O‘ZBEKISTON RESPUBLIKASI OLIY VA O‘RTA MAXSUS TA’LIM VAZIRLIGI

OLIY MATEMATIKADAN INDIVIDUAL TOPSHIRIQLAR TO‘PLAMI

To‘rt qismdan iborat

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3 Qism

Belorussiya Xalq ta’limi vazirligi
tomonidan oliy o‘quv yurtlarining muxandis texnik
mutaxassisliklar talabalar uchun
o‘quv qo‘llanma sifatida
tavsiya etilgan

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Tarjimonlar: O'zMU Mexanika va matematik modellashtirish kafedrasi mudiri, professor,f.-m.f.d. A.B.Axmedov, Toshkent davlat texnika universiteti Oliy matematika kafedrasi dosentlari f-m.f. nomzodlari: [G.Shodmonov], A.Abdukarimov, D.N.Shamsiyev, E.E.Esonov.

Ushbu kitob oliy matematikadan o'quv qo'llanmalar jamlanmasining tarkibiy qismi bo'lib oliy ta'lim muassalari talabalarining mustaqil faoliyatini rivojlantirish va faollashtirishga yo'naltirilgan.
Bu yerda qatorlar, karrali va egri chiziqli integrallar hamda maydonlar nazariyasi elementlariga oid nazariy ma'lumotlar hamda auditoriya topshiriqlari va mustaqqil ishlar uchin masala va mashqlar keltirilgan.

Oliy texnika oquv yurtlari uchin mo'ljallangan.

Книга является составной частью комплекса учебных пособий по курсу высшей математики, направленных на развитие и активизацию самостоятельной работы студентов вузов. Содержатся теоретические сведения и набора задач для аудиторных и индивидуальных заданий по рядам, кратным и криволинейным интегралам и элементам теории поля.

Для студентов инженерно-технических специальностей вузов.

The book is an integral part of a set of textbooks for the course of higher mathematics, aimed at the development and activation of independent work of university students. Contains theoretical information and a set of tasks for classroom and individual tasks for series, multiple and curvilinear integrals and elements of field theory.

For students of engineering and technical specialties of universities.

Taqrizchilar:

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So‘z boshi

Qo‘lingizdagi ushbu kitob, “Oliy matematikadan individual topshiriqlar” umumiy nomli o‘quv qo‘llanmalar majmuasining uchinchi qismi bo‘lib, u oliy o‘quv yurtlarining muhandis–texnik yo‘nalishlari uchun mo‘ljallangan 380-450 soatlik dastur asosida yozilgan. Shuningdek, mazkur majmuadan, oliy matematika fanini o‘qitish uchun ajratilgan soatlar anchagina kam bo‘lgan boshqa yo‘nalishdagi mutaxassislar tayyorlaydigan oliy o‘quv yurtlarining talabalari ham foydalanishlari mumkin. (Buning uchun taqdim etilayotgan materiallardan keraklilarini tanlab olinishi lozim). Shu bilan bir qatorda kitobdan oliy o‘quv yurtlarining kechki va sirtqi ta’lim talabalari ham foydalanishlari mumkin.

Ushbu o‘quv qo‘llanma o‘qituvchilar va talabalar uchin tavsiya etilgan bo‘lib, ular auditoriyada amaliy mashg‘ulotlar va mustaqil (nazorat) ishlarni o‘tkazish uchun hamda oliy matematikaning barcha bo‘limlari bo‘yicha individual uy topshiriqlarini bajarish uchun mo‘ljallangan.

O‘quv majmuaning uchinchi qismida qatorlar, karrali va egri chiziqli integrallar hamda maydonlar nazariyasi elementlariga bag‘ishlangan mavzular bo‘yicha materiallar keltirilgan.

Kitobning ucunchi qismi tuzilishi ham uning birinchi qismiga aynan o‘xshash ko‘rinishda yozilgan. Boblar, paragraflar va rasmlarning raqamlanishi birinchi qismga mos ravishda davom ettirilgan.

Kitobning yaxshilanishi borasidagi bebaho ko‘rsatma va maslahatlarini ayamaganliklari uchun mualliflar jamoasi, mazkur majmuaning taqrizchilari bo‘lgan, FA muxbir a’zosi, fizika-matematika fanlari doktori, professor S.I. Poxojaev rahbarligidagi Moskva energetika instituti “Oliy matematika” kafedrasining jamoasiga, Minsk radiotexnika institutining “Oliy matematika” kafedrasining mudiri, fizika-matematika fanlari doktori, professor L.A. Cherkasga hamda shu kafedraning dotsentlari, fizika-matematika fanlari nomzodlari L.A. Kuznetsov, P.A. Shmelyov, A.A. Karpuklarga, o‘zlarining minnatdorchiliklarini bildiradilar.

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Mualliflar

USLUBIY TAVSIYALAR

Tavsiya etilayotgan qo‘llanmaning shakli, undan foydalanish uslubi, talabaning ko‘nikma va bilimlarini baholash mezonlarini tavsiflab chiqamiz.

Oliy matematika kursi bo‘yicha barcha ma’lumotlar boblarga taqsimlangan bo‘lib, ularning har birida masala va misollarni yechish uchun zarur bo‘ladigan nazariy bilimlar (asosiy ta’riflar, tushunchalar, teoremalar va formulalar) keltirilgan. Ushbu ma’lumotlar yechilgan mashqlar yordamida mustahkamlanadi. (Misollar yechishning boshlanishi - ► va oxiri - ◄ belgilar yordamida berilgan.) So‘ngra auditoriya mashg’ulot (AT) va o‘tkazilayotgan mashg‘ulotlarda 10-15 minutga mo‘ljallangan mustaqil (kichik-nazoratli) ishlar uchun javoblari bilan birgalikda masala va misollar tanlab olingan. Va nihoyat 30 variantdan iborat haftalik individual uy topshiriqlari (IUT), namunaviy misollar yyechimi bilan birgalikda berilgan. IUT ma’lum qismining javoblari ham keltirilgan. Har bobning nihoyasida amaliy ahamiyatga molik, darajasi yuqori qiyinchilikka ega bo‘lgan qo‘shimcha topshiriqlar joylashtirilgan.

Ilovada muhim mavzular bo‘yicha bir va ikki soatga mo‘ljallangan (har biri 30 variantlik) nazorat ishlari ishlari keltirilgan.

AT topshiriqlarining raqamlanishi uzluksiz bo‘lgan ikki sondan iborat: birinchisi- qismi bobni aniqlasa, ikkinchisi- ushbu bobdagi AU tartib raqamini belgilaydi. Masalan AT 12.1 shifri o‘n ikkinchi bobga tegishli birinchi topshiriqni aniqlaydi. Qo‘llanmaning uchinci qismida 21 AT va 10 IUT berilgan.

IUT uchun ham boblar bo‘yicha raqamlash kiritilgan. Masalan IUT 12.2 belgisi o‘n ikkinchi bobdagi ikkinchi IUT ekanligini ta’kidlaydi. Har bir IUT ning ichida esa quyidagicha raqamlash kiritilgan: birinchi son topshiriqdagi masalaning tartib raqamiga tegishli bo‘lsa, ikkinchisi variantning tartib raqamini aniqlaydi. Shunday qilib, IUT 12.2:16 shifri talabaning IUT12.2 dan 16 variantdagi topshiriqlarini bajarishini belgilab, ushbu variantda 1.16, 2.16, 3.16, 4.16 masalalar borligini ta’kidlaydi. IUT bo‘yicha variantlarni tanlab olishda oldingi topshiriqdan keyingisiga o‘tganida tasodifiy yoki boshqa usulda almashtirish usulini qo‘llash mumkin. Bundan tashqari, ixtiyoriy talabaga IUT berilishida bir xil turdagi masalalarni har xil variantlardan olish mumkin. Masalan, IUT -12.2:1.2; 2.4; 3.6; 4.1; 5.15 shifri talaba IUT -12.2 dan birinchi masalani 2 - variantdan, ikkinchisini 4 - variantdan, uchinchisini 6 – variantdan, to‘rtinchisini 1-variantdan, beshinchisini 15- variantdan yechishini ta’kidlaydi. Bu ko‘rinishdagi kombinatsion usul 30 ta variantdan keng qamrovli ko‘p variantlar hosil qilishni ta’minlaydi.

IUT larni ba’zi oliy texnika o‘quv yurtlari (Belorussiya qishloq xo‘jaligini mexanizatsiyalash instituti, Belorussiya politexnika instituti, Uzoq sharq politexnika instituti v.b) ning o‘quv jarayonida qo‘llanilishi, IUT ni har bir haftalik auditoriya topshiriqlaridan keyin alohida har safar berishning o‘rniga, ikki haftada bir marta, ikki haftalik auditoriya mashg‘ulotlari mazmuniga mos ravishda berish maqsadga muvofiq ekanligini ko‘rsatdi. Ushbu qo‘llanmaga muvofiq talabalar bilan ishlashni tashkil etish bo‘yicha umumiy tavsiyalarni beramiz.

1. Oliy o‘quv yurtlarining 25 talik guruhlari uchun har haftada ikkita auditoriya mashg‘ulotlari, talabalar erkin qatnashadigan maslahat darslari rejalashtiriladi va haftalik IUT beriladi. Ushbu tadbirlarni samarali tashkil etish maqsadida, talabalar bilimini, xato va kamchiliklarini aniqlash va tuzatish yo‘llarini ko‘rsatgan holda, tizimli baholash uchun kafedra tomonidan oldindan tayyorlangan professor-o‘qituvchilarga IUT ning javoblar varaqasi va yyechimlar majmuasi beriladi (talabalar mustasno). Javoblar varaqasi har bir topshiriqlar uchun tayyorlansa, yyechimlar majmuasi faqat yechish usulini, amallar ketma-ketligi va hisoblashlardagi ko‘nikmalarning to‘g‘riligini tekshirish uchun zarur bo‘lgan muhim bo‘lgan masala va variantlarga ishlab chiqiladi. Kafedra tomonidan yyechimlar varaqasi qaysi IUT lar uchun zarurligini belgilanadi. Yyechimlar varaqasi (bitta variant bitta varaqda joylashadi) talabalar tomonidan bajarilgan topshiriqlar bajarilishida o‘z o‘zini nazorat qilish uchun, talabalar o‘rtasida o‘zaro nazorat tashkil etishda ishlatiladi. Lekin ko‘pchilik hollarda

yyechimlar varaqasi yordamida o‘qituvchi usulning to‘g‘riligini tekshirsa, talabalar o‘zining hisob-kitoblari to‘g‘riligini nazoratdan o‘tkazishi mumkin. Ushbu usullar 25 talabaning IUT larini 15-20 minut davomida tekshirib baholash imkonini beradi.

2. Oliy o‘quv yurtlarining 15 talik guruhlarida esa har haftada ikkita auditoriya mashg‘ulotlari, guruhlar dars jadvalida mustaqil tayyorlanish uchun, o‘qituvchi nazorati ostida haftalik yuklamaga kiritilgan ikki soatlik maslahat darslari rejalashtiriladi. Dars jarayonini ushbu taxlitda tashkil etish (Belorussiya qishloq xo‘jaligini mexanizatsiyalash instituti), talabalarning mustaqil va ijodiy ishlashlari hamda bilim sifatini o‘qituvchilar tomonidan tezkor ravishda nazorat qilish darajasi sezilarli tarzda oshishi kuzatiladi. Yuqorida tavsiya etilgan usullar bu yerda ham o‘zining samarasini beradi. Lekin, ushbu guruhlarda AT va IUT larni tekshirish tezlashadi va topshiriqlarni bajarishda nazariy bilimlarni nazorat qilish imkoni oshadi, o‘zlashtirmovchi talabalardan mavjud qarzdorliklarni kamaytirish imkoniyati paydo bo‘ladi. Shuningdek, yana IUT, mustaqil va nazorat ishlari bo‘yicha baholar jamlamasi yordamida o‘quv jarayonini boshqarish, nazorat qilish, talabalar olgan bilimlari sifatini baholash imkoni ham paydo bo‘ladi.

Yuqorida aytilgan tadbirlarni amalga oshirish natijasida semestr mobaynida o‘rganilgan bilimlar bo‘yicha an’anaviy semestr (yillik) imtihonlardan voz kechish, hamda talabalar ko‘nikmalari va bilimlarini baholash bo‘yicha blokli-siklik (modulli-siklik) deb ataluvchi usuldan foydalanish mumkin bo‘ladi. Ushbu usulning mohiyati quyidagilardan iborat: Fanning semestrdagi (yillik) yuklamasi 3-5 ta blok (modul) larga bo‘linadi va ularning har biri bo‘yicha AT, IUT bajarilib, sikl yakunida esa ikki soatlik yozma nazorat o‘tkazilib, bu yerda 2-3 ta nazariy savollar, 5-6 ta masala va misolllar beriladi. AT, IUT va yakuniy nazorat ballarining yig‘indisi talabalarning har bir blok (modul) va semestr (o‘quv yilida) hamma bloklar (modullar) bo‘yicha olgan bilimlarini ham alohida obektiv baholash imkonini beradi. Shunga o‘xshash usul Belorussiya qishloq xo‘jaligini mexanizatsiyalash institutida tadbiq qilingan.

Fikrimiz yakunida, ushbu qo‘llanma o‘rtacha imkoniyatli talabalarga mo‘ljallanganligini va bu yerdagi bilimlarni egallash oliy matematika fanidan qoniqarli va yaxshi ko‘nikmalarga ega bo‘lishlarini ta’minlashini ta’kidlashimiz mumkin. Iqtidorli va a’lo bahoga o‘quvchi talabalar uchun esa, rag‘batlantirishning chora-tadbirlarini e’tiborga olgan holda alohida murakkab topshiriqlar (ta’limda individual yondoshuv) tayyorlanishi zarur. Masalan, bu talabalarga, o‘z ichiga ushbu qo‘llanmadagi yuqori murakkablikka ega masalalar va nazariy mashqlar (ushbu maqsad uchun, xususan, har bir bob oxiridagi qo‘shimcha topshiriqlar mo‘ljallangan) butun semestr uchun ishlab chiqilishi lozim. O‘qituvchi ushbu topshiriqlarni semestr boshida berib, ularning bajarilish ketma-ketligini belgilab (o‘zining shaxsiy nazoratida), talabalarga oliy matematikadan ma’ruza va amaliyot darslarida erkin qatnashishga ruxsat berishi mumkin va hamma topshiriqlar muvaffaqiyatli bajarilgandan so‘ng sessiyada a’lo baho qo‘yiladi.

12. QATORLAR

12.1 Sonli qatorlar. Sonli qatorlarning yaqinlashish belgilari.

Quyidagi ko‘rinishdagi ifoda

$$u_1 + u_2 + \cdots + u_n + \cdots = \sum_{n=1}^{\infty} u_n \qquad (12.1)$$

sonli qator deb ataladi. Bu erda $u_n \in R$, u_1, u_2,… u_n,… sonlar qatorning hadlari deb ataladi, u_n – qatorning umumiy hadi.

$$S_1 = u_1, \qquad S_2 = u_1 + u_2, \ldots S_n = u_1 + u_2 + \cdots + u_n$$

yig‘indilar qatorning xususiy yig‘indilari deb ataladi. S_n esa (12.1) qatorning n - chi xususiy yig‘indisi deyiladi.

Agar $\lim S_n$ mavjud bo‘lib, biror S soniga teng bo‘lsa, ya’ni

$S = \lim S_n$, u holda (12.1) qator yaqinlashuvchi deyiladi, S esa qatorning yig‘indisi deyiladi. Agar $S = \lim S_n$ mavjud bo‘lmasa (xususiy holda cheksiz), u holda (12.1) qator uzoqlashuvchi deyiladi.

$$r_n = u_{n+1} + u_{n+2}, \ldots + u_{n+k} + \cdots$$

yig‘indi (12.1) qatorning n-chi qoldig‘i deb ataladi.

Agar (12.1) qator yaqinlashuvchi bo‘lsa, u holda

$$\lim_{n\to\infty} r_n = \lim_{n\to\infty}(S - S_n) = 0$$

o‘rinli bo‘ladi.

1- misol. $\sum_{n=1}^{\infty}\frac{1}{n(n+1)}$ qator berilgan. Qatorning yaqinlashuvchi ekanligini ko‘rsating va uning yig‘indisini toping.

►Qatorning birinchi n-ta xususiy yig‘indisini yozamiz va uning shaklini almashtiramiz :

$$S_n=\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\cdots+\frac{1}{n(n+1)}=\left(\frac{1}{1}-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\cdots+\left(\frac{1}{n}-\frac{1}{n+1}\right)=1-\frac{1}{n+1}$$

$$S=\lim_{n\to\infty}S_n=\lim_{n\to\infty}\left(1-\frac{1}{n+1}\right)=1.$$

U holda berilgan qator yaqinlashuvchi va uning yig‘indisi S=1. ◄

Quyidagi ko‘rinishdagi qator

$$a+aq+aq^2+\ldots+aq^{n-1}+\ldots \qquad (12.2)$$

mahraji q bo‘lgan geometrik progressiyaning hadlari yig‘indisini ifodalaydi. Ma’lumki, $|q|<1$bo‘lsa, (12.2) qator yaqinlashuvchi va uning yig‘indisi S= $\frac{a}{1-q}$. Agar $|q|\geq 1$ bo‘lsa, u holda (12.2) qator uzoqlashuvchidir.

1- Teorema. (Qator yaqinlashishining zaruriy sharti).

Agar (12.1) sonli qator yaqinlashuvchi bo‘lsa, u holda $\lim_{n\to\infty}u_n=0$. Teskari tasdiq noto‘g‘ri.

Masalan, $1+\frac{1}{2}+\cdots+\frac{1}{n}+\cdots=\sum_{n=1}^{\infty}\frac{1}{n}$ garmonik qator umumiy hadi nolga intiladi, ammo qator uzoqlashuvchi.

2- Teorema. (Qator uzoqlashishining yetarli sharti). Agar $\lim_{n\to\infty}u_n\neq 0$ bo‘lsa, u holda (12.1) qator uzoqlashuvchidir.

Agar sonli qatorning ihtiyoriy chekli sondagi hadlari tashlab yuborilsa, uning uzoqlashuvchi yoki yaqinlashuvchi ekanligi saqlanib qoladi, ammo uning yig‘indisi, agar u mavjud bo‘lsa, o‘zgaradi.

2- misol. $\sum_{n=1}^{\infty} \frac{n}{3n+1}$ qator yaqinlashishini tekshiring.

►Qatorning umumiy hadini yozamiz.

$u_n = \frac{n}{3n+1}$. U holda, $\lim_{n\to\infty} u_n = \lim_{n\to\infty} \frac{n}{3n+1} = \frac{1}{3} \neq 0$,

ya’ni qator uzoqlashuvchi.◄

Musbat hadli sonli qatorlar yaqinlashishining ba’zi yetarlilik alomatlarini ko‘rib chiqamiz.

3- Teorema. (Taqqoslash alomatlari).

Agar ikkita qator berilgan bo‘lsa,

$$u_1 + u_2 + \cdots + u_n + \cdots \quad (12.3)$$

$$v_1 + v_2 + \cdots + v_n + \cdots \quad (12.4)$$

va barcha $n \geq n_0$ lar uchun $0 < u_n \leq v_n$ tengsizlik bajarilsa, u holda:

1) (12.4) qatorning yaqinlashuvchiligidan (12.3) qatorning yaqinlashuvchiligi kelib chiqadi;

2) (12.3) qator uzoqlashuvchiligidan (12.4) qatorning uzoqlashuvchi ekanligi kelib chiqadi.

Qatorlarni taqqoslash uchun taqqoslanuvchi qatorlar sifatida, $\sum_{n=0}^{\infty} aq^n$ geometrik qatorni hamda garmonik (uzoqlashuvchi) qatorni olish maqsadga muvofiqdir.

3- misol. Qatorning yaqinlashuvchi ekanligini isbotlang.

$$\sum_{n=1}^{\infty} \frac{1}{n\cdot 3^n} = \frac{1}{1\cdot 3} + \frac{1}{2\cdot 3^2} + \cdots + \frac{1}{n\cdot 3^n} + \cdots \quad (1)$$

► (1) qatorning yaqinlashuvchi ekanligini ko‘rsatish uchun quyidagi tengsizlikdan foydalanamiz

$$u_n = \frac{1}{n \cdot 3^n} < \frac{1}{3^n} \quad (n \geq 2).$$

va berilgan qatorni yaqinlashuvchi bo‘lgan $\sum_{n=1}^{\infty} \frac{1}{3^n}$, $q = \frac{1}{3} < 1$ qator bilan taqqoslaymiz. Taqqoslash alomatiga asosan (3 – teoremaga qarang) (1) qator yaqinlashuvchidir.◄

4- misol. $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n^2-1}}$ qatorni yaqinlashishga tekshiring.

► Ixtiyoriy $n \geq 2$ da $\frac{1}{\sqrt{n^2-1}} > \frac{1}{n}$ bo‘lganligi uchun berilgan qatorning hadlari ularga mos bo‘lgan uzoqlashuvchi garmonik qatorning hadlaridan katta. Demak, berilgan qator uzoqlashuvchi.◄

4- Teorema. (Dalamber alomati). Aytaylik, (12.1) qator uchun (biror n$> n_0$ hadidan boshlab) $u_n > 0$ va $\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = q$ limit mavjud bo‘lsin. U holda, berilgan qator:

1) q< 1 bo‘lganda yaqinlashuvchi;

2) q> 1 da uzoqlashuvchidir.

$q = 1$ da Dalamber alomati qatorning yaqinlashuvchi yoki uzoqlashuvchi ekanligiga javob bera olmaydi, qator yaqinlashuvchi ham, uzoqlashuvchi ham bo‘lishi mumkin. Bunday holda qatorning yaqinlashuvchiligi boshqa alomatlar yordamida tekshiriladi.

5- misol. $\sum_{\mathrm{n}=1}^{\infty} \frac{\mathrm{n}^2}{2^{\mathrm{n}-1}}$ qatorni yaqinlashishiga tekshiring.

►$u_n = \frac{n^2}{2^{n-1}}$, $u_{n+1} = \frac{(n+1)^2}{2^n}$, u holda, $\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = \lim_{n\to\infty} \frac{(n+1)^2 \cdot 2^{n-1}}{2^n \cdot n^2} =$

$= \frac{1}{2} \lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^2 = \frac{1}{2} < 1$. Demak, berilgan qator yaqinlashuvchi. ◄

5- Teorema. (Koshining radikal alomati). Agar biror n= n_0 dan boshlab, $u_n > 0$ va $\lim_{n\to\infty} \sqrt[n]{u_n} = q$ bo‘lsa, u holda $q < 1$ bo‘lganda (12.1) qator yaqinlashuvchi, $q > 1$ bo‘lganda uzoqlashuvchi bo‘ladi.

$q = 1$ uchun Koshi alomatini qo‘llab bo‘lmaydi.

6- misol. Qatorni yaqinlashishga tekshiring

$$\sum_{n=1}^{\infty}\left(\frac{n+1}{8n-1}\right)^n.$$

► Koshining radikal alomatidan foydalanamiz.

$$q = \lim_{n\to\infty} \sqrt[n]{\left(\frac{n+1}{8n-1}\right)^n} = \lim_{n\to\infty}\frac{n+1}{8n-1} = \lim_{n\to\infty}\frac{1+1/n}{8-1/n} = \frac{1}{8} < 1.$$

Demak, berilgan qator yaqinlashuvchi.◄

6- Teorema. (Koshining integral alomati). Faraz qilaylik, (12.1) qatorning hadlari monoton kamaysin va $y = f(x)$ funksiya $x \geq a \geq 1$ da uzluksiz bo‘lib, $f(n) = u_n$ bo‘lsin. U holda (12.1) qator va $\int_a^{\infty} f(x)dx$ integral bir vaqtda yaqinlashuvchi yoki uzoqlashuvchi bo‘ladi.

Masalan, $\int_a^{\infty}\frac{1}{x^{\alpha}}dx\ (\propto \epsilon R)$ integral $\alpha > 1$ da yaqinlashuvchi va $\alpha \leq 1$ da uzoqlashuvchi ekanligi uchun Dirixle qatori $\sum_{n=1}^{\infty}\frac{1}{n^{\alpha}}, \alpha > 1$ da yaqinlashuvchi va $\alpha \leq 1$ da uzoqlashuvchidir.

Ko‘pgina qatorlarning yaqinlashuvchiligini ularga mos Dirixle qatoriga taqqoslash yo‘li bilan tekshirish mumkin.

7- misol. Qatorni yaqinlashishiga tekshiring

$$\sum_{n=1}^{\infty}\frac{2n}{(n^2+1)^2}.$$

►$f(x) = \frac{2x}{(x^2+1)^2}$ deb olaylik. Bu funksiya Koshining integral alomatining barcha shartlarini qanoatlantiradi. U holda xosmas integral uchun:

$$\int_1^{\infty} \frac{2x}{(x^2+1)^2} dx = \lim_{B \to \infty} \int_1^{B} \frac{2x}{(x^2+1)^2} dx = -\lim_{B \to \infty} \frac{1}{x^2+1}\Big|_1^B = \frac{1}{2},$$

ya'ni xosmas integral yaqinlashuvchi. Demak berilgan qator ham yaqinlashuvchi.◄

Agar ixtiyoriy N (n > N) hadidan boshlab u_n hadlarining ishoralari har xil bo'lsa, (12.1) qator o'zgaruvchan ishorali qator deb ataladi.

Agar

$$|u_1| + |u_2| + \cdots + |u_n| + \cdots \qquad (12.5)$$

qator yaqinlashuvchi bo'lsa, (12.1) qator ham yaqinlashuvchi bo'ladi va u mutloq yaqinlashuvchi deb ataladi.

Agar (12.5) qator uzoqlashuvchi (12.1) qator esa yaqinlashuvchi bo'lsa, (12.1) qator shartli (mutloq bo'lmagan) yaqinlashuvchi deyiladi.

Qatorni mutloq yaqinlashishga tekshirishda musbat hadli qatorlarning yaqinlashish alomatlaridan foydalaniladi.

8- misol. Qator yaqinlashishini tekshiring.

$$\sum_{n=1}^{\infty} \frac{sinn\alpha}{n^2} \qquad (\alpha \epsilon R)$$

► Berilgan qatorning absolyut qiymatlaridan tuzilgan qatorni qaraymiz, ya'ni $\sum_{n=1}^{\infty} \frac{|sinn\alpha|}{n^2}, (\alpha \in R)$. $|sinn\alpha| \le 1$ bo'lganligi uchun berilgan qatorning hadlari yaqinlashuvchi bo'lgan $\sum_{n=1}^{\infty} \frac{1}{n^2}$ $(\alpha = 2)$ Dirixle qatorining hadlaridan katta emas. Bundan kelib chiqadiki, taqqoslash alomatiga asosan (3 – teoremaga qarang) berilgan qator mutloq yaqinlashuvchi.◀

Quyidagi ko‘rinishdagi qator

$$u_1 - u_2 + u_3 - \cdots + (-1)^{n-1}u_n + \cdots$$

(12.6)

bu erda $u_n \geq 0$, ishoralari almashinuvchi qator deb ataladi.

7- Teorema. (Leybnits alomati). Agar ishoralari almashinuvchi (12.6) qator uchun $u_1 > u_2 > \cdots > u_n > \cdots$va $\lim_{n\to\infty} u_n = 0$ o‘rinli bo‘lsa, u holda (12.6) qator yaqinlashuvchi bo‘ladi va uning yig‘indisi $S, 0 < S < u_1$ shartni qanoatlantiradi.

Natija. (12.6) qatorning r_n qoldig‘i har doim $|r_n| < u_{n+1}$ shartni qanoatlantiradi.

Masalan,

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots + (-1)^{n-1}\frac{1}{n} + \cdots$$

qator yaqinlashuvchi, chunki Leybnits alomatining shartlari bajariladi. $1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} + \cdots$ qator uzoqlashuvchi bo‘lganligi uchun u shartli yaqinlashuvchidir.

Mutloq yaqinlashuvchi qatorlar (shartli yaqinlashuvchidan farqli ravishda) chekli sondagi qoshiluvchilar yig‘indisining xossalariga ega (masalan, qo‘shiluvchilarning o‘rinlari almashgani bilan yig‘indi o‘zgarmaydi)

Quyidagi o‘rinlidir.

8- Teorema. Agar sonli qator shartli yaqinlashuvchi bo‘lsa, u holda ixtiyoriy a sonini olib, qatorning hadlarini shunday o‘rin almashtirish mumkinki, uning yig‘indisi a soniga teng buladi. Bundan tashqari shartli yaqinlashuvchi qatorning hadlarini shunday almashtirish mumkinki, natigada hosil bo‘lgan qator uzoqlashadi.

Shartli yaqinlashuvchi qator hadlarini shunday o‘rin almashtirish mumkinki, almashtirishdan keyingi hosil bo‘lgan qator uzoqlashuvchi bo‘ladi. 8 – teorema tatbiqini misolda ko‘raylik. Quyidagi shartli yaqinlashuvchi qatorni qaraymiz.

$$1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}\dots+(-1)^{n-1}\frac{1}{n}+\cdots=S.$$

Uning hadlarini shunday almashtiraylik-ki, har bir musbat haddan keyin ikkita manfiy had tursin.Quyidagiga ega bo‘lamiz:

$$1-\frac{1}{2}-\frac{1}{4}+\frac{1}{3}-\frac{1}{6}-\frac{1}{8}+\frac{1}{5}-\frac{1}{10}-\frac{1}{12}+\cdots+\frac{1}{2k-1}-\frac{1}{4k-2}-\frac{1}{4k}+\cdots$$

Endi har bir musbat hadni o‘zidan keyingi manfiy hadga qo‘shamiz.

$$\frac{1}{2}-\frac{1}{4}+\frac{1}{6}-\frac{1}{8}+\frac{1}{10}-\frac{1}{12}+\cdots+\frac{1}{4n-2}-\frac{1}{4k}+\cdots=$$

$$=\frac{1}{2}\left(1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+\cdots+\frac{1}{2k-1}-\frac{1}{2k}+\cdots\right)=\frac{1}{2}S.$$

Ko‘rinib turibdiki, berilgan qatorning yig‘indisi ikki marta kamayadi.

9- misol. Qatorni yaqinlashishga tekshiring.

$$\sum_{n=1}^{\infty}(-1)^{n-1}\frac{2n+1}{n(n+1)}, \qquad (1)$$

► Berilgan ishorasi almashinuvchi qatorning hadlari monoton kamayuvchi va $\lim_{n\to\infty}\frac{2n+1}{n(n+1)}=0$, u holda Leybnits alomatiga ko‘ra (1) qator yaqinlashuvchi. (1) qatorning absolyut qiymatlaridan tuzilgan qatorni qaraylik.

$$\sum_{n=1}^{\infty}\frac{2n+1}{n(n+1)}, \qquad (2)$$

Bu qatorning umumiy hadi $x = n$ bo‘lganda $f(x) = \frac{2x+1}{x(x+1)}$ funksiya orqali beriladi. Quyidagini topamiz.

$$\int_1^\infty \frac{2x+1}{x(x+1)}dx = \lim_{B\to\infty}\int_1^B \left(\frac{1}{x}+\frac{1}{x+1}\right)dx = \lim_{B\to\infty}(ln|x| + ln|x+1|)\Big|_1^B =$$

$$= \lim_{B\to\infty}\left(\ln\left(B\cdot(B+1)\right) - ln2\right) = \infty$$

Demak, (2) qator uzoqlashuvchi, shuning uchun (1) qator shartli yaqinlashuvchidir.◄

10- misol. Qatorning yig‘indisini $\delta = 0{,}001$ aniqliqda hisoblang .

$$\frac{1}{2}+\frac{1}{2!}\cdot\left(\frac{1}{2}\right)^2+\frac{1}{3!}\left(\frac{1}{2}\right)^3+\cdots+\frac{1}{n!}\left(\frac{1}{2}\right)^n+\cdots$$

► Yaqinlashuvchi qatorning har qanday n – chi xususiy yig‘indisi absolyut qiymati bo‘yicha bu qator qoldig‘idan oshmaydigan aniqlikda, qatorning yig‘indisiga yaqinlashadi.$|r_n| \le \delta$ tengsizlik n – chi xususiy yig‘indining nechinchi hadida o‘rinli bo‘lishini ko‘ramiz.

Berilgan qator uchun,

$$r_n = \frac{1}{(n+1)!}\left(\frac{1}{2}\right)^{n+1} + \frac{1}{(n+2)!}\left(\frac{1}{2}\right)^{n+2} + \cdots$$

Ma’lumki $(n+1)! < (2n+2)! < (2n+3)! < \cdots$ u holda,

$r_n \le \frac{1}{(n+1)!}\left(\frac{1}{2}\right)^{n+1}\left(1+\frac{1}{2}+\left(\frac{1}{2}\right)^2+\cdots\right) = \frac{1}{(n+1)!}\cdot\left(\frac{1}{2}\right)^n$ tanlash yo‘li bilan n = 4 bo‘lganda ,$r_n < \frac{1}{120\cdot16} < 0.001$ ekanligini topamiz. Demak berilgan qatorning yig‘indisi ($\delta = 0{,}001$ aniqliqda)

$$S \approx S_4 = \frac{1}{2}+\frac{1}{8}+\frac{1}{48}+\frac{1}{384} = 0.648$$◄

11- misol. Qatorning yig‘indisini $\delta = 0{,}001$ aniqlikda hisoblang.

$$\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{n^2\cdot 2^n}$$

► Berilgan qator ishora almashinuvchi, yaqinlashuvchi bo‘lganligidan qoldiqni hisoblashda tashlab yuborilgan kattalik ham ishora almashinuvchi qator bo‘ladi va birinchi tashlab yuborilgan haddan katta bo‘lmaydi (Leybnits alomati natijasiga asosan). Kerakli hadlar soni n ni $\frac{1}{n^2\cdot 2^n}\leq 0.001$ tengsizlikdan tanlash yo‘li bilan topamiz. n = 6 da tengsizlik o‘rinli bo‘ladi. Demak, berilgan qatorda oltinchi hadidan boshlab, barcha hadlarni tashlab yuborsak, talab qilingan aniqlikka erishamiz. $S\approx S_5=\frac{1}{2}-\frac{1}{16}+\frac{1}{72}-\frac{1}{256}+\frac{1}{800}=0.449$◄

AT – 12.1

1. Qatorning yaqinlashuvchiligini isbotlang va uning yig‘indisini toping:

a) $\sum_{n=1}^{\infty}\frac{1}{(3n-2)(3n+1)}$; b) $\sum_{n=1}^{\infty}\frac{5^n+2^n}{10^n}$ (Javob: a) $\frac{1}{3}$ b) 5/4.)

2. Quyidagi qatorlarni yaqinlashishiga tekshiring:

$a)\ \sum_{n=1}^{\infty}\frac{n^2}{2n^2-1}$; $b)\ \sum_{n=1}^{\infty}\frac{3n-1}{(\sqrt{2})^n}$

$c)\ \sum_{n=1}^{\infty}\frac{3^n}{2^n(n+2)}$; $d)\ \sum_{n=1}^{\infty}\frac{1}{n^2}\left(\frac{n+2}{n+1}\right)^{n^2+2n}$

$e)\ \sum_{n=1}^{\infty}ntg\frac{\pi}{2^{n+1}}$; $f)\sum_{n=1}^{\infty}\frac{n!}{n^n}$

3. Quyidagilarni isbotlang:

$a)\ \lim\limits_{n\to\infty}\frac{a^n}{n!}=0$; $b)\ a>1$ bo‘lganda $\lim_{n\to\infty}\frac{(2n)!}{a^{n}!}=0$.

4. Quyidagi qatorlarni Koshining integral alomati yordamida yaqinlashishga tekshiring:

$$a)\ \sum_{n=1}^{\infty}\frac{1}{n^2+2n+5} \qquad b)\sum_{n=1}^{\infty}\frac{n}{n^2+1} \qquad c)\ \sum_{n=2}^{\infty}\frac{1}{nln^2n}$$

Mustaqil ish.

1. 1. Qatorning yaqinlashuvchiligini isbotlang va yig‘indisini toping $\sum_{n=1}^{\infty}\frac{3^n+5^n}{15^n}$. (Javob : ¾)

2. Qatorni yaqinlashishga tekshiring $\sum_{n=1}^{\infty}\frac{n^2+1}{n^3}$.

2. 1. Qatorning yaqinlashuvchiligini isbotlang va yig‘indisini toping $\sum_{n=1}^{\infty}\frac{1}{(2n-1)(2n+1)}$. (Javob: ½)

2. Qatorni yaqinlashishiga tekshiring $\sum_{n=1}^{\infty}\frac{n}{(n^2+4)^2}$.

3. 1.Qatorning yaqinlashuvchiligini isbotlang va yig‘indisini toping $\sum_{n=1}^{\infty}\frac{1}{(3n-1)(3n+2)}$ (Javob: 1/6)

2. Qatorni yaqinlashishga tekshiring $\sum_{n=1}^{\infty}\frac{n^2}{3^n\cdot n!}$.

AT – 12.2

1. Quyidagi qatorlarni shartli va mutloq yaqinlashishga tekshiring:

$$a)\ \sum_{n=1}^{\infty}(n-1)^{n-1}\frac{1}{\sqrt{n}}; \qquad b)\ \sum_{n=1}^{\infty}(-1)^{n-1}\mathrm{n}\cdot 2^{-h};$$

$$c)\ \sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{n^2-9}; \qquad d)\ \sum_{n=1}^{\infty}(-1)^{n-1}\frac{n}{6n+5};$$

$$e)\ \sum_{n=1}^{\infty}\frac{\cos(2n\alpha)}{n^2+1}; \qquad f)\ \sum_{n=1}^{\infty}\frac{(-1)^n}{n-\ln n}.$$

2. Quyidagi ikkita uzoqlashuvchi qatorlarning ayirmasini tuzing va hosil bo‘lgan qatorni yaqinlashishga tekshiring:

$$\sum_{n=1}^{\infty}\frac{1}{2n-1} \text{ va } \sum_{n=1}^{\infty}\frac{1}{2n}$$

3. Quyidagi qatorning yig‘indisini $\delta = 0.01$ aniqlikda hisoblang:

$\sum_{n=1}^{\infty}\frac{1}{2^n n^2}$ (Javob: 0,58)

4. Qatorning nechta birinchi hadini olganda uning yig‘indisi qator yig‘indisidan 10^{-6} dan kichik bo‘lgan kattalikka farq qiladi:

$$a)\ \sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{n^2}; \qquad b)\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{n}$$

(Javob : a) n = 10^3 ; b) n = 10^6)

Mustaqil ish

1. 1. Qatorni shartli va mutloq yaqinlashishga tekshiring.

$$\sum_{n=1}^{\infty}(-1)^n\frac{1}{nln^2n}.$$

2. Qatorning uchta hadi bilan chegaralangan yig‘indisini toping:

$$\sum_{n=1}^{\infty}(-1)^{n-1}\frac{(0.6)^n}{n^2+1}.$$

Hisoblashning absolyut xatoligini baholang (Javob: S=0.266; δ =0,01).

2. 1.Qatorni shartli va mutloq yaqinlashishga tekshiring.

$$\sum_{n=1}^{\infty}(-1)^n\frac{\ln n}{n}.$$

2. Qatorning uchta hadi bilan chegaralangan yig‘indisini toping:

$$\sum_{n=1}^{\infty}(-1)^{n-1}\frac{(0.7)^n}{(n-1)!}.$$

Hisoblashning absolyut hatoligini baholang (Javob: S= 0.56 ; δ = 0,1).

3. 1. Qatorni shartli va mutloq yaqinlashishga tekshiring

$$\sum_{n=1}^{\infty}(-1)^{n}\frac{n^2}{3^n}.$$

2. Quyidagi qatorning birinchisidan boshlab nechta hadini olganda uning yig‘indisi qator yig‘indisidan 0,001 dan oshmaydigan kattalikka farq qiladi?

$$\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{n\cdot 2^n}.$$

12.2 Funksional va darajali qatorlar

Faraz qilaylik, $u_i(x)$ (i =1,2,…,n,…) funksiyalar D_x sohada aniqlangan bo‘lsin. U holda

$$u_1(x)+u_2(x)+\cdots+u_n(x)+\cdots=\sum_{n=1}^{\infty}u_n(x) \qquad (12.7)$$

ko‘rinishidagi ifoda funksional qator deb ataladi.

Agar $\sum_{n=1}^{\infty}\mathrm{u}_n(x_0)$ sonli qator yaqinlashsa, (12.7) qator $x = x_0$ nuqtada yaqinlashuvchi deyiladi. (12.7) qator yaqinlashuvchi bo‘ladigan x ning qiymatlari to‘plami fuksional qatorning aniqlanish sohasi deyiladi. Uni D_s bilan belgilaylik, ma’lumki D_s soha D_x soha bilan ustma- ust tushmaydi, ya’ni $D_s \subset D_x$.

1- misol. Fuksional qatorning yaqinlashish sohasini toping :

$$\ln x + ln^2 x + \cdots + ln^n x = \sum_{n=1}^{\infty} ln^n x$$

► Berilgan qator maxraji q = *lnx* bo‘lgan geometrik progressiya hadlarining yig‘indisidir. Agar$|q|$ = ln|x|< 1 ya’ni -1 <lnx< 1 bo‘lganda bunday qator yaqinlashuvchi bo‘ladi. Shuning uchun izlanayotgan qatorning yaqinlashish sohasi D_s *: (1/e) <x<e* intervaldan iborat bo‘ladi. ◄

Har bir $x \in D_s$ qiymatga sonli qatorning yig‘indisi bo‘lgan biror son mos keladi, shu moslik D_s sohasidagi (12.7) qatorning yig‘indisi deb ataluvchi S (x) funksiyani aniqlaydi.

Agar *S(x)* qatorning yig‘indisi, $S_n(x) = u_1(x) + u_2(x) + \cdots + u_n(x)$ esa (12.7) qatorning xususiy yig‘indisi bo‘lsa, u holda qatorning n – chi qoldig‘i quyidagi tenglik bilan aniqlanadi.

$$r_n(x) = S(x) - S_n(x) = u_{n+1}(x) + u_{n+2}(x) + \cdots$$

Qatorning aniqlanish sohasida $\lim_{n\to\infty} S_n(x) = S(x)$, $\lim_{n\to\infty} r_n(x) = 0$ o‘rinlidir.

Funksional qator yig‘indisining ta’rifini quyidagicha berish ham mumkin. Agar ixtiyoriyε>0 son uchun shunday $N_0 = N_0(x)$ nomer mavjud bo‘lib, barcha $n>N_0$ uchun

$$|r_n(x)| < \varepsilon \quad (x \in D) \tag{12.8}$$

Tengsizlik o‘rinli bo‘lsa, biror D sohada S(*x*) funksiya (12.7) qatorning yig‘indisi deb ataladi.

Umumiy holda N_0 natural son *x* ga bog‘liqdir, ya’ni har bir $x \in D$ uchun berilgan ε> 0 da N_0 natural son turlicha bo‘ladi. Agar bitta N_0 nomer mavjud bo‘lib, $n>N_0$ da barcha $x \in D$ uchun (12.8) tengsizlik o‘rinli bo‘lsa, u holda (12.8) qator D sohada tekis yaqinlashuvchi deb ataladi. Tekis yaqinlashuvchi funksional qator uchun uning n – xususiy yig‘indisi barcha x∈D uchun qatorning yig‘indisi bo‘la oladi.

Agar shunday

$$\sum_{n=1}^{\infty} \alpha_n \quad (\alpha_n > 0) \tag{12.9}$$

yaqinlashuvchi sonli qator mavjud bo‘lib, barcha x∈D uchun

$$|u_k(x)| \leq \alpha_k \quad (k = 1, 2, \dots)$$

Tengsizlik o‘rinli bo‘lsa, (12.7) funksional qator biror D sohada kuchaytirilgan (majorirlanuvchi) qator deb ataladi.

Masalan,

$$\frac{\cos x}{1} + \frac{\cos 2x}{2^2} + \frac{\cos 3x}{3^2} + \cdots + \frac{\cos nx}{n^2} + \cdots$$

funksional qator $|\cos nx| \leq 1$ bo‘lganligi uchun

$$1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots + \frac{1}{n^2} + \cdots$$

qator bilan kuchaytiriladi.

Tekis yaqinlashuvchi qatorlar ba’zi bir umumiy xossalarga ega :

1) agar tekis yaqinlashuvchi qatorning hadlari biror kesmada uzluksiz bo‘lsa, uning yig‘indisi ham bu kesmada uzluksizdir;

2) agar (12.7) qatorning hadlari [a, b] kesmada uzluksiz va bu kesmada qator tekis yaqinlashsa, u holda $[\alpha;\ \beta] \subset [a, b]$ bo‘lganda,

$$\int_{\alpha}^{\beta} S(x)dx = \sum_{n=1}^{\infty} \int_{\alpha}^{\beta} u_n(x)\, dx,$$

bu yerda $S(x)$ - (12.7) qatorning yig‘indisi;

3) agar (12.7) qator [a, b] kesmada uzluksiz hosilalarga ega bo‘lgan funksiyalardan tuzilgan bo‘lib, shu kesmada S (x) yig‘indisiga yaqinlashsa

va $u'(x) + u'_2(x) + \cdots + u'_n(x) + \cdots$ qator bu kesmada tekis yaqinlashuvchi bo‘lsa, u holda

$$u'(x) + u'_2(x) + \cdots + u'_n(x) = S'(x)$$

o‘rinlidir.

$$\sum_{n=0}^{\infty} a_n (x - x_0)^n,$$

ko‘rinishdagi funksional qatorga darajali qator deyiladi. Bu erda a_0 ,a_1, a_2 ,.. a_n o‘zgarmas sonlar bo‘lib, qatorning koeffitsientlari deb ataladi. x_0 – berilgan qiymat.

$x_0 = 0$ da quyidagi darajali qatorga ega bo‘lamiz.

$$\sum_{n=0}^{\infty} a_n x^n \qquad (12.10)$$

1- Teorema (Abel teoremasi).

1. Agar (12.10) darajali qator x ning biror $x = x_1 \neq 0$ qiymatida yaqinlashsa, u holda bu qator $|x|<|x_1|$ shartni qanoatlantiruvchi x ning ixtiyoriy qiymatida mutloq yaqinlashadi.

2. Agar (12.10) darajali qator x ning biror $x = x_2$ qiymatida uzoqlashsa, u holda bu qator $|x| > |x_2|$ shartni qanoatlantiruvchi x ning ixtiyoriy qiymatida uzoqlashadi.

(12.10) darajali qator barcha $|x| < R$ da yaqinlashuvchi, $|x| > R$ da esa uzoqlashuvchi bo‘ladigan, manfiy bo‘lmagan R soni qatorning *yaqinlashish radiusi* deb ataladi. (-R; R) oraliq (12.10) ning *yaqinlashish oralig‘i* deb ataladi.

Agar biror $n > n_0$ dan boshlab, barcha $a_n \neq 0$ bo‘lsa, u holda (12.10) darajali qatorning yaqinlashish radiusi

$$R = \lim_{n\to\infty} \left|\frac{a_n}{a_{n+1}}\right| \text{ yoki } R = \lim_{n\to\infty} \frac{1}{\sqrt[n]{a_n}} \qquad (12.11)$$

formulalar bilan aniqlanadi. (12.11) formulalarni Dalamber yoki Koshining radikal alomatlaridan foydalanib hosil qilish mumkin.

2-misol. Darajali qatorning yaqinlashishi sohasini toping $\sum_{n=1}^{\infty}\frac{2^n\cdot x^n}{3^n\sqrt{n}}$.

► $a_n=\frac{2^n}{3^n\sqrt{n}},\quad a_{n+1}=\frac{2^{n+1}}{3^{n+1}\sqrt{n+1}}$ ekanligidan

$$R=\lim_{n\to\infty}\frac{2^n\cdot 3^{n+1}\sqrt{n+1}}{3^n\sqrt{n}\cdot 2^{n+1}}=\frac{3}{2}\lim_{n\to\infty}\sqrt{1+\frac{1}{n}}=\frac{3}{2}$$

Demak, darajali qator (-3/2 ; 3/2) oraliqda yaqinlashadi. Bu oraliqning chetki nuqtalarida qator yaqinlashishi yoki uzoqlashishi mumkin. Bizning misolimizda x = -3/2 da berilgan qator $\sum_{n=1}^{\infty}(-1)^n\frac{1}{\sqrt{n}}$ ko‘rinishida bo‘ladi. Leybnits alomatiga asosan bu qator yaqinlashuvchidir. x = 3/2 da $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}$ qatorni hosil qilamiz. Bu qatorning hadlari uzoqlashuvchi bo‘lgan garmonik qatorning mos hadlaridan kattadir. Demak, x = 3/2 da darajali qator uzoqlashadi. Shunday qilib, berilgan darajali qatorning yaqinlashish sohasi [-3/2 ; 3/2) yarim oraliqdan iborat ekan.◄

Agar qator $\sum_{n=1}^{\infty}a_n(x-x_0)^n$ ko‘rinishida berilsa, uning yaqinlashish radiusi R ham (12.11) formula bilan aniqlanadi, yaqinlashish oralig‘i esa markazi $x = x_0$ nuqtada bo‘lgan oraliq bo‘ladi. (x_0- R; x_0 + R).

3- misol. Darajali qatorning yaqinlashish sohasini toping.

$$\sum_{n=0}^{\infty}(-1)^n\frac{(x-2)^n}{2^n\sqrt{n+1}}$$

► Berilgan qatorning yaqinlashishi radiusini topamiz:

$$R=\lim_{n\to\infty}\frac{2^{n+1}\sqrt{n+2}}{2^n\sqrt{n+1}}=2\lim_{n\to\infty}\sqrt{\frac{n+2}{n+1}}=2,$$

ya'ni, qator (0; 4) oraliqda yaqinlashuvchi ekan. x =0 da $\sum_{n=0}^{\infty}\frac{1}{\sqrt{n+1}}$ qatorni hosil qilamiz. Bu qator uzoqlashuvchidirchunki uning hadlari uzoqlashuvchi bo'lgan garmonik qatorning mos hadlaridan katta. x = 4 da esa, $\sum_{n=0}^{\infty}(-1)^n\cdot\frac{1}{\sqrt{n+1}}$ qatorni hosil qilamiz. $\lim_{n\to\infty}\frac{1}{\sqrt{n+1}}=0$, Leybnits alomatiga ko'ra bu qator yaqinlashuvchidir. Berilgan qatorning yaqinlashish sohasi (0; 4].◄

4-misol. Qatorning yaqinlashish sohasini toping.

$$\sum_{n=0}^{\infty}\frac{x^n}{n!}$$

► Qatorning yaqinlashish radiusini topamiz:

$$R=\lim_{n\to\infty}\left(\frac{1}{n!}:\frac{1}{(n+1)!}\right)=\lim_{n\to\infty}(n+1)=\infty$$

Berilgan qator barcha sonlar to'g'ri chizig'ida yaqinlashuvchi. Bundan, xususiy holda, qator yaqinlashishining zaruriy shartini e'tiborga olsak (12.1, 1– teoremaga qaralsin) ixtiyoriy chekli x uchun $\lim_{n\to\infty}\frac{x^n}{n!}=0$ ni hosil qilamiz.◄

Yaqinlashish oralig'ining ichida yotuvchi har qanday [α, β] kesmada darajali qator tekis yaqinlashadi, shuning uchun yaqinlashish oralig'ida uning yig'indisi uzluksiz funksiya bo'ladi. Darajali qatorni ularning yaqinlashish oraliqlarida hadma – had integrallash va differensiallash mumkin. Bunday holda yaqinlashish radiusi o'zgarmaydi.

5- misol. Qatorning yig'indisini toping

$$x+\frac{x^3}{3}+\frac{x^5}{5}+\cdots+\frac{x^{2n-1}}{2n-1}+\cdots$$

► R = 1, bo'lgani uchun$|x|$< 1 da berilgan qator yaqinlashadi, demak, uni yaqinlashish oralig'ida hadma – had differensiallash mumkin. Qatorning yig'indisini S(x) bilan belgilab quyidagiga ega bo'lamiz :

$$S'(x) = 1 + x^2 + x^4 + \cdots + x^{2n-2} + \cdots$$

$|x| < 1$ bo‘lgani uchun hosil qilingan qator maxraji $q = x^2$ bo‘lgan kamayuvchi geometrik progressiyaning yig‘indisidir va uning yig‘indisi $S'(x) = \frac{1}{1-x^2}$; buni integrallab, berilgan qatorning yig‘indisini topamiz

$$S(x) = \int_0^x \frac{1}{1-x^2}dx = \frac{1}{2}ln\left|\frac{x+1}{x-1}\right|, \ (|x| < 1). \blacktriangleleft$$

AT– 12.3

1. Quyidagi qatorlarning har biri uchun yaqinlashish sohasini toping

$$a)\ \sum_{n=0}^{\infty}\frac{x^n}{(n+1)2^n}; \qquad b)\ \sum_{n=0}^{\infty}\frac{n}{n+1}\left(\frac{x}{2}\right)^n$$

$$c)\ \sum_{n=0}^{\infty}\frac{2^n x^n}{n^2+1}; \qquad d)\ \sum_{n=0}^{\infty}\frac{4^n x^n}{3^n\sqrt{(n+1)^3}};$$

$$e)\ \sum_{n=0}^{\infty}\frac{(x+2)^n}{(2n-1)4^n}; \qquad f)\ \sum_{n=0}^{\infty}\frac{2^{n-1}x^{2(n-1)}}{\sqrt{n^3-1}}.$$

(Javob: $a)\ -2 \le x < 2;\quad b) -1 < x < 1;\quad c) -{}^1\!/_2 \le x \le 1/2$

$$d) -\frac{3}{2} \le x \le \frac{3}{2};\quad e) -8 \le x < 2\,;\quad f) -{}^{\sqrt{2}}\!/_2 \le x \le {}^{\sqrt{2}}\!/_2\Big)$$

2. Quyidagi qatorlarning tekis yaqinlashish sohasini toping

$$a)\ \sum_{n=0}^{\infty}\frac{\sin nx}{n!}; \qquad b)\ \sum_{n=0}^{\infty}\frac{cosnx}{2^n}$$

3. Hadlab integrallash va differensiallashdan foydalanib, quyidagi qatorlarning yig‘indisini toping:

$$a)\ \sum_{n=1}^{\infty}\frac{x^n}{n}; \qquad b)\ \sum_{n=1}^{\infty}(n+1)x^n$$

(Javob: $a) - \ln(1-x)\ (-1 \le x \le 1)$; $b)\ \frac{1}{(x-1)^2}\ (|x| < 1)$.

Mustaqil ish

1. 1. Qatorning yaqinlashish sohasini toping

$$\sum_{n=0}^{\infty} \frac{7^{n-1}}{5^n\sqrt{n^2-1}}.$$

(Javob: $-\frac{5}{7} \le x < \frac{5}{7}$)

2. Qatorning yig‘indisini toping

$$\frac{1}{x} + \frac{2}{x^2} + \frac{3}{x^3} + \cdots \frac{n}{x^n} + \cdots$$

(Javob: $\frac{x}{(x-1)^2} \cdot (|x| > 1)$

2. 1. $\sum_{n=0}^{\infty} \frac{2^n(x-3)^n}{5^n\sqrt{n^3-0.5}}$ qatorning yaqinlashish oralig‘ini toping va oraliqning chekka nuqtalarida yaqinlashishga tekshiring.

(Javob: (1/2; 11/2), qator x=1/2 va x=11/2 da yaqinlashadi).

2. Qatorning yaqinlashish sohasini toping $\sum_{n=1}^{\infty} \frac{e^{-n^2x^2}}{n^2}$

3. 1. $\sum_{n=1}^{\infty} 10^n x^{n-1}$qatorning yaqinlashish oralig‘ini toping va oraliqning chekka nuqtalarida yaqinlashishini tekshiring.

(Javob: (-1/10; 1/10), qator $x=\pm 1/10$ da uzoqlashuvchi).

2. Qatorning yaqinlashish sohasini toping $\sum_{n=0}^{\infty} \frac{1}{x^n}$.

12.3. Teylor va Makloren qatorlarining formulalari. Funksiyalarning darajali qatorlarga yoyilmasi.

Agar $y=f(x)$ funksiya $x=x_0$ nuqtaning atrofida $(n+1)$ – tartibgacha hosilaga ega bo‘lsa, u holda shunday $c = x_0 + \theta(x - x_0)$ $(0 < \theta < 1)$ nuqta mavjud bo‘ladiki,

$$f(x) = f(x_0) + \frac{f'(x_0)}{1!}(x - x_0) + \frac{f'(x_0)}{2!}(x - x_0)^2 + \cdots +$$

$$+\frac{f^{(n)}(x_0)}{n!}(x - x_0)^n + R_n(x), \qquad (12.12)$$

bu erda $R_n(x) = \frac{f^{(n+1)}(c)}{(n+1)!}(x - x_0)^{n+1}$.

(12.12) formula $y=f(x)$ funksiyaning x_0 nuqta atrofida aniqlangan *Teylor formulasi* deb ataladi.

$R_n(x)$ esa Teylor formulasining Lagranj ko‘rinishidagi qoldiq hadi deyiladi.

$$P_n(x) = f(x_0) + \frac{f'(x_0)}{1!}(x - x_0) + \cdots + \frac{f^{(n)}(x_0)}{n!}(x - x_0)$$

ko‘phad $y=f(x)$ funksiyaning Teylor ko‘phadi deyiladi. $x_0=0$ da (12.12) formulaning xususiy holiga ega bo‘lamiz.

$$f(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \cdots + \frac{f^{(n)}(0)}{n!}x^n + R_n(x), \qquad (12.13)$$

bu erda $R_n(x) = \frac{f^{(n+1)}(c)}{(n+1)!}x^n, \quad c = \theta x \quad (0 < \theta < 1)$

(12.13) formula $y=f(x)$ funksiyaning *Makloren formulasi* deyiladi.

1- misol. $y = x^4 - 3x^2 + 2x + 2$ funksiyani $x-1$ ayirmaning daraja-

lari bo‘yicha yoying.

► $x_0= 1$ da Teylor formulasidan foydalanish uchun quyidagilarni topamiz:

$$y(1) = 2; \quad y'(1) = (4x^3 - 6x + 2)\Big|_{x=1} = 0$$

$$y''(1) = (12x^2 - 12x)\Big|_{x\,=\,1} = 0, \qquad y'''(1) = (24x - 12)\Big|_{x\,=\,1} = 12,$$

$$y^{IV}_{(1)} = 24 \quad y^{V}(x) = 0 \text{ va hokazo.}$$

Shunday qilib,

$$x^4 - 3x^2 + 2x - 2 = 2 + 2(x-1)^3 + (x-1)^4. \blacktriangleleft$$

2- misol. $y = \frac{1}{x}$ funksiyaning x_0=1 nuqtadagi Teylor ko‘phadini yozing.

►Berilgan funksiyaning hosilalarini va ularning x_0=1 nuqtadagi qiymatlarini topamiz:

$$y(x)\Big|_{x\,=\,1} = 1; \quad y'(1) = -\frac{1}{x^2}\Big|_{x\,=\,1} = -1$$

$$y''(1) = \frac{2}{x^3}\Big|_{x\,=\,1} = 2, \;\; y'''(1) = \frac{-1\cdot 2\cdot 3}{x^4}\Big|_{x\,=\,1} = -6$$

$$y^{IV}(1) = \frac{1\cdot 2\cdot 3\cdot 4}{x^5}\Big|_{x\,=\,1} = 24, \ldots y^{(n)}(1) = (-1)^n\frac{n!}{x^{n+1}}\Big|_{x=1} = (-1)^n n!$$

Bundan kelib chiqadiki,

$$P_n(x) = 1 - \frac{(x-1)}{1!} + \frac{2}{2!}(x-1)^2 - \frac{6}{3!}(x-1)^3 + \cdots + (-1)\frac{n!}{n!}(n-1)^n =$$

$$= 1 - (x-1) + (x-1)^2 - (x-1)^3 + \cdots + (-1)^n(x-1)^n$$

Berilgan funksiya uchun Teylor formulasining qoldiq hadi quyidagi ko‘rinishda bo‘ladi:

$$R_n(x) = (-1)^{n+1}\frac{(x-1)^{n+1}}{(1+\theta(x-1))^{n+2}}(0 < \theta < 1).\blacktriangleleft$$

Funksiyaning Teylor qatoriga yoyilmasining shartini ifodalaymiz.

Agar *f(x)* funksiya x_0 nuqtaning atrofida istalgan tartibda differensiallanuvchi bo‘lsa va x_0 nuqtaning biror atrofida

$$\lim_{n\to\infty} R_n(x) = 0 \quad \text{yoki}$$

$$\lim_{n\to\infty} \frac{f^{(n+1)}(x_0+\theta(x-x_0))}{(n+1)!}(x-x_0)^{n+1} = 0 \qquad (12.14)$$

bo‘lsa, u holda

$$f(x) = f(x_0) + \frac{f'(x_0)}{1!}(x-x_0) + \cdots + \frac{f^{(n)}(x_0)}{n!}(x-x_0)^n + \cdots \qquad (12.15)$$

Xususiy holda x_0=0 bo‘lganda

$$f(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f'(0)}{1!}x^2 + \cdots + \frac{f^{(n)}(0)}{n!}x^n + \cdots \ (12.16)$$

(12.15) qator *Teylor qatori*, (12.16) qator esa *Makloren qatori* deyiladi.

(12.14) shart (12.15) yoki (12.16) ko‘rinishdagi qatorlarning $x=x_0$ nuqtaning biror atrofida *f(x)* funksiyaga yaqinlashishining zaruriy va etarli sharti hisoblanadi. Har bir aniq hollar uchun qatorning berilgan funksiyaga yaqinlashish oralig‘ini topish zarurdir.

3- misol. *chx* funksiyani Makloren qatoriga yoying va berilgan funksiyaga yaqinlashish oralig‘ini toping.

► *f(x)=ch(x)* funksiyaning hosilalarini topamiz:

$$f'(x) = shx, \qquad f''(x) = chx, \qquad f'''(x) = shx, \ldots$$

Shunday qilib, agar *n*-juft bo‘lsa, $f^{(n)}(x) = chx$ va *n*-toq bo‘lsa $f^{(n)}(x) = shx$, x_0=0 desak, quyidagilarga ega bo‘lamiz:

$f(0) = 1,\ f'(0) = 0,\ f'' = 1, \ldots, \text{n} - \text{juft bo‘lsa}, f^{(n)}(0) = 1$ va n-toq bo‘lsa $f^{(n)}(0) = 0$. Topilgan hosilalarni (12.16) qatorga qo‘yib, quyidagini hosil qilamiz:

$$chx = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots + \frac{x^{2n}}{(2n)!} + \cdots$$

(1)

(12.14) shartdan foydalanib, (1) qatorning berilgan funksiyaga yaqinlashish oralig‘ini aniqlaymiz.

Agar *n*-toq bo‘lsa, u holda

$$R_n(x) = \frac{x^{n+1}}{(n+1)!} ch\theta x$$

Agar *n*-juft bo‘lsa, u holda

$$R_n(x) = \frac{x^{n+1}}{(n+1)!} sh\theta x.$$

$0 < \theta < 1$ bo‘lgani uchun $|ch\theta x| = \left|e^{\theta x} + e^{-\theta x}\right|/2 \leq e^{|x|}$va $|sin\theta x| \leq e^{|x|}$. Demak,

$$|R_n(x)| \leq \frac{|x|^{n+1}}{(n+1)!} e^{|x|}.$$

12.2 ning 4-misolida ixtiyoriy *x* da

$$\lim_{n\to\infty} \frac{x^{n+1}}{(n+1)!} = 0$$

ekanligi ko‘rsatilgan edi. Bundan kelib chiqadiki, ixtiyoriy x da

$$\lim_{n\to\infty} R_n(x) = 0$$

va (1) qator chx funksiyaga yaqinlashadi.◄

Boshqa funksiyalarining darajali qatorga yoyilmalarini ham shu kabi hosil qilish mumkin:

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!} + \cdots (-\infty < x < \infty) \qquad (12.17)$$

$$cosx = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots + (-1)^{n-1} \frac{x^{2n}}{(2n)!} + \cdots (-\infty < x < \infty) \qquad (12.18)$$

$$sinx = 1 - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots + (-1)^{n-1}\frac{x^{2n-1}}{(2n-1)!} + \cdots (-\infty < x < \infty) \quad (12.19)$$

$$ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots + (-1)^{n-1}\frac{x^n}{n} + \cdots \quad (-1 < x \leq 1)(12.20)$$

$$(1+x)^m = 1 + \frac{m}{1!} + \frac{m(m-1)}{2!}x^2 + \cdots +$$

$$+\frac{m(m-1)\dots(m-n+1)}{n!}x^n + \cdots (-1 < x < 1)$$

(12.21)

Har bir hol uchun qavs ichida darajali funksiyaning unga mos funksiyaga yaqinlashish oralig‘i ko‘rsatilgan. Binomial qator deb ataluvchi oxirgi qator $m\epsilon R$ ga bog‘liq ravishda yaqinlashish oralig‘ining chetki nuqtalarida turlicha bo‘ladi. $m \geq 0$ da $x = \pm 1$ nuqtalarda mutloq yaqinlashadi. $-1 < m < 0$ bo‘lganda $x = -1$ nuqtada uzoqlashadi va $x = 1$ nuqtada shartli yaqinlashadi. m ≤ -1 bo‘lganda $x = \pm 1$ nuqtalarda uzoqlashadi.

Umumiy holda darajali qatorga yoyish Teylor yoki Makloren qatorlaridan foydalanishga asoslangan. Ammo amaliyotda ko‘pgina funksiyalarning darajali qatorini (12.17) - (12.21) qatorlardan foydalanib yoki geometrik progressiya hadlari yig‘indisi formulalaridan foydalanib topish mumkin.

Ba’zi hollarda qatorni hadma-had differensiallash yoki integrallashdan foydalanib, yoyilmani topish foydalidir. Yaqinlashish oralig‘ida qatorlar mos funksiyalarga yaqinlashadi.

Masalan, $cos\sqrt{x}$ funksiyani darajali qatorga yoyishda (12.18) formulada *x* o‘rniga $\sqrt{x}$ qo‘yamiz. U holda,

$$cos\sqrt{x} = 1 - \frac{x}{2!} + \frac{x^2}{4!} - \cdots + (-1)^n\frac{x^n}{(2n)!} + \cdots$$

Hosil qilingan qator ixtiyoriy $x\epsilon R$ da yaqinlashuvchi, ammo $cos\sqrt{x}$ funksiya x<0 da aniqlanmagan. Shuning uchun hosil qilingan qator $cos\sqrt{x}$ funksiyaga faqat $0 \leq x < \infty$ yarim oraliqda yaqinlashadi.

Xuddi shu kabi $f(x) = e^{-2x}$ va $f(x) = \frac{sinx}{x}$ funksiyalarning darajali qatorlarini yozish mumkin:

$$e^{-2x} = 1 - \frac{2x}{1!} + \frac{4x^2}{2!} - \frac{8x^3}{3!} + \cdots + (-1)^n \frac{2^n x^n}{n!} + \cdots$$

$$\frac{sinx}{x} = 1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \cdots + (-1)^{n-1} \frac{x^{2n-2}}{(2n-1)!} + \cdots$$

4- misol. $f(x) = \frac{3}{(1-x)(1+2x)}$ funksiyani Makloren qatoriga yoying.

►Berilgan funksiyani sodda ratsional kasrlar yig‘indisi ko‘rinishida yozamiz:

$$\frac{3}{(1-x)(1+2x)} = \frac{1}{1-x} + \frac{2}{1+2x}$$

Ma’lumki,

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \ (|x| < 1),$$

(1)

$$\frac{1}{1+2x} = \sum_{n=0}^{\infty} (-1)^n 2^n x^n \quad (|2x| < 1)$$

(2)

U holda

$$\frac{3}{(1-x)(1+2x)} = \sum_{n=0}^{\infty} x^n + 2\sum_{n=0}^{\infty} (-1)^n 2^n x^n =$$

$$= \sum_{n=0}^{\infty} (1 + (-1)^n 2^{n+1})\, x^n.$$

(1) qator $|x| < 1$ da (2) qator esa $|x| < 1/2$ da yaqinlashadi, u holda (3) qator berilgan funksiyaga$|x| < \frac{1}{2}$ da yaqinlashadi.◄

5- misol. *f(x)=arctgx* funksiyani darajali qatorga yoying.

► Ma'lumki,

$$\frac{1}{1+x^2} = \frac{1}{1-(-x^2)} = 1 - x^2 + x^4 - x^6 + \cdots + (-1)^{n-1}x^{2n-1} + \cdots$$

Hosil qilingan qator [-1, 1] oraliqning ichida yaqinlashuvchi, demak uni ihtiyoriy [0; *x*] ⊂ (-1;1) oraliqda hadma-had integrallash mumkin.

Bundan,

$$\int_0^x \frac{dt}{1+t^2} = \int_0^x \sum_{n=1}^{\infty} (-1)^{n-1} t^{(2n-1)} dt,$$

$$arctgx = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^{2n-1}}{2n-1}.$$

Yani, $|x| < 1$ da berilgan funksiyaga yaqinlashuvchi qatorni hosil qildik.◄

AT - 12.4

1. $f(x) = x^5 - 4x^4 + 2x^3 + 2x + 1$ ko'phadni (*x*+1) ning darajalari bo'yicha qatorga yoying.

2. $y = \frac{1}{x+1}$ funksiyani Makloren qatorini qo'llab, *x* ning darajalari bo'yicha qatorga yoying.

3. Quyida ko'rsatilgan funksiyalarni *x* ning darajalari bo'yicha qatorga yoying va hosil bo'lgan qatorning yaqinlashish oralig'ini toping.

$a)\ \ e^{-x};\qquad b)\ x\cos 2x;\qquad c)\ {}^{1}/_{\sqrt{4-x^2}}$

$$d)\ arc\sin x; \quad e)\ \frac{3x+5}{x^2-3x+2}; \qquad f)\ cos^2 x$$

4. $f(x) = \frac{1}{x^2+4x+7}$ funksiyani *x+2* ning darajalari bo‘yicha qatorga yoying.

5. $y = ln(2+x)$ funksiyani *1+x* ning darajalari bo‘yicha qatorga yoying.

6. Agar *x*=0 da *y*=1 ekanligi ma’lum bo‘lsa, $xy+e^x=y$ tenglama bilan berilgan funksiyaning darajali qatorga yoyilmasining birinchi uchta hadini toping.

Mustaqil ish

1. 1.$f(x) = \sqrt{x}$ funksiyaning *x-4* ning darajalari bo‘yicha qatorga yoyilmasining birinchi uchta hadini toping.

2. $f(x) = \ln(1-3x)$ funksiyani darajali qatorga yoying va bu qatorning yaqinlashish sohasini toping.

(Javob: $-1/3 \le x < 1/3$)

2. 1. *y=xsin2x* funksiyaning darajali qatorga yoyilmasini toping.

2. $f(x) = \frac{3}{(1+x)(1-2x)}$ funksiyani darajali qatorga yoying va bu qatorning yaqinlashish sohasini toping. (Javob: $|x| < 1/2.$)

3. 1. $f(x) = x^4 + 3x^2 - 6x^2 + 3$ ko‘pxadni x+1 ning darajalari bo‘yicha qatorga yoying.

2. $f(x) = \ln(1+2x)$ funksiyani darajali qatorga yoying va bu qatorning yaqinlashish sohasini toping.(Javob: $-1/2 < x \le 1/2$).

12.4. Darajali qatorlar yordamida taqribiy hisoblash.

Funksiyaning qiymatini hisoblash. Faraz qilaylik, *f(x)* funksiyaning darajali qatorga yoyilmasi berilgan bo‘lsin. Bu funksiyaning qiymatini topish masalasi, argumentning berilgan qiymatida qatorning yig‘indisini topishga keltiriladi.

Funksiyaning qiymatini, qatorning ma’lum hadlari bilan chegaralangan holda, Teylor yoki Makloren qatorlarining $R_n(x)$ qoldiq hadini yoki sonli qatorning qoldiq hadini berilgan aniqlikda baholash yo‘li bilan topiladi.

1- misol. *ln2* ni δ = 0,0001 aniqlikda hisoblang.

► Ma’lumki,

$$ln\,(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots + (-1)^{n-1}\frac{x^n}{n} + \cdots$$

(1)

darajali qator *x*=1 da shartli yaqinlashadi. (§12.1. 8 − misolga qarang) *ln2* ni δ =0,0001 aniqlikda (1) qator yordamida hisoblash uchun uning kamida 10.000 hadini olishi kerak bo‘ladi. Shuning uchun *ln*(1+*x*) va *ln*(1-*x*) funksiyalarinig darajali qatorga yoyilmalarining ayirmasidan hosil bo‘lgan qatordan foydalanamiz.

$$ln\frac{1+x}{1-x} = 2(x + \frac{x^3}{3} + \frac{x^5}{5} + \ldots + \frac{x^{2n-1}}{2n-1} + \ \ldots) \qquad (2)$$

Dalamber alomati yordamida bu qatorning yaqinlashish radiusi R = 1 ekanligini ko‘rsatish mumkin. |x|< 1 bo‘lganda (2) qator shartli yaqinlashadi.

$x = \frac{1}{3}$ da $\frac{1+x}{1-x} = 2$ ekanligidan, *x* ning bu qiymatini (2) qatorga qo‘yib, quyidagiga ega bo‘lamiz:

$$ln2 = 2(\frac{1}{3} + \frac{1}{3*3^3} + \frac{1}{5*3^5} + \ \ldots + \frac{1}{(2n-1)3^{2n-1}} + \cdots)$$

$ln2$ ni berilgan aniqlikda hisoblash uchun yig‘indining qoldig‘i $|r_n|<\delta$ bo‘ladigan, $S_n$xususiy yig‘indi hadlarining n sonini topish zarur.

Bizning misolimizda,

$$r_n = 2\left(\frac{1}{(2n+1)3^{2n+1}} + \frac{1}{(2n+3)3^{2n+3}} + \cdots\right) \qquad (3)$$

2n+3, 2n+5, ... sonlari 2n+1 dan katta bo‘lganligi uchun ularni 2n+1 bilan almashtirib, (3) formuladagi har bir kasrni kattalashtiramiz. Shuning uchun,

$$r_n < \frac{2}{2n+1}\left(\frac{1}{3^{2n+1}} + \frac{1}{3^{2n+3}} + \cdots\right) = \frac{2}{(2n+1)3^{2n+1}} \cdot \frac{1}{1-\frac{1}{9}} = \frac{1}{4*(2n+1)*3^{2n+1}} .$$

n ning qiymatini tanlash yo‘li bilan topamiz. n=3 uchun $r_n < 0{,}00015$, shu bilan birgalikda *ln2*= 0,6931.◄

2-misol. $\sqrt{e}$ ni $\delta = 0{,}001$ aniqlikda hisoblanig.

►e^xfunksiyaning darajali qatorga yoyilmasidan foydalanamiz. (12.17 formulaga qarang). *x*= $\frac{1}{2}$ deb olsak, quyidagiga ega bo‘lamiz:

$$\sqrt{e} = 1+\frac{1}{2} + \frac{1}{2!2^2} + \cdots + \frac{1}{n!2^n} + \cdots$$

Bu qatorning qoldiq hadi,

$$r_n = \sum_{K=1}^{\infty} \frac{1}{(n+k)! \cdot 2^{n+k}} < \frac{1}{(n+1)! \cdot 2^n} \sum_{K=1}^{\infty} \frac{1}{2^n} = \frac{1}{(n+1)! * 2^n}$$

(n+1)!<(n+2)!<… bo‘lganligi uchun n=4 da $r_n < \frac{1}{5! \cdot 24} < 0{,}001.$

Shunday qilib,

$$e^{½} \approx 1 + \frac{1}{2} + \frac{1}{8} + \frac{1}{48} + \frac{1}{384} \approx 1{,}674$$

Berilgan aniqlikda hisoblashni ta’minlovchi qator hadlari sonini aniqlash uchun Makloren formulasining qoldiq hadidan foydalanish mumkin.

$$R_n(x) = \frac{e^{\theta x}}{(n+1)!} x^{n+1}$$

bu erda 0<θ<1; x=½. U holda n = 4 da $|R_n(\frac{1}{2})| < \frac{2(½)^{n+1}}{(n+1)!} < 0{,}001$ ◄

3-misol. $Sin\frac{1}{2}$ ni δ= 10^{-3}aniqlikda hisoblang.

► x=½ qiymatni (12.19) formulaga qo‘yib quyidagini hosil qilamiz.

$$\sin\frac{1}{2} = \frac{1}{2} - \frac{1}{3!2^3} + \frac{1}{5!2^5} + \cdots + (-1)^{n-1}\frac{1}{(2n-1)!2^{2n-1}} + \cdots$$

Ma’lumki, ishora almashinuvchi qatorning qoldiq hadi $|r_n| \le u_{n+1}$ (Leybnits alomatining natijasiga va (12.6) qatorga qarang), u holda u_{n+1}uchun $u_{n+1} < \delta$ bo‘lgan birinchi hadni topish etarlidir. Ma’lumki, qatorning uchinchi hadi $\frac{1}{5!25} < 10^{-3}$, shuning uchun sin $\frac{1}{2}$ ning $\delta = 10^{-3}$ aniqlikdagi qiymati

$$\sin\frac{1}{2} \approx \frac{1}{2} - \frac{1}{48} \approx 0{,}470. ◄$$

4- misol. $\sqrt[5]{34}$ ni $\delta = 10^{-3}$ aniqlikda hisoblang.

► Ma’lumki, $\sqrt[5]{34}$ =$\sqrt[5]{32+2}$ =$2(1+\frac{1}{16})^{\frac{1}{5}}$.$m$ =1/5, x =1/6 bo‘lganda binomial qatordan foydalanamiz.((12.21) formulaga qarang).

$$(1+\frac{1}{16})^{\frac{1}{5}} = 1 + \frac{1}{5}\cdot\frac{1}{16} + \frac{\frac{1}{5}(\frac{1}{5}-1)}{2}\cdot\frac{1}{16^2} + \frac{\frac{1}{5}\left(\frac{1}{5}-1\right)(\frac{1}{5}-2)}{3!}\cdot\frac{1}{16^3} + \cdots = 1 + \frac{1}{80} -$$
$$-\frac{1}{320} + \cdots = 1+0{,}0125-0{,}0003+\cdots \approx 1.012$$

Uchinchi hadi $\delta = 10^{-3}$dan kichik bo‘lganligi uchun uchinchi haddan boshlab tashlab yuborish mumkin (Leybnits teoremasining natijasiga qarang). Demak,

$$\sqrt[5]{34} = 2(1+\frac{1}{16})^{\frac{1}{5}} \approx 2{,}024◄$$

Integrallarni hisoblash. Darajali qatorlar o‘zining yaqinlashish sohasining ichida yotuvchi har qanday kesmada tekis yaqinlashuvchi bo‘ladi, shuning uchun funksiyalarni darajali qatorlarga yoyishdan foydalanib, aniqmas integrallarni darajali qatorlar ko‘rinishida yozib olib, ularga mos bo‘lgan aniq integrallarni taqribiy hisoblash mumkin.

5- misol. $\int_0^1 \sin(x^{2)}\, dx\, ni \;\; \delta = 10^{-3}$aniqlikda hisoblang.

►(12.19) formuladan foydalanamiz. x ni x^2 ga almashtirib quyidagiga esa bo‘lamiz.

$$\sin(x^2) = x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \cdots + (-1)^{n-1}\frac{x^{4n-2}}{(2n-1)!} + \cdots$$

Bu qator barcha sonlar to‘g‘ri chizig‘ida yaqinlashuvchidir, shuning uchun uni hamda-had integrallash mumkin.

$$\int_0^1 \sin(x^2) = \int_0^1 (x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} + \cdots + (-1)^{n-1}\frac{x^{4n-2}}{(2n-1)!} + \cdots)\text{dx}=$$

$$= (\frac{x}{3} - \frac{x^7}{7*3!} + \quad \frac{x^{11}}{11*5!} - \cdots + (-1)^{n-1}\frac{1}{(4n-1)(2n1)!} + \cdots)\Big|_0^1 \approx$$

$$\approx \frac{1}{3} - \frac{1}{7*3!} = 0{,}3 - 0{,}0381 = \quad = 0{,}295.$$

Qatorning uchinchi hadi $\delta = 10^{-3}$ dan kichik bo‘lgani uchun ikkita hadi bilan chegaralandik. ◄

6- misol. $\int \frac{\sin x}{x}$integralni darajali qator ko‘rinishida yozing va uning yaqinlashish sohasini ko‘rsating

►(12.19) formuladan foydalanib, integral ostidagi ifoda uchun quyidagi qatorni hosil qilamiz.

$$\frac{1}{x}\sin x = 1 - \frac{x^2}{3} + \frac{x^4}{5!} - \cdots (-1)^{n-1}\frac{x^{2n-2}}{(2n-1)!} + \ldots$$

Bu qator barcha sonlar to‘g‘ri chizig‘ida yaqinlashuvchidir, shuning uchun uni hadma- had integrallash mumkin:

$$\int \frac{\sin x}{x}dx = C + x - \frac{x^3}{3\cdot 3!} + \frac{x^5}{5\cdot 5!} - \ldots + (-1)^n \frac{x^{2n-1}}{(2n-1)\cdot(2n-1)!} + \cdots$$

Darajali qatorni integrallashda uning yaqinlashish sohasini o‘zgarmaydi, shuning uchun hosil qilingan qator ham barcha sonlar to‘g‘ri chizig‘ida yaqinlashuvchidir. ◄

Differensial tenglamalarini taqribiy yechish. Differensial tenglamani elementar funksiyalar yordamida aniq integrallash mumkin bo‘lmasa, uning yyechimini darajali qator, masalan Teylor yoki Makloren qatorlari ko‘rinishida izlash qulaydir. Quyidagi,

$$y'=f(x,y), \quad y(x_0) = y_o \qquad (12.22)$$

Koshi masalasini yechishda, Teylor qatoridan foydalaniladi.

$$\sum_{n=o}^{\infty} \frac{y^{(n)}(x_0)}{n!}(x - x_0) \qquad (12.23)$$

Bu erda $y(x_0) = y_o, y'=f(x_o, yo)$. Qolgan $y^{(n)}(x_0)$ (n=2,3,..) hosilalar (12.22) tenglamani ketma-ket differensiallash va bu hosilalar uchun olingan ifodalarga boshlang‘ich shartlarni qo‘yish yo‘li bilan topiladi.

7- misol. $y(1)=1$ boshlang‘ich shartni qanoatlantiruvchi $y'=x^2+y^2$ differensial tenglama yyechimining darajali qatorga yoyilmasining birinchi beshta hadni topining.

► Berilgan tenglamadan, $y'(1) = 1 + 1 = 2$ ekanligini topamiz. Berilgan tenglamani differensiallaymiz.

$$y'' = 2x + yu', \quad y''(1) = 6, \quad y'''= 2+2y'^2+2uy'', y'''(1) = 22$$

$$y^{(IV)} = 4y' \cdot y'' + 2y' \cdot y'' + 2yy''', y^{(IV)}(1) = 116 \text{ va hokazo.}$$

Hosilalarning topilgan qiymatlarini (12.23) qatorga qo‘yib, quyidagini hosil qilamiz:

$$y(x) = 1 + 2(x-1) + \frac{6(x-1)^2}{2} + \frac{22}{6}(x-1)^3 + \frac{116}{24}(x-1)^4 + \cdots = 1 + 2(x-1) + 3(x-1)^2 + \frac{11}{3}(x-1)^3 + \frac{29}{6}(x-1)^4 + \cdots \text{ ◄}$$

8- misol. $y(0)=-2$, $y'(0)=2$ boshlang‘ich shartlarda $y''-(1+x^2)y=0$ differensial tenglama yyechimining darajali qatorga yoyilmasining birinchi hadini toping.

►Tenglamaga boshlang‘ich shartni qo‘yib, quyidagiga ega bo‘lamiz:

$$y''(0)=1\cdot(-2)=-2.$$

Berilgan tenglamani ketma-ket differensiallaymiz:

$$y'''=2xy+(1+x^2)y',\; y'''(0)=2;$$

$$y^{(IV)}=2y+2xy'+2xy'+(1+x^2)y',\; y^{(IV)}(o)=-6$$

$$y^{(V)}=6y'+6xy'+(1+x^2)y''',\; y^{(V)}(0)=14.$$

Hosilalarning topilgan qiymatlarini Makloren qatoriga qo‘yib quyidagini hosil qilamiz:

$$y(x)=-2+2x-x^2+\frac{1}{3}x^3-\frac{1}{4}x^4+\frac{7}{60}x^5+\cdots$$ ◄

Differensial tenglama uchun Koshi masalasi yyechimi $u=\varphi(x)$ ning darajali qatoriga yoyilmasini $a_i, i=1,2,...$ noma’lum koeffisiyentlar bilan quyidagi ko‘rinishida izlash mumkin

$$y=\varphi(x)=a_o+a_1(x-x_o)+a_2(x-x_o)^2+\cdots+a_n(x-x_o)^n+\cdots \quad (12.24)$$

9- misol. (12.24) qatordan foydalanib $y'=x+y-1$, $y(1)=2$ Koshi masalasi yyechimining noldan farqli birinchi to‘rtta hadini yozing.

► Bizning misolimizda $x_o=1$. Shuning uchun boshlang‘ich shartni e’tiborga olib (12.24) da x=1 deb olsak, $a_o=2$ bo‘ladi. (12.24) qatorni differensiallaymiz, hosil qilingan y' hosila qiymatini va y ning (12.24) qator ko‘rinishidagi qiymatlarni berilgan tenglamaga qo‘yamiz. U holda

$$y'=a_1+2a_2(x-x_o)+3a_3(x-x_o)^2+\cdots=$$
$$=x-1+[a_o+a_1(x-x_o)+a_2(x-x_o)^2+\cdots]^2.$$

Endi tenglamaning chap va o‘ng tomonlaridagi (x-1) ayirmaning bir xil darajalari oldidagi koeffitsientlarini tenglashtiramiz (ya’ni $(x\text{-}1)^0$, $(x-1)^1$ va $(x-1)^2$ va hokazo). Quyidagi rekkurent formulalarni hosil qilamiz.

$$a_1 = a^2{}_0, 2a_2 = 1 + 2a_0a_1, 3a_3 = a_1{}^2 + 2a_0a_2,$$

bundan, $a_0 = 2$ ekanligini e’tiborga olib, $a_1 = 4, a_2 = \frac{17}{2}$, $a_3 = 50\,/3.$ ni topamiz. Demak, yyechimning izlanayotgan yoyilmasi quyidagi ko‘rinishga ega bo‘ladi

$$y = 2+4(x\text{-}1) + \frac{17}{2}(\underline{x}\text{-}1)^2 + \frac{50}{3}(x-1)^3 + \cdots ◀$$

AT-12.5

1.Quyidagi kattaliklarni darajali qatorlar yordamida $\delta = 0{,}001$ aniqlikda taqribiy hisoblang:

a)$\sqrt[3]{e}$; b) $\sqrt[3]{10}$; c) $cos10^o$; d) $\sqrt[10]{2027}$; $e)$ $ln\frac{3}{2}$.

(Javob: a)1,396; b) 2,154; c) 0,985; d) 2,001; e) 0,405.)

2 Quyidagi aniq integrallarni $\delta = 0{,}001$ aniqlikda darajali qatorlar yordamida hisoblang:

a) $\int_0^{1/2} \sqrt{1 + x^2}\, dx$; $b)$ $\int_0^1 cos\sqrt{x}\, dx$; c) $\int_0^4 e^{1/x}\, dx$; d) $\int_0^{1/4} e^{-x^2}\, dx.$

(Javob: a) 0.508; b) 0.764; c) 2.835; d) 0.245.)

3.Aniqlamas integralni darajali qator ko‘rinishida ifodalang va u qatorning yaqinlashish sohasini ko‘rsating:

a) $\int \frac{cosx}{x} dx$; b) $\int \frac{e^x}{x} dx.$

4.Berilgan boshlang‘ich shartni qanoatlantiruvchi differensial tenglama yyechimining darajali qatorga yoyilmasining birinchi beshta hadini yozing:

a) $y' = e^y + xy$, y(0)=0;

b) $y' = 1 + x + x^2 - 2y^2$, $y(1) = 1$;

c) $y'' = x^2$y-y', y(0)=1,$y'(0) = 0$;

d) $y'' = x + y^2$, $y(0) = 0$; $y'(0) = 1$.

Mustaqil ish.

1. 1. sin1 ni $\delta = 0{,}001$ aniqlikda darajali qator yordamida xhisoblang . (Javob: 0.841.)

2. y(1)=1 boshlang‘ich shartni qanoatlantiruvchi $\boldsymbol{y' = x^2 - y}$ differensial tenglama, yyechimining darajali qatorga yoyilmasining birinchi uchta hadini toping.

2. 1. $\sqrt[3]{70}$ ni $\delta = 0{,}001$ aniqlikda darajali qator yordamida hisoblang (Javob:4.125.)

2. Agar y(0) = 1, $y'(0) = 1$ bo‘lsa, $y'' = x^2 - y$ differensial tenglama yyechimining darajali qatorga yoyilmasining birinchi to‘rtta hadini toping.

3. 1. $\int_0^{0.5} \frac{sin2x}{x} dx$ ni $\delta = 0{,}001$ aniqlikda darajali qator yordamida hisoblang. (Javob:0.946.)

2. Agar y(0) = 1 bo‘lsa, $y' = x^2 y + y^3$ differensial tenglama yyechimining darajali qatorga yoyilmasining birinchi uchta hadini toping.

12.5 Furye qatorlari

Quyidagi ko‘rinishdagi funksional qator $f(x)$ funksiyaning Furye qatori deb ataladi.

$$\frac{\boldsymbol{a_o}}{\boldsymbol{2}} + \sum_{n=1}^{\infty}(a_n cosnx + b_n\, sinnx) \qquad (12.25)$$

bu erda a_n, b_n koeffisiyentlar

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)cosnxdx \quad b_n = \int_{-\pi}^{\pi} f(x)sin\, nxdx \qquad (12.26)$$

formulalar bilan aniqlanadi. Har doim $b_o = 0$ bo‘ladi.

[a,b] kesmani n ta chekli sondagi (a,x_1), $(x_1, x_2), \dots .(x_{n-1}, b)$ oraliqlarga shunday bo‘lish mumkin bo‘lsaki, ularning har birida $f(x)$ funksiya monoton bo‘lsa, u holda $f(x)$ funksiyani [a,b] kesmada *bo‘lakli-monoton* deb ataladi.

1- Teorema. Agar $f(x)$ funksiya $[-\pi; \pi]$ kesmada davriy (davri $\omega = 2\pi$) bo‘lakli–monoton va chegaralangan bo‘lsa, u holda uning Furye qatori ihtiyoriy $x \epsilon R$ da yaqinlashuvchi va yig‘indisi

$$S(x) = \frac{f(x-0)+f(x+0)}{2}$$

bo‘ladi.

Teoremadan, $f(x)$ funksiya uzluksiz bo‘lgan nuqtalarda $S(x) = f(x)$ va $f(x)$ funksiya birinchi tur uzilishga ega bo‘lgan nuqtalarda S(x) yig‘indi chap va o‘ng limitlarning o‘rta arifmetigiga teng ekanligi kelib chiqadi.

1- misol. $f(x) = \begin{cases} o, -\pi < x < o, \\ x, o \leq x \leq \pi \end{cases}$ davriy funksiyani (davri 2π) Furye qatoriga yoying.

► Berilgan funksiya chegaralangan bo‘lakli-monoton bo‘lganligi uchun Furye qatoriga yoyilmasi mavjud. Qatorning koeffitsiyentlarini topamiz.

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx = \frac{1}{\pi}\int_{0}^{\pi} xdx = \frac{1}{\pi}\frac{x^2}{2}\Big|_{o}^{\pi} = \frac{\pi}{2}$$

$$a_n = \frac{1}{\pi}\int_{o}^{\pi} xcosnxdx = \left| \begin{matrix} u = x, dv = cosndx \\ du = dx, v = \dfrac{sinnx}{n} \end{matrix} \right| = \frac{1}{\pi}\left(\frac{x}{n}sinnx\Big|_{o}^{\pi} - \int_{o}^{n}\frac{1}{n}sinnxdx\right)$$

$$= \frac{1}{\pi} \cdot \frac{1}{n^2} cosnx \Big|_{o}^{\pi} = \frac{1}{\pi n^2}((-1)^{n-1}),$$

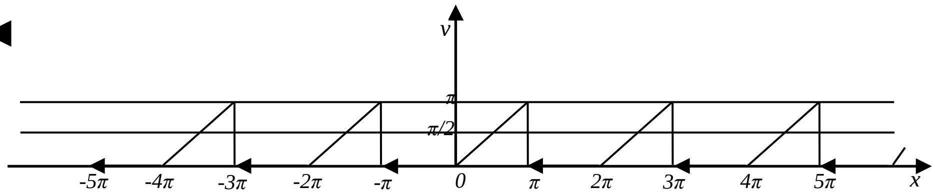
$$b_n = \frac{1}{\pi} \int_{o}^{\pi} xsinnxdx = \frac{1}{\pi}\left(-\frac{x}{n} cosnx \Big|_{o}^{\pi} + \frac{1}{n^2} sinnx \Big|_{o}^{\pi}\right) =$$

$$= -\frac{\pi}{n} cos\pi n = \frac{(-1)^{n-1}}{n} \quad (n\epsilon N)$$

Topilgan koeffitsiyentlarni (12.25) qatorga qo‘yib, quyidagiga ega bo‘lamiz.

$$f(x) = \frac{\pi}{4} + \sum_{n=1}^{\infty}(-\frac{2}{\pi(2n-1)^2} cos\big((2n-1)x\big) + \frac{(-1)^{n-1}}{n} sinnx)$$

Bu qator barcha $x \neq$ (2n-1)π da davri 2π bo‘lgan berilgan davriy funksiyaga yaqinlashadi. x=(2n-1)π nuqtalarda qatorning yig‘indisi (n+0)/2 $= \frac{\pi}{2}$ ga teng. (12.1 rasm) ◄



12.1-rasm

Agar $y = f(x)$ funksiyaning davri 2ℓ bo‘lsa, uning Furye qatori quyidagi ko‘rinishda yoziladi.

$$f(x) = \frac{a_o}{2} + \sum_{n=1}^{\infty}(a_n cos\left(\frac{n\pi}{l}x\right) + b_n sin(\frac{n\pi}{l}x)) \qquad (12.27)$$

bu erda

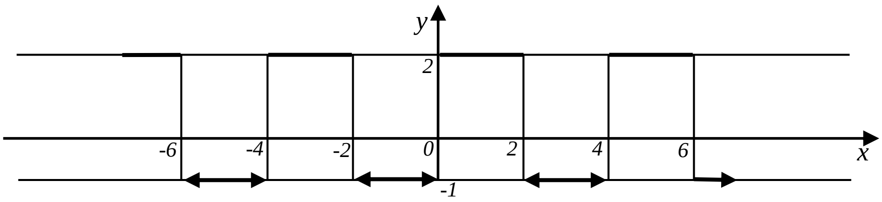
$$a_n = \frac{1}{l}\int_{-l}^{l} f(x) \cos\left(\frac{\pi n}{l}x\right) dx, \quad b_n = \frac{1}{l}\int_{-l}^{l} f(x) \sin\left(\frac{\pi n}{l}x\right) dx$$

(12.28)

2-Teorema. Agar davri 2ℓ ga teng davriy funksiya [-ℓ;ℓ] kesmada chegaralangan va bo‘lakli monoton bo‘lsa, u holda ihtiyoriy $x\epsilon R$ uchun uning Furye qatori (12.27) *S(x)* =$(f(x-0) + f(x+0))/2$

yig‘indiga yaqinlashadi. (1- teorema bilan solishtiring.)

2- misol. Davri 4 ga teng bo‘lgan, davriy funksiyaning Furye qatoriga yoyilmasini toping, $f(x) = \begin{cases} -1, & -2 < x < 0 \\ 2, & 0 \le x \le 2 \end{cases}$.



12.2-rasm

►Qatorning koeffitsiyentlarning topamiz.

$$a_o = \frac{1}{2}\left(\int_{-2}^{2}(-1)dx + \int_{o}^{2} 2dx\right) = \frac{1}{2}\left(-x\Big|_{-2}^{o} + 2x\Big|_{o}^{2}\right) = \frac{1}{2}(-2+4) = 1$$

$$a_n = \frac{1}{2}\left(\int_{-2}^{0}(-1)\cos(\frac{\pi n}{2}x\right)dx + \int_{0}^{2} 2\cos(\frac{\pi n}{2}x)dx) =$$

$$= \frac{1}{2}\left(-\frac{2}{\pi n}\sin\left(\frac{\pi n}{2}x\right)\Big|_{-2}^{0} + \frac{4}{\pi n}\sin\left(\frac{\pi n}{2}x\right)\Big|_{0}^{2}\right) = 0.$$

$$b_n = \frac{1}{2}(\int_{-2}^{0}(-1)sin\left(\frac{\pi n}{2}x\right)dx + \int_{0}^{2} 2sin\left(\frac{\pi n}{2}x\right)dx) = \frac{1}{2}(\frac{2}{\pi n}\cos\left(\frac{\pi n}{2}x\right)\Big|_{-2}^{0}$$

$$-\frac{4}{\pi n}(cos\pi n - 1)) = \frac{3}{\pi n}(cos\pi n - 1) = \frac{3}{\pi n}((-1)^n - 1).$$

Topilgan koeffitsiyentlarni (12.27) qatorga qo‘yib quyidagini hosil qilamiz

$$f(x) = \frac{1}{2} - \frac{6}{\pi}\sum_{n=1}^{\infty}\frac{1}{2n-1}\sin\left(\frac{(2n-1)\pi}{2}\cdot x\right).$$ ◄

Agar davriy funksiya juft bo‘lsa, u Furye qatoriga faqat kosinuslar bo‘yicha yoyiladi, shu bilan birgalikda

$$a_n = \frac{2}{l}\int_{o}^{l} f(x)\cos\left(\frac{\pi n}{l}x\right)\mathrm{dx}.$$

Agar davriy funksiya toq bo‘lsa u Furye qatorga faqat sinuslar bo‘yicha yoyiladi va

$$b_n = \frac{2}{l}\int_{o}^{l} f(x)cos\,(\frac{\pi n}{l}x)dx.$$

Ixtiyoriy $\lambda \epsilon R$ va davri 2*l* bo‘lgan har qanday $f(x)$ davriy funksiya uchun quyidagi tenglik o‘rinlidir.

$$\int_{-l}^{l} f(x)dx = \int_{\lambda-l}^{\lambda+l} f(x)dx$$

U holda Furye koeffitsiyentlarini quyidagi formulalar bilan hisoblash mumkin:

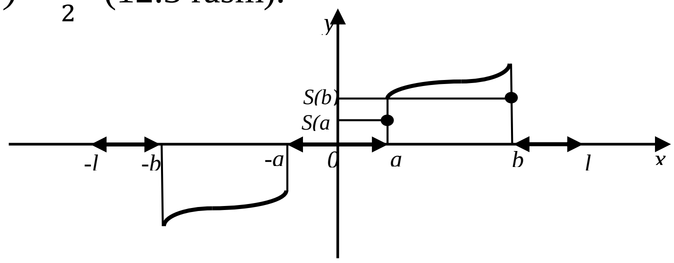
$$a_n = \frac{1}{l}\int_{0}^{2l} f(x)cos(\frac{\pi n}{l}x)dx, \qquad b_n = \frac{1}{l}\int_{0}^{2l} f(x)sin(\frac{\pi n}{l}x)dx,$$

bu erda $n = 0, 1, 2, \ldots$

Faraz qilaylik, $f(x)$ funksiya [a,b]⊂(-l; l) kesmada chegaralangan va bo‘lakli-monoton bo‘lsin. Bu funksiyani Furye qatoriga yoyish uchun (-l; l) oraliqda ixtiyoriy ravishda shunday davom ettiramizki, u(-l; l) da chegaralangan va bo‘lakli monotonligicha qolsin. Topilgan funksiyani [a;b] kesmada berilgan funksiyaga yaqinlashuvchi bo‘lgan Furye qatoriga yoyamiz. Agar berilgan funksiyani (-l; l) da juft holda davom ettirsak, u holda uning faqat kosinuslar bo‘yicha yoyilmasini hosil qilamiz. Masalan, [a,b]⊂(-l; l) da aniqlangan va (-l; l) da quyidagi tengliklar bilan davom ettirilgan $f(x)$ funksiya

$$f(x)=\begin{cases}0, & -l<x<b,\\ -f(x), & -b\le x\le -a,\\ 0, & a<x<a,\\ f(x), & a\le x\le b,\\ 0, & 0<x<l,\end{cases}$$

faqat sinuslar bo‘yicha yoyiladi.**[**a,b] kesma ichida bunday funksiyaning Furye qatori yig‘indisi S(x), $f(x)$ ga teng bo‘ladi, 2- teoremaga asosan esa *S(a)=*$f(a)$*/2, S(b)=*$\frac{f(b)}{2}$ (12.3 rasm).



