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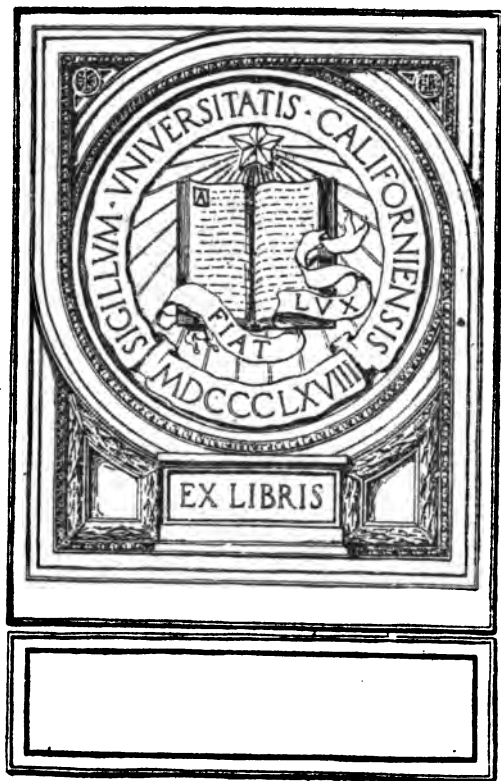
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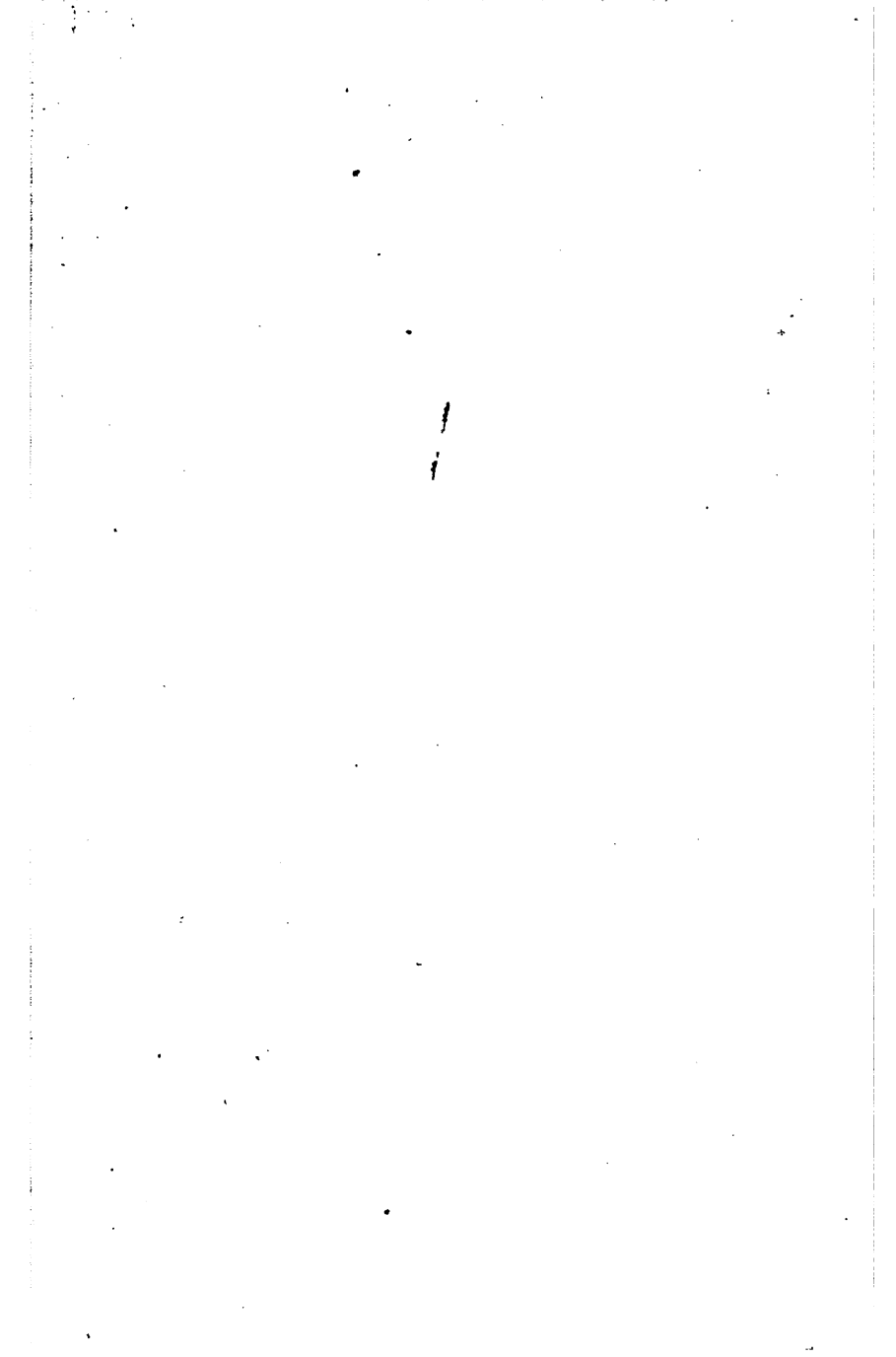
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# DESCRIPTIVE GEOMETRY

MILLER

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# DESCRIPTIVE GEOMETRY

BY

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THE MANUAL ARTS PRESS  
PEORIA, ILLINOIS

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## PREFACE

Believing that no one study plays a larger part than Descriptive Geometry in the shaping of the student's mind into the analytic thinking machine, necessary to success in any engineering profession, the author has outlined and written the text with this as its chief aim.

To accomplish such an end, the following fixed arrangement has been used in the case of each problem: (1) Statement of given data and requirements; (2) Discussion; (3) Analysis or brief statement of general order of procedure in solution; (4) Construction of some general case of the problem.

The author, after graduating from college, learned the trade of boilermaking. Before the expiration of two years at his work, from observation and experience, the conclusion became reasonable to him that three-fourths of the errors in construction were due, either to the inability of the foreman to give his orders so clearly as to admit of no misinterpretation, or to the general inability of workmen to understand an order when given distinctly.

Students entering college are notoriously deficient in the ability to answer a question concisely and plainly, to express their ideas in any definite shape whatever, or to obey explicitly an order. To rectify these defects in the student, analyses have been given first place in the text, inasmuch as they constitute a definite outline of procedure necessary to the correct solution of each problem.

The text has been written in a conversational style, because, in numerous experiences with notes, the author has found this style to be the most effective in exciting the student's interest and in giving instruction clearly.

# DESCRIPTIVE GEOMETRY

## CHAPTER 1

### ABBREVIATIONS, NOTATION, CONVENTIONS, ETC.

(1) It is of the utmost importance that each student make this chapter a constant handbook of reference; for, in the conscientious mastery of correct notation and conventions, lies half the victory in Descriptive Geometry.

From a close observation of the many failures in this subject, it has been estimated that at least three fourths of them were due to carelessness in notation and conventions as well as general carelessness, thereby making Descriptive Geometry, one of the most beautifully definite subjects known, a nightmare of indefiniteness and intangibility.

It is not desired that any student attempt to memorize this chapter of symbols and abbreviations, but that each man refer constantly to it for information on how to letter points, how to designate certain angles and the character of line to be used for each different purpose. In a short while all these will become fixed in each man's mind by their constant use, and Descriptive Geometry will have been made a far easier subject for his small amount of trouble.

### DRAWING ROOM PLATES

(2) **Drawing Room Plates** are to be left untrimmed,  $8\frac{1}{2}''$  x  $11''$  outside.

**Border Line** to be a heavy ink line,  $8''$  x  $10\frac{1}{2}''$ .

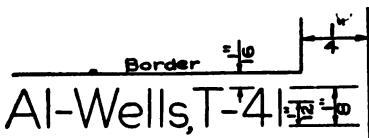
All work to be executed in pencil, (6H) unless other-

wise directed. The work calls for a finished **pencil** drawing.

**No thumb tack holes** to appear on the finished plate.

Place the **section, division, name, initials, and your number**, in the lower margin at the right of sheet as shown in figure.

To be lettered in **ink**, **freehand vertical** style of letter.



All plates to be handed in, finished or unfinished, at the end of the period.

The **left border line** and the **ground line** are reference lines for the **system of coordinates** by which the location of points on the sheet is determined.

**HOME PLATES**

(3) **Home Plates** must be handed in at the **beginning** of the period at which they are due in order to receive credit.

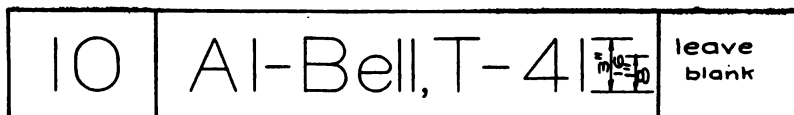
Letter all **notation** neatly in **vertical** letters (Caps. 1/8", l. c. 1/12").

All work must be completed in **pencil** (4H) using the drawing instruments where necessary.

Where **numbers**, e.g. (46), are given with the problems, they refer to the articles in the text which govern the conditions and requirements. Other problems will be handled by placing with the given graphic data a statement of requirements as follows:

Reqd: Plane thru P // AB.

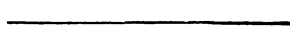
Fill in the form in the lower right corner as in figure.



## STANDARD CONVENTIONAL LINES

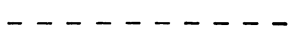
## (4) Ground lines

Given lines


 Solid, medium weight.

Projection lines

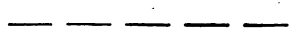
Construction lines

 1/16" dash, 1/16" space,  
medium weight.


Auxiliary lines

 3/8" dash, 1/16" space,  
medium weight.

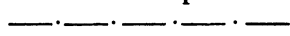
Hidden lines

 3/16" dash, 1/16" space,  
medium weight.

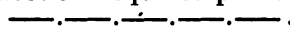
Required lines

 Solid, heavy weight.

Traces of Given planes

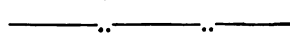
 1/4" dash and dot.  
medium weight.

Traces of Required planes

 1/4" dash and dot.  
heavy weight.

Axes of Solids

Axes of Revolution

 1/2" dash, double dot,  
medium weight.

## ABBREVIATIONS

- (5) H = Horizontal plane of projection.  
 V = Vertical " " "  
 P = Profile " " "  
 GL = Ground Line.  
 G<sub>1</sub>L<sub>1</sub> = Profile Ground Line.

**SYMBOLS**

- (6)  $\perp$  = "perpendicular to"  
 $\parallel$  = "parallel to"  
 $\sphericalangle$  = "angle"  
 $\backslash$  = "inclined to"  
 $X$  = **Angle** between two intersecting **Lines**.  
 = " between two intersecting **Planes**.  
 = " between a **Line** and a **Plane**.  
 $\odot$  = **Angle** which any **Line** makes with **H**.  
 = " " " **Plane** " " **H**.  
 $\Phi$  = " " " **Line** " " **V**.  
 = " " " **Plane** " " **V**.  
 $\pi$  = " " " **Line** " " **P**.  
 = " " " **Plane** " " **P**.

**NOTATION**

- (7)  $a = H$  projection }  
 $a' = V$  " " } of point A.  
 $a'' = P$  " " }  
 $tTt'$  = traces of plane T.  
 $tT$  = H trace of plane T.  
 $Tt'$  = V trace of plane T.  
 $t''t''$  = P trace of plane T.  
 $h = H$  projection } of the H piercing point of  
 $h' = V$  " " } every line.  
 $v = H$  projection } of the V piercing point of  
 $v' = V$  " " } every line.  
 $p = H$  projection } of the piercing point of  
 $p' = V$  " " } every line in either the P  
 plane or any given oblique  
 plane.

$o = H$	projection	} of an <b>assumed</b> point in any plane.
$o' = V$	"	
$o_1 =$		} revolved positions of Point O.
$o_2 =$		

### SYSTEM OF COORDINATES

(8) The **left hand border line** and the **ground line** are the reference lines for the system of coordinates by which the location of points on the sheet is determined.

$z =$  distance of projecting line from left hand border line.

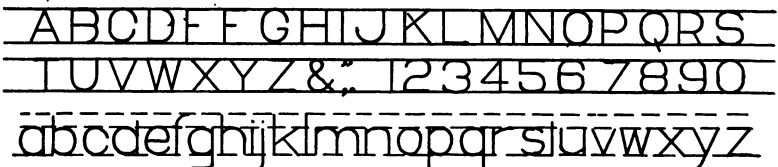
$y =$  distance of point above or below H, as  $y$  is  $+$  or  $-$ .

$x =$  distance of point in front or behind V, as  $x$  is  $+$  or  $-$ .

The coordinates are always given in this order and the distances are stated in inches; e. g. the point located by  $(6, 3, -2\frac{1}{2})$  is 6" to the right of the left hand border line, 3" above H and  $2\frac{1}{2}$ " behind V. If  $z = 0$ , the projecting line is either at the border line or may be taken at will.

### LETTERING ON PLATES

(9) It is of the greatest importance that all points, lines and planes shall not only be lettered, but lettered carefully. The following alphabet of **vertical** letters (Caps.  $1/8''$ , l. c.  $1/12''$ ) will be used thruout the course. Each man should, for some weeks, practice faithfully on these letters until both the manner of forming them and their height have become a habit instead of an effort.



## ADVICE ON STUDY METHODS

(10) It may not be amiss to give here a few suggestions on the study methods that seem to give the best results in Descriptive Geometry.

During the first two or three weeks it will perhaps be difficult for some to see points, lines, and planes in space without the aid of some mechanical devices. Therefore it is advisable that each man make himself two sets of cardboard coordinate planes; one set that can be folded together, the other set fixed rigidly by adding a profile plane. This last set can readily be had by using the bottom back and right end of any cardboard box. Pencils or wires answer well for lines.

After the first two weeks or more of preliminary work we have to deal with a number of characteristic geometric problems, involving points, lines and planes; e. g. one of the problems is that of finding the distance from a point in space to a given plane. It is readily seen that as both the point and the plane can be in any number of positions with reference to the coordinate planes, it would be absurd for anyone to memorize the solution of the problem with the point and plane in certain positions and expect to be able to solve all similar problems by means of his memorized solution. However the basic principle of the solution is exactly the same in every possible variation of the problem and once this principle is understood and memorized, the problem becomes elementary. This basic principle we call the **analysis** of the problem, and a perfect familiarity with all these analyses is absolutely necessary to a thoro understanding of Descriptive Geometry.

A fixed order of statement and arrangement is used on



all of the problems in the text and the following method of study is recommended :

(a) Read carefully the statement of the problem which is given in heavy type, until you understand in the abstract just what is given and required.

(b) Read thru the analysis carefully several times until you catch its meaning and understand the successive steps of solution suggested therein.

(c) Take up the construction as given, and work thru it carefully until you understand it. **By all means try to look at the explanation in a general way;** i. e. keep before you the fact that the example given is only one of a multitude of such problems that can be given and that perhaps not a single other case of this problem will, when solved, have the same appearance as this solution. The **principle** of the solution is what you must understand and memorize, and **NOT** the **picture** of this one solution.

(d) Then close the book and see if, with your understanding of the problem, you can formulate an analysis of your own. Be especially careful in your choice of English. If you can formulate an analysis of the problem it is a good indication that you understand it.

## CHAPTER 2

### REPRESENTING THE POINT AND LINE.

(11) **Descriptive Geometry** is that branch of mathematics which has for its object the solution of all geometric problems which arise in representing objects or geometric magnitudes by drawings.

The geometric magnitudes mentioned above are represented by drawings that are known as **projections** which are made on three conventional intersecting planes.

### COORDINATE PLANES OR PLANES OF PROJECTION

(12) The three conventional planes mentioned in the preceding article are called the **planes of projection** or **coordinate planes**, and are: a Vertical plane, a Horizontal plane, and an End or Profile plane which is perpendicular to both the Vertical and Horizontal planes.

For convenience, we call the Vertical plane the V plane, the Horizontal plane the H plane, and the Profile plane the P plane.

In Fig. 2 these three planes are shown in their relative positions.

### GROUND LINE

(13) The line of intersection of the H and V planes is known as the **ground line** and for convenience is abbreviated to GL.

The H and V planes form four diedral angles, each  $90^\circ$ . These angles are named

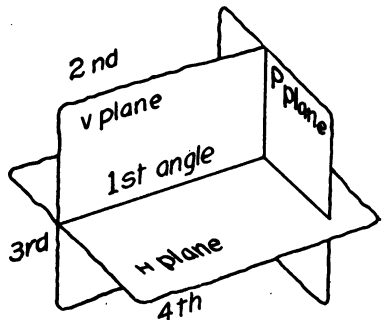


Fig. 2.

the 1st, 2nd, 3rd and 4th and are taken in the order shown in Fig. 2.

### PROJECTIONS

(14) The **projections** of objects on these coordinate planes are such **representations** as will have the *same appearance* as the *object* itself to the eye of an observer when the observer is in a *certain* position.

The *choice* of positions in which the observer may be placed (these positions being called **points of sight** or **view points**) of course gives rise to *two* kinds of projections.

#### SCENOGRAPHIC PROJECTION OR PERSPECTIVE

(15) When the *view point* or the eye of the observer is at a finite distance, e.g. 100 ft. from the projection planes, no two *lines of sight* from the *view point* to different points of the object are *parallel* and the projection is either *smaller* or *larger* than the object itself. Such projections are **scenographic**. Any picture, photograph or painting is a *scenographic projection*.

#### ORTHOGRAPHIC PROJECTION

(16) In Orthographic Projection however, the *point of sight* or *view point* is always at an *infinite* distance from the coordinate planes; hence for convenience, we will assume that the *lines of sight* from the *view point* to all points of an object, however large, are *parallel* and *perpendicular* to the planes of projection. e.g. To obtain the projection on the *horizontal* plane (the H projection) we imagine the *point of sight* to be at an infinite distance *above* the H plane; to obtain the projection on the V plane (the V projection) we imagine the *point of sight* to be at an infinite distance in *front* of the V plane. A similar condition holds

for the profile plane, the *point of sight* being at an infinite distance to the left of the profile plane. Thus we see that in obtaining the **orthographic** projections of an object on the H and V planes, we must have *two* points of sight. One of the projections (H) has the appearance of a *top* view of the object and the other (V) the appearance of a *front* view.

### PROJECTING LINES

(17) In obtaining the projections of an object it will be seen that the H projection of a particular point A, will be the *point* in which the *line of sight* thru A pierces the H plane. A similar condition holds for all points. The portions of these *lines of sight* between the *object* and the *plane of projection* are called **projecting lines**. The *projecting lines* of a point might then be defined as the **perpendiculars** dropped from the point to the planes of projection.  $Aa'$  and  $Aa$ , Fig. 3 are *projecting lines*.

### PROJECTIONS OF A POINT

(18) Summarizing the conclusions of the last several articles, we may say that the H projection of a point A, is the H piercing point of a perpendicular dropped from A to the H plane. The V projection is the V piercing point of a perpendicular dropped from A to the V plane. The P projection is the P piercing point of a perpendicular dropped from A to the P plane. These **projections** are lettered as in Fig. 3; the H projection,  $a$ , V projection  $a'$ , and P projection,  $a''$ . **The projections of all points are lettered in a similar manner.**

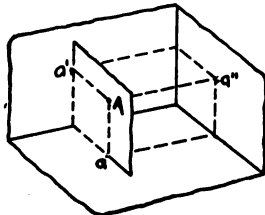


Fig. 3.

## ALPHABET OF A POINT

(19) A point may be placed in nine (9) different positions with respect to the coordinate planes and GL; these nine positions constitute what is known as the **alphabet of a point**. These positions are respectively:

- |   |                            |   |                          |
|---|----------------------------|---|--------------------------|
| 1 | In space in the 1st angle. | 5 | In the H plane before V. |
| 2 | In space in the 2nd angle. | 6 | In the H plane behind V. |
| 3 | In space in the 3rd angle. | 7 | In the V plane above H.  |
| 4 | In space in the 4th angle. | 8 | In the V plane below H.  |
|   |                            | 9 | In the Ground Line.      |

These positions are shown in Fig. 4.

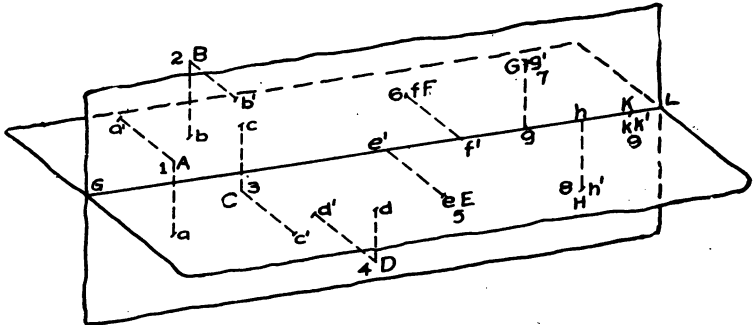


Fig. 4.

## POINT DETERMINED BY ITS PROJECTIONS

(20) If thru the H projection,  $a$ , of a point A, Fig. 5, a perpendicular be erected to the H plane, it will contain the point A; likewise a perpendicular to the V plane thru the V projection,  $a'$ , will contain point A; hence the point is determined, being the intersection of these two perpendiculars.

## LOCATION OF POINTS

(21) If thru the projecting lines  $Aa'$  and  $Aa$ , Fig. 5, a plane be passed, it will be perpendicular to both H and

V and intersect H and V in lines perpendicular to GL; these lines with  $Aa'$  and  $Aa$  form a rectangle in which the distance from  $a'$  to GL equals  $Aa$  and the distance from  $a$  to GL equals  $Aa'$ . From the above we deduce the following axioms:

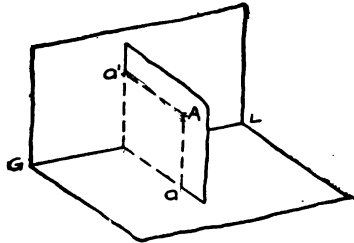


Fig. 5.

(a) **The distance from a point to the H plane is shown by the distance from its V projection to GL.**

(b) **The distance from a point to the V plane is shown by the distance from its H projection to GL.**

(c) **If a point lies in V its H projection is in GL; it cannot be in GL otherwise.**

(d) **If a point lies in H its V projection is in GL; it cannot be in GL otherwise.**

#### REVOLUTION OF THE COORDINATE PLANES

(22) Probably the question has already entered our minds of how to represent these three coordinate planes of projection on a single sheet of paper or blackboard. The answer to this question is, that two of the planes are revolved about certain axes until they coincide with V. This will be more easily understood by reference to Fig. 6. **Revolution:** Using  $GL$ , the line of intersection of H and V, as an axis, the part of H in front of V is revolved *down*, the part behind, *up*, of course, until H coincides with V. Using  $G_1L_1$  as an axis the portion of the Profile plane in

*front* of V is revolved to the *right*, the portion *behind*, to the *left*, until P coincides with V.

In the revolution just explained, the V projections of points are not affected; i.e., the V projections of points in the 1st and 2nd angles are *above* GL, and *below* GL for the 3rd and 4th, regardless of any revolution of planes; however, the H projections of all points in *front* of V, either in the 1st or 4th angles, being in the portion of H which was revolved *down*, now come *below* GL; see Fig. 5, then Fig. 7 (1). The H projections of all points *behind* V, in either the 2nd or 3rd angles, are found *above* GL since that part of H was revolved *up*.

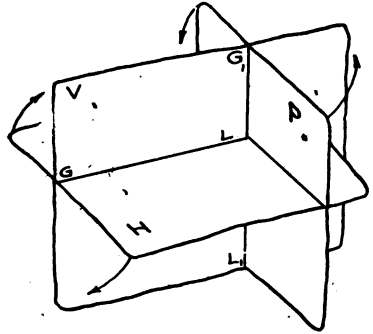


Fig. 6

As the H plane is revolved about GL, as an axis, the H projection of a point moves in a plane which is perpendicular to GL; hence after the revolution the H projection is found on the perpendicular to GL, dropped from the V projection of the point.

#### AXIOM

**The two projections of a point must be on the same perpendicular to GL.**

The orthographic projections of the nine positions which constitute the *alphabet* of a point, are shown in Fig. 7.

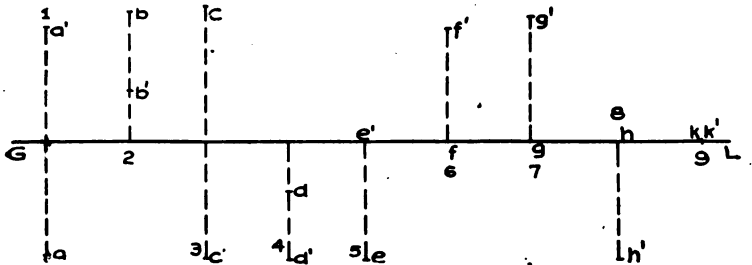


Fig. 7.

## LINES

(23) Since a *right line* is generated by the *motion of a point in some fixed direction*, the *H projection of a right line* is made up of the *H projections of the successive positions of the generating point*. The *H projecting lines* of all these points, being perpendicular to *H*, constitute a *plane* which is perpendicular to *H*; the *line of intersection* of this *plane* with *H* is the *H projection of the given line*. From this we see that the *H projection of a line* may be considered the *line of intersection with H*, of a *plane* which passes thru the given line and is perpendicular to *H*. This auxiliary plane is called the **H Projecting Plane** of the line. The *V projection of a right line* is obtained by passing thru the line a *V projecting plane* and finding its *line of intersection with V*.

Since the *H (V) projection of a right line* is a *right line*, the *projection* may be determined by *dropping perpendiculars* from the *extremities of the line* to *H (V)*; the line joining the *H (V) piercing points of these perpendiculars*, Fig. 8, is the required *H (V) projection*.

## LOCATION OF A LINE

(24) The two projections of a line being given, the



line is in general completely determined. For, if thru the H projection we pass a plane perpendicular to H, and thru the V projection a plane perpendicular to V, the line will lie in both of these planes, hence is their line of intersection.

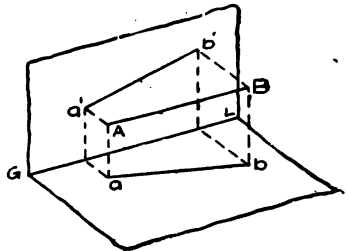


Fig. 8.

**TO PASS A LINE THRU A GIVEN POINT**

(25) Since the projections of a line are made up of the projections of all points of the line, the projection of a line which passes thru a given point, pass thru the projections of the point.

**ALPHABET OF A LINE**

(26) In finding the different positions in which a line may be placed with respect to the coordinate planes, we limit our choice of location to one angle at a time and name the alphabet to correspond to the angle, viz., *alphabet of a line in the 1st angle*, etc.

These positions in the 1st angle are shown in Fig. 9.

