No. —. Graphic Layout No. 53.
783. Lay out the patterns for Motor Boat Funnel No. —.
784. Lay out the patterns for Ventilator Hood No. —.
785. Lay out the patterns for Sink Drainer No. —.
786. Lay out the patterns for Offset Reducing Pipe No. —,  $C = \text{—}$ ,  $\theta = \text{—}$ .
787. Lay out the patterns for the Furnace Hood No. —. Make the plan of the conical helix a Spiral of Archimedes, making one complete revolution. Omit lap and rivet holes.
788. Lay out the patterns for Chimney Transition Piece No. —. Graphic Layout No. 54.





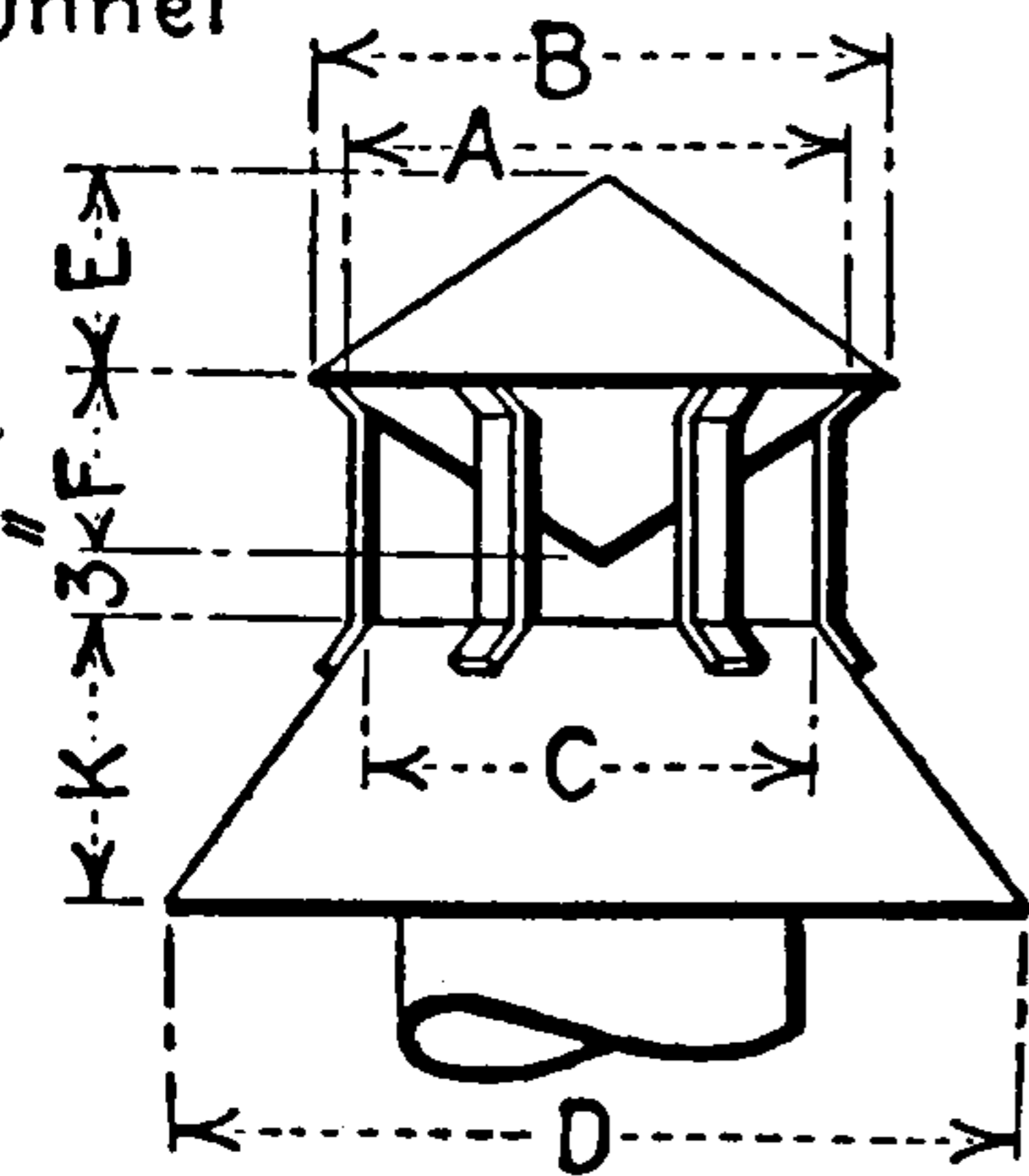
	A	B	C	D	E
1	4	1	1/2	4	4
2	6	1 1/4	3/4	4	5
3	8	2 1/2	1	6	8
4	10	3	1 1/4	5	7

Funnel



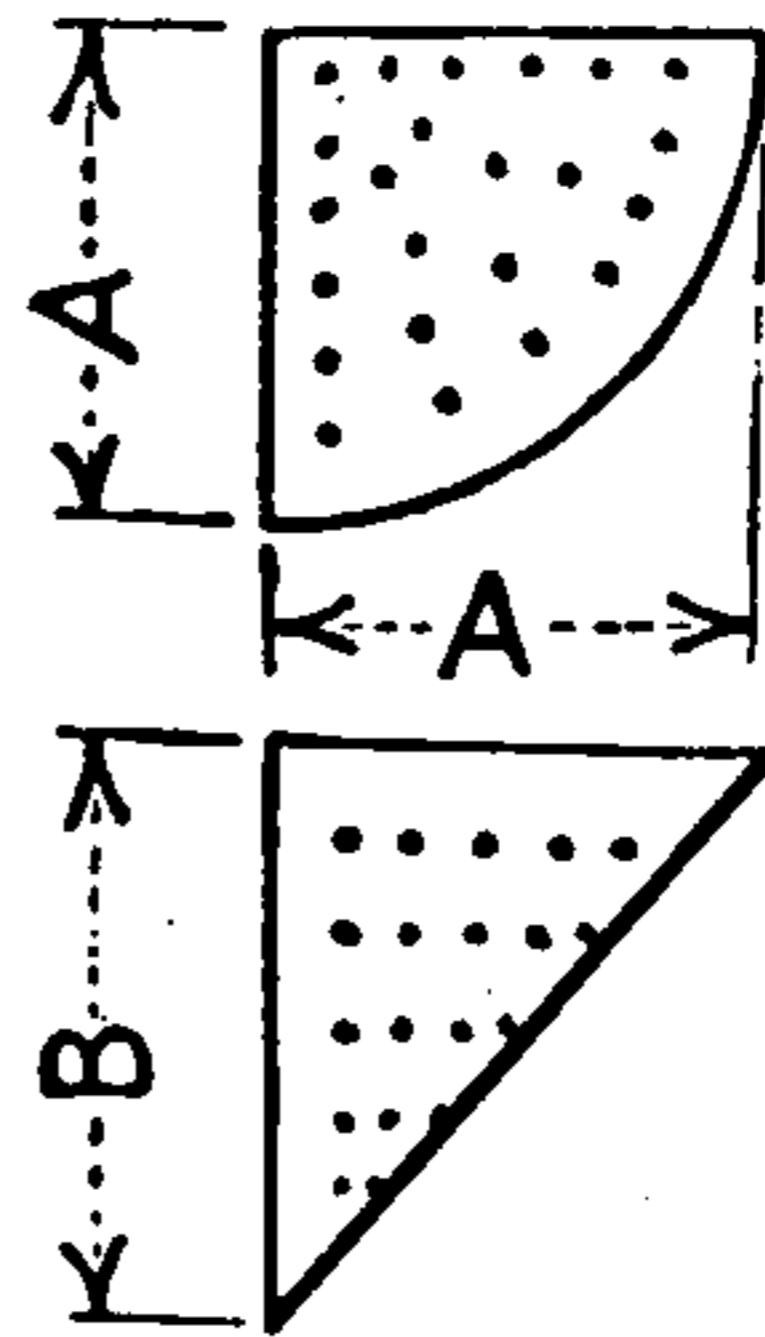
	A	B	C	theta
1	8	3	10, 12, 16	30°, 45°, 60°, 67 1/2°
2	8	4	"	"
3	10	5	12, 15, 18	"
4	10	6	"	"
5	12	6	"	"

Offset Reducing Pipe



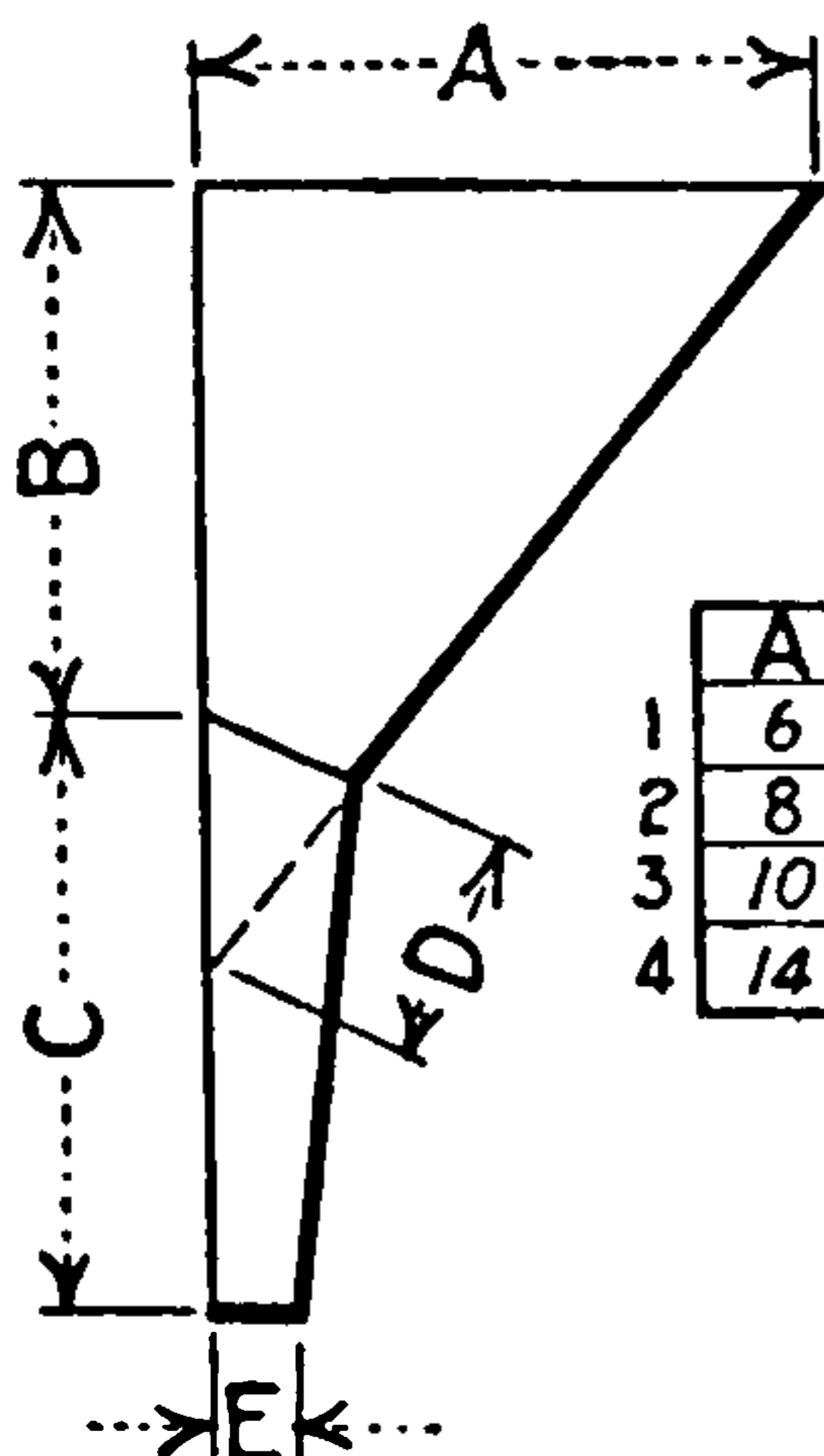
	A	B	C	D	E	F	K
1	27	24	20	36	6	5	8
2	32	28	26	40	7	6	9
3	36	32	28	44	8	7	10
4	40	35	30	48	9	8	12
5	45	40	36	54	10	9	14

Ventilator Hood



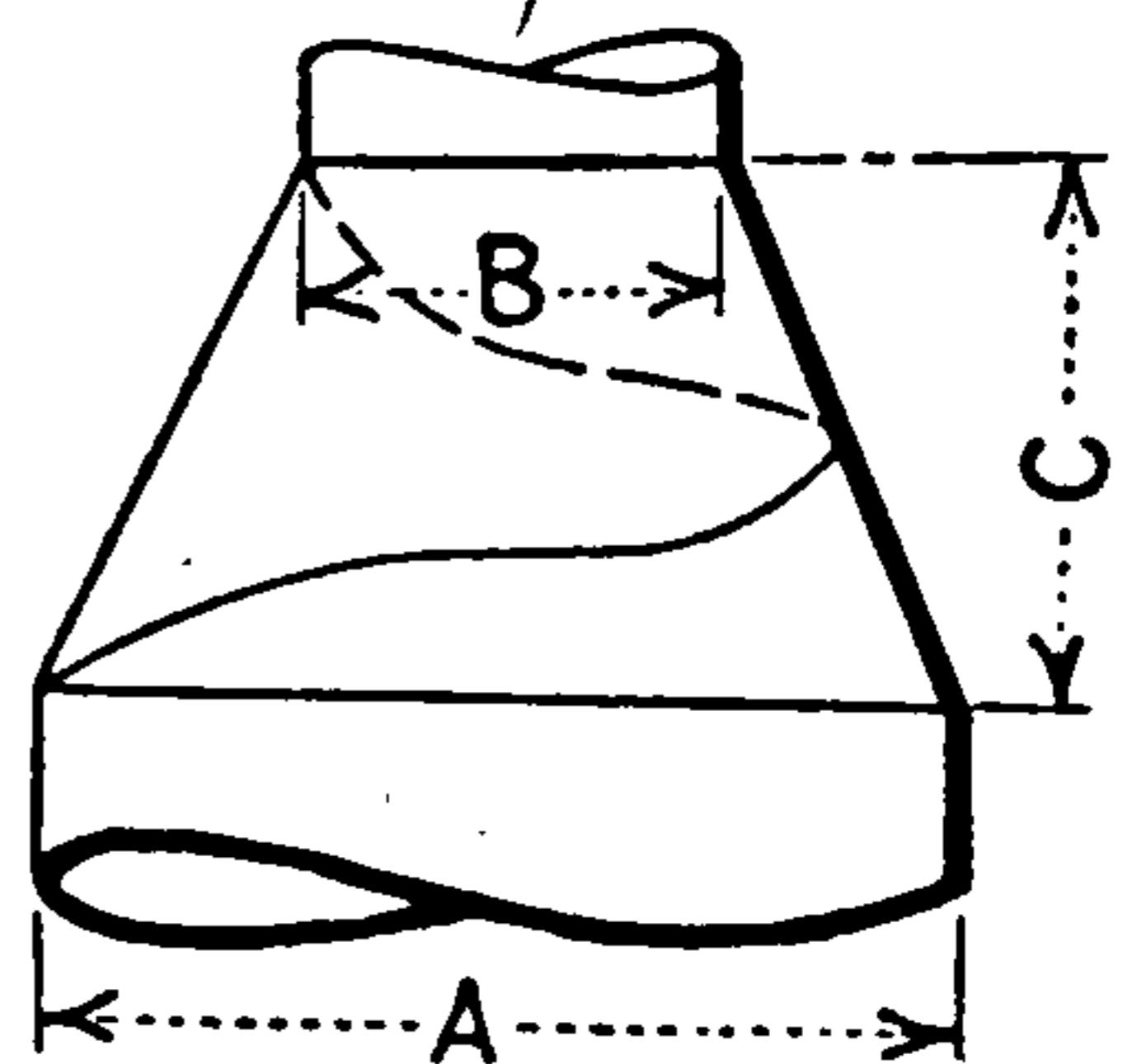
	A	B
1	6	8
2	6	10
3	8	10
4	8	12
5	10	12

Sink Drainer



	A	B	C	D	E
1	6	8	8	4	3/4
2	8	10	10	5	7/8
3	10	12	14	6	1
4	14	18	16	10	1 1/4

Motor Boat Funnel



	A	B	C
1	24	12	20
2	28	12	24
3	30	13 1/2	26
4	32	15	28
5	40	18	30

Furnace Hood



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## THE CYLINDER

**185. Definition.**—A cylinder is a single-curved surface generated by a line moving along any curved-line directrix, always remaining parallel to a given line. It is a cone with its apex at infinity.

The Surface of a cylinder is usually taken to be that portion which is included between two plane bases. The bases may be parallel or not. Theoretically the cylinder is infinite in extent.

**Axis.**—The axis is a line through the center of a right section, parallel to all the elements. It is a useful line in defining the cylinder in space.

**Varieties of Cylinders.**—The characteristic distinction between cylinders is the curve of their right section. This is at some variance to the cone distinctions, which are founded on base and axial peculiarities. The reason for this is that it is absolutely necessary to know the right section of a cylinder in most of the problems arising; as, for instance, in its development, the right section always rolls out in a straight line. Also, in most tangent problems, the right section has to be considered. So, when a cylinder is spoken of as circular, elliptical, etc., a right section is understood.

**Limits.**—When the directrix becomes a straight line, the cylinder becomes a plane, and when the directrix becomes a point the cylinder becomes a line.

**186. Tangent Planes to Cylinders.**—A plane which contains two consecutive elements is tangent. **The Tangent Plane is parallel to the axis, and to every element.** The two consecutive elements contained in the tangent plane form the **Element of Contact**. This element of contact is one of the two tangent lines necessary to determine the tangent plane. From this fact we derive the special rule for passing planes tangent to cylinders.

**Rule.**—A tangent line and a line parallel to the axis, intersecting the tangent, determine a tangent plane to a cylinder.

**Axiom.**—When the base of a cylinder is in  $H$ , the  $H$ -trace of any plane, tangent to the cylinder, will be tangent to the base curve of the cylinder.

**Axiom.**—A plane cannot in general be tangent to two cylinders unless the axes are parallel. The exceptions, which are not frequent, will only be found when two outside elements of the two cylinders are in the same plane. That plane will be the

tangent plane. If two cylinders have parallel axes, four planes tangent to both can usually be passed. Two cylinders of equal diameter, with axes intersecting, can have two common tangent planes outside.

**187. Development.**—As was before mentioned, the right section of a cylinder rolls out into a straight line. This is true, because the right section is perpendicular to all the elements, and when they are rolled out they remain parallel, and the plane perpendicular to them cuts their plane in a straight line perpendicular to them. Therefore, the first thing to do is to **rectify a right section**. Then lay off perpendiculars, properly spaced, with the true lengths of the various elements laid off in order.

### PROBLEMS RELATING TO CYLINDERS

**188. Problem 72.**—To locate a point on the surface.

**Analysis.**—1. Assume one projection of the point.

2. Draw a line through that projection parallel to the axis. (This will be the element on which it lies.)

3. Find the intersection of this element with the base.

4. Project this intersection to the other projection of the base and draw the other projection of the element.

5. Project the point to the second projection of the element.

Let the student make the construction.

**189. Problem 73.**—To find the intersection of a plane and a cylinder.

**Analysis.**—1. Draw a sufficient number of elements.

2. Obtain the piercing points of these elements with the plane.

3. Draw a smooth curve through these piercing points in order.

**190. Problem 74.**—To find the points in which a straight line pierces a cylinder.

There are two methods: 1. to use when the base of the cylinder lies in one of the planes of projection; 2. to use when the base is in space.

**Analysis. First Method.**—When the base is in  $H$ ,  $V$  or  $P$ .

1. Pass a plane through the line parallel to the axis.

2. At the points in which the trace of this plane crosses the base, draw elements of the cylinder.

3. The points in which these elements are intersected by the given line will be the required points.





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diameter, whose axis is AB (Fig. 142). Let it be required to pass a tangent plane through M, a point outside the surface.

1. Draw the line MN, parallel to AB through M.
2. Pass a plane Q through A, perpendicular to AB. (This will be the plane of the base, cutting a circle from the surface.)
3. Find the piercing point, N, of MN with Q.
4. Revolve A and N into  $H$  about HQ.
5. Draw a  $\frac{3}{4}$ -in. circle about  $a_1$  as a center.
6. From  $n_1$  draw a tangent to the circle.
7. Counter-revolve the tangent  $n_1o$  into the plane Q.
8. Pass the plane S through NO and MN. It will be the required tangent plane.

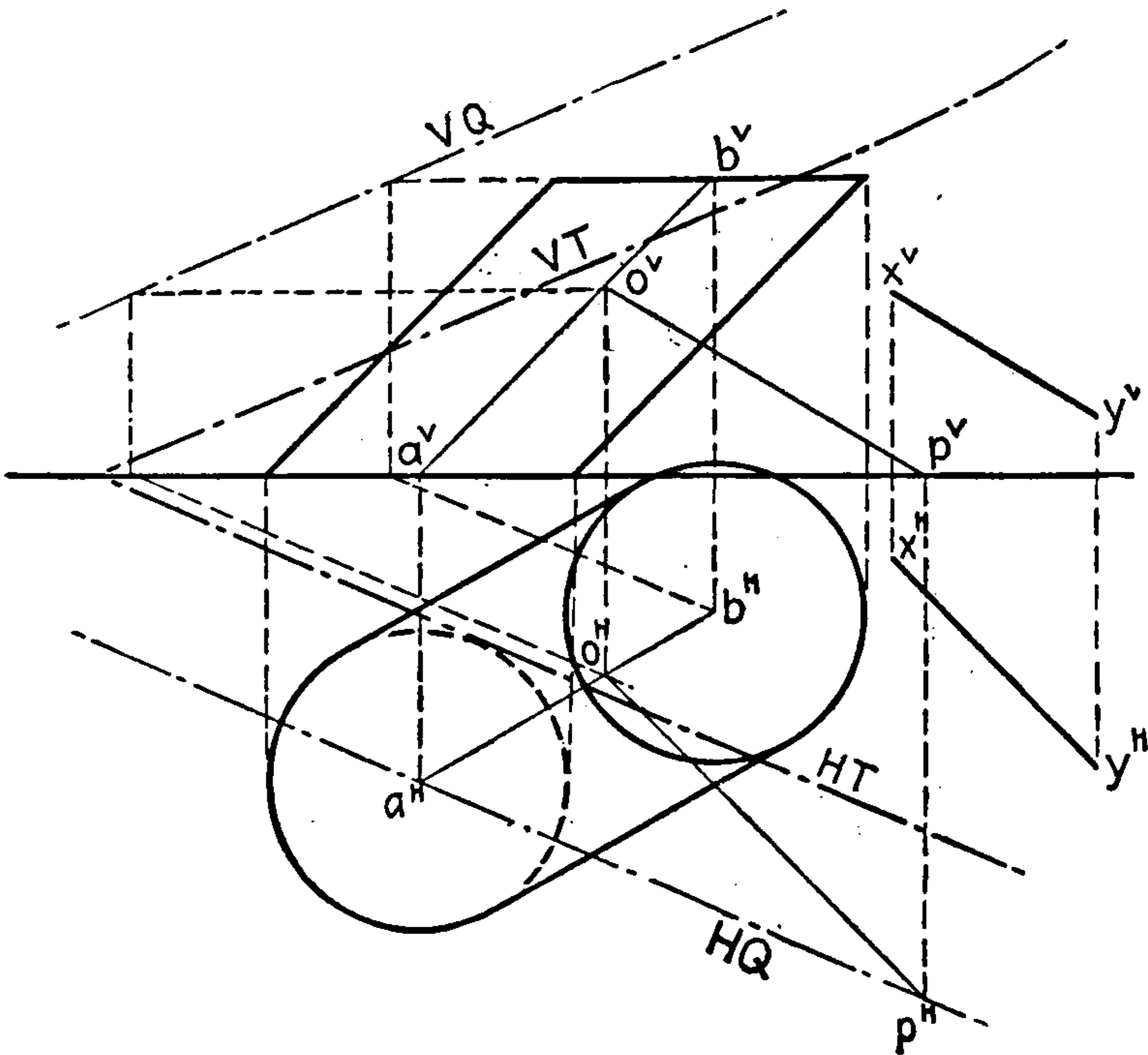


FIG. 143.

**193. Problem 77.**—To pass a plane tangent to a cylinder and parallel to a given line.

**Analysis.**—1. Pass a plane parallel to the axis and the given line. Why?

2. Find the intersection of this plane and the plane of the base.
3. Draw a tangent to the base parallel to the intersection obtained in operation (2).

4. Pass the required tangent plane through this tangent line, parallel to the auxiliary plane, passed in operation (1).

**Construction.**—Let AB, Fig. 143, be the axis of the cylinder, and let it be required to pass the tangent plane parallel to XY.

1. Pass the auxiliary plane  $Q$  through  $AB$  parallel to  $XY$ .
2. Draw the required plane  $T$ , tangent to the cylinder and parallel to  $Q$ . The  $H$ -trace of  $T$  will be tangent to the base in  $H$ , and parallel to  $HQ$ , and the  $V$ -trace of  $T$  will be parallel to  $VQ$ .

**194.** If the cylinder be circular and its axis oblique, the best method for the foregoing problem is the "Normal" method. Its analysis is as follows:

1. Pass a plane through the axis parallel to the given line.
2. Pass a plane parallel to this plane at a distance equal to the radius of the cylinder.

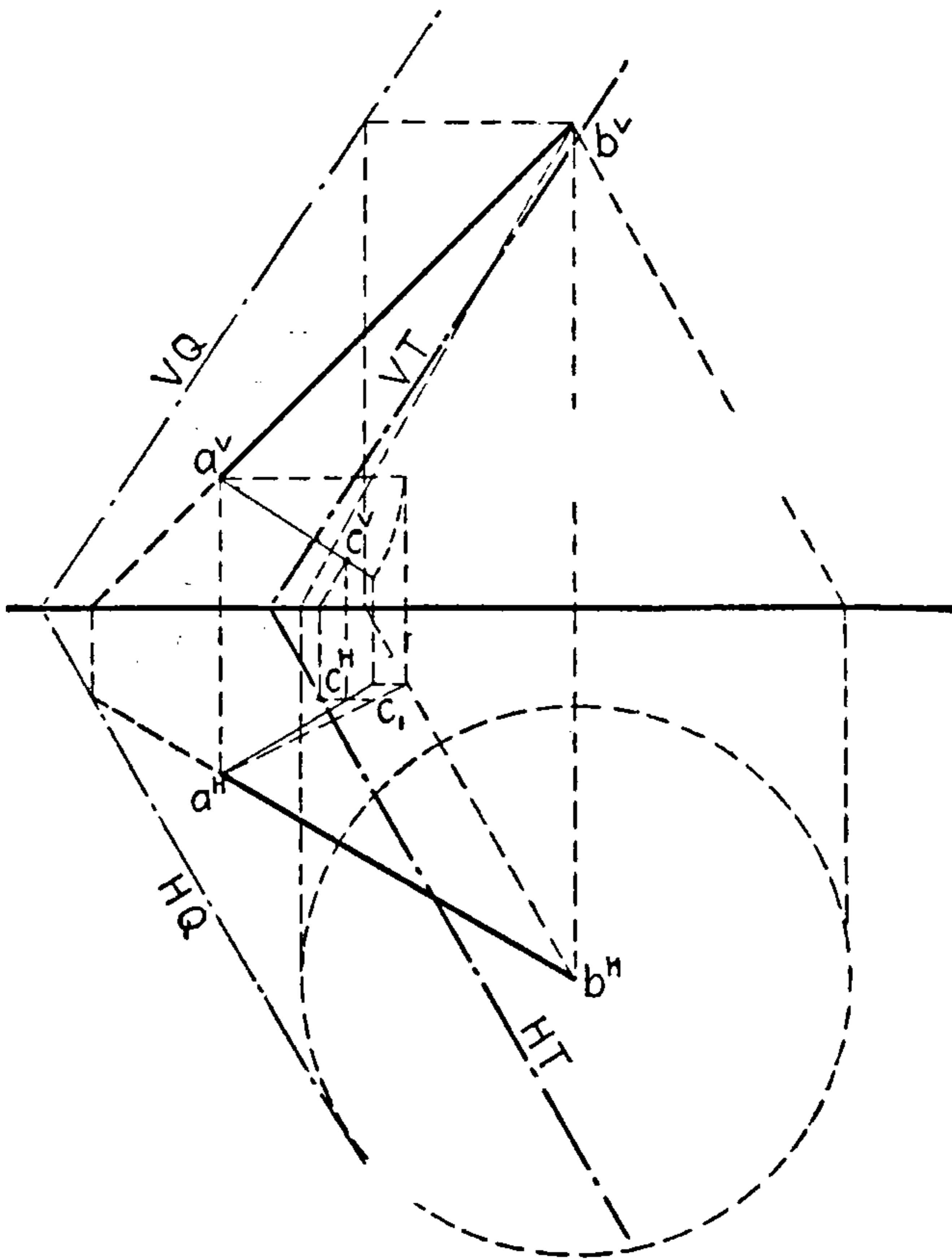


FIG. 144.

The problem may be done in this way without drawing any elements or bases of the cylinder. Let the student prove that the "normal" method is correct.

**195. Problem 78.**—To pass a plane, inclined at any given angle to  $H$  or  $V$ , tangent to a cylinder.

**Limitations.**—The limits of this angle must be between  $90^\circ$  and the angle between the elements and the stipulated plane.

**Analysis.**—1. Pass an auxiliary plane through the axis making the given angle with  $H$  or  $V$ , as required; see Problem 40.



2. If the base of the cylinder is in  $H$ , draw the  $H$ -trace of the required plane, tangent to the base and parallel to the  $H$ -trace of the auxiliary plane.

3. Draw the  $V$ -trace of the required plane parallel to the  $V$ -trace of the auxiliary plane.

**196. Normal Method.**—When this problem is required of a circular cylinder, oblique to  $H$  and  $V$ , the normal method, like that in Art. 194, is the easiest and best.

Fig. 144 shows the construction of such a problem, in which  $AB$  is the axis and  $AC$  the radius of the given cylinder. The elements of the auxiliary cone make the given angle with  $H$ .  $T$  is one of the four possible required planes.

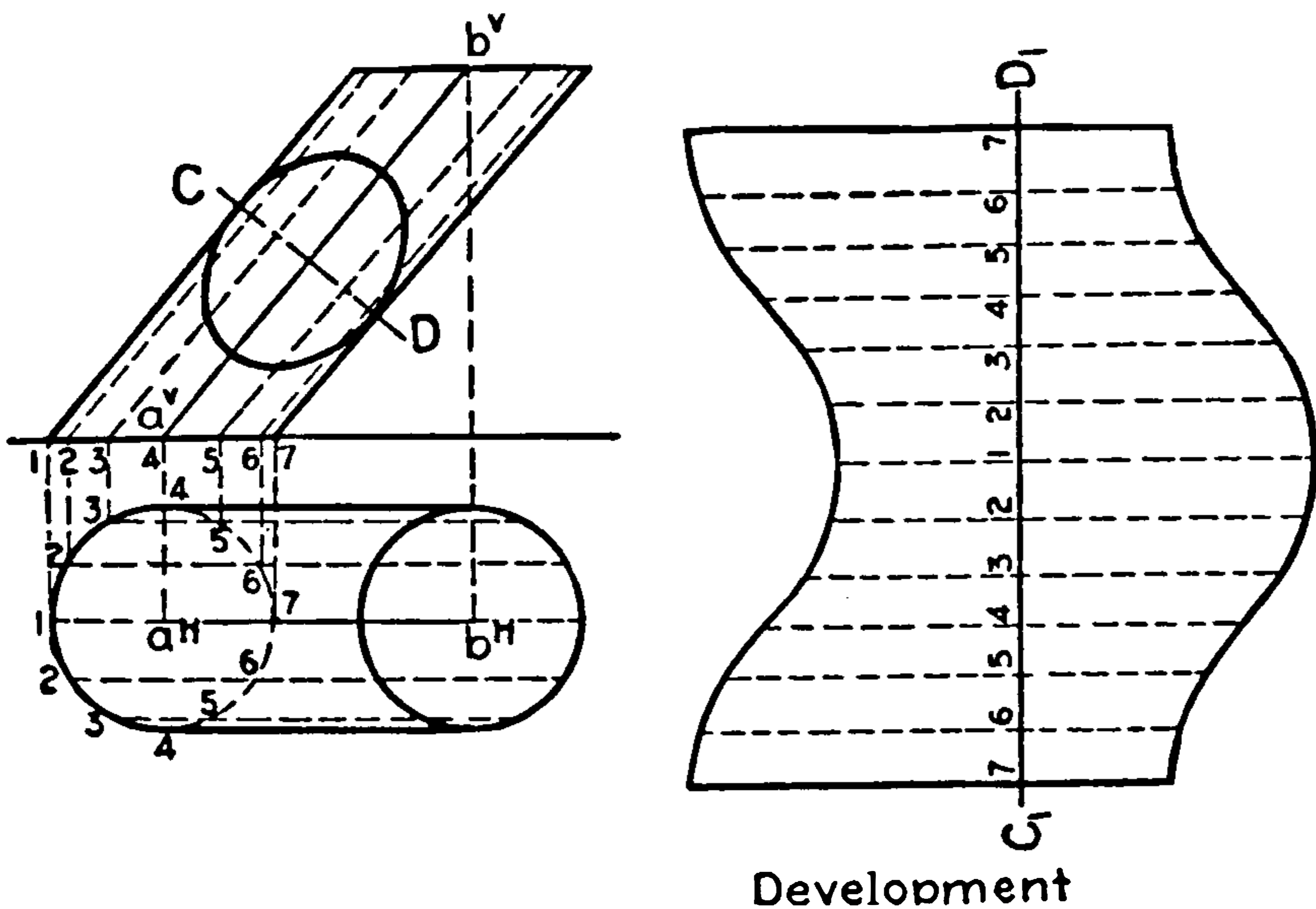


FIG. 145.

**197.** When the axis is parallel to  $H$ , and the angle between the tangent plane and  $H$  is given, the problem is very easily solved by projecting on a plane perpendicular to the axis.

Note.—Four planes can be passed tangent to any cylinder, making the given angle, if the problem is possible.

**198. Problem 79.**—To develop the surface of a cylinder.

**Construction.**—To develop the elliptical cylinder, whose axis is  $AB$  (Fig. 145).

1. Draw a sufficient number of elements.
2. Pass a plane perpendicular to the axis, as, for example, along the line  $CD$ .
3. Revolve this right section to show its true size.
4. Lay off the base line,  $C_1D_1$ , equal in length to the length of



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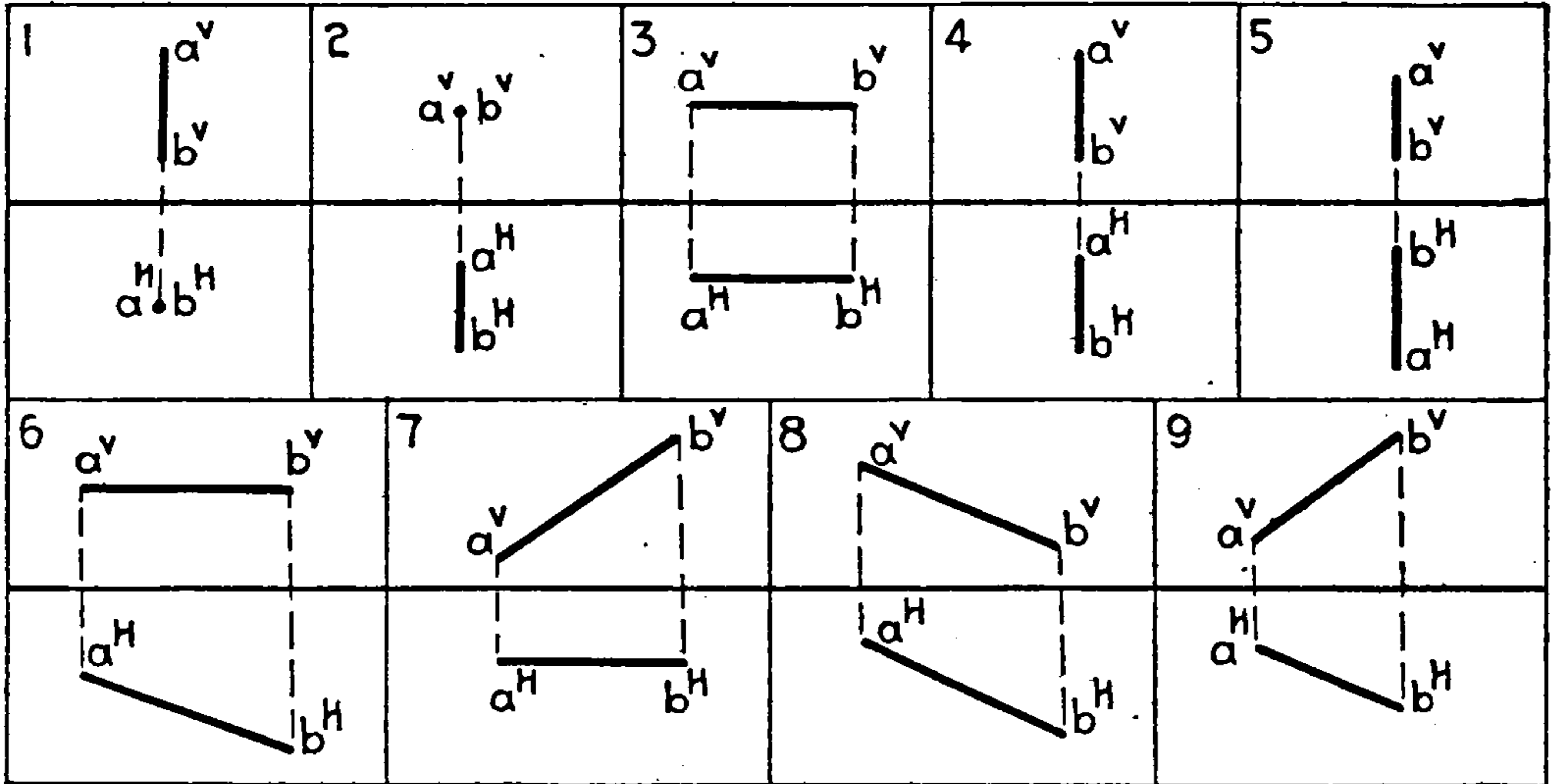
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(See Fig. 146.) Lay off a  $90^\circ$  arc on a 10-in. radius, and divide the arc into arcs of  $11\frac{1}{4}^\circ$ . The first of these radii will be the projection of the first joint, and the odd radii, the third, fifth, and seventh, will be the projections of the other joints. Pipes laid out perpendicularly to the even radii will give a pipe bend of uniform right section.

200.

## EXERCISES



Graphic Layout No. 55.—Axes for circular or elliptical cylinders, with bases perpendicular to the axes.

Bases.—Circles 1 in.,  $1\frac{1}{2}$  in., 2 in.; ellipses 1 in.  $\times$   $1\frac{1}{2}$  in., 1 in.  $\times$  2 in.,  $1\frac{1}{2}$  in.  $\times$  2 in.,  $1\frac{1}{2}$  in.  $\times$   $2\frac{1}{2}$  in.

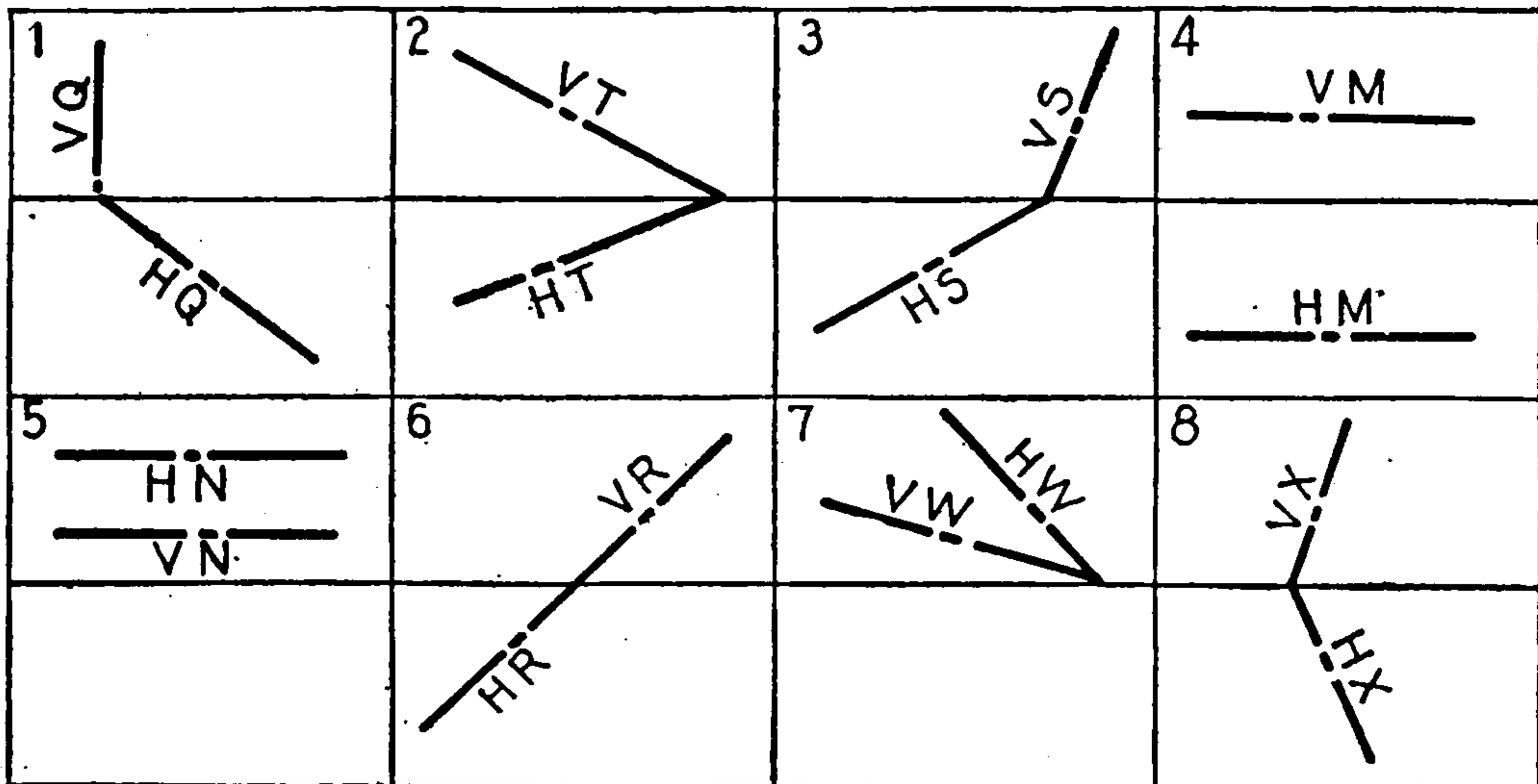
Altitudes.— $1\frac{1}{2}$  in., 2 in.,  $2\frac{1}{2}$  in., 3 in.