12.3-rasm

3- misol. $f(x)=|\,x\,|$ (-2$\le x\le 2$) funksiyani Furye qatoriga yoying.

► Berilgan funksiya juft funksiya bo‘lganligi uchun u Furye qatoriga faqat kosinuslar bo‘yicha yoyiladi, ya’ni $b_n=0$.

$$a_n=\frac{2}{2}\int_o^2 xdx=\frac{x^2}{2}\Big|_o^2=2,$$

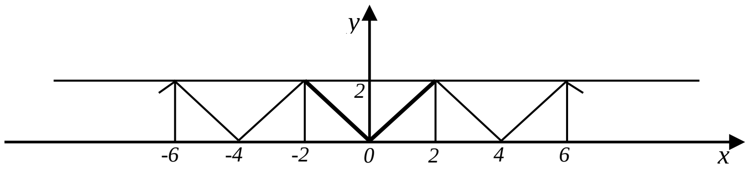
$$a_n=\frac{2}{l}\int_0^l f(x)cos(\frac{\pi n}{l}x)dx=\int_0^2 x\cdot cos(\frac{\pi n}{2}x)dx\ =$$

$$=\frac{2x}{\pi n}sin\left(\frac{\pi n}{2}x\right)\Big|_o^2+\frac{4}{\pi^2n^2}cos\left(\frac{\pi n}{2}x\right)\Big|_o^2==\frac{4}{\pi^2n^2}((-1)^n-1).$$

Bundan, agar n juft bo‘lsa, $a_n=0$,n toq bo‘lsa $a_n=\frac{-8}{n^2\pi^2}$. Berilgan funksiyaning Furye qatori :

$$f(x)=1-\frac{8}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}\cos\left(\frac{(2n-1)\pi}{2}x\right).$$

Buning yig‘indisi [-2,2] kesmada berilgan funksiyaga teng, barcha sonlar to‘g‘ri chizig‘ida esa bu yig‘indi davri $\omega = 4$ bo‘lgan davriy funksiyani aniqlaydi.(12.4 rasm).◄

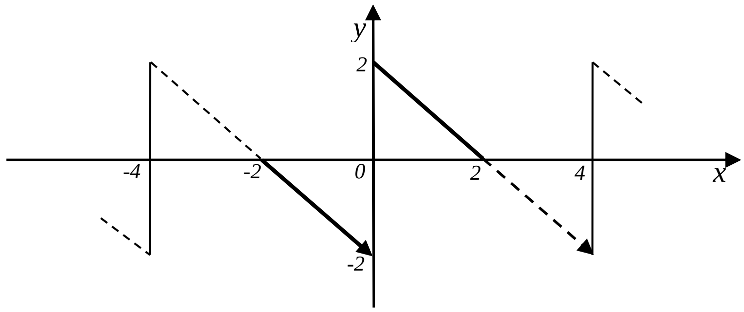


12.4-rasm

4- misol. $f(x) = 2 - x$ funksiyani [0;2] kesmada sinuslar bo‘yicha qatorga yoying.

► Berilgan funksiyani [-2;0] kesmada toq ravishda davom ettiramiz (12.5rasm) ya’ni,

$$f(x) = \begin{cases} -2 - x, & -2 \leq x < 0 \\ 2 - x, & 0 \leq x \leq 2 \end{cases}$$



12.5-rasm

U holda, n = 0,1,2,… bo‘lganda $a_n = 0$,

$$b_n = \frac{2}{l}\int_0^l f(x)sin\left(\frac{\pi n}{\ell}x\right)dx = \int_0^2 (2-x)sin(\frac{\pi n}{2}x)dx =$$

$$= \begin{vmatrix} u = 2-x & du = -dx \\ dv = sin\left(\frac{\pi n}{2}x\right)dx & v = -\frac{2}{\pi n}cos(\frac{\pi n}{2}x \end{vmatrix} =$$

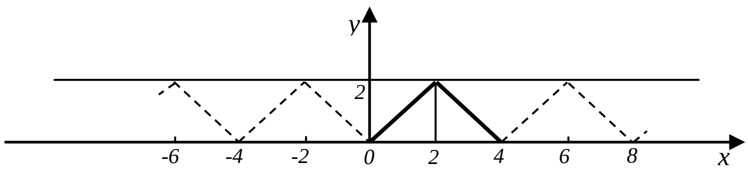
$$= -\frac{2(2-x)}{\pi n}cos\left(\frac{\pi n}{2}x\right)\Big|_0^2 - \int_0^2 \frac{2}{\pi n}cos(\frac{\pi n}{2}x)dx =$$

$$= \frac{4}{\pi n} - \frac{4}{\pi^2 n^2}sin\left(\frac{\pi n}{2}x\right)\Big|_0^2 = \frac{4}{\pi n}$$

Topilgan koeffitsiyentlarni Furye qatoriga qo‘yib quyidagiga ega bo‘lamiz.

$$f(x) = \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}sin(\frac{\pi n}{2}x). \blacktriangleleft$$

5- misol. 12.6 rasmda tutash chiziq ko‘rinishda tasvirlangan funksiyani Furye qatorga yoying.



12.6- rasm

► Berilgan funksiyani [-2;0] kesmada juft ravishda davom ettiramiz va $f(x) = x,\ \ x\epsilon$ [0;2] funksiyani kosinuslar bo‘yicha yoyamiz, ya’ni

$$f(x) = \frac{a_o}{2} + \sum_{n=1}^{\infty} a_n cos(\frac{\pi n}{2}x)$$

$$a_o = \frac{2}{2}\int_0^2 xdx = \frac{x^2}{2}\Big|_o^2 = 2$$

$$a_n = \int_o^2 xcos(\frac{\pi n}{2}x)dx = \frac{2x}{4\pi}sin\left(\frac{\pi n}{2}x\right)\Big|_o^2 - \frac{2}{\pi n}\int_o^2 sin(\frac{\pi n}{2})dx \ =$$

$$= \frac{4}{\pi^2 n^2} cos(\frac{\pi n}{2} x) \Big|_{o}^{2} = \frac{4}{\pi^2 n^2}((-1)^n - 1).$$

Izlanayotgan Furye qatori quyidagi ko‘rinishda bo‘ladi.

$$f(x) = 1 - \frac{8}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} cos\left(\frac{(2n-1)\pi}{2} \cdot x\right).$$

[0;2] kesmada bu berilgan funksiyani ifodalaydi, barcha son o‘qida esa davri $\omega = 4$ bo‘lgan davriy funksiyani ifodalaydi. (12.6 rasmga qarang, shtrixlangan va tutash chiziqlar). ◄

Funksiya uzluksiz bo‘lgan nuqtalarda Furye qatori unga mos funksiyaning qiymatlariga yaqinlashadi , shunig uchun ko‘p hollarda sonli qator yig‘indisini topish uchun Furye qatori qo‘llaniladi. Masalan, agar 5-misolda aniqlangan funksiyaning Furye qatorida *x=2* desak quyidagiga ega bo‘lamiz:

$$2 = 1 - \frac{8}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} sos\pi$$

$$\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$$

6- misol. y = x^2 funksiyani [0;π] kesmada kosinuslar bo‘yicha Furye qatoriga yoying va hosil qilingan qator yordamida

$$\sum_{n=1}^{\infty} \frac{1}{n^2} \text{va} \sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n^2}$$

sonli qatorlarning yig‘indisini toping.

► Berilgan funksiyani (-π; 0) oraliqda juft ravishda davom ettiramiz va 2π davr bilan sonli to‘g‘ri chiziqda kosinuslar bo‘yicha qatorga yoyamiz u holda :

$$a_o = \frac{2}{\pi}\int_0^{\pi} x^2 dx = \frac{2}{\pi}\cdot\frac{x^3}{3}\Big|_0^{\pi} = \frac{2\pi^2}{3},$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} x^2 cosnxdx = \frac{2}{\pi}\Big(\frac{x^2}{\pi} sinnx\Big|_0^{\pi} -$$

$$-\int_0^{\pi} 2x\frac{1}{n} sinnxdx\Big) = -\frac{4}{\pi n}\Big(-\frac{x}{n}cosnx\Big|_0^{\pi} - \int_0^{\pi}\frac{cosnx}{n}dx\Big)$$

$$= \frac{4}{n^2}cosnx\Big|_o^{\pi} = \frac{4\cdot(-1)^n}{2}.$$

Quyidagi Furye qatorini hosil qildik

$$f(x) = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}(-1)^n\frac{cosnx}{n^2}.$$

Davom ettirilgan funksiya uzluksiz bo‘lgani uchun x ning har qanday qiymatida uning Furye qatori berilgan funksiyaga yaqinlashadi. Shuning uchun $x = 0$ da

$$0 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}(-1)^n\frac{1}{n^2},$$

ya’ni,

$$\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{n^2} = \frac{\pi^2}{12},$$

$x = \pi$ da $\pi^2 = \frac{n^2}{3} + 4\sum_{n=1}^{\infty}\frac{1}{n^2}$, $\sum_{n=1}^{\infty}\frac{1}{n^2} = \frac{\pi^2}{6}$. ◄

AT-12.6.

1. Davri 2π bo‘lgan quyidagi funksiyani Furye qatoriga yoying.

$$f(x) = \begin{cases} x, & -\pi < x \le 0 \\ 2x, & 0 < x \le \pi \end{cases}$$

(Javob: $\frac{\pi}{4}\ -\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{cos(2n-1)x}{(2n-1)^2}+3\sum_{n=1}^{\infty}(-1)^{n-1}\frac{sinnx}{n}.$)

2. Funksiyani Furye qatoriga yoying.

$$f(x)=\begin{cases}\pi+2x, & -\pi<x\leq 0\\ -\pi, & 0\ <x\leq\pi\end{cases}$$

(Javob: $-\frac{\pi}{2}+2\sum_{n=1}^{\infty}(\frac{2}{\pi(2n-1)^2}\ \ cos\frac{\pi(2n-1)}{2}x-\frac{1}{n}sinnx$).

3. Agar $f(x)=\begin{cases}1+x, & -2<x\leq 0,\\ -1, & 0\ <x\leq 2\end{cases}$

bo‘lsa, davriy funksiyani (davr $\omega=4$) Furye qatoriga yoying.

(Javob: $-\frac{1}{2}+\frac{2}{\pi}\sum_{n=1}^{\infty}(\frac{2}{\pi(2n-1)^2}sos\frac{\pi(2n-1)}{2}x-\frac{1}{n}sin\frac{n\pi x}{2}$).

4. y = x^2 funksiyaning $[-\pi;\pi]$ kesmadagi Furye qatoriga yoyilmasini toping. Qator yig‘indisinig va berilgan funksiyaning grafigini chizing.

(Javob: $\frac{\pi^2}{3}+4\sum_{n=1}^{\infty}(-1)^n\frac{sosnx}{n^2}$.)

Mustaqil ish.

1. $f(x)=-x$ funksiyaning [-2;2] kesmasidagi Furye qatoriga yoyilmasini toping. Berilgan funksiyaning va qator yig‘indisining grafigini chizing

(Javob: $2\sum_{n=1}^{\infty}\frac{(-1)^n}{n}sinnx$).

2.$f(x)=\begin{cases}-2, & -\pi<x\leq 0,\\ 1, & 0\ x\leq\pi\end{cases}$ funksiyaning Furye qatoriga yoyilmasini toping. Qator yig‘indisining va berilgan funksiyaning grafigini chizing.

(Javob: $-1+\frac{6}{\pi}\sum_{n=1}^{\infty}\frac{1}{2n-1}sin(2n-1)x)$.

3. $f(x)=\begin{cases}-x, & -\pi<x\leq 0\\ 0, & 0<x\leq\pi\end{cases}$funksiyani Furye qatoriga yoying. Qator yig‘indisining va berilgan funksiyaning grafigini chizing.

(Javob: $\frac{1}{4}+\sum_{n=1}^{\infty}(\frac{(-1)^n-1}{\pi n^2}cosnx+\frac{(-1)^n}{n}\ sinnx).)$

AT-12.7

1. $f(x)=x^2$ funksiyani (0;π) oraliqda sinuslar bo‘yicha Furye qatoriga yoying. Qator yig‘indisining va berilgan funksiyaning grafigini chizing.

(Javob: $\frac{2}{\pi}\sum_{n=1}^{\infty}(-1)^{n-1}\left(\frac{\pi^2}{n}+\frac{2}{n^2}((-1)^n-1)\right)sinnx)$

2. y = sinx funksiyani (0;π) kesmada kosinuslar bo‘yicha Furye qatoriga yoying. (Javob:$\frac{2}{\pi}+\sum_{n=1}^{\infty}\frac{cos2nx}{1-(2n)^2}$.)

3. y =1$-\frac{x}{2}$ funksiyani [0;2] kesmada sinuslar bo‘yicha Furye qatoriga yoying. (Javob: $\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{\pi}sin\frac{\pi nx}{2}$.)

4. y =1$-2x$ funksiyani [0;1] kesmada kosinuslar bo‘yicha Furye qatoriga yoying . (Javob: $\frac{8}{\pi^2}\sum_{n=1}^{\infty}\frac{cos(2n-1)x}{(2n-1)^2}$.)

5.$f(x)=1$ funksiyani [0; π] kesmada sinuslar bo‘yicha Furye qatoriga yoying va $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+(-1)^{n-1}\frac{1}{2n-1}+\cdots$qatorning yig‘indisini toping.

Mustaqil ish.

1. $f(x)=1-x$ funksiyani [0; 2] kesmada kosinuslar bo‘yicha Furye qatoriga yoying. (Javob: $\frac{8}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}\cdot cos\frac{(2n-1)\pi}{2}x$.)

2.$f(x)\ =\pi-x$ funksiyani [0; π] kesmada sinuslar bo‘yicha Furye qatoriga yoying. (Javob: $2\sum_{\mathrm{n}=1}^{\infty}\frac{\mathrm{sinnx}}{\mathrm{n}}$.)

3. $f(x) = \frac{\pi}{4} - \frac{2}{x}$ funksiyani $[0;\ \pi]$kesmada kosinuslar bo‘yicha Furye qatoriga yoying. (Javob: $\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{Cos((2n-1)x)}{(2n-1)^2}$.)

12.6. 12 BOBGA DOIR INDIVIDUAL UY TOPSHIRIQLARI

IUT 12.1

1.Qatorning yaqinlashuvchi ekanligini isbotlang va uning yig‘indisini toping.

1.1. $\sum_{n=1}^{\infty}\frac{1}{n(n+2)}$. (Javob: S $=\frac{3}{4}$.)

1.2. $\sum_{n=1}^{\infty}\frac{3^n+4^n}{12^n}$. (Javob: S $=\frac{5}{6}$.)

1.3. $\sum_{n=0}^{\infty}\frac{1}{(2n+5)(2n+7)}$. (Javob: S $=\frac{1}{10}$.)

1.4. $\sum_{n=1}^{\infty}\frac{2^n+5^n}{10^n}$. (Javob: S $=\frac{5}{4}$.)

1.5. $\sum_{n=0}^{\infty}\frac{1}{(n+5)(n+6)}$. (Javob: S $=\frac{1}{5}$.)

1.6. $\sum_{n=1}^{\infty}\frac{5^n-2^n}{10^n}$. (Javob: S $=\frac{3}{4}$.)

1.7. $\sum_{n=0}^{\infty}\frac{1}{(2n+7)(2n+9)}$. (Javob: S $=\frac{1}{14}$.)

1.8. $\sum_{n=1}^{\infty}\frac{4^n-3^n}{12^n}$. (Javob: S $=\frac{1}{6}$.)

1.9. $\sum_{n=1}^{\infty}\frac{1}{(n+6)(n+7)}$. (Javob: S $=\frac{1}{7}$.)

1.10. $\sum_{n=1}^{\infty}\frac{3^n+5^n}{15^n}$. (Javob: S $=\frac{3}{4}$.)

1.11. $\sum_{n=1}^{\infty}\frac{1}{(n+9)(n+10)}$. (Javob: S $=\frac{1}{10}$.)

1.12. $\sum_{n=1}^{\infty}\frac{5^n-3^n}{15^n}$. (Javob: S$=\frac{1}{4}$.)

1.13. $\sum_{n=1}^{\infty}\frac{1}{(n+7)(n+8)}$. (Javob: S $=\frac{1}{8}$.)

1.14. $\sum_{n=1}^{\infty}\frac{2^n+7^n}{14^n}$. (Javob: S $=\frac{7}{6}$.)

1.15. $\sum_{n=0}^{\infty}\frac{1}{(n+2)(n+3)}$. (Javob: S = $\frac{1}{2}$.)

1.16. $\sum_{n=1}^{\infty}\frac{7^n-2^n}{14^n}$. (Javob: S =$\frac{5}{6}$.)

1.17. $\sum_{n=0}^{\infty}\frac{1}{(n+3)(n+4)}$. (Javob: S =$\frac{1}{3}$.)

1.18. $\sum_{n=1}^{\infty}\frac{4^n+5^n}{20^n}$. (Javob: S =$\frac{7}{12}$.)

1.19. $\sum_{n=1}^{\infty}\frac{1}{(n+4)(n+5)}$. (Javob: S =$\frac{1}{5}$.)

1.20. $\sum_{n=1}^{\infty}\frac{5^n-4^n}{20^n}$. (Javob: S =$\frac{1}{12}$.)

1.21. $\sum_{n=0}^{\infty}\frac{1}{(2n+1)(2n+3)}$. (Javob: S =$\frac{1}{2}$.)

1.22. $\sum_{n=1}^{\infty}\frac{7^n+3^n}{21^n}$. (Javob: S = $\frac{2}{3}$.)

1.23. $\sum_{n=0}^{\infty}\frac{1}{(2n+3)(2n+5)}$. (Javob: S = $\frac{1}{6}$.)

1.24. $\sum_{n=1}^{\infty}\frac{7^n-3^n}{21^n}$. (Javob: S = $\frac{1}{3}$.)

1.25. $\sum_{n=1}^{\infty}\frac{1}{(3n-1)(2n+2)}$. (Javob: S = $\frac{1}{6}$.)

1.26. $\sum_{n=1}^{\infty}\frac{3^n+8^n}{24^n}$. (Javob: S = $\frac{9}{14}$.)

1.27. $\sum_{n=1}^{\infty}\frac{1}{(3n+1)(3n+4)}$. (Javob: S =$\frac{1}{12}$.)

1.28. $\sum_{n=1}^{\infty}\frac{8^n-3^n}{24^n}$. (Javob: S = $\frac{5}{14}$.)

1.29. $\sum_{n=1}^{\infty}\frac{1}{(3n+2)(3n+5)}$. (Javob: S = $\frac{1}{15}$.)

1.30. $\sum_{n=1}^{\infty}\frac{9^n-2^n}{18^n}$. (Javob:S = $\frac{7}{8}$.)

Quyidagi musbat hadli qatorlarni yaqinlashishga tekshiring.

2

2.1. $\sum_{n=1}^{\infty}\frac{3^n(n+2)}{n^5}$. (Javob: uzoqlashadi.)

2.2. $\sum_{n=1}^{\infty}\frac{7n-1}{5^n(n+1)}$. (Javob: yaqinlashadi.)

2.3. $\sum_{n=1}^{\infty}\left(\frac{7}{8}\right)^n\left(\frac{1}{n}\right)^7$. (Javob: yaqinlashadi.)

2.4. $\sum_{n=1}^{\infty}(2n+1)tg\frac{\pi}{3^n}$. (Javob: yaqinlashadi.)

2.5. $\sum_{n=1}^{\infty}\frac{n^{n/2}}{3^n}$. (Javob: uzoqlashadi.)

2.6. $\sum_{n=1}^{\infty}\frac{4\cdot5\cdot6\cdots(n+3)}{5\cdot7\cdot9\cdots(2n+3)}$. (Javob: yaqinlashadi.)

2.7. $\sum_{n=1}^{\infty}\left(\frac{9}{10}\right)^n\cdot n^7$. (Javob: yaqinlashadi.)

2.8. $\sum_{n=1}^{\infty}\frac{1\cdot7\cdot13\cdots(6n-5)}{2\cdot3\cdot4\cdots(n+1)}$. (Javob: uzoqlashadi.)

2.9. $\sum_{n=1}^{\infty}\frac{3n(n+1)}{5^n}$. (Javob: yaqinlashadi.)

2.10. $\sum_{n=1}^{\infty}\frac{(n+2)!}{n^n}$. (Javob: yaqinlashadi.)

2.11. $\sum_{n=1}^{\infty}n\sin\frac{2\pi}{3^n}$. (Javob: yaqinlashadi.)

2.12. $\sum_{n=1}^{\infty}\frac{(n+1)^{n/2}}{3^n}$. (Javob: yaqinlashadi.)

2.13. $\sum_{n=1}^{\infty}\frac{n!}{5^n(n+3)!}$. (Javob: yaqinlashadi.)

2.14. $\sum_{n=1}^{\infty}\frac{1\cdot6\cdot11\cdots(5n-4)}{3\cdot7\cdot11\cdots(4n-1)}$. (Javob: uzoqlashadi.)

2.15. $\sum_{n=1}^{\infty}\frac{n^n}{(n+3)\,!}$. (Javob: uzoqlashadi.)

2.16. $\sum_{n=1}^{\infty}n^3 tg\frac{2\pi}{5^n}$. (Javob: yaqinlashadi.)

2.17. $\sum_{n=1}^{\infty}\frac{(n^2+3)}{(n+1)!}$. (Javob: yaqinlashadi.)

2.18. $\sum_{n=1}^{\infty}\frac{n}{(2n+3)!}$. (Javob: yaqinlashadi.)

2.19. $\sum_{n=1}^{\infty}\frac{(n+1)^n}{n!}$. (Javob: uzoqlashadi.)

2.20. $\sum_{n=1}^{\infty}\frac{2\cdot5\cdot8\cdots(3n-1)}{3\cdot7\cdot11\cdots(4n-1)}$ (Javob: yaqinlashadi.)

2.21. $\sum_{n=1}^{\infty}(3n-1)\sin\frac{\pi}{4^n}$. (Javob: yaqinlashadi.)

2.22. $\sum_{n=1}^{\infty}\frac{n+2}{n!}$. (Javob: yaqinlashadi.)

2.23. $\sum_{n=1}^{\infty}\frac{3n-1}{\sqrt{n\cdot 7^n}}$. (Javob: yaqinlashadi.)

2.24. $\sum_{n=1}^{\infty}\frac{1\cdot 5\cdot 9\cdots(4n-3)}{1\cdot 4\cdot 7\cdots(3n-2)}$. (Javob: uzoqlashadi.)

2.25. $\sum_{n=1}^{\infty}\frac{5^n}{4n!}$. (Javob: yaqinlashadi.)

2.26. $\sum_{n=1}^{\infty}\frac{1\cdot 3\cdot 5\cdots(2n-1)}{2\cdot 7\cdot 12\cdots(5n-3)}$. (Javob: yaqinlashadi.)

2.27. $\sum_{n=1}^{\infty}\frac{n^n}{(n+1)!}$. (Javob: uzoqlashadi.)

2.28. $\sum_{n=1}^{\infty}\frac{(2n-1)^3}{(2n)!}$. (Javob: yaqinlashadi.)

2.29. $\sum_{n=1}^{\infty}\frac{2^n}{5^n(2n-1)}$. (Javob: yaqinlashadi.)

2.30. $\sum_{n=1}^{\infty}\frac{2n+1}{\sqrt{n\cdot 2^n}}$. (Javob: yaqinlashadi.)

3

3.1. $\sum_{n=1}^{\infty}\frac{10^n}{\left(\frac{n+1}{n}\right)^n}$. (Javob: uzoqlashadi.)

3.2. $\sum_{n=1}^{\infty}\left(\frac{5n-1}{5n}\right)^{n^2}$. (Javob: yaqinlashadi.)

3.3. $\sum_{n=1}^{\infty}\left(arctg\,\frac{1}{2n+1}\right)^n$. (Javob: yaqinlashadi.)

3.4. $\sum_{n=1}^{\infty}\frac{1}{(ln(n+2))^n}$. (Javob: yaqinlashadi.)

3.5. $\sum_{n=1}^{\infty}\left(arcsin\,\frac{1}{2n}\right)^{3n}$. (Javob: yaqinlashadi.)

3.6. $\sum_{n=1}^{\infty}\left(\frac{n^2+5n+8}{3n^2-2}\right)^n$. (Javob: yaqinlashadi.)

3.7. $\sum_{n=1}^{\infty}\left(arctg\,\frac{1}{5^n}\right)^n$. (Javob: yaqinlashadi.)

3.8. $\sum_{n=1}^{\infty}\frac{(n/(n+1))^{n^2}}{2^n}$. (Javob: yaqinlashadi.)

3.9. $\sum_{n=1}^{\infty}\frac{1}{(ln(n+1))^{2n}}$. (Javob: yaqinlashadi.)

3.10. $\sum_{n=1}^{\infty}\left(tg\,\frac{\pi}{5^n}\right)^{3n}$. (Javob: yaqinlashadi.)

3.11. $\sum_{n=1}^{\infty}\frac{1}{(ln(n+3))^n}$. (Javob: yaqinlashadi.)

3.12. $\sum_{n=1}^{\infty}\left(\frac{3n^2+4n+5}{6n^2-3n-1}\right)^{n^2}$. (Javob: yaqinlashadi.)

3.13 . $\sum_{n=1}^{\infty}\left(\frac{2n-1}{2n}\right)^{n^2}$. (Javob: yaqinlashadi.)

3.14. $\sum_{n=1}^{\infty}\left(sin\frac{\pi}{n^3}\right)^{2n}$. (Javob: yaqinlashadi.)

3.15. $\sum_{n=1}^{\infty}\left(\frac{n+1}{4^n}\right)^{3n}$. (Javob: yaqinlashadi.)

3.16. $\sum_{n=1}^{\infty}\frac{4^n}{((n+1)/n)^{n^2}}$. (Javob: yaqinlashadi.)

3.17. $\sum_{n=1}^{\infty}\frac{1}{(\ln(n+1))}$. (Javob: yaqinlashadi.)

3.18 $\sum_{n=1}^{\infty}\left(\frac{3n-1}{3^n}\right)^{n^2}$. (Javob: yaqinlashadi.)

3.19. $\sum_{n=1}^{\infty}\left(arcsin\frac{1}{3^n}\right)^{n}$. (Javob: yaqinlashadi.)

3.20. $\sum_{n=1}^{\infty}\left(\frac{n+1}{2^n}\right)^{n^2}$. (Javob: yaqinlashadi.)

3.21. . $\sum_{n=1}^{\infty}\left(\frac{3n^2-n-1}{7n^2+3n+4}\right)^{n}$ (Javob: yaqinlashadi.)

3.22. $\sum_{n=1}^{\infty}\left(\frac{n}{3n+1}\right)^{n}$. (Javob: yaqinlashadi.)

3.23. $\sum_{n=1}^{\infty}\left(arcsin\frac{1}{3^n}\right)^{2n}$. (Javob: yaqinlashadi.)

3.24. $\sum_{n=1}^{\infty}\left(\frac{n+1}{2n}\right)^{5n}$. (Javob: yaqinlashadi.)

3.25. $\sum_{n=1}^{\infty}\frac{((n+1)/n)^{n^2}}{5^n}$. (Javob: yaqinlashadi.)

3.26. $\sum_{n=1}^{\infty}\left(tg\frac{\pi}{2n+1}\right)^{n}$. (Javob: yaqinlashadi.)

3.27.$\sum_{n=1}^{\infty}\left(sin\frac{\pi}{5n+1}\right)^{n}$. (Javob: yaqinlashadi.)

3.28. $\sum_{n=1}^{\infty}\left(arctg\frac{1}{2n-1}\right)^{2n}$. (Javob: yaqinlashadi.)

3.29. $\sum_{n=1}^{\infty}\frac{10^n}{(In(n+5))^2}$. (Javob: yaqinlashadi.)

3.30. $\sum_{n=1}^{\infty}\left(arcsin\frac{n+3}{2n+5}\right)^{n}$. (Javob: yaqinlashadi.)

4

4.1. $\sum_{n=1}^{\infty}\left(\frac{2n+1}{4n^2+1}\right)^2$

4.2. $\sum_{n=1}^{\infty}\frac{1}{(3n+2)In(3n+2)}$.

4.3.$\sum_{n=1}^{\infty}\frac{1}{(2n+1)In^2(2n+1)}$.

4.4. .$\sum_{n=1}^{\infty}\frac{1}{\sqrt[4]{(4n+5)^3}}$.

4.5. $\sum_{n=1}^{\infty}\frac{1}{(3n+4)In^2(3n+4)}$.

4.6. $\sum_{n=1}^{\infty}\frac{1}{\sqrt[4]{(7n-5)^5}}$.

4.7. $\sum_{n=1}^{\infty}\left(\frac{7+n}{49+n^2}\right)^2$

4.8.$\sum_{n=1}^{\infty}\frac{1}{(3n-1)In(3n-1)}$.

4.9. $\sum_{n=2}^{\infty}\frac{1}{\sqrt{n}}ln\frac{n+1}{n-1}$.

4.10. $\sum_{n=1}^{\infty}\frac{1}{(5n-2)In(5n-2)}$.

4.11.$\sum_{n=1}^{\infty}\frac{6+n}{36+n^2}$.

4.12. $\sum_{n=1}^{\infty}\frac{1}{\sqrt[7]{(3-+7n)^{10}}}$.

4.13.$\sum_{n=1}^{\infty}\frac{1}{\sqrt[5]{(3n-1)^4}}$.

4.14.$\sum_{n=1}^{\infty}\frac{1}{(n+2)In(n+2)}$.

4.15. $\sum_{n=1}^{\infty}\frac{1}{(10n+5)In(10n+5)}$.

4.16. $\sum_{n=1}^{\infty}\frac{1}{\sqrt[4]{(2n+3)^7}}$.

4.17. $\sum_{n=1}^{\infty}\frac{5+n}{(25+n^2)}$.

4.18.$\sum_{n=1}^{\infty}\frac{1}{(n+3)In(n+3)In(In(n+3))}$.

4.19.$\sum_{n=1}^{\infty}\frac{1}{(3+2n)In^5(3+2n)}$.

4.20. $\sum_{n=1}^{\infty}\frac{1}{\sqrt[8]{(2n+3)^5}}$.

4.21.$\sum_{n=1}^{\infty}\frac{1}{(9n-4)In^2(9n-4)}$.

4.22. $\sum_{n=1}^{\infty}\frac{3+n}{9+n^2-2n}$.

4.23. $\sum_{n=1}^{\infty}\frac{1}{(5n+8)In^2(5n+8)}$.

4.24. $\sum_{n=1}^{\infty}\frac{1}{\sqrt[8]{(7n-5)^3}}$.

4.25. $\sum_{n=1}^{\infty}\frac{1}{(n+4)In(n+4)In(In(n+4))}$.

4.26. $\sum_{n=1}^{\infty}\frac{1}{(3n+8)In^3(3+8n)}$.

4.27. $\sum_{n=1}^{\infty}\frac{1}{\sqrt{(4n-3)^3}}$.

4.28. $\sum_{n=1}^{\infty}\frac{1}{(10n+3)In^2(10n+3)}$.

4.29. $\sum_{n=1}^{\infty}\frac{2+n}{4+n^2-n}$.

4.30. $\sum_{n=1}^{\infty}\frac{1}{(n+5)In(n+5)In(In(n+5))}$.

5

5.1. $\sum_{n=1}^{\infty}\frac{1}{\sqrt{(n^3+2}}$. (Javob: yaqinlashadi.)

5.2. $\sum_{n=1}^{\infty}\frac{1}{\sqrt[3]{n^5}}$. (Javob: yaqinlashadi.)

5.3. .$\sum_{n=1}^{\infty}\frac{1}{5n+2}$. (Javob: uzoqlashadi.)

5.4. $\sum_{n=1}^{\infty}\frac{1}{\sqrt{(n^3+3n}}$. (Javob: uzoqlashadi.)

5.5. $\sum_{n=1}^{\infty}\frac{1}{\sqrt{(n^2+n}}$. (Javob: uzoqlashadi.)

5.6. $\sum_{n=1}^{\infty}\frac{1}{In(n+2)}$. (Javob: uzoqlashadi.)

5.7. $\sum_{n=1}^{\infty}\frac{1}{\sqrt[3]{n}}$. (Javob: uzoqlashadi.)

5.8. $\sum_{n=1}^{\infty}\frac{1}{3n-1}$. (Javob: uzoqlashadi.)

5.9. $\sum_{n=1}^{\infty}tg\,\frac{n}{3^n}$. (Javob: yaqinlashadi.)

5.10. $\sum_{n=1}^{\infty}\frac{n+3}{n(n-1)}$. (Javob: uzoqlashadi.)

5.11. $\sum_{n=1}^{\infty}\frac{3n-1}{n^2+1}$. (Javob: uzoqlashadi.)

5.12. $\sum_{n=1}^{\infty}\frac{1}{In(n+3)}$. (Javob: uzoqlashadi.)

5.13. $\sum_{n=1}^{\infty}\frac{2n-1}{3n^2+5}$. (Javob: uzoqlashadi.)

5.14. $\sum_{n=1}^{\infty}\frac{1}{3n^2-n+1}$. (Javob: yaqinlashadi.)

5.15. $\sum_{n=1}^{\infty}sin\,\frac{n}{2^{n-1}}$. (Javob:yaqinlashadi.)

5.16. $\sum_{n=1}^{\infty}\frac{n+2}{n(n+4)}$. (Javob: uzoqlashadi.)

5.17. $\sum_{n=1}^{\infty}sin\,\frac{2n}{3^n}$. (Javob: yaqinlashadi.)

5.18. $\sum_{n=1}^{\infty}\frac{1}{(n+1)(n+3)}$. (Javob:yaqinlashadi.)

5.19. $\sum_{n=1}^{\infty}\frac{1}{n\cdot 3^{2n}}$. (Javob: yaqinlashadi.)

5.20. $\sum_{n=1}^{\infty}\frac{1}{(2n+1)\cdot 3^n}$. (Javob: yaqinlashadi.)

5.21. $\sum_{n=1}^{\infty}\frac{n+2}{n\sqrt[3]{n}}$. (Javob: uzoqlashadi.)

5.22. $\sum_{n=1}^{\infty}sin\,\frac{n}{2n-1}$. (Javob: uzoqlashadi.)

5.23. $\sum_{n=1}^{\infty}\frac{n^2}{n^3+2}$. (Javob: uzoqlashadi.)

5.24. $\sum_{n=1}^{\infty}sin\,\frac{\pi}{4\pi}$. (Javob: uzoqlashadi.)

5.25. $\sum_{n=1}^{\infty}\frac{n}{n^3+1}$. (Javob: yaqinlashadi.)

5.26. $\sum_{n=1}^{\infty}\frac{1}{2n^2+5}$. (Javob: yaqinlashadi.)

5.27. $\sum_{n=1}^{\infty}\frac{1}{n^2+4}$. (Javob: yaqinlashadi.)

5.28. $\sum_{n=1}^{\infty}\frac{2n+1}{n^2+4}$. (Javob: uzoqlashadi.)

5.29. $\sum_{n=1}^{\infty}\frac{1}{5n^2+3}$. (Javob: yaqinlashadi.)

5.30. $\sum_{n=1}^{\infty}\frac{1}{(n+1)(n+6)}$. (Javob:yaqinlashadi.)

6

6.1. $\sum_{n=1}^{\infty}\frac{n}{(n+1)^3}$. **6.2**. $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n(n-1)}}$.

6.3. $\sum_{n=1}^{\infty}\frac{2n-1}{2n^2+1}$. **6.4.** $\sum_{n=1}^{\infty}\frac{n\,(n+1)}{3^n}$.

6.5. $\sum_{n=1}^{\infty}\frac{2^n}{1+2^{2n}}$. **6.6**. $\sum_{n=2}^{\infty}\frac{1}{nLn^7n}$.

6.7. $\sum_{n=1}^{\infty}\frac{n^3}{(n+1)!}$. **6.8**. $\sum_{n=1}^{\infty}\frac{2}{1^2+3}$.

6.9. $\sum_{n=1}^{\infty}\frac{n\,!}{7^2}$. **6.10**. $\sum_{n=1}^{\infty}\frac{1}{(5n-1)(6n+3)}$.

6.11. . $\sum_{n=1}^{\infty}\frac{1}{\sqrt{3n+1}}$. **6.12**. $\sum_{n=1}^{\infty}\frac{1}{5^n}\left(\frac{n}{n+3}\right)^{n^2}$

6.13. $\sum_{n=1}^{\infty}\frac{1}{3^n+n}$. **6.14**. $\sum_{n=1}^{\infty}\frac{n+2}{n^2}$.

6.15. $\sum_{n=1}^{\infty}\frac{2n!}{3^n}$. **6.16**. $\sum_{n=1}^{\infty}\frac{5^n}{n^5}$.

6.17. $\sum_{n=1}^{\infty}\frac{1}{n\sqrt{n+1}}$. **6.18**. $\sum_{n=1}^{\infty}\frac{2n-1}{n!}$.

6.19. $\sum_{n=1}^{\infty}\frac{n+1}{2n+5}$. **6.20**. $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n(n+3)}}$.

6.21. $\sum_{n=1}^{\infty}\frac{1}{n^3+1}$. **6.22**. $\sum_{n=1}^{\infty}\frac{(n+1)!}{(2n)!}$.

6.23. $\sum_{n=1}^{\infty}\frac{1}{(3n-2)(7n-1)}$. **6.24**. $\sum_{n=1}^{\infty}\frac{1}{2^n}\left(\frac{n+1}{n}\right)^{n^2}$

6.27. $\sum_{n=1}^{\infty}\frac{n-7}{3n^4+5n-2}$. **6.28**. $\sum_{n=1}^{\infty}\frac{1}{(4n-1)(4n+5)}$.

6.29. $\sum_{n=1}^{\infty}\left(\frac{n}{n+7}\right)^{n^2}$. **6.30**. $\sum_{n=1}^{\infty}\frac{6^n}{(n-1)!}$.

Ishora almashinuvchi qatorlarni yaqinlashishga va mutloq yaqinlashishga tekshiring.

7

7.1. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{(n+1)\cdot 3^n}$. (Javob: mutloq yaqinlashadi.)

7.2. $\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{2n+1}}$. (Javob:shartli yaqinlashadi.)

7.3. $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{\ln n}$. (Javob: shartli yaqinlashadi.)

7.4. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n}{6n+5}$. (Javob: uzoqlashadi.)

7.5. $\sum_{n=1}^{\infty}(-1)^n \frac{1}{\sqrt[4]{n^5}}$. (Javob: uzoqlashadi.)

7.6. $\sum_{n=1}^{\infty}(-1)^{n+1} \frac{1}{\sqrt{n}}$. (Javob: shartli yaqinlashadi.)

7.7. $\sum_{n=1}^{\infty}(-1)^{n-1} \frac{1}{n^2}$. (Javob: mutloq yaqinlashadi.)

7.8. $\sum_{n=1}^{\infty}(-1)^{n+1} \frac{1}{(2n+1)n}$. (Javob: mutloq yaqinlashadi.)

7.9. $\sum_{n=1}^{\infty}(-1)^{n+1} \frac{1}{\sqrt{n+1}}$. (Javob: shartli yaqinlashadi.)

7.10. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n\sqrt[3]{n}}$. (Javob: mutloq yaqinlashadi.)

7.11. $\sum_{n=1}^{\infty}(-1)^{n+1} \frac{2n+1}{n(n+1)}$. (Javob: shartli yaqinlashadi.)

7.12. $\sum_{n=1}^{\infty}(-1)^n \frac{n+5}{3^n}$. (Javob: mutloq yaqinlashadi.)

7.13. $\sum_{n=1}^{\infty}(-1)^{n+1} \frac{n}{3n-1}$. (Javob: uzoqlashadi.)

7.14. $\sum_{n=1}^{\infty} \frac{(-1)^n}{2n-1}$. (Javob: shartli yaqinlashadi.)

7.15. $\sum_{n=1}^{\infty} \frac{(-1)^n}{(2n-1)3^n}$. (Javob: mutloq yaqinlashadi.)

7.16. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n}$. (Javob: shartli yaqinlashadi.)

7.17. $\sum_{n=1}^{\infty}(-1)^{n+1} \frac{2n+1}{n}$. (Javob: uzoqlashadi.)

7.18. $\sum_{n=1}^{\infty} \frac{(-1)^n}{3n^2+1}$. (Javob: mutloq yaqinlashadi.)

7.19. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n\sqrt{n}}$. (Javob: mutloq yaqinlashadi.)

7.20. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n \cdot 5^n}$. (Javob: mutloq yaqinlashadi.)

7.21. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n!}$. (Javob: mutloq yaqinlashadi.)

7.22. $\sum_{n=1}^{\infty}(-1)^n \frac{3}{ln(n+1)}$. (Javob: shartli yaqinlashadi.)

7.23. $\sum_{n=1}^{\infty}(-1)^{n+1} \frac{2n+1}{5n(n+1)}$. (Javob: shartli yaqinlashadi.)

7.24. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n+1}$. (Javob: shartli yaqinlashadi.)

7.25. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} \cdot 3^n}{(2n+1)^n}$. (Javob: mutloq yaqinlashadi.)

7.26. $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{\sqrt{n+5}}$. (Javob: shartli yaqinlashadi.)

7.27. $\sum_{n=1}^{\infty}(-1)^{n}\frac{n+5}{3^{n}}$. (Javob: mutloq yaqinlashadi.)

7.28. $\sum_{n=1}^{\infty}(-1)^{n+1}\left(\frac{1}{2n+7}\right)^{n}$. (Javob: mutloq yaqinlashadi.)

7.29. $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{(3n-2)!}$. (Javob: mutloq yaqinlashadi.)

7.30. $\sum_{n=1}^{\infty}(-1)^{n}nln\left(1+\frac{1}{n^2}\right)$. (Javob: shartli yaqinlashadi.)

8.

8.1. $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{(2n-1)^3}$.

8.2. $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{(2n+1)!}$.

8.3. $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2+1}$.

8.4. $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{ln(n+1)}$.

8.5. $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n\cdot 2^n}$.

8.6. $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}\cdot 2^n}{n^4}$.

8.7. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{2n-1}{3^n}$.

8.8. $\sum_{n=1}^{\infty}(-1)^{n}\frac{n^2+1}{n^3}$.

8.9. $\sum_{n=1}^{\infty}\frac{(-1)^{n}}{n^3+1}$.

8.10. $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{\left(ln(n+1)\right)^n}$.

8.11. $\sum_{n=1}^{\infty}\frac{(-1)^{n}}{n(\ln n)^2}$.

8.12.$\sum_{n=1}^{\infty}(-1)^{n}\left(\frac{n}{2n+1}\right)^{n}$.

8.13. $\sum_{n=2}^{\infty}\frac{(-1)^{n+1}}{n\ln n}$.

8.14. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n}{(n+1)!}$.

8.15. $\sum_{n=1}^{\infty}(-1)^{n}\frac{n}{12^n}$.

8.16. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{(n+1)^{3/2}}$.

8.17. $\sum_{n=1}^{\infty}(-1)^{n}\frac{n}{9n-1}$.

8.18. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{2n+1}{n(n+1)}$.

8.19. $\sum_{n=1}^{\infty}\frac{(-1)^{n}}{(5n+1)^n}$.

8.20. $\sum_{n=1}^{\infty}(-1)^{n}\frac{\ln n}{7^n}$.

8.21. $\sum_{n=1}^{\infty}(-1)^{n}\frac{n+1}{n^2}$.

8.22. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n^3}{n^2+1}$.

8.23. $\sum_{n=1}^{\infty}(-1)^{n+1}sin\frac{\pi}{8^n}$.

8.24.$\sum_{n=1}^{\infty}(-1)^{n}\frac{3^n}{2n+2}$.

8.25. $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{(n+1)(n+4)}$.

8.26. $\sum_{n=1}^{\infty}(-1)^{n}sin^{n}\frac{\pi}{6n}$.

8.27. $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{2n+1}{n(n+2)}$.

8.28. $\sum_{n=1}^{\infty}(-1)^{n}\frac{n-3}{n^2-1}$.

8.29. $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{\sqrt{n}\sqrt[5]{(n+1)^3}}$.

8.30. $\sum_{n=1}^{\infty}\left(-\frac{4n}{5n+1}\right)^{n}$.

Namunaviy variantning yyechimi

1. $\sum_{n=1}^{\infty}\frac{2n+1}{n^2(n+1)^2}$ qatorning yaqinlashishini isbotlash va yig‘indisini toping.

►Berilgan qatorning $a_n=\frac{2n+1}{n^2(n+1)^2}$ umumiy hadini sodda kasrlarning yig‘indisi ko‘rinishida yozib olamiz.

$$a_n=\frac{2n+1}{n^2(n+1)^2}=\frac{A}{n}+\frac{B}{n^2}+\frac{C}{n+1}+\frac{D}{(n+1)^2},$$

2n + 1=An(n+ 1)^2+B (n+1)^2+S$n^2(n+1)+Dn^2$

$$\left.\begin{array}{c|c} n=0 & B=1 \\ n=-1 & D=-1 \\ n^3 & 0=A+C \\ n & 2=A+2B \end{array}\right\}\Rightarrow A=O, C=O.$$

Shuning uchun, $a_n=\frac{1}{n^2}-\frac{1}{(n+1)^2}$.

Qatorning birinchi *n* ta hadining yig‘indisini topamiz.

$$\mathrm{S_n}=1-\frac{1}{4}+\frac{1}{4}-\frac{1}{9}+\frac{1}{9}-\frac{1}{16}+\cdots+\frac{1}{(\mathrm{n}-1)^2}-\frac{1}{\mathrm{n}^2}+\frac{1}{\mathrm{n}^2}-$$

$$-\frac{1}{(\mathrm{n}+1)^2}=1-\frac{1}{(\mathrm{n}+1)^2}.$$

Qatorning yig‘indisini topamiz.

$$\lim_{n\to\infty}S_n=\lim_{n\to\infty}\left(1-\frac{1}{(n+1)^2}\right)=1.$$

Demak, qator yaqinlashuvchi va uning yig‘indisi S=1.◄

Quyidagi musbat hadi qatorlarni yaqinlashishga tekshiring.

2. $\sum \frac{n!}{n^n}$.

►Dalamber alomatidan foydalanib, quyidagiga ega bo‘lamiz.

$$a_n = \frac{n!}{n^n};\ a_{n+1} = \frac{(n+1)!}{(n+1)^{n+1}},$$

$$\lim_{n\to\infty} \frac{(n+1)!\,n^n}{(n+1)^{n+1}n!} = \lim_{n\to\infty} \frac{(n+1)\cdot n^n}{(n+1)^n(n+1)} = \lim_{n\to\infty} (\frac{n}{n+1})^n =$$

$$= \lim_{n\to\infty} \frac{1}{(1+\frac{1}{n})^n} = \frac{1}{e} < 1,$$

ya’ni berilgan qator yaqinlashuvchi.◄

3. $\sum_{n=3}^{\infty} \frac{(n+1)^{n^2}}{n^{n^2}\cdot 3^n}$.

► Koshining radikal alomatiga asosan quyidagiga ega bo‘lamiz.

$$a_n = \frac{(n+1)^{n^2}}{n^{n^2}\cdot 3^n}, \quad \lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \sqrt[n]{\frac{(n+1)^{n^2}}{n^{n^2}\cdot 3^n}} = \lim_{n\to\infty} \frac{(n+1)^n}{n^n\cdot 3} = \frac{1}{3}\lim_{n\to\infty}(1+\frac{1}{n})^n = \frac{e}{3} < 1$$

ya’ni, berilgan qator yaqinlashuvchi. ◄

4. $\sum_{n=1}^{\infty} \frac{n}{2^{n^2}}$.

►Koshining integral alomatidan foydalanamiz. Buning uchun quyidagi xosmas integralni tekshiramiz^

$$\int_1^{\infty} \frac{xdx}{2^{x^2}} = \lim_{\beta\to\infty} \int_1^{\beta} x\cdot 2^{-x^2}\,dx = \lim_{\beta\to\infty}\left(-\frac{1}{2}\int_1^{\beta} 2^{x^2} d(-x^2))\right) = \lim_{\beta\to\infty}\left(-\frac{1}{2}\cdot\frac{2^{-x^2}}{ln2}\Big|_1^{\beta}\right) =$$

$$= \lim_{\beta\to\infty}\left(-\frac{1}{2\cdot ln2\cdot 2^{\beta^2}} + \frac{1}{4ln2}\right) = \frac{1}{4ln2}.$$

Bu integralning yaqinlashuvchiligidan berilgan qator xam yaqinlashuvchi bo‘ladi.◄

5. $\sum_{n=1}^{\infty} tg^2 \frac{\pi}{4\sqrt{n}}$.

►Berilgan qatorni taqqoslash yo‘li bilan tekshiramiz. Agar $\lim_{n\to\infty} \frac{a_n}{b_n} = k,\ \ k\epsilon R, k\ \neq 0,$ bo‘lsa, bunday qatorlar yoki ikkalasi xam yaqinlashuvchi, yoki ikkalasi xam uzoqlashuvchi bo‘ladi. $a_n = tg^2 \frac{\pi}{\sqrt{n}}$. Berilgan qatorni umumiy hadi $b_n = \frac{1}{n}$ bo‘lgan uzoqlashuvchi garmonik qator bilan taqqoslaymiz. U holda,

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{tg^2 \frac{\pi}{\sqrt[4]{n}}}{(\frac{\pi^2}{16n}) \cdot \frac{16}{\pi^2}} = \frac{\pi^2}{16} = k\ \neq 0$$

(Biz bu yerda birinchi ajoyib limitdan foydalandik.) Shunday qilib, qaralayotgan qator uzoqlashuvchi.◄

6. $\sum_{n=1}^{\infty} \left(1 - Sin\frac{1}{n}\right)$

►Bu qator uchun qator yaqinlashishining zaruriy sharti bajarilmaydi.

($\lim_{n\to\infty} a_n = 0$). Haqiqatan ham,

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \left(1 - sin\frac{1}{n}\right) = 1\ \neq 0,$$

ya’ni, berilgan qator uzoqlashuvchi .◄

7. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n \cdot 7^n}$.

►Leybnits alomatidan foydalansak,

$$a_n = \frac{1}{n \cdot 7^n}, \qquad \lim_{n\to\infty} \frac{1}{n \cdot 7^n} = 0,$$

ya'ni, berilgan qator yaqinlashuvchi. Berilgan qatorning absolyut qiymatlaridan tuzilgan qatorni tekshiramiz.

$$\sum_{n=1}^{\infty} \frac{1}{n \cdot 7^n}. \tag{1}$$

Dalamber alomatidan foydalanamiz.

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{n \cdot 7^n}{(n+1)7^{n+1}} = \frac{1}{7} \lim_{n\to\infty} \frac{n}{n+1} = \frac{1}{7} < 1,$$

ya'ni, (1) qator yaqinlashuvchi. Demak, berilgan qator mutloq yaqinlashuvchi.

8. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{2+(-1)^n}{n} = 2\sum_{n=1}^{\infty} \frac{(-1)^n}{n} + \sum_{n=1}^{\infty} \frac{1}{n}.$

►$\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ qator uchun Leybnits alomati bajariladi, u holda $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ qator shartli yaqinlashuvchidir. $\sum_{n=1}^{\infty} \frac{1}{n}$ qator uzoqlashuvchi bo'lgan garmonik qator. Yaqinlashuvchi va uzoqlashuvchi qatorlarning yig'indisi uzoqlashuvchi qatordir. Demak, qaralayotgan qator uzoqlashuvchi.◄

IUT 12. 2

Qatorning yaqinlashish sohasini toping.

1

1.1. $\sum_{n=1}^{\infty} \frac{2^n x^n}{n^2+1}$. (Javob:$\left[-\frac{1}{2}; \frac{1}{2}\right]$.)

1.2. $\sum_{n=1}^{\infty} \frac{n x^{n-1}}{2^{n-1} \cdot 3^n}$. (Javob: -6; 6).)

1.3. $\sum_{n=1}^{\infty} \frac{x^{3n}}{8^n}$. (Javob: -2;2).)

1.4. $\sum_{n=1}^{\infty}\frac{x^2}{n\cdot 2^n}$. (Javob: -2; 2).)

1.5. $\sum_{n=1}^{\infty}\frac{x^n}{n}$. (Javob: -1; 1).)

1.6. $\sum_{n=1}^{\infty}\frac{x^{3n}}{8^n}$. (Javob: -2; 2).)

1.7. $\sum_{n=1}^{\infty}\frac{2^n x^n}{2n-1}$. (Javob:$\left[-\frac{1}{2};\frac{1}{2}\right].$)

1.8. $\sum_{n=1}^{\infty}(lnx)^n$. (Javob: $(\frac{1}{e};e).$)

1.9. . $\sum_{n=1}^{\infty}\frac{x^n}{n(n+1)}$. (Javob: -1; 1).)

1.10. .$\sum_{n=1}^{\infty}\frac{x^{3n}}{8^n(n^2+1)}$. (Javob: -2; 2).)

1.11. .$\sum_{n=1}^{\infty}n(n+1)x^n$. (Javob: -1; 1).)

1.12. $\sum_{n=1}^{\infty}x^n tg\frac{x}{2^n}$. (Javob: -2; 2).)

1.13. $\sum_{n=1}^{\infty}\frac{10^3 x^n}{\sqrt{n}}$. (Javob: $\left[-\frac{10}{10};\frac{1}{10}\right].$)

1.14 $\sum_{n=1}^{\infty}\frac{n!x^n}{n^n}$. (Javob: (-e; e).)

1.15. $\sum_{n=1}^{\infty}\frac{x^{n+1}}{5^{n+1}n}$. (Javob: -5; 5).)

1.16. $\sum_{n=1}^{\infty}\frac{x^2}{n^2}$. (Javob: -1; 1).)

1.17. $\sum_{n=1}^{\infty}\frac{(0,1)^n x^{2n}}{n}$. (Javob: -$\sqrt{10}$; $\sqrt{10}$).)

1.18. $\sum_{n=1}^{\infty}(lgx)^n$. (Javob: $\frac{1}{10}$; 10).)

1.19. $\sum_{n=1}^{\infty}\frac{x^n}{5^n}$. (Javob: -5; 5).)

1.20. $\sum_{n=1}^{\infty}\frac{5^n x^n}{(2n+1)^2\sqrt{3^n}}$. (Javob: $\left[-\frac{\sqrt{3}}{5};\ \frac{\sqrt{3}}{5}\right]$)

1.21. $\sum_{n=1}^{\infty}\frac{x^n}{\sqrt{n}}$. (Javob: $[-1;1].$)

1.22. $\sum_{n=1}^{\infty}\frac{2^n x^n}{\sqrt{n}}$. (Javob: $\left[-\frac{1}{2};\frac{1}{2}\right].$)

1.23. $\sum_{n=1}^{\infty}\frac{(-x)^{n+1}}{n^3}$. (Javob: $[-1;1].$)

1.24. $\sum_{n=1}^{\infty}\frac{3^n x^n}{\sqrt[3]{n}}$. (Javob: $\left[-\frac{1}{3};\frac{1}{3}\right].$)

1.25. $\sum_{n=1}^{\infty} \frac{x^n}{2^n\sqrt{3n-1}}$. (Javob: $[-2; 2].$)

1.26. $\sum_{n=1}^{\infty} \frac{2^n x^n}{\sqrt{2n}-1}$. (Javob: $\left[-\frac{1}{2}; \frac{1}{2}\right].$)

1.27. $\sum_{n=1}^{\infty} \frac{(n+1)^2 x^n}{2^n}$. (Javob: -2; 2).

1.28. $\sum_{n=1}^{\infty} \frac{5^n x^n}{6^n \sqrt[3]{n}}$. (Javob: $\left[-\frac{6}{5}; \frac{6}{5}\right].$)

1.29. $\sum_{n=1}^{\infty} x^n tg\frac{1}{n}$. (Javob: -1; 1).)

1.30. $\sum_{n=1}^{\infty} (\frac{n}{n+1})^{n^2} \frac{x^n}{5^n}$. (Javob: $(-5e; 5e).$)

2

2.1. $\sum_{n=1}^{\infty} \frac{\sqrt{n}x^n}{n!}$.

2.2 $\sum_{n=1}^{\infty} \frac{n^{n/2}x^n}{(n+1)!}$.

2.3. $\sum_{n=1}^{\infty} \frac{ln^n x}{n^n}$.

2.4. $\sum_{n=1}^{\infty} (nx)^n$.

2.5. $\sum_{n=1}^{\infty} \frac{(x-3)^n}{n!}$.

2.6. $\sum_{n=1}^{\infty} \frac{(x-1)^n}{(n+1)!}$.

2.7. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^{2n-1}}{(2n-1)(2n-1)!}$.

2.8. $\sum_{n=1}^{\infty} sin\frac{x}{2^n}$.

2.9. $\sum_{n=1}^{\infty} e^{-n^2 x}$.

2.10. $\sum_{n=1}^{\infty} tg\frac{x}{2^n}$.

2.11. $\sum_{n=1}^{\infty} \frac{x}{n!}$.

2.12. $\sum_{n=1}^{\infty} \frac{n^3}{x^n}$.

2.13. $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n}x^n}$.

2.14. $\sum_{n=1}^{\infty} \frac{1}{n(x-2)^n}$.

2.15. $\sum_{n=1}^{\infty} \frac{(-1)^n}{x^n nInn}$.

2.16. $\sum_{n=1}^{\infty} sin\frac{x}{2^n}$.

2.17. $\sum_{n=1}^{\infty} \frac{x^n}{2^{n\sqrt{2n+1}}}$.

2.18. $\sum_{n=1}^{\infty} \frac{1}{(nx)^n}$.

2.19. $\sum_{n=1}^{\infty} \frac{1}{n^x}$.

2.20. $\sum_{n=1}^{\infty} \frac{\sin(2n-1)x}{(2n-1)^2}$.

2.21. $\sum_{n=1}^{\infty} 2^n sin\frac{x}{3^n}$.

2.22. $\sum_{n=1}^{\infty} \frac{n!}{x^n}$.

2.23. $\sum_{n=1}^{\infty} \frac{1}{n!x^n}$.

2.24. $\sum_{n=1}^{\infty} n!\, x^n$.

2.25. $\sum_{n=1}^{\infty} \frac{x^n}{n^n}$.

2.26. $\sum_{n=1}^{\infty} \frac{\sin nx}{n^2}$.

2.27. $\sum_{n=1}^{\infty} e^{-n^2}x$.

2.28. $\sum_{n=1}^{\infty} \frac{nx}{e^{nx}}$.

2.29. $\sum_{n=1}^{\infty}\frac{1}{x^n}$.

2.30. $\sum_{n=1}^{\infty}\frac{cosnx}{n^2}$.

3

3.1. $\sum_{n=1}^{\infty}\frac{(x-4)^{2n-1}}{2n-1}$. (Javob: 3<x< 5.)

3.2 $\sum_{n=1}^{\infty}\frac{(x-2)^n}{n^n In(1+1/n)}$. (Javob: 1<x< 3.)

3.3. $\sum_{n=1}^{\infty}\frac{(x-4)^{2n-1}}{2n-1}$. (Javob: 0<x< 4.)

3.4 $\sum_{n=1}^{\infty}\frac{(x-1)^n}{n^2}$. (Javob: 0<x< 2.)

3.5. $\sum_{n=1}^{\infty}\frac{(x-8)^n}{n^2}$. (Javob: -9≤x≤ $x-7$.)

3.6.$\sum_{n=1}^{\infty}(2+x)^n$. (Javob: -3<x< -1.)

3.7. $\sum_{n=1}^{\infty}\frac{(x-1)^n}{2^n(n+3)}$. (Javob: -1≤x< 3.)

3.8. $\sum_{n=1}^{\infty}\frac{(x+5)}{\sqrt[3]{n+1\sqrt{n^2}+1}}$. (Javob: -6≤x≤ $x-4$.)

3.9. $\sum_{n=0}^{\infty}2^{n^2}(2+x)^{n^2}$. (Javob: -2,5<x< $-1{,}5$.)

3.10. $\sum_{n=1}^{\infty}\frac{(x-1)^n}{2^n In(n+1)}$. (Javob: -1≤x< 3.)

3.11. $\sum_{n=1}^{\infty}\frac{n!(x+10)^n}{n^n}$. (Javob: -e-10<x< $e-10$.)

3.12. $\sum_{n=1}^{\infty}\frac{(x+5)^{n^2}}{(n+1)^n}$. (Javob: -6≤x≤ -4.)

3.13. $\sum_{n=1}^{\infty}\frac{\sqrt{In^3}(n+1)}{(n+1)}x-1)^n$. (Javob: 0≤x< 2.)

3.14. $\sum_{n=0}^{\infty}(2-x)^n sin\frac{\pi}{2^n}$. (Javob: 0<x< 4.)

3.15. $\sum_{n=1}^{\infty}\frac{(3-2x)^n}{n-In^2n}$. (Javob: 1<x≤ 2.)

3.16. $\sum_{n=1}^{\infty}\frac{(3n-2)(x-3)^n}{(n+1)^2 2^{n+1}}$. (Javob: 1≤x< 5.)

3.17. $\sum_{n=1}^{\infty}\frac{(3-2x)^n}{n-In^2n}$. (Javob: 1<x≤ 2.)

3.18. $\sum_{n=1}^{\infty}\frac{(x-2)^n}{(2n-1)\cdot 2^n}$. (Javob: 0≤x< 3.)

3.19. $\sum_{n=1}^{\infty}(-1)^n\frac{\sqrt[3]{n}+2}{n+1}(x-2)^n$.(Javob: 1<x≤ 3.)

3.20. $\sum_{n=1}^{\infty}\frac{(x+5)^{n-1}}{2n\cdot 4^n}$. (Javob: -7<x< -3.)

3.21. $\sum_{n=1}^{\infty}\frac{(2n-1)^n(x+1)^n}{2^{n-1}n^2}$. (Javob: -2<x< 0.)

3.22. $\sum_{n=1}^{\infty}\frac{(x+3)^n}{n^2}$. (Javob: -4≤x≤ -2.)

3.23. $\sum_{n=1}^{\infty}\frac{(x+2)^{n^2}}{n^n}$. (Javob: -3≤x≤ -1.)

3.24. $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{(x-2)^{2n}}{2n}$. (Javob: 1≤x≤ 3.)

3.25. $\sum_{n=1}^{\infty}\frac{(x-1)^{2n}}{n\cdot 9^n}$. (Javob: 2<x< 4.)

3.26. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(x-2)^n}{(n+1)In(n+1)!2n}$. (Javob: 1<x≤ 3.)

3.27. $\sum_{n=1}^{\infty}\frac{(x-2)^n}{n\cdot 5^n}$. (Javob: -2≤x< 8.)

3.28. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(2n-1)^{2n}(x-1)^n}{(3n-2)^{2n}}$. (Javob: -$\frac{5}{4}$ <x< $3<\frac{13}{4}$.)

3.29. $\sum_{n=1}^{\infty}\frac{(x-3)^{2n}}{(n+1)In(n+1)}$. (Javob: 2<x< 4.)

3.30. $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{(x-5)^n}{n\cdot 3^n}$. (Javob: 2<x≤ 8.)

4

f(x) funksiyani Makloren qatoriga yoying. Hosil qilingan qatorning bu funksiyaga yaqinlashish oralig‘ini ko‘rsating.

4.1. $f(x)=cos5x$ (Javob: $\sum_{n=1}^{\infty}\frac{(-1)^n\cdot 5^{2n}x^{2n}}{(2n)!}, |x\,|<\infty.$)

4.2. $f(x)=x^3arctgx.$ (Javob: $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}\cdot x^{2n+2}}{2n-1}, |x\,|<1.$)

4.3. $f(x)=sinx^2$ (Javob: $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}\cdot x^{4n-2}}{(2n-1)!}, |x|<\infty.$)

4.4. $f(x)=\frac{x^2}{1+x}$ (Javob: $\sum_{n=1}^{\infty}(-1)^n x^{n+2},\ |x|<1.$)

4.5. $f(x)=cos\frac{x^3}{3}$ (Javob: $\sum_{n=1}^{\infty}\frac{(-1)^n\cdot 2^{2n}x^{6n}}{3^{2n}(2n)!},\ |x|<\infty.$)

4.6. $f(x)=\frac{2}{1-3x^2}$ (Javob: $2\sum_{n=1}^{\infty}3^n x^{2n},\ |x|<\frac{1}{\sqrt{3}}.$)

4.7. $f(x)=e^{3x}$ (Javob: $\sum_{n=1}^{\infty}\frac{3^n x^n}{n!},\ |x|<\infty.$)

4.8. $f(x) = \frac{1}{1-x}$ (Javob: $\sum_{n=1}^{\infty} -1)^n x^n, |x| < 1.$)

4.9. $f(x) = ch(2x^2)$ (Javob: $\sum_{n=1}^{\infty} \frac{2^n x^{6n}}{n!}, |x| < \infty.$)

4.10. $f(x) = \frac{1}{\sqrt{e^x}}$ (Javob: $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{2^n n!}, |x| < \infty.$)

4.11. $f(x) = shx$ (Javob: $\sum_{n=1}^{\infty} \frac{x^{2n-1}}{(2n-1)!}, |x| < \infty.$)

4.12. $f(x) = e^{-x^4}$ (Javob: $\sum_{n=0}^{\infty} \frac{(-1)^n x^{4n}}{n!}, |x| < \infty.$)

4.13. $f(x) = 2^{-x^2}$ (Javob: $\sum_{n=0}^{\infty} \frac{(-1)^n ln^n \cdot 2}{n!} x^{2n}, |x| < \infty.$)

4.14. $f(x) = 5^x$ (Javob: $\sum_{n=0}^{\infty} \frac{x^n ln^n \cdot 5}{n!}, |x| < \infty.$)

4.15. $f(x) = x \cos \sqrt{x}$ (Javob: $\sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{(2n)!}, 0 \leq x < \infty.$)

4.16. $f(x) = \frac{\sin 3x}{x}$ (Javob: $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} \cdot 3^{2n-1}}{(2n-1)!} x^{2n-2}, |x| < \infty.$)

f(x) funksiyani ko‘rsatilgan nuqta atrofida Teylor qatoriga yoying. Hosil qilingan qatorning bu funksiyaga yaqinlashish oralig‘ini ko‘rsating.

4.17. $f(x) = \frac{1}{x}, x_0 = -2.$ (Javob:$-\frac{1}{2}\sum_{n=0}^{\infty} \frac{(x+2)^n}{2^n}, -4 < x < 0.$)

4.18. $f(x) = \frac{1}{x+3}, x_0 = -2.$ (Javob:$\sum_{n=0}^{\infty}(-1)^n (x+2)^n, -3 < x < -1.$)

4.19. $f(x) = e^x, x_0 = 1.$ (Javob: $\sum_{n=0}^{\infty} \frac{(x-1)^n}{n!}, \ |x| < \infty.$)

4.20. $f(x) = \frac{1}{2x+5}, x_0 = 3.$

(Javob:$\frac{1}{11}\sum_{n=0}^{\infty}(-1)^n \left(\frac{2}{11}\right)^n (x - 3)^n, -\frac{5}{2} < x < \frac{17}{2}.$)

4.21. $f(x) = \frac{1}{(x-3)^2}, x_0 = 1.$ (Javob: $\frac{1}{4}\sum_{n=0}^{\infty} \frac{n+1}{2^n}(x-1)^n, \ -1 < x < 3.$)

4.22. $f(x) = sin\frac{\pi x}{4}, x_0 = 2.$ (Javob: $\sum_{n=0}^{\infty}(-1)^n \left(\frac{\pi}{4}\right)^{2n} \frac{(x-2)^{2n}}{(2n)!}, \ |x| < \infty.$)

4.23. $f(x) = \ln(5x+3), x_0 = \frac{2}{5}.$

(Javob: $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} \cdot 5^n}{n} \left(x + \frac{2}{5}\right)^n, \ -\frac{7}{5} < x \leq \frac{3}{5}.$)

4.24. $f(x) = ln\frac{1}{x^2-2x+2}, x_0 = 1.$ (Javob:$\sum_{n=1}^{\infty}\frac{(-1)^n}{n}(x-1)^{2n}, 0 \le x \le 2.)$

4.25. $f(x) = \frac{1}{\sqrt{4+x}},\ x_0 = -3.$

(Javob: $1 + \sum_{n=0}^{\infty}\frac{(-1)^n(2n-1)}{2^n n!}(x+3)^n,\ -4 < x \le -2.)$

4.26. $f(x) = cosx, x_0 = \frac{\pi}{4}.$ (Javob: $\sum_{n=0}^{\infty}\frac{cos\left(\frac{\pi}{4}+n\cdot\frac{\pi}{2}\right)}{n!}\left(x-\frac{\pi}{4}\right)^n,\ |x| < \infty.)$

4.27. $f(x) = \frac{1}{\sqrt{x-1}},\ x_0 = 2.$

(Javob: $1 + \sum_{n=1}^{\infty}\frac{(-1)^n(2n-1)!!}{2^n n!}(x-2)^n,\ 1 < x \le 3.)$

4.28. $f(x) = \frac{1}{x^2-4x+3}, x_0 = -2.$

(Javob: $\sum_{n=0}^{\infty}\left(\left(\frac{1}{6\cdot 3^n} - -\frac{1}{10\cdot 5^n}\right)(x+2)^n\right),\ -5 < x < 1.)$

4.29. $f(x) = sinx,\ x_0 = a.$

(Javob: $\sum_{n=0}^{\infty}\frac{sin\left(a+\frac{n\pi}{2}\right)}{n!}(x-a)^n,\ |x| < \infty.)$

4.30. $f(x) = \ln(5x+3),\ x_0 = 1.$

(Javob:$\ln 8 + \sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}\left(\frac{5}{8}\right)^n(x-1)^n,\ -\frac{3}{5} < x \le \frac{13}{5}.)$

5. Tanlab olingan funksiyning unga mos darajali qatorga yoyilymasidan foydalalib berilfan kattalikni α aniqlikda hisoblansin.

5.1. $e, \alpha = 0{,}0001.$ (Javob: 2,7183.)

5.2. $\sqrt[5]{250}, \alpha = 0{,}01.$ (Javob: 3,017.)

5.3. $\sin 1, \alpha = 0{,}00001.$ (Javob: 0,84147.)

5.4. $\sqrt{1{,}3}, \alpha = 0{,}001.$ (Javob: 1,140.)

5.5. $arctg\frac{\pi}{10}, \alpha = 0{,}001.$ (Javob: 0,304.)

5.6. $\ln 3,\ \alpha = 0{,}0001.$ (Javob: 1,0986.)

5.7. $ch\, 2,\ \alpha = 0{,}0001.$ (Javob: 3,7622.)

5.8. $\lg e, \alpha = 0{,}0001.$ (Javob: 0,4343.)

5.9. $\pi,\ \alpha = 0{,}00001.$ (Javob: 3,14159.)

5.10. e^2, $\alpha = 0{,}001$. (Javob: 7,389.)

5.11. $\cos 2°$, $\alpha = 0{,}001$. (Javob: 0,999.)

5.12. $\sqrt[3]{80}$, $\alpha = 0{,}001$. (Javob: 4,309.)

5.13. $\ln 5$, $\alpha = 0{,}001$. (Javob: 1,609.)

5.14. $arctg\frac{1}{2}$, $\alpha = 0{,}001$. (Javob: 0,464.)

5.15. $\sqrt[6]{738}$, $\alpha = 0{,}001$. (Javob: 3,006.)

5.16. $\sqrt[3]{e}$, $\alpha = 0{,}00001$. (Javob: 1,3956.)

5.17. $\sin 1°$, $\alpha = 0{,}0001$. (Javob: 0,0175.)

5.18. $\sqrt[3]{8{,}36}$, $\alpha = 0{,}001$. (Javob: 2,030.)

5.19. $\ln 10$, $\alpha = 0{,}0001$. (Javob: 2,3026.)

5.20. $\arcsin\frac{1}{3}$, $\alpha = 0{,}001$. (Javob: 0,340.)

5.21. $\lg 7$, $\alpha = 0{,}001$. (Javob: 0,8451.)

5.22. $\sqrt{e}$, $\alpha = 0{,}0001$. (Javob: 1,6487.)

5.23. $\cos 10°$, $\alpha = 0{,}0001$. (Javob: 0,9848.)

5.24. $\frac{1}{\sqrt[3]{30}}$, $\alpha = 0{,}001$. (Javob: 0,302.)

5.25. $\sqrt[10]{1080}$, $\alpha = 0{,}001$. (Javob: 2,031.)

5.26. $\frac{1}{e}$, $\alpha = 0{,}0001$. (Javob: 0,3679.)

5.27. $\sin\frac{\pi}{100}$, $\alpha = 0{,}0001$. (Javob: 0,0314.)

5.28. $\sqrt[4]{90}$, $\alpha = 0{,}001$. (Javob: 3,079.)

5.29. $\frac{1}{\sqrt[7]{136}}$, $\alpha = 0{,}001$. (Javob: 0,496.)

5.30. $\frac{1}{\sqrt[3]{e}}$, $\alpha = 0{,}001$. (Javob: 0.716.)

6. Integral ostidagi funksiyaning darajali qatorga yoyilmasidan foydalanib aniq integralni 0,001 aniqlikda hisoblang.

6.1. $\int_0^{0,25} \ln(1+\sqrt{x})\,dx.$ (Javob:0,070)

6.2. $\int_0^1 arctg\left(\frac{x^2}{2}\right)dx.$ (Javob: 0,162.)

6.3 $\int_0^{0.2} \sqrt{x}\, e^{-x}dx.$ (Javob: 0,054)

6.4. $\int_0^{0.5} \frac{arctg}{x}dx.$ (Javob: 0,487)

6.5. $\int_0^{0,2} \sqrt{x}\cos xdx.$ (Javob:0,059)

6.6. $\int_1^{0.5} In(1+x^3)dx$ (Javob: 0,015.)

6.7. $\int_0^1 x^2 \sin xdx.$ (Javob: 0,223.)

6.8. $\int_0^1 e^{-x^2/2}dx.$ (Javob: 0,855.)

6.9. $\int_0^{0.5} \sqrt{1+x^2}dx.$ (Javob:0,480.)

6.10. $\int_0^{0.5} \frac{dx}{1+x^5}.$ (Javob: 0,484.)

6.11. $\int_0^1 \sqrt[3]{1+x^2/4dx.}$ (Javob:1,027)

6.12. $\int_0^{0.5} \frac{\sin x^2}{x}dx.$ (Javob:0,493)

6.13. $\int_{0}^{0.1} \frac{e^2 - 1}{x} dx$ (Javob:0,103.)

6.14. $\int_{0}^{0.5} x^2 \cos\ 3x\, dx.$ (Javob: 0,018.)

6.15. $\int_{0}^{0.5} In(1 + x^2) dx.$ (Javob: 0,385)

6.16. $\int_{0}^{0.4} \sqrt{x}\, e^{-x/4} dx$ (Javob:0,159.)

6.17. $\int_{0.3}^{0.5} \frac{1 + \cos x}{x^2} dx$ (Javob: 2,568.)

6.18. $\int_{0}^{0.5} \frac{arctgx^2}{x^2} dx.$ (Javob: 0,498.)

6.19. $\int_{0}^{0.8} \frac{1 - \cos x}{x} dx.$ (Javob:0,156.)

6.20. $\int_{0}^{1} \sin x^2 dx.$ (Javob: 0,310.)

6.21. $\int_{0}^{0.1} \frac{In\ (1 + x)}{x} dx.$ (Javob:0,098.)

6.22. $\int_{0}^{1} \cos \sqrt[3]{x}\, dx$ (Javob: 0,718.)

6.23. $\int_{0}^{1} \sqrt{x \sin x dx.}$ (Javob: 0,364.)

6.24. $\int_0^{25} \frac{e^{-2x^2}}{\sqrt{x}} dx.$ (Javob: 0.976.)

6.25. $\int_0^{1} \cos\frac{x^2}{4} dx.$ (Javob: 0,994.)

6.26. $\int_0^1 arctg\left(\frac{\sqrt{x}}{2}\right) dx.$ (Javob: 0,318)

6.27. $\int_0^{0,5} \frac{\text{x}-\text{arctg x}}{x^2} dx.$ (Javob:0.039.)

6.28. $\int_0^{0.4} \sqrt{1-x^3}dx.$ (Javob: 0.397.)

6.29. $\int_0^{0.5} e^{-x^2} dx.$ (Javob: 0.461.)

6.30. $\int_0^{0.5} \sqrt{1+x^3}\, dx.$ $\text{Javob: } 0.508.)$

7. Differensial tenglamaning yyechimini *x* ning darajalari bo‘yicha darajali qatorga yoying. (Bu yoyilmaning noldan farqli birinchi uchta hadini yozing.)

7.1. $y' = yx + e^y,\ y(0) = 0.$ (Javob: $y = x + \frac{1}{2}x^2 + \frac{2}{3}x^3 + \dots$)

7.2. $y' = x^2y^2 + 1,\ \ y(0) = 1.$ (Javob: $y = 1 - x + \frac{1}{3}x^3 + \dots$)

7.3. $y' = x^2 - y^2,\ y(0) = \frac{1}{2}.$ (Javob: $y = \frac{1}{2} - \frac{1}{4}x - \frac{1}{8}x^2 + \dots$)

7.4. $y' = x^3 + y^2,\ y(0) = \frac{1}{2}$. (Javob: $y = \frac{1}{2} + \frac{1}{4}x + \frac{1}{8}x^2 + \dots$)

7.5. $y' = x + y^2,\ y(0) = -1.$ (Javob: $y = -1 + x + 3x^2 + \dots$)

7.6. $y' = x + x^2 + y^2,\ y(0) = 1.$ (Javob: $y = 1 + x + \frac{3}{2}x^2 + \dots$)

7.7. $y' = 2\, cosx - xy^2, y(0)$ =1. (Javob:y=1+2x$-\frac{1}{2}x^2+\dots$)

7.8. $y' = e^x - y^2, y\ (0) = 0.$ (Javob: $y = x + \frac{1}{2}x^2 - \frac{1}{6}x^3 + \ldots$)

7.9. $y' = x + y + y^2,\ y(0) = 1.$ (Javob: $y = 1 + 2x + \frac{7}{2}x^2 + \ldots$)

7.10. $y' = x^2 + +y^2,\ y(0) = 1.$ (Javob: $y = 1 + x + x^2 + \cdots$)

7.11. $y' = x^2y^2 + y\sin\ x,\ y(0) = \frac{1}{2}.$ (Javob: $y = \frac{1}{2} + \frac{1}{4}x^2 + \frac{x^3}{12} + \ldots$)

7.12. $y' = 2y^2 + ye^x,\ y(0) = \frac{1}{3}.$ (Javob: $y = \frac{1}{3} + \frac{5}{9}x + \frac{26}{27}x^2 + \ldots$)

7.13. $y' = e^{3x} + 2xy^2,\ y(0) = 1.$ (Javob: $y = 1 + x + \frac{5}{3}x^2 + \ldots$)

7.14. $y' = x + e^y,\ \ y(0) = 0.$ (Javob:y=x+$x^2 + \frac{1}{2}x^3$+…)

7.15. $y' = y\cos x + 2\cos y, y(0) = 0.$ (Javob: $y = 2x + x^2 - x^3$+…)

7.16. $y' = x^2 + 2y^2, y(0) = 0.2.$ (Javob: $y = 0.2 + 0.08x + 0.032x^2 + \ldots$)

7.17. $y' = x^2 + xy + y^2, y(0) = 0.5.$

(Javob: $y = 0.5 + 0.25 + \ 0.375x^2 + \ldots$)

7.18. $y' = e^{\sin x} + x,\ \ y(0) = 0.$ (Javob: $y = x + x^2 + x^3 + \ldots$)

7.19. $y' = xy - y^2,\ y(0) = 0.2.$ (Javob: $y = 0.2 - 0.04x + 0.108x^2 + \ldots$)

7.20. $y^{'} = 2x + y^2 + e^x,\ \ y(0) = 1.$ (Javob: $y = 1 + 2x + x^2 + \ldots$)

7.21.$y' = x\sin x - y^2,\ \ \ y(0) = 1.$ (Javob: $y = 1 - x + x^2 + \ldots$)

7.22. $y' = 2x^2 - xy,\ y(0) = 0.$ (Javob: $y = \frac{4x^3}{3!} - \frac{16x^5}{5!} + \frac{96x^7}{7!} - \ldots$)

7.23. $y' = x - 2y^2,\ y(0) = 0.5.$ (Javob: $y = 0.5 - 0.5x + x^2$+…)

7.24. $y' = xe^x + 2y^2,\ y\ (0) = 0.$ (Javob: $y = \frac{1}{2}x^2 + \frac{1}{3}x^3 + \frac{1}{8}x^4 + \ldots$)

7.25. $y' = xy + x^2 + y^2,\ y(0) = 1.$ (Javob: $y = 1 + x + \frac{3}{2}x^2 + \dots$)

7.26. $y' = xy + e^x,\ y(0) = 0.$ (Javob: $y = x + \frac{1}{2}x^2 + \frac{1}{2}x^3 + \dots$)

7.27. $y' = ye^x,\ y(0) = 1.$ (Javob: $y = 1 + x + x^2 + \dots$)

7.28. $y' = 2\sin x + xy,\ y(0) = 0.$

(Javob: $y = x^2 + \frac{1}{6}x^4 + \ + \frac{11}{360}x^6 + \dots$)

7.29. $y' = x^2 + e^y,\ y(0)=0.$ (Javob: $y = x + \frac{1}{2}x^2 + \frac{2}{3}x^3 + \dots$)

7.30. $y' = x^2 + y,\ y(0) = 1.$ (Javob: $y = 1 + x + \frac{x^2}{2!} + \dots$)

8. Ko‘rsatilgan boshlang‘ich shartlarda, ketma-ket differensiallash usulida differensial tenglama yechimining darajali qatorga yoyilmasining birinchi *k* ta hadini toping.

8.1. $y' = \arcsin y + x,\ y(0) = \frac{1}{2}, k = 4.$ (Javob: $y = \frac{1}{2} + \frac{\pi x}{6} +$
$+ \frac{1}{2}\left(1 + \frac{\pi}{\sqrt[3]{x}}\right)x^2 + \frac{1}{6}\left(\frac{2}{\sqrt{3}} + \frac{2\pi}{9} + \frac{\pi^2}{27\sqrt{3}}\right)x^3 + \dots$)

8.2. $y' = xy + \ln(y + x),\ y(1) = 0,\ k = 5.$

(Javob: $y = \frac{(x-1)^2}{2} + \frac{(x-1)^3}{6} + \frac{(x-1)^4}{6} + \dots$)

8.3. $y' = x + y^2$, y (0)=1, *k*=3. (Javob: $y = x + \frac{1}{2!}x^2 + \frac{2}{3!}x^3 + \dots$)

8.4. $y' = x + \frac{1}{y}, y(0) = 1,$ *k*=3. (Javob: $y = 1 + x + \frac{x3}{3} + \frac{x^4}{3} + \dots$)

8.5. $y^{IV} = xy + y'x^2, y\ (0) = y'(0) = y''(0) = 1,\ y'''(0)=1,$ *k*=7.
(Javob: $y = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{4x^6}{6!} + \dots$)

8.6. $y' = 2x - 0{,}1y^2,\ y(0) = 1,\ k = 3.$

(Javob: $y = 1 - 0.1x + 0.01x^2 + \ldots$)

8.7. $y''' = y'' + y'^2 + y^3 + x,\ y(0) = 1,\ y'(0) = 2,\ y''(0) = 0.5, k = 6$. (Javob: $y = 1 + 2x + \frac{x^2}{4} + \frac{11}{12}x^3 + \frac{29}{48}x^4 + \frac{25}{48}x^5 + \ldots$)

8.8. $y' = x^2 - xy,\ y(0) = 0.1, k = 3$.

(Javob: $y = 0.1 - 0.05x^2 + 0.333x^3 + \ldots$)

8.9. $y'' = 2yy'$, y(0)=0, $y'(0)$ =1, k=3. (Javob: $y = x + \frac{2x^3}{3!} + \frac{12x^5}{5!} + \ldots$)

8.10. $y' = 2x + \cos y$, y(0)=0, k=5. (Javob: $y = x^2 - \frac{x^3}{6} - \frac{x^4}{4} + \ldots$)

8.11. $y''' = ye^x - xy'^2,\ y(0) = 1, y'(0) = y''(0) = 1,$

$k = 6$. (Javob: $y = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + 0.x^5 + \ldots$)

8.12. $y' = 3x - y^2,\ y(0) = 2,\ k = 3$. $\Big($Javob: $y = 2 - 4x - \frac{13}{2}x^2 - \ldots\Big)$

8.13. $y''' = xyy', y(0) = y'(0) = 1,\ k = 6$.

(Javob: $y = 1 + x + \frac{x^3}{3!} + \frac{2x^4}{4!} + \frac{3x^5}{5!} + \ldots$)

8.14. $y' = x^2 - 2y,\ y(0) = 1, k = 3$. (Javob: $y = 1 - 2x + 2x^2 + \ldots$)

8.15. $y'' = \frac{y'}{y} - \frac{1}{x},\ y(1) = 1.\ \ y'(1) = 0,\ k = 4$.

(Javob: $y = 1 + \frac{(x-1)^2}{2!} - \frac{(x-1)^4}{4!} + \frac{4(x-1)^5}{5!} + \ldots$)

8.16. $y' = x^2 + 0.2y^2,\ y(0) = 0{,}1\ k = 3$.

(Javob: $y = 0.1 + 0.002x + 0.00004x^2 + \ldots$)

8.17. $y'' = y'^2 + xy,\ y(0) = 4,\ y'(0) = -2, k = 5.$

(Javob: $y = 4 - 2x + 2x^2 - 2x^3 + \frac{19}{6}x^4 + \ldots$)

8.18. $y' = xy + y^2,\ y(0) = 0.1,\ k = 3.$

(Javob: $y = 0.1 + 0.01x + 0.051x^2 + \ldots$)

8.19. $y'' = e^y \sin y',\ \ y(\pi) = 1,\ y'(\pi) = \frac{\pi}{2},\ k = 3.$

(Javob: $y = 1 + \frac{\pi}{2}(x - \pi) + \frac{e}{2}(x - \pi)^2 +,,$)

8.20. $y' = 0.2x + y^2, y(0) = 1, k = 3.$ (Javob: $y = 1 + x + 1.1x^2 + \ldots$)

8.21. $y'' = x^2 + y^2, y(-1) = 2, y'(-1) = 0.5, k = 4.$

(Javob: $y = 2 + \frac{1}{2}(x + 1) + \frac{5}{2}(x + 1)^2 + \frac{15}{16}(x + 1)^4 + \cdots$)

8.22. $y' = x^2 + xy + e^{-x}$, y(0)=0, k=3. (Javob: $y = x - \frac{x^2}{2!} + \frac{5x^3}{3!} + \ldots$)

8.23. $y' = \frac{1-x^2}{y} + 1, y(0) = 1, k = 5.$

(Javob: $y = 1 + 2x - x^2 + \frac{4}{3}x^3 - \frac{17}{9}x^4 + \ldots$)

8.24. $y'' + y = 0. y(0) = 0, y'(0) = 1,\ k = 3.$

(Javob: $y = x - \frac{x^3}{3!} - \frac{x^5}{5!} + \ldots$)

8.25. $y'' = ycosy' + x,\ y(0) = 1,\ y'(0) = \frac{\pi}{3}, k = 3.$

(Javob: $y = 1 + \frac{\pi}{3}x + \frac{1}{4}x^2 + \ldots$)

8.26. $y' = \cos x + x^2, y(0) = 0, k = 3.$ (Javob: $y = x + \frac{x^3}{3!} + \frac{x^5}{5!} \ldots$)

8.27. $y' - 4y + 2xy^2 - e^{3x}, y(0) = 2, k = 4.$

(Javob: $y = 2 + 9x + \frac{31}{2}x^2 - \frac{11}{6}x^3 + \ldots$)

8.28. $(1-x)y'' + y = 0,\ \ y(0) = y'(0) = 1,\ \ k = 3.$

(Javob: $y = 1 + x - \frac{x^2}{2} + \ldots$)

8.29. $4x^2y'' + y = 0,\ \ y(1) = 1,\ \ y'(1) = \frac{1}{2},\ k = 3.$

(Javob: $y = 1 + \frac{1}{2}(x-1) - \frac{1}{8}(x-1)^2 + \ldots$)

8.30. $y' = 2x^2 + y^3,\ \ y(1) = 1,\ k = 3.$

(Javob: $y = 1 + 3(x-1) + \frac{13}{2}(x-1)^2 + \ldots$)

Namunaviy variantning yechimi

Qatorning yaqinlashish sohasini toping:

1. $\sum_{n=1}^{\infty}\sqrt{\frac{x^n}{n^2+1}}.$

►Dalamber alomatidan foydalanamiz.

$$u_n = \sqrt{\frac{x^n}{n^2+1}};\quad u_{n+1} = \sqrt{\frac{x^{n+1}}{(n+1)^2+1}},$$

$$\lim_{n\to\infty}\left|\frac{u_{n+1}}{u_n}\right| = \lim_{n\to\infty}\left|\frac{\sqrt{x^{n+1}}\cdot\sqrt{n^2+1}}{\sqrt{(n+1)^2+1}\,\sqrt{x^n}}\right| =$$

$$= \sqrt{x}\lim_{n\to\infty}\sqrt{\frac{n^2+1}{n^2+2n+2}} = \sqrt{x}.$$

Yaqinlashish sohasi $\sqrt{x} < 1$ tengsizlik bilan aniqlanadi, bundan $0 < x < 1$. Bu oraliqning chegaraviy nuqtalarni tekshiramiz. $x = 0$ da hadlari nollardan iborat sonli qator hosil qilamiz. Bu qator yaqinlashuvchidir, $x = 0$ nuqta uning yaqinlashish sohasiga kiradi. $x=1$ da $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+1}}$ sonli qatorni hosil qilamiz. Musbat hadli qatorlarni taqqoslash alomatidan foydalanib, bu qatorni umumiy hadi $v_n = 1/n$ bo‘lgan, uzoqlashuvchi garmonik qator bilan solishtiramiz:

$$\lim_{n\to\infty} \frac{u_n}{v_n} = \lim_{n\to\infty} \frac{n}{\sqrt{n^2}+1} = 1 = k \neq O.$$

Demak, $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+1}}$ sonli qator uzoqlashuvchi va $x = 1$ nuqta yaqinlashish sohasiga kirmaydi. Shunday qilib, qarayotgan qatorning yaqinlashish sohasi o$\leq x \leq 1$.◄

2. $\sum_{n=1}^{\infty} \frac{n^2+1}{n^2} \left(\frac{x^2-3x+2}{x^2+3x+2}\right)^n$

►Dalamber alomatiga asosan quyidagiga ega bo‘lamiz:

$$\lim_{n\to\infty} \left|\frac{u_{n+1}}{u_n}\right| = \lim_{n\to\infty} \frac{\frac{n^2+2n+2}{n^2+2n+1}}{\frac{n^2+1}{n^2}} \cdot \frac{\left|\frac{x^2-3x+2}{x^2+3x+2}\right|^{n+1}}{\left|\frac{x^2-3x+2}{x^2+3x+2}\right|^{n}} =$$

$$= \left|\frac{x^2-3x+2}{x^2+3x+2}\right| \cdot \lim_{n\to\infty} \frac{n^2(n^2+2n+2)}{(n^2+1)(n^2+2n+1)} =$$

$$= \left|\frac{x^2-3x+2}{x^2+3x+2}\right| < 1,$$

$$-1 < \frac{x^2-3x+2}{x^2+3x+2} < 1.$$

Hosil qilingan tengsizlikni yechamiz:

$$-1 < \frac{x^2-3x+2}{x^2+3x+2}, \qquad \frac{x^2-3x+2}{x^2+3x+2} + 1 > 0, \qquad \frac{2x^2+4}{x^2+3x+2} > O.$$

Bundan, $x^2+3x+2>0, \quad x\epsilon(-\infty;-2)\cup(-1;\infty)$

Shu kabi,

$$\frac{x^2-3x+2}{x^2+3x+2}<1, \qquad <\frac{x^2-3x+2}{x^2+3x+2}-1<0, \qquad \frac{-6x}{x^2+3x+2}<0,$$

$$\frac{x}{x^2+3x+2}>O.$$

Bundan, $x\epsilon(-2;-1)\cup(0;\infty)$. $x=0$ da $\sum\frac{n^2+1}{n^2}$ sonli qatorni hosil qilamiz. Bu qator uchun qator yaqinlashishining zaruriy sharti bajarilmaydi, ya'ni. $\lim\limits_{n\to\infty}u_n=\lim\limits_{n\to\infty}\frac{n^2+1}{n^2}=1\neq 0.$

demak, bu qator uzoqlashuvchi. Qaralayotgan qatorning yaqinlashish sohasi: 0$<x<\infty$.◄

3. $\sum_{n=1}^{\infty}(3-x^2)^n.$

► Koshining radikal alomatidan foydalanamiz:

$$u_n=(3-x^2)^n$$

$$\lim_{n\to\infty}\sqrt[n]{(3-x^2)^n}=|3-x^2|<1, \qquad -<3-x^2<1$$

Hosil qilingan tengsizlikni yechamiz:

$$3-x^2>-1, \quad x^2-4<0, \qquad x\,\epsilon(-2;2);$$

$$3-x^2<1, \quad x^2-2>0, \quad x\,\epsilon\left(-\infty;-\sqrt{2}\right)\cup(\sqrt{2}\,;\infty\,)$$

Topilgan yyechimlarning kesishmasi qaralayotgan qatorning yaqinlashish sohasini beradi. $x\epsilon\left(-2;\sqrt{2}\right)\cup(\sqrt{2};2)$. Bu intervallarning chetki nuqtalarida qatorning yaqinlashishini tekshiramiz. $x=\pm\,2$ da $\sum_{n=1}^{\infty}(-1)^n$ sonli qatorni hosil qilamiz. Bu o‘zgaruvchan ishorali qator uzoqlashuvchidir, chunki $\lim\limits_{n\to\infty}u_{\mathrm{n}}=0$ shart bajarilmaydi. $x=\pm\sqrt{2}$ bo‘lganda $\sum_{n=1}^{\infty}1^n$ qatorni hosil qilamiz.

Bu qator ham uzoqlashuvchi, chunki qator yaqinlashishining zaruriy sharti bajirlmaydi. Demak, qaralayotgan qatorning yaqinlashish sohasi: $\left(-2;-\sqrt{2}\right)\cup$ $(\sqrt{2}\,;2)$. ◄

4. y=cos^2x funksiyani $x_o=\pi/3$ nuqta atrofida Teylor qatoriga yoying. Hosil qilingan qatorning bu funksiyaga yaqinlashish oralig‘ini toping.

►Berilgan funksiyani quyidagicha shaklini almashtiramiz.

$$y=\cos^2x=\frac{1}{2}+\frac{1}{2}cos2x.$$

Hosil qilingan funksiyani Teylor qatoriga yoyamiz. Buning uchun funksiyaning va uning n tartibgacha hosilalarining $x_o=\pi/3$ nuqtadagi qiymatlarini topamiz.

$$f(x)=\frac{1}{2}+\frac{1}{2}cos2x,\qquad f(x_o)=f\left(\frac{\pi}{3}\right)=\frac{1}{2}+\frac{1}{2}cos\frac{2\pi}{3}=\frac{1}{2}-\frac{1}{4}=\frac{1}{4};$$

$$f'(x)=-sin2x,\qquad f'\left(\frac{\pi}{3}\right)=-sin\frac{2\pi}{3}=-\frac{\sqrt{3}}{2};$$

$$f''(x)=-2cos2x,\ \ f''\left(\frac{\pi}{3}\right)=-2cos\frac{2\pi}{3}=1$$

$$f'''(x)=4sin2x, f'''\left(\frac{\pi}{3}\right)=4sin\frac{2\pi}{3}=2\sqrt{3};$$

$$f^{(n)}(x)=-2^{n-1}sin\,(\,2x+(n-1)\frac{\pi}{2})\,,$$

$$f^{(n)}(\frac{\pi}{3})=-2^{n-1}\cdot sin\left(\frac{2\pi}{3}+(n-1)\frac{\pi}{2}\right).$$

Topilgan hosilalarining qiymatlarini Teylor qatoriga qo‘yamiz:

$$cos^2x = \frac{1}{4} - \frac{1}{1!}\frac{\sqrt{3}}{2}\left(x - \frac{\pi}{3}\right) + \frac{1}{2!}(x - \frac{\pi}{3})^2 + \frac{1}{3!}2\sqrt{3}\,(x - \frac{\pi}{3})^3 + \cdots$$

$$+\frac{1}{n!}(-2^{n-1}\sin\left(\frac{2\pi}{3} + (n-1)\frac{\pi}{2}\right))(x - \frac{\pi}{3})^n + \ \ldots.. \ =$$

$$= \frac{1}{4} - \sum_{n=1}^{\infty}\frac{2^{n-1}}{n!}\sin(\frac{2\pi}{3} + (n-1)\frac{\pi}{2})(x - \frac{\pi}{3})^n.$$

Hosil qilingan qatorning yaqinlashish sohasini topish uchun Teylor qatorining qoldiq hadi *x* ning qanday qiymatlarida nolga intilishini ko‘rsatishimiz kerak. Uning ko‘rinishi quyidagicha bo‘ladi.

$$R_n(x) = \frac{-2^n}{(n+1)!}sin\left(2\xi + n\cdot\frac{\pi}{2}\right)(x - \frac{\pi}{3})^{n+1},$$

bu erda $\xi \epsilon\ (x, x_o)\cdot|sin(2\xi + n\cdot\frac{\pi}{2})| \leq 1$,bo‘lganligi uchun umumiy hadi $\frac{2^n}{(n+1)!}(x - \frac{\pi}{3})^{n+1}$bo‘lgan qatorning yaqinlashish sohasini topish yetarlidir. Dalamber alomatiga asosan,

$$\lim_{n\to\infty}\left|\frac{2^{n+1}(x - \frac{\pi}{3})^{n+2}\cdot(n+1)!}{(n+2)!\,2^n\cdot\left(x - \frac{\pi}{3}\right)^{n+1}}\right| = \lim_{n\to\infty}\frac{2|x - \frac{\pi}{3}|}{n+2} = 0 < 1.$$

Hosil qilingan qator *x* ning ihtiyoriy qiymatida yaqinlashuvchi. Demak, uning $f(x) = cos^2x$ funksiyaga yaqinlashish sohasi quyidagicha: $\infty < x < \infty$.◄

5. $y = e^x$ funksiyaning darajali qatorga yoyilmasidan foydalanib, 1/$\sqrt{e}$ ni $\alpha = 0{,}0001$ aniqlikda taqribiy hisoblang.