- |   |                                      |   |  |
|---|--------------------------------------|---|--|
| 1 | Perpendicular to H                   | } | V projection a line, perpendicular to GL.<br>H projection a point. |
| 2 | Perpendicular to V                   | } | V projection a point.<br>H projection a line perpendicular to GL.  |
| 3 | Lying in a plane perpendicular to GL | } | Both projections perpendicular to GL.                              |

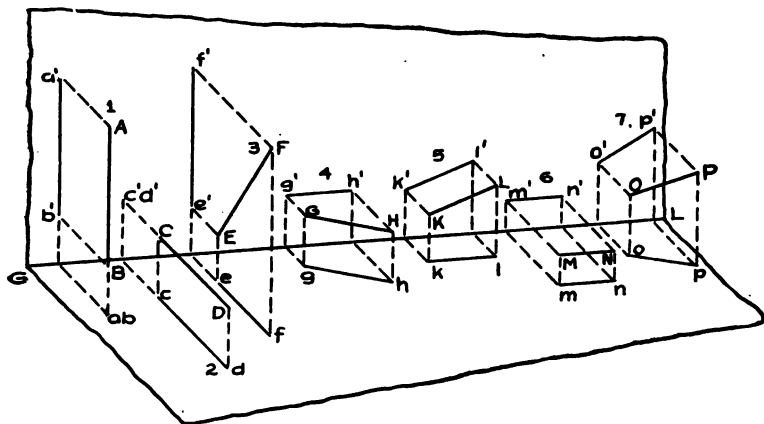


Fig. 9.

- |   |                               |  |
|---|-------------------------------|--|
| 4 | Parallel to H & inclined to V | } V projection parallel to GL.<br>H projection inclined to GL. |
| 5 | Parallel to V & inclined to H |  |
| 6 | Parallel to both H and V      | } Both projections parallel to GL.                             |
| 7 | Inclined to both H and V      |  |

**TO ASSUME A RIGHT LINE**

(27) In assuming a right line, the projections may be assumed at pleasure; e.g., Fig. 10, to assume a right line in the first angle, draw the V projection in any position and in any direction above GL; select two points,  $a'$  and  $b'$ , on this projection as the V projections of two points, A and B, of the line. Drop perpendiculars from  $a'$  and  $b'$  to GL; any line,  $ab$ , below GL connecting any point

$a$  on the one perpendicular with any point  $b$  on the other perpendicular, will be the H projection of the required line.

#### TO ASSUME A POINT ON A LINE

(28) Since the projections of the point must lie on the corresponding projections of the line and on the same perpendicular to  $GL$ , assume any point  $c'$  on the V projection of  $AB$ , Fig. 10, and from  $c'$  drop a perpendicular to  $GL$ ; the point  $c$  in which this perpendicular intersects  $ab$  is the H projection of point  $C$ .

(29) If two lines intersect, their projections will intersect, and the points of intersection of their projections will be the projections of their point of intersection.

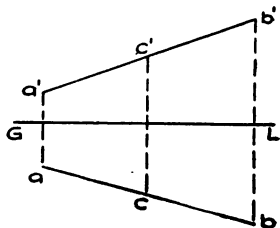


Fig. 10.

Since the point of intersection is common to both lines, its V projection lies on the V projection of each line, hence at their intersection; likewise the H projection of the point of intersection is the intersection of the H projections. These two points, being the projections of the same point, must lie on the same perpendicular to  $GL$ .

#### TO ASSUME TWO INTERSECTING LINES

(30) Assume at will both projections of one line  $AB$ , Fig. 11, and one projection,  $c'd'$ , of the other line  $CD$ , intersecting  $a'b'$  at any point  $o'$ . From  $o'$  drop a perpendicular to  $GL$  until it intersects  $ab$  at  $o$ ; thru  $o$  draw the H projection of  $CD$  in any desired direction.

**IF TWO LINES ARE PARALLEL THEIR PROJECTIONS ARE PARALLEL.**

(31) Thru the lines  $AB$  and  $CD$ , Fig. 12, pass the two H

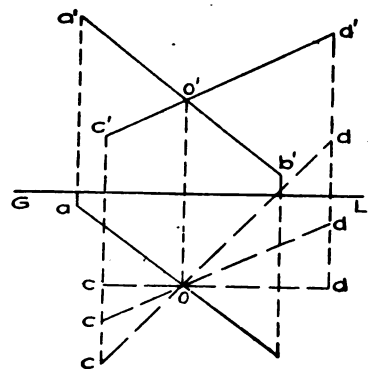


Fig. 11.

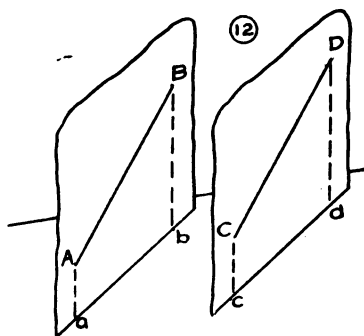


Fig. 12.

projecting planes. These two planes are parallel since the one contains two intersecting lines,  $AB$  and  $Bb$ , parallel respectively to the two intersecting lines,  $CD$  and  $Cd$ , of the other. Then we have two parallel planes cut by a third plane (the  $H$  plane); the lines of intersection,  $ab$  and  $cd$ , are parallel.

## CHAPTER 3

### PLANES

(32) **Planes** are represented on the coordinate planes of projection by their *lines of intersection* with these planes. These lines of intersection are called the **traces** of the plane; e.g., the line of intersection of a plane T with V is called the *V trace* of T, and the intersection of T with H is called the *H trace*. In Fig. 13a, plane T is shown in perspective. In Fig. 13b plane T is shown as represented in Orthographic Projection; likewise the manner of lettering the respective traces and the conventional line used to represent a plane trace ·  $Tt'$  is the V trace and  $Tt$ , the H trace.

**THE TRACES OF A PLANE MUST INTERSECT ON GL** (unless the plane is parallel to GL).

(33) Since plane T, Fig. 13a, is oblique to GL it must intersect GL in some point; this point is common to V and T, hence lies on their line of intersection,  $Tt'$ ; it is also common to H and T, hence lies on their line of intersection,  $Tt$ . The point in which T intersects GL has been proven common to  $Tt'$  and  $Tt$ , hence  $Tt'$  and  $Tt$  must intersect in a point on GL.

Further we see that the *traces* of a plane intersect in the *point* in which GL pierces that plane. Therefore if a plane is *parallel* to GL this *piercing point* is at *infinity*, i.e. traces intersect at infinity and for our purposes are *parallel* to GL.

### LOCATION OF A PLANE

(34) It is readily seen that the plane is definitely determined by its traces, for thru these traces (two inter-

secting lines) one and only one plane can be passed, and that the required plane.

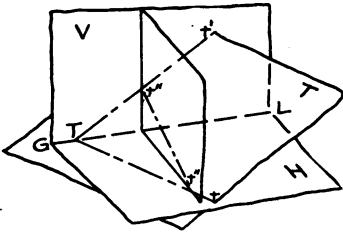


Fig. 13a.

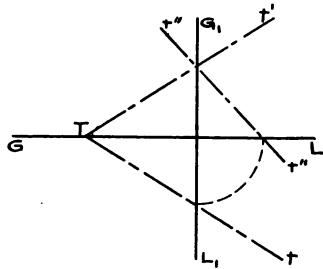


Fig. 13b.

**ALPHABET OF A PLANE**

(35) As with the line, in placing a plane in its different possible positions with respect to H and V, we limit our choice to one angle at a time.

**ALPHABET OF A PLANE IN THE FIRST ANGLE**

- 1 Perpendicular to both H & V — Both traces perpendicular to GL.
- 2 Perpendicular to H { V trace perpendicular to GL.  
and inclined to V { H " inclined to GL.
- 3 Perpendicular to V { V trace inclined to GL.  
and inclined to H { H trace perpendicular to GL.
- 4 Parallel to V — H trace parallel to GL.
- 5 Parallel to H — V trace parallel to GL.
- 6 Parallel to GL and inclined to H & V — Both traces parallel to GL.
- 7 Inclined to H, V, and GL — Both traces inclined to GL.

**PLANES ARE UNLIMITED IN EXTENT**

(36) Plane T, Fig. 14, is *not* limited to the portion of the plane in the *first angle*. Producing the traces thru GL, we have immediately, the traces of the portions of the plane in all of the other angles. From this we deduce that: The **traces** of a plane are **NOT** limited by GL.

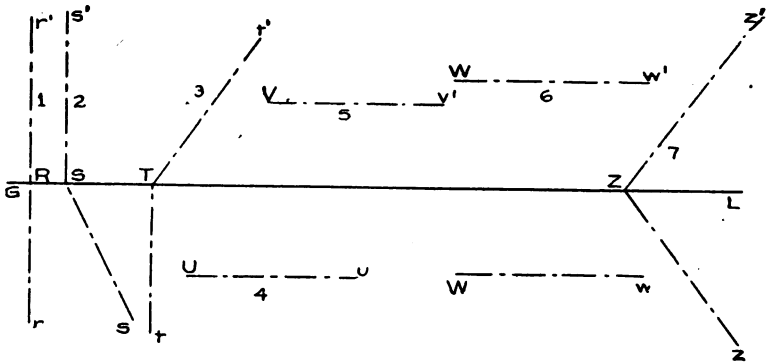


Fig. 14.

**TO ASSUME A PLANE**

(37) In assuming a plane in space, the traces may be drawn at pleasure; however, they *must intersect* on GL unless the plane is *parallel* to GL.

**AXIOMS**

(a) If two planes are parallel their respective traces are parallel.

For if two parallel planes are cut by a third plane, (the H plane) the lines of intersection, (the H traces) are parallel; likewise the V traces.

(b) If a plane is perpendicular to H its V trace is perpendicular to GL.

(c) If a plane is perpendicular to V its H trace is perpendicular to GL.

PROFILE PLANE

(38) As explained in Art. 22, to represent the Profile plane on the same plane with V, it is necessary to revolve P about its line of intersection,  $G_1L_1$ , with V, until it coincides with V. The portion in front of V, 1st and 4th angles, is revolved to the right, the 2nd and 3rd angles to the left. The angles of the Profile plane are then as indicated in Fig. 15.

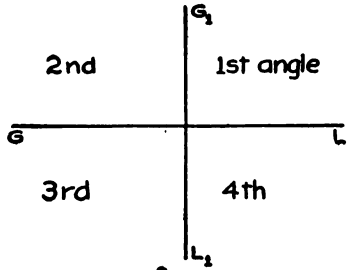


Fig. 15.

H AND V PROJECTIONS OF POINTS AND LINES IN THE PROFILE PLANE

(39) When we are dealing with the H and V projections of any point or line which lies in the Profile plane, we consider that plane to be in its *original* position, i. e., perpendicular to GL. In this case  $G_1L_1$  represents both the intersection of P with V and P with H; hence *both* the H and V projections of the point or line lie on  $G_1L_1$ .

Whenever in any problem, it is required to find the Profile *piercing point* of a line, let it be understood that the H and V projections of that piercing point are desired and **not** the Profile projection unless so specified.

PROFILE PROJECTIONS OF POINTS, ETC.

(40) When we are dealing with the **Profile** projections of points and lines, the Profile plane is considered as *revolved* about  $G_1L_1$  until it coincides with V. In this case  $G_1L_1$  represents the intersection of P and V, and GL, the



intersection of P and H; or for convenience,  $G_1L_1$  may be considered the V plane, and GL, the H plane viewed edge-wise.

As explained in Art. 18, the P projection of a point A is obtained by dropping a projecting line thru A perpendicular to P; the piercing point of this perpendicular in P is the P projection of A. It will be seen that this projecting line, Fig. 3, pierces P at a distance above H equal to  $Aa$ , which in turn equals the distance from  $a'$  to GL. Likewise it pierces P at a distance be-

fore V equal to  $Aa'$  which is equal to the distance from the H projection,  $a$ , to GL. Then after revolving the Profile plane about  $G_1L_1$  into V, Fig. 16,  $a''$  will be found above GL a distance equal to the distance from  $a'$  to GL, or on a line parallel to GL thru  $a'$ ; likewise since A is in the 1st angle,  $a''$  will be to the right of  $G_1L_1$ , a distance equal to the

distance that the H projection,  $a$ , is below GL. The construction for finding the Profile projections of points in the various angles, from their H and V projections, is shown in Fig. 16.

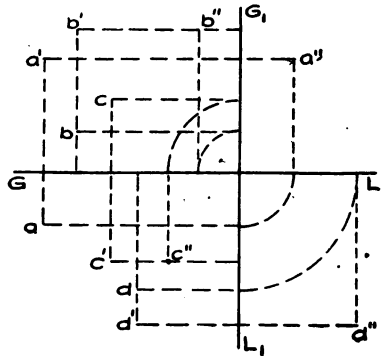


Fig. 16.

## CHAPTER 4

### SIMPLE RELATIONS OF THE POINT, LINE, AND PLANE

41) Problem — 1 — **To find the H and V piercing points of a given right line.**

**Analysis:**

(e.g., the H piercing point)

*Since the H piercing point of a line lies in H, its V projection lies in GL; its V projection lies also on the V projection of the line, therefore at the intersection of the V projection of the line and GL. The intersection of a perpendicular to GL thru this point, and the H projection of the line, is the required H projection, since the H projection of the piercing point must lie on the H projection of the line.*

*If we substitute H for V and V for H in the above analysis, we have the analysis for the V piercing point.*

From the above analysis, a rule may be deduced which perhaps is more easily remembered than the analysis.

**Rule:** To find the H piercing point of a given line, produce the V projection until it intersects GL; this point is the V projection of the H piercing point. The intersection of a perpendicular to GL thru this point, and the H projection of the line, is the H projection of the H piercing point.

**Construction:**

Given: Line AB, Fig. 17.

Reqd: H and V piercing points of AB.

Produce  $a'b'$  to GL; this point of intersection is

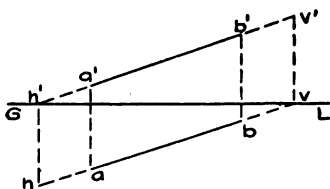


Fig. 17.

$h'$ , the V projection of the H piercing point. At  $h'$ , a perpendicular to GL is drawn, intersecting  $ab$  in  $h$ , the H projection of the H piercing point.  $ab$  produced, intersects GL at  $v$ , the H projection of the V piercing point; the intersection of a perpendicular to GL at  $v$  with  $a'b'$ , locates  $v'$ , the V projection of the V piercing point.

The H and V piercing points of lines whose **projections** are **perpendicular** to GL, may easily be determined by referring the line to the *Profile* plane; i.e. by producing the Profile projection until it intersects GL and  $G_1L_1$ , thereby obtaining the Profile projections of the H and V piercing points.

The H and V piercing points of every right line used in descriptive geometry are lettered as in Fig. 17,  $hh'$ ,  $vv'$ , and each man will save himself unlimited mistakes and poor grades, if he holds himself rigidly to the practice of lettering these points immediately upon locating them.

#### TO ASSUME A LINE IN A GIVEN PLANE

(42) If a line lies in a plane T, Fig. 18, it must pierce the V plane in a point common to both planes, therefore in a point on their line of intersection, the V trace. For similar reasons it must pierce H in a point on the H trace. Hence to *assume a line in a given plane assume any point on the V trace as the V piercing point*, and any point on the H trace as the H piercing point of the re-

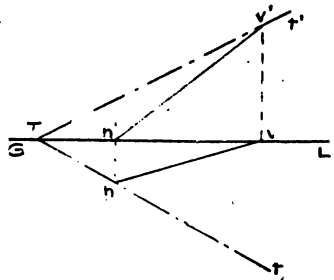


Fig. 18.

*quired line*; the line joining these two points is the required line.

To assume a point in a given plane, first assume a right line as explained. Any point of this line is the required point.

#### A HORIZONTAL OF A PLANE

Any line which lies in a given plane  $T$ , Fig. 19, and is *parallel* to  $H$ , is called a **horizontal** of that plane. Being parallel to  $H$ , it has only one piercing point ( $V$ ), and its  $V$  projection is *parallel* to  $GL$ . By a simple application of Plane Geometry, it can be proven also, that the  $H$  projection of a horizontal is *parallel* to the  $H$  trace of the plane.

A horizontal of a plane may be **assumed** by assuming its  $V$  piercing point, and drawing the  $V$  projection thru  $v'$  parallel to  $GL$  and the  $H$  projection thru  $v$  parallel to the  $H$  trace.

#### A VERTICAL OF A PLANE

Any line which lies in a plane  $T$ , Fig. 19, and is *parallel* to  $V$ , is called a **vertical** of that plane. Being parallel to  $V$ , it has only the  $H$  piercing point and its  $H$  projection is parallel to  $GL$ . The  $V$  projection of a vertical is *parallel* to the  $V$  trace. The method of assuming a vertical is similar to that of a horizontal.

Obviously, a horizontal or a vertical is the easiest and simplest line that can be assumed in a plane, and its value as a working tool cannot be overestimated.

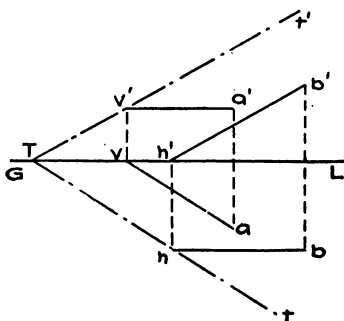


Fig. 19.

## TO PASS A PLANE THRU A GIVEN LINE

(43) Since by Art. 42, a line which lies in a plane  $T$ , pierces  $V$  and  $H$  in points which lie on the  $V$  and  $H$  traces of  $T$ , obviously the reverse of this proposition is true; i.e., if a plane passes thru or contains a given line,  $AB$ , its traces must pass thru the respective piercing points of  $AB$ , and unless drawn parallel to  $GL$ , these traces must intersect on  $GL$ .

(44) Problem — 2 — **To pass a plane thru two intersecting lines**

**Discussion:** (1)

Since the plane is to contain the two given intersecting lines, its traces will pass thru or contain the respective piercing points of these lines.

**Analysis:** (1)

*Find the  $H$  and  $V$  piercing points of the given intersecting lines and thru these piercing points pass the respective traces of the required plane.*

**Construction:**

Given: Intersecting lines  $AB$  and  $CD$ , Fig. 20.

Reqd: Traces of the plane of  $AB$  and  $CD$ .

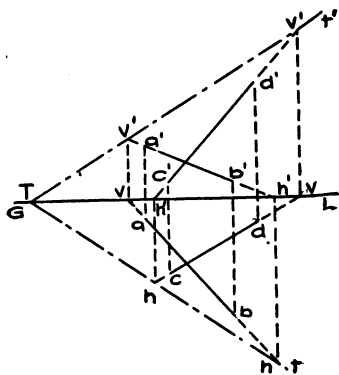


Fig. 20.

By Problem 1, the V piercing points of AB and CD are  $v'$  and  $v''$ , and the H piercing points are  $h$  and  $h'$ . The V trace of the required plane T, passes thru  $v'$  and  $v''$  and the H trace thru  $h$  and  $h'$ . The traces must intersect on GL.

**Discussion: (2)**

Frequently one or more of the desired piercing points are inaccessible, or perhaps all may coincide in a point on GL. If then a third line be drawn from some point of one given line to a point of the second given line, this auxiliary line will lie in the plane of the given lines.

**Analysis: (2)**

*Connect any convenient point of the one given line with any point of the second given line. The piercing points of this auxiliary line are points of the traces of the required plane.*

**(45) To pass a plane thru two parallel lines**

**Analysis:**

*Two lines cannot be parallel unless they lie in some common plane. The traces of this plane are drawn thru the respective piercing points of the given lines.*

**(46) To pass a plane thru three points**

**Analysis:**

*Connect any one of the given points with the other two by right lines. The plane of these two intersecting right lines is the required plane.*

**(47) To pass a plane thru a given point and given right line.**

**Analysis:**

*Connect the given point with any point of the given line by a right line. The plane of these two intersecting lines is the required plane.*

(48) **Thru one given line, to pass a plane parallel to a second given line.**

**Analysis:**

*If thru any point of the first given line, an auxiliary line parallel to the second given line be drawn, the plane of these two intersecting lines is the required plane.*

(49) **Thru a given point, to pass a plane parallel to a given line.**

**Analysis:**

*Thru the given point pass a line parallel to the given line. Any plane passing thru this line will pass thru the given point and be parallel to the given line.*

(50) **Thru a given point, to pass a plane parallel to two given lines.**

**Analysis:**

*Thru the given point pass two lines parallel respectively to the two given lines. The plane of these intersecting lines is the required plane.*

(51) **Thru a given point, to pass a plane parallel to a given plane.**

**Discussion:**

By Art. 37, if two planes are parallel their respective traces are parallel. Also we have learned that to pass a plane thru a given point, it is first necessary to pass a line thru the point and pass the plane thru the line. In this problem it is necessary that this auxiliary line be parallel to the given plane, otherwise it is impossible to pass

thru it a plane parallel to the given plane. e. g., if a line AB be drawn thru the given point perpendicular to the given plane, it would be impossible to pass thru AB, a plane parallel to the given plane.

### Analysis:

*Thru the given point, pass a line parallel to the given plane. The required plane will pass thru this line, its traces parallel respectively to the traces of the given plane.*

### Construction:

Given: Plane T and point O, Fig. 21.

Reqd: Plane thru O parallel to T.

Thru O, a line parallel to T is drawn; for the sake of simplicity a Horizontal is used. The V piercing point of this Horizontal is at  $v'$  and the V trace of the required plane S, passes thru  $v'$  parallel to  $Tt'$ , the H trace thru S parallel to  $Tt'$ .

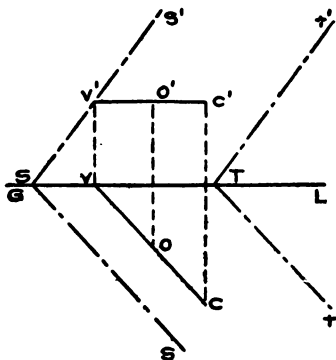


Fig. 21.

(52) Problem — 3 — **To find the line of intersection of two given intersecting planes.**

### Analysis:

*The line of intersection of two planes is a line common to both planes; therefore its H piercing point lies in the H trace of each plane, hence at their point of intersection. The V piercing point of the line of intersection, lies at the intersection of the V traces. The line joining these two piercing points is the required line of intersection.*



**Construction:**

Given: Planes S and T, Fig. 22.

Reqd: Line of intersection of S and T.

The point of intersection  $v'$ , of the V traces of S and T, is the required V piercing point of the line of intersection; its H projection,  $v$ , on GL. The H projection,  $h$ , of the H piercing point of the required line, is the intersection of  $S_s$  and  $T_t$ ; its V projection,  $h'$  is on GL;  $vh$  and  $v'h'$  are the H and V projections of the required line.

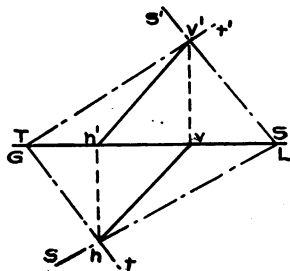


Fig. 22.

**CAUTION**

(53) As stated before, a large percentage of the difficulties experienced by students in Descriptive Geometry, is due to carelessness; fully one half of this carelessness is in the failure to properly letter the H and V piercing points of lines, and the lines of intersection of intersecting planes. For this reason each student will do well, to hold himself rigidly to the immediate and proper lettering of the lines of intersection of planes, and piercing points of lines, until it becomes a fixed habit. Except in a few special cases, which will be given, the line of intersection is always indicated by its piercing points, as in Fig. 22.

**SPECIAL CASES OF LINE OF INTERSECTION OF PLANES**

(54) 1—Either pair of traces do not intersect on limits of drawing.

**Analysis:**

*Draw an auxiliary plane parallel to the V plane. This*

plane will cut from the given planes, lines which will intersect in a point of the required line of intersection. A second such auxiliary plane will in the same manner give a second point on the required line. The line passing thru these two points is the required line of intersection.

**Construction:**

Given: Planes S and T,  
Fig. 23.

Reqd: Line of intersection.

The auxiliary plane R, parallel to V, cuts from S and T, right lines whose H projections coincide as  $hh$  and whose V projection,  $h'o'$  and  $h'o'$  are parallel respectively to the V traces of S and T. These lines intersect at O, a point on the required line. A second plane M cuts from S and T, two more lines which intersect at C, a second point on the required line. The line CO is the required line of intersection.

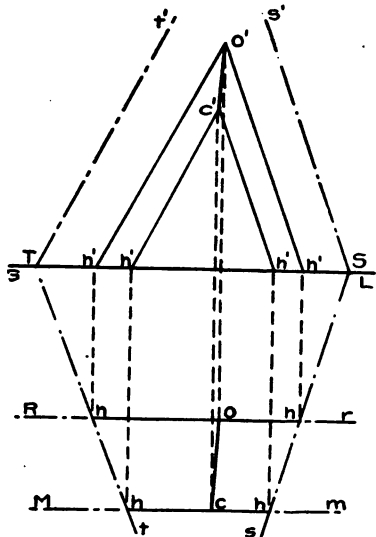


Fig. 23.

**2—All traces intersect on GL.**

**Discussion:**

In this case both the H and V piercing points of the line of intersection are at the intersection of the traces with GL; hence both projections of the line of intersection pass thru this point.

**Analysis:**

*In any convenient position draw an auxiliary plane parallel to one of the given planes, and find its line of intersection with the other given plane. The required line of intersection is parallel to this line.*

**Construction:**

Given: Planes S and T, Fig. 24.

Reqd: Line of intersection.

The auxiliary plane R is drawn parallel to T;  $v'h'$  and  $vh$  are the projections of the line of intersection of R and S;  $h'c'$  and  $hc$ , the projections of the required line of intersection, are drawn parallel to  $h'v'$  and  $hv$ .

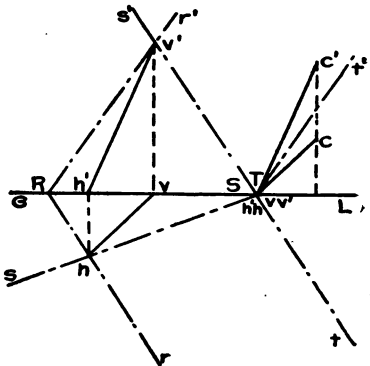


Fig. 24.

(55). Problem — 4 — To find the point in which a given right line pierces a given plane.

**Analysis:**

*Thru the given line pass an auxiliary plane, (preferably either the H or V projecting plane of the given line) and find its line of intersection with the given plane. Since the given line lies in the auxiliary plane, it must pierce the given plane in a point common to the two planes, i. e., in a point on their line of intersection. The intersection of the given line and this line of intersection is the required piercing point.*

**Construction:**

Given: Line AB and plane T, Fig. 25.

Reqd: Piercing point of AB in T.

Thru  $AB$  is passed the  $H$  projecting plane  $S$ . The projections of the line of intersection of  $S$  and  $T$  are  $h'v'$  and  $hv$ . Since the  $H$  projections,  $ab$  and  $hv$ , coincide, to find the point of intersection or piercing point,  $a'b'$  is produced until it intersects  $h'v'$  at point  $p'$ . The  $H$  projection  $p$  is on  $hv$  or  $ab$ .

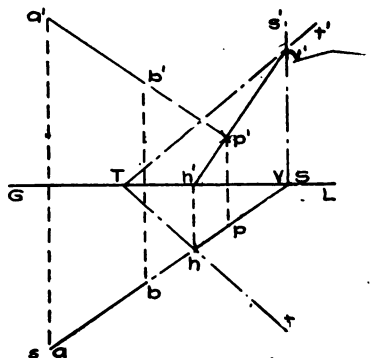


Fig. 25.