Note.—The signs for these lines may be reversed, and the cylinders drawn in *III*, if desired.

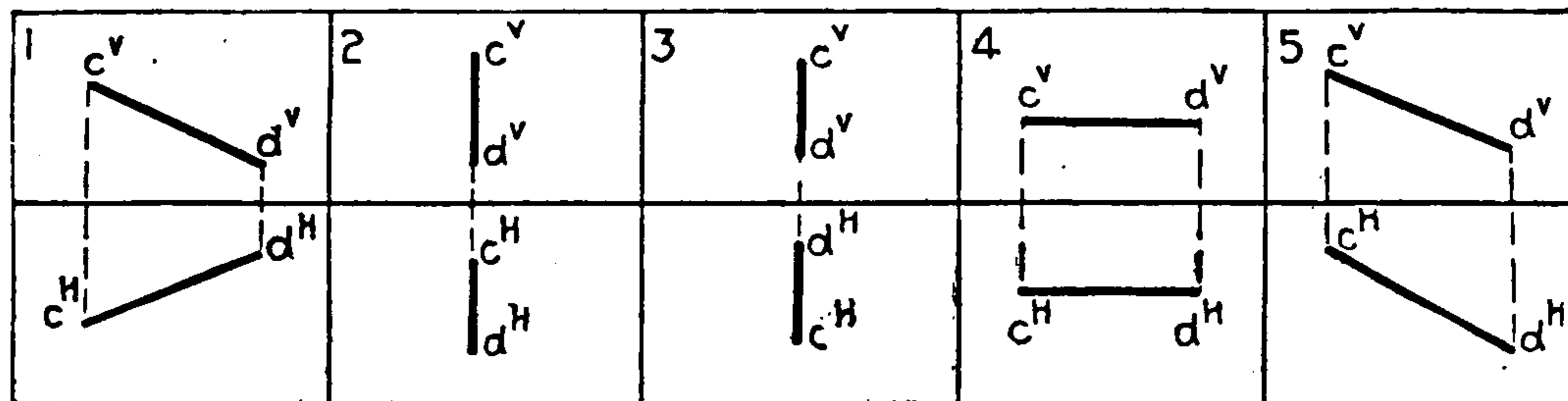
Dimensioned Layout.—Center lines for circular cylinders or elliptical cylinders, with bases perpendicular to the axes.

- (1)  $A(3, 0 - 1\frac{1}{2}) B(3 + 3 - 1\frac{1}{2})$ .
- (2)  $A(3 + 1, 0) B(3 + 1 - 3)$ .
- (3)  $A(2 + 1\frac{1}{2} - 1) B(4 + 1\frac{1}{2} - 1)$ .
- (4)  $A(2 + 1 - 1) B(4 + 1 - 2)$ .
- (5)  $A(2 + 2 - 1) B(4 + 1 - 1)$ .
- (6)  $A(3 + 2 - \frac{1}{2}) B(3 + 1 - 2)$ .
- (7)  $A(3 + 2\frac{1}{2} - 2) B(3 + 1 - 1)$ .
- (8)  $A(1 + 2\frac{1}{2} - 1) B(3\frac{1}{2} + 1 - 2)$ .
- (9)  $A(2 + 2 - 2\frac{1}{2}) B(4 + 1 - 1)$ .

Note.—The foregoing axes may be drawn in *III* by reversing the signs  $+$  and  $-$ . In this event, the signs on the plane traces and piercing lines should be reversed, if they enter the exercise.



Graphic Layout No. 56.—Cutting planes. (Note.—If the cylinders are drawn in *III*, reverse the lettering of these traces.)



Graphic Layout No. 57.—Piercing lines. (Note.—If the axes are drawn in *III*, draw these lines in *III*, and in any case, draw them so that they are likely to pierce the cylinder.)

**Cutting Planes.**

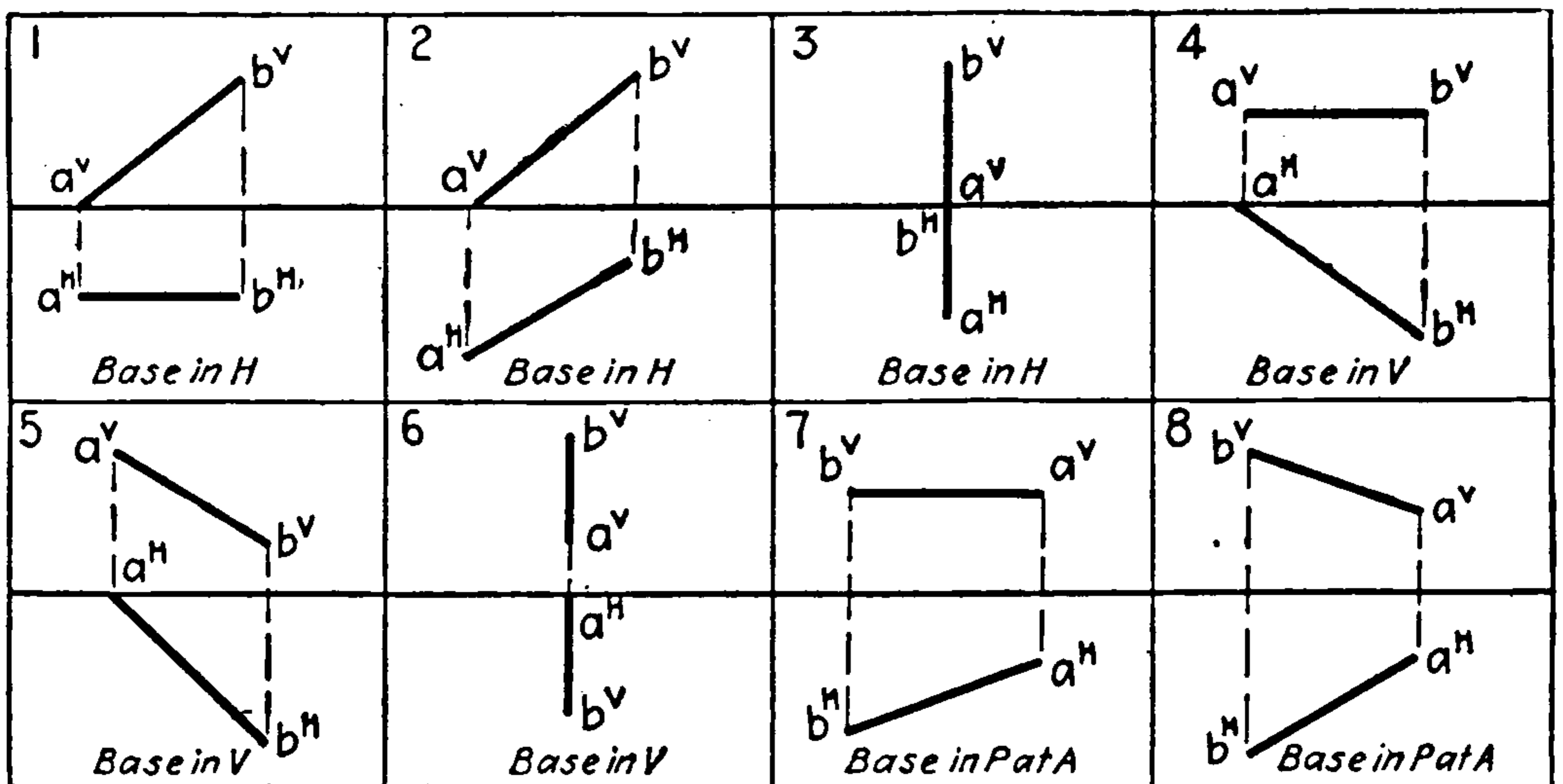
- (1)  $Q(1\frac{1}{2} + 3) 1\frac{1}{2}(5 - 2)$ .
- (2)  $R(1 + 2) 4(4 - 3)$ .
- (3)  $S(5 + 3) 1(5 - 2)$ .
- (4)  $T(1 + 3) 2(5 - 2)$ .
- (5)  $U(+ 2) \infty (- 1)$ .
- (6)  $W(+ \frac{1}{2}) \infty (+ 3)$ .
- (7)  $X(1 + 1) 5(1 + 2\frac{1}{2})$ .
- (8)  $Y(1 + 3) 3(5 - 3)$ .
- (9)  $Z(4 + 3) 3(4 - 2)$ .
- (10)  $P(3\frac{1}{2} + 3) 3\frac{1}{2}(3\frac{1}{2} - 3)$ .

**Piercing Lines.**

- (1)  $A(2, 0 - 1\frac{1}{4}) B(4 + 2 - 2)$ .
- (2)  $C(2, 0 - 2) D(4 + 2 - \frac{1}{2})$ .
- (3)  $E(3 + 2 - \frac{1}{2}) F(3 + 1 - 2\frac{1}{2})$
- (4)  $G(3 + 2 - 2) K(3 + \frac{1}{2} - 1)$ .
- (5)  $M(1 + 1 - 1\frac{1}{2}) N(4 + 1 - 1\frac{1}{2})$ .

792. Cylinder —, axis —, base, or right section —; locate the projections of a point on the — (front or rear) surface.

793. Cylinder —, axis —, base, or right section—; draw the intersection, and its true size, made by plane —.
794. Draw the development of the cylinder in Ex. 793, included between one of the bases and the cutting plane.
795. Cylinder —, axis —, base, or right section —; find the points in which it is pierced by line —.
796. Draw the shortest line on the surface between the two piercing points found in Ex. 795.
797. Draw the  $H$ -,  $V$ -, and  $P$ -intersections of cylinder —, axis —, base, or right section —, continuing the cylinder to meet the planes, if necessary.
798. Draw the traces of the plane tangent to cylinder —, axis —, base, or right section —, at a point assumed on the — (front or rear) surface, — in. from  $H$ .
799. Draw the traces of — (one or two) tangent planes to cylinder —, axis —, base, or right section —, through a point without the surface, in — ( $I$ ,  $II$ ,  $III$  or  $IV$ ).
800. Draw the traces of — (one or two) tangent planes to cylinder —, axis —, base, or right section —, parallel to  $A( \quad )$   $B( \quad )$ .
801. Draw the traces of — (one, two, three, four) tangent planes to cylinder —, axis —, base, or right section, inclined —° to — ( $H$ ,  $V$ , or  $P$ ).



Graphic Layout No. 58.—Axes for elliptical cylinders, with circular or elliptical bases in  $H$ ,  $V$ , or  $P$ , as given.

Bases.—1 in.,  $1\frac{1}{2}$  in.,  $1\frac{3}{4}$  in., 2 in., circles; 1 in.  $\times$   $1\frac{1}{2}$  in., 1 in.  $\times$  2 in.,  $1\frac{1}{2}$  in.  $\times$  2 in., ellipses.

#### Dimensioned Layout for Axes of Foregoing Cylinders.

(1)  $A(1\frac{1}{2}, 0 - 1\frac{1}{2})$   $B(3\frac{1}{2} + 2 - 1\frac{1}{2})$ . Base in  $H$ .

(2)  $A(2 + 1\frac{1}{2}, 0)$   $B(4 + \frac{1}{2} - 2)$ . Base in  $V$ .

(3)  $A(3 + 2, 0)$   $B(3, 0 - 1\frac{1}{2})$ . Base in  $H$ .

(4)  $A(3 + 1\frac{3}{4}, 0)$   $B(3, 0 - 2)$ . Base in  $V$ .



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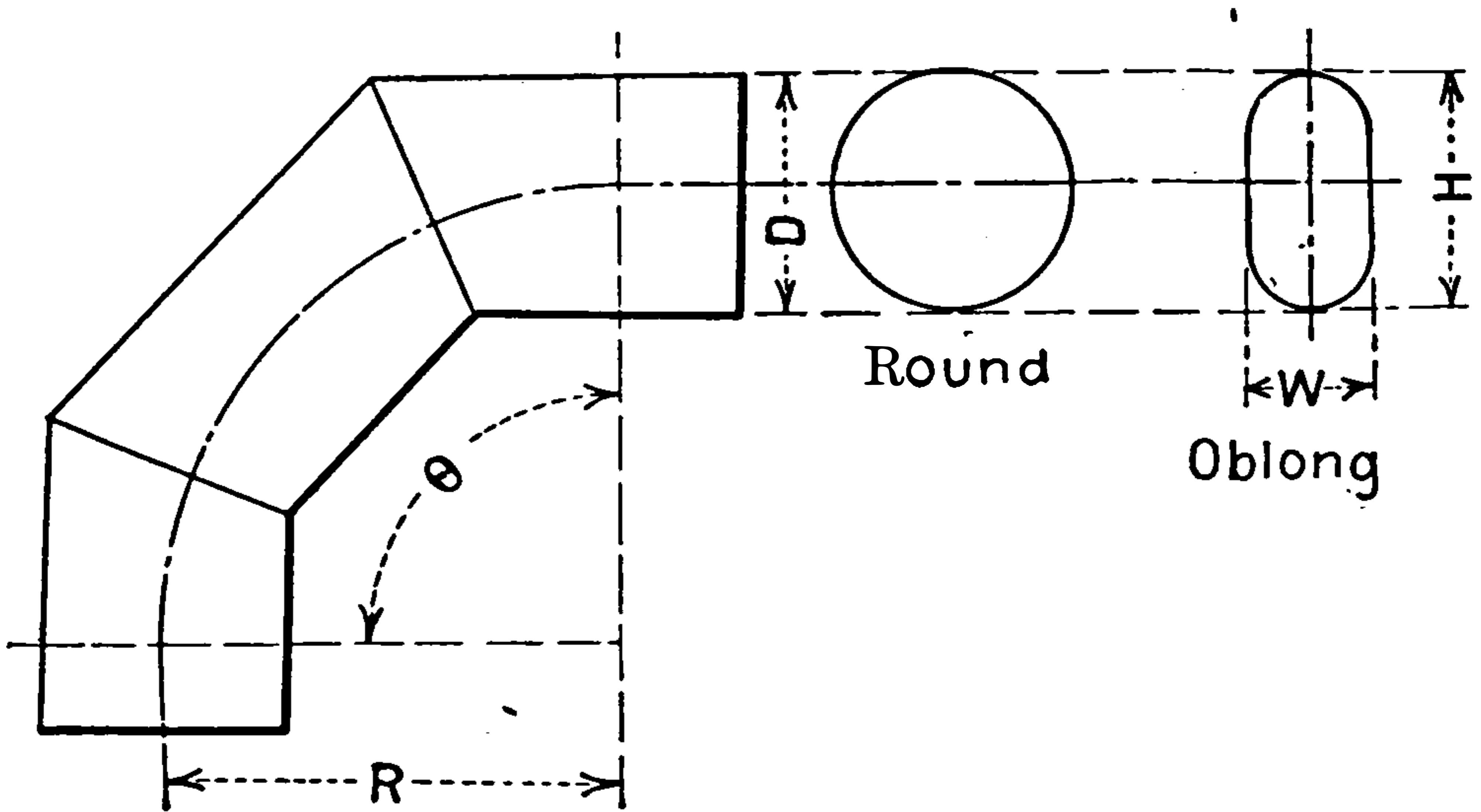
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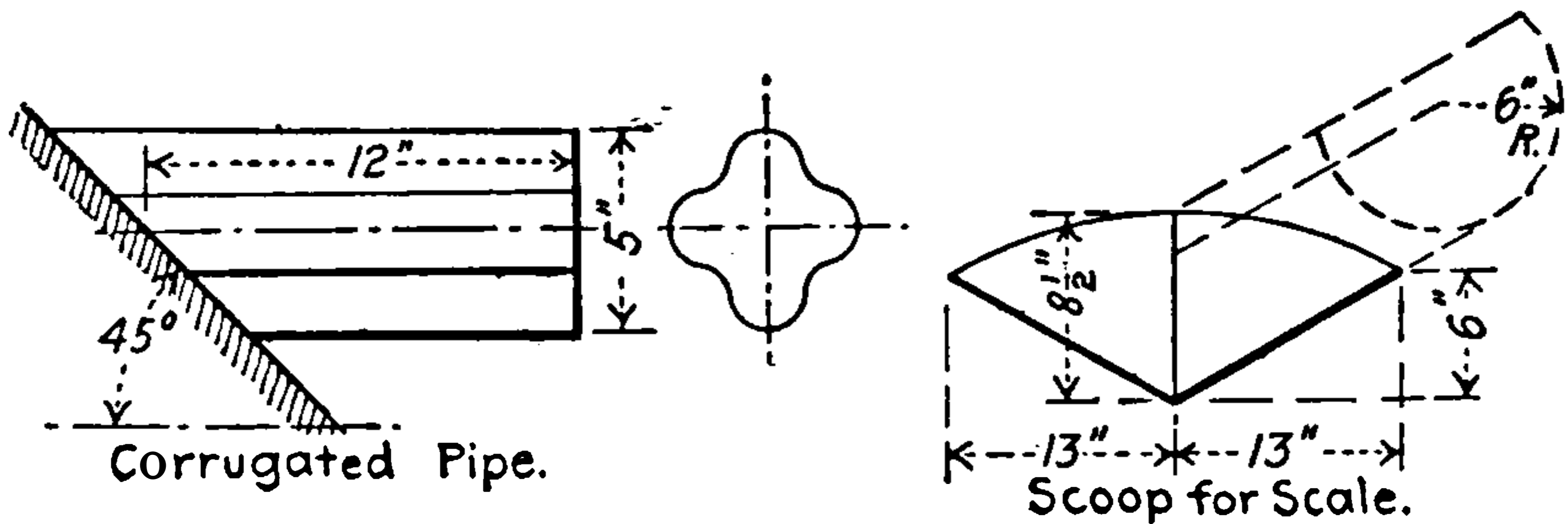


	Number of Pieces	$\theta$	R	D	W	H
1	3, 4, 5, 6, 7	$60^\circ, 75^\circ, 90^\circ, 105^\circ, 120^\circ$	8	4	2	4
2	"	"	10	5	2	6
3	"	"	12	6	2	8
4	"	"	16	8	3	6
5	"	"	20	10	5	8
6	"	"	7	4	2	5
7	"	"	9	5	2	7
8	"	"	11	6	2	9
9	"	"	14	8	3	9
10	"	"	18	10	4	8

Multi-Joint Pipe Elbow

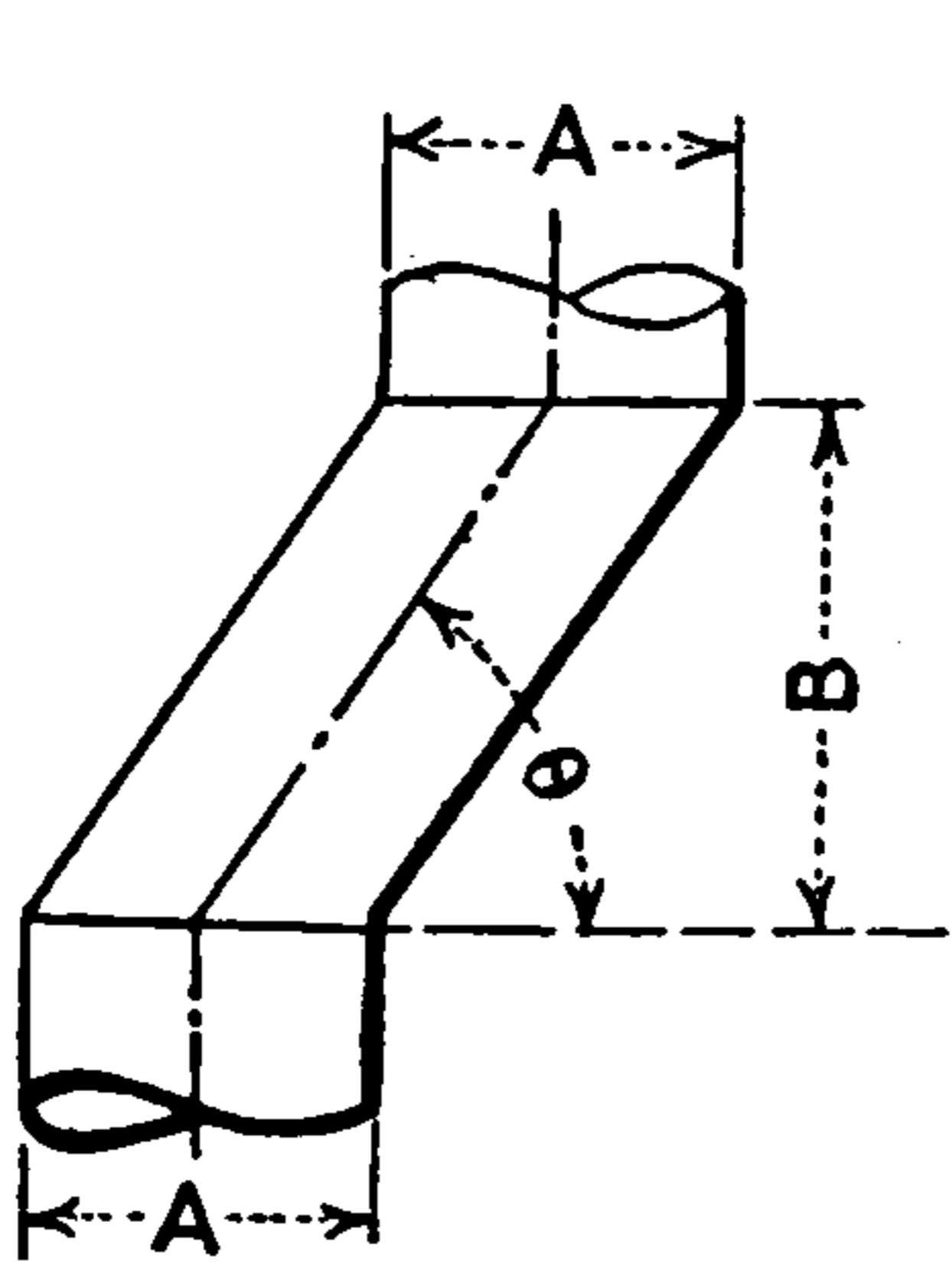
Graphic Layout No. 60.—Dimensions in inches.

- 815. Lay out the patterns for the Multi-joint Pipe Elbow —,  $\theta = \text{---}^\circ$ , number of pieces —.
- 816. Lay out the patterns for the Oblong Pipe Elbow —,  $\theta = \text{---}^\circ$ , number of pieces —.



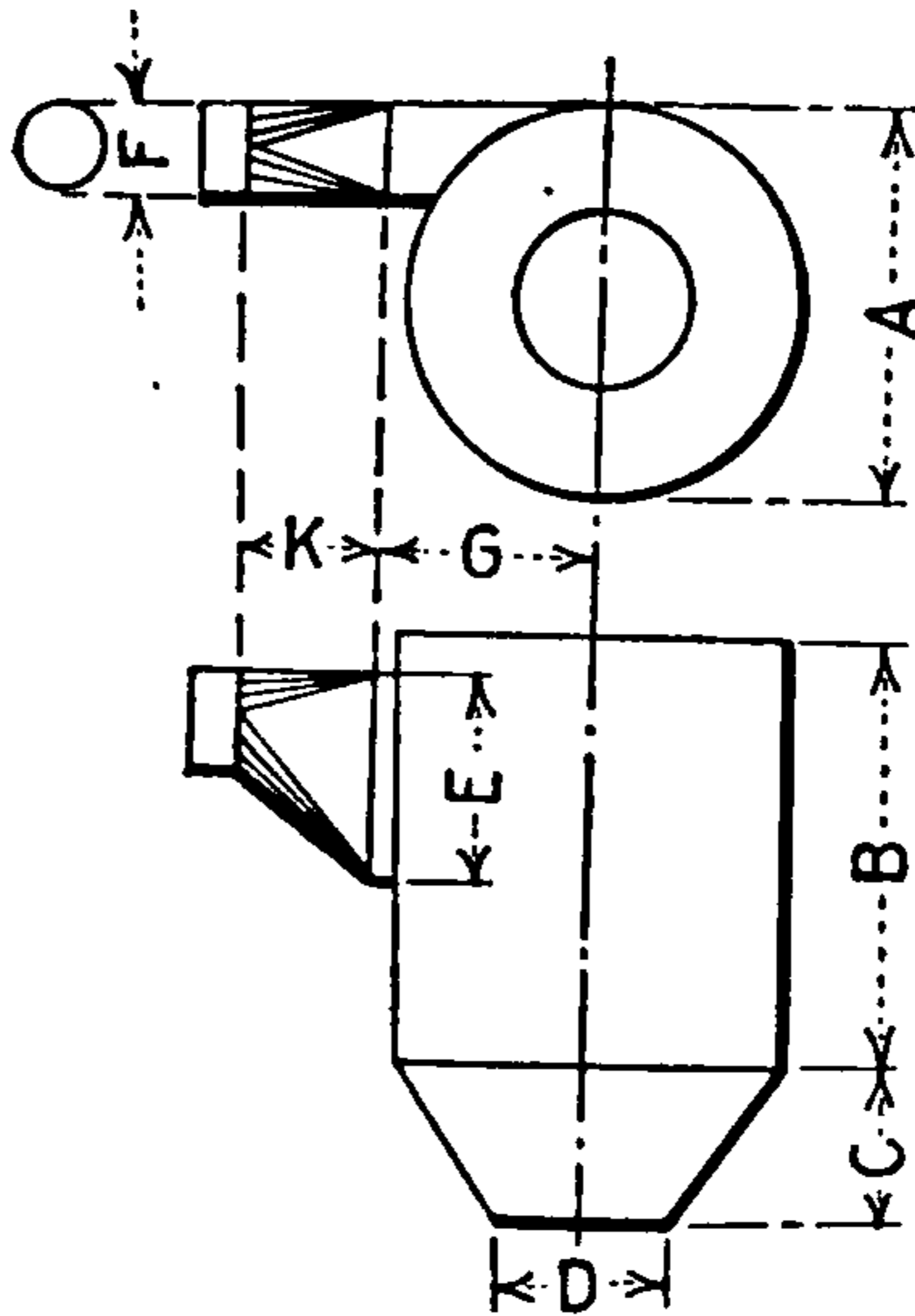
Graphic Layout No. 61.

- 817. Lay out the patterns for the Corrugated Pipe.
- 818. Lay out the patterns for the Scoop.



	A	B	$\theta$
1	4	6	$45^\circ, 60^\circ, 67\frac{1}{2}^\circ, 75^\circ$
2	4	7	"
3	4	8	"
4	6	8	"
5	6	10	"
6	6	12	"
7	8	12	"
8	8	14	"
9	8	16	"

Oblique Connector for Circular Pipe



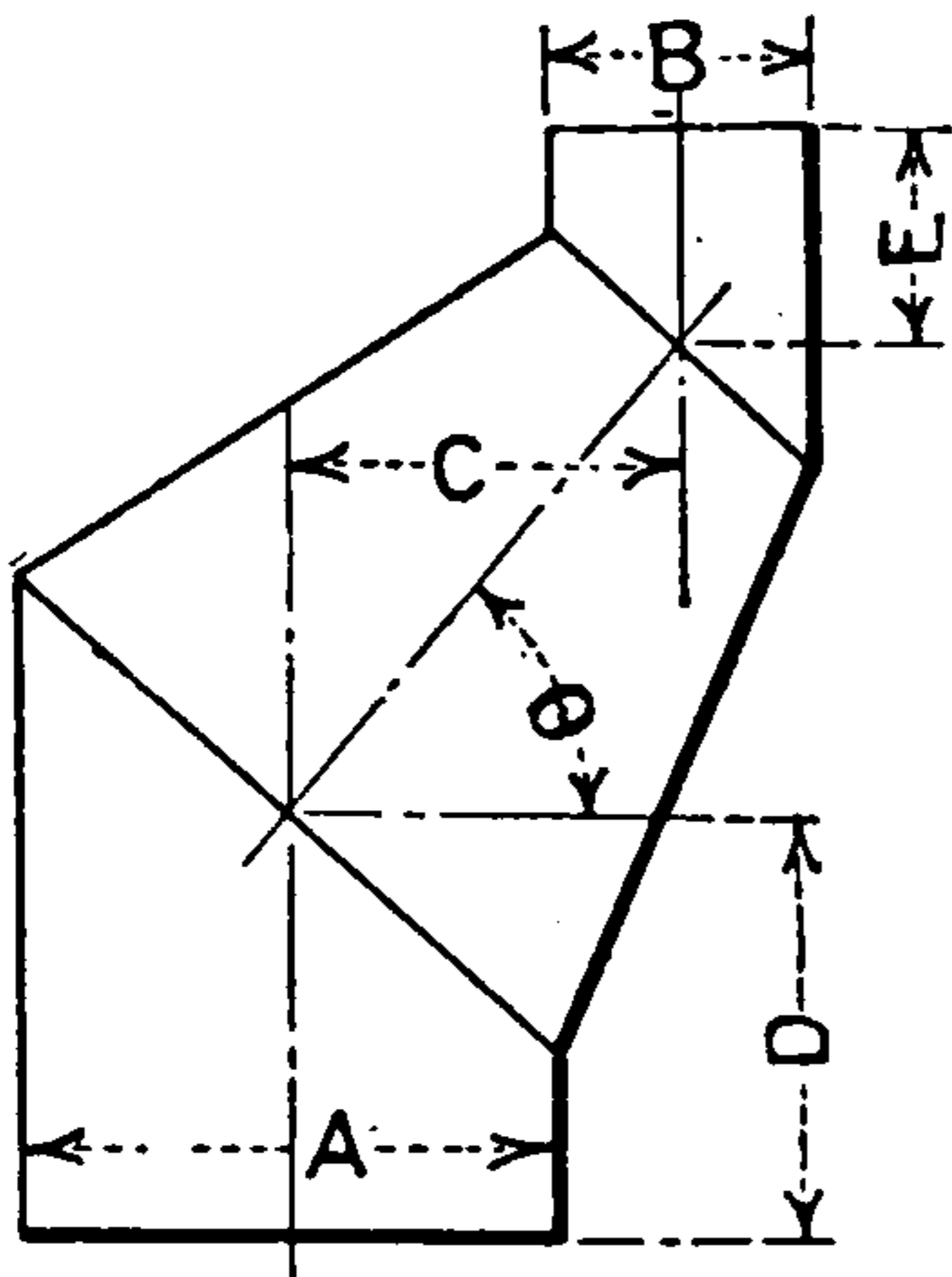
	A	B	C	D	E	F	G	K
1	$89\frac{1}{2}$	109	80	8	64	20	50	36
2	$73\frac{1}{2}$	$89\frac{1}{2}$	64	8	$53\frac{1}{2}$	16	43	32
3	$61\frac{1}{2}$	74	53	7	44	13	38	28
4	41	49	35	5	29	8	24	24
5	21	23	$17\frac{1}{2}$	3	$13\frac{1}{2}$	$3\frac{1}{4}$	17	16

Morse Rarefied Dust Collector

Graphic Layout No. 62.—Dimensions in inches.

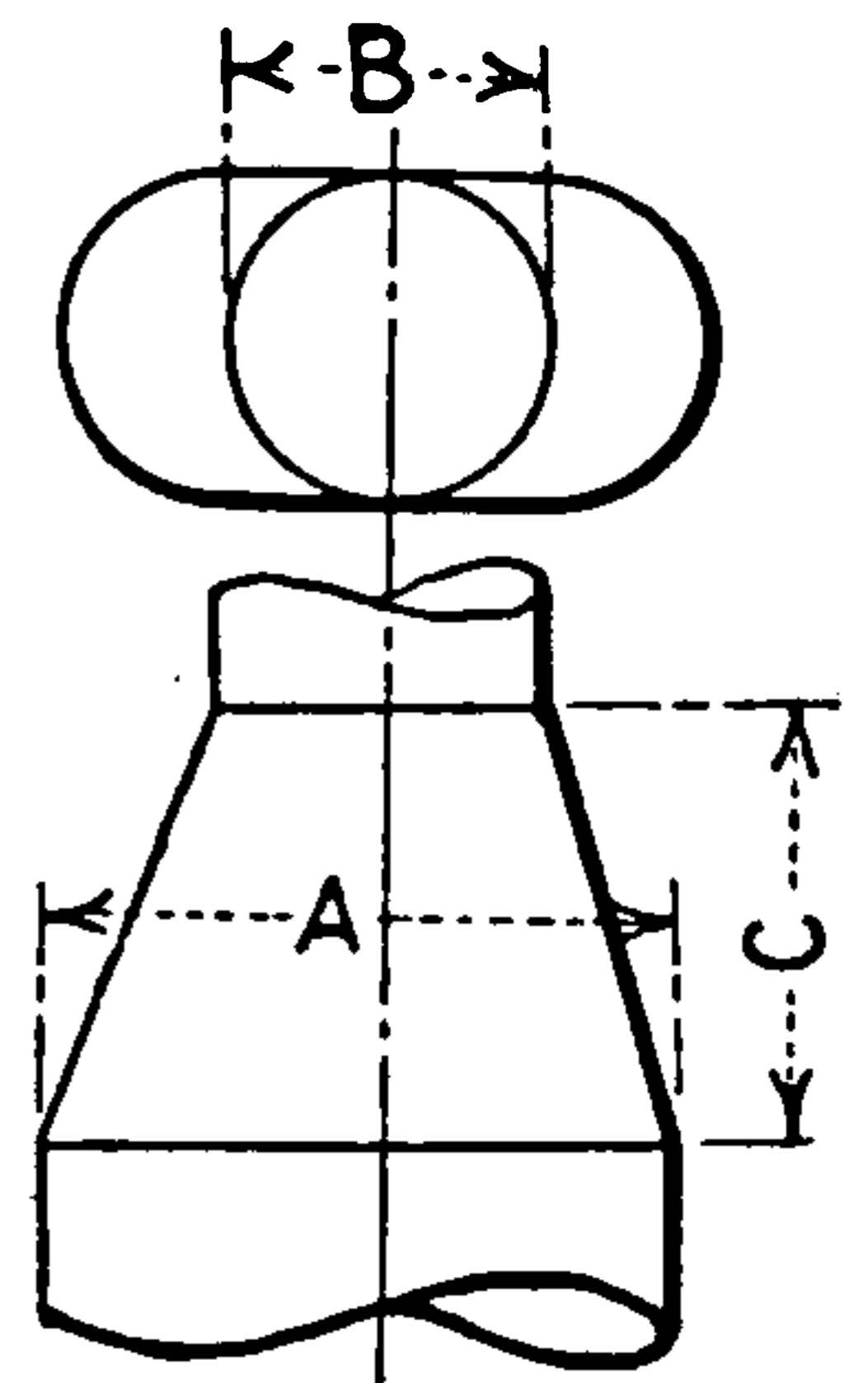
819. Lay out the patterns for Oblique Connector —,  $\theta = \text{---}^\circ$ .

820. Lay out the patterns for the Dust Collector —.



	A	B	C	D	E	$\theta$
1	8	4	6	10	5	$30^\circ, 45^\circ, 52\frac{1}{2}^\circ, 60^\circ$
2	8	3	8	10	5	
3	8	5	10	10	5	
4	10	4	8	10	5	
5	10	6	10	10	5	
6	12	4	10	10	5	
7	12	6	12	10	5	
8	12	5	14	10	5	

Offset Pipe Reducer



	A	B	C
1	16	10	10
2	18	10	10
3	20	12	12
4	24	12	12
5	26	12	14
6	28	14	14
7	30	14	16
8	32	14	16
9	36	16	18
10	40	16	18

Oblong Hood

Graphic Layout No. 63.—Dimensions in inches.

820. Lay out the patterns for Offset Pipe Reducer —,  $\theta = \text{---}^\circ$ .

821. Lay out the patterns for Oblong Hood —.



## THE CONVOLUTE

**201.** The third class of single-curved surfaces, the **Convolute**, is generated by the tangent line to any space curve, as it moves along the curve. The tangent line to a curve contains two consecutive points of the curve. This means that two consecutive tangents have one point in common; thus, A, B, and C are consecutive points in a curve, and AB and BC are therefore two consecutive tangents with B as their intersection. Thus all consecutive tangents to any space curve intersect two and two, and therefore become elements of a single-curved surface.

**202. Helical Convolute.**—The only convolute of importance is that generated by the tangent to the cylindrical helix. Fig. 147 shows both projections.

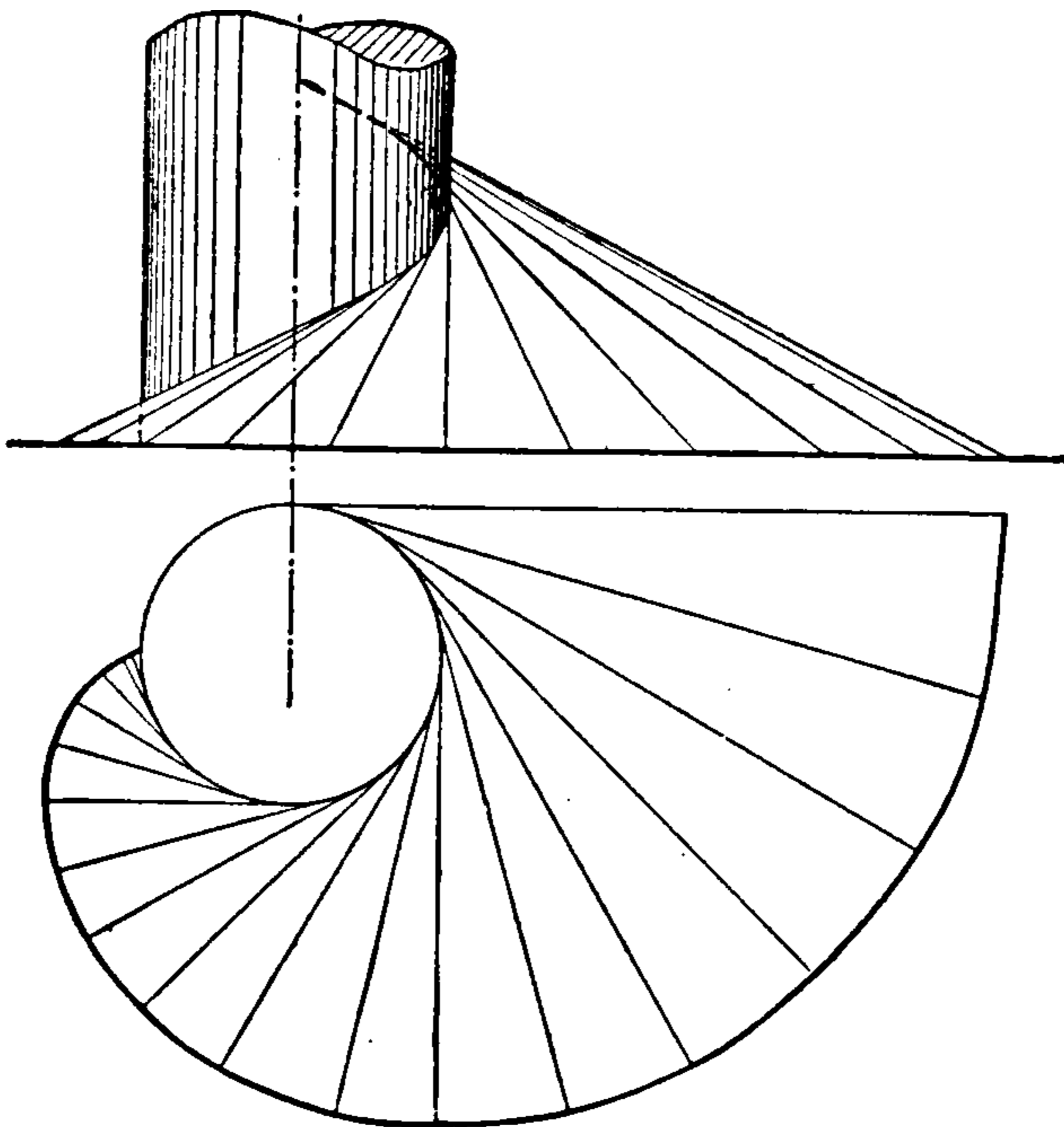


FIG. 147.—Projections of a convolute.

