AMALIY MASHG`ULOT TOPSHIRIQLARI
“DIFFERENSIAL GEOMETRIYA”

Skalyar argumentli vektor funksiya.

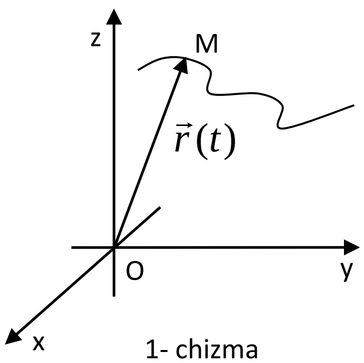
Reja:

1. Ta'rifi va belgilanishi
2. Vektor funktsiya $\vec{r}(t)$ ning koordinatalari
3. Misollar

Tayanch tushuncha va iboralar:

Vektor funksiya, cheksiz kichik vektor, vektor–funksiya-ning koordinatalari, o`zgaruvchi vektorning limiti, ketma-ketlikning limiti, vektor–funksiyaning uzluksizligi, vektor–funksiyaning orttirmasi, hosila, differensiallanuvchi vektor-funksiya.

R to`g`ri chiziqdagi bog`lanishli nuqtalar to`plami I bo`lsin. Agar $t \in I$ ning har bir qiymatiga biror qoida asosida $\vec{r}(t)$ vektor mos keltirilsa u holda I to`plamda $\vec{r} = \vec{r}(t)$ vektor – funksiya berilgan deyiladi.



1- chizma

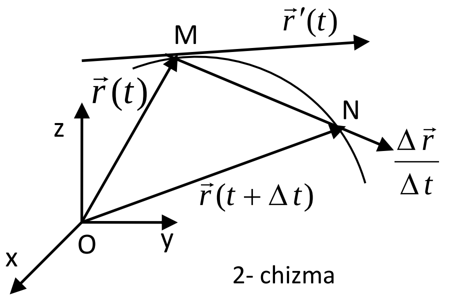
Fazoda dekart koordinatalar sistemasi o`rnatilgan bo`lib, koordinatalar boshi O nuqtadan barcha $\overline{OM} = \vec{r}$ vektorlarni qo`ysak, u holda $t \in I$ parametrning o`zgarishi bilan vektorning M oxiri egri chiziq tashkil etadi, uni $\vec{r}(t)$ vektor-funksiyaning **godografi** deyiladi. (1- chizma)

Vektor- funksiyaning limiti va uzluksizligiga skalyar analizdagi kabi ta`rif beriladi.

**1-Ta`rif.** Har qanday $\varepsilon > 0$ son uchun $\delta > 0$ son mavjud bo`lsaki, barcha $t \in I$ uchun $0 < |t - t_0| < \delta$ bajarilganda $|\vec{r}(t) - \vec{a}| < \varepsilon$ o`rinli bo`lsa, u holda $\vec{a}$

o`zgarmas vektorni $\vec{r}(t)$ **vektor-funksiyaning limiti** deb ataladi. $\lim_{t\to t_0}\vec{r}(t)=\vec{a}$ yoki $\vec{r}(t)\underset{t\to t_0}{\to}\vec{a}$ belgilanadi.

2-Ta`rif.** Agar $\lim_{t\to t_0}\vec{r}(t)=\vec{r}(t_0)$ o`rinli bo`lsa, u holda $\vec{r}(t)$ vektor-funksiyani $t_0\in I$ nuqtada **uzluksiz deyiladi.



2- chizma

**3-Ta`rif.** $\lim_{\Delta t\to 0}\frac{\vec{r}(t+\Delta t)-\vec{r}(t)}{\Delta t}$ mavjud bo`lsa, u holda $\vec{r}(t)$ vektor-funksiya $t\in I$ nuqtada hosilaga ega deyiladi va uni $\frac{d\vec{r}}{dt}$ yoki $\vec{r}\,'(t)$ belgilanadi.