(56) Axiom: If a line is perpendicular to a plane, its projections are perpendicular to the traces of the plane.

**Proof:**

Given: Plane  $T$  and line  $AB$ , Fig. 26.

Thru  $AB$  is drawn the  $H$  projecting plane  $S$ ; its  $H$  trace,  $Ss$  coincides with  $ab$ , the  $H$  projection of  $AB$ . Plane  $S$  is perpendicular to  $T$  and also to  $H$ , hence to the line of intersection,  $Tt$ , of  $T$  and  $H$ ; we may then say that the  $H$  trace of  $Tt$  is perpendicular to plane  $S$ . If  $Tt$  is perpendicular to plane  $S$ , it is perpendicular to every line in plane  $S$ , passing thru its piercing point,  $h$ , in  $S$ .  $Ss$  is one of these lines; therefore  $Tt$  is perpendicular to  $Ss$  or to  $ab$ , which coincides with  $Ss$ . In

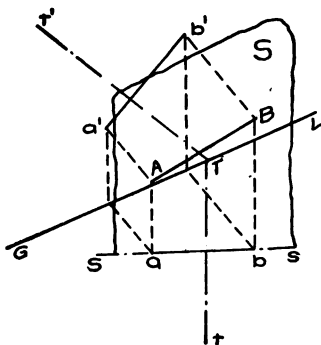


Fig. 26.

similar manner,  $a'b'$  can be proven perpendicular to the V trace,  $Tt'$ .

The converse of this proposition is true except in cases in which the projections of the line are perpendicular to GL.

(57) Problem —5—**To find the projection of a given line on a given plane.**

**Analysis:**

*Thru any two points of the given line, (preferably its extremities) drop perpendiculars to the given plane. The line connecting the piercing points of these perpendiculars in the given plane, is the required projection.*

**Construction:**

Given: Plane T and line AB, Fig. 27.

Reqd: Projection of AB on T.

From points A and B are drawn the perpendiculars to plane T. The projections of the piercing points in T are  $pp'$  and  $pp'$ ; the projections of the projection of AB on T are then  $p'p'$  and  $pp$ .

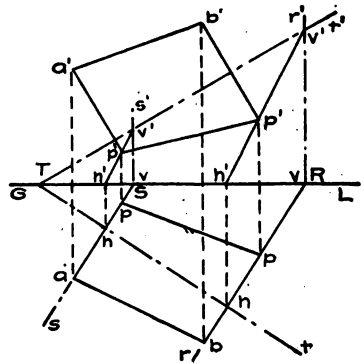


Fig. 27.

(58) Problem — 6 —

**Thru a given point, to pass a plane perpendicular to a given line.**

**Analysis:**

*Since the plane is to be perpendicular to the given line, its traces will be perpendicular respectively to the projections of the line, Art. 56. Then thru the given point pass a Horizontal and a Vertical of the required plane*

and find their *H* and *V* piercing points. Thru these piercing points, draw the traces of the plane, perpendicular to the corresponding projections of the given line.

### Construction:

Given: Line *AB* and point *O*, Fig. 28.

Reqd: Plane thru *O* perpendicular to *AB*.

Thru *O* pass the Horizontal, *OC*; its *H* projection, being parallel to the *H* trace of the required plane which is perpendicular to *ab*, is also perpendicular to *ab*; its *V* piercing point is at *v'*. Thru *v'* is drawn *Tt'*, perpendicular to *a'b'*, and thru *T* the *H* trace *Tt* is drawn perpendicular to *ab*.

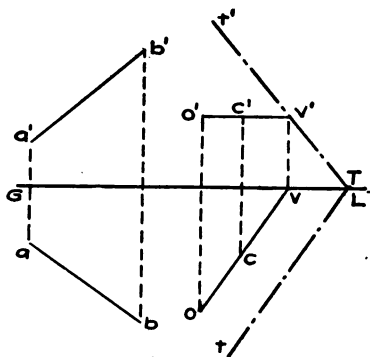


Fig. 28.

(59) Thru a given point, to pass a plane perpendicular to a given plane.

### Discussion:

The only requirement that a plane *T* must fill in order that it be perpendicular to a plane *S* is, that it contain one line perpendicular to plane *S*. If the respective traces of two planes are perpendicular to each other, the **PLANES CANNOT** be perpendicular to each other unless the one plane is parallel and the other perpendicular to *GL*.

### Analysis:

Thru the given point pass a line perpendicular to the

*given plane. Any plane passing thru this line will be perpendicular to the given plane.*

**(60) Thru a given line, to pass a plane perpendicular to a given plane.**

**Analysis:**

*Thru any point of the given line, pass a line perpendicular to the given plane. The plane of this perpendicular and the given line is the required plane.*

**(61) Thru a given point, to pass a plane perpendicular to two given planes.**

**Discussion: (1)**

If a plane T is perpendicular to a line AB which lies in a plane S, plane T is perpendicular to plane S.

**Analysis: (1)**

*Thru the given point, pass a plane perpendicular to the line of intersection of the two given planes.*

**Discussion: (2)**

If a plane S contains one line perpendicular to plane T, plane S is perpendicular to plane T.

**Analysis: (2)**

*Thru the given point pass two right lines perpendicular respectively to the given planes. The plane of these two lines is the required plane.*

## CHAPTER 5

### HIGHER RELATIONS OF THE POINT, LINE, AND PLANE

(62) Problem —7— To revolve a given point  $O$ , about a given line  $AB$ , which lies in the  $H$  plane, and find the points in which the path of  $O$  pierces the  $H$  and  $V$  planes.

#### Analysis:

*Thru point  $O$ , pass a plane perpendicular to the given line  $AB$ , Art. 58. It will intersect the line in a point which is the center of the circular path in which  $O$  travels.  $O$  will pierce  $H$  in the  $H$  trace of this plane and at a distance from the center of its path, equal to the radius of revolution.*

#### Construction:

Given: Point  $O$  and line  $AB$  in  $H$ , Fig. 29.

Reqd:  $H$  and  $V$  piercing points of path of  $O$ , as it revolves about  $AB$  as an axis.

Pass the plane  $T$  thru  $O$ , perpendicular to  $AB$ .  $Tt$  will be the  $H$  trace perpendicular to  $ab$ , and  $Tt'$  perpendicular to  $GL$ . Revolve this plane about its  $H$  trace as an axis, into the  $H$  plane.  $O$  will fall at  $o_1$  ( $oo_1$  being perpendicular to  $Tt$  and equal in length to the distance from  $o'$  to  $GL$ ). and  $o_1c$  is the radius of the circular path in which  $O$  moves. The length  $o_1c$ , will be seen to be the hypotenuse of a right triangle, having for one of its legs the distance from the  $H$  projection of the point to the axis ( $ab$ ), and for the other the distance from the  $V$  projection of the point to  $GL$ . (Why is this so?) (In case the axis is in the  $V$  plane, the words "Horizontal" and "Vertical" will simply be interchanged). Draw the circle with  $c$  as



a center. It cuts  $Tt$  at the points " $h$ ", which are the H projections of the piercing points in the H plane of the path of  $O$ , and show in their true positions since they are in the axis of revolution; (axis of revolution of the plane, not the point). Since these points lie in the H plane, their V projections fall in  $GL$ .

As the plane  $T$  is revolved into  $H$ , the V trace  $Tt'$  will take the position  $Tt'_1$ , being a line of the plane  $T$  and perpendicular to  $Tt$  at the point  $T$ . This cuts the

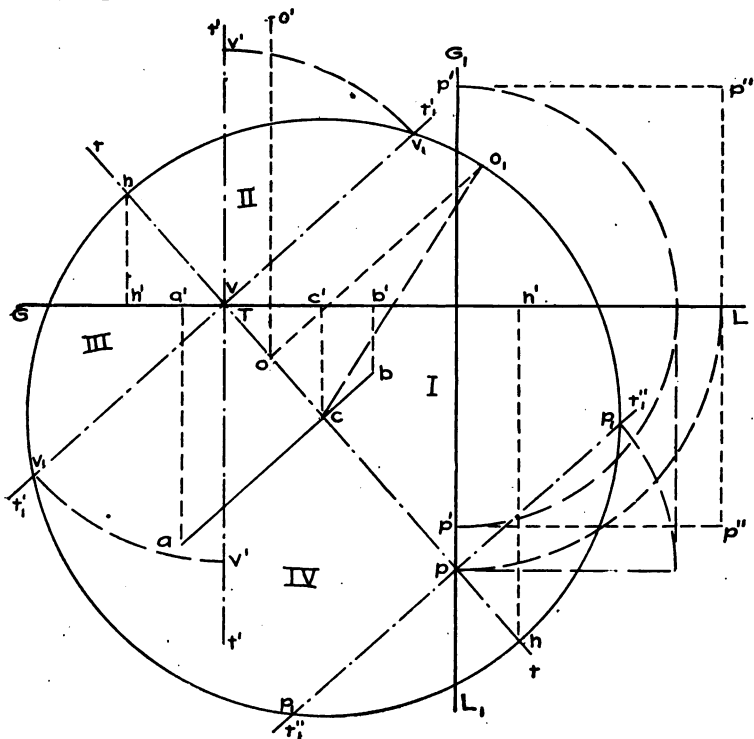


Fig. 29.

circle at the points  $v_1$  and  $v_1$  which must therefore be the revolved positions of the points in which the path of  $O$  pierces the  $V$  plane. Revolving plane  $T$  back into its original position, the  $v_1$  to the right, comes into its true position  $v'$ , on  $Tt'$ , a distance above  $GL$  equal to  $Tv_1$ , and the other point into the position  $v'$  the same distance below  $GL$ ; the  $H$  projections,  $v$ , both fall at  $T$ .

The construction for finding the points in which the path of  $O$  pierces  $P$  is also given in Fig. 29. The  $H$  projections of these two points coincide as  $p$ , similar to the

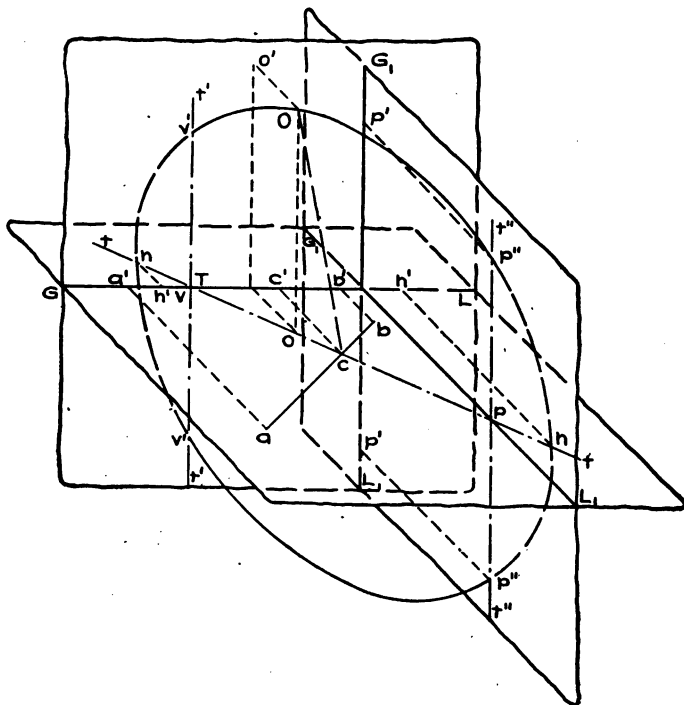


Fig. 30.

case of the V piercing points; the V projections of these piercing points are  $p'$  and  $p''$ , the Profile projections  $p''$  and  $p'''$ .

#### RULES OF REVOLUTION

(63) (1) **To determine the angles represented by the different sections of the circle.**

(1) The part of the circle in which the triangle is constructed is the same as that in which the point is found. To determine the other angles is then quite simple; e.g., suppose the angle determined by the triangle to be the *1st* angle; the opposite angle is the *3rd*. The circle being divided into four parts by the two plane traces, to move on the circumference of the circle from one angle to another, it is necessary to pass thru a plane trace, either H or V. The two angles adjacent to the first are the *2nd* and *4th*. In moving from the *1st* to the *4th* angle, we pass thru the H trace, and from the *1st* to the *2nd* thru a V trace. Thus the angles of the circle adjacent to the one first located are definitely determined. By reference to Figures 29 and 30, this explanation will be easily understood.

(2) In revolving a point into a position with reference to H or V, **never** measure any distance **along** or **parallel** to the trace which is the axis of the circle; but **always** measure given distances either **perpendicular** to this trace or **perpendicular** to **GL**.

(64) Problem — 7a — **To revolve a given point about a given line in H or V, thru a given angle "X", and find the projections of the point in this position.**

#### **Analysis:**

*Thru the given point, pass a plane perpendicular to the given line. The circle of revolution, whose radius*

*equals the distance from the point to the given line, lies in this plane. Move the point from its given position, along the circumference of the circle in the desired direction, until it has moved thru an arc of the circle equal to the given angle "X". The point is now in the desired' position and its projections may be found.*

**Construction:**

Given: Line AB and point O, Fig. 31.

Reqd: To revolve O about AB thru  $90^\circ$ , into the 4th angle.

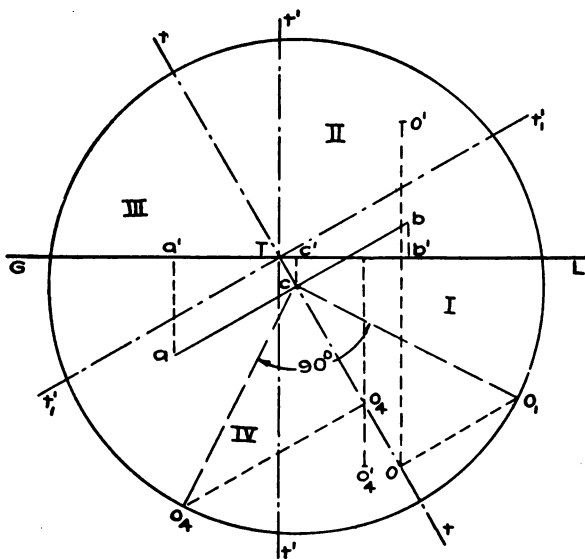


Fig. 31.

Pass plane T thru O, perpendicular to AB. Its H trace, Tt, passes thru O and perpendicular to  $ab$ , the V trace perpendicular to GL. Revolving this plane about Tt into H, and constructing the revolved position of the circle

of revolution as in the previous problem, we find  $O$  in the position  $o_1$ , in the first angle. Then moving  $o_1$  along the circumference of the circle in the direction of the 4th angle thru an arc of  $90^\circ$ , we find the required position at  $o_4$ . As  $O$  moves along the circle, its  $H$  projection moves along the  $H$  trace of  $T$ , always perpendicularly below or above the moving point. When  $O$  comes into the position  $o_4$ , its  $H$  projection is the foot of the perpendicular from  $o_4$  to  $Tt$ , or  $o_4$ . (To avoid an excess of notation, the position of the point on the circle, and its  $H$  projection are lettered alike; the sub numeral should correspond to the angle in which the point is, or into which it is revolved, e.g.,  $o_4$  for 4th angle, etc.) The distance  $o_4o_4$  represents the distance that  $O$  is below  $H$ ; dropping a projecting line from  $o_4$ , perpendicular to  $GL$ , and measuring below  $GL$  on this perpendicular, a distance equal to  $o_4o_4$ , the  $V$  projection  $o_4$  is located and the point determined.

(65) Problem — 7b — **To revolve a given point about a given line in  $H$  or  $V$ , into a position in a given angle and a given distance from  $H$  or  $V$ .**

**Discussion: (1)**

As a point revolves about a given line in  $H$ , the distances from the several positions of the moving point to  $H$ , are equal to the distances from these positions of the point to the  $H$  trace of the plane of revolution.

**Analysis:**

*Thru the given point, pass the plane of the circle of revolution perpendicular to the given line. Move the given point along the circumference of the circle, until it is in the given angle and the given distance from  $H$  or  $V$ .*

**Construction: (1)**

Given: Line AB and point O, Fig. 32.

Reqd: To revolve O about AB into the 4th angle,  
 $\frac{1}{2}$ " below H.

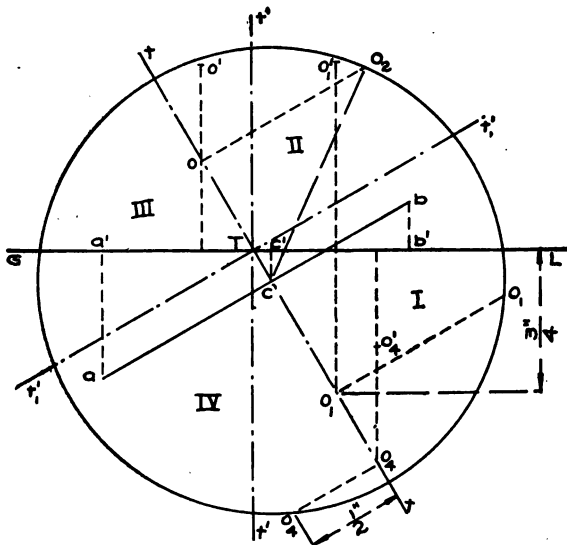


Fig. 32.

Thru O pass plane T perpendicular to AB; Tt passes thru o perpendicular to ab, Tt' perpendicular to GL. Revolving plane T about Tt into H and constructing the circle of revolution, radius  $o_2c$ , we find O in the position

Since plane T has been revolved into H about its H line, the distances from any points on the circumference of the circle to Tt represent the distances which the true positions of these same points are from H. To find the point of the circle, in the 4th angle, which is  $\frac{1}{2}$ " below H, erect a perpendicular to Tt,  $\frac{1}{2}$ " long and below Tt, and thru its extremity draw a line parallel to Tt; this

line intersects the circle at  $o_4$ , the required point. The H projection of  $o_4$  is  $o_4$ , and the V projection  $o'_4$ , below GL a distance equal to  $o_4o_4$ .

**Discussion: (2)**

As a point revolves about a line in H, no ordinates nor distances in the circle indicate the distances which points are from the V plane. However, we know that as the point revolves about the line, its H projection always lies in the H trace of the plane of revolution, and that the distances from the H projections of the moving point to GL, indicate the distances from the point to V.

**Construction: (2)**

Given: Line AB and point O, Fig. 32.

- . Reqd: To revolve O about AB into the 1st angle,  $\frac{3}{4}$ " before V.

Since the point is to be  $\frac{3}{4}$ " before V, its H projection will be  $\frac{3}{4}$ " below GL and on the H trace Tt. Erect a perpendicular to GL  $\frac{3}{4}$ " long, and thru its extremity draw a line parallel to GL until it intersects the H trace Tt at  $o_1$ , the H projection of the required position. Erecting a perpendicular to Tt at  $o_1$ , we find the position of the point on the circle at  $o_1$ ;  $o_1o_1$  is then the distance from this position of the point to H; the V projection,  $o'_1$ , will then be found above GL a distance equal to  $o_1o_1$ , and on the perpendicular to GL from  $o_1$ .

(66) Problem — 8 — To find the true length of a given line.

**Analysis: (1)**

*Revolve the given line about its H or V projection, into the corresponding coordinate plane. In this revolved position the line shows in its true length.*

**Construction: (1)**

Given: Line AB, Fig. 33.

Reqd: True Length of AB.

Revolving AB about its V projection into V, is equivalent to revolving the points A and B about  $a'b'$ , (a line in V) as an axis, until their paths pierce V at  $a_1$  and  $b_1$ , the radii of revolution being respectively the distances which the H projections,  $a$  and  $b$ , are from GL. In actual construction this amounts merely to the erecting of perpendiculars to  $a'b'$ , at  $a'$  and  $b'$ , equal in length to the distances from  $a$  and  $b$  to GL. The line  $a_1b_1$ , connecting their extremities, is the required revolved line and *true length*.

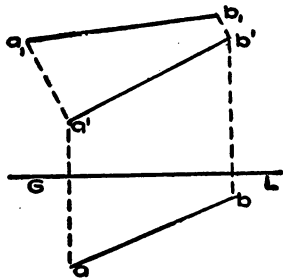


Fig. 33.

**Rule**—To find the true length of a given line, to the V projection at its extremities, erect perpendiculars, equal in length to the distances from the H projections of these points to GL. The line joining the extremities of these perpendiculars is the required true length.

**Analysis: (2)**

*Revolve the given line about the H projecting line of one extremity until it is parallel to V; its true length will then be shown on the V plane.*



**Construction: (2)**

Given: Line AB, Fig. 34.

Reqd: True length of AB.

As AB revolves about the H projecting line of B, the H projection  $ab$  moves with  $b$  as a center into the position  $a_1b$  parallel to GL. A remains the same distance above H in this revolution, therefore its V projection  $a'$  moves from  $a'$  along a parallel to GL to  $a'_1$ .  $a'_1b'$  is the required true length.

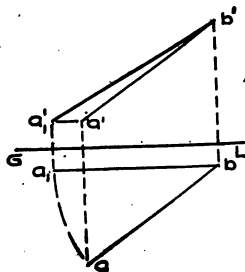


Fig. 34.

(67) Problem (9)—To find the distance from a given point to a given plane.

**Analysis:**

*From the given point drop a perpendicular to the given plane and find its piercing point in the plane. The true length of the portion of the perpendicular between the given point and the piercing point is the required distance.*

**Construction:**

Given: Plane T and point O, Fig. 35.

Reqd: Distance from O to T.

Thru O draw the perpendicular to plane T;  $p$  and  $p'$  are the projections of its piercing points in T;  $o'p'_1$  is the

true length of  $OP$  and the required distance.

(68) To find the distance between two parallel planes.

**Analysis:**

From any point in the one plane  $S$ , drop a perpendicular to plane  $T$  and find its piercing point in  $T$ . The true length of the perpendicular from the assumed point in  $S$  to the piercing point in  $T$  is the required distance.

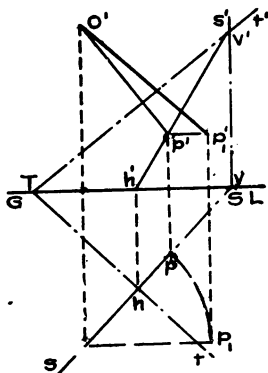


Fig. 35.

(69) Problem —10—To find the distance from a given point to a given line.

**Analysis:**

Pass a plane thru the given point and line and revolve this plane about either of its traces into the corresponding coordinate plane; the perpendicular distance from the revolved position of the point to the revolved position of the line is the required distance.

**Construction:**

Given: Line  $AB$  and point  $O$ , Fig. 36.

Reqd: Distances from  $O$  to  $AB$ .

Plane  $T$ , of the given point and line, is located by means of  $AB$  and the auxiliary line  $OA$ . Its  $H$  trace is  $Tt$ . Revolving plane  $T$  about  $Tt$  into  $H$ ,  $O$  falls at  $o_1$ ,

and  $AB$  at  $a_1b_1$ ; the perpendicular  $o_1m_1$  from  $o_1'$  to  $a_1b_1$ , is the required true distance. The projections of this perpendicular are  $om$  and  $o'm'$ ;  $m$  being located from  $m_1$  by the intersection with  $ab$  of a perpendicular to  $Tt$  from  $m_1$ .

(70) To find the true distance between two given parallel lines.

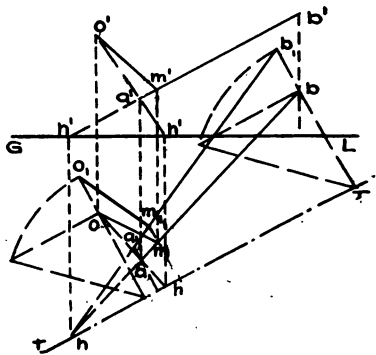


Fig. 36.

### Analysis:

*If the plane of the two parallel lines be revolved about either of its traces into the corresponding coordinate plane, the lines will show in their true relative positions. The perpendicular distance between the revolved positions of the parallel lines is the required distance.*

(71) Problem — 11 — To find the projections and true length of the common perpendicular to two non intersecting and non parallel lines  $AB$  and  $CD$ :

### Analysis:

*Thru the line  $AB$  pass a plane  $T$ , parallel to  $CD$ , and find the projection of  $CD$  on plane  $T$ . The perpendicular to plane  $T$ , at the point in which the projection of  $CD$  on  $T$  intersects  $AB$ , will intersect  $CD$  and is the required common perpendicular.*

### Construction:

Given: Lines  $AB$  and  $CD$ , Fig. 37.

Reqd: Common Perpendicular to  $AB$  and  $CD$ .

Thru point  $A$  pass  $AE$  parallel to  $CD$ ; the plane of

$AB$  and  $AE$  is the required plane thru  $AB$  parallel to  $CD$ . From  $C$  and  $D$  drop perpendiculars to  $T$ ; their piercing points are  $P$  and  $P'$ , and the projection of  $CD$  on  $T$  is  $p'p'$ ,  $pp$ . At point  $O$ , at which this projection intersects  $AB$ , erect the required perpendicular to  $T$ ; it intersects  $CD$  at  $M$ .  $o'm'$  and  $om$  are then the projections, and  $o'm_1'$  the true length of the common perpendicular.

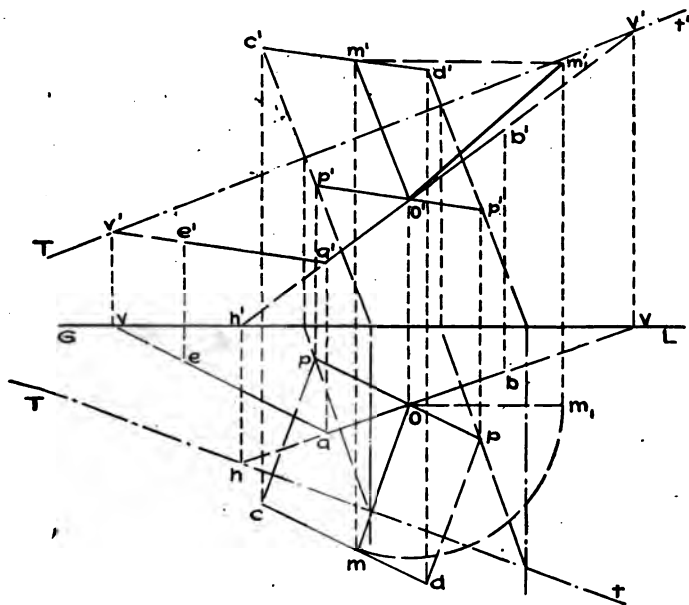


Fig. 37.

(72) Problem —12—To find the true value of the angle "X" between two intersecting lines.

### Analysis:

Pass a plane thru the two intersecting lines and revolve this plane about either of its traces into the corres-

ponding coordinate plane. In this revolution the relative positions of the lines do not change and the angle between the revolved positions of the given lines is the true value of the required angle "X".

### Construction:

Given: Lines AB and CD, Fig. 38.

Reqd: Angle "X".