It is sometimes called the “Developable Helicoid,” and its most frequent application is in screw conveyors.

Theoretically, the surface is unlimited, the portion shown here being a “three-quarter turn.” In practical design it is always limited by a cylinder and upper and lower bases, or by two cylinders. The *H*-trace of the surface is an involute. (See Fig. 147.)

**203. Problems.**—The problems that relate to this surface do not differ materially from those of other single-curved surfaces; therefore the only problem given a solution here is the problem of the development of the surface.



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**Construction of the Development.**—Draw the circle with the radius  $b^v c^v$  and draw an involute to it. Make the arc of the developed involute equal to the arc of the helix, since the helix from any point to the origin is equal to the tangent from that point to the plane of the origin. This, then, is accomplished by measuring the tangent and laying off its length on the involute in the development.

**Mathematical Value.**—The mathematical value for the radius of the development is the expression  $\frac{R}{\cos^2 \theta}$ , in which  $R$  is the radius of the helical cylinder, and  $\theta$  is the inclination of the elements to the horizontal.

### EXERCISES

822. Draw a  $270^\circ$  convolute to the helix, whose axis is  $A(1\frac{1}{2}, 0 - 1\frac{1}{4})$   $B(1\frac{1}{2} + 2 - 1\frac{1}{4})$ , 1-in. diameter, 2-in. pitch. Find the traces of the tangent plane containing  $C(2\frac{1}{2}, x - 2)$  on the surface.
823. Draw the convolute in Ex. 822. Draw the traces of the tangent plane through  $D(1\frac{3}{4} + \frac{1}{4}, y)$  on the front surface.
824. Draw a  $270^\circ$  convolute to the "left-hand" helix, whose axis is  $E(4\frac{1}{2}, 0 - 1)$   $F(4\frac{1}{2} + 3 - 1)$ ,  $1\frac{1}{8}$ -in. diameter,  $2\frac{1}{2}$ -in. pitch. Draw the traces of a plane tangent at the point  $G(4\frac{3}{4}, x - 1\frac{3}{4})$  on the surface.
825. Draw the convolute in Ex. 824. Draw the traces of the plane tangent at  $K(3\frac{1}{2} + 1, y)$  on the front surface.
826. Draw the convolute in Ex. 822. Draw the traces of a plane tangent containing  $M(1 + 2 - 2\frac{1}{2})$ .
827. Convolute in Ex. 824. Draw the traces of the tangent plane containing  $N(4\frac{1}{4} + 1 - 2\frac{1}{2})$ .
828. Convolute in Ex. 822. Find its intersection with the plane  $S(1 + 1) 4(1 - 3)$ .
829. Convolute in Ex. 824. Find its intersection with the plane  $T(+ 1\frac{1}{2}) \infty(- 3)$ .
830. Develop the surface of the convolute in Ex. 822.
831. Develop the surface of the convolute in Ex. 824.
832. Find the shortest path on the surface of the convolute in Ex. 822 from  $O(1\frac{3}{4}, x - 2)$  to  $P(3, y - 1)$ .
833. Find the shortest path on the surface of the convolute in Ex. 824 from  $C(5, x - 1\frac{3}{4})$  to  $D(3, y - \frac{1}{2})$ .

## CHAPTER VII

### WARPED SURFACES

**204. Discussion.**—According to the definition in Art. 167, a warped surface is generated by moving a straight line so that no two consecutive positions shall be in the same plane. It thus falls into the classification with single-curved surfaces as a **ruled surface**. Planes may be passed through any point of a warped surface that will cut straight lines from the surface, always one, and, in the case of several surfaces, two straight lines. Other planes cut curves from them.

Warped surfaces are of many varieties and are common in practical work. In many cases, when they occur in practical work, it is not necessary for the draftsman to recognize their peculiarities, but very often, as in arches, sheet metal work, turbine blades, propeller blades, skew gears, conveyors, etc., it is highly important that engineers and draftsmen know their properties and methods of generation.

**205. Methods of Generation.**—Warped surfaces may be generated:

1. By revolving one line about another not in the same plane.
2. By moving a straight line, the generatrix, along two lines, directrices, either straight, curved, or both, keeping parallel to some plane, called the **Plane Director**.
3. By moving a straight line along three directrices, straight, curved, or both.
4. By moving a straight line along an axis at a constant angle with the axis, following a space curve as a directrix.

Other methods might be invented, but these cover all the cases met in practical work.

**206. Tangent Planes.**—In Art. 170 we find the general rule for tangent planes to all Ruled Surfaces, as follows: **One tangent line and the element at any point determine the tangent plane at that point.**

In general, a tangent plane is only tangent **at one point** of a warped surface. There are certain points, as in conoids and warped cones, where the tangent plane will be tangent all along the element, but such exceptions will be readily recognized.

All planes tangent to warped surfaces (except in the cases just noted) intersect the surface, because they are not tangent all along the element, and, conversely, any plane that intersects a warped surface in an element will, in general, be tangent to the surface at some point.

**207. Double-Ruled Surfaces.**—Two surfaces, the **Hyperboloid of Revolution of One Nappe** and the **Hyperbolic Paraboloid**, are called double-ruled surfaces from the fact, as will be shown later, that they are capable of generation by two different straight lines. This yields the phenomenon that through every point in the surface two straight lines (elements) can be drawn lying wholly within the surface. From this fact we have the special rule for tangent planes to these surfaces: **The two elements of a double-ruled surface through any point of the surface determine the tangent plane at that point.**

**208. Representation.**—Some warped surfaces can be shown in outline in orthographic projection with sufficient clarity, and others must be shown by drawing a sufficient number of elements. In practical drafting, where the surface is always limited, the outlines are usually sufficient.

**209. Location of Points.**—To locate a point on any warped surface, one projection being given, find the piercing point of the projector of the point with the surface. This is done by passing a plane through the projector, and finding its intersection with the surface. Where the projector intersects this line will be the location of the point. In many cases it will be easy to draw the element through the point, simplifying the operation.

**210. Developments.**—Strictly speaking, a warped surface cannot be developed, because no two consecutive elements lie in the same plane, and cannot, therefore, be rolled into a plane without “warping” them; that is, destroying the relation between them. However, there are approximate developments that give very satisfactory results, almost as good as the development of a cone by triangulation. These warped developments also are obtained by triangulation. A sufficient number of elements are drawn, dividing the surface into minute warped quadrilaterals, in which the amount of warp is so slight as to be negligible. Diagonals are then drawn across each of these quadrilaterals, dividing them into triangles. These triangles are then measured and placed in order in their (approximate)



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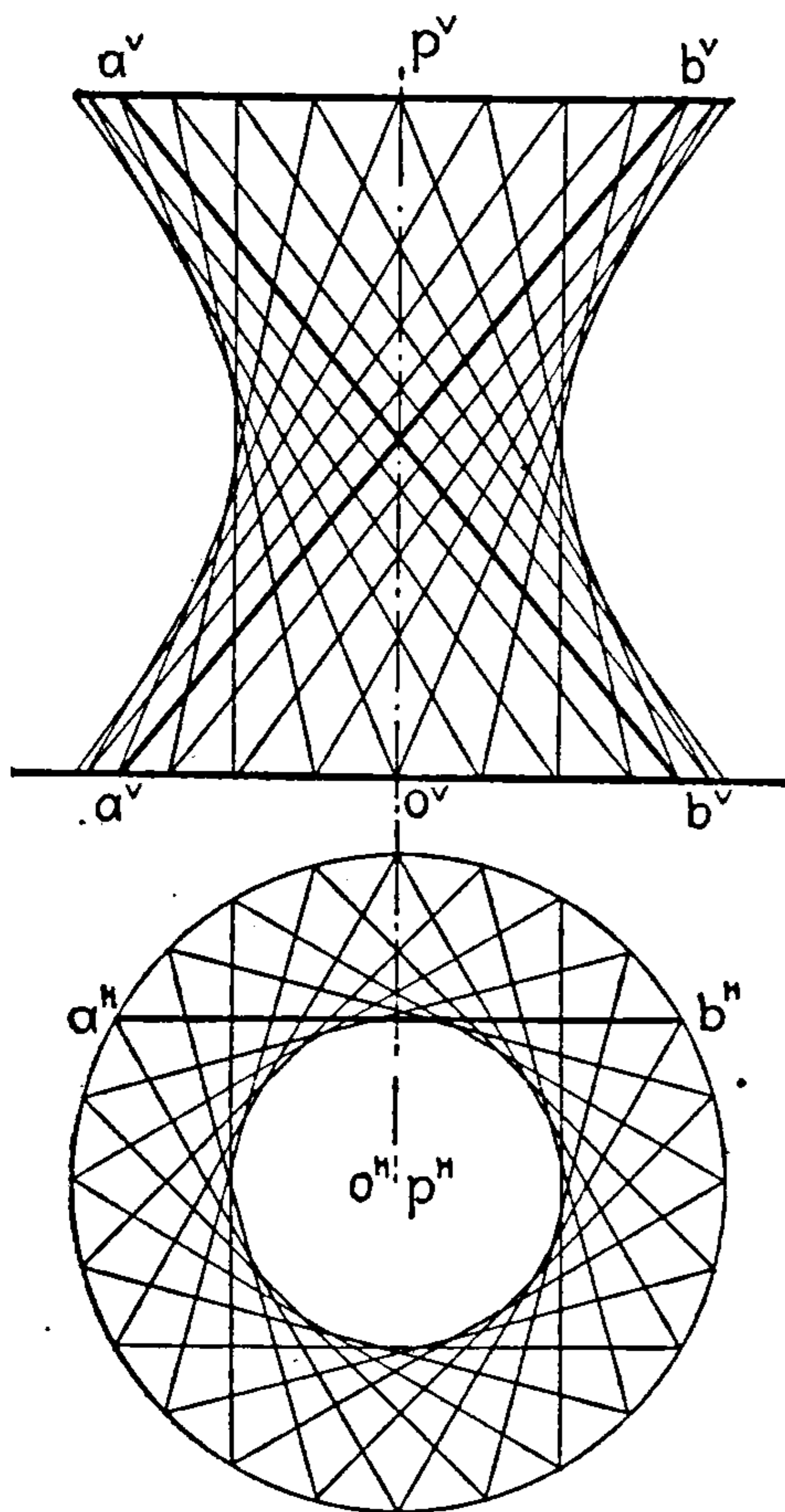
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through the point and locating the other projection on the element. If the  $V$ -projection of the point is given, pass a plane through the point parallel to  $H$ . This will cut the elements in a curve on which the  $H$ -projection will be located.

The helicoid is the surface of the  $V$ -thread, the square thread, spiral springs, screw conveyors, certain fire-escapes, the twist drill, the spiral runway, helicoidal arch, and numerous other surfaces frequent in machine and structural design.



Hyperboloid of Revolution.

FIG. 150.

**Helicoidal Arch.**—In the design of skew arches, that is, arches with their axes oblique to one or both of the openings, warped surfaces are often encountered, cylindroids or cow's horns being the usual surfaces in such arches. The arch called the **helicoidal arch** takes its name not from the character of its surface, as might be supposed, but from the surface of the "coursing joints." The line of the joint is laid out as a straight line on the developed surface of the arch, and, when brought into its actual position, it becomes a helix. The surface of the joint is generated by a line normal to the surface of the

arch moving along the helix, radial to the axis of the helix. This surface is therefore identified as a helicoid. Inasmuch as the developed helix is a straight line the work of laying out the blocks is very simple. The seam, being helicoidal, must be laid out with care, to ensure good-fitting blocks.

### THE HYPERBOLOID OF REVOLUTION OF ONE NAPPE

**212.** This peculiar surface is the only warped surface that is a surface of revolution. It is generated by Method 1, Art. 205, by revolving a straight line about an axis not in the same plane. Fig. 150 shows this surface represented by its elements. Either projection of all the elements could be inverted, and there would

be no change in the surface. If  $AB$  be considered an element, its  $V$ -projection might be either of the lines marked  $a^v b^v$ , and the same surface would be generated by revolving either line about the axis. This makes it a double-ruled surface, and a tangent plane is determined at any point by two elements through that point.

**213. To Draw the Elements.**—All points in the generatrix move in circles, hence there is a vertex to the curve of the outline, a smallest circle, called the **Gorge Circle**, generated by the point nearest the axis. All elements intersect the gorge circle and their  $H$ -projections are all tangent to its  $H$ -projection. Two limiting planes, **bases**, are always necessary, always taken perpendicular

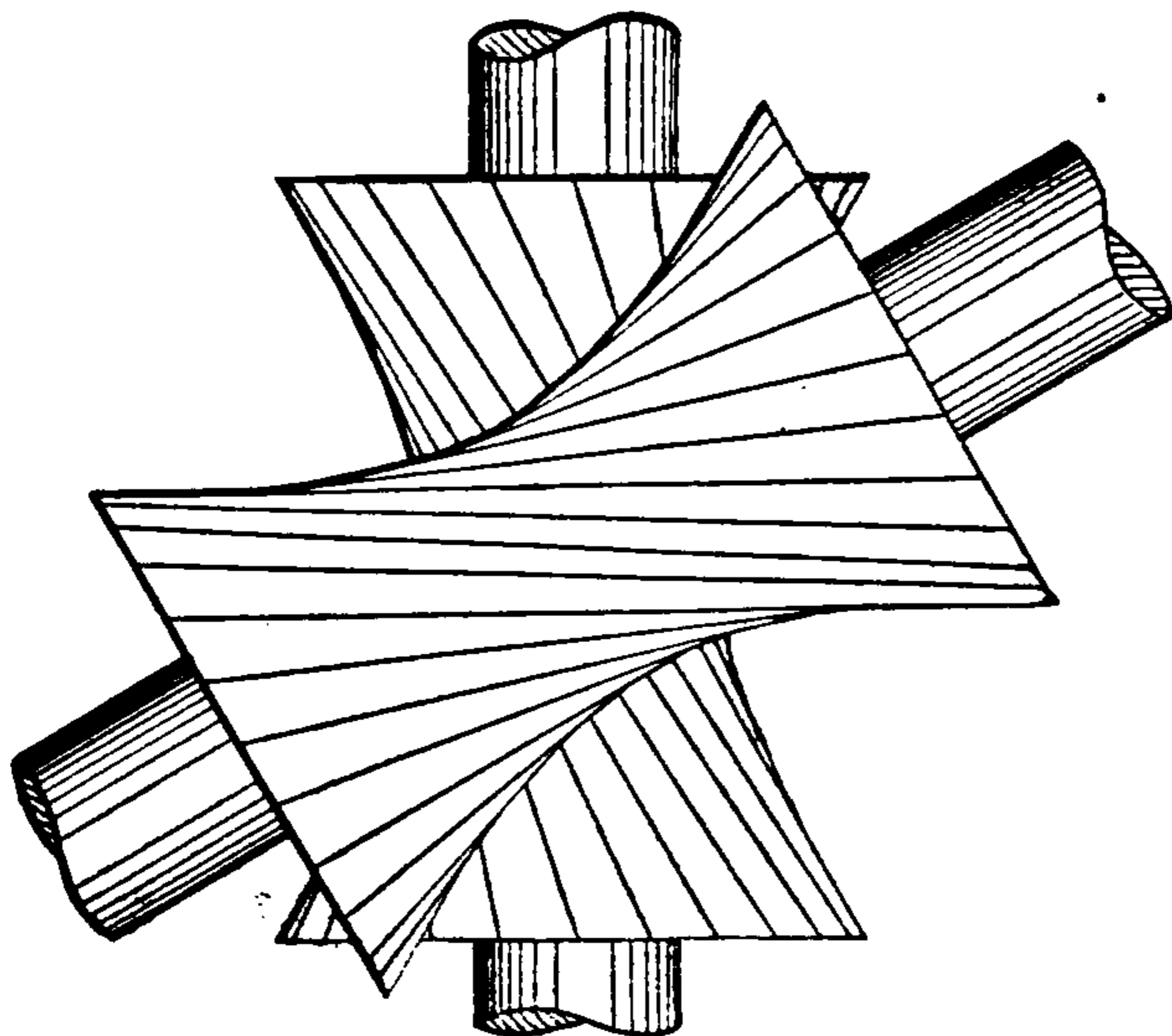


FIG. 151.—Rolling hyperboloids. (Both surfaces are generated by revolving the same straight line about two different axes, and they thus have one element in common, and are tangent to each other along this element.)

to the axis, and usually at equal distances from the gorge circle. Being perpendicular to the axis, these bases are circular. Why? One base is usually in  $H$ , and the elements terminate in the bases. Hence, 1. Draw the gorge and base circles. 2. Draw as many elements as desirable, 16 or 24, the  $H$ -projections tangent to the gorge circle and terminating in the base circles. 3. Project the terminals of the elements to the  $V$ -traces of the respective base planes.

Note.—When the bases are symmetrically placed, there will be only one circle projected on  $H$ , as in Fig. 150.

**214. To Pass a Plane Tangent at any Point on the Surface.**—Draw the two elements through the given point. They will determine the tangent plane.



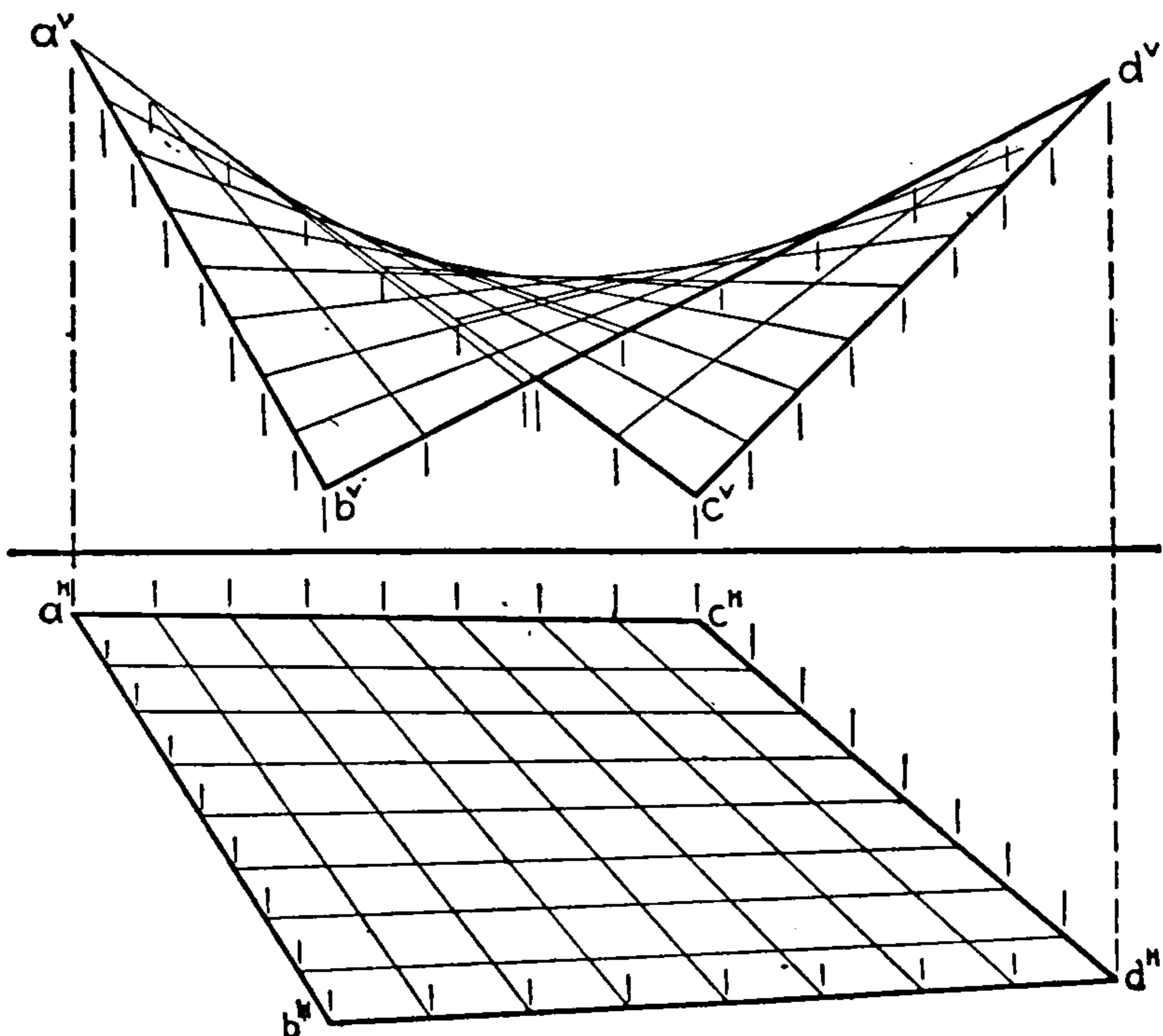
Tangent planes to this surface, as to all surfaces of revolution, are perpendicular to the plane of the axis and the point of tangency, called the **Meridian Plane**.

The **Meridian Section** of this surface is a hyperbola, and the same surface can be generated by revolving a hyperbola about its vertical axis. If proof of this is desired, it can be found in Church and Bartlett, page 150.

The most important function of the hyperboloid is found in the pitch surfaces of skew gears. If one line be successively revolved about two axes, not in its plane, the line will be simultaneously an element of both hyperboloids; that is, the two hyperboloids generated will be tangent. These, if used as the basis of gears, form an ideal transmission medium between shafts not in the same plane.

### THE HYPERBOLIC PARABOLOID

**215.** This surface is generated by moving a straight line along two straight lines, not in the same plane, keeping it always parallel to a plane director.



Hyperbolic Paraboloid, Showing Double Ruling

FIG. 152.

allel to a plane director. The elements are all parallel to this plane, but not to each other.



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## THE WARPED CONE

**221.** This surface, often found in reducing hoods and pipe connections, has three directrices; as follows, two circles or ellipses (or one of each) in planes not parallel, and the line joining their centers, called the axis.

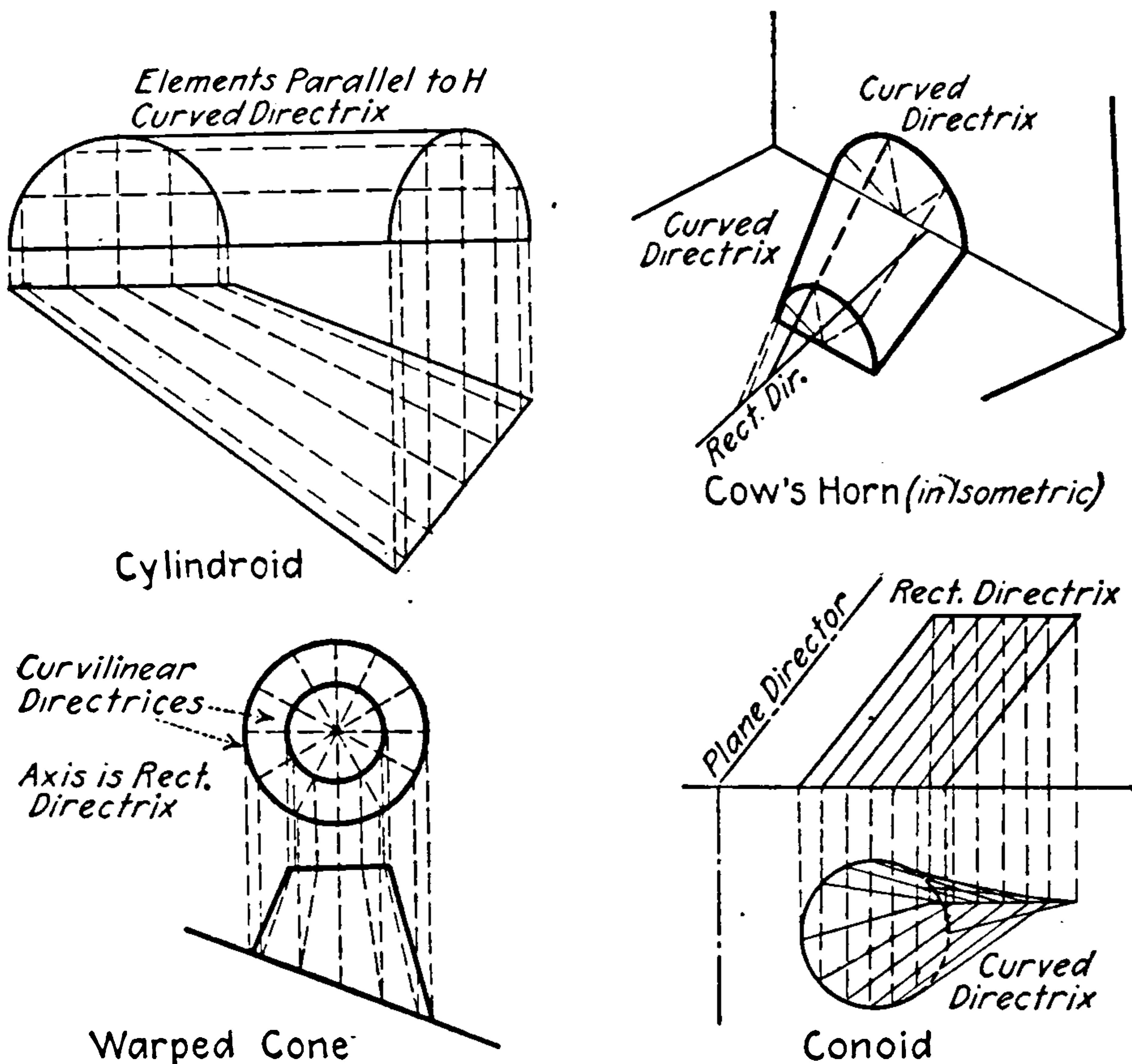


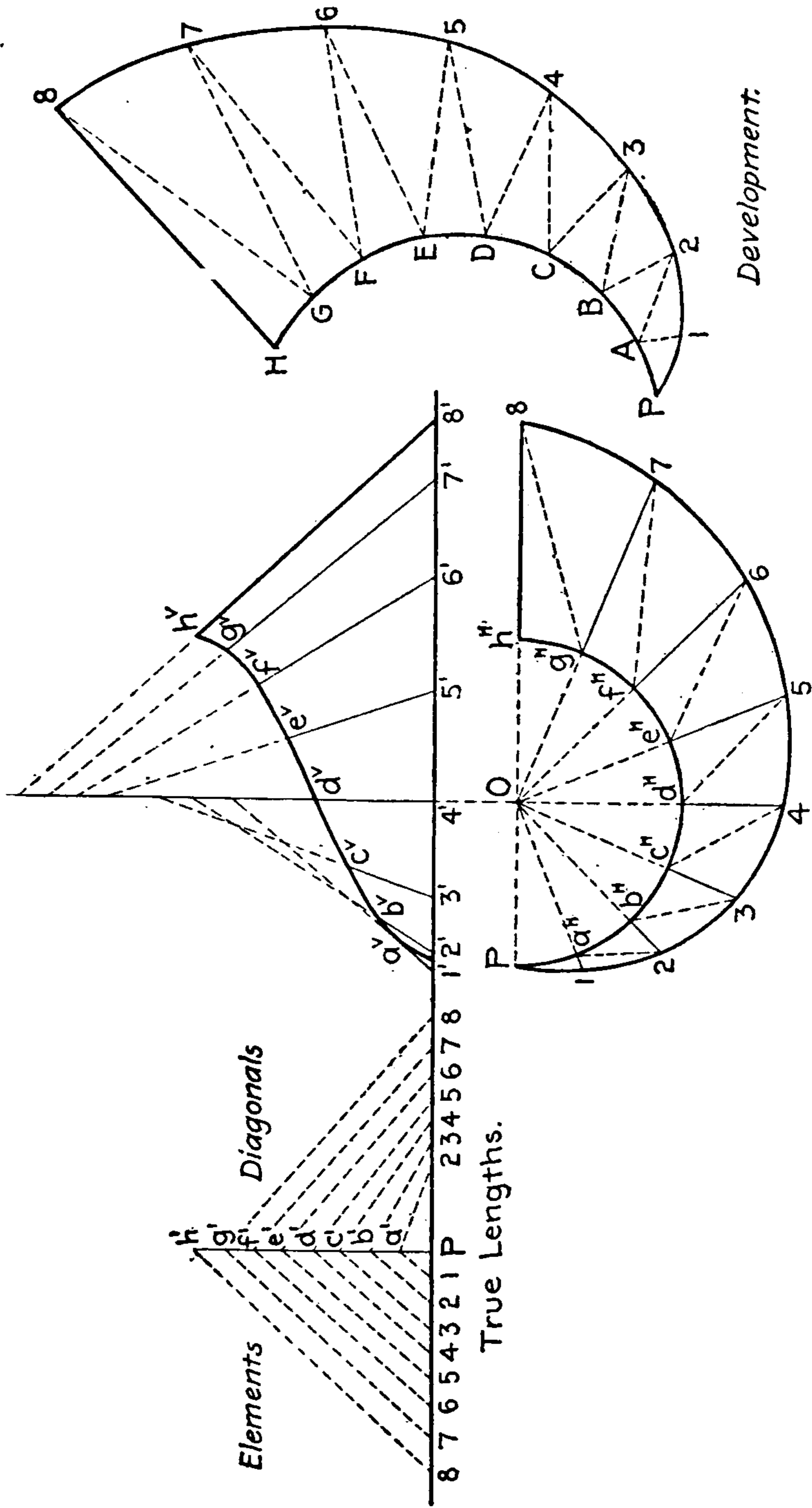
FIG. 153.—Minor warped surfaces and their methods of generation.

**222. Other Warped Surfaces.**—Besides the foregoing, there are many forms, which have no specific names. They may have three straight directrices, three curved directrices, or combinations of these with plane directors. Whenever they are met, they will present no difficulties to the student, who is familiar with those described.

**223. The Development of a Specimen Warped Surface.**—Develop the helicoid shown in Fig. 154, for a half turn, between  $H$  and the helical directrix.

1. Draw the elements  $A_1, B_2$ , etc.
2. Draw diagonals  $A_2, B_3$ , etc., of the quadrilaterals.
3. Measure the true lengths of the various elements and diagonals. The construction on the left shows a simple arrange-

ment for accomplishing this. Lay off the heights of the various points,  $a'$ ,  $b'$ , etc., on a vertical line, and lay off the lengths of the various  $H$ -projections on a horizontal line, elements on one



Approximate Development of a Typical Warped Surface (Helicoid.)

FIG. 154.

side, diagonals on the other. Their true lengths will be the hypotenuses,  $a'1$  or  $a'2$ , etc.

4. Rectify the various arcs of the helix and spiral. The arcs

of the helix are equal in length to corresponding arcs of a circle whose radius  $= \frac{OP}{\cos^2 \theta}$ . (See the graphical solution given in Art. 204.) The lengths of the spiral arcs are obtained by stepping off small chords with the dividers.

5. With these materials, construct the triangles in order, as shown on the right.

This method, with such modifications as may be necessary, is employed in the development of all warped surfaces. It is laborious, but there is no shorter method. The important operation in this, as in all problems relating to surfaces, is to get the elements.

224.

## EXERCISES

## Helicoid

Helicoid dimensions: pitch of helix, 1 in., 2 in.,  $2\frac{1}{2}$  in., 3 in.; diameter of helix, 1 in.,  $1\frac{1}{4}$  in.,  $1\frac{1}{2}$  in. Angle of elements with axis,  $30^\circ$ ,  $45^\circ$ ,  $60^\circ$ ,  $75^\circ$ ,  $90^\circ$ .

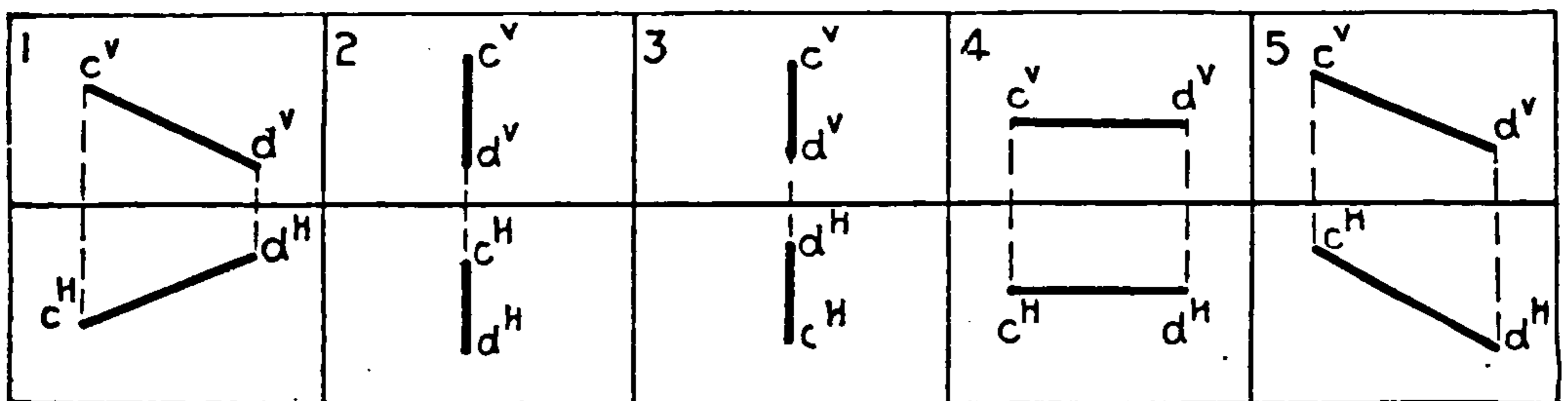
834. Right-hand helix, axis  $A(2, 0 - 1\frac{1}{2}) B(2 + 3 - 1\frac{1}{2})$ , pitch —, diameter —, elemental angle —°.

(a) Draw the projections of as much of the helicoid as possible, showing its  $H$ -trace.

(b) Locate  $E(1\frac{1}{4}, x - 2)$  and  $F(2\frac{1}{2} + 1, y)$  on the surface.

835. Helicoid in Ex. 834. Draw the traces of the plane tangent at  $E$  or  $F$ .

836. Helicoid in Ex. 834. Locate the points in which the surface is pierced by line —.



Graphic Layout No. 64.—Piercing lines to use for all the given surfaces in these exercises.

## Dimensioned Layout for the Same.

- (1)  $A(2, 0 - 1\frac{1}{4}) B(4 + 2 - 2)$ .
- (2)  $C(2, 0 - 2) B(4 + 2 - \frac{1}{2})$ .
- (3)  $E(3 + 2 - \frac{1}{2}) F(3 + 1 - 2\frac{1}{2})$ .
- (4)  $G(3 + 2 - 2) J(3 + \frac{1}{2} - 1)$ .
- (5)  $M(1 + 1 - 1\frac{1}{2}) N(4 + 1 - 1\frac{1}{2})$ .



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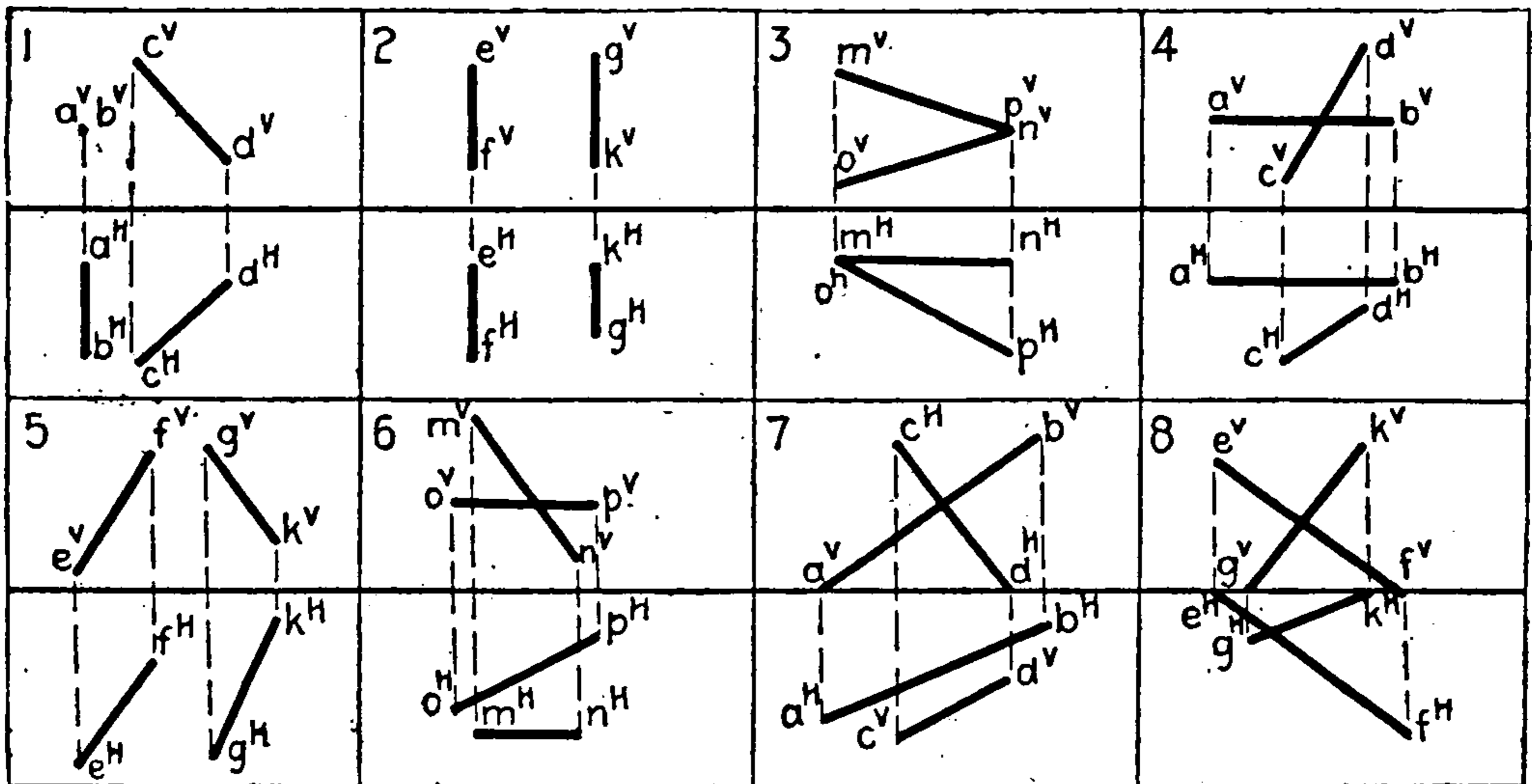
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844. Hyperboloid; gorge diameter —, elemental inclination —°. Draw the projections of its surface (24 elements), and locate  $C(2\frac{1}{2} + \frac{3}{4}, x)$  and  $D(3\frac{1}{2}, y - 2)$  on its surface.
845. Hyperboloid in Ex. 844. Draw the traces of the plane tangent at — (C or D).
846. Hyperboloid in Ex. 844. Locate the points in which the surface is pierced by the line —.
847. Hyperboloid in Ex. 844. Draw the projections of the curve cut by plane —.
848. Hyperboloid in Ex. 844. Develop the surface of the truncated portion between the lower base and the section made by plane—.

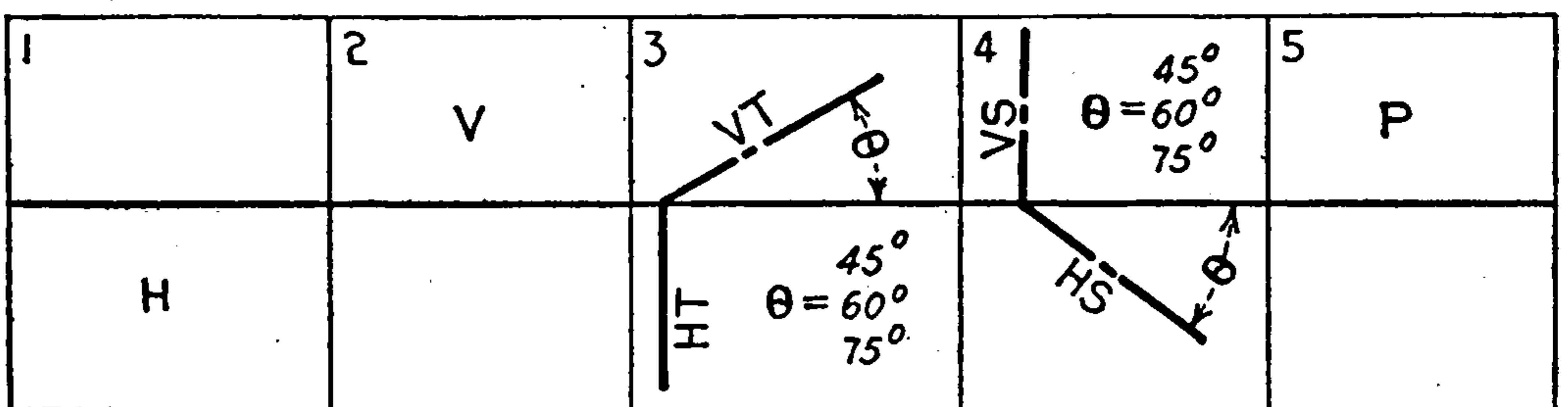
**Hyperbolic Paraboloid.**



Graphic Layout No. 66.—Directrices for paraboloids.