►(12.17) qatordan foydalanamiz. 1/$\sqrt{e} = e^{-\frac{1}{2}}$, u holda

$$e^{-½} = 1 - \frac{1}{2} + \frac{1}{4\cdot 2!} - \frac{1}{8\cdot 3!} + \frac{1}{16\cdot 4!} - \frac{1}{32\cdot 5!} + \cdots$$

ishorasi almashinuvchi qator hosil bo‘ldi. Funksiyaning qiymatini 0,0001 aniqlikda hisoblash uchun tashlab yuborilgan birinchi had 0,0001 dan kichik bo‘lishi kerak. (Leybnits alomatining natijasiga asosan).

$$a_7 = \frac{1}{64 \cdot 6!} = \frac{1}{64 \cdot 720} = \frac{1}{46080} < 0{,}0001.$$

Demak, funksiyaning berilgan aniqlikdagi qiymati:

$$e^{-½} \approx 1 - \frac{1}{2} + \frac{1}{8} - \frac{1}{48} + \frac{1}{384} - \frac{1}{3840},$$

$$\frac{1}{\sqrt{e}} \approx 1 - 0{,}5 + 0{,}125 - 0{,}02083 + 0{,}0026 - 0{,}00026 \approx 0{,}6065.◄$$

6. Aniq integralni integral ostidagi funksiyaning qatorga yoyilmasidan foydalanib, 0,001 aniqlikda hisoblang:

$$\int_{-1}^{0} \frac{dx}{\sqrt[3]{8-x^3}}.$$

►Binomial qatordan foydalanamiz. ((12.21) formulaga qarang). U holda

$$\frac{1}{\sqrt[3]{8-x^3}} = \frac{1}{2}(1 - \left(\frac{x}{2}\right)^3)^{-\frac{1}{3}}.$$

$(1+z)^m$ ko‘rinishdagi binomni hosil qildik, bu erda m = $-1/3$, $z = -(x/2)^3$.

$$\frac{1}{\sqrt[3]{8-x^3}} = \frac{1}{2}\left(1 + \frac{1}{3}\left(\frac{x}{2}\right)^3 + \frac{4}{9} \cdot \frac{1}{2!}\left(\frac{x}{2}\right)^6 + \frac{28}{27} \cdot \frac{1}{3!}\left(\frac{x}{2}\right)^9 + \cdots\right) =$$

$$= \frac{1}{2}\left(1 + \frac{x^3}{24} + \frac{x^6}{288} + \frac{7x^9}{18176} + \cdots\right),$$

$$\int_{-1}^{o} \frac{dx}{\sqrt[3]{8-x^3}} \approx \ \frac{1}{2}\int_{-1}^{0} (1 + \frac{x^3}{24} + \frac{x^6}{288} + \frac{7x^9}{18176} + \cdots)\, dx \approx$$

$$\approx \frac{1}{2}\left(x+\frac{x^4}{24}+\frac{x^7}{7\cdot 288}+\frac{7\cdot x^{10}}{10\cdot 18176}+\cdots\right)\Big|_0^{\pi} =$$

$$=\frac{1}{2}\left(1-\frac{1}{96}+\frac{1}{2016}-\frac{7}{18176}+\cdots\right).$$

$\frac{1}{2016}<0{,}001$ bo‘lganligi uchun, berilgan aniq integralning 0,001 aniqlikdagi qiymati:

$$\int_{-1}^{o}\frac{dx}{\sqrt[3]{8-x^3}}\approx\frac{1}{2}-\frac{1}{192}\approx 0{,}5-0{,}0052\approx 0{,}495. \blacktriangleleft$$

7. $y'=2x-y^3, y(1)=1$ differensial tenglama yechimining (*x-1)* ning darajalari bo‘yicha darajali qatorga yoyilmasini toping. (bu yoyilmaning noldan farqli, birinchi uchta hadini yozing).

►Berilgan tenglama uchun *x* =1 nuqta maxsus nuqta emas, shuning uchun uning yechimini quyidagi qator ko‘rinishida izlash mumkin.

$$y=f(1)+\frac{f'(1)}{1!}(x-1)+\frac{f''(2)}{2!}(x-1)^2+\frac{f'''(1)}{3!}(x-1)^3+\cdots$$

Quyidagilarga ega bo‘lamiz:

$$f(1)=1,\ f'(1)=2+1^3=3,\ f''(x)=2+3y^2\cdot y',$$

$$f'''(1)=2+3\cdot 1^2\cdot 3=11.$$

Hosilalarning topilgan qiymatlarini izlanayotgan qatorga qo‘yib, berilgan tenglamaning yyechimini hosil qilamiz:

$$y=1+\frac{3}{1!}(x-1)+\frac{11}{2!}(x-1)^2+\cdots. \blacktriangleleft$$

8. Ketma – ket differensiallash usuli bilan quyidagi $y(1)=1$, $y'(1)=\frac{1}{2}$ boshlang‘ich shartlarini qanoatlantiruvchi $4x^2y''+y=0$ differensial tenglama yechimining darajali qatorga yoyilmasining birinchi 5 ta hadini toping.

►Berilgan tenglamaning yechimini quydagi ko‘rinishda izlaymiz:

$$y = f(1) + \frac{f'(1)}{1!}(x-1) + \frac{f''(1)}{2!}(x-1)^2 + \frac{f'''(1)}{3!}(x-3)^3 +$$

$$+\frac{f^{1V}(1)}{4!}(x-1)^4 + \cdots,$$

$$f(1) = 1, \ \ f'(1) = \frac{1}{2}; f''(x) = -\frac{y}{4x^2}, \ f''(1) = -\frac{1}{4};$$

$$f'''(x) = -\frac{y'x^2 - 2xy}{4x^2}, f'''(1) = -\frac{(½)\cdot 1 - 2\cdot 1}{4} = \frac{3}{8};$$

$$f^{IV}(x) = -((y''x^2 + 2xy' - 2y - 2xy')x^4 - 4x^3(y'x^2 - 2xy))/(4x^8);$$

$$f^{IV}(1) = -\frac{15}{16}.$$

Hosilalarining topilgan qiymatlarini qatorga qo‘yib, differensial tenglamaning izlanayotgan yechimini hosil qilamiz:

$$y = 1 + \frac{1}{2}(x-1) - \frac{1}{4\cdot 2!}(x-1)^2 + \frac{3}{8\cdot 3!}(x-1)^3 - \frac{15}{16\cdot 4!}(x-1)^4 + \cdots,$$

$$y = 1 + \frac{x-1}{2} - \frac{(x-1)^2}{8} + \frac{(x-1)^3}{16} - \frac{5(x-1)^4}{128} + \cdots ◀$$

IUT 12.3

1.[-π, π] oraliqda berilgan (davri $\omega = 2\pi$ bo‘lgan) $f(x)$ davriy funksiyani Furye qatoriga yoying.

1.1. $f(x) = \begin{cases} 0, & -\pi \le x < 0 \\ x-1, & 0 \le x \le \pi \end{cases}$

$$\left(\text{Javob: } f(x) = \frac{\pi-2}{4} - \frac{2}{\pi}\sum_{k=1}^{\infty}\frac{\cos((2k-1)x)}{(2k-1)^2} + \right.$$

$$\left. \frac{\pi-2}{\pi}\sum_{k=1}^{\infty}\frac{\sin((2k-1)x)}{2k-1} - \sum_{k=1}^{\infty}\frac{\sin(2kx)}{2k}.\right)$$

1.2. $f(x)=\begin{cases}2x-1, -\pi\le x\le 0\\ 0, \quad 0<x\le\pi\end{cases}$

$$\left(\text{Javob}: f(x)=-\frac{\pi+1}{2}+\quad+\frac{4}{\pi}\sum_{k=1}^{\infty}\frac{\cos\big((2k-1)x\big)}{(2k-1)^2}+\right.$$

$$\left.\frac{2(\pi+1)}{\pi}\sum_{k=1}^{\infty}\frac{\sin\big((2k-1)x\big)}{2k-1}-2\sum_{k=1}^{\infty}\frac{\sin(2kx)}{2k}.\right)$$

1.3. $f(x)=\begin{cases}0, & -\pi\le x<0,\\ x+2, & 0\le x\le\pi.\end{cases}$

$$\left(\text{Javob}: f(x)=\frac{\pi+4}{4}-\frac{2}{\pi}\sum_{k=1}^{\infty}\frac{\cos\big((2k-1)x\big)}{(2k-1)^2}+\right.$$

$$\left.+\frac{\pi+4}{\pi}\sum_{k=1}^{\infty}\frac{\sin\big((2k-1)x\big)}{2k-1}\sum_{k=1}^{\infty}\frac{\sin(2kx)}{2k}.\right)$$

1.4. $f(x)=\begin{cases}-x+\frac{1}{2}, & -\pi\le x\le 0,\\ 0, & 0<x\le\pi.\end{cases}$

$$\left(\text{Javob}: f(x)=\frac{\pi+1}{4}-\frac{2}{\pi}\sum_{k=1}^{\infty}\frac{\cos\big((2k-1)x\big)}{(2k-1)^2}-\right.$$

$$\left.\frac{\pi+1}{\pi}\sum_{k=1}^{\infty}\frac{\sin\big((2k-1)x\big)}{2k-1}+\sum_{k=1}^{\infty}\frac{\sin(2kx)}{2k}.\right)$$

1.5. $f(x)=\begin{cases}-0, & -\pi\le x<0,\\ \frac{x}{2}+1, & 0\le x\le\pi.\end{cases}$

$$\left(\text{(Javob}: f(x)=\frac{\pi-4}{8}-\frac{1}{\pi}\sum_{k=1}^{\infty}\frac{\cos\big((2k-1)x\big)}{(2k-1)^2}+\right.$$

$$\left.\frac{\pi-4}{2\pi}\sum_{k=1}^{\infty}\frac{\sin\big((2k-1)x\big)}{2k-1}-\sum_{k=1}^{\infty}\frac{\sin(2kx)}{2k}.\right)$$

1.6. $f(x) = \begin{cases} 2x+3, & -\pi \le x \le 0, \\ 0, & 0 < x \le \pi. \end{cases}$

$$\left(\text{Javob: } f(x) = \frac{3-\pi}{2} + \frac{4}{\pi}\sum_{k=1}^{\infty}\frac{\cos\big((2k-1)x\big)}{(2k-1)^2} + \right.$$

$$\left. + \frac{2(\pi-3)}{\pi}\sum_{k=1}^{\infty}\frac{\sin\big((2k-1)x\big)}{2k-1} - -2\sum_{k=1}^{\infty}\frac{\sin(2kx)}{2k}.\right)$$

1.7. $f(x) = \begin{cases} 0, & -\pi \le x < 0, \\ 3-x, & 0 \le x \le \pi. \end{cases}$

$$\left(\text{Javob: } f(x) = \frac{6-x}{4} + \frac{\pi}{2}\sum_{k=1}^{\infty}\frac{\cos\big((2k-1)x\big)}{(2k-1)^2} + \right.$$

$$\left. + \frac{6-\pi}{\pi}\sum_{k=1}^{\infty}\frac{\sin\big((2k-1)x\big)}{2k-1} + \sum_{k=1}^{\infty}\frac{\sin(2kx)}{2k}.\right)$$

1.8. $f(x) = \begin{cases} x-2, & -\pi \le x < 0, \\ 0, & 0 \le x \le \pi. \end{cases}$

$$\left((\text{Javob: } f(x) = -\frac{\pi+4}{2} + \frac{2}{\pi}\sum_{k=1}^{\infty}\frac{\cos\big((2k-1)x\big)}{(2k-1)^2} + \right.$$

$$\left. + \frac{4+\pi}{\pi}\sum_{k=1}^{\infty}\frac{\sin\big((2k-1)x\big)}{2k-1} - \sum_{k=1}^{\infty}\frac{\sin(2kx)}{2k}.\right)$$

1.9. $f(x) = \begin{cases} 0, & -\pi \le x < 0, \\ 4x-3, & 0 \le x \le \pi. \end{cases}$

$$\left(\text{Javob: } f(x) = \frac{2\pi-3}{2} - \frac{8}{\pi}\sum_{k=1}^{\infty}\frac{\cos\big((2k-1)x\big)}{(2k-1)^2} + \right.$$

$$\left. \frac{2(2\pi-3)}{\pi}\sum_{k=1}^{\infty}\frac{\sin\big((2k-1)x\big)}{2k-1} - 4\sum_{k=1}^{\infty}\frac{\sin(2kx)}{2k}.\right)$$

1.10. $f(x) = \begin{cases} 5 - x, -\pi \le x < 0, \\ 0, \quad 0 \le x \le \pi. \end{cases}$

$$\left(\text{Javob: } f(x) = \frac{\pi + 10}{4} - \frac{2}{\pi}\sum_{k=1}^{\infty}\frac{\cos\big((2k-1)x\big)}{(2k-2)^2} - \right.$$

$$\left. \frac{\pi + 10}{\pi}\sum_{k=1}^{\infty}\frac{\sin\big((2k-1)x\big)}{2k-1} + \sum_{k=1}^{\infty}\frac{\sin(2kx)}{2k}.\right)$$

1.11. $f(x) = \begin{cases} 0, \quad -\pi \le x < 0, \\ 3x - 1, \ 0 \le x \le \pi. \end{cases}$

$$\left(\text{Javob: } f(x) = \frac{3\pi - 2}{4} - \frac{6}{\pi}\sum_{k=1}^{\infty}\frac{\cos\big((2k-1)x\big)}{(2k-1)^2} + \right.$$

$$\left. \frac{3\pi - 2}{\pi}\sum_{k=1}^{\infty}\frac{\sin\big((2k-1)x\big)}{2k-1} - 3\sum_{k=1}^{\infty}\frac{\sin(2kx)}{2k}.\right)$$

1.12 . $f(x) = \begin{cases} 3 - 2x, -\pi \le x < 0, \\ 0, \quad 0 \le x \le \pi. \end{cases}$

$$\left(\text{Javob: } f(x) = \frac{\pi + 3}{2} - \frac{4}{\pi}\sum_{k=1}^{\infty}\frac{\cos\big((2k-1)x\big)}{(2k-1)^2} - \right.$$

$$\left. \frac{2(\pi + 3)}{\pi}\sum_{k=1}^{\infty}\frac{\sin\big((2k-1)x\big)}{2k-1} + 2\sum_{k=1}^{\infty}\frac{\sin(2kx)}{2k}.\right)$$

1.13. $f(x) = \begin{cases} 0, \quad -\pi \le x < 0, \\ \frac{\pi - x}{2}, \ 0 \le x \le \pi. \end{cases}$

$$\left(\text{Javob: } f(x) = \frac{\pi}{8} + \frac{1}{\pi}\sum_{k=1}^{\infty}\frac{\cos\big((2k-1)x\big)}{(2k-1)^2} + \frac{1}{2}\sum_{k=1}^{\infty}\frac{\sin kx}{k}.\right)$$

1.14. $f\ (x) = \begin{cases} 5x + 1, -\pi \le x < 0, \\ 0, \quad 0 \le x \le \pi. \end{cases}$

$$\left(\text{Javob: } f(x) = \frac{2-5x}{4} + \frac{10}{\pi}\sum_{k=1}^{\infty}\frac{\cos((2k-1)x)}{(2k-1)^2}\right.$$

$$\left. + \frac{5\pi-2}{\pi}\sum_{k=1}^{\infty}\frac{\sin((2k-1)x)}{2k-1} - 5\sum_{k=1}^{\infty}\frac{\sin(2kx)}{2k}.\right)$$

1.15. $f(x) = \begin{cases} 0, & -\pi \le x < 0, \\ 1-4x, & 0 \le x \le \pi. \end{cases}$

$$\left(\text{Javob: } f(x) = \frac{1-2\pi}{2} + \frac{8}{\pi}\sum_{k=1}^{\infty}\frac{\cos((2k-1)x)}{(2k-1)^2}\right.$$

$$\left. + \frac{2-4\pi}{\pi}\sum_{k=1}^{\infty}\frac{\sin((2k-1)x)}{2k-1} + 4\sum_{k=1}^{\infty}\frac{\sin(2kx)}{2k}.\right)$$

1.16. $f(x) = \begin{cases} 3x+2 & -\pi \le x \le 0, \\ 0, & 0 < x \le \pi. \end{cases}$

$$\left(\text{Javob: } f(x) = \frac{4-3\pi}{4} + \frac{6}{\pi}\sum_{k=1}^{\infty}\frac{\cos((2k-1)x)}{(2k-1)^2}\right.$$

$$\left. + \frac{3\pi-4}{\pi}\sum_{k=1}^{\infty}\frac{\sin((2k-1)x)}{2k-1} - 3\sum_{k=1}^{\infty}\frac{\sin(2kx)}{2k}.\right)$$

1.17. $f(x) = \begin{cases} 0, & -\pi \le x < 0, \\ 4-2x, & 0 \le x \le \pi. \end{cases}$

$$\left(\text{Javob: } f(x) = = \frac{4-\pi}{2} + \frac{4}{\pi}\sum_{k=1}^{\infty}\frac{\cos((2k-1)x)}{(2k-1)^2} + \right.$$

$$\left. + \frac{2(4-\pi)}{\pi}\sum_{k=1}^{\infty}\frac{\sin((2k-1)x)}{2k-1} + 2\sum_{k=1}^{\infty}\frac{\sin(2kx)}{2k}.\right)$$

1.18. $f(x) = \begin{cases} x+\frac{\pi}{2}, & -\pi \le x \le 0, \\ 0, & 0 < x \le \pi. \end{cases}$

$$\left(\text{Javob: } f(x)=\frac{2}{\pi}\sum_{k=1}^{\infty}\frac{\cos\big((2k-1)x\big)}{(2k-1)^2}--\sum_{k=1}^{\infty}\frac{\sin(2kx)}{2k}.\right)$$

1.19. $f(x)=\begin{cases}0, & -\pi\le x<0,\\ 6x-5, & 0\le x\le\pi.\end{cases}$

$$\left(\text{Javob: } f(x)==\frac{3\pi-5}{2}-\frac{12}{\pi}\sum_{k=1}^{\infty}\frac{\cos\big((2k-1)x\big)}{(2k-1)^2}+\right.$$

$$\left.\frac{2(3\pi-5)}{\pi}\sum_{k=1}^{\infty}\frac{\sin((2k-1)x)}{2k-1}--6\sum_{k=1}^{\infty}\frac{\sin(2kx)}{2k}.\right)$$

1.20. $f(x)=\begin{cases}7-3x, & -\pi\le x\le 0,\\ 0, & 0<x\le\pi\end{cases}$

$$\left(\text{Javob: } f(x)=\frac{3\pi+14}{4}-\frac{6}{\pi}\sum_{k=1}^{\infty}\frac{\cos\big((2k-1)x\big)}{(2k-1)^2}-\right.$$

$$\left.-\frac{3\pi+14}{4}\sum_{k=1}^{\infty}\frac{\sin\big((2k-1)x\big)}{2k-1}+3\sum_{k=1}^{\infty}\frac{\sin(2kx)}{2k}\right)$$

1.21 .$f(x)=\begin{cases}0, & -\pi\le x<0\\ \frac{\pi}{4}-\frac{x}{2}, & 0\le x\le\pi\end{cases}$

$$\left(\text{Javob: } f(x)=\frac{1}{\pi}\sum_{k=1}^{\infty}\frac{\cos\big((2k-1)x\big)}{(2k-1)^2}+\frac{1}{2}\sum_{k=1}^{\infty}\frac{\sin(2kx)}{2k}\right)$$

1.22. $f(x)=\begin{cases}6x-2, & -\pi\le x\le 0\\ 0, & 0<x\le\pi\end{cases}$

$$\left(\text{Javob: } f(x)=-\frac{3\pi+2}{2}-\frac{12}{\pi}\sum_{k=1}^{\infty}\frac{\cos\big((2k-1)x\big)}{(2k-1)^2}+\right.$$

$$\left.+\frac{2(3\pi+2)}{\pi}\sum_{k=1}^{\infty}\frac{\sin\big((2k-1)x\big)}{2k-1}-6\sum_{k=1}^{\infty}\frac{\sin(2kx)}{2k}\right)$$

1.23. $f(x)=\begin{cases}0, & -\pi\le x<0\\ 4-9x, & 0\le x\le\pi\end{cases}$

$$\left(\text{Javob: } f(x) = \frac{8-9\pi}{4} + \frac{18}{\pi}\sum_{k=1}^{\infty}\frac{\cos\big((2k-1)x\big)}{(2k-1)^2} + \right.$$

$$\left. + \frac{8-9\pi}{\pi}\sum_{k=1}^{\infty}\frac{\sin\big((2k-1)x\big)}{2k-1} + 9\sum_{k=1}^{\infty}\frac{\sin(2kx)}{2k}\right)$$

1.24. $f(x) = \begin{cases} x/3 - 3, & -\pi \le x \le 0 \\ 0, & 0 < x \le \pi \end{cases}$

$$\left(\text{Javob: } f(x) = -\frac{\pi+18}{12} - \frac{2}{3\pi}\sum_{k=1}^{\infty}\frac{\cos\big((2k-1)x\big)}{(2k-1)^2} + \right.$$

$$\left. + \frac{18+\pi}{9\pi}\sum_{k=1}^{\infty}\frac{\sin\big((2k-1)x\big)}{2k-1} - \frac{1}{9}\sum_{k=1}^{\infty}\frac{\sin(2kx)}{2k}\right)$$

1.25. $f(x) = \begin{cases} 0, & -\pi \le x < 0 \\ 10x - 3, & 0 \le x \le \pi \end{cases}$

$$\left(\text{Javob: } f(x) = \frac{5\pi-3}{2} + \frac{20}{\pi}\sum_{k=1}^{\infty}\frac{\cos\big((2k-1)x\big)}{(2k-1)^2} + \right.$$

$$\left. + \frac{2(5\pi-3)}{\pi}\sum_{k=1}^{\infty}\frac{\sin\big((2k-1)x\big)}{2k-1} - 10\sum_{k=1}^{\infty}\frac{\sin(2kx)}{2k}\right)$$

1.26. $f(x) = \begin{cases} 1 - x/4, & -\pi \le x \le 0 \\ 0, & 0 < x \le \pi \end{cases}$

$$\left(\text{Javob: } f(x) = \frac{\pi+8}{16} - \frac{1}{2\pi}\sum_{k=1}^{\infty}\frac{\cos\big((2k-1)x\big)}{(2k-1)^2} - \right.$$

$$\left. - \frac{\pi+8}{4\pi}\sum_{k=1}^{\infty}\frac{\sin\big((2k-1)x\big)}{2k-1} + \frac{1}{4}\sum_{k=1}^{\infty}\frac{\sin(2kx)}{2k}\right)$$

1.27. $f(x) = \begin{cases} 0, & -\pi \le x < 0 \\ \frac{x}{5} - 2, & 0 \le x \le \pi \end{cases}$

$$\left(\text{Javob: } f(x)=\frac{\pi-20}{20}+\frac{2}{5\pi}\sum_{k=1}^{\infty}\frac{\cos\big((2k-1)x\big)}{(2k-1)^2}+\right.$$

$$\left.+\frac{\pi-20}{5\pi}\sum_{k=1}^{\infty}\frac{\sin\big((2k-1)x\big)}{2k-1}-\frac{1}{5}\sum_{k=1}^{\infty}\frac{\sin(2kx)}{2k}\right)$$

1.28. $f(x)=\begin{cases}2x-11, & -\pi\le x\le 0\\ 0, & 0<x\le\pi\end{cases}$

$$\left(\text{Javob: } f(x)=-\frac{\pi+11}{2}-\frac{4}{\pi}\sum_{k=1}^{\infty}\frac{\cos\big((2k-1)x\big)}{(2k-1)^2}+\right.$$

$$\left.+\frac{2(\pi+11)}{\pi}\sum_{k=1}^{\infty}\frac{\sin\big((2k-1)x\big)}{2k-1}-2\sum_{k=1}^{\infty}\frac{\sin(2kx)}{2k}\right)$$

1.29. $f(x)=\begin{cases}0, & -\pi\le x<0\\ 3-8x, & 0\le x\le\pi\end{cases}$

$$\left(\text{Javob: } f(x)=\frac{3-4\pi}{2}+\frac{16}{\pi}\sum_{k=1}^{\infty}\frac{\cos\big((2k-1)x\big)}{(2k-1)^2}+\right.$$

$$\left.+\frac{2(3-4\pi)}{\pi}\sum_{k=1}^{\infty}\frac{\sin\big((2k-1)x\big)}{2k-1}+8\sum_{k=1}^{\infty}\frac{\sin(2kx)}{2k}\right)$$

1.30. $f(x)=\begin{cases}7x-1, & -\pi\le x\le 0\\ 0, & 0<x\le\pi\end{cases}$

$$\left(\text{Javob: } f(x)=-\frac{7\pi+2}{4}+\frac{14}{\pi}\sum_{k=1}^{\infty}\frac{\cos((2k-1)x)}{(2k-1)^2}+\right.$$

$$\left.+\frac{7\pi+2}{\pi}\sum_{k=1}^{\infty}\frac{\sin((2k-1)x)}{(2k-1)}-7\sum_{k=1}^{\infty}\frac{\sin((2kx)}{2k}\right)$$

2. $(0,\pi)$ oraliqda berilgan $f(x)$ funksiyani toq yoki juft ravishda davom ettirib Furye qatoriga yoying. Har bir davom ettirilgan funksiyaning grafigini chizing.

2. 1. $f(x)=e^x$. $\left(\text{Javob: } e^x=\frac{e^\pi-1}{\pi}+\frac{2}{\pi}\sum_{\pi=1}^{\infty}\frac{((-1)^n e^\pi-1)cosnx}{1+n^2},\right.$

$$e^{x}=\frac{2}{\pi}\sum_{\pi=1}^{\infty}\left(1-(-1)^{n}e^{\pi}\right)\frac{n\,sin\,nx}{n^{2}+1}\Bigg)\Bigg).$$

2.2. $f(x)=x^2$. $\Bigg($ Javob: $x^{2}=\frac{\pi^{2}}{3}+4\sum_{n=1}^{\infty}\frac{(-1)^{n+1}\cos nx}{n^{2}},$

$$x^{2}=\frac{2}{\pi}\sum_{k=1}^{\infty}\frac{\pi^{2}-4(2k-1)^{2}}{(2k-1)^{3}}\sin\big((2k-1)x\big)-2\pi\sum_{k=1}^{\infty}\frac{\sin(2kx)}{2k}\Bigg).$$

2.3. $f(x)=2^x$. $\Bigg($ Javob: $2^{x}=\frac{2^{3}-1}{\pi 1n2}+\frac{2ln2}{\pi}\sum_{\pi=1}^{\infty}\frac{2^{3}(-1)^{n}-1}{n^{2}+ln^{2}2}\cos nx$

$$2^{x}=\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}2^{\pi}+1}{n^{2}+1n^{2}2}n\sin nx\Bigg).$$

2.4. $f(x)=ch\,x$. $\Bigg($ Javob: $ch\,x=\frac{sh\,\pi}{\pi}\left(1+2\sum_{n=1}^{\infty}(-1)^{n}\frac{\cos nx}{1+n^{2}}\right),$

$$ch\,x=\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1-(-1)^{n}ch\pi}{1+n^{2}}n\sin n\,x\Bigg).$$

2.5. $f\ (x)=e^{-x}$. $\Bigg($ Javob: $e^{-x}=\frac{1-e^{-\pi}}{\pi}+\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1-(-1)^{n}e^{-\pi}}{1+n^{2}}\cos nx$

$$e^{-x}=\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1-(-1)^{n}e^{-\pi}}{1+n^{2}}nsin\,nx\Bigg).$$

2.6. $f(x)=(x-1)^2$. $\Bigg($ Javob: $(x-1)^{2}=\frac{\pi^{2}-3\pi+3}{3}+$

$$+\frac{4}{\pi}\sum_{k=1}^{\infty}\frac{2-\pi}{(2k-1)^{2}}\cos\big((2k-1)x\big)+4\sum_{k=1}^{\infty}\frac{\cos(2kx)}{(2k)^{2}},$$

$$(x-1)^{2}=\frac{2}{\pi}\sum_{k=1}^{\infty}\left(\frac{\pi^{2}-2\pi+2}{2k-1}+\frac{4}{(2k-1)^{3}}\right)\sin\big((2k-1)x\big)+2(2$$

$$-\pi)\sum_{k=1}^{\infty}\frac{\sin(2kx)}{2k}\Bigg).$$

2.7. $f(x)=3^{-\frac{x}{2}}$.

$$\left(\text{Javob: } 3^{-\frac{x}{2}} = \frac{2\left(1-3^{-\frac{\pi}{2}}\right)}{\pi \ln 3} + \frac{4\ln 3}{\pi}\sum_{n=1}^{\infty}\frac{1-(-1)^n \cdot 3^{-\frac{\pi}{2}}}{4n^2+(\ln 3)^2},\right.$$

$$\left. 3^{-x/2} = \frac{8}{\pi}\sum_{n=1}^{\infty}\frac{1-(-1)^n \cdot 3^{-\pi/2}}{4n^2+(\ln 3)^2} n \sin nx\right).$$

2. 8. $f(x) = sh\ 2x$

$$\left(\text{Javob: } sh\ 2x = \frac{ch\ 2\pi}{2\pi} + + \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{ch\ 2\pi \cdot (-1)^n - 1}{4+n^2}\cos nx,\right.$$

$$\left. sh\ 2x = \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1} sh\ 2\pi}{n^2+4} n \sin nx\right).$$

2.9. $f(x) = e^{2x}$.

$$\left(\text{Javob: } f(x) = \frac{e^{2\pi}-1}{2\pi} + \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n e^{2\pi}-1}{4+n^2}\cos nx,\right.$$

$$\left. e^{2x} = \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1-(-1)^n e^{2\pi}}{4+n^2} n \sin nx\right).$$

2.10 $f(x) = (x-2)^2$.

$$\left(\text{Javob: } (x-2)^2 = \frac{\pi^2-6\pi+12}{3} + \frac{4(4-\pi)}{\pi}\sum_{k=1}^{\infty}\frac{\cos(2k-1)x}{(2k-1)^2} + 4\sum_{k=1}^{\infty}\frac{\cos 2kx}{(2k)^2},\right.$$

$$\left. (x-2)^2 = \frac{2}{\pi}\sum_{n=1}^{\infty}\left(\frac{4n^2-2}{n^3} + (-1)^n\frac{2-n^2(2-n)^2}{n^3}\right)\sin nx\right).$$

2.11. $f(x) = 4^{x/3}$.

$$\left(\text{Javob: } 4^{x/3} = \frac{3(4^{\frac{\pi}{3}}-1)}{\pi} + \frac{6ln4}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n \cdot 4^{\frac{\pi}{3}}-1}{9n^2+(\ln 4)^2}\cos nx,\right.$$

$$\left. 4^{x/3} = \frac{18}{\pi}\sum_{n=1}^{\infty}\frac{1-(-1)^n \cdot 4^{\frac{\pi}{3}}}{9n^2+(\ln 4)^2} n \sin nx\right).$$

2.12. $f(x) = ch\ \frac{x}{2}$.

$$\left(\text{Javob: } ch\ \frac{x}{2} = \frac{2\ sh\ (\pi/2)}{\pi} + \frac{4\sin(\pi/2)}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n \cos nx}{1+4n^2},\right.$$

$$\left. ch\ \frac{x}{2} = \frac{8ch(\pi/2)}{\pi}\sum_{n=1}^{\infty}\frac{1-(-1)^n}{1+4n^2} n \sin nx\right).$$

2.13. $f(x) = e^{4x}$.

$$\left(\text{Javob: } e^{4x} = \frac{e^{4\pi}-1}{4\pi} + \frac{8}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n e^{4\pi}-1}{n^2+16}\cos nx,\right.$$

$$\left. e^{4x} = \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1-(-1)^n e^{4\pi}}{n^2+16} n \sin nx\right).$$

2.14. $f(x) = (x+1)^2$.

$$\left(\text{Javob: } (x+1)^2 = \frac{\pi^2+3\pi+3}{3} - \frac{4(\pi+2)}{\pi}\sum_{k=1}^{\infty}\frac{\cos(2k-1)x}{(2k-1)^2} + 4\sum_{k=1}^{\infty}\frac{\cos(2kx)}{(2k)^2},\right.$$

$$\left. (x+1)^2 = \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(2-n^2)+(-1)^n((\pi-1)^2 n^2-2)}{n^3}\sin nx\right).$$

2. 15. $f(x) = 5^{-x}$.

$$\left(\text{Javob: } 5^{-x} = \frac{1-5^{-\pi}}{\pi \ln 5} + \frac{2\ln 5}{\pi}\sum_{n=1}^{\infty}\frac{1-5^{-\pi}(-1)^n}{n^2+(\ln 5)^2}\cos nx,\right.$$

$$\left. 5^{-x} = \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1-(-1)^n\cdot 5^{-\pi}}{n^2+(\ln 5)^2} n \sin nx\right).$$

2.16. $f(x) = sh\ 3x$.

$$\left(\text{Javob: } sh\ 3x = \frac{ch\ 3\pi - 1}{3\pi} + \frac{6}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n ch\ 3\pi - 1}{n^2+9}\cos nx,\right.$$

$$sh\,3x = \frac{2sh\,3}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2+9}n\sin nx\Bigg).$$

2.17. $f(x) = e^{-x/4}$.

$$\left(\text{Javob: } e^{-\frac{x}{4}} = \frac{4\left(1-e^{-\frac{\pi}{4}}\right)}{\pi} + \frac{8}{\pi}\sum_{n=1}^{\infty}\frac{1-(-1)^n e^{-\frac{\pi}{4}}}{16n^2+1}\cos nx\,,\right.$$

$$\left. e^{-x/4} = \frac{32}{\pi}\sum_{n=1}^{\infty}\frac{1-(-1)^n e^{-\pi/4}}{16n^2+1}n\sin nx.\right)$$

2.18. $f(x) = (2x-1)^2$.

$$\left(Javob: (2x-1)^2 = \frac{4\pi^2-6\pi+3}{3} + \frac{8}{\pi}\sum_{k=1}^{\infty}\frac{(-1)^n(2\pi-1)^2+1}{n^2}\cos nx\,,\right.$$

$$\left.(2x-1)^2 = \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{n^2-8+(-1)^n(8-(1-2n)^2)}{n^3}\sin nx\right).$$

2. 19. $f(x) = 6^{x/4}$.

$$\left(Javob: 6^{\frac{x}{4}} = \frac{4\left(6^{\frac{\pi}{4}}-1\right)}{\pi\ln 6} + \frac{8\ln 6}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n\cdot 6^{\frac{\pi}{4}}-1}{16n^2+(\ln 6)^2}\cos nx\,,\right.$$

$$\left.6^{x/4} = \frac{32}{\pi}\sum_{n=1}^{\infty}\frac{1-(-1)^n\cdot 6^{\frac{\pi}{4}}}{16n^2+(\ln 6)^2}\,n\sin nx\right).$$

2.20. $f(x) = ch\,4x$.

$$\left(Javob: ch\,4x = \frac{sh\,(4\pi)}{4\pi} + \frac{8\,\text{sh}(4\pi)}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n\cos nx}{n^2+16}\,,\right.$$

$$ch\, 4x = \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1-(-1)^n ch\, 4\pi}{n^2+16} n \sin nx\Bigg).$$

2.21. $f(x) = e^{-3x}$.

$$\Bigg(Javob\!: e^{-3x} = \frac{1-e^{-3\pi}}{3\pi} + \frac{6}{\pi}\sum_{n=1}^{\infty}\frac{1-(-1)^n e^{-3\pi}}{n^2+9}\cos nx\,,$$

$$e^{-3x} = \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1-(-1)^n e^{-3\pi}}{n^2+9} n \sin nx\Bigg).$$

2.22. $f(x) = x^2 + 1$.

$$\Bigg(Javob\!: x^2+1 = \frac{\pi^2+3}{3} + 4\sum_{k=1}^{\infty}\frac{(-1)^n}{n^2}\cos nx\,,$$

$$x^2+1 = \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(n^2-2)+(2-n)^2(\pi^2+1)(-1)^n}{n^3}\sin nx\Bigg).$$

2.23. $f(x) = 7^{-x/7}$.

$$\Bigg(Javob\!: 7^{-x/7} = \frac{7(1-7^{-\pi/7})}{\pi \ln 7} + \frac{14\ln 7}{\pi}\sum_{n=1}^{\infty}\frac{(1-(-1)^n)\cdot 7^{-\pi/7}}{49n^2+(\ln 7)^2}\cos nx\,,$$

$$7^{-x/7} = \frac{98}{\pi}\sum_{n=1}^{\infty}\frac{1-(-1)^n\cdot 7^{-\pi/7}}{49n^2+(\ln 7)^2}\, n \sin nx\Bigg).$$

2.24. $f(x) = sh\,\frac{x}{5}$.

$$\Bigg(Javob\!: sh\,\frac{x}{5} = \frac{5(ch\frac{\pi}{5}-1)}{\pi} + \frac{10}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n ch\,\pi/5 - 1}{25n^2+1}\cos nx\,,$$

$$sh\,\frac{x}{5} = \frac{50 sh\,\pi/5}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{25n^2+1} n \sin nx\Bigg).$$

2.25. $f(x) = e^{-2x/3}$.

$$\left(Javob: e^{-2x/3} = \frac{3(1-e^{-2\pi/3})}{2\pi} + \frac{12}{\pi}\sum_{n=1}^{\infty}\frac{1-(-1)^n e^{-2\pi/3}}{9n^2+4}\cos nx,\right.$$

$$\left. e^{-2x/3} = \frac{18}{\pi}\sum_{n=1}^{\infty}\frac{1-(-1)^n e^{-2\pi/3}}{9n^2+4} n\sin nx.\right).$$

2.26. $f(x) = (x-\pi)^2$

$$\left(Javob: (x-\pi)^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}\frac{\cos nx}{n^2},\right.$$

$$\left.(x-\pi)^2 = \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(n^2\pi^2+2)(-1)^n - 1}{n^3}\sin nx.\right)$$

2.27. $f(x) = 10^{-x}$.

$$\left(Javob: 10^{-x} = \frac{1-10^{-\pi}}{\pi \ln 10} + \frac{2\ln 10}{\pi}\sum_{n=1}^{\infty}\frac{1-10^{-\pi}(-1)^n}{n^2+ln^2 10}\cos nx\ ,\right.$$

$$\left.10^{-x} = \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1-(-1)^n\cdot 10^{-\pi}}{n^2+ln^2 10} n\sin nx\right).$$

2.28. $f(x) = ch\frac{x}{\pi}$.

$$\left(Javob: ch\ \frac{x}{\pi} = sh\ 1 + 2\ sh\ 1\sum_{n=1}^{\infty}\frac{(-1)^n}{1+n^2\pi^2}\cos nx\ ,\right.$$

$$\left.ch\ \frac{x}{\pi} = 2\pi\sum_{n=1}^{\infty}\frac{1-(-1)^n ch\ 1}{1+n^2\pi^2} n\sin nx\right).$$

2.29. $f(x) = e^{4x/3}$.

$$\left(Javob: e^{\frac{4x}{3}} = \frac{3\left(e^{\frac{4\pi}{3}} - 1\right)}{4\pi} + \frac{24}{\pi}\sum_{n=1}^{\infty} \frac{(-1)^n e^{\frac{4\pi}{3}} - 1}{9n^2 + 16}\cos nx,\right.$$

$$\left. e^{4x/3} = \frac{18}{\pi}\sum_{n=1}^{\infty} \frac{1 - (-1)^n e^{4\pi/3}}{9n^2 + 16} n \sin nx\right).$$

2.30. $f(x) = (x - 5)^2$.

$$\left(Javob: (x - 5)^2 = \frac{\pi^2 - 15\pi + 75}{3} + \frac{4}{\pi}\sum_{n=1}^{\infty} \frac{(\pi - 5)^2(-1)^n + 5}{n^2}\cos nx,\right.$$

$$\left.(x - 5)^2 = \frac{2}{\pi}\sum_{n=1}^{\infty} \frac{(25n^2 - 2)(-1)^n(2 - n^2(5 - \pi)^2)}{n^3}\sin nx\right).$$

3. Davri $\omega = 2l$ bo‘lgan davriy funksiyani ko‘rsatilgan intervalda Furye qatoriga yoying.

3.1. $f(x) = |x|, \quad -1 < x < 1, \ l = 1.$

$$\left(Javob: |x| = \frac{1}{2} - \frac{4}{\pi^2}\sum_{n=0}^{\infty} \frac{\cos\big((2n + 1)\pi x\big)}{(2n + 1)^2}\right)$$

3.2. . $f(x) = 2x, \quad -1 < x < 1, \ l = 1.$

$$\left(Javob: 2x = 1 - \frac{2}{\pi}\sum_{n=0}^{\infty} \frac{\sin(2\pi nx)}{n}\right)$$

3.3. $f(x) = e^x, \quad -2 < x < 2, \ l = 2.$

$$\left(Javob: e^x = sh\, 2(\frac{1}{2} + 2\sum_{n=0}^{\infty}(-1)^n \frac{2\cos\frac{n\pi x}{2} - \pi n \sin\frac{n\pi x}{2}}{4 + n^2\pi^2}\ )\right)$$

3.4. $f(x) = |x| - 5, \quad -2 < x < 2.$

$$\left(Javob\colon |x|-5=-4-\frac{8}{\pi^2}\sum_{n=0}^{\infty}\frac{1}{(2n+1)^2}\cos\left(\frac{(2n+1)\pi x}{2}\right).\right)$$

3.5. $f(x)=\begin{cases}1, & -1\le x<0\\ x\,, & 0<x\le 1\end{cases}\ ,\quad l=1$

$$\left(Javob\colon\ f(x)=\frac{3}{4}-\frac{1}{\pi}\sum_{n=1}^{\infty}\frac{\cos(\pi(2n-1)x)}{\pi(2n-1)^2}+\sin(\pi nx).\right)$$

3.6. $f(x)=x,\quad 1<x<3,\quad l=1.$

$$\left(Javob\colon x=2+\frac{2}{\pi}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\sin(n\pi x)}{n}.\right)$$

3.7. $f(x)=\begin{cases}0, & -2\le x<0,\\ x, & 0\le x<1,\\ 2-x, & 1\le x\le 2,\end{cases}\ l=2.$

$$\left(Javob\colon\ f(x)=\frac{1}{2}-\frac{4}{\pi^2}\sum_{n=1}^{\infty}\frac{\cos\big((2n-1)\pi x\big)}{(2n-1)^2}+\right.$$

$$\left.+\frac{8}{\pi^2}\sum_{n=0}^{\infty}\frac{(-1)^n\sin\left(\frac{(2n+1)\pi x}{2}\right)}{(2n-1)^2}.\right)$$

3.8. $f(x)=10-x,\quad 5<x<15,\quad l=5.$

$$\left(Javob\colon 10-x=\frac{10}{\pi}\sum_{n=1}^{\infty}(-1)^n\frac{\sin(n\pi x/5)}{n}.\right)$$

3.9. $f(x)=\begin{cases}1, & -1\le x<0,\\ \frac{1}{2}, & x=0,\\ x, & 0<x\le 1,\end{cases}\ l=1.$

$$\left(Javob:\ f(x)=\frac{3}{4}-\frac{2}{\pi^2}\sum_{n=1}^{\infty}\frac{\cos\big((2n-1)\pi x\big)}{(2n-1)^2}-\frac{1}{\pi}\sum_{n=1}^{\infty}\frac{\sin(n\pi x)}{n}.\right)$$

3.10. $f(x)=5x-1, \quad -5<x<5, \quad l=5.$

$$\left(Javob: 5x-1=-1+\frac{50}{\pi}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n}\sin(n\pi x/5).\right)$$

3.11. $f(x)=\begin{cases}0, & -3<x\le 0\\ x, & 0<x<3\end{cases}, \quad l=3$

$$\left(Javob:\ f(x)=\frac{3}{4}-\frac{6}{\pi^2}\sum_{n=1}^{\infty}\frac{\cos\left(\frac{(2n-1)\pi x}{3}\right)}{(2n-1)^2}-\frac{3}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\sin(n\pi x/3).\right)$$

3.12. $f(x)=3-x, \quad -2<x<2, \quad l=2.$

$$\left(Javob: 3-x=2+\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\sin(n\pi x/2).\right)$$

3.13. $f(x)=\begin{cases}1, & 0<x<1\\ -1, & 1<x<2\end{cases}, \quad l=1$

$$\left(Javob:\ f(x)=\frac{4}{\pi}\sum_{n=0}^{\infty}\frac{\sin((2n+1)\pi x)}{2n+1}.\right)$$

3.14. $f(x)=\begin{cases}0, & -2<x<0\\ 2, & 0<x<2\end{cases}, \quad l=2$

$$\left(Javob: f(x)=1+\sum_{n=1}^{\infty}\frac{4}{\pi(2n-1)}\sin(\frac{(2n-1)\pi x}{2}).\right)$$

3.15. $f(x)=\begin{cases}x, & 0\le x\le 1,\\ 1, & 1<x<2,\\ 3-x, & 2\le x\le 3,\end{cases} \quad l=3.$

$$\left(Javob: f(x)=\frac{2}{3}-\frac{9}{2\pi^2}\sum_{n=1}^{\infty}\frac{\cos\left(\frac{2\pi nx}{3}\right)}{n^2}+\frac{1}{2\pi^2}\sum_{k=1}^{\infty}\frac{\cos(2\pi kx)}{k^2}.\right)$$

3.16. . $f(x)=2x-3, \quad -3<x<3, \quad l=3.$

$$\left(Javob: 2x-3=-3+\frac{12}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\sin(n\pi x/3).\right)$$

3.17. $f(x)=\begin{cases}1, & 0<x<\frac{3}{2}\\ -1\,, & \frac{3}{2}<x<3\end{cases}, \quad l=3$

$$\left(Javob: \; f(x)=\frac{4}{\pi}\sum_{n=0}^{\infty}(-1)^n\frac{\cos((2n+1)\pi x/3)}{2n+1}.\right)$$

3.18. $f(x)=3-|x|, \quad -5<x<5, \quad l=5.$

$$\left(Javob: 3-|x|=\frac{1}{2}-\frac{20}{\pi^2}\sum_{n=0}^{\infty}\frac{1}{(2n+1)^2}\cos\left(\frac{(2n+1)\pi x}{5}\right).\right)$$

3.19. $f(x)=\begin{cases}-x, & -4<x<0,\\ 1, & x=0,\\ 2, & 0<x<4,\end{cases} \; l=4.$

$$\left(Javob: \; f(x)=2-\frac{8}{\pi^2}\sum_{n=1}^{\infty}\frac{\cos\left(\frac{(2n-1)\pi x}{4}\right)}{(2n-1)^2}+\right.$$

$$\left.+\sum_{n=1}^{\infty}\frac{(-1)^n\sin(n\pi x/4)}{n}+\frac{4}{\pi}\sum_{k=1}^{\infty}\frac{\sin((2k-1)\pi x/4)}{2k-1}.\right)$$

3.20. $f(x)=1+x, \quad -1<x<1, \quad l=1.$

$$\left(Javob: 1+\;x=1+\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}\sin(n\pi x)}{n}.\right)$$

3.21. $f(x)=\begin{cases}-1, & -2<x<0,\\ -\frac{1}{2}, & x=0, l=2.\\ \frac{x}{2}, & 0<x<2,\end{cases}$

$$\left(f(x)=-\frac{1}{4}-\frac{2}{\pi^2}\sum_{n=1}^{\infty}\frac{\cos\left(\frac{(2n-1)\pi x}{2}\right)}{(2n-1)^2}+\frac{3}{\pi}\sum_{n=1}^{\infty}\frac{\sin\left(\frac{(2n-1)\pi x}{2}\right)}{2n-1}-\right.$$

$$\left.-\frac{1}{\pi}\sum_{k=1}^{\infty}\frac{\sin(k\pi x)}{2k}.\right)$$

3.22. $f(x)=2x+2, \quad -1<x<3, \quad l=2.$

$$\left(Javob:\ 2x+2=2-\frac{8}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n\sin(n\pi x/2)}{n}.\right)$$

3.23. $f(x)=\begin{cases}3, & -3<x<0,\\ \frac{3}{2}, & x=0,\ l=3.\\ -x, & 0<x<3,\end{cases}$

$$\left(Javob: f(x)=\frac{3}{4}-\frac{6}{\pi^2}\sum_{n=1}^{\infty}\frac{\cos\left(\frac{(2n-1)\pi x}{3}\right)}{(2n-1)^2}-\right.$$

$$\left.-\frac{9}{\pi}\sum_{n=1}^{\infty}\frac{\sin\left(\frac{(2n-1)\pi x}{3}\right)}{2n-1}--\frac{3}{\pi}\sum_{k=1}^{\infty}\frac{\sin(2k\pi x/3)}{2k}.\right)$$

3.24. $f(x)=1-|x|, \quad -3<x<3, \quad l=3.$

$$\left(Javob: 1-|x|=-\frac{1}{2}-\frac{12}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}\cos\left(\frac{(2n-1)\pi x}{3}\right).\right)$$

3.25. $f(x)=\begin{cases}-2, & -4<x<0,\\ -1/2, & x=0, l=4.\\ 1+x, & 0<x<4,\end{cases}$

$$\left(Javob: f(x) = -\frac{1}{2} + \frac{32}{\pi^2}\sum_{n=1}^{\infty}\frac{\cos\left(\frac{(2n-1)\pi x}{4}\right)}{(2n-1)^2} + \right.$$

$$\left.\frac{10}{\pi}\sum_{n=1}^{\infty}\frac{\sin\left(\frac{(2n-1)\pi x}{4}\right)}{2n-1} - -\frac{4}{\pi}\sum_{k=1}^{\infty}\frac{\sin(k\pi x/2)}{2k}.\right)$$

3.26. $f(x) = 4x - 3, \quad -5 < x < 5, \quad l = 5.$

$$\left(Javob:\ 4x - 3 = -3 + \frac{40}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\sin\left(\frac{n\pi x}{5}\right).\right)$$

3.27. $f(x) = \begin{cases} x+2, & -2 < x < -1, \\ 1, & -1 \le x \le 1, l = 2. \\ 2-x, & 1 < x < 2, \end{cases}$

$$\left(Javob: f(x) = \frac{3}{4} + \frac{4}{\pi^2}\sum_{n=1}^{\infty}\frac{\cos\left(\frac{(2n-1)\pi x}{2}\right)}{(2n-1)^2} - \frac{8}{\pi^2}\sum_{k=1}^{\infty}\frac{\cos\left(\frac{2(2k-1)\pi x}{2}\right)}{(2(2k-1))^2}.\right)$$

3.28. $f(x) = \begin{cases} -\frac{1}{2}, & -6 < x < 0 \\ 1, & 0 < x < 6 \end{cases}, \quad l = 6.$

$$\left(Javob:\ f(x) = \frac{1}{4} + \frac{3}{\pi}\sum_{n=1}^{\infty}\frac{1}{2n-1} sin\frac{(2n-1)\pi x}{6}.\right)$$

3.29. $f(x) = \begin{cases} -2x, & -2 < x < 0, \\ 2, & x = 0, \quad l = 2. \\ 4, & 0 < x < 2, \end{cases}$

$$\left(f(x) = 3 - \frac{8}{\pi^2}\sum_{n=1}^{\infty}\frac{\cos\left(\frac{(2n-1)\pi x}{2}\right)}{(2n-1)^2} + \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\left(\frac{\pi n x}{2}\right).\right)$$

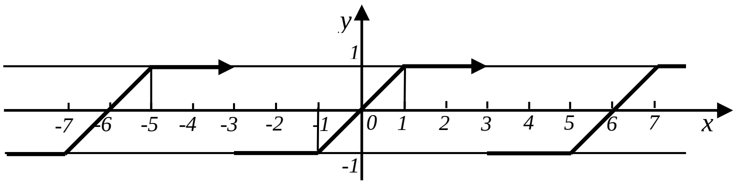
3.30. $f(x) = |x| - 3, \quad -4 < x < 4, \quad l = 4.$

$$\left(Javob\colon |x| - 3 = -1 - \frac{16}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}\cos\left(\frac{(2n-1)\pi x}{2}\right) -\right.$$

$$\left.\frac{8}{\pi}\sum_{n=1}^{\infty}\frac{\sin(n\pi x)}{2n} - - \frac{4}{\pi}\sum_{k=1}^{\infty}\frac{1}{2k-1}\sin\frac{(2k-1)\pi x}{2}.\right)$$

4. Grafik yordamida berilgan funksiyani Furye qatoriga yoying.

4.1.



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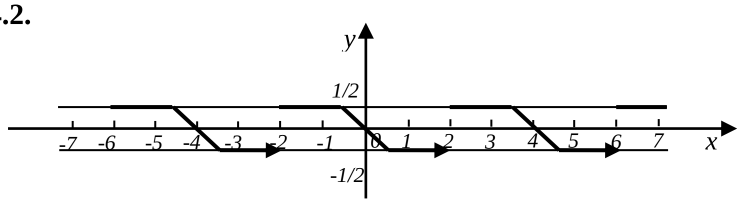

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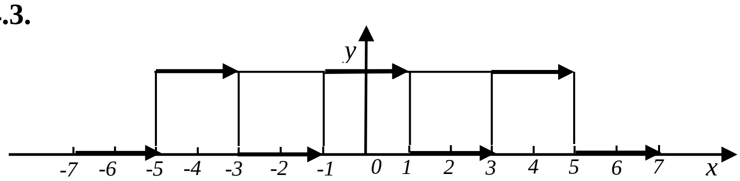

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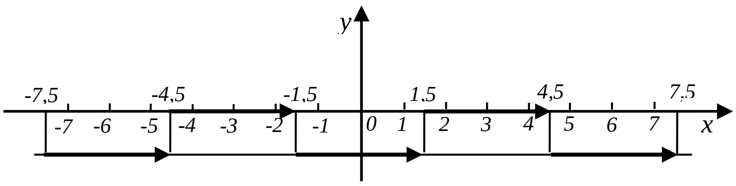

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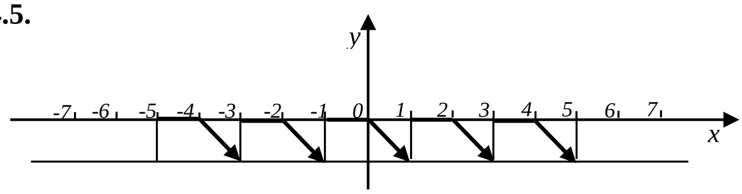

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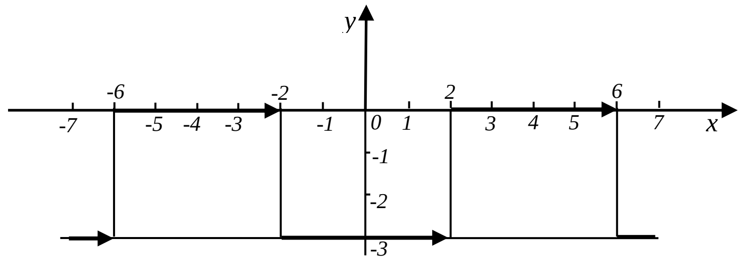

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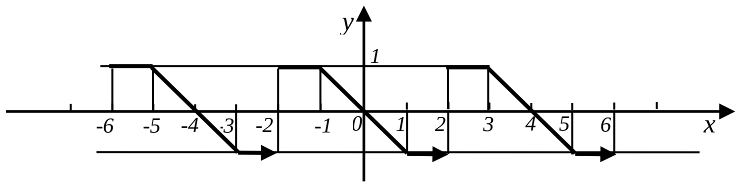

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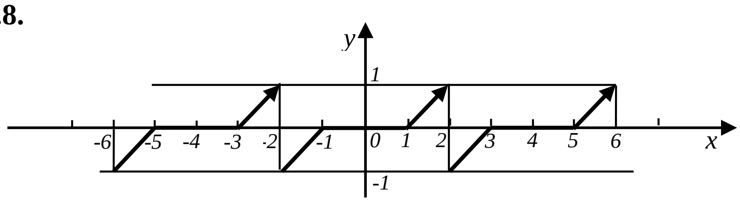

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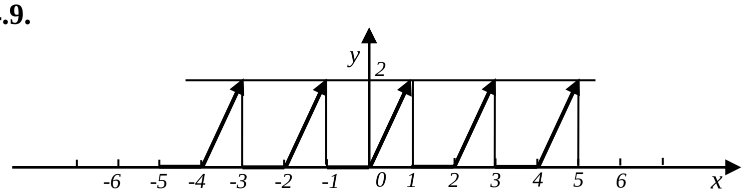

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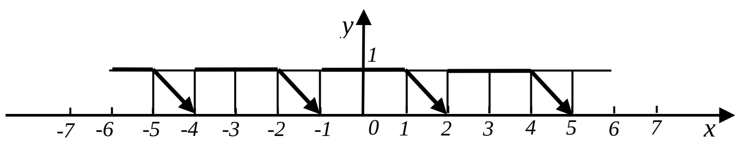

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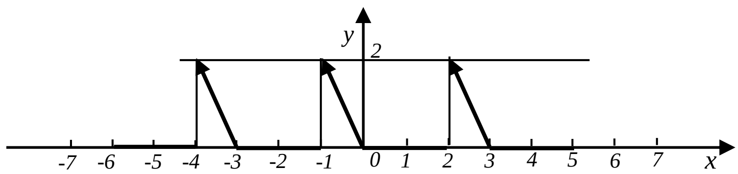

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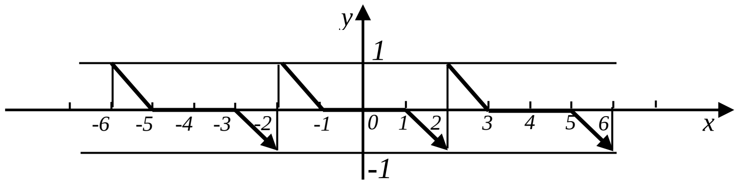

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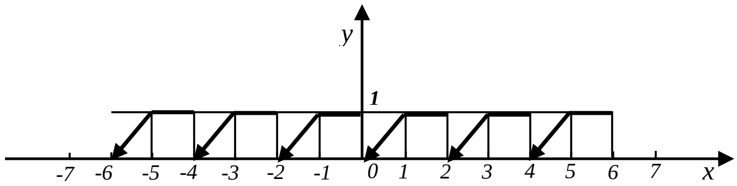

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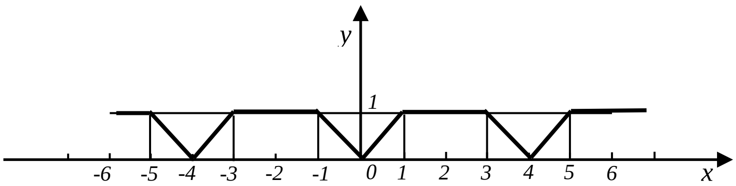

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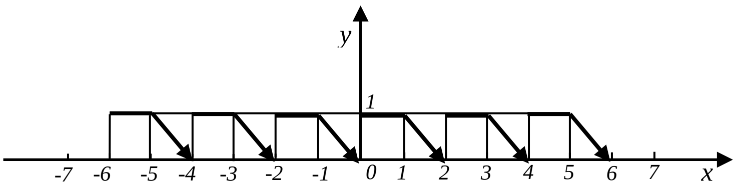

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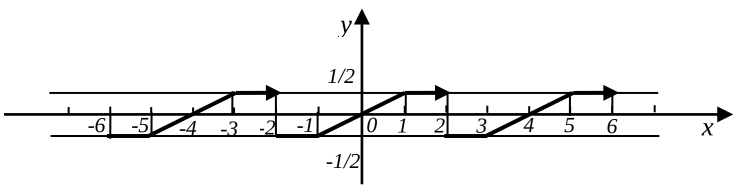

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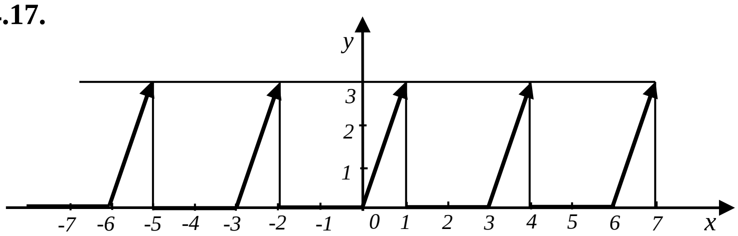

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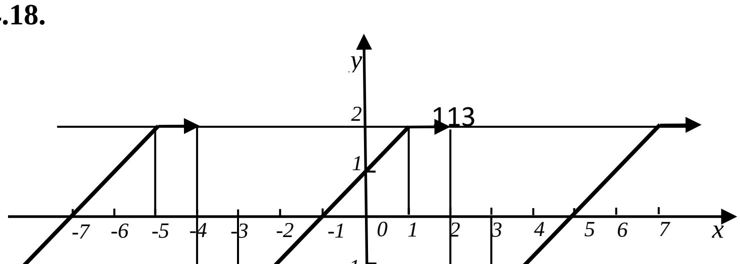

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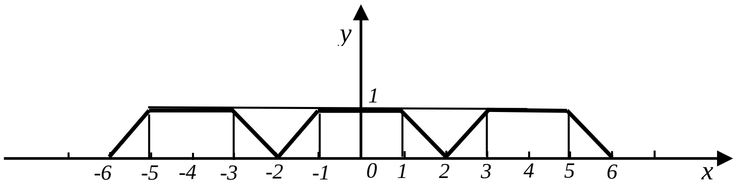

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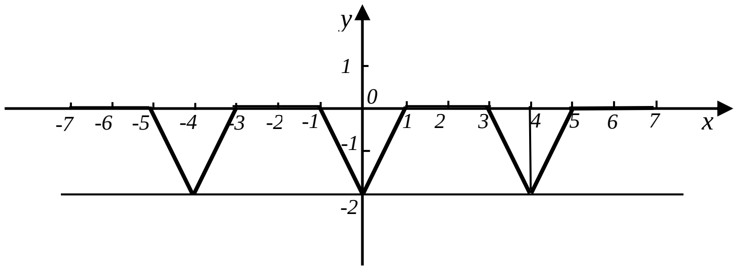

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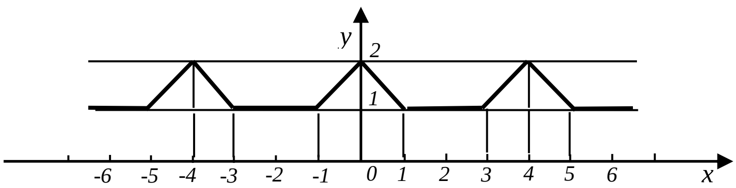

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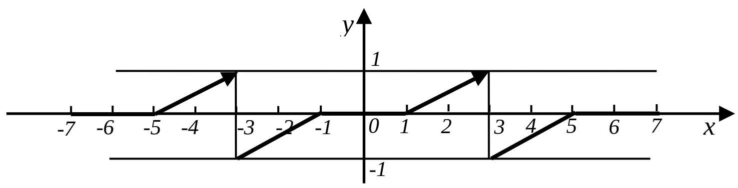

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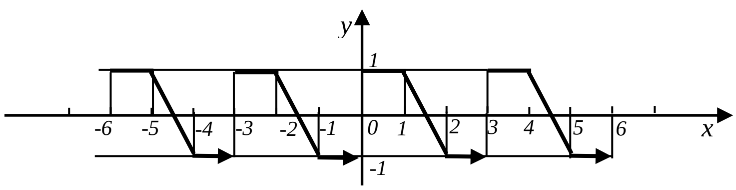
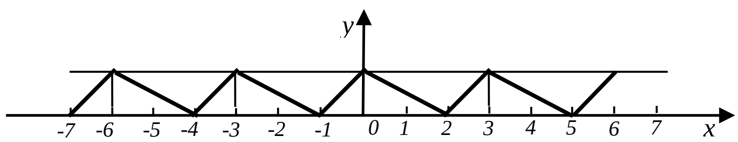


4.24



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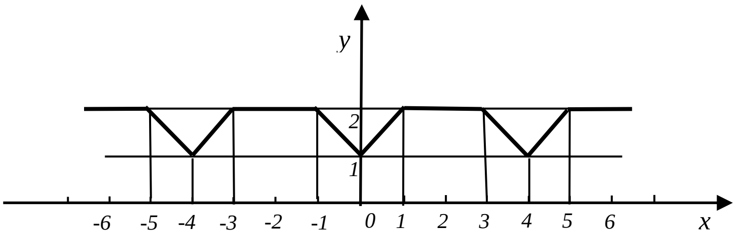

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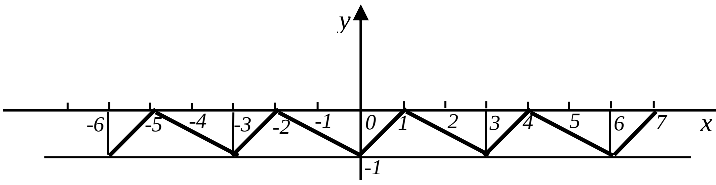

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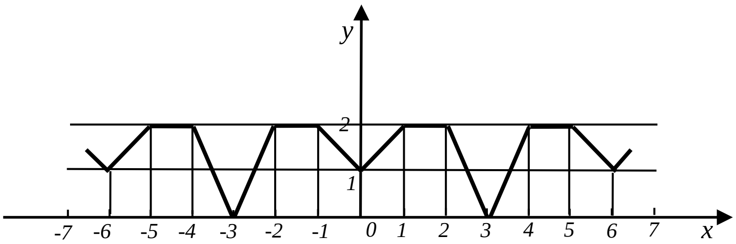

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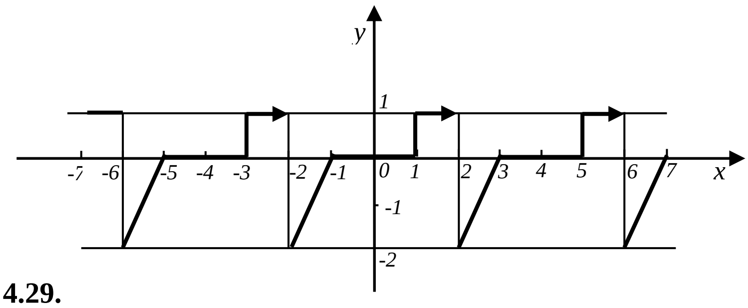

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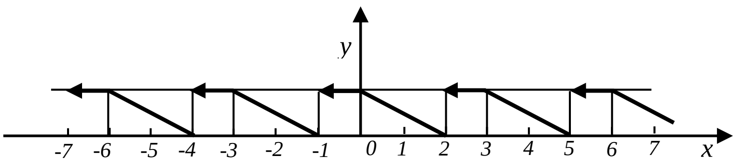
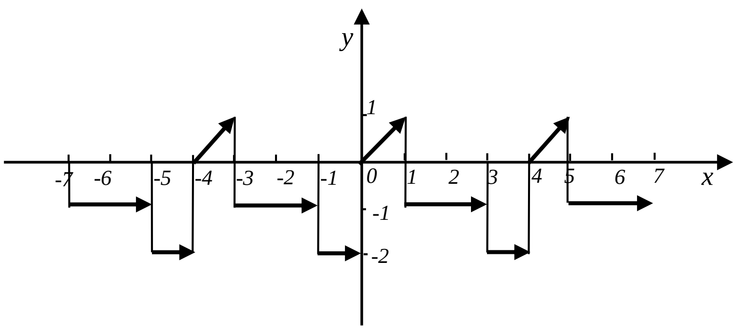


4.30.



5. $f(x)$ funksiyaning ko'rsatilgan oraliqda Furye qatoriga yoyilmasidan foydalanib berilgan sonli qatorning yoyilmasini toping.

5.1. $f(x) = |x|$, (-π; π), $\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}$. (Javob: $\frac{\pi^2}{8}$.)

5.2. $f(x)$=$|\sin x|$, (-π; π), $\sum_{n=1}^{\infty}\frac{1}{4n^2-1}$. (Javob: $\frac{1}{2}$.)

5.3 . $f(x)$=x^2, [-π; π], $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n^2}$. (Javob: $\frac{\pi^2}{12}$.)

5.4 . $f(x)$= x, [0; π], kosinus bo'yicha, $\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}$. (Javob: $\frac{\pi^2}{8}$.)

5.5. $f(x) = \begin{cases} -x, -\pi \le x \le 0, \\ x^2/\pi\ , 0 < x \le \pi, \end{cases}$ $\sum_{n=1}^{\infty}\frac{3-(-1)^n}{n^2}$. (Javob: $\frac{7\pi^2}{12}$.)

5.6. $f(x)$= $\begin{cases} -1, -\pi < x < 0, \\ 1,\ \ 0 < x < \pi, \\ 0,\ \ x = -\pi,\ \ x = 0,\ \ x = \pi, \end{cases}$ $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{2n-1}$. (Javob: $\frac{\pi}{4}$.)

5.7. $f(x) = \frac{\pi}{4}$, (0; π), $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{2n-1}$. (Javob: $\frac{\pi}{4}$.)

5.8. $f(x)$= $\cos x$, [0; $\frac{\pi}{2}$], $\sum_{k=1}^{\infty}\frac{(-1)^k}{(2k-1)(2k+1)}$. (Javob: $\frac{2-\pi}{4}$.)

5.9. $f(x)$ =x, (0; π), $\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}$. (Javob: $\frac{\pi^2}{8}$.)

5.10. $f(x)$ = x^2, (- π; π), $\sum_{n=1}^{\infty}\frac{1}{n^2}$. (Javob: $\frac{\pi^2}{6}$.)

5.11. $f(x)$=$x(\pi - x)$, (0; π), sinus bo'yicha, $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{(2n-1)^3}$. (Javob: $\frac{\pi^3}{32}$.)

5.12. $f(x)$=$|\sin x|$, (-π; π), $\sum_{n=1}^{\infty}\frac{(-1)^n}{4n^2-1}$. (Javob: $\frac{2-\pi}{4}$.)

5.13. $f(x)$ = $\begin{cases} 0, -3 < x \le 0, \\ x,\ \ 0 < x < 3, \end{cases}$ $\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}$. (Javob: $\frac{\pi^2}{8}$.)

5.14. $f(x)$ = $\begin{cases} 1, -1 \le x < 0, \\ x,\ \ 0 \le x \le 1, \end{cases}$ $\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}$. (Javob: $\frac{\pi^2}{8}$.)

5.15. $f(x) = |x|$, (-1; 1), $\sum_{n=0}^{\infty}\frac{1}{(2n+1)^2}$. (Javob: $\frac{\pi^2}{8}$.)

5.16. $f(x)$ = x^2, (- π; π), $\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}$. (Javob: $\frac{\pi^2}{8}$.)

5.17. $f(x)=\begin{cases} 1, & -1 \le x < 0, \\ 1/2, & x = 0, \\ x, & 0 < x \le 1, \end{cases}$ $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$. (Javob: $\frac{\pi^2}{8}$.)

5.18. $f(x) = \begin{cases} 1, 0 < x < 1, \\ -1, \ 1 < x < 2, \end{cases}$ $\sum_{n=1}^{\infty} \frac{(-1)^n}{2n+1}$. (Javob: $\frac{\pi}{4}$.)

5.19. $f(x)=\begin{cases} -x, & -4 < x < 0, \\ 1, & x = 0, \\ 2, & 0 < x < 4, \end{cases}$ $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$. (Javob: $\frac{\pi^2}{8}$.)

5.20. $f(x) = \begin{cases} 1, \ 0 \le x < 3/2, \\ -1, \ 3/2 < x < 3, \end{cases}$ $\sum_{n=1}^{\infty} \frac{(-1)^n}{2n+1}$. (Javob: $\frac{\pi}{4}$.)

5.21. $f(x)=\begin{cases} -1, & -2 < x < 0, \\ -1/2, & x = 0, \\ x/2, & 0 < x < 2, \end{cases}$ $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$. (Javob: $\frac{\pi^2}{8}$.)

5.22. $f(x)=\begin{cases} -2x, -2 < x < 0, \\ 2, \quad x = 0, \\ 4, \quad 0 < x < 2, \end{cases}$ $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$. (Javob: $\frac{\pi^2}{8}$.)

5.23. $f(x) = \begin{cases} 0, & -\pi \le x < 0, \\ x-1, & 0 \le x \le \pi, \end{cases}$ $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$. (Javob: $\frac{\pi^2}{8}$.)

5.24. $f(x) = \begin{cases} -2x, & -\pi \le x \le 0, \\ 3x, & 0 < x \le \pi, \end{cases}$ $\sum_{n=1}^{\infty} \frac{(1-(-1)^n)}{n^2}$. (Javob: $\frac{7\pi^2}{20}$.)

5.25. $f(x) = \pi^2 - x^2$, (- π; π), $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$. (Javob: $\frac{\pi^2}{12}$.)

5.26. $f(x) = x\sin x$, [- π; π], $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2-1}$. (Javob: $\frac{1}{4}$.)

5.27. $f(x) = \begin{cases} 0, & -\pi \le x < 0, \\ 1, & 0 \le x \le \pi, \end{cases}$ $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n-1}$. (Javob: $\frac{\pi}{4}$.)

5.28. $f(x) = \begin{cases} -a, & -\pi \le x < 0, \\ a, & 0 \le x \le \pi, \end{cases}$ $\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{2n+1}$. (Javob: $\frac{\pi}{4}$.)

5.29. $f(x) = |\cos x|$, [- π; π], $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{4n^2-1}$. (Javob: $\frac{\pi-2}{4}$.)

5.30. $f(x) = \left|\cos\frac{x}{2}\right|$, [- π; π], $\sum_{n=1}^{\infty} \frac{(-1)^n}{1-4n^2}$. (Javob: $\frac{\pi-2}{4}$.)

Namunaviy variantning yechimi

1.Davriy funksiyaning Furye qatoriga yoying (davri ω $=2\pi$)

$$f(x) = \begin{cases} \pi + x, & -\pi \le x < 0, \\ 0, & 0 \le x \le \pi, \end{cases}$$

►Furye koeffitsiyentini hisoblaymiz:

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{0}(\pi + x)dx = \frac{1}{\pi}\frac{(\pi+x)^2}{2}\Big|_{-\pi}^{0} = \frac{1}{\pi}\frac{\pi^2}{2} = \frac{\pi}{2},$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{0}(\pi + x)\cos nx\, dx = \begin{vmatrix} u = \pi + x, & du = dx, \\ dv = \cos nx\, dx, & v = \frac{1}{n}\sin nx, \end{vmatrix} =$$

$$= \frac{1}{n}\left(\left(\frac{\pi+x}{n}\sin nx\right)\Big|_{-\pi}^{0} - \frac{1}{n}\int_{-n}^{0}\sin nx\, dx\right) = \frac{1}{\pi n^2}\cos nx\,\Big|_{-\pi}^{0} =$$

$$= \frac{1}{\pi n^2}(1 - (-1)^n) = \frac{2}{\pi(2n-1)^2},$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{0}(\ \pi + x)\sin nx\, dx = \begin{vmatrix} u = \pi + x, & du = dx, \\ dv = \sin nx\, dx, & v = -\frac{1}{n}\cos nx \end{vmatrix} =$$

$$= \frac{1}{n}\left(\left(-\frac{\pi+x}{n}\cos nx\right)\Big|_{-\pi}^{0} + \frac{1}{n}\int_{-\pi}^{0}\cos nx\, dx\right) = \frac{1}{\pi}\left(-\frac{\pi}{n} + \frac{1}{n^2}sin\, nx\Big|_{-\pi}^{0}\right) = -\frac{1}{n}.$$

Berilgan funksiya uchun Furye qatori quyidagicha ko‘rinishga ega bo‘ladi

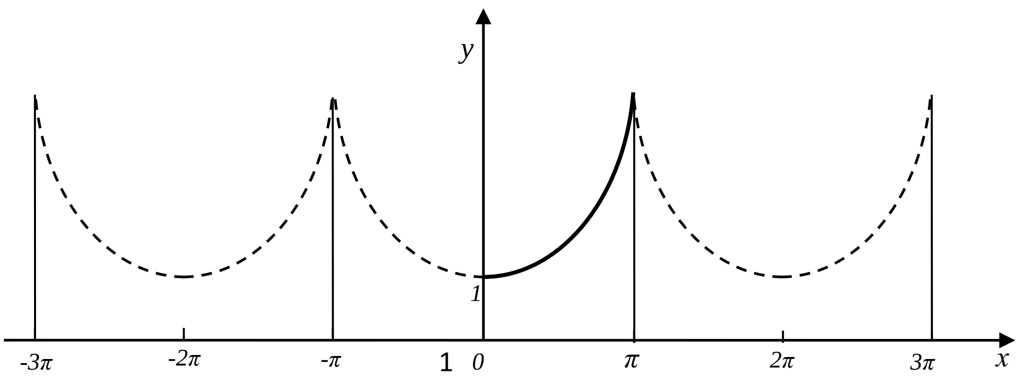
$$f(x) = \frac{\pi}{4} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\cos((2n-1)\pi x)}{(2n-1)^2} - \sum_{n=1}^{\infty}\frac{\sin(n\pi x)}{n}.$$ ◄

2. (0; π) oraliqda berilgan quyidagi $f(x) = 8^{x/2}$ funksiyani, juft va toq ko‘rinishda davom ettirib Furye qatoriga yoying. Juft va toq ko‘rinish uchun grafikni aniqlang.

►Funksiyani juft qilib davom ettiramiz (12.7-rasm). U holda:

$$a_0 = \frac{2}{\pi}\int_0^{\pi} 8^{x/2} = \frac{2}{\pi}\cdot 2\cdot\frac{8^{x/2}}{\ln 8}\Big|_0^{\pi} = \frac{4}{\pi\ln 8}\left(8^{\pi/2} - 1\right),$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} 8^{x/2}\cos nx\, dx.$$



12.7- rasm

Ushbu $\int 8^{x/2} \cos nx\, dx$, integralni ikki martta bo‘laklab integrallab hisoblaymiz:

$$\int 8^{x/2} \cos nx\, dx = \begin{vmatrix} u = 8^{x/2}, \; du = \frac{1}{2} \cdot 8^{x/2} \ln 8\, dx, \\ dv = \cos nx\, dx, \;\; v = \frac{1}{n} \sin nx \end{vmatrix} =$$

$$= \frac{1}{n} 8^{x/2} \sin nx - \frac{\ln 8}{2n} \int 8^{x/2} \sin nx\, dx =$$

$$= \begin{vmatrix} u = 8^{x/2}, \; du = \frac{1}{2} \cdot 8^{x/2} \ln 8\, dx, \\ dv = \sin nx\, dx, \;\; v = -\frac{1}{n} \cos nx\,, \end{vmatrix} = \frac{1}{n} \cdot 8^{x/2} \sin nx +$$

$$+ \frac{\ln 8}{2n^2} \cdot 8^{x/2} \cos nx - \frac{\ln^2 8}{4n^2} \int 8^{x/2} \cos nx\, dx,$$

$$\left(1 + \frac{\ln^2 8}{4n^2}\right) \int 8^{x/2} \cos nx\, dx = \frac{1}{n} \cdot 8^{x/2} \sin nx + \frac{\ln 8}{2n^2} \times 8^{x/2} \cos nx,$$

$$\int 8^{x/2} \cos nx\, dx = \frac{4n^2}{4n^2 + \ln^2 8} \left(\frac{1}{n} \cdot 8^{x/2} \sin nx + \frac{\ln 8}{2n^2} \cdot 8^{x/2} \cos nx\right).$$

a_n koeffitsiyentlarini hisoblaymiz:

$$a_n = \frac{8n^2}{\pi(4n^2 + (\ln 8)^2)} \left(\frac{1}{n} \cdot 8^{x/2} \sin nx + \frac{\ln 8}{2n^2} \times 8^{x/2} \cos nx\right) \Big|_0^{\pi} =$$

$$= \frac{4 \ln 8\, (8^{\pi/2}(-1)^n - 1)}{\pi(4n^2 + (\ln 8)^2)}.$$

Berilgan funksiyani kosinuslar bo‘yicha yoyilmasi quyidagi ko‘rinishda bo‘ladi

$$8^{x/2} = \frac{2(8^{\pi/2} - 1)}{\pi \ln 8} + \frac{4 \ln 8}{\pi} \sum_{n=1}^{\infty} \frac{8^{\pi/2} \cdot (-1)^n - 1}{4n^2 + (\ln 8)^2} \cos nx\,.$$

Berilgan funksiyani toq ko‘rinishda davom ettiramiz (12.8-rasm). U holda:

$$b_n = \frac{2}{\pi} \int 8^{x/2} \sin nx\, dx,$$

$$\int 8^{x/2}\sin nx\,dx = \left|\begin{matrix} u = 8^{x/2}, du = \frac{1}{2}\cdot 8^{x/2}\ln 8\,dx, \\ dv = \sin nx\,dx,\ \ v = -\frac{1}{n}\cos nx \end{matrix}\right| =$$

$$= -\frac{1}{n}\cdot 8^{x/2}\cos nx + \frac{\ln 8}{2n}\int 8^{x/2}\cos nx\,dx =$$

$$= \left|\begin{matrix} u = 8^{x/2},\ du = \frac{1}{2}\cdot 8^{x/2}\ln 8\,dx, \\ dv = \cos nx\,dx,\quad v = \frac{1}{n}\sin nx\,dx \end{matrix}\right| =$$

$$= -\frac{1}{n}\cdot 8^{x/2}\cos nx + \frac{\ln 8}{2n^2}\cdot 8^{x/2}\sin nx - \frac{\ln^2 8}{4n^2}\int 8^{x/2}\sin nx\,dx,$$

$$b_n = \frac{8n^2}{\pi(4n^2 + (\ln 8)^2)}\left(-\frac{1}{n}\cdot 8^{x/2}\cos nx + \frac{\ln 8}{2n^2}\cdot 8^{x/2}\sin nx\right)\Big|_0^{\pi} =$$

$$= \frac{8n(\cdot 8^{\pi/2}(-1)^{n+1} + 1)}{\pi((4n^2 + (\ln 8)^2)}.$$

Demak berilgan funksiyani sinuslar bo‘yicha yoyilmasi

$$8^{x/2} = \frac{8}{\pi}\sum_{n=1}^{\infty}\frac{8^{\pi/2}(-1)^{n+1}}{4n^2 + \ln^2 8}n\sin nx$$

bo‘lar ekan. ◀

3. Davri $\omega = 2$ ga teng bo‘lgan funksiyani Furye qatoriga yoying

$$f(x)=\begin{cases}1, & -1\le x<0,\\ 0.5, & x=0,\\ x, & 0<x\le 1.\end{cases}$$

►Furye koeffitsiyentlarini hisoblaymiz:

$$a_0=\int_{-1}^{0}dx+\int_{0}^{1}xdx=x|_{-1}^{0}+\frac{x^2}{2}\Big|_0^1=1+\frac{1}{2}=\frac{3}{2},$$

$$a_n=\int_{-1}^{0}\cos(n\pi x)\,dx\;+\int_{0}^{1}x\cos(n\pi x)\,dx=$$

$$=\begin{vmatrix}u=x,\;\; du=dx,\\ dv=\cos(n\pi x)\,dx,\;\; v=\frac{\sin(n\pi x)}{n\pi}\end{vmatrix}=$$

$$=\frac{1}{n\pi}\sin(n\pi x)\,|_{-1}^{0}+\frac{1}{n\pi}x\sin(n\pi x)\,\Big|_0^1-$$

$$-\frac{1}{n\pi}\int_{0}^{1}\sin(n\pi x)\,dx=\frac{1}{n^2\pi^2}\cos(n\pi x)\,|_0^1=\frac{1}{n^2\pi^2}((-1)^n-1),$$

$$a_n=\frac{-2}{\pi^2(2n-1)^2},$$

$$b_n=\int_{-1}^{0}\sin(n\pi x)\,dx+\int_{0}^{1}x\sin(n\pi x)\,dx=$$

$$=\begin{vmatrix}u=x,\;\; du=dx,\\ dv=\sin(n\pi x)\,dx,\;\; v=\frac{1}{n\pi}\cos(n\pi x)\end{vmatrix}=$$

$$=-\frac{1}{n\pi}\cos(n\pi x)\,\Big|_{-1}^{0}-\frac{x}{n\pi}\cos(n\pi x)\,\Big|_0^1+\frac{1}{n\pi}\int_{0}^{1}\cos(n\pi x)\,dx=$$

$$-\frac{1}{n\pi}(1-(-1)^n)-\frac{1}{n\pi}(-1)^n-\frac{1}{n^2\pi^2}\sin(n\pi x)\,\Big|_0^1=-\frac{1}{n\pi}.$$

Natijada quyidagi

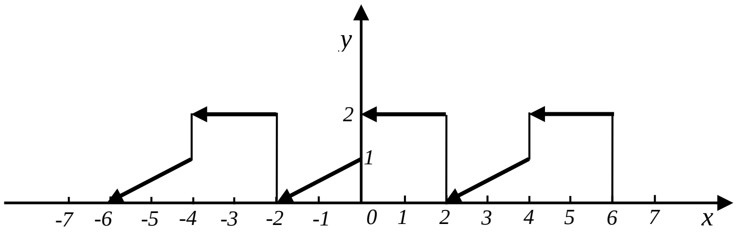
$$f(x)=\frac{3}{4}-\frac{2}{\pi^2}\sum_{n=1}^{\infty}\frac{\cos\big((2n-1)\pi x\big)}{(2n-1)^2}-\frac{1}{\pi}\sum_{n=1}^{\infty}\frac{\sin(n\pi x)}{n}.$$

Furye qatoriga ega bo‘lamiz. ◄

4. Grafik ko‘rinishda berilgan funksiyani Furye qatoriga yoying(12.9-rasm).

►Funksiyani analitik ko‘rinishda yozib olamiz:

$$f(x)=\begin{cases}0{,}5x+1, -2<x\le 0,\\ 2, \qquad\qquad 0<x\le 2,\end{cases}\ \omega=4.$$



12.9- rasm

Furye koeffitsiyentlarini hisoblaymiz:

$$a_0=\frac{1}{2}\int_{-2}^{0}\left(\frac{1}{2}x+1\right)dx+\frac{1}{2}\int_{0}^{2}2dx=\frac{1}{2}\left(\frac{x^2}{4}+x\right)\Big|_{-2}^{0}+$$

$$+x|_0^2=-\frac{1}{2}(1-2)+2=\frac{5}{2},$$

$$a_n=\frac{1}{2}\int_{-2}^{0}\left(\frac{1}{2}x+1\right)\cos\frac{n\pi x}{2}dx+\int_{0}^{2}\cos\frac{n\pi x}{2}dx=$$

$$=\left|\begin{matrix}u=x/2+1, du=(1/2)dx,\\ dv=\cos\frac{n\pi x}{2}dx,\ \ v=\frac{2}{n\pi}\sin\frac{n\pi x}{2}\end{matrix}\right|=$$

$$=\frac{x/2+1}{n\pi}\sin\frac{n\pi x}{2}\Big|_{-2}^{0}-\frac{1}{2n\pi}\int_{-2}^{0}\sin\frac{n\pi x}{2}dx+\frac{2}{n\pi}\sin\frac{n\pi x}{2}\Big|_0^2=$$

$$=\frac{1}{n^2\pi^2}\cos\frac{n\pi x}{2}\Big|_{-2}^{0}=\frac{1}{n^2\pi^2}((-1)^{n+1}+1)=\frac{2}{\pi^2(2n-1)^2},$$

$$b_n=\frac{1}{2}\int_{-2}^{0}\left(\frac{1}{2}x+1\right)\sin\frac{n\pi x}{2}dx+\int_{0}^{2}\sin\frac{n\pi x}{2}dx=$$

$$=\left|\begin{matrix}u=x/2+1, du=(1/2)dx,\\ dv=\sin\frac{n\pi x}{2}dx,\ \ v=-\frac{2}{n\pi}\cos\frac{n\pi x}{2}\end{matrix}\right|=$$

$$= -\frac{x/2+1}{n\pi}\cos\frac{n\pi x}{2}|_{-2}^{0} + \frac{1}{2n\pi}\int_{-2}^{0}\cos\frac{n\pi x}{2}dx -$$

$$-\frac{2}{n\pi}((-1)^n - 1) - \frac{2}{n\pi}\cos\frac{n\pi x}{2}|_0^2 =$$

$$= -\frac{1}{n\pi} + \frac{1}{n^2\pi^2}\sin\frac{n\pi x}{2}|_{-2}^{0} - \frac{2}{n\pi}(-1)^n + \frac{2}{n\pi} = \frac{1}{n\pi} - \frac{2}{n\pi}(-1)^n$$

$$= \frac{(1+2(-1)^{n+1})}{n\pi}.$$

Demak

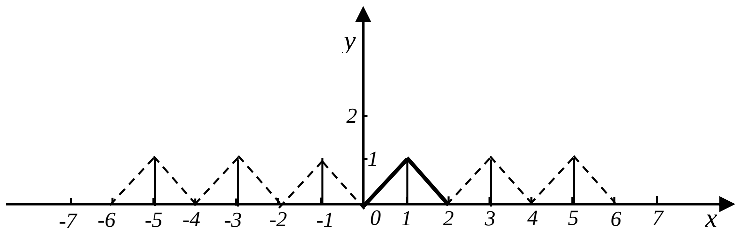
$$f(x) = \frac{5}{4} + \frac{2}{\pi^2}\sum_{n=1}^{\infty}\frac{\cos\big((2n-1)\,\pi x/2\big)}{(2n-1)^2} + \frac{1}{\pi}\sum_{n=1}^{\infty}\frac{(1+2(-1)^{n+1})}{n}\sin\frac{n\pi x}{2}$$

kabi Furye qatoriga ega bo‘lamiz.◄

5.Ushbu fuksiyani [0; 2] kesmada Furye qatoriga kosinuslar bo‘yicha yoying:

$$f(x) = \begin{cases} x, & 0 \le x \le 1, \\ 2-x, & 1 < x \le 2, \end{cases}$$

va $\sum_{n=1}^{\infty}\frac{1}{(2n+1)^2}$ qatorning yig‘indisini toping.



12.10- rasm

►Funksiyani juft ko‘rinishda davom ettiramiz va Furye koeffitsiyentlarini hisoblaymiz:

$$a_0 = \int_0^1 x dx + \int_1^2 (2-x)dx = \frac{x^2}{2}|_0^1 + \left(2x - \frac{x^2}{2}\right)\Big|_1^2 = \frac{1}{2} + 2 - \frac{3}{2} = 1,$$

$$a_n = \int_0^1 x\cos\frac{n\pi x}{2}dx + \int_1^2 (2-x)\cos\frac{n\pi x}{2}dx =$$

$$= \begin{vmatrix} u = x, \quad du = dx, \\ dv = \cos\frac{n\pi x}{2}dx, v = \frac{2}{n\pi}\sin\frac{n\pi x}{2} \end{vmatrix} + \begin{vmatrix} u = 2 - x, du = -dx, \\ dv = \cos\frac{n\pi x}{2}dx, \ \ v = \frac{2}{n\pi}\sin\frac{n\pi x}{2} \end{vmatrix} =$$

$$= \frac{2x}{n\pi}\sin\frac{n\pi x}{2}|_0^1 - \frac{2}{n\pi}\int_0^1 \sin\frac{n\pi x}{2}dx + \frac{2(2-x)}{n\pi}\sin\frac{n\pi x}{2}|_1^2 + \frac{2}{n\pi}\int_1^2 \sin\frac{n\pi x}{2}dx =$$

$$= \frac{2}{n\pi}\sin\frac{n\pi}{2} + \frac{4}{n^2\pi^2}\cos\frac{n\pi x}{2}|_0^1 - \frac{2}{n\pi}\sin\frac{n\pi}{2} - \frac{4}{n^2\pi^2}\cos\frac{n\pi x}{2}|_1^2$$

$$= -\frac{4}{\pi^2(2n+1)^2}.$$

Demak,

$$f(x) = \frac{1}{2} - \frac{4}{\pi^2}\sum_{n=1}^{\infty}\frac{\cos(2n+1)\pi x}{(2n+1)^2}.$$

$x = 0$ deb faraz qilib quyidagiga ega bo‘lamiz:

$$0 = \frac{1}{2} - \sum_{n=1}^{\infty}\frac{1}{(2n+1)^2},$$

$$\sum_{n=1}^{\infty}\frac{1}{(2n+1)^2} = \frac{\pi^2}{8}.$$

Furye qatori yordamida sonli qatorning yig‘indisini hisobladik. ◄

12-bo‘limga doir qo‘shimcha masalalar

1.Qatorning yig‘indisini toping

$\sum_{n=1}^{\infty}\frac{1}{(n+1)(n+2)(2n+1)(2n+5)}$. (Javob:$1/90.$)

2.Qatorning yaqinlashishini tekshiring

$\frac{1}{2} + \frac{3}{7} + (\frac{5}{10})^{3/2} + \ldots + (\frac{2n-1}{2n+1})^{n/2} + \ldots$ (Javob: yaqinlashadi.)

3.Agar $\sum_{n=1}^{\infty} a_n$ absolyut yaqinlashsa, $\sum_{n=1}^{\infty}\frac{n+1}{n}a_n$ qator ham absolyut yaqinlashishini isbotlang.

4.Qatorni absolyut yaqinlashish va yaqinlashishga tekshiring $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}\cdot 3^n}{(2n+1)^n}$. (Javob: absolyut yaqinlashadi.)

5.Quyidagi $1-\sum_{n=1}^{\infty}(\frac{3}{2})^n$ va $1+\sum_{n=1}^{\infty}(\frac{3}{2})^{n-1}\cdot\left(2^n+2^{-(n+1)}\right)$, qatorlarni ko‘paytirishdan hosil bo‘lgan qator absolyut yaqinlashishini ko‘rsating.

6.Ushbu, $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n\cdot 2^n}$ qatorning yig‘indisi “S”ni xususiy S_n-yig‘indiga almashtirganda absolyut xatolik $\alpha=10^{-3}$ dan oshmaslik uchun, yani $|S-S_n|=|r_n|\le\alpha$ bo‘lishi uchun nechta had olish kerak? (Javob:n≥7.)

7.Berilgan $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{2n-1}{n^2}$ qatorning yig‘indisini 0,01 aniqlikda hisoblash uchun nechta had olish kerak? (Javob: n=200.)

8.Hadlab differinsiallash va integrallash yo‘li bilan ushbu qatorning yig‘indisini toping $1-3x^2+5x^4+\ldots+(-1)^{n-1}(2n-1)x^{2n-2}$. (Javob: $S(x)=\frac{1-x^2}{(1+x^2)^2}, |x|<1$.)

9.Agar $0\le x\le\pi$ bo‘lsa $\sum_{n=1}^{\infty}\frac{\cos nx}{n^2}=\frac{3x^2-6\pi x+2\pi^2}{12}$ tenglikni isbotlang.

10.Shunday ikkita qator tanlangki, ularning yig‘indisi yaqinlashuvchi, ayirmasi uzoqlashuvchi bo‘lsin.

11.Berilgan $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^n}{\sqrt{n}}$ qatorning [0;1] kesmada tekis yaqinlashuvchi bo‘lishini ko‘rsating.

12.Umumiy hadi $u_n=\int_0^{1/n}\frac{\sqrt{x}dx}{x^2+1}$ bo‘lgan qatorni yaqinlashishga tekshiring.

(Javob: yaqinlashadi,$u_n\le\frac{2}{3n^{3/2}}$.)

13.Ushbu funksiya $y=\sum_{n=1}^{\infty}\frac{x^{2n}}{2^n n!}$ quyidagi $y'-xy=0$ differensial tenglamani yyechimi bo‘lishini isbotlang.

13. KARRALI INTEGRALLAR

13.1. Ikki o‘lchovli integrallar va ularni hisoblash

Oxy tekislikda L yopiq egri chiziq bilan chegaralangan yopiq D sohada $z = f(x,y)$ funksiya berilgan bo‘lsin. Ixtiyoriy egri chiziqlar bilan D sohani yuzi ΔS_i ga teng bo‘lgan elementar S_i- sohalarga bo‘lamiz ($i = \overline{1,n}$) (13.1 rasm). Har bir S_i- sohadan ixtiyoriy $P(x_i, y_i)$ nuqtani tanlaymiz. S_i-sohaning diametri deb shu sohaning chegaraviy nuqtalarini tutashiruvchi vatarlarning eng kattasiga aytiladi.

Quyidagi ifoda

$$I_n = \sum_{i=1}^{n} f(x_i, y_i)\Delta S_i \qquad (13.1)$$

$z = f(x,y)$ funksiya uchun D sohada n - integral yig‘indi deyiladi.

Sohani ixtiyoriy ravishda bo‘lganimiz va nuqtalarni ulardan tasodifiy tanlaganimiz uchun cheksiz ko‘p integral yig‘indi tuzishimiz mumkin. Ammo mavjudlik va yagonalik teoremasiga ko‘ra, agar $z = f(x,y)$ funksiyamiz D sohada uzluksiz va L egri chiziq bo‘lakli silliq bo‘lsa, bu yig‘indilarning limiti $d_i \to 0$ mavjud va yagona bo‘ladi.

Berilgan $z = f(x,y)$ funksiyadan D-soha bo‘yicha ikki o‘lchovli integral deb ushbu $lim_{d_i \to 0} I_n$ limitga aytiladi va quyidagicha belgilanadi

$$\iint_D f(x,y)dS = \lim_{d_i \to 0} \sum_{i=1}^{n} f(x_i, y_i)\, \Delta S_i. \qquad (13.2)$$

Bu yerda va keyinchalik $z = f(x,y)$ funksiyani D sohada uzluksiz, L egri chiziqni bo‘lakli-silliq deb faraz qilamiz, shu sababli (13.2) formulada limit doim mavjud bo‘ladi.

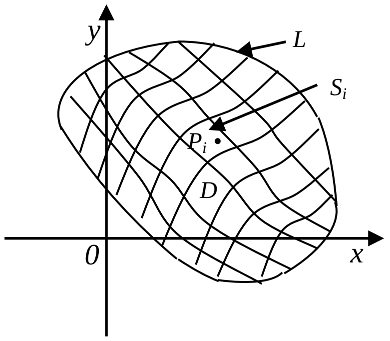
Ikki o‘lchovli integralning asosiy xossalari va uning geometrik va fizik ma’nosini keltirib o‘tamiz.

1.$\iint_D dS = S_D$, bu yerda S_D-integrallanuvchi sohaning yuzi.

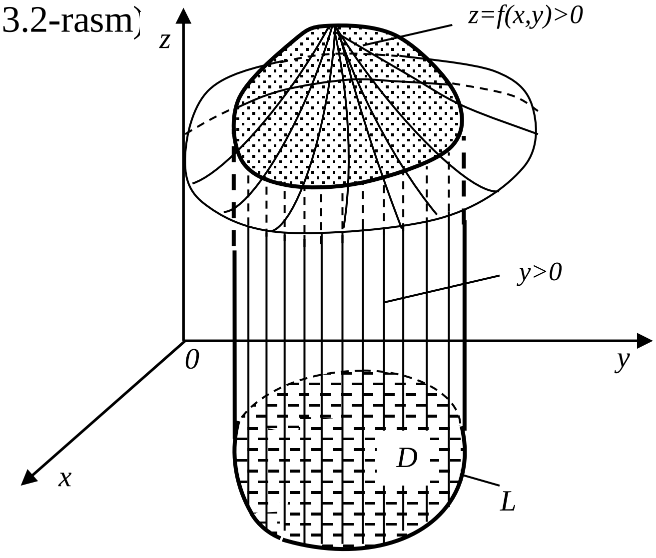
2.Agar $z = f(x,y) = \mu(x,y)$ D sohani egallovchi material plastinkaning zichligi bo‘lsa, u holda plastinkaning massasi quyidagi formula orqali aniqlanadi

$$m = \iint_D \mu(x,y)dS \qquad (13.3)$$

3.Agar D sohada $f(x,y) \geq 0$ bo‘lsa, u holda ikki o‘lchovli integral (13.2), pastki asosi Oxy tekislikda yotgan D soha, yuqori qismi D sohaga akslanuvchi $z = f(x,y)$ sirt, yon tomoni D sohaning chegarasi bo‘lgan L egri chiziqdan o‘tuvchi “ Oz” o‘qiga parallel bo‘lgan to‘g‘ri chiziqlardan iborat silindrik jismning V hajmiga teng bo‘ladi (13.2-rasm).