The lines AB and CD pierce V at  $v'$  and  $v'$ ; the V trace,  $Rr'$ , of plane R passes thru  $v'$  and  $v'$ ; AB and CD pierce H at  $h$  and  $h$ . Revolve plane R about  $Rr'$  into V; point O falls at  $o_1$ ; the points  $v'$  and  $v'$  of AB and CD, being on the axis of revolution, are fixed; hence the lines AB and CD revolve into the positions  $o_1v'$  and  $o_1v'$ ;  $v'o_1v'$  is then the true value of angle "X".

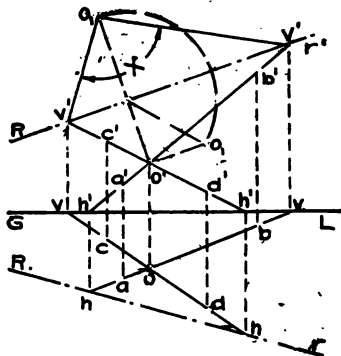


Fig. 38.

(73) To find the projections of the bisector of the angle "X" between two intersecting lines.

Caution—The bisectors of the projections of the angle are NOT the projections of the bisector.

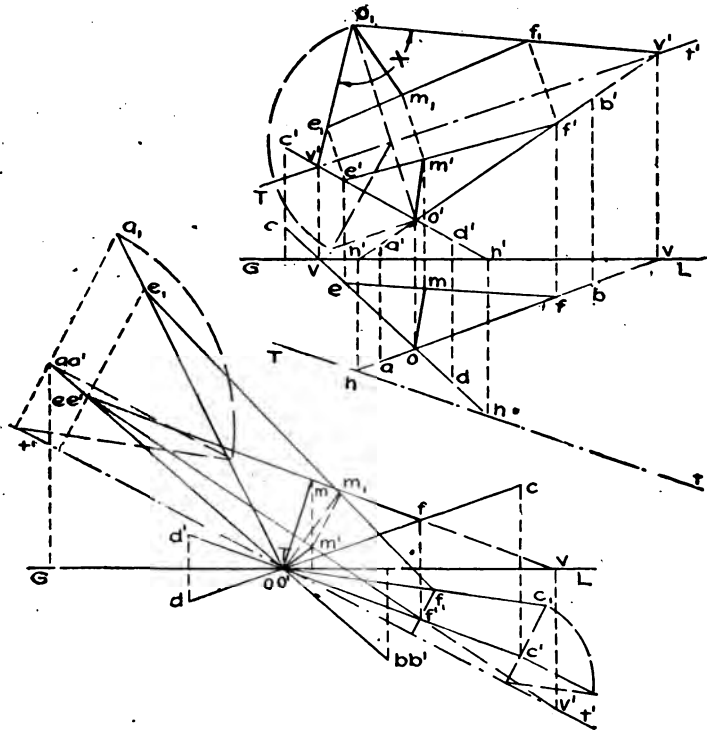
### Analysis:

If the plane of the given lines be revolved about either of its traces into the corresponding coordinate plane, the angle "X" will show in its true value and may be bisected by a right line. If any point of this bisector be revolved back into its true position, a right line thru this point and the vertex is the required bisector.

**Construction:**

**Given:** Lines AB and CD, Fig. 39.

**Reqd:** Bisector of angle "X".



**Fig. 39.**

The traces of the plane T of AB and CD are respectively  $Tt'$  and  $Tt$ . Connect any point E on CD with any point F on AB by a right line. Revolving plane T about  $Tt'$  into V, E falls at  $e_1$ , OE into the position  $o_1e_1$  and OF into the position  $o_1f_1$ ;  $e_1o_1f_1$  is the true value of angle "X". The bisector of angle  $e_1o_1f_1$  intersects  $e_1f_1$  at  $m_1$  a point

of the required bisector; the V projection of M falls at  $m'$  on  $e'f'$  and the H projection at  $m$ ;  $o'm'$  and  $om$  are the projections of the required bisector.

(74) Problem—13—**Thru a given point on a given line to pass a line making a given angle with the given line.**

**Analysis:**

*If an H projecting plane be passed thru the given line and revolved about its H trace into H, the revolved position of the required line may be drawn making the given angle with the revolved position of the given line. If then we revolve the H projecting plane and the line just drawn back into their true positions the required line is determined.*

(75) Problem—14—**To find the angle "X" which a given line makes with a given plane.**

**Discussion:**

The angle which a line makes with a plane is the angle which the line makes with its projection on that plane. If from some point of the line we drop a perpendicular to the plane, it will pierce the plane in the projection of the line; the given line, its projection on the plane, and the perpendicular form a right triangle of which one acute angle is the required angle "X", and the other the complement of angle "X". Since it is easier to find the angle "X" by finding its complement between the given line and a perpendicular than to find "X" directly, this method will be used.

**Analysis:**

*Thru any point of the given line drop a perpendicular to the given plane. The complement of the angle between*

*the given line and this perpendicular is the required angle "X".*

**Construction:**

Given: Plane T and line AB, Fig. 40.

Reqd: Angle "X":

Thru point B on AB drop the perpendicular BC to plane T. The V trace of the plane of AB and BC is  $Ss'$ ; revolve plane S about  $Ss'$  into V; AB falls at  $a_1b_1$ , BC at  $b_1c_1$ , and  $a_1b_1c_1$  is the true value of angle ABC. Its complement,  $m_1b_1c_1$ , is the required angle "X".

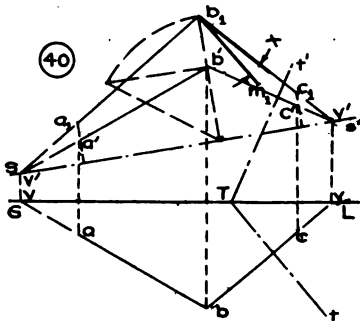


Fig. 40.

(76) To find the angles "Θ" and "Φ" of a given line.

**Discussion:**

Since the angle "Θ" (Φ) of a given line is the angle between the line and its H (V) projection, either of the two methods for finding the true length of a line will obtain the angle "Θ" (Φ).

**To Find Angle "Θ".**

**Analysis:**

(1) Revolve the given line about its H projection into the H plane; the angle between the revolved position of the line and the H projection is the required angle "Θ".

(2) Revolve the given line about the H projecting line of one extremity until the line is parallel to V; the angle is then projected in its true value on the V plane.



(77) The “ $\Theta$ ” Cone; the “ $\Phi$ ” Cone.

A “ $\Theta$ ” cone is a *right circular cone* whose axis is *perpendicular* to *H* and whose *base angle* is equal to angle “ $\Theta$ ” of a *line* or *plane*, either given or required.

A “ $\Phi$ ” cone is a *right circular cone* whose axis is *perpendicular* to *V* and whose *base angle* is equal to angle “ $\Phi$ ” of a *line* or *plane*.

(78) Problem—15—Given one projection of a line and the angle “ $\Theta$ ” or “ $\Phi$ ”; required the other projection.

**Analysis:**

(e.g. Given angle “ $\Theta$ ”)

If we make any point of the given line the apex of a right circular cone whose axis is perpendicular to *H* and whose base angle is equal to “ $\Theta$ ”, the given line must be an element of this cone and is determined.

**Construction:**

Given: *V* projection of line *AB* and “ $\Theta$ ” =  $45^\circ$ , Fig. 41.

Reqd: *H* projection of *AB*.

Select any point *O* on *AB* as the apex of the “ $\Theta$ ” cone; the *H* projection of *O* may be at any distance above or below *GL* on the perpendicular from *o'* to *GL*. The *V* projection of the “ $\Theta$ ” cone is the isosceles triangle whose vertex is *o'*, base angles  $45^\circ$ , and the *H* projection is the circle whose center is *o* and whose diameter is equal to the base of the triangle.

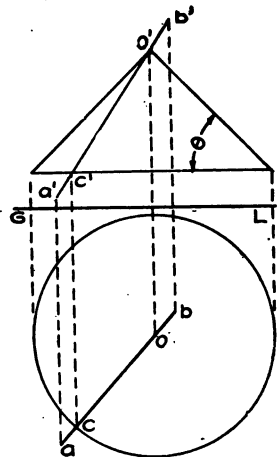


Fig. 41.

Since the line is an element of this cone the point  $c'$  in which  $a'b'$  intersects the base of the triangle is the V projection of the point in which the element AB intersects the base of the cone; its H projection is at  $c$  and the H projection of AB is determined.

(79) In a given plane to draw a line making a given angle " $\Theta$ " or " $\Phi$ " with H or V.

**Analysis:**

(Given angle " $\Theta$ ")

*Select any point in the given plane as the apex of a right circular cone whose axis is perpendicular to H, base angle equal to " $\Theta$ ", and base resting in H. Either of the elements cut from this cone by the given plane will be the required line.*

(80) Problem—16—Thru a given point to pass a line making given angles " $\Theta$ " and " $\Phi$ " with H and V.

**Analysis:**

*Make the given point the apex of two right circular cones whose elements are of the same length; the one cone with its axis perpendicular to H and base angle equal to " $\Theta$ ", the other with its axis perpendicular to V and base angle equal to " $\Phi$ ". Either of the lines of intersection of the surfaces of these two cones is the required line.*

**Construction:**

Given: Point O, " $\Theta$ "=45°, " $\Phi$ "=30°, Fig. 42.

Reqd: Line thru O at " $\Theta$ " and " $\Phi$ " to H and V.

Select any length (e.g. 1") as the common length of elements. With  $O$  as the apex draw the " $\Theta$ " cone, its elements 1" long; the  $V$  projection is the isosceles triangle whose vertex is  $o'$  and base angle  $45^\circ$ , its  $H$  projection the circle whose center is  $o$  and diameter equal to base of triangle. Then with  $O$  as the apex draw the " $\Phi$ " cone, its elements also 1" long; its  $H$  projection is the isosceles triangle whose vertex is  $o$  and base angle  $30^\circ$ , its  $V$  projection the circle whose center is  $o'$  and diameter equal to base of  $30^\circ$  triangle. Since the elements of these two cones are equal, the circumferences of their bases lie on the surface of the same sphere (whose center is  $O$  and radius 1") and intersect in points on the required lines of intersection of the cones.  $cc'$  and  $dd'$  are these points and  $CO$  and  $DO$  are the required lines.

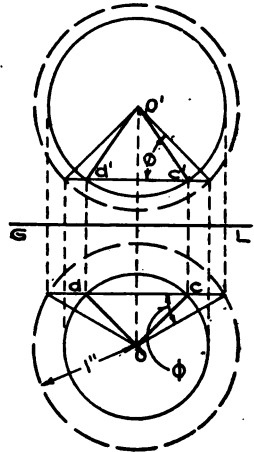


Fig. 42.

(81) Problem—17—To find the angle " $X$ " between two intersecting planes.

**Discussion:** (1)

The angle between two intersecting planes is measured by the angle between two right lines, one in each plane and perpendicular to the line of intersection at the same point. These two lines may be considered lines cut from the given planes by a third plane which is perpendicular to their line of intersection. The construction will be greatly simplified and the problem more easily understood if this auxiliary plane be drawn thru some

chosen point on the line of intersection rather than at random.

### Analysis:

*Thru any chosen point on the line of intersection of the given planes pass a plane perpendicular to this line. The angle between the lines cut by this plane from the given planes is the required angle "X".*

### Construction:

Given: Planes S and T, Fig. 43.

Reqd: Angle "X".

The projections of the line of intersection of planes S and T are  $v'h'$  and  $vh$ . Select any point O on this line and thru O pass plane R perpendicular to VH; its H trace is  $Rr$ ; the V trace is not required and is not drawn. The point of intersection  $h$  of  $Ss$  and  $Rr$  is the H piercing point of the intersection of S and R, and the point of intersection  $h$  of  $Rr$  and  $Tt$  is the H piercing point of the intersection of R and T;  $oh$  and  $oh$  are then the H projections of the lines cut by R, S and T. Revolve plane R about  $Rr$  into H; O falls at  $o_1$  and  $ho_1h$  is the true value of the required angle "X".

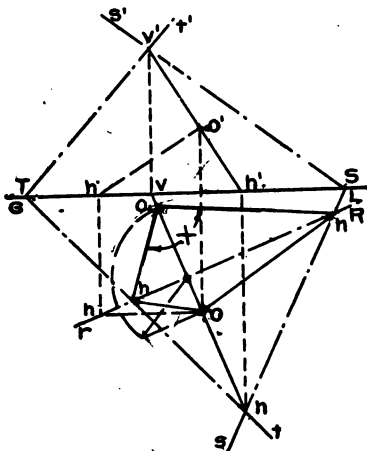


Fig. 43.

### (82) Discussion: (2)

If thru any point in space lines be drawn perpendicu-

lar respectively to the two intersecting planes and a plane be passed thru these perpendiculars, it will cut from the given planes right lines which with the two perpendiculars form a quadrilateral; two of the angles of this quadrilateral are right angles, therefore the remaining angles, i. e. the angle between the perpendiculars and the angle between the two lines lying in the planes, (angle "X"), are supplementary.

However the two intersecting planes make with each other **two** angles "X" which are supplementary; therefore the angle between the perpendiculars, being the supplement of one of these angles "X", is equal to the other.

### Analysis:

*Thru any chosen point in space pass two lines perpendicular respectively to the given planes. The angle between these lines is one of the required angles "X".*

### Construction:

Given: Planes S and T, Fig. 44.

Reqd: Angle "X".

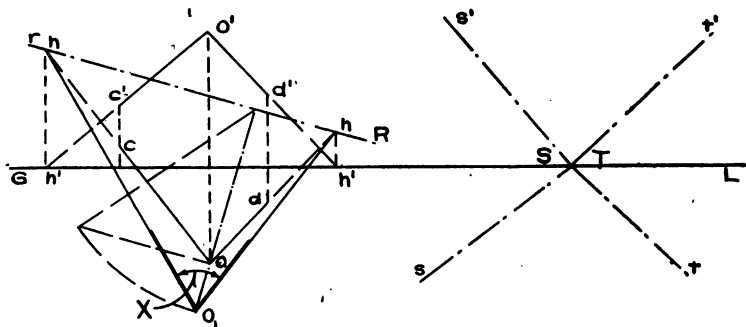


Fig. 44.

Assume point O in any convenient position and thru it pass OC perpendicular to S, and OD perpendicular

to T.  $Rr$  is the H trace of the plane of COD. Revolve plane R about  $Rr$  into H.  $ho_1h$  is the true value of COD, one of the required angles "X".

(83) Thru a given line in a given plane T, to pass a plane S making a given angle "X" with the given plane.

**Analysis:**

*Thru any point O of the given line pass a plane R perpendicular to this line and find its line of intersection with the given plane T; then thru point O draw a line in plane R making angle "X" with the line of intersection just found. The plane of this line and the given line is the required plane S.*

**Construction:**

Given: Plane T and line VH, Fig. 45.

Reqd: Plane thru VH making  $90^\circ$  with T.

Thru any point O on VH pass plane R perpendicular to VH; its H trace  $Rr$  passes thru  $h$  of the Vertical and is perpendicular to  $vh$ .  $oh$  is the H projection of the line of intersection of R and T. Revolving plane R about  $Rr$  into H, O falls at  $o_1$  and  $ho_1$  is the revolved position of the intersection of R and T. Thru  $o_1$  draw  $o_1h$  making  $90^\circ$  with  $o_1h$ ; since OH is a line of plane R its H piercing point lies on  $Rr$ , and at the point  $h$ . If a plane pass thru

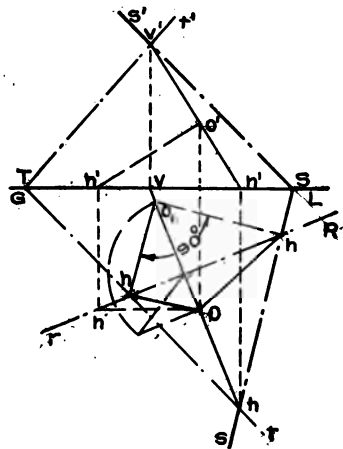


Fig. 45.

If a plane pass thru

OH its H trace will pass thru this point  $h$ ; the H trace of the required plane then passes thru  $h$  of line OH and  $h$  of line VH; the V trace thru S and  $v'$  of VH.

(84) To find the angles " $\Theta$ " and " $\Phi$ " of a given plane.

### Analysis:

(e.g. Reqd. " $\Theta$ ")

If we make any point of the V trace of the given plane, the apex of a right circular cone which is tangent to the given plane, its axis perpendicular to H and its base resting on H, the base angle of this cone is equal to the required angle " $\Theta$ ".

For angle " $\Phi$ " substitute in the above analysis, H for V, V for H, and " $\Phi$ " for " $\Theta$ ".

### Construction:

Given: Plane T, Fig. 46.

Reqd: " $\Theta$ " and " $\Phi$ ".

Select any point O on the V trace as the apex of the right cone; the center of its base is at  $o$  and the radius of the base is the perpendicular distance from  $o$  to Tt. The H projection of this cone is then the circle whose center is  $o$  and radius  $oh$ ; the V projection is the isosceles triangle whose vertex is  $o'$  and base equal to the diameter of circle. The base angle of this triangle is the required angle " $\Theta$ ".

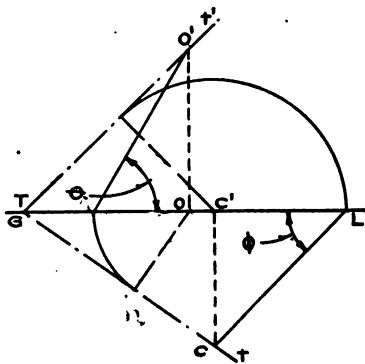


Fig. 46.

base angle of this triangle

(85) Problem—18—Given one trace of a plane and the angle “ $\Theta$ ” or “ $\Phi$ ”, required the other trace.

**Analysis:**

(e.g. Given V trace and “ $\Theta$ ”.)

Select any point on the given V trace as the apex of a right circular cone which is tangent to the plane, its axis perpendicular to H, base angle equal to “ $\Theta$ ”, and base resting in H. The V trace of the plane passes thru the apex of this cone and the H trace is tangent to the circumference of the base.

(Given V trace and “ $\Phi$ ”.)

Select any point on the given V trace as the point of tangency with the base of the “ $\Phi$ ” cone which is tangent to the plane, axis in H and perpendicular to V, base resting in V. The required H trace will pass thru the apex of this cone and the intersection of the given trace with GL.

**Construction:**

Given: V trace of plane T and angle “ $\Theta$ ”, Fig. 47.

Reqd: H trace of Plane T.

Select any point  $o'$  on  $Tt'$  as the V projection of the apex of the “ $\Theta$ ” cone  $o'c'd'$  is the V projection of this cone, the circle  $cod$  the H projection. Since the base of the cone rests in the H plane the required H trace  $Tt$  is tangent to the circumference of the base. Hence  $Tt$  is drawn thru T and tangent to the circle  $cod$ .

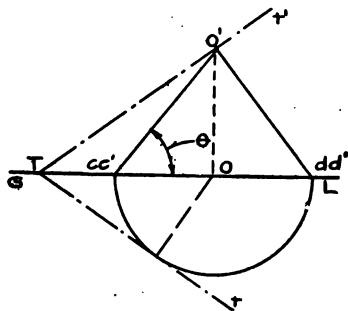


Fig. 47.



(86) Problem—19—To draw the traces of a plane which makes given angles " $\Theta$ " and " $\Phi$ " with H and V.

**Analysis:**

Assume any point on GL as the center of a sphere of any radius. Then draw, tangent to this sphere, two right circular cones; the first with its axis in V and perpendicular to H, the base resting in H, and base angle equal to " $\Theta$ ". (Any plane tangent to this cone will make with H an angle equal to " $\Theta$ "). The second cone with its axis in H and perpendicular to V, its base in V and base angle equal to " $\Phi$ ". (Any plane tangent to this cone will make with V an angle equal to " $\Phi$ "). Since both cones are tangent to the same sphere it is possible to draw a plane tangent to the surfaces of both; this plane is the required plane.

**Construction:**

Given: Angles " $\Theta$ " and " $\Phi$ ", Fig. 48.

Reqd: Traces of plane T.

Assume the center of the sphere at  $oo'$  and the diameter of any length. The projections of the first cone (axis in V and perpendicular to H, base in H, and base angle equal to " $\Theta$ ") are: V projection the triangle  $v'd'c'$  tangent

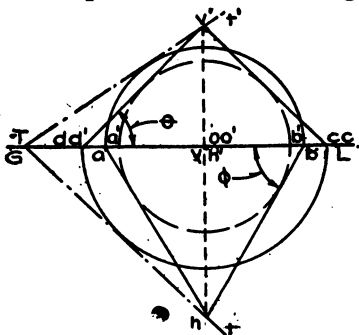


Fig. 48.

to the V projection of the sphere; H projection the circle whose center is  $v$  and radius  $vd$ . Since the vertex of this cone is in V and base in H, any plane tangent to it will have its V trace passing thru the V projection of the vertex and its H trace tangent to the circle which is the H projection of the base of the cone.

The projections of the second cone (axis in H and perpendicular to V, base in V and base angle equal to " $\Phi$ ") are: H projection the triangle  $hab$  tangent to the H projection of the sphere; V projection the circle whose center is  $h'$  and radius  $h'b'$ . Since the vertex of this cone is in H and the base in V, any plane tangent to it will have its H trace passing thru the H projection of the apex and its V trace tangent to the circle which is the V projection of the base of the cone.

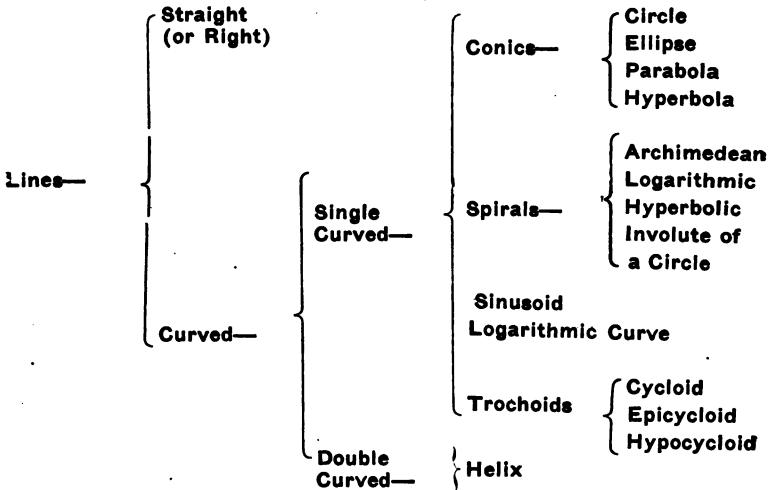
From the two deductions above the V trace of the required plane, which is tangent to both cones, passes thru  $v'$  tangent to the small circle  $h'a'b'$ ; the H trace passes thru T and  $h$  and is tangent to the large circle  $vde$ ;  $tTt'$  is the required plane.

## PART 2

### CHAPTER 6

#### LINES AND SURFACES

#### CLASSIFICATION OF LINES



(87) *The projections of a curve are composed of the projections of the component points of the curve. If all the points of a curve be projected upon the H plane and these projections joined by lines the resultant curve is the H projection of the given curve.*

Likewise if the V projections of all points of a curve be joined by lines the resultant curve is the V projection of the given curve.

(88) *The two projections of a curve being given, the curve will in general be completely determined. For if*

from some point in one projection of a curve a perpendicular to GL be produced until it intersects the other projection of the curve and perpendiculars be erected to H and V at these two projections just located, the intersection of these perpendiculars is the true position of that point in space.

(89) If the plane of a curve of single curvature is perpendicular to H (V), the projection of the curve on that plane will be a right line coinciding with the H (V) trace of the plane.

If the plane of the curve be perpendicular to GL both projections of the curve will be right lines and perpendicular to GL.

If the plane of a curve be parallel to either H or V its projection on that plane will be an identical curve, the other projection a right line parallel to GL.

#### TANGENTS AND NORMALS TO CURVES OF SINGLE CURVATURE.

(90) A line which lies in the plane of a given curve and has two consecutive points in common with it, is called a **tangent** to that curve.

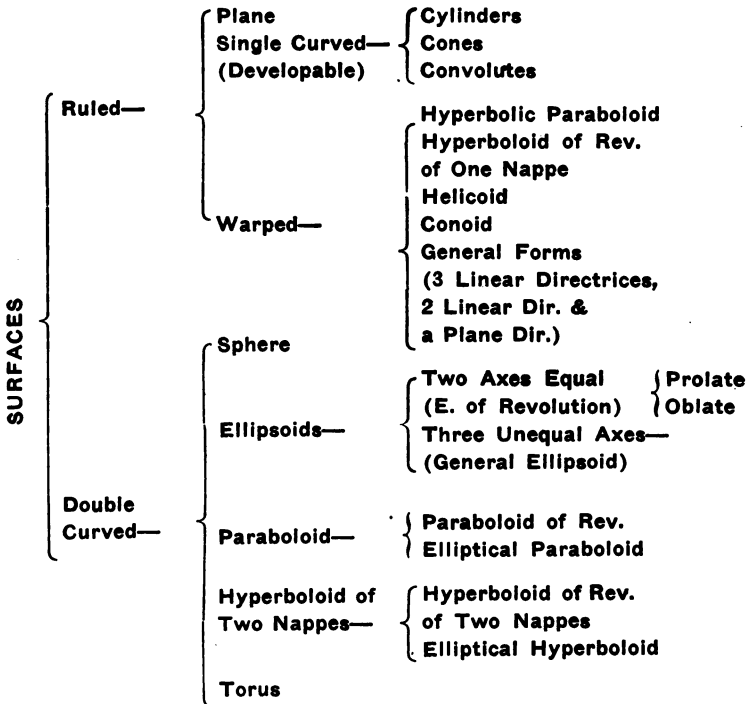
If then a second line be drawn in the plane of the curve and perpendicular to the tangent line at one of these consecutive points of tangency we have a **normal** of the curve.

If two curves lie in the same plane and have two consecutive points in common or are tangent to the same right line at the same point they are called **tangent curves**.

(91) If two lines are tangent in space their projections are tangent. For if H projecting lines be dropped from the several points of the two curves only the lines from the two common consecutive points coincide; hence the H projections of the curves can have only two points in

common and these points consecutive; therefore the H projections of the given curves are tangent. Likewise the V projections; and the *projections of the consecutive points of tangency are the points of tangency of the projections.*

**CLASSIFICATION OF SURFACES**



SURFACES—(1) Ruled (2) Double Curved

(92) — 1 — **Ruled Surfaces**—*Generated by the motion of a right line.*

(a) **Planes**—*Generated by a right line moving so as*

to touch another right line and having all of its positions parallel to the first position.

(b) **Single Curved**—Generated by a right line, moving so that it constantly touches a given curve and any two consecutive positions are in the same plane.

Single curved surfaces are divided into three classes, two of which are:

**Cylinders**—All positions of the generating line parallel.

**Cones**—All positions of the generating line intersecting in a common point.

#### CYLINDERS

(93) A cylinder may be generated by the motion of a right line which is always parallel to its first position and constantly touches a given curve.

If the cylinder be intersected by a plane the *curve of intersection* may be considered the **base** of the *cylinder*. In general this plane is a horizontal plane.

If we wish to consider a definite portion of the surface a second plane is drawn parallel to the first, cutting a second base from the cylinder, one being called the **lower base**, the other the **upper base**. The portion of the surface between these two bases, with the two bases is termed the "**cylinder**".