**Dimensioned Layout for Same.**

- (1)  $A(1 + 2 - 2)$   $B(4, 0 - 1)$  and  $C(3, 0 - 3)$   $D(4\frac{1}{2} + 2 - 1)$ .
- (2)  $E(4 + 1 - 1)$   $F(4 + 1 - 3)$  and  $G(1 - 1 - 1)$   $K(3 - 3 - 3)$ .
- (3)  $M(1 + 2 - 1)$   $N(3 + 2 - 1)$  and  $O(2 + 3 - 3)$   $P(3 + 1 - 2)$ .
- (4)  $O(1\frac{1}{2}, 0 - 2)$   $P(1\frac{1}{2} + 1\frac{1}{2}, 0)$  and  $X(3\frac{1}{2} + 2, 0)$   $Y(3\frac{1}{2}, 0 - 1)$ .
- (5)  $A(1\frac{1}{2}, 0 - 2)$   $B(1\frac{1}{2} + 2, 0)$  and  $C(4, 0, 0)$   $D(4 + 1\frac{1}{2} - 2)$ .
- (6)  $E(1\frac{1}{2} + 2 - \frac{1}{2})$   $F(4 + 2 - 2)$  and  $G(4 + 2, 0)$   $K(2, 0 - 1)$ .
- (7)  $M(1\frac{1}{2} + 3 - 1)$   $N(1\frac{1}{2}, 0 - 1)$  and  $O(4 + 2, 0)$   $P(4, 0 - 2\frac{1}{2})$ .
- (8)  $X(1 + 2\frac{1}{2} - \frac{1}{2})$   $Y(3 + 1 - 2)$  and  $A(3\frac{1}{2} + \frac{1}{2} - \frac{1}{2})$   $B(5 + 2 - 2\frac{1}{2})$



Graphic Layout No. 67.—Plane directors.

849. **Hyperbolic Paraboloid.**—Directrices No. ———.
- (a) Draw the projections of the surface within the limits of the given directrices, without using a plane director.
- (b) Draw the traces of its plane director.
- (c) Locate the projections of the points  $C(2, x - 1\frac{1}{2})$  and  $D(3 + 1, y)$  on the surface.
850. Hyperbolic Paraboloid in Ex. 849. Draw the traces of a plane tangent to the surface at ——— (C or D).
851. Hyperbolic Paraboloid in Ex. 849. Locate the point or points in which it is pierced by the line ———.
852. Hyperbolic Paraboloid in Ex. 849. Draw the projections of the curve cut from the surface by plane ———.
853. Hyperbolic Paraboloid in Ex. 849. Develop the surface included between the given directrices.
854. Hyperbolic Paraboloid in Ex. 852. Develop the surface included between the directrix ——— and the curve cut by the given plane.
855. **Hyperbolic Paraboloid.**—Directrices No ———, Plane Director ———.
- (a) Draw eight elements of the surface.
- (b) Locate the projections of the points  $E(2 + 1, x)$  and  $F(3, y - 1\frac{1}{4})$  on the surface.
856. Hyperbolic Paraboloid in Ex. 855. Draw the traces of a plane tangent to it at ——— (E or F).
857. Hyperbolic Paraboloid in Ex. 855. Find the point or points in which it is pierced by line ———.
858. Hyperbolic Paraboloid in Ex. 855. Draw the projections of the curve cut by the plane ———.
859. Hyperbolic Paraboloid in Ex. 855. Develop the surface included between the directrices.
860. Hyperbolic Paraboloid in Ex. 855. Develop the surface included between directrix ——— and the cutting plane ———.

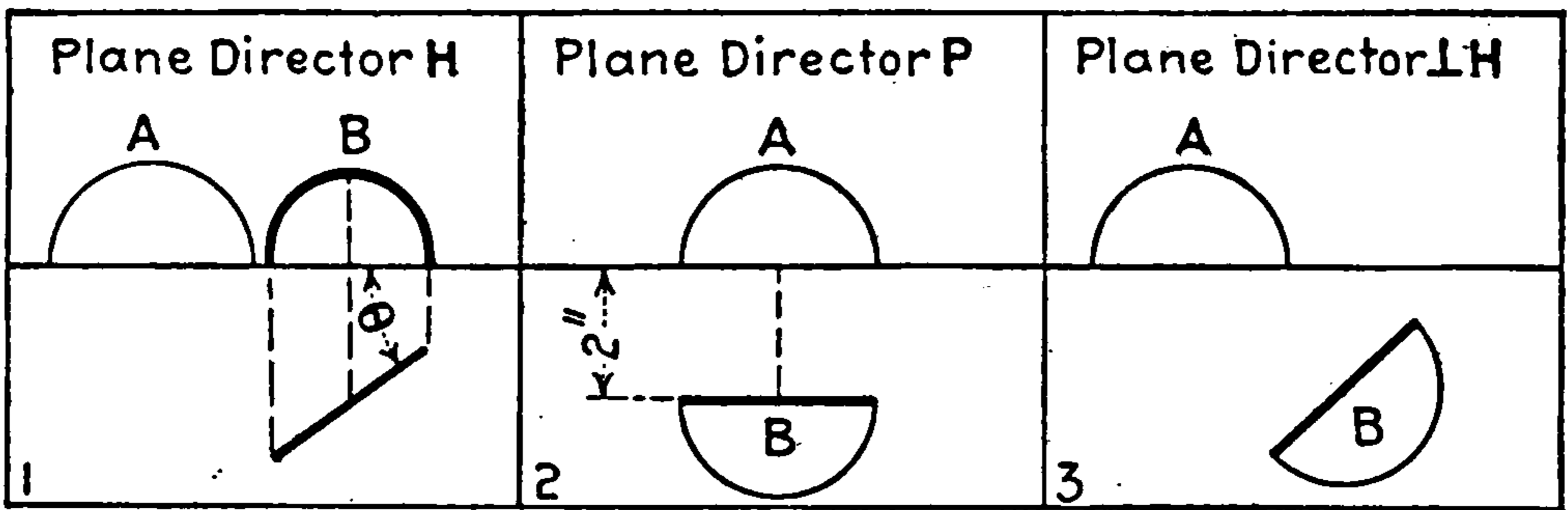
### MINOR WARPED SURFACES

**Conoids, Directrices, one a Straight Line, as given, and either a Circle in  $H$ , of 1 in.,  $1\frac{1}{2}$  in., 2 in., or  $2\frac{1}{2}$  in. diam., or an Ellipse in  $H$ , 1 in.  $\times$   $1\frac{1}{2}$  in., 1 in.  $\times$  2 in., or  $1\frac{1}{2}$  in.  $\times$  2 in. Plane directors for (2), (3), or (4), to be chosen from (3) in Graphic Layout No. 67.**

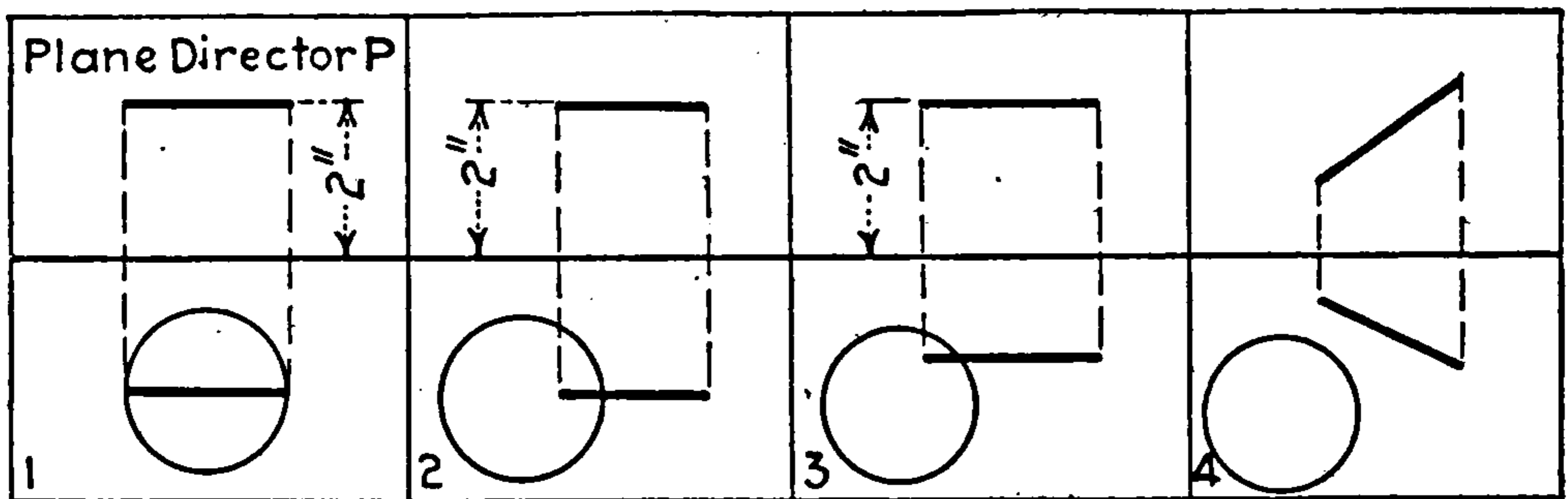
**Cow's Horn.** Circular Directrices, 1 in.,  $1\frac{1}{4}$  in.,  $1\frac{1}{2}$  in.,  $1\frac{3}{4}$  in., 2 in.,  $2\frac{1}{2}$  in., 3 in., 4 in. diams. (Any combination may be used.) Straight Directrix, CD, lies in  $H$ .

**Warped Cone.** Diams. A,  $1\frac{1}{4}$  in.,  $1\frac{1}{2}$  in.,  $1\frac{3}{4}$  in., 2 in.,  $2\frac{1}{2}$  in. Diams. B,  $\frac{1}{2}$  in.,  $\frac{3}{4}$  in., 1 in.,  $1\frac{1}{8}$  in.  $\theta = 22\frac{1}{2}^\circ, 30^\circ, 37\frac{1}{2}^\circ, 45^\circ$ .





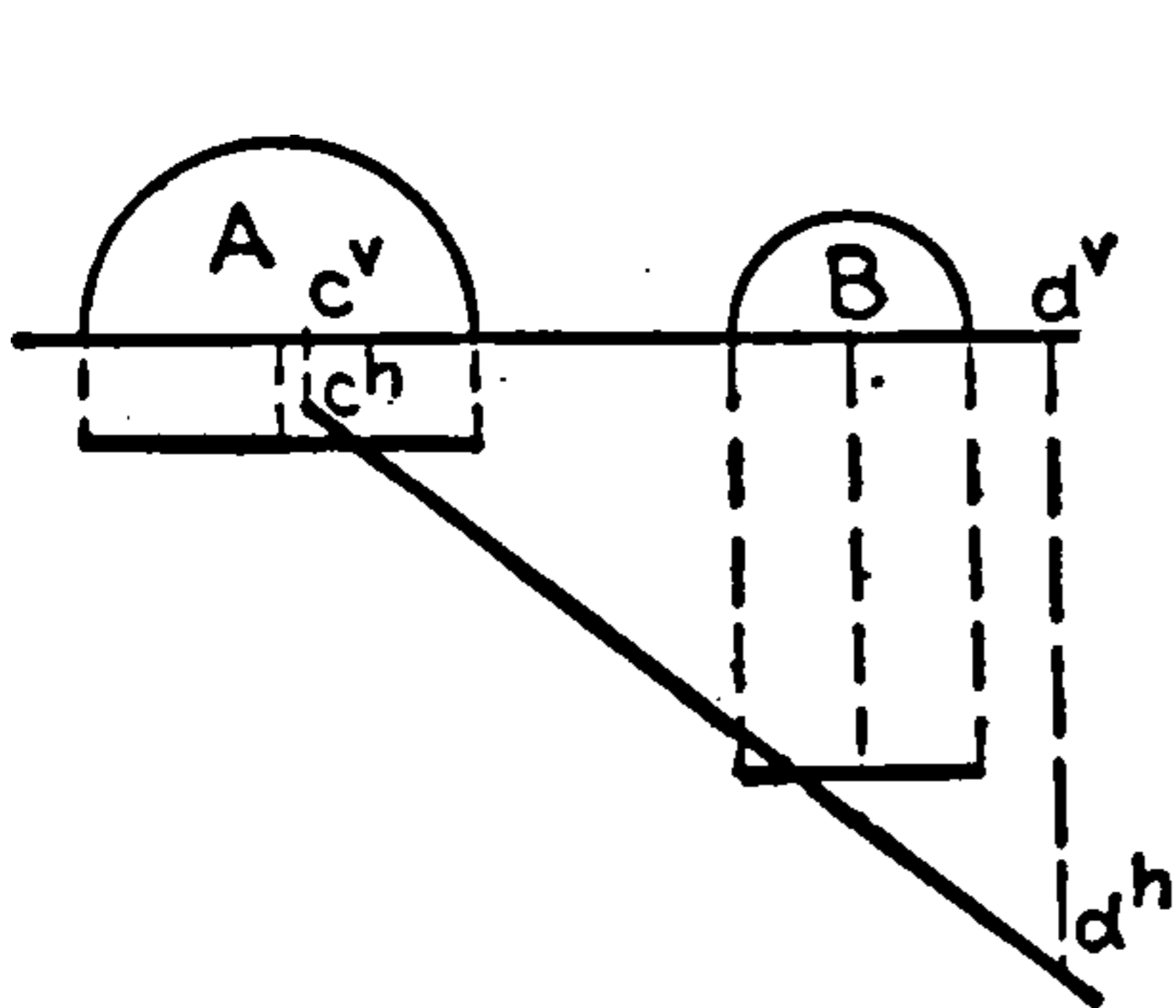
Cylindroids; Directrices are Equal Semi-Circles



Conoids; Rectilinear Directrix Given

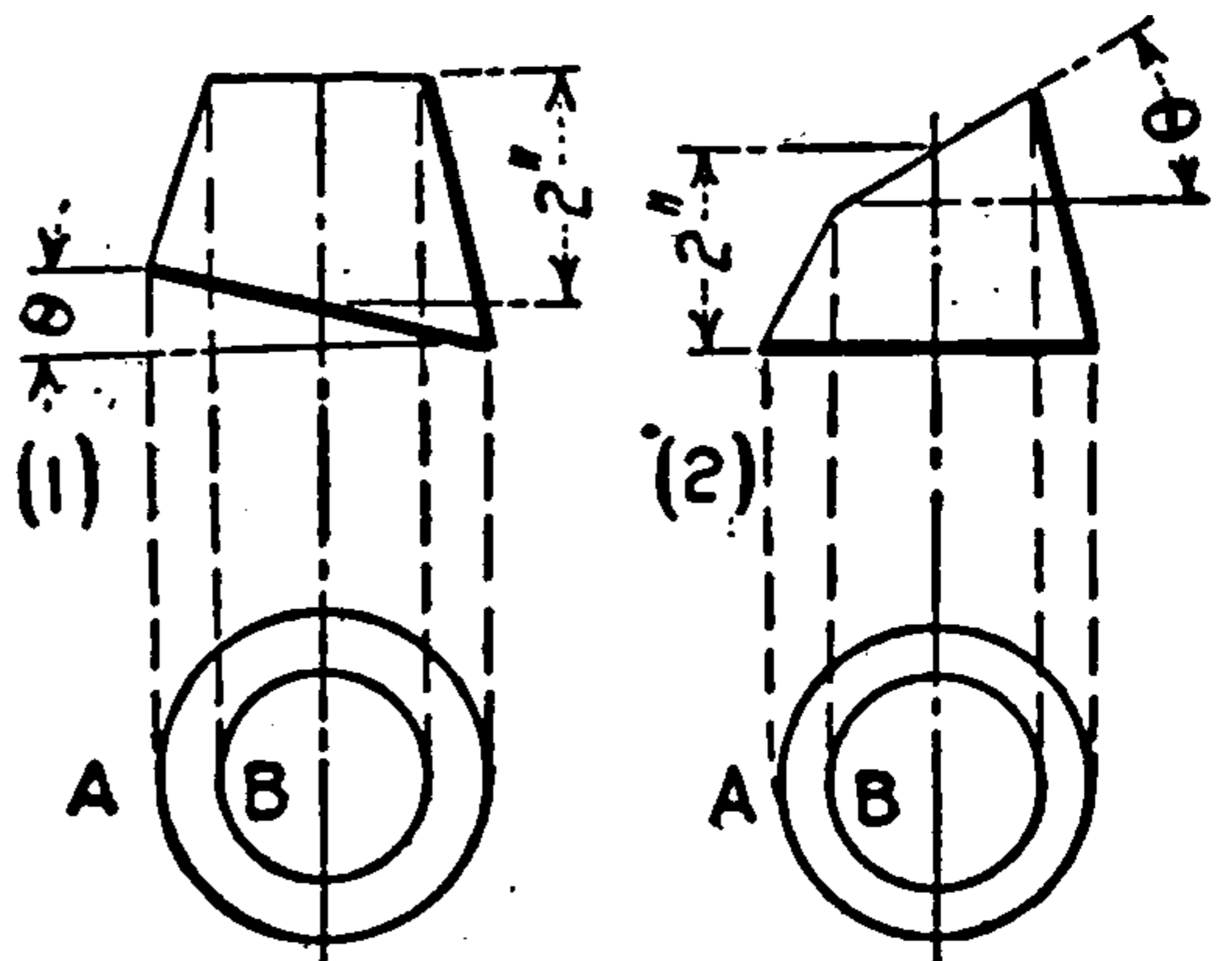
Curved Directrix: Circle in H, Diameter  $1\frac{1}{2}''$ ,  $1\frac{3}{4}''$ ,  $2''$ ,  $2\frac{1}{2}''$  or  
 Ellipse in H,  $1 \times 1\frac{1}{2}''$ ,  $1 \times 2''$ ,  $1\frac{1}{2} \times 2''$

Plane Director for 2, 3 or 4, Selected from Graphic Layout #67, Plane #3



Cow's Horn

Circular Directrices  
 Diams,  $1''$ ,  $1\frac{1}{2}''$ ,  $1\frac{3}{4}''$ ,  $2''$ ,  $2\frac{1}{2}''$ ,  $3''$ ,  $4''$   
 Straight Directrix, CD in H



Warped Cone

Diams, A -  $1\frac{1}{4}''$ ,  $1\frac{1}{2}''$ ,  $1\frac{3}{4}''$ ,  $2''$ ,  $2\frac{1}{2}''$   
 " B -  $\frac{1}{2}''$ ,  $\frac{3}{4}''$ ,  $1''$ ,  $1\frac{1}{8}''$   
 $\theta = 22\frac{1}{2}^\circ$ ,  $30^\circ$ ,  $37\frac{1}{2}^\circ$ ,  $45^\circ$



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876. Develop the surface of the automobile fender shown in Fig. 155.  
 877. Develop the surface of the reducing hood (warped cone), Fig. 155.  
 878. Develop the surface of Edison horn for talking machine (cow's horn), Fig. 155.

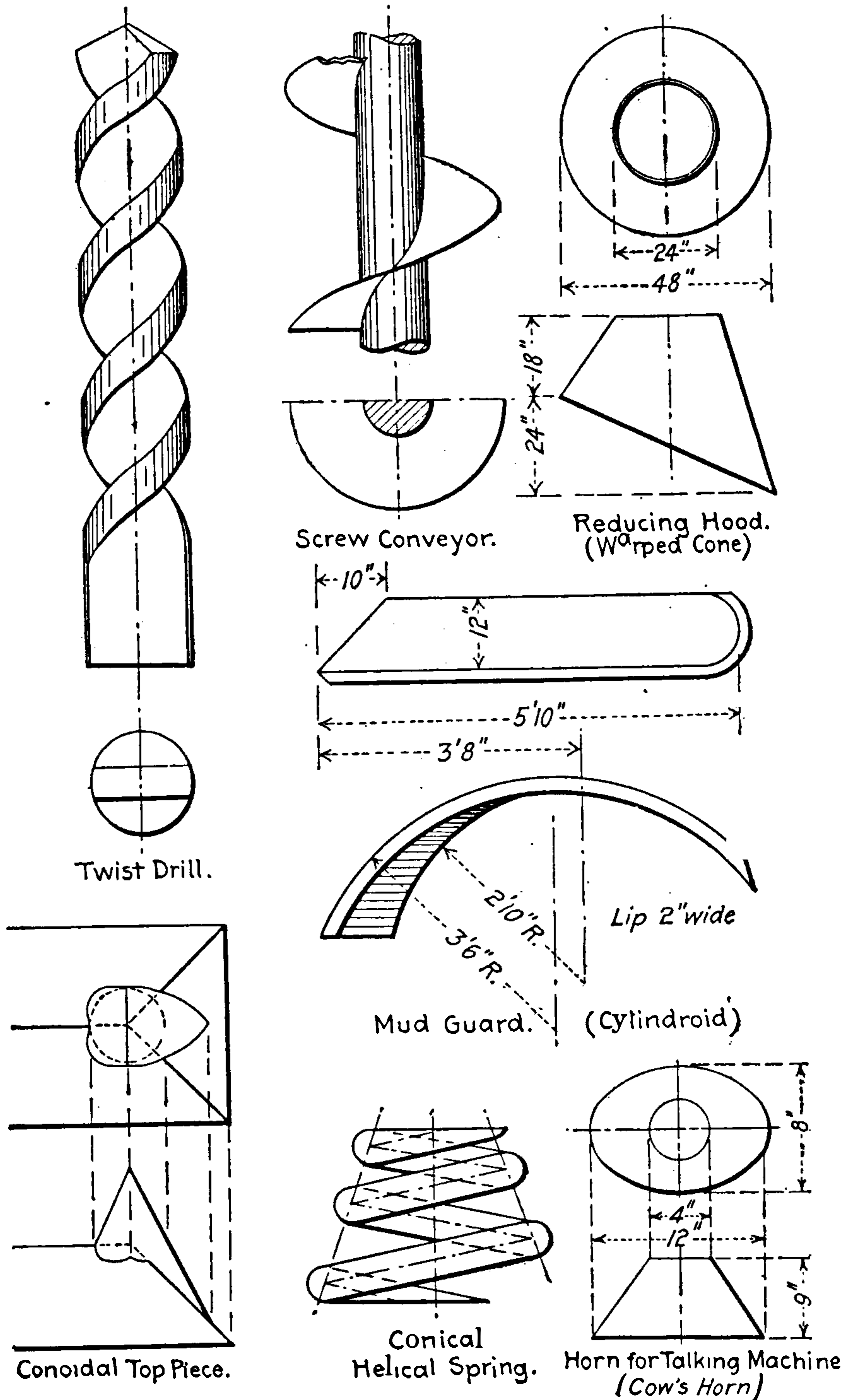


FIG. 155.

879. What is the surface of concrete form B, Fig. 95, Art. 121? Develop the surface.  
 880. What is the surface of concrete form A, Fig. 95, Art. 121? Develop the surface.

## CHAPTER VIII

### DOUBLE-CURVED SURFACES AND SURFACES OF REVOLUTION

#### DOUBLE-CURVED SURFACES

**225.** A double-curved surface is generated by a curve following a curve in such a way as to generate a surface that is neither single-curved or warped. A double-curved surface contains no **straight lines**. It may be either double convex, like a sphere or ellipsoid, or it may be concavo-convex, like a torus, or like the turned surfaces of numberless articles.

As all important double-curved surfaces are also **surfaces of revolution**, they will be considered under that head.

#### SURFACES OF REVOLUTION

**226.** Any line revolving about an axis generates a surface of revolution.

**Single-Curved Surfaces of Revolution.**—If the generatrix is a straight line and in the plane of the axis, the surface will be single-curved. The **Cylinder of Revolution** and the **Cone of Revolution** are the only examples.

**Warped Surfaces of Revolution.**—The only example in this class is the **Hyperboloid of Revolution of One Nappe**.

**Double-Curved Surfaces of Revolution.**—Any plane curve revolving about an axis generates this surface, except in the case of the **Hyperbola**.

**Sphere.**—Generated by a circle revolving about one of its diameters.

**Ellipsoids.** 1. **Prolate Spheroid.**—Generated by revolving an ellipse about its major axis. Its shape resembles a foot ball.

2. **Oblate Spheroid.**—Generated by revolving an ellipse about its minor axis. Its shape resembles a door knob.

**Paraboloids.**—Generated by revolving a parabola about either axis. When revolved about the transverse axis, it generates the surface used in search-light reflectors. Why?

**Torus.**—Generated by revolving a circle about an axis that is not a diameter, usually completely outside the circle. In this case it is called a ring, or annular torus. As a matter of fact, a torus is often generated by other curves than the circle, or by a combination of curves. The architectural detail is of this character. In this work the word torus will be used to mean the annular torus.

**Hyperboloid of Revolution of Two Nappes.**—Generated by revolving a hyperbola about its transverse axis. It is not often encountered in drafting.

Many surfaces generated by combination curves will be found, some with names and some without.

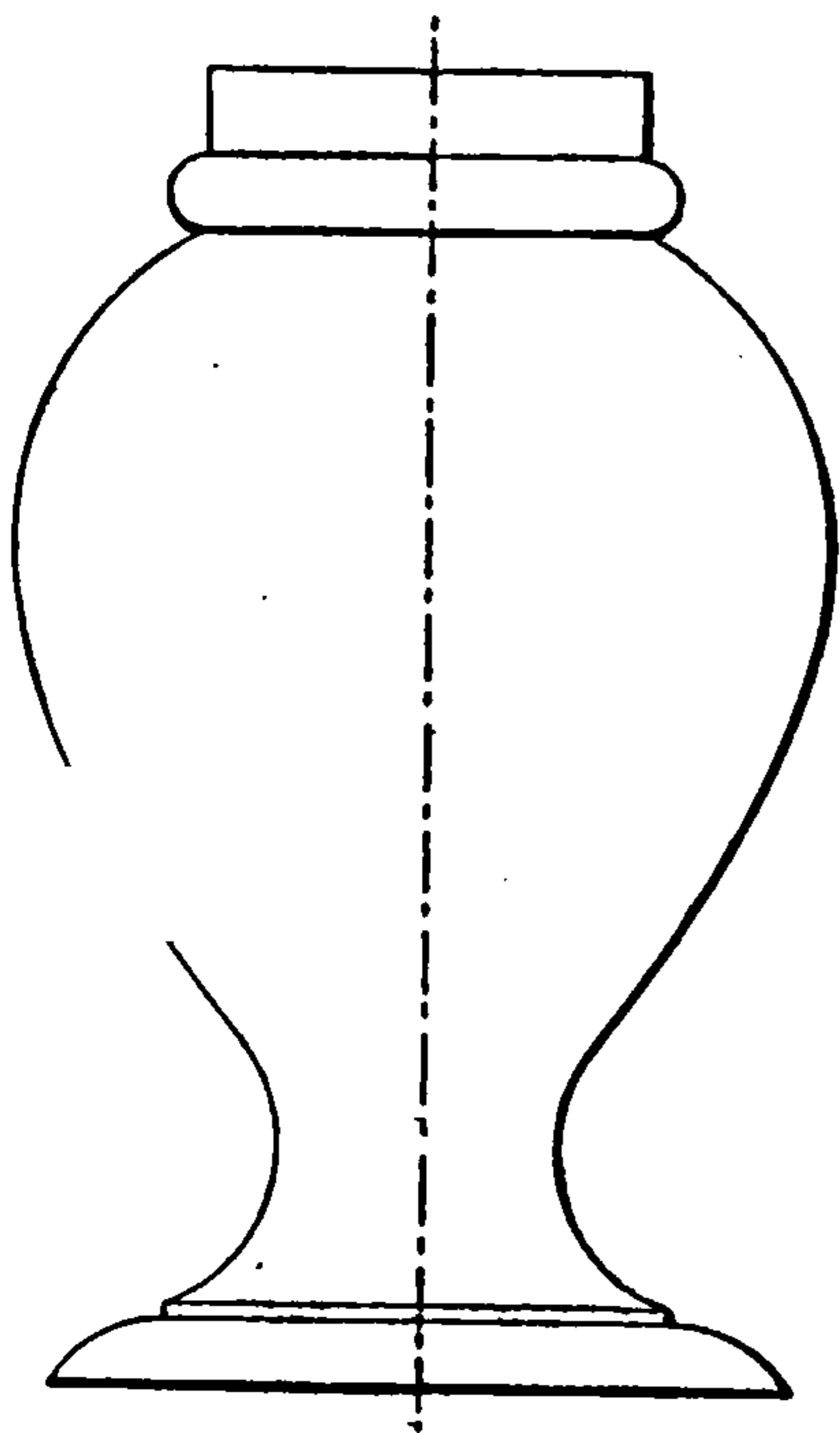


FIG. 156.

**227. Meridian and Right Sections.**—Every plane perpendicular to the axis cuts a surface of revolution in a circle. Hence, we may even conceive a surface to be generated by a circle moving in the direction of the axis, and changing its diameter according to some law. Any surface turned in a lathe is generated in this way. (See Fig. 156.)

Every plane through the axis, called a **Meridian Plane**, cuts the surface in a **Meridian Line**, which has the identical form of the line used to generate the surface.

Any two surfaces of revolution having a common axis will intersect in a circle or circles, perpendicular to the axis. When the axis is parallel to the plane of projection these intersections appear as straight lines. (See Fig. 156.)

**228. Representation.**—Two views of double-curved surfaces of revolution are all that are necessary, and usually one is sufficient, and that in outline. A center line on  $V$ , perpendicular to  $GL$ , is almost always drawn, and the meridian section parallel to  $V$  constitutes the  $V$ -projection. The  $H$ -projection is merely the circle of a diameter equal to the largest dimension of the generatrix, with as many other circles, visible or invisible, as may be necessary to show the various intersections of the surface. Fig. 157 shows a torus, which will fairly represent all such surfaces.



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**231. Problem 82.**—To pass a plane tangent to a sphere at a point on the surface.

**Analysis.**—Pass a plane through the point perpendicular to the radius.

Let the student make the construction.

**232. Problem 83.**—To pass a plane tangent to any double-curved surface of revolution at a given point on the surface.

Let the given point be  $O$  on the surface of the torus, Fig. 157.

**Analysis.**—1. Draw a tangent to the right section through the point.

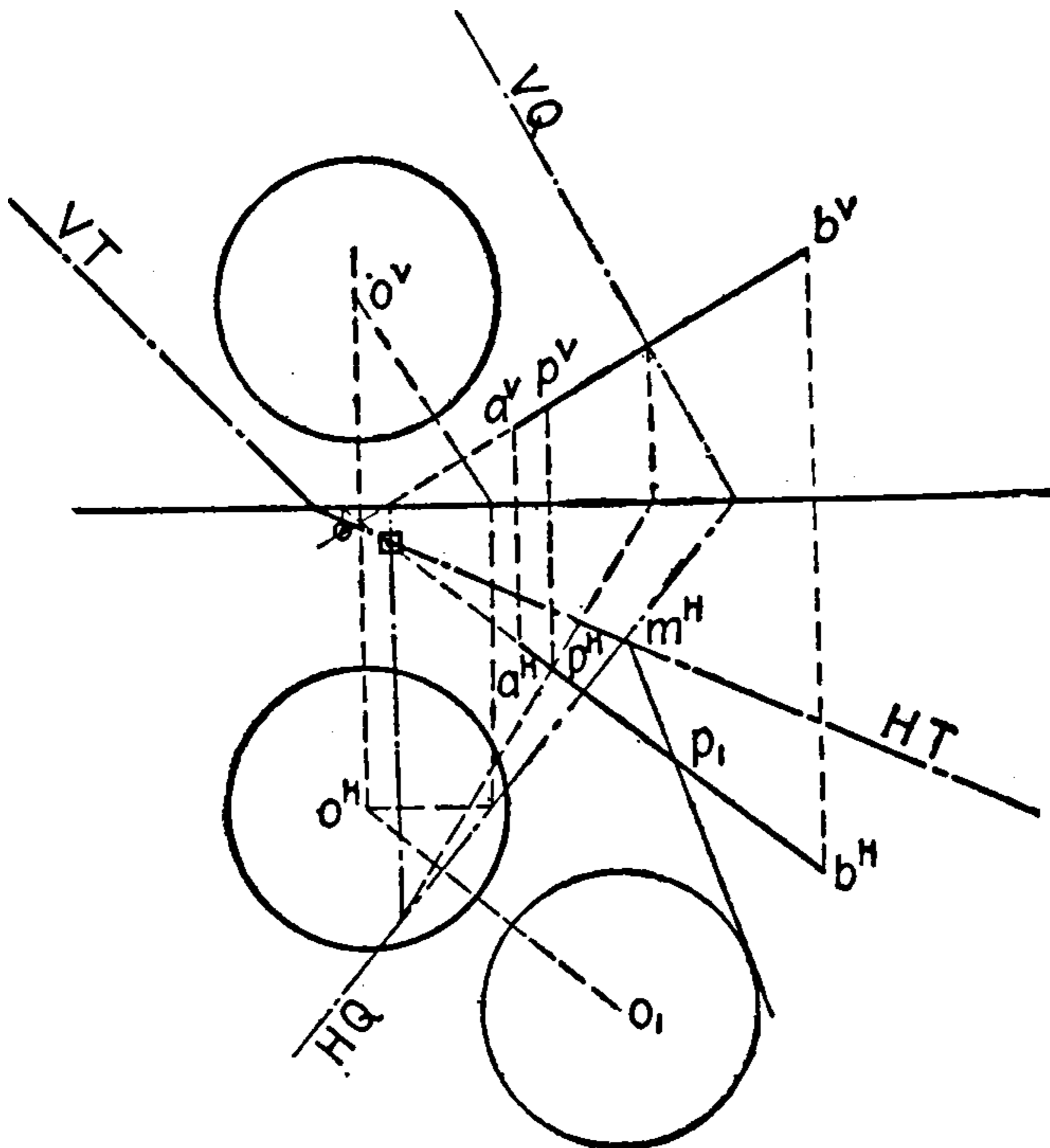


FIG. 158.

2. Draw a tangent to the meridian section at the point.

3. These two tangents will determine the plane.

**Construction.**—1. Draw the tangent  $OM$  to the circle through  $O$ , and locate its  $V$ -piercing point  $M$ .

2. Revolve  $O$  to  $o_1 o_1^V$ , so that the meridian plane will be parallel to  $V$ .

3. Draw the tangent  $o_1^V n_1^V$  to the meridian curve, in revolved position, and locate its  $H$ -piercing point  $N$ .

4. Counter-revolve  $N$  to its original position  $n^V n^H$ , into the trace of the meridian plane through  $O$ .

5. The two tangents will now determine the tangent plane,  $HT$  being parallel to  $o^H m^H$ , and  $VT$  being drawn through  $m^V$ .

**233. Problem 84.**—To pass a plane tangent to a sphere through a given straight line.

**Analysis.**—Regard the sphere as being enveloped by a cylinder parallel to the given line, and pass the plane tangent to the cylinder. The plane will be tangent also to the sphere.

**Construction.**—Let  $AB$  (Fig. 158) be the given line, and  $O$  be the center of the sphere.

1: Pass a plane,  $Q$ , through  $O$  perpendicular to  $AB$ .

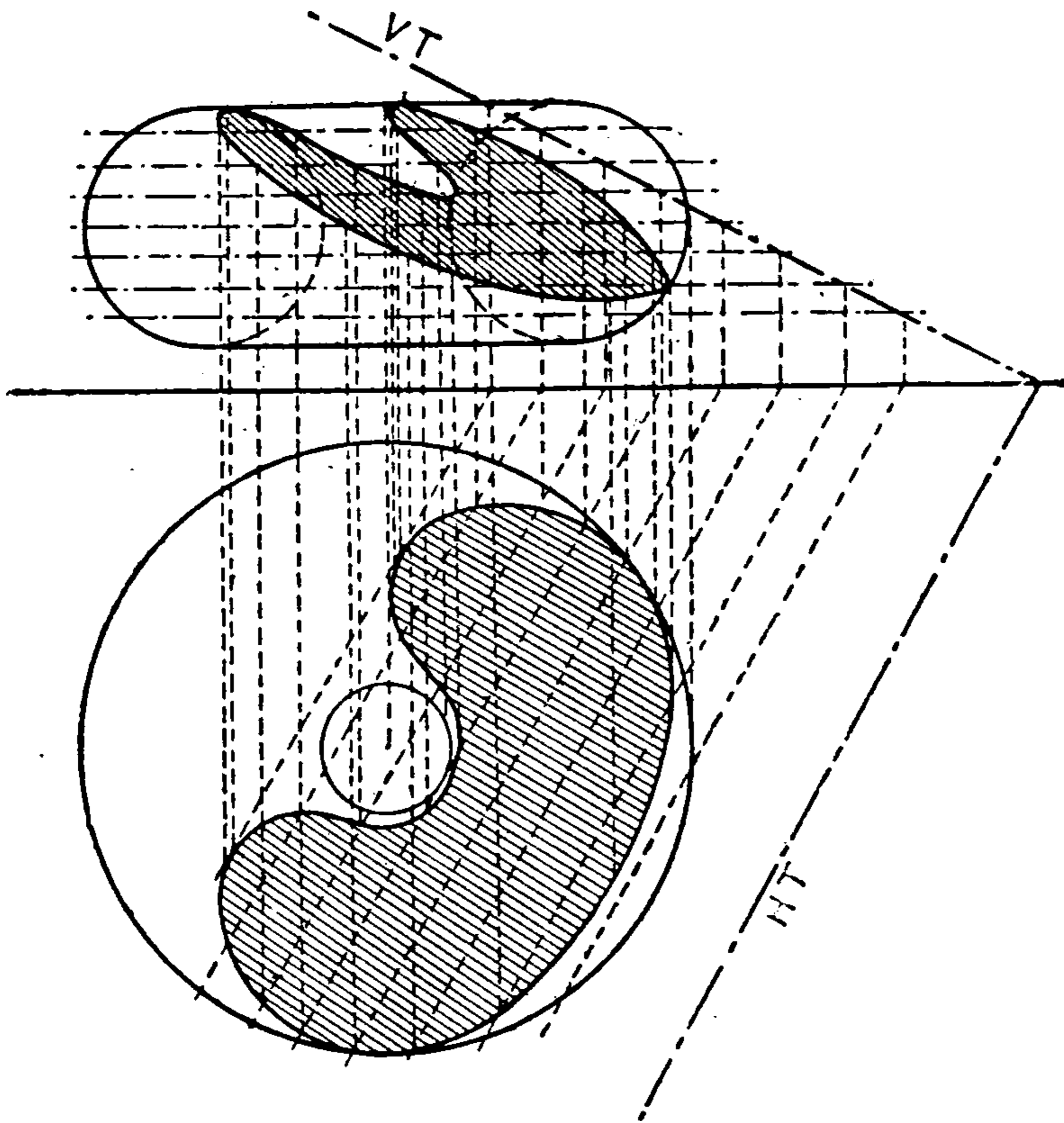


FIG. 159.

2. Find the point,  $P$ , in which  $AB$  pierces  $Q$ .

3. Revolve  $P$  and  $O$  about the  $H$ -trace of  $Q$  into  $H$  at  $p_1$  and  $o_1$ , and draw a circle at  $o_1$  of the same radius as that of the sphere.

4. Draw a tangent from  $p_1$  to this circle.

5. Find its  $H$ -piercing point  $m^H$  (where it crosses  $HQ$ ).

6. Draw the traces of  $T$ , the required tangent plane, through  $m^H$  and the piercing points of  $AB$ .

**Note 1.**—This problem usually has two solutions, but if the line  $AB$  pierces the sphere, the problem is impossible.