$\vec{r}(t)=\vec{r}(t_0)+\vec{r}\,'(t_0)\Delta t+\frac{(\Delta t)^2}{2!}\vec{r}\,''(t_0)+...+\frac{(\Delta t)^n}{n!}\vec{r}^{\,(n)}(t_0)+\vec{\varepsilon}(t_0,\Delta t)(\Delta t)^n$ ifodaga vektor-funksiyaning Teylor qatoriga yoyilmasi deyiladi.

Masala yechish namunalari

1. O`zgarmas $\vec{a}$ vektor $\vec{r}(t)$ vektor-funksiyaning limiti bo`lishi uchun $t_0\in[a,b]$ da $\vec{a}$ vektor koordinatalari $\vec{r}(t)$ vektor koordinatalarining limiti bo`lishi zarur va yetarli.

Isboti. (zaruriyligi) $\lim_{t\to t_0}|\vec{r}(t)-\vec{a}|=0$ o`rinli bo`lsin,

$\vec{a}\{\alpha,\beta,\gamma\},\ \vec{r}(t)\{x(t),y(t),z(t)\}$ vektorlarning mos koordinatalari uchun quyidagi tengsizliklar bajariladi.

$$|x(t)-\alpha|\le|\vec{r}(t)-\vec{a}|,\quad |y(t)-\beta|\le|\vec{r}(t)-\vec{a}|,\quad |z(t)-\beta|\le|\vec{r}(t)-\vec{a}|\ \Rightarrow \lim_{t\to t_0}|x(t)-\alpha|=0,\ \lim_{t\to t_0}|y(t)-\beta|=0,$$

$$\lim_{t\to t_0}|z(t)-\gamma|=0\ \Rightarrow \lim_{t\to t_0}x(t)=\alpha,\ \lim_{t\to t_0}y(t)=\beta,\ \lim_{t\to t_0}z(t)=\gamma \qquad (*)$$

(yetarliligi). (*) tengliklar o`rinli bo`lsin.

$$|\vec{r}(t)-\vec{a}|=\sqrt{(x(t)-\alpha)^2+(y(t)-\beta)^2+(z(t)-\gamma)^2}\le|x(t)-\alpha|+|y(t)-\beta|+|z(t)-\gamma|\to 0\ \Rightarrow$$

$\Rightarrow\ \lim_{t\to t_0}\vec{r}(t)=\vec{a}$ teorema isbotlandi.

2. $\vec{r}(t)$ vektor-funksiya $Б=\{\vec{i}_1,\vec{i}_2,...,\vec{i}_n\}$ kanonik bazisda $\{x_1(t),x_2(t),...,x_n(t)\}$ koordinatalarga ega bo`lsa, $x_1,x_2,...,x_n$ -larni aniqlang.

Yechish: $\vec{r}(t)=x_1(t)\cdot\vec{i}_1+x_2(t)\cdot\vec{i}_2+...+x_n(t)\cdot\vec{i}_n$ tengliklarning chap va o`ng tomonlarini mos ravishda $\vec{i}_1,\vec{i}_2,...,\vec{i}_n$ larga skalyar ko`paytiramiz

$$(\vec{i}_p\vec{i}_q)=\begin{cases}1,\ agar\ p=q,\\ 0,\ agar\ p\ne q\end{cases}\qquad (**)$$ qiymatlardan foydalansak

$$x_1(t)=(\vec{r}(t),\vec{i}_1),\ \ x_2(t)=(\vec{r}(t),\vec{i}_2),\ \ ...\ ,\ \ x_n(t)=(\vec{r}(t),\vec{i}_n)$$

3. $\lim_{t\to t_0}\vec{r}_1(t)=\vec{a}_1,\ \lim_{t\to t_0}\vec{r}_2(t)=\vec{a}_2$ bo`lsa, $\lim_{t\to t_0}[\vec{r}_1(u),\vec{r}_2(u)]=[\vec{a}_1,\vec{a}_2]$ ni isbotlang.