If the *elements* of the *cylinder* are *perpendicular* to the *plane* of the *base*, the cylinder is a **right cylinder** and the base a **right section**.

A cylinder whose *elements* are *oblique* to the *plane* of the *base* is termed an **oblique cylinder**.

(94) A cylinder is represented in general by one or more curves of its surface or bases; the *projections* of the *cylinder* are then represented by the *projections* of these *bases* and the *principle* or *limiting elements*.

(95) **To assume a point on the surface of a cylinder.**

### Analysis:

*First assume one projection of the point, e.g. the H projection and thru it draw a right line parallel to the elements of the cylinder. This right line is the H projection of the element which passes thru the point. The V projection of this element may then be determined and the V projection of the point.*

### CONES

(96) Cones are generated by the motion of a right line which constantly passes thru a given point and touches a given curve.

The *given curve* is called the **directrix**, the *given point* the **apex** and the different *positions* of the *moving line* the **elements** of the cone

If a plane be drawn *intersecting all* the *elements* of the cone, this *curve of intersection* is called the **base** of the cone; the cone may then take its *name* from the *shape* of this *base*, e.g. **circular, elliptical**, etc.

If *all* the *elements* of the cone make the *same angle* with a *right line passing thru* the *apex*, the cone is termed a **right cone** and the *right line passing thru* the *apex*, the **axis**.

(97) **To assume a point on the surface of a cone.**

**Analysis:**

Assume either projection of the point, e.g. the *H* projection and thru it draw a right line to the *H* projection of the apex of the cone. This right line is the *H* projection of the element passing thru the point. The *V* projection of the element may then be determined, and the *V* projection of the point.

(98) — 2 — **Double Curved Surfaces**—Generated by the motion of a curve of single curvature.

**SURFACES OF REVOLUTION**

A surface of revolution may be generated by revolving a given curve about a given right line as an axis.

From the manner of generation of this curve we see that any plane perpendicular to the axis of revolution cuts from the surface a circle.

**(99) SPHERES AND ELLIPSOIDS OF REVOLUTION.**

If a circle be revolved about one of its diameters as an axis the surface generated will be a sphere.

If an ellipse be revolved about its major axis the surface generated is a prolate ellipsoid of revolution. If revolved about the minor axis the surface is an oblate ellipsoid of revolution.

(100) To assume a point on the surface of a sphere or ellipsoid of revolution.

**Analysis:**

Draw an auxiliary plane perpendicular to the axis of revolution. This plane cuts from the surface a circle whose projections are easily determined. Any point of this circle will lie on the given surface.



## CHAPTER 7

### PLANES TANGENT TO SINGLE AND DOUBLE CURVED SURFACES

(101) Heretofore planes have been determined in various ways, e.g., as perpendicular to a given line, parallel to a given plane, etc. In dealing with planes tangent to surfaces however, invariably they will be determined by two intersecting lines. When the surfaces are *single curved* these two intersecting lines will be tangent to the surface at points on the same element. When *double curved*, the two intersecting lines will be tangent to the surface at the same point, i.e., their point of intersection.

(102) Problem—20—Thru a given point on the surface of a cone to pass a plane tangent to the surface of the cone.

#### Analysis:

*Thru the given point pass the element of the cone and produce this element until it pierces the plane of the base. Thru this piercing point draw a line tangent to the base of the cone. The required plane is determined by this tangent line and the element.*

#### Construction:

Given: Cone X and the V projection of the point O on surface of cone. Fig. 49.

Reqd: Plane thru O tangent to cone.

Thru  $o'$  and  $x'$  draw the V projection of the element of the cone and produce until it intersects the base at  $p'$ ;  $p'$  is the V projection of the point in which the element  $OX$  pierces the plane of the base. Its H projection is at  $p$ ; thru  $p$  draw the H projection  $ph$  of the tangent line, tangent to the circle; since this tangent line lies in the plane of the base its V projection  $p'h'$  coincides with the V projection of the base of the cone. The required plane  $R$  passes thru the two intersecting lines  $OP$  and  $PH$ .

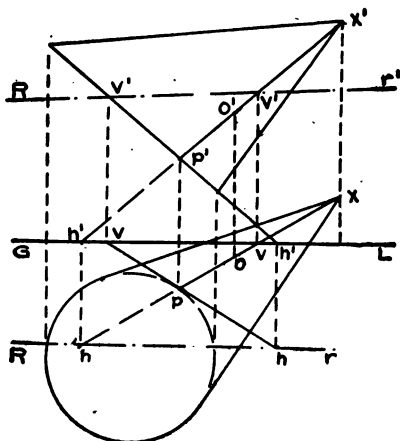


Fig. 49.

(103) Problem—21—Thru a given point in space to pass a plane tangent to the surface of a given cone.

### Analysis:

*Thru the given point and the apex of the cone draw a right line and produce this line until it pierces the plane of the base of the cone. Thru this piercing point draw a line tangent to the base. This tangent line and the line thru the given point determine the required plane.*

### Construction:

Given: Cone "X" and point O, Fig. 50.

Reqd: Plane thru O tangent to surface of cone.

Thru  $O$  and  $X$  pass a right line. To find the point in which  $OX$  pierces the plane of the base, produce  $o'x'$  until it intersects the  $V$  trace  $Ss'$  of the plane of the base at  $p'$ ;  $p$  is then found on the  $H$  projection of  $OX$ . Thru  $P$  pass the line  $PC$ ,  $pc$  and  $p'c'$ , tangent to the base of cone. The required plane  $T$  passes thru  $PO$  and  $PC$ .

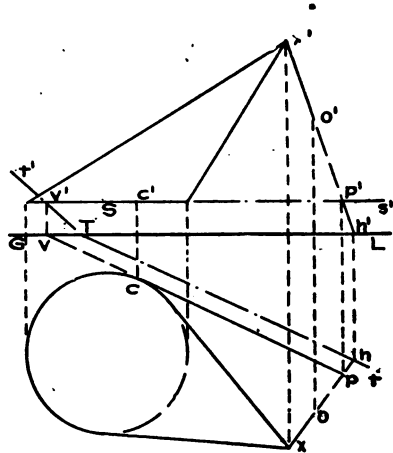


Fig. 50.

(104) Problem—22—To pass a plane tangent to the surface of a given cone and parallel to a given line.

### Analysis:

*Thru the apex of the cone pass a right line parallel to the given line and find its piercing point in the plane of the base of the cone. Thru this piercing point draw a right line tangent to the base of the cone. The required plane passes thru this tangent line and the line drawn thru the apex.*

### Construction:

Given: Cone "X" and line AB, Fig. 51.

Reqd: Plane tangent to cone and parallel to AB.

Thru the apex  $X$  pass the line  $XP$  parallel to the given line  $AB$ .  $p$  and  $p'$  are the projections of the piercing point  $P$  of this line in the plane  $S$  of the base of the cone. Thru  $P$  draw the line  $PC$  tangent to the base of the cone at  $C$ . The required plane  $T$  passes thru the two lines  $PX$  and  $PC$ .

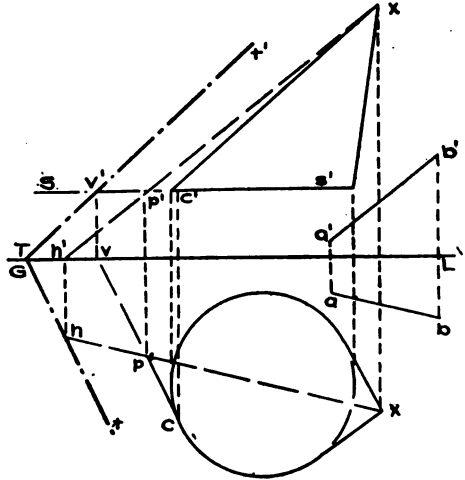


Fig. 51.

(105) Problem—

23 — Thru a given point on the surface of a given sphere to pass a plane tangent to the sphere.

**Discussion:**

If a plane be drawn tangent to the surface of a sphere, the radius of the sphere, which passes thru the point of tangency, is perpendicular to the tangent plane. Conversely if a plane be drawn thru the extremity of, and perpendicular to any radius of a sphere, this plane is tangent to the surface of the sphere.

**Analysis:**

*Thru the given point pass the required plane perpendicular to the radius which passes thru this point.*

**Construction:**

Given: Sphere  $O$  and point  $P$  on surface of sphere,  
Fig. 52.

Reqd: Plane thru  $P$  tangent to surface of sphere.

The V projection  $p'$  of point P is given. To determine the H projection draw thru  $p'$  the V trace of a plane S parallel to H. S cuts from the sphere a circle whose V projection is  $c'c'$  and H projection a circle with center at  $o$  and diameter equal to  $c'c'$ . Since point P lies on this circle its H projection is at  $p$ . Draw the radius OP of the sphere. The required plane T passes thru point P and perpendicular to this radius OP.

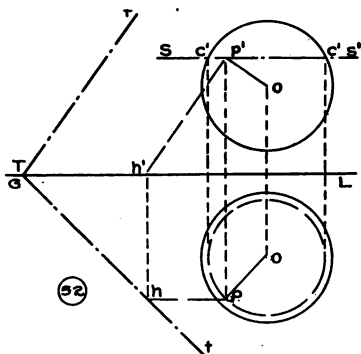


Fig. 52.

(106) Problem—24—Thru a given point on the surface of a given ellipsoid of revolution to pass a plane tangent to the ellipsoid.

### Discussion:

If thru the given point two right lines be drawn tangent to the surface of the ellipsoid, and thru these lines a plane be passed, the plane will be tangent to the surface of the given ellipsoid.

### Analysis:

*Thru the given point pass a plane perpendicular to the axis of revolution; it will cut from the surface a circle whose circumference passes thru the given point. If thru the point a line be drawn tangent to this circle it will be one of the two required tangent lines. If then a right circular cone be drawn, tangent to the surface of the ellipsoid on the circle just cut from the surface, apex on the axis of revolution, the element of the cone which passes*

thru the given point will be the **second** required tangent line. The plane which passes thru these two tangent lines passes thru the given point and is tangent to the surface of the given ellipsoid.

### Construction:

Given: Ellipsoid of Rev. and point P on surface of ellipsoid, Fig. 53.

Reqd: Plane thru P tangent to surface of ellipsoid.

Thru point P pass plane S perpendicular to the axis of revolution. It cuts from the surface a circle whose

H projection is the chord  $oo$  and V projection the circle whose center is  $x'$  and diameter equal to  $oo$ . Draw the line PH thru P tangent to this circle.

The right circular cone X is drawn with its apex on the axis of revolution and tangent to the ellipsoid on the circle cut from the surface by plane S. Its H projection is the isosceles triangle  $xoo$ , the sides  $xo$  and  $xo$  being tangent to the ellipse at  $o$  and  $o$ ;

the V projection is the circle  $o'x'o'$ . Since all of the elements of this cone are tangent to the ellipsoid, the element XP is the second of the two required tangent lines. The required plane T passes thru the tangent lines PH and PX.

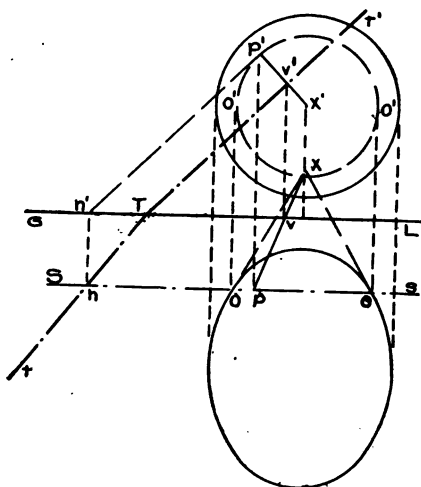


Fig. 53.

## CHAPTER 8

### INTERSECTIONS OF PLANES WITH SINGLE AND DOUBLE CURVED SURFACES.

(107) Problem—25—To find the line of intersection of a given plane and the surface of a given cone or cylinder.

#### **Analysis:**

*Divide the base or one projection of the base of the cylinder or cone into a number of equal arcs (12 is most convenient in practical work) and thru these division points draw the elements of the cylinder or cone. The line joining the piercing points of these elements in the given plane is the required line of intersection.*

#### **Construction:**

Given: Plane T and Cone X, Fig. 54 .

Reqd: Line of intersection.

(1) Divide the base of the cone into 12 equal arcs and thru these division points draw the elements of the cone. (It is a good practice to number these elements carefully) By passing V projecting planes thru these elements in succession the points in which they pierce plane T are obtained as shown. The line passing thru these points is the required line of intersection.

(2) The construction for the line of intersection of a given plane and a given cylinder is similar to the construction just given for the cone.

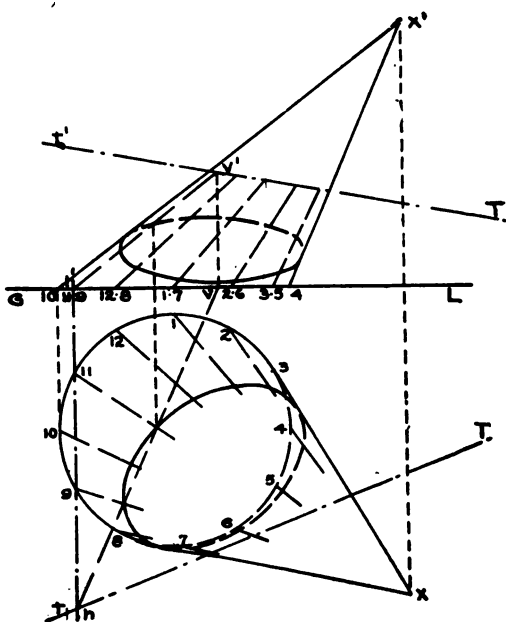


Fig. 54.

(108) Problem—26—To find the line of intersection of a given plane and a given ellipsoid of revolution.

**Analysis:**

*Draw a number of auxiliary planes perpendicular to the axis of revolution of the ellipsoid. These planes will cut circles from the surface of the ellipsoid and right lines from the given plane. The points of intersection of these circles and the corresponding right lines are points which lie both on the surface of the ellipsoid and in the given plane, hence in the required line of intersection. The line drawn thru these points so determined is the required line of intersection.*



**Construction:**

Given: Ellipsoid of revolution and plane T, Fig. 55.

Reqd: Line of intersection.

At intervals of about  $\frac{1}{4}$ " draw a number of planes R, S and W, etc., perpendicular to the axis of revolution; since this axis is perpendicular to H all of these planes are parallel to H and their lines of intersection with plane T are Horizontals of plane T. Each plane cuts from the ellipsoid a circle whose V projection coincides with the trace of the plane and whose H projection is a circle with diameter equal to the chord of the ellipse. The points of intersection of these circles and the corresponding Hori-

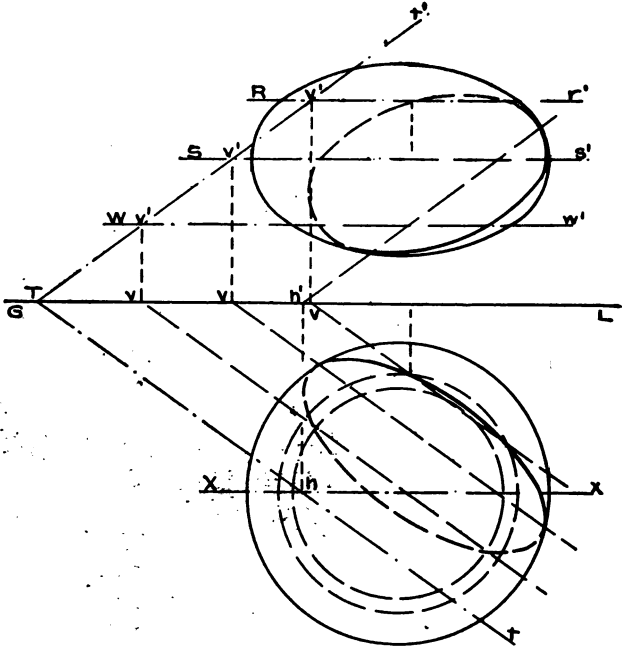


Fig. 55.

izontals of plane T are points on the required line of intersection.

It will be seen that the projections of this line of intersection are ellipses and except when the line of intersection is either entirely visible or hidden these ellipses are tangent at two points to the corresponding projections of the ellipsoid. These points of tangency are the *division* points between the *visible* and *hidden* portions of the line of intersection as viewed perpendicular to H and V respectively. The construction for finding these points of tangency in each projection is shown in Fig. 55; planes X and S are drawn in the proper positions for locating these points.

(109) Problem—27—To find the line of intersection of the surfaces of two given cylinders.

**Analysis:**

*Draw at regular intervals a number of auxiliary planes parallel to the elements of the two given cylinders. These planes will cut from the cylinders elements which will intersect in points on the required line of intersection.*

**Discussion:**

For purposes of development, should the occasion arise, the 12 point method should be used as far as possible.

**Construction:**

Given: Two intersecting cylinders, Fig. 56.

Reqd: Line of intersection.

The line 1-7 may be considered the V projection of a right section of the oblique cylinder; the H projection is both difficult to draw accurately and is unnecessary, hence is omitted. Using 1-7 as an axis the right section is revolved parallel to V; it then shows on V in its true shape and size. With 1 and 7 as two division points, divide the circle into 12 equal arcs and connect the corresponding division points by chords parallel to 1-7 as shown; these cords may be considered

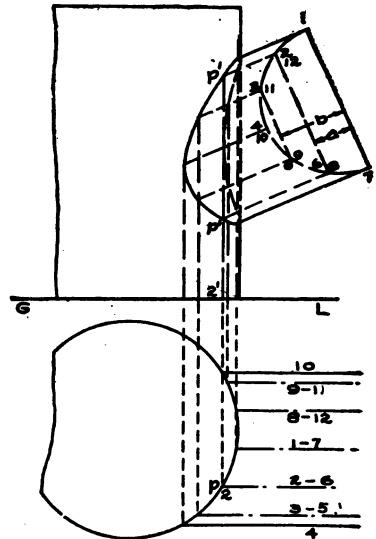


Fig. 56.

chords cut from the right section by the auxiliary planes. Thru the points 1, 2, 3, etc., draw the V projections of the elements of the oblique cylinder as shown. To obtain the H traces of the auxiliary planes and the elements cut by them from the vertical cylinder, measure distances  $a$  and  $b$  on both sides of the H projection of the axis of the oblique cylinder; plane 2-6 cuts element 2 from the vertical cylinder; element 2 of the vertical cylinder intersects elements 2 and 6 of the oblique cylinder in points P and P, two points on the required line of intersection. The remaining ten points are located in a similar manner.

**Note**—Fig. 56 may seem to illustrate a very particular and simple case of the above problem. However, in practical work when making drawings preliminary to laying out iron for a joint similar to this, it will always be found

easily possible to place the cylinders in such simple relative positions with respect to the coordinate planes.

(110) Problem—28—To find the line of intersection of the surfaces of a given cone and a given cylinder.

**Analysis:**

*Thru the apex of the given cone pass a line parallel to the elements of the cylinder. If thru this line a series of auxiliary planes be passed, they will cut from the given cone and cylinder elements which will intersect in points on the required line of intersection.*

**Construction:**

Given: Cone and cylinder, Fig. 57.

Reqd: Line of intersection.

Thru the apex X pass the line XH parallel to the elements of the cylinder; this line is perpendicular to H and its H piercing point is at *h*. The H traces of all planes passing thru this line will pass thru *h*. Thru *h* draw Tt the H trace of the auxiliary plane T; T cuts from the cylinder the element 1, H projection 1 and V projection 1'; from the cone T cuts the elements 1 and 2. Elements 1 and 2 of the cone intersect element 1 of the cylinder in the points P; these points

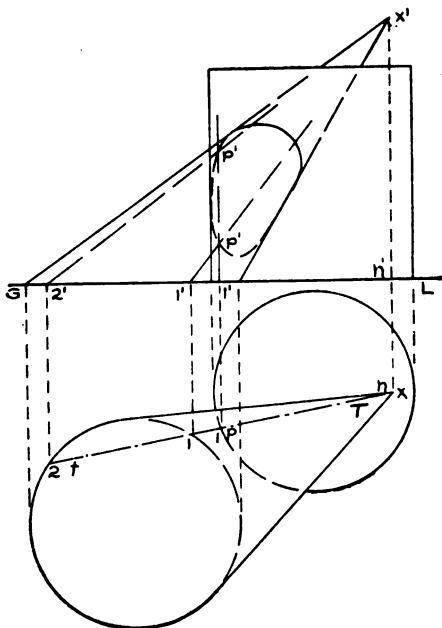


Fig. 57.

are then two points of the required line of intersection. The auxiliary planes should be drawn at as small angular intervals as convenient so that the general directions of the different portions of the curve, likewise the points of change of curvature may be accurately determined.

(111) Problem—29—To find the line of intersection of the surfaces of two given cones.

**Analysis:**

*Thru the apexes of the given cones draw a right line.*

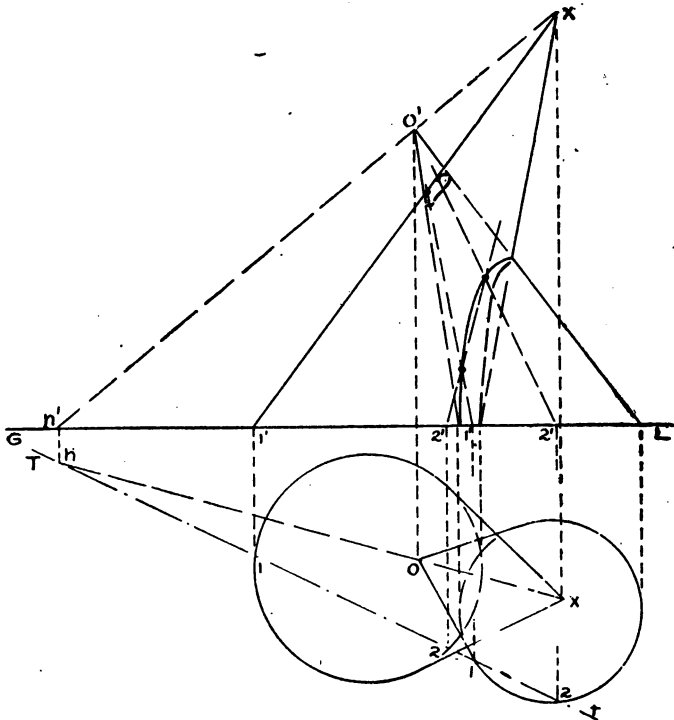


Fig. 58.

*If thru this right line a series of auxiliary planes be drawn, each plane will cut from the given cones elements which intersect in points on the required line of intersection.*

**Construction:**

Given: Cones X and O, Fig. 58.

Reqd: Line of intersection.

Thru the apexes of the cones pass the line OX; its H piercing point is at *h*. The H traces of all planes passed thru line OX will pass thru *h*. Draw the auxiliary plane T; its H trace is *Tt*. Plane T cuts from cone X the elements 1 and 2 and from cone O the elements 1 and 2; elements 1 and 2 of cone O intersect elements 1 and 2 of cone X in four points of the required lines of intersection; these points are indicated in the V projection, two of the four points on the larger curve, the other two on the smaller curve.

## CHAPTER 9

### DEVELOPMENTS OF SURFACES OF CYLINDERS AND CONES.

(112) The development of the surface of a cylinder or cone is the surface of that body rolled out on a plane. In other words we may say that a plane is drawn tangent to the cylinder or cone on one of its elements and the surface is then split on another of its elements and rolled out on this tangent plane. Thus a funnel which is in reality a cone can be made from a flat sheet of metal.

(113) Problem—30—To rectify a given arc.

#### Discussion:

In Fig. 59 is given an approximate method for rectifying a given arc. It must be remembered that the method is merely approximate and its degree of accuracy depends upon the length of the arc to be rectified, likewise upon the degree of curvature; i.e., for curves with radius of less than 1", the accuracy of the method could not be depended upon for arcs subtended by chords longer than  $\frac{1}{2}$ ".

#### Construction:

Given: Arc AB, Fig. 59.

Reqd: The approximate length of Arc AB.

Connect the limits, A and B, of the arc, by a right line, AB, and produce this chord one half its length, to a



Fig. 59.

point, C. At point A, draw a tangent to the arc and with C as a center and radius CB, describe an arc cutting the tangent at B<sub>1</sub>. AB<sub>1</sub> is the approximate length of the arc AB.

(114) Problem—31—To develop the surface of a right circular cylinder with a circular base, and trace upon the development the curve of intersection of the cylinder and a given oblique plane.

**Analysis:**

*Since the plane of the base is perpendicular to the tangent development plane, the development of the circumference of the base will be a right line whose length equals the circumference of the base. This length and the positions of the elements on the development may be determined by stepping off the correct number of times on the right line development of the base, a distance equal to the rectified length of one of the equal arcs of the base.*

*Points of the development of the curve of intersection are obtained by laying off from the rectified base along the developments of the respective elements, distances equal to the distances from those points to the base of the cylinder.*

**Construction:**

**Given:** The right circular cylinder and its line of intersection with plane T, Fig. 60.

**Reqd:** Development of surface and line of intersection.

Divide the base of the cylinder into 12 equal arcs; thru these division points draw the elements of the cylinder and find their piercing points in plane T as shown. For convenience let us assume GL as the development of the base and lay off twelve times as shown, a distance



equal to the rectified length 4-3 of the arc 4-3. The large rectangle in the figure is then the development of the sur-

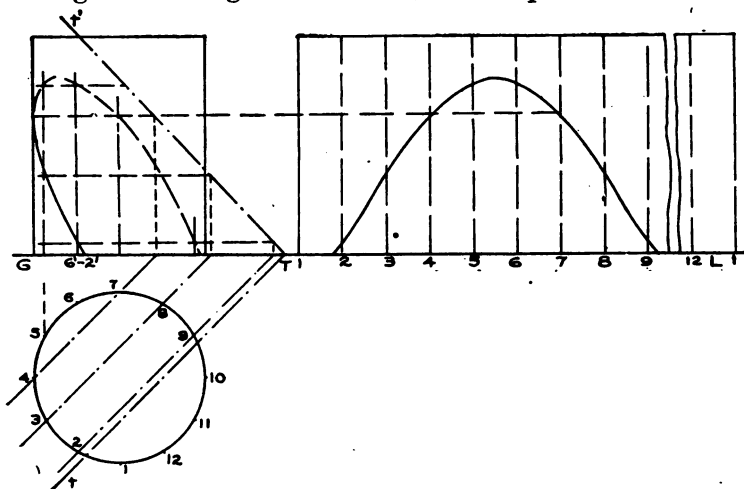


Fig. 60.

face of the cylinder. To trace the curve of intersection on this development, lay off on elements 1 2 3 4 5 etc., from the rectified base, distances equal to the distances above the base of the cylinder, at which the line of intersection intersects those elements, as shown in the V projection.

(115) Problem—32—To develop the surface of a right circular cone and trace upon the development the curve of intersection of the cone and a given oblique plane.