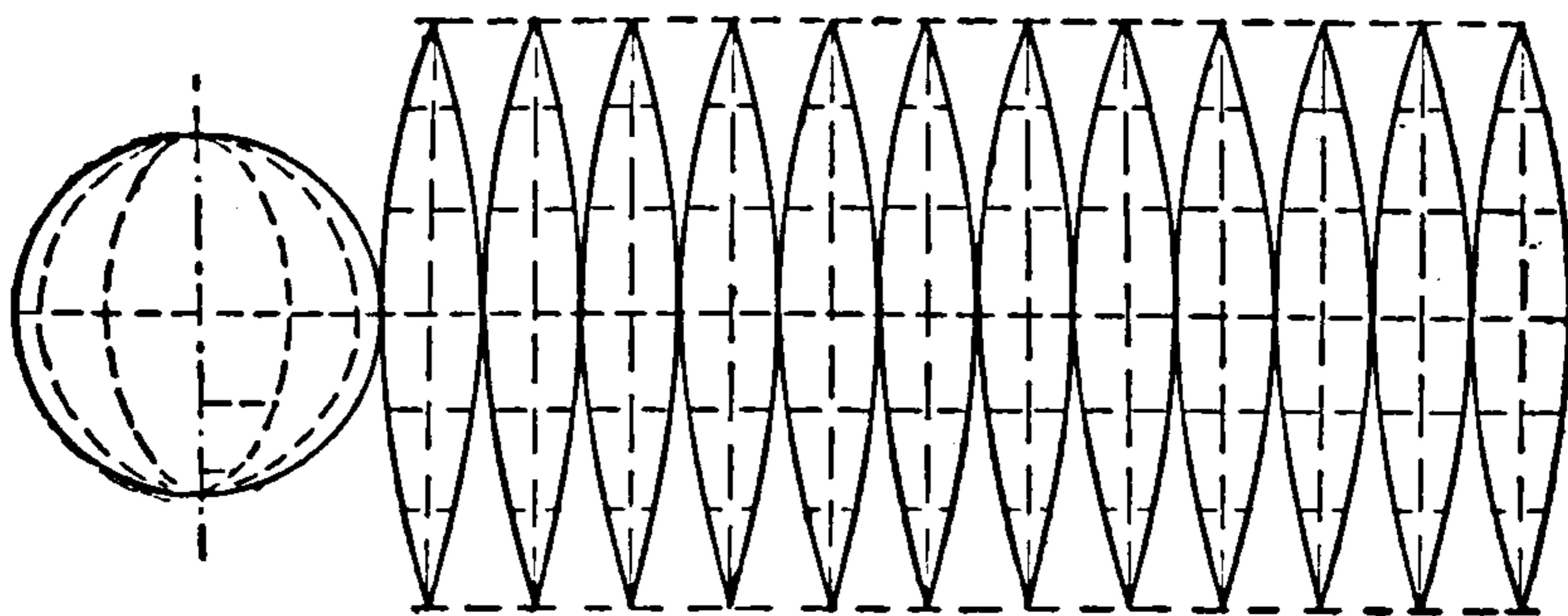
**Note 2.**—The projections of the cylinder were not drawn, as it is merely an imaginary assistance to the analysis. It is not even necessary to imagine



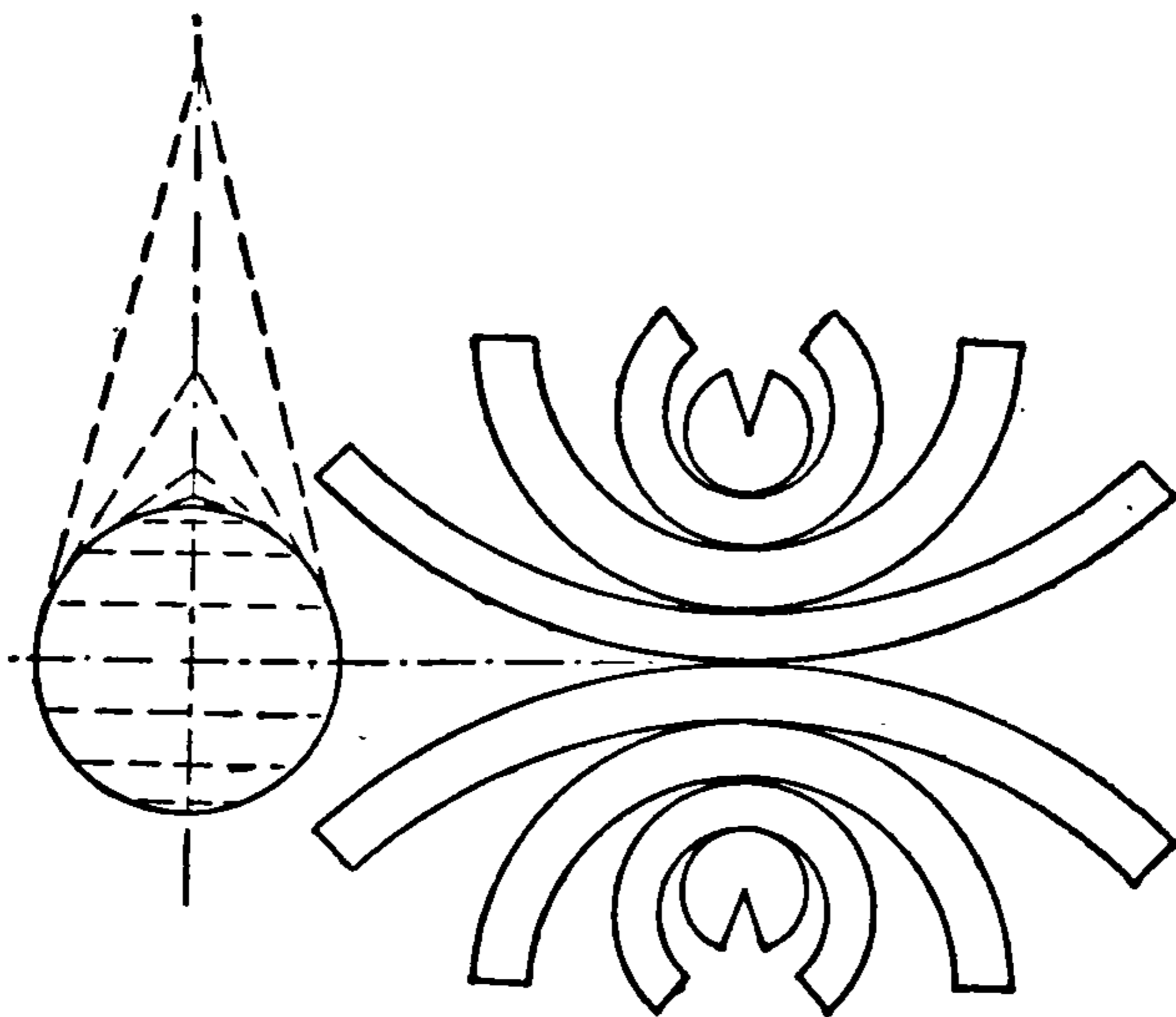
it, as the normal plane,  $Q$ , will have to pass through the center of the sphere, although it does not have to be perpendicular to  $AB$ . This is convenient, however, and would be necessary if the cylinder idea were carried out.

**234. Problem 85.**—To obtain the intersection of a plane with any surface of revolution.

**Analysis.**—The simplest curve that can be cut from a surface of revolution is the circle, which is cut by a plane perpendicular to the axis. Therefore a series of planes perpendicular to the axis will cut straight lines from the plane and circles from the



*Meridian Method.*



*Zone Method.*

FIG. 160.

given surface. The curve of intersection will pass through the various intersections of the straight lines and circles. Fig. 159 shows both projections of the line of intersection of the plane  $T$  with a torus.

The vertices (high and low points) of the curve may be exactly located by projecting the torus and plane on a plane perpendicular to the  $H$ -trace of the given plane. The vertices always lie in the meridian plane which is perpendicular to the given plane.



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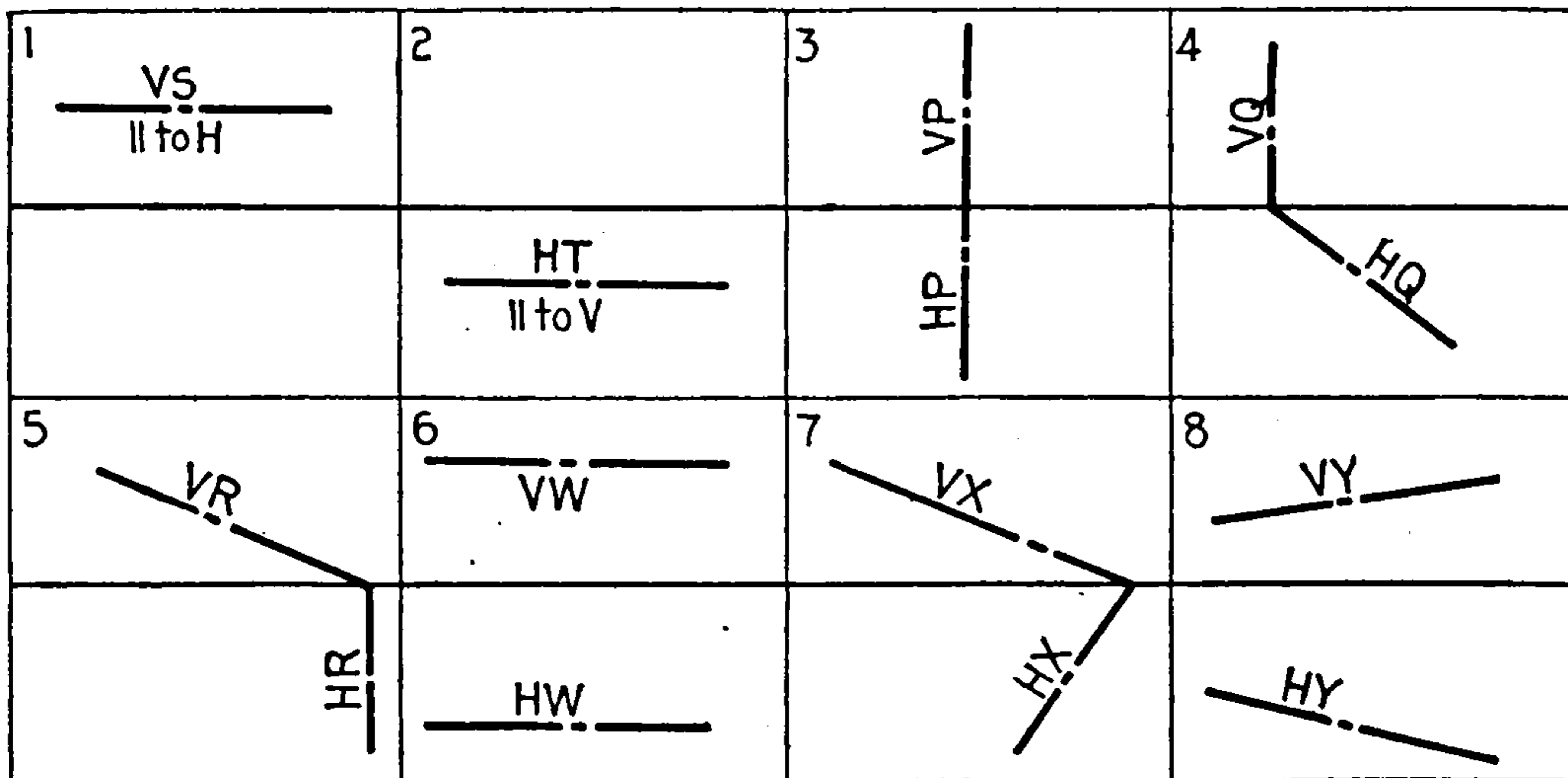
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## Dimensioned Layout for Piercing Lines.

- (1)  $A(2, 0 - 1\frac{1}{4}) B(4 + 2 - 2)$ .
- (2)  $C(2, 0 - 2) D(4 + 2 - \frac{1}{2})$ .
- (3)  $E(3\frac{1}{2} + 2 - \frac{1}{2}) F(3\frac{1}{2} + 1 - 2\frac{1}{2})$ .
- (4)  $G(3\frac{1}{4} + 2 - 2) K(3\frac{1}{4} + \frac{1}{2} - 1)$ .
- (5)  $M(1 + 1 - 1\frac{1}{2}) N(4 + 1 - 1\frac{1}{2})$ .
- (6)  $E(1 + 1 - \frac{1}{2}) F(4 + 1 - 2\frac{1}{2})$ .



Graphic Layout No. 71.—Cutting planes.

## Dimensioned Layout for Cutting Planes.

- (1)  $S(1 + 1) (5 + 2) (1 - 1\frac{1}{2}) (5 - 3)$ .
- (2)  $T(-1\frac{3}{4}) || \text{ to } V$ .
- (3)  $P(3\frac{1}{2} + 3) 3\frac{1}{2}(3\frac{1}{2} - 3)$ .
- (4)  $Q(1 + 3) 1(5 - 3)$ .
- (5)  $R(5 + 2) 1(1 - 3)$ .
- (6)  $W(+1) \infty (-3)$ .
- (7)  $X(+1\frac{1}{2}) \infty (+3)$ .
- (8)  $Y(2 + 3) 6(5 - 3)$ .

881. Sphere —; A —. Draw its projections, and locate  $C(2\frac{1}{2} + 1, x)$  on — (front or rear) surface, and  $D(3\frac{1}{2}, y - 1)$  on — (upper or lower) surface.
882. Sphere in Ex. 881. Find the points in which it is pierced by line —.
883. Sphere in Ex. 881. Draw the projections and true size of the section made by plane —.
884. Sphere in Ex. 881. Draw the traces of the plane tangent at — (C or D).
885. Sphere in Ex. 881. Draw the traces of a tangent plane which contains the line  $X(3 + 3 - 1\frac{1}{2}) Y(5 + 1 - 2)$ .
886. Sphere in Ex. 881. Draw the traces of a tangent plane which contains the line  $M(4 + 2 - 1) N(5 + 1 - 2)$ .
887. Sphere in Ex. 881. Develop the surface by the Zone Method.
888. Sphere in Ex. 881. Develop the surface by the Meridian Method.

889. **Prolate Spheroid**; A —, B —. Draw its projections, and locate  $C(2\frac{5}{8}, x - 1)$  on the — (upper or lower) surface, and  $D(3\frac{1}{4} + 1, y)$  on the — (front or rear) surface.
890. Spheroid in Ex. 888. Find the points in which it is pierced by line —.
891. Spheroid in Ex. 888. Draw the projection and true size of the curve cut by plane —.
892. Spheroid in Ex. 888. Draw the traces of the tangent plane at — (C or D).
893. Spheroid in Ex. 888. Develop the surface by the Zone Method.
894. Spheroid in Ex. 888. Develop the surface by the Meridian Method.
895. Spheroid in Ex. 888. Draw the traces of a tangent plane which contains  $M(3 + 3 - 1\frac{1}{2})$   $O(4 + 1 - 2)$ .
896. Spheroid in Ex. 888. Draw the traces of the tangent plane which contains  $X(4 + 2 - 1)$   $Y(5 + 1 - 2)$ .
897. **Oblate Spheroid**; A —, B —. Draw its projections, and locate  $C(2, x - 1)$  on the — (upper or lower) surface, and  $D(3\frac{3}{4} + \frac{3}{4}, y)$  on the — (front or rear) surface.
899. Spheroid in Ex. 897. Draw the traces of the plane tangent at — (C or D).
900. Spheroid in Ex. 897. Locate the points in which it is pierced by the line —.
901. Spheroid in Ex. 897. Draw the projections and true size of the curve cut by the plane —.
902. Spheroid in Ex. 897. Develop its surface by the Zone Method.
903. Spheroid in Ex. 897. Develop its surface by the Meridian Method.
904. **Torus**; A —, C —. Draw its projections, and locate  $C(2, x - 1)$  on the — (upper or lower) and  $D(3\frac{1}{2} + \frac{7}{8}, y)$  on the — (front or rear) surface.
905. Torus in Ex. 904. Draw the traces of the plane tangent at — (C or D).
906. Torus in Ex. 904. Locate the points in which it is pierced by the line —.
907. Torus in Ex. 904. Draw the projections and true size of the curve cut by the plane —.
908. Torus in Ex. 904. Develop the fragment of the surface — (below or above) the cutting plane — (by Zone Method).
909. Torus in Ex. 904. Develop the fragment of the surface — (below or above) the cutting plane — (by Meridian Method).
910. **Ovoid**; B —, C —. Draw the projections of the surface, and locate  $C(2 + 1, x)$  on the — (front or rear) and  $D(3\frac{3}{4}, y - 1)$  on the — (upper or lower) surface.
911. Ovoid in Ex. 910. Draw the traces of the plane tangent at — (C or D).
912. Ovoid in Ex. 910. Draw the traces of a tangent plane which contains  $P(3 + 3 - 1\frac{1}{2})$   $X(5, 0 - 2)$ .
913. Ovoid in Ex. 911. Draw the traces of a tangent plane which contains  $Y(3 + 2, 0)$   $Z(5, 0 - 1\frac{3}{4})$ .

914. Ovoid in Ex. 911. Locate the points in which the surface is pierced by the line —.
915. Ovoid in Ex. 911. Draw the projections and true size of the curve cut by the plane —.
916. Ovoid in Ex. 911. Develop the fragment of its surface — (above or below) the cutting plane —, by — (Zone or Meridian) Method.
917. Find the plane tangent to three spheres, all resting on  $H$ , whose respective centers are  $A(2 + 1 - 1)$ ,  $B(3 + \frac{1}{2} - 1\frac{1}{2})$ , and  $C(3\frac{1}{2} + \frac{5}{8} - \frac{1}{2})$ .
918. Develop the surface of the ornamental sheet metal surface of revolution, shown in Fig. 161. Any convenient scale.
- Note.—One strip, or meridian, will be sufficient.

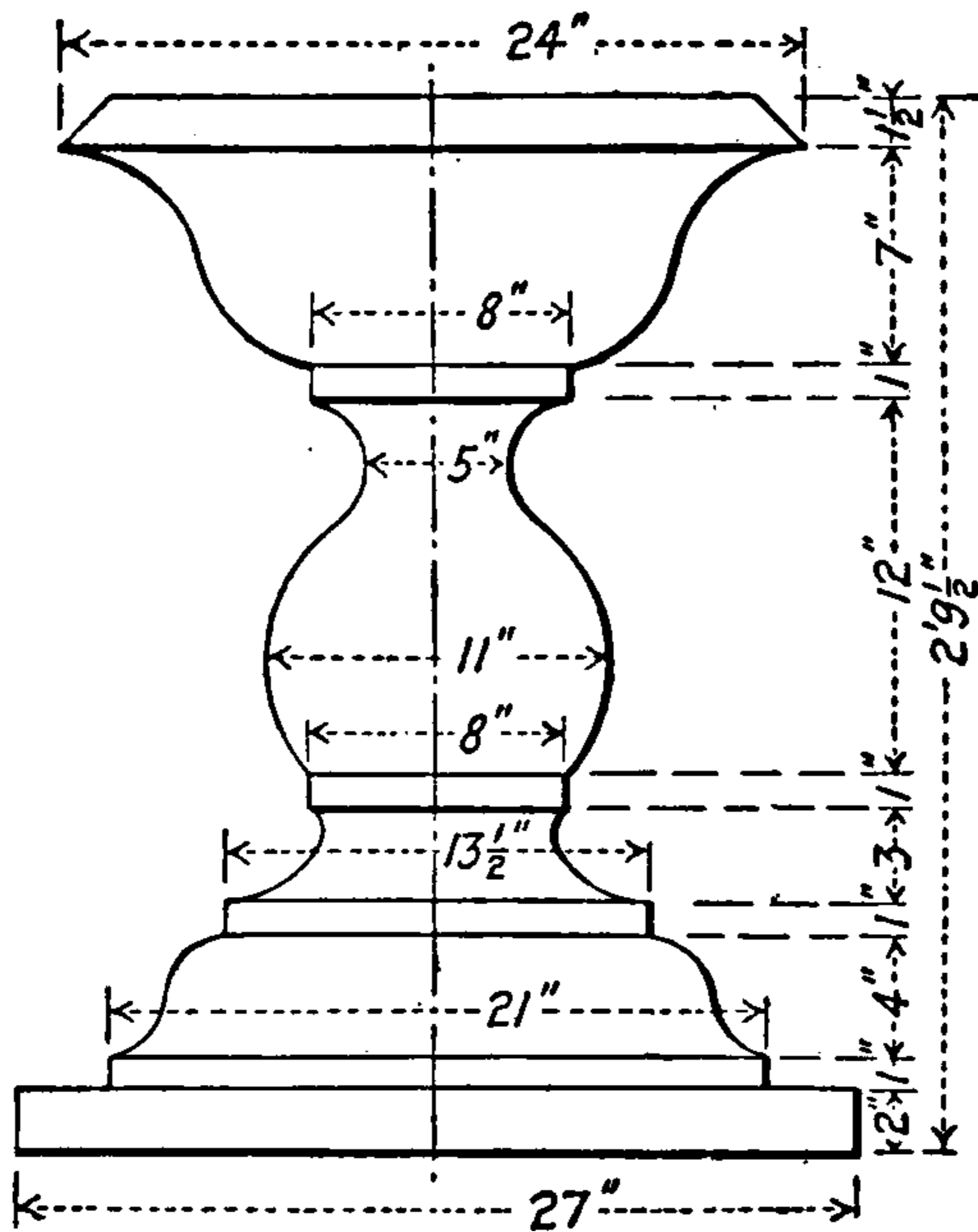


FIG. 161.



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from a cone by passing a plane through the apex, and from a cylinder by passing a plane parallel to the axis. In the case of a cone intersecting a cylinder, then, a natural procedure would be to pass a series of planes through the apex of the cone and parallel to the axis of the cylinder. This is one example. The accompanying table may be of use in deciding what sort of plane to use in the various contingencies that arise.

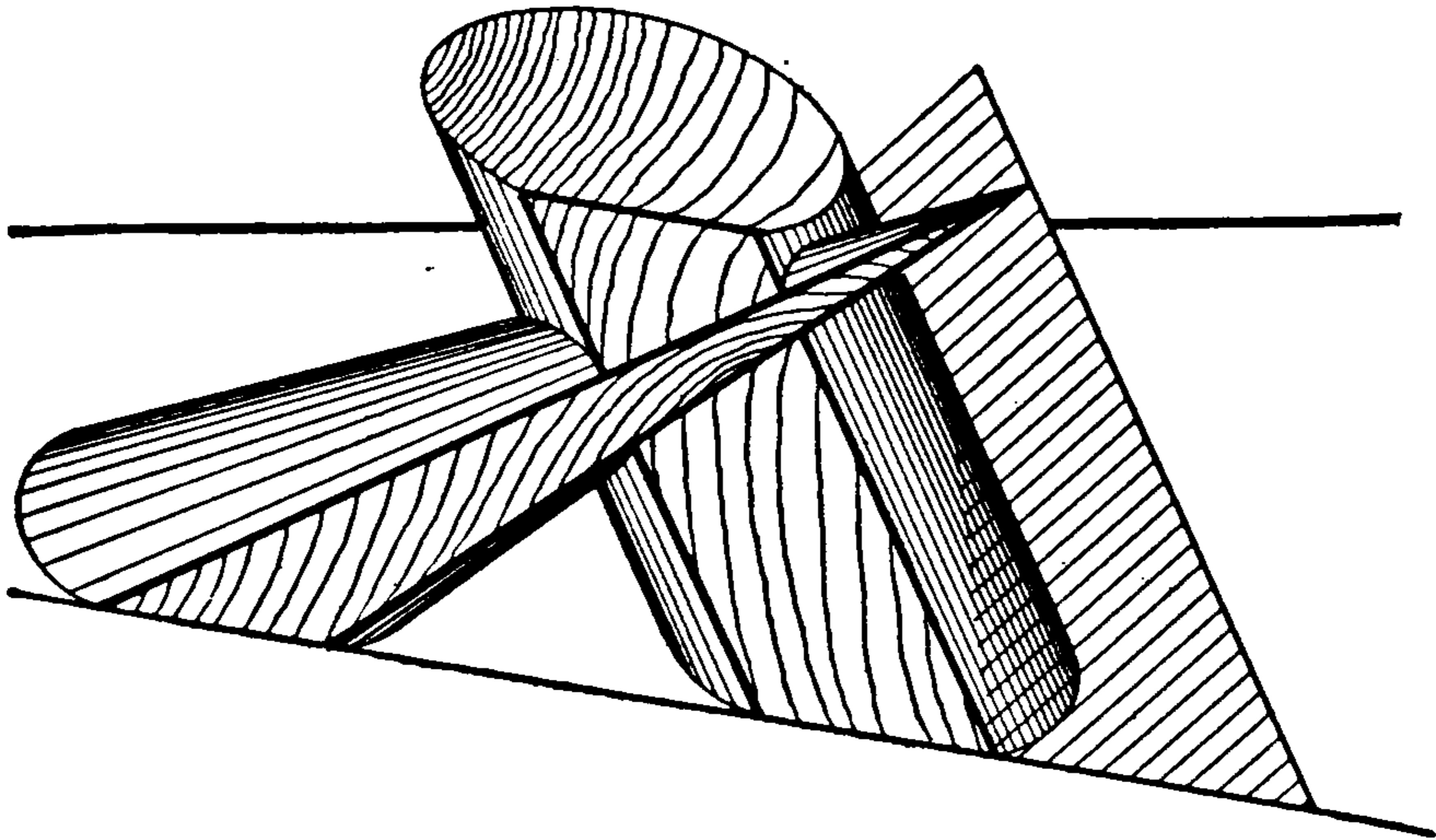


FIG. 162.—Showing straight lines cut from intersecting cone and cylinder by passing a plane through the cone apex parallel to the cylinder axis.

<i>Surface</i>	<i>Intersection</i>	<i>Kind of Plane</i>
Cone	Straight line	Through the apex.
Cone	Circle	Parallel to circular base.
Cylinder	Straight line	Parallel to the axis.
Cylinder	Circle	Parallel to a circular base.
Convolute	Straight line	Tangent to helical cylinder.
Convolute	Involute	Perpendicular to the axis.
Helicoid	Straight line	Through the axis.
Warped surface	Straight line	According to its generation.
Surface of revolution	Circle	Perpendicular to the axis.
Surface of revolution	Meridian curve	Through the axis.

It may be noted that planes cutting straight lines from warped surfaces are seldom easy to use because of the variety of their directions. There are cases of surfaces having a plane director where it is convenient to get straight line intersections.

Frequently it is not possible to use the planes in the foregoing list for both of the intersecting surfaces, but it is always possible to use them for one. In such cases the second surface may be cut in somewhat complicated curves. It is such cases as these that make it impossible to lay down rules for all cases.

The draftsman must use his judgment and instinct for the right auxiliary surfaces.

### CONCENTRIC SPHERES METHOD

**240.** When two surfaces of revolution have intersecting axes the easiest method of obtaining their intersection is to use concentric spheres. By employing a sphere, as an auxiliary surface, with its center at the intersection of the axes, circles are cut from the two surfaces, because the intersections of two surfaces of revolution having a common axis will be circles. A typical construction will be shown in Problem 89.

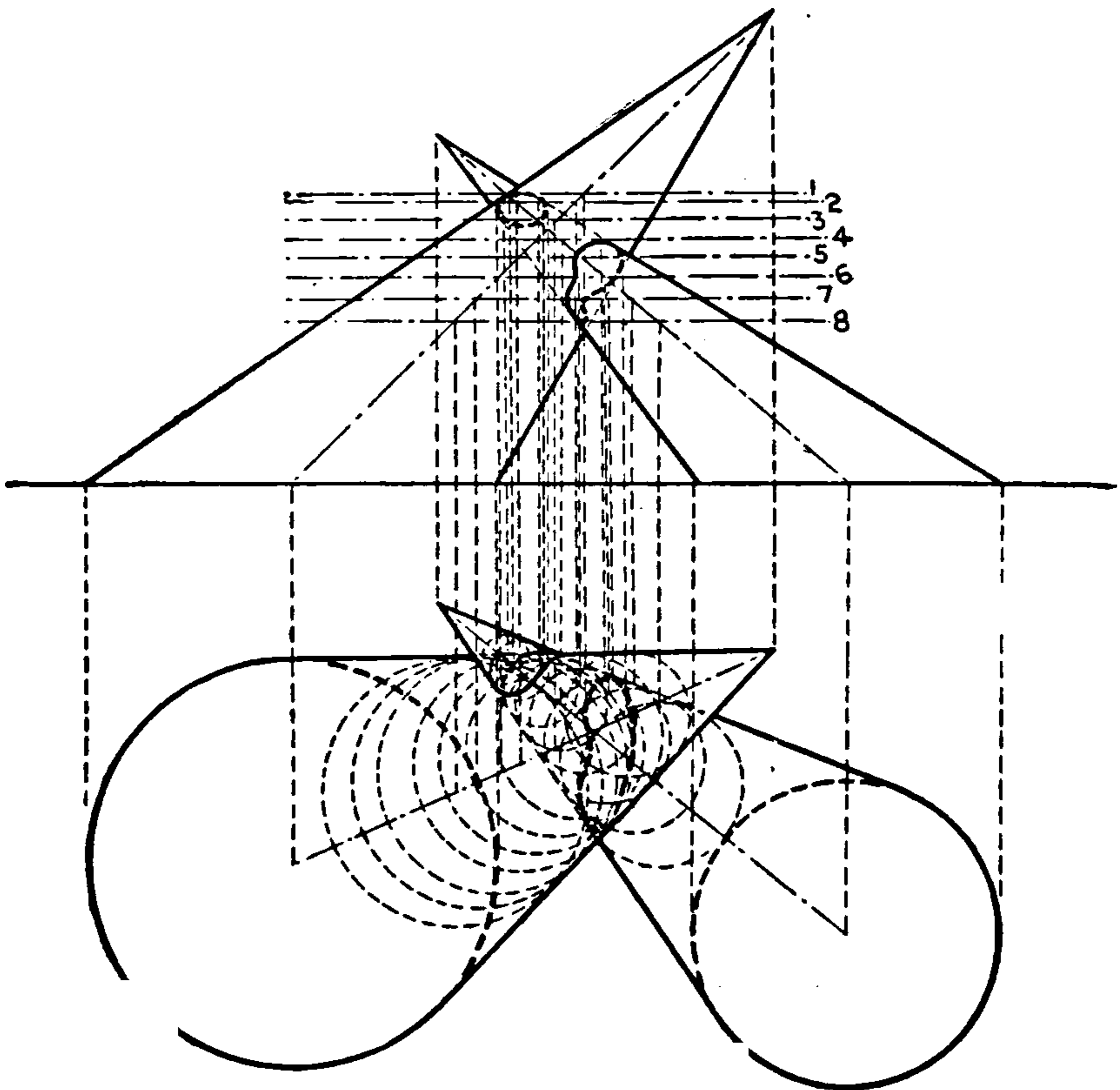


FIG. 163.

### PROBLEMS IN INTERSECTIONS

A few representative problems of the many that might be given are selected for illustration.

**241. Problem 86.**—To find the intersection between two cones.

Even in so specific a problem as this, the position and character of the two cones have much to do with the choice of auxiliary planes to be used. We give two representative cases, and both could be successfully worked by using another system of auxiliary planes.



**First Case.**—When the cones are oblique and their bases are circles in the same plane.

**Analysis.**—Pass a series of planes parallel to the bases, intersecting the cones in circles. Study Fig. 163 for the solution and construction. Let the student also work this by passing planes through the apices of the cones. The advantage of the latter method is that it gives definitely the outside elements of the cones that are involved in the intersection.

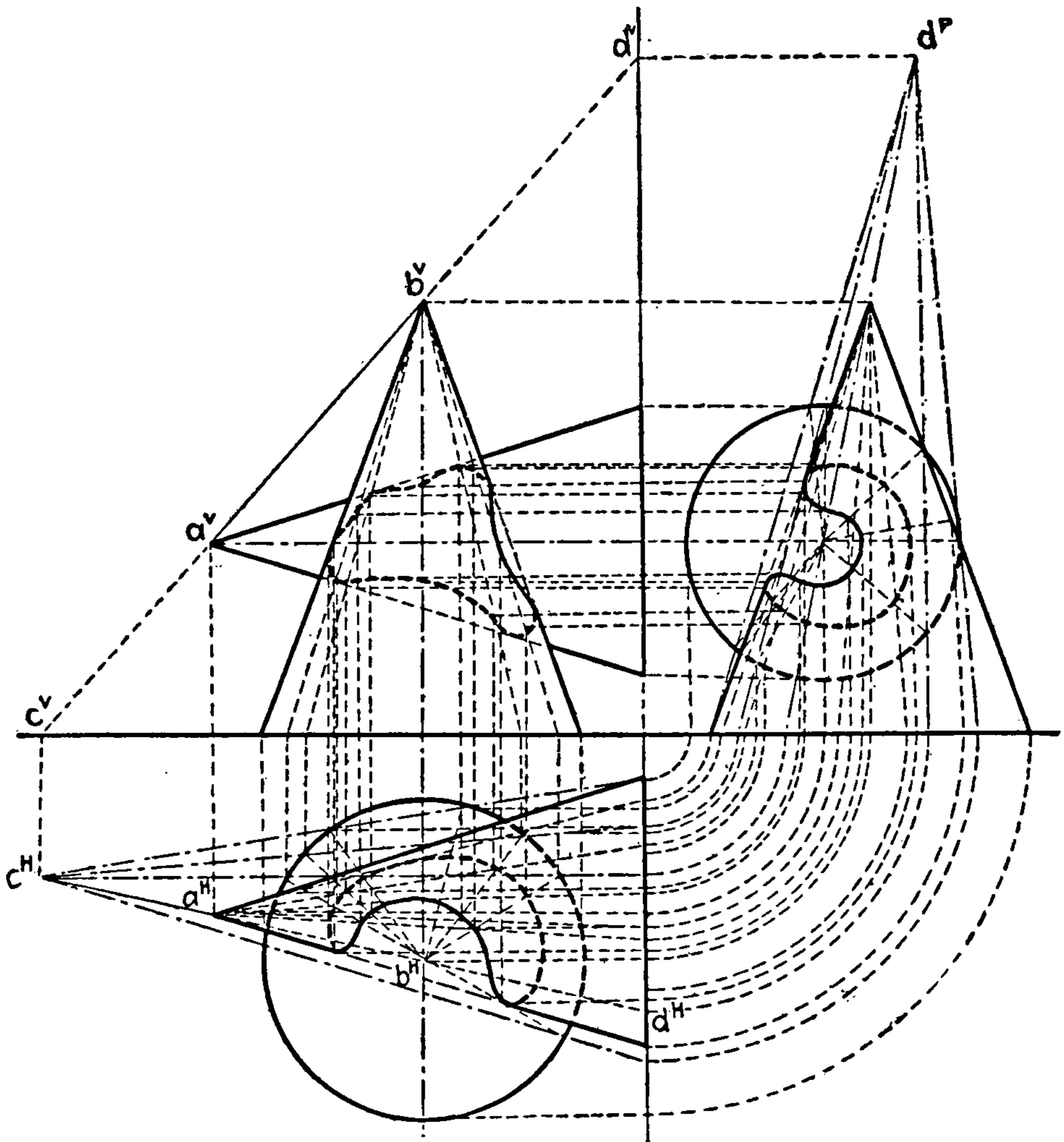


FIG. 164.

**242. Second Case.**—When the cones are right circular, and their bases are in different planes.

**Analysis.**—1. Pass a series of planes through the apices of the cones, cutting elements from each.

2. Connect the intersections of the elements in order.

**Construction.**—To find the intersection between the two right circular cones, whose apices are  $A$  and  $B$ , and whose bases are in  $P$  and  $H$  respectively (Fig. 164).



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3. Through M, N, O, and P draw elements parallel to the respective axes.

4. These four elements lie in the same plane, and will intersect each other in four points on the curve. Locate these points.

5. Draw a series of traces parallel to HQ, and repeat operations (2), (3), (4) with each.

6. Draw a smooth curve through all of the points, taking them in order.

Note.—Do not divide either circle into any number of equal arcs. Let the planes determine the elements.

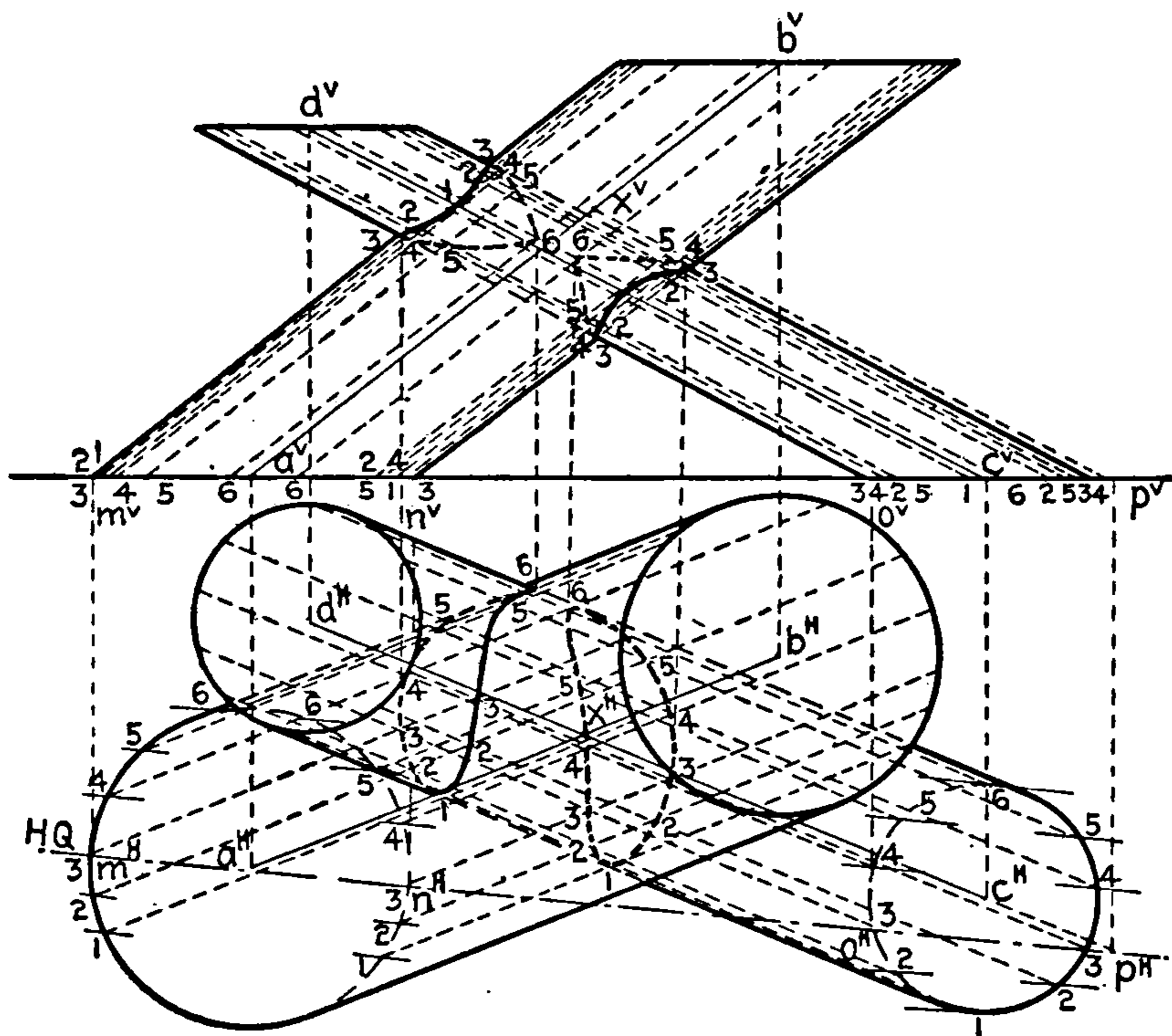


FIG. 165.

**244. Problem 88.—To find the intersection of a cone and cylinder.**

The following methods of passing planes may be employed:

1. Through the apex of the cone, parallel to the axis of the cylinder.

2. Parallel to the bases in any plane. (This method is very convenient if both surfaces have circular bases in *H*, *V*, or *P*.)

3. Parallel to the elements of the cylinder and perpendicular to the axis of the cone, if it be a right circular cone, if the conditions are right.

**Concentric Sphere Method.**—If both are surfaces of revolution with axes intersecting, the process in Problem 89 is usually the best.

**Auxiliary Plane Method.**—There are certain conditions when the easiest and neatest solution is made by projecting both sur-

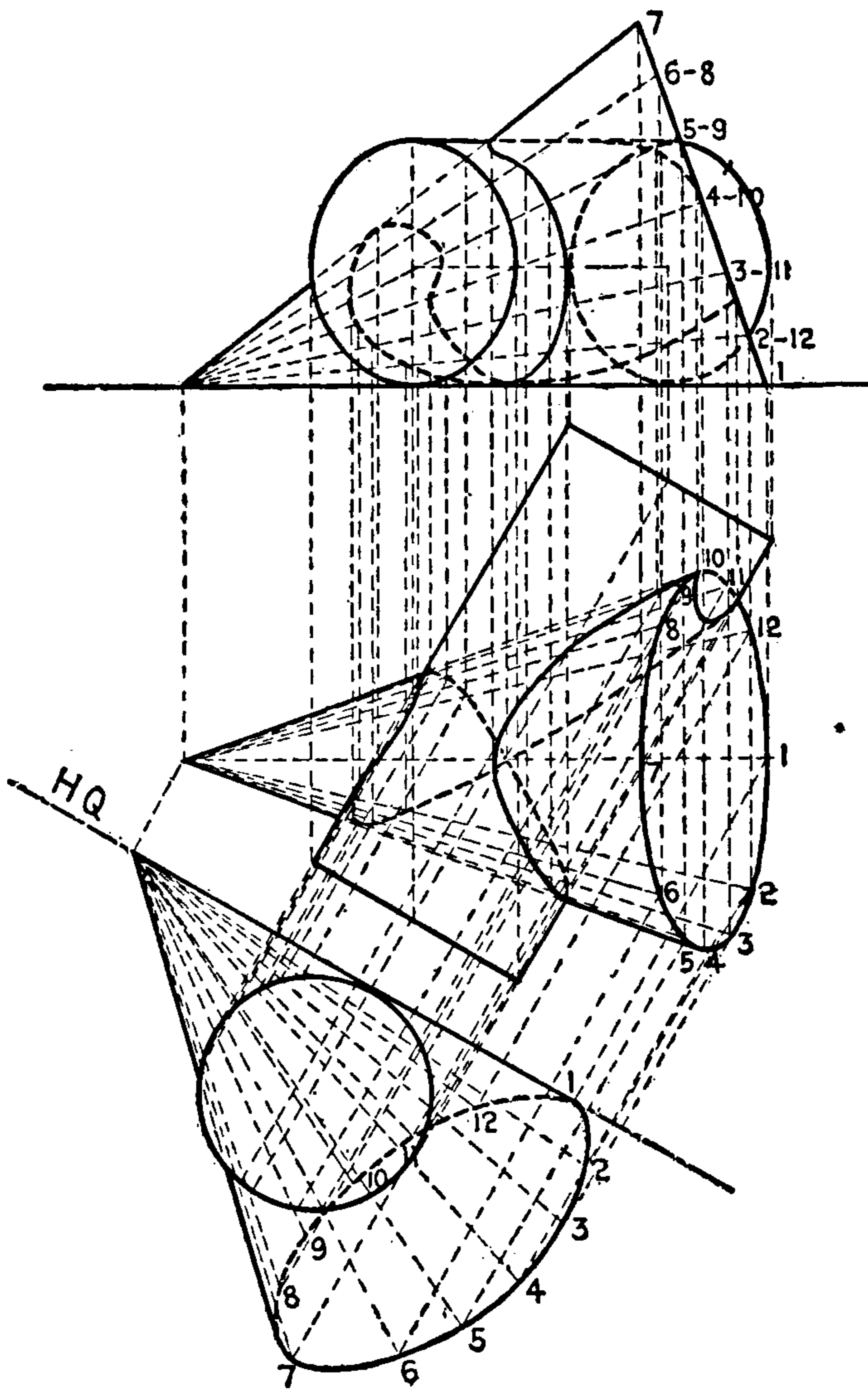


FIG. 166.

faces on a plane perpendicular to the axis of the cylinder. The specimen solution given here is for such a condition.