Isboti. $\vec{r}_i(t) = \vec{a}_i + \vec{\alpha}_i(t)$, bunda $\vec{\alpha}_i(t)$ cheksiz kichik o`zgaruvchi vektor, ya`ni $|\vec{\alpha}_i(t)| \to 0$

$[\vec{r}_1(t)\,\vec{r}_2(t)] = [(\vec{a}_1 + \vec{\alpha}_1(t))(\vec{a}_2 + \vec{\alpha}_2(t))] = [\vec{a}_1\vec{a}_2] + [\vec{a}_1\vec{\alpha}_2(t)] + [\vec{\alpha}_1(t)\,\vec{a}_2] + [\vec{\alpha}_1(t)\,\vec{\alpha}_2(t)]$.

$\lim\limits_{t\to t_0}[\vec{a}_1\vec{a}_2] = [\vec{a}_1\vec{a}_2]$; $\lim\limits_{t\to t_0}[\vec{a}_1\vec{\alpha}_2(t)] = 0$, $\lim\limits_{t\to t_0}[\vec{\alpha}_1(t)\,\vec{a}_2] = 0$, $\lim\limits_{t\to t_0}[\vec{\alpha}_1(t)\,\vec{\alpha}_2(t)] = 0$ bo`lganidan

$$\lim_{t\to t_0}[\vec{r}_1(t)\,\vec{r}_2(t)] = \left[\lim_{t\to t_0}\vec{r}_1(t)\cdot\lim_{t\to t_0}\vec{r}_2(t)\right] = [\vec{a}_1\,\vec{a}_2]$$

4. $\vec{r} = \vec{r}(t)$ vektor funksiya uzluksiz bo`lishi uchun koordinatalar bo`yicha uzluksiz bo`lishi zarur va yetarlidir.

(**zaruriyligi**).

$|\Delta\vec{r}|^2 = \Delta x^2 + \Delta y^2 + \Delta z^2$, $\Delta t \to 0$ *da* $\Delta x \to 0$, $\Delta y \to 0$, $\Delta z \to 0 \Rightarrow |\Delta\vec{r}| \to 0$

(**yetarliligi).** $|\Delta\vec{r}| \to 0$ bo`lsin.

$|\Delta x| \le |\Delta\vec{r}|$, $|\Delta y| \le |\Delta\vec{r}|$, $|\Delta z| \le |\Delta\vec{r}| \Rightarrow \Delta x \to 0$, $\Delta y \to 0$, $\Delta z \to 0$

Ko`ramizki, vektor funksiyaning uzluksizligidan koordinatalar bo`yicha uzluksizligi kelib chiqadi.

5. $\vec{r}(t)$ vektor funksiyaning $t \in [t_1, t_2]$ kesmada tekis uzluksizligini isbotlansin.

Isboti. $\vec{r}(t)$ vektor funksiya $[t_1, t_2]$ kesmaning har bir nuqtasida uzluksiz bo`lsin. U holda $\vec{r}(t)$ ning koordinatalari ham uzluksiz va $|\Delta t| < \varepsilon$ tengsizlikni qanoatlantiruvchi cheksiz kichik $\varepsilon$ sonni aniqlash mumkin bo`lib, $|\Delta x| < \frac{\delta}{\sqrt{3}}$, $|\Delta y| < \frac{\delta}{\sqrt{3}}$, $|\Delta z| < \frac{\delta}{\sqrt{3}}$;

u holda $|\Delta\vec{r}|^2 = \Delta x^2 + \Delta y^2 + \Delta z^2 < \delta^2 \Rightarrow |\Delta\vec{r}| < \delta$ ya`ni $|\Delta t| < \varepsilon$ va $\forall t \in [t_1, t_2] \Rightarrow |\Delta\vec{r}| < \delta$

6. $\vec{r}_1(t)$ va $\vec{r}_2(t)$ vektor funksiya hosilalari mavjud bo`lsa, $[\vec{r}_1(t)\,\vec{r}_2(t)]$ ning hosilasi hisoblansin.