**Analysis:**

*Since the plane of the base is everywhere equidistant from the apex, as the surface of the cone is rolled out, each point of the development of the base will be in the cir-*

*circumference of a circle whose center is the development of the apex and radius equal to the length of the elements. To obtain an arc of this circle approximately equal to the circumference of the base of the cone, with a chord length equal to the chord subtended by the equal arcs of the base of the cone, lay off a corresponding number of arcs on the circumference of the development. Joining the ends of this arc with the center we have the development of the surface. Laying off on the proper elements the true distances from the apex to the different points of the curve of intersection and tracing a curve thru these points, we have the development of the curve of intersection.*

### **Construction:**

Given: Cone X and its line of intersection with plane T, Fig. 61.

Reqd: Development of cone and line of intersection.

Divide the base into 12 equal arcs and thru these division points draw the elements of the cone; their piercing points in T give the line of intersection as shown. Assume any point X as the development of the apex of the cone; with X as a center and radius equal to true length of elements, describe an arc. With a chord length equal to chord 3-4 of the base of the cone, step off 12 arcs on the circle just drawn; number these arcs carefully; joining 11 and 11 with X we have the development of the surface of the cone. Laying off from X on elements 1, 2, 3, 4, 5, 6, and 7 the distances from the apex to the corresponding points of the curve of intersection and tracing a curve thru these points, we have the required development of the curve of intersection.

To obtain the true length of element 6 from X to P a short cut may be used: By revolving the cone about its axis each element may in turn be brought parallel to

V when its V projection is the isosceles side of the triangle, and shows in its true length. Hence to find the

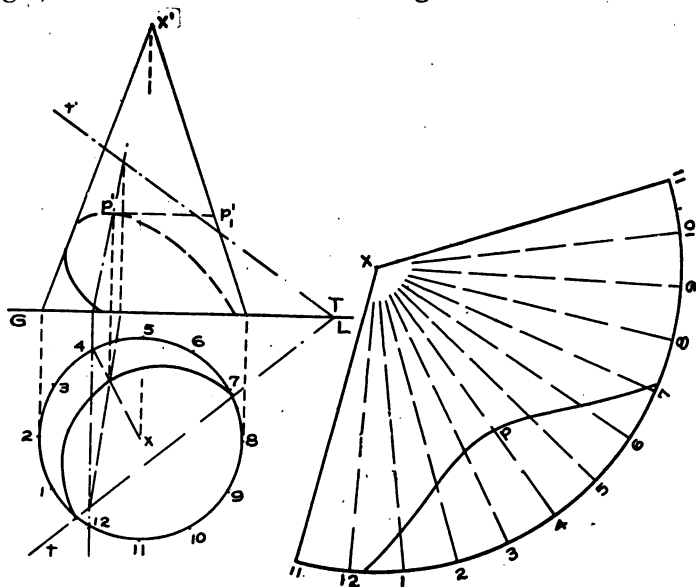


Fig. 61.

length of  $XP$ , thru  $p'$  draw a line parallel to  $GL$  until it intersects the side of the triangle at  $p'_1$ ;  $x'p'_1$  is the required true length.

(116) Problem—33—To develop the surface of an oblique cylinder.

#### Analysis:

*If a plane be drawn perpendicular to the elements of the cylinder, it will cut from the surface a right section whose development will be a right line. Divide this right section into a number of equal arcs and thru these division points draw the elements of the cylinder. By rec-*

*tifying one of the equal arcs of the right section, the rectified length of the right section may be obtained, and the developments of the several elements drawn at the proper intervals and perpendicular to the rectified right section. Lay off on these elements from the developed right section the distances from the respective points of the right section to the bases; tracing curves thru these points we have the developments of the bases and the entire surface of the cylinder.*

**Construction:**

Given: Oblique cylinder, Fig. 62.

Reqd: Development of surface.

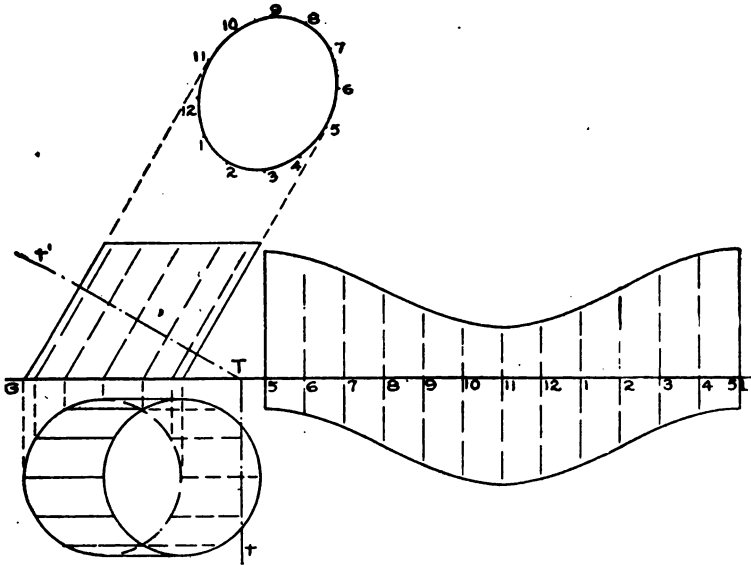


Fig. 62.

Draw plane T perpendicular to the elements of the cylinder; it cuts from the cylinder the right section shown.

Revolve plane T about  $Tt'$  into V; the right section revolves into the position shown, appearing now in its true size. Divide this true size into 12 approximately equal arcs, 1-2, etc., by equal chords and draw the elements of the cylinder. The rectified circumference of the right section is the right line 5-5.

At the several points of this rectified right section draw the elements and measure **up** on each, e.g. No. 7, a distance equal to the length of the element from the right section (whose V projection coincides with  $Tt'$  to the upper base; and **down** a distance equal to the length of the element from the right section to the lower base.

#### IMPORTANT

Should the cylinder be placed in a position other than parallel to V, a new V plane may be drawn parallel to the elements of the cylinder, and after obtaining the projection of the cylinder on this new V plane, the development of the surface may be obtained as in Fig. 61.

(117) Problem—34—**To develop the surface of an oblique cone.**

#### **Analysis:**

*If the circumference of the base of the cone be divided into a large number (e.g. 12 or 16) of equal arcs and the elements of the cone be drawn thru these points, we may conceive these elements and the chords of the base to compose an inscribed pyramid of 12 or 16 faces, whose surface may easily be developed. For practical purposes the development of this pyramid may be considered the development of the surface of the cone.*

#### **Construction:**

Given: Oblique cone X, Fig. 63.

Reqd: Development of surface.

Divide the base into 12 parts as shown and draw the elements of the cone. For the sake of symmetry divide the surface on element XO. Assume any point X as the common vertex of the developments of the triangular

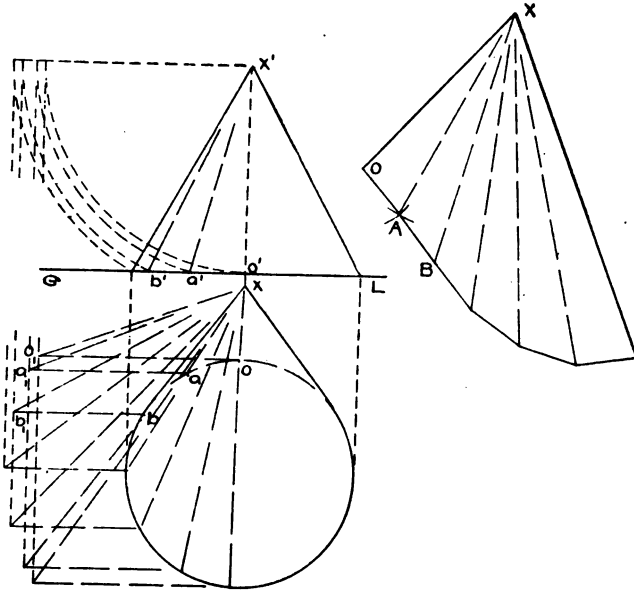


Fig. 63.

faces. From X draw the line XO equal to the true length  $xo_1$  of element XO. Then with X as a center and radius equal to the true length  $xa_1$ , of element XA, describe a short arc; and with O as a center and radius equal to chord  $oa$ , describe a second arc cutting the first at A; this triangle XOA is the development of the triangular face XOA. So in succession the triangular faces XAB, etc., may be added until the entire surface is developed.'

## PART 3

### CHAPTER 10

#### SHADES AND SHADOWS

(118) In work of an architectural nature, quite frequently for the sake of natural effects, it is desirable to show the shades and shadows cast by the different parts of an object. However, it is not the intention of the author to treat the subject from any but a strictly mathematical standpoint, leaving the more elaborate discussions of degrees of shade, etc., to other works. Likewise only such theory will be given as seems to be necessary for ordinary practical purposes.

#### LIGHT—SOURCE—RAYS

(119) Shades and Shadows, indicating the absence of a certain amount of light from some surface, of course always presume the *existence* and a *source* of light of some sort. Ordinarily, for *out door* work this source is the *sun*; and for the easy treatment of the subject in physics, etc., a *shaft* or *beam* of *sunlight* is imagined to be composed of countless *parallel rays*, these rays being merely *mathematical lines*.

#### DIRECTION OF LIGHT

(120) Upon the assumption of a *source* of light and the existence of light *rays*, the *direction* of these rays must be either determined or assumed. Shadows being used

chiefly for effects, there can be no objection to the assumption of the direction of the rays. However, in assuming this direction, some care should be taken to make it approximately what it is theoretically for that latitude. A reasonably good rule is to assume both projections of the light rays to make with GL an angle equal to the *complement* of the *latitude* of that section of the country. The angle commonly used in the northern half of the United States is  $45^\circ$ . As it has been found best to work with the light coming from the *left*, the projections of the rays will slope *toward* GL to the *right*, when objects are placed in the *1st* angle.

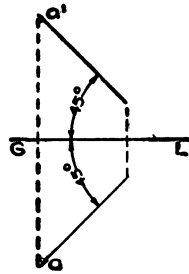


Fig. 64.

#### DIRECT LIGHT—INDIRECT LIGHT

(121) All light received from the *source* is known as **direct** light, and any portion of any surface from which the *direct light* is excluded by some body is said to be **in shadow**. However, it must not be imagined that these shaded surfaces receive no light at all, for in some cases it may be almost impossible to distinguish with the eye between the *shaded* and *illuminated* portions of a surface. There are always countless surfaces which reflect the direct light upon the shaded surfaces, possibly lighting them more brilliantly than they could be by direct light. This *reflected* light is known as **indirect** light and gives rise to *degrees* of *shade* mentioned before.

#### SHADES—SHADOWS

(122) Any portion of the surface of an object from which the *direct* light is excluded by some part of the



*same* object is said to be **in shade**, or for convenience may be called a **shade**.

Any portion of a plane or the surface of an object from which the **direct** light is excluded by some part of **another** object is said to be **in shadow**, and for convenience may be called a **shadow**.

#### LINE OF SHADE—SHADOW

(123) The line which *separates* the *shaded* from the *illuminated* portions of any surface is known as the **line of shade**; and the line which *limits* a *shadow* on any surface is called a **line of shadow**. See  $c_1b_1$ , Fig. 65.

#### SHADOW RAY

(124) In finding the shadow of an object on a surface considerable labor can be saved if one determines by inspection what lines of the object will determine the limits of the shadow. e.g. in Fig. 65 it is easily seen that points A and G will play no part whatever in determining the shadow, while points E, F, C, B and D will. It is seen that by passing a ray of light thru B, the point  $b_1$  is located; likewise  $c_1$  etc. These are significant points in the line of shadow and when located carefully may save much labor. The line  $Bb_1$  is in reality a ray of light; however, inasmuch as it has been used in determining a point on the *line of shadow* it will be known in this work as a **shadow ray**; the term may

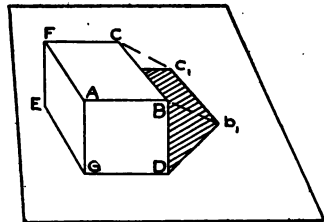


Fig. 65.

be absurd, however, its value as a working tool cannot be doubted. Any other such ray which may be used to *determine* any *point* either on a *line of shade* or a *line of shadow* will be known as a **shadow ray**.

#### 1st VERSUS 3rd ANGLE

(125) Inasmuch as only one view of an object is ordinarily retained when shadows are used, the writer sees no objection but an advantage in placing the object in the *first* rather than in the *third* angle. The V plane may then easily be conceived to be a wall of any building upon which shadows may be cast; also the student seems to find it more natural to imagine the H plane to be the ground or plane of the base of the object than to use the extra plane when the object is in the third angle.

#### SHADES AND SHADOWS—HOW REPRESENTED

(126) It is easily understood that any given point can have only *one* shade or *one* shadow; likewise, that an object can be viewed from only *one* point at one time. Hence, when objects are viewed from *before* V, i.e., lines of sight are perpendicular to V, as far as the observer is concerned there are *no* shades nor shadows on *horizontal* planes, and unless so specified in the requirements of the problem such shades and shadows need not be found.

Concerning shades and shadows on *oblique* planes, even tho this whole subject is here kept within the realm of orthographic projection, it is seen that when observed from *before* V the shade or shadow of a given point on a given *oblique* plane is represented by the V *projection* of that shade or shadow; as far as the observer is concerned an H *projection does not exist* and need not be found. When viewed from above H the H *projection*

of the shade or shadow represents that shade or shadow and the V projection does not exist and need not be found.

(127) Problem — 35 — **To find the shadow of a given point on a given surface,**

**Analysis:**

*Thru the given point pass a shadow ray and find the point in which it pierces the given surface. This piercing point is the required shadow.*

**Construction:**

Given: Point O, Fig. 66.

Reqd: Shadow of O on  
V or on H.

Thru  $o'$  and  $o$  draw the projections of the shadow ray at  $45^\circ$  to GL; this ray pierces V at  $o_v$  the required shadow on V, or H at  $o_h$  the shadow on H.

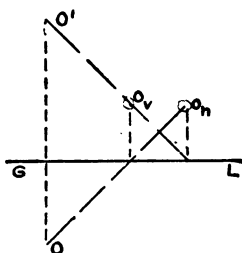


Fig. 66.

**Construction: (2)**

Given: Point O and plane  
R, Fig. 67.

Reqd: Shadow of O on R.

Thru the shadow ray from O pass an H projecting plane; its line of intersection with R is VH. The shadow ray intersects this line at O,  $o_1$  and  $o'_1$ . If shadows are required in elevation,  $o'_1$  is the desired shadow; in plan  $o_1$  is used.

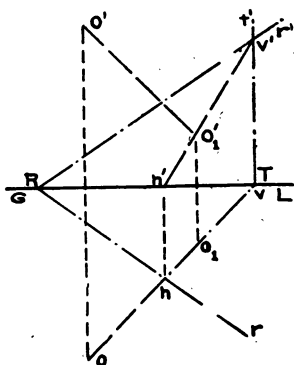


Fig. 67.

**Construction: (3)**

Given: Point O and Cone X, Fig. 68.

Reqd: Shadow of O on surface of cone.

Thru O pass the shadow ray; also connect O with the apex X. Plane T of these two lines cuts the element CX from the cone; the shadow ray intersects this element in  $O_1$ . In dealing with shadows in elevation  $o'_1$  is the required shadow; in plan  $o_1$  is used.

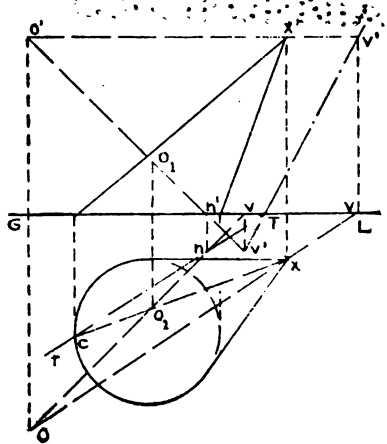


Fig. 68.

**Construction: (4)**

Given: Point O and the Ellipsoid X, Fig. 69.

Reqd: Shadow of O on Ellipsoid.

Thru OC pass the H projecting plane T; it cuts from the Ellipsoid an ellipse whose minor axis is  $aa$ . Laying off the distance  $e$  on the V projection as shown, the major axis  $d'd'$  is obtained. Using the axis of the Ellipsoid as an axis, revolve plane T parallel to V. OC comes into the position  $O_2C_1$  and the V projection of the revolved position of the ellipse may be constructed in the position shown, with minor axis equal to  $aa$  and major axis equal to  $d'd'$ ; the ordinary methods of constructing an ellipse may be used. The revolved position of the required shadow is then found at  $o'_3$  and the shadows themselves at  $o'_1$  and  $o_1$  respectively.

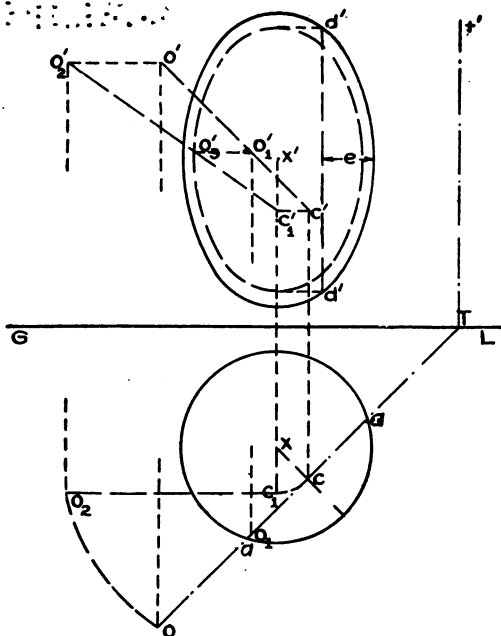


Fig. 69.

**BRILLIANT POINTS—LINES**

(128) If on a surface of double curvature there is a *point* at which a ray of *direct* light is *reflected* to the eye, this point appears more brilliant than any other point of the illuminated surface and is known as a **brilliant point**. On a surface of *single* curvature instead of a point there will be a *line* along which *direct* light will be reflected to the eye, (if reflected at all); this line is known as the **brilliant line**.

It can be readily seen that such *brilliant points* or *lines* do not necessarily exist for every position of the

point of sight and any assumed direction of light; however, if the surface be either a sphere or an ellipsoid of revolution, such a brilliant point does exist and can readily be found.

(129) Problem — 36 — **To find the brilliant point on the surface of a given ellipsoid of revolution.**

### Discussion:

Before proceeding with the construction it is necessary to fix well in mind a number of facts of the problem.

(1) Assuming that it is desired to locate the brilliant point in *elevation*, the eye is an infinite distance before the  $V$  plane; hence the *reflected ray* is *perpendicular* to  $V$ . (2) The *bisector* of the *angle* between the *incident* and *reflected rays* is *normal* to the *surface* of the ellipsoid at the *brilliant point*. (3) A *normal* to the *surface* of an *ellipsoid* passes thru the *axis of revolution*.

### Analysis:

*Thru any convenient point in space pass a ray of light and a line of sight (perpendicular to  $V$ ), and bisect the angle between these right lines. (This bisector is parallel to the normal at the brilliant point). Then thru the axis of the ellipsoid pass a plane parallel to the bisector just found. If a line be drawn in this plane perpendicular to the direction of the normal (or the bisector) and tangent to the ellipse cut by the plane from the surface, the brilliant point is located, being the point of tangency.*

### Construction:

Given: The Ellipsoid X, Fig. 70.

Reqd: The brilliant point in elevation.

Thru any point O pass the ray of light OB and the line of sight OA. Revolving OB parallel to H about OA

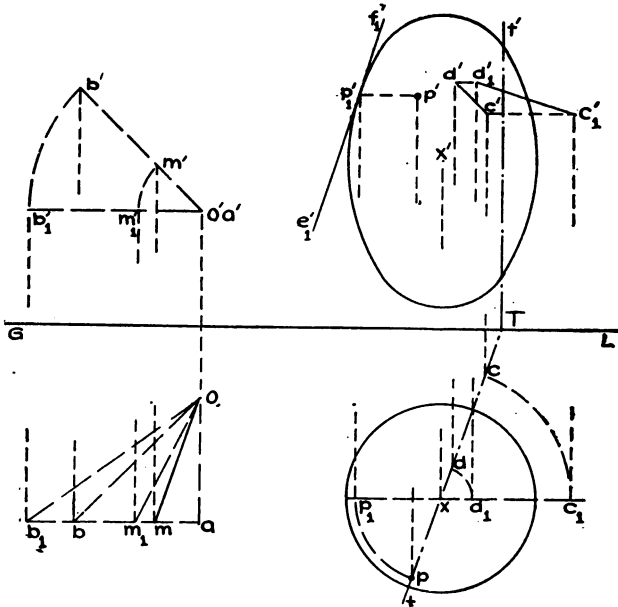


Fig. 70.

as an axis, the bisector  $om_1$  is located, thence  $om$  and  $o'm'$  the projections of the bisector. Thru the axis of the ellipsoid pass plane  $T$  parallel to  $OM$ ;  $Tt$  is parallel to  $om$ . Thru any point  $C$  in  $T$  pass the line  $CD$  parallel to  $OM$ ; revolving  $T$  about the axis of the ellipsoid as an axis until it is parallel to  $V$ , the  $V$  projection of the ellipse cut from the ellipsoid by  $T$  now coincides with the  $V$  projection of the ellipsoid;  $CD$  revolves into the position  $c'_1 d'_2, c_1 d_1$ . The tangent line  $e'_1 f'_1$ , perpendicular to  $c'_1 d'_1$  locates the  $V$  projection of the revolved position of

the brillian point at  $p'$ , and the required brilliant point is found at  $p'$ .

(130) Problem — 37 — **To find the shade and shadow of a given ellipsoid of revolution on H,**

**Analysis:**

*If a cylinder of light rays be drawn tangent to the given ellipsoid, the line of tangency is the line of shade; the line of intersection with H of this cylinder is the required line of shadow.*

**Construction:**

Given: Ellipsoid O, Fig. 71.

Reqd: Shade and shadow of ellipsoid on H.

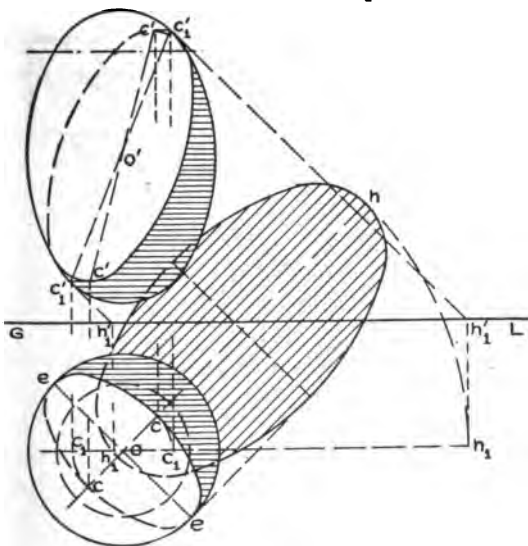
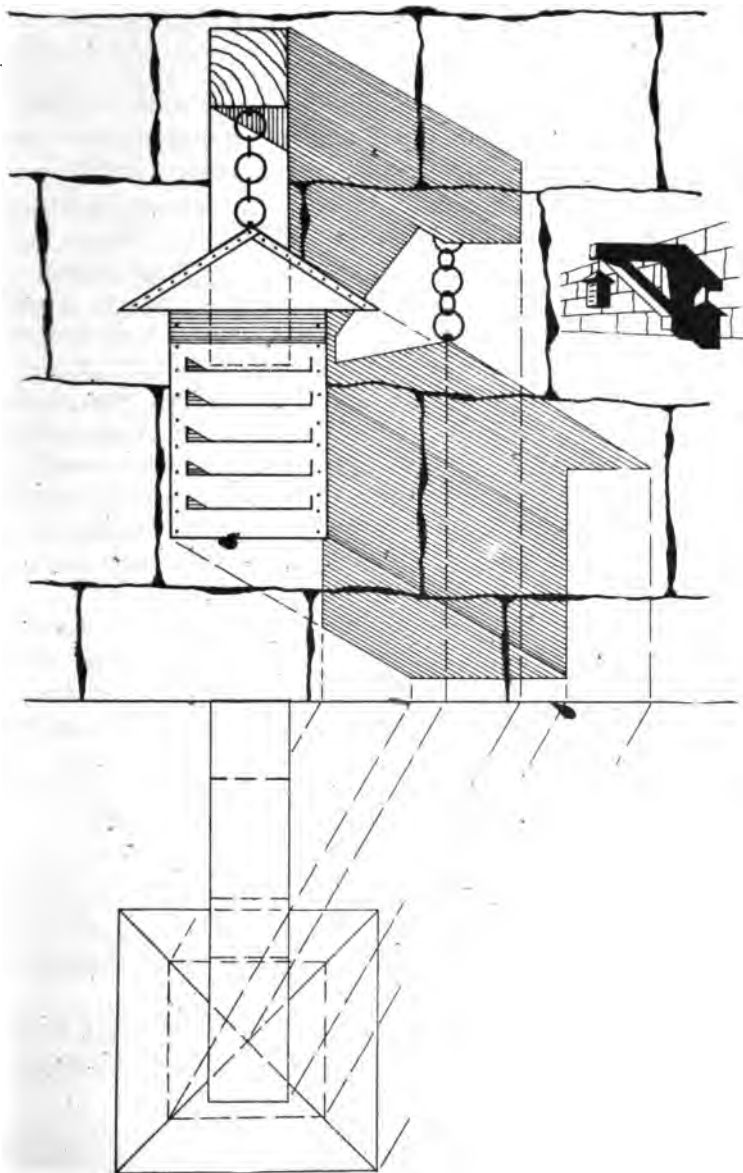


Fig. 71.

Revolving the ellipsoid and the cylinder of tangent rays about the axis of the ellipsoid until the elements of



the cylinder are parallel to  $V$ , the upper and lower limiting elements come into the positions  $C_1H_1$ , projections  $c'_1h'_1$ ,  $c'_1h'_1$  and  $c_1h_1$ ,  $c_1h_1$ . The axes of the elliptical line of shadow on  $H$  are determined as equal to  $h'_1h'_1$  (major) and the diameter of the circular  $H$  projection of the ellipsoid (minor); the line of shadow is then determined as shown.  $CC$  is the major axis of the elliptical line of shade; however, its  $H$  projection  $c_1c_1$  becomes the minor axis of the elliptical  $H$  projection of the line of shade;  $c_1c_1$  revolves back into the position  $cc$ ; the chord  $ee$  perpendicular to  $cc$  at its center is the major axis.  $c'_1$  and  $c'_1$  revolve back into the positions  $c'$  and  $c'$  giving two points on the  $V$  projection of the line of shade. Other points may be determined by cutting circles from the ellipsoid by auxiliary planes perpendicular to the axis, and finding the points of intersection of these circles with the  $H$  projection of the line of shade.



## CHAPTER 11

### PERSPECTIVE

(131) A *perspective drawing* of any object is such a **representation** as will have the *same appearance* as the *object itself* when viewed from some *definite point*. An object can be viewed from an infinite number of positions, each view differing slightly from the others; hence, any number of perspectives can be made of the same object.

But little use is made of perspective except in architectural work. Working drawings, composed of several views and made with a corresponding number of view points at infinite distances from the coordinate planes, of course represent little but the bare details of construction. Isometric and oblique projections, tho representing an object by a single view, serve little but to convey a conception of the relation of the several parts of an object to each other. Despite all care and allowances such projections never appear quite natural.