**Construction.**—See Fig. 166.

1. Project the cone and cylinder on Q, perpendicular to the axis of the cylinder.
2. The various elements, 1, 2, etc., of the cone will intersect the circle in their piercing points with the surface of the cylinder.

3. Project these points back to the  $H$ - and  $V$ -projections of the elements, and draw the curve through them.

Note.—Draw elements of the cone only.

Hint.—To avoid confusion, number your points and elements, because the following out of the correct order of these points is often very difficult.

245. Problem 89.—To find the intersection of any two surfaces of revolution whose axes intersect.

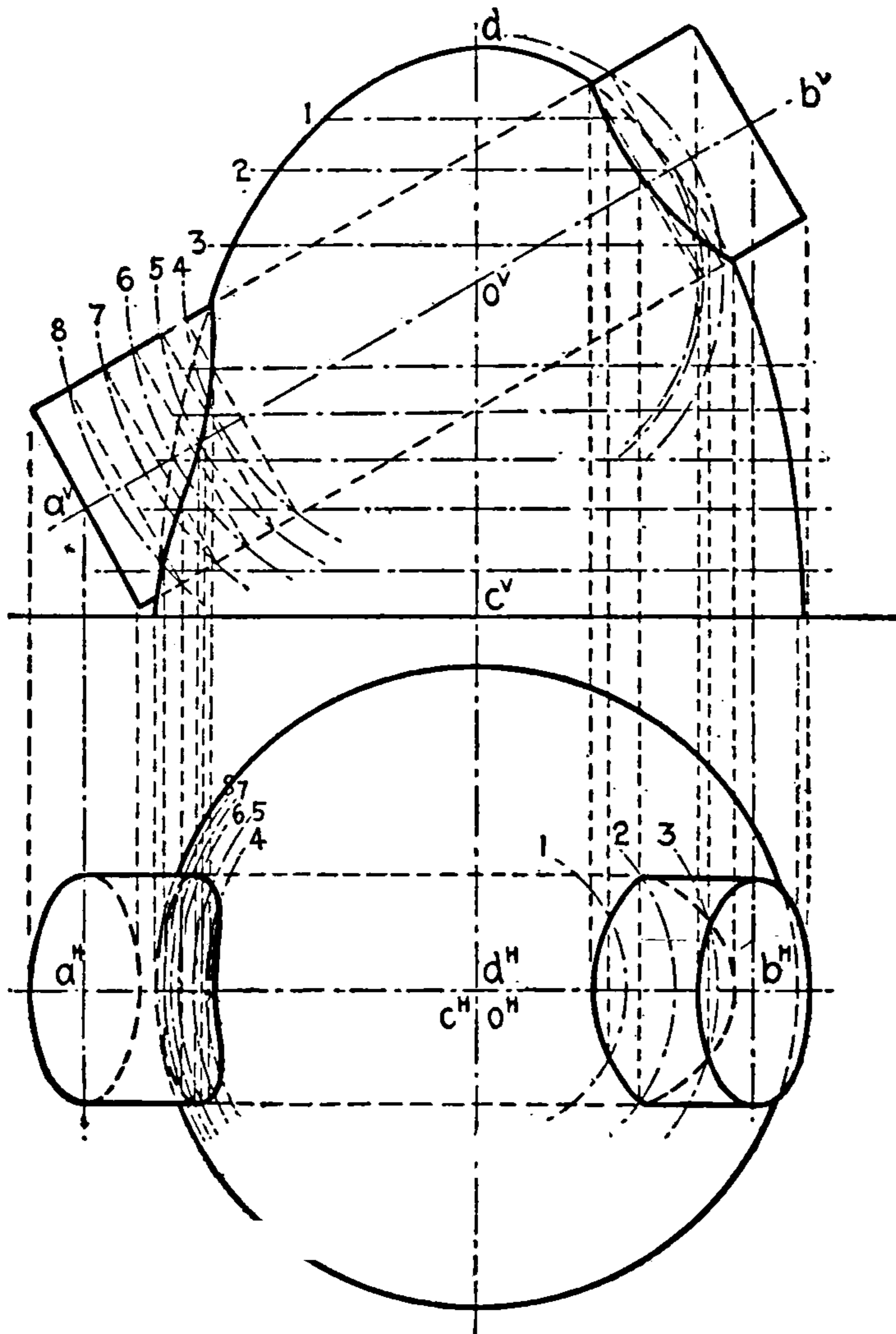


FIG. 167.

Analysis.—1. At the intersection of the axes as a center draw the projections of a number of concentric spheres.

2. These spheres will cut circles from the two surfaces.

3. The intersections of the proper circles will yield points on the required intersection.



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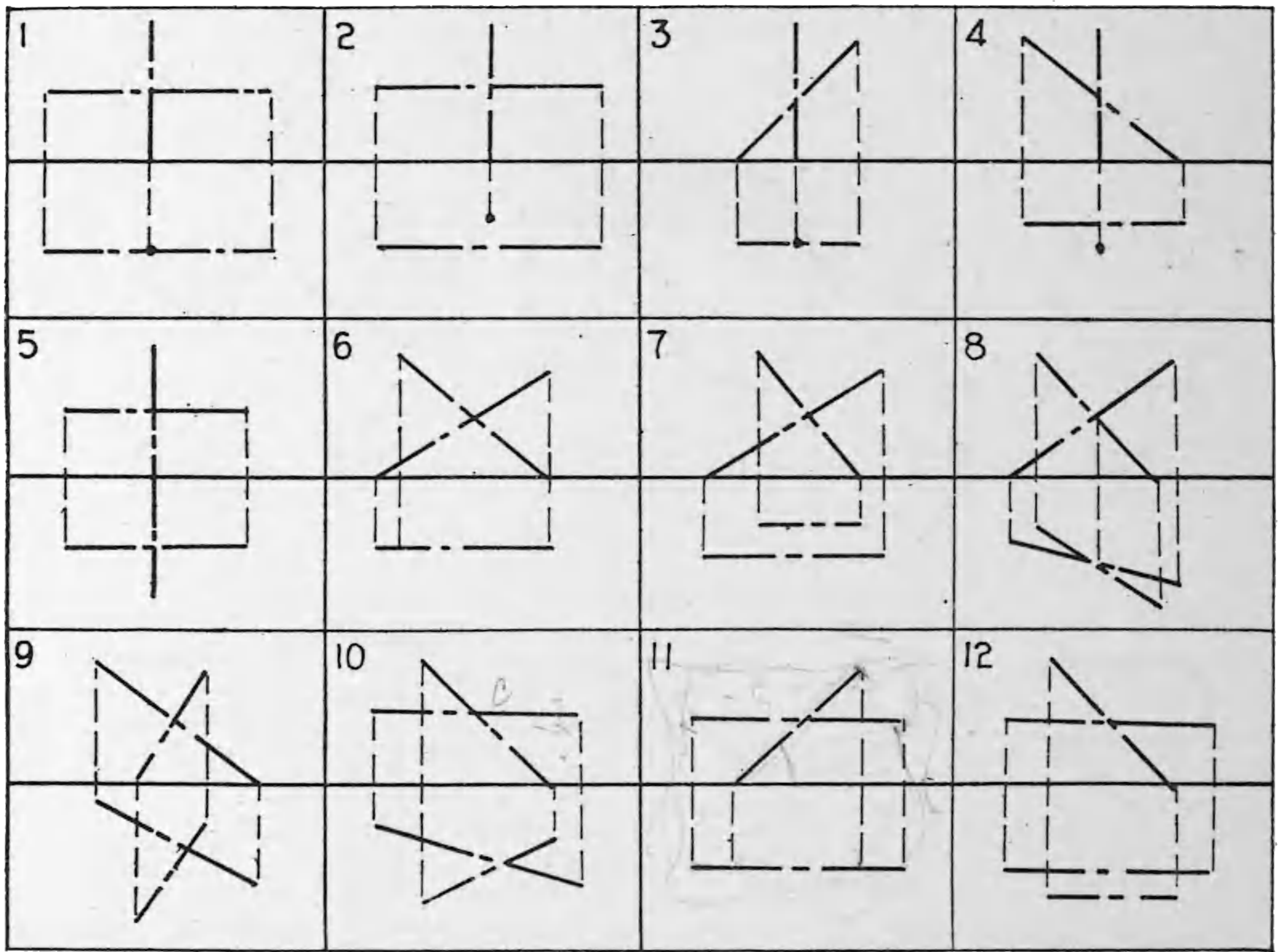
nearly all such problems are given in this position, and the difficult positions will hardly ever be encountered.

246.

EXERCISES

Directions.—In making up the data for the exercises from the following layouts:

1. State what each surface is to be, whether cone, sphere, prism, torus, etc.
2. Give one of the combinations of axes.
3. Give the base or generator of each surface. (Graphic Layout No. 72.)
4. State whether the base is oblique or perpendicular to the axis, if necessary.
5. Give the altitude, if necessary, 2 in., 2½ in., 3 in.



Graphic Layout No. 73.—Axes for intersecting solids. (May be located in *I* or *III*.)

Data	Specimen Exercise
1. Surfaces <u>A</u> , <u>B</u> .	A-Prolate Spheroid, B-Oblique Cone.
2. Axes <u>1, 11</u> .	No. 3.
3. Base or Generator, <u>1</u> , <u>2</u> .	No. 10. <span style="float: right;">No. 2.</span>
4. Oblique or Perpendicular <u>P</u> ,	Perpendicular. Base in <i>H</i> .
5. Altitude <u>3</u> , <u>2½</u> .	— <span style="float: right;">2½ in.</span>

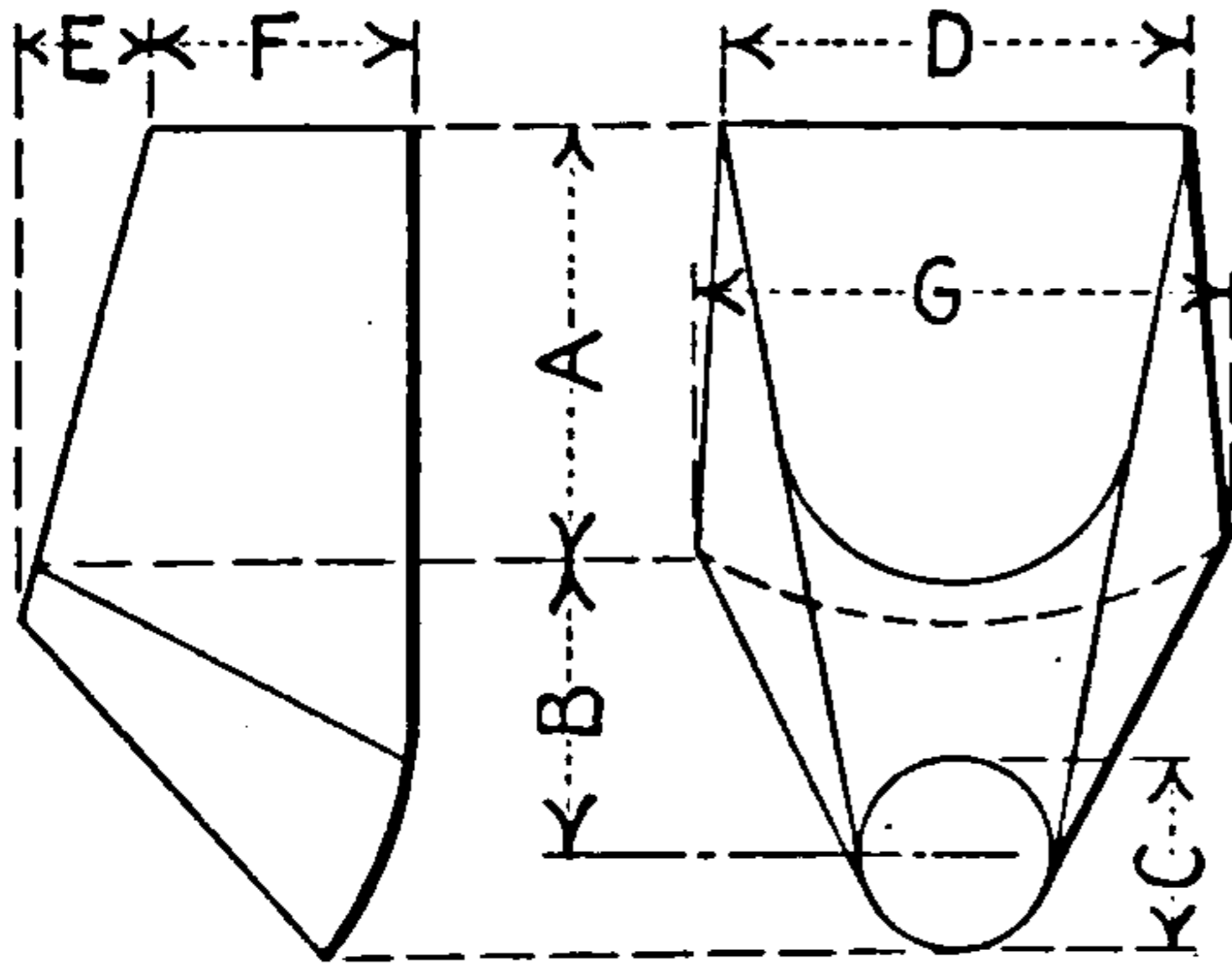
919. Draw the line of intersection between solids A and B, as specified in the data.

920. Develop the surface of solid A, as given in Ex. 919.

921. Develop the surface of solid B, as given in Ex. 919.

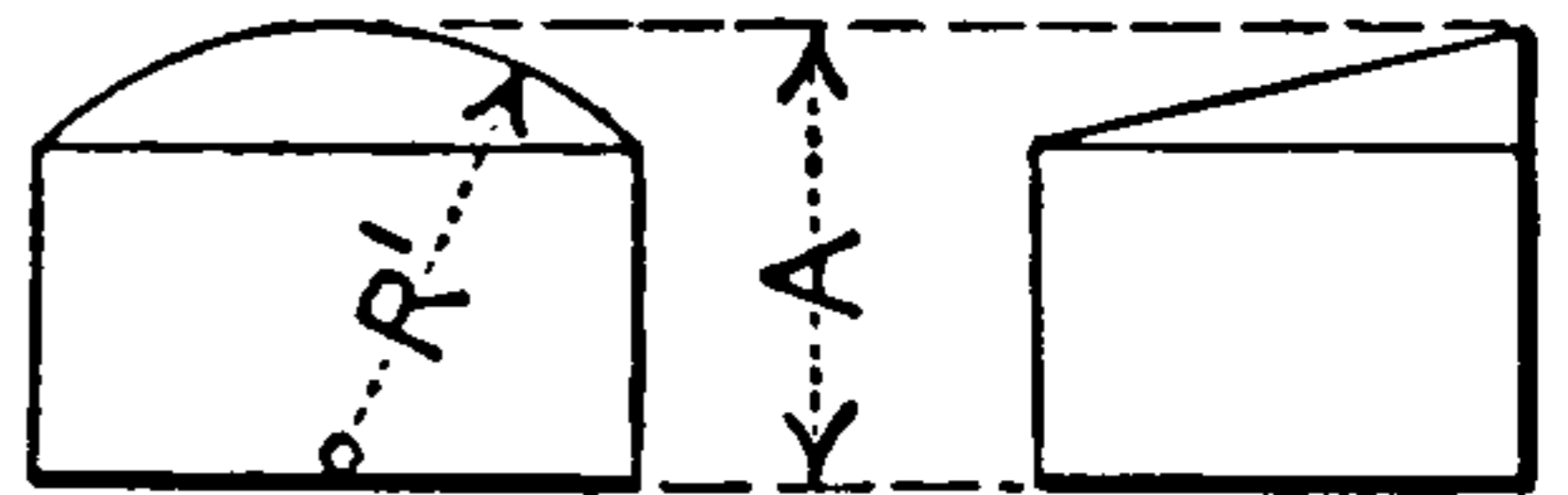
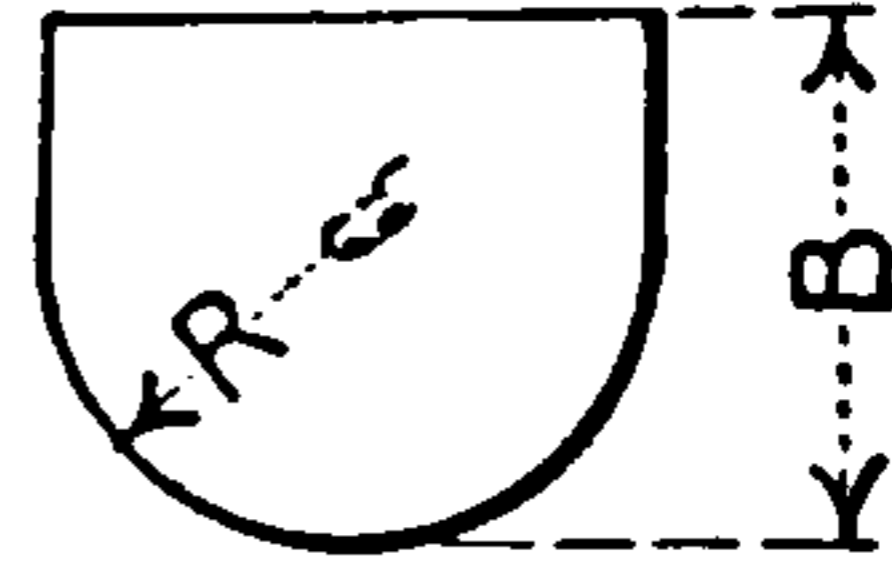
PRACTICAL EXERCISES IN VARIOUS SURFACES

922. Draw the line of intersection between the 3-in. sphere, center at  $O(3 + 1\frac{1}{2} - 2)$  and a  $1\frac{1}{2}$ -in. circular cylinder, axis  $A(1 + 1\frac{1}{2} - 2)$   $B(4 + 1\frac{1}{2} - \frac{1}{2})$ .



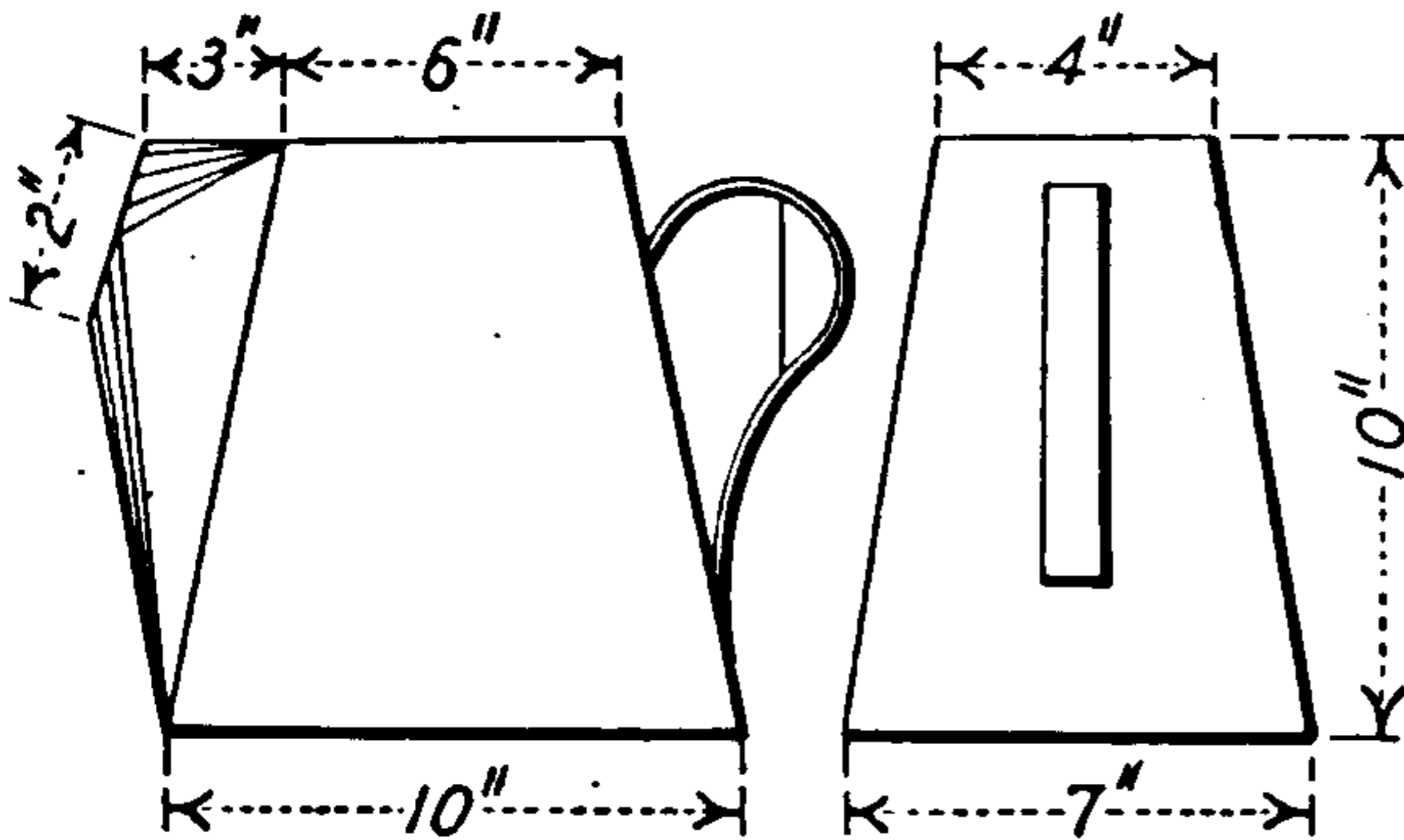
	A	B	C	D	E-F	G
1	20	20	8	12	8	16
2	30	28	9	14	9	18
3	40	36	10	16	12	21
4	50	40	12	20	14	28

Loading Skip for Mixer

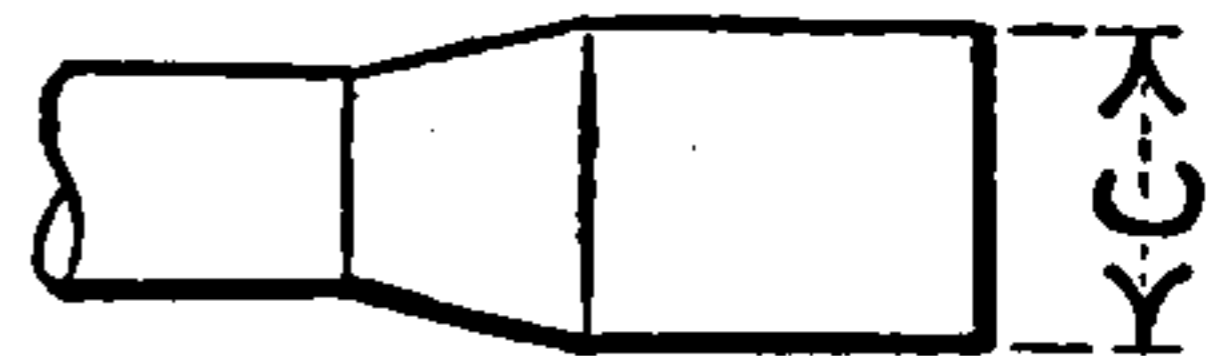
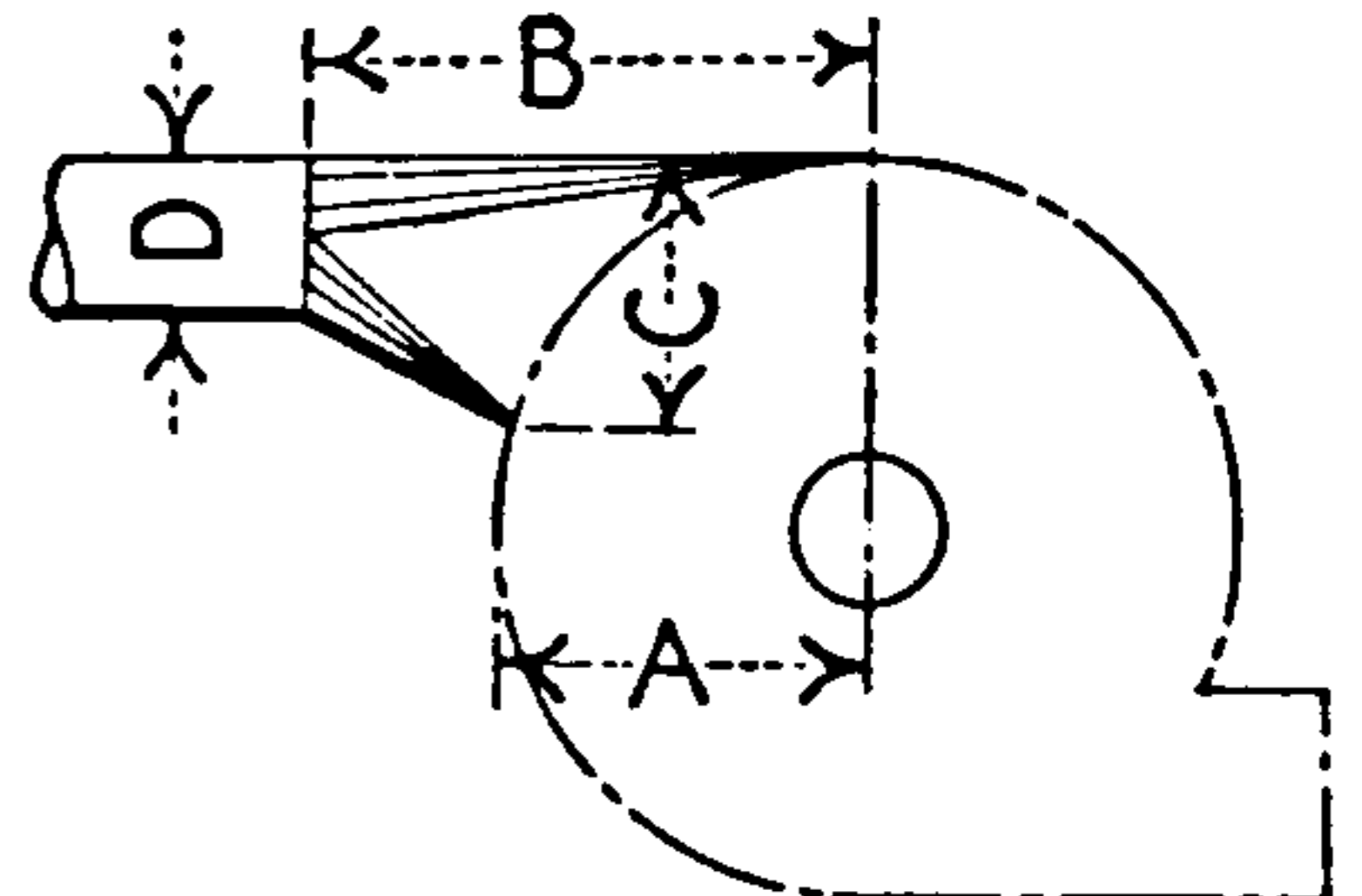


	A	B	R	R'
1	9	9	4 1/2	8
2	12	12	6	10
3	16	16	8	14
4	20	20	10	18

Cylindroidal Hood

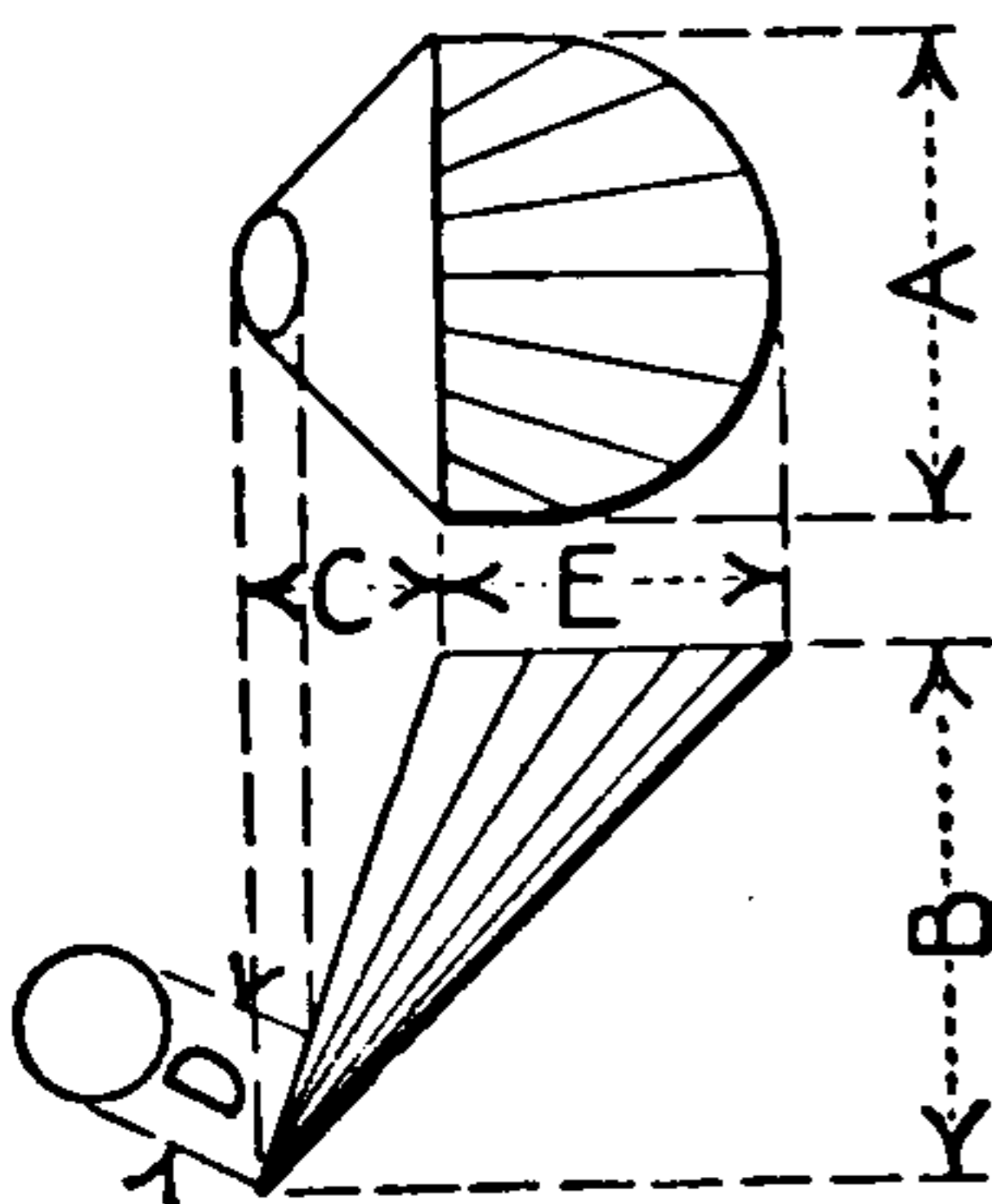


Coffee Receiver



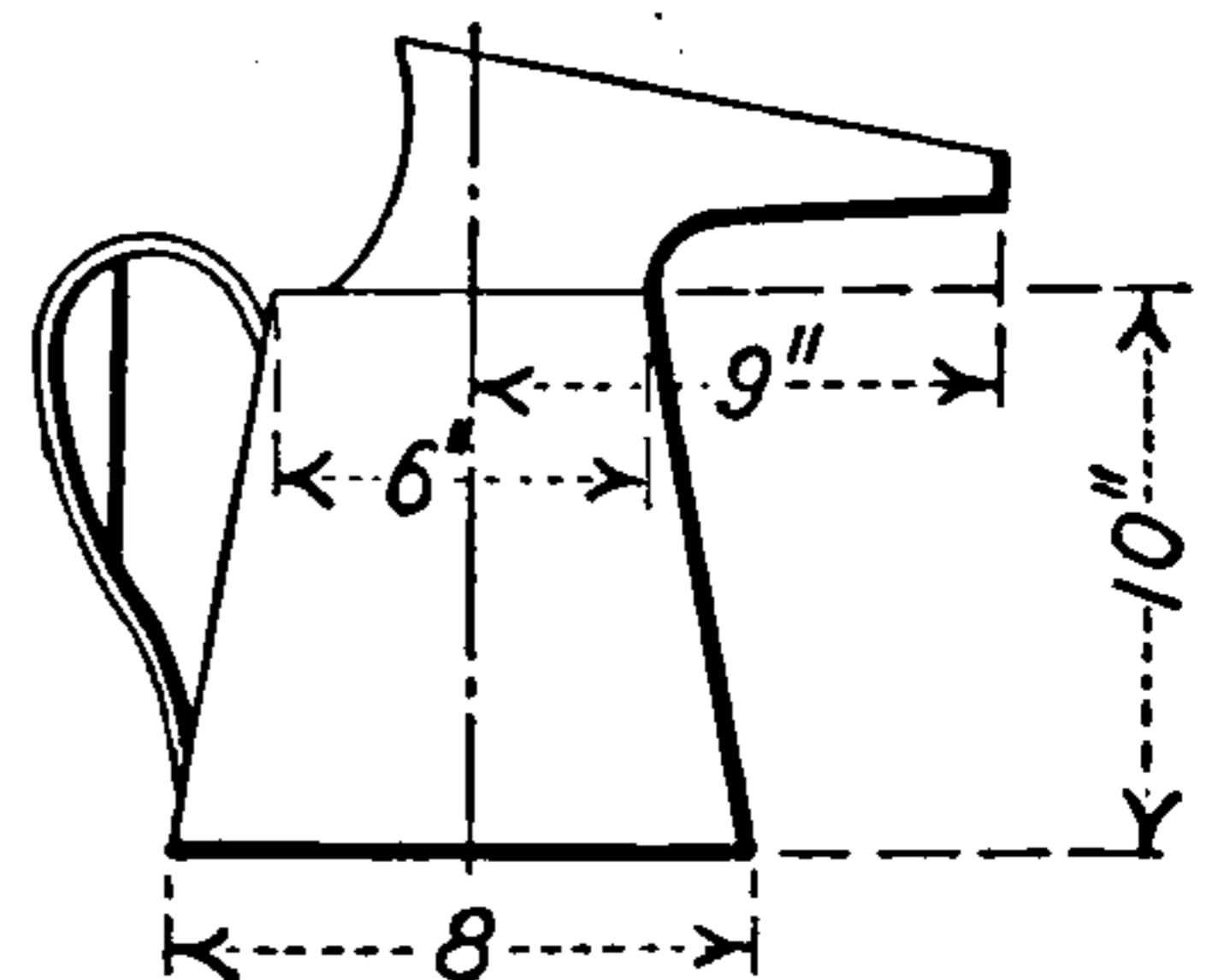
	A	B	C	D
1	20	13	14	6
2	30	50	20	8
3	40	64	30	9
4	48	72	40	10
5	60	96	44	14

Fan Transition Piece



	A	B	C	D	E
1	18	40	10	10	16
2	20	40	10	10	16
3	24	50	12	12	20
4	30	60	15	16	26

Concrete Hopper

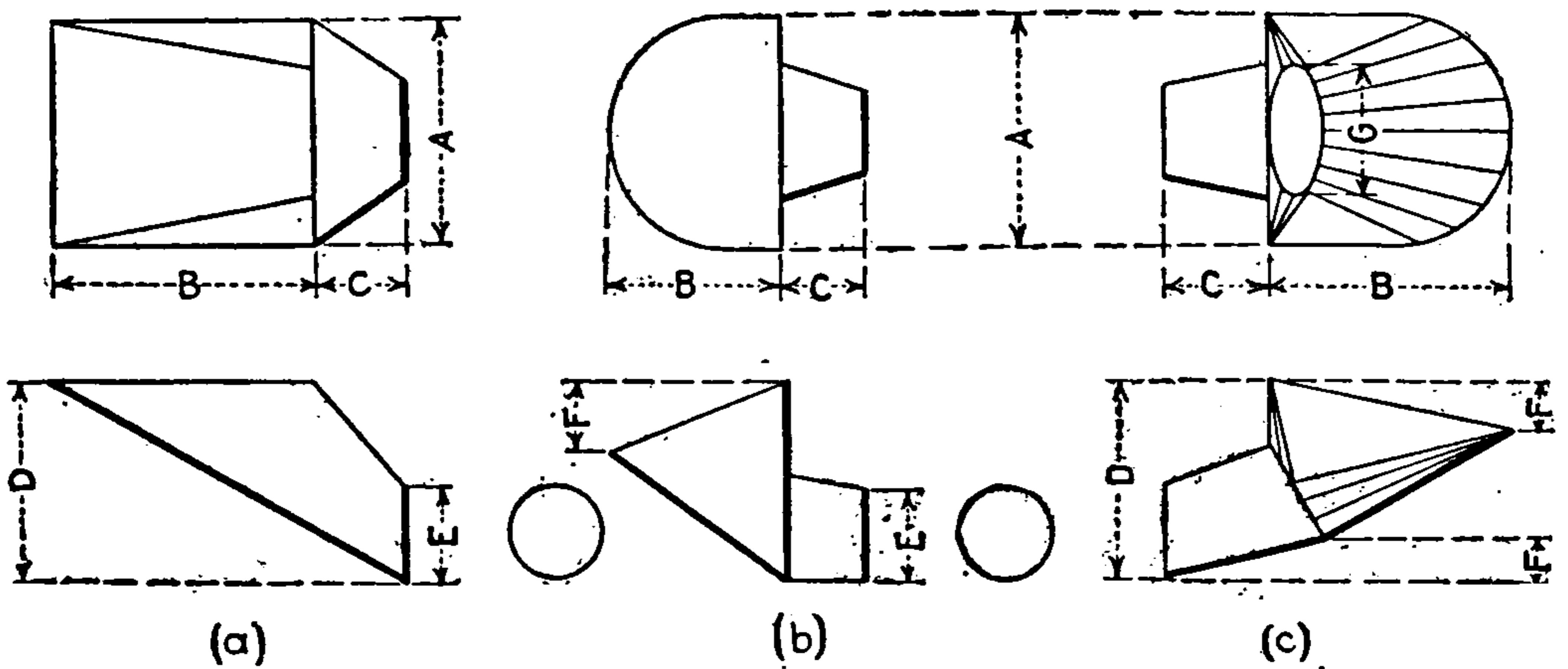


Gasoline Measure

FIG. 168.—Exercises in various surfaces.