Yechish.

$$[\vec{r}_1(t)\,\vec{r}_2(t)]'_t = \lim_{\Delta t\to 0}\frac{\Delta[\vec{r}_1\,\vec{r}_2]}{\Delta t} = \lim_{\Delta t\to 0}\frac{[(\vec{r}_1 + \Delta\vec{r}_1)(\vec{r}_2 + \Delta\vec{r}_2)] - [\vec{r}_1\,\vec{r}_2]}{\Delta t} = \lim_{\Delta t\to 0}\left[\frac{\Delta\vec{r}_1}{\Delta t}(\vec{r}_2 + \Delta\vec{r}_2)\right] + \lim_{\Delta t\to 0}\left[\vec{r}_1\,\frac{\Delta\vec{r}_2}{\Delta t}\right] =$$
$$[\vec{r}_1'(t)\,\vec{r}_2] + [\vec{r}_1\,\vec{r}_2'(t)]$$

7. $\vec{r}(t) = \vec{a}t + \vec{b}$ chiziqli vektor funksiya hosilasi topilsin.

$$\Delta\vec{r}(t) = [\vec{a}(t + \Delta t) + \vec{b}] - (\vec{a}t + \vec{b}) = \vec{a}\Delta t \Rightarrow \frac{\Delta\vec{r}(t)}{\Delta t} = \vec{a} \Rightarrow \vec{r}'(t) = \lim_{\Delta t\to 0}\frac{\Delta\vec{r}}{\Delta t} = \lim_{\Delta t\to 0}\vec{a} = \vec{a}$$

Mustaqil yechish uchun masala va mashqlar

1. $\lim\limits_{t\to t_0}\vec{r}_i(t) = \vec{a}_i$, $(i = 1,2,3,...)$ va $\lim\limits_{t\to t_0}\lambda(t) = \lambda_0$ bo`lsa, quyidagi formulalar isbotlansin:

a) $\lim\limits_{t\to t_0}(\vec{r}_1(t) + \vec{r}_2(t) + \vec{r}_3(t)) = \vec{a}_1 + \vec{a}_2 + \vec{a}_3$;

b) $\lim\limits_{t\to t_0}\left(\lambda(t)\,\vec r_i(t)\right)=\lambda_0\vec a_i$

c) $\lim\limits_{t\to t_0}\left(\vec r_1(t)\ \vec r_2(t)\right)=\left(\vec a_1\,\vec a_2\right);$

d) $\lim\limits_{t\to t_0}\left(\vec r_1\,\vec r_2\,\vec r_3\right)=\left(\vec a_1\,\vec a_2\,\vec a_3\right);$

2. $\vec r_i(t)$ va $f(t)$ funksiyalar $M_0(t_0)$ nuqtada uzluksiz bo`lsa:

a) $\vec r_1\ (t)\pm\vec r_2(t),$ b) $\left(f(t)\vec r_i(t)\right)$ c) $\left(\vec r_1(t)\vec r_2(t)\right),$ d) $\left[\vec r_1(t)\vec r_2(t)\right]$ e) $\left(\vec r_1(t)\vec r_2(t)\vec r_3(t)\right)$ funksiyalarning shu nuqtada uzluksizligini isbotlansin.

3. $\vec r=\vec r(t)$ vektor-funksiya uzluksiz bo`lsa, $|\vec r|=|\vec r(t)|$ funksiya uzluksiz bo`ladimi? Teskarisi har vaqt bajarilmasligini ko`rsating.

4. $\vec r_i(t),\ (i=1,2,3)$ va f(t) funksoyalar differensiallanuvchi bo`lsa, u holda quyidagi qoidalar isbotlansin.

a) $\left(\vec r_1(t)\pm\vec r_2(t)\right)'_t=\vec r_1'(t)\pm\vec r_2'(t),$

b) $\left(f(t)\vec r_i(t)\right)'_t=f_t'(t)\vec r_i(t)+f(t)\vec r_i'(t),$

c) $\left(\vec r_1(t)\ \vec r_2(t)\right)'_t=\vec r_t'(t)\vec r_2(t)+\vec r_1(t)\vec r_2'(t),$

d) $\left(\vec r_1(t)\vec r_2(t)\vec r_3(t)\right)'_t=\left(\vec r_1'(t)\vec r_2(t)\vec r_3(t)\right)+\left(\vec r_1(t)\vec r_2'(t)\vec r_3(t)\right)+\left(\vec r_1(t)\vec r_2(t)\vec r_3'(t)\right),$

5. Vektor-funksiya differnsiallanuvchi bo`lsa, u holda ushbu vektor-funksiyaning uzluksizligini isbotlang.

6. Koordinatalari differensiallanuvchi bo`lgan vektor-funksiyaning differensiallanuvchi bo`lishini isbotlang.

7. Quyidagi vektorlarning hosilalarini toping.

1) $\vec r^{\,2}(t),$ 2) $\vec r_t'^{\,2}(t),$ 3) $\left[\vec r_t'(t)\,\vec r_{t^2}''(t)\right],$ 4) $\left(\vec r_t'(t)\,\vec r_{t^2}''(t)\,\vec r_{t^3}'''(t)\right),$ 5) $\sqrt{\vec r^{\,2}(t)}$

8. $|\vec r'(t)|=|\vec r(t)|'_t$

9. $\vec r_u'=\vec r_v'=0$ bo`lsa, $\vec r=\vec r(u,v)$ vektor-funksiyaning o`zgarmas vektor bo`lishini ko`rsating.

10. $\vec r(t)=const$ vektor-funksiya uchun $\vec r(t)$ va $\vec r'(t)$ vektorlarning orthogonal bo`lish shartini aniqlang.

11. $\vec r(t)$ vektor-funksiya yo`nalishining doimiy (o`zgarmas) lik shartini aniqlang.

12. $\vec r(t)$ vektor-funksiya moduli (uzunligi)ning o`zgarmaslik shartini aniqlang.

13. $\vec r=\vec r_0+t\,\vec r_1+t^2\vec r_2$ vektor-funksiya orqali berilgan chiziqning godografi (grafigi) parabola bo`lishini ko`rsating, bunda $\vec r_0(t),\vec r_1(t),\vec r_2(t)$ o`zgarmas vektorlar. Boshqacha bo`lishi mumkinmi?

14. $\vec{r}=\vec{r}_0+\cos t\cdot\vec{r}_1+\sin t\cdot\vec{r}_2$ vektor-funksiya orqali berilgan chiziq godografi ellips bo`lishini isbotlang, bunda $\vec{r}_0, \vec{r}_1, \vec{r}_2$ o`zgarmas vektorlar. Boshqacha bo`lishi mumkinmi?

15. $\vec{r}=\vec{r}_0+ch(t)\cdot\vec{r}_1+sh(t)\cdot\vec{r}_2$ vektor-funksiya orqali berilgan chiziq godografi giperbola bo`lishini isbotlang, bunda $\vec{r}_0, \vec{r}_1, \vec{r}_2$ o`zgarmas vektorlar. Boshqacha bo`lishi mumkinmi?

16. $\vec{r}=\vec{r}_0+x\cdot\vec{r}_1+f(x)\cdot\vec{r}_2$ vektor-funksiya godografini aniqlang.

17. Tekislikning vektor ko`rinishdagi parametrik tenglamalarini yozing.

18. $\vec{r}_0, \vec{r}_1, \vec{r}_2, \vec{r}_3$ vektorlar o`zgarmas bo`lib, ulardan $\vec{r}_1, \vec{r}_2, \vec{r}_3$ komplanar bo`lmasin. Quyidagi formulalar orqali berilgan sirtlarning nomini ayting va shaklini tasvirlang.

a) $\vec{r}=\vec{r}_0+u\vec{r}_1+u^2\vec{r}_2+v\vec{r}_3$; Javob: parabolik silindr;

b) $\vec{r}=\vec{r}_0+\cos u\vec{r}_1+\sin u\vec{r}_2+v\vec{r}_3$; Javob: elliptik silindr;

c) $\vec{r}=\vec{r}_0+\left(u+\frac{1}{u}\right)\vec{r}_1+\left(u-\frac{1}{u}\right)\vec{r}_2+v\vec{r}_3$; Javob: giperbolik silindr;

d) $\vec{r}=\vec{r}_0+u\cos v\vec{r}_1+u\sin v\vec{r}_2+u^2\vec{r}_3$; Javob: elliptik paraboloid.

19. Umumlashgan doiraviy funksiyalarni $\vec{E}(\varphi)=\vec{a}\cos\varphi+\vec{b}\sin\varphi$, $\vec{G}(\varphi)=-\vec{a}\sin\varphi+\vec{b}\cos\varphi$, tengliklar orqali berish mumkin. $\frac{d\vec{E}(\varphi)}{d\varphi}=\vec{G}(\varphi)$ va $\frac{d\vec{G}(\varphi)}{d\varphi}=-\vec{E}(\varphi)$ - isbotlansin.

20. $\vec{e}(\varphi+\alpha)=\vec{e}(\varphi)\cos\alpha+\vec{g}(\varphi)\sin\alpha$ ayniyatni isbotlang.

Foydalanilgan adabiyotlar

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3.Сборник задач по дифференциал`ной геометрии. Под ред. Феденко А.С. М., 1979

12-13. Regulyar chiziq. Chiziq tenglamalari.

Reja:

1. Ta’riflar
2. Chiziq tenglamasi
3. Misollar

Tayanch tushuncha va iboralar:

Uzluksiz almashtirish, topologik almashtirish, elementar yoy, elementar chiziq, chiziqning chegaraviy nuqtalari, sodda chiziq, umumiy chiziq, regulyar va silliq chiziq.

Tekislikda dekart reper $B=\{0,\vec{i},\vec{j}\}$ o`rnatilgan bo`lsin. M(x,y) nuqtalarning L to`plamiga tegishli istalgan nuqtalarning B reperdagi (x,y) koordinatalarini $x=\varphi(t),\ y=\psi(t),\ \alpha\le t\le\beta$ (1) ko`rinishda ifodalash mumkin bo`lib, $\varphi(t),\ \psi(t)$ funksiyalar I=$[\alpha,\beta]$ kesmada uzliksiz bo`lsa va t parametrning $[\alpha,\beta]$ kesmadagi turli qiymatlarida L to`plamning turli nuqtalari mos qo`yilsa, u holda L nuqtalar to`plamiga **tekis elementar egri chiziq** deb ataladi. (1) ifodalarni **elementar chiziqning parametrik tenglamalari** deyiladi. t-parametr fizik miqdor – vaqt bo`lishi ham mumkin.

Tekis elementar chiziqni y=f(x) yoki x=t , y=f(t), (2) ko`rinishda ham ifodalash mumkin. $t\in$ I bo`lsin. I- ni $[t_{i-1},t_i]$, $\alpha=t_0<t_1<...<t_n\to\beta$ segmentlarga ajrataylik

Ta`rif.** I bog`lanishli to`plamning har bir $[t_{i-1},t_i]$ segmentida (1) ko`rinishda ifodalangan nuqtalar to`plami elementar egri chiziq tashkil etsa, u holda L to`plam barcha egri chiziqlarning birlashmasidan iborat bo`lib, uni **soda chiziq deb ataladi.

Soda egri chiziq uchun o`z-o`zini kesish nuqtalari, ustma-ust qo`yilgan (yopishgan) tarmoqlari mavjud bo`lmaydi. Soda egri chiziqni $\vec{r}=\vec{r}(t)$ (3) ko`rinishda ifodalash ham mumkin, bunda $\vec{r}(t)$ vektor-funksiyalar I sohada uzluksiz bo`lib, uzluksiz funksiyalarning C^k sinfiga tegishlidir.

Ta`rif.** Agar $\vec{r}(t)$ vektor-funksiyalar I sohaning ixtiyoriy $t_0$ nuqtasida aniqlangan va uzluksiz bo`lib, shu nuqtada $\vec{r}_t'\neq 0$ bo`lsa, u holda L egri chiziqni **regulyar chiziq (k=1 uchun silliq) deyiladi.

Elementar va sodda fazoviy egri chiziq tushunchasi, ta`rifi va tenglamalarini tekis egri chiziq tushunchasiga o`xshash tarzida yoritish mumkin.

Fazoviy egri chiziq L ga tegishli istalgan M nuqtaning (x,y,z) koordinatalarini

$$x=\varphi(t),\ \ y=\psi(t),\ \ z=\lambda(t),\ \ \alpha\le t\le\beta \qquad (4)$$

ko`rinishda ifodalash mumkin bo`lib, $\varphi(t),\ \psi(t),\ \lambda(t)$ funksiyalar $[\alpha,\beta]$ uzluksiz va t-ning $[\alpha,\beta]$ dagi turli qiymatlariga L to`plamning turli nuqtalari mos kelsa, u holda L-ni **elementar fazoviy egri chiziq** deyiladi. (4) L-ning parametrik tenglamalaridir.

Fazoviy soda egri chiziq tushunchasini va ta`rifini soda egri chiziqdagi kabi kiritish mumkin.

Sodda fazoviy egri chiziqni Φ_1 va Φ_2 sirtlarning kesishish chizig`i sifatida olish ham mumkin. Ushbu egri chiziq ixtiyoriy M(x,y,z) nuqtasining koordinatalari

$$\text{L: } \begin{cases} F_1(x,y,z)=0 \\ F_2(x,y,z)=0 \end{cases} \qquad (5)$$

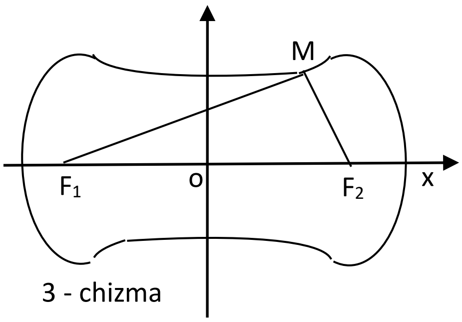
sistemani qanoatlantiradi. (5) sistema fazoviy egri chiziqni aniqlashi uchun

$$M=\begin{Vmatrix} F'_{1x} & F'_{1y} & F'_{1z} \\ F'_{2x} & F'_{2y} & F'_{2z} \end{Vmatrix} \qquad (6)$$

matritsaning rangini 2 ga teng bo`lishi talab qilinadi.

Masala yechish namu`nalari

1-masala. Bernulli lemniskatasining dekart va qutb koordinatalr bo`yicha tenglamasi tuzilsin.

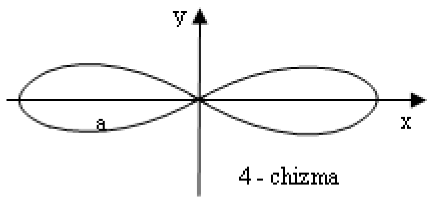


3 - chizma

Yechish. Bernulli lemniskatasi kassini ovalining xususiy holi bo`lib, oval tenglamasidan kelib chiqadi.

Kassining ovali. Har biridan F_1 va F_2 nuqtalargacha bo`lgan masofalar ko`paytmasi o`zgarmas son $b^2$ ga teng bo`lgan tekislikdagi nuqtalar to`plamiga kassining ovali deyiladi.

Bu chiziq tenglamasini aniqlash uchun (F_1F_2) to`g`ri chiziqni OX o`q deb qabul qilamiz. $F_1F_2$ kesma o`rtasidan F_1F_2 ga perpendikulyar qilib, OY o`q o`tkazamiz. OXY dekart sistemada F_1 va F_2 nuqtalar koordinatalarini aniqlash uchun F_1F_2 kesma uzunligini 2c ga teng bo`lsin deb faraz qilamiz. U holda $F_1(-c,0)$, $F_2(c,0)$. Ovalning ixtiyoriy M nuqtasi M(x,y) koordinatalarga ega bo`lib,