It is then left entirely to perspective to *represent objects* as they *actually appear* to us. It is true, as mentioned before, that the chief use of perspective is in architectural work; however, the principles of construction, depending upon the principles of Descriptive Geometry, are easily acquired and can be applied in a mechanical way by the engineer without any special artistic training.

### PICTURE PLANES

(132) Perspective drawings are ordinarily made from working drawings, and as working drawings are 3rd angle projections we will assume that always in making a perspective the object is placed in the 3rd angle.

Any *plane parallel* to either the V plane or the H plane may be used as the **picture plane**; however, in this treatise,

for simplicity of construction, the V and H planes will be termed and used as the picture planes; i.e. the *planes* on which the *representations* or *perspective drawings* are made.

When the representation or drawing is made on the H plane it is termed a **bird's eye view**; such drawings are made frequently of large areas of ground, cities, groups of buildings, etc. If, however, the drawing is made on the V plane it is termed a **perspective**; such drawings of course have the wider range of utility.

#### POINT OF SIGHT—PRINCIPAL POINT

(133) The *position* of the *eye* of the *observer* is

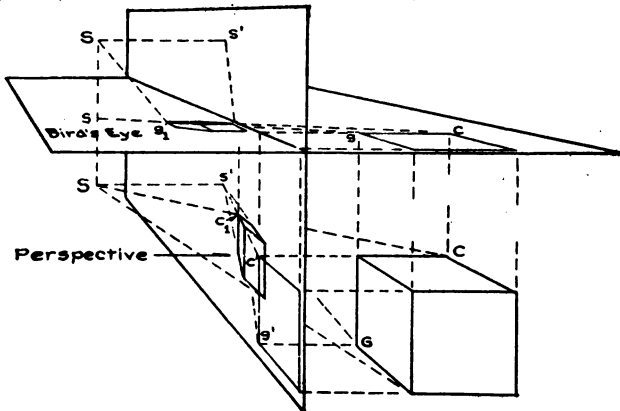


Fig. 72.

known as the **point of sight** and is lettered S. This point of sight is invariably on the *opposite* side of the *picture plane* from the *object*, that the drawing may be *smaller* than the object; however, if it is desired to make the drawing larger than the object, a new picture plane may be drawn in such position as to bring both the object and the point of sight on the same side.

In making a *bird's eye view* the *point of sight* will be *above* the H plane and, ordinarily in the 1st angle; while for a perspective, S is taken in *front* of V and usually in the 4th angle.

The V projection of S will be lettered  $s'$ , the H projection  $s$ .  $s'$  is of prime importance in construction hence is known as the **principal point**; its use will be explained later.

There is more to the locating of the point of sight with reference to the object than appears on first thought. The principles of construction can be applied as easily with S in any one position as in any other; however, unless care is exercised the drawings, tho mathematically correct, may have a most distorted appearance; e.g. one can imagine how absurd the perspective of a house would appear when made with the view point only a few feet from one of the walls. Suggestions for placing the point of sight will be given later.

#### VISUAL RAY—PLANE—CONE

(134) A line from the *point of sight* to any *point* of an object is known as a **visual ray**. A plane passing thru the *point of sight* and any *right line* of the object is a **visual plane**. A cone of *visual rays* with the *apex* at the *point of sight* and the surface of the cone *tangent* to a *given curved surface* (sphere, etc.,) or all *rays intersecting* a *given curved line* is known as a **visual cone**.

#### PERSPECTIVE BY PIERCING POINTS

(135) Problem — 38 — **To find the perspective of a given point.**

#### **Analysis:**

*Thru the point of sight and the given point pass a*

**visual ray.** The piercing point of this ray in  $V$  (the picture plane) is the required perspective.

**Construction:**

Given: Point  $O$ , 3rd angle, Fig. 73.

Reqd: Perspective of  $O$ .

$s'o'$  and  $so$  are the projections of the visual ray  $SO$ . This ray pierces  $V$  at  $V$ , projections  $v'$  and  $v$ . The point  $v'$  is the required perspective and will be lettered  $o_1$  if lettered at all.

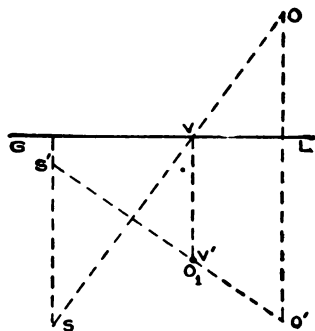


Fig. 73.

**Defect:** The theory of the problem just given is sufficient to construct completely the perspective of any object; however, in practical work, with few exceptions the method cannot be used; for the point of sight is invariably in such position as to bring its  $H$  projection  $s$  without the limits of the drawing. This theory is however the foundation of problems to follow.

(136) Problem — 39 — To find the perspective of a given line.

**Discussion:**

From the previous problem it is seen that if a point lies in  $V$ , the piercing point of the visual ray and the given point coincide; i. e. the *point* is its *own perspective*. Then if a given line be produced until it pierces  $V$ , this point is one point of the perspective of the given line.

**Analysis:**

*Produce the given line until it pierces  $V$  (the picture plane). Also connect the point of sight with some point*

of the given line by a visual ray, and find the piercing point of this ray in the V plane. The line passing thru these two piercing points is the required perspective.

**Construction:**

Given: Line AB, 3rd angle, Fig. 74.

Reqd: Perspective of AB.

AB produced pierces V at  $v'$  and the visual ray from S to A pierces V at  $a_1$ . The perspective of B lies on the V projection of the visual ray from S to B, hence  $b_1$  is at the intersection of  $s'b'$  and  $a_1v'$ .  $a_1b_1$  is the required perspective.

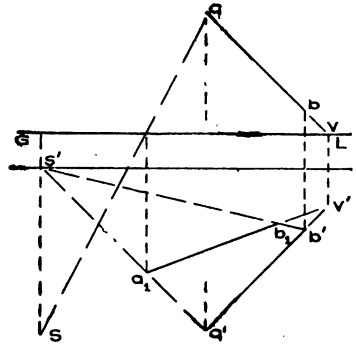


Fig. 74.

**Analysis: (2)**

Thru the point of sight and the given line pass a visual plane. The V trace of this plane is the required perspective of the given line.

**SPECIAL LINES**

From Analysis (2) above, it is seen that if a line is parallel to the picture plane (V), its perspective is parallel to the given line, likewise to the V projection of the given line.

The perspective of a vertical line is a vertical line.

(137) Problem — 40 — To find the vanishing point of a given group of parallel lines.

**Discussion:**

If a group of parallel lines be produced an infinite distance they appear to meet in a point known as their

**vanishing point.** A visual ray parallel to this group of lines likewise appears to pass thru this vanishing point, hence its piercing point in  $V$  is the perspective of this point. In perspective this *piercing point* is known as the **vanishing point** of this group of lines. The perspective of each line of the group passes thru the vanishing point.

It is seen from the figure that after the vanishing point of a group of parallel lines is found, it is necessary to find the perspective of only *one* point of each line; the vanishing point, being common to all of the perspectives, furnishes a second point.

### Analysis:

*Thru the point of sight pass a visual ray parallel to any line of the given group. The piercing point of this ray in  $V$  (the picture plane) is the required vanishing point.*

### Construction:

**Given:** Point of sight and lines,  $AB$ ,  $CD$ , and  $EF$ , Fig. 75.

**Reqd:** Vanishing point of given lines.

The projections of the ray are drawn thru  $s$  and  $s'$  parallel to the respective projections of the given lines. Its piercing point in  $V$  (the picture plane) is  $V$  the required vanishing point.

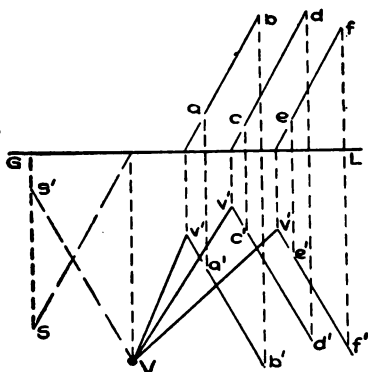


Fig. 75.

PERPENDICULARS—HORIZONTALS—DIAGONALS—POINTS OF DISTANCE

(138) Lines *perpendicular* to the  $V$  picture plane



are known as **perpendiculars**; such lines are of course horizontal.

If a group of perpendiculars be produced indefinitely they appear to meet in a point on what we commonly call the *horizon*. A visual ray parallel to these perpendiculars pierces  $V$  in  $s'$ ; hence  $s'$  is the *vanishing point* of all perpendiculars. A horizontal line thru  $s'$  is the *perspective* of the horizon mentioned above; this horizontal line thru  $s'$  is known in perspective as the **horizon**. All horizontal lines vanish in points of the horizon.

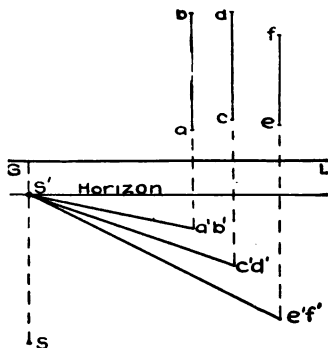


Fig. 76.

Any horizontal line making  $45^\circ$  with the picture plane is known as a **diagonal**. e.g. AB and BC, Fig. 77. *Diagonals* vanish in points of the horizon; on inspection of Fig. 77 it will be seen that these vanishing points  $D_r$  and  $D_l$  of *diagonals* are on the horizon at distances to the right and left of  $s'$  equal to the distance that  $s$  is below  $GL$ , or that  $S$  is before the picture plane. Hence  $D_r$  and  $D_l$ , also known as **points of distance**, are readily located whether  $s$  is without the limits of the drawing or not.

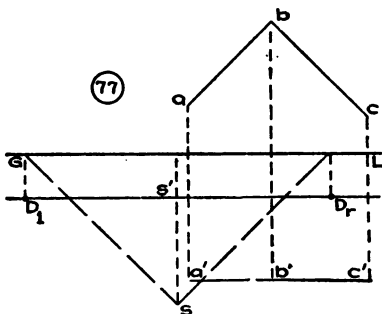


Fig. 77.

(139) Problem — 41 — **To find the perspective of a given point by diagonals and perpendiculars.**

**Discussion:**

If thru any given point two lines be passed and the perspectives of these two lines be found, the point of intersection of the perspectives is the perspective of the given point.

**Analysis:**

*Thru the given point pass a perpendicular and a diagonal and find their perspectives. The intersection of the perspectives of the perpendicular and diagonal is the perspective of the given point.*

**Construction:**

Given: Point O, 3rd angle, Fig. 78.

Reqd: Perspective of O by a perpendicular and a diagonal.

Locate  $D_r$  and  $D_1$  at equal distances from  $s'$ . The diagonal thru O pierces V at  $v'$  and its perspective is  $v'D_r$ . The perpendicular thru O pierces V at  $o'$  and its perspective is  $o's'$ ;  $v'D_r$  and  $o's'$  intersect at  $o_1$  the required perspective.

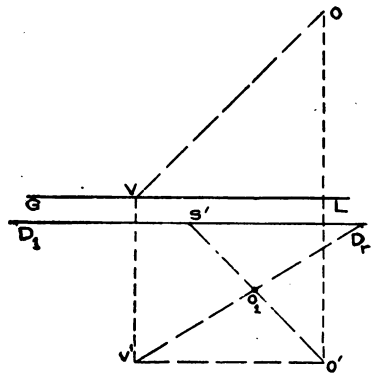


Fig. 78.

**Defect:** Tho the construction explained above can and generally is used in perspective construction, problems will arise in which even the points  $D_r$  and  $D_1$  fall without the limits of the drawing. In such cases a Geometrical modification of the method is used; i.e. points

known as **reduced points of distance** are used instead of  $D_r$  and  $D_1$ ; these *reduced points of distance* are located on the *horizon* at distances to the *right* and *left* of  $s'$  equal to some *simple fractional part* of  $s'D_r$ ; e.g.  $\frac{1}{2}$ ,  $\frac{1}{4}$ , etc. Such points are lettered  $D_{2r}$  and  $D_{4r}$ . See Fig. 79.

(140) Problem —42—To find the perspective of a given point by perpendiculars and diagonals using reduced points of distance.

**Construction:**

Given: Point O, 3rd angle, Fig. 79.

Reqd: Perspective of O.

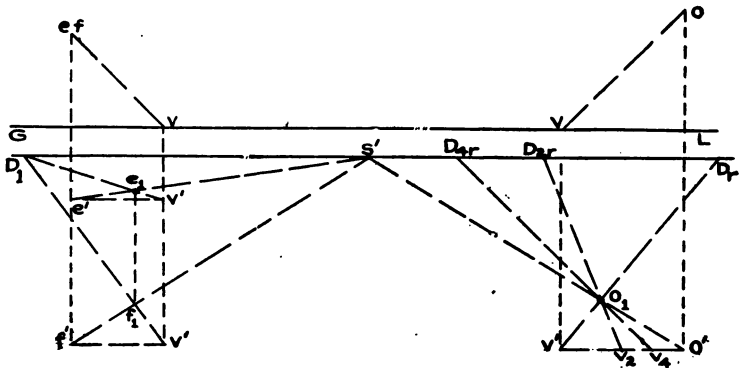


Fig. 79.

The perspective of the perpendicular thru O is  $o's'$ . The V piercing point of the diagonal is at  $v'$ . The perspective of the diagonal is  $v'D_r$  and the perspective of O is  $o_1$ . If  $D_r$  is inaccessible and  $D_{2r}$  can be used, the line from  $v_2$ , midway between  $o'$  and  $v'$ , is connected with  $D_{2r}$  giving  $o_1$  as before; if  $D_{2r}$  is likewise inaccessible the line  $v_4D_{4r}$  may be used to locate  $o_1$ .

## POINTS NEAR THE LEVEL OF THE HORIZON

If as in the case of point E, Fig. 79, a point is near the level of the horizon, by the method above it is possible to get but a very unsatisfactory point of intersection for  $e_1$ . Knowing that the *perspective* of a *vertical line* is a *vertical line*, it may be imagined that E is *raised* or *dropped* on a vertical line to F and the perspective of F satisfactorily located at  $f_1$ ; a vertical line thru  $f_1$  intersects  $e's'$  in  $e_1$ , the required perspective.

## CIRCLES IN PERSPECTIVE

(141) If the *plane* of a *given circle* is *parallel* to the *picture plane* the *perspective* is a *true circle*; its radius may be found by finding the perspectives of the center and one point of the circumference. If the *plane* of the *circle* is *oblique* to the *picture plane* its *perspective* is an *ellipse*. The major and minor axes of this ellipse can be determined, however, for practical work this is unnecessary; a construction known as the *8 point method* is more easily used and with perfect satisfaction.

**Problem — 43 — To find by the eight point method the perspective of a given circle whose plane is oblique to the picture plane.**

**Analysis:**

*Circumscribe about the circle a rhombus, two of whose sides are parallel to the picture plane and two parallel to H; find the perspective of this rhombus and draw its diagonals and center lines. Then on the perspective of one of the sides parallel to V, construct a square and draw its diagonals, center lines and inscribed circle. From the 8 points of intersection of these diagonals and center lines with the circle, 8 points of the perspective of the given.*

circle are located. The required perspective may then be traced freehand thru these 8 points.

**Construction:**

Given: Circle with center O, Fig. 80.

Reqd: Perspective of circle.

Circumscribe the rhombus ABCD; the two sides AD and BC, parallel to V, project on V in their true length; the sides AB and DC, parallel to H project on H in their true length. The perspective of the rhombus is  $a_1b_1c_1d_1$ , found by perpendiculars and diagonals. Since AD and BC are parallel to V their perspectives are parallel to their V projections. Then with  $a_1d_1$  as a side the square

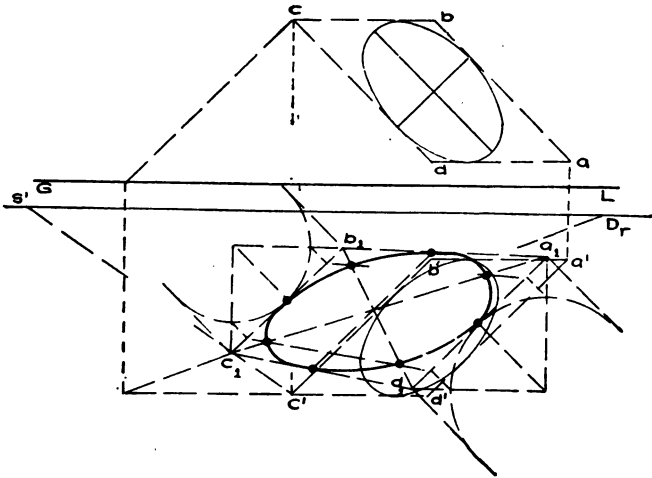


Fig. 80.

is constructed and the diagonals, center lines and inscribed circle drawn. Inasmuch as the vanishing point of the lines  $a_1b_1$  and  $c_1d_1$  comes without the limits of the drawing it is necessary to construct a second square on  $b_1c_1$  to

locate 4 of the 8 points. No further explanation is necessary as the construction is evident from the figure.

(142) Problem — 44 — **To find the perspective of a given sphere.**

**Discussion:**

If with the eye as an apex a cone of tangent visual rays be drawn to the sphere, the line of tangency, tho not a meridian circle, will to the observer be the bounding line of the body and is known as the line of **apparent contour**. The perspective of this *line of contour* is the bounding line of the perspective.

**Analysis:**

*With the eye as an apex draw a cone of visual rays tangent to the sphere, and find the line of apparent con-*

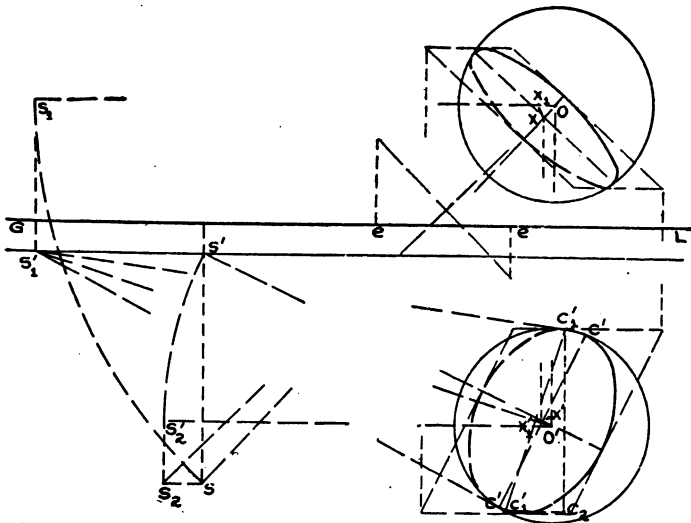


Fig. 81.

*tour. The perspective of this line of contour (circle), is the required perspective of the given sphere.*

**Construction:**

Given: Sphere O, Fig. 81.

Reqd: Perspective of the sphere.

Connect the point of sight with O the center of the sphere. Revolving S about the vertical axis of the sphere until SO is parallel to V,  $s'$  moves into the position  $s'_1$  and the projection of the revolved cone may be drawn.  $c'_1c'_1$  is the V projection of the line of apparent contour in this revolved position and equal to the diameter of the circle. Angle  $o's_1's'$  equals angle "Θ" of the axis of the cone, hence, projecting  $c'_1c'_1$  upon a horizontal line thru  $c'_1$  we obtain  $c_2c'_1$ , the minor axis of the elliptical H projection of the circle.  $x$  is the center of this ellipse; the axes of the ellipse may then be drawn thru  $x$  as follows: the major perpendicular to  $so$  and equal to  $c'_1c'_1$ , the minor coinciding with  $so$  and equal to  $c_2c'_1$ . The major axis of the elliptical V projection of the circular line of contour may be drawn thru  $x'$  perpendicular to  $s'o'$  and equal to  $c'_1c'_1$ . To determine the minor axis, SO is revolved about a perpendicular V thru O until it is parallel to H. The angle between  $s_2o$  and GL equals angle "Φ" of the axis of the cone, hence a perpendicular to  $s_2o$ , equal in length to  $c'_1c'_1$ , when projected on GL gives  $ee$ , the required minor axis. Measuring along  $s'o'$  on either side of  $x'$  one half of  $ee$ , the minor axis is located.

To find the perspective of the line of contour as represented by these ellipses, the 8 point method of Problem 42 can be used. In circumscribing the projections of the rhombus it is seen that in both projections, one pair of sides are parallel to the major axis while the other pair are perpendicular to the minor.

## SHADOWS IN PERSPECTIVE

(143) Before taking up any problems in finding the perspectives of shadows it may be well to mention a few facts of the principles of shadows.

(1) The *shadow* of a *given point* on a *given surface* is the *piercing point* in that *surface* of a *shadow ray* thru the *given point*.

(2) The *shadow* of a *given point* on a *given plane* is also the *intersection* of the *shadow ray* with the *projection* of the *shadow ray* on the *given plane*.

(3) The *vanishing point* of rays of *light* may be *determined* exactly as the *vanishing point* of any *other group* of *parallel lines*. See V, Fig. 82.

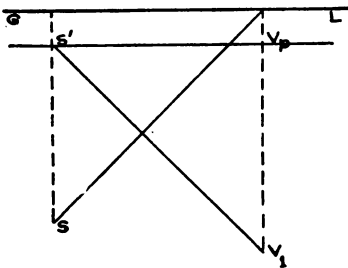


Fig. 82.

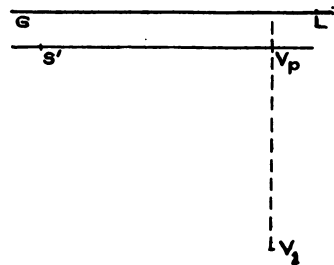


Fig. 83.

(4) It being permissible to *assume* the *direction* of *light* the draftsman has a perfect right to *assume* at will the *vanishing point* of rays. This has the added advantage of placing the shadow more nearly where it is wanted. See Fig. 83.

(5) Shadows are most frequently wanted on horizontal planes. The projections of rays on horizontal planes are horizontal lines, hence have their vanishing



point in the horizon. Since the ray and its projection lie in the same vertical plane the vanishing points of rays and projections are on the same vertical line; hence on the location of the vanishing point of rays the vanishing point of projections on horizontal planes is immediately determined as the intersection with the horizon of a perpendicular to GL from  $V_1$ . See Fig. 83.

(144) Problem — 45 — To find the perspective of the shadow of a given point on a given horizontal plane.

### Analysis:

*The intersection of the perspective of the shadow ray from the given point and the perspective of the projection of this shadow ray on the given horizontal plane, is the required perspective of the shadow of the given point.*

### Construction:

Given: Point O and plane T, Fig. 84.

Reqd: Perspective of shadow of O on T.

Assuming the vanishing point of light at  $V_1$  the vanishing point of horizontal projections is located at  $V_p$ . By a perpendicular and a diagonal the perspective of O is  $o_1$ ;  $o_1V_1$  is then the perspective of the shadow ray from O. The projection of O on T is  $O_t$ , V projection  $o'_t$ , H projection coinciding with  $o$ . The perspective of  $O_t$  is  $o_t$ , and the perspective of the projection,  $o_tV_p$ . The required perspective of the

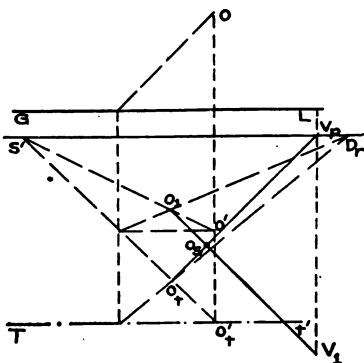


Fig. 84.

required perspective of the

shadow of  $O$  on  $T$  is then  $o_s$ , the intersection of  $o_1V_1$  and  $o_tV_p$ .

(145) Problem — 46 — To find the perspective of the shadow of a given point on any oblique plane.

**Analysis:**

*The intersection of the perspective of the shadow ray from the given point with the perspective of the projection of this shadow ray on the given oblique plane, is the required perspective of the shadow of the given point.*

(146) Problem — 47 — To find the perspective of the shadow of a given point on a given surface.

**Analysis:**

*The intersection of the perspective of the shadow ray*

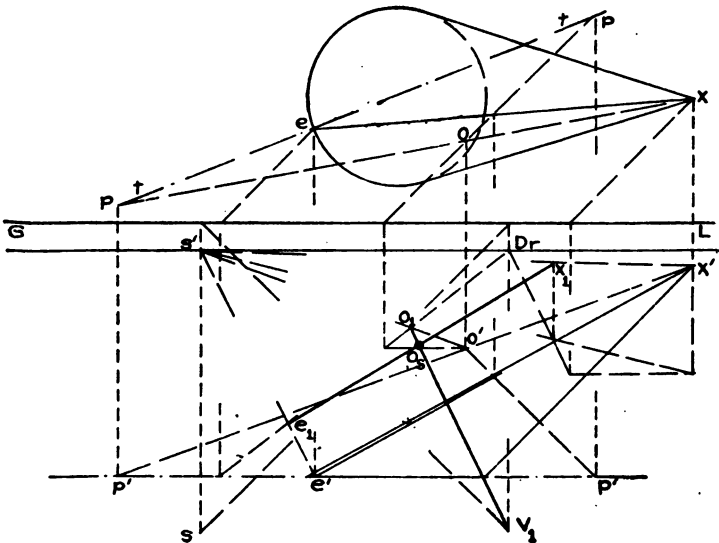


Fig. 85.

from the given point with the perspective of line cut from the surface by a plane thru the shadow ray, is the required perspective of the shadow of the given point.

### Construction:

Given: Point O and Cone X, Fig. 85.

Reqd: Perspective of shadow of O on X.

The assumed projections of a ray of light thru S locate the vanishing point of light at  $V_1$ . Thru O pass a shadow ray, also a line to the apex X. These lines pierce the plane of the base of the cone in points P and P. The plane of the two lines cuts from the plane of the base the trace  $tt$ , and from the surface of the cone the element EX. The perspective of the element EX is  $e_1x_1$ , and the perspective of the shadow ray thru O is  $o_1V_1$ . These perspectives intersect at  $o_s$ , the required perspective of the shadow of O on the given cone X.