923. Lay out the patterns for the Loading Skip for Mixer, No.—, Fig. 168.



	A	B	C	D	E	F	G
1	20	24	10	36	8	12	11
2	30	30	16	45	10	12	15
3	40	44	20	60	12	12	18
4	60	56	30	96	16	16	20
5	72	80	30	108	20	18	28

Concrete Mixer Hopper

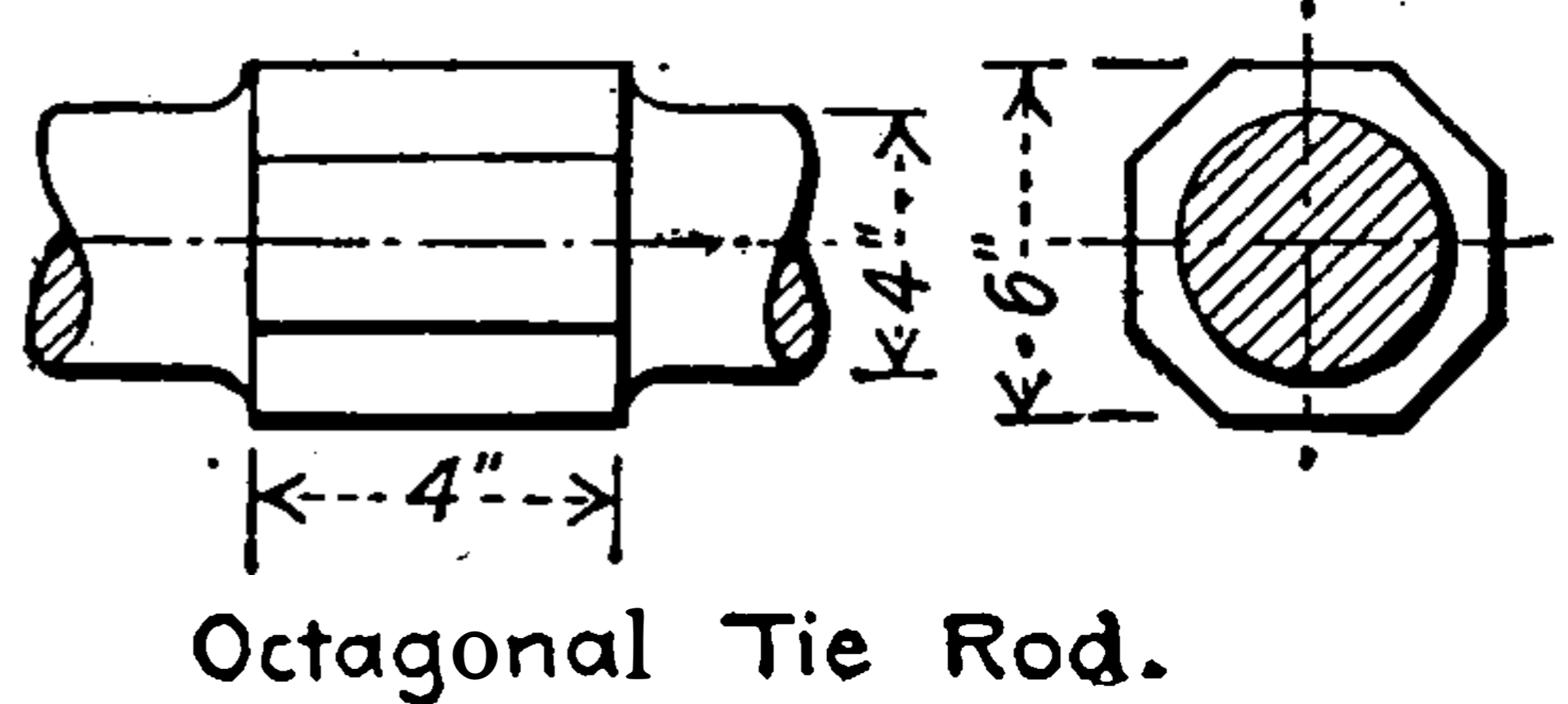
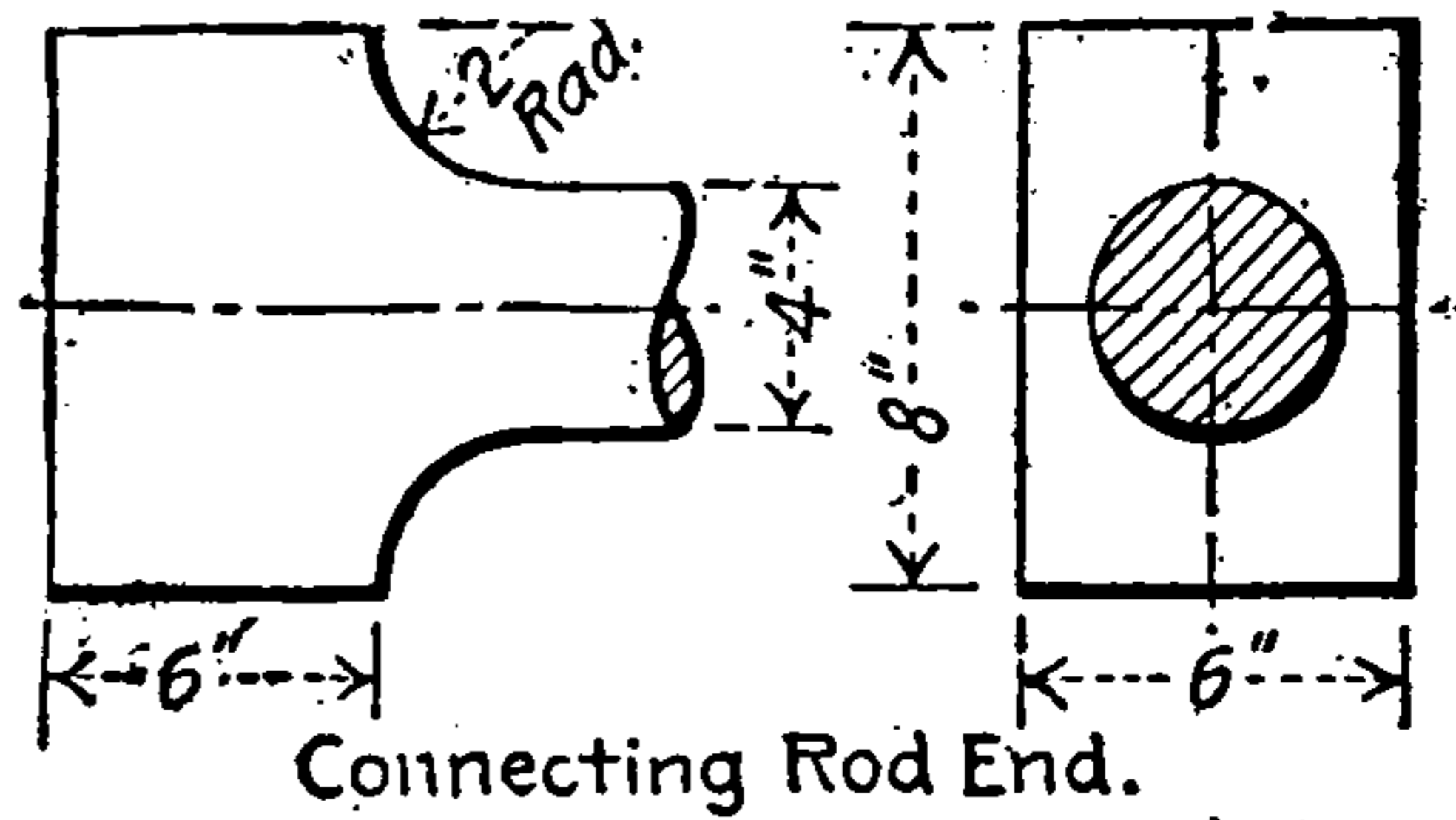
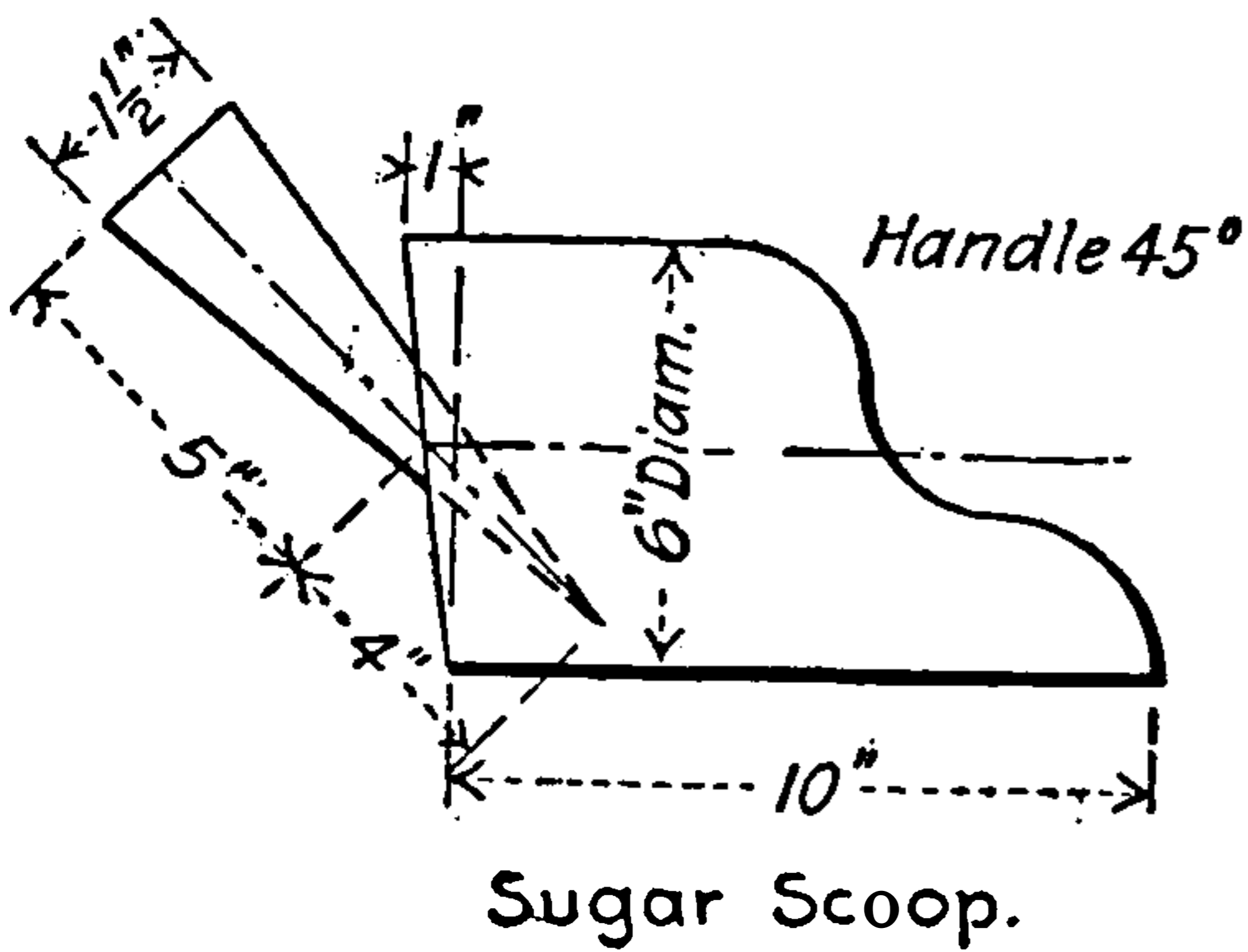
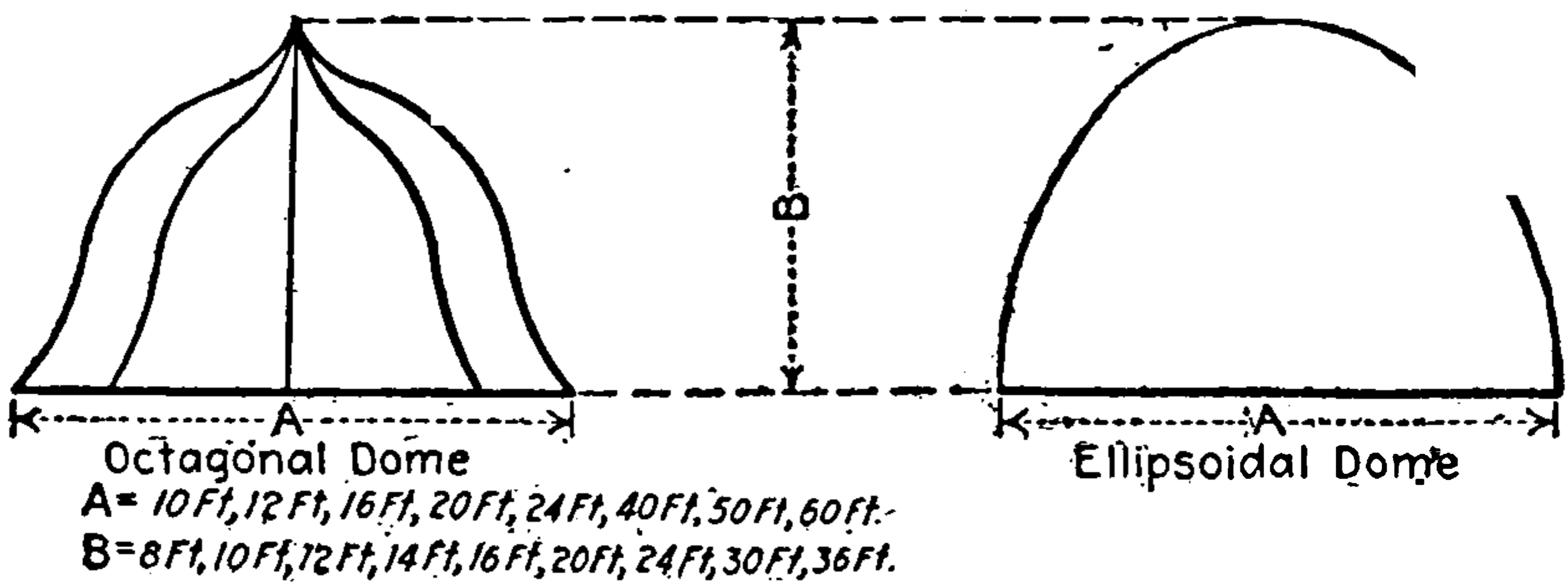


FIG. 169.—Practical exercises.

924. Lay out the patterns for the Cylindroidal Hood, No.—, Fig. 168.

925. Lay out the patterns for the Receiver for Coffee Grinder, Fig. 168.



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930. Develop the surface of the Ellipsoidal Dome, Fig. 169. Let A = —, and B = —.
931. Lay out the patterns of the Sugar Scoop, Fig. 169.
932. Draw the intersection of surfaces in the Connecting Rod End, Fig. 169.
933. Draw the line of intersection of the surfaces in the Octagonal Tie Rod, Fig. 169.
934. Lay out the patterns for the Two-way Boiler Breeching, Fig. 170. Supply the necessary dimensions.
935. Lay out the patterns for the Smoke Connector for Two Boilers, Fig. 170.
936. Lay out the patterns for the Oblong Outlet, Fig. 170.
937. Lay out the patterns for the Kewaunee Flue Attachment, Fig. 170.
938. Lay out the patterns for the Breeching for Kewaunee Boilers, Fig. 170.
939. Lay out the patterns for the Tudor Boiler Breeching, Fig. 170. Supply the necessary dimensions.
940. Lay out the patterns for the Oval Three-way Breeching, Fig. 170. Supply the necessary dimensions.
941. Lay out the patterns for the Ames Boiler Stack Connection, Fig. 170.

## CHAPTER X

### PICTORIAL PROJECTION PERSPECTIVE

**247.** One of the most important and interesting applications of Descriptive Geometry is the art of **Perspective**, or, as it is sometimes called, **Conical Projection**. As was pointed out in the beginning of the work, Perspective Projection differs from Orthographic in that it represents the object by an image which is practically identical with that formed on the retina of the eye of the observer. The title, Conical Projection, is a correct one, since the picture is made by the projection of rays that converge in a point.

Numerous methods are given by various writers of text-books, and educators make use of all of them. The method presented here is offered, for two reasons: (1) its extreme simplicity and reasonableness; and (2) because it is a pure application of Descriptive Geometry, and, being such, it can be given to the student already grounded in the fundamentals without much detailed illustration and complicated directions.

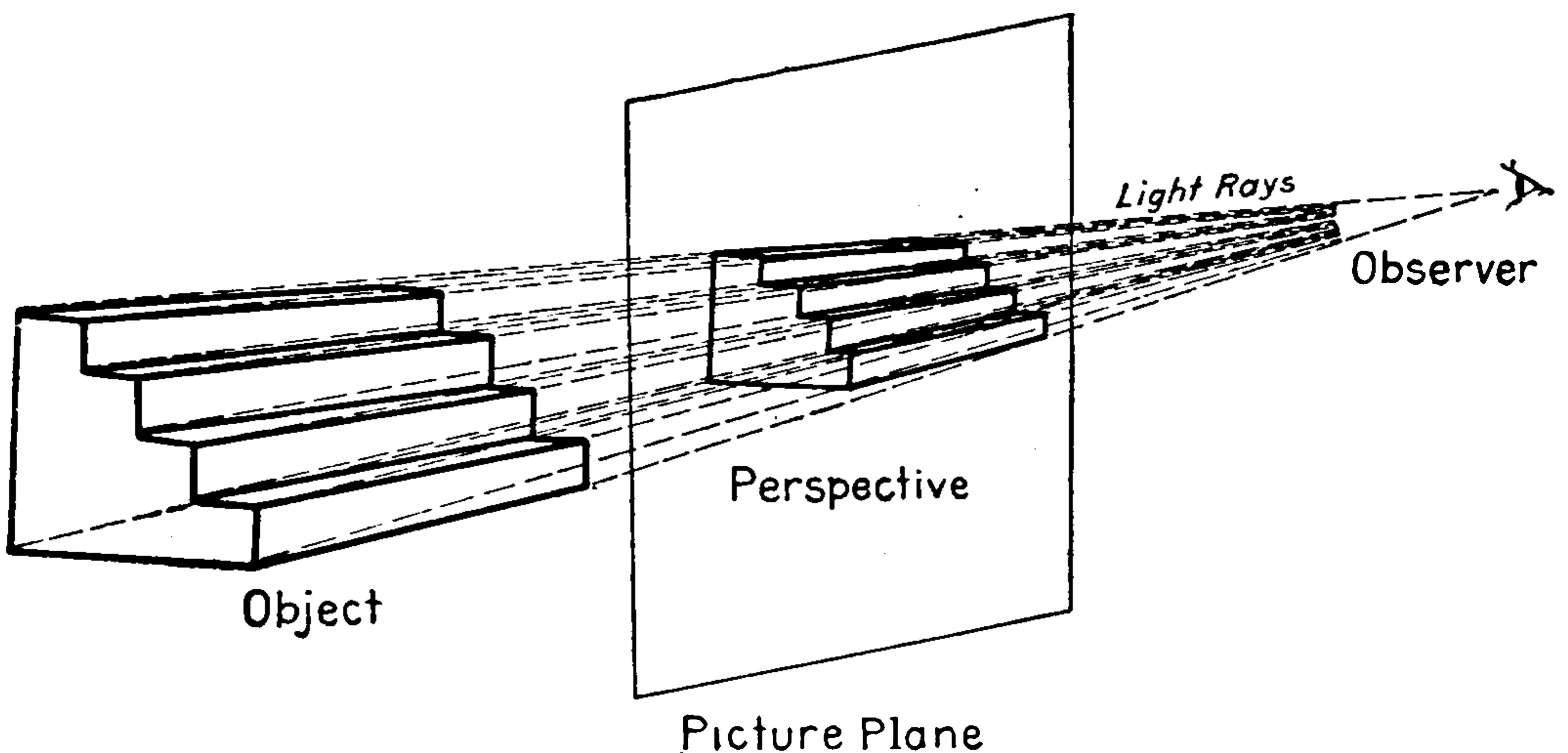


FIG. 171.—How the pictorial image is made.

**248**

### CONICAL PROJECTION

If a plane, called the **Picture Plane**, be so placed as to intersect the rays coming from an object to the eye, the piercing points of

the rays will yield a perfect visual image of the object. This is the exact process of the camera, so that a photograph is an approximately perfect perspective.

### 249. The Construction of a Perspective from a Working Drawing.

In Fig. 172 a set of steps is shown in third angle projection, somewhat below the horizon.

Note.—The horizon should not be confused with  $GL$ , as it has not the same function.

The point of view for the perspective may be taken from any convenient position, but it is customary to view the object “from

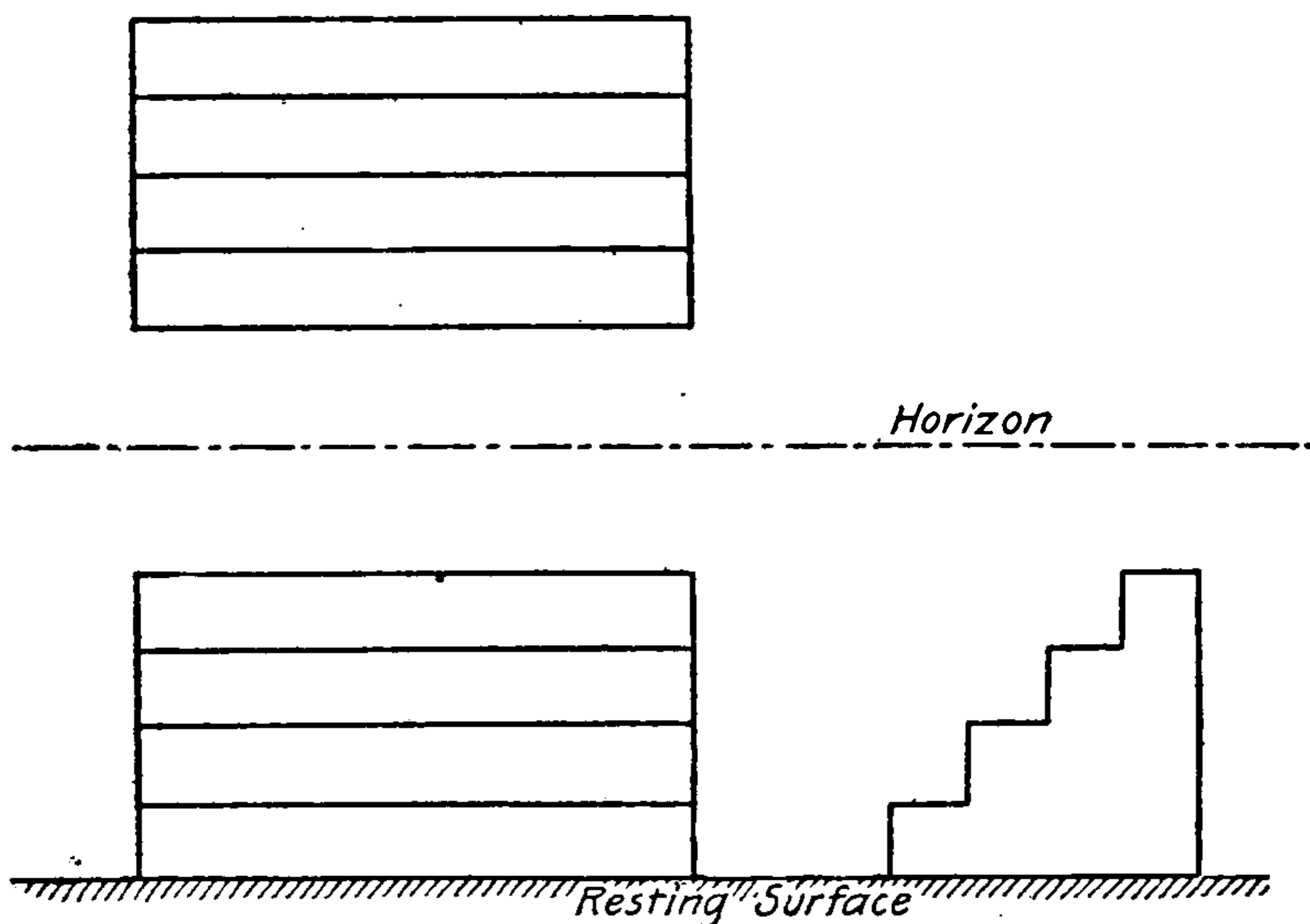


FIG. 172.—Object in orthographic projection.

an angle,’ so as to show two sides. This takes away the uncompromising appearance of the front elevation, as it appears in the working drawings, and offers opportunity to the draftsman of taking the most interesting view. In making interior views of buildings, and in stage scenery, etc., the point of sight is often taken directly in front of the scene.

To draw the picture of the steps, place the plan and elevation (or side view, if preferred) in a convenient position on the drawing, with the horizon at its proper elevation; see Fig. 173. Locate the point of sight,  $s^H$ ,  $s^V$ , at a suitable position, where both front and side are nicely visible. Note that  $s^V$  is always on the horizon. The picture plane, HP, is placed so as to give the desired scale, and should be about perpendicular to the central line of sight. The scale is determined by the ratio of the distances of the object



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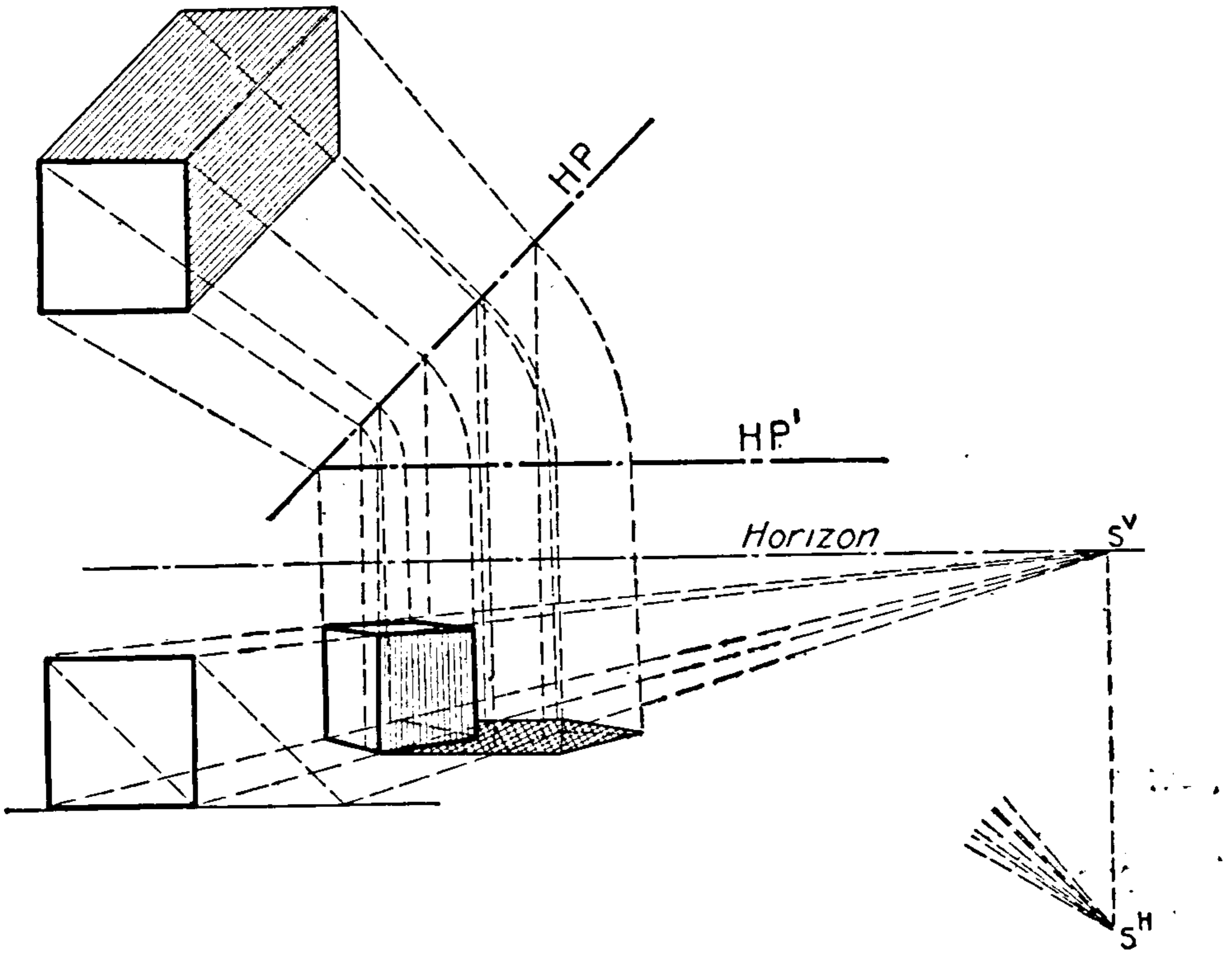
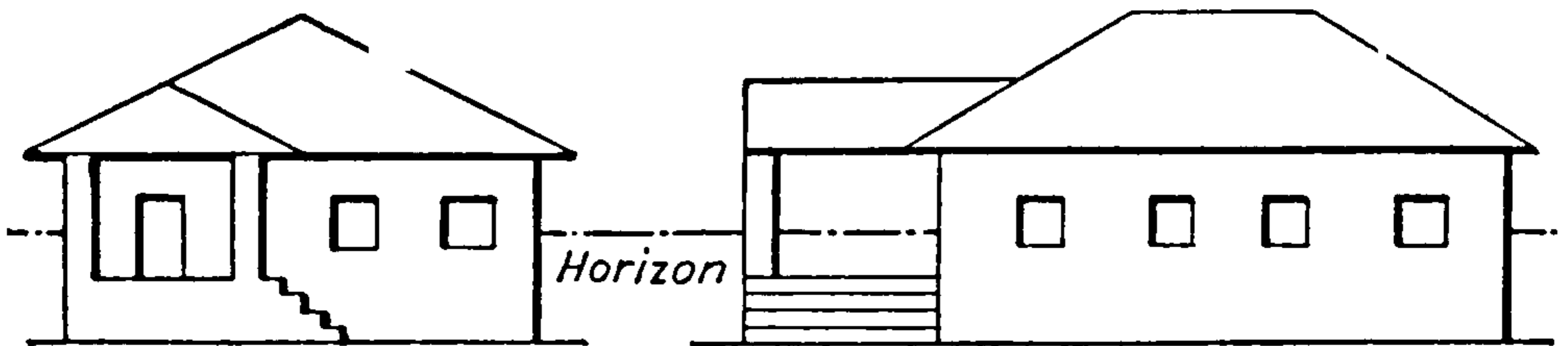
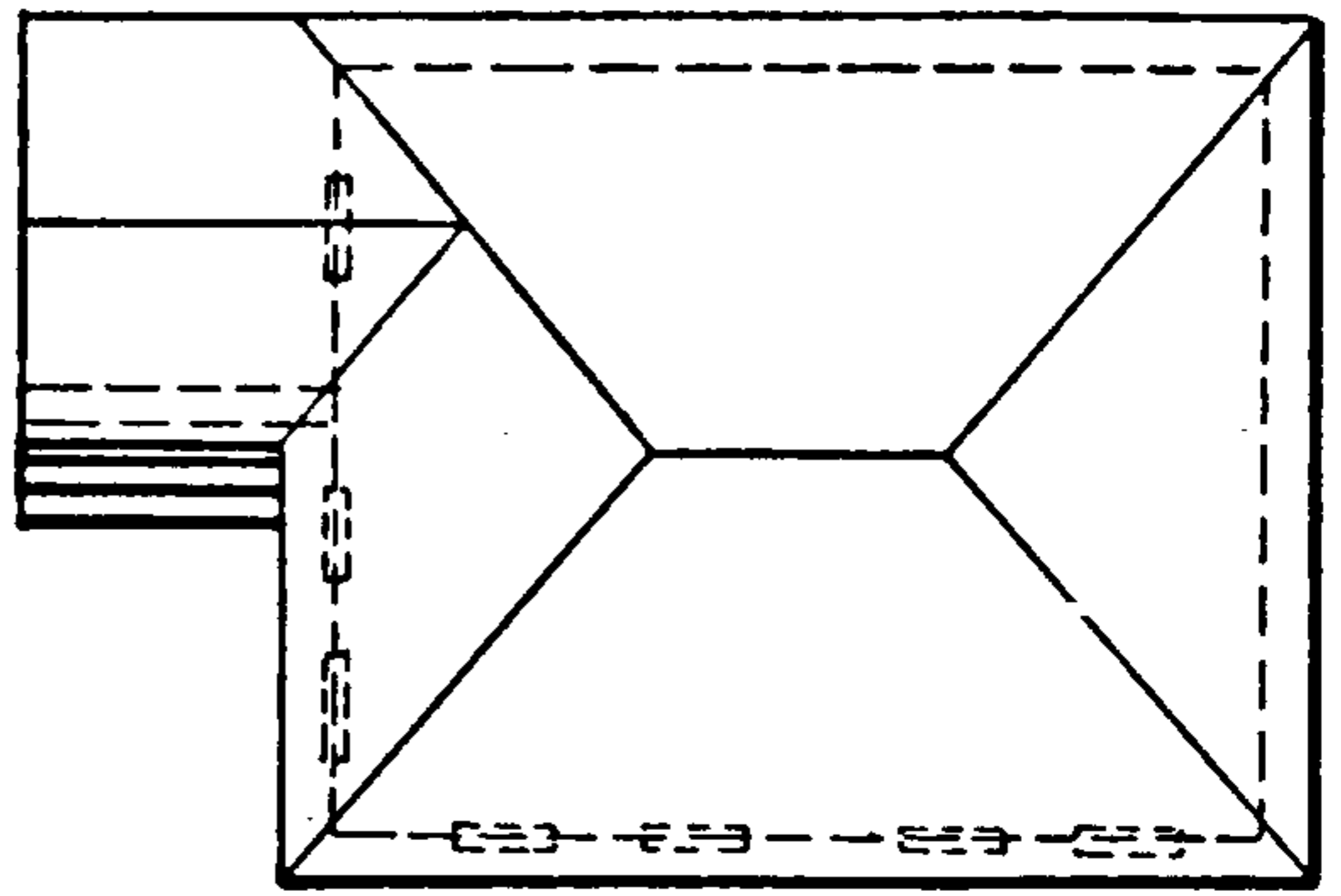


FIG. 174.—Perspective of an object, showing its shade and shadow.



Projections of a Bungalow

FIG. 175.

As an illustration of the application to more complicated objects, a bungalow is shown in Fig. 176, drawn in perspective by

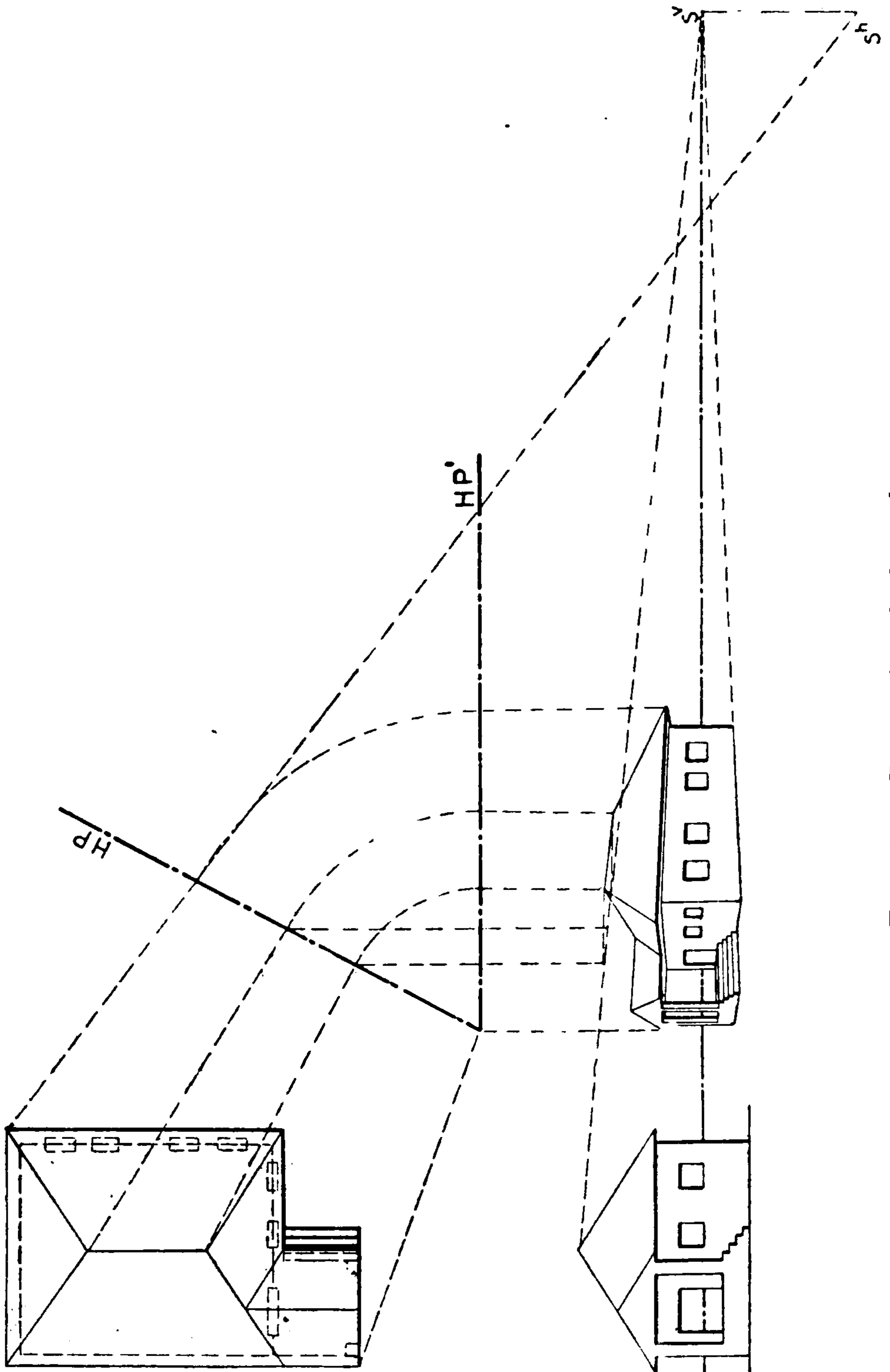


FIG. 176.—Perspective of a bungalow.

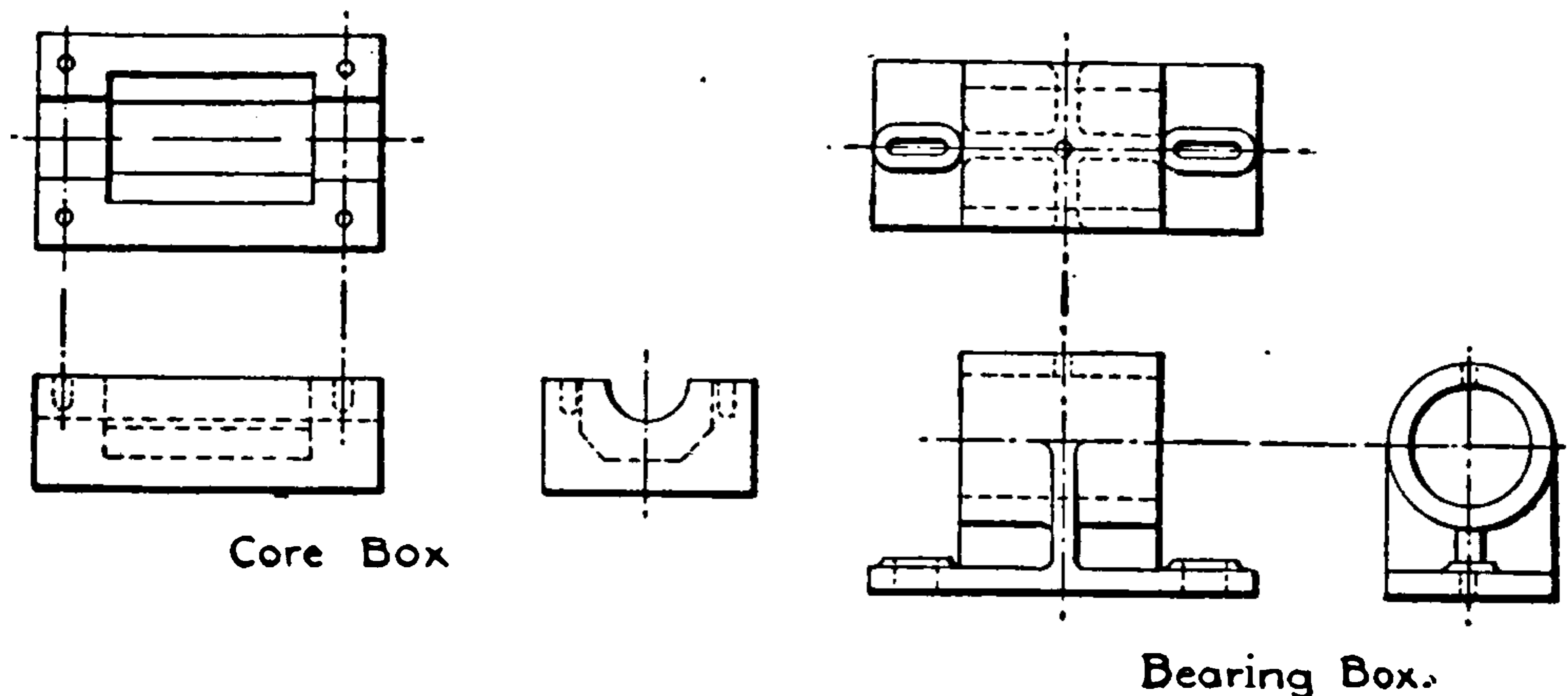
conical projection from the working drawing as given in Fig. 175. In Fig. 176 only a few construction lines are shown.



251.

## EXERCISES

Draw the perspective of the following objects, to the scale designated by the instructor, one-half, three-quarters, full, or double the scale of the working drawings. Make the working drawings as large as the space will allow



Graphic Layout No. 74.

to give plenty of room for the perspective and its construction. Place all small objects below the horizon.

942.  $1\frac{1}{2}$ -in. cube. Scale —.

Graphic Layout No. 75.

943. Frustum of a square pyramid, lower base  $1\frac{1}{2}$  in., upper base  $\frac{3}{4}$  in., altitude (base to base) 1 in. Scale —.

944. Wooden box 20 in. long, 12 in. wide, 6 in. deep, walls 1 in. thick, without top. Scale —.

945. Place a  $1\frac{1}{4}$ -in. equilateral triangular prism,  $1\frac{1}{4}$  in. long, on a 1-in.



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projection run in three directions at equal angles ( $120^\circ$ ). Thus, in **Isometric**, the  $90^\circ$  angles of the three systems of parallels of the cube (called the **isometric axes**) radiate at  $120^\circ$  from each other.