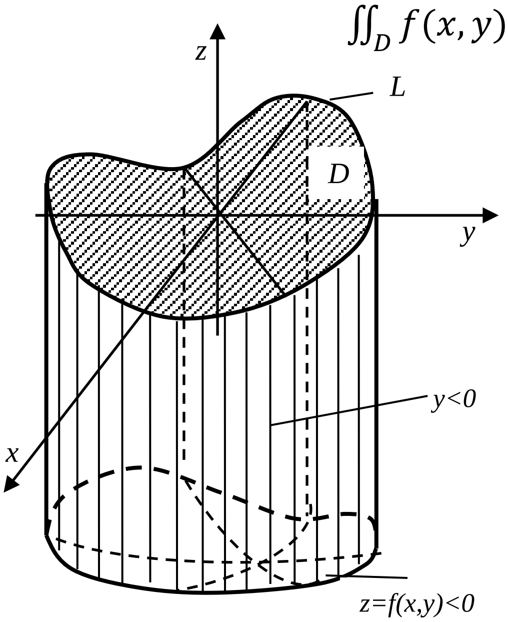
13.1- rasm



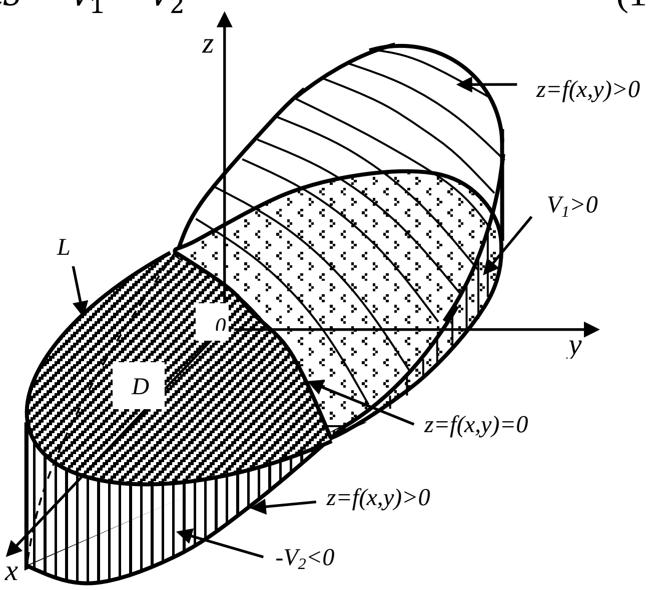
13.2- rasm

Agar $f(x,y) \leq 0$ bo‘lsa ikki o‘lchovli integral Oxy tekislikning past qismida joylashgan silindrik jismning hajmiga teng bo‘ladi (13.3-rasm) va ishorasi “-“ olinadi, (-V). Agar $f(x,y)$ funksiya D sohada ishorasini o‘zgartirsa, ikki o‘lchovli integral Oxy tekislikdan yuqorida va pastda joylahsgan silindrik jismlarning hajmlari ayirmasiga teng, ya’ni (13.4-rasm)

$$\iint_D f(x,y)dS = V_1 - V_2 \qquad (13.4)$$



13.3- rasm



13.4- rasm

Bu xossa ikki o‘lchovli integralning geometrik ma’nosini anglatadi.

4.Agar $z = f_i(x, y)$ $(i = \overline{1, k})$ funksiyalar D sohada uzluksiz bo‘lsa quyidagi formula o‘rinli bo‘ladi

$$\iint_D \left(\sum_{i=1}^k f_i(x, y)\right) dS = \sum_{i=1}^k \iint_D f_i(x, y) dS.$$

5.O‘zgarmas ko‘paytuvchi C ni ikki o‘lchovli integral belgisidan tashqariga chiqarish mumkin:

$$\iint_D Cf(x, y)dS = C\iint_D f(x, y)dS$$

6.Agar D sohani chekli sondagi $D_1, D_2, \dots, D_k$ umumiy ichki nuqtalarga ega bo‘lmagan sohalarga bo‘lib chiqsak, u holda D soha bo‘yicha integral:

$$\iint_D f(x, y)dS = \iint_{D_1} f(x, y)dS + \iint_{D_2} f(x, y)dS + \dots + \iint_{D_k} f(x, y)dS$$

7.(**O‘rta qiymat haqida teorema)**. D- sohada uzluksiz bo‘lgan $z = f(x, y)$ funksiya uchun, kamida bitta shunday nuqta topiladiki $P(\xi, \eta) \in D$ **(**bu yerda S_D- D sohaning yuzi) quyidagi tenglik o‘rinli bo‘ladi

$$\iint_D f(x, y)dS = f(\xi, \eta)S_D.$$

$f(\xi, \eta)$- son $f(x, y)$ funksiyaning *D* sohadagi o‘rta qiymati deyiladi.

8.Agarda *D* sohada berilgan uzluksiz $f(x, y)$, $f_1(x, y)$, $f_2(x, y)$ funksiyalar uchun quyidagi tengsizlik o‘rinli bo‘lsa $f_1(x, y) \le f(x, y) \le f_2(x, y)$, u holda

$$\iint_D f_1(x, y)dS \le \iint_D f(x, y)dS \le \iint_D f_2(x, y)dS$$

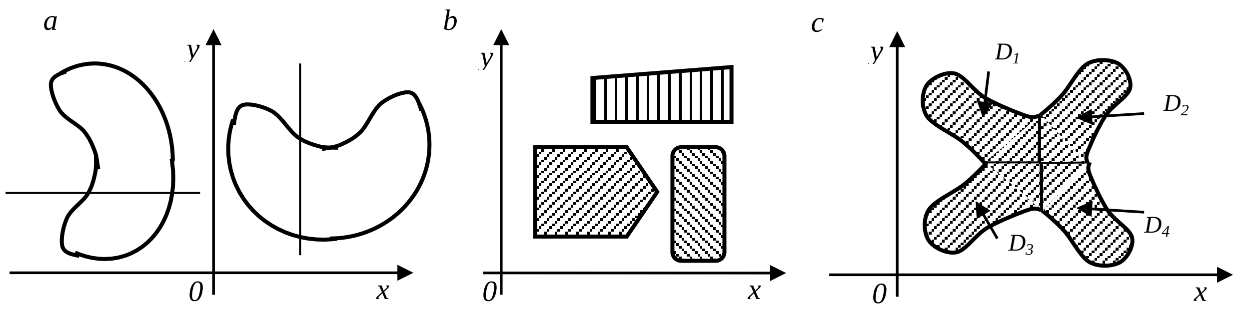
9.Agar D sohada $z = f(x, y) \ne const$ va uzluksiz, $M = max\, f(x, y)\,, m = min\, f(x, y)$ bo‘lsa, u holda

$$mS_D < \iint_D f(x, y)dS < MS_D.$$

Eslatma Xususiy I_n −integral yig‘indi (13.1, 13.2 formulalar) D sohani elementar S_i −sohalarga bo‘lish usuliga bog‘liq bo‘lmaganligi uchun (mavjudlik va yagonalik teoremasi), dekart koordinatalar sistemasida D sohani koordinatalar o‘qiga parallel to‘g‘ri chiziqlar bilan elementar “S_i” sohalarga to‘rtburchaklar bilan bo‘lish qulay. Natijada $dS = dxdy$ va

$$\iint_D f(x,y)dS = \iint_D f(x,y)dxdy$$

D –integrallash sohasi $Ox(Oy)$ o‘qiga nisbatan to‘g‘ri deyiladi, agarda $Ox(Oy)$ o‘qiga parallel to‘g‘ri chiziq uning chegarasi L egri chiziqni ikkitadan ortiq bo‘lmagan nuqtada kesib o‘tsa (13.5,a-rasm) D soha to‘g‘ri soha deyiladi agarda uning butun chegarasi yoki bir qismi to‘g‘ri chiziqdan iborat bo‘lsa (13.5,b-rasm).



13.5- rasm

Ikki o‘lchovli integralni to‘g‘ri sohalar bo‘yicha hisoblash usullarini ko‘rib o‘tamiz, chunki har bir sohani to‘g‘ri sohalar birlashmasi ko‘rinishda tasvirlash mumkin (13.5,c-rasm), u holda ikki o‘lchovli integralning 6-xossasiga ko‘ra bu usullar har qanday soha bo‘yicha hisoblash uchun to‘g‘ri bo‘ladi.

Ikki o‘lchovli integralni hisoblash uchun integral ostidagi funksiyani o‘zgaruvchilarning biri bo‘yicha integrallaymiz (D- to‘g‘ri sohada o‘zgarishi bo‘yicha), ikkichi o‘zgaruvchini o‘zgarmas deb hisoblaymiz. Olingan natijani ikkinchi o‘zgaruvchi bo‘yicha uning D sohadagi maksimal o‘zgarish oralig‘ida integrallaymiz. U holda $f(x,y)dxdy$ ko‘paytma ikki o‘lchovli integralda yig‘indida bir martadan qatnashadi, D sohaga tegishli bo‘lmagan ko‘paytmalar qatnashmaydi.

Agar D-soha Oy o‘qiga nisbatan to‘g‘ri bo‘lib, Ox o‘qidagi $[a,b]$ kesmaga akslansa, u holda uning chegarasi L chiziq ikkita $AmB, y = \varphi_1(x)$ va $AnB, y = y = \varphi_2(x)$ chiziqlarga ajraladi(13.6-rasm). U holda D soha quyidagi tengsizliklar sistemasi bilan aniqlanadi:

$$D: a \le x \le b, \varphi_1(x) \le y \le \varphi_2(x),$$

va ikki o‘lchovli integral ushbu qoida asosida hisoblanadi (ichki integral y o‘zgaruvchi bo‘yicha, tashqi integral x bo‘yicha)

$$\iint_D f(x,y)dxdy = \int_a^b dy \int_{\varphi_1(x)}^{\varphi_2(x)} f(x,y)dy. \qquad (13.5)$$

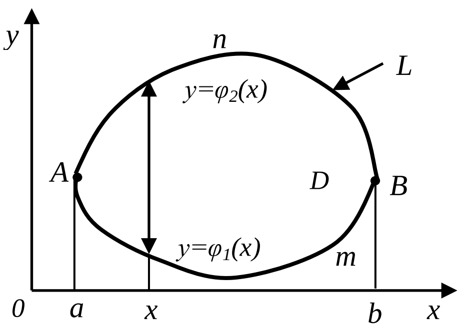
Agar D soha Ox o‘qi yo‘nalishi bo‘yicha to‘g‘ri bo‘lsa va Oy o‘qida [c,d] kesmaga akslansa, u holda uning chegarasi L egri chiziq ikkita $CpD^*, x = \varphi_1(y)$va $CqD^*, x = \varphi_2(y)$ egri chiziqlarga ajraladi (13.7-rasm). U holda D soha tengsizliklar sistemasi bilan aniqlanadi:

$$D: c \le y \le d, \;\; \varphi_1(y) \le x \le \varphi_2(y),$$

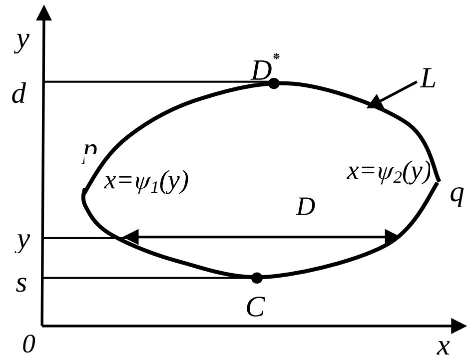
va ikki o‘lchovli integral quyidagi qonun asosida hisoblanadi (ichki integral x – o‘zgaruvchi bo‘yicha, tashqi integral- y o‘zgaruvchi bo‘yicha)

$$\iint_D f(x,y)dxdy = \int_c^d dx \int_{\varphi_1(y)}^{\varphi_2(y)} f(x,y)dx. \qquad (13.6)$$

Yuqorida (13.5), (13.6) formulaning o‘ng tomonida ifodalar ikki karrali integral deyiladi.



13.6- rasm



13.7- rasm

Yuqoridagi (13.5) va (13.6) tengliklardan ushbu formulaga ega bo‘lamiz:

$$\int_a^b dx \int_{\varphi_1(x)}^{\varphi_2(x)} f(x,y)dy = \int_c^d dy \int_{\varphi_1(y)}^{\varphi_2(y)} f(x,y)dx. \qquad (13.7)$$

(13.7)- formulaning chap tomonidan o‘ng tomoniga o‘tish va aksincha ikki karrali integralda tartibni o‘zgartirish deyiladi.

1- misol. Oxy tekislikda o‘zgaruvchilarning ikki o‘lchovli

$I = \int_0^4 dx \int_{\frac{3x^2}{8}}^{3\sqrt{x}} dy$ integral chegarasiga asosan D sohani quring. Integrallash tartibini o‘zgartiring va integralni berilgan chegarada va o‘zgargan tartibda hisoblang.

►Integrallash sohasi D $x = 0$ va $x = 4$ to‘g‘ri chiziqlar orasida joylashgan quyidan $y = 3x^2/8$ parabola bilan, yuqoridan $y = 3\sqrt{x}$ parabola bilan chegaralangan.(13.8-rasm). Demak,

$$I = \int_0^4 (y|_{3x^2/8}^{3\sqrt{x}})\, dx = \int_0^4 (3\sqrt{x} - 3x^2/8)dx = \left(2x^{3/2} - x^3/8\right)|_0^4 = 8.$$

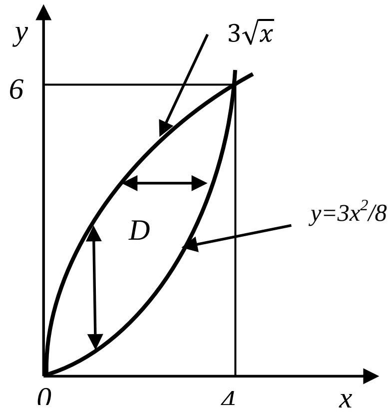
Boshqa tomondan, integrallash sohasi D $y = 0$ va $y = 6$ to‘g‘ri chiziqlar orasida joylashgan, o‘zgaruvchi x, y ning har bir o‘zgarmas qiymatida $x = y^2/9$ parabolaning qiymatlaridan $x = \sqrt{8y/3}$ parabolaning nuqtalarigacha o‘zgaradi, (13.7-rasm) formulaga ko‘ra,

$$I = \int_0^6 dy \int_{y^2/9}^{\sqrt{8y/3}} dx = \int_0^6 (\sqrt{\frac{8y}{3}} - \frac{y^2}{9})dy = \left(2\sqrt{\frac{2}{3}} \cdot \frac{2}{3} y^{3/2} - y^3 \cdot \frac{1}{27}\right)|_0^6 = 8.$$◄

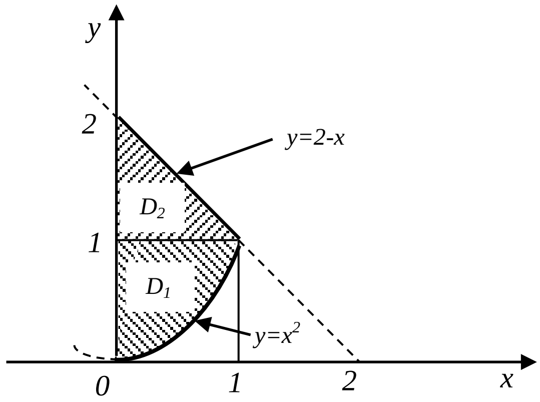
2-misol. Karrali integralda integrallash tartibini o‘zgartiring.

$$\int_0^1 dx \int_{x^2}^{2-x} f(x, y)\, dy.$$

►Integrallash sohasi D $x = 0,\ x = 1,\ y = x^2$ va $y = 2 - x$ (13.9-rasm) chiziqlar bilan chegaralangan sohaning o‘ng chegarasi ikkita chiziq bilan berilgan $y = 1$ to‘g‘ri chiziq uni $D_1: 1 \le y \le 2, 0 \le x \le \sqrt{y}$ va $D_2: 1 \le y \le 2,\ 0 \le x \le 2 - y$ sohalarga ajratadi. Natijada:



13.8- rasm



13.9- rasm

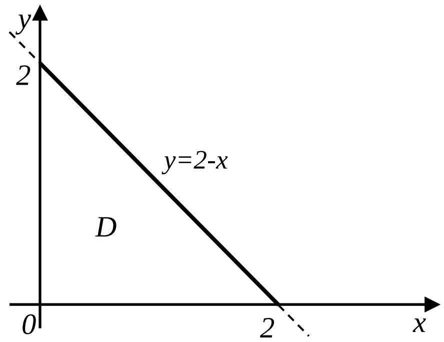
$\int_0^1 dx \int_{x^2}^{2-x} f(x,y)\,dy = \int_0^1 dy \int_0^{\sqrt{y}} f(x,y)dx + \int_1^2 dy \int_0^{2-y} f(x,y)dx.$◄

3-misol. Agar

D soha $x+y=2,\ x=0,\ y=0$ chiziqlar bilan chegaralangan bo‘lsa

$\iint_D (x+y+3)dxdy,$

Ikki o‘lchovli integralni hisoblang.



13.10- rasm

►Integrallanish sohasi $y=2-x$ to‘g‘ri chiziq va koordinatalar o‘qi bilan chegaralangan (13.10-rasm). Demak:

$$\iint_D (x+y+3)dxdy = \int_0^2 dx \int_0^{2-x} (x+y+3)\,dy =$$

$$\int_0^2 \frac{(x+y+3)^2}{2}\Big|_{y=0}^{y=2-x} dx = \frac{1}{2}\int_0^2 (25-(x+3)^2)\,dx =$$

$$= \frac{1}{2}\left(25x - \frac{(x+3)^3}{3}\right)\Big|_0^2 = \frac{26}{3}.◄$$

4-misol. $y=x,\ y=3x,\ x=2$ to‘g‘ri chiziqlar bilan chegaralangan uchburchakda aniqlangan $z=x+6y$ funksiyaning o‘rta qiymatini toping.

►D sohada aniqlangan $z=f(x,y)$ funksiyaning o‘rta qiymati (ikki o‘lchovli integralning 7-xossasi)

$$\bar{f} = \frac{1}{S_D}\iint_D f(x,y)dxdy.$$

Dastlab D sohaning yuzini hisoblaymiz:

$$S_D = \iint_D dxdy = \int_0^2 dx \int_x^{3x} dy = \int_0^2 (3x-x)dx = x^2\Big|_0^2 = 4.$$

Integralni hisoblaymiz:

$$\iint_D (x+6y)dxdy = \int_0^2 dx \int_x^{3x} (x+6y)dy = \int_0^2 \frac{1}{12}(x+6y)^2|_x^{3x} dx =$$

$$\frac{1}{12}\int_0^2 ((19x)^2-(7x)^2)dx = \frac{1}{12}\int_0^2 312x^2\,dx = 26\int_0^2 x^2dx = \frac{26}{3}x^3|_0^2 = \frac{208}{3}.$$

Demak,

$$\bar{f} = \frac{1}{4}\cdot\frac{208}{3} = \frac{52}{3}. \blacktriangleleft$$

AT-13.1

1.Quyidagi karrali integrallarni hisoblang:

a) $\int_0^2 dx \int_0^1 (x^2+2y)\,dx$; b) $\int_{-3}^8 dy \int_{y^2-4}^5 (x+2y)\,dx$; c) $\int_1^2 dx \int_{1/x}^{x} \frac{x^2dy}{y^2}$.

(Javob: a) $14/3$; b) 50,4; c) 2,25.)

2. Agar integrallash sohasi D ma'lum bo'lsa ikki o'lchovli $\iint_D f(x,y)dxdy$ ni chegarasini qo'ying

a) to'g'ri chiziqlar $x=1, x=4, 3x-2y+4=0, 3x-2y-1=0$ bilan chegaralangan

b) egri chiziq $x^2+y^2-4x=0$ bilan chegaralangan;

c) uchlari O(0;0), A(1;3), B(1,5) nuqtada bo'lgan uchburchak soha;

d) ushbu egri chiziqlar bilan $y=x^3+1,\ x=0; x+y=4$.

3. Berigan ikki karrali integralda integrallash tartibini o'zgartiring:

a) $\int_{-2}^2 dx \int_0^{\sqrt{4-x^2}} f(x,y)dy$; b) $\int_0^1 dx \int_{2x}^{5x} f(x,y)dy$; c) $\int_0^1 dy \int_{-\sqrt{1-y^2}}^{1-y} f(x,y)dx$.

4. Agar D soha $y=x^2$ va $y^2=x$ egri chiziqlar bilan chegaralangan bo'lsa $\iint_D (x^2+y)dxdy$ ni hisoblang. (Javob: $33/140$.)

5. D–soha $x^2+y^2=9$ egri chiziq bilan chegaralangan bo'lsa $\iint_D x^3y^2\,dxdy$ ni hisoblang. (Javob: 0.)

6.Agar D soha $y=0,\ x=\pi,\ y=x$ chiziqlar bilan chegaralangan bo‘lsa $\iint_D x\cos(x+y)\,dxdy$ ni hisoblang. (Javob: $-\pi/2$.)

7. D soha $x=a(t-\sin t), y=a(1-\cos t)$ sikloidaning birinchi arkasi va Ox o‘qi bilan chegaralangan bo‘lsa $\iint_D ydxdy$, ni hisoblang. (Javob:$\frac{5}{2}\pi a^3$.)

Mustaqil ish

1. 1. Agar D soha $y=2x,\ x=0,\ y+x=3$ chiziqlar bilan chegaralangan bo‘lsa, ikki o‘lchovli $\iint_D f(x,y)dxdy$ integralni x va y o‘zgaruvchi bo‘yicha tartibini o‘zgartirib ikki karrali integralga keltiring,

2. Agar D soha $y=x^2$ va $y=2x$ chiziqlar bilan chegaralangan bo‘lsa, $\iint_D xdxdy$ integralni hisoblang. (Javob: 4/3.)

2. 1.Karrali integralda integrallash tartibini o‘zgartiring

$$\int_0^4 dx \int_{x^2/2-3}^{2x-3} f(x,y)dy.$$

2. Agar D soha quyidagi chiziqlar bilan chegaralangan bo‘lsa $x=0$, $y=0,\ y=\sqrt{4-x^2}$, ushbu integralni hisoblang $\iint_D xdxdy$. (Javob: 8/3.)

3. 1. Ikki karrali integralda, integrallash tartibini o‘zgartiring

$$\int_{-4}^{8} dy \int_{(y+4)/2}^{\sqrt{3y+12}} f(x,y)dx\ .$$

2. Agar D soha $y=x,\ y=\frac{1}{x}, x=2$ chiziqlar bilan chegaralangan bo‘lsa $\iint_D x^2dxdy$ integralni hisoblang. (Javob: 2.)

13.2 Ikki o‘lchovli integralda o‘zgaruvchularni almashtirish. Qutb koordinatalari sistemasida ikk o‘lchovli integrallar

O‘zgaruvchilar x va y lar u va v o‘zgaruvchilar bilan $x=\varphi(u,v),\ y=\psi(u,v)$ ko‘rinishda bog‘langan bo‘lsin va bu funksiyalar uzluksiz, differensiallanuvchi Oxy tekislikdagi D sohani, Ouv tekislikdagi D* sohaga o‘zaro bir qiymatli akslantirsin, hamda Yakobian

$$J = J(x,y) = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}$$

D sohada o‘z ishorasini o‘zgartirmasin. U holda ikki o‘lchovli integralda quyidagi o‘zgaruvchini almashtirish formulasi o‘rinli bo‘ladi.

$$\iint_D f(x,y)dxdy = \iint_{D^*} f(\varphi(u,v),\psi(u,v))|J|dudv \qquad (13.8)$$

Yangi integralda, integrallash chegarasi D* sohani hisobga olib qo‘yiladi.

1-misol. Ikki o‘lchovli integralni hisoblang

$$\iint_D (x+y)dxdy$$

D-soha quyidagi chiziqlar bilan chegaralangan: $y = x - 1$, $y = x + 2$, $y = -x - 2$, $y = -x + 3$

►Quyidagi almahtirishni bajaramiz

$$\left.\begin{matrix} u = y - x \\ v = y + x \end{matrix}\right\} \qquad (1)$$

U holda $y = x - 1$ va $y = x + 2$ to‘g‘ri chiziqlar $O'uv$ tekislikda $u = -1$, $u = 2$ to‘g‘ri chiziqlarga, $y = -x - 2$, $y = -x + 3$ to‘g‘ri chiziqlar $v = -2$, $v = 3$ to‘g‘ri chiziqlarga o‘tadi. D-soha $O'uv$ tekislikda D to‘g‘ri to‘rtburchakka akslantiriladi, bu yerda $-1 \le u \le 2, -2 \le v \le 3$.

Yuqoridagi (1)-sistemadan:

$$\left.\begin{matrix} x = (-u + v)/2, \\ y = (\ u + v)/2. \end{matrix}\right\}$$

Demak,

$$J = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{vmatrix} = -\frac{1}{2},$$

$|J| = \frac{1}{2}$. U holda (13.8) formulaga ko‘ra

$$\iint_D (x+y)dxdy = \iint_{D'} v\cdot\frac{1}{2}dudv = \frac{1}{2}\int_{-1}^{2} du\int_{-2}^{3} vdv = \frac{15}{4}.◄$$

Ma'lumki (x,y) dekart koordinatalar sistemasi va (ρ,φ) qutb koordinatalar sistemasi o'zaro quyidagicha bog'langan:

$$x=\rho\cos\varphi, \qquad y=\rho\sin\varphi, \qquad (\rho\geq 0, 0\leq\varphi\leq 2\pi).$$

Agar ikki o'lchovli integralda dekart koordinatalar sistemasidan qutb koordinatalar sistemasiga o'tsak, quyidagi formulaga ega bo'lamiz ($\mathrm{J}=\rho$)

$$\iint_D f(x,y)dxdy = \iint_{D'} f(\rho\cos\varphi,\rho\sin\varphi)\rho d\rho d\varphi. \qquad (13.9)$$

Umumlashgan qutb koordinatalar sistemasida

$$x=a\rho\cos\varphi, y=b\rho\sin\varphi \ (\rho\geq 0,\ 0\leq\varphi\leq 2\pi) \qquad (13.10)$$

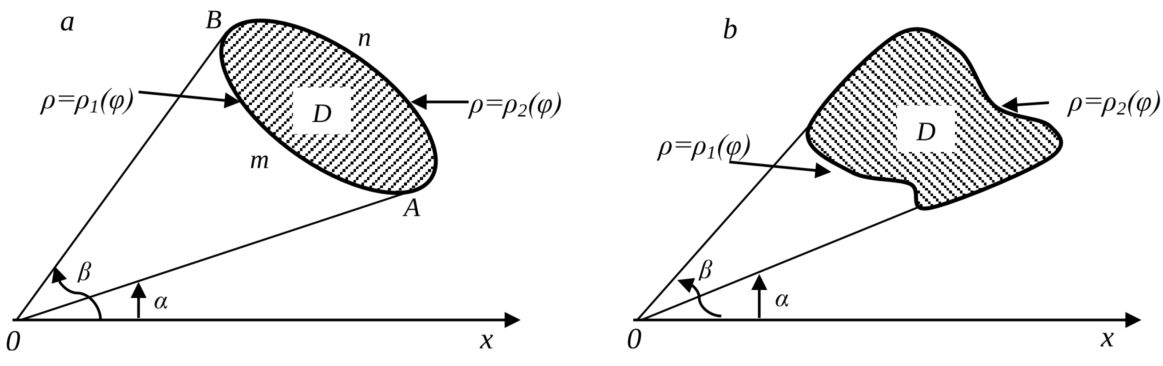
Ushbu formulaga ega bo'lamiz ($J=ab\rho$):

$$\iint_D f(x,y)dxdy = ab\iint_{D'} f(a\rho\cos\varphi, b\rho\sin\varphi)\,\rho d\rho. \qquad (13.11)$$

Ikki o'lchovli integraldan ikki karrali integralga o'tish uchun (13.9) va (13.11) formulalarning o'ng tomonining chegarasi O qutb nuqta D sohaning ichida, tashqarisida va chegarasida joylashganiga bog'liq.

1. Agar O qutb $\varphi=\alpha,\varphi=\beta$ nurlar $(\alpha<\beta)$ va tenglamalari $\rho=\rho_1(\varphi),\ \rho=\rho_2(\varphi),\ \rho_1(\varphi)\leq\rho_2(\varphi),\varphi\epsilon[\alpha,\beta]$ bo'lgan AmB, AnB egri chiziqlar bilan chegaralangan D sohadan tashqarida joylashgan bo'lsa u holda ikki o'lchovli integral ikki karrali integralga quyidagicha keltiriladi (13.11-rasm.)

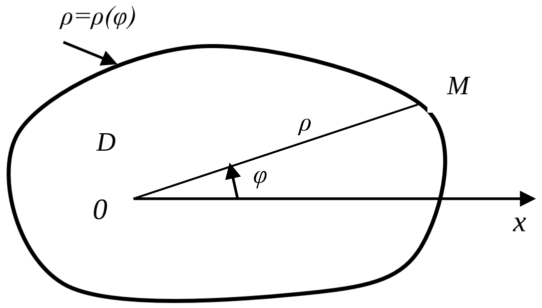
$$\iint_D f(x,y)dxdy = \int_\alpha^\beta d\varphi\int_{\rho_1(\varphi)}^{\rho_2(\varphi)} f(\rho\cos\varphi,\rho\sin\varphi)\,\rho d\rho. \qquad (13.12)$$



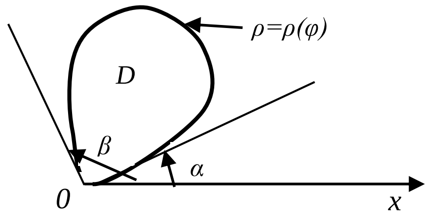
13.11- rasm

3. Agar qutb tenglamasi $\rho=\rho(\varphi)$ bo‘lgan D sohaning ichida yotsa, u holda (13.12) formulada $\alpha=0,\ \beta=2\pi,\ \rho_1(\varphi)=0,\ \rho_2(\varphi)=\rho(\varphi)$ (13.12-rasm).

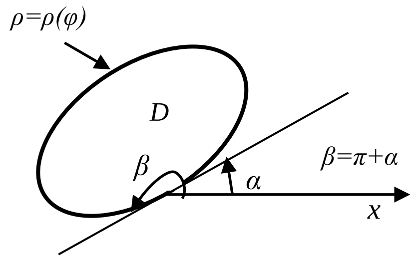
4. Agar qutb tenglamasi $\rho=\rho(\varphi)$ bo‘lgan D sohaning chegarasida joylashgan bo‘lsa, u holda (13.12) formulada $\rho_1(\varphi)=0,\ \rho_2(\varphi)=\rho(\varphi)$ hamda α va β har xil qiymatlar qabul qiladi (13.13, 13.14-rasmlar).



13.12- rasm



13.13- rasm



13.14- rasm

Ushbu formulalar umumlashgan qutb koordinatalar uchun ham o‘rinli.

2-misol. Agar D soha markazi koordinata boshida radiusi R ga teng bo‘lgan doira bo‘lsa, $\iint_D \sqrt{(x^2+y^2)^3}dxdy$ integralni hisoblang.

►Agar D soha doira yoki uning qismi bo‘lsa ko‘p integrallarni qutb koordinatalar sistemasida hisoblash oson. Yuqoridagi (13.9) va (13.12) formulalarga ko‘ra (2-hol):

$$\iint_D \sqrt{(x^2+y^2)^3}dxdy=\iint_D \sqrt{(\rho^2\sin^2\varphi\ +\rho^2\cos^2\varphi\ )^3}\rho d\rho d\varphi=$$

$$=\iint_D \rho^4 d\rho d\varphi=\int_0^{2\pi}d\varphi\int_0^R \rho^4 d\rho=2\pi\frac{R^5}{5}.$$◄

3-misol. Ellips $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ bilan chegaralangan shaklning yuzini hisoblang.

►Dekart koordinatalar sistemasida ellipsning yuzi $\iint_D dxdy$ integral bilan ifodalanadi, umumlashgan qutb koordinatalar sistemasiga o'tamiz (13.10) formulaga ko'ra ellipsning tenglamasi $\rho = 1$ ko'rinishda bo'ladi. Demak (13.11) formulaga ko'ra

$$\iint_D dxdy = \iint_D ab\rho d\rho d\varphi = ab\int_0^{2\pi} d\varphi \int_0^1 \rho d\rho = \pi ab.$$◄

AT-13.2

1. Agar D soha $2x + y = 1,\ 2x + y = 3,\ x - y = -1,\ x - y = 2$ to'g'ri chiziqlar bilan chegaralangan bo'lsa $\iint_D (x + y)dxdy$ integralni hisoblang. (Javob: 2,5.)

2. Agar D soha $x^2 + y^2 = 4x$-aylana bilan chegaralangan bo'lsa, qutb koordinatalaridan foydalanib ikki o'lchovli $\iint_D (x^2 + y^2)dxdy$ integralni hisoblang. (Javob: $24\,\pi$.)

3. Quyidagi chiziqlar bilan chegaralangan $x^2 + y^2 = 4x,\ x^2 + y^2 = 6x,$ $y = \frac{1}{\sqrt{3}}x,\ y = \sqrt{3}x$ shaklning yuzini hisoblang. (Javob: $5\pi/6.$)

4. Agar D soha quyidagi chiziqlar bilan chegaralangan $x^2 + y^2 = 1,\ x^2 + y^2 = 9,\ y = \frac{1}{\sqrt{3}}x,\ y = \sqrt{3}x$ halqaning qismi bo'lsa $\iint_D arctg\frac{y}{x}dxdy,$ integralni hisoblang. (Javob: $\pi^2/6$.)

5. Agar D soha $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ellips va $x = 0, y = 0$ to'g'ri chiziqlar bilan chegaralangan bo'lsa $\iint_D xydxdy$ integralni hisoblang, (Javob: $a^2b^2/8$.)

6. $\iint_D e^{-x^2-y^2}\,dxdy$ integralning qiymatidan foydalanib, bu yerda D-soha $x^2 + y^2 = R^2$ aylana bilan chegaralangan, ushbu $\int_{-\infty}^{\infty} e^{-x^2}dx,$ xosmas integralni hisoblang. (Javob: $\sqrt{\pi}$.)

Mustaqil ish.

1. Agar D soha $x^2 + y^2 = 9$ aylana bilan chegaralangan bo'lsa, $\iint_D (12 - x - y)dxdy$ integralni hisoblang. (Javob: 108π.)

2. D soha $x^2+y^2=4$ aylana bilan chegaralangan bo'lsa $\iint_D (6-2x-3y)dxdy$ integralni hisoblang. (Javob: 24π.)

3. Agar D-soha $x^2+y^2=2x$ aylana bilan chegaralangan bo'lsa $\iint_D (4-x-y)dxdy$ integralni hisoblang. (Javob: 3π.)

13.3. Ikki o'lchovli integrallarning tadbiqlari

Yassi shaklning yuzini hisoblash. Bir nechta misol ko'ramiz.

1- misol. Ushbu chiziqlar $y=x^2-2x$ va $y=x$ chegaralangan shaklning yuzini hisoblang.

►Chegaralarning tenglamasiga asosan shaklni chizib olamiz (13.15-rasm). D sohani chegaralovchi chiziqlar $O(0;0)$ va $M_0(3;3)$ nuqtada kesishgani uchun, quyidagi tengsuzlik o'rinli: $0\le x\le 3, x^2-2x\le y\le x$.

Ikki o'lchovli integralning 1-xossasiga asosan sohaning yuzasi

$$S=\iint_D dxdy=\int_0^3 dx\int_{x^2-2x}^{x} dy=\int_0^3 (x-x^2+2x)dx=\left(\frac{3}{2}x^2-\frac{x^3}{3}\right)\Big|_0^3=\frac{9}{2}.◄$$

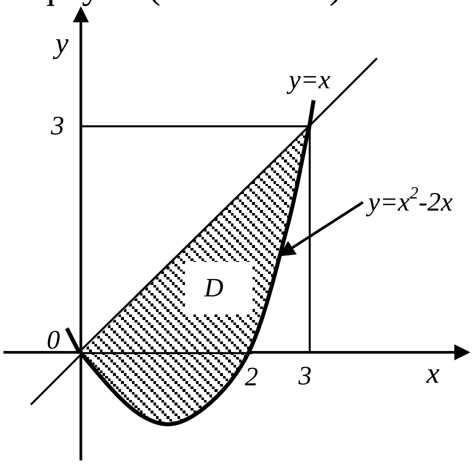
2- misol. Ushbu $(x^2+y^2)^2=a^2(x^2-y^2)$, $a>0$ egri chiziq bilan chegaralangan shaklning yuzini toping.

►Qutb koordinatalar sistemasiga o'tamiz, u holda egri chiziqning tenglamasi quyidagi ko'rinishda bo'ladi:

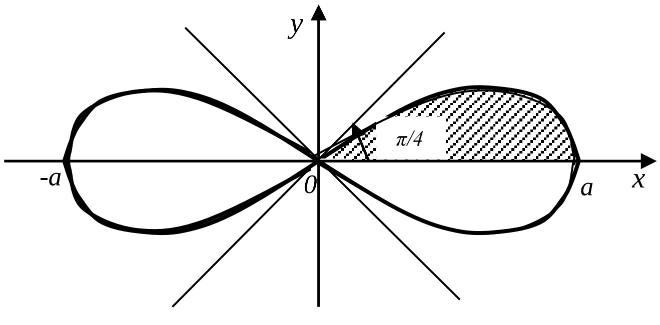
$$\rho^4=a^2\rho^2(\cos\varphi^2-\sin\varphi^2),$$

$$\rho^2=a^2\cos 2\varphi, \qquad \rho=a\sqrt{\cos 2\varphi}.$$

Oxirgi tenglama Bernulli lemniskamasi deb ataladigan egri chiziqni aniqlaydi. (13.16-rasm).



13.15- rasm



13.16- rasm

Egri chiziqning tenglamasidan va 13.16-rasmdan ko‘rinib turibdiki, chiziq koordinatalar o‘qiga nisbatan simmetrik va shaklning yuzi ikki o‘lchovli integral orqali ifodalanadi $S = 4\iint_D \rho d\rho d\varphi$. Bu yerda D-birinchi chorakda yotuvchi shakl (soha) va $0 \le \varphi \le \pi/4\,, 0 \le \rho \le a\sqrt{\cos 2\varphi}$. Demak,

$$S = 4\int_0^{\pi/4} d\varphi \int_0^{a\sqrt{\cos 2\varphi}} \rho d\rho = 4\int_0^{\pi/4} \frac{\rho^2}{2}\Big|_0^{a\sqrt{\cos 2\varphi}} d\varphi =$$

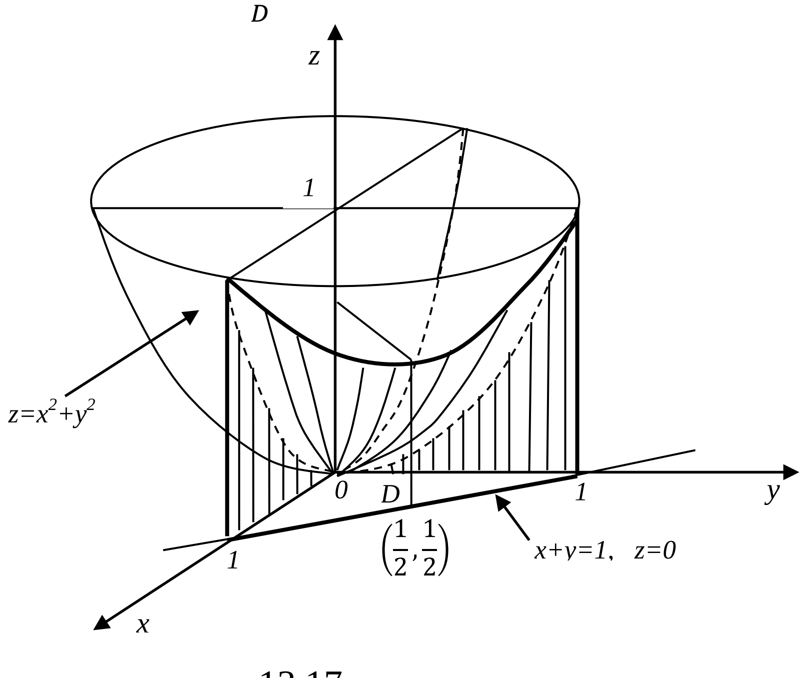
$$2a^2\int_0^{\pi/4} \cos 2\varphi\, d\varphi = a^2 \sin 2\varphi\,\Big|_0^{\pi/4} = a^2.◄$$

Jismning hajmini hisoblash. Quyidagi misollarni ko‘rib chiqamiz.

3- misol. Ushbu sirtlar bilan chegaralangan jismning hajmini hisoblang $z = x^2 + y^2,\ x + y = 1,\ x = 0,\ y = 0,\ z = 0.$

►Ushbu jism koordinatalar tekisligi, Oz o‘qiga parallel $x + y = 1$ tekislik va $z = x^2 + y^2$ aylanma paraboloid bilan chegaralangan (13.17-rasm). Ikki o‘lchovli integralning geometrik ma’nosiga asosan v hajmni ushbu formula orqali hisoblash mumkin

$$v = \iint_D (x^2 + y^2)\, dxdy,$$



13.17- rasm

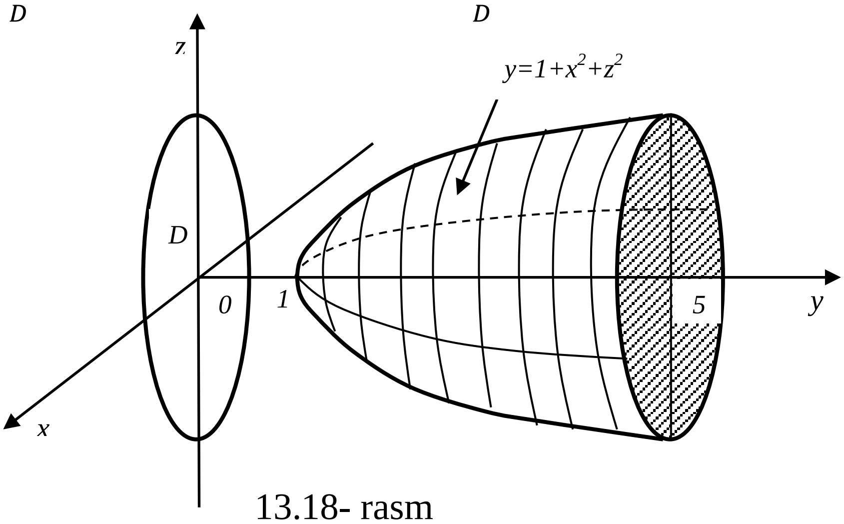
bu yerda D soha uchburchakdan iborat bo‘lib $0 \leq x \leq 1,\ 0 \leq y \leq 1 - x$. Demak,

$$v = \int_0^1 dx \int_0^{1-x} (x^2 + y^2)\, dy = \int_0^1 (x^2 y + \frac{y^3}{3})|_0^{1-x} dx = \int_0^1 \left(x^2 - x^3 + \frac{(1-x)^3}{3}\right) dx = \left(\frac{x^3}{3} - \frac{x^4}{4} - \frac{(1-x)^4}{12}\right)|_0^1 = \frac{1}{6}.◄$$

4- misol. Ushbu sirtlar bilan chegaralangan jismning hajmini hisoblang: $y = 1 + x^2 + z^2,\ y = 5.$

►Qaralayotgan jism Oy o‘qi atrofidagi aylanma paraboloid va Oy o‘qiga perpendikulyar $y = 5$ tekislik bilan chegaralangan (13.18-rasm). Uning Oxz tekislikka proyeksiyasi, $y = 0, x^2 + z^2 \leq 4$ tenglama bilan aniqlanadi. Jismning hajmi

$$y = \iint\limits_D (5 - 1 - x^2 - z^2)\, dxdz = \iint\limits_D (4 - x^2 - z^2)\, dxdz.$$



13.18- rasm

Qutb koordinatalar sistemasiga $x = \rho\cos\varphi,\ y = \rho\sin\varphi$ formulalar yordamida o‘tamiz, u holda $dxdz = \rho d\rho d\varphi$ va

$$v = \iint\limits_D (4 - \rho^2)\, \rho d\rho d\varphi = \int\limits_0^{2\pi} d\varphi \int\limits_0^2 (4\rho - \rho^3)\, d\rho =$$

$$= 2\pi\left(2\rho^2 - \frac{\rho^4}{4}\right)|_0^2 = 8\pi.◄$$

Sirt yuzalarini hisoblash. Aytaylik Oxy tekislikdagi D_z-sohada $z = f(x,y)$ funksiya berilgan bo‘lsin va u o‘zining xususiy hosilalari bilan birgalikda uzluksiz bo‘lsin. Ushbu funksiya bilan aniqlangan sirt silliq deyiladi. Ma’lumki,

D_z-soha shu sirtning Oxy tekislikka proyeksiyasi bo‘ladi. Berilgan $z = f(x,y), (x,y)\epsilon D_z$ sirtning yuzi Q_z quyidagi formula yordamida hisoblanadi:

$$Q_z = \iint_{D_z} \sqrt{1 + (\frac{\partial z}{\partial x})^2 + (\frac{\partial z}{\partial y})^2}\, dxdy \qquad (13.13)$$

Agar silliq sirt $x = f(y,z)$ funksiya (D_x sohada) yoki $y = f(x,z)$ (D_y sohada) funksiya ko‘rinishida b o‘lsa, bu sirtning yuzi quyidagi formulalar yordamida hisoblanadi

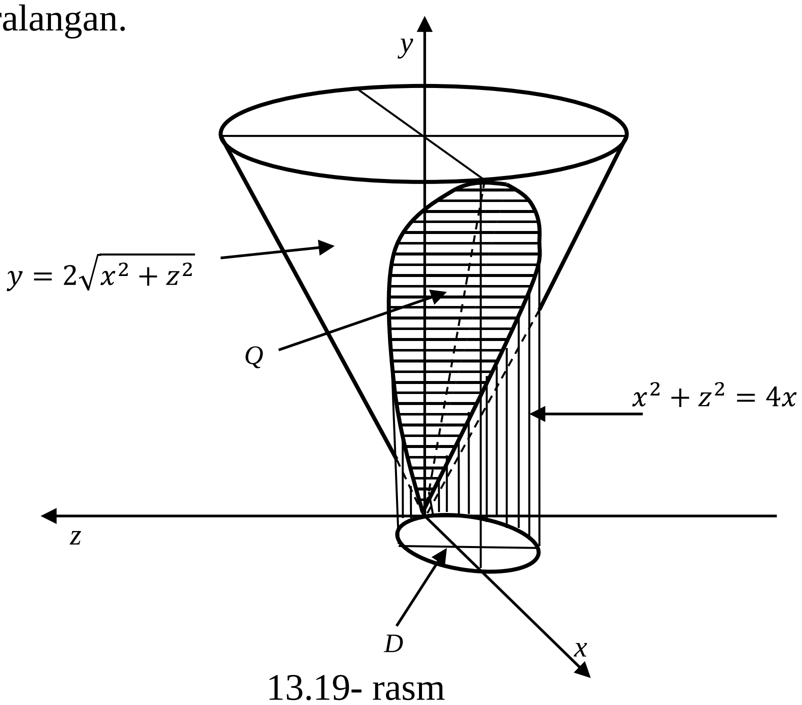
$$Q_x = \iint_{D_x} \sqrt{1 + (\frac{\partial x}{\partial y})^2 + (\frac{\partial x}{\partial z})^2}\, dydz \qquad (13.14)$$

yoki

$$Q_y = \iint_{D_y} \sqrt{1 + (\frac{\partial y}{\partial x})^2 + (\frac{\partial y}{\partial z})^2}\, dydz. \qquad (13.15)$$

5- misol. $y = 2\sqrt{x^2 + z^2}$ konusning $x^2 + z^2 = 4x$ silindr ichida joylashgan qismining yuzini hisoblang.

►Funksiya $y = f(x,z)$ ko‘rinishida berilganligi uchun sirtning yuzi Q_y (13.15) formula yordamida hisoblanadi, bu yerda D_y-sirtning Oxz tekislikka proyeksiyasi (13.19-rasm). Proyeksiya doiradan iborat bo‘lib $(x-2)^2 + z^2 = 4$ aylana bilan chegaralangan.



13.19- rasm

Demak,

$$\frac{\partial y}{\partial x} = \frac{2x}{\sqrt{x^2 + z^2}}, \qquad \frac{\partial y}{\partial z} = \frac{2z}{\sqrt{x^2 + z^2}}$$

u holda yuza

$$Q_y = \iint\limits_{D_y} \sqrt{1 + \frac{4x^2}{x^2 + z^2} + \frac{4z^2}{x^2 + z^2}}\,dxdz = \sqrt{5}\iint\limits_{D_y} dxdy =$$

$$= \begin{vmatrix} z = \rho\cos\varphi, & dxdz = \rho d\rho d\varphi, \\ x = \rho\sin\varphi, & \rho = 4\sin\varphi \end{vmatrix} = \sqrt{5}\int\limits_0^{\pi} d\varphi \int\limits_0^{4\sin\varphi} \rho d\rho =$$

$$= 8\sqrt{5}\int\limits_0^{\pi} \sin\varphi^2\, d\varphi = 4\sqrt{5}\int\limits_0^{\pi}(1 - \cos 2\varphi)d\varphi = 4\sqrt{5}\left(\varphi - \frac{1}{2}\sin 2\varphi\right)\Big|_0^{\pi} =$$

$= 4\pi\sqrt{5}$. ◄

Moddiy plastinkani massasini hisoblash. Massani hisoblashni misolda ko‘rib chiqamiz.

6- misol. Agar moddiy plastinkaning zichligi $\mu = y$ va $x = (y-1)^2, y = x - 1$ egri chiziqlar bilan chegaralangan bo‘lib, Oxy tekislikda yotsa, uning massasini hisoblang,.

►D sohani chegaralovchi egri chiziqlarning kesishish nuqtasini topamiz: A(1;0), B(4;3) (13.20-rasm). U holda iki o‘lchovli integralning fizik ma’nosiga ko‘ra (13.1§, 2 xossa) qidiralayotgan massa

$$m = \iint_D ydxdy = \int_0^3 dy \int_{(y-1)^2}^{y+1} ydx = \int_0^3 y(y + 1 - (y-1)^2)\,dy =$$

$$\int_0^3 (3y^2 - y^3)\,dy = \left(y^3 - \frac{y^4}{4}\right)\Big|_0^3 = \frac{27}{4}. \blacktriangleleft$$

Moddiy plastinkaning statistik momenti va og‘irlik markazining koordinatalarini hisoblash. Agar Oxy tekislikda D moddiy plastinka uzluksiz sirt zichligi $\mu = (x, y)$ bilan berilgan bo‘lsa, uning og‘irlik markazining koordinatalari quyidagi formula bilan aniqlanadi:

$$x_c = \frac{\iint_D x\mu(x,y)dxdy}{\int_D \mu(x,y)dxdy}, \quad y_c = \frac{\iint_D y\mu(x,y)dxdy}{\int_D \mu(x,y)dxdy} \tag{13.16}$$

$$M_x = \iint_D y\mu(x,y)dxdy, \; M_y = \iint_D x\mu(x,y)dxdy \tag{13.17}$$

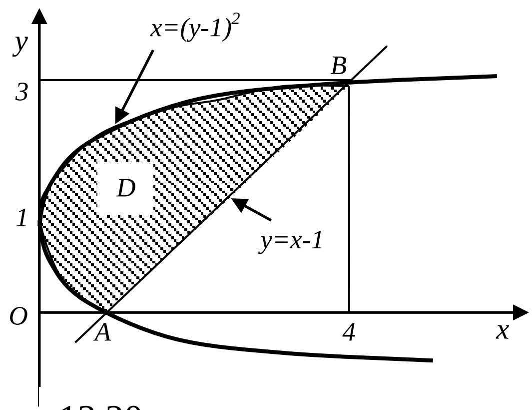
kattaliklar D plastinkaning Ox va Oy o‘qiga nisbatan *statistik momentlari* deb ataladi.

7- misol. Agar Oxy tekislikda yotgan D plastinka $y = x, y = 2x, x = 2$ chiziqlar bilan chegaralangan va zichligi $\mu(x, y) = xy$ bo‘lsa, uning massasini toping (13.21-rasm).

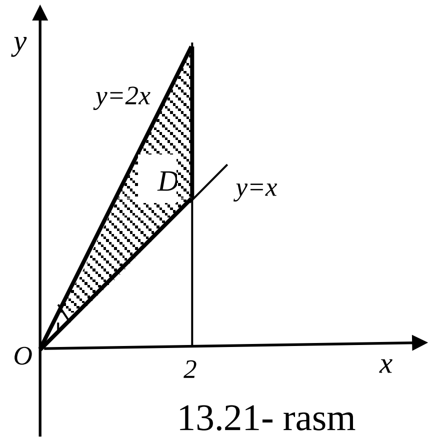
►Dastlab D plastinkaning massasini topamiz:

$$m = \iint_D xdxdy = \int_0^2 xdx \int_x^{2x} ydy = \int_0^2 x \cdot \frac{y^2}{2}\Big|_x^{2x} dx =$$

$$= \frac{1}{2}\int_0^2 x(4x^2 - x^2)\, dx = \frac{3}{2}\int_0^2 x^3\, dx = \frac{3}{8}x^4\Big|_0^2 = 6.$$



13.20- rasm



13.21- rasm

Yuqoridagi (13.16) formulaga ko‘ra og‘irlik markazining koordinatalarini aniqlaymiz

$$x_c = \frac{1}{m}\iint_D x^2 y\, dxdy = \frac{1}{6}\int_0^2 x^2\, dx \int_x^{2x} ydy =$$

$$= \frac{1}{6}\int_0^2 x^2 \frac{1}{2}(4x^2 - x^2)dx = \frac{1}{4}\int_0^2 x^4 dx = \frac{x^5}{20}\Big|_0^2 = \frac{8}{5},$$

$$y_c = \frac{1}{m}\iint_D xy^2\, dxdy = \frac{1}{6}\int_0^2 x\, dx \int_x^{2x} y^2 dy =$$

$$= \frac{1}{6}\int_0^2 x \cdot \frac{y^3}{3}\Big|_x^{2x} = \frac{7}{18}\int_0^2 x^4 dx = \frac{112}{45}. \blacktriangleleft$$

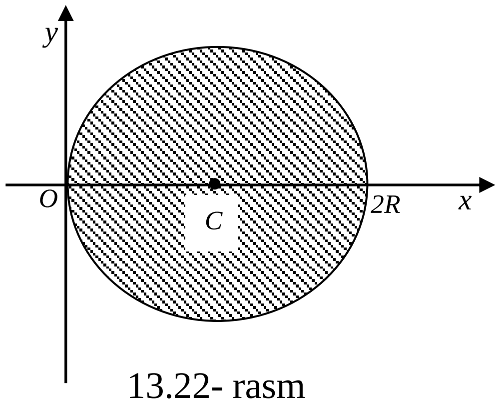
Moddiy plastinkaning inertsion momentini hisoblash. Sirt zichligi $\mu(x,y)$ kabi tekis taqsimlangan Oxy tekislikda joylashgan D plastinkaning koordinatalar boshi va koordinata o‘qlari Ox, Oy ga nisbatan inertsiya momentlarini quyidagi formulalar yordamida hisoblaymiz:

$$I_0 = \iint_D (x^2 + y^2)\, \mu(x,y) dxdy,$$

$$I_x = \iint_D y^2\, \mu(x,y) dxdy, \qquad I_y = \iint_D x^2\, \mu(x,y) dxdy \qquad (13.18)$$

8- misol. Bir jinsli doiraning radiusi R ga, og‘irligi P ga teng. Uning chegara nuqtasi va diametriga nisbatan inertsiya momentini hisoblang.

►Koordinata boshini doiraning chegara nuqtasida, doiraning markazini esa C(R;0) nuqtada joylashtiramiz (13.22-rasm). U holda masala koordinata boshi va Ox o‘qiga nisbatan unertsiya momentini topishga keladi.



13.22- rasm

Doira bir jnsli bo‘lgani uchun uning zichligi o‘zgarmas va $\mu = P/(g\pi R^2)$ ga teng. Aylananing dekart koordinatalar sistemasidagi tenglamasi $(x-R)^2 + y^2 = R^2$, qutb koordinatalar sistemasida esa $\rho = 2R\cos\varphi$. Berilgan doira uchun $-\pi/2 \le \varphi \le \pi/2$, $0 \le \rho \le 2R\cos\varphi$.

Natijada (13.18) formulaga ko‘ra:

$$I_0 = \mu \iint_D (x^2 + y^2)\, dxdy = \mu \int_{-\pi/2}^{\pi/2} d\varphi \int_0^{2R\cos\varphi} \rho^3\, d\rho =$$

$$= 4\mu R^4 \int\limits_{-\pi/2}^{\pi/2} \cos\varphi^4\, d\varphi = 8\mu R^4 \int\limits_0^{\pi/2} \left(\frac{1+\cos 2\varphi}{2}\right)^2 d\varphi =$$

$$= 2\mu R^4 \int\limits_0^{\pi/2} \left(1 + 2\cos 2\varphi + \frac{1+\cos 4\varphi}{2}\right) d\varphi =$$

$$= 2\mu R^4 \left(\varphi + \sin 2\varphi + \frac{1}{2}\varphi + \frac{1}{8}\sin 4\varphi\right)\Big|_0^{\pi/2} = \frac{3}{2}\mu\pi R^4 = \frac{3}{2}\frac{P}{g}R^2,$$

$$I_x = \mu \iint\limits_D y^2 dxdy = \mu \int\limits_{-\pi/2}^{\pi/2} d\varphi \int\limits_0^{2R\cos\varphi} \rho^3 \sin^2\varphi\ d\rho =$$

$$= 4\mu R^4 \int\limits_{-\pi/2}^{\pi/2} \cos^4\varphi\ \sin^2\varphi\ d\varphi = 8\mu R^4 \int\limits_0^{\pi/2} \frac{1}{4}\sin^2 2\varphi \cdot \frac{1+\cos 2\varphi}{2} d\varphi =$$

$$= \mu R^4 \int\limits_0^{\pi/2} \sin^2 2\varphi\, d\varphi + \mu R^4 \int\limits_0^{\pi/2} \sin^2 2\varphi \cos 2\varphi\, d\varphi =$$

$$= \mu R^4 \int\limits_{-\pi/2}^{\pi/2} \frac{1}{2}(1 - \cos 4\varphi)\, d\varphi + \mu R^4 \frac{sin^3 2\varphi}{6}\Big|_0^{\pi/2} =$$

$$= \frac{1}{2}\mu R^4 \left(\varphi - \frac{1}{4}\sin 4\varphi\right)\Big|_0^{\pi/2} = \frac{\pi}{4}\mu R^4 = \frac{1}{4}\frac{P}{g}R^2. \blacktriangleleft$$

AT-13.3

1. Quyidagi chiziqlar bilan chegaralangan shakllarning yuzini toping:

a) $y = \sqrt{x},\ y = 2\sqrt{x},\ x = 4;$

b) $y^2 = 10x + 25,\ y^2 = -6x + 9;$ c) $\rho = a\sin 2\varphi,\ a > 0.$

(Javob: a) $\frac{16}{3}$; b) $\frac{16}{3}\sqrt{15}$; c) $\frac{1}{2}\pi a^2$.)

2. Ushbu sirtlar bilan chegaralangan jismning hajmini toping:

a) paraboloid $z = 1 + x^2 + y^2$ va $x = 0,\ y = 0,\ z = 0,\ x = 4,\ y = 4$ tekisliklar bilan;

b) silindrlar $x^2 + y^2 = R^2,\ x^2 + z^2 = R^2;$

c) paraboloid $z = x^2 + y^2$ va $z = 0,\ y = 1,\ y = 2x,\ y = 6 - x$ tekisliklar bilan;

d) silindrlar $x^2 + y^2 = 4$ va $z = 0,\ z = x + y + 10$ tekisliklar bilan;

e) elliptik silindr $\frac{x^2}{4} + \frac{y^2}{1} = 1$ va $z = 12 - 3x - 4y,\ z = 1$ tekisliklar bilan.

(Javob: a)$186\frac{2}{3}$; b) $\frac{16}{3}R^3$; c) $78\frac{15}{32}$; d) 40π; e) 22π.)

3. Ushbu $6x + 3y + 2z = 12$ tekislikning birinchi oktantda joylashgan qismining yuzini hisoblang. (Javob: 14.)

4. Berilgan $x^2 + y^2 = 4x$ silindrning ichida joylashgan $z = \sqrt{x^2 + y^2}$ konus qismining yuzini hisoblang. (Javob: $4\sqrt{2}\pi.$)

5. Silindr $x^2 + y^2 = 1$ ning ichida joylashgan $2z = x^2 + y^2$ paraboloidning sirti yuzi hisoblansin. (Javob: $\frac{2}{3}\pi\left(\sqrt{8} - 1\right)$.)

6. Tomoni a ga teng bo‘lgan kvadrat plastinkaning massasini hisoblang, agar uning zichligi ixtiyoriy M nuqtada shu nuqtadan diagonallari kesishish nuqtasigacha bo‘lgan masofaning kvadratiga propolsional bo‘lsa va kvadratning uchlaridagi nuqtalarda zichlik 1 ga teng. (Javob: $a^2/3$.)

Mustaqil ish

1. Egri chiziqlar bilan chegaralangan shaklning yuzini hisoblang $y = 2 - x,\ y^2 = 4x + 4$. (Javob:64/3.)

2. Sirtlar bilan chegaralangan jismning hajmini toping $x^2 + y^2 = 1,\ z = 0$, $x + y + z = 4$. (Javob:4π.)

3. Silindr $z = y^2/2$ va $2x + 3y = 12,\ x = 0,\ y = 0,\ z = 0$ tekisliklar bilan chegaralangan jismning hajmini toping. (Javob: 16.)

AT-13.4

1. Bir jinsli Oxy tekislikda yotuvchi va $y^2 = 4x + 4,\ \ y^2 = -2x + 4$ egri chiziqlar bilan chegaralangan yassi shakl massasi markazining koordinatalari topilsin. (Javob: $x_c = 2/5, y_c = 0$.)

2. Zichligi $\mu(x,y) = xy$ va $y = x^2$, $y^2 = x$ chiziqlar bilan chegaralangan shakl massasi markazi koordinatalarini toping. (Javob: $x_c = 9/14$, $y_c = 3/56$.)

3. Bir jinsli, kardoida $\rho = a(1 + \cos\varphi)$ bilan chegaralangan yassi shaklning massasi markazining koordinatalari topilsin. (Javob: $x_c = \frac{5}{6}a$, $y_c = 0$.)

4. Zichligi $\mu(x,y) = 3{,}5$ va $x^2 + y^2 - 2x = 0$ egri chiziq bilan chegaralangan shaklning koordinata boshiga nisbatan inertsiya momentini toping. (Javo b: 21π/4.)

5. Zichligi $\mu(x,y) = x^2 y$, Oxy tekislikda yotuvchi va $y = x^2$, $y = 1$ chiziqlar bilan chegaralangan plastinkaning koordinata boshi va koordinata o‘qlariga nisbatan inertsiya momentlarini toping. (Javob: $I_0 = 104/495$, $I_x = 4/33$, $I_y = 4/45$.)

6. Kardioda $\rho = a(1 - \cos\varphi)$ bilan chegaralangan, zichligi $\mu = 1{,}6$ ga teng plastinkani markaziga nisbatan inertsiya momentini hisoblang. (Javob: $7\pi a^4/2$.)

7. Yarim qirralari *a* va *b* bo‘lgan elliptik plastinka ($\mu(x,y) = 1$) markaziga nisbatan inertsiya momentini hisoblang. (Javob: $\pi ab\,(a^2 + b^2)/4$.)

Mustaqil ish

1. Zichligi $\mu(x,y) = 1$ va $x + y = 2$, $x = 2$, $y = 2$ chiziqlar bilan chegaralangan shaklning koordinata boshiga nisbatan inertsiya momentini toping. (Javob: 4.)

2. Bir jinsli, $y = -x^2 + 2x$, $y = 0$ chiziqlar bilan chegaralangan Oxy tekislikda yotuvchi shaklning massasi markazining koordinatalarini toping. (Javob: $x_c = 1$, $y_c = 1/4$.)

3. Tomonlari 4 va 6 ga teng, zichligi $\mu(x,y) = 2$ bo‘lgan to‘rtburchakli plastinkaning diagonallari kesishish nuqtasiga nisbatan inertsiya momentini toping. (Javob: 208.)

13.4. Uch o‘lchovli integral va uni hisoblash

Faraz qilaylik $u = f(x, y, z)$ funksiya bo‘lakli silliq S sirt bilan chegaralangan $V\epsilon R^3$ yopiq sohada uzluksiz bo‘lsin. Ixtiyoriy silliq sirtlar yordamida V-sohani n -ta elementar V_i-sohalarga ($i = \overline{1, n}$) bo‘lamiz va ularning hajmini Δv_i-deb belgilaymiz. Har bir V_i-sohada ixtiyoriy $M_i(x_i, y_i, z_i)$ nuqtani tanlaymiz va quyidagi yig‘indi tuzib olamiz

$$I_n = \sum_{i=1}^{n} f(x_i, y_i, z_i)\, \Delta v_i. \tag{13.19}$$

Elementar sohalarning maksimal diametrini d_i-deb belgilaymiz. Yuqoridagi (13.19) yig‘indi $f(x, y, z)$ funksiyaning V sohadagi n-integral yig‘indi deb ataladi.

Integral yig‘indi (13.19) ning $d_i \to 0$ shartda topilgan limiti $f(x, y, z)$ funksiyaning V soha bo‘yicha uch o‘lchovli integrali deyiladi va quyidagicha belgilanadi $\iiint_V f(x, y, z)\, dv$.

Demak ta’rif bo‘yicha

$$\iiint_V f(x, y, z)\, dv = \lim_{d_i \to 0} \sum_{i=1}^{n} f(x_i, y_i, z_i)\, \Delta v_i, \tag{13.20}$$

Agar integral ostidagi $f(x, y, z)$ funksiya V sohada uzluksiz bo‘lsa, (13.20) integral mavjud va V ni V_i elementar sohalarga bo‘lish va M_i nuqta tanlashga bog‘liq bo‘lmaydi.

Yuqorida §13.1 da keltirilgan ikki o‘lchovli integrallarning hossalari, uch o‘lchovli integral uchun ham o‘rinli, shuning uchun ikki o‘lchovli integraldan farqli bo‘lgan xossalarini keltirib o‘tamiz.

1. Agar V sohada $f(x, y, z) = 1$ ga teng bo‘lsa u holda

$$\iiint_V dv = v, \tag{13.21}$$

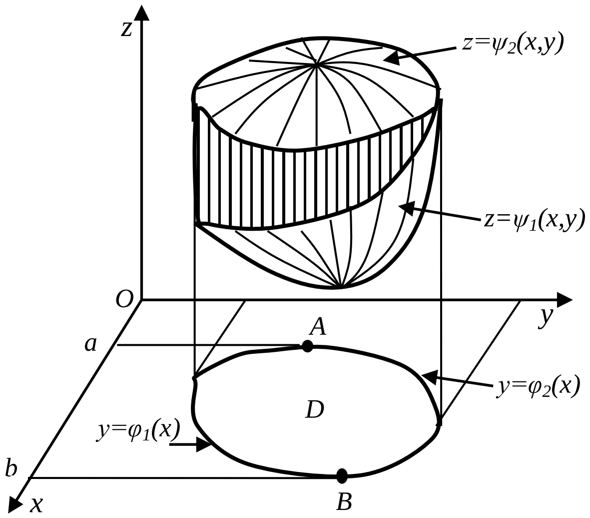
2. Integral ostidagi funksiya $f(x, y, z)$, jismning zichligi $\delta(x, y, z)$ bo‘lsa u holda uch o‘lchovli integral V sohani egallovchi jismning massasini aniqlaydi

$$m = \iiint_V \delta(x, y, z) dv \tag{13.22}$$

Shuni qayd qilish kerakki, dekart koordinatalar sistemasida V sohani elementar sohalarga ajratish uchun koordinata tekisliklariga parallel tekisliklardan foydalanish muhim, u holda elementar hajm $dv = dxdydz$ ga teng bo‘ladi.

V –sohani to‘g‘ri soha deyiladi, agarda koordinata o‘qlariga parallel to‘g‘ri chiziqlar uning chegarasini ikkitadan ortiq nuqtada kesib o‘tmasa.

To‘g‘ri V-soha uchun (13.23-rasm) quyidagi tengsizlik o‘rinli: $a \le x \le b$, $\varphi_1(x) \le y \le \varphi_2(x)$, $\psi_1(x,y) \le z \le \psi_2(x,y)$, u holda uch o‘lchovli integralni hisoblash uchun quyidagi formulaga ega bo‘lamiz:



13.23- rasm

$$\iiint_V f(x,y,z)dxdydz = \int_a^b dx \int_{\varphi_1(x)}^{\varphi_2(x)} dy \int_{\psi_1(x,y)}^{\psi_2(x,y)} f(x,y,z)dz. \quad (13.23)$$

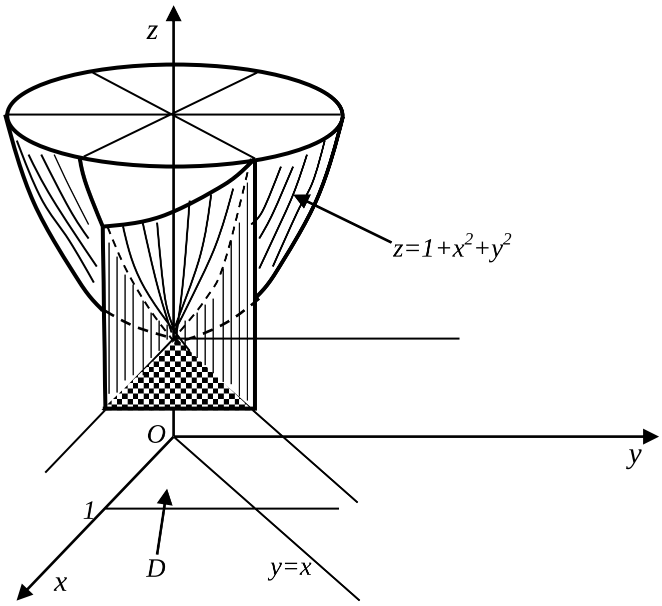
Demak uch o‘lchovli integralni hisoblashda oddiy to‘g‘ri V soha uchun dastlab $f(x,y,z)$ funksiya biron bir o‘zgaruvchi bo‘yicha (masalan z) integrallanadi, qolgan ikkita o‘zgaruvchi o‘zgarmas deb hisoblanadi, so‘ngra natija ikkinchi o‘zgaruvchi bo‘yicha integrallanadi (masalan y), uchinchi o‘zgaruvchining ixtiyoriy o‘zgarmas qiymatida va oxirida uchinchi o‘zgaruvchining (masalan x) maksimal oralig‘ida integrallanadi.

Murakkab sohalar chekli sondagi sohalarga ajratiladi va bu sohalar bo‘yicha natijalarning yig‘indilari olinadi. Xususan V soha to‘g‘ri burchakli parallelepiped bo‘lsa $V = \{a \le x \le b, c \le y \le d, p \le z \le q\}$ u holda

$$\iiint_V f(x,y,z)dxdydz = \int_a^b dx \int_c^d dy \int_p^q f(x,y,z)dz \quad (13.24)$$

1- misol. Ushbu uch o‘lchovli $I = \iiint_V (2x+y)dxdydz$ integralni hisoblang, V soha quyidagi sirtlar bilan chegaralangan $y = x$, $y = 0$, $x = 1$, $z = 1$, $z = 1 + x^2 + y^2$.

►Berilgan sirt bo‘yicha integrallash sohasini aniqlaymiz (13.24-rasm). V-sohada quyidagi tengsizliklar o‘rinli: $0 \le x \le 1,\ 0 \le y \le x,\ 1 \le z \le 1 + x^2 + y^2$. U holda



13.24- rasm

$$I = \int_0^1 dx \int_0^x dy \int_1^{1+x^2+y^2} (2x+y)dz = \int_0^1 dx \int_0^x (2x+y)z\,\Big|_1^{1+x^2+y^2} dy =$$

$$= \int_0^1 dx \int_0^x (2x+y)(x^2+y^2)dy = \int_0^1 dx \int_0^x (2x^3 + y^3 + 2xy^2 + x^2 y)\,dy =$$

$$= \int_0^1 (2x^3 y + \frac{1}{2}x^2 y^2 + \frac{2}{3}xy^3 + \frac{1}{4}y^4)dx = \int_0^1 \frac{41}{12}x^4 dx = \frac{41}{60}.$$ ◄

Faraz qilaylik

$$\left.\begin{aligned} x &= \varphi(u,v,\omega), \\ y &= \psi(u,v,\omega), \\ z &= \theta(u,v,\omega). \end{aligned}\right\} \quad (13.25)$$

uzluksiz va uzluksiz xususiy hosilalarga ega bo‘lsin, yakobian

$$J = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} & \frac{\partial x}{\partial \omega} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} & \frac{\partial y}{\partial \omega} \\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} & \frac{\partial z}{\partial \omega} \end{vmatrix} \ne 0$$

va u, v, ω o‘zgaruvchilarning o‘zgarish sohasi V' da ishorasini o‘zgartirmasin. Yuqoridagi (13.25) funksiya V sohani V' sohaga o‘zaro bir qiymatli akslantiradi. U holda quyidagi formula o‘rinli bo‘ladi

$$\iiint\limits_{V} f(x,y,z)dxdydz = \iiint\limits_{V`} f(\varphi(\mathrm{u,v,\omega}), \psi(\mathrm{u,v,\omega}), \theta(\mathrm{u,v,\omega}))|\mathrm{J}|dudvd\omega.$$

Silindrik koordinatalar sistemasida ρ, φ, z (13.25-rasm)

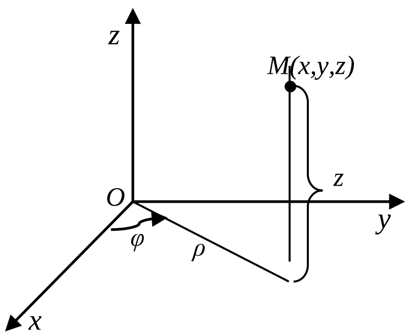
$$\left.\begin{array}{c} x = \rho\cos\varphi\,, \;\; y = \rho\sin\varphi\,, \;\; z = z, \\ 0 \le \varphi \le 2\pi, \;\; 0 \le \rho \le \infty, \;\; -\infty \le z \le \infty, \\ J = \rho, \;\; dxdydz = \rho d\rho d\varphi dz. \end{array}\right\} \qquad (13.26)$$

Sferik koordinatalar sistemasida ρ, φ, z (r-radius vektor, φ-uzunlik, θ-kenglik yoki og‘ish) (13.26-rasm) quyidagiga ega bo‘lamiz

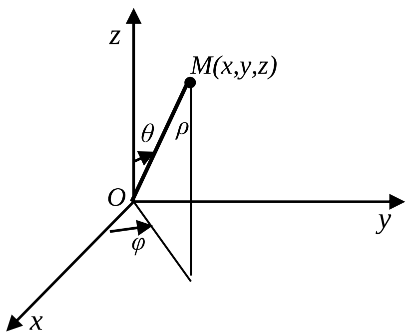
$$\left.\begin{array}{c} x = \rho\sin\theta\cos\varphi\,, y = \rho\sin\theta\sin\varphi\,, z = r\cos\theta\,, \\ 0 \le r < \infty, \;\; 0 \le \varphi \le 2\pi, \;\; 0 \le \theta \le \pi \\ J = r^2\sin\theta\,, dxdydz = r^2\sin\theta\, drd\varphi d\theta. \end{array}\right\} \qquad (13.27)$$

Umumlashgan sferik koordinatalar sistemasida

$$\left.\begin{array}{c} x = ar\sin\theta\cos\varphi\,, y = br\sin\theta\sin\varphi\,, z = cr\cos\theta\,, \\ J = abcr^2\sin\theta\,, dxdydz = abcr^2\sin\theta\, drd\varphi d\theta. \end{array}\right\} \qquad (13.28)$$



13.25- rasm



13.26- rasm

Yuqoridagi (13.26)-(13.28) formulalar uch o‘lchovli integralda dekart koordinatalar sistemasidan silindrik, sferik va umumlashgan sferik koordinatalar sistemasiga o‘tishga imkon beradi. Uch o‘lchovli integrallarni dekart koordinatalar sistemasida hisoblash uchun (13.23) formula, silindrik va sferik koordinatalar sistemasi uchun ham o‘rinli.

2- mislol. Ushbu $I = \iiint_V \sqrt{x^2+y^2}\, dxdydz$ integralni hisoblang, integrallash sohasi V: $x^2+y^2=4,\ z=1,\ z=2+x^2+y^2$ sirtlar bilan chegaralangan.

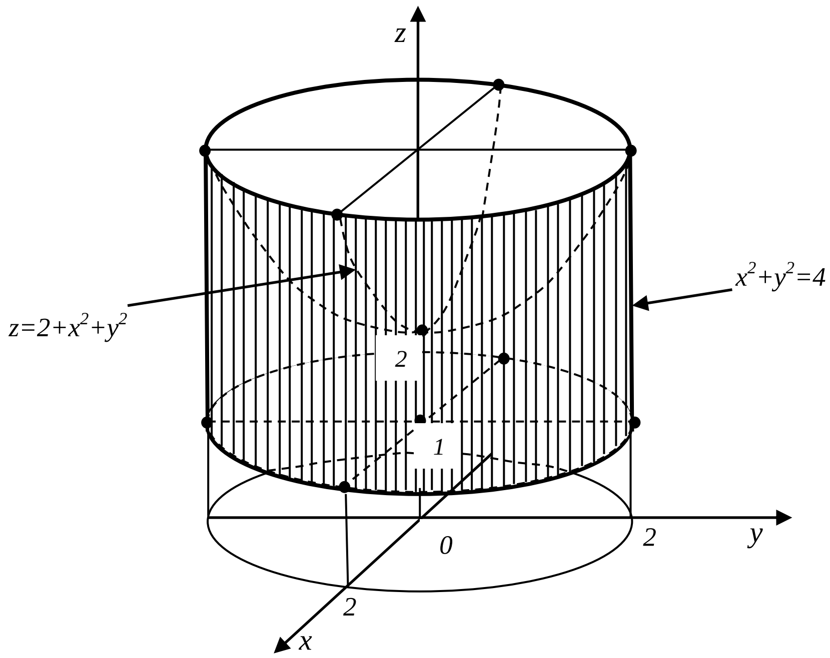
►Masala shartidagi sirtlar yordamida V sohani qurib olamiz (13.27-rasm). Berilgan integralda silindrik koordinatalar sistemasiga o‘tib olsak

$$I = \iiint\limits_{V`} \rho\rho d\rho d\varphi dz = \int\limits_0^{2\pi} d\varphi \int\limits_0^2 \rho^2\, d\rho \int\limits_1^{2+\rho^2} dz =$$

$$= \int\limits_0^{2\pi} d\varphi \int\limits_0^2 \rho^2\,(1+\rho^2) d\varphi = \varphi|_0^{2\pi} \int\limits_0^2 (\rho^2+\rho^4)\, d\rho =$$

$$= 2\pi\left(\frac{\rho^3}{3}+\frac{\rho^5}{5}\right)|_0^2 = \frac{272}{15}\pi.$$

Demak $I = \frac{272}{15}\pi$.



13.27- rasm

3- misol. Agar integrallash sohasi $x^2+y^2+z^2=4$ sfera va $y=0$ $(y\geq 0)$ tekislik bilan chegaralangan bo‘lsa, berilgan $I=\iiint_V\sqrt{x^2+y^2+z^2}\,dxdydz$ integralni hisoblang,.

►V soha yarim shar bo‘lib, Oxz tekislikdan o‘ngda joylashgan $(y\geq 0)$, sferik koordinatalar sistemasida $r,\varphi,\theta,$ V sohada quyidagicha o‘zgaradi: $0\leq z\leq 2, 0\leq\varphi\leq\pi, 0\leq\theta\leq\pi$. Demak

$$I=\iiint_{V`} r^3r^2\sin\theta\, drd\varphi d\theta=$$

$$=\int_0^\pi d\varphi\int_0^\pi\sin\theta\,d\theta\int_0^2 r^5\,dr=\varphi|_0^\pi\cdot(-\cos\theta)|_0^\pi\cdot\frac{r^6}{6}|_0^2=\frac{64}{3}\pi.$$◄

AT-13.5

1. Agar V soha $0\leq x\leq 1,\ 0\leq y\leq x,\ 0\leq z\leq xy$ tengsizliklar bilan aniqlangan bo‘lsa, ushbu $\iiint_V x^2y^2z\,dxdydz$ integralni hisoblang. (Javob:1/110.)

2. Ushbu $\iiint_V\frac{dxdydz}{(1+x+y+z)^3}$ integralni hisoblang, agar V-soha $x=0,y=0,z=0,x+y+z=1$ tekisliklar bilan chegaralangan bo‘lsa. (Javob: $\frac{1}{2}\left(\ln 2-\frac{5}{8}\right)$.)

3. Quyidagi $y=x^2, y+z=4, z=0$ sirtlar bilan chegaralangan jismning hajmini hisoblang. (Javob: 256/15.)

4. Agar V soha $x^2+y^2=1, z=0, z=x^2+y^2$ sirtlar bilan chegaralangan bo‘lsa, $\iiint_V x^2y^2\,dxdydz$ integralni hisoblang. (Javob: π/32.)

5. Ushbu sirtlar $x^2+y^2=10x,\ x^2+y^2=13x,\ z=\sqrt{x^2+y^2}, z=0,$ $y\geq 0$ bilan chegaralangan jismning hajmini hisoblang. (Javob: 266.)

6. Agar V soha $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ ellipsoidning ichki qismi bo‘lsa,

$\iiint_V(\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2})\,dxdydz$ -integralni hisoblang. (Javob: $\frac{4}{5}\pi abc$.)

7. Quyidagi $z^2=x^2+y^2$-konusning ichida joylashgan $x^2+y^2+z^2=1$ shar qismining hajmini hisoblang. (Javob: $\frac{4}{3}\pi\left(1-\frac{\sqrt{2}}{2}\right)$.)

Mustaqil ish

1. 1. Agar V-soha $x=0, y=0, z=0, 2x+3y+4z=12$ tekisliklar bilan chegaralangan bo‘lsa $\iiint_V f(x,y,z)\,dxdydz$ −integralning chegarasini qo‘ying.

2. V-soha $z=x^2+y^2, z=1$ sirtlar bilan chegaralangan bo‘lsa $\iiint_V \sqrt{x^2+y^2}\,dxdydz$-integralni hisoblang. (Javob: $4\pi/15$.)

2. 1. Ushbu $\iiint_V f(x,y,z)\,dxdydz$-integralda, V-soha $y=x, y=2x, z=0, x+z=2$ sirtlar bilan chegaralangan bo‘lsa, integrallash chegaralarini qo‘ying.

2. Agar V-soha $y=x^2+z^2, z=1$-sirtlar bilan chegaralangan bo‘lsa $\iiint_V \sqrt{x^2+y^2}\,dxdydz$-integralni hisoblang. (Javob: $4\pi/15$.)

3. 1. Quyidagi $\iiint_V f(x,y,z)\,dxdydz$ integralda V-soha $y=x^2,\ z=0,$ $y+z=4$-sirtlar bilan chegaralangan bo‘lsa, integrallash chegarasini qo‘ying.

2. Agar jism $x^2+y^2=9,\ z=1,\ x+y+z=11$ sirtlar bilan chegaralangan bo‘lsa, uning hajmini hisoblang. (Javob: 90π.)

13.5. Uch o‘lchovli integralning tadbiqlari

Jism hajmini hisoblash. V-sohaning hajmi v (jism hajmi) –(13.21) formula orqali hisoblanadi va bu yerda agar hisoblashga qulay bo‘lsa boshqa koordinatalarga o‘tiladi (silindrik, sferik va boshqalar).

1- misol. Agar jism $z=1,\ z=5-x^2-y^2$ sirtlar bilan chegaralangan bo‘lsa, uning hajmini hisoblang.

►Sirtlarning tenglamasiga ko‘ra dekart koordinatalar sistemasida V sohani chizib olamiz (13.28-rasm). U holda silindrik koordinatalar sistemasida hajm

$$v=\iiint\limits_{V`} \rho d\rho d\varphi dz,$$

bu yerda $V`$: $(0\le\varphi\le 2\pi,\ 0\le\rho\le 2,\ 1\le z\le 5-\rho^2)$.

Demak

$v=\int_0^{2\pi}d\varphi\int_0^2\rho d\rho\int_1^{5-\rho^2}dz=2\pi\int_0^2\rho(5-\rho^2-1)d\rho=2\pi\left(2\rho^2-\frac{\rho^4}{4}\right)|_0^2=$ $8\pi.$◄

2- misol. Ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ bilan chegaralangan jism hajmini hisoblang.

►Umumlashgan sferik koordinatalar sistemasining (13.26) formulasiga asosan, qidiralayotgan hajm

$$v=\iiint\limits_{V`}abcr^2\sin\theta\,drd\varphi d\theta,$$

bu yerda V'-sferik koordinatalar sistemasiga o‘tganimizda ellipsoidning ichki qismi akslangan soha. V'-sohani chegaralovchi sirtning tenglamasi, ellipsoidning tenglamasiga x,y,z o‘zgaruvchilarning (13.28) formuladagi qiymatlarini qo‘yib chiqishdan hosil bo‘ladi:

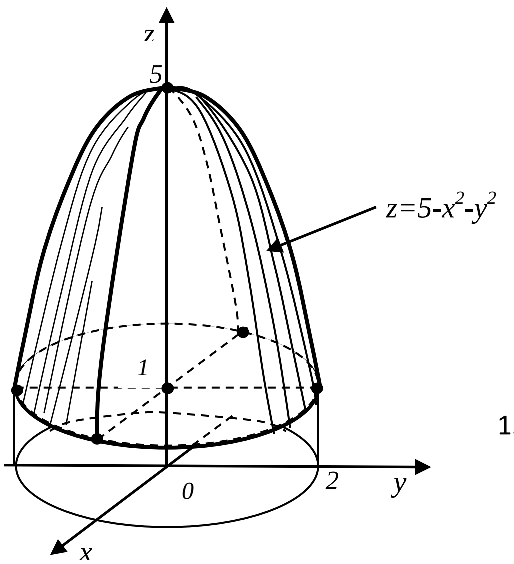
$$r^2\sin\theta^2\cos\varphi^2+r^2\sin\theta^2\sin\varphi^2+r^2\cos\theta^2=1,$$

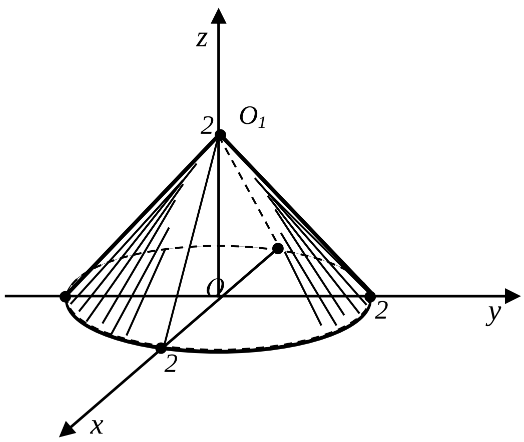
ya’ni $r=1$. Demak,

$$v=abc\int\limits_0^{2\pi}d\varphi\int\limits_0^{\pi}\sin^2\theta\,d\theta\int\limits_0^1 r^2dz=\frac{4}{3}\pi abc.$$

Jism massasini hisoblash. Jism massasi *m* (13.22) formula yordamida hisoblanadi.◄

3- misol. Agar u, $(z-2)^2=x^2+y^2$ konusning sirti va $z=0$ tekislik bilan chegaralangan va zichligi $\delta(x,y,z)=z$ ga teng bo‘lsa, jismning massasini hisoblang.





►Konusning uchi $O_1(0,0,2)$ nuqtada joylashgan va $z=0$ tekislik bilan kesimda $x^2+y^2=4$ aylana hosil bo'ladi (13.29-rasm). Qaralayotgan jismning sirtida $z=2-\sqrt{x^2+y^2}$. U holda jismning massasi

$$m=\iiint\limits_V zdxdydz=\iiint\limits_V z\rho d\rho d\varphi dz=\int\limits_0^{2\pi} d\varphi\int\limits_0^2 \rho d\rho\int\limits_0^{2-\rho} dz=$$

$$=2\pi\int_0^2 \rho(2-\rho)d\rho=2\pi\left(\rho^2-\frac{\rho^3}{3}\right)\Big|_0^2=\frac{8}{3}\pi.$$◄

Jism massa markazining koordinatalarini hisoblash. Uch o'lchovli R^3-fazoda uzluksiz taqsimlangan hajm zichligi $\delta=\delta(x,y,z)$ bo'lgan V-jism berilgan bo'lsin. U holda bu jismning massasi markazi koordinatalari quyidagi formulalar orqali hisoblanadi:

$$x_c=\frac{\iiint_V x\delta(x,y,z)dv}{\iiint_V \delta(x,y,z)dv},\ y_c=\frac{\iiint_V y\delta(x,y,z)dv}{\iiint_V \delta(x,y,z)dv},\ z_c=\frac{\iiint_V z\delta(x,y,z)dv}{\iiint_V \delta(x,y,z)dv}.$$

Ushbu miqdorlar

$$M_x=\iiint\limits_V x\delta(x,y,z)dv,\qquad M_y=\iiint\limits_V y\delta(x,y,z)dv,$$

$$M_z=\iiint\limits_V z\delta(x,y,z)dv$$

jismning Oyz, Oxz va Oxy koordinatalar tekisligiga nisbatan *statistik momenti* deyiladi. Agar zichlik $\delta(x,y,z)=\text{const}$ o'zgarmas bo'lsa jism massasi markazining koordinatalari jism zichligiga bog'liq bo'lmaydi.

4- misol. Bir jinsli $x=y^2+z^2$, $x=4$ sirtlar bilan chegaralangan V-jismning massasi markazining koordinatalari hisoblansin.

►Berilgan sirtlar bilan chegaralangan jismni chizib olamiz (13-30 rasm). V-soha $x=4$ tekislik bilan kesilgan paraboloidning sirti bilan chegaralangan.

Uning Oyz tekislikka proyeksiyasi radiusi 2 ga teng bo‘lgan $y^2+z^2=4$ aylana bilan chegaralangan doiradan iborat. Dastlab silindrik koordinatalar sistemasida jismning massasini uning zichligi $\delta=1$ ga teng deb hisoblaymiz

$$m=\iiint\limits_V dxdydz=\int\limits_0^{2\pi} d\varphi\int\limits_0^2 \rho d\rho\int\limits_{\rho^2}^4 dx=2\pi\int\limits_0^2 \rho(4-\rho^2)\,d\rho=$$

$$=2\pi\left(2\rho^2-\frac{\rho^2}{4}\right)\Big|_0^2=8\pi.$$

U holda

$$x_C=\frac{1}{m}\iiint\limits_V xdxdydz=\frac{1}{8\pi}\int\limits_0^{2\pi} d\varphi\int\limits_0^2 \rho d\rho\int\limits_{\rho^2}^4 xdx=\frac{1}{8\pi}\cdot 2\pi\int\limits_0^2 \rho\left(\frac{1}{2}x^2\right)\Big|_{\rho^2}^4 d\rho=$$

$$\frac{1}{8}\int\limits_0^2 \rho(16-\rho^4)\,d\rho=\frac{1}{8}\left(8\rho^2-\frac{\rho^5}{5}\right)\Big|_0^2=\frac{16}{5}.$$

Xuddi shuningdek y_C va z_C aniqlanadi, jism bir jinsli va Ox o‘qiga nisbatan simmetrik bo‘lgani uchun, birdaniga $y_C=0$ va $z_C=0$ deb qabul qilsa bo‘ladi. ◄

Jismning inertsiya momentini hisoblash. Koordinata boshiga nisbatan $V\epsilon R^3$ inertsiya momenti quyidagi formula yordamida hisoblanadi.

$$I_0=\iiint\limits_V (x^2+y^2+z^2)\delta(x,y,z)\,dxdydz$$

Ox, Oy, Oz koordinata o‘qlariga nisbatan inertsiya momenti mos ravishda quyidagicha aniqlanadi:

$$I_x=\iiint\limits_V (y^2+z^2)\delta(x,y,z)\,dxdydz,$$

$$I_y = \iiint\limits_V (x^2+z^2)\delta(x,y,z)\,dxdydz,$$

$$I_z = \iiint\limits_V (x^2+y^2)\delta(x,y,z)\,dxdydz.$$

Oxy, Oyz, Oxz koordinata tekisliklariga nisbatan esa mos ravishda quyidagicha aniqlanadi:

$$I_{xy} = \iiint\limits_V z^2\delta(x,y,z)\,dxdydz,$$

$$I_{yz} = \iiint\limits_V x^2\delta(x,y,z)\,dxdydz,$$

$$I_{xz} = \iiint\limits_V y^2\delta(x,y,z)\,dxdydz.$$

5- misol. Radiusi R va og‘irligi P bo‘lgan bir jinsli sharning, uning markazi va diametriga nisbatan inertsiya momenti hisoblansin.

►Sharning hajmi $V = \frac{4}{3}\pi R^3$, u holda uning o‘zgarmas zichligi $\delta = 3P/(4g\pi R^3)$ ga teng. Sharning markazini koordinata boshiga joylashtiramiz, u holda shart sirtining tenglamasi $x^2+y^2+z^2=R^2$ tenglama bilan aniqlanadi. Sharning, shar markaziga nisbatan inertsiya momenti sferik koordinatalar sistemasida hisoblash oson:

$$I_0 = \delta\iiint\limits_V (x^2+y^2+z^2)\delta(x,y,z)\,dxdydz = \delta\iiint\limits_{V`} r^4\sin\theta\,drd\varphi d\theta =$$

$$= \delta\int\limits_0^{2\pi} d\varphi \int\limits_0^{\pi}\sin^3\theta\,d\theta\int\limits_0^R r^4dr = \delta\cdot 2\pi\cdot 2\frac{R^5}{5} = \frac{3}{5}\frac{P}{g}R^2.$$

Shar simmetrik va bir jinsli bo‘lgani uchun uning inertsiya momenti ixtiyoriy diametr uchun bir xil, masalan Oz o‘qida yotgan diametr uchun inertsiya momentini hisoblaymiz:

$$I_z = \delta\iiint\limits_V (x^2+y^2)\,dxdydz = \delta\iiint\limits_{V`} r^2\sin^2\theta r^2\sin\theta\,drd\varphi d\theta =$$

$$= \delta \int_0^{2\pi} d\varphi \int_0^{\pi} \sin^3\theta \, d\theta \int_0^{R} r^4 dr = -\delta \cdot 2\pi \frac{R^5}{5}(1-\cos^2\theta)d(\cos\theta) =$$

$$= -\delta \cdot 2\pi \frac{R^5}{5}\left(\cos\theta - \frac{1}{3}\cos^3\theta\right)\Big|_0^{\pi} = \frac{2}{5}\frac{P}{g}R^2. ◄$$

AT-13.6

1. Quyidagi sirtlar $z = \sqrt{x^2+y^2},\ 2-z = x^2+y^2$ chegaralangan jismning hajmini hisoblang. (Javob:4π/3.)

2. Agar jism $x+y+z=1, x=0, y=0, z=0$ tekisliklar bilan chegaralangan va zichligi $\delta(x,y,z) = 1/(x+y+z+1)^4$ ga teng bo‘lsa, uning massasini hisoblang. (Javob: 1/48.)

3. Agar jism silindr $x = y^2$ va $x+z=1,\ z=0$ tekisliklar bilan chegaralangan bo‘lsa, uning hajmini hisoblang. (Javob: 8/15.)

4. Sferalar $x^2+y^2+z^2=1, x^2+y^2+z^2=16$ va $z^2=x^2+y^2$ konus bilan chegaralangan (jism konusning ichida joylashgan) jismning hajmini toping. (Javob: $\frac{28\pi}{3}\left(1-\frac{\sqrt{2}}{2}\right)$.)

5. Bir jinsli markazi koordinata boshida radiusi R ga teng bo‘lgan sharning Oxy tekislikdan yuqorida joylashgan qismini massasi markazi koordinatalarini toping. (Javob: $C\left(0{,}0,\frac{3}{8}R\right)$.)

6. $x+y+z=a,\ x=0,\ y=0,\ z=0$ tekisliklar bilan chegaralangan bir jinsli jism massasi markazining koordinatalarini toping. (Javob: $(\frac{1}{4}a, \frac{1}{4}a, \frac{1}{4}a)$.)

7. Og‘irligi P, balandligi H asosining radiusi R ga teng bo‘lgan bir jinsli doiraviy to‘g‘ri konusning o‘qiga nisbatan inertsiya momenti topilsin. (Javob: $\frac{3}{10}\frac{P}{g}R^2$.)

Mustaqil ish.

1. Agar jism $z=x^2, 3x+2y=12,\ y=0, z=0$ sirtlar bilan chegaralangan bolsa, uning hajmi hisoblansin. (Javob: 32.)

2 $x+2y-z=2, x=0, y=0, z=0$ tekisliklar bilan chegaralangan va zichligi $\delta(x,y,z)=x$ bo‘lgan jismning Oyz tekislikka nisbatan inertsiya momenti hisoblansin. (Javob:4/15.)

3. $2z=4-x^2-y^2, z=0$ sirtlar bilan chegaralangan bir jinsli jism massasi koordinata markazini hisoblang. (Javob: (0,0,2/3).)

13.6 13 BOBGA DOIR INDIVIDUAL UY TOPSHIRIQLARI

IUT-13.1

1. Agar D soha quyidagi chiziqlar bilan berilgan bo‘lsa, ikki o‘lchovli $\iint_D f(x,y)dxdy$ integralni x va y o‘zgaruvchilar boyicha tashqi integral ko‘rinishda karrali integralda keltiring.

1.1. D: $y=\sqrt{4-x^2},\ y=\sqrt{3x},\ x\geq 0$.

1.2. D: $x^2=2y,\ 5x-2y-6=0$.

1.3. D: $x=\sqrt{8-y^2},\ y\geq 0,\ y=x$.

1.4. D: $x\geq 0, y\geq 0, y\leq 1,\ y=\ln x$.

1.5. D: $x^2=2-y,\ x+y=0$.

1.6. D: $y=\sqrt{2-x^2},\ y=x^2$.

1.7. D: $y=x^2-2,\ y=x$.

1.8. D: $x \geq 0,\ y \geq 1,\ y \leq 3,\ y = x$.

1.9. D: $y^2 = 2x,\ x^2 = 2y,\ x \leq 1$.

1.10. D: $x \geq 0,\ y \geq x,\ y = \sqrt{9 - x^2}$.

1.11. D: $y^2 = 2 - x, y = x$.

1.12. D: $x = \sqrt{2 - y^2},\ x = y^2,\ y \geq 0$.

1.13. D: $y \geq 0,\ x + 2y - 12 = 0,\ y = \lg x$.

1.14. D: $x \leq 0,\ y \geq 1,\ y \leq 3,\ y = -x$.

1.15. D: $y = 0,\ y \geq x,\ y = -\sqrt{2 - x^2}$.

1.16. D: $y \geq 0,\ x = \sqrt{y},\ y = \sqrt{8 - x^2}$.

1.17. D: $y = -x,\ y^2 = x + 3$.

1.18. D: $y = \sqrt{4 - x^2},\ x \geq 0,\ x = 1,\ y = 0$.

1.19. D: $x = -1,\ x = -2,\ y \geq 0,\ y = x^2$.

1.20. D: $y \leq 0,\ x^2 = -y,\ x = \sqrt{1 - y^2}$.

1.21. D: $y \geq 0,\ y \leq 1,\ y = x,\ x = -\sqrt{4 - y^2}$.

1.22. D: $x \leq 0,\ y = 1,\ y = 4,\ y = -x$.

1.23. D: $y = 3 - x^2,\ y = -x$.

1.24. D: $x = 0,\ x = -2,\ y \geq 0,\ y = x^2 + 4$.

1.25. D: $x = 0,\ y = 0,\ y = 1,\ (x - 3)^2 + y^2 = 1$.

1.26. D: $x = \sqrt{9 - y^2},\ y = x,\ y \geq 0$.

1.27. D: $x + 2y - 6 = 0,\ y = x,\ y \geq 0$.

1.28. D: $y = -x,\ 3x + y = 3,\ y = 3$.

1.29. D: $x \geq 0,\ y = 1,\ y = -1,\ y = \log_{1/2} x$.

1.30. D: $x \geq 0,\ y \geq 0,\ y = 1,\ x = \sqrt{4 - y^2}$.

2. Quyidagi chiziqlar bilan chegaralangan D soha bo‘yicha ikki o‘lchovli integralni hisoblang.

2.1. $\iint_D (x^2 + y)dxdy,\ \ D:\ y = x^2,\ \ x = y^2$.

2.2. $\iint_D x^2 y dxdy,\ \ D:\ y = x^2,\ \ y = 2x$.

2.3. $\iint_D (x+y)dxdy$, D: $y^2 = x$, $y = x$.

2.4. $\iint_D x^2 y dxdy$, D: $y = 2 - x$, $y = x$, $x \geq 0$.

2.5. $\iint_D (x^3 - 2y)dxdy$, D: $y = x^2 - 1$, $x \geq 0$, $y \leq 0$.

2.6. $\iint_D (y - x)dxdy$, D: $y = x$, $y = x^2$.

2.7. $\iint_D (1 + y)dxdy$, D: $y^2 = x$, $5y = x$.

2.8. $\iint_D (x+y)dxdy$, D: $y = x^2 - 1$, $y = -x^2 + 1$.

2.9. $\iint_D x(y-1)dxdy$, D: $y = 5x$, $y = x$, $x = 3$.

2.10. $\iint_D (x-2)ydxdy$, D: $y = x$, $y = \frac{1}{2}x$, $x = 2$.

2.11. $\iint_D (x - y^2)dxdy$, D: $y = x^2$, $y = 1$.

2.12. $\iint_D x^2 y dxdy$, D: $y = 2x^3$, $y = 0$, $x = 1$.

2.13. $\iint_D (x^2 + y^2)dxdy$, D: $x = y^2$, $x = 1$.

2.14. $\iint_D xy dxdy$, D: $y = x^3$, $y = 0$, $x \leq 2$.

2.15. $\iint_D (x+y)dxdy$, D: $y = x^3$, $y = 8$, $y = 0$, $x = 3$.

2.16. $\iint_D x(2x+y)dxdy$, D: $y = 1 - x^2$, $y \geq 0$.

2.17. $\iint_D y(1-x)dxdy$, D: $y^3 = x$, $y = x$.

2.18. $\iint_D xy^3 dxdy$, D: $y^2 = 1 - x$, $x \geq 0$.

2.19. $\iint_D x(y+5)dxdy$, D: $y = x + 5$, $x + y + 5 = 0$, $x \leq 0$.

2.20. $\iint_D (x-y)dxdy$, D: $y = x^2 - 1$, $y = 3$.

2.21. $\iint_D (x+1)y^2 dxdy$, D: $y = 3x^2$, $y = 3$.

2.22. $\iint_D xy^2 dxdy$, D: $y = x$, $y = 0$, $x = 1$.

2.23. $\iint_D (x^3 + y)dxdy$, D: $x + y = 1$, $x + y = 2$, $x \leq 1$, $x \geq 0$.

2.24. $\iint_D xy^3 dxdy$, D: $y = x^3$, $y \geq 0$, $y = x$.

2.25. $\iint_D (x^3 + 3y)dxdy$, D: $x + y = 1$, $y = x^2 - 1$, $x \geq 0$.

2.26. $\iint_D xy dxdy$, D: $y = \sqrt{x}$, $y = 0$, $x + y = 2$.

2.27. $\iint_D \frac{y^2}{x^2} dxdy$, D: $y = x$, $xy = 1$, $y = 2$.

2.28. $\iint_D y(1 + x^2)dxdy$, D: $y = x^3$, $y = 3x$.

2.29. $\iint_D y^2(1+2x)dxdy\,,\ \ D\colon\ x=2-y^2,\ \ x=0.$

2.30. $\iint_D e^y dxdy\,,\ \ D\colon\ y=\ln x\,,\ \ y=0,\ x=2.$

3. Qutb koordinatalar sistemasida ikki o‘lchovli integralni hisoblang.

3.1. $\int_0^1 dx\int_0^{\sqrt{1-x^2}}\sqrt{\frac{1-x^2-y^2}{1+x^2+y^2}}\,dy.$

3.2. $\int_{-\sqrt{3}}^0 dx\int_0^{\sqrt{3-x^2}}\frac{dy}{\sqrt{1+x^2+y^2}}.$

3.3. $\int_0^R dx\int_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}}\frac{\tan\sqrt{x^2+y^2}}{-\sqrt{x^2+y^2}}\,dy.$

3.4. $\int_0^1 dx\int_0^{\sqrt{1-x^2}}\ln(1+x^2+y^2)dy.$

3.5. $\int_{-2}^2 dy\int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}}\sqrt{1-x^2-y^2}\,dx.$

3.6. $\int_{-\sqrt{2}}^{\sqrt{2}} dx\int_{-\sqrt{2-x^2}}^0\frac{xy}{x^2+y^2}\,dy.$

3.7. $\int_{-R}^0 dx\int_0^{\sqrt{R^2-x^2}}\cos\sqrt{x^2+y^2}\,dy.$

3.8. $\int_{-R}^R dx\int_0^{\sqrt{R^2-x^2}}\tan(x^2+y^2)\,dy.$

3.9. $\int_0^R dx\int_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}}\cos(x^2+y^2)\,dy.$

3.10. $\int_{-R}^R dx\int_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}}\sin\sqrt{x^2+y^2}\,dy.$

3.11. $\int_{-\sqrt{3}}^{\sqrt{3}} dx\int_0^{\sqrt{3-x^2}}\sqrt{1+x^2+y^2}\,dy.$

3.12. $\int_{-\sqrt{2}}^{\sqrt{2}} dx\int_{-\sqrt{2-x^2}}^{\sqrt{2-x^2}}(1+x^2+y^2)dy.$

3.13. $\int_0^2 dx\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\frac{dy}{1+x^2+y^2}.$

3.14. $\int_0^1 dx\int_0^{\sqrt{1-x^2}}\frac{dy}{1+\sqrt{x^2+y^2}}.$

3.15. $\int_{-R}^R dx\int_{-\sqrt{R^2-x^2}}^0\frac{\sin\sqrt{x^2+y^2}}{\sqrt{x^2+y^2}}\,dy.$

3.16. $\int_0^R dx\int_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}}\frac{dy}{\sqrt{x^2+y^2}\cos^2\sqrt{x^2+y^2}}.$

3.17. $\int_{-R}^R dx\int_{-\sqrt{R^2-x^2}}^0\frac{dy}{\sqrt{x^2+y^2}\sin^2\sqrt{x^2+y^2}}.$

3.18. $\int_0^2 dx \int_0^{\sqrt{4-x^2}} \frac{xy}{\sqrt{x^2+y^2}} dy.$

3.19. $\int_{-R}^0 dx \int_0^{\sqrt{R^2-x^2}} \frac{dy}{\sqrt{x^2+y^2}\,\text{ctg}^2\sqrt{x^2+y^2}}.$

3.20. $\int_{-3}^3 dx \int_{-\sqrt{9-x^2}}^0 \frac{xy}{x^2+y^2} dy.$

3.21. $\int_{-R}^0 dx \int_{-\sqrt{9-x^2}}^0 \cos(x^2+y^2)\, dy.$

3.22. $\int_{-R}^0 dx \int_0^{\sqrt{R^2-x^2}} \sin(x^2+y^2)\, dy.$

3.23. $\int_{-1}^1 dx \int_0^{\sqrt{1-x^2}} \sqrt{1+x^2+y^2}\, dy.$

3.24. $\int_{-2}^2 dx \int_0^{\sqrt{4-x^2}} \sqrt{x^2+y^2}\, e^{x^2+y^2} dy.$

3.25. $\int_0^3 dx \int_0^{\sqrt{9-x^2}} \ln(1+x^2+y^2)\, dy.$

3.26. $\int_{-\sqrt{2}}^{\sqrt{2}} dx \int_{-\sqrt{2-x^2}}^{\sqrt{2-x^2}} e^{-(x^2+y^2)} dy.$

3.27. $\int_0^1 dx \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{\ln\left(1+\sqrt{x^2+y^2}\right)}{\sqrt{x^2+y^2}} dy.$

3.28. $\int_0^2 dx \int_0^{\sqrt{4-x^2}} \cos\sqrt{x^2+y^2}\, dy.$

3.29. $\int_0^R dx \int_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}} \sin(x^2+y^2)\, dy.$

3.30. $\int_0^R dx \int_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}} \frac{\tan\sqrt{x^2+y^2}}{\sqrt{x^2+y^2}} dy.$

4. Berilgan yassi chiziqlar bilan chegaralangan D sohani yuzini hisoblang.

4.1. D: $y^2 = 4x,\ x+y=3,\ y \geq 0.$ (Javob: 10/3.)

4.2. D: $y = 6x^2,\ x+y=2,\ x \geq 0.$ (Javob: 5/8.)

4.3. D: $y^2 = x+2,\ x=2.$ (Javob: 32/3.)

4.4. D: $x = -2y^2,\ x = 1-3y^2,\ x \leq 0,\ y \geq 0.$ (Javob: 16/3.)

4.5. D: $y = 8/(x^2+4),\ x^2 = 4y.$ (Javob: 2π-4/3.)

4.6. D: $y = x^2+1,\ x+y=3.$ (Javob: 9/2.)

4.7. D: $y^2 = 4x,\ x^2 = 4y.$ (Javob: 16/3.)

4.8. D: $y = \cos x,\ y \leq x+1,\ y \geq 0.$ (Javob: 3/2.)

4.9. D: $x = \sqrt{4-y^2},\ y = \sqrt{3x},\ x \geq 0.$ (Javob: 2π-$\sqrt{3}$/6.)

4.10. D: $y = x^2 + 2,\ x \geq 0,\ x = 2,\ y = x.$ (Javob: 14/3.)

4.11. D: $y = 4x^2,\ 9y = x^2,\ \ y \leq 2.$ (Javob: $20\sqrt{2}/3$.)

4.12. D: $y = x^2,\ y = -x.$ (Javob: 1/6.)

4.13. D: $x = y^2,\ x = \frac{3}{4}y^2 + 1.$ (Javob: 8/3.)

4.14. D: $y = \sqrt{2 - x^2},\ y = x^2.$ (Javob: π/2+1/3.)

4.15. D: $y = x^2 + 4x,\ y = x + 4.$ (Javob: 125/6.)

4.16. D: $2y = \sqrt{x},\ x + y = 5,\ x \geq 0.$ (Javob: 28/3.)

4.17. D: $y = 2^x,\ y = 2x - x^2,\ x = 2,\ x = 0.$ (Javob: $\frac{3}{\ln 2} - \frac{4}{3}$.)

4.18. D: $y = -2x^2 + 2,\ \ y \geq -6.$ (Javob: 64/3.)

4.19. D: $y^2 = 4x,\ x = 8/(y^2 + 4).$ (Javob: 2π-4/3.)

4.20. D: $y = 4 - x^2,\ y = x^2 - 2x.$ (Javob: 9.)

4.21. D: $x = y^2 + 1,\ x + y = 3.$ (Javob: 9/2.)

4.22. D: $x^2 = 3y,\ \ y^2 = 3x.$ (Javob: 3.)

4.23. D: $x = \cos y,\ x \leq y + 1,\ x \geq 0.$ (Javob: ½.)

4.24. D: $x = 4 - y^2,\ x - y + 2 = 0.$ (Javob: 125/6.)

4.25. D: $x = y^2,\ x = \sqrt{2 - y^2}.$ (Javob: π/2+1/3.)

4.26. D: $\frac{x^2}{4} + \frac{y^2}{1} = 1,\ y \leq \frac{1}{2}x,\ y \geq 0.$ (Javob: π/4.)

4.27. D: $y^2 = 4 - x,\ y = x + 2,\ y = 2,\ y = -2.$ (Javob: 56/3.)

4.28. D: $y = x^2,\ y = \frac{3}{4}x^2 + 1.$ (Javob: 8/3.)

4.29. D: $x = y^2,\ y^2 = 4 - x.$ (Javob: $16\sqrt{2}/3$.)

4.30. D: $xy = 1,\ x^2 = y,\ y = 2,\ x = 0.$ (Javob: 2/3+ln 2.)

5. Qutb koordinatalar sistemasida ikki o‘lchovli integral yordamida berilgan chiziqlar bilan chegaralangan yassi shaklning yuzini toping.

5.1. $(x^2 + y^2)^2 = a^2(4x^2 + y^2).$

5.2. $(x^2 + y^2)^3 = a^2x^2y^2.$

5.3. $(x^2 + y^2)^3 = a^2x^2(4x^2 + 3y^2).$

5.4. $(x^2 + y^2)^2 = a^2(3x^2 + 2y^2).$

5.5. $x^4 - y^4 = (x^2 + y^2)^3$. 5.6. $\rho = a\sin^2 2\varphi$.

5.7. $\rho = a\sin^2\varphi$. 5.8. $\rho = a(1 - \cos\varphi)$.

5.9. $(x^2 + y^2)^2 = a^2(2x^2 + 3y^2)$.

5.10. $(x^2 + y^2)^2 = a^2(5x^2 + 3y^2)$.

5.11. $(x^2 + y^2)^2 = a^2(7x^2 + 5y^2)$.

5.12. $(x^2 + y^2)^2 = 2a^2xy$.

5.13. $(x^2 + y^2)^3 = 4x^2y^2$. 5.14. $(x^2 + y^2)^2 = a^4y^2$.

5.15. $(x^2 + y^2)^3 = a^4x^2$. 5.16. $\rho = a\cos^2\varphi$.

5.17. $\rho^2 = a^2(1 + \sin^2\varphi)$. 5.18. $(x^2 + y^2)^3 = a^2x^4$.

5.19. $(x^2 + y^2)^2 = 4(3x^2 + 4y^2)$.

5.20. $(x^2 + y^2)^3 = a^2x^2y^2$.

5.21. $(x^2 + y^2)^3 = a^2(x^4 + y^4)$.

5.22. $(x^2 + y^2)^3 = 2ay^3$.

5.23. $(x^2 + y^2)^3 = 4a^2xy(x^2 - y^2)$.

5.24. $\rho = a\sin 2\varphi$.

5.25. $\rho = a\cos 5\varphi$. 5.26. $\rho = 4(1 + \cos\varphi)$.

5.27. $\rho = 2a(2 + \cos\varphi)$. 5.28. $\rho^2 = a^2\cos 3\varphi$.

5.29. $\rho^2 = a^2\cos 2\varphi$. 5.30. $\rho = a\sin 3\varphi$.

6. Berilgan sirtlar bilan chegaralangan jismning hajmini toping.

6.1. $z = x^2 + y^2,\ x + y = 1,\ x \geq 0,\ y \geq 0,\ z \geq 0$. (Javob: 1/6.)

6.2. $z = 2 - (x^2 + y^2),\ x + 2y = 1,\ x \geq 0,\ y \geq 0,\ z \geq 0$. (Javob: 53/96.)

6.3. $z = x^2,\ x - 2y + 2 = 0,\ x + y - 7 = 0,\ z \geq 0$. (Javob: 32.)

6.4. $z = 2x^2 + 3y^2,\ y = x^2,\ y = x,\ z \geq 0$. (Javob: 29/140.)

6.5. $z = 2x^2 + y^2,\ y \leq x,\ y = 3x,\ x = 2,\ z \geq 0$. (Javob: 152/3.)

6.6. $z = x,\ y = 4,\ x = \sqrt{25 - y^2},\ x \geq 0,\ y \geq 0,\ z \geq 0$. (Javob: 118/3.)

6.7. $y = \sqrt{x},\ y = x,\ x + y + z = 2,\ z \geq 0$. (Javob: 11/60.)

6.8. $y = 1 - x^2,\ x + y + z = 3,\ y \geq 0,\ z \geq 0$. (Javob: 104/30.)

6.9. $z = 2x^2 + y^2,\ x + y = 4,\ x \geq 0,\ y \geq 0,\ z \geq 0.$ (Javob: 64.)

6.10. $z = 4 - x^2,\ x^2 + y^2 = 4,\ x \geq 0,\ y \geq 0,\ z \geq 0.$ (Javob: 3π.)

6.11. $2x + 3y - 12 = 0,\ 2z = y^2,\ \ x \geq 0,\ y \geq 0,\ z \geq 0.$ (Javob: 16.)

6.12. $z = 10 + x^2 + 2y^2,\ y = x,\ x = 1,\ y \geq 0,\ z \geq 0.$ (Javob: 65/12.)

6.13. $z = x^2,\ x + y = 6,\ y = 2x,\ x \geq 0,\ y \geq 0,\ z \geq 0.$ (Javob: 4.)

6.14. $z = 3x^2 + 2y^2 + 1,\ y = x^2 - 1,\ y = 1,\ z \geq 0.$ (Javob: $264\sqrt{2}/35$.)

6.15. $3y = \sqrt{x},\ y \leq x,\ x + y + a = 10,\ y = 1,\ z = 0.$ (Javob: 303/20.)

6.16. $y^2 = 1 - x,\ x + y + z = 1,\ x = 0,\ z = 0.$ (Javob: 49/60.)

6.17. $y = x^2,\ x = y^2,\ z = 3x + 2y + 6,\ z = 0.$ (Javob: 11/4.)

6.18. $x^2 = 1 - y,\ x + y + z = 3,\ y \geq 0,\ z \geq 0.$ (Javob:52/15.)

6.19. $x = y^2,\ \ x = 1,\ x + y + z = 4,\ z = 0.$ (Javob: 68/15.)

6.20. $z = 2x^2 + y^2,\ x + y = 1,\ x \geq 0,\ y \geq 0,\ z \geq 0.$ (Javob: 1/4.)

6.21. $y = x^2,\ y = 4,\ z = 2x + 5y + 10,\ z \geq 0.$ (Javob: 704/3.)

6.22. $y = 2x,\ x + y + z = 2,\ x \geq 0,\ \ z \geq 0.$ (Javob: 4/9.)

6.23. $y = 1 - z^2,\ y = x,\ y = -x,\ y \geq 0,\ z \geq 0.$ (Javob: 8/15.)

6.24. $x^2 + y^2 = 4y,\ z^2 = 4 - y,\ z \geq 0.$ (Javob: 256/15.)

6.25. $x^2 + y^2 = 1,\ z = 2 - x^2 - y^2,\ z \geq 0.$ (Javob: $\frac{3}{2}\pi$.)

6.26. $y = x^2,\ z = 0,\ y + z = 2.$ (Javob: $\frac{32}{15}\sqrt{2}$.)

6.27. $z^2 = 4 - x,\ x^2 + y^2 = 4x,\ z \geq 0.$ (Javob:256/15.)

6.28. $z = x^2 + 2y^2,\ y = x,\ \ x \geq 0,\ y = 1,\ z \geq 0.$ (Javob:7/12.)

6.29. $z = y^2,\ x + y = 1,\ x \geq 0,\ \ z \geq 0.$ (Javob: 1/12.)

6.30. $y^2 = x,\ x = 3,\ z = x,\ z \geq 0.$ (Javob: $36\sqrt{3}/5$.)

Namunaviy variantni yechish

1. Agar D soha $x = \sqrt{y},\ x = \sqrt{2 + y},\ x = 0,\ y = 0$ chiziqlar bilan chegaralangan bo‘lsa, ikki o‘lchovli $\iint_D f(x,y)dxdy$ integralni x va y o‘zgaruvchi tashqi integral ko‘rinishida karrali integralda keltirilsin.

►D soha 13.31 rasmda keltirilgan va $x^2 = y + 2,\ x^2 = y$ parabolaning yoylari, $x = 0,\ x = 2$ to‘g‘ri chiziqlar bilan chegaralangan.

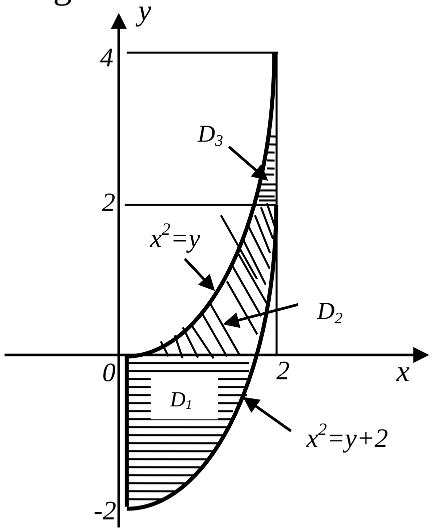
Demak,

$$\iint\limits_D f(x,y)dxdy = \int\limits_0^2 dx \int\limits_{x^2-2}^{x^2} f(x,y)\,dy =$$

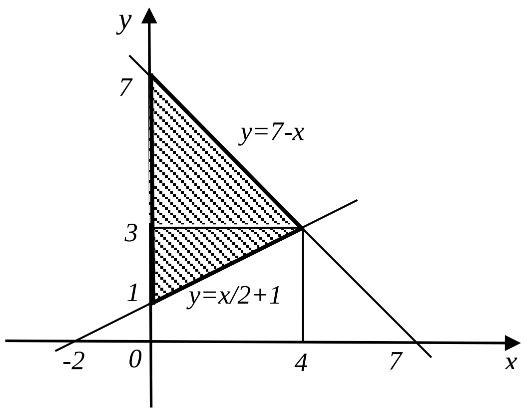
$$= \int\limits_{-2}^{0} dy \int\limits_{0}^{\sqrt{y+2}} f(x,y)\,dx + \int\limits_{0}^{2} dy \int\limits_{\sqrt{y}}^{\sqrt{y+2}} f(x,y)\,dx + \int\limits_{2}^{4} dy \int\limits_{\sqrt{y}}^{2} f(x,y)dx$$

bo‘lar ekan.

2. Agar D soha $x = 0,\ y = 7 - x,\ y = \frac{1}{2}x + 1$ chiziqlar bilan chegaralangan bo‘lsa, ushbu $\iint_D (x - 2y)dxdy$ integralni D soha bo‘yicha hisoblang.



13.31- rasm



13.32- rasm

►D soha 13.32- rasmda keltirilgan. Agar ichki integralni *y* bo‘yicha, tashqi integralni *x* bo‘yicha hisoblasak, u holda ikki o‘lchovli integral, bitta ikki karrali integral orqali ifodaladi:

$$\iint\limits_D (x-2y)dxdy = \int\limits_0^4 dx \int\limits_{\frac{1}{2}x+1}^{7-x} (x-2y)\,dy =$$

$$= \int\limits_0^4 (xy - y^2)\,\Big|_{\frac{1}{2}x+1}^{7-x} dx = \int\limits_0^4 (7x - x^2 - 49 + 14x - x^2 - \frac{1}{2}x^2 -$$

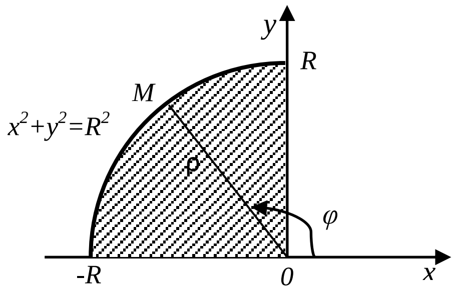
$$-x+\frac{1}{4}x^2+x+1)dx=\int_0^4\left(-\frac{9}{4}x^2+21x-48\right)dx=$$

$$=\left(-\frac{3}{4}x^3+\frac{21}{2}x^2-48x\right)\Big|_0^4=-72. \blacktriangleleft$$

3. Ikki o‘lchovli integralni qutb koordinatalari yordamida hisoblang. Uning $R=1$ da sonli qiymatini toping.

$$I=\int_{-R}^{0}dx\int_0^{\sqrt{R^2-x^2}}\frac{\ln\left(1+\sqrt{x^2+y^2}\right)}{\sqrt{x^2+y^2}}dy.$$

►Integrallash sohasi ikkinchi chorakda joylashgan doiraning chorak qismini tashkil qiladi (13.33-rasm).



13.33- rasm

Qutb koordinatalar sistemasiga o‘tamiz $x=\rho\cos\varphi$, $y=\rho\sin\varphi$, $x^2+y^2=\rho^2$, bu yerda $0\le\rho\le R$; $\frac{\pi}{2}\le\varphi\le\pi$.

U holda

$$I=\int_{\pi/2}^{\pi}d\varphi\int_0^R\frac{\ln(1+\rho)}{\rho}\rho d\rho=$$

$$=\left|\begin{matrix}u=\ln(1+\rho), & du=d\rho/(1+\rho), \\ dv=d\rho, & v=\rho,\end{matrix}\right|=$$

$$=\rho|_{\pi/2}^{\pi}(\rho\ln(1+\rho)\,|_0^R-\int_0^R\frac{\rho}{(1+\rho)}d\rho=$$

$$=\frac{\pi}{2}(R\ln(1+R)-\rho|_0^R+\ln(1+\rho)\,|_0^R)=$$

$$=\frac{\pi}{2}(R\ln(1+R)-R+\ln(1+R)).$$

$R = 1$ qiymatda $I = \frac{\pi}{2}(2\ln 2 - 1)$ ga ega bo‘lamiz. ◄

4. Quyidagi egri chiziqlar bilan chegaralangan shaklning yuzini toping $y = x^2 - 3x$ va $3x + y - 4 = 0$.

►Berilgan shakl pastdan $y = x^2 - 3x$ parabola bilan, yuqoridan $3x + y - 4 = 0$ to‘g‘ri chiziq bilan chegaralangan(13.34-rasm).

Demak,

$$S = \iint_D dxdy \int_{-2}^{2} dx \int_{x^2-3x}^{4-3x} dy = \int_{-2}^{2} (4 - 3x - x^2 + 3x)\, dx =$$

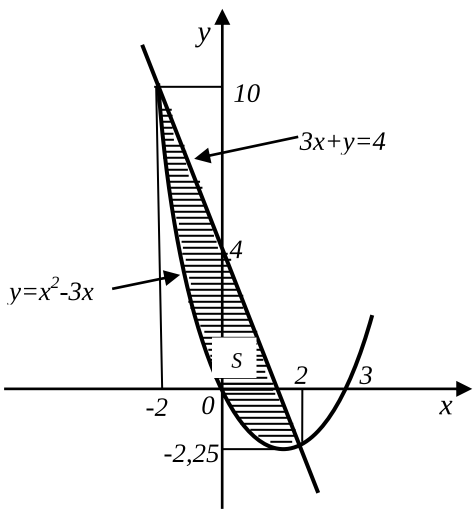
$$= \left(4x - \frac{x^3}{3}\right)\Big|_{-2}^{2} = \frac{32}{3}.$$ ◄

5. Qutb koordinatalar sistemasida ikki o‘lchovli integral yordamida $(x^2 + y^2)^2 = 2y^3$ egri chiziq bilan chegaralangan shaklning yuzini toping.

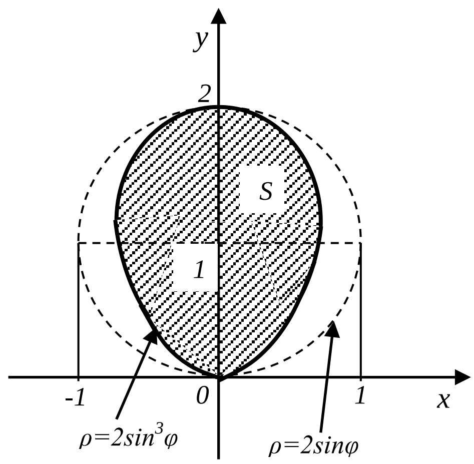
►Qutb koordinatalar egri chiziq bilan chegaralangan D soha 13.35 rasmda keltirilgan. Qutb O nuqta D sohaning chegarasida joylashgan shuning uchun 13.12 formulaga ko‘ra (qarang §13.2, 2-misol, 3-hol):

$$S = \iint_D \rho d\rho d\varphi = \int_0^{\pi} d\varphi \int_0^{2\sin^3\varphi} \rho d\rho = \int_0^{\pi} d\varphi \frac{\rho^2}{2}\Big|_0^{2\sin^3\varphi} =$$

$$= 2\int_0^{\pi} \sin^6\varphi d\varphi = \frac{1}{4}\int_0^{\pi} (1 - \cos 2\varphi)^3\, d\varphi =$$



13.34- rasm



13.35- rasm

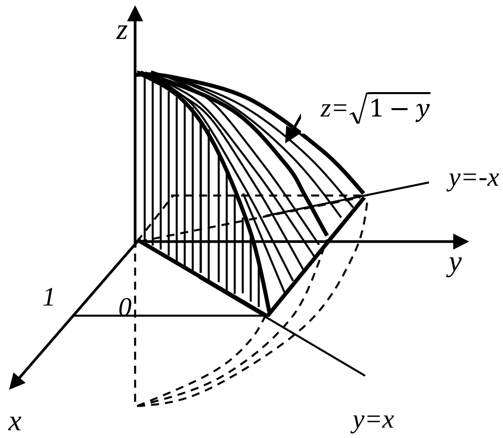
$$=\frac{1}{4}\int_0^{\pi}(1-3\cos 2\varphi+3\cos^2 2\varphi-\cos^3 2\varphi)d\varphi=$$

$$=\frac{1}{4}\left(\pi-\frac{3}{2}\sin 2\varphi\,|_0^{\pi}+\frac{3}{2}\int_0^{\pi}(1+\cos 4\varphi\right)d\varphi-$$

$$-\int_0^{\pi}\cos 2\varphi\,(1-\sin^2 2\varphi)d\varphi=\frac{5}{8}\pi.$$

ni hosil qilamiz. ◄

6. Agar jism $z=\sqrt{1-y},\ y=x,\ y=-x,\ z=0$ sirtlar bilan chegaralangan bo‘lsa, uning hajmini hisoblang,.



13.36- rasm

►Ushbu jism yuqoridan $z=\sqrt{1-y}$ sirt bilan chegaralangan (13.36-rasm), demak

$$v=\iint_D\sqrt{1-y}\,dxdy=2\int_0^1 dy\int_0^y\sqrt{1-y}\,dx=2\int_0^1\sqrt{1-y}\,x|_0^y dy=$$

$$=2\int_0^1 y\sqrt{1-y}dy=\left|\begin{matrix}\sqrt{1-y}=t,\ \ y=1-t^2,\\ dy=-2tdt,\ \ y=0,\ \ t=1\\ y=1,\ \ t=0\end{matrix}\right|=$$

$$=2\int_0^1(1-t^2)t(-2tdt)=-4\int_1^0(t^2-t^4)dt=$$

$$= -4\left(\frac{t^3}{3} - \frac{t^5}{5}\right)\Big|_1^0 = \frac{8}{15}. \blacktriangleleft$$

IUT-13.2

1. Agar V soha berilgan sirtlar berilgan chegaralangan bo‘lsa $\iiint_V f(x,y,z)dxdydz$ uch o‘lgovli integrallash chegarasini qo‘ying. Integrallash sohasini chizing.

1.1 *V:* $x=2, y=4x, y=3\sqrt{x}\,; z\geq 0, z=4$.

1.2 *V:* $x=1; y=3x, y\geq o, z\geq 0, z=2(x^2+y^2)$.

1.3 *V:* $x=1, y=4x,\ z\geq 0, z=\sqrt{3y}$.

1.4 *V:* $x=3, y=x, y\geq 0, z\geq 0, z=3x^2+y^2$.

1.5 *V:* $y=2x, y=2, z\geq 0, z=2\sqrt{x}$.

1.6 *V:* $x=0, y=x, y=5, z\geq 0, z=2x^2+y^2$.

1.7 *V:* $x\geq 0, y=2x, y=1, z\geq 0, x+y+z=3$.

1.8 *V:* $x\geq 0, y=3x, y=3, z\geq 0, x=3\sqrt{z}$.

1.9 *V:* $x=5, y=x/5, y\geq 0, z\geq 0, z=x^2+5y^2$.

1.10 *V:* $x=2, y=4x, z\geq 0, y=2\sqrt{z}$.

1.11 *V:* $x=3, y=\frac{1}{3}x, y\geq 0,\ z\geq 0, z=\frac{1}{2}(x^2+y^2)$.

1.12 *V:* $x=4, y=x/4, z\geq 0, z=4y^2$.

1.13 *V:* $x\geq 0, y=3x, y=3, z\geq 0, z=2(x^2+y^2)$.

1.14 *V:* $x\geq 0, y=4x, y=8, z\geq 0, z=3x^2+y^2$.

1.15 *V:* $x\geq 0, y=5x, y=10, z\geq 0, z=x^2+y^2$.

1.16 *V:* $y=x, y=-x, y=2\ z\geq 0, z=3(x^2+y^2)$.

1.17 *V:* $x=1, y=2x, y=3x, z\geq 0, z=2x^2+y^2$.

1.18 *V:* $y=x, y=-2x, y=1, z\geq 0, z=x^2+4y^2$.

1.19 *V:* $x\geq 0, y\geq 0, z\geq 0, x+y=1, z=3x^2+2y^2$.

1.20 *V:* $x\geq 0, y\geq 0, z\geq 0, 3x+2y=6, z=x^2+y^2$.

1.21 *V:* $x\geq 0, y\geq 0, z\geq 0, x+y=2, z=4-x^2-y^2$.

1.22 *V:* $x\geq 0, y\geq 0, z\geq 0, x+y=3, z=9-x^2-y^2$.

1.23 V: $x \geq 0, y \geq 0, z \geq 0, 3x + 4y = 12, z = 6 - x^2 - y^2$.

1.24 V: $x \geq 0, z \geq 0, y = x, y = 3, z = 18 - x^2 - y^2$.

1.25 V: $x=2, y \geq 0, z \geq 0, y = 3x, z = 4(x^2 + y^2)$.

1.26 V: $x \geq 0, y = 2x, y = 4, z \geq 0, z = 10 - x^2 - y^2$.

1.27 V: $x=3, y \geq 0, z \geq 0, y = 2x, z = 4\sqrt{y}$.

1.28 V: $x \geq 0, y \geq 0, z \geq 0, 2x + 3y = 6, z = 3 + x^2 + y^2$.

1.29 V: $x \geq 0, y \geq 0, z \geq 0, x + y = 4, z = 16 - x^2 - y^2$.

1.30 V: $x \geq 0, y \geq 0, z \geq 0, 5x + y = 5, z = x^2 + y^2$.

2. Berilgan uch o‘lchovli integrallarni hisoblang.

2.1 $\iiint_V (2x^2 + 3y + z)dxdydz, V: 2 \leq x \leq 3, -1 \leq y \leq 2,\ 0 \leq z \leq 4$.

2.2 $\iiint_V x^2yzdxdydz,\ V: -1 \leq x \leq 2, 0 \leq y \leq 3, 2 \leq z \leq 3$.

2.3 $\iiint_V (x + y + 4z^2)dxdydz, V: -1 \leq x \leq 1, 0 \leq y \leq 2, -1 \leq z \leq 1$.

2.4 $\iiint_V (x^2 + y^2 + z^2)dxdydz, V: 0 \leq x \leq 3, -1 \leq y \leq 2, 0 \leq z \leq 2$.

2.5 $\iiint_V x^2y^2zdxdydz, V: -1 \leq x \leq 3, 0 \leq y \leq 2, -2 \leq z \leq 5$.

2.6 $\iiint_V (x + y + z)dxdydz, V: 0 \leq x \leq 1, -1 \leq y \leq 0, 1 \leq z \leq 2$.

2.7 $\iiint_V (2x + y^2 - z)dxdydz, V: 1 \leq x \leq 5, 0 \leq y \leq 2, -1 \leq z \leq 0$.

2.8 $\iiint_V 2xy^2zdxdydz, V: 0 \leq x \leq 3, -2 \leq y \leq 0, 1 \leq z \leq 2$.

2.9 $\iiint_V 5xyz^2dxdydz, V: -1 \leq x \leq 0, 2 \leq y \leq 3, 1 \leq z \leq 2$.

2.10 $\iiint_V (x^2 + 2y^2 - z)dxdydz, V: 0 \leq x \leq 1, 0 \leq y \leq 3, -1 \leq z \leq 2$.

2.11 $\iiint_V (x + 2yz)dxdydz, V: -2 \leq x \leq 0, 0 \leq y \leq 1, 0 \leq z \leq 2$.

2.12 $\iiint_V (x + yz^2)dxdydz, V: 0 \leq x \leq 1, 0 \leq y \leq 2, -1 \leq z \leq 3$.

2.13 $\iiint_V (xy + 3z)dxdydz, V: -1 \leq x \leq 1, 0 \leq y \leq 1, 1 \leq z \leq 2$.

2.14 $\iiint_V (xy + z^2)dxdydz, V: 0 \leq x \leq 2, 0 \leq y \leq 1, -1 \leq z \leq 3$.

2.15 $\iiint_V (x^3 + yz)dxdydz, V: -1 \leq x \leq 2, 0 \leq y \leq 1, 0 \leq z \leq 1$.

2.16 $\iiint_V (x^3 + y^2 - z)dxdydz, V: 0 \leq x \leq 2, -1 \leq y \leq 0, 0 \leq z \leq 1$.

2.17 $\iiint_V (2x^2 + y - z^3)dxdydz, V: 0 \leq x \leq 1, -2 \leq y \leq 1, 0 \leq z \leq 1$.

2.18 $\iiint_V x^2yz^2dxdydz, V: 0 \leq x \leq 2, 1 \leq y \leq 2, -1 \leq z \leq 0$.

2.19 $\iiint_V (x+y-z)dxdydz, V: 0 \le x \le 4, 1 \le y \le 3, -1 \le z \le 5.$

2.20 $\iiint_V (x+2y-3z^2)dxdydz, V: -1 \le x \le 2, 0 \le y \le 1, 1 \le z \le 2.$

2.21 $\iiint_V (3x^2+2y+z)dxdydz, V: 0 \le x \le 1, 0 \le y \le 1, 0 \le z \le 3.$

2.22 $\iiint_V (xy-z^3)dxdydz, V: 0 \le x \le 1, -1 \le y \le 2, 0 \le z \le 3.$

2.23 $\iiint_V (x^3yz)dxdydz, V: -1 \le x \le 2, 1 \le y \le 3, 0 \le z \le 1.$

2.24 $\iiint_V (xy^2z)dxdydz, V: -2 \le x \le 1, 0 \le y \le 2, 0 \le z \le 3.$

2.25 $\iiint_V (xyz^2)dxdydz, V: 0 \le x \le 2, -1 \le y \le 0, 0 \le z \le 4.$

2.26 $\iiint_V (x+yz)dxdydz, V: 0 \le x \le 1, -1 \le y \le 4, 0 \le z \le 2.$

2.27 $\iiint_V (x+y^2-z^2)dxdydz, V: -2 \le x \le 0, 1 \le y \le 2, 0 \le z \le 5.$

2.28 $\iiint_V (x+y+z^2)dxdydz, V: -1 \le x \le 0, 0 \le y \le 1, 2 \le z \le 3.$

2.29 $\iiint_V (x+y^2-2z)dxdydz, V: 1 \le x \le 2, -2 \le y \le 3, 0 \le z \le 1.$

2.30 $\iiint_V (x-y-z)dxdydz, V: 0 \le x \le 3, 0 \le y \le 1, -2 \le z \le 1.$

3. Uch o‘lchovli integralni sferik yoki silindrik koordinatalar sistemasida hisoblang.

3.1 $\iiint_V (x^2+y^2+z^2)dxdydz, V: (x^2+y^2+z^2=4, x \ge 0, y \ge 0,$ $z \ge 0$ $(Javob: 16\pi/5.)$

3.2 $\iiint_V y\sqrt{(x^2+y^2}\, dxdydz, V:\ z \ge 0, z=2, y \ge \pm x,\ z^2 = 4(x^2+$ $+y^2).$ $(Javob: \sqrt{2}/10.)$

3.3 $\iiint_V z^2\, dxdydz, V: 1 \le x^2+y^2 \le 36, y \ge x, x \ge 0,\ z \ge 0.$ $(Javob: 1555\pi/12)$

3.4 $\iiint_V y\, dxdydz, V:\ x^2+y^2+z^2=32, y^2=x^2+z^2, y \ge 0.$ $(Javob: 128\pi.)$

3.5 $\iiint_V x\, dxdydz, x^2+y^2+z^2=8, x^2=\ y^2+z^2, x \ge 0.$ $(Javob: 8\pi.)$

3.6 $\iiint_V ydxdydz,$ V: $4 \le x^2+y^2+z^2 \le 16, y \le \sqrt{3x}, y \ge 0, z \ge 0.$ $(Javob: 15\pi/2.)$

3.7 $\iiint_V y dxdydz, V: z=\sqrt{8-x^2-y^2}, z=\sqrt{x^2+y^2}, y\ge 0.$

$(Javob: 8(\pi/2-1).)$

3.8 $\iiint_V \frac{y^2 dxdydz}{x^2+y^2+z^2}, V: , x\ge 0, z\ge 0, y\ge \sqrt{3}x, 4\le x^2+y^2+z^2\le 36.$

$\left(Javob: \frac{52}{27}(2\pi+3\sqrt{3}).\right)$

3.9 $\iiint_V \frac{y^2 dxdydz}{\sqrt{(x^2+y^2)^3}}, V: y\ge 0, y\le \sqrt{3}x, z=3(x^2+y^2), z=3.$

$(Javob: 3(4\pi-3\sqrt{3})/20.)$

3.10 $\iiint_V \frac{x^2 dxdydz}{\sqrt{(x^2+y^2+z^2)^3}}, V:\ x^2+y^2+z^2=16,\ z\ge 0. (Javob:\ 16\pi/3.)$

3.11 $\iiint_V \frac{xz dxdydz}{\sqrt{x^2+y^2}}, V: z=2(x^2+y^2), y\ge 0, y\le \frac{1}{\sqrt{3}}x, z=18.$

$(Javob: 81.)$

3.12 $\iiint_V \frac{xy dxdydz}{\sqrt{(x^2+y^2)^3}}, V: z=x^2+y^2,\ y\ge 0, y\le x, z=4. (Javob: 4/3.)$

3.13 $\iiint_V \frac{z dxdydz}{\sqrt{x^2+y^2}}, V:\ x^2+y^2=4y, y+z=4, z\ge 0. (Javob: 1472/45.)$

3.14 $\iiint_V \frac{y dxdydz}{\sqrt{x^2+y^2}}, V: x^2+y^2=2x, x+z=2, y\ge 0, z\ge 0. (Javob: 4/5.)$

3.15 $\iiint_V \frac{x dxdydz}{\sqrt{x^2+y^2}}, V:\ x^2+y^2=16y,\ y+z=16, x\ge 0, z\ge 0.$

$(Javob: 2048/5.)$

3.16 $\iiint_V \sqrt{x^2+y^2}\, dxdydz, V: x^2+y^2=2x, x+z=2, z\ge 0.$

$(Javob:\ 128/45.)$

3.17 $\iiint_V xy dxdydz, V: 2\le x^2+y^2+z^2\le 8, z^2=x^2+y^2, x\ge 0,$

$y\ge 0,\ z\ge 0. (Javob: 31(4\sqrt{2}-5)/15.)$

3.18 $\iiint_V \frac{y dxdydz}{\sqrt{x^2+y^2}}, V:\ x^2+y^2=2y, x^2+y^2=4y, x\ge 0, z\ge 0, z=6.$

$(Javob: 24)$

3.19 $\iiint_V \sqrt{x^2+y^2+z^2}\, dxdydz, V: x^2+y^2+z^2=36, y\ge 0, z\ge 0,$

$y\le -x. (Javob: 81\pi.)$

3.20 $\iiint_V \frac{xdxdydz}{\sqrt{x^2+y^2}}, V:\ x^2+y^2=2x, x^2+y^2=4x, z\geq 0, z=4, y\geq 0,$ $y\leq x. (Javob: 10\sqrt{2})$

3.21 $\iiint_V \frac{zdxdydz}{\sqrt{x^2+y^2+z^2}}, V: 1\leq x^2+y^2+z^2\leq 9, y\geq 0, y\leq \frac{1}{\sqrt{3}}x, z\geq 0.$ $(Javob: 13\pi/8.)$

3.22 $\iiint_V \sqrt{x^2+y^2}\, dxdydz, V: x^2-2x+y^2=0, y\geq 0, z\geq 0,$ $x+z=2. (Javob:\ 64/45.)$

3.23 $\iiint_V x^2 dxdydz, V: 1\leq x^2+y^2+z^2\leq 16, y\geq 0, y\leq x, z\geq 0.$ $(Javob:\ 341(\pi+2)/20.)$

3.24 $\iiint_V \frac{dxdydz}{\sqrt{x^2+y^2}}, V:\ x^2+y^2=4y, y+z=4, z\geq 0.\ (Javob: 64/3\,.)$

3.25 $\iiint_V \frac{ydxdydz}{\sqrt{x^2+y^2+z^2}}, V: 4\leq x^2+y^2+z^2\leq 16, y\leq \sqrt{3}x, y\geq 0, z\geq 0.$ $(Javob: 7\pi/3\,.)$

3.26 $\iiint_V z\sqrt{x^2+y^2}\, dxdydz, V: x^2+y^2=2x,, y\geq 0, z\geq 0, z=3.$ $(Javob: 8.)$

3.27 $\iiint_V \frac{xdxdydz}{\sqrt{x^2+y^2+z^2}}, V: 1\leq x^2+y^2+z^2\leq 4, x\geq 0, y\leq x, y\geq 0,$ $z\geq 0. (\ Javob: 7\sqrt{2}\pi/24.)$

3.28 $\iiint_V xdxdydz, V:\ x^2=2(y^2+z^2), x=4, x\geq 0. (Javob: 32\pi.)$

3.29 $\iiint_V \frac{xdxdydz}{\sqrt{x^2+y^2+z^2}}, V: 1\leq x^2+y^2+z^2\leq 9, y\leq x, y\geq 0, z\geq 0.$ $(Javob: 13\sqrt{2}\pi/2.)$

3.30 $\iiint_V xdxdydz, V:\ z=\sqrt{18-x^2-y^2}, z=\sqrt{x^2+y^2}, x\geq 0.$ $(Javob: \frac{81}{2}\left(\frac{\pi}{2}-1\right).)$

4. Uch o‘lchovli integral yordamida berilgan sirtlar bilan chegaralangan jism hajmini toping. Rasmini chizing.

4.1 $z^2=4-x, x^2+y^2=4x.$ $(Javob: 512/15\,.)$

4.2 z=4-$y^2, x^2+y^2=4, z\geq 0.$ $(Javob: 12\pi.)$

4.3 $x^2+y^2=1, z=2-x-y, z\geq 0.$ $(Javob: 2\pi.)$

4.4 $z=y^2, x \geq 0, z \geq 0, x + y = 2.$ $(Javob: 4/3\,.)$

4.5 $y \geq 0, z \geq 0, z = x, x = \sqrt{9 - x^2}, x = \sqrt{25 - y^2}.$ $(Javob: 98/3\,.)$

4.6 $x^2 + y^2 = 4, z = 4 - x - y, z \geq 0\,.$ $(Javob: 16\pi.)$

4.7 $z \geq 0, z = x^2, x - 2y + 2, x + y = 7.$ $(Javob: 32.)$

4.8 $x \geq 0, z \geq 0, z = y, x = 4, y = \sqrt{25 - x^2}.$ $(Javob: 118/3\,.)$

4.9 $z \geq 0, z = 4 - x, x = 2\sqrt{y}, y = 2\sqrt{x}.$ $(Javob: 176/15\,.)$

4.10 $y \geq 0, z \geq 0, 2x - y = 0, x + y = 9, z = x^2.$ $(Javob: 1058/2\,.)$

4.11 $y \geq 0, z \geq 0, x = 4, y = 2x, z = x^2.$ $(Javob: 128.)$

4.12 $x \geq 0, z \geq 0, y = 2x, y = 3, z = \sqrt{y}.$ $(Javob: 9\sqrt{3}/5\,.)$

4.13 $y \geq 0, z \geq 0, x = 3, y = 2x, z = y^2.$ $(Javob: 54.)$

4.14 $z \geq 0, y^2 = 2 - x, z = 3x.$ $(Javob: 32\sqrt{2}/5\,.)$

4.15 $z \geq 0, y = \sqrt{9 - x^2}, z = 2y.$ $(Javob: 36.)$

4.16 $x \geq 0, y \geq 0, z \geq 0, x + y = 2, z = x^2 + y^2.$ $(Javob: 8/3\,.)$

4.17 $z \geq 0, x^2 + y^2 = 9, z = 5 - x - y.$ $(Javob: 45\pi.)$

4.18 $z \geq 0, z = x, x = \sqrt{4 - y^2}.$ $(Javob: 16/3\,.)$

4.19 $y \geq 0, z \geq 0, x + y = 2, z = x^2.$ $(Javob: 4/3\,.)$

4.20 $y \geq 0, z \geq 0, y = 4, z = x, x = \sqrt{25 - y^2}.$ $(Javob: 118/3\,.)$

4.21 $z \geq 0, x^2 + y^2 = 9, z = y^2.$ $(Javob: 81/8\pi\,.)$

4.22 $x \geq 0, z \geq 0, y \geq x, z = 1 - x^2 - y^2.$ $(Javob: \pi/16\,.)$

4.23 $z \geq 0, x^2 + y^2 = 4, z = x^2 + y^2.$ $(Javob: 8\pi.)$

4.24 $z \geq 0, y = 2, y = x, z = x^2.$ $(Javob: 4/3\,.)$

4.25 $z \geq 0, y + z = 2, x^2 + y^2 = 4.$ $(Javob: 8\pi.)$

4.26 $y \geq 0, z \geq 0, x - y = 0, 2x + y = 2, 4z = y^2.$ $(Javob: 1/162\,.)$

4.27 $x \geq 0, y \geq 0, z \geq 0, 2x + y = 2, z = y^2.$ $(Javob: 2/3\,.)$

4.28 $z \geq 0, x = y^2, x = 2y^2 + 1, z = 1 - y^2.$ $(Javob: 8/5\,.)$

4.29 $x \geq 0, y \geq 0, z \geq 0, y = 3 - x, z = 9 - x^2.$ $(Javob: 135/4\,.)$

4.30 $x \geq 0, z \geq 0, x + y = 4, z = 4\sqrt{y}.$ $(Javob: 512/15\,.)$

Namunaviy variantni yechish

1. Agar V soha x=1, y=x, z=0, z=y^2-sirtlar bilan chegaralangan bo‘lsa, uch o‘lchovli $\iiint_V f(x,y,z)$ integralda integrallash chegaralarini qo‘ying va integrallash sohasini chizing.

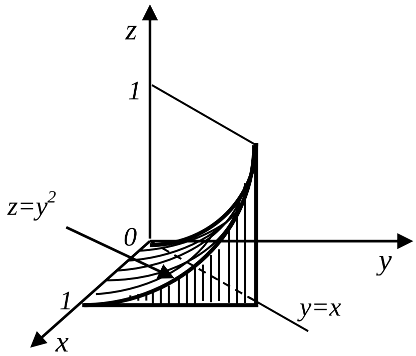
► (13.23)-formulaga ko‘ra, quyidagiga ega bo‘lamiz:

$$\iiint\limits_V f(x,y,z)\,dxdydz = \int\limits_0^1 dx \int\limits_0^x dy \int\limits_0^{y^2} f(\,x,y,z)dz.$$

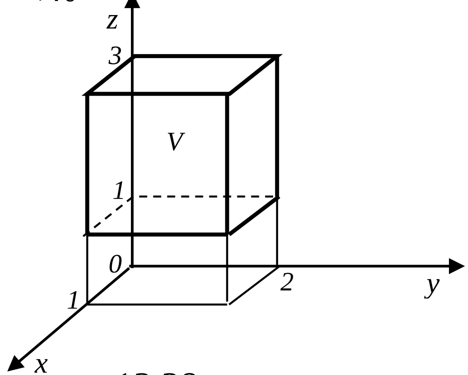
Integrallash sohasi 13.37 rasmda keltirilgan. ◄

2. Ushbu $\iiint_V (3x+2y-z^3)dxdydz$ integralni hisoblang, V: 0$\le x \le 1, 0 \le y \le 2, 1 \le z \le 3$.

►Ushbu V soha uchun (13.38-rasm), (13.24)-formulaga ko‘ra hisoblaymiz. $\iiint_V (3x+2y-z^3)dxdydz = \int_0^1 dx \int_0^2 dy \int_1^3 (\,3\,x+2y-z^3)dz = \int_0^1 dx \int_0^2 \left(3xz+2yz-\frac{z^4}{4}\right)|_1^3 dy = \int_0^1 dx \int_0^2 (6x+4y-20)dy = \int_0^1 (6xy + 2\,y^2-20y)|_0^2 dx = \int_0^1 (12x-32)dx = (6\,x^2$-32x)$|_0^1$ = −26. ◄



13.37- rasm



13.38- rasm

3. Ucn lni silindrik koordina a hisoblang $\iiint_V \frac{xzdxdydz}{x^2+y^2-R^2}$ agar V-soha birinchi oktantda joylashgan bo‘lib x=0, y=0, z=h va $z^2=\frac{h^2}{R^2}(x^2+y^2)$ konus bilan chegaralangan bo‘lsa.

►Integrallash sohasi V va uning Oxy tekislikka proyeksiyasi D 13.39-rasmda berilgan. Silindrik koordinatalar ρ, φ, z ga (13.26) formulaga ko‘ra o‘tib, berilgan soha uchun 0$\le \varphi \le \frac{\pi}{2}, 0 \le \rho \le R$: $z^2=\frac{h^2 * \rho^2}{R^2}$, z=$\frac{h*\rho}{R}$,

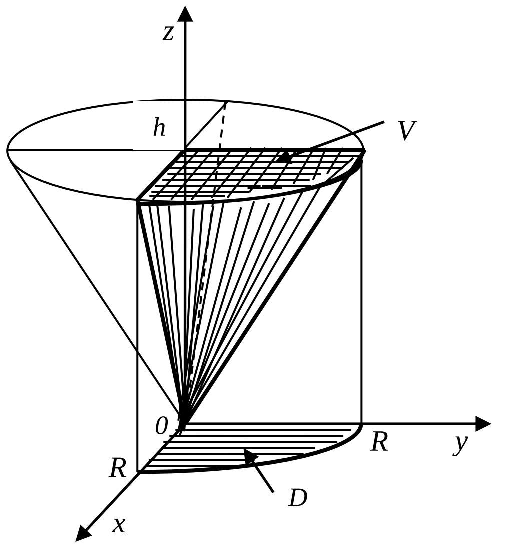
$$\iiint\limits_V \frac{xzdxdydz}{x^2+y^2-R^2} = \iiint\limits_V \frac{\rho^2 \cos\varphi d\varphi d\rho dz}{\rho^2 - R^2} =$$

$$= \int_0^{\frac{\pi}{2}} \cos\varphi d\varphi \int_0^R \frac{\rho^2}{\rho^2-R^2} d\rho \int_{\frac{h\rho}{R}}^{h} zdz = \int_0^{\frac{\pi}{2}} \cos\varphi d\varphi \int_0^R \frac{\rho^2}{\rho^2-R^2} * \frac{z^2}{2}\Big|_{\frac{h\rho}{R}}^{h}$$

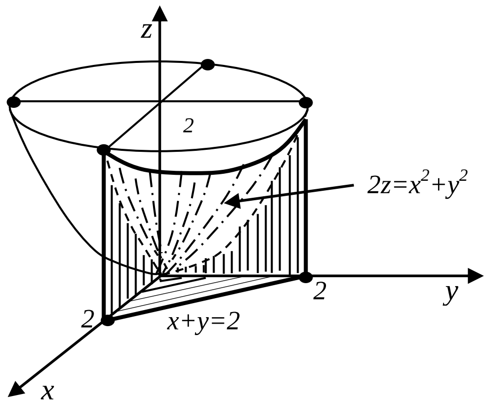
$$= \frac{1}{2}\int_0^{\frac{\pi}{2}} \cos\varphi d\varphi \int_0^R \frac{\rho^2}{\rho^2-R^2}\left(h^2 - \frac{h^2}{R^2}\rho^2\right) d\rho =$$

$$= -\frac{h^2}{2R^2}\int_0^{\frac{\pi}{2}} \cos\varphi d\varphi \int_0^R \rho^2 d\rho = -\frac{h^2}{2R^2}\sin\varphi\Big|_0^{\pi/2} * \frac{\rho^3}{3}\Big|_0^R = -\frac{1}{6}Rh^2$$

ni hosil qilamiz. ◄



13.39- rasm



13.40- rasm

4. Uch o‘lchovli integral yordamida ushbu sirtlar bilan chegaralangan: x=0, y=0, x+y=2, 2z=x^2+y^2 jisimning hajmini hisoblang.

►Ushbu 2z=x^2+y^2 teglama aylanma paraboloid, qolgan sirtlar esa tekislikdir. Hajmi hisoblanayotgan jism 13.40- rasmda keltirilgan. Uning hajmi V (13.21) va (13.23) formulaga ko‘ra hisoblanadi:

$$V=\iiint_V dxdydz=\int_0^2 dx\int_0^{2-x}dy\int_0^{\frac{(x^2+y^2)}{2}}dz=\int_0^2 dx\int_0^{2-x} z\Big|_0^{\frac{(x^2+y^2)}{2}}dy=$$

$$\frac{1}{2}\int_0^2 dx\int_0^{2-x}(x^2+y^2)dy=\frac{1}{2}\int_0^2(x^2y+\frac{y^3}{3})\Big|_0^{2-x}dx=\frac{1}{2}\int_0^2\Big(x^2(2-x)+$$

$$\frac{1}{3}(2-x)^3\Big)dx=\frac{1}{2}(\frac{2}{3}x^3-\frac{x^4}{4}-\frac{1}{12}(2-x)^4\Big|_0^2=\frac{4}{3}.\ ◄$$

IUT-13.3

1. Bir jinsli bo‘lmagan sirt zichligi har bir nuqtasida $\mu=\mu(x,y)$ ga teng bo‘lgan va berilgan chiziqlar bilan chegaralangan D plastinkaning massasini hisoblang.

1.1. D: $y^2=x, x=3, \mu=x$. $\Big(Javob: \frac{36\sqrt{3}}{5}.\Big)$

1.2. D: $x=0,\ y=0,\ x+y=1,\ \mu=x^2$. (Javob:1/12.)

1.3. D: $x=0,\ y=0,\ 2x+3y=6,\ \mu=y^2/2$. (Javob: 1.)

1.4. D: $x^2+y^2=4x,\ \mu=4-x$. (Javob: 8π.)

1.5. D: $x=0,\ y=1,\ y=x,\ \mu=x^2+2y^2$. (Javob: 7/12.)

1.6. D: $x^2+y^2=1,\ \mu=2-x-y$. (Javob: 2π.)

1.7. D: $x^2+y^2=4y,\ \mu=\sqrt{4-y}$. (Javob: 256/15.)

1.8. D: $y=x,\ y=-x,\ y=1,\ \mu=\sqrt{1-y}$. (Javob: 8/15.)

1.9. D: $x=0,\ y=2x,\ x+y=2,\ \mu=2-x-y$. (Javob: 4/9.)

1.10. D: $x=1,\ x=y^2,\ \mu=4-x-y$. (Javob: 68/15.)

1.11. D: $y=0,\ x^2=1-y,\ \mu=3-x-y$. (Javob: 14/5.)

1.12. D: $y=x^2,\ x=y^2,\ \mu=3x+2y+6$. (Javob: 11/4.)

1.13. D: $y=x^2,\ y=4,\ \mu=2x+5y+10$. (Javob: 752/3.)

1.14. D: $x=0,\ y=0,\ x+y=1,\ \mu=2x^2+y^2$. (Javob: 1/4.)

1.15. D: $x=0,\ y^2=1-x,\ \mu=2-x-y$. (Javob: 32/15.)

1.16. D: $y=\sqrt{x},\ y=x,\ \mu=2-x-y$. (Javob: 51/60.)

1.17. D: $y=x^2-1,\ y=1,\ \mu=3x^2+2y^2+1$. (Javob: $264\sqrt{2}$/35.)

1.18. D: $x=1,\ y=0,\ y=x,\ \mu=x^2+2y^2+10$. (Javob: 65/12.)

1.19. D: $y=0,\ y=2x,\ x+y=6,\ \mu=x^2$. (Javob: 104.)

1.20. D: $x \geq 0,\ y \geq 0,\ x^2 + y^2 = 4,\ \mu = 4 - x^2.$ (Javob: 3π.)

1.21. D: $y = x^2,\ \ y = 2,\ \ \mu = 2 - y.$ (Javob: $32\sqrt{2}/15$.)

1.22. D: $x = 0,\ \ y = 0,\ x + y = 1,\ \mu = x^2 + y^2.$ (Javob: 1/6.)

1.23. D: $y = x^2 + 1,\ x + y = 3,\ \mu = 4x + 5y + 2.$ (Javob: 351/6.)

1.24. D: $y = x^2 - 1,\ x + y = 3,\ \mu = 2x + 5y + 8.$ (Javob: 45.)

1.25. D: $x = 0,\ \ y = 0,\ y = 4,\ x = \sqrt{25 - y^2},\ \mu = x.$ (Javob: 118/3.)

1.26. D: $x = 2,\ \ y = x,\ y = 3x,\ \ \mu = 2x^2 + y^2.$ (Javob: 152/3.)

1.27. D: $y = x,\ y = x^2,\ \ \mu = 2x + 3y.$ (Javob: 11/30.)

1.28. D: $x = 0,\ x + 2y + 2 = 0,\ x + y = 1,\ \mu = x^2.$ (Javob: 32/3.)

1.29.D:$x = 0,\ \ y = 0,\ \ x + 2y = 1,\ \mu = 2 - (x^2 + y^2).$(Javob: 43/96.)

1.30. D: $x = 0,\ \ y = 0,\ \ x + y = 2,\ \mu = x^2 + y^2.$ (Javob: 8/3.)

2. Qutb koordinatalar sistemasida berilgan chiziqlar bilan chegaralangan bir jinsli D plastinkaning inertsion momentini ko‘rsatilgan o‘qqa nisbatan hisoblang.

2.1. D: $x^2 + y^2 - 2ay = 0,\ x - y \leq 0,\ Ox.$

2.2. D: $x^2 + y^2 - 2ax = 0,\ x + y \leq 0,\ Oy.$

2.3. D: $x^2 + y^2 + 2ay = 0,\ x - y \geq 0,\ Ox.$

2.4. D: $x^2 + y^2 + 2ax = 0,\ x + y \geq 0,\ Ox.$

2.5. D: $x^2 + y^2 + 2ax \geq 0,\ x^2 + y^2 + 2ay \leq 0,\ x \leq 0,\ Ox.$

2.6. D: $x^2 + y^2 - 2ay \geq 0,\ x^2 + y^2 + 2ax \leq 0,\ y \geq 0,\ Oy.$

2.7. D: $x^2 + y^2 - 2ay \leq 0,\ x^2 + y^2 - 2ax \geq 0,\ x \geq 0,\ Ox.$

2.8. D: $x^2 + y^2 - 2ax \leq 0,\ x^2 + y^2 + 2ay \geq 0,\ y \leq 0,\ Oy.$

2.9. D: $x^2 + y^2 - 2ax \geq 0,\ x^2 + y^2 + 2ay \leq 0,\ x \geq 0,\ Ox.$

2.10. D: $x^2 + y^2 + 2ax \leq 0,\ x^2 + y^2 + 2ay \geq 0,\ y \leq 0,\ Oy.$

2.11. D: $x^2 + y^2 - 2ay \leq 0,\ x^2 + y^2 + 2ax \geq 0,\ x \leq 0,\ Ox.$

2.12. D: $x^2 + y^2 - 2ay \geq 0,\ x^2 + y^2 - 2ax \leq 0,\ y \geq 0,\ Oy.$

2.13. D: $x^2 + y^2 + 2ay = 0,\ x^2 + y^2 + ay = 0,\ x \leq 0,\ Ox.$

2.14. D: $x^2 + y^2 - 2ax = 0,\ x^2 + y^2 - ax = 0,\ y \geq 0,\ Oy.$

2.15. D: $x^2 + y^2 + 2ay = 0,\ x^2 + y^2 + ay = 0,\ x \geq 0,\ Ox.$

2.16. D: $x^2+y^2-2ay=0,\ x^2+y^2-ay=0,\ x\ge 0,\ Ox$.

2.17. D: $x^2+y^2-2ay=0,\ x^2+y^2-ay=0,\ x\le 0,\ Ox$.

2.18. D: $x^2+y^2+2ax=0,\ x^2+y^2+ax=0,\ y\ge 0,\ Oy$.

2.19. D: $x^2+y^2-2ax=0,\ x^2+y^2-ax=0,\ y\le 0,\ Oy$.

2.20. D: $x^2+y^2+2ax=0,\ x^2+y^2+ax=0,\ y\le 0,\ Oy$.

2.21. D: $x^2+y^2+2ay=0,\ x+y\le 0,\ x\ge 0,\ Ox$.

2.22. D: $x^2+y^2-2ay=0,\ y-x\ge 0,\ x\ge 0,\ Ox$.

2.23. D: $x^2+y^2+2ax=0,\ y-x\ge 0,\ y\le 0,\ Oy$.

2.24. D: $x^2+y^2-2ay=0,\ x+y\ge 0,\ x\le 0,\ Ox$.

2.25. D: $x^2+y^2+2ax=0,\ x+y\le 0,\ y\ge 0,\ Oy$.

2.26. D: $x^2+y^2-2ax=0,\ y-x\le 0,\ y\ge 0,\ Ox$.

2.27. D: $x^2+y^2-2ax=0,\ y-x\le 0,\ x+y\ge 0,\ Oy$.

2.28. D: $x^2+y^2-2ay=0,\ y-x\ge 0,\ x+y\ge 0,\ Ox$.

2.29. D: $x^2+y^2+2ax=0,\ x+y\le 0,\ y-x\ge 0,\ Oy$.

2.30. D: $x^2+y^2+2ay=0,\ y-x\le 0,\ x+y\le 0,\ Ox$.

3. Bir jinsli V-sohani egallagan jism massasi markazi koordinatalarini hisoblang.

3.1. V: $x=6(y^2+z^2),\ y^2+z^2=3,\ x=0$. (Javob: (6,0,0).)

3.2. V: $y=3\sqrt{x^2+z^2},\ x^2+z^2=36,\ y=0$. (Javob: (0,27/4,0).)

3.3. V: $x=7(y^2+z^2),\ x=28$. (Javob: (56/3, 0, 0).)

3.4. V: $z=2\sqrt{x^2+y^2},\ \ z=8$. (Javob: (0, 0, 6).)

3.5. V: $z=5(x^2+y^2),\ \ x^2+y^2=2,\ z=0$. (Javob: (0, 0, 10/3).)

3.6. V: $x=6\sqrt{y^2+z^2},\ y^2+z^2=9,\ x=0$. (Javob: (27/4, 0, 0).)

3.7. V: $z=8(x^2+y^2),\ z=32$. (Javob: (0, 0, 64/3).)

3.8. V: $y=3\sqrt{x^2+z^2},\ y=9$. (Javob: (0, 27/4, 0).)

3.9. V: $9y=x^2+z^2,\ x^2+z^2=4,\ y=0$. (Javob: (0, 4/27, 0).)

3.10. V: $3z=\sqrt{x^2+y^2},\ x^2+y^2=4,\ z=0$. (Javob: (0, 0, 1/4).)

3.11. V: $x^2+z^2=6y,\ y=8$. (Javob: (0, 16/3, 0).)

3.12. V: $8x = \sqrt{y^2 + z^2},\ x = 1/2.$ (Javob: (3/8, 0, 0).)

3.13. V: $2x = y^2 + z^2,\ y^2 + z^2 = 4,\ x = 0.$ (Javob: (2/3, 0, 0).)

3.14. V: $4y = \sqrt{x^2 + z^2},\ x^2 + z^2 = 16,\ y = 0.$ (Javob: (0, 3/8, 0).)

3.15. V: $y^2 + z^2 = 8x,\ x = 2.$ (Javob: (4/3, 0, 0).)

3.16. V: $z = 9\sqrt{x^2 + y^2},\ z = 36.$ (Javob: (0, 0, 27).)

3.17. V: $z = 3(x^2 + y^2),\ x^2 + y^2 = 9,\ z = 0.$ (Javob: (0, 0, 9).)

3.18. V: $x = 2\sqrt{y^2 + z^2},\ y^2 + z^2 = 4,\ x = 0.$ (Javob: (3/2, 0, 0).)

3.19. V: $x^2 + z^2 = 4y,\ y = 9.$ (Javob: (0, 6, 0).)

3.20. V: $x = 5\sqrt{y^2 + z^2},\ x = 20.$ (Javob: (15, 0, 0).)

3.21. V: $y = x^2 + z^2,\ x^2 + z^2 = 10,\ y = 0.$ (Javob: (0, 10/3, 0).)

3.22. V: $y = 3\sqrt{x^2 + z^2},\ x^2 = z^2 = 16,\ y = 0.$ (Javob: (0, 9/2, 0).)

3.23. V: $y^2 + z^2 = 3x,\ x = 9.$ (Javob: (6, 0, 0).)

3.24. V: $y = \sqrt{x^2 + z^2},\ y = 4.$ (Javob: (0, 3, 0).)

3.25. V: $x = y^2 + z^2,\ y^2 + z^2 = 9,\ x = 0.$ (Javob: (3, 0, 0).)

3.26. V: $x = 0,\ y = 0,\ z = 0,\ x + y + z = 3.$ (Javob: (3/4, 3/4, 3/4).)

3.27. V: $z = 2\sqrt{x^2 + y^2},\ x^2 + y^2 = 9,\ z = 0.$ (Javob: (0, 0, 9/4).)

3.28. V: $x^2 + y^2 = 2z,\ z = 3.$ (Javob: (0, 0, 2).)

3.29. V: $z = \sqrt{x^2 + y^2},\ z = 4.$ (Javob: (0, 0, 3).)

3.30. V: $z = x^2 + y^2,\ x^2 + y^2 = 4,\ z = 0.$ (Javob: (0, 0, 4/3).)

4. Bir jinsli berilgan sirtlar bilan chegaralangan V-sohani egallovchi jismning inertsiya momentini ko‘rsatilgan koordinata o‘qlariga nisbatan hisoblang. Jism zichligi $\delta = 1$.

4.1. V: $y^2 = x^2 + z^2,\ y = 4, Oy.$ (Javob: 512π/5.)

4.2. V: $x = y^2 + z^2,\ x = 2,\ Ox.$ (Javob: 4π/3.)

4.3. V: $y^2 = x^2 + z^2,\ y = 2, Oy.$ (Javob: 16π/5.)

4.4. V: $x = y^2 + z^2,\ x = 9,\ Ox.$ (Javob: 243π/2.)

4.5. V: $x^2 = y^2 + z^2,\ x = 2,\ Ox.$ (Javob: 16π/5.)

4.6. V: $y = x^2 + z^2,\ y = 2,\ Oy.$ (Javob: 4π/3.)

4.7. V: $x^2 = y^2 + z^2,\ x = 3,\ Ox.$ (Javob: 243π/10.)

4.8. V: $x = y^2 + z^2,\ x = 3,\ Ox.$ (Javob: 9π/2.)

4.9. V: $y = 2\sqrt{x^2 + z^2},\ y = 2,\ Oy.$ (Javob: π/5.)

4.10. V: $y = x^2 + z^2,\ y = 3,\ Oy.$ (Javob: 9π/2.)

4.11. V: $x^2 = y^2 + z^2,\ y^2 + z^2 = 1,\ x = 0,\ Ox.$ (Javob: 2π/5.)

4.12. V: $x = y^2 + z^2,\ y^2 + z^2 = 1,\ x = 0,\ Ox.$ (Javob: π/3.)

4.13. V: $z^2 = x^2 + y^2,\ z = 3,\ Oz.$ (Javob: 243π/10.)

4.14. V: $z = x^2 + y^2,\ z = 3,\ Oz.$ (Javob: 9π/2.)

4.15. V: $y^2 = x^2 + z^2,\ x^2 + z^2 = 4,\ y = 0, Oy.$ (Javob: 64π/5.)

4.16. V: $2y = x^2 + z^2,\ y = 2,\ Oy.$ (Javob: 16π/3.)

4.17. V: $x^2 = y^2 + z^2,\ x = 2,\ Ox.$ (Javob: 16π/5.)

4.18. V: $2z = x^2 + y^2,\ z = 2,\ Oz.$ (Javob: 16π/3.)

4.19. V: $x^2 = y^2 + z^2,\ y^2 + z^2 = 4,\ x = 0,\ Ox.$ (Javob: 64π/5.)

4.20. V: $2z = x^2 + y^2,\ x^2 + y^2 = 4,\ z = 0,\ Oz.$ (Javob: 32π/3.)

4.21. V: $z = 2(x^2 + y^2),\ z = 2,\ Oz.$ (Javob: π/3.)

4.22. V: $x = 1 - y^2 - z^2,\ x = 0,\ Ox.$ (Javob: π/6.)

4.23. V: $y = 4 - x^2 - z^2,\ y = 0,\ Oy.$ (Javob: 32π/3.)

4.24. V: $x = 3(y^2 + z^2),\ x = 3,\ Ox.$ (Javob: π/2.)

4.25. V: $z = 9 - x^2 - y^2,\ z = 0,\ Oz.$ (Javob: 243π/2.)

4.26. V: $z = 4\sqrt{x^2 + y^2},\ z = 2,\ Oz.$ (Javob: π/80.)

4.27. V: $z = 3(x^2 + y^2),\ z = 3,\ Oz.$ (Javob: π/2.)

4.28. V: $x = 2\sqrt{x^2 + y^2},\ x = 2,\ Ox.$ (Javob: π/5.)

4.29. V: $y = 3(x^2 + z^2),\ y = 3,\ Oy.$ (Javob: π/2.)

4.30. V: $z = 3 - x^2 - y^2,\ z = 0,\ Oz.$ (Javob: 9π/2.)

Namunaviy variantni yechimi

1. Bir jinsli bo‘lmagan, har bir nuqtasida sirt zichligi $\mu = x^2 + 2xy$ ga teng bo‘lgan, $y=2x-x^2$, $y=x$ chiziqlar bilan chegaralangan D plastinaning M massasi hisoblansin.

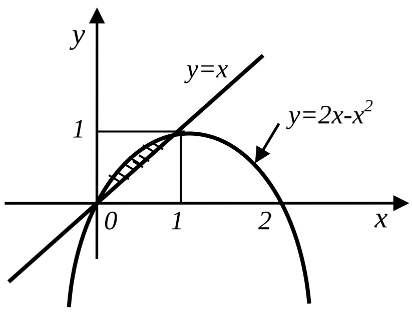
►Sirt zichligi μ bo‘lsa yassi plastinkaning massasi M ni hisoblash uchun ikki o‘lchovli integralning fizik ma‘nosidan (13.1, 2-xossa) va M=$\iint_D(x^2+2xy)dxdy$ formuladan foydalanamiz, integrallash sohasi D 13.41-rasmda keltirilgan. Ushbu hol ikki o‘lchovli integralni karrali integral ko‘rinishga keltirishga yordam beradi:

$$M=\int_0^1 dx\int_x^{2x-x^2}(x^2+2xy)dy=\int_0^1(x^2y+xy^2)|_x^{2x-x^2}dx=$$

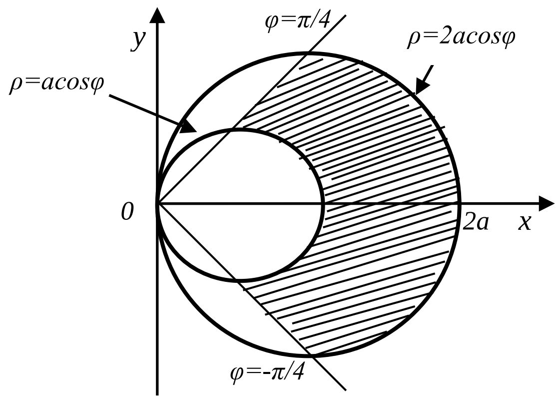
$$=\int_0^1(2x^3-x^4-x^3+4x^3-4x^4+x^5-x^3)dx=\int_0^1(x^5-5x^4+$$

$$+4x^3)dx=\int_0^1(x^5-5x^4+4x^3)dx=\left(\frac{x^6}{6}-x^5+x^4\right)|_0^1=\frac{1}{6}.◄$$

2. Bir jinsli $x^2+y^2-2ax=0,\ x^2+y^2-ax=0, y-x=0, y+x=0$ chiziqlar bilan chegaralangan (13.42-rasm) D plastinaning qutb koordinatalar sistemasida Oy-o‘qiga nisbatan inversiya momentini hisoblang. Plastinaning sirt zichligi $\mu=2$.



13.41- rasm



13.42- rasm

►Berilgan D plastinaning Oy o‘qiga inersiya momenti (13.17)-fotmula yordamida aniqlanadi. Qutb koordinatar sistemasida D soha D' sohaga akslantiriladi: $a\cos\varphi\le\rho\le 2a\cos\varphi, -\frac{\pi}{4}\le\varphi\le\frac{\pi}{4}$. U holda

$$M_y=\iint_{D'}2\rho\cos\varphi\rho d\rho d\varphi=2\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\cos\varphi d\varphi\int_{a\cos\varphi}^{2a\cos\varphi}\rho^2 d\rho=2\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\cos\varphi\cdot$$

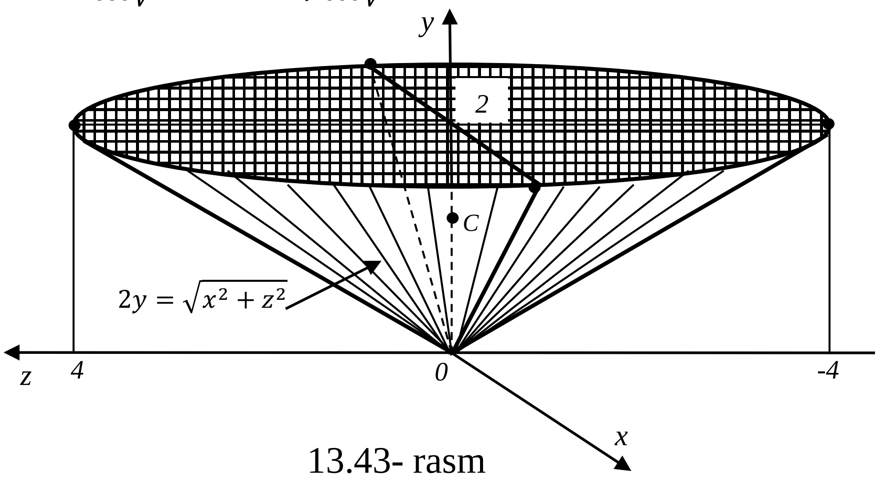
$$\cdot\frac{\rho^3}{3}|_{a\cos\varphi}^{2a\cos\varphi}d\varphi=2\cdot\frac{7a^2}{3}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\cos^4\varphi d\varphi=\frac{28}{3}a^3\int_0^{\frac{\pi}{4}}\frac{(1+\cos 2\varphi)^2}{4}d\varphi=$$

$$= \frac{7}{3}a^3 \int_0^{\frac{\pi}{4}}(1 + 2\cos 2\varphi + \cos^2 2\varphi)d\varphi = \frac{7a^3}{3}((\varphi + \sin 2\varphi)\Big|_0^{\frac{\pi}{4}} +$$

$$+ \int_0^{\frac{\pi}{4}}(\frac{1}{2} + \frac{1}{2}\cos 4\varphi)d\rho) = \frac{7}{3}a^3\left(\frac{3\pi}{8} + 1\right). ◄$$

3. $y = \frac{1}{2}\sqrt{x^2 + z^2}, y = 2$ sirtlar bilan chegaralangan V-sohani egallovchi bir jinsli jism massasi markazi koordinatalarini hisoblang.

►Berilgan jism Oy o‘qiga nisbatan simmetrik (13.43-rasm), demak $x_c = z_c = 0$ va $y_c = \iiint_V ydxdydz/\iiint_V dxdydz.$



13.43- rasm

Silindrik koordinatalar sistemasiga o‘tamiz, (13.26) formulaga ko‘ra: x=$\rho\cos\varphi, z = \rho\sin\varphi, y = y$. U holda

$$\iiint_V ydxdydz = \iiint_{V'} y\rho d\rho d\varphi dy = \int_0^{2\pi} d\varphi \int_0^4 \rho d\rho \int_{\rho/2}^2 ydy =$$

$$= \frac{1}{2}\int_0^{2\pi} d\varphi \int_0^4 \rho\left(4 - \frac{1}{4}\rho^2\right)d\rho = \frac{1}{2}\int_0^{2\pi}(2\rho^2 - \frac{\rho^4}{16})\Big|_0^4 d\varphi = \frac{1}{2}\cdot 16\varphi\Big|_0^{2\pi} = 16\pi.$$

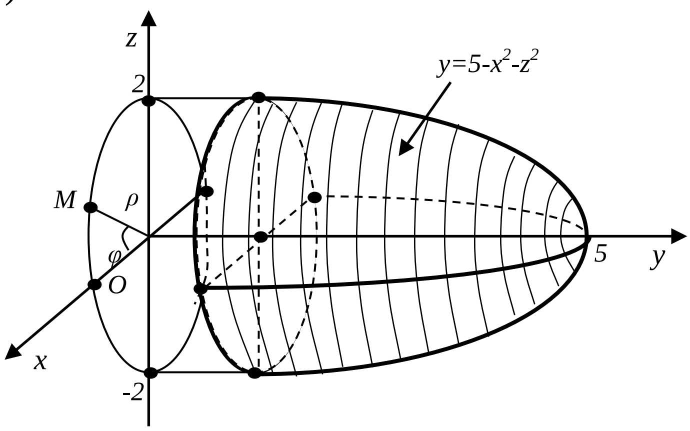
$$\iiint_V dxdydz = \iiint_{V'} y\rho d\rho d\varphi dy = \int_0^{2\pi} dx \int_0^4 \rho d\rho \int_{\frac{\rho}{2}}^2 dy =$$

$$= \int_0^{2\pi} d\varphi \int_0^4 \rho\left(2 - \frac{1}{2}\rho\right)d\varphi = \int_0^{2\pi}(\rho^2 - \frac{1}{6}\rho^3)_0^4 d\varphi = \varphi_0^{2\pi}\cdot\frac{16}{3} = \frac{32\pi}{3}.$$

Demak, $y_c = \frac{16\pi * 3}{32\pi} = \frac{3}{2}$ va massa markazi C(0, $3/2$, 0).◄

4. y=5-$x^2 - z^2$ sirt va y=1 tekislik bilan chegaralangan bir jinsli (zichlik $\delta = const$) V sohani egallagan jismning Oy o‘qiga nisbatan inersiya momentini hisoblang.

►Hisoblanayotgan inersiya momenti (13.18) formulaga ko‘ra $I_y = \iiint_V \delta(x,y,z)(x^2 - z^2)dxdydz = \delta \iiint_V (x^2 - z^2)dxdydz$. (V soha 13.44-rasmda keltirilgan.)



13.44- rasm

Silindrik koordinatalar sistemasiga $x = \rho\cos\varphi, z = \rho\sin\varphi, y = y$ formulalar orqali o‘tamiz. U holda

$$I_y = \delta \iiint_V \rho^2 \rho d\rho d\varphi dy = \delta \int_0^{2\pi} d\varphi \int_0^2 \rho^3 d\rho \int_1^{5-\rho^2} dy =$$

$$= \delta \int_0^{2\pi} d\varphi \int_0^2 y \Big|_1^{5-\rho^2} \rho^3 d\rho = \delta \int_0^{2\pi} d\varphi \int_0^2 \rho^3(5 - \rho^2 - 1)d\rho =$$

$$= \delta \int_0^{2\pi} (\rho^4 - \frac{\rho^6}{6})_0^2 d\varphi = \delta(2^4 - \frac{2^6}{6}) \int_0^{2\pi} d\varphi = \frac{32}{3}\pi\delta.$$◄

13.7 13-bo‘limga qo‘shimcha masalalar.

1. Tenglikni isbot qiling $\iint_D x^2 dxdy = \iint_D y^2 dxdy = \frac{1}{2}\iint_D (x^2 + y^2)dxdy$ agar D-soha quyidagi tengsizliklar bilan aniqlansa x$> 0, y > 0, x^2 + y^2 < a^2$.

2. Qutb koordinatalaridan foydalanib $\iint_D \sqrt{a^2-x^2-y^2}\,dxdy$, integralni hisoblang D soha $(x^2+y^2)^2 = a^2(x^2-y^2), x \geq 0$ lemniskataning yaprog‘i. (Javob:$\frac{\pi}{3} - \frac{16\sqrt{2}-20}{9})\frac{a^2}{2}$.)

3. Quyidagi integral orqali ifodalanadigan $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\varphi \int_0^{a(1+\cos\varphi)} \rho d\rho$ yuzaning sohasini chizing.

4. Berilgan egri chiziq bilan chegaralangan $(\frac{x^2}{4}+\frac{y^2}{2}) = \frac{x^2}{4} - \frac{y^2}{4}$ shaklning yuzini toping. (Javob:6)

5. Ushbu egri chiziqlar bilan chegaralangan shaklning yuzini toping $(x^2+y^2-ax)^2 = a^2(x^2+y^2)$ va $x^2+y^2 = ay\sqrt{3}$. $(Javob: \frac{3a^2\sqrt{3}}{2})$

6. $x^2+y^2-z^2 = a^2$ giperboloid, $x^2+y^2+z^2 \leq 3a^2$sharning hajmini qanday nisbatda bo‘ladi. (Javob:$3\sqrt{3}-2/2$.)

7. Quyidagi z=$e^{-x^2-y^2}$ va $z=0$ sirtlar bilan chegaralangan jismning hajmi π ga tengligini isbot qiling.

8. Kardioida $\rho = \alpha(1+\cos\varphi)$ bilan chegaralangan bir jinsli plastinaning massasi markazi koordinatalarini hisoblang. (Javob:($\frac{5}{6}$a,0).)

9. Bir jinsli $x^4+y^4 = x^2+y^2$ egri chiziq bilan chegaralangan plastinaning Ox o‘qiga nisbatan inertsiya momentini hisoblang.(Javob:($3\pi/2\sqrt{2}$).)

10. Silindrik koordinatalar yordamida $\int_0^2 dx \int_0^{\sqrt{2x-x^2}} dy \int_0^a z\sqrt{x^2+y^2}\,dz$ integralni hisoblang. (Javob: $8a^2/9$.)

11. Sferik koordinatalar yordamida $\int_{-R}^{R} dx \int_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}} dy \int_0^{\sqrt{R^2-x^2-y^2}} (x^2+y^2)dz$ integralni hisoblang. (Javob:$4\pi R^2/15$.)

12. Radiusi R va balandligi H bo‘lgan to‘g‘ri doiraviy silindr bilan chegaralangan, zichligi ixtiyoriy nuqtada, shu nuqtadan silindr asosining

markazigacha bo‘lgan masofaning kvadratiga teng bo‘lgan jismning massasini hisoblang. (Javob:$\frac{\pi R^2 H}{6}(3R^2+2H^2)$.)

13. Bir jinsli $y=\sqrt{x}, y=2\sqrt{x}, z=0$ va $x+z=6$ sirtlar bilan chegaralangan jism massasi markazi koordinatalarini hisoblang. (Javob:$(\frac{14}{15},\frac{26}{15},\frac{8}{3})$.)

14. $x^2+y^2=z$ va $x+y+z=0$ sirtlar bilan chegaralangan bir jinsli jism massasi markazining koordinatalarini hisoblang. (Javob:$(-\frac{1}{2},-\frac{1}{2}, \frac{5}{6})$.)

15. $z^2=x^2-y^2$ konus va $x^2+y^2+z^2=R^2$ sfera bilan chegaralangan bir jinsli jismning koordinata boshiga nisbatan inertsiya momentini toping. (Javob:$2\pi(2-\sqrt{2)}\,R^5/5)$

16. Zichligi o‘zgarmas $\gamma=const$ bo‘lgan asosini radiusi R, balandligi H bo‘lgan doiraviy konusning asosini diametriga nisbatan inertsiya momentini toping.(Javob: $\pi\gamma HR^2\,(2H^2+3R^2)/60.$)

17. Agar sharning massasi uning markaziga joylashgan bo‘lsa, bir jinsli shardan tashqaridagi moddiy nuqtaga ta’sir qiluvchi tortishish kuchining o‘zgarmasligini ko‘rsating.

18. Ikkita konsentrik sferalar bilan chegaralangan bir jinsli jism berilgan . Jisimning ichki qismida joylashgan nuqtaga bu sferik qatlamning tortishish kuchi ta’siri 0 tenligini isbotlang.

19. Radiusi R bo‘lgan yarim sharning ixtiyoriy nuqtadagi massa taqsimlanishi zichligi, shu nuqtadan shar asosidagi O nuqtagacha bo‘lgan masofaga (k-proporsionallik koeffisienti) proporsional bo‘lsa, shu yarim sharning massasini toping.(Javob:$4k\pi R^2/5.$)

20. Radiusi R bo‘lgan sharning markazi, radiusi $R/2$ bo‘lgan doiraviy silindrning sirtiga joylashgan bo‘lsa, ularning umumiy qismi hajmini toping. (Javob:$\frac{4}{3}\pi R^3\left(\frac{\pi}{2}-\frac{2}{3}\right).$)

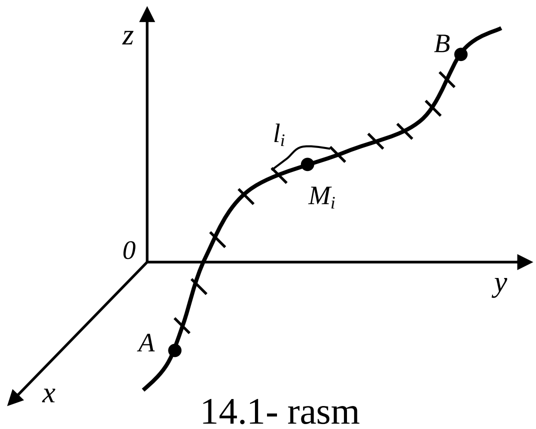
21. Radiusi R bo‘lgan sferaning markazi, radiusi $R/2$ bo‘lgan doiraviy silindrning sirtiga joylashgan bo‘lsa, bu sferik sirtning silindrik sirt bilan kesishidan hosil bo‘lgan qismining yuzini hisoblang.(Javob:$2R^2(2\pi - 2).$)

14. EGRI CHIZIQLI INTEGRALLAR

14.1 Egri chiziqli integrallar va ularni hisoblash

Birinchi tur egri chiziqli integrallar (yoy bo‘yicha). R^3 fazoda L egri chiziqning silliq L_{AB} yoyi berilgan bo‘lib, uning har bir nuqtasida uzluksiz biror $u = f(x, y, z)$ funksiya aniqlangan bo‘lsin. L_{AB} chiziqni ixtiyoriy usulda har bo‘lakning uzunligi Δl_i $(i = 1, n)$ bo‘lgan n ta bo‘lakka bo‘lamiz. Har bir elementar l_i yoyda ixtiyoriy tarzda $M_I(x_i, y_{i,} z_i)$ nuqtani (14.1-rasm) tanlaymiz va quyidagi integral yig‘indi tuzib olamiz

$$I_n = \sum_{i=1}^{n} f\left(x_{i,} y_{i,} z_i\right) \Delta l_i.$$



14.1- rasm

U holda $\lim_{max\Delta l_i \to 0} I_n$ har doim mavjud bo‘lib u *birinchi tur egri chiziqli integral* yoki L_{AB} *yoy uzunligi bo‘yicha integral* deyiladi va $\int_{L_{AB}} f(x, y, z) dl$ ko‘rinishda belgilanadi. Demak, ta’rif bo‘yicha

$$\int_{L_{AB}} f(x, y, z) dl = \lim_{max\Delta l_{i \to 0}} \sum_{i=1}^{n} f(x_{i,} y_{i,} z_i) \Delta l_i$$

Agar *L*-egri chziq *Oxy* tekislikda yotsa va bu egri chiziqda uzluksiz *f(x,y)* funksiya berilgan bo‘lsa, u holda

$$\int_{L_{AB}} f(x,y)dl = \lim_{max\Delta l_{i\to 0}} \sum_{i=0}^{n} f(x_i, y_i)\Delta l_i \qquad (14.1)$$

Agar *L* egri chiziq, R^3 fazoda $x(t), y(t), z(t)$ parametrik tenglamalar orqali berilgan va nuqta *L* egri chiziqda A nuqtadan B nuqtagacha harakat qilganda t parametr, $[\alpha, \beta]$ $(\alpha < \beta)$ kesmada monoton o‘zgarsa, u holda egri chiziqli integral quyidagi formula bo‘yicha hisoblanadi

$$\int_{L_{AB}} f(x,y,z)dl =$$

$$= \int_{\alpha}^{\beta} f\big(x(t), y(t), z(t)\big)\sqrt{\big(x'(t)\big)^2 + \big(y'(t)\big)^2 + \big(z'(t)\big)^2}dt \qquad (14.2)$$

Yassi chiziq holida (14.2) formula soddalashadi

$$\int_{L_{AB}} f(x,y)dl = \int_{\alpha}^{\beta} f\big(x(t), y(t)\big)\sqrt{\big(x'(t)\big)^2 + \big(y'(t)\big)^2}dt \qquad (14.3)$$

Agar yassi chiziq ρ, φ qutb koordinatalari sistemasida $\rho = \rho(\varphi)$ tenglama bilan berilgan bo‘lib $\rho(\varphi)$ va unung hosilasi $\rho'(\varphi)$ uzluksiz bo‘lsa, (14.3) formulaning φ parametr uchun xususiy holi hosil bo‘ladi

$$\int_{L_{AB}} f(x,y)dl = \int_{\varphi_A}^{\varphi_B} f(\rho(\varphi)cos\varphi, \rho(\varphi)sin\varphi)\sqrt{\rho^2 + \rho'^2}d\varphi \qquad (14.4)$$

(bu yerda φ_A va φ_B lar φ ning A va B nuqtalarni chiziqda aniqlovchi qiymatlari)

Agar yassi chiziq $[a, b]$ kesmada uzluksiz differensiallanuvchi $y = y(x)$ funksiya bilan berilgan bo‘lib a va b A va B nuqtalarning abssissalari bo‘lsa, u holda

$$\int_{L_{AB}} f(x,y)dl = \int_{a}^{b} f(x, y(x))\sqrt{1 + \big(y'(x)\big)^2}dx \qquad (14.5)$$

Sunday qilib, barcha hollarda birinchi tur egri chiziqli integralni hisoblash aniq integralni hisoblashga keltirilar ekan.

1-misol. Agar L - $x = tcost, y = tsint, z = t, 0 \le t \le 2\pi$ konus vint chizig‘ining birinchi o‘rami bo‘lsa

$$I = \int_L \left(2z - \sqrt{x^2 + y^2}\right) dl,$$

integralni hisoblang.

► (14.2) formuladan foydalanamiz

$$dl = \sqrt{\left(x'(t)\right)^2 + \left(y'(t)\right)^2 + \left(z'(t)\right)^2} =$$

$$= \sqrt{(cost - tsint)^2 + (sint + tcost)^2 + 1}dt = \sqrt{2 + t^2}dt$$

Demak

$$I = \int_0^{2\pi} (2t - t)\sqrt{2 + t^2}dt = \int_0^{2\pi} t\sqrt{2 + t^2}dt = \frac{1}{3}(2 + t^2)^{3/2} \Big|_0^{2\pi} =$$

$$= \frac{2\sqrt{2}}{3}\left((1 + 2\pi^2)^{3/2} - 1\right). \blacktriangleleft$$

2-misol. Agar L , $y=2x-2$ to‘g’ri chiziqning $A(0,-2),B(1,0)$ nuqtalar orasidagi qismi bo‘lsa, ushbu integralni hisoblang

$$I = \int_L \frac{dl}{x + 2y + 5}.$$

►U holda dl=$\sqrt{1 + y'^2}$ dx= $\sqrt{1 + 4}$ dx=$\sqrt{5}$dx

Demak

$$I=\int_0^1 \frac{\sqrt{5}\,dx}{x+2(2x-2)+5} = \sqrt{5}\int_0^1 \frac{dx}{5x+1} = \frac{\sqrt{5}}{5}\ln(5x + 1)\,{}^1/_0 = \frac{\sqrt{5}}{5}\ln 6. \blacktriangleleft$$

Yuqorigagi (14.2)-(14.5) formulalarga ko‘ra birinchi tur egri chiziqli integral, aniq integral orqali ifodalangani uchun, uning aniq integral xossalarini umumlashturuvchi xossalarini keltirib o‘tamiz.

1. $\int_{L_{AB}} dl = l_{AB}$, bu yerda $l_{AB} - AB$ yoyning uzunligi (egri chiziqli integralning geometrik ma’nosi).

2. Agar $f(x,y,z)=\delta(x, y. z) - L_{AB}$ yoyning chiziqli zichligi bo‘lsa, uning massasi m quyidagi formula yordamida hisoblanadi:

$$m=\int_{L_{AB}} \delta(x, y, z)dl \qquad (14.6)$$

(birinchi tur egri chiziqli integralning mexanik ma’nosi).

3. Chiziqli zichligi $\delta = \delta(x, y, z)$ bo‘lgan moddiy L_{AB} yoyning massa markazi koordinatalari ushbu formulalar yordamida aniqlanadi:

$x_c = \frac{1}{m}\int_{L_{AB}} x\delta(x,y,z)dl, \quad y_c = \frac{1}{m}\int_{L_{AB}} y\delta(x,y,z)dl, z_c = \frac{1}{m}\int_{L_{AB}} z\delta(x,y,z)dl,$

(14.7)

bu yerda *m*- L_{AB} yoyning massasi.

4. Zichligi $\delta = \delta(x,y,z)$ bo‘lgan material L_{AB} yoyning koordinata boshi O, koordinata o‘qlari *Ox*,*Oy*,*Oz* va koordinata tekisliklari *Oxy*,*Oxz*,*Oyz* ga nisbatan inersiya momenti quyidagi formulalar yordamida hisoblanadi:

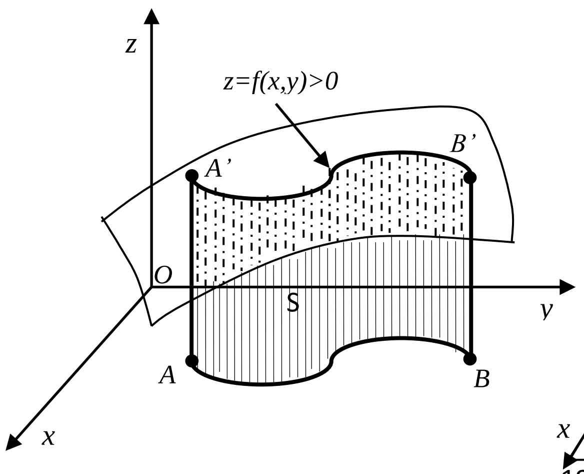
$$\left.\begin{array}{c} I_o = \int_{L_{AB}} (x^2+y^2+z^2)\delta dl \ , \ I_x = \int_{L_{AB}} (y^2+z^2)\delta dl \,, \\ I_y = \int_{L_{AB}} (x^2+z^2)\delta dl \ , \ \ I_z = \int_{L_{AB}} (x^2+y^2)\delta dl \,, \\ I_{xy} = \int_{L_{AB}} z^2\delta dl. \ I_{xz} = \int_{L_{AB}} y^2\delta dl, \ \ I_{yz} = \int_{L_{AB}} x^2\delta dl. \end{array}\right\} \qquad (14.8)$$

Inersiya momentlari quyidagi tengliklar bilan bog‘langan $2I_O = I_x + I_y + I_z$, $I_{O} = I_{xy} + I_{xz} + I_{yz}.$ Agar L_{AB} yoy *Oxy* tekislikda yotsa, y holda I_O, I_x, I_y momentlar qaraladi (z=0 shartda)

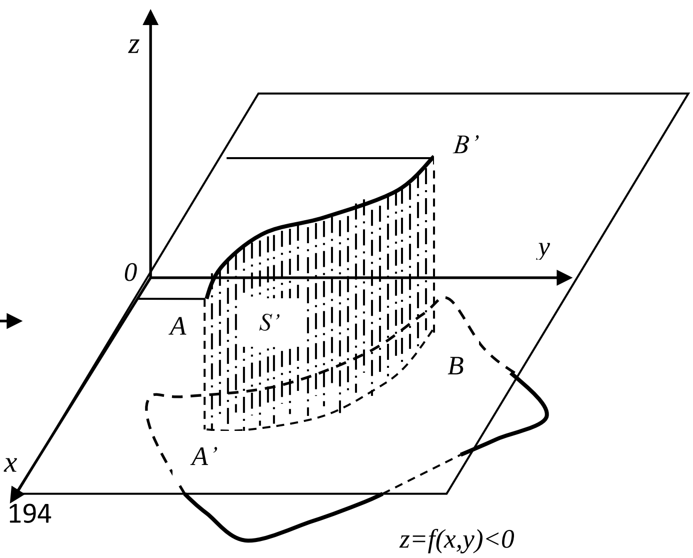
5. Agar $z = f(x,y)$ funksiya uzunlik o‘lchamiga ega va L_{AB} yoyning O*xy* tekislikda yotuvchi barcha nuqtalarida f(x,y)>0 bo‘lsa, y holda

$$\int_{L_{AB}} f(x,y)dl = S$$

bu yerda S - pastdan L_{AB} egri chiziq, yuqoridan *z=f(x,y)* sirt bilan, yon tomondan *A* va *B* nuqtalardan o‘tib *Oz* o‘qiga parallel to‘g‘ri chiziq bilan chegaralangan yasovchilari *Oz* o‘qiga parallel silindrik sirt bo‘lagi yuzasi. Silindrik ABBˈAˈ sirtning tasviri 14.2-rasmda keltirilgan. Agar L_{AB} yoyning hamma nuqtalarida f(x,y)<0 bo‘lsa u holda $L_{AB} f(x,y)dl = -S$ (14.3-rasm).



14.2- rasm



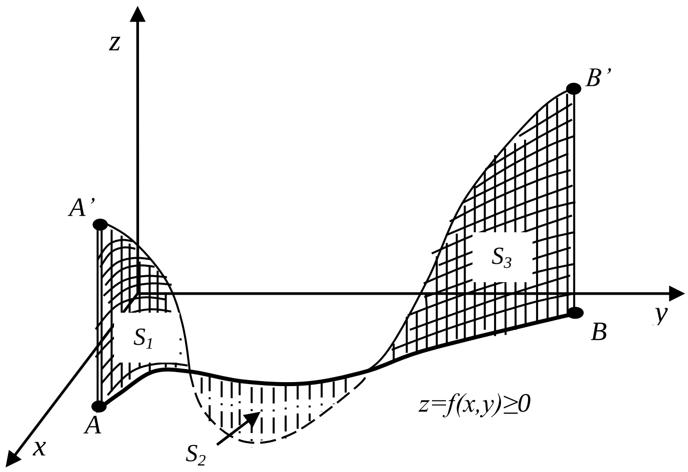
14.3- rasm

Agar *z=f(x,y)* funksiya L_{AB} yoyning ishoralarini almashtirsa, u holda $\int_{L_{AB}} f(x,y)dl$ integral, silindrik sirtning *Oxy* tekislikdan yuqorida va pastda joylashgan qismlari yuzining ayirmasini ifodalaydi (14.4-rasm)

$$\int\limits_{L_{AB}} f(x,y)dl = S_1 - S_2 + S_3$$

3-misol. Zichligi $\delta(x,y) = y\sqrt{1+x}$ boʻlgan yassi $y=\frac{2}{3}x^{\frac{3}{2}}, 0 \le x \le 1$ moddiy yoyning massasi *m* va massa markazining koordinatalari x_c, y_c −topilsin.

►Yuqoridagi (14.5) va (14.6) formulalarga asosan yassi yoy uchun quyidagiga ega boʻlamiz



14.4- rasm

$$m = \int_0^1 \delta\big(x, y(x)\big)\sqrt{1 + \big(y'(x)\big)^2}dx =$$

$$= \frac{2}{3}\int_0^1 \sqrt{1+x}\sqrt{1+x}\,dx = \frac{2}{3}\int_0^1 \left(x^{\frac{3}{2}} + x^{\frac{5}{2}}\right)dx = \frac{16}{35}$$

(14.7)-formulalarga koʻra $x_c = \frac{35}{16}\int_0^1 \left(x^{\frac{5}{2}} + x^{\frac{7}{2}}\right)dx = \frac{10}{9}$,

$y_c = \frac{35}{16}\int_0^1 \frac{2}{3}x^{3/2}\left(x^{\frac{3}{2}} + x^{\frac{5}{2}}\right)dx = \frac{35}{24}\int_0^1 (x^3 + x^4)dx = \frac{21}{32}$.◄

4-misol. $x^2 + y^2 = 4$ silindrik sirtning $z = 2 + \frac{x^2}{2}$ sirt va *Oxy* tekislik orasida joylashgan qismining yuzini hisoblang (14.5-rasm).

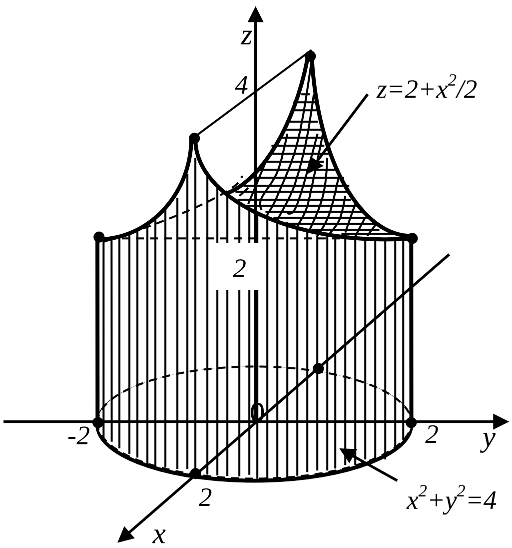
►Silindrik sirtning S yuzi quyidagi integral orqali ifodalanadi

$$S = \int\limits_L \left(2 + \frac{x^2}{2}\right) dl,$$

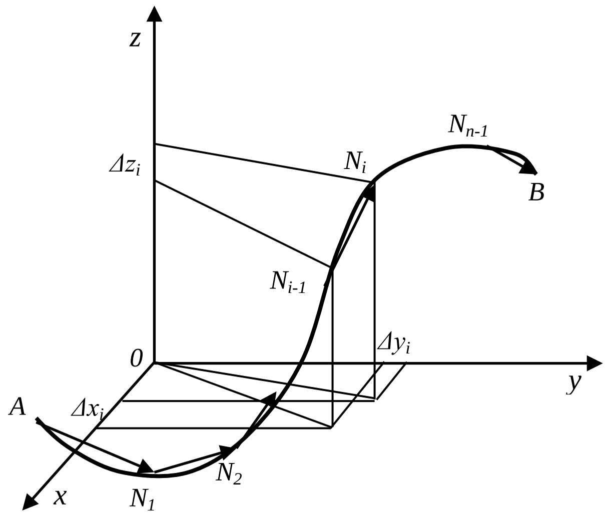
bu yerda L Oxy tekislikdagi $x^2 + y^2 = 4, z = 0$ aylana, uning parametrik ko‘rinishi $x = 2\cos t, y = 2\sin t$ kabi bo‘ladi. U holda dl=2dt va

$$S = \int_0^{2\pi} \left(2 + \frac{1}{2} \cdot 4 \cdot \cos^2 t\right) 2dt = 4\int_0^{2\pi} (1 + \cos^2 t)dt =$$

$$= 4\int_0^{2\pi} (1 + \cos^2 t)dt = 4\int_0^{2\pi} (1 + \frac{1}{2} + \frac{1}{2}\cos 2t)dt = 12\pi. \blacktriangleleft$$



14.5- rasm



14.6- rasm

Ikkinchi tur egri chiziqli integrallar (koordinatalar bo‘yicha).

Faraz qilaylik R^3 fazoda $\vec{a} = P(x,y,z)\vec{\imath} + Q(x,y,z)\vec{\jmath} + R(x,y,z)\vec{k}$ vektor berilgan bo‘lib, uning koordinatalari yonalishi aniqlangan, L_{AB}chiziqda uzluksiz bo‘lsin. Egri L_{AB}-chiziqni A nuqtadan B nuqta yo‘nalishda n ta l_i bo‘lakka bo‘lamiz va $\overrightarrow{\Delta l_\imath} = \Delta x_i\vec{\imath} + \Delta y_i\vec{\jmath} + \Delta z_i\vec{k}$ vektor qurib olamiz, bu yerda Δx_i, Δy_i, Δz_i, Δl_i vektorning koordinata o‘qlariga proyeksiyasi. Bu vektorning boshi elementar l_i yoyning boshi, oxiri shu l_i yoylarning oxiri bilan ustma ust tushadi. (14.6-rasm). Har bir elementar $l_i$qisimda ihtiyoriy$M_i(x_i, y_i, z_i)$ nuqta tanlab olamiz va integral yig‘indi tuzamiz

$$I_n = \sum_{i=0}^{n} P(x_i, y_i, z_i)\Delta x_i + Q\big(x_i, y_{i,} z_i\big)\Delta y_i + R(x_i, y_i, z_i)\Delta z_i =$$

$$= \sum_{i=1}^{n} \vec{a}(x_i, y_i, z_i)\,\overrightarrow{\Delta l_i} \qquad (14.9).$$

Yuqoridagi (14.9) yig‘indining hamma $\left|\overline{\Delta l_i}\right| \to 0$ dagi limiti *ikkinchi tur egri chiziqli integrali* deyiladi yoki L_{AB} egri chiziqdan $\bar{a}(x, y, z)$ funksiyaning koordinatalar bo‘yicha *ikkinchi tur integralli* deyiladi va quyidagicha belgilanadi

$$\int_{L_{AB}} \vec{a}(x, y, z) d\vec{l} = \int_{L_{AB}} P(x, y, z)dx + Q(x, y, z)dy + R(x, y, z)dz =$$

$$\lim_{\overline{\Delta l_i} \to 0} \sum_{i=1}^{n} \vec{a}(x_i, y_i, z_i)\overline{\Delta l_i} \qquad (14.10).$$

Agar *P(x,y,z), Q(x,y,z), R(x,y,z*) funksiyalar silliq L_{AB} chiziqning nuqtalarida uzluksiz bo‘lsa, u holda (14.8) yig‘indining limiti mavjud bo‘ladi, ya‘ni ikkinchi tur (14.10) integral mavjud bo‘ladi.

Ikkinchi tur egri chiziqli integrallarning xossalari aniq integralning xossalari bilan bir xil bo‘ladi (chiziqlilik,additivlik). Ikkinchi tur egri chiziqli integralning ta’rifidan, uning qiymati yonalishiga bog‘likligi bevosita kelib chiqadi, ya‘ni integral yonalish o‘zgarganda ishorasi o‘zgaradi

$$\int_{L_{AB}} \vec{a} d\vec{l} = -\int_{L_{BA}} \vec{a} d\vec{l}.$$

Agar integrallash chizig‘i yopiq bo‘lsa, ikkinchi tur egri chiziqli integral $\oint_L adl$ ko‘rinishda belgilanadi. Ushbu holda sirtni yonalishini aniqlanadi, musbat yonalish deb shunday yonalish tanlanadiki bunda L egri chiziqda harakatlanganda, L chiziq bilan chegaralangan soha chapda joylashadi (ya‘ni L yopiq kontur bo‘yicha harakat soat strelkasiga qarshi bo‘ladi).

Agar yassi L yopiq kontur bilan chegaralangan D sohani L_1 va L_2 chiziqlar bilan chegaralangan ikki qismga ajratsak va hosil bo‘lgan sohalar umumiy ichki nuqtalarga ega bo‘lmasa, quyidagi tenglik o‘rinli bo‘ladi

$$\oint_L \vec{a} d\vec{l} = \int_{L_1} \vec{a} d\vec{l} + \oint_{L_2} \vec{a} d\vec{l}$$

bu yerda L_1 va L_2 kontur bo‘yicha yo‘nalish yoki musbat yoki manfiy.

Agar silliq L_{AB} chiziq *x=x(t), y=y(t), z=z(t)* parametrik tenglamalar bilan berilgan bo‘lib, *x(t), y(t), z(t)*-funksiyalar uzluksiz differesiallanuvchi bo‘lsa, hamda $A(x(\alpha), y(\alpha), z(\alpha))$ va $B(x(\beta), y(\beta), z(\beta))$ nuqtalar egri chiziqning boshlang‘ich va oxirgi nuqtasi bo‘lsa, u holda ikkinchi tur egri chiziqli integral quyidagi formula yordamida hisoblanadi:

$$\int_{L_{AB}} P(x,y,z)dx + Q(x,y,z)dy + R(x,y,z)dz =$$

$$= \int_{\alpha}^{\beta} (P\big(x(t),y(t),z(t)\big)x'(t) + Q\big(x(t),y(t),z(t)\big)y'(t) +$$

$$+R\big(x(t),y(t),z(t)\big)z'(t))dt \qquad (14.11)$$

Agar L_{AB}-chiziq *Oxy* tekislikda yotsa , $\vec{a} = P(x,y)\vec{\imath} + Q(x,y)\vec{\jmath}$, u holda $\mathrm{R(x,y,z)} \equiv 0,\ z(t) \equiv 0$ va (14.11) formula soddalashadi:

$$\int_{L_{AB}} P(X,Y)dx + Q(x,y)dy = \int_{\alpha}^{\beta} (P\big(x(t),y(t)\big)x'(t) + Q(x(t),y(t)y'(t)dt \qquad (14.12)$$

Agar L_{AB}- chiziq *Oxy* tekislikda yotsa va *y=f(x)* tenglama bilan berilgan bo‘lib va $f'(x)$ hosila $[a,b]$ kesmada uzluksiz bo‘lsa, u holda

$$\int_{L_{AB}} P(x,y)dx + Q(x,y)dy = \int_{a}^{b} (P\big(x,f(x)\big) + Q(x,f(x))f'(x)\,dx \qquad (14.13)$$

5-misol. Hisoblang $I = \int_{L_{AB}} ydx + (x+z)dy + (x-y)dz$, bu yerda L_{AB} A(1,-1,1) va B(2,3,4) nuqtalarni tutashtiruvchi tog‘ri chiziq kesmasi.

►To‘g‘ri chiziqning parametrik teglamasini tuzamiz:

$x = 1 + t, y = -1 + 4t, z = 1 + 3t$. *AB* kesmada parametr 0$\leq t \leq 1$.

Demak (14.11) formulaga ko‘ra

I=$\int_0^1 \big((1+4t) + (2+4t)4 + (2-3t)3\big)dt = \int_0^1 (13+11t)dt = 18.5.$◄

6-misol. Agar L egri chiziq $x^2 + y^2 = 4$ silindr va *x+y-z=0* tekislikning kesishishidan hosil bo‘lib, yonalishi tekislikning tanlangan yuqori qismida musbat bo‘lsa, ushbu I=$\oint_L ydx - x^2dy + (x+y)dz$, integralni hisoblang.

►Chiziqning parametrik tenglamalarini topamiz. Chiziqning *Oxy* tekislikka proyeksiyasi $x^2 + y^2 = 4$, $z = 0$ aylana bo‘lib parametrik tenglamasi x=2cos t , $y = 2\sin t$ ko‘rinishda bo‘ladi.

U holda tekislikning tenglamasidan z=2(cos t + sin t). Demak

$$\begin{cases} x = 2\cos t \\ y = 2\sin t \\ z = 2(\cos t + \sin t), t\epsilon[0; 2\pi] \end{cases} \rightarrow \begin{cases} dx = -2\sin t dt \\ dx = 2\cos t\, dt \\ dz = 2(-\sin t + \cos t)dt \end{cases}$$

Yuqoridagi (14.11) formulaga ko‘ra

$$I = \int_0^{2\pi}(-4\sin^2 t - 8\cos^3 t + 4(\cos^2 t - \sin^2 t))dt = \int_0^{2\pi}(-2 +$$

$$+2\cos 2t - 8\cos t + 8\sin^2 t\cos t + 4\cos 2t)dt = -4\pi.◄$$

7-misol. Agar L_{AB}- chiziq *y*=x^2 parabolaning *A(0;0)* va *B(2;4)* nuqtalar orasida joylashgan yoyi bo‘lsa, ushbu I=$\int_{L_{AB}} xydx + (x^2 + y)dy$, integralni hisoblang.

►Bu holda f(x)=$x^2, f'(x) = 2x, x \in [0; 2]$, demak (14.13) formulaga ko‘ra I=$\int_0^2(x \cdot x^2 + (x^2 + x^2)2x)dx = \int_0^2 5x^3dx = \frac{5x^4}{4}\,{}^2/_0 = 20.$◄

AT-14.1

1. Hisoblang: $\int_L \frac{dl}{x-y}$, L:- y=$\frac{1}{2}x - 2$ to‘g‘ri chiziqning *A(0;-2)* va *B(4;0)* nuqtalar orasida kesmasi. (Javob:$\sqrt{5}\ln 2$).

2. Hisoblang: $\oint xydl$, L- uchlari A(0,0), B(4,0), C(4,2), D(0,2) nuqtalarda bo‘lgan to‘g‘ri to‘rtburchak konturi. (Javob:24).

3. Hisoblang: $\int_L \sqrt{2y}dl$, L- $x = a(\mathrm{t} - \sin t)$, $y = a(1 - \cos t)$, $(a > 0)$ sikloidaning birinchi arkasi. (Javob:$4\pi a\sqrt{a}$).

4. Hisoblang: $\int_L xyzdl$, L- A (1,0,1) va B(2,2,3) nuqtalar orasidagi to‘g‘ri chiziq kesmasi. (Javob:12).

5. Ushbu $x^2 + y^2 + z^2 = R^2$sferaning ichida joylashgan $x^2 + y^2 = Rx$ silindrning yon sirti yuzini hisoblang. (Javob:$4R^2$).

6. Hisoblang:$\int_{L_{AB}}(x^2 - 2xy)dx + (2xy + y^2)dy$, bu yerda L_{AB} A(1,1) va B(2,4) nuqtalar orasidagi parabolaning yoyi. (Javob:$40\frac{19}{30}$).

7. Hisoblang: $\int_{L_{AB}} xdx + ydy + (x + y - 1)dz$, bu yerda L_{AB}- A(1,1,1) va B(2,3,4) nuqtalarni tutashtiruvchi tog‘ri chiziq kesmasi (Javob:13).

8. Hisoblang: $\int_{L_{AB}} yzdx + zxdy + xydz,$ bu yerda L- x=$Rcos\, t, y = R\, sin\, t,\ z = at/2\pi$ vint chizig‘ining z=0 va z=a tekislik bilan kesishishidan hosil bo‘lgan yoy bo‘lagi. (Javob:0).

9. Hisoblang: $\int_{L_{AB}} xydx + (y - x)dy$, L_{AB}- chiziq *A(0;0)* va *B(1;1)* nuqtalarni tutashtirib quyidagi tenglamalar bilan berilgan: a)y=x; b)y=x^2; c)$y^2 = x,\ d)y = x^3$. Javob: a)$\frac{1}{3}; b)\ \frac{1}{12}; c)\ \frac{17}{30}; d) - \frac{1}{20};$).

10. x=a(t-sin $t), y = a(1 - \cos t)$ sikloida birinchi yarim arkasining massasi markazi koordinatalarini toping, t$\epsilon[0;\pi]$. (Javob:$4a/3\, , 4a/3$).

Mustaqil ish

1. Hisoblang:

a) $\int_L xdl$ L: A(0, 0) va B(1,2) nuqtalarni tutashtiruvchi to‘gri chiziq kesmasi;

b) $\int_{L_{AB}} (x + y)dx + (x - y)dy,\quad L_{AB} : y = x^2$ parabolaning A(-1,1) va B(1,1) nuqtalar orasidagi yoy bo‘lagi. (Javob: a)$\frac{\sqrt{5}}{2}$; b)2.)

2. Hisoblang:

a) $\int_L x^2ydl$, L: $x^2 + y^2 = 9$ aylananing birinchi chorakdagi qismi;

b) $\int_{L_{AB}} (x - y)dx + (x + y)dy,\quad L_{AB}$: A(2,3) va B(3,5) nuqtalarni tutashtiruvchi to‘g‘ri chiziq kesmasi. (Javob: a)27 : b)$23/2$).

3. Hisoblang:

a) $\int_L \frac{dl}{x+y}$, L: *A(2, 4)* va *B(1, 3)* nuqtalarni tutashtiruvchi *y=x+2* to‘g‘ri chiziq;

b) $\int_{L_{AB}} (y + x^2)dx + (2x - y)dy$, bu yerda $L_{AB}, y = 2x - x^2$ parabolaning A(1, 1) va B(3, -3) nuqtalar orasidagi yoyning bo‘lagi. (Javob: a) ($\frac{\sqrt{2}}{2}$) ln 2; b) 12.)

14.2. Egri chiziqli integrallarning tatbiqlari.

Birinchi tur egri chiziqli integral yordamida, yoy uzunligini, material yoyning massasini, uning massasi markazini, silindrik sirtlarning yuzini va boshqa miqdorlarni hisoblash mumkin.

1-misol. Zichligi $\delta = (1 + 4x^2 + y^2)^{1/2}$ bo‘lgan, x=$\frac{t^2}{2}$, y=t, z=$\frac{t^3}{3}$ tenglama berilgan L-chiziqning massasini hisoblang, 0$\leq t \leq 2$.

►Yuqoridagi (14.6) formulaga ko‘ra m-massa quyidagi integral orqali ifodalanadi

$m = \int_L \sqrt{1 + 4x^2 + y^2}$dt=$\int_0^2 \sqrt{1 + t^4 + t^2}\sqrt{t^2 + 1 + t^4}$dt=$\int_0^2 (1 + t^2 + t^4)dt = 116/15.$◄

2-misol. $x^2 + y^2 = R^2$ aylana birinchi chorakda joylashgan yoyining inersionya momentlari I_0, I_x, I_y va massasi markazining koordinatalarini hisoblang.

► y=x to‘g‘ri chiziq aylana yoyini simmetriya o‘qi bo‘lgani uchun $x_c = y_c$ tenglik o‘rinli bo‘ladi. Avval keltirilgan (14.7) formulani x_c ni hisoblash uchun qo‘llaymiz $x_c = \int_L x\delta dl / \int_L \delta dl = \int_L xdl / \int_L dl$, chunki δ –o‘zgarmas.

Ushbu integral $\int_L dl = \frac{1}{2}\pi R$ qaralayotgan aylananing uzunligini chorak qismiga teng. Quyidagi $\int_L xdl$, integralni hisoblaymiz bu yerda x=R$\cos t$, y=R$\sin t$, 0$\leq t \leq \frac{\pi}{2}$;

$$dt = \sqrt{(x'^2(t) + y'^2(t)}dt = Rdt.$$

Demak,

$$\int_L xdl = \int_0^{\frac{\pi}{2}} R\cos tRdt = R^2 .$$

Bundan, $$x_c = y_c = \frac{R^2}{\pi R/2} = \frac{2R}{\pi}.$$

I_0, I_x, I_y larni hisoblashda yassi yoy uchun (z=0) (14.8) va (14.3) formulalardan foydalanamiz va $I_x = I_y$ ekanligini hisobga olamiz:

$$I_0 = \int_L (x^2 + y^2)\delta dl = \delta \int_0^{\frac{\pi}{2}} R^2 Rdt = R^3\delta\pi/2,$$

$$I_x = \int_L y^2\delta dl = \delta \int_0^{\frac{\pi}{2}} R^2 \sin^2 tRdt = \frac{R^3\delta}{2}\int_0^{\frac{\pi}{2}}(1 - \cos 2t)dt = \pi R^3\delta/4.$$ ◄

(14.9) ikkinchi tur chiziqli integral, agar $\bar{a} = \bar{F}$ kuchni ifodalasa va bu kuch ta'siri ostida jism L_{AB} yoyi bo'yicha qo'zg'algandagi bajarilgan ishni ifodalaydi. Bu ikkinchi tur integralning fizik ma'nosidan iborat.

3-misol. Agar B(1, 1, 1) va C(2, 3, 4) bo'lsa, to'g'ri chiziqning BC kesmasi bo'yicha $\bar{F} = yz\bar{\imath} + xz\bar{\jmath} + xy\bar{k}$ kuchning bajargan ishi A ni hisoblang.

►To'g'ri chiziqning parametrik tenglamasini yozamiz *BC: x=1+t, y=1+2t, z=1+3t*, bu yerda $0\leq t \leq 1$. U holda $\bar{F}$ kuch *BC* yol bo'yicha bajargan *A* ish quyidagi formula yordamida hisoblanadi

A=$\int_{L_{BC}} yzdx + xzdy + xydz = \int_0^1 (1+2t)(1+3t)dt + (1+z)(1+3z)2dt + (1+t)(1+2t)3dt = \int_0^1 (18t^2 + 22t + 6)dt = 23.$◄

Grin teoremasi. Agar *P(x,y)* va *Q(x,y)* funksiyalar *Oxy* tekislikda yotuvchi, bo'lakli silliq *L* chiziq bilan chegaralang bir bog'lamli yopiq D sohada xususiy hosilalari bilan birgalikda uzluksiz bo'lsa, u holda

$$\oint_L Pdx + Qdy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dxdy, \qquad (14.14)$$

formula o'rinli bo'ladi, bu yerda *L* kontur bo'yicha integrallash musbat yonalishda bajariladi. Yuqoridagi (14.14) formula *Grin formulasi* deyiladi.

Agar biror sohada Grin formulasining shartlari bajarilsa, quyidagi tasdiqlar teng kuchli.

1. $\oint_L Pdx + Qdx = 0$, bu yerda *L*, D sohada yotuvchi ixtiyoriy yopiq kontur.
2. Integral $\int_{L_{AB}} Pdx + Qdy$, A va B nuqtalarni tutashtiruvchi yo'llarga bog'liq emas, bu yerda $L_{AB} \in D$.
3. *Pdx+Qdy=du(x, y),* bu yerda *du(x, y)* ifoda *u(x, y)* funksiyaning to'la differensiali.
4. D-sohaning hamma nuqtalarida quyidagi tenglik o'rinli

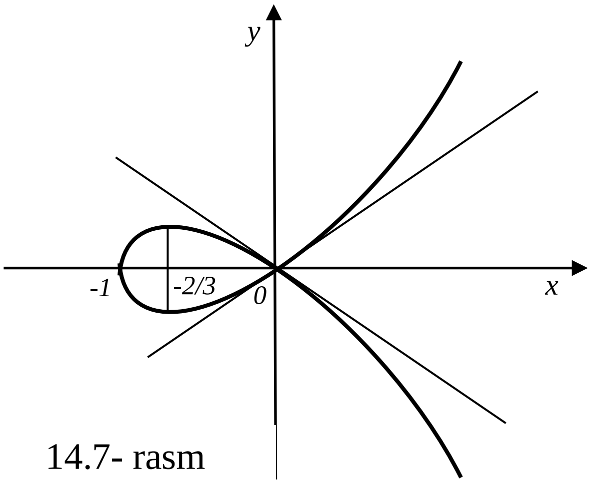
$$\frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y}. \qquad (14.15)$$

Grin formulasidan *D* sohaning *S* yuzini 2-tur egri chiziqli integral yordamida hisoblash mumkinligi kelib chiqadi:

$$S_D = \frac{1}{2}\oint_L -ydx + xdy,$$

bu yerda L kontur bo‘yicha integrallash musbat yonalishda bajariladi.

4-misol. $x^3 + x^2 - y^2 = 0$ chiziq bilan chegaralangan shaklning yuzini toping (14.7 rasm)



14.7- rasm

Egri chiziqning tenglamasidan, $y = \pm x\sqrt{x+1}$, ya’ni egri chiziq Ox o‘qiga nisbatan simmetrik va uni x=0, x=-1 nuqtalarda kesib o‘tadi. $x \geq -1$ da aniqlangan va x$\to \infty, y \to \pm\infty$. Bu egri chiziqning parametrik tenglamasiga o‘tamiz: y=xt. Tenglamada y ning o‘rniga $y=xt$ ni qo‘yib $x^3 + x^2 - y^2 = 0$, $x^3 + x^2 = x^2t^2$, $x = t^2 - 1$, $y = t^3 - t$ ga ega bo‘lamiz, bu yerda shakl uchun -1$\leq t \leq 1$.

Demak, qidirilayotgan yuza

S=$\frac{1}{2}\int_{-1}^{1}[-(t^3 - t)2t + (t^2 - 1)(3t^2 - 1)]dt = \int_0^1 (t^4 - 2t^2 + 1)dt = \frac{8}{15}$. ◄

5-misol. Ushbu integralni hisoblang

$$I = \oint_L y(1 - x^2)dx + (1 + y^2)xdy$$

bu yerda L- kontur $x^2 + y^2 = 4$ aylanani musbat yo‘nalishda aylanib chiqish.

►Integralni hisoblash uchun Grin (14.14) formulasidan foydalanamiz:

$$I = \iint_D (1 + y^2 - 1 + x^2)dxdy = \iint_D (x^2 + y^2)dxdy,$$

bu yerda D- $x^2 + y^2 \leq 4$ tengsizlik bilan aniqlanadigan doira. Bundan

$$I = \iint_D (x^2 + y^2)dxdy = \left| \begin{matrix} x = \rho\cos\varphi, \varphi dxdy = \rho d\rho d\varphi \\ y = \rho\sin\varphi, 0 \leq \varphi \leq 2\pi, 0 \leq \rho \leq 2 \end{matrix} \right|$$

$=\iint_{D'} \rho^3 d\rho d\varphi = \int_0^{2\pi} d\varphi \int_0^2 \rho^3 d\rho = 8\pi.$◄

Ikkinchi tur egri chiziqli integrallar nazariyasi asosida quyidagi masalalarni yechish mumkin. P(x,y)dx+Q(x,y)dy to‘la differensial ifoda ma‘lum bo‘lsin va *u(x,y)* funksiyaning to‘la differensiali bo‘lsin, shu funksiyani topish talab qilinsin.

Qidirilayotgan funksiya quyidagi formulalar yordamida topiladi

$$u(x,y) = \int_{x_0}^{x} P(x,y_0)dx + \int_{y_0}^{y} Q(x,y)dy + C \qquad (14.16)$$

yoki

$$u(x,y) = \int_{x_0}^{x} P(x,y)dx + \int_{y_0}^{y} Q(x_0,y)dy + C \qquad (14.17)$$

bu yerda $M_0(x_0,y_0)$ va *M(x,y)* nuqtalar D sohaga tegishli, va bu sohada *P(x,y)* va *Q(x,y)* funksiyalar o‘zining xususiy hosilalari bilan uzluksiz, C-ixtiyoriy o‘zgarmas.◄

6-misol. Quyidagi differensial ifoda $\frac{x}{y}dy + (\frac{1}{1+x^2} - \frac{1}{x} + \ln y)dx$ biror u(x,y) funksiyaning to‘la differensali ekanini ko‘rsating va shu funksiyani toping.

►Demak, $P(x,y) = (\frac{1}{1+x^2} - \frac{1}{x} + \ln y)dx$, Q(x,y)=$\frac{x}{y}$, u holda $\frac{\partial P}{\partial y} = \frac{1}{y}$ va $\frac{\partial Q}{\partial x} = \frac{1}{y}$. Bundan ifoda koordinata o‘qlaridagi nuqtalardan tashqari Oxy tekislikning barcha nuqtalarida, (14.14) formulaga ko‘ra u(x,y) funksiyaning to‘la differensiali. Endi umumiy (14.16) yoki (14.17) formulalardan foydalanamiz, bu yerda $M_0(1,1)$.

Yuqoridagi (14.16) formulaga asosan

$$u(x,y) = \int_1^x \left(\frac{1}{1+x^2} - \frac{1}{x}\right)dx + \int_1^y \frac{x}{y} + C = (arctg\, x - ln|x|)_1^x +$$

$$x\ln|y|_1^y + +C = arctg\, x = \ln|x| + x\ln|y| + C,$$

bu yerda C ixtiyoriy o‘zgarmas.◄

AT-14.2

1. Agar $y = lnx$ yoyning zichligi $\delta = x^2$ va yoyning chekkalari x ning quyidagi qiymatlari bilan aniqlansa, $x_1 = \sqrt{3}, x_2 = \sqrt{8}$, uning massasini toping (Javob: $19/3.$)

2. O‘qlari perpendikuliar bo‘lgan, radiusi R bo‘lgan silindirlarning kesishishidan hosil bo‘lgan sirtning yuzini hisoblang. (Javob: $8R^2$.)

3. Ikkinchi tur egri chiziqli integral yordamida ushbu egri chiziqlar bilan chegaralangan shaklning yuzini hisoblang:

a) $x = a\cos^3 t, y = a\sin^3 t$-egri chiziq (astroida);

b) Sikloidaning birinchi arkasi $x = a(t - \sin t), y = a(1 - \cos t)$ va Ox o‘qi bilan. (Javob: a) $3\pi a^2/8\,; b)\ 3\pi a^2.$)

4. To‘la differensial orqali u(x,y) funksiyani toping:

a) du=4$(x^2 - y^2)(xdx - ydy)$;

b) du=(2x$\cos y - y^2 \sin x)dx + (2y\cos x - x^2 \sin y)dy$;

c) du=(3y-x)dx+$(\mathrm{y} - 3\mathrm{x})\mathrm{dy}/(x + y)^3$.

5. $\bar{F} = (x^2 + y^2 + 1)\bar{\imath} + 2xy\bar{\jmath}$ kuchning y=x^3parabola A(0,0) va B(1,1) nuqtalar orasida yoyi bo‘yicha bajargan ishini toping. (Javob: $196/105.$)

6. Grin formulasi yordamida integralni hisoblang $\oint_L y^2dx + (x + y)^2dy$, bu yerda L uchlari A(3,0), B(3,3) va C(0,3) nuqtalarda bo‘lgan uchburchakning konturi (Javob: 18.)

7. Ushbu differensial tenglamaning umumiy integralini toping

$(4x^3y^3 - y^2)dx + (3x^4y^2 - 2xy)dy = 0.$ (Javob: $x^4y^3 - xy^2 = C.$)

Mustaqil ish

1. 1. Ikkinchi tur egri chiziqli integral yordamida y=x^2 va y=$\sqrt{x}$ chiziqlar bilan chegaralangan D sohaning yuzini toping. (Javob: $1/3.$)

2. Agar du(x,y)=(2xy+$x^3 - 5)dx + (x^2 - y^3 + 5)dy$ bo‘lsa, u(x,y) funksiyani toping

2. 1. Koordinata o‘qlari va $\frac{x^2}{a^2}+\frac{y^2}{b^2}$ ellips yoyi bilan chegaralangan 1-chorakda joylashgan shaklning yuzini toping. (Javob: $\pi ab/4.$)

2. Agar du(x,y)=(2xy+x^2-y^2)dx+$(x^2+y^2-2xy)dy$ bo‘lsa, u(x,y)-funksiyani toping,

3. 1. Ushbu F(x,y)=2xy$\bar{\imath}+x^2\bar{\jmath}$ kuchning A(0, 0) va B(2, 1) nuqtalarni tutashtiruvchi yo‘l bo‘yicha bajargan ishini toping (Javob: 4.)

2. Agar du=$\frac{2x(1-e^y)}{(1+x^2)^2}dx+\left(\frac{e^y}{1+x^2}+1\right)dy$ bo‘lsa, u(x,y)-funksiyani toping.

14-BOBGA DOIR INDIVIDUAL UY TOPSHIRIQLARI

14.1-IUT

Berilgan egri chiziqli integrallar hisoblansin.

1.

1.1. . $\int_{L_{AB}}(x^2-2xy)dx+(y^2-2xy)dy,$ bu yerda $L_{AB}: y=x^2$ parabolaning A(-1;1) nuqtadan B (1;1) gacha bo‘lgan yoyi. (Javob: -6)

1.2. $\int_{L_{AB}}\frac{x^2dy-y^2dx}{\sqrt[3]{x^5+\sqrt[3]{y^5}}},$ bu yerda $L_{AB}: x=2cos^3t,\ \ y=2sin^3t$ astroidaning A(2;0) nuqtadan B(0;2) gacha bo‘lgan yoyi. (Javob: $\frac{3\sqrt[3]{2\pi}}{8}$)

1.3. $\int_{L_{OA}}(x^2+y^2)dx+2xydy,$ bu yerda $L_{OA}:\ y=x^3$ kubik parabolaning O(0;0) nuqtadan A(1;1) nuqtagacha bo‘lgan yoyi.(Javob: $\frac{4}{3}$)

1.4. . $\oint_L(x+2y)dx+2xydy,$ bu yerda $L: x=2cost \quad y=2sint$ musbat yo‘nalishdagi aylana (Javob: -4π)

1.5. $\oint_L(x^2y-x)dx+(y^2x-2y)dy,$ bu yerda $L:\ L\ x=3cost, y=2sint$ ellips yoyi (Javob:-7,5π)

1.6. . $\int_{L_{AB}}(xy-1)dx+2x^2ydy,$ bu yerda $L_{AB}:\ x=cost, y=sint$ ellipsning $A(1:0)$ nuqtadan B(0;2) nuqtagacha bo‘lgan yoyi.(Javob:$\frac{5}{6}$)

1.7. $.\int_{L_{OBA}} 2xydx - x^2dy,$ bu yerda $L_{OBA}, O(0;0), B(2;0)$ va $A(0;2)$ nuqtalarni birlashtiruvchi OBA siniq chiziq. (Javob: 4).

1.8. $\int_{L_{AB}}(x^2+y^2)dx + xydy,$ bu yerda L_{AB}: $A(1;1)$ va $B(3;4)$ nuqtalardan o‘tuvchi to‘g’ri chiziqning AB kesmasi (Javob:$11\frac{1}{6}$.)

1.9. $\int_{L_{AB}} cosydx - sinxdy,$

bu yerda L_{AB}: $A(2\pi, -2\pi)$ va $B(-2\pi, 2\pi)$ nuqtalardan o‘tuvchi to‘g’ri chiziqning AB kesmasi. (Javob:0)

1.10. $\int_{L_{AB}} \frac{ydx+xdy}{x^2+y^2},$ bu yerda L_{AB}: $A(1;2)$va$B(3;6)$nuqtalardan o‘tuvchi to‘g’ri chiziqning AB kesmasi (Javob:$\frac{4}{5}\ln 3$)

1.11. $\int_{L_{AB}} xydx + (y-x)dy,$ bu yerda L_{AB}: $A(0,0)$ va $B(1,1)$ nuqtalardan o‘tuvchi y=x^2 kubik parabolaning AB yoyi. (Javob:$\frac{1}{4}$)

1.12. $\int_{L_{ABC}}(x^2+y^2)dx + (x+y^2)dy$ bu yerda L_{ABC}: $A(1;2)$ $B(3;2)$, $C(3;5)$ nuqtalardan o‘tuvchi ABC siniq chiziq (Javob: $64\frac{2}{3}$)

1.13. $\int_{L_{OB}} xy^2dx + yz^2dy - zx^2dz,$ bu yerda, L_{OB} , $O(0,0,0)$ va $B(-2,4,5)$ nuqtalardan o‘tuvchi to‘g’ri chiziqning OB kesmasi. (Javob:91).

1.14. $\int_{L_{OA}} ydx + xdy,$ bu yerda L_{OA}: $x = Rcost, y = Rsint$ aylananing O(R;0) va A(0;R) nuqtalarni birlashtiruvchi yoyi.(Javob:0)

1.15. $\int_{L_{OA}} xydx + (y-x)dy,$ bu yerda L_{OA}: $y^2 = x$ parabolaning (0;0) va A(1;1) nuqtalari orasidagi joylashgan yoyi.(Javob: $\frac{17}{30}$)

1.16. $\int_{L_{AB}} xdx + ydy + (y-x+1)dz,$

bu yerda L_{AB}: $A(1,1,1)$ va $B(2,3,4)$ nuqtalardan o‘tuvchi to‘g’ri chiziqning kesmasi.(Javob:7)

1.17. $\int_{L_{AB}}(xy-1)dx + x^2ydy,$ bu yerda L_{AB}: $y^2 = 4-4x$ parabolaning A(1;0) va B(0;2) nuqtalarini birlashtiruvchi yoyi. (Javob: $\frac{17}{15}$)

1.18. $\int_{L_{OB}} xydx + (y - x)dy$, bu yerda L_{OB}: $y = x^2$ parabolaning O(0;0) B(1;1) nuqtalarini birlashtiruvchi yoyi. (Javob: $\frac{1}{12}$)

1.19. $\int_{L_{OB}} (xy - y^2)dx + xdy$, bu yerda L_{OB}: $y = x^2$ parabolaning O(0;0) B(1;1) nuqtalarini birlashtiruvchi yoyi. (Javob: $\frac{43}{60}$)

1.20. . $\int_{L_{AB}} xdx + ydy$, bu yerda L_{AB}: $x = 2cos^3 t$ $y = 2sin^3 t$ astroidaning A(2;0) nuqtadan B(0;2) nuqtagacha bo‘lgan yoyi.(Javob:$\frac{3\pi}{4}$)

1.21. $\int_{L_{AB}} (xy - x)dx + \frac{1}{2}x^2 ydy$, *bu yerda* L_{AB}: $y^2 = 4x$ parabolaning A(0;0) va B(1;2) nuqtalari orasida joylashgan yoyi. (Javob:0,5)

1.22. $\int_{L_{AB}} (xy - 1)dx + x^2 ydy$, bu yerda L_{AB}: $A(1;0)$ va $B(0;2)$ nuqtalardan o‘tuvchi to‘g’ri chiziqning AB kesmasi. (Javob:1.)

1.23. . $\int_{L_{AB}} 2xydx + y^2 dy + z^2 dz$,

bu yerda L_{AB}: $A(1;0;0)$ *va* $B(1;0;4\pi)$ uqtalardan o‘tuvchi *x=cost* , *y=sint* *z=2t* vint chiziq bitta o‘ramining yoyi . (Javob:$\frac{64\pi^3}{3}$.)

1.24. $\int_{L_{AB}} \frac{y}{x}dx + xdy$,

bu yerda L_{AB}: $y = lnx$ chiziqning $A(1;0)$ va $B(e;1)$ nuqtalar orasidagi yoyi. (Javob: $e - \frac{1}{2}$)

1.25. . $\oint_L ydx - xdy$, bu yerda L : musbat yo'nalishli $x = 3cost, y = 2sint$ ellips yoyi . (Javob: -12π)

1.26. $\int_{L_{OA}} 2xydx - x^2 dy$, L_{OA}: $y = \frac{x^2}{4}$ parabola $O(0;0)$ va $A(2;1)$ nuqtalari orasidagi yoyi. (Javob: 0)

1.27. $\int_{L_{AB}} (x^2 + y^2)dx + (x^2 + y^2)dy$, bu yerda L_{AB}: $A(-1;1)$ va $B(2;2)$ nuqtalardan o‘tuvchi y=|x| siniq chiziq. (Javob:6)

1.28. $\int_{L_{OA}} 2xydx - x^2 dy + z^2 dz$, bu yerda L_{OA}: $O(0;0;0)$ va $A(2;1;-1)$ nuqtalarni birlashtiruvchi to‘g’ri chiziqning kesmasi.(Javob $\frac{11}{6}$)

1.29. $\oint_L xdy - ydx,$ bu yerda L : uchlari $A(1;0), B(1;0)$va $C(0;1)$ nuqtalarda bo‘lgan uchburchakning musbat yo‘nalishdagi konturi. (Javob:2)

1.30. $\int_{L_{ACB}}(x^2+y)dx+(x+y^2)dy,$ L_{ACB}: $A(2;0)$ va $C(5;0)$ $B(5;3)$ nuqtalarni birlashtiruvchi ABC siniq chiziq.(Javob:63)

2.

2.1. $\int_L \sqrt{2-z^2}\,(2z-\sqrt{x^2+y^2})\ dl,$ bu yerda $L, x = tcost, y = tsint, z = t$ $(0 \le t \le 2\pi)$ egri chiziq yoyi . (Javob:$4\pi^2(1+\pi^2)$)

2.2. $\oint_L (x^2+y^2)dl,$ bu yerda $L : x^2+y^2 = 4$ aylana. (Javob: 16π

2.3. $\int_{L_{OB}} \frac{dl}{\sqrt{8-x^2-y^2}},$ bu yerda L_{OB}: $O\ (0;0)$ va $B(2;2)$ nuqtalardan o‘tuvchi to‘g’ri chiziq kesmasi . (Javob:$\frac{\pi}{2}$)

2.4. $\int_{L_{AB}}(4\sqrt[3]{x}-3\sqrt{y}\,)\,dl,$ bu yerda $L_{AB}, A(-1;0)$ va $B(4;0)$ nuqtalardan o‘tuvchi AB to‘g’ri chiziq kesmasi. (Javob:-5$\sqrt{2}$)

2.5. $\int_{L_{AB}} \frac{dl}{\sqrt{5}(x-y)},$ bu yerda L_{AB}: $A\ (0;4)$ va $B(4;0)$ nuqtalar orasidagi kesma. (Javob:0)

2.6. $\int_L \frac{y}{\sqrt{x^2+y^2}}dl,$ bu yerda L: $\rho = 2(1+cos\varphi)(0 \le \varphi \le \frac{\pi}{2})$ kardioida yoyi. (Javob:$\frac{16}{3}$)

2.7. $\int_{L_{AB}} y\,dl,$ bu yerda $L_{AB}, x = cos^3t, y = sin^3t$ astroidaning $A(1;0)$va $B(0;1)$ nuqtalar orasidagi yoyi. (Javob: 0,6)

2.8. $.\int_{L_{OB}} y\,dl, bu\ yerda\ L_{OB}, y^2 = \frac{2}{3}x,$ parabolaning $O(0;0)$ va $B(\frac{\sqrt{35}}{6};\frac{\sqrt{35}}{3})$ nuqtalar orasidagi yoyi. (Javob: $7\frac{26}{27}$)

2.9. $\int_L (x^2+y^2+z^2)dl,$ bu yerda L: $x = cos, y = sint, z = \sqrt{3}\,t$ $(0 \le t \le 2\pi)$ egri chiziq yoyi. (Javob: $4\pi(1+4\pi^2)$)

2.10. $\int_L arctg\frac{y}{x}dl,$ bu yerda, $L, \rho = (1+cos\varphi)(0 \le \varphi \le \frac{\pi}{2})$ kardioida yoyi. (Javob: $(\pi+2)\sqrt{2}$-8)

2.11. $\int_L \sqrt{2y}\,dl$, bu yerda L , $x = 2(t - sint), y = 2(1 - cost)$ sikloidaning birinchi arkasi. (Javob: $8\pi\sqrt{2}$)

2.12. $\int_{L_{OA}} \frac{dl}{\sqrt{x^2+y^2+4}}dl$, bu yerda L_{OA}: $O(0;0)$va$A(1;2)$ nuqtalardan o'tuvchi to'g'ri chiziqning kesmasi. (Javob: $\ln(\frac{\sqrt{5}+3}{2})$)

2.13. $\int_L \frac{(x^2-y^2)xy}{(x^2+y^2)^2}dl$, $bu\ yerda, L, \rho = (9sin2\varphi)(0 \le \varphi \le \frac{\pi}{4})$ egri chiziq yoyi. (Javob:- $\frac{9}{8}$)

2.14. $\int_{L_{OABC}} xy\,dl$, bu yerda L_{OABC}, uchlari $O(0;0), A(4;0), B(4;2), C(0;2)$ nuqtalarda bo'lgan to'g'ri to'rtburchak konturi. (Javob:24)

2.15. $\int_{L_{ABO}} (x+y)\,dl$, bu yerda L_{ABO} uchlari , $A(1;0), B(0;1)$, va $O(0;0)$ nuqtalarda bo'lgan uchburchakning konturi. (Javob: $-\sqrt{2}$)

2.16. $\int_L \frac{z^2}{x^2+y^2}dl$, bu yerda, $L, x = 2cost, y = 2sint, z = 2t$ vint chizig'ining birinchi o'rami. (Javob: $\frac{16}{3}\sqrt{2}\pi^3$)

2.17. $\int_{L_{OAB}} (x+y)\,dl$, bu yerda L_{OAB}, uchlari $O(0;0), A(-1;0)$ va $B(0;1)$ nuqtalarda bo'lgan uchburchakning konturi. (Javob:0)

2.18. $\int_L (x+y)\,dl$, bu yerda L , $\rho^3 = (cos2\varphi)(-\frac{\pi}{4} \le \varphi \le \frac{\pi}{4})$ Bernulli lemnskatasining yoyi. (Javob: $\sqrt{2}$.)

2.19. $\oint_L \sqrt{x^2+y^2}dl$, bu yerda $L : x^2 + y^2 = 2y$ aylana. (Javob: 8)

2.20. $\int_{L_{OABC}} xy\,dl$, bu yerda L_{OABC}, uchlari $O(0;0), A(5;0), B(5;3), C(0;3)$ nuqtalarda bo'lgan to'g'ri to'rtburchakning konturi. (Javob:-15)

2.21. $\oint_L (x^2+y^2)dl$, bu yerda $L : x^2 + y^2 = 4x$ aylana. (Javob: 32π)

2.22. $\int_{L_{AB}} (4\sqrt[3]{x} - \sqrt[3]{y})dl$, bu yerda $L_{AB}, x = cos^3t, y = sin^3t$ astroidaning A(1;0) va B(0;1) nuqtalari orasidagi yoyi. (Javob:1)

2.23. $\int_L xydl$, bu yerda, L, tomonlari $x = \pm1$ va $y = \pm1$ bo'lgan kvadratning konturi. (Javob:0)

2.24. $\int_L y^2 dl$, bu yerda L, $x = t - sint, y = 1 - cost$ sikloidaning birinchi arkasi. (Javob: $17\frac{1}{15}$)

2.25. $\int_{L_{ABCD}} xy\, dl$, bu yerda L_{ABCD}, uchlari $A(2;0), B(4;0), C(4;3) D(2;3)$ nuqtalarda bo‘lgan to‘ri to‘rtburchakning konturi. (Javob:45)

2.26. $\int_L ydl$, bu yerda, $L, y^2 = 2x$ parabolaning $x^2 = 2y$ parabola kesgan yoyi. (Javob: $\frac{5\sqrt{5}-1}{3}$)

2.27. $\int_{L_{AB}} \frac{dl}{x-y}$, bu yerda, L_{AB}, $A(4;0)$va $B(6;1)$ nuqtalardan o‘tuvchi to‘g’ri chiziq kesmasi. (Javob: $\sqrt{5}\ln(\frac{5}{4})$)

2.28. $\int_L (x^2 + y^2)^2 dl$, bu yerda, $L, \rho = 2$ aylananing birinchi chorakdagi yoyi. (Javob:16π)

2.29. $\int_{L_{AB}} \frac{dl}{\sqrt{x^2+y^2+z^2}}$, bu yerda, L_{AB}, $A(1;1;1)$va $B(2;2;2)$ nuqtalarni birlashtiruvchi to‘g’ri chiziq kesmasi. (Javob: $ln\, 2$)

2.30. $\oint_L (x-y)dl$, bu yerda $L : x^2 + y^2 = 2x$ aylana. (Javob: 2π)

3.

3. 1. $\oint\limits_L \sqrt{2y^2 + z^2} dl$, bu yerda $L - x^2 + y^2 + z^2 = a^2, x = y$ aylana. (Javob: $2\pi a^2$)

3. 2. $\int\limits_L xyzdl$, bu yerda $L - x^2 + y^2 + z^2 = R^2,\ x^2 + y^2 = \frac{R^2}{4}$ aylananing birinchi oktantdagi chorak qismi. (Javob: $\frac{R^4\sqrt{3}}{32}$)

3. 3. $\int\limits_L arctg\frac{y}{x} dl$ bu yerda $L - \rho = 2\varphi$ Arximed spiralining markazi qutbda bo′lib, radiusi R dan iborat bo‘lgan doira ichidagi yoyining bo‘lagi.(Javob: $\frac{(R^2+4)^{\frac{3}{2}}-8}{12}$)

$\mathbf{3.4.} \int_L (x^2+y^2+z^2)dl$ bu yerda $L-$ $x = acost, y = asin\,t$ $z = bt$

$(0 \le t \le 2\pi)$ egri chiziq yoyi.(Javob: $\frac{2\pi\sqrt{a^2+b^2}(3a^2+4\pi^2b^2)}{3}$)

$\mathbf{3.5.} \int_L \left(2z - \sqrt{x^2+y^2}\right) dl$ bu yerda $L-$ $x = tcost, y = tsin\,t, z = t$

konus vint chiziqning birinchi o‘rami.(Javob: $\frac{2\sqrt{2}((1+\pi^2)^{\frac{3}{2}}-1}{3}$)

$\mathbf{3.6.} \int_L (x+z)dl,$ bu yerda $L - x = t, y = \frac{3}{\sqrt{2}}t^{\,2}, z = t^3$ $(0 \le t \le 1)$

egri chiziq yoyi.(Javob: $\frac{56\sqrt{7}-1}{54}$)

$\mathbf{3.7.} \int_L \left(x\sqrt{x^2+y^2}\right) dl,$ bu yerda $L - (x^2+y^2)^2 = a^2(x^2-y^2),$

$x \ge 0$ egri chiziq yoyi.(Javob: $\frac{2a\sqrt[3]{2}}{3}$)

$\mathbf{3.8.} \int_L (x+y)dl, bu\ yerda, L-$ $\rho^{\,2} = a^2cos2\varphi$ lemniskataning

birinchi o‘rami.(Javob: $a^2\sqrt{2}$)

$\mathbf{3.9.} \int_L xydl,$ bu yerda, $L-$ $\frac{x^2}{a^2} + \frac{y^2}{b^2}$ ellipsning birinchi chorakdagi

bo‘lagi.(Javob: $\frac{ab(a^2+ab+b^2)}{3(a+b)}$)

$\mathbf{3.10.} \int_L (x+y)dl$ $L-$ $x^2+y^2+z^2 = R^2, y = x$ aylananing I-

oktantdagi chorak bo‘lagi.(Javob: $R^2\sqrt{2}$)

$\mathbf{3.11.} \int_{L_{AB}} \frac{dl}{x-z},$ bu yerda, $L_{AB} - z = -\frac{x}{2}, y = 0$ tog′ri chiziqning $A(0;0;$

$B(4;0;0)$ nuqtalarni birlashtiruvchi kesmasi. (Javob :$\sqrt{5}\ln 2$)

$\mathbf{3.12.} \int_L (\sqrt{2y})dl$ bu yerda $L-$ $x = a(t - sint), y = a(1 - cost)$

sikloidaning birinchi arkasi. (Javob:$4\pi a\sqrt{a}$)

3.13 .$\oint_L (x-y)dl$, bu yerda $L-$ $x^2+y^2=$ ax aylana.(Javob: $\frac{\pi a^2}{2}$)

3.14. $\int_L \frac{dl}{x^2+y^2+z^2}$, bu yerda, $L-$ $x=acost, y=asint, z=bt$ vint chiziqning birinchi o‘rami.(Javob: $\frac{\sqrt{a^2+b^2}}{ab} arctg\frac{2\pi b}{a}$)

3.15. $\int_L \frac{z^2 dl}{x^2+y^2}$, bu yerda, $L-$ $x=acost, y=asint, z=at$ vint chizig’ining birinchi o‘rami.(Javob: $\frac{8a\pi^3\sqrt{2}}{3}$)

3.16. $\int_L \left(\sqrt{x^2+y^2}\right) dl$ bu yerda $L-$ $x=a(cost+sint)$, $y=a(sint-cost) 0\le t\le 2\pi$ aylananing yoyilmasi . (Javob: $\frac{4a^2(1+4\pi^2)^{\frac{3}{2}}-1}{3}$)

3.17. $\int_{L_{AB}} \frac{dl}{\sqrt{x^2+y^2}}$, bu yerda, L_{AB} $-A(0;-2)$ va $(4;0)$ nuqtalardan o‘tuvchi to‘g’ri chiziqning kesmasi .(Javob: Ln $\frac{3\sqrt{5}-7}{2}$)

3.18. $\int_L \frac{dl}{x^2+y^2+z^2}$, bu yerda, $L-x=5cost, y=5sint, z=t$ vint chiziqning birinchi o‘rami.(Javob: $\frac{\sqrt{26}}{5} arctg\frac{2\pi}{5}$)

3.19. $\int_{L_{OABC}} yzdl$, bu yerda, $L_{OABC}-O(0;0;0), A(0;4;0), B(0;4;2)$, $C(0;0;2)$ nuqtalarda bo‘lgan to‘g’ri to‘rtburchakning konturi.(Javob: 24)

3.20. $\int_L x^2 dl$, bu yerda, $L-x^2+y^2=a^2$ aylananing yuqori yarim bo‘lagi.(Javob: $\frac{\pi a^3}{2}$).

3.21. $\int_L (x^2+y^2+z^2)\, dl$ bu yerda, $L-x=4cost, y=sint, z=3t$

vint chiziqning birinchi o‘rami.(Javob: $\frac{10\pi(48+36\pi^2)}{3}$)

$\mathbf{3.22.}\int_L \ ydl,$ bu yerda, $L - y^2 = 6x$ parabolaning $x^2 = 6$y parabola bilan kesishishdan hosil bo‘lgan yoyi.(Javob: $3(5\sqrt{5}$-1))

$\mathbf{3.23.}\int_{L_{AB}} xdl,$ bu yerda, $L_{AB} - y = x^2$ parabolaning $A(2;4)$nuqtadan $B(1;1)$nuqtagacha bo‘lgan yoyi.(Javob: $\frac{17\sqrt{17}-5\sqrt{5}}{12}$)

$\mathbf{3.24.}\int_L (x+y)$ bu yerda, $L - \rho^2 = 7cos2\varphi$ lemniskadaning birinchi o‘rami.(Javob: $7\sqrt{2}$)

$\mathbf{3.25.}\oint_L (z^2+y^2)$ bu yerda, $L - (z^2+y^2) = 4$ aylana. (Javob: 256π)

$\mathbf{3.26.}\int_L y^2dl$ bu yerda, $L - \ x = 3(t - sint), y = 3(1 - cost)$ sikloidaning birinchi arkasi.(Javob: $458\frac{4}{5}$)

$\mathbf{3.27.}\int_L \sqrt{x^2+y^2} \ \ dl$ bu yerda $L - \ x = 6(cost + tsint), y = 6(sint - tcost)$, $(0 \le t \le 2\pi)$ aylananing yoyilmasi.

(Javob: $12((1+4\pi^2)^{\frac{3}{2}} - 1)$.)

$\mathbf{3.28.}\int_L \frac{z^2dl}{x^2+y^2}$, bu yerda, $L - x = 9cost, y = 9sint, z = 9t$ vint chiziqning birinchi o‘rami.(Javob: $24\pi^3\sqrt{2}$)

$\mathbf{3.29.}\oint_L (x^2+y^2)^2dl$ bu yerda, $L - x = 3cost, y = sint$ aylana. (Javob: 486π)

3.30. $\int_L \; ydl$ bu yerda, $L - y^2 = 12x$ parabolaning $x^2 = 12y$ parabola bilan kesilgan yoyi. (Javob: $12(5\sqrt{5} \quad - 1)$

4

4.1. $\int_{L_{OA}} (xy - y^2)dx + xdy$ bu yerda, $L_{OA} - y = 2x^2$ parabolaning $O(0;0), A(1;2)$ nuqtalar orasidagi yoyi. (Javob: $\frac{31}{30}$)

4.2. $\int\limits_{L_{OBA}} 2yzdy - y^2\, dz$ bu yerda, $L_{OBA} - \; y =, 2x^2$ parabolaning $O(0;0;0)$, $B(0;2;0)$ va A(0;1;1) nuqtalarni birlashtiruvchi siniq chiziq. (Javob:-4)

4.3. $\int_L \; \frac{x}{y}dx + \frac{1}{y-a}dy,$ bu yerda, $L - x = a(t - sint), y = a(1 - cost),$ $\frac{\pi}{6} \leq t \leq \frac{\pi}{3}$ sikloidaning yoyi. (Javob: $\frac{a\pi^2}{24} + \frac{a}{2}(1 - \sqrt{3}) - \frac{1}{2}\ln 3$)

4.4. $\int_L \; yzdx + z\left(\sqrt{R^2 - y^2}\right)$ bu yerda, $L - x = Rcost\,, y = Rsint,$ $z = \frac{at}{2\pi}$ egri chiziqning z=0 tekislik bilan kesishish nuqtasidan z=a tekislik bilan kesishish nuqtasigacha bo‘lgan oraliqdagi yoyi. (Javob:0)

4.5. $\int\limits_{L_{OA}} 2xzdy - y^2dz$ bu yerda, $L_{OA} - z = \dfrac{x^2}{4}$ parabolaning $O(0;0;0), A(2;0;1)$ nuqtalar orasidagi yoyi. (Javob:0)

4.6. $\int\limits_{L_{AB}} \left(x - \dfrac{1}{y}\right) dy$, bu yerda, $L_{AB} - y = x^2$ parabolaning $A(1;1), B(2;4)$ nuqtalar orasidagi yoyi. (Javob: $\frac{14}{3} - \ln 4$)

$\mathbf{4.7.} \int\limits_{L_{AB}} coszdx - sinxdz,$ bu yerda, L_{AB} − to′g′ri chiziqning

$A\ (2;0;-2), B(-2;0;2)$ nuqtalarni birlashtiruvchi kesmasi .(Javob: -2sin2)

$\mathbf{4.8.} \int\limits_{L} ydx - xdy,$ bu yerda, $L - x = Rcost, y = Rsint$ aylana yoyining

birinchi kvadratdagi chorak qismi yo‘nalish soat miliga teskari . (Javob:0)

$\mathbf{4.9.} \int_{L_{OA}} (xy - x)dx - \frac{x^2}{y}dy,$ bu yerda, $L_{OA} - y = 2\sqrt{x}$ parabolaning

O(0;0) A(1;2) nuqtalar orasidagi yoyi.(Javob: $\frac{1}{2}$)

$\mathbf{4.10.} \oint\limits_{L} ydx - xdy$, bu yerda, $L - x = acost, y = bsint$

ellipsning o‘ng yo‘nalishdagi yoyi.(Javob: $-2\pi ab$.)

$\mathbf{4.11.} \oint_{L} xdy$, bu yerda, $L - y = x, x = 2$ va $y = 0$ to‘g’ri chiziqlar

hosil qilgan uchburchakning musbat yo‘nalishdagi konturi. (Javob:2)

$\mathbf{4.12.} \int\limits_{L} xdy,$ bu yerda, $L - y = sinx$ sinusoidaning $(\pi;0)$ va $(0;0)$

nuqtalar orasidagi yoyi. (Javob:2)

$\mathbf{4.13.} \int\limits_{L} y^2dx + x^2dy,$ bu yerda, $L - x = acost\ , y = bsint,$ ellipsning

soat mili yo‘nalishidagi yuqori yarim pallasi.(Javob: $\frac{4ab^2}{3}$)

$\mathbf{4.14.} \int\limits_{L_{OB}} (xy - y^2)dx + xdy,$ bu yerda, $L - y = 2\sqrt{x}$ parabolaning

O(0;0), B(1;2) nuqtalar orasidagi yoyi. (Javob: $-\frac{8}{15}$)

$\mathbf{4.15.} \int\limits_{L} xdx + xydy,$ bu yerda, $L - y^2 + x^2 = 2x$, aylananing musbat yo‘nalishdagi yuqori yarim bo‘lagi. (Javob: $-\frac{4}{3}$)

$\mathbf{4.16.}\ \int\limits_L (x-y)dx+dy$, bu yerda, $L-y^2+x^2=R^2$, aylananing o‘ng yo‘nalishida bo‘lgan yuqori yarim bo‘lagi.(Javob: $\frac{\pi R^2}{2}$)

$\mathbf{4.17.}\oint\limits_L (x^2-y)dx$, bu yerda, $L-\ x=0, y=0\ x=1$ va $y=2$ to‘g’ri chiziqlardan tashkil topgan to‘g’ri to‘rtburchakning o‘ng yo‘nalishli konturi. (Javob: 2)

$\mathbf{4.18.}\oint_{L_{OB}} 4xsin^2dx+ycosxdy$, bu yerda $L_{OB}-$ to'g'ri chiziqning $O(0;0)$va $B(3;6)$ nuqtalar orasidagi kesmasi. (Javob:18)

$\mathbf{4.19.}\ \int\limits_L ydx-xdy$, bu yerda, $L-x=6cost$, $y=4sint$ ellipsning o‘ng yo‘nalishdagi yoyi.(Javob: -48π)

$\mathbf{4.20.}\ \int\limits_{L_{OA}} 2xydx-x^2dy$, bu yerda, $L_{OA}-x=2y^2$ parabolaning O(0;0) va A(2;1) nuqtalar orasidagi yoyi. (Javob:2,4)

$\mathbf{4.21.}\ \int\limits_{L_{AB}} xye^xdx+(x-1)e^xdy$, bu yerda, $L_{AB}-A(0;2)$ va $B(1;2)$ nuqtalarni birlashtiruvchi to‘g’ri chiziq. (Javob:2)

$\mathbf{4.22.}\oint\limits_L (x^2+y^2)dx+(x^2-y^2)dy$ bu yerda, $L-$ uchlari $A(0;0), B(1;0)$ va $C(0;1)$ nuqtalarda bo‘lgan uchburchakning musbat yo‘nalishdagi konturi.(Javob: $-\frac{1}{3}$)

4.23. $\int\limits_{L_{ABO}} (xy - x)dx + \frac{x^2}{2}dy$, bu yerda, $L_{ABO} - O(0;0), A(1;2)$

va $B(\frac{1}{2};3)$ nuqtalardan tashkil topgan ABO siniq chiziqning musbat yo‘nalishdagi konturi.(Javob: $-\frac{1}{2}$)

4.24. $\int\limits_{L_{OA}} (xy - y^2)dx + xdy$, bu yerda, $L_{OA} - O(0;0)$ va $A(1;2)$ nuqtalardan o‘tuvchi to‘g’ri chiziq kesmasi.(Javob: $\frac{1}{3}$)

4.25. $\int\limits_{L_{OA}} xdy - ydx$, bu yerda, $L_{OA} - y = x^3$ kubik parabolaning $O(0;0)$ va $A(2;8)$ nuqtalar orasidagi yoyi. (Javob:8)

4.26. $\int\limits_{L_{AB}} 2ysin2xdy - cosydx$, bu yerda, $L_{AB} - A\left(\frac{\pi}{4};2\right)$ nuqtadan $B\left(\frac{\pi}{6};1\right)$ nuqtagacha bo‘lgan ixtiyoriy chiziq.(Javob: $-\frac{1}{2}$)

4.27. $\int\limits_{L_{OB}} (xy - x)dx + \frac{x^2}{2}dy$, bu yerda, $L_{OB} - y = 4x^2$ parabolaning O(0;0) va B(1;4) nuqtalar orasidagi yoyi.(Javob: $\frac{3}{2}$)

4.28. $\int\limits_{L_{AB}} (x + y)dx + (x - y)dy$, bu yerda $L_{AB} - y = x^2$ parabolaning A(1;1) va B(1;1) nuqtalar orasidagi yoyi. (Javob:2)

4.29. $\int_{L_{AB}} xdy$, bu yerda, $L_{AB} - x^2 + y^2 = a^2$ aylananing $A(0;-a)$ va $B(0;a)$ nuqtalar orasidagi o‘ng yarim bo‘lagining yoyi.(Javob: $\frac{\pi a^2}{2}$)

4.30. $\int\limits_{L} y^2dx + x^2dy$, bu yerda L $- x = 5cost, y = 2sint$

ellipsning yuqori yarim pallasining musbat yo‘nalishdagi yoyi.(Javob: $\frac{80}{3}$)

Namunaviy variantning yechimi

Berilgan egri chiziqli integrallar hisoblansin.

1. $\oint\limits_{L} (x^2+y^2)^n dl,$ bu yerda $L,$ $x^2+y^2 = a^2$ aylana

►$x^2+y^2 = a^2$ aylana tenglamasini parametrik shaklda yozib olamiz: $x = acost,$ $y = asint, 0 \le t \le 2\pi$

U holda: $x_t' = -asint,$ $y_t' = acost,$ $dl = \sqrt{x_t'^2 + y_t'^2}dt$ bo‘lganligidan,

$dl = \sqrt{a^2sin^2t + a^2cos^2t}dt = adt.$ Natijada,

$\oint_L (x^2+y^2)^n dl = a^{2n+1}\int_0^{2\pi} dt = 2\pi\, a^{2n+1}.$◄

2. $\int_{L_{OB}} xdl,$ bu yerda $L_{OB} -$ to‘g’ri chiziqning $O(0;0)$ va $B(1;2)$ nuqtalarini birlashtiruvchi kesmasi.

►To‘g’ri chiziqning tenglamasi $y = 2x$ ga ko‘ra, $dl = \sqrt{1+(y_t')^2}dx$ dan $dl = \sqrt{5}dx$ ni topamiz. U holda:

$\int_{L_{OB}} xdl = \sqrt{5}\int_0^1 xdx = \sqrt{5}\frac{x^2}{2}\Big|_0^1 = \frac{\sqrt{5}}{2}$◄

3. $I = \oint 2x(y-1)dx + x^2dy,$ bu yerda $L -$ $y = x^2$

parabola bilan $y = 9$ to‘gri chiziq tashkil qilgan figuraning musbat yo‘nalishidagi konturi.

►Ikkinchi turdagi egri chiziqli integrallarning xossalariga binoan, quyidagini yozamiz:

$$I = \int\limits_{L_1} 2x(y-1)dx + x^2dy + \int\limits_{L_2} 2x(y-1)dx + x^2dy,$$

bu yerda L_1, $y = x^2$ parabola yoyi bo‘lib, $y = 9$ to‘g’ri chiziq kesmasidir. Parabola bilan to‘g’ri chiziq (-3;9) va (3;9) nuqtalarda kesishganliklari uchun, $I = \int_{-3}^{3}(4x^3 - 2x)dx + 16\int_{3}^{-3} xdx = 0.$◄

4. $I = \int_L \left(\sqrt[3]{x} + y\right)dx - \left(\sqrt[3]{y} + x\right)dy,$ bu yerda L , $x = 8cos^3t, y = 8sin^3t$ astroidaning (8;0) nuqta bilan (-8;0) nuqta orasidagi yuqori yoyi.

►$dx = 24cos^2t(-sint)dt,\ \ dy = 24sin^2cost\ dt$ va $0 \le t \le \pi$ bo′lganligidan,

$$I = 24\int\limits_0^{\pi} (2cost + 8cos^3t)(-cos^2tsint)dt - (2sint + 8cos^3t)sin^2tcostdt =$$

$$= \int\limits_0^{\pi} (-48sintcost - 48sin^2 2t)dt = 12cos2t\,\Big|_0^{\pi} -$$

$$-24\int_0^{\pi}(1 - cos4t)dt = -24\left(t - \frac{1}{4}sin4t\right)\Big|_0^{\pi} = -24$$◄

14.2-IUT

1. Berilgan ifoda $u = u(x;y)$ funksiyaning to‘la differensiali ekanligi isbotlanib, funksiyaning o‘zi topilsin.

1.1. $(2x - 3y^2)dx + (2 - 6xy)dy.$ (Javob: $x^2 + x + 2y - 3xy^2 + c.$)

1.2. $\left(\frac{2xy^2}{1 + x^2y^2} - 3\right)dx + \left(\frac{2x^2y}{1 + x^2y^2} - 5\right)dy.$

$(Javob: ln(1 + x^2y^2) - 3x - 5y + c\ .)$

1.3. $-\left(\frac{1}{2}cos2y + ysin2x\right)dx + (xsin2y + cos^2x + 1)dy$

$(Javob: ycos^2x - \frac{x}{2}\ cos2y + y + c.)$

1.4. $\left(y^2e^{xy^2} + 3\right)dx + \left(2xye^{xy^2} - 1\right)dy$ (Javob: $3x + e^{xy^2} - y + c.$)

1.5. $\left(\frac{1}{x + y} + cosxcosy - 3x^2\right)dx + \left(\frac{1}{x + y} + sinxsiny + 4y\right)dy.$

(Javob: $\ln(x+y)+sinxcosy-x^3+2y^2+c$.)

1.6. $\left(\frac{y}{x}+lny+2x\right)dx+\left(lnx+\frac{x}{y}+1\right)dy.$

($Javob$: $x^2+y\ln x+xlny+y+c$.)

1.7. $\left(e^{x+y}\ \ -cosx\right)dx+\left(e^{x+y}\ \ +siny\right)dy$

($Javob$: $e^{x+y}\ \ -cosy-sinx+c$.)

1.8. $\left(\frac{y}{\sqrt{1-x^2y^2}}+2x\right)dx+\left(\frac{x}{\sqrt{1-x^2y^2}}+6y\right)dy.$

($Javob$: $arcsinxy+x^2+3y^2+c$.)

1.9. $(e^{xy}+xye^{xy}+2)dx+(x^2e^{xy}+1)dy$

($Javob$: $xe^{xy}+2x+y+c$.)

1.10. $\left(ye^{xy}\ \ +y^2\right)dx+\left(xe^{xy}\ \ +2xy\right)dy$ ($Javob$: $e^{xy}\ \ +xy^2+c$.)

1.11. $(ycos(xy)+2x-3y)dx+(xcos(xy)-3x+4y)dy$

($Javob$: $sin(xy)+x^2-3xy+2y^2+c$.)

1.12. $(ysin(x+y)+xycos(x+y)-9x^2)dx$

$+(xsin(x+y)+xycos(x+y)+2y)dy$

($Javob$: $xysin(x+y)-3x^3+y^2+c$.)

1.13. $(5y+cosx+6xy^2)dx+(5x+6x^2y)dy$

($Javob$: $sinx+5xy+3x^2y^2+c$.)

1.14. $\left(y^2\ \ e^{xy}\ \ -3\right)dx+e^{xy}\ \ (1+xy)dy$ ($Javob$: $ye^{xy}\ \ -3x+c$.)

1.15. $\left(1+cos(xy)\right)ydx+\left(1+cos(xy)\right)xdy$

($Javob$: $xy+\sin(xy)+c$.)

1.16. $(y-sinx)dx+(x-2ycosy^2)dy$

($Javob$: $cosx+xy-\sin y^2+c$.)

1.17. $\left(sin2x-\frac{1}{x^2y}\right)dx-\frac{1}{x^2y}dy.$ $\left(Javob: \frac{1}{xy}-\frac{1}{2}cos2x+c.\right)$

1.18. $\frac{x+y}{xy}dx+\frac{y-x}{y^2}dy$ $\left(Javob: \ln(xy)+\frac{x}{y}+c.\right)$

1.19. $(20x^3-21x^2y+2y)dx+(3+2x-7x^3)dy.$

$(Javob: 5x^4 + 7x^3y + 2xy + 3y + c.)$

$\mathbf{1.20.} \left(y \quad e^{xy} \quad - 2sinx\right) dx + \left(xe^{xy} \quad + cosy\right) dy$

$(Javob: e^{xy} \quad + 2cosx + siny + c.)$

$\mathbf{1.21.} y\left(e^{xy} \quad + 5\right) dx + x\left(e^{xy} \quad + 5\right) dy$

$(Javob: e^{xy} \quad + 5xy + c.)$

$\mathbf{1.22.} \left(x - \frac{y}{x^2 - y^2}\right) dx + \left(\frac{x}{x^2 - y^2} - y\right) dy.$

$(Javob: \frac{x^2}{2} + arctg\frac{y}{x} - \frac{y^2}{2} + c\ .)$

$\mathbf{1.23.} \frac{xlny + y}{x} dx + \frac{ylnx + x}{y} dy \qquad (Javob: ylnx + xlny + c.)$

$\mathbf{1.24.} e^{x-y} \quad (1 + x + y)dx + e^{x-y} \quad (1 - x - y)dy$

$(Javob: e^{x-y} \quad (x + y) + c.)$

$\mathbf{1.25.} (3x^2 - 2xy + y)dx + (x - x^2 - 3y^2 - 4y)dy.$

$(Javob: x^3 - x^2y - y^3 + xy - 2y^2 + c.)$

$\mathbf{1.26.} \left(2xe^{x^2-y^2} \quad - sinx\right) dx + \left(siny - 2ye^{x^2-y^2} \quad\right) dy.$

$(Javob: e^{x^2-y^2} \quad + cosx - cosy + c.)$

$\mathbf{1.27.} \left(\frac{y}{\sqrt{1 - x^2y^2}} + x^2\right) dx + \left(\frac{x}{\sqrt{1 - x^2y^2}} + y\right) dy.$

$(Javob: \frac{x^3}{3} + +arcsin(xy) + \frac{y^2}{2} + c\ .)$

$\mathbf{1.28.} \frac{1 - y}{x^2y} dx +. \frac{1 - 2x}{xy^2} dy. \qquad (Javob: \frac{2x - 1}{xy} + \frac{1}{x} + c\ .)$

1.29. $\left(\frac{1}{y - 1} - \frac{y}{(x - 1)^2} - 2\right) dx + \left(\frac{1}{x - 1} - \frac{x}{(y - 1)^2} + 2y\right) dy$

$(Javob: \frac{y}{x - 1} + \frac{x}{y - 1} - 2x + y^2 + c\ .)$

$\mathbf{1.30.} (3x^2 - 2xy + y^2)dx + (2xy - x^2 - 3y^2)dy.$

$(Javob: x^3 - x^2y + xy^2 + y^3 + c.)$

2. Quyidagi masalalar yechilsin

2.1. Zanjir chiziq $y = \frac{(e^x + e^{-x})}{2}$, $(x\epsilon[0;1])$ ning yoyi uzunligi topilsin. (Javob: $\frac{e^2-1}{2e}$)

2.2. Tenglamasi $2x + y = 1$ bo′lgan bir jinsli to′g′ri chiziqning koordinata o′qlari orasida joylashgan kesmasining koordinata o′qlariga nisbatan inersiya momentlari hisoblansin. $\left(Javob: J_x = \frac{\sqrt{5}}{6}, J_y = \frac{\sqrt{5}}{24}.\right)$

2.3. Tenglamasi $x^2 + y^2 = a^2$ bo′lgan bir jinsli aylananing birinchi kvadrantida yotuvchi chorak bo′lagining og′irlik markazi koordinatalari topilsin. $\left(Javob: \left(\frac{2a}{\pi}; \frac{2a}{\pi}\right).\right)$

2.4. Agar $y = lnx$ egri chiziq yoyining har bir nuqtasidagi zichligi, shu nuqta absissasining kvadratiga teng bo‘ladigan bo‘lsa, uning absissalari $x = \sqrt{3}$ bilan $x = \sqrt{8}$ bo‘lgan nuqtalari orasidagi yoyining massasi hisoblansin. (Javob: 19/3)

2.5. $y^2 = x^3$yarim kubik parabolaning absissalari $x = 0$ va $x = \frac{4}{3}$ bo‘lgan nuqtalari orasidagi joylashgan yoyining *Oy* o‘qqa nisbatan inersiya momenti hisoblansin. $\left(Javob: J_y = \frac{107*2^{10}}{105*3^6} \approx 1{,}13.\right)$

2.6. Tomonlari $x = \pm a \; va \; y = \pm a$ bo‘lgan kvadrat konturining koordinata boshiga nisbatan inersiya momenti hisoblansin. (kvadratning zichligini o‘zgarmas deb olinsin). $\left(Javob: J_O = \frac{32}{3}.\right)$

2.7. $x = 2 - \frac{t^2}{4}$, $y = \frac{t^6}{6}$ egri chiziqning koordinata o‘qlari bilan kesishish nuqtalari orasidagi yoyining uzunligi hisoblansin. (Javob: $\frac{13}{3}$.)

2.8. Tenglamasi $x^2 + y^2 = 4$bo‘lgan bir jinsli aylananing *Ox* o‘qiga nisbatan simmetrik bo‘lagining og’irlik markazi koordinatalari topilsin. $\left(Javob: \left(\frac{4}{\pi}; 0\right).\right)$

2.9. $x = t - sint,\ y = 1 - cost$ sikloidaning bir jinsli bir arkasi yoyining og'irlik markazi koordinatalari hisoblansin. $\left(Javob: \left(\pi; \frac{4}{3}\right).\right)$

2.10. Agar to'g'ri chiziqning har bir nuqtasidagi chiziqli zichligi 1 ga teng bo'ladigan bo'lsa, uning A(2;0) va B(0;1) nuqtalari orasida joylashgan kesmaning koordinata boshiga nisbatan inersiya momenti hisoblansin. $\left(Javob: J_O = \frac{5\sqrt{5}}{3}.\right)$

2.11. $x^2 + y^2 + z^2 = 1, x \geq 0,\ y \geq 0, z \geq 0$ lardan iborat sferik uchburchak bir jinsli konturning og'irlik markazi koordinatalari hisoblansin.$\left(Javob: \left(\frac{4}{3\pi}; \frac{4}{3\pi}; \frac{4}{3\pi}\right).\right)$

2.12. $x = 2cos^3 t, y = 2sin^3 t$astroidaning birinchi chorakda joylashgan yoyi bo'lagining koordinata o'qlariga nisbatan statik momentlari hisoblansin. $\left(Javob: M_x = 2{,}4,\ M_y = 2{,}4.\right)$

2.13. Agar $y = 2 - x$ to'g'ri chiziqning har bir nuqtasidagi chiziqli zichligi y nuqta abscissa kvadratiga proporsional bo'lib $(2; 0)$ nuqtada 4 ga teng bo'ladigan bo'lsa, uning koordinata o'qlari orasidagi kesmasining massasi hisoblansin. $\left(Javob: \frac{8\sqrt{2}}{3}.\right)$

2.14. $\rho^2 = a^2 cos2\varphi$ Bernulli lemniskatasining bir jinsli bo'lgan birinchi o'ramidan iborat yoyining *Oy* o'qqa nisbatan statik momentlari hisoblansin. $\left(Javob: M_y = a^2\sqrt{2}.\right)$

2.15. $\vec{F} = x\vec{i} + (x + y)\vec{j}$ kuchning $\frac{x^2}{16} + \frac{y^2}{9} = 1$ ellips yoyi bo'ylab *m* nuqtaviy massa ko'chganda bajargan ishini hisoblansin. $(Javob: 12\pi m.)$

2.16. $x = 2cost, y = 2sint, z = t$ vint chizig'i birinchi o'ram bir jinsli yoyining *Oz* o'qqa nisbatan inersiya momenti hisoblansin.$\left(Javob: I_z = 8\sqrt{5}\pi.\right)$

2.17. $\rho \quad = 3sin\varphi, \left(\ \varphi\epsilon \left[0; \frac{\pi}{4}\right]\right)$ egri chiziqning har bir nuqtasidagi chiziqli zichligi qutbgacha bo'lgan masofaga proporsional bo'lib, $\varphi = \frac{\pi}{4}$ da 3 ga teng bo'ladigan bo'lsa, egri chiziq yoyining massasi hisoblansin. . $\left(Javob: \frac{9(2-\sqrt{2})}{2}.\right)$

2.18. $x = cost, y = sint, z = 2t$ vint chizig'i birinchi o'ram bir jinsli yoyining og'irlik markazining koordinatalari hisoblansin. $(Javob: (0; 0; 2\pi).)$

2.19. Birinchi chorakda yotuvchi $x = 2cost, y = 2sint$ aylana yoyi chorak qismining koordinata o'qlariga nisbatan inersiya momentlari hisoblansin. $\left(Javob: J_x = 2\pi,\ J_y = 2\pi.\right)$

2.20. Agar $. x = 2cost, y = 2sint, z = t$ vint chizig'ining har bir nuqtasidagi chiziqli zichligi shu nuqta applikatasiga proporsional bo'lib, $t = \pi$ da 1 ga teng bo'lsa, uning birinchi o'ram yoyi og'irlik markazi koordinatalari hisoblansin. $\left(Javob: \left(0; -\frac{2}{\pi}; \frac{4\pi}{3}\right).\right)$

2.21. Agar $\frac{x^2}{4} + y^2 = 1$ ellipsning har bir nuqtasidagi chiziqli zichlik, shu nuqta koordinatalari ko'paytmasiga teng bo'lsa, uning birinchi kvadrantida yotuvchi chorak qismining massasi hisoblansin**.** $\left(Javob: \frac{14}{9}.\right)$

2.22. Agar moddiy nuqta y=x to'g'ri chiziq bo'ylab harakatlanayotgan bo'lsa, u holda, $\vec{F} = xy\vec{\imath} + (x + y)\vec{\jmath}$ kuchning (0;0) nuqtadan (1;1) nuqtagacha ko'chgandagi ishining miqdori hisoblansin. $\left(Javob: \frac{4}{3}.\right)$

2.23. Tenglamasi $y = \frac{(e^x + e^{-x})}{2}$, $\left(x\epsilon \left[0; \frac{1}{2}\right]\right)$ bo'lgan zanjir chiziq bir jinsli yoyining *Ox* o'qqa nisbatan statik momenti hisoblansin. $\left(Javob: \frac{\frac{e-1}{e}+2}{8}.\right)$

2.24. Agar moddiy nuqta, $x = \pm 1$ va $y = \pm 1$to'g'ri chiziqlar tashkil etgan kvadratning konturi bo'ylab harakatlanayotgan bo'lsa, $\vec{F} = (x - y)\vec{\imath} + x\vec{\jmath}$ kuchning bajargan ishi hisoblansin. $(Javob: 8.)$

2.25. $\rho \ = a(1 + cos\varphi)$ kardioida bir jinsli yoyining *Ox* o'qqa nisbatan statik momenti hisoblansin.$\left(Javob: \frac{32a^2}{5}.\right)$

2.26. $x = 3(t - sint),\ y = 3(1 - cost)$ sikloida bir arkasi yoyining uzunligi hisoblansin. $(Javob: 24.)$

2.27. Agar moddiy nuqta soat mili bo‘yicha $x = 2cost, y = 2sint$ aylana bo‘ylab harakatlanayotgan bo‘lsa, $\vec{F} = (x - y)\vec{\imath} - x\vec{\jmath}$ kuchning bajargan ishi hisoblansin. $(Javob: 8\pi.)$

2.28. Agar moddiy nuqta $y = x^2$ parabola bo‘ylab koordinata boshidan (1;1) nuqtaga tomon harakatlanayotgan bo‘lsa, $\vec{F} = y\vec{\imath} + (x + y)\vec{\jmath}$ kuchning bajargan ishi hisoblansin. $(Javob: 17/2.)$

2.29. Moddiy nuqta $y = -3x^2$ parabola bo‘ylab koordinata boshidan (1;-3) nuqtaga tomon harakatlanayotgan bo‘lsa, $\vec{F} = (x - y)\vec{\imath} + 2y\vec{\jmath}$ kuchning bajargan ishi hisoblansin. $(Javob: 10{,}5.)$

2.30. $y = 2x$ to‘g’ri chiziqning (1;2) va (2;4) nuqtalari orasidagi bir jinsli kesmaning koordinata o‘qlariga nisbatan inersiya momentlari hisoblansin. $\left(Javob: J_x = \frac{28\sqrt{5}}{3},\ J_y = \frac{7\sqrt{5}}{3}.\right)$

Namunaviy variantning yechimi

1. $\left(\frac{y}{1+x^2y^2} - 1\right)dx + \left(\frac{x}{1+x^2y^2} - 10\right)dy$ kabi ifoda $u = u(x;y)$ funksiyaning to‘la differensiali ekanligi ko‘rsatilib, $u(x;y)$ funksiyaning ko‘rinishi topilsin.

►$u(x;y)$ funksiya uchun to‘la differensiallik sharti $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$ ni tekshiramiz:

$P(x;y) = \frac{y}{1+x^2y^2} - 1$ va $Q(x;y) = \frac{x}{1+x^2y^2} - 10$ bo‘lganligi uchun:

$$\frac{\partial P}{\partial y} = \frac{\partial}{\partial y}\left(\frac{y}{1 + x^2y^2} - 1\right) = \frac{1 + x^2y^2 - y * 2x^2y}{(1 + x^2y^2)^2} = \frac{1 - x^2y^2}{(1 + x^2y^2)^2},$$

$$\frac{\partial Q}{\partial x} = \frac{\partial}{\partial x}\left(\frac{x}{1 + x^2y^2} - 10\right) = \frac{1 + x^2y^2 - x * 2xy^2}{(1 + x^2y^2)^2} = \frac{1 - x^2y^2}{(1 + x^2y^2)^2}$$

Demak, berilgan ifoda $u(x;y)$ uchun to‘la differensial bo‘lar ekan. (14.16) formulada $x_0 = 0$ va $y_0 = 0$ deb olib, $u(x;y)$ ni aniqlaymiz:

$$u(x;y) = \int_0^x (-1)\,dx + \int_0^y \left(\frac{x}{1 + x^2y^2} - 10\right)dy + C =$$

$$-x\Big|_0^x+(arctgxy-10y)\Big|_0^y+C==-x+arctgxy-10y+C$$

Agar $\frac{\partial u(x;y)}{\partial x}=P(x;y),\ \frac{\partial u(x;y)}{\partial y}=Q(x;y)$ shart bajarilsa, hisoblash natijasi to‘g’ri bo‘ladi. Tekshiramiz:

$$\frac{\partial}{\partial x}(-x+arctgxy-10y+c)=-1+\frac{y}{1+x^2y^2},$$

$$\frac{\partial}{\partial y}(-x+arctgxy-10y+c)=\frac{x}{1+x^2y^2}-10.$$

Demak, $u(x;y)=arctgxy-x-10y+c$.◄

2. Tenglamasi $4x+2y-3=0$ bo‘lgan to‘g’ri chiziqning $\left(0;\frac{3}{2}\right)$ va $\left(2;-\frac{5}{2}\right)$ nuqtalari orasida joylashgan bir jinsli kesmasining koordinata o‘qlariga nisbatan inersiya momentlari hisoblansin.

►Inersiya momentlarini hisoblash formulalarini ketma-ket qo‘llab quyidagilarni topamiz:

$$J_x=\int_L y^2dl,\qquad \text{bu yerda}, L: y=-2x+\frac{3}{2}\text{va } dl=\sqrt{5}dx.$$

$$J_x=\sqrt{5}\int_0^2\left(-2x+\frac{3}{2}\right)^2dx=-\frac{\sqrt{5}}{2}*\frac{x^3}{3}\Big|_0^2=-\frac{\sqrt{5}}{2}*\frac{\left(-2x+\frac{3}{2}\right)^3}{3}\Big|_0^2=$$

$$=-\frac{\sqrt{5}}{2}\left(-\frac{125}{8}+\frac{27}{8}\right)=\frac{49\sqrt{5}}{24};$$

$$J_y=\int_L x^2dl,\ J_y=\sqrt{5}\int_0^2x^2dl=\sqrt{5}*\frac{x^3}{3}\Big|_0^2=\frac{8\sqrt{5}}{3}.◄$$

14.4. 14 – bobga doir qo‘shimcha masalalar

1. Tenglamasi $x=ae^t cost,\ y=ae^t sint, z=ae^t$ bo‘lgan konussimon vint chizig’ining *O(0;0;0)* va *A(a;0;a)* nuqtalar orasida joylashgan yoyining uzunligi topilsin. ($Javob$: $a\sqrt{3}$)

2. Agar tenglamasi $y=ach(x/a)$ bo‘lgan zanjir chizig’ining har bir nuqtasidan chiziqli zichligi y nuqta ordinatasiga teskari proparsional bo‘lib,

$(0;a)$ nuqtada esa γ ga teng bo‘ladigan bo‘lsa, uning absissalari $x_1 = 0\; va\; x_2 = a$ bo‘lgan nuqtalari orasida joylashgan bo‘lagining massasi topilsin. ($Javob$: γa)

3. Har bir nuqtadagi chiziqli zichligi $|y|$ ga teng bo‘lgan $\frac{x^2}{9} + \frac{y^2}{4} = 1$ ellipsning massasi topilsin. ($Javob$: $4 + \frac{18\sqrt{5}}{5} arcsin \frac{\sqrt{5}}{3}$)

4. Har bir nuqtadagi chiziqli zichligi o‘zgarmas bo‘lgan $x = acost\,, y = sint, z = bt$ vint chizig’ining birinchi yarim o‘rami og’irlik markazining koordinatalari topilsin. ($Javob$: $(0; \frac{2a}{\pi}; \frac{b\pi}{2})$)

5. $y = 2cost\,, z = 2sint$ bir jinsli aylananing *Oyz* tekislik birinchi kvadrantida joylashgan chorak qismning koordinata o‘qlari hamda koordinata boshiga nisbatan inersiya momentlari hisoblansin. ($Javob$: $J_x = J_y = 2\pi, J_0 = 4\pi$)

6. $x = acost,\; y = asint\,, z = \frac{ht}{2\pi}$ vint chizig’i birinchi o‘ramining *Ox* o‘qqa nisbatan inersiya momenti hisoblansin. ($Javob$: $(\frac{a^2}{2} + \frac{h^2}{3})\sqrt{4\pi^2 a^2 + h^2}$)

7. Quyidagi integral $\oint_L (x + y)dx - 2xdy$ uchun Grin formulasining bajarilishi tekshirilsin. Bu yerda, $L-$ tomonlari $x = 0, y = 0$ va $x + y = a$ bo'lgan uchburchakning konturidir.

8. Grin formulasidan foydalanib, $\oint_{L_{ABC}} y^2dx + (x + y)^2dy$ hisoblansin.

Bu yerda $L_{ABC} -$ uchlari $A(2;0), B(2;2)$ va $C(0;2)$ nuqtalarda bo‘lgan *ABC* uchburchak konturidan iborat. ($Javob$: $\frac{16}{3}$)

9. $\int_L (yx^3 + e^y)dx + (xy^3 + xe^y - 2y)dy = 0$ ekanligi isbotlansin. Bu yerda, $L -$ koordinata Boshiga nisbatan simmetrik bo‘lgan yopiq egri chiziq.

10. Agar $L-$ ixtiyoriy yopiq kontur bo‘ladigan bo‘lsa, $\int_L (2xy - y)dx + x^2dy$ ning qiymati shu kontur chegaralagan sohaning yuzasiga teng ekanligi isbotlansin.

11. Agar L, o‘ng yo‘nalishdagi va koordinata boshini o‘zichiga oladigan ixtiyoriy yopiq kontur bo‘ladigan bo‘lsa, u holda, $\oint \frac{xdy-ydx}{x^2+y^2}$ ning qiymati 2π ga teng L ekanligi isbotlansin.

12. $du = e^{\frac{y}{z}}dx + \left(\frac{x+1}{z}e^{\frac{y}{z}} + ze^{\frac{y}{z}}\right)dy + \left(ye^{yz} + e^{-z} - \frac{(x+1)y}{z^2}e^{\frac{y}{z}}\right)dz$

to‘la differensianalga nisbatan $u(x;y)$ funksiya topilsin.

(Javob: $a^{\frac{y}{z}}(x+1) + e^{yz} - e^{-z}.$)

15. MAYDONLAR NAZARIYASI ELEMENTLARI

15.1. Skalyar argumentli vektor funksiya.Yo‘nalish bo‘yicha hosila va gradient

Har bir *t*∈*T*∈*R* songa biror qoida bo‘yicha bitta $\vec{r}$ vektorni mos qilib qo‘yadigan akslantirishni *t* skalyar argumentli *vektor funksiya* deb ataladi va uni $\vec{r} = \vec{r}(t)$ deb belgilash qabul qilingan. Bu yerda, *T* to‘plamni $\vec{r}(t)$ funksiyaning aniqlanish sohasi deb yuritiladi. Odatda, *T* to‘plam uchun son o‘qidan $[a;b]$ kesma yoki (a;b) oraliq olinadi. Shuningdek, *t* sonni parametr deb ham yuritiladi. Boshqa vektorlarda bo‘lgani kabi, *t* ning har qanday tayinli qiymati uchun $\vec{r}(t)$ ni $\vec{i}, \vec{j}, \vec{k}$ bazislar bo‘yicha bir qiymatli yoyish mumkin, ya’ni:

$$\vec{r} = \vec{r}(t) = x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k} \qquad (15.1)$$

Ravshanki, $\vec{r} = \vec{r}(t)$ vektor funksiyaning koordinatalari bo‘lgan *x, y, z* lar ushbu bazisda $x(t), y(t), z(t)$ kabi funksiyalar bo‘lib, ularning aniqlanish sohalari ham *T* to‘plam bilan ustma – ust tushadi. Shu boisdan quyidagi uchta skalyar tengliklar o‘rinlidir:

$$x = x(t),\ y = y(t),\ z = z(t) \qquad (15.2)$$

Agar $\vec{r}$ vektorni *t*∈*T* ning turli xildagi qiymatlari bo‘yicha bitta O nuqtaga qo‘yiladigan bo‘lsa, u holda, uning *M(t)* uchi fazoda biror chiziqni chizidaki, u chiziqni $\vec{r} = \vec{r}(t)$vektor funksiyaning *godografi* deb ataladi. O nuqta esa, godografining *qutbi* deb yuritiladi. Bu holda, (15.1) ni godografning vektor-

parametrik tenglamasi deb atalib, (15.2) ni esa uning parametrik tenglamalari deb yuritiladi (15.1 - rasm)

Bir necha misollar keltiramiz :

1.Vektor – parametrik tenglama $\vec{r} = \vec{r}(t) = \overrightarrow{r_o} + \vec{s}t$ ifodalovchi godograf (bu yerda, $\overrightarrow{r_o} - M_0(x_0; y_0; z_0)$ nuqtaning radius-vektori bo‘lib, $\vec{s}$ esa, biror berilgan vektordir), berilgan M_0 nuqtadan o‘tuvchi hamda, $\overrightarrow{s}$ yo‘naltiruvchi vektorli fazodagi to‘g’ri chiziqdan iboratdir (Mazkur qo‘llanmaning birinchi qismidagi (3.6) tenglama bilan 3.1- rasmga qaralsin).

2. $x = acost, y = asint, t = bt$ $(t\epsilon(-\infty; \infty),$ a va b lar o‘zgarmas sonlar) parametrik tenglamalar bilan aniqlangan godograf, Oz o‘qli a radiusli doiraviy silindrga joylashgan vint chizig’idan iboratdir (Mazkur qo‘llanmaning I-qismidagi 4.3 ga qaralsin).

Agarda, t vaqtni ifodalab, $x(t), y(t), z(t)$ lar uzunlik o‘lchovlari bo‘lsalar, u holda (15.1) bilan (15.2) lar mos ravishda vektor-parametrik va nuqta harakatining parametrik tenglamalari deb yuritilib, ularga mos keluvchi godograf esa, harakat trayektoriyasi deb ataladi.

Agar, $\lim_{t\to t_0} x(t) = x_0, \lim_{t\to t_0} y(t) = y_0, \lim_{t\to t_0} z(t) = z_0$ bo‘lsa, $\overrightarrow{r_0} = x_0\vec{\imath} + y_0\vec{\jmath} + +z_0\vec{k}$ vektorni $\vec{r}(t)$ vektor funksiyaning $t = t_0$ nuqtadagi limiti deb ataladi va $\lim_{t\to t_0} \vec{r}(t) = \vec{r}_0$ bo‘lsa, $\vec{r}(t)$vektor-funksiya $t = t_0$ nuqtada uzluksiz funfsiya deb ataladi. Agar $\Delta t \neq 0,$ parametrning ixtiyoriy orttirmasi bo‘lsa,

$$\Delta\,\vec{r}(t) = \vec{r}(t + \Delta t) - \vec{r}(t)$$

ni $\vec{r}(t)$ vektor-funksiyaning orttirmasi deyiladi.

Agarda

$$\lim_{\Delta t\to 0}\frac{\Delta\vec{r}(t)}{\Delta t}=\lim_{\Delta t\to 0}\frac{\vec{r}(t+\Delta t)-\vec{r}(t)}{\Delta t}$$

kabi limit mavjud bo‘lsa, u holda uni $\vec{r}(t)$vektor-funksiyaning t nuqtadagi *hosilasi* deb atalib, hosilani $\vec{r}\,'(t)$ yoki $\dot{\vec{r}}(t)$ yoki $d\vec{r}(t)/dt$ kabi belgilarning biri orqali beriladi.

$\vec{r}\,'(t)$ vektor, har doim $\vec{r}(t)$ funksiya godografiga o‘tkazilgan urinmaning t parametr o‘sishi bo‘ylab yo‘nalgan bo‘ladi. Mexanik jihatdan esa, $\vec{r}'(t)$ hosila, $\vec{r}=\vec{r}(t)$ funksiya godografi bo‘lgan trayektoriya bo‘ylab harakatlanayotgan moddiy nuqtaning t vaqt momentidagi *M(t)* nuqtadagi oniy tezligini ifodalaydigan vektorni tasvirlaydi (15.1-rasmga qaralsin).

Agar, $x'(t), y'(t)$ va $z'(t)$ hosilalar mavjud bo‘lsalar, $\vec{r}'(t)$ ham mavjud bo‘ladi va u quyidagicha yoziladi:

$$\vec{r}'(t)=x'(t)\vec{i}+y'(t)\vec{j}+z'(t)\vec{k} \qquad (15.3)$$

Agar $\vec{r}'(t_0)$ vektor (15.2) tenglama bilan aniqlanadigan egri chiziqning $M_0(t_0)$ nuqtasiga o‘tkazilgan urinma bo‘ylab yo‘nalishini inobatga oladigan bo‘lsak, u holda, bu egri chiziqning M_0 nuqtasiga o‘tkazilgan urinma tenglamasi quyidagicha yoziladi:

$$\frac{x-x(t_0)}{x'(t_0)}=\frac{y-y(t_0)}{y'(t_0)}=\frac{z-z(t_0)}{z'(t_0)} \qquad (15.4)$$

Egri chiziqning $M_0(t_0)$ nuqtasiga o‘tkazilgan urinmaga perpendikulyar bo‘lib, urinish nuqtasidan o‘tadigan tekislikni egri chiziqning *normal tekisligi* deb yuritiladi va uning tenglamasi quyidagi ko‘rinishga ega bo‘ladi:

$$x'(t_0)\big(x-x(t_0)\big)+y'(t_0)\big(y-y(t_0)\big)+z'(t_0)\big(z-z(t_0)\big)=0 \qquad (15.5)$$

Skalyar argumentli vektor funksiyalar uchun quyidagicha differensiallash qoidalari o‘rinli bo‘ladi:

1) $\big(\boldsymbol{r_1}(t)+\boldsymbol{r_2}(t)\big)'=\boldsymbol{r'_1}(t)+\boldsymbol{r'_2}(t)$;

2) $\big(\boldsymbol{Cr_1}(t)\big)'=C\boldsymbol{r'_1}(t)$;

3) $\big(\boldsymbol{r_1}(t)\cdot\boldsymbol{r_2}(t)\big)'=\boldsymbol{r'_1}(t)\cdot\boldsymbol{r_2}(t)+\boldsymbol{r_1}(t)\cdot\boldsymbol{r'_2}(t)$;

4) $\left(\boldsymbol{r_1}(t)\times\boldsymbol{r_2}(t)\right)'$=$\boldsymbol{r'_1}(t)\times\boldsymbol{r_2}(t)$+$\boldsymbol{r_1}(t)\times\boldsymbol{r'_2}(t)$.

1- misol.$\vec{r}(t)=(cost-1)\vec{\imath}+sin^2t\vec{\jmath}+tgt\vec{k}$ vektor funksiyaning $t_0=\frac{\pi}{4}$ nuqtadagi hosilasi topilsin.

► (15.3) formulaga binoan, $\vec{r}'(t)=-sint\vec{\imath}+2sintcost\vec{\jmath}+\frac{1}{cos^2t}\overrightarrow{k}$ bo‘lganligidan, $\vec{r}'\left(\frac{\pi}{4}\right)=-\frac{1}{\sqrt{2}}\vec{\imath}+\vec{\jmath}+2\vec{k}$ ni topamiz.◄

2- misol. Agar egri chiziq o‘zining $x=t^3+t-1,\ y=2t^2+3t+2,\ z==t^2+1$ kabi parametrik tenglamasi bilan berilgan bo‘lsa, uning $t_0=1$ qiymatga mos keladigan M_0 nuqtasiga o‘tkazilgan urinma to‘g’ri chiziqning kanonik tenglamasi va normal tekislikning umumiy tenglamasi yozilsin.

► $\vec{r}'(t)=\left(x'(1),y'(1),z'(1)\right)=(4{,}7{,}2)$ ni topamiz. Agar $t_0=1$ parametrga egri chiziqning $M_0\left(x(1),y(1),z(1)\right)$ nuqtasi, ya’ni, $M_0(1{,}7{,}2)$ mos kelishini inobatga olsak, (15.4) va (15.5) formulalarga ko‘ra, urinma va normal tekislik tenglamalari mos ravishda quyidagicha aniqlanadi:

$$\frac{x-1}{4}=\frac{y-7}{7}=\frac{z-2}{2}$$

$$4(x-1)+7(y-7)+2(z-2)=0.$$

Funksiyaning biror yo‘nalishi bo‘yicha hosilasi tushunchasini kiritishdan oldin shuni ta’kidlaymizki, fazodagi yo‘nalishini ixtiyoriy $\vec{s}^0=(cos\alpha,cos\beta,cos\gamma)$ kabi birlik vektor orqali berilishi mumkin (bu yerda, α,β,γlar $\vec{s}^0$ vektorning mos ravishda *Ox*, *Oy*, va *Oz* o‘qlari bilan tashkil qilgan burchaklar).◄

Ta’rif: Agar $u=f(x,y,z)$ funksiya biror $M_0(x_0,y_0,z_0)$ nuqta atrofida aniqlangan bo‘lib, u nuqtaning radius-vektori $\vec{r}_0=(x_0,y_0,z_0)$ bo‘lib,

$$\lim_{t\to 0}\frac{f(\vec{r}_0+\vec{s}^0t)-f(\vec{r}_0)}{t}$$

kabi limit mavjud bo‘ladigan bo‘lsa, u holda uni $u=f(x,y,z)$ funksiyaning $\vec{S}^0$ vektor yo‘nalishi bo‘yicha $M_0(x_0,y_0,z_0)$ nuqtadagi hosilasi deb ataladi va $\frac{\partial u(M_0)}{\partial\vec{S}^0}$ deb belgilanadi. Demak, ta’rifga binoan:

$$\frac{\partial u(M_0)}{\partial \vec{S}^0} = \lim_{t\to 0}\frac{f(\vec{r}_0+\vec{S}^0 t)-f(\vec{r}_0)}{t}$$

Shuningdek, quyidagi formula o‘rinlidir:

$$\frac{\partial u(M_0)}{\partial \vec{S}^0} = \frac{\partial u(M_0)}{\partial x}cos\alpha + \frac{\partial u(M_0)}{\partial y}cos\beta + \frac{\partial u(M_0)}{\partial z}cos\gamma \qquad (15.6)$$

Agar (15.6) formuladan ikkita argumentga bog’liq bo‘ladigan bo‘lsa, ya’ni $f(x,y)$ bo‘lsa, u yanada soddalashadi

$$\frac{\partial u(M_0)}{\partial \vec{S}^0} = \frac{\partial u(M_0)}{\partial x}cos\alpha + \frac{\partial u(M_0)}{\partial y}cos\beta, \qquad (15.7)$$

bu yerda $\vec{s}^0 = (cos\alpha, cos\beta), \beta = \frac{\pi}{2} - \alpha$

Uch argumentli $u = f(x,y,z)$ funksiyaning xususiy hosilalari uning koordinata o‘qlari yo‘nalishi bo‘yicha xususiy hosilalari bo‘ladi. Fizikaviy nuqtai nazardan qaralganda $\frac{\partial u}{\partial \vec{s}}$ hosilani funksiyaning berilgan yo‘nalish bo‘yicha berilgan nuqtadagi o‘zgarish tezligi deb talqin etish mumkin bo‘ladi.

Biror L egri chiziq bo‘ylab hosila deganda, shu egri chiziqning biror nuqtaga o‘tkazilgan urinmaning ko‘rsatilgan tomoni bo‘yicha urinish nuqtaga hisoblangan hosilaga aytiladi.

Differensiallanuvchi har qanday funksiyaga koordinatalari $\frac{\partial u(M)}{\partial x}, \frac{\partial u(M)}{\partial y}, \frac{\partial u(M)}{\partial z}$ bo‘lgan vektor mos keladiki u vektorni $u = u(x;y;z)$ funnksiyaning biror M nuqtadagi gradient deb atalib uni $\overrightarrow{gradu}$ bilan belgilanadi

Demak, ta’rifga ko‘ra,

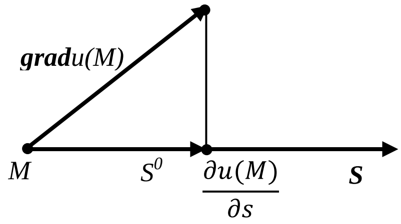
$$\overrightarrow{gradu} = \left(\frac{\partial u(M)}{\partial x}, \frac{\partial u(M)}{\partial y}, \frac{\partial u(M)}{\partial z}\right) = \frac{\partial u}{\partial x}\vec{i} + \frac{\partial u}{\partial y}\vec{j} + \frac{\partial u}{\partial z}\vec{k} \qquad (15.8)$$

Agar $\vec{s}^0 = (cos\alpha, cos\beta, cos\gamma)$ ekanligini e’tiborga olsak, (15.6) bilan (15.8) formulalardan:

$\frac{\partial u(M)}{\partial \vec{S}^0} = \overrightarrow{gradu} \quad \vec{s}^0 = pr_{\vec{s}^0}\overrightarrow{gradu}(M)$ $d$eb yozish mumkin.

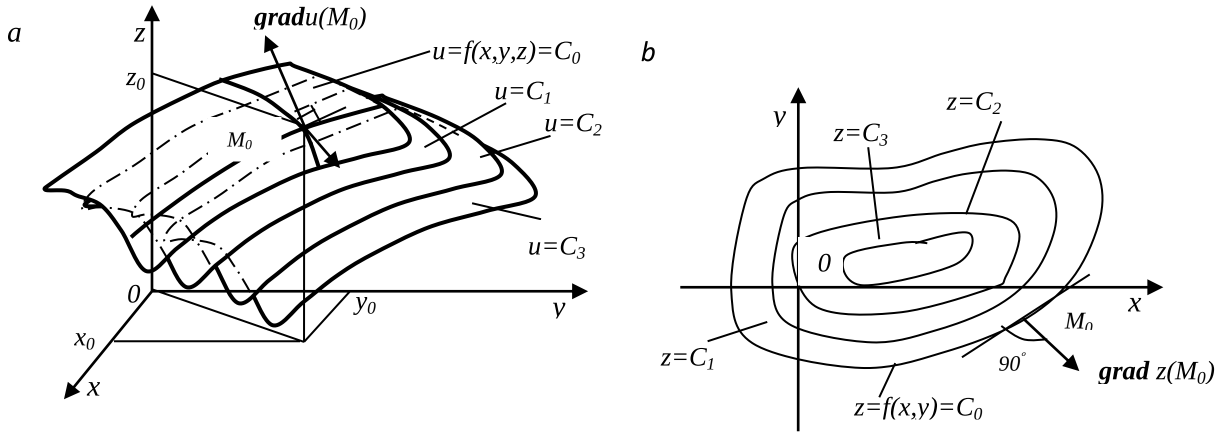
$u = f(x, y, z)$ (yoki $z = f(x, y)$) funksiyaning yo‘nalish bo‘yicha hosilasi bilan uning gradientini bog’laydigan ushbu tenglikdan quyidagilarni yozish mumkin bo‘ladi:

1) $u = f(x, y, z)$ (yoki $z = f(x, y)$) funksiyaning gradienti, uning qiymatlarining maksimal darajada o‘sish tomoniga qarab yo‘nalgan, ya’ni, $\frac{\partial u}{\partial \vec{s}}$ $(yoki\ \frac{\partial z}{\partial \vec{s}}\ )$ eng katta qiymatga gradient bo‘ylab yo‘nalishda erishadi (15.2 rasmga qaralsin)



15.2- rasm

2) Agar $\vec{S}^0 \perp \overrightarrow{gradu}$ bo‘lsa (yoki $\vec{S}^0 \perp \overrightarrow{gradz}$), u holda har doim $\frac{\partial u}{\partial \vec{s}} = 0$ (yoki $\frac{\partial z}{\partial \vec{s}} = 0$) bo‘ladi (15.2 rasmga qaralsin).



15.3 rasm

3) $\overrightarrow{gradu}(M)$ $(yoki\ \overrightarrow{gradz}(M))$ vektori, $u\ (yoki\ Z)$ funksiyaning sirt sathi (yoki chizig’ sathi) dagi M nuqtada yo‘nalish normaliga ega (15.3 a, b rasmlariga qaralsin)

Har qanday differensiallanuvchi funksiyaning gradienti quyidagi xossalarga ega:

1) $\overrightarrow{grad}(u_1 + u_2) = \overrightarrow{grad}u_1 + \overrightarrow{grad}u_2;$

2) $\overrightarrow{grad}(Cu) = C\overrightarrow{grad}u, \quad C = const;$

3) $\overrightarrow{grad}(u_1 * u_2) = u_2\overrightarrow{grad}u_1 + u_1\overrightarrow{grad}u_2;$

3- misol. $u = \sqrt{x^2 + y^2 + z^2}$ funksiyaning $M_1(-2; 3; 6)$ nuqtadagi $M_2(-1; 1; 4)$ nuqtaga tomon yo‘nalish bo‘yicha hosilasi topilsin.

►Funksiyaning M_1 nuqtadagi xususiy hosilalari:

$$\frac{\partial u(M_1)}{\partial x} = \frac{x}{\sqrt{x^2 + y^2 + z^2}}\Bigg|_{M_1} = -\frac{2}{7}$$

$$\frac{\partial u(M_1)}{\partial y} = \frac{y}{\sqrt{x^2 + y^2 + z^2}}\Bigg|_{M_1} = \frac{3}{7}$$

$$\frac{\partial u(M_1)}{\partial z} = \frac{z}{\sqrt{x^2 + y^2 + z^2}}\Bigg|_{M_1} = \frac{6}{7}$$

$\overrightarrow{M_1M_2}$ vektor yo‘nalishi bo‘yicha yo‘nalgan birlik $\vec{S}^0$ vektor quyidagichadir:

$\vec{S}^0 = \frac{\overrightarrow{M_1M_2}}{|\overrightarrow{M_1M_2}|} = \left(\frac{1}{3}, -\frac{2}{3}, -\frac{2}{3}\right)$, U holda, (15.6) formulaga binoan

$\frac{\partial u(M_1)}{\partial \vec{S}} = -\frac{2}{7} \cdot \frac{1}{3} + \frac{3}{7} \cdot \left(-\frac{2}{3}\right) + \frac{6}{7} \cdot \left(-\frac{2}{3}\right) = -\frac{20}{21}$.◄

4- misol. $z = arctag(xy)$ funksiyaning $y = x^2$ parabolada yotuvchi $M_0(1; 1)$ nuqtada parabola yo‘nalishi bo‘yicha (absissaning o‘sish tomoni yo‘nalishi) hosilasi hisoblansin.

► $y = x^2$ parabolaning $M_0(1,1)$ nuqtadagi yo‘nalishi $\vec{s}^0$ uchun shu nuqtadagi urinmaning Ox o‘qi bilan α burchak tashkil etadigan yo‘nalishni olamiz. U holda: $y' = 2x$ $tg\alpha = y'(1) = 2$ va

$$cos\alpha = \frac{1}{\sqrt{1 + tg^2\alpha}} = \frac{1}{\sqrt{5}}, \quad sin\alpha = \frac{tg\alpha}{\sqrt{1 + tg^2\alpha}} = \frac{2}{\sqrt{5}}.$$

Funksiyaning M_0 nuqtadagi xususiy hosilalarini topamiz:

$$\frac{\partial z(M_0)}{\partial x} = \frac{y}{1 + x^2y^2}\Bigg|_{M_0} = \frac{1}{2} \frac{\partial z(M_0)}{\partial y} = \frac{x}{1 + x^2y^2}\Bigg|_{M_0} = \frac{1}{2}$$

Yuqorida hosil qilingan qiymatlarni keltirib (15.7) formulaga qo‘yib quyidagini hosil qilamiz:

$$\frac{\partial u(M_0)}{\partial \vec{S}} = \frac{1}{2} \cdot \frac{1}{\sqrt{5}} + \frac{1}{2} \cdot \frac{2}{\sqrt{5}} = \frac{3}{2\sqrt{5}}.◄$$

15.1 AT

1. $\vec{r} = 4(t^2 + t)\vec{\imath} + arctgt\vec{\jmath} + ln(t^2 + 1)\overrightarrow{k}$ vektor funksiya hosilasining $t = 1$ dagi qiymati topilsin. (Javob: $\vec{r}'(1) = 12\vec{\imath} + \frac{1}{2}\vec{\jmath} + \vec{k}$)

2. Agar $\vec{r} = \vec{r}(t) = (2t^2 + 3)\vec{\imath} + 3t^2\vec{\jmath} + (4t^2 - 5)\vec{k}$ biror M nuqta harakatining vektori parametrik tenglamasi bo‘lsa, nuqta harakatining *t=0,5* vaqt momentidagi tezligi $|\vec{v}|$ va tezlanishi $|\vec{w}|$ lar hisoblansin.

(Javob: $|\vec{v}| = \sqrt{29}, |\vec{w}| = 2\sqrt{29}$)

3. Moddiy nuqta harakatining tenglamasi $\vec{r} = 2cost\vec{\imath} + 2sint\vec{\jmath} + 3t\vec{k}$ berilgan. U nuqta harakatining ixtiyoriy t momentidagi harakat trayektoriyasi tezligi $|\vec{v}|$ va tezlanishi $|\vec{w}|$ lar aniqlansin. (Javob:$x = 2cost, y = 2sint, z = 3t, |\vec{v}| = \sqrt{13},\ |\vec{w}| = 2$)

4. Tenglamasi $\vec{r} = t\vec{\imath} + t^2\vec{\jmath} + t^3\vec{k}$ bo‘lgan egri chiziqning t=3 nuqtadagi urinmasining kanonik tenglamasi hamda normal tekislik tenglamasi yozilsin.

$$\text{(Javob:}\frac{x-1}{1} = \frac{y-9}{6} = \frac{z-27}{27},\ \ x + 6y + 27z = 768)$$

5. $z = x^2 + y^2$ va $y = x$ tenglamalar bilan berilgan egri chiziqning $M_0(1,1,2)$ nuqtasidagi urinmasining kanonik tenglamasi hamda normal tekislik tenglamasi yozilsin.

$$\text{(Javob: }\frac{x-1}{1} = \frac{y-1}{1} = \frac{z-2}{4},\ \ x + y + 4z = 10)$$

6. Agar $|\vec{r}| = const$ bo‘lsa, $\vec{r}$ vektorning $\vec{r}'$ vektorga perpendikulyar ekanligi isbotlansin.

7. $u = \ln(3 - x^2) + xy^2z$ funksiyaning $M_1(1,3,2)$ nuqtadagi $M_2(0,5,0)$ nuqta tomon yo‘nalishi bo‘yicha hosilasi hisoblansin. (Javob: $-\frac{11}{3}$)

8. $z = \sqrt{x^2 + y^2}$ funksiyaning$M_0(3,4)$ nuqtadagi quyidagi yo‘nalishlar bo‘yicha hosilalari hisoblansin.

a) $\vec{r} = (1; 1)$ vektor yo‘nalishi; b) M_0 nuqtaning radius vektor yo‘nalishi

c) $\vec{s} = (4; 3)$ vektor yo‘nalishi. (Javob: a)$\frac{7\sqrt{2}}{2}$ b) 1 c) 0)

9. $z = arctg\frac{y}{z}$ funksiyaning $x^2 + y^2 = 4x$ aylananing $M_0(2, -2)$ nuqtasidagi shu aylana yoyi bo‘ylab yo‘nalishidagi hosilasi hisoblansin. (Javob: $\pm\frac{1}{4}$)

10. $u = ln(xy + xz + yz)$ funksiyaning $M_0(0,1,1)$ nuqtadagi $x = cost$, $y = sint, z = 1$ aylana bo‘ylab yo‘nalishidagi hosilasi hisoblansin. (Javob: ± 2)

11. $(z^2 - x^2)xyz - y^5 = 5$ sirtining narmali bo‘ylab yo‘nalgan birlik vektorning $M_0(1,1,2)$ nuqtasidagi koordinatalari hisoblansin. (Javob; $\pm(\frac{2}{3\sqrt{14}}, \frac{1}{3\sqrt{14}}, \frac{11}{3\sqrt{14}})$)

12. Agar $u = x^2yz - xy^2z + xyz^2$ berilgan bo‘lsa, $M_0(1,1,1)$ nuqtadagi $\overrightarrow{gradu}$ topilsin. (Javob: $\overrightarrow{gradu} = 2\vec{i} - 2\vec{j} + 2\vec{k}$)

13. $u = \frac{3}{2}x^2 + 3y^2 - 2z^2$ bilan $v = x^2yz$ funksiyalarning $M_0\left(1, \frac{1}{3}, \frac{\sqrt{3}}{2}\right)$ nuqtadagi gradientlari orasidagi φ burchak topilsin. (Javob: $\varphi = \frac{\pi}{2}$)

14. $z = \frac{2x^2}{y^3}$ sirtning $(2,1,8)$ nuqtadagi φ ko‘tarilishdagi eng katta egriligi aniqlansin. ($Javob$: $tg\varphi = 8\sqrt{10}$, $\varphi = 87°\, 40'$)

Mustaqil ish

1. 1. $u = x + \ln(y^2 + z^2)$ funksiyaning $\vec{s} = -2\vec{i} + \vec{j} - \vec{k}$ vektor yo‘nalishi bo‘yicha $M_0(2; 1; 1)$ nuqtadagi hosilasi hisoblansin. (Javob: $-\frac{\sqrt{6}}{3}$)

2. $xy + xz + yz = 3$ sirtga perpendikulyar bo‘lgan birlik vektorning $M_0(1; 1; 1)$ nuqtadagi koordinatalari hisoblansin. (Javob: $\pm\left(\frac{1}{\sqrt{3}}; \frac{1}{\sqrt{3}}; \frac{1}{\sqrt{3}}\right)$)

2. 1. $z = artg(x^2y)$ funksiyaning $y = x^2$ parabola bo‘ylab uning $M_0(1; 4)$ nuqtasidagi hosilasi hisoblansin. (Javob: $\pm\frac{2\sqrt{5}}{17}$)

2. $z = 5x^2 - 2xy + y^2$ sirtning $M_0(1; 1; 4)$ nuqtadagi ko‘tarilishdagi eng katta egriligi aniqlansin(Javob: $tg\varphi = 8, \varphi \approx 83^0$)

3. 1. Vektor-parametrik tenglamasi $\vec{r} = cos^2 t\vec{i} + sin^2 t\vec{j} - tgt\vec{k}$ bo‘lgan chiziqning $t = \frac{\pi}{4}$ nuqtasidagi urinmaning kanonik tenglamasi bilan normal tekislik tenglamasi yozilsin.(Javob: $\frac{x-0,5}{-1} = \frac{y-0,5}{1} = \frac{z-1}{2}$, $x - y - 2z + 2 = 0$)

2. $z = x^3 y + xy^2$ sirtning $M_0(1;3;12)$ nuqtadagi ko‘tarilishdagi eng katta egriligi aniqlansin(Javob: $tg\varphi = \sqrt{373}, \varphi \approx 87^0$)

15.2. Skalyar va vektor maydonlar

Agar R^3 fazoning (yoki uning biror V qismi) har bir *M(x;y;z)* nuqtasida *u=f(x;y;z)* skalyar miqdor aniqlangan bo‘lsa, u holda R^3 (yoki V) da *u=u(M) skalyar maydon* berilgan deb ataladi. Ya’ni R^3 fazoning biror V sohasida berilgan har qanday sonli funksiya shu sohadagi skalyar maydonni aniqlaydi.

Xususan 2 argumentli z=f(x,y) funksiya Oxy tekislikning biror D sohasidagi tekis skalyar maydonni aniqlaydi. Skalyar maydonni grafik jihatdan f(x,y,z)=C bilan ifodalanadigan sirt sathi yoki f(x;y)=C chiziq sathi orqali talqin etish mumkin.(15.3-rasmga qaralsin).

$M_0(x_0, y_0, z_0)$ nuqtada differensiallanuvchi bo‘lgan har qanday *u=f(x;y;z)* funksiya uchun $\frac{\partial u(M_0)}{\partial \vec{S}}$ soni , skalyar maydonning $\vec{S}^0 = (cos\alpha; cos\beta; cos\gamma)$ vektor yo‘nalishi bo‘yicha o‘zgarish tezligini aniqlaydi.(15.6-formulaga qaralsin).

Agarda R^3 fazo (yoki uning qismi V)ning har bir M(x;y;z) nuqtasida kordinatalari skalyar funksiyalar *P=P(x;y;z), Q=Q(x;y;z)* va *R=R(x;y;z)* dan iborat bo‘lgan biror $\vec{a}$=(*P;Q;R*) vektor aniqlangan bo‘lsa, u holda, bu fazoda (yoki V da) $\vec{a} = \vec{a}(M)$ vektor maydoni berilgan deb yuritiladi. Agar *P=P(x;y;z), Q=Q(x;y;z)* va *R=R(x;y;z)* lar uzluksiz funksiyalar bo‘lsa,$\vec{a}$ vektor maydon uzluksiz deyiladi. Suyuqlik oqimining tezliklar maydoni, qattiq jism nuqtalarining tezliklar maydoni, elektr yoki magnit kuchlanishlari va boshqalar vektor maydonlarga misol bo‘la oladi.

Agar $\vec{a} = \vec{a}(M)$ vektor maydonning $\vec{a}(M)$ vektori, biror chiziqning har bir M nuqtasiga o‘tkazilgan urinma bo‘ylab yo‘naladigan bo‘lsa, u chiziqni vektor maydoning vektor (kuch) chizig’i deb ataladi.Vektor chiziqlarga, suyuqlikning oqish chizig’ini, magnit maydonning kuch chiziqlarini misol qilib keltirish mumkin.

Fazoning faqat vektor chiziqlardan tashkil topgan sohasini vektor trubka deb ataladi. Vektor trubka sirtidagi har bir M nuqta $\vec{a}$vektor, shu trubkaning M nuqtasiga o‘tkazilgan urinma tekislikda yotadi. Koordinatalari vaqtga bog’liq bo‘lmaydigan vektor (yoki skalyar) maydonning statsionar yoki turg’un maydon deb yuritiladi. Agar $\vec{r}(t)$, $\vec{a} = \vec{a}(M)$ vektor maydon vektor chizig’ining radius-vektori bo‘ladigan bo‘lsa,vektor chiziqlarning tenglamalari quyidagi differensial tenglamalar sistemasidan aniqlanadi:

$$\frac{dz}{P} = \frac{dy}{Q} = \frac{dz}{R} \qquad (15.9)$$

1- misol.$\overline{a}(M) = -y\vec{\imath} + x\vec{\jmath} + b\vec{k}$ vektor maydonning $M_0(1,0,0)$ nuqtadan o‘tuvchi vektor chizig’i aniqlansin.

►Yuqorida keltirilgan (15.9) formulaga binoan, quyidagi differensial tenglamalar sistemasini hosil qilamiz:

$$\frac{dx}{-y} = \frac{dy}{x} = \frac{dz}{b}.$$

Uni yechamiz: $\frac{dz}{-y} = \frac{dy}{x}$, $xdx + ydy = 0$,$x^2+y^2 = C_1^2$,yoki parametrik ko‘rinishda, $x = C_1 cost, y = C_1 sint$; $\frac{dy}{x} = \frac{dz}{b}, \frac{dz}{b} = \frac{C_1 costdt}{C_1 sint}$, $dz = bdt, z = bt + C_2$.

Vektor chiziq $M_0(1; 0; 0)$ nuqtadan o‘tishi lozimligidan foydalanib,$C_1 = 1$ va $C_2 = 0$ larni aniqlaymiz. Natijada berilgan vektor maydonning vektor chizig’ning tenglamasi *x=cost,y=sint,z=bt* (vint chizig’i) kabi bo‘ladi. ◄

Skalyar maydon *u(M)=f(x;y;z)* (yoki *z(M)=f(x;y)*) ning gradient orqali hosil bo‘lgan vektor maydonni gradient maydon deyiladi. Gradientning 3-xossasiga binoan, vektor chiziqlar $\overrightarrow{grad}\ U(M)$(yoki $\overrightarrow{grad}\ Z(M)$) shunday egri chiziqlarki, ular bo‘ylab *u=f(x;y;z)* (*z=f(x;y*)) funksiya maksimal darajada o‘sadi(kamayadi). Bu chiziqlar *u(M)* (yoki *z(M))* skalyar maydonning sirt sathi (yoki sirt chizig’i) ga har doim $\overrightarrow{grad}\,u(M)$ vektor chiziqlarni aniqlaydigan differensial tenglamalar quyidagilardir:

$$\frac{dx}{u_x'} = \frac{dy}{u_y'} = \frac{dz}{u_z'} \qquad (15.10.)$$

2-misol. Agar $u = \frac{x^2+y^2+z^2}{2}$ bo‘lsa $\overrightarrow{grad u}$ maydonning vektor chiziqlari topilsin.

►Gradientning formulasi bo‘lgan (15.8) ga binoan, $\overrightarrow{grad U} = y\vec{\imath} + x\vec{\jmath} + z\vec{k}$ bolganligi uchun,(15.10) formulaga ko‘ra qralayotgan maydonning vektor chiziqlarini quyidagi

$$\frac{dx}{x} = \frac{dy}{y} = \frac{dz}{z}$$

kabi differensial tenglamar sistemasidan aniqlaymiz.

$$\frac{dx}{x} = \frac{dy}{y}, \ln|y| = \ln|x| + \ln C_1, y = C_1 x,$$

$$\frac{dz}{z} = \frac{dx}{x}, \ln|z| = \ln|x| + \ln C_2, z = C_2 x.$$

Hosil qilingan $y = C_1 x$ va $z = C_2 x$ yechimlarni

$$\frac{x}{1} = \frac{y}{C_1} = \frac{z}{C_2}$$

ko‘rinishda ifodalash mumkin, ya’ni berilgan $\overrightarrow{grad}\,U(M)$ maydonning vektor chiziqlari, koordinata boshidan o‘tuvchi hamda $x^2 + y^2 + z^2 = 2C$ sirtlar sathi (sfera) turlariga ortogonal bo‘lgan to‘g’ri chiziqlar oilasidan iboratdir.◄

15.2-AT.

1. Quyidagi berilgan funksiyalar bilan aniqlangan skalyar maydonlarning sath sirtlari tenglamalari yozilib hamda ular tafsiflansin:

$a)\ u = arccos\frac{z}{\sqrt{x^2+y^2}}$; $\quad b)\ u = \ln(x^2 + y^2 + z^2)$; $\quad c) u = \frac{z}{x^2+y^2}$.

2. Tekislikdagi $z = xy$ skalyar maydonning sath chiziqlari tasvirlansin.

3. Agar $\vec{c}$ o‘zgarmas vektor bolib,$\vec{r} - M(x;y;z)$ nuqtaning radius-vektori bo‘lsa, $u = \vec{c} \cdot \vec{r}$ skalyar maydon gradienti topilsin va ushbu maydon sath sirtlari tenglamalari yozilib, ularning $\vec{c}$ vektorga nisbatan qanday joylashganligi tushuntirilsin.

4. Agar $2x^2 + 12x + 5y^2 + z^2 - 3z - 58 = 0$ normal Oz o‘q bilan o‘tkir burchak tashkil etadigan bo‘lsa, $u = x^2 + y^2 - \sqrt{x^2 + z^2}$ skalyar maydonning M(-3;0;4) nuqtadagi shu normal yo‘nalish bo‘yicha hosilasi topilsin.(Javob: $\frac{-4}{5}$)

5.Agar $\omega \epsilon R, \omega \neq 0$ bo'lsa, $\vec{a}(M) = \omega y\,\vec{i} + \omega x\vec{j}$ vektor maydonning vektor chiziqlari tenglamalari topilsin.(Javob: $x^2 - y^2 = C_1 \; va \; z = C_2$).

6.Quyidagi berilgan vektor maydonlarning vektor chiziqlari topilsin:

$a)\vec{a}(M) = 5x\,\vec{i} + 10y\vec{j}$; $b)\vec{a}(M) = 4z\,\vec{j} - 9y\vec{k}$

(Javob: $a)\ x^2 = C_1 y \; va \; z = C_2;\ b)\ 9y^2 + 4z^2 = C_1^2, x = C_2$).

7. Agar $u = x^2 - 2y + 5z^2$bo‘lsa, $\overrightarrow{grad u}$ maydonning vektor chiziqlari aniqlansin.(Javob: $x = C_1 e^{-y} \; va \; z = C_2 e^{-y}$)

Mustaqil ish

1. 1. $\vec{a}(M) = (x + y)\vec{i} - 10x\vec{j} - x\vec{k}$ vektor maydonning vektor chiziqlari topilsin .($Javob: x^2 + y^2 + z^2 = C_2^2, y - z = C_1$).

2. $z = x^2 + y^2 \; sirtga \; M_0(-1;1;2)$ nuqtada perpendikulyar bo‘lib, Oy o‘q bilan o‘tkir burchak hosil qiladigan birlik vektorning koordinatalari topilsin.

$\left(Javob: \left(\frac{-2}{3}; \frac{2}{3}; \frac{-1}{3}\right)\right)$.

2. 1. $Agar\, u = x + y^2$ bo‘lsa $\overrightarrow{grad u}$ donning vektor chiziqlari aniqlansin. ($Javob: x = \frac{1}{2}\ln y + C_1, z = C_2$).

2. u=2x-3y+6z-5 skalyar maydonning sath sirtlariga perpendikulyar bo‘lib, Oz o‘q bilan o‘tmas burchak tashkil etadigan birlik $\vec{n}^0$ vektorning koordinatalari anilansin. $\left(Javob: \vec{n}^0 = \left(\frac{-2}{3}; \frac{3}{7}; \frac{-6}{7}\right)\right)$.

3. 1. $\vec{a}(M) = 2x\vec{i} + 8z\vec{k}$ vektor maydonning vektor chiziqlari aniqlansin. ($Javob: z = C_1\, x^4, y = C_2$).

$2.\, u = x^2 + y^2 + z^2 + 4$ skalyar maydonning sath sirtlariga ortoganal bo‘lgan $\vec{n}^0$ birlik vektor yozilsin. $\left(Javob: \vec{n}^0 = \left(\frac{x}{\sqrt{x^2+y^2+z^2}}; \frac{y}{\sqrt{x^2+y^2+z^2}}; \frac{z}{\sqrt{x^2+y^2+z^2}}\right)\right)$.

15.3. Sirt integrallari

Analitik *f(x,y,z)* funksiya biror silliq $S \epsilon R^3$ sirtning nuqtalarida uzluksiz bo‘lsin. S sirtni, bo‘lak- bo‘lak silliq chiziqlar orqali *n* ta S_i bo‘laklarga ajratamiz hamda ularning yuzalarini $\Delta S_i (i = \overline{1,n})$ va diametrlarini $\emptyset S_i$ deb belgilaymiz. Har bir S_i bo‘lakda ixtiyoriy $M_i(x_i, y_i, z_i)$ nuqta tanlab, $f(x_i, y_i, z_i)$ni hisoblaymiz hamda quyidagi integral yig’indini tuzamiz:

$$I_n = \sum_{i=1}^{n} f\ (x_i, y_i, z_i) \cdot \Delta S_i$$

Agar ushbu integral yig’indining $\Delta S_i \to 0$ da biror limiti mavjud bo‘ladigan bo‘lsa, u limitni *f(x;y;z)* funksiyaning S sirt bo‘yicha birinchi tur sirt integrali deb ataladi va quyidagicha belgilanadi:

$$\iint_S f(x,y,z)ds = \lim_{\emptyset S_i \to 0} \sum_{i=1}^{n} f(x_i, y_i, z_i) \cdot \Delta S_i \qquad (15.11)$$

Birinchi turdagi sirt integrallari o‘z navbatida chiziqlilik, additivlik xossalariga ega, hamda ular uchun o‘rta qiymat haqidagi teorema o‘rinli bo‘lib, ularning qiymati sirt tomoninig tanlanishiga bo‘g’liq bo‘lmaydi. Shuningdek, agar $\delta(x,y,z)$, S sirtning zichligi bolsa, $\iint_S \ \delta(x,y,z)ds$ qiymati S sirtning massasisini ifodalab, $\iint_S \ ds$ esa sirtning yuzasiga teng bo‘ladi.

Agar S sirtning *Oxy* tekislikdagi proyeksiyasi bo‘lgan *D*, bir qiymatli bo‘lsa, ya’ni *Oz* o‘qqa parallel bo‘lgan har qanday to‘g’richiziq *S* sirtni faqatgina bitta nuqtada kesib o‘tadigan bo‘lsa, u holda S sirtni z=F(x,y) tenglama bilan berish mumkin bo‘ladi, hamda birinchi sirt integralni ikki o‘lchovli integralga keltirib yyechimini ifodalaydigan quyidagi tenglik o‘rinli bo‘ladi:

$$\iint_S f(x,y,z)ds = \iint_D f(x,y,F(x,y)\sqrt{1+(F_x')^2+(F_y')^2}\,dxdy. \quad (15.12)$$

1- misol. Agar *S*, *z=0* bilan *z=2* tekisliklar orasida joylashgan $x^2+y^2=z^2$ konus sirtning qismi bo‘ladigan bo‘lsa $\iint_S \sqrt{x^2+y^2}\,ds$ hisoblansin.

►Sirtning tenglamasidan ko‘rinmoqdaki, uning qaralayotgan qismi $z=\sqrt{x^2+y^2}$ ning *Oxy* tekislikdagi proyeksiyasi $\sqrt{x^2+y^2}\le 4$ doiradan iboratdir.

$$F_x' = \frac{x}{\sqrt{x^2+y^2}}, F_y' = \frac{y}{\sqrt{x^2+y^2}}$$

kabi bo‘lganliklari uchun,(15.12) formulaga binoan

$$\iint_S \sqrt{x^2+y^2}\,ds = \iint_S \sqrt{x^2+y^2}\sqrt{1+\frac{x^2+y^2}{x^2+y^2}}dxdy = \sqrt{2}\iint_D \sqrt{x^2+y^2}\,dxdy =$$

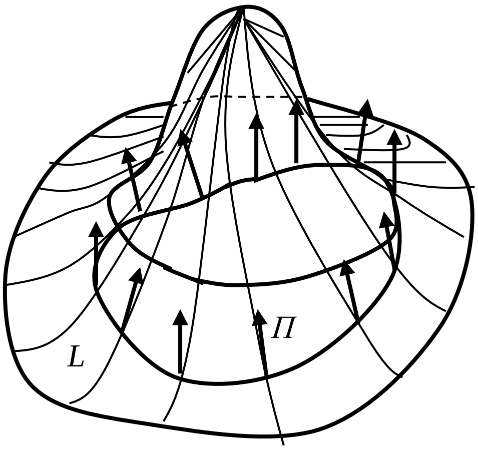
$$= \begin{vmatrix} x=\rho cos\varphi \\ y=\rho sin\varphi \end{vmatrix} = \sqrt{2}\iint_D \rho^2 d\rho d\varphi = \sqrt{2}\int_0^{2\pi} d\varphi \int_0^2 \rho^2 d\rho = \sqrt{2}\cdot 2\pi\cdot\frac{8}{3} = \frac{16\sqrt{2}}{3}\pi$$

ni hosil qilamiz.◄

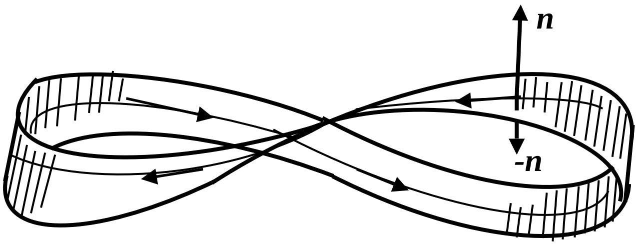
Silliq S sirtning biror tomoni tanlanib, uning har bir nuqtasidan $\vec{n}$ normal vektor chiqariladigan bo‘lsa, u tomonni musbat tomon deb ataladi, sirtning boshqa tomonini esa (agar u mavjud bo‘lsa), manfiy tomon deyiladi. Xususan agar S yopiq sirt bo‘lib fazodagi biror *V* sohani chegaralaydigan bo‘lsa, u holda sirtning musbat yoki tashqi tomoni deb, uning norml vektorlari *V* sohadan chiqaradigan tomoniga aytiladi va agar normal vektorlari *V* sohaga tomon yo‘naladigan bo‘lsa u tomonni manfiy yoki ichki tomon deb yuritiladi. Ham musbat(tashqi) ham manfiy (ichki) tomonlari mavjud bo‘lgan sirtlarni ikki tomonli sirtlar deb ataladi.

Ikki tomonli sirtlar quyidagi xususiyatga ega, ya’ni: Ikki tomonli sirtning normal vektori $\vec{n}$ ning asosini shu sirtda yotuvchi ixtiyoriy yopiq L kontur bo‘yicha uzluksiz ko‘chiriladigan bo‘lsa, u nuqta aylanib yana avvalgi holatiga qaytganda $\vec{n}$ vektorning yo‘nalishi dastlabki yo‘nalish bilan ustma-ust tushadi

(15.4-rasm).Tekisliklar, barcha ikkinchi tartibli sirtlar, tor va boshqalar ikki tomonli sirtlarga misol bo‘ladi.



15.4- rasm



15.5- rasm

Bir tomonli sirtlar esa, $\vec{n}$ normal yuqoridagicha ko‘rinib, dastlabki nuqtaga qaytganda “antinormal”, ya’ni, $-\vec{n}$ ga o‘zgaradi. Bir tomonli sirtga Myobius varag’i misol bo‘ladi.(15.5-rasm)

Aniq tomoni tanlangan sirtni yo‘naltirilgan sirt deb ataladi. Agar S sirt *z=f(x,y)* tenglama bilan berilgan bo‘lsa, *Oz* o‘q bilan γ o‘tkir burchak tashkil etadigan $\vec{n}$ normal vektor, $\vec{n} = (f_x', f_y', 1)$ dek aniqlanadi, normalning birlik $\vec{n}^0$ vektorining kordinatalari esa, uning yo‘naltiruvchi kosinuslariga teng bo‘ladi,ya’ni:

$$\vec{n}^0 = \left(-\frac{f_x'}{|\vec{n}|}, -\frac{f_y'}{|\vec{n}|}, \frac{1}{|\vec{n}|}\right) = (cos\alpha, cos\beta, cos\gamma)$$

$$|\vec{n}| = \sqrt{1 + f_x'^2 + f_y'^2}$$

Agar *S* sirt *F*(*x*,*y*,*z*) =0 tenglama bilan berilgan bo‘lsa, $\vec{n}^0 = \frac{g\vec{ra}dF}{|g\vec{ra}dF|}$. Agar bu yerda, γ burchak o‘tkir bo‘lsa, ”+” ishora agar o‘tmas bo‘lsa, ”−“ ishora olinadi.

Faraz qilaylik, $V \epsilon R^3$ sohada, koordinatalari $P = P(x, y, z), Q = Q(x, y, z), R = R(x, y, z)$ funksiyalardan iborat bo‘lgan $\vec{a} = P\vec{i} + Q\vec{j} + R\vec{k}$ vektor funksiya aniqlangan bo‘lsin. Shuningdek, musbat tomoni tanlangan (ya’ni, $\vec{n}^0$ birlik vektor yo‘nalishi tanlangan), *V* sohada yotuvchi S silliq sirt ham qaralayotgan bo‘lsin. S srtni, shu sirtda yotuvchi bo‘lak-bo‘lak silliq chiziqlar yordamida yuzalari $\Delta S_i(i=\overline{1,n})$ lardan tashkil topgan S_i ajratamiz va ular ichida yotadigan $M_i(x_i,y_i,z_i)$ nuqtalarni ixtiyoriy tanlaymiz. U holda quyidagicha limit mavjud bo‘ladiki, ya’ni

$$\lim_{\Delta S_i\to 0}\sum_{i=1}^{n}\vec{a}(x_i,y_i,z_i)\cdot\ \vec{n}^0(x_i,y_i,z_i)\cdot\Delta S_i \qquad (15.13).$$

Uni $\vec{a}$ funksiyaning S sirt bo‘yicha 2-tur sirt integrali deb ataladi va

$$\iint_S \vec{a}\ \cdot\vec{n}^0 dS$$

ko‘rinishda belgilanadi. Demak, ta’rifga binoan:

$$\iint_S \vec{a}\ \cdot\vec{n}^0 dS=\iint_S (Pcos\alpha+Qcos\beta+Rcos\gamma)dS \qquad (15.14)$$

2-tur sirt integrali ham chizqlilik va additivlik xossalariga ega. Agar sirtning tomoni teskariga o‘zgarsa, ya’ni $\vec{n}^0$ ni $-\vec{n}^0$ ga almashtirilsa (15.14) integralning ishorasi ham o‘zgaradi .

Agar $cos\alpha ds=dydz, cos\beta ds=dzdx, cos\gamma ds=dydx$ ekanligini inobatga olsak. (15.14) integralni quyidagicha yozish mumkin:

$$\iint_S \vec{a}\ \cdot\vec{n}^0 dS=\iint_D Pdydz+Qdxdz+Rdydx \qquad (15.15)$$

Yuqorida keltirilgan (15.14) integralni ikki o‘lchovli integralga keltirib integrallanadi, ya’ni,

$$\iint_S \vec{a}\ \cdot\vec{n}^0 dS=\iint_{D_z} \vec{a}(x,y;,z)\cdot\vec{n}(x,y,z)dxdy \qquad (15\ .16)$$

Bu yerda D_z, S sirtning Oxy tekislikdagi proyeksiyasi bo‘lib, S sirt $z=f_3(x,y)$ funksiya orqali beriladi hamda, $\vec{n}=\pm g\overrightarrow{rad}(z-f_3(x,y))$ dir. Shuningdek, ikki o‘lchovli integralda z ni $f_3(x,y)$ bilan almashtiriladi.

Ikkinchi turdagi sirt integralni hisoblach uchun quyidagi ikkita formulani ham keltiramiz:

$$\iint_S \vec{a} \cdot \vec{n}^0 dS = \iint_{D_x} \vec{a}(x,y,z) \cdot \vec{n}(x,y,z) dydz =$$

$$= \iint_{D_y} \vec{a}(x,y,z) \cdot \vec{n}(x,y,z) dxdz \qquad (15.17)$$

Bu yerda, $D_x \, va \, D_y$ lar, S sirtning mos ravishda *Ozy* va *Oxz* tekisliklardagi proyeksiyalaridir; S sirt esa $x = f_1(y,z)$ va $y = f_2(x,z)$ funksiyalar orqali beriladi. $D_x$soha bo‘yicha ikki o‘lchovli integraldagi integral ostidagi ifodada *x* ni $f_1(y,z)$ bilan almashtirilib, $\vec{n} = \pm g\overrightarrow{rad}(x - f_1(y,z))$ deb olinadi; D_y soha bo‘yicha ikki o‘lchovli integral esa mos ravishda $y = f_2(x,z)$ va $\vec{n} = \pm g\overrightarrow{rad}(y - f_2(x,z))$ lar olinadi. Ta’kidlash lozimki, $\vec{n}$ vektor uchun "+" yoki " $-$ " ishorani tanlashda sirtdagi tanlangan orientatsiya hisobga olinadi (sirtning tomoni). (15.14) va (15.15) formularning o‘ng tomnlaridagi integrallarni uchta integrallarining yig’indisi deb qaraladiki, ularning har birini hisoblash uchun (15.16) yoki (15.17) formulalarning bittasini qo‘llash mumkin bo‘ladi.

2-misol. $I = \iint_S zdydz - 4ydxdz + 8x^2dydx$ ni hisoblansin, bu yerda, S, $z = x^2 + y^2 + 1$ sirtning *z=2* tekislik bilan kesishgan qismi bo‘lib, S sirtning normal $\vec{n}$ vektori Oz o‘q bilan γ o‘tmas burchak tashkil etadi.

► Gradient orqali berilgan sirtning tanlangan tomoniga o‘tkazilgan normal vektorni aniqlaymiz: $\vec{n} = (2x, 2y, -1)$, chunki $cos\gamma < 0$.
Masala shartiga ko‘ra, $\vec{a} = (z, -4y, 8\,x^2)$ bo‘lganligidan (15.15) va (15.16) formulalardan (15.6-rasm).

$$\iint_{D_z} \vec{a} \cdot \vec{n}\, dxdy = \iint_{D_z} (2xz - 8y^2 - 8x^2) dxdy =$$

$$= \iint_{D_z} (2x(y^2 + x^2 + 1) - 8(y^2 + x^2)\, dxdy =$$

$$= \begin{vmatrix} x = \rho cos\varphi, 0 \le \varphi \le 2\pi \\ y = \rho sin\varphi, 0 \le \rho \le 1 \\ dxdy = \rho d\rho d\varphi \end{vmatrix} = \iint\limits_{D_z} (2\rho cos\varphi(\rho^2+1) - 8\rho^2)\, \rho d\rho d\varphi =$$

$$= \int_0^1 \rho d\rho \int_0^{2\pi} (2\rho cos\varphi(\rho^2+1) - 8\rho^2) d\varphi = \int_0^1 16\pi\rho^3 d\rho = -4\pi$$

larni hosil qilamiz.◄

3- misol. Agar S- $x^2+y^2+z^2=1$ sferaning birinchi oktantda joylashlashgan qismining tashqi tomonidan iborat bo‘lsa,

$$I = \iint\limits_S (xdydz + dxdz + xz^2)dydx \text{ ni hisoblansin.}$$

►Agar S isrtning Oyz, Oxz va Oxy koordinata tekisliklaridagi proyeksiyalarini mos ravishda $D_x, D_y\ va\ D_z$ lar bilan belgilab berilgan I integralni

$$I_1 = \iint\limits_S xdydx, I_2 = \iint\limits_S dxdz, I_3 = \iint\limits_S xz^2dydx,$$

integrallarning yig’indisi deb qaraladigan bo‘lsa, ularning har biriga (15.16) va (15.17) formulalarning birini qo‘llab quyidagilarni hosil qilamiz:

$$I_1 = \iint\limits_{D_x} \sqrt{1-y^2-z^2}\, dydz, I_2 = \iint\limits_{D_y} dxdz\,, I_3 = \iint\limits_{D_z} x(1-x^2-y^2)dxdy,$$

bu yerda, I_1 uchun P=x, Q=R=0, I_2 uchun Q=1, P=R=0 va I_3 uchun P= Q=0, R= xz^2. Shuningdek, $D_x, D_y\ va\ D_z$ lar, mos ravishda koordinata tekisliklarida joylashgan birlik doiralarning chorak qismlaridir. Shuning uchun $I_2 = S_{D_y} = \frac{\pi}{4}$.

Qolgan I_1 va I_3 integrallarni hisoblash uchun qutb koordinatalariga o‘tamiz:

I_1uchun $y = \rho cos\varphi, z = \rho sin\varphi, dydz = \rho d\rho d\varphi$, I_3 uchun esa $x = \rho cos\varphi, y = \rho sin\varphi, dxydy = \rho d\rho d\varphi$ deb olamiz. Har ikkala holda ham $0 \leq \varphi \leq \frac{\pi}{2}, 0 \leq \rho \leq 1$ bo'ladi. U holda:

$$I_1 = \iint\limits_{D_x} \sqrt{1-\rho^2}\,\rho d\rho d\varphi = -\int\limits_0^{\frac{\pi}{2}} d\varphi \int\limits_0^1 ((1-\rho^2)^{\frac{1}{2}}\frac{1}{2}\,d(1-\rho^2) = -\frac{\pi}{4}\frac{3}{2}(1-\rho^2)^{\frac{3}{2}}\Big|_0^1 =$$

$$= \frac{\pi}{6}$$

$$I_3 = \int\limits_0^{\frac{\pi}{2}} d\varphi \int\limits_0^1 (\rho cos\varphi(1-\rho^2)\quad \rho d\rho = sin\varphi|_0^{\frac{\pi}{2}} \cdot \left(\frac{\rho^3}{3} - \frac{\rho^5}{5}\right)\Big|_0^1 = \frac{2}{15}$$

Natijada: $I = I_1 + I_2 + I_3 = \frac{\pi}{6} + \frac{\pi}{4} + \frac{2}{15} = \frac{5\pi}{12} + \frac{2}{15}$.◀

Agarda *S,V* sohani chegaralaydigan yopiq silliq sirt bo‘ladigan bo‘lib, *P=P(x,y,z), Q=Q(x,y,z)* va *R=(x,y,z)* lar, o‘zlarining birinchi tartibli xususiy hosilalari bilan birgalikda *V* yopiq sohada uzluksiz funksiyalar bo‘lsalar, u holda, Ostrogradiskiy-Gauss formulasi o‘rinlidir:

$$\iint\limits_S (Pdydz + Qdxdz + Rdxdy)dS = \iiint\limits_V (\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z})dxdydz, \qquad (15.18)$$

yoki boshqacha ko‘rinishda :

$$\iint_S (Pcos\alpha + Qcos\beta + Rcos\gamma)dS = \iiint_V (\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z})dxdydz, \qquad (15.19)$$

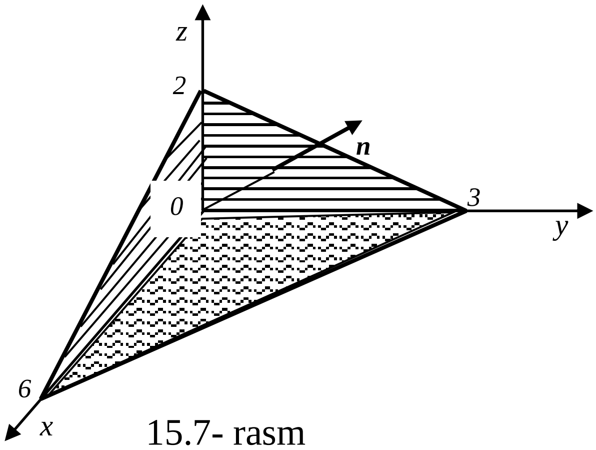
bu yerda, $cos\alpha, cos\beta\ va\ cos\gamma$ lar S sirt tashqi normalining yo‘naltiruvchi kosinuslaridir, Ostrogradiskiy-Gauss formulasi, ko‘plab sirt integrallarining hisoblash ishlarini soddalashtiradi.

4- misol. Agar *S, x=0, y=0, z=0 va x+2y+3z=6* tenglamalar bilan chegaralangan jism sirtining tashqi tomoni bo‘lsa,

$$I = \iint_S (x+y)dydz + (y+z)dxdz + (z+x)dxdy$$

ni hisoblansin.

►(15.18) formulaga binoan,



15.7- rasm

$$\iiint_V (1+1+1)dxdydz = 3\iiint_V dxdydz = 18,$$

Chunki, oxirgi uch o‘lchovli integral tetraedrning hajmiga teng(15.7-rasm)◄

15.3-AT

1. Agar$S$ sirt, $\left(\frac{x^2}{16}+\frac{y^2}{16}=\frac{z^2}{9}\right)$ konusning z=0 va z=3 tekislikning orasida joylashgan qismi bo‘ladigan bo‘lsa, birinchi tur sirt integrali

$$\iint_S \sqrt{x^2+y^2}\,dS$$

ni hisoblansin.(Javob: $\frac{160\pi}{3}$)

2. $\iint_S xyzdS$ kabi birinchi tur sirt integrali hisoblansin. Bu yerda S,

x+y+z=1 tekislikning birinchi oktantda yotuvchi bo‘lagidan iborat. (Javob: $\frac{\sqrt{3}}{120}$).

3. Har bir nuqtasidagi sirt zichligi $\delta = x^2y^2$ bo'lgan $z=\sqrt{4-x^2-y^2}$ yarim sferaning massasi hisoblansin. (Javob: $\frac{128\pi}{15}$)

4. Har bir nuqtasidagi sirt zichligi $\delta = x^2 + y^2$ bo'lgan $z = \sqrt{a^2 - x^2 - y^2}$ yarim sferaning massasi hisoblansin. (Javob: $\frac{4\pi a^4}{3}$)

5. Agar S, x+2y+z-6=0 sirtning birinchi oktantda joylashgan yuqori qismi bo‘ladigan bo‘lsa , quyidagi ikkinchi turdagi sirt integrali hisoblansin

$$\iint_S (xdydz + ydzdy + zdxdy) \quad \text{(Javob: 54).}$$

6. $\iint_S (x+y)dydz + (y-z)dxdz + (z-2)dxdy$ hisoblansin. Bu yerda S, sirt $x^2 + y^2 - z^2 = 0$ ning *z=0* va *z=1* tekisliklar bilan kesilgan bo‘lagi bo‘lib, unga o‘tkazilgan normal *Oz* o‘q bilan o‘tmas burchak tashkil etadi. (Javob: $\frac{8\pi}{3}$)

7. Agar S, $x^2 + y^2 - z^2 = 1$ sferaning tashqi tomoni bo‘lsa,

$$\iint_S xdydz + z^3dxdy \text{ ni hisoblansin. (Javob: } \frac{32\pi}{15}\text{)}$$

8. Agar S, asoslari z=0 va z = H bo‘lgan $x^2 + y^2 = R^2$ silindrning tashqi tomoni bo‘lsa,

$$\iint_S (xdydz + ydzdx + zdxdy) \text{ hisoblansin. (Javob: } 3\pi R^2 H\text{).}$$

9. Agar S, silindrning tashqi tomoni bo‘ladigan bo‘lsa, S sirt bilan chegaralangan jismning hajmi,

$$V = \frac{1}{3}\iint_S (xdydz + ydzdx + zdxdy) \text{ ekanligi isbotlansin.}$$

10. Agar S, $x^2 + y^2 = R^2$ silandr hamda x=0 ,y=0,z=0,z=H tekisliklardan tashkil topib,birinchi oktanda joylashgan sirtning tashqi tomoni bo‘lsa,

$$\iint_S (yzdxdy + xzdydz + xydxdz) \text{ ni hisoblansin. (Javob: } R^2H^2(\frac{2R}{3} + \frac{\pi H}{8})\text{)}$$

11. Agar S, yon yoqlari x=0,y=0,z=0 va x+y+z=1 tekisliklardan iborat bo‘lgan piramidaning tashqi tomoni bo‘lsa,

$$\iint_S (yzdxdy + xzdydz + xydxdz) \text{ ni hisoblansin. (Javob: } \frac{1}{8})$$

Mustaqil ish

1. Agar S, birinchi oktantda joylashgan 6x+3y+2z=6 tekislikning yuqori qismidan iborat bo‘lsa, $\iint_S (y + 2z)dxdy$ hisoblansin. (Javob: $\frac{8}{3}$)

2. Agar S, z = 1 tekislik bilan kesilgan $z = x^2 + y^2$ paraboloid sirtining qismi bo‘lsa, $\iint_S xyzdS$ hisoblansin. (Javob: 0)

3. Agar S , z = 0, $x^2 + y^2 = 1, z = x^2 + y^2 + 2$ sirtlar bilan chegaralangan jism sirtining tashqi tomoni bo‘lsa,

$$\iint_S zdydz + (3y - x)dxdz - zdxdy \text{ hisoblansin. (Javob: } 5\pi)$$

15.4. Vektor maydonning sirt bo‘yicha oqimi. Vektor maydon divergensiyasi

Biror S sirt bo‘yicha ikkinchi tur sirt integrali (15.14) ni, S sirtning birlik normal vektori

$$\vec{n}^0 = (\cos\alpha, \cos\beta, \cos\gamma).$$

bo′yicha $\vec{a}(M), (M(x, y, z)\epsilon S)$ vektor maydonining oqimi deb yuritiladi. Agar $\vec{a} = (P, Q, R)$ vektor biror oqayotgan suyuqlikning tezliklar maydonini aniqlaydigan bo‘lsa, (15.14) integral qiymat jihatdan, S sirtning $\vec{n}^0$ normal bo‘yicha oqib o‘tgan suyuqlikning hajmi P ga teng bo‘ladi , ya’ni:

$$\Pi = \iint_S \vec{a}(M)\vec{n}^0 dS \qquad (15.20)$$

Bu esa o‘z navbatida (15.14) integralning fizik ma’nosidan iboratdir.

Ushbu (15.20) integraldan ko‘rinyaptiki, P skalyar miqdor bo‘lib, agar $\psi = (\vec{a}\ \ \vec{n}^0) < \frac{\pi}{2}$ bo‘lsa, $\Pi > 0$ bo‘ladi,agar $\psi > \frac{\pi}{2}$ bo‘lsa, $\Pi < 0$ va $\psi = \frac{\pi}{2}$ bo‘lganda Π =0 bo‘ladi.

Sirtning orientatsiyasi o‘zgarganda Π ning ishorasi qarama-qarshi ishoraga o‘zgaradi (ikkinchi tur sirt integraliningxossasiga binoan).

Aytaylik, S tashqi normal birlik vektori $\vec{n}^0$ bo‘lak-bo‘lak silliq yopiq sirt bo‘lsin. Uholda, $\vec{a} = (P; Q; R)$ vektorning S sirt bo‘yicha oqimini Ostrogradiskiy-Gauss formulasi (15.18) yordamida hisoblash mumkin:

$$\Pi = \iint_S \vec{a}(M)\vec{n}^0 dS = \iiint_V (\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z})dxdydz \qquad (15.21)$$

Faraz qilaylik $\vec{a}(M)$ suyuqlik oqimining tezliklar maydoni bo‘lsin. Agar $\Pi > 0$ bo‘lsa, (15.21) formulaga binoan ayta olamizki,V sohadan oqib chiqadigan suyuqlik miqdoridan ko‘proq bo‘lar ekan. Bu degani V soha ichida suyuqlik manbalari mavjud ekan, ya’ni shunday nuqtalar mavjudki, ulardan suyuqlik oqib chiqadi. Agar $\Pi < 0$ bo‘lsa V sohadan oqib chiqadigan suyuqlik miqdori unga oqib kiradigan suyuqlik miqdoriga nisbatan kamroq bo‘ladi. Bu holatda aytiladiki, V soha ichida oqimlar mavjud, ya’ni shunday nuqtalr mavjudki unga suyuqlik oqib kiradi. Agar Π =0 bo‘lsaV sohaga oqib kiradigan suyuqlik miqdori bilan oqib chiqadigan suyuqlik miqdori bir xil bo‘ladi.

Aytaylik, V sohada shunday bir $\vec{a} = (P, Q, R)$ vektor maydoni berilgan bo‘lsin, P(x,y,z), Q(x,y,z), R(x,y,z) lar V sohaningi xtiyoriy M(x;y;z) nuqtasida $\frac{\partial P}{\partial x}; \frac{\partial Q}{\partial y}$ va $\frac{\partial R}{\partial z}$ kabi xususiy hosilalarga ega bo‘lsin. U holda $\vec{a}(M)$ vektor maydonning M(x,y,z) nuqtadagi *divergensiyasi* deb, M(x,y,z) nuqqtada hisoblangan ushbu hususiy hosilalarining yig’indisiga aytiladi va $div\vec{a}(M)$ deb belgilanadi. Demak, ta’rifga binoan:

$$div\vec{a}(M) = (\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z})\Big|_M \qquad (15.22)$$

Fizikaviy nuqtai nazardan qaralganda, $div\vec{a}(M)$ o‘z navbatida, M nuqtadagi manbalar zichligini yoki $\vec{a}(M)$ vektor maydon oqimini ifodalaydi. Agar

$\mathrm{div}\vec{a}(M) > 0$ bo‘lsa, M manba bo‘lib, agar $\mathrm{div}\vec{a}(M) < 0$ bo‘lganda oqim bo‘ladi. Agar $\mathrm{div}\vec{a}(M) = 0$ bolganda, M nuqtada na manba, na oqim mavjud bo‘lmaydi.

Vektor maydon divergensiyasining asosiy xossalarini keltiramiz:

1) $div(\vec{a}+\vec{b}) = div\vec{a} + div\vec{b}$

2) $div\vec{c} = 0,$ agar $\vec{c}$ o′zgarmas vektor bo′lsa;

3) $div(f\vec{\boldsymbol{a}}) = f div\vec{\boldsymbol{a}} + \vec{\boldsymbol{a}}\, gr\vec{a}df,$ bu yerda $f(x, y, z)$ - skalyar funksiya.

Yuqoridagi (15.21) bilan (15.22) formulalardan quyidagini hosil qilamiz:

$$\Pi = \iint_S \vec{a}\vec{n}^0 dS = \iiint_V (div\vec{a}(M))dxdydz \qquad (15.23)$$

ya’ni, vektor maydon $\vec{a}(M)$ ning yopiq S sirt tashqi tomoni bo‘yicha oqimi, miqdor bo‘yicha, shu maydon divergensiyasining S sirt bilan chegaralangan V soha bo‘yicha uch o‘lchovli integraliga teng bo‘lar ekan.

1- misol . $\vec{a}(M) = (x^2+y)\vec{\imath} + (y^2+z)\vec{\jmath} + (z^2+x)\vec{k}$ vektor maydonning $M_0(1, -2{,}3)$ nuqtadan divergensiyasi hisoblansin.

►(15.22) formulaga binoan,

$$div\vec{a}(M) = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} = 2x + 2y + 2z \quad \text{deb yozamiz.}$$

M_0 nuqtada esa $div\vec{a}(M) = 4 > 0$ bo‘lganligi uchun M_0 nuqta maydonga manba bo‘ladi.◄

2 − misol. Tenglamasi x+2y+3z-6=0 bo‘lgan tekislikning birinchi oktantda joylashgan yuqori qismi bo‘yicha $\vec{a} = x\vec{\imath} + 2y\vec{\jmath} + z\vec{k}$ vektor maydonining oqimi hisoblansin.

►Tekislik tenglamsidan $z = 2 - \frac{1}{2}x - \frac{1}{3}y$ ni topamiz.Bu tekislikka,Oz o‘q bilan o‘tkir burchak tashkil etuvchi normal vektor $\vec{n} = (\frac{1}{3}, \frac{2}{3}, 1)$ kabi bo‘ladi. U holda, (15.20) bilan (15.16) formulalarga ko‘ra, quyidagini hosilqilamiz.

$$\Pi = \iint_S \vec{a}\vec{n} \; dS = \iint_{D_z} \frac{1}{3}(x - 4y + 3z)dxdy = \frac{1}{3}\iint_{D_z}(6 - 6y)\,dxdy$$

$$= 2\int_0^3 dy \int_0^{6-2y} (1 - y)dx = 2\int_0^3 (1 - y)(6 - 2y)dy =$$

$= 2\int_0^3 (2y^2 - 8y + 6)dy = 36$ ◄

3- misol. $x^2 + y^2 + z^2 = a^2$ shar sirtining tashqi tomoni bo‘yicha $\vec{a}(M) = (xz^2)\vec{\imath} + (yx^2)\vec{\jmath} + (zy^2)\vec{k}$ vektor maydon oqimi hisoblansin.

►Qaralayotgan sirt yopiq bo‘lgani uchun $\vec{a}(M)$ vektor maydonning oqimi Π ni (15.23) formula orqali topamiz:

$$\Pi = \iint_S \vec{a}\vec{n}^0 dS = \iiint_V div\vec{a}\,(M)dxdydz = \iiint_V (x^2 + y^2 + z^2)\,dxdydz$$

Ushbu uch o‘lchovli integralni hisoblash uchun sferik koordinatalarga o‘tamiz:

$$x = \rho sin\theta cos\varphi; y = \rho sin\theta sin\varphi; z = \rho cos\theta$$

$$dxdydz = \rho^2 sin\theta d\rho d\varphi d\theta; 0 \le \rho \le a; 0 \le \varphi \le 2\pi; 0 \le \theta \le \pi$$

U holda $\Pi = \iiint_V \rho^4 sin\theta d\rho d\varphi d\theta = \int_0^a \rho^4 d\rho \int_0^\pi sin\theta d\theta \int_0^{2\pi} d\varphi = \frac{4\pi a^5}{5}$◄

4- misol $x^2 + y^2 + z^2 = R^2$ sferaning markaziga joylashtirilgan q nuqtaviy zaryad elektrostatik maydon oqimi topilsin.

►Ma’lumki , nuqtaviy zaryad maydoni, kuchlanish vektori $\vec{E} = \frac{q\vec{r}}{|\vec{r}|}$ orqali beriladi. Bu yerda: $\vec{r} = x\vec{\imath} + y\vec{\jmath} + z\vec{k}$ sferaga normal vektorning yo‘naltiruvchi kosinuslarini topamiz:

$$\vec{n}^0 = \frac{\vec{n}}{|\vec{n}|}, \vec{n} = (2x, 2y, 2z),$$

$$|\vec{n}| = \sqrt{4x^2 + 4y^2 + 4z^2} = 2R, \qquad \vec{n}^0 = (\frac{x}{R}, \frac{y}{R}, \frac{z}{R})$$

Ya’ni: $cos\alpha = \frac{x}{R}, cos\beta = \frac{y}{R}, cos\gamma = \frac{z}{R}$. Shuning uchun,

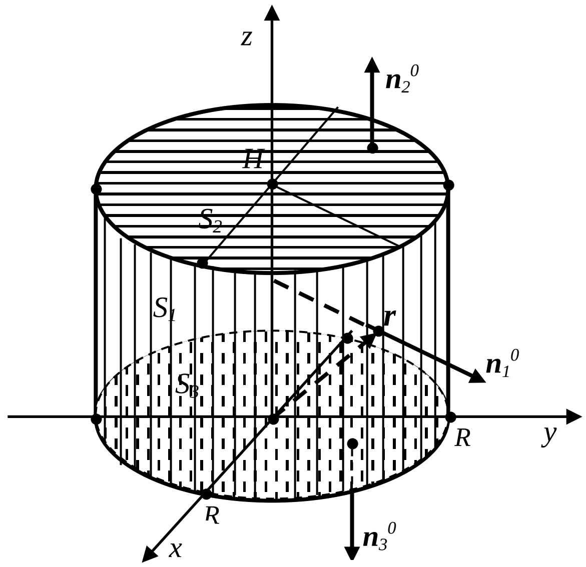
$$\vec{E}\cdot\vec{n}^0=\left(\frac{q}{|\vec{r}|^3}\right)\cdot(\vec{r}\cdot\vec{n}^0)=\frac{q}{R^3}\left(x\vec{i}+y\vec{j}+z\vec{k}\right)\cdot\left(\frac{x}{R}\vec{i}+\frac{y}{R}\vec{j}+\frac{z}{R}\vec{k}\right)$$

$$=\frac{q}{R^3}\cdot\frac{x^2+y^2+z^2}{R}==\frac{q}{R^3}\cdot\frac{R^2}{R}=\frac{q}{R^2}=const.$$

Demak, $\Pi=\iint_S \vec{a}\vec{n}^0dS=\iint_S \frac{q}{R^2}dS=\frac{q}{R^2}\cdot 4\pi R^2=4\pi q$◄

5- misol . Agar asosi Oxy tekislikda joylashgan R radiusli va H balandlikdagi to‘g’ri silindrning o‘qi Oz o‘q bilan ustma-ust bo‘lsa, uning S sirt bo‘yicha $\vec{a}(M)=x\vec{i}+y\vec{j}+z\vec{k}$ vektor maydonning oqimi hisoblansin. Bu yerda, normal silindrning tashqi tomoniga yo‘nalgan.

►15.8-rasmdan ko‘rinadiki silindrning yon sirti S_1 uchun $\vec{a}\vec{n}_1^0=pr_{\vec{n}_1^0}\vec{a}=R$ kabi



15.8- rasm

tenglik o‘rinlidir. Silindrning yuqori asosi S_2 uchun $\vec{a}\,\vec{n}_2^0=pr_{\vec{n}_2^0}\vec{a}=H$ bo‘lib, pastki asos S_3 uchun $\vec{a}\,\vec{n}_3^0=0$ dir. Shu sababdan:

$$P=\iint\limits_S \vec{a}\,\vec{n}^0dS=\iint\limits_{S_1}\vec{a}\,\vec{n}_1^0dS+\iint\limits_{S_2}\vec{a}\,\vec{n}_2^0dS+\iint\limits_{S_3}\vec{a}\,\vec{n}_3^0dS=$$

$$=\iint\limits_{S_1}RdS+\iint\limits_{S_2}HdS+\iint\limits_{S_3}0dS=R2\pi RH+H\pi R^3=3\pi R^2H$$

Ostrogradiskiy-Gauss formulasini qo‘llab hisoblashlarni ancha qisqartirish mumkin.(15.18. formulaga qaralsin). Silindrning hajmi

$$\iiint\limits_V dxdydz = \pi R^2 H$$

bo‘lganligi bois, quyidagiga ega bo‘lamiz:

$$P = \iiint_V (1+1+1)dxdydz = 3\pi R^2 H. ◄$$

15.4-AT

1. $\vec{a}(M) = (xy+z^2)\vec{\imath} + (yz+x^2)\vec{\jmath} + (zx+y^2)\vec{k}$ vektor yig′indining M$(1;3;-5)$ nuqtadagi divergensiyasi hisoblansin.(Javob:-1)

2. $\vec{a}(M) = (x-3z)\vec{\imath} + (x+2y+z)\vec{\jmath} + (4x+y)\vec{k}$ vektor maydonning birinchi oktantda yotuvchi *x+y+z=1* tekislikning yuqori qismi bo‘yicha oqimi hisoblansin. (Javob:$\frac{26}{3}$)

3. $\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{16} = 1$ ellipsning birinchi oktantda yotuvchi sirti bo′yicha uning tashqi normali yo‘nalishidagi $\vec{a}(M) = 2x\vec{\imath} + y\vec{\jmath} + 3z\vec{k}$ vektor maydonining oqimi hisoblansin. (Javob: 24π)

4. $x^2+y^2=1, z=0$ va $z=2$ $s$irtlarbilan chegaralangan silindrik jism sirti bo′yicha uning tashqi normali yo‘nalishidagi $\vec{a}(M) = (x-y)\vec{\imath} + (x+y)\vec{\jmath} + z^2\vec{k}$ vektor maydonining oqimi hisoblansin.(Javob:-4π)

5. Hajmi v bo‘lgan V jismni chegaralaydigan sirtning tashqi tomoni bo‘yicha radius-vektor $\vec{r} = x\vec{\imath} + y\vec{\jmath} + z\vec{k}$ ning oqimi P, 3v ga teng ekani isbotlansin.

6. Cheksiz uzunlikdagi o‘tkazgich bo‘ylab o‘tadigan J tok hosil qiluvchi magnit maydoni $H = \frac{2J}{r}(-y\vec{\imath} + x\vec{\jmath})$ ning vektor kuchlanishi divergensiyasi hisoblansin.(Javob:$div\,\vec{H} = 0$)

7. $\vec{a}(M) = x^3\vec{\imath} + y^3\vec{\jmath} + z^3\vec{k}$ vektor maydonining $x^2+y^2+z^2 = R^2$ Shar sirti bo‘yicha tashqi normal yo‘nalishdagi oqimi Π topilsin.(Javob.($\frac{12\pi R^5}{5}$))

8. $\vec{a}(M) = 8x\vec{\imath} + 11y\vec{\jmath} + 17z\vec{k}$ vektor maydonining *x+2y+3z=1* tekislikning birinchi oktantda joylashgan qismi boyicha oqimi *P* hisoblansin. (Normal Oz o‘q bilan o‘tkir burchaktashkil etadi deb olinsin).(Javob:1)

9. $1-z=x^2+y^2$ va z=0 bo‘lgan sirtlar bilan chegaralangan yopiq S sirt bo‘yicha uning tashqi normali yo‘nalishidagi $\vec{a}=x\vec{\imath}-2y\vec{\jmath}-z\vec{k}$ vektorining P oqimi topilsin.(Javob: $-\pi$)

10. Birinchi oktantda yotuvchi $z^2=4-x-y$ sirt qismi hamda shu sirt kesishidan hosil bo‘ladigan koordinata tekisliklarining qismlaridan tashkil topgan sirt bo‘yicha uning tashqi normal yo‘nalishidagi $\vec{a}=x^2\vec{\imath}+z^2\vec{k}$ vektor oqimi P hisoblansin. (Javob.($19\frac{53}{105}$))

Mustaqil ish

1. 1. Agar $\mathrm{u}=\ln(x^2+y^2+z^2)$ bo‘lsa, $\overrightarrow{grad}u$ maydonning divergensiyasi topilsin.

2. $x+y+z=1$ tekislikning birinchi oktantda yuqori qismi bo‘yicha $\vec{a}(M)=x\vec{\imath}+3y\vec{\jmath}+2z\vec{k}$ vektor maydonining oqimi P hisoblansin.(Javob.1)

2. 1. $\vec{a}(M)=xy^2\vec{\imath}+yx^2\vec{\jmath}+z^3\vec{k}$ vektor maydonining M(1,-1,3) nuqtadagi divergensiyasi hisoblansin.

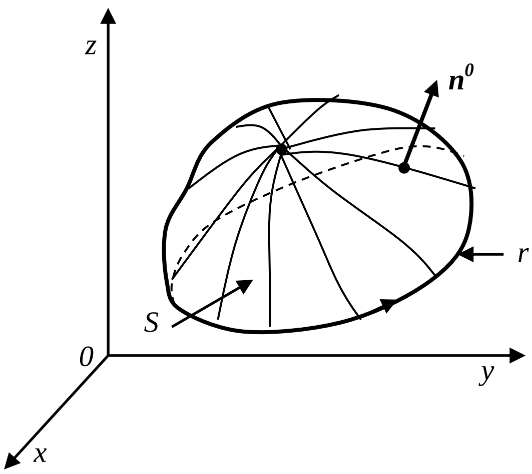
2. Biror jismni $9-z=x^2+y^2$, x=0, y=0 va z=0 sirtlar chegaralaydigan bo‘lsa,u jism sirti bo‘yicha tashqi normal yo‘nalishdagi $\vec{a}(M)=3x\vec{\imath}-y\vec{\jmath}-z\vec{k}$ vektor maydon oqimi hisoblansin.(Javob: $\frac{81\pi}{8}$)

3. 1. $div\left(\overrightarrow{grad}\sqrt{x^2+y^2+z^2}\right)$ ni hisoblansin.

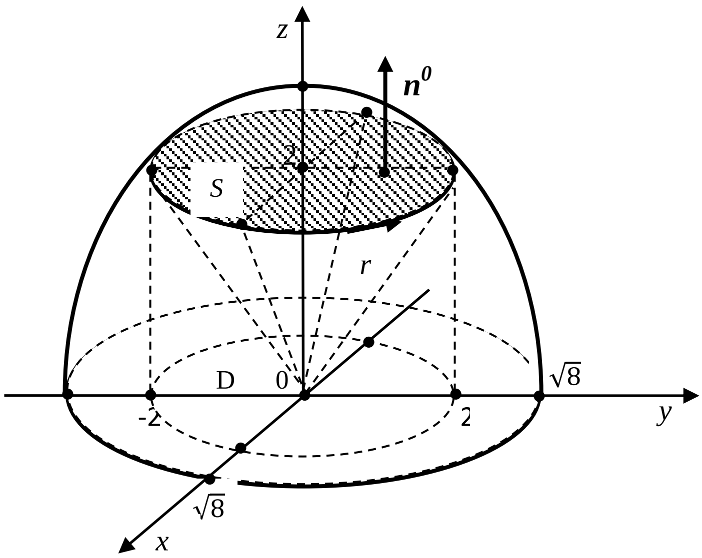
2. Tenglamalari $z=3x^2+2y^2,\ x^2+y^2=4\ va\ z=0$ bo‘lgan sirtlar bilan chegaralangan jismning sirti bo‘yicha uning tashqi normali yo‘nalishidagi $\vec{a}(M)=2x\vec{\imath}+z^2\vec{k}$ vektor maydon oqimi hisoblansin. (Javob:20)

15.5. Vektor maydon sirkulasiyasi.Vektor maydon rotori.

Aytaylik Γ, R^3 fazodagi bo‘lak-bo‘lak silliq yopiq egri chiziq bo‘lib, S esa Γ egri chiziq chegaralaydigan silliq sirt bo‘lsin. Γ egri chiziqning musbat yo‘nalishi uchun shunday yo‘nalish , qabul qilinadiki, bu egri chiziq bilan chegaralangan soha, S sirtning musbat tomoniga nisbatan chap S sirtning shunday tomoniki ,uning nuqtalaridan birlik nirmal vektor $\vec{n}^0 = (cos\alpha, cos\beta, cos\gamma)$ chiqarilgan bo‘ladi faraz qilaylik, S sirtning atrofida shunday $\vec{a} = (P, Q, R)$ berilgan bo‘lsinki uning koordinatalari P,Q,R lar x,y va z o‘zgaruvchilarga bog’liq funksiyalar bo‘lib, ular o‘zlarining birinchi tartibli xususiy hosilalari bilan birgalikda uzluksiz funksiyalar bo‘lsin.U holda, egri chiziqli integral bilan sirt integralini bog’laydigan Stoks formulasi o‘rinli bo‘ladi.(15.9-rasm):



15.9- rasm



15.10- rasm

$$\oint_G Pdx + Qdy + Rdz =$$

$$= \iint_S \left(\left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z} \right) cos\alpha + \left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x} \right) cos\beta + \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) cos\gamma \right) dS, \qquad (15.24)$$

bu yerda, G egri chiziq bo‘ylab yo‘nalishi musbat yo‘nalishdir.

Ta’kidlash lozimki, Grin formulasi bo‘lgan (4.14) formula, Stoks formulasining xususiy holi bo‘ladi (G egri chiziq bilan S sirt *Oxy* tekislikda yotadigan hol). Shu bilan birgalikda, (15.24) Stoks formulasi, har qanday S sirt uchun ham o‘rinlidir, agar uni tenglamalari $z = f(x, y)$ bo‘lgan bo‘laklarga ajratish mumkin bo‘lsa.

1- misol. $J = \oint_G (z^2 - x^2)dx + (x^2 - y^2)dy + (y^2 - z^2)dz$ hisoblansin

Bu yerda G, $x^2+y^2+z^2=8,\ x^2+y^2=z^2, z>0$ lardan tashkil topgan yopiq kontur bo‘lib, koordinata boshi O nuqtadagi kuzatuvchining nuqta’i nazardan qaralganda yo‘nalish saot mili bo‘yicha olinadi.

►Integrallsh konturi G, $x^2+y^2=4, z=2$ tekislikda yotadi hamda $x^2+y^2+z^2=8$ sferaning $x^2+y^2=z^2$ konusning kesishishidan hosil bo‘ladi (15.10-rasm). S sirt sifatida G chegarasi $x^2+y^2\le 4, z=2$ bo‘lgan doirani olamiz. $P=z^2-x^2, Q=x^2-y^2, R=y^2-z^2$ bo‘lganligi uchun,

$\frac{\partial R}{\partial y}-\frac{\partial Q}{\partial z}=2y, \frac{\partial P}{\partial z}-\frac{\partial R}{\partial x}=2z, \frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}=2x$ ni hosil qilamiz. U holda, Stoks formulasiga hamda masalaning shartiga binoan $\vec{n}^0=(0{,}0{,}1)$ bo‘ladi (shu bilan G kontur bo‘ylab harakatning musbat yo‘nalishligi ta’minlanadi).

Natijada quyidagini hosil qilamiz:

$$J=\iint_D 2xdxdy=\begin{vmatrix} x=\rho cos\varphi, & dxdy=\rho d\rho d\varphi \\ y=\rho sin\varphi, & 0\le\varphi\le 2\pi,\ \ 0\le\rho\le 2 \end{vmatrix}=$$

$$=2\int_0^{2\pi} cos\varphi d\varphi\int_0^2 \rho^2 d\rho=0.$$

Agar $\vec{a}\,(M)=(P,Q,R)$ vektor maydoni hamda R^3 fazoda biror bo‘lak-bo‘lak siliq yopiq G egri chiziq berilgan bo‘lsa, u holda, quyidagi egri chiziqli integral

$$C=\oint_G \vec{a}\cdot\vec{\tau}^0 dt=\oint_G Pdx+Qdy+Rdz \qquad (15.25)$$

ni $\vec{a}\,(M)$ vektor maydonining G kontur bo‘ylab *sirkulyasiyasi* deb ataladi. Bu yerdagi $\vec{\tau}^0$, G egri chiziqqa o‘tkazilgan urinma bo‘ylab yo‘nalgan birlik vektor bo‘lib, u G kontur yo‘nalishini ko‘rsatadi.

Agar $\vec{a}$, kuch vektori bo‘ladigan bo‘lsa, yuqoridagi (15.25) sirkulyasiyasi, ushbu kuchning G yopiq egri chiziq bo‘ylab bajargan ishini ifodalaydi.◄

2- misol. Agar G chiziq $\frac{x^2}{16}+\frac{y^2}{9}=1$ silindr bilan $z=x+2y+2$ tekislikning kesishishidan hosil bo‘ladigan bo‘lsa,$\vec{a}\,(M)=x\vec{\imath}-2z^2\vec{\jmath}+y\vec{k}$

vektor maydonning G chiziq bo‘ylab sirkulyasiyasi hisoblanadi. (Bu yerda musbat yo‘nalish tekislikning normal vektori $\vec{n}\,(-1;-2;1)$ ga nisbatan olinadi).

►$\frac{x^2}{16}+\frac{y^2}{9}=1$ silindrning parametrik tenglamasi, $x=4cost, y=3sint$ kabi bo‘lganligidan, G egri chiziqning (tekislik kesimi-ellips) parametrik tenglamasi $x=4cost, y=3sint$ va $z=4cost+6sint+2$ kabi yoziladi. Shuning uchun vektor maydoning ellips bo‘yicha musbat yo‘nalishdagi sirkulyasiyasi quyidagicha hisoblanadi: $C=\oint_G xdx-2z^2dy+ydz=$ $\int_0^{2\pi}-16\cos t\, sint-2(4cost++6sint+2)^2\,3cost+3sint(-4sint+$ $6cost))dt=\int_0^{2\pi}(-16costsint-96cos^3t-216sin^2tcost-24cost-$ $-288cos^2tsint-96cos^2t-144costsint-12sin^2t+18costsint)dt=$ $=-\int_o^{2\pi}(96cos^2t+12sin^2t)dt=-\int_o^{2\pi}48(1+cos2t)dt-6\int_o^{2\pi}(1-$ $-cos2t)dt=-48\cdot 2\pi-6\cdot 2\pi=-108\pi.$◄

Vektor maydon $\vec{a}\,(M)=(P,Q,R)$ ning *rotori* yoki *quyuni* (vixr) deb quyidagi vektorga aytiladi

$$rot\vec{a}\,(M)=\left(\frac{\partial R}{\partial y}-\frac{\partial Q}{\partial z}\right)\vec{\imath}+\left(\frac{\partial P}{\partial z}-\frac{\partial R}{\partial x}\right)\vec{\jmath}+\left(\frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}\right)\vec{k} \qquad (15.26)$$

Rotor va sirkulyasiyasi tushunchalaridan foydalanib Stoks formulasini vektor shaklda yozish mumkin:

$$C=\oint_G \vec{a}\,\cdot\vec{\tau}^0dt=\iint_S rot\vec{a}\cdot\vec{n}^0dS, \qquad (15.27)$$

ya’ni, $\vec{a}\,(M)$ vektor maydonning G yopiq kontur bo‘yicha sirkulyasiyasi, chegarasi G bo‘lgan har qanday silliq S sirt bo‘yicha shu maydon rotorining oqimiga teng bo‘lar ekan. S sirt tomoni va uning chegarasi G ning yo‘nalishi bir paytda yoki musbat yoki manfiydir.

$$C(M)=pr_{\vec{n}^0}\vec{a}\,(M)$$

sonni vektor maydoni $\vec{a}\,(M)$ ning M nuqtadagi $\vec{n}^0$ vektor yo‘nalishdagi sirkulyasiyasining zichligi deb ataladi. Ushbu zichlik, $rot\,\vec{a}\,(M)$ yo‘nalishida maksimumga erishib, u max $C(M)=|\mathbf{rot}\vec{a}\,(M)|$ ga teng bo‘ladi.

Vektor maydon rotorining ayrim hossalarini keltiramiz:

1) $rot\left(\vec{a}+\vec{b}\right)=rot\vec{a}\ +rot\vec{b}$;

2) $rot\vec{c}=0,$ agar $\vec{c}$ doimiy vektor (skalyar)bo′lsa;

3) Agar $\varphi(x,y,z)$ skalyar funksiya bo‘lsa, $rot\ (\varphi\vec{a})=\varphi rot\vec{a}+\overrightarrow{grad}\ \varphi\cdot\vec{a}$;

Agar $\boldsymbol{rot}\vec{a}\ \neq 0$ bo‘lsa, bu vektor maydon $\vec{a}\ (M)$ ning aylanishini ifodalaydi.

3- misol. Chiziqli tezlik vektori $\vec{v}\ =\vec{\omega}\cdot\vec{r}$ ning fazodagi ixtiyoriy $M(x,y,z)$ nuqtadagi rotori topilsin.

$$►\vec{v}\ =\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ \omega_x & \omega_y & \omega_z\\ x & y & z\end{vmatrix}=\left(z\omega_y-y\omega_z\right)\vec{i}+(x\omega_z-z\omega_x)\vec{j}+\left(y\omega_x-x\omega_y\right)\vec{k}$$

kabi bo‘lganligidan, rotorning ta’rifidan quyidagini topamiz:

$\boldsymbol{rot}\vec{v}=\left(2\omega_x,2\omega_y,2\omega_z\right)=2\vec{\omega}$.◄

4- misol. Vektor maydoni ,$\vec{a}\ (M)=y\vec{i}+x^2\vec{j}-z\vec{k}$ ning $x^2+y^2=4,z=3$ aylana G bo‘yicha birlik $\vec{k}$ vektorga nisbatan musbat yo‘nalishdagi sirkulyasiyasini 1) (15.25) ta’rifdan foydalanib; 2) Stoks formulasi (15.27) ni qo‘llab sirt integralidan foydalanib hisoblansin.

►1) Parametr t, 0 dan 2π gacha o‘zgarganida, aylana bo‘ylab harkat $\vec{k}=(0;0;1)$ birlik vektorga nisbatan soat miliga teskari yo‘nalishda bo‘ladi va orietirlangan Γ egri chiziqning parametrik tenglamalari

$x=2cost,y=2sint,\ z=3\ \ (t\epsilon[0;2\pi])$ kabi ko‘rinishda yoziladi. U holda:

$$C=\oint\limits_G ydx+x^2dy-zdz=\int\limits_0^{2\pi}2si\,n(-2sintdt)+4cos^2t\cdot 2costdt-3\cdot 0=$$

$$=8\int\limits_0^{2\pi}cos^3tdt-4\int\limits_0^{2\pi}sin^2tdt\,8\int\limits_0^{2\pi}(1-sin^2t)d(sint)-\ 2\int\limits_0^{2\pi}(1-cos2t)dt=$$

$$=-4\pi.$$

2) Chegarasi G egri chiziq bo‘ylab S sirt sifatida $x^2+y^2\leq 4,z=3$ doirani olamiz (15.11-rasm). U holda, $\vec{n}^0=\vec{k}$ bo‘lib, $\boldsymbol{rot}\vec{a}\ =(2x-1)\vec{k}$

bo‘ladi. 15.11-rasm chiziladi. Natijada quyidagini hosil qilamiz: $C = \iint_S rot\vec{a} * \vec{n}^0 dS = \iint_D (2x-1)dxdy = \iint_D (2\rho cos\varphi - 1)\rho d\rho d\varphi =$
$\int_0^{2\pi} d\varphi \int_0^2 (2\rho cos\varphi - 1)\rho d\rho =$
$= -2\pi * \frac{\rho^2}{2}\Big|_0^2 = -4\pi$ ◄

15.5-AT

1. $\vec{a}(M) = xyz\vec{i} + (x+y+z)\vec{j} + (x^2+y^2+z^2)\vec{k}$ vektor maydonning M(1,-1,2) nuqtadagi rotori hisoblansin. (Javob: $\boldsymbol{rot}\vec{a}(M) = -3\vec{i} - 3\vec{j} - \vec{k}$)

2. Stoks formulasi yordamida $\oint_G (z^2+y^2)dx + (x^2+z^2)dy + (x^2+y^2)dz$ integralning chegarasi Γ yopiq kontur bo‘lgan S sirt bo‘yicha integralga o‘zgartirilsin.

3. Bir pallali giperboloid $2x^2 - y^2 + z^2 = R^2$ bilan y=x tekislik kesishidan hosil bo‘lgan ellips bo‘yicha $\vec{a}(M) = y\vec{i} - 2z\vec{j} + x\vec{k}$ vektor maydonning sirkulatsiyasi topilsin .Natijani Stoks formulasi yordamida tekshirilsin. (Javob: $\pm 3\pi R^2$)

4. $x^2 + y^2 = 4$, z=0 kontur bo‘yicha G bo‘yicha $\vec{n}^0 = \vec{k}$ ortga nisbatan musbat yo‘nalishdagi, $\vec{a}(M) = z\vec{i} + x\vec{j} + y\vec{k}$ vektor maydonning sirkulatsiyasini ham bevosita,ham Stoks formulasi yordamida hisoblansin. (Javob: 4π)

5. $\vec{a}\,(M) = z^2\vec{i} + x^2\vec{j} + y^2\vec{k}$ vektor maydoning, $x^2 + y^2 + z^2 = R^2$ sfera bilan x+y+z=R tekislikning kesisish chizig'i bo'ylab, $\vec{n}$ (1,1,1) vektorga nisbatan musbat yo'nalishdagi sirkulatsiyasi hisoblansin.(Javob: $\frac{3\pi R^2}{2}$).

6. Birinchi oktantdagi $x^2 + y^2 = Rz$ paraboloidning $x=0$,$y=0$ va $z=R$ tekisliklar bilan kesilganda hosil bo'ladigan kontur bo'yicha $\vec{a}\,(M) = y^2\vec{i} + xy\vec{j} + (x^2 + y^2)\vec{k}$ vektor maydonining shu kontur chegaralaydigan sirt tashqi normaliga nisbatan musbat yo'nalishdagi sirkulatsiyasi hisoblansin. (Javob: $\frac{R^3}{3}$)

7. $\vec{a}\,(M) = zy^2\vec{i} + xz^2\vec{j} + yx^2\vec{k}$ vektor maydonining ,$x = y^2 + z^2$ paraboloid bilan x=9 tekislik kesisishidan hosil bo'lgan kontur bo'yicha $\vec{n}^0 = \vec{i}$ ortga nisbatan musbat yo'nalishdagi sirkulatsiyasi hisoblansin. (Javob: 729π)

8. $\vec{a}\,(M) = -y\vec{i} + 2\vec{j} + \vec{k}$ vektor maydonining $x^2 + y^2 - z^2 = 0$ konus bilan z=1 tekislikning kesishidan hosil bo'lgan G chiziq bo'yicha $\vec{n}^0 = \vec{k}$ ortga nisbatan musbat yo'nalishdagi sirkulatsiyasi hisoblansin. (Javob: π)

Mustaqil ish

1. Agar G, $x^2 + y^2 - z^2 = 4$ sfera bilan $z = \sqrt{x^2 + y^2}$ konusning kesisishidan hosil bo'lgan chiziq bo'ladigan bo'lsa, $\vec{n}^0 = \vec{k}$ ortga nisbatan musbat yo'nalishdagi G chiziq bo'yicha $\vec{a}\,(M) = y\vec{i} - x\vec{j} + z\vec{k}$ vektor maydon sirkulatsiyasi hisoblansin.

2. Yarim sfera $z = \sqrt{25 - x^2 - y^2}$ ning $x^2 + y^2 = 16$ silindr bilan kesisishidan hosil bo'lgan G chiziqning $\vec{n}^0 = \vec{k}$ ortga nisbatan musbat yo'nalishi bo'yicha $\vec{a}\,(M) = yz\vec{i} + 2xz\vec{j} + y^2\vec{k}$ vektor maydonining sirkulatsiyasi hisoblansin.

3. Agar $\vec{n}^0 = \vec{k}$ bo'lsa, $\vec{a}\,(M) = (x - y)\vec{i} + x\vec{j} - z\vec{k}$ vektor maydonining $x^2 + y^2 = 1$ silindr bilan z=2 tekislik bilan kesisishidan hosil bolgan Γ chiziq bo'yicha sirkulatsiyasi hisoblansin.

15.6 Ikkinchi tartibli differensial amallar. Vektor maydonlarning nurlari

Differensial amallar. Vektor analizning yuqorida kiritilgan asosiy tushunchalari bo‘lgan gradient, divergensiya, rotorlarni

$$\vec{\nabla}=\frac{\partial}{\partial x}\vec{i}+\frac{\partial}{\partial y}\vec{j}+\frac{\partial}{\partial z}\vec{k} \qquad (\nabla - \text{nabla deb o'qiladi})$$

belgilanadigan hamda Gamilton operatori deb ataladigan differensial operator orqali ifodalash ancha qulay bo‘ladi.

Asosiy differensial amallarni $\vec{\nabla}$ operatori orqali ifodalaymiz:

$$\vec{\nabla}u(M) = \frac{\partial u}{\partial x}\vec{i}+\frac{\partial u}{\partial y}\vec{j}+\frac{\partial u}{\partial z}\vec{k} = \overrightarrow{grad}\, u(M),$$

$$\vec{\nabla}\cdot\vec{a}(M) = \frac{\partial P}{\partial x}\vec{i}+\frac{\partial Q}{\partial y}\vec{j}+\frac{\partial R}{\partial z}\vec{k} = div\vec{a}(M),$$

$$\vec{\nabla}\times\vec{a}(M) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix} = rot\vec{a}(M)$$

Gradiet, divergensiya va rotorlarni topish amallarini *birinchi tartibli differensial amallar* deb ataladi.

Ikkinchi tartibli differensial amallarning asosiy xossalarini keltiramiz:

$$div\overrightarrow{grad}u(M) = \frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2} = \Delta u(M), \qquad \text{bu yerda}$$

$$\Delta = \frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2} = \vec{\nabla}*\vec{\nabla} = \vec{\nabla}^2$$

bo‘lib, uni Laplas operatori deb yuritamiz.

$rot\ \overrightarrow{grad}u(M) = (\vec{\nabla}*\vec{\nabla})u(M) = 0$

$div\ rot\vec{a}(M) = \vec{\nabla}*(\vec{\nabla}\times\vec{a}(M)) = 0$

$\overrightarrow{grad}\ div\vec{a}(M) = \vec{\nabla}*(\vec{\nabla}*\vec{a}(M)),$

$rot\ rot\vec{a}(M) = \vec{\nabla}\times(\vec{\nabla}\times\vec{a}(M)) = \overrightarrow{grad}\ div\vec{a}(M) - \Delta\vec{a}(M),$

Solenoidal vektor maydon

Agar fazoning V sohasidagi $\vec{a}(M)$ vektor maydonning har bir nuqtasida

$$div\vec{a}(M) = 0$$

tenglik o'rinli bo'ladigan bo'lsa, u holda $\vec{a}(M)$ vektor maydonni yoki *solenoidal* yoki *naysimon maydon* deb ataladi.

Har doim $div\, rot\vec{a}(M) = 0$ bo'lganligi uchun har qanday $\vec{a}(M)$ vektor maydonning rotori solenoidal maydon bo'ladi.

$\vec{a}(M)$ning vektor chiziqlari bo'ylab yo'nalishdagi bektor naylarining har bir kesimi bo'yicha oqimi, Ostrogradskiy-Gauss formulasiga binoan bir xil bo'ladi. Naysimon maydon manba'alarga ham oqimlarga ham ega emas.

Har bir $\vec{a}(M)$ solenoidal maydonga shunday bir $\vec{b}(M)$ vektor maydoni mavjudki, ular uchun $\vec{a}(M) = rot\, \vec{b}(M)$ dir. Bu yerdagi $\vec{b}(M)$ vektorni berilgan $\vec{a}(M)$ maydonning vektor-potensiali deb yuritiladi.

Potensial vektor maydon.

Fazoning bir bog'lamli V sohasidagi har bir nuqtada

$$rot\,\vec{a}(M) = 0$$

tenglik o'rinli bo'ladigan bo'lsa, $\vec{a}(M) = (P, Q, R)$ vektor maydonning *potensial maydon* (yoki quyunsiz maydon) deb ataladi.

Rotorning ta'rifiga ko'ra $\vec{a}(M) = (P, Q, R)$ vektor maydon potensial maydon bo'lishliligining zaruriy va yetarli shartlari quyidagi tengliklar bilan ifodalanadi:

$$\frac{\partial R}{\partial y} = \frac{\partial Q}{\partial z}, \quad \frac{\partial P}{\partial z} = \frac{\partial R}{\partial x}, \quad \frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y} \qquad (15.28)$$

Har qanday skalyar maydon $u = u(x, y, z)$ uchun $rot\,\overrightarrow{grad}\,u(M) = 0$ bo'lganligidan, gradient maydoni patensial maydondir. Vektor maydonning $\vec{a}(M)$, biror V sohada potensial maydon bo'lishligi uchun $\vec{a} = \overrightarrow{grad}\,u(M)$ tenglikni qanoatlantiradigan shunday bir ikki marta uzluksiz differensiallanuvchi $u = u(x, y, z)$ funksiyaning mavjudligi zarur va yetarlidir. Mazkur funksiyani $\vec{a}(M)$ maydonning potensial funksiyasi yoki potensiali deb yuritiladi.

Agar (15.28) shartlar bajarilganda ikkinchi turdagi egri chiziqli integral M_0 va M_1 nuqtalarni birlashtiruvchi chiziqqa bog'liq bo'lmas edi,shu sababdan $\vec{a}(M) = P\vec{i} + Q\vec{j} + R\vec{k}$ potensial maydon uchun uning potensial funksiyasini aniqlaydigan quyidagi formula o'rinli bo'ladi:

$$u(x;y;z) = \int_{M_0 M_1} Pdx + Qdy + Rdz + C \qquad (15.29)$$

Bu yerda, $M_0(x_0, y_0, z_0)$ *V* sohadagi biror tayinli nuqta bo'lib $M(x, y, z)$ esa *V* dagi ixtiyoriy nuqtadir. C ixtiyoriy o'zgarmas sondir.

Ushbu (15.29) formulaga binoan, integrallash yo'liga bog'liq bo'lmagan ikkinchi turdagi egri chiziqli integralni integrallash formulasi bo'lgan quyidagi formulani hosil qilamiz.

$$\int_{AB} Pdx + Qdy + Rdz = u(B) - u(A) \qquad (15.30)$$

Bu yerda, u(A) bilan u(B) lar, potensial *u* ning boshlang'ich A nuqtadagi va oxirgi B nuqtadagi qiymatlaridir.

Garmonik vektor maydoni. Agar $\vec{a}(M)$ vektor maydonga nisbatan $div\vec{a}(M) = 0$ va $rot\,\vec{a}(M) = 0$ lar bajariladigan bo'lsa, y vektor maydonini *garmonik vektor maydoni* deb ataladi. Garmonik bo'lgan *u* maydon potensiali o'z navbatida Laplas tenglamasi deb ataluvchi

$$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0 \qquad (15.31)$$

tenglamaning yyechimi bo'ladi. Shuningdek, Laplas tenglamasi (15.31) ni qanoatlantiradigan $u = u(x, y, z)$ funksiyani garmonik funksiya deb ataladi.

1- misol $\vec{a}\,(M) = (2xy + z)\vec{i} + (x^2 - y)\vec{j} + x\vec{k}$ maydonning solenoidal maydon emas, balki potensial maydon ekanligi isbotlanib, berilgan maydonning potensiali *u* topilsin.

►Shartga ko'ra, $P = 2xy + z, Q = x^2 - y, R = x$ bo'lganligi uchun, quyidagiga ega bo'lamiz:

$$\boldsymbol{rot}\vec{a}(M) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ 2xy+z & x^2-2y & x \end{vmatrix} = (0-0)\vec{i} + (1-1)\vec{j} + (2x-2x)\vec{k} = 0,$$

$\vec{a}(M)$ potensial maydon ekan .

$$div\vec{a} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} = 2y - 2 + 0 \neq 0$$

ligi uchun ayta olamizki, $\vec{a}(M)$ solenoidal maydon emas. (15.29) formulaga muvofiq,

$$u(x,y,z) = \int\limits_{M_0 M_1} (2xy+z)dx + (x^2-y)dy + xdz + C$$

ni hosil qilamiz.

$P(x,y,z), Q(x,y,z), R(x,y,z)$ lar R^3 fazoning barcha nuqtalarida ham uzliksiz ham uzluksiz xususiy hosilalarga ega bo‘lganliklari uchun $M_0(x_0, y_0, z_0)$ nuqta uchun koordinata boshi $O(0,0,0)$ ni va $M(x,y,z)$ nuqta uchun esa, fazodagi ixtiyoriy nuqtani olishimiz mumkin.

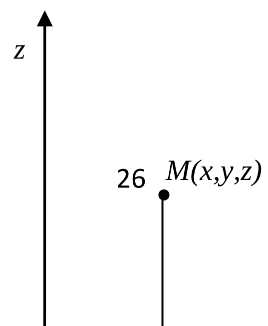
Yuqorida ta’kidlanganidek, ikkinchi tur egri chiziqli integrallash yo‘liga bog’liq bo‘lmas edi, shu boisdan uni $OABM$ siniq chiziq bo‘yicha hisoblash mumkin (15.12-rasm)

$$u(X,Y,Z) = \int\limits_{OM} + C == \int\limits_{OA} + \int\limits_{AB} + \int\limits_{BM} + C =$$

$$= \begin{vmatrix} OA: y=0, z=0, & dy=0, dz=0, & 0 \leq x \leq X \\ AB: x=X, z=0, & dx=0, dz=0, & 0 \leq y \leq Y \\ BM: x=X, y=Y, & dx=0, dy=0, & 0 \leq z \leq Z \end{vmatrix} =$$

$$= \int\limits_0^X 0dx + \int\limits_0^Y (X^2-y)dy + \int\limits_0^Z Xdz = X^2Y - Y^2 + XZ.$$

Oxirgi ifodada X,Y,Z larni x,y,z lar bilan almashtirib maydonning potensialini yozamiz: $u(x;y;z) = x^2y - y^2 + xz + C$.◄



2- misol. $\vec{a} = (yz - xy)\vec{i} + (xz - \frac{x^2}{2} + yz^2)\vec{j} + (xy + y^2z)\vec{k}$ maydoning potensial maydon ekanligi tekshirilib uning potensiali topilsin, hamda $A(1,1,1)$ va $B(2,-2,3)$ nuqtalrni birlashtiruvchi chiziq bo'yicha mos ikkinchi tur egri chiziqli integral hisoblansin.

►$P = yz - xy, Q = xz - \frac{x^2}{2} + xz^2, R = xy + y^2z$ ekanliklaridan, quyidagini topamiz:

$$rot\vec{a}(M) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz - xy & z - \frac{x^2}{2} + xz^2 & xy + y^2z \end{vmatrix} =$$

$$= (x + 2yz - x - 2yz)\vec{i} + (y - y)\vec{j} + +(z - x - z + x)\vec{k} = 0$$

Demak, $\vec{a}$ potensial maydon va uning potensiali mavjuddir ((15.29) formula va 1- misolga qaralsin):

$$u(X,Y,Z) = \int_{M_0M} Pdx + Qdy + Rdz + C =$$

$$= \int_0^X 0dx + \int_0^Y (-\frac{x^2}{2})dy + \int_0^Z (xy + y^2z)dz + C = -\frac{X^2Y}{2} + XYZ + \frac{Y^2Z^2}{2} + C$$

X,Y,Z larni x,y,z lar bilan almashtirib, quyidagiga ega bo'lamiz,

$$u = xyz - \frac{x^2y}{2} + \frac{y^2z^2}{2} + C$$

bu yerda ham,potensial maydondagi 2-tur egri chiziqli integralning A va B nuqtalarni birlashtiruvchi yo'lga bog'liq emasligini inobatga oladigan bo'lsak, (15.30) formulaga binoan quyidagini hosil qilamiz:

$$\int_{AB}(yz-xy)dx+\left(xz-\frac{x^2}{2}+yz^2\right)dy+(xy+y^2z)dz=$$
$$=u(B)-u(A)=9.◄$$

3- misol. Agar $r=\sqrt{x^2+y^2+z^2}$ bo‘lsa, $U=\frac{1}{r}$ funksiyaning garmonik funksiya ekanligi va $\vec{a}(M)=\overrightarrow{grad}\,u(M)$ vektor maydonining garmonik maydon ekanligi isbotlansin.

►Eng avvalo berilgan funksiya uchun (15.31) Laplas tenglamasi o‘rinli bo‘lish yoki bo‘lmasligini tekshiramiz. Buning uchun $\frac{\partial^2 u}{\partial x^2}, \frac{\partial^2 u}{\partial y^2}, \frac{\partial^2 u}{\partial z^2}$ va Δu larni hisoblaymiz:

$$\frac{\partial u}{\partial x}=-\frac{x}{r^3}, \quad \frac{\partial^2 u}{\partial x^2}=-\frac{1}{r^3}+\frac{3x^2}{r^3}; \quad \frac{\partial u}{\partial y}=-\frac{y}{r^5}, \frac{\partial^2 u}{\partial y^2}=-\frac{1}{r^3}+\frac{3y^2}{r^5};$$

$$\frac{\partial u}{\partial z}=-\frac{z}{r^3}, \frac{\partial^2 u}{\partial z^2}=-\frac{1}{r^3}+\frac{3z^2}{r^5};$$

$$\Delta u=-\frac{3}{r^3}+3\frac{x^2+y^2+z^2}{r^5}=-\frac{3}{r^3}+\frac{3}{r^3}=0$$

Demak, $\Delta u=0$ Laplass tenglamasi bajarilmoqda va $u=\frac{1}{r}$ garmonik funksiya ekan, $\vec{a}(M)=\overrightarrow{grad}u(M)=-r^3(x\vec{\imath}+y\vec{\jmath}+z\vec{k})$ ni aniqlaganimizdan so‘ng $rot\vec{a}(M)=rot\overrightarrow{grad}u(M)=0$ tenglik har qanday u funksiya uchun o‘rinli bo‘lganligidan ayta olamizki, $\vec{a}(M)$ garmonik ta’rifidagi bitta shart bajarilmoqda $div\vec{a}=div\overrightarrow{grad}u(M)=\Delta u(M)=0$ bo‘lganligi uchun ikkinchi shart ham bajarilmoqda. ◄

15.6-AT

1. Agar G, har qanday yopiq kontur bo‘lganda ham $\oint_G yzdx+xzdy+xydz=0$ ekanligini Stoks formulasi orqali isbotlansin. Natijani uchlari $A(0,0,0), B(1,1,0), C(1,1,1)$ nuqtalarda bo‘lgan ABC uchburchak konturi bo‘yicha integralni hisoblab tekshirilsin.

2. Agar $\vec{a}\,(M)=x^3\vec{\imath}+y^3\vec{\jmath}+z^3\vec{k}$ bo‘lsa $\overrightarrow{grad}\,div\vec{a}(M)$ ni hisoblansin.

3. Biror muhit qattiq jism sifatida Oz o‘qi atrofida $\vec{\omega} = \omega\vec{k}$ burchak tezlik bilan aylanmoqda. Agar $\vec{r}$ harakatlanayotgan $M(x; y; z)$ nuqtaning radius vektori bo‘ladigan bo‘lsa, chiziqli tezliklar maydoni $\vec{v} = \vec{\omega} \times \vec{r}$ ning rotori topilsin. (Javob: $2\omega\vec{k}$)

4. Avvalgi topshiriqda tasvirlangan $\vec{v}$ tezlik maydonining $\vec{k}$ ortga nisbatan musbat yo‘nalishda bo‘lgan $x^2 + y^2 = R^2$, $z = 0$ aylana bo‘ylab sirkulyasiyasi hisoblansin. (Javob: $2\pi R^2$)

5. Har qanday $\vec{a}\,(M)$ maydon uchun $div\, rot\vec{a}(M) = 0$ ligi isbotlansin.

6. Quyidagi berilganlarga ko‘ra, $\vec{a}\,(M)$ Maydonning potensialligi tekshirilib uning potensiali U topilsin.

a) $\vec{a}\,(M) = 2xy\vec{i} + (x^2 - 2yz)\vec{j} - \vec{k}$

b) $\vec{a}\,(M) = (3x^2y - y^3)\vec{i} + (x^3 - 3xy^2\,)\vec{j}$

c) $\vec{a}\,(M) = (y + 2)\vec{i} + (x + z)\vec{j} + (y + x)\vec{k}$

(Javob: $a)u = x^2y - y^2z + c$; $b)u = x^3y - xy^3 + c; u = xy + yz + xz + c$)

7. Agar $r = \sqrt{x^2 + y^2}$ bo‘lsa, $u = lnr$ ning garmonikligi tekshirilsin.

8. Quyida berilgan $\vec{a}\,(M)$ maydonlarning potensilliklari isbotlanib ularning potensiallari U topilsin.

a) $\vec{a}\,(M) = e^{\frac{y}{z}}\vec{i} + \left(\frac{e^{\frac{y}{z}}(x+1)}{z} + ze^{yz}\right)\vec{j} + \left(-\frac{e^{\frac{y}{z}}(x+1)y}{z^2} + ye^{yz} + e^{-z}\right)\vec{k}$

b) $\vec{a}\,(M) = yzcos(xy)\vec{i} + xzcos(xy)\vec{j} + \sin(xy)\vec{k}.$

(Javob: $a)\ u = e^{\frac{y}{z}}(x + 1) + e^{yz} - e^{-z} + c$; $b)u = zsin(xy) + c;$)

9. Agar $\vec{a}\,(M) = \frac{\gamma_m}{|\vec{r}|^3}\vec{r}\ (bu\ yerda\ , \vec{r} = x\vec{i} + y\vec{j} + z\vec{k})$vektor maydoni, koordinata boshida joylashgan nuqtaviy m massa hosil qiladigan gravitatsiya maaydoni ($\gamma -$ tortilishning Nyuton doimiysi) ni ifodalaydigan bo‘lsa, uning garmonik (potensial va quynsiz) maydon ekanligi isbotlansin hamda uning potensiali u topilib, u Laplas tenglamasini qanoatlantirishi tekshirilsin . (Javob: $u = \gamma_m/|\vec{r}|.$)

10. $rot\,\overrightarrow{grad}\,U(M) = 0$ ekanligi isbotlansin.

11. $\vec{a}\,(M) = (yz+1)\vec{\imath} + xz\vec{\jmath} + xy\vec{k}$ maydonning potensiali *u* topilsin va $\int_{(1,1,1)}^{(2,3,2)}(yz+1)dx + xzdy + xydz$ hisoblansin. (Javob: $u = x + xyz + C; 12.$)

Mustaqil ish

Quyidagi berilganlarga binoan. $\vec{a}\,(M)$ vektor maydonining potensialligi tekshirilib, uning potensiali topilsin hamda *A* va *B* nuqtalarni birlashtiruvchi chiziq yoyi bo‘yicha mos 2-tur egri chiziqli integralning qiymati hisoblansin (*A*-yoyning boshlanishi *B*-uning oxiri).

1. $\vec{a}\,(M) = 2xyz\vec{\imath} + x^2z\vec{\jmath} + x^2y\vec{k}, A(1,-1,2), B(-2,4,2)$ (Javob: 34)

2. $\vec{a}\,(M) = (x^2 - 2yz)\vec{\imath} + (y^2 - 2xz)\vec{\jmath} + (z^2 - -xy)\vec{k}$

$A(1,-1,1), B(-2,2,3)$. (Javob: $\frac{92}{3}$)

3. $\vec{a}\,(M) = (2xy + z^2)\vec{\imath} + (2xy + x^2)\vec{\jmath} + (2xz + y^2)\vec{k}$

$A(0,1,-2), B(2,3,1)$. (Javob: 25)

15.7. 15 BOBGA DOIR INDIVIDUAL UY TOPSHIRIQLARI

15.1 IUT

1. u(M) = u(x,y,z) funksiya va M_1, M_2 nuqtalar berilgan. Quyidagilar hisoblansin:

1) $\overrightarrow{M_1\,M_2}$ vektor yo‘nalishi bo‘yicha u(M) = u(x,y,z) funksiyaning M_1 nuqtadagi hosilasi;

2) grad u(M_1).

1.1. u(M) = $x^2y + y^2z + z^2x$. $M_1(1,-1,2)$, $M_2(3,4,-1)$.

1.2. u(M) = $5xy^3z^2$, $M_1(2,1,-1)$, $M_2(4,-3,0)$.

1.3. u(M) = ln($x^2 + y^2 + z^2$), $M_1(-1,2,1)$, $M_2(3,1,-1)$.

1.4. u(M) = z$e^{x^2+y^2+z^2}$, $M_1(0,0,0)$, $M_2(3,-4,2)$.

1.5. u(M) =ln(xy +yz +xz), $M_1(-2,3,-1)$, $M_2(2,1,-3)$.

1.6. u(M) = $\sqrt{1 + x^2 + y^2 + z^2}$ $M_1(1,1,1,)$, $M_2(3,2,1)$.

1.7. u(M) = $x^2y + x\,z^2 - 2$, $M_1(1,1,-1)$, $M_2(2,-1,3)$.

1.8. u(M) =x$e^y + ye^x - z^2$, $M_1(3,0,2)$, $M_2(4,1,3)$.

1.9. u(M) = $3xy^2 + z^2 - xyz$, $M_1(1,1,2)$, $M_2(3,-1,4)$.

1.10. u(M) = $5x^2yz + xy^2z + yz^2$, $M_1(1,1,1)$, $M_2(9,-3,9)$.

1.11. u(M) = x/($x^2 + y^2 + z^2$), $M_1(1,2,2)$, $M_2(-3,2,-1)$.

1.12. u(M) = $y^2z - 2xyz + z^2$, $M_1(3,1,-1)$, $M_2(-2,1,4)$.

1.13. u(M) = $x^2 + y^2 + z^2 - 2xyz$, $M_1(1,-1,2)$, $M_2(5,-1,4)$.

1.14. u(M) = ln(1+ x+ $y^2 + z^2$), $M_1(1,1,1)$, $M_2(3,-5,1)$.

1.15. u(M) = $x^2 + 2y^2 - 4z^2 - 5$, $M_1(1,2,1)$, $M_2(-3,-2,6)$.

1.16. u(M) = ln($x^2 + y^2 + z + 1$), $M_1(1,3,0)$, $M_2(-4,1,3)$.

1.17. u(M) = x – 2y +e^z, $M_1(-4,-5,0)$, $M_2(2, 3,4)$.

1.18. u(M) = $x^y - 3xyz$, $M_1(2,2,-4)$, $M_2(1,0,-3)$.

1.19. u(M) = $3x^2yz^3$, $M_1(-2,-3,1)$, $M_2(5,-2,0)$.

1.20. u(M) =e^{xy+z^3}, $M_1(-5,0,2)$, $M_2(2,4,-3)$.

1.21. u(M) = x^{yz} , $M_1(3,1,4)$, $M_2(1,-1,-1)$.

1.22. u(M) = ($x^2 + y^2 + z^2)^3$, $M_1(1,2,-1)$, $M_2(0,-1,3)$.

1.23. u(M) =$(x-y)^z$, $M_1(1,5,0)$, $M_2(3,7,-2)$.

1.24. u(M) = $x^2y + y^2z - 3z$, $M_1(0,-2,-1)$, $M_2(12,-5,0)$.

1.25. u(M) = 10/($x^2 + y^2 + z^2 + 1$), $M_1(-1,2,-2)$, $M_2(2,0,1)$.

1.26. u(M) = ln(1+ x$-\, y^2 + z^2$), $M_1(1,1,1)$, $M_2(5-4,8)$.

1.27. u(M) = $\frac{x}{y}+\frac{y}{z}-\frac{z}{x}$. $M_1(-1,1,1)$, $M_2(2,3,4)$.

1.28. u(M) = $x^3 + xy^2 - 6xyz$, $M_1(1,3,-5)$, $M_2(4,2,-2)$.

1.29. u(M) = $\frac{x}{y}-\frac{y}{z}-\frac{x}{z}$, $M_1(2,2,2), M_2(-3,4,1)$.

1.30. u(M) =e^{x-yz}, $M_1(1,0,3), M_2(2,-4,5)$.

2. S sirt bo‘yicha I – tur sirt integrali hisoblansin, bu erda S - koordinata tekisliklari bilan kesilgan (p) tekislikning qismi.

2.1. $\iint_S (2x+3y+2z)dS$, (p): x+3y+z =3. (Javob:$15\sqrt{11}/2$)

2.2. $\iint_S (2+y-7x+9z)dS$, (p): 2x-y -2z = -2. (Javob: 12)

2.3. $\iint_S (6x+y+4z)dS$, (p): 3x+3y+z =3. (Javob: $19\sqrt{19}/6$)

2.4. $\iint_S (x+2y+3z)dS$, (p): x+y+z =2. (Javob: $8\sqrt{3}$)

2.5. $\iint_S (3x-2y+6z)dS$, (p): 2x+y+2z = 2. (Javob: $5/2$)

2.6. $\iint_S (2x+5y-z)dS$, (p): x+2y+z = 2. (Javob: $7\sqrt{6}/3$)

2.7. $\iint_S (5x-8y-z)dS$, (p): 2x -3y+z =6. (Javob: $25\sqrt{14}$)

2.8. $\iint_S (3y-x-z)dS$, (p): x - y+z = 2. (Javob: -$20\sqrt{3}/3$)

2.9. $\iint_S (3y-2x-2z)dS$, (p): 2x – y - 2z = -2. (Javob: 3)

2.10. $\iint_S (2x-3y+z)dS$, (p): x+2y+z = 2. (Javob: $\sqrt{6}$)

2.11. $\iint_S (5x+y-z)dS$, (p): x+2y+ 2z = 2. (Javob: 5)

2.12. $\iint_S (3x+2y+2z)dS$, (p): 3x+2y+ 2z = 6. (Javob: $9\sqrt{17}$)

2.13. $\iint_S (2x+3y-z)dS$, (p): 2x+y+z = 2. (Javob: $2\sqrt{6}$)

2.14. . $\iint_S (9x+2y+z)dS$, (p): x+3y+z =3. (Javob: $40\sqrt{6}$)

2.15. $\iint_S (5x+8y+8z)dS$, (p): x+4y+ 2z = 8. (Javob: $96\sqrt{21}$)

2.16. $\iint_S (4y-x+4z)dS$, (p): x - 2y+2z = 2. (Javob: -1)

2.17. $\iint_S (7x+y+2z)dS$, (p): 3x - 2y+ 2z = 6. (Javob: $17\sqrt{17}/2$)

2.18. $\iint_S (2x+3y+z)dS$, (p): 2x +3y+ z = 6. (Javob: $18\sqrt{14}$)

2.19. $\iint_S (4x-\ y+z)dS$, (p): x - y+ z = 2. (Javob: $8\sqrt{3}$)

2.20. $\iint_S (6x-\ y+8z)dS$, (p): x + y+ 2z = 2. (Javob: $6\sqrt{6}$)

2.21. $\iint_S (4x-4y-z)dS$, (p): x + 2y+ 2z = 4. (Javob: 44)

2.22. $\iint_S (2x+5y+z)dS$, (p): x +y+ 2z = 2. (Javob: $5\sqrt{6}$)

2.23. $\iint_S (7x+y+2z)dS$, (p): 3x - 2y+ 2z = 6. (Javob: 44)

2.24. $\iint_S (5x+2y+2z)dS$, (p): x + 2y+ z = 2. (Javob: $16\sqrt{3}/6$)

2.25. $\iint_S (2x+5y+10z)dS$, (p): 2x + y+ 3z = 6. (Javob: $56\sqrt{14}$)

2.26. $\iint_S (2x+15y+z)dS$, (p): x+ 2y+ 2z = 2. (Javob: 10)

2.27. $\iint_S (3x+10y-z)dS$, (p): x +3y+ 2z = 6. (Javob: $35\sqrt{14}$)

2.28. . $\iint_S (2x+3y+z)dS,$ (p): 2x + 2y+ z = 2. (Javob: $7/6$)

2.29. $\iint_S (5x-y+5z)dS,$ (p): 3x + 2y+ z = 6. (Javob: $37\sqrt{14}$)

2.30. $\iint_S (\mathrm{x}+3\mathrm{y}+2\mathrm{z})\mathrm{dS},$ (p): 2x +y+ 2z = 2. (Javob: $9/2$)

3. II – tur egri chiziqli integral hisoblansin.

3.1. $\iint_S (y^2+z^2)dxdz,$ bu erda S – x = 0 tekislik bilan kesilgan x = 9- $-y^2 - z^2$ paraboloid sirtining qismi. ($\overrightarrow{n}$ - i ort bilan o‘tkir burchak hosil qiluvchi normal vektor.) (Javob: 81π /2)

3.2. $\iint_S \mathrm{z}^2\mathrm{dxdy},$ bu erda S - $x^2+y^2+2z^2=2$ ellipsoid sirtining tashqi tomoni. (Javob: 0.)

3.3. $\iint_S zdxdy+ydxdz+xdydz,$ bu erda S - x=0, y = 0, z = 0, x = 1, y = 1, z = 1 tekisliklar bilan chegaralangan kub sirtining tashqi tomoni. (Javob: 3.)

3.4. $\iint_S (z+1)dxdy,$ bu S - $x^2+y^2+z^2=16$ sfera sirtining tashqi tomoni. (Javob: 256π /3.)

3.5. $\iint_S yzdydz+xzdxdz+xydxdy,$ bu erda S – koordinata tekisliklari bilan kesilgan x+ y + z = 4 tekislikning yuqori tomoni. (Javob: 32.)

3.6. $\iint_S x^2dydz+y^2dxdz+z^2dxdy,$ bu erda S – birinchi oktantda yotuvchi $x^2+y^2+z^2=16$ sferaning tashqi tomoni. (Javob: 96π .)

3.7. $\iint_S xdydz+ydxdz+zdxdy,$ bu erda S - $x^2+y^2+z^2=1$ sferaning tashqi tomoni. (Javob: (4π .)

3.8. $\iint_S xzdxdy+xydydz+yzdxdz,$ bu erda S - koordinata tekisliklari bilan kesilgan, x+ y + z = 1 tekislikning yuqori qismi. (Javob: 1/8.)

3.9. $\iint_S yzdxdy+xzdydz+xydxdz,$ bu erda S - z = 0 va z = 5 tekisliklar bilan kesilgan $x^2+y^2=1$ silindrning tashqi sirti. (Javob: 25π .)

3.10. $\iint_S y^2zdxdy+xzdydz+x^2ydxdz,$ bu erda S – $x^2+y^2=1$ silindr bilan kesilgan z = x^2+y^2 paraboloid sirtining qismi. ($\overrightarrow{\boldsymbol{n}}$ - $\boldsymbol{k}$ ort bilan o‘tmas burchak hosil qiluvchi normal vektor.) (Javob: $\pi/8$.)

3.11. $\iint_S (x^2+y^2)zdxdy$, bu erda S – $x^2+y^2+z^2=9$ sferaning quyi yarim qismining tashqi tomoni. (Javob: $324\pi/5$.)

3.12. $\iint_S x^2dydz + y^2dxdy$, bu erda S – z = 0 va z = 1 tekisliklar orasida yotuvchi $z^2=x^2+y^2$ konus sirtining qismi. ($\overrightarrow{\boldsymbol{n}}$ - $\boldsymbol{k}$ ort bilan o‘tmas burchak hosil qiluvchi normal vektor.) (Javob: - $\pi/2$.)

3.13. $\iint_S (2y^2-z)dxdy$, bu erda S – z = 2 tekislik bilan kesilgan $z^2=x^2+y^2$ paraboloid sirtining qismi. ($\overrightarrow{\boldsymbol{n}}$ - $\boldsymbol{k}$ ort bilan o‘tmas burchak hosil qiluvchi normal vektor.) (Javob: 0.)

3.14. $\iint_S \frac{dxdy}{\sqrt{x^2+y^2+1}}$, bu erda S – z = 0, z = $\sqrt{3}$ tekisliklar bilan kesilgan $x^2+y^2=z^2+1$ giperboloid sirtining qismi. ($\overrightarrow{\boldsymbol{n}}$ - $\boldsymbol{k}$ ort bilan o‘tmas burchak hosil qiluvchi normal vektor.) (Javob: - $2\sqrt{3}\pi$.)

3.15. $\iint_S xydydz + yzdxdz + xzdxdy$, bu erda S – birinchi oktantda yotuvchi $x^2+y^2+z^2=1$sferaning tashqi qismi. (Javob: $3\pi/16.$)

3.16. $\iint_S x^2dydz + zdxdy$, bu erda S – z = 4 tekislik bilan kesilgan $z^2=x^2+y^2$ paraboloid sirtining qismi. ($\overrightarrow{\boldsymbol{n}}$ - $\boldsymbol{k}$ ort bilan o‘tmas burchak hosil qiluvchi normal vektor.) (Javob: 8π.)

3.17. $\iint_S x^2dydz + y^2dxdz - zdxdy$, bu erda S – z = 0 va z = 3 tekisliklar bilan kesilgan z = x^2+y^2 konus sirtining qismi. ($\overrightarrow{\boldsymbol{n}}$ - $\boldsymbol{k}$ ort bilan o‘tkir burchak hosil qiluvchi normal vektor.) (Javob: - 18π.)

3.18. $\iint_S x^2dydz - z^2dxdz + zdxdy$, bu erda S – z = 0 tekislik bilan kesilgan $z=3-x^2+y^2$ paraboloid sirtining qismi. ($\overrightarrow{\boldsymbol{n}}$ - $\boldsymbol{k}$ ort bilan o‘tmas burchak hosil qiluvchi normal vektor.) (Javob: $9\pi/2$.)

3.19. $\iint_S yzdydz - x^2dxdz - y^2dxdy$, bu erda S - y = 0 va y = 1 tekisliklar bilan kesilgan x^2 + $z^2=y^2$ konus sirtining qismi. ($\overrightarrow{\boldsymbol{n}}$ - $\boldsymbol{k}$ ort bilan o‘tmas burchak hosil qiluvchi normal vektor.) (Javob: $\pi/4$.)

3.20. $\iint_S x^2dydz + 2y^2dxdz - zdxdy$, bu erda S – z = 4 tekislik bilan kesilgan z = $x^2 + y^2$ paraboloid sirtining qismi. ($\overrightarrow{\boldsymbol{n}}$ - $\boldsymbol{k}$ ort bilan o‘tkir burchak hosil qiluvchi normal vektor.) (Javob: -$\pi/2$.)

3.21. $\iint_S 2xdydz + (1 - z)dxdy$, bu erda S - z = 0 va z = 1 tekisliklar bilan kesilgan $x^2 + y^2 = 4$ silindrning tashqi tomoni (Javob: -8π .)

3.22. $\iint_S 2xdydz - ydxdz + zdxdy$, bu erda S – 3z = $x^2 + y^2$ paraboloid va z = $\sqrt{4 - x^2 - y^2}$ yarim sferadan tashkil topgan yopiq sirtning tashqi tomoni. (Javob: 19$\pi/3$.)

3.23. $\iint_S 4xdydz + 2ydxdz - zdxdy$, bu erda S – $x^2 + y^2 + z^2 = 4$ sferaning tashqi tomoni (Javob: 160$\pi/3$.)

3.24. $\iint_S (x + z)dydz + (z + y)dxdy$, bu erda S - z = 0 va z = 2 tekisliklar bilan kesilgan $x^2 + y^2 = 1$ silindrning ichki tomoni. (Javob: 2π .)

3.25. $\iint_S 3xdydz - ydxdz - zdxdy$, bu erda S – z = 0 tekislik bilan kesilgan $9 - z = x^2 + y^2$ paraboloid sirtining qismi. ($\overrightarrow{\boldsymbol{n}}$ - $\boldsymbol{k}$ ort bilan o‘tkir burchak hosil qiluvchi normal vektor.) (Javob: 243$\pi/2$.)

3.26. $\iint_S (y - x)dydz + (z - y)dxdz + (x - z)dxdy$, bu erda S – x = 1 tekislik va x^2 = $y^2 + z^2$ konusdan tashkil topgan yopiq sirtning ichki tomoni. (Javob: π.)

3.27. $\iint_S 3x^2dydz - y^2dxdz - zdxdy$, bu erda S – 1– z= $x^2 + y^2$ paraboloid sirtining qismi ($\overrightarrow{\boldsymbol{n}}$ - $\boldsymbol{k}$ ort bilan o‘tkir burchak hosil qiluvchi normal vektor.) (Javob: -$\pi/2$.)

3.28. $\iint_S (1 + 2x^2)dydz + y^2dxdz + zdxdy$, bu erda S – z = 0 va z = 4 tekisliklar bilan kesilgan $x^2 + y^2$ =z^2 konus sirtining qismi. ($\overrightarrow{\boldsymbol{n}}$ - $\boldsymbol{k}$ ort bilan o‘tmas burchak hosil qiluvchi normal vektor.) (Javob: 128$\pi/3$.)

3.29. $\iint_S x^2dydz + z^2dxdz + ydxdy$, bu erda S – z = 0 tekislik bilan kesilgan $x^2 + y^2 = 4 - z$ paraboloid sirtining qismi. ($\overrightarrow{\boldsymbol{n}}$ - $\boldsymbol{k}$ ort bilan o‘tkir burchak hosil qiluvchi normal vektor.) (Javob: 0.)

3.30. $\iint_S (y^2+x^2)dydz - y^2dxdz + 2yz^2dxdy$, bu erda S – y = 0 va y = 1 tekisliklar bilan kesilgan $x^2+z^2 = y^2$ konus sirtining qismi. ($\overrightarrow{\boldsymbol{n}}$ - $\mathbf{j}$ ort bilan o‘tmas burchak hosil qiluvchi normal vektor.) (Javob: $\pi/2$.)

4. *a(M)* vektor maydonining tekisligini (*r*) va koordinata tekisliklarini hosil qilgan piramidaning tashqi yuzasi orqali oqimini ikki yo‘l bilan hisoblang: a) oqim ta’rifidan foydalanib; b) Ostrogradskiy-Gauss formulasidan foydalangan holda

4.1. *a(M)=3x**i**+(y+z)**j**+(x-z)**k**, (p): x+3y+z=3* (Javob: 9/2)

4.2. *a(M)=(3x-1)**i**+(y-x+z)**j**+4z**k**, (p): 2x-y-2z=2* (Javob: 8/3)

4.3. *a(M)=x**i**+(x+z)**j**+(y+z)**k**, (p): 3x+3y+z=3* (Javob: 1)

4.4. *a(M)=(x+z)**i**+(z-x)**j**+(x+2y+z)**k**, (p): x+y+z=2* (Javob: 8/3)

4.5. *a(M)=(y+2z)**i**+(x+2z)**j**+(x-2y)**k**, (p): 2x+y+2z=2* (Javob: 0)

4.6. *a(M)=(x+z)**i**+2y**j**+(x+y-z)**k**, (p): x+2y+z=2* (Javob: 4/3)

4.7. *a(M)=(3x-y)**i**+(2y+z)**j**+(2z-x)**k**, (p): 2x-3y+z=6* (Javob: 42)

4.8. *a(M)=(2y+z)**i**+(x-y)**j**-2z)**k**, (p): x-y+z=2* (Javob: -4)

4.9. *a(M)=(x+y)**i**+3y**j+(y**-z)**k**, (p): 2x-y-2z=-2* (Javob: -1)

4.10. *a(M)=(x+y-z)**i**+2y**j**+(x+2z)**k**, (p): x+2y+z=2* (Javob: 2/3)

4.11. *a(M)=(y-z)**i**+(2x+y)**j**+z**k**, (p): 2x+y+z=2* (Javob: 4/3)

4.12. *a(M)=x**i**+(y-2z)**j**+(2x-y+2z)**k**, (p): x+2y+2z=2* (Javob: 4/3)

4.13. *a(M)=(x+2z)**i**+(y-3z)**j**-z**k**, (p): 3x+2y+2z=6* (Javob: 9)

4.14. *a(M)=4x**i**+(x-y-z)**j**+(3y+2z)**k**, (p): 2x+y+z=4* (Javob: 80/3)

4.15. *a(M)=(2z-x)**i**+(x+2y)**j**+3z**k**, (p): x+4y+2z=8* (Javob: 128/3)

4.16. *a(M)=4z**i**+(x-y-z)**j**+(3y+z)**k**, (p): x-2y+2z=2* (Javob: 0)

4.17. *a(M)=(x+y)**i**+(y+z)**j**+2(z+x)**k**, (p): 3x-2y+2z=6* (Javob: 12)

4.18. *a(M)=(x+y+z)**i**+2z**j**+(y-7z)**k**, (p): 2x+3y+z=6* (Javob: -36)

4.19. *a(M)=(2x-z)**i**+(y-x)**j**+(x+2z)**k**, (p): x-y+z=2* (Javob: 20/3)

4.20. *a(M)=(2y-z)**i**+(x+y)**j**+x**k**, (p): x+2y+2z=4* (Javob: 8/3)

4.21. *a(M)=(2z-x)**i**+(x-y)**j**+(3x+z)**k**, (p): x+y+2z=2* (Javob: -2/3)

4.22. *a(M)=(x+z)**i**+(x+3y)**j**+y**k**, (p): x+y+2z=2* (Javob: 8/3)

4.23. *a(M)=(x+z)**i**+z**j**+(2x-y)**k**, (p): 2x+2y+z=4* (Javob: 8/3)

4.24. *a(M)=(3x+y)**i**+(x+z)**j**+y**k**, (p): x+2y+z=2* (Javob: 2)

4.25. *a(M)=(y+z)**i**+(2x-z)**j**+(y+3z)**k**, (p): 2x+y+3z=6* (Javob: 18)

4.26. *a(M)=(y+z)**i**+(x+6y)**j**+y**k**, (p): x+2y+z=2* (Javob: 2)

4.27. *a(M)=(2y-z)**i**+(x+2y)**j**+y**k**, (p): x+3y+2z=6* (Javob: 12)

4.28. *a(M)=(y+z)**i**+x**j**+(y-2z)**k**, (p): 2x+2y+z=2* (Javob: -2/3)

4.29. *a(M)=(x+z)**i**+z**j**+(2x-y)**k**, (p): 3x+2y+z=6* (Javob: 6)

4.30. *a(M)=z**i**+(x+y)**j**+y**k**, (p): 2x+y+2z=2* (Javob: 1/3)

Namunaviy variantning yechimi

1. $u(M)=\frac{\sqrt{x}}{z}-\frac{\sqrt{y}}{x}+2xyz$ funksiya va $M_1(1,1,-1)$, $M_2(-2,-1,1)$ nuqtalar berilgan. 1) $\overrightarrow{M_1M_2}$ vektor yo‘nalishi bo‘yicha M_1 nuqtada funksiyaning hosilasini; 2) grad $u(M_1)$ ni hisoblang.

►1. $u(M)=u(x,y,z)$ funksiyaning hosilasini $\overrightarrow{M_1M_2}=(-3,-2,2)$ vektor yo‘nalishi bo‘yicha M_1 nuqtada hosilasini topamiz:

$$\frac{du(M_1)}{\partial\overrightarrow{M_1M_2}}=\frac{\partial u(M)}{\partial x}\bigg|_{M_1}\cdot\cos\alpha+\frac{\partial u(M)}{\partial x}\bigg|_{M_1}\cdot\cos\beta+\frac{\partial u(M)}{\partial z}\bigg|_{M_1}\cdot\cos\gamma,$$

$$\frac{\partial u(M)}{\partial x}=\frac{1}{2z\sqrt{x}}+\frac{\sqrt{y}}{x^2}+2yz,\quad \frac{\partial u(M)}{\partial x}\bigg|_{M_1}=-\frac{3}{2},$$

$$\frac{\partial u(M)}{\partial y}=\frac{1}{2z\sqrt{y}}+2xz,\quad \frac{\partial u(M)}{\partial y}\bigg|_{M_1}=-\frac{5}{2},$$

$$\frac{\partial u(M)}{\partial z}=\frac{\sqrt{x}}{z^2}+2xy,\quad \frac{\partial u(M)}{\partial z}\bigg|_{M_1}=1,$$

$$\cos\alpha=-\frac{3}{\sqrt{17}},\quad \cos\beta=-\frac{2}{\sqrt{17}},\quad \cos\gamma=\frac{2}{\sqrt{17}},$$

$$\frac{du(M_1)}{\partial\overrightarrow{M_1M_2}}=-\frac{3}{2}\left(-\frac{3}{\sqrt{17}}\right)-\frac{5}{2}\left(-\frac{2}{\sqrt{17}}\right)+1\cdot\frac{2}{\sqrt{17}}=\frac{23}{2\sqrt{17}}.$$

2. Ta’rifga ko‘ra

$$\overrightarrow{grad}\,\mathrm{u}(M_1)=\frac{\partial u}{\partial x}\bigg|_{M_1}\boldsymbol{i}+\frac{\partial u}{\partial y}\bigg|_{M_1}\boldsymbol{j}+\frac{\partial u}{\partial z}\bigg|_{M_1}\boldsymbol{k}=-\frac{3}{2}\boldsymbol{i}-\frac{5}{2}\boldsymbol{j}+\boldsymbol{k}\,.$$ ◄

2. $\iint(3x-y+z)dS$ birinchi turdagi sirt integralini S sirt bo‘yicha hisoblang, bu erda S - koordinata tekisliklari bilan kesilgan $x+z-2y=2$ tekislikni bir qismi.

►$z=2-x+2y$, $z'_x=-1$, $z'_y=2$ tekislik tenglamasidan

$$dS=\sqrt{1+z'^2_x+z'^2_y}dxdy=\sqrt{6}dxdy$$

Biz D soha bo‘yicha hisoblashni kamaytirish uchun ikki karrali sirt integralni birlashtiramiz, bu erda D – *Oxu* tekislikda S sirtning proeksiyasi bo‘lgan *AOV* uchburchak. U holda (15.13 rasm)

$$\iint_S(3x-y+z)dS==\iint_D(3x-y+2-x+2y)\sqrt{6}dxdy=$$

$$=\iint_D(2x+y+2)\sqrt{6}\,dxdy=\sqrt{6}\int_{-1}^{0}dy\int_{0}^{2+2y}(2x+y+2)dx=$$

$$=\sqrt{6}\int_{-1}^{0}(x^2+(y+2)x)\Big|_{0}^{2+2y}dy=$$

$$=\sqrt{6}\int_{-1}^{0}(4+8y+4y^2+2y+2y^2+4+4y)dy=$$

$$=\sqrt{6}\int_{-1}^{0}(6y^2+14y+8)dy=\sqrt{6}\,(2y^3+7y^2+8y)\Big|_{-1}^{0}=3\sqrt{6}$$

bo‘ladi. ◄

3. Ikkinchi tartibli sirt integralini hisoblang

$\iint_S(x^2+z^2)dxdz+x^2dydz-2z^2dxdy$, bu erda *S:* $4-y=x^2+z^2$ paraboloidni kesib o‘tuvchi $y=0$ tekislikning bir qismi (*n* - vektor j birligi bilan o‘tkir burchak hosil qiluvchi normal vektor)

►Ushbu sirt integralini koordinatalar ustida uchta integralning yig’indisi deb faraz qilamiz va paraboloid tenglamasidan foydalanib, ularning har birini D soha
bo‘yicha ikki karrali integralga aylantiramiz.

$$I = \iint_S (x^2+z^2)dxdz + x^2dydz - 2z^2dxdy = I_1 + I_2 + I_3,$$

bu erda

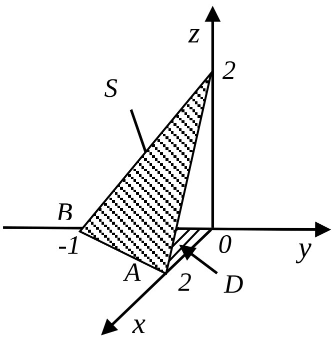
$$I_1 = \iint\limits_S (x^2+z^2)dxdz, I_2 = \iint\limits_S x^2\, dydz,\ \ I_3 = \iint\limits_S -2\, z^2 dxdy$$

I_1, I_2, I_3 integrallarni ketma-ket hisoblaymiz:

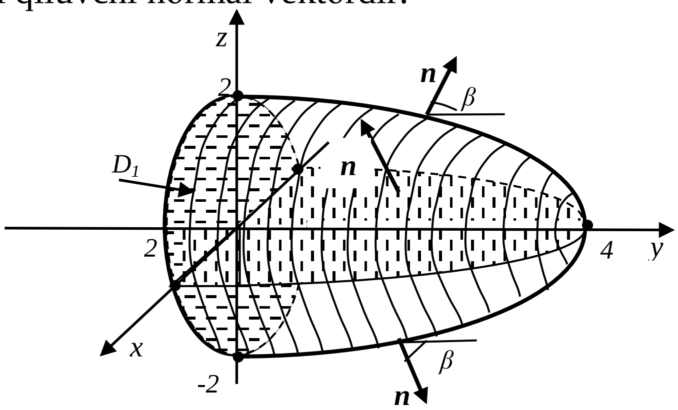
$$I_1 = \iint\limits_S (x^2+z^2)dxdz = \ |x = \rho cos\varphi, z = \rho sin\varphi, dxdz = \rho d\rho d\varphi|$$

$$= \int\limits_0^{2\pi} d\varphi \int\limits_0^2 \rho^3 d\rho = \varphi| \begin{matrix} 2\pi \\ 0 \end{matrix} \cdot \frac{\rho^4}{4}\Big|\begin{matrix} 2 \\ 0 \end{matrix} = 8\pi,$$

bu erda D soha – *Oxz* tekislikda $x^2+z^2 = 4, y = 0$ paraboloid sirtning proeksiyasidir. I_1 integraldan oldin “+” belgisi qo‘yiladi, chunki n - *Ou* o‘qi bilan tekislik β o‘tkir burchak hosil qiluvchi normal vektordir.



15.13-rasm



15.14- rasm

Keyin

$$I_2 = \iint\limits_S x^2 dydz = \iint\limits_{D_2} \left(\sqrt{4-y-z^2}\right)^2 dydz - \iint\limits_{D_2} (-\sqrt{4-y-z^2})^2 dydz =$$

$$= \iint\limits_{D_2} (4-y-z^2)dydz - \iint\limits_{D_2} (4-y-z^2)dydz = 0.$$

Koordinata tekisligi *Oyz* paraboloid sirtini ikki qism $x = \sqrt{4-y-z^2}$ va $x = -\sqrt{4-y-z^2}$ ga bo‘ladi. Koordinata tekisligi *Oyz* da D_2 sohaga proeksiyasi tushadi. Shuning uchun I_2 integralni ikkita integralni yig’indisi ko‘rinishda tasvirlash mumkin, birinchi integral oldiga “+” belgisi qo‘yiladi, *n* -

Ox o‘qi bilan o‘tkir burchak hosil qiluvchi paraboloid sirtining bir qismi bo‘lgan normal vektor, ikkinchi integral oldiga “-” belgisi qo‘yiladi, chunki normal vektor *n* – *Ox* o‘qi bilan o‘tmas burchak tashkil qiladi.

Xuddi shu kabi

$$I_3 = \iint_S -2z^2 dxdy =$$

$$= -2\iint_{D_3} \left(\sqrt{4-y-z^2}\right)^2 dxdy + 2\iint_{D_3} (-\sqrt{4-y-z^2})^2 dxdy = 0$$

Shunday qilib,

$$I = \iint_S (x^2+z^2)dxdz + x^2 dydz - 2z^2 dxdy = 8\pi$$

ni hosil qilamiz. ◄

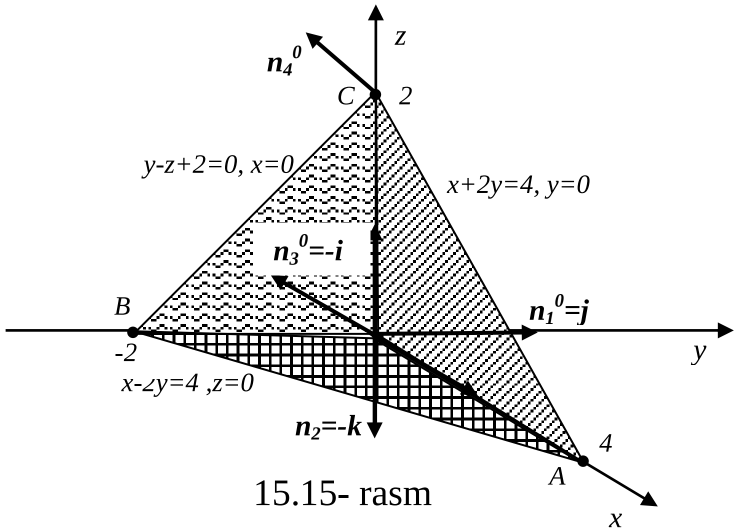
4.Piramidaning tashqi yuzasi orqali $a(M)=(x+z)\boldsymbol{i}+(2y-x)\boldsymbol{j}+z\boldsymbol{k}$, *(p):* $x-2y+2z=4$ vektor maydonining oqimini ikki xil usulda hisoblang: 1) Oqim ta’rifidan foydalanib; 2) Ostrogradskiy – Gauss formulasi yordamida.

► 1. Sirt integrali yordamida vektor maydonining oqimini hisoblaymiz

$$P = \iint_S a \cdot n^0 dS,$$

bu erda *S* - *ABCO* piramida sirtining tashqi tomoni(15.15 – rasm)

Boshida, biz piramidaning har bir to‘rt tomoni orqali oqimni hisoblaymiz. *AOC* tomoni *u=0* tekislikda yotibdi.



15.15- rasm

Bu tomonning normal vektori $\boldsymbol{n^0} = \boldsymbol{j}, dS = dxdz$. U holda *a(M)* vektorli maydon oqimi *AOS* tomoni orqali

$$P_1 = -\iint\limits_{\Delta AOC} xdS = -\iint\limits_{\Delta AOC} xdxdz =$$

$$= -\int\limits_0^4 xdx \int\limits_0^{2-\frac{x}{2}} dz = \int\limits_0^4 x\left(2 - \frac{x}{2}\right)dx = -\left(x^2 - \frac{x^3}{6}\right)\Bigg|_0^4 = -\frac{16}{3}$$

bo‘ladi.

AOB tomoni *z=0* tekislikda yotibdi, bu tomonning normal vektori $\boldsymbol{n_2^0} = -\boldsymbol{k}, dS = dxdy,$

$$P_2 = -\iint\limits_{\Delta AOB} 0 \cdot dxdy = 0.$$

BOC tomon *x=0* tekislikda yotibdi, bu tomonning normal vektori $n_3^0 = -i, dS = dydz,$

$$P_3 = -\iint\limits_{\Delta BOC} zdydz = -\iint\limits_0^2 zdz \iint\limits_{z-2}^0 dy = -\int\limits_0^2 z(-z+2)dz =$$

$$= -\left(-\frac{z^3}{3} + z^2\right)\Bigg|_0^2 = -\frac{4}{3}$$

Va nihoyat, *ABC* tomon $x - 2y + 2z - 4 = 0$ tekislikda yotibdi, bu tomonning normal vektori

$$\boldsymbol{n_4^0} = \frac{\boldsymbol{i} - 2\boldsymbol{j} + 2\boldsymbol{k}}{\sqrt{1+4+4}} = \frac{\boldsymbol{i} - 2\boldsymbol{j} + 2\boldsymbol{k}}{3}$$

$$dS = \sqrt{1 + {z'_x}^2 + {z'_y}^2}dxdy, \quad z = -\frac{1}{2}x + y + 2$$

$$z'_x = -\frac{1}{2}, \qquad z'_y = 1$$

Shuning uchun

$$\text{dS} = \sqrt{1 + \tfrac{1}{4} + 1}dxdy = \tfrac{3}{2}dxdy,$$

$$P_4 = \frac{1}{3} \cdot \frac{3}{2} \iint\limits_{\Delta ABC} \big((x+z) - 2(2y - x) + 27\big) dxdy =$$

$$= \frac{1}{2} \iint\limits_{\Delta ABC} (x + z - 4y + 2x + 2z) dxdy = \frac{1}{2} \iint\limits_{\Delta ABC} (3x - 4y + 3z) dxdy =$$

$$= \frac{1}{2} \iint\limits_{\Delta AOB} \left(3x - 4y - \frac{3}{2}x + 3y + 6\right) dxdy = \frac{1}{2} \iint\limits_{\Delta AOB} \left(\frac{3}{2}x - y + 6\right) dxdy =$$

$$= \frac{1}{2} \int\limits_{-2}^{0} dy \int\limits_{0}^{2y+4} \left(\frac{3}{2}x - y + 6\right) dx = \frac{1}{2} \int\limits_{-2}^{0} dy \left(\frac{3}{4}x^2 + (6 - y)x\right) \Bigg|_{0}^{2y+4} =$$

$$= \frac{1}{2} \int\limits_{-2}^{0} \left(\frac{3}{4}(2y+4)^2 + (6 - y)(2y + 4)dy\right) dy =$$

$$= \frac{1}{2} \int\limits_{-2}^{0} (3(y^2 + 4y + 4) + 12y + 24 - 2y^2 - 4y) dy =$$

$$= \frac{1}{2} \int\limits_{-2}^{0} (y^2 + 20y + 36) dy = \frac{1}{2} \left(\frac{y^3}{3} + 10y^2 + 36y\right) \Bigg|_{-2}^{0} = \frac{52}{3}.$$

Keyinchalik, biz *ABCO* piramidaning butun yuzasi orqali oqimni topamiz.

$$P = P_1 + P_2 + P_3 + P_4 = \frac{32}{3}$$

2. Ostrogradskiy-Gauss formulasi bo‘yicha *ABCO* piramida sirti orqali oqimni hisoblang:

$$P = \iiint\limits_{V} \left(\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}\right) dxdydz.$$

$$\frac{\partial P}{\partial x} = \frac{\partial (x+z)}{\partial x} = 1, \; \frac{\partial Q}{\partial y} = \frac{\partial (2y - x)}{\partial y} = 2, \; \frac{\partial R}{\partial z} = \frac{\partial z}{\partial z} = 1$$

tengliklarga ega bo‘lamiz.

$$\iiint_V dxdydz$$

integral *AVSO* to‘g’ri burchakli piramidaning hajmiga teng bo‘lgani uchun

$$P = \iiint\limits_{V} (1 + 2 + 1) dxdydz = 4 \iiint\limits_{V} dxdydz = \frac{32}{3}$$

bo‘ladi. ◄

15.2 IUT

1. $a(M)$ vektor maydonning $(p): Ax + By + Cz = D$ tekislik bilan koordinata tekisliklari kesishishi natijasida hosil bo'lgan uchburchak konturi bo'ylab *n*(A, B, C) normal vektorning musbat aylanishidagi sirkulyasiyasini ikki usulda toping: 1) sirkulyasiya tarifidan foydalanib 2) Stoks formulasi yordamida.

1.1 $a(M) = zi + (x+y)j + yk,\ (p): 2x + y + 2z = 2\ (Javob: 5/2)$

1.2 $a(M) = (x+z)i + zj + (2x-y)k, (p): 3x + 2y + z = 6$
$(Javob: -24)$

1.3 $a(M) = (y+z)i + xj + (y-2z)k, (p):\ 2x + 2y + z = 2$
$(Javob: 2)$

1.4 $a(M) = (2y-z)i + (x+2y)j + yk,\ (p): x + 3y + 2z = 3$
$(Javob: -12)$

1.5 $a(M) = (x+z)i + (x+6y)j + yk,\ (p): x + 2y + 2z = 2$
$(Javob: 3/2)$

1.6 $a(M) = (y+z)i + (2x-z)j + (y+3z)k,$
$(p): 2x + y + 3z = 6\ (Javob: 24)$

1.7 $a(M) = (3x+y)i + (x+z)j + yk,\ (p): x + 2y + z = 2$
$(Javob: 0)$

1.8 $a(M) = (x+z)i + zj + (2x-y)k,\ (p): 2x + 2y + z = 4$
$(Javob: -12)$

1.9 $a(M) = (x+z)i + (x+3y)j + yk,\ (p): x + y + 2z = 2$
$(Javob: 4)$

1.10 $a(M) = (2y-z)i + (x+y)j + xk,\ (p): x + 2y + 2z = 4$
$(Javob: -12)$

1.11 $a(M) = (2z-x)i + (x-y)j + (3x+z)k,$
$(p): x + y + 2z = 2\ (Javob: 1)$

1.12 $a(M) = (2x-z)i + (y-x)j + (x+2z)k,$
$(p): x - y + z = 2\ (Javob: 2)$

1.13 $a(M) = (x+y+z)i + 2zj + (y-7z)k,$

$(p): 2x + 3y + z = 6 \quad (Javob: 0)$

1.14 $a(M) = (x + y)i + (y + z)j + 2(x + z)k,$

$(p): 3x - 2y + 2z = 6 \quad (Javob: -3/2)$

1.15 $a(M) = 4zi + (x - y - z)j + (3y + z)k,$

$(p): x - 2y + 2z = 2 \quad (Javob: -1)$

1.16 $a(M) = (2z - x)i + (x + 2y)j + 3zk, \quad (p): x + 4y + 2z = 8$ $(Javob: 40)$

1.17 $a(M) = 4xi + (x - y - z)j + (3y + 2z)k,$

$(p): 2x + y + z = 4 \quad (Javob: 36)$

1.18 $a(M) = (x + 2z)i + (y - 3z)j + zk,$

$(p): 3x + 2y + 2z = 6 \ (Javob: 39/2)$

1.19 $a(M) = xi + (y - 2z)j + (2x - y + 2z)k,$

$(p): x + 2y + 2z = 2 \ (Javob: -3/2)$

1.20 $a(M) = (y - z)i + (2x + y)j + zk, \quad (p): 2x + y + z = 2$ $(Javob: 0)$

1.21 $a(M) = (x + y - z)i - 2yj + (x + 2z)k,$

$(p): x + 2y + z = 2 \quad (Javob: -5)$

1.22 $a(M) = (x + y)i + 3yj + (y - z)k, \quad (p): 2x - y - 2z = -2$ $(Javob: -2)$

1.23 $a(M) = (2y + z)i + (x - y)j + 2zk, \quad (p): x - y + z = 2$ $(Javob: -4)$

1.24 $(M) = (3x - y)i + (2y + z)j + (2z - x)k,$

$(p): 2x - 3y + z = 6 \quad (Javob: 12)$

1.25 $a(M) = (x + z)i + 2yj + (x + y - z)k, \quad (p): x + 2y + z = 2$ $(Javob: 1)$

1.26 $a(M) = (y + 2z)i + (x + 2z)j + (x - 2y)k,$

$(p): 2x + y + 2z = 2 \quad (Javob: -7/2)$

1.27 $a(M) = (x + z)i + (z - x)j + (x + 2y + z)k,$

$(p): x + y + z = 2. (Javob: 0)$

1.28 $a(M) = xi + (x+z)j + (y+z)k,\ \ (p): 3x + 3y + z = 3$
$(Javob: 3/2)$

1.29 $a(M) = (3x-1)i + (y-x+z)j + 4zk,$
$(p): 2x - y - 2z = -2\ \ (Javob: 0)$

1.30 $a(M) = 3xi + (y+z)j + (x-zz)k,\ \ (p): x + 3y + z = 3$
$(Javob: -6)$

2. $u(x,y,z)$ funksiyaning $M_0(x_0, y_0, z_0)$ nuqtadagi qiymatini va eng katta o‘zgarish yo‘nalashini toping.

2.1. $u(M) = xyz,\ M_0(0,1,-2).$ $(Javob: 2)$

2.2. $u(M) = x^2yz,\ M_0(2,0,2).$ $(Javob: 12)$

2.3. $u(M) = xy^2z,\ M_0(1,-2,0).$ $(Javob: 4)$

2.4. $u(M) = xyz^2,\ M_0(3,0,1,).$ $(Javob: 3)$

2.5. $u(M) = x^2y^2z,\ M_0(-1,0,3).$ $(Javob: 0)$

2.6. $u(M) = x^2yz^2,\ M_0(2,1,-1).$ $(Javob: 4\sqrt{6})$

2.7. $u(M) = xy^2z^2,\ M_0(-2,1,1).$ $(Javob: \sqrt{33})$

2.8. $u(M) = y^2z - x^2,\ M_0(0,1,1).$ $(Javob: \sqrt{5})$

2.9. $u(M) = x^2y - y^2z,\ M_0(0,-2,1).$ $(Javob: 4\sqrt{2})$

2.10. $u(M) = x(y+z),\ M_0(0,1,2).$ $(Javob: 3)$

2.11. $u(M) = xy - xz,\ M_0(-1,2,1).$ $(Javob: \sqrt{3})$

2.12. $u(M) = x^2yz,\ M_0(1,-1,1).$ $(Javob: \sqrt{6})$

2.13. $u(M) = xyz,\ M_0(2,1,0).$ $(Javob: 2)$

2.14. $u(M) = xyz^2,\ M_0(4,0,1).$ $(Javob: 4)$

2.15. $u(M) = 2x^2yz,\ M_0(-3,0,2).$ $(Javob: 36)$

2.16. $u(M) = x^2yz,\ M_0(1,0,4,).$ $(Javob: 4)$

2.17. $u(M) = (x+y)z^2,\ M_0(0,-01,4).$ $(Javob: 24)$

2.18. $u(M) = (x+z)y^2,\ M_0(2,2,2).$ $(Javob: 12\sqrt{2})$

2.19. $u(M) = x^2(y^2+z),\ M_0(4,1,-3).$ $(Javob: 16\sqrt{2})$

2.20. $u(M) = (x^2+z)y^2\ M_0(-4,0,1).$ $(Javob: \sqrt{33})$

2.21. $u(M) = x^2(y + z^2)$, $M_0(3,0,1)$. $(Javob: 21)$

2.22. $u(M) = (x^2 - y)z^2$, $M_0(1,3,0)$. $(Javob: 0)$

2.23. $u(M) = x(y^2 + z^2)$, $M_0(1, -2,1)$. $(Javob: \sqrt{15})$

2.24. $u(M) = x^2 + y^2 - z^2$, $M_0(,0,0,1)$. $(Javob: 2)$

2.25. $u(M) = x^2z - y^2$, $M_0(1,1, -2)$. $(Javob: \sqrt{21})$

2.26. $u(M) = xz^2 + y$, $M_0(2,21)$. $(Javob: 3\sqrt{2})$

2.27. $u(M) = x^2y - z$, $M_0(-2,2,1)$. $(Javob: 9)$

2.28. $u(M) = xy^2 - z$, $M_0(-1,2,1)$. $(Javob: \sqrt{33})$

2.29. $u(M) = y(x + z)$, $M_0(0,2, -2)$. $(Javob: 2\sqrt{3})$

2.30. $u(M) = z(x + y)$, $M_0(1, -1,0)$. $(Javob: 2)$

3. $a(M) = (x, y, z)$ vektor maydonning $M_0(x_0, y_0, z_0)$ nuqtadagi sikulyasiyasi eng katta zichligini toping.

3.1. $a(M) = x^2i - xy^2j - z^2k$, $M_0(0,1, -2)$ $(Javob: 1)$

3.2. $a(M) = xyi + yzj + xzj + xzk$, $M_0(2,0,3)$ $(Javob: \sqrt{13})$

3.3. $a(M) = xy^2i + yz^2j - x^2k$, $M_0(1, -2,0)$ $(Javob: 2\sqrt{5})$

3.4. $a(M) = xzi + zj + yzk$, $M_0(3,0,1)$ $(Javob: 3)$

3.5. $a(M) = xyi + xyzj - xk$, $M_0(-1,0,3)$ $(Javob: \sqrt{2})$

3.6. $a(M) = yzi - z^2j + xyzk$, $M_0(2,1, -1)$ $(Javob: \sqrt{21})$

3.7. $a(M) = y^2i - xyj + z^2k$, $M_0(-2,1,1)$ $(Javob: 1)$

3.8. $a(M) = xzi - xyzj + x^2zk$, $M_0(0,1,1)$ $(Javob: 1)$

3.9. $a(M) = xyi - y^2zj - xzk$, $M_0(0, -2,1)$ $(Javob: \sqrt{17})$

3.10. $a(M) = xzi - yj - zyk$, $M_0(0,1,2)$ $(Javob: 2)$

3.11. $a(M) = yi - xy^2j + z^2k$, $M_0(-1,2,1)$ $(Javob: 8)$

3.12. $a(M) = xyi - xy^2j + z^2k$, $M_0(1, -1,1)$ $(Javob: 2)$

3.13. $a(M) = (x + y)i + yzj + xzk$, $M_0(2,1,0)$ $(Javob: \sqrt{2})$

3.14. $a(M) = xyi - (y + z)j + xzk$, $M_0(4,0,1)$ $(Javob: 3\sqrt{2})$

3.15. $a(M) = xi - zyj + x^2zk$, $M_0(-3,0,2)$ $(Javob: 12)$

3.16. $a(M) = (x + y^2)i + zyj - x^2k$, $M_0(1,0,4)$ $(Javob: 2)$

3.17. $a(M) = xzi - yj + yzk,\ M_0(0,-1,4)$ $(Javob: 4)$

3.18. $a(M) = xyi - zyj + x^2zk,\ M_0(-3,0,2)$ $(Javob: 12)$

3.19. $a(M) = (x+y)i + xyzj - xk,\ M_0(4,1,-3)$ $(Javob: \sqrt{33})$

3.20. $a(M) = (x-y)i + yzj - yk,\ M_0(-4,1,0,)$ $(Javob: \sqrt{5})$

3.21. $a(M) = (y-z)i + z^2j + xyzk,\ M_0(3,0,1)$ $(Javob: 3\sqrt{3})$

3.22. $a(M) = yzi + z^2j - (x+y)zk,\ M_0(1,3,0)$ $(Javob: 3)$

3.23. $a(M) = z^2i - xzj + z^2k,\ M_0(1,-2,1)$ $(Javob: \sqrt{6})$

3.24. $a(M) = xyi + (x-y)j + (y-x)k,\ M_0(0,0,1)(Javob: \sqrt{6})$

3.25. $a(M) = xzi + (x-y)j + x^2zk,\ M_0(1,1,-2)$ $(Javob: \sqrt{26})$

3.26. $a(M) = (x-z)i + xyj + y^2zk,\ M_0(2,2,1)$ $(Javob: \sqrt{21})$

3.27. $a(M) = (x-z)i + xyzj + xk,\ M_0(-2,2,1)$ $(Javob: \sqrt{24})$

3.28. $a(M) = (y-z)i + yj - z^2k,\ M_0(-1,2,1)$ $(Javob: \sqrt{2})$

3.29. $a(M) = (x-y)i - zyj + xzk,\ M_0(0,2,2)$ $(Javob: 2)$

3.30. $a(M) = (x-z)i - yj + xyk,\ M_0(1,-1,0)$ $(Javob: 0)$

4.

$a(M) = (x,,y,z)$ vektor maydonni solenoidal ekanligi yoki bo‘lmasligini aniqlang.

4.1. $a(M) = (\alpha-\beta)xi - (\gamma-\alpha)j + (\beta-\gamma)zk,$

4.2. $a(M) = x^2yi - 2xy^2j + 2xyzk,$

4.3. $a(M) = (yz-2x)i + (xz+2y)j + xyk.$

4.4. $a(M) = (x^2-z^2)i - 3xyj + (x^2+z^2)k.$

4.5. $a(M) = 2xyzi - y(yz+1)j + zk.$

4.6. $a(M) = 2x - 3yi + 2xy - z^2k.$

4.7. $a(M) = (x^2-y^2)i - (y^2-z^2) + (z^2-x^2)k.$

4.8. $a(M) = yzi + (x-y)j - z^2k.$

4.9. $a(M) = (y+z)i + (x+z)j + (x+y)k.$

4.10. $a(M) = 3x^2yi - 2xy^2j - 2xyzk.$

4.11. $a(M) = (x+y)i - 2(y+z)j + (z-x)k.$

$a(M) = (x, , y, z)$ vektor maydonni potensial ekanligi yoki bo‘lmasligini aniqlang.

4.12. $a(M) = (yz - 2x)i + (xz + zy)j + xyk$.

4.13. $a(M) = yzi + xzj - xyk$.

4.14. $a(M) = 6xyi + (3x^2 - 2y)j + zk$.

4.15. $a(M) = (2x - yz)i + (2x - xy)j + yzk$.

4.16. $a(M) = (y - z)i + 3xyzj + (z - x)k$.

4.17. $a(M) = (y - z)i + (x + z)j + (x^2 - y^2)k$.

4.18. $a(M) = (x + y)i - 2xzj - 3(y + z)k$.

4.19. $a(M) = z^2 i + (xz + y)j + x^2 yk$.

4.20. $a(M) = xy(3x - 4y)i + x^2(x - 4y)j + 3z^2 k$.

4.21. $a(M) = 6x^2 i + 3\cos(3x + 2z)j + \cos(3y + 2z)k$.

4.22. $a(M) = (x + y)i + (z - y)j + 2(x + z)k$.

4.23. $a(M) = 3(x - z)i + (x^2 - y^2)j + 3zk$.

4.24. $a(M) = (2x - yz)i + (xz - 2y)j + 2xyzk$.

4.25. $a(M) = 3x^2 i + 4(x - y)j + (x - z)k$.

$a(M) = (x, y, z)$ vektor maydonni garmonik ekanligi yoki bo‘lmasligini aniqlang.

4.26. $a(M) = x^2 zi + y^2 j - xz^2 k$.

4.27. $a(M) = (x + y)i + (y + z)j + (x + z)k$.

4.28. $a(M) = \frac{x}{y} i + \frac{y}{z} j + \frac{z}{x} k$.

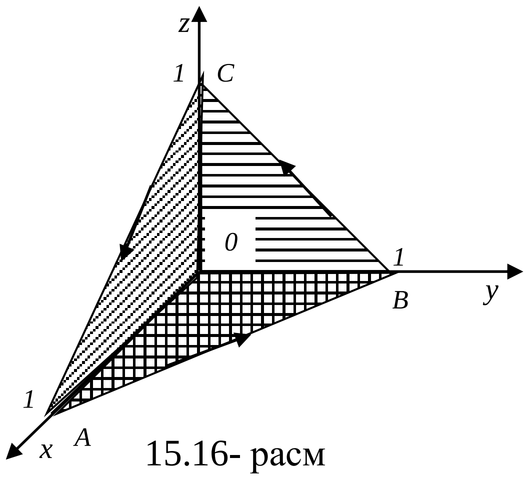
4.29. $a(M) = yzi + xzj + xyk$.

4.30. $a(M) = (y - z)i + (z - x)j + (x - y)k$.

Namunaviy variant yechimi

1. $a(M) = (x - 2z)\boldsymbol{i} + (x + 3y + z)\boldsymbol{j} + (5x + y)\boldsymbol{k}$ vektor maydonning $(p): x + y + z = 1$ tekislik bilan koordinata tekisliklari kesishishi natijasida hosil bo‘lgan uchburchak konturi bo‘ylab $\boldsymbol{n}(1, 1, 1)$ normal vektorning musbat aylanishidagi sirkulyasiyasini ikki usulda toping: 1) sirkulyasiya tarifidan foydalanib 2) Stoks formulasida yordamida.

► (p) tekislikning koordinata tekisliklari bilan kesishishi natijasida ABC uchburchakni hosil qilamiz va unda masalaning shartiga mos ravishda ABCA konturni aylanib o‘tishning musbat yo‘nalishini ko‘rsatamiz.



15.16- расм

1. $dl = \overrightarrow{r^0}dl$ deb belgilab (15.25) formula buyicha berilgan maydonning S sirkulyasiyasini hisoblaymiz.

C = $\oint_{ABCA} \boldsymbol{a}\cdot\boldsymbol{dl} = \int_{AB} \boldsymbol{a}\cdot\boldsymbol{dl} + \int_{BC} \boldsymbol{a}\cdot\boldsymbol{dl} + \int_{CA} \boldsymbol{a}\cdot\boldsymbol{dl}$

AB kesmada quyidagilarga egamiz: z = 0, x+y = 1, y = 1-x, dy = -dx,

$\boldsymbol{a} = x\vec{\imath} + (x+3y)\vec{\jmath} + (5x+y)\vec{k}, \boldsymbol{dl} = dx\vec{\imath} + dy\vec{\jmath},$

$\boldsymbol{a}\cdot\boldsymbol{dl} =$ xdx +(x+3y)dy

$\int_{AB} \boldsymbol{a}\cdot\boldsymbol{dl} = \int_{AB} xdx + (x+3y)dy = \int_1^a \big(x - x - 3(1-x)\big)dx = \int_1^a (3x-3)dx$

$= \left(\frac{3x^2}{2} - 3x\right)\Big|_1^0 = \frac{3}{2}$

BC kesmada quyidagi munosabatlar o‘rinli: x = 0, y+z = -1, z = 1-y, dz = -dy

$\boldsymbol{a} = -\boldsymbol{2z}\vec{\imath} + (3y+z)\vec{\jmath} + y\vec{k}, \boldsymbol{dl} = dy\vec{\jmath} + dz\vec{k},$ $\mathbf{a}\cdot\boldsymbol{dl} =$ (3y+z)dy +ydz.

$\int_{BC} \boldsymbol{a}\cdot\boldsymbol{dl} = \int_{BC}(3y+z)dy + ydz = \int_1^a (3y+1-y-y)dy = \int_1^a (1+y)dy =$

$= \frac{(y+1)^2}{2}\Big|_1^0 = -\frac{3}{2}.$

CA kesmada quyidagilarga egamiz: y= 0, x+z = 1, dz = -dx,

$\boldsymbol{adl} = ($x – 2z)dx + 5xdz

$\int_{CA} \boldsymbol{a}\cdot\boldsymbol{dl} = \int_{CA}(x-2z)dx + 5xdz = \int_0^2 (x-2+2x-5x)dx =$

$= \int_0^1 (= 2x - z)dx = (x^2 - 2x)\Big|_0^1 = -3.$

2. Stoks formulasi yordamida (15.27) berilgan maydonning sirkulyasiyasini hisoblaymiz. Buning uchun quyidagini aniqlaymiz:

$$rot\ a(M) = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial x} & \frac{\partial}{\partial x} \\ x-2z & x+3y+z & 5x+y \end{vmatrix} = -7j + k.$$

Stoks formulasida S sirt sifatida OABC piramidaning yon sirtini olamiz:

$$S = S_{OCA} + S_{OAB} + S_{OBC}.$$

Stoks formulasi buyicha quyidagiga ega bo'lamiz:

$$S = \iint_S rot\vec{a} \cdot n^0 dS = \iint_S rot\vec{a}\boldsymbol{dS},$$

bu erda, dS = dydz$i$ +$dxdzj$+$dxdyk$. ($rot\vec{a}\boldsymbol{dS}$) = -7dxdz + dxdy.

Bundan, quyidagiga ega bo'lamiz:

$$C = \iint_S -7dxdz + dxdy = -7\iint_{S_{OAC}} dxdz + \iint_{S_{OAB}} dxdy = -3. ◄$$

2. u(M) = $5x^2yz - 7xy^2z + 5xyz^2$ funksiyaning $M_0(1,1,1)$ nuqtadagi qiymatini va eng katta o'zgarish yo'nalashini toping.

► u(M) funksiyaning ihtiyoriy M(x,u, z) nuqtadagi va M_0 nuqtadagi hususiy hosilalarini topamiz:

$\frac{\partial u(M)}{\partial x} = 10xyz - 7y^2z + 5yz^2$, $\frac{\partial u(M_0)}{\partial x} = 10-7+5 = 8$,

$\frac{\partial u(M)}{\partial y} = 5x^2z - 14xyz + 5xz^2$. $\frac{\partial u(M_0)}{\partial y} = 5-14+5 = -4$,

$\frac{\partial u(M)}{\partial z} = 5x^2y - 7xy^2 + 10xyz$, $\frac{\partial u(M_0)}{\partial z} = 5-7+10 = 8$.

U holda, $M_0(1,1,1)$ nuqtada quyidagiga ega bo'lamiz, $\overrightarrow{gradu}(M_0) = 8\vec{\imath} - 4\vec{\jmath} + 8\vec{k}$ M_0 nuqtadagi maydon o'zgarishining eng katta tezligiga $\overrightarrow{gradu}(M_0)$ yo'nalishida erishiladi va u son jihatdan $|\overrightarrow{gradu}(M_0)|$ ga teng bo'ladi:

$$rot\,\vec{a}(M) = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial x} & \frac{\partial}{\partial x} \\ xy^2z^2 & x^2yz^2 & xyz \end{vmatrix} = (xz-2x^2yz)\vec{\imath} - (yz-2xy^2z)\vec{\jmath}.$$

$rot\,\vec{a}(M_0) = 10\vec{\imath} + 5\vec{\jmath}$, $|rot\,\vec{a}(M_0)| = \sqrt{10^2 + 5^2} = 5\sqrt{5}$. ◄

4. $\vec{a}(M)$= (y+z) $\vec{i}$+xy$\vec{j}$- xz$\vec{k}$ vektor maydonning solenoidal maydon bo‘la olish yoki bo‘la olmasligi ko‘rsatilsin.

► $\vec{a}(M)$ -vektor maydonning har bir nuqtasida div$\vec{a}(M)$ =0 bo‘lsa, u solenoidal maydon bo‘ladi. div $\vec{a}(M)$ = $\frac{\partial P}{\partial x}+\frac{\partial Q}{\partial y}+\frac{\partial R}{\partial z}$ = $\frac{\partial}{\partial x}(x+y)+\frac{\partial}{\partial y}(xy)+\frac{\partial}{\partial z}(-xz)=0-x+x=0.$◄

15.8. 15- bobga doir qo‘shimcha masalalar.

1. x^2+y^2 =$\pm ax$ silindr tashqarisida joylashgan $x^2+y^2+z^2=a^2$ shar sirti qismining yuzasi topilsin. (Javob: $8a^2$.)

2. Agar sirt zichligi M(x,y,z) nuqtada xyz ga teng bo‘lsa, $0\le x\le 1, 0\le y\le 1, 0\le z\le 1$ kub sirtining massasi hisoblansin. (Javob: $\frac{3}{4}$)

3. Agar konus sirtining zichligi , har bir nuqtasida bu nuqtadan konus o‘qigacha bo‘lgan masofaga proporsional bo‘lsa, $z^2=\ \ x^2+y^2$ konus sirt massasi markazining koordinatalari hisoblansin. (Javob: $(0,0,\frac{3}{4})$.)

4. u(M) = $x^3+y^3+z^3-3xyz$ skalyar maydon gradiyenti fazoning qanday nuqtalarida: a) Oz o‘qiga perpendikulyar; b) nulga teng? (Javob: a) $z^2=xy$; $b)\ x=y=z.$)

5. u(M) = $x^2y+y^2z+z^2x$ skalyar maydonning $M_0(2,1,2)$ nuqtadagi o‘sishining eng katta tezligi hisoblansin. (Javob: $\sqrt{209}$)

6. z = $x^2+3x^2+6xy+y^2$ funksiyaning A(4, -12) nuqtadagi hosilasi ixtiyoriy yo‘nalish bo‘yicha nulga tengligi ko‘rsatilsin.

7. Moddiy nuqtaning harakat tenglamasi: x = t, y = t^2, z = t^3. Shu nuqtadan koordinata boshigacha bo‘lgan masofa qanday tezlikda kattalashadi. (Javob: $\frac{1+2t^3+3t^4}{\sqrt{1+t^2+t^4}}$.)

8. A punktdan bir vaqtda chiqqan ikki paraxoddan biri shimolga, ikkinchisi shimoliy – sharqqa qarab harakat qilmoqda. Paroxodlarning harakat tezliklari 20 km/soat va 40 km/soat. Ular orasidagi masofa qanday tezlikda ortadi. (Javob: $20\sqrt{5-2\sqrt{2}}$ km/soat.)

9. $\vec{a}(M) = x\vec{i} + y\vec{j} + 2z\vec{k}$ vektor maydonning kuch chiziqlari tenglamasi yozilsin. (Javob: y = C_1x, z = C_2x^2.)

10. Vektor maydon, moduli quyilgan nuqtasidan OXU tekislikkacha bo‘lgan masofaga teskari proporsional bo‘lgan kuch bilan aniqlanadi.Shu maydon dvirgensiyasi topilsin. (Javob: - k/(z$\sqrt{x^2+y^2+z^2}$), bu yerda k-proporsionallik koeffisienti.)

11. Qattiq jism $\vec{\omega}$ burchak tezlik bilan Oz o‘qi atrofida aylanadi. Chiziqli tezlik vektori koordinata o‘qlarida quyidagi proeksiyalarga ega: $\vartheta_x = -\omega y,\ \vartheta_y = \omega x\ , \vartheta_z = 0.$ Quyidagilar topilsin: a) V vektorning rotori; b) V vektorning $x^2 + y^2 = a^2$ aylana bo‘yicha $\vec{k}$ ortga nisbatan musbat yo‘nalishdagi sirkulyasiyasi. (Javob: a) (0, 0, 2ω); b) $2\pi a^2\omega$.)

ILOVA

“Ko‘p o‘lchovli integral va egri chiziqli integrallar” bo‘yicha yozma ish

1. Integrallash tartibini o‘zgartiring**.**

1.1. $\int_0^2 dx \int_{4-2x^2}^{4-x^2} f(x,y)dy.$

1.2. $\int_0^3 dx \int_{\sqrt{9-x^2}}^{\sqrt{25-x^2}} f(x,y)dy.$

1.3. $\int_0^4 dy \int_{\sqrt[3]{y/2}}^{\sqrt{25-y^2}} f(x,y)dx\,.$

1.4. $\int_0^1 dy \int_{2y+1}^{4-y^2} f(x,y)dx.$

1.5. $\int_0^4 dy \int_{4+1}^{7-y} f(x,y)dx.$

1.6. $\int_0^5 dx \int_0^{\sqrt{25-x^2}} f(x,y)dy.$

1.7. $\int_0^2 dx \int_{x^2/4}^{2\sqrt{x}} f(x,y)dy.$

1.8. $\int_{-2}^4 dy \int_{y^2/2}^{y+4} f(x,y)dx.$

1.9. $\int_{-2}^1 dy \int_{y^2}^4 f(x,y)dx.$

1.10. $\int_0^2 dy \int_{y^2}^{y^2+4} f(x,y)dx.$

1.11. $\int_{0}^{2} dx \int_{2x}^{x^2/2+2} f(x,y)dy.$

1.12. $\int_{0}^{1} dx \int_{x^3}^{2-x} f(x,y)dy.$

1.13. $\int_{0}^{\pi/4} dy \int_{y}^{\pi/2-y} f(x,y)dx.$

1.14. $\int_{0}^{2} dx \int_{3x^2}^{6x} f(x,y)dy.$

1.15. $\int_{0}^{1} dy \int_{\sqrt{1-y^2}}^{1-y} f(x,y)dx.$

1.16. $\int_{0}^{1} dx \int_{-1}^{x^2+1} f(x,y)dy.$

1.17. $\int_{-1}^{2} dx \int_{x^2}^{x+2} f(x,y)dy.$

1.18. $\int_{0}^{1} dx \int_{-x^2}^{x^2} f(x,y)dy.$

1.19. $\int_{0}^{1} dy \int_{2y^2}^{3-y} f(x,y)dx.$

1.20. $\int_{-1}^{0} dy \int_{2x^2}^{3-x} f(x,y)dx.$

1.21. $\int_{0}^{1} dy \int_{-\sqrt{1-y^2}}^{1-y} f(x,y)dx.$

1.22. $\int_{0}^{1} dx \int_{-1-y}^{11+y} f(x,y)dy.$

1.23. $\int_{0}^{1} dy \int_{1-y}^{2-2y} f(x,y)dx.$

1.24. $\int_{0}^{1} dy \int_{y/2+1}^{y/2+4} f(x,y)dx.$

1.25. $\int_{-1}^{0} dx \int_{-\sqrt{1+x}}^{1+x} f(x,y)dy.$

1.26. $\int_{0}^{4/5} dy \int_{1+y}^{3-3y/2} f(x,y)dx.$

1.27. $\int_{0}^{1} dy \int_{-\sqrt{y}}^{y} f(x,y)dx.$

1.28. $\int_{0}^{1} dx \int_{-x}^{\sqrt{1-(x-1)^2}} f(x,y)dy$

1.29. $\int_{-1}^{0} dy \int_{-2-y}^{2y+1} f(x,y)dx.$

1.30. $\int_{0}^{3} dx \int_{0}^{\sqrt{4-x}} f(x,y)dy.$

2. Berilgan sirtlar bilan chegaralangan V soha bo‘yicha uch o‘lchovli integralni hisoblang.

$\mathbf{2.1.}\iiint\limits_V z\sqrt{x^2+y^2}dxdydz;$ $\quad$ $\mathrm{V}: \mathrm{y}=0, \mathrm{z}=0, x^2+y^2=2x.$

$\mathbf{2.2.}\iiint\limits_V (x^2+y^2)dxdydz;$ $\quad$ $\mathrm{V}: \mathrm{y}=2, \quad x^2+z^2=2y.$

$\mathbf{2.3.}\iiint\limits_V zdxdydz$ $\quad$ $V: z=\sqrt{x^2+y^2}, \ z=2$

$\mathbf{2.4.}\iiint\limits_V ydxdydz$ $\quad$ $V: y=4(x^2+z^2), y=4$

$\mathbf{2.5.}\iiint\limits_V ydxdydz$ $\quad$ $V: y^2= x^2+z^2, \ y=2$

$\mathbf{2.6.}\iiint\limits_V (4-x-y)dxdydz$ $\quad$ $V: x^2+y^2=4, \ z=0, \ z=1$

$\mathbf{2.7.}\iiint\limits_V dxdydz$ $\quad$ $V: z=\sqrt{4-x^2-y^2}, \ x^2+y^2=3z$

$\mathbf{2.8.}\iiint\limits_V \sqrt{x^2+y^2+z^2}dxdydz$ $\quad$ $V: x^2+y^2+z^2 \geq a^2, x^2+y^2+z^2 \leq$

$\leq 4a^2$

$\mathbf{2.9.}\iiint\limits_V xdxdydz$ $\quad$ $V: \ z-1-\sqrt{x^2+y^2}, \quad z \geq 0$

$\mathbf{2.10.}\iiint\limits_V ydxdydz$ $\quad$ $V: \ z=1-(x^2+y^2), \quad z \geq 0$

$\mathbf{2.11.}\iiint\limits_V dxdydz$ $\quad$ $V: \ z=\sqrt{a^2-x^2-y^2}, \ z=\sqrt{x^2+y^2}$

$\mathbf{2.12.}\iiint\limits_V 5dxdydz$ $\quad$ $V: \ z=2-(x^2+y^2), \quad z=x^2+y^2$

$$\mathbf{2.13.} \iiint\limits_V (x^2+1)dxdydz \qquad V: x^2+y^2=1, \quad z=x^2+y^2, \ z\geq 0$$

$$\mathbf{2.14.} \iiint\limits_V (z^2+1)dxdydz \qquad V: x^2+y^2=z^2, \quad z\geq 0, \quad z\leq 1$$

$$\mathbf{2.15.} \iiint\limits_V \frac{e^{\sqrt{x^2+y^2}}}{y^2+z^2}dxdydz \qquad V: \ y^2+z^2=1, \ x^2=y^2+z^2, x\geq 0$$

$$\mathbf{2.16.} \iiint\limits_V (x^2+y^2+z)dxdydz \qquad V: x^2+y^2=9, \quad z\geq 0, z\leq 3$$

$$\mathbf{2,17.} \iiint\limits_V \frac{z\, e^{\sqrt{x^2+y^2}}}{\sqrt{(x^2+y^2+z^2)^3}}dxdydz \quad V: x^2+y^2+z^2=1, \qquad z\geq 0$$

$$\mathbf{2.18.} \iiint\limits_V y^2\, dxdydz \qquad V: x^2+y^2=1\,, \quad x^2+y^2=z^2\,, \qquad z\geq 0$$

$$\mathbf{2.19.} \iiint\limits_V \frac{z^2dxdydz}{\sqrt{x^2+y^2+z^2}} \quad V: \ x^2+y^2+z^2\geq 1, \ x^2+y^2+z^2\leq 4, \ z$$

$$\geq 0$$

$$\mathbf{2.20.} \iiint\limits_V dxdydz \qquad V: \ x^2+y^2=4, \quad z=5-(x^2+y^2), z\geq 0$$

$$\mathbf{2.21.} \iiint\limits_V \frac{zdxdydz}{\sqrt{1-x^2-y^2}} \qquad V: \ z=\sqrt{1-x^2-y^2}, \quad z\geq 0$$

$$\mathbf{2.22.} \iiint\limits_V (x-2)\, dxdydz \quad V: \ x=6(y^2+z^2), \ y^2+z^2=3, \quad x=0$$

$$\mathbf{2.23.} \iiint\limits_V (y+1)\, dxdydz \quad V: y=3\sqrt{x^2+z^2},$$

$$x^2+z^2=36, \quad y=0$$

$$\mathbf{2.24.} \iiint\limits_V zdxdydz \qquad V: \ z=5\,(x^2+y^2), \ x^2+y^2=2, \ z=0$$

$$\mathbf{2.25.}\iiint\limits_V (x+3)dxdydz \quad V:\ 2x=y^2+z^2,\quad y^2+z^2=4\,,x=0$$

$$\mathbf{2.26.}\iiint\limits_V (x^2+z^2)dxdydz \quad V:\ y^2=x^2+z^2,\ \ y=4$$

$$\mathbf{2.27.}\iiint\limits_V (y^2+z^2)dxdydz \qquad V:\ x=y^2+z^2,\quad x=9$$

$$\mathbf{2.28.}\iiint\limits_V (x^2+y^2)dxdydz \quad V:\ 2z=x^2+y^2,\ \ x^2+y^2=4,\quad z=0$$

$$\mathbf{2.29.}\iiint\limits_V (x+4)\,dxdydz \quad V:\ 2x=y^2+z^2,\ \ y^2+z^2=4,x=0$$

$$\mathbf{2.30.}\iiint\limits_V (y-3)dxdydz \quad V:\ 4y=\sqrt{x^2+z^2},\ \ x^2+z^2=16,y=0$$

3. Berilgan ifoda biror $u(x,y)$ funksiyaning to‘la differensiali bo‘lishini tekshiring. $u(x,y)$ funksiyani toping.

$\mathbf{3.1.}\ (sin^2y - y\sin 2x + 1/2)dx + (x\sin 2y + cos^2x + 1)dy.$

$\mathbf{3.2.}\ (y/x + lny + 2x) + (lnx + x/y + 1)dy.$

$\mathbf{3.3.}\ (x^2 - 2xy)dx + (y^2 - 2xy)dy.$

$\mathbf{3.4.}\ (y/\sqrt{1-x^2y^2} + x^2)dx + \left(x/\sqrt{1-x^2y^2} + y\right)dy.$

$\mathbf{3.5.}\ (\frac{x}{x^2+y^2} + 2x)dx + \left(\frac{y}{x^2+y^2} - 2y\right)dy.$

$\mathbf{3.6.}\ (\frac{x}{1+x^2y^2} - 1)dx + (\frac{y}{1+x^2y^2} + 10)dy.$

$\mathbf{3.7.}\ (y^2e^{xy^2} + 3)dx + \left(2xye^{xy^2} - 1\right)dy.$

$\mathbf{3.8.}\ (\sin x + \cos x\, cosy/sin^2x)dx + (siny/sinx - cosy)dy.$

$\mathbf{3.9.}\ \frac{1-y}{x^2y}dx + \frac{1-2x}{xy^2}dy.$

$\mathbf{3.10.}\ (\frac{y^2}{(x+y)^2} - \frac{1}{x})dx + (\frac{x^2}{(x+y)^2} + \frac{1}{y})dy.$

$\mathbf{3.11.}\ (3x^2 - 2xy + y^2)dx + (2xy - x^2 - 3y^2)dy$

$\mathbf{3.12}.\left(\frac{1}{y}-\frac{y}{x^2}\right)dx+\left(\frac{1}{x}-\frac{x}{y^2}\right)dy.$

$\mathbf{3.13}.\,(\frac{y}{x^2+y^2}-1)dx-\frac{x}{x^2+y^2}dy.$

$\mathbf{3.14}.\,(3x^2-2xy+y^2)dx+(2xy-x^2-3y^2)dy.$

$\mathbf{3.15}.\,(\sin 2x-2sinxsiny-12x^2y)dx+(sin2y+2cosxcosy$

$-4x^3)dy.$

$\mathbf{3.16}.\,(12x^2y+1/y^2)dx+(4x^2-2/y^3)dy.$

$\mathbf{3.17}.\,(2xy-1/x^2)dx+(x^2-2/y^3)dy.$

$\mathbf{3.18}.\,(\cos 3x-\frac{2}{x^2y^2})dx+(sin3y-\frac{1}{x^2y^2})dy.$

$\mathbf{3.19}.\,(2/x^2+cos^2y)dx+(y-xsin2y)dy.$

$\mathbf{3.20}.\,(\cos x-2xy)dx+(-3siny-x^2)dy.$

$\mathbf{3.21}.\,(2xy-14e^ysinxcosx)dx+(x^2+7e^ycos^2x)dy$

$\mathbf{3.22}.\,(\frac{1}{cos^2x}+y^2)dx+3xy^2dy$

$\mathbf{3.23}.\,(\frac{1}{x}+siny)dx+xcosydy$

$\mathbf{3.24}.\left(\frac{1}{x^2}+\frac{1}{y}\right)dx=(\frac{1-x}{y^2})dy$

$\mathbf{3.25}.\,(x+ysin^2y)dx+(1+xsin^2ysin2y)dy$

$\mathbf{3.26}.\,(e^{x-y}+ycosxy-6x)dx+(xcosxy-e^{x-y})dy$

$\mathbf{3.27}.\left(\frac{2x}{3+x^2+y^2}-12x^2y^2+3\right)dx+(\frac{2y}{3+x^2+y^2}-8x^3y+4)dy$

$\mathbf{3.28}.\,(cosy+ycosx-6xy^2)dx+(sinx-xsiny-6x^2y)dy$

$\mathbf{3.29}.\,(e^{xy}-2xsin(x^2-y^2))dx+(xe^{xy}-2ysin(x^2-y^2))dy$

$\mathbf{3.30}.\left(\frac{x}{\sqrt{1+x^2+y^2}}+6x^2y^3-3\right)dx+\left(\frac{y}{\sqrt{1+x^2+y^2}}\right.$

$+6x^3y^2+8y)dy$

4. Berilgan L yoy bo‘yicha egri chiziqli integralni hisoblang.

4.1. $\int_L xdy-ydx\,, L\colon x=acos^2t,\;\; y=asin^2t(0\le t\le 2).$

4.2. $\int_{L_{AB}}(x^2+y^2)\,dx+(x^2-y^2)dy,$

L_{AB} $A(-1,1)$ nuqtadan $B(2,2)$ gacha fo'lgan to'g'ri chiziq kesmasi.

4.3. $\int_{L_{AB}}(x^2-2xy)\,dx+(y^2-2xy)dy,$

$L_{AB}: y=x^2,$ $A(-1,1)$ nuqtadan $B(1,1)$ nuqtagacha.

4.4. $\int_{L_{AB}} sinydx-sinxdy,$

L_{AB}: $A(0,\pi)$ va $B(\pi,0)$ nuqtalar orasidagi to'g'ri chiziq kesmasi.

4.5. $\int_{L_{AB}} xdy-ydx$, $L_{AB}: x=a(t-sint), y=a(1-cost),$

$A(2\pi a,0)$ nuqtadan $B(0,0)$ nuqtagacha fo'lgan yoy.

4.6. $\int_{L_{ABS}} xdy+ydx$, $L_{ABS}: A(-1,0),$ $B(1,0),$ $C(0,1)$ uchburchak.

4.7. $\int_{L_{AB}} \frac{y}{x}dx+xdy,$ $L_{AB}: y=lnx,$

$A(1,0)$ nuqtadan $B(e,1)$ nuqtagacha.

4.8. $\int_{L_{AB}} xe^{x^2}dy+ydx,$ $L_{OA}: y=x^2,$

$O(0,0)$ nuqtadan $A(1,1)$ nuqtagacha.

4.9. $\int_{L_{AB}}(x^2+y)dx+(x+y^2)dy,$

L_{AB}: $A(1,2)$ va $B(3,5)$ nuqtalar orasidagi to'g'ri chiziq kesmasi.

4.10. $\int_{L_{AB}}(xy-1)dx+x^2ydy,$

L_{AB}: $A(1,0)$ va $B(0,2)$ nuqtalar orasidagi to'g'ri chiziq kesmasi.

4.11. $\int_{L_{AB}} cosydx-sinxdy,$

L_{AB}: $A(2,-2)$ va $B(-2,2)$ nuqtalar orasidagi to'g'ri chiziq kesmasi.

4.12. $\int_{L_{OAB}} xdy+ydx$, $L_{OAB}: O(0,0),$ $A(3,0),$ $B(0,2)$ nuqtalar tutashtirilishidan hosil bo'lgan uchburchak.

4.13. $\int_{L_{OAB}}(x+y)dl$, $L_{OAB}: O(0,0),$ $A(2,0),$ $B(0,2)$ nuqtalar tutashtirilishidan hosil bo'lgan uchburchak.

4.14. $\int_L (x+y)dl$, $L: \rho^2=a^2cos2\varphi$ Bernulli lemniskatasining birinchi yaprog'i.

4.15. $\oint_L \sqrt{x^2+y^2}\,dl,$ $L: x^2+y^2=ax$ aylana.

4.16. $\int_L y^2 dl\,,\ L: x = a(t - sint), y = a(1 - cost)$ sikloidaning birinchi arkasi.

4.17. $\int_{L_{OB}} xydx + (y - x)dy\ \ L_{OB}: y = x^2\ O(0,0)$ dan $B(1,1)$ gacha.

4.18. $\int_{L_{OA}} \frac{dl}{\sqrt{x^2+y^2+4}},\quad L_{OA}: O(0,\ 0)\ va\ \ A(1,2)$ nuqtalar orasidagi kesma.

4.19. $\int_{L_{AB}} 2xdy + ydx\,,\ L_{AB}: x = y^2,$

$A(1,1)$ nuqtadan $B(4,2)$ nuqtagacha.

4.20. $\int_{L_{OA}} \frac{dl}{x^2+y^2+z^2},\ L: x = 4cost, y = 4sint, z = 3t$ vint chizig'i birinchi o‘rami.

4.21. $\oint_L y\,e^x dl,\quad L:\ x^2 + y^2 = 3$ aylana.

4.22. $\oint_L (2x + y^2)\,dl,\quad L:\ x^2 + y^2 = 1$ aylana.

4.23. $\oint_L (\,x^2 + y^2)\,dl,\quad L: x = 2cost, y = 2sint$ aylana.

4.24. $\oint_L \frac{x^2 dl}{\sqrt{x^2+16y^2}} dl,\quad L: x = 4cost, y = sint$ ellips.

4.25. $\int_{L_{OAB}} (\,x^2 + y^2)dx + (\,x^2 - y^2)dy,\ L_{OAB}: O(0,0),\ A(1,0),\ B(0,1)$ nuqtalar tutashtirilishidan hosil bo‘lgan uchburchak.

4.26. $\int_L (arcsiny -\ x^2)dl\,,\ L: x = cost, y = sint$ aylana yoyi $(0 \le t \le \pi/4)$.

4.27. $\int_{L_{AB}} x^2 ydx + ye^{x+2}dy,\quad L_{AB}: A(1,1)$ va $B(2,3)$ nuqtalar orasidagi kesma.

4.28. $\int_{L_{AB}} ydx + \frac{x}{y}dy,\quad L_{AB}: y = e^{-x}$ egri chiziqning A(0, 1) va V(1, 2) nuqtalar orasidagi yoyi.

4.29. $\int_{L_{OA}} 2xydx + x^2 dy,\quad L_{OA}: y = x^3$ egri chiziqning O(0, 0) va A(1, 1) nuqtalar orasidagi yoyi.

4.30. $\int_{L_{AB}} (xy + x^2)dl,\quad L_{AB}: A(1,1)$ va $B(3,3)$ nuqtalar orasidagi to‘g’ri chiziq kesmasi.

TAVSIYA ETILAYOTGAN ADABIYOTLAR

Darslik va o‘quv qo‘llanmalar

1. *Бугров Я. С., Никольский С. М.* Дифференциальные уравнения. Кратные интегралы. Ряды. Функции комплексного переменного. М.: Наука, 1981.- 448 с.
2. *Жевняк Р. М., Карпук А. А.* Высшая математика: В 5 ч.- Мн.:Выш шк., 1984-1988.- Ч.3. 208 с, Ч-4.- 1987. – 240 с.
3. *Ильин В. А., Позняк Э. Г.* Основы математического анализа: В 2 ч.- М.: Наука, 1971-1973. Ч 2. -1973. – 448 с.
4. *Кудрявцев Л. Д.* Курс математического анализа: В 2 т. М.: Высш. шк., 1981.- Т. 2.- 576 с.
5. *Курант Р.* Курс дифференциального и интегрального исчисления: В 2 т.- М.: Наука, 1967-1970. Т. 2.-1970.- 671 с.
6. *Пискунов Н. С.* Дифференциальное и интегральное исчисление.. В 2 т.-М.: Наука, 1985.-Т. 2.-576 с.

Masala va mashqlar to‘plamlari

7. *Берман Г. Н.* Сборник задач по курсу математического анализа. – М.: Наука, 1985.- 416 с.
8. *Данко П. Е., Попов А. Г., Кожевникова Т. Я.* Высшая математика в упражнениях и задачах: В 2 ч. – М.: Высш. шк., 1986.- Ч. 2.-464 с.
9. *Кузнецов Л. А.* Сборник задач по высшей математике: Типовые расчеты.- М.: Высш. шк., 1983. 176 с.
10. *Лихолетов И. И., Мацкевич И. П.* Руководство к решению задач по вышей математике, теории вероятностей и математической статистике.- Мн.: Выш шк., 1976.- 456 с.
11. Сборник задач по курсу вышей математики. Г. И. Кручкович, Н. И. Гутарина, П. Е. Дюбюк и др.: Под ред. Г. И. Кручковича.- М.: Высш. Шк., 1073.- 576 с.
12. Сборник задач по математике для втузов. В 2 ч./ В. А. Болгов, Б. П. Демидович, В. А. Ефименко и др.: Под ред. А. Ф. Ефимова, Б. П. Демидовича. – М.: Наука, 1981.- Ч 2.-368 с.

MUNDARIJA

15. Maydonlar nazariyasi elementlari