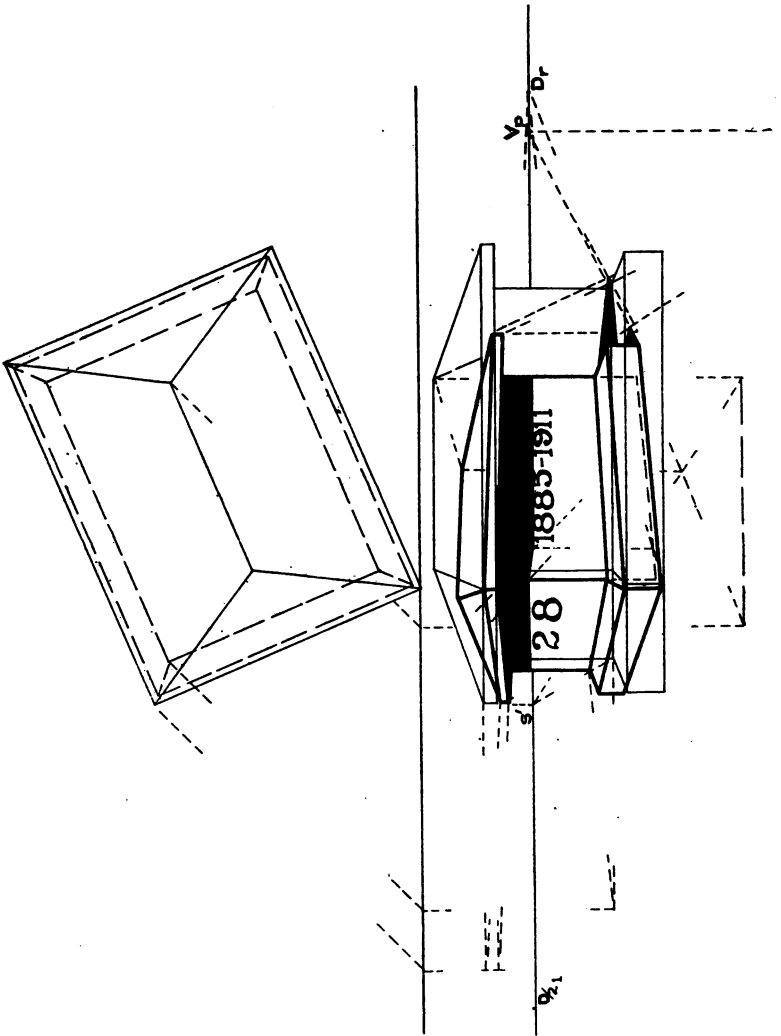
### POSITION OF POINT OF SIGHT

(147) Tho no fixed rules for placing the point of sight will be given, a few suggestions will be helpful.

With beginners there is a tendency to simplify construction by placing the point of sight *too close* to the object, perhaps within the limits of the drawing. The result is invariably a "steep" perspective having a most unnatural appearance. Remember always that *too great a distance produces less serious faults than too small a distance*. With large objects the point of sight should be before the picture plane a distance of at least three or four times the greatest dimension of the object.

More natural perspectives are produced with the point of sight nearly in *front* of the object. With the point too far to the side trouble is likewise experienced with too long intersections.

With small objects any rule but that of "common sense" is hard to give. The best solution of the matter is to ask ones self the question : If this object were before me, how far from it should the eye be, and in just what position to see all of the details to the best advantage? A few minutes thought usually suffices to answer the question satisfactorily.



**PART 4****QUIZ SHEETS**

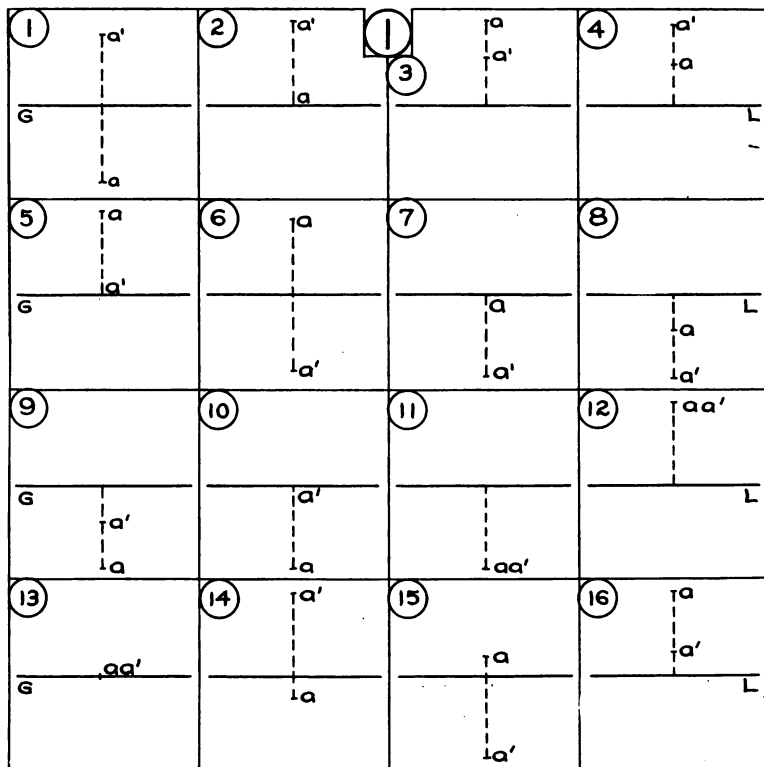
The following quiz sheets are given for the convenience of both instructor and student in assigning problems for oral recitation. In connection with each sheet is given a number of requirements which may be made of any problem on the sheet.

It is suggested that the problems be assigned numerically as follows: 1—6—2; this assignment reads, Quiz Sheet 1, Problem 6, to be solved for Requirement 2.

# QUIZ SHEET NO. 1

## Requirements

- 1—Required: A line thru point A.
- 2— “ A line from point A passing thru ? angle into ? angle.
- 3— “ A line thru point A,  $Z''$  long and at angle “ $\Theta$ ” or “ $\Phi$ ” with H or V.
- 4— “ Two lines thru point A at  $90^\circ$  to each other.
- 5— “ A plane thru point A.
- 6— “ A plane thru point A at angle “ $\Theta$ ” or “ $\Phi$ ” with H or V.
- 7— “ A point on GL,  $Z''$  from point A.
- 8— “ Five (5) points in space each  $Z''$  from point A.
- 9— “ A line thru point A at given angles “ $\Theta$ ” and “ $\Phi$ ” with H and V.
- 10— “ A plane thru point A at given angles “ $\Theta$ ” and “ $\Phi$ ” with H and V.

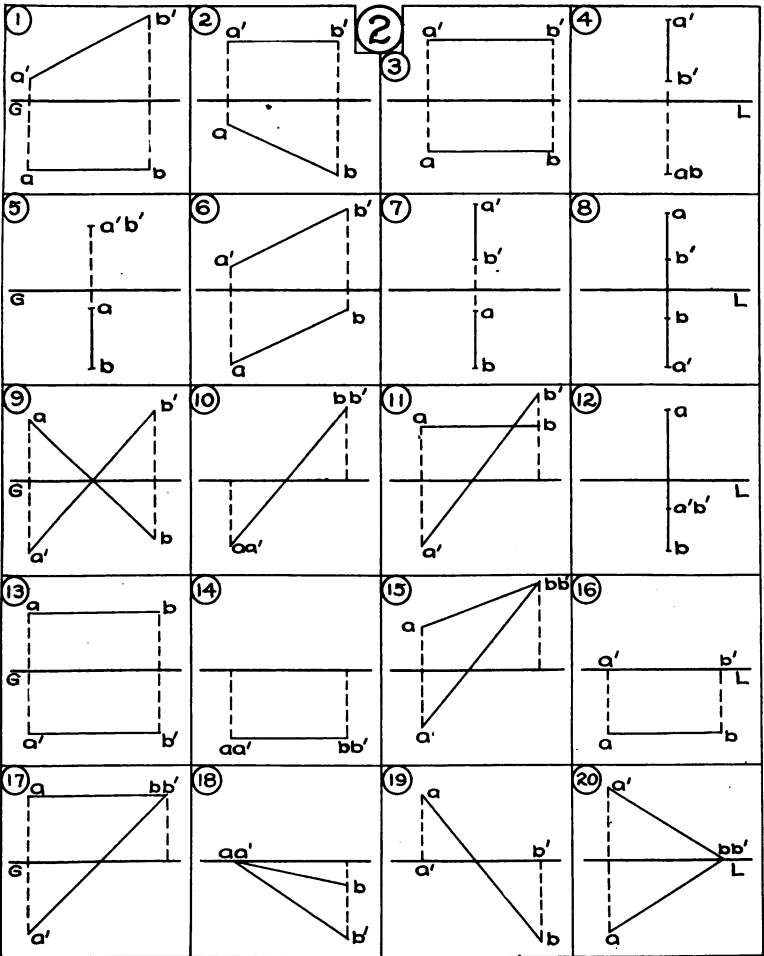




## QUIZ SHEET NO. 2

### Requirements

- 1—Required: Two (2) points, each  $Z''$  from line  $AB$ .
- 2— “ A line  $BC$  making  $90^\circ$  with  $AB$ .
- 3— “ A plane thru  $A$  perpendicular to line  $AB$ .
- 4— “ True length of line  $AB$ .
- 5— “ A line thru  $A$  at angle “ $X$ ” with line  $AB$ .
- 6— “ A plane thru  $A$  at angle “ $X$ ” with line  $AB$ .
- 7— “ Angles “ $\Theta$ ” and “ $\Phi$ ” of line  $AB$ .
- 8— “ A plane thru  $AB$  at given angle “ $\Theta$ ” or “ $\Phi$ ” with  $H$  or  $V$ . (Given angle equal to or greater than corresponding angle of the given line)
- 9— “ A point on  $GL$ ,  $Z''$  from line  $AB$ .
- 10— “  $P$  proj. of  $AB$ .



## QUIZ SHEET NO. 3

### Requirements

- 1—Required: A plane thru point O parallel to line AB.
- 2— “ A plane thru point O and line AB.
- 3— “ A plane thru point O perpendicular to line AB.
- 4— “ The distance from point O to line AB.
- 5— “ A line thru point O at given angle “X” with line AB.
- 6— “ A plane thru point O at given angle “X” with line AB.



## QUIZ SHEET NO. 4

### Requirements

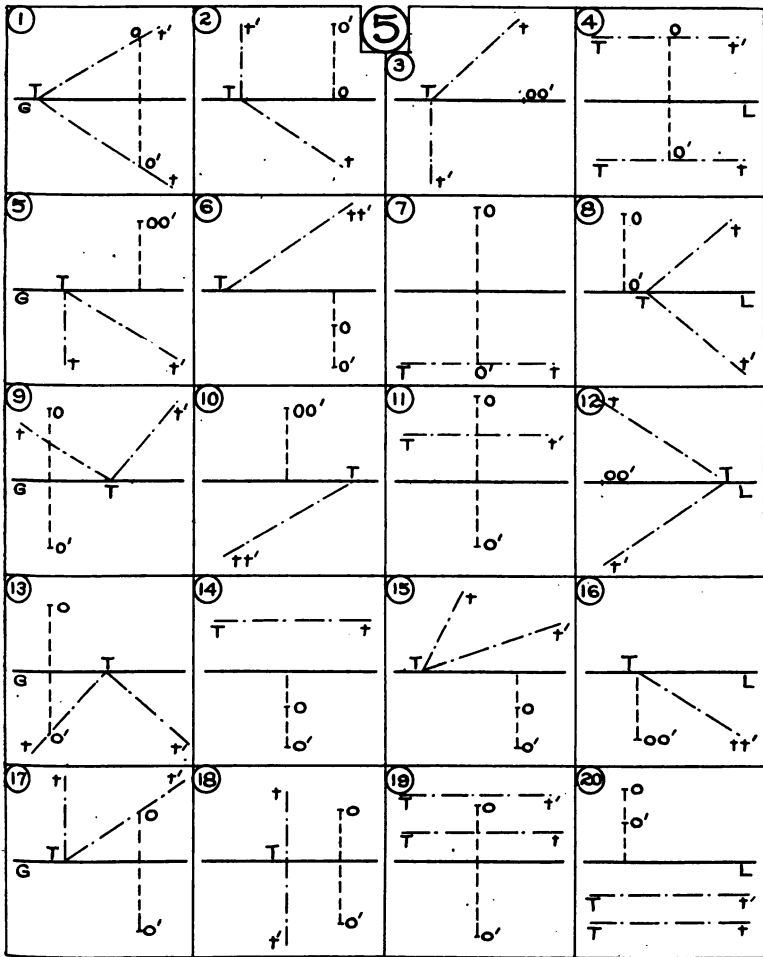
- 1—Required: A plane thru lines AB and BC.
- 2— “ True value of angle “X”.
- 3— “ Projections of the bisector of angle “X”.
- 4— “ Locus of points equidistant from lines AB and BC.



## QUIZ SHEET NO. 5

### Requirements

- 1—Required: Line thru point O parallel to plane T.
- 2— “ A plane thru point O parallel to plane T.
- 3— “ The projection of point O on plane T.
- 4— “ The distance from point O to plane T.
- 5— “ A line thru point O at given angle “X” with plane T.
- 6— “ A plane thru point O at given angle “X” with plane T.
- 7— “ Angles “ $\Theta$ ” and “ $\Phi$ ” of plane T.
- 8— “ A plane thru Tt or Tt' at given angle “X” with plane T.
- 9— “ A point in plane T, Z" from point O.

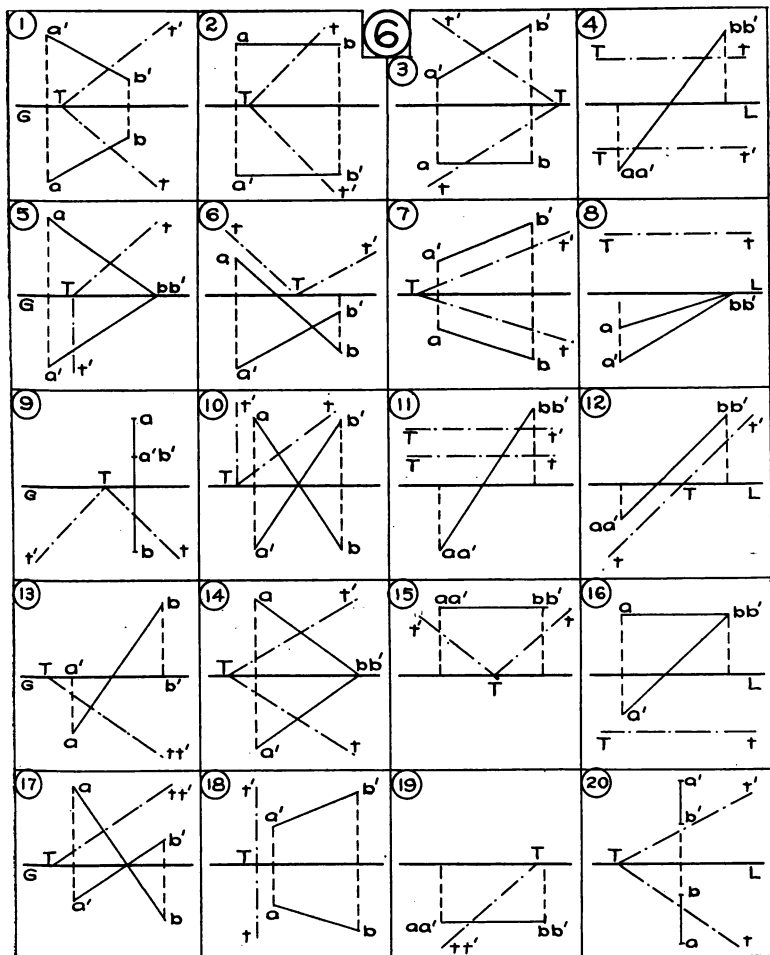




# QUIZ SHEET NO. 6

## Requirements

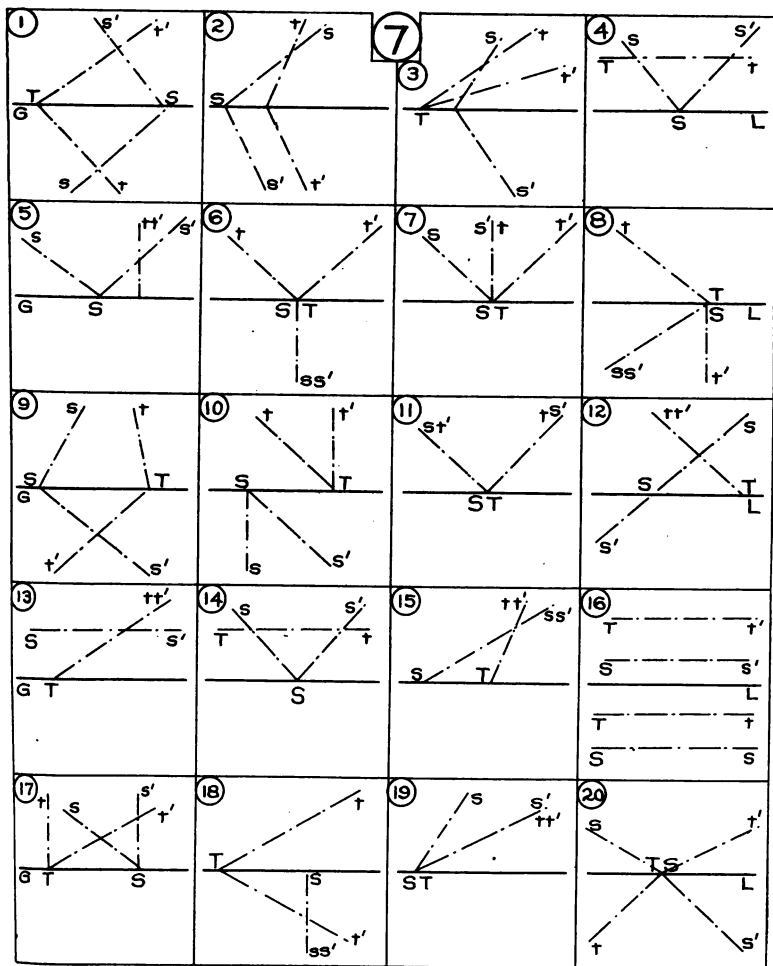
- 1—Required: The piercing point of line AB in plane T.
- 2— “ The projection of line AB on plane T.
- 3— “ Angle “X”.
- 4— “ A line in plane T at given angle “X” with line AB. (Given angle “X” equal to or greater than angle “X” of line AB.)
- 5— “ A plane thru line AB perpendicular to plane T.



# QUIZ SHEET NO. 7

## Requirements

- 1—Required: Angle "X".
- 2— " Plane bisecting angle "X".
- 3— " A plane perpendicular to S and T.



**QUIZ SHEET NO. 8**

<p>① Tt makes <math>60^\circ</math> with Tt'. Req'd: Tt</p>	<p>② Req'd: Line in H at <math>60^\circ</math> to AB</p>	<p>③ Req'd: Projs. of O lies in T. Req'd: Projs. <math>o_1</math> of O.</p>	<p>④ Req'd: H. proj. of O &amp; H trace of T</p>
<p>⑤ Req'd: Point in T, Z'' from O. O lies in t'. T.</p>	<p>⑥ O is Z'' from T. Req'd: Projs. <math>o'</math> of O.</p>	<p>⑦ AB &amp; CD are // &amp; Z'' apart. Req'd: H Proj. of CD.</p>	<p>⑧ O is Z'' from T. Req'd: Tt'.</p>
<p>⑨ AB is side of equi-lat. <math>\Delta</math>. <math>\theta</math> of <math>\Delta = 60^\circ</math>. Req'd: <math>\Delta</math></p>	<p>⑩ O is Z'' from AB. Req'd: H Proj. of O.</p>	<p>⑪ Revolve O about AB into H. equi-dist. from H &amp; V.</p>	<p>⑫ Req'd: Pt. in H Z'' from AB.</p>
<p>⑬ Req'd: Traces of T &amp; projs. of O. t' &amp; B, O lies in T.</p>	<p>⑭ S &amp; T are // &amp; Z'' apart. Req'd: H traces</p>	<p>⑮ Req'd: 3 pts. in T Z'' from O.</p>	<p>⑯ O is Z'' from T. Req'd: H Proj. <math>o'</math> of O.</p>
<p>⑰ AB is Z'' from O. Req'd: H Proj.</p>	<p>⑱ ab makes <math>60^\circ</math> with a'b'. Req'd: ab.</p>	<p>⑲ <math>\angle X = 60^\circ</math>. Req'd: ab.</p>	<p>⑳ <math>\angle X = 90^\circ</math>. Req'd: Ss.</p>



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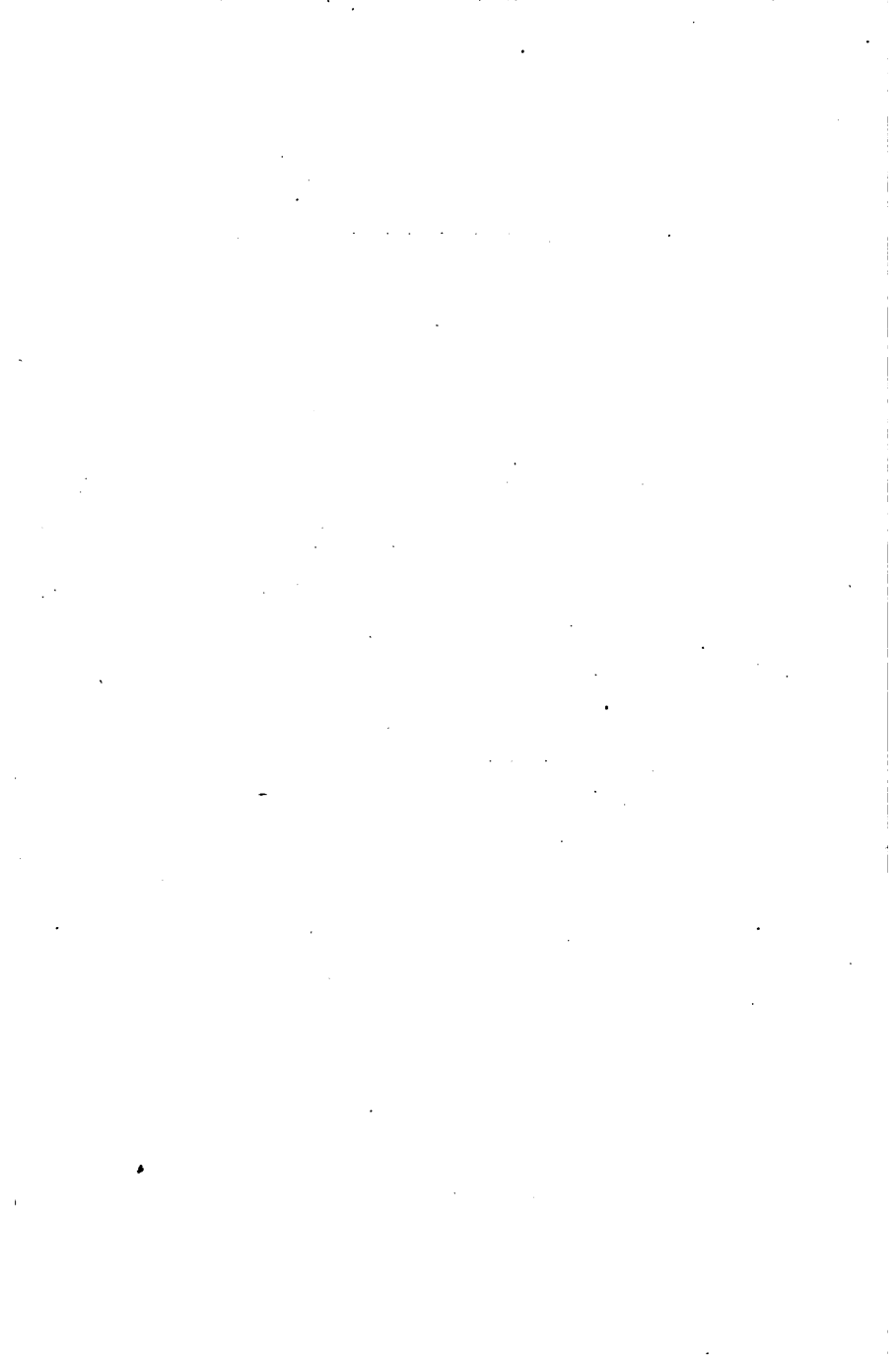
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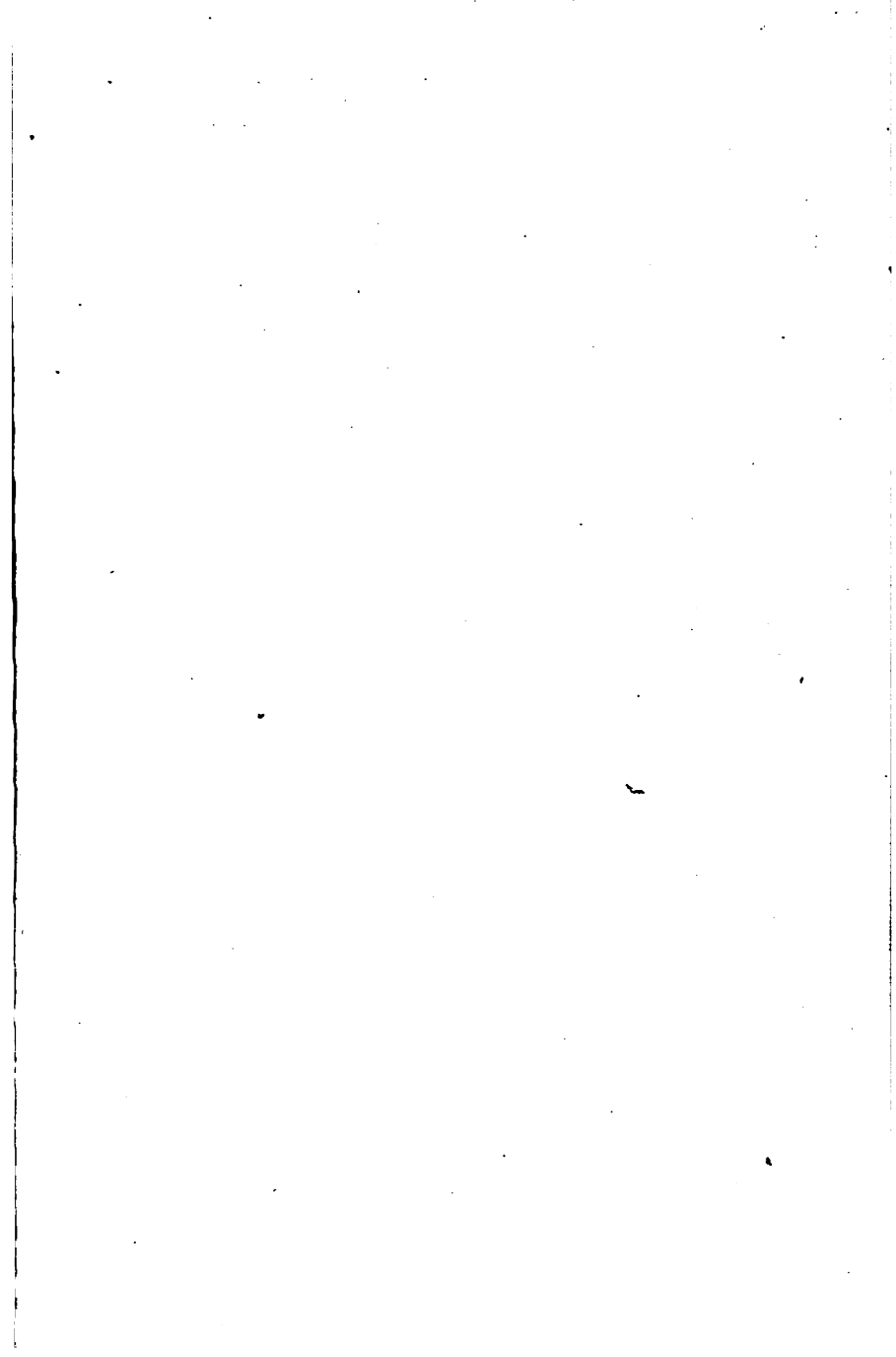
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