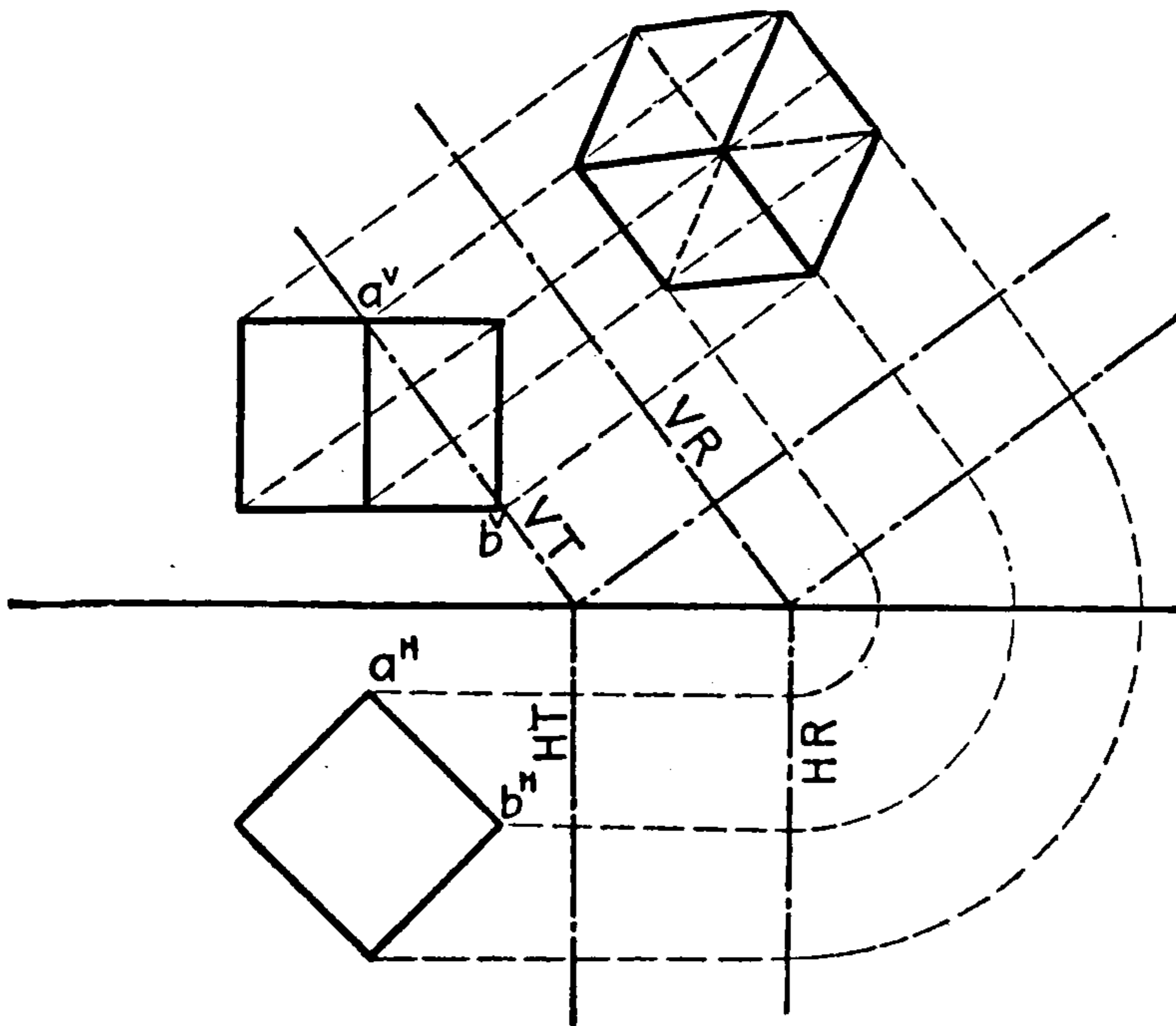


FIG. 177.

### ISOMETRIC DRAWING

**253. Isometric Drawing** differs from isometric projection only in that the latter would require all "isometric lines" to be drawn to a reduced  $\left(\frac{81}{10^3}\right)$  scale. Since most working drawings are themselves drawn to a reduced scale, and the isometric is chiefly for pictorial purposes, this distinction is of no importance, and the isometric lines may be drawn the full length of the working drawing.

### ISOMETRIC AXES

Lay off  $AO$ ,  $BO$ , and  $CO$ , as shown in Fig. 178, radiating at  $120^\circ$ ,  $AO$  and  $CO$  being drawn at  $30^\circ$  from the horizontal. These three lines represent three of the adjacent edges of the cube,  $BO$  a vertical and  $AO$  and  $CO$  horizontal lines at right angles to each other. In isometric all objects are drawn so that the rectangular lines run in these directions. Other lines (non-isometric) are laid off by joining points on the isometric lines, or by erecting ordinates from known points.

**254. Isometric drawing of a cube.**

**Construction.**—Given the cube in Fig. 179.

1. Lay off the isometric axes, as in Fig. 178.

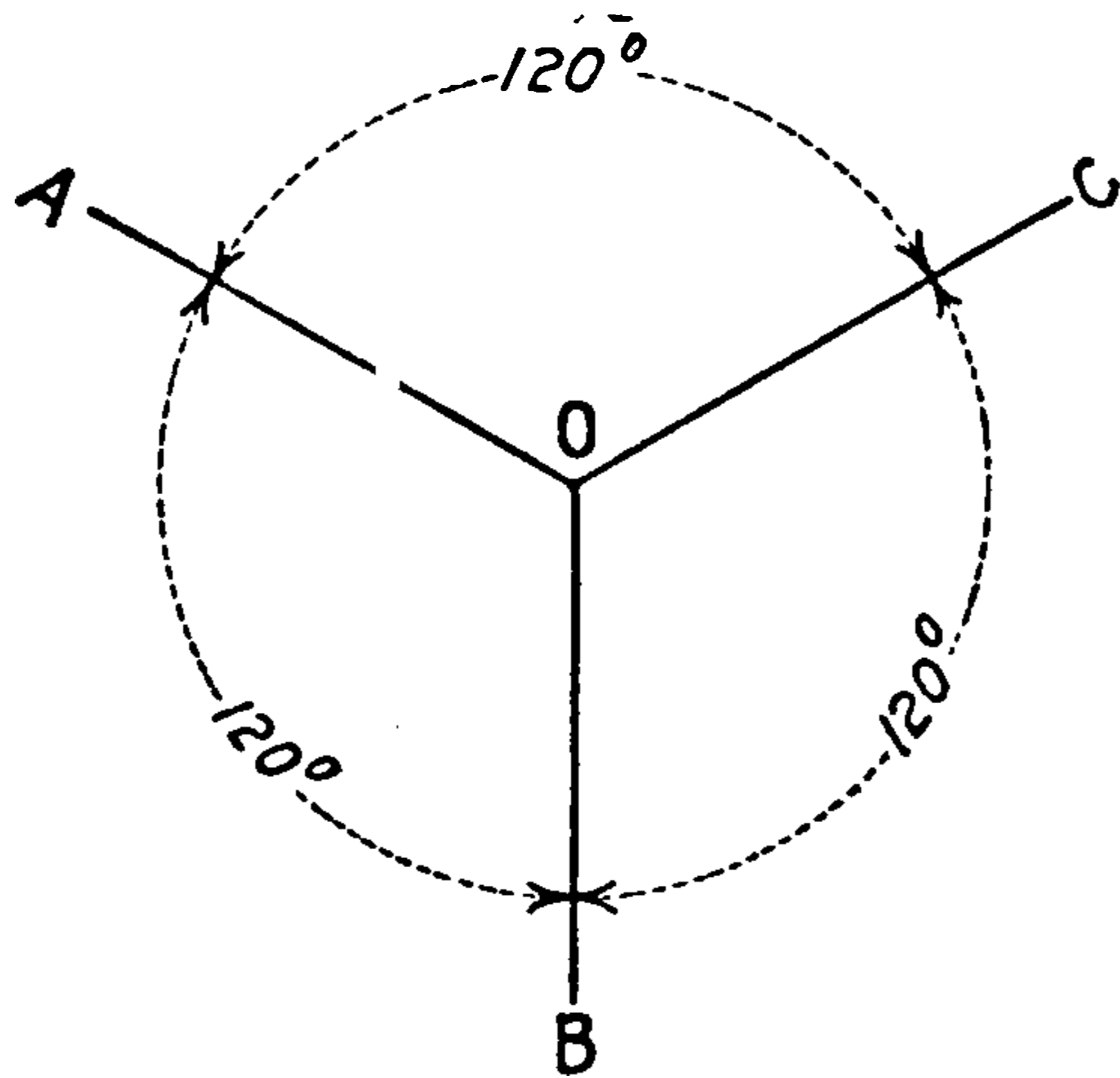
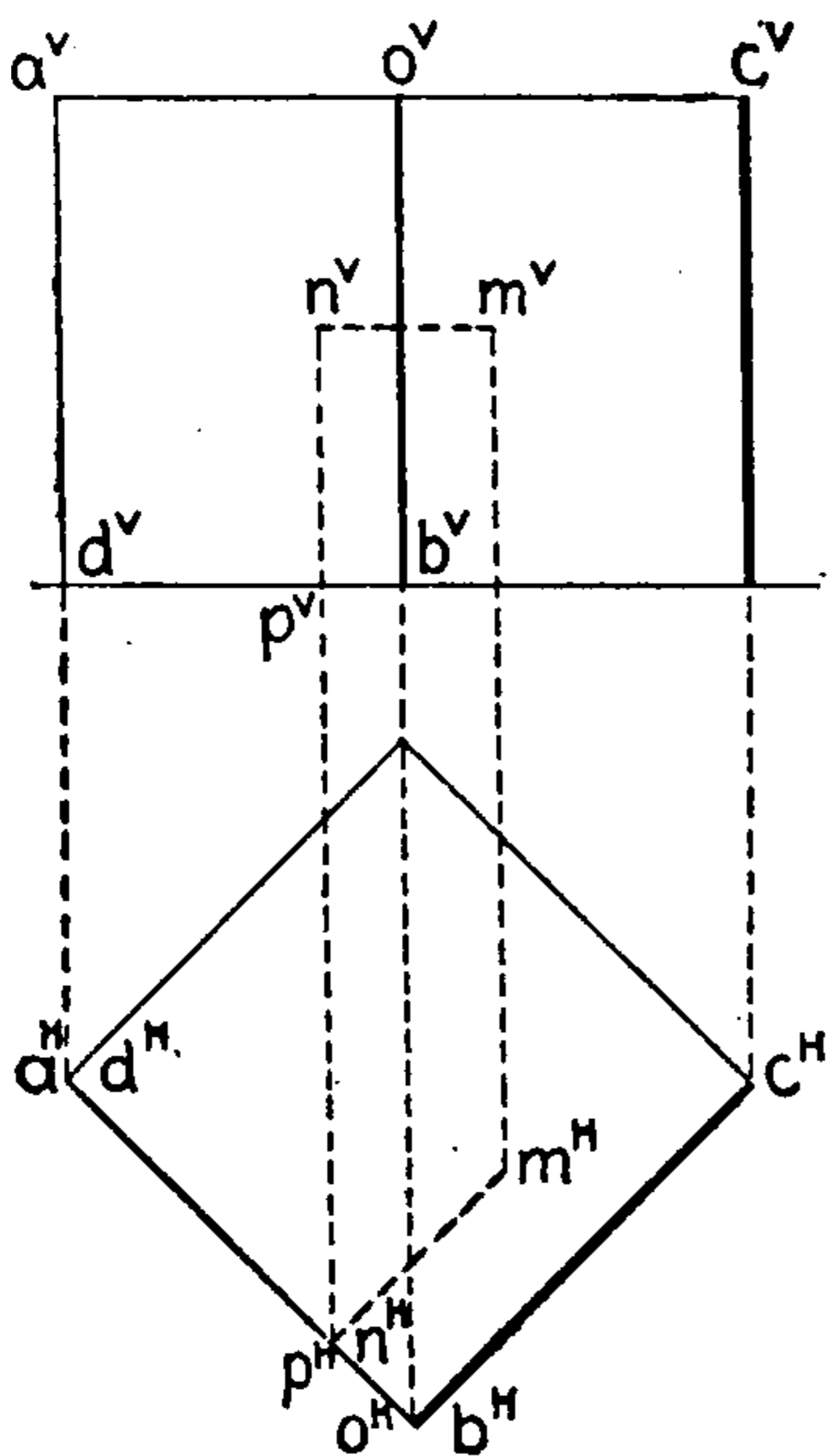


FIG. 178.—Isometric axes.

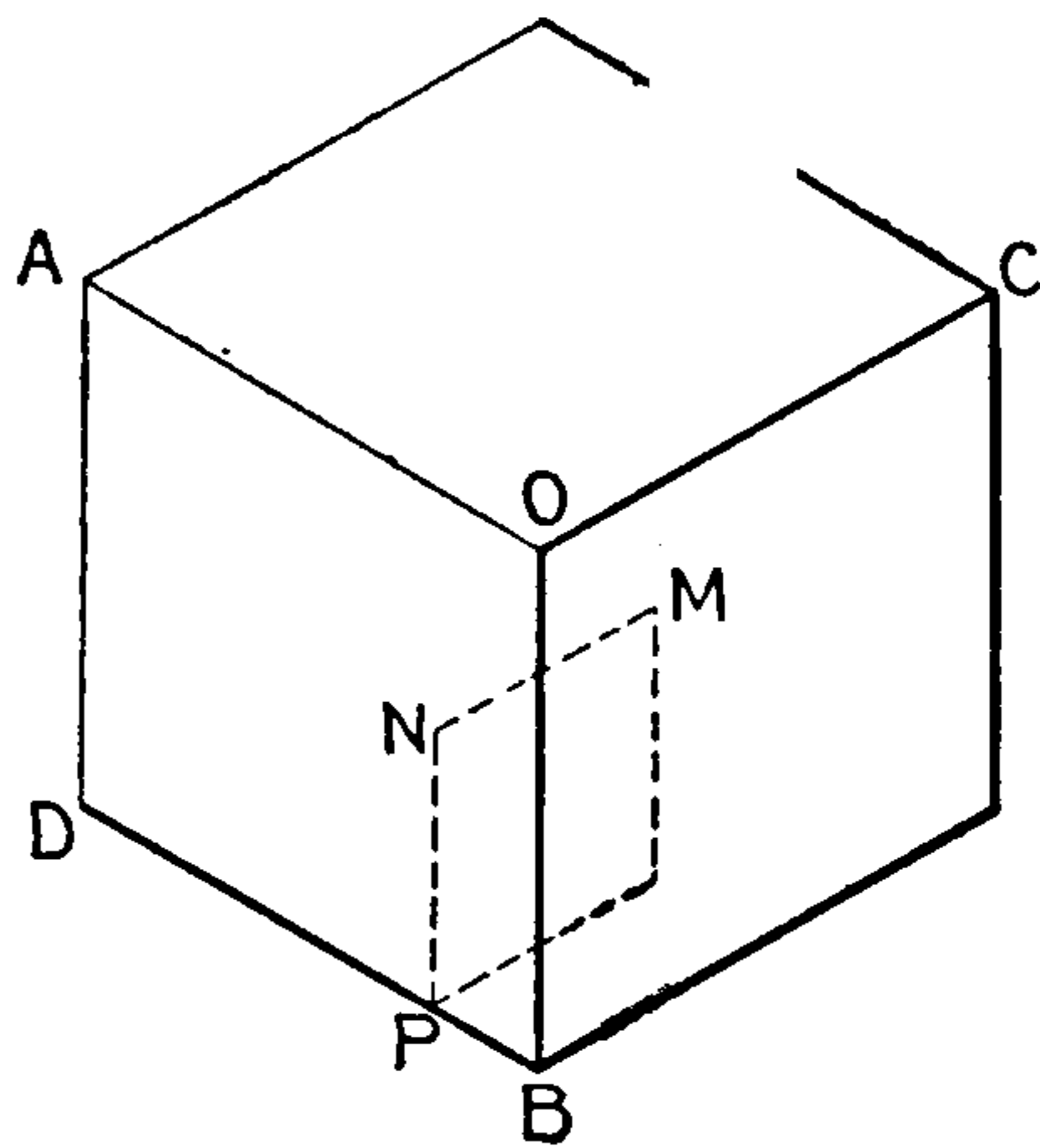
2. Make each edge, AO, BO, CO, equal to the true length of the cube edges.

3. Complete the three parallelograms on these edges.



Orthographic Projection of a Cube.

FIG. 179.



Isometric Drawing of the same Cube.

FIG. 180.

**255. To locate a point isometrically.**

Refer to Figs. 179 and 180. Let M, inside the cube, be the required point, given by its projections  $m^H$ ,  $m^V$ .

1. Draw isometric ordinates, as follows: (a) a perpendicular MN to the face AOBD; (b) a perpendicular NP to the edge DB.

2. Lay off these ordinates in their true length on the proper isometric lines in Fig. 180, thus locating M in its proper place within the cube.

**255. To draw an exact circle.**

**Construction.**—Refer to Figs. 181 and 181a. On the left face of the cube (Fig. 181) is inscribed a circle, with squares in each corner touching the circle at E, G, H, and K.

1. Draw each of these squares in isometric, Fig. 181a.
2. Draw an ellipse in the face of the cube, Fig. 181a, touching  $E'$ ,  $G'$ ,  $H'$ , and  $K'$ , and the center point of each of the four edges.

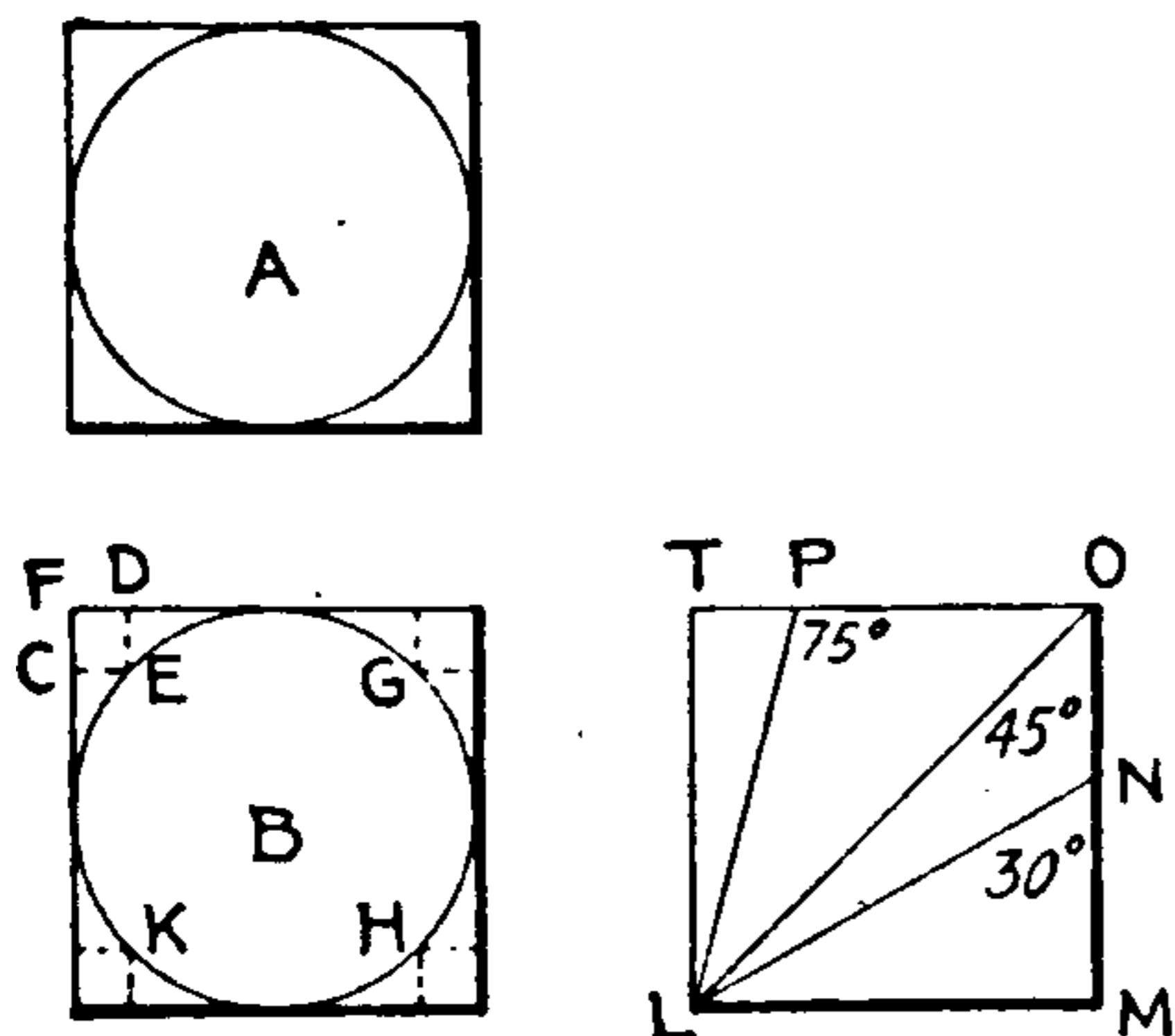


FIG. 181.

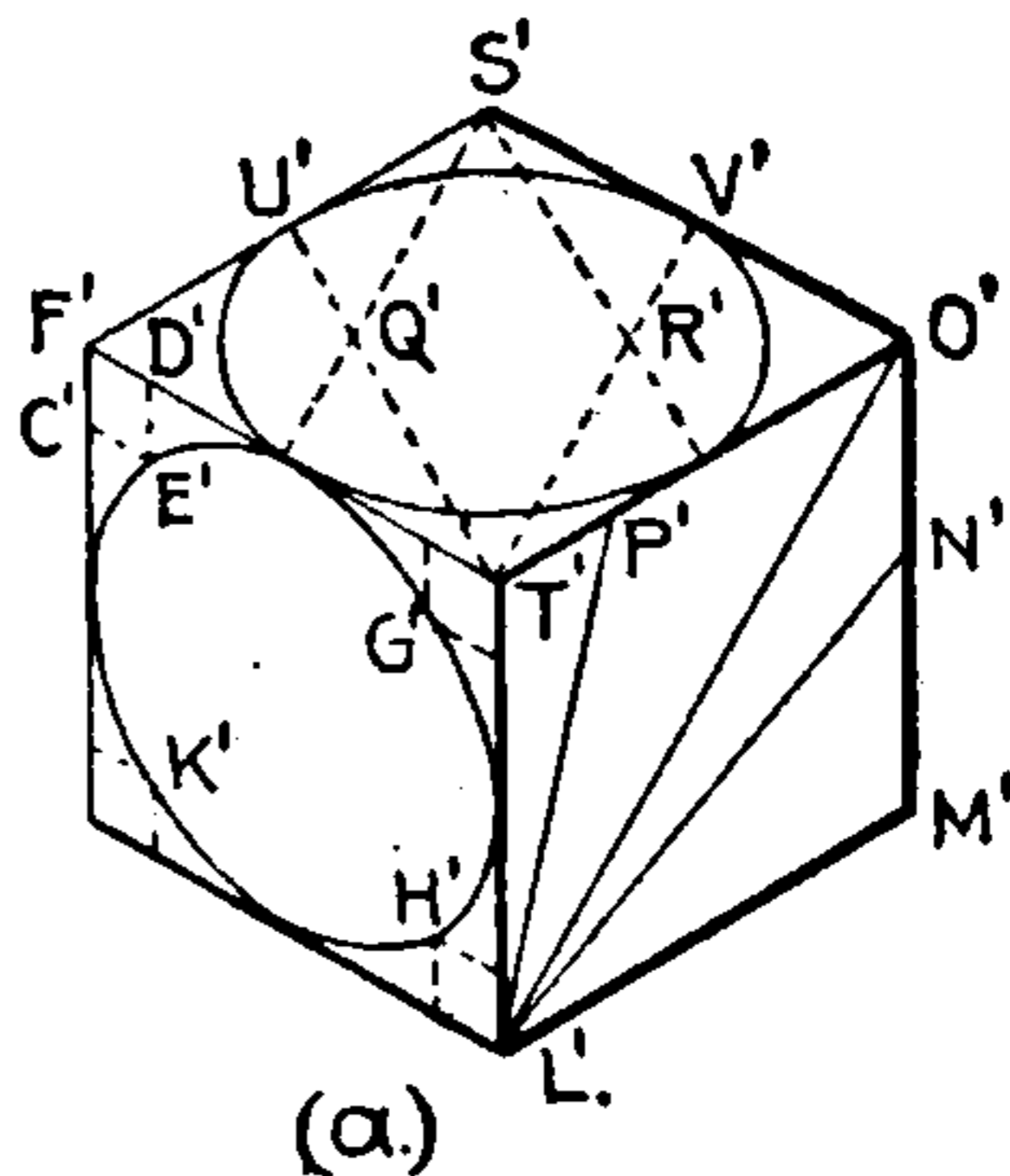


FIG. 181a.

**256. To draw an approximate circle.**

**Construction.**—Refer to Fig. 181a.

1. Connect  $S'$  and  $T'$  with the middle points of the opposite edges.
2. Using  $T'U'$  as a radius, draw arcs from  $T'$  and  $S'$  as centers, from middle point to middle point.
3. Using  $Q'$  and  $R'$  as centers, and  $Q'U'$  as radius, complete the oval.

**Note.**—This system of drawing the circle isometrically is almost exclusively employed in commercial drafting, since it can be done with the compasses and is clean cut in appearance.

**257. To draw lines making any desired angle with any given line.**

**Construction.**—Required to draw lines at  $30^\circ$ ,  $45^\circ$ , and  $75^\circ$  to LM, Fig. 181.

1. Draw LN, LO, and LP in Fig. 181 at their true angles to LM.
2. From  $M'$  (Fig. 181a) lay off the ordinates  $M'N'$  and  $M'O'$  equal to MN and MO.



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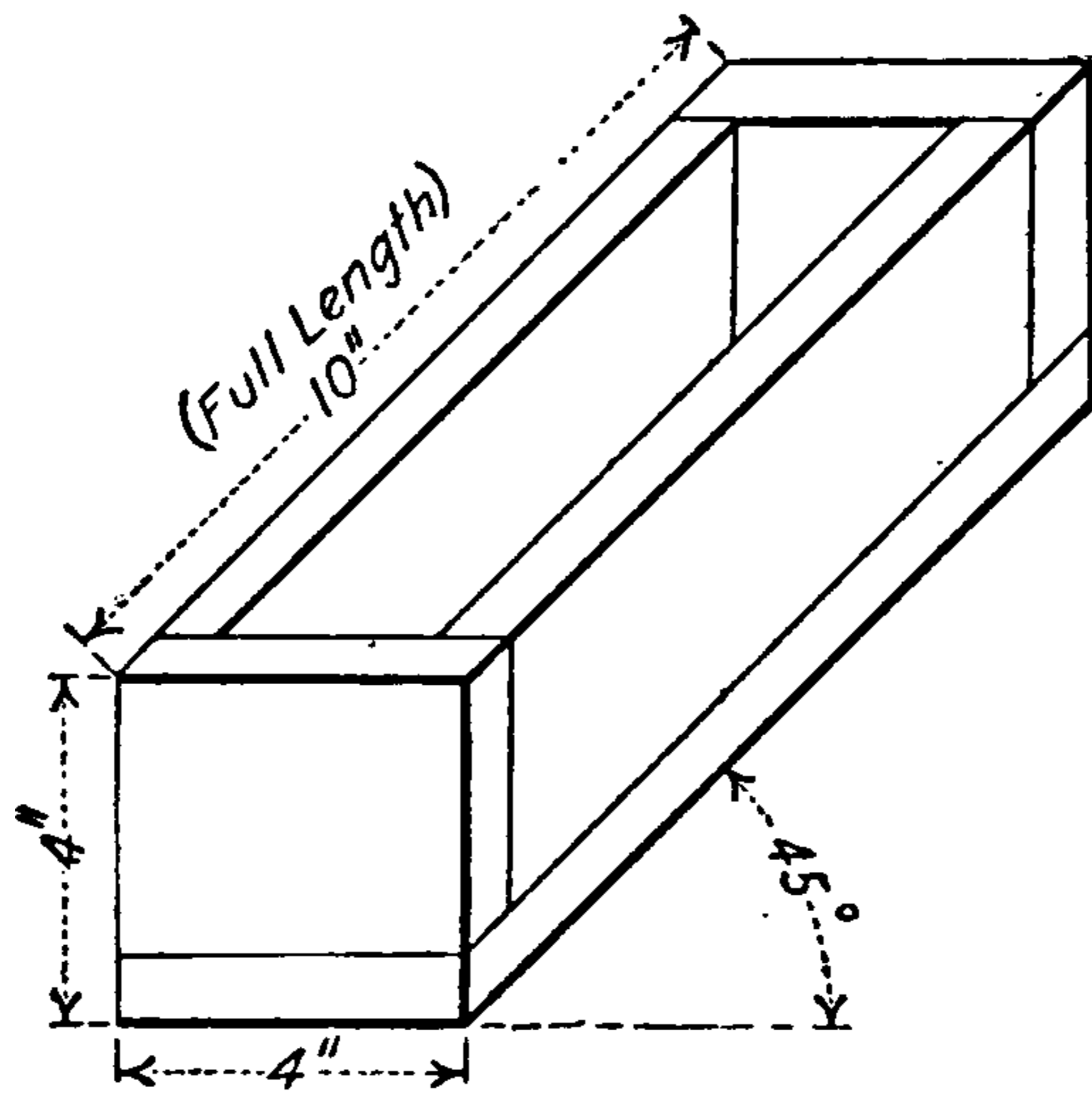
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because one face is the same as orthographic projection would make it.

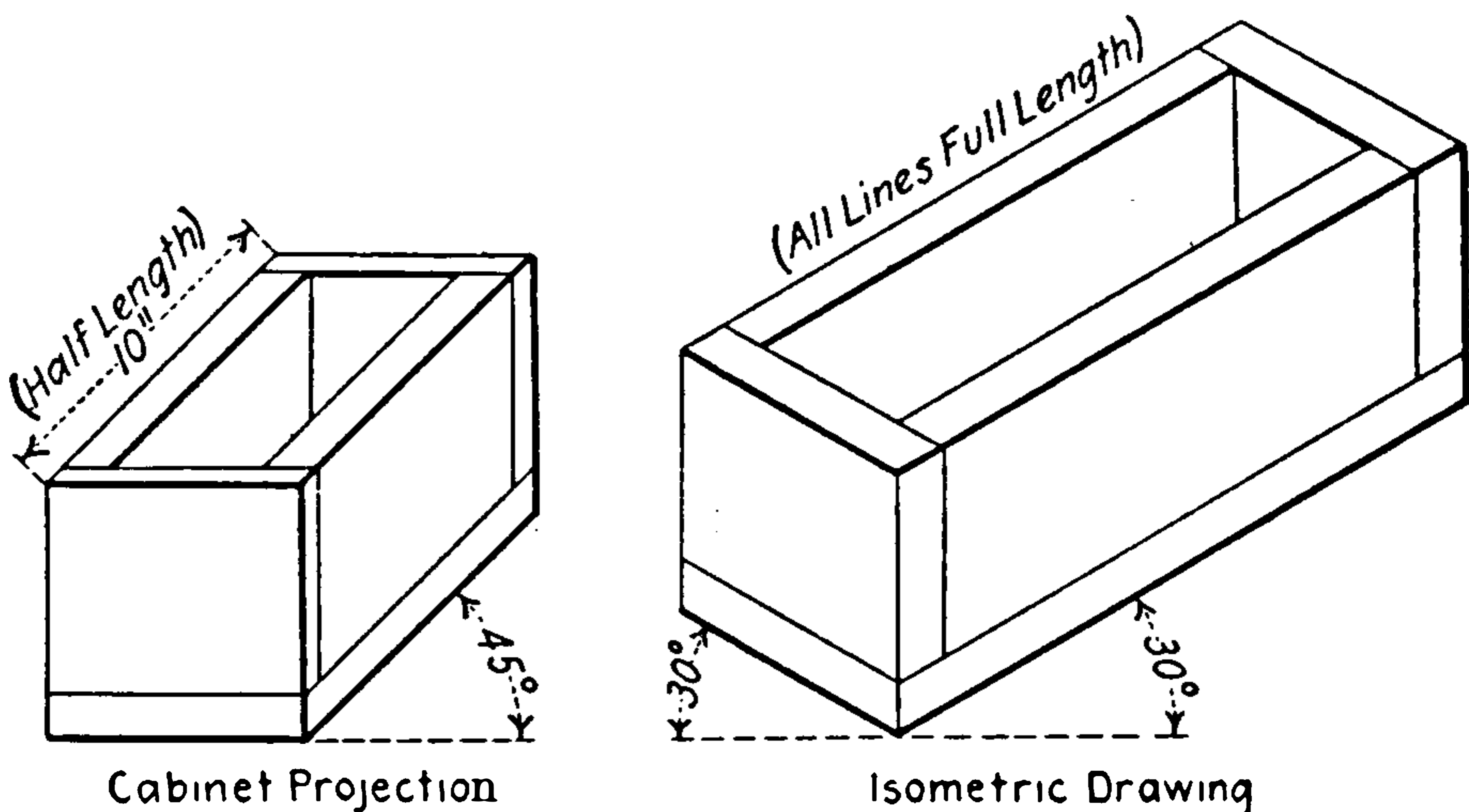
**Cabinet Projection.**—To further reduce the distortion, the receding lines are shortened one-half. Otherwise the drawing is made like cavalier projection.



Cavalier Projection

FIG. 183.

Fig. 183 shows a box drawn by the cavalier method, and Fig. 184 shows the same box drawn by the cabinet and isometric methods.



Cabinet Projection

Isometric Drawing

FIG. 184.

### EXERCISES IN SINGLE PLANE PROJECTION

**260.** Make——(isometric, cavalier, cabinet) drawings of the following object, making them to reduced scale, if necessary.

**952.** Wooden box, without cover, 10 in. long, 6 in. wide, 4 in. deep, made of  $\frac{3}{4}$ -in. boards. Stand on end, open to the front.

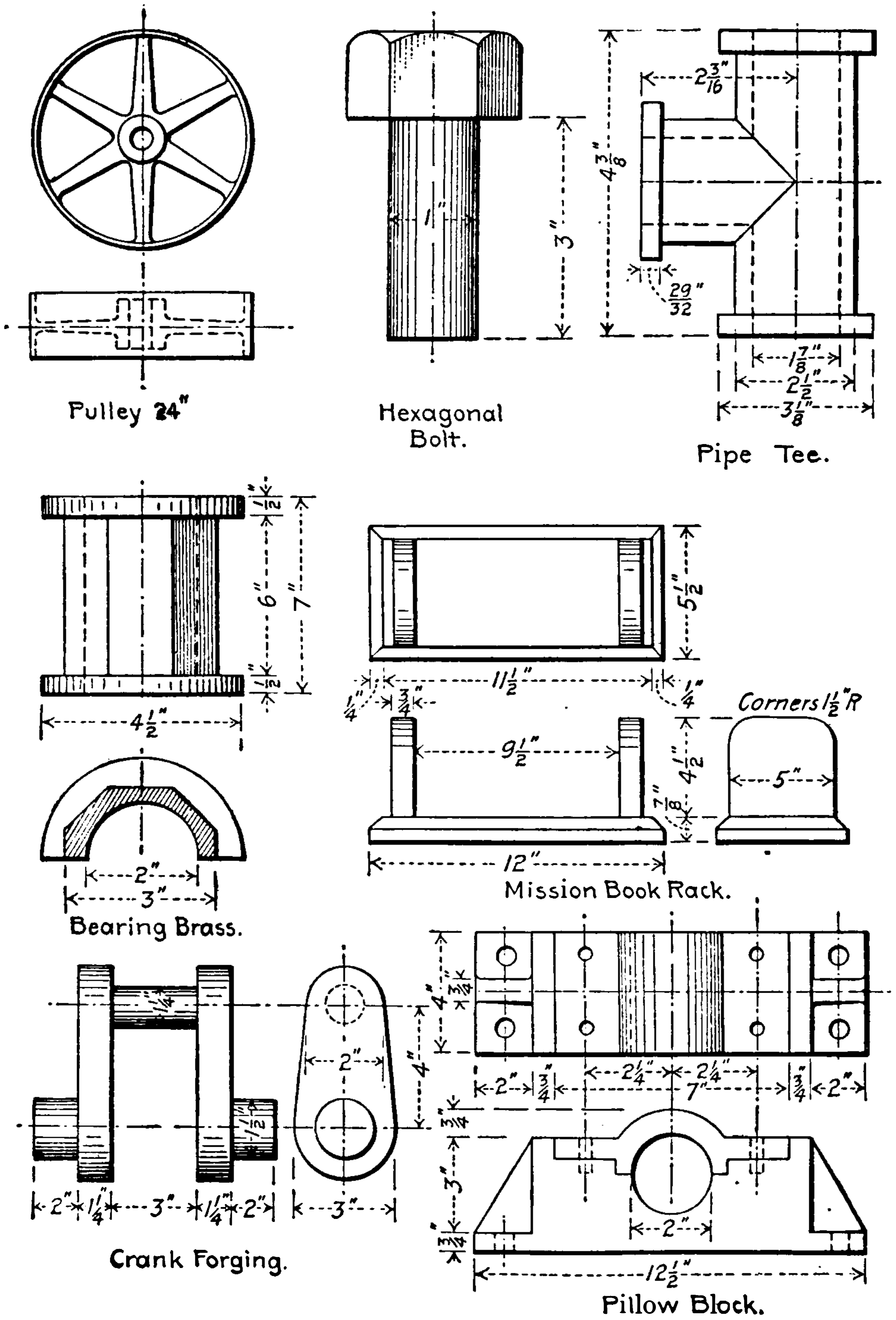


FIG. 185.



953. Same box, in natural position, with a cover open  $120^\circ$ .  
 954. Pentagonal plinth, 2-in. diameter,  $\frac{1}{2}$  in. thick, surmounted by a 1-in. cylinder,  $1\frac{1}{2}$  in. high.  
 955. Bolt shown in Fig. 185.  
 956. Pulley, Fig. 185.  
 957. Book rack, Fig. 185.  
 958.  $1\frac{1}{2}$ -in. pipe tee, Fig. 185.

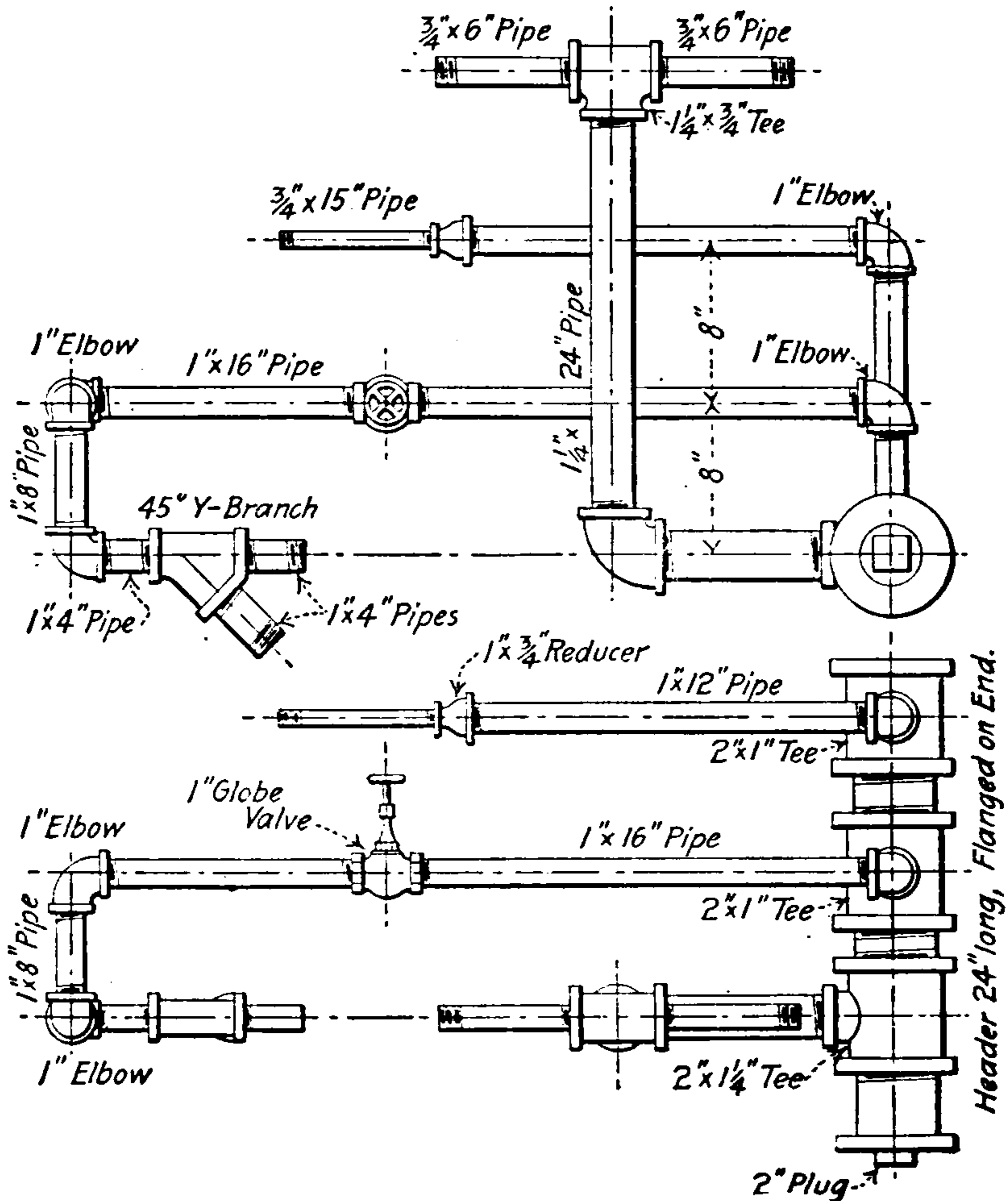


FIG. 186.

959. Bearing brass, Fig. 185.  
 960. Crank forging, Fig. 185.  
 961. Pillow block, Fig. 185.  
 962. Frustum of pyramid, given in Ex. 943.  
 963. Saw horse, Fig. 86, Chap. III.  
 964. Stand, Fig. 89, Chap. III.  
 965. Piping layout, Fig. 186. This is intended for a large sheet, and makes a fine drawing if traced.

Note.—First make the layout of the center lines, making necessary allowances for all connections.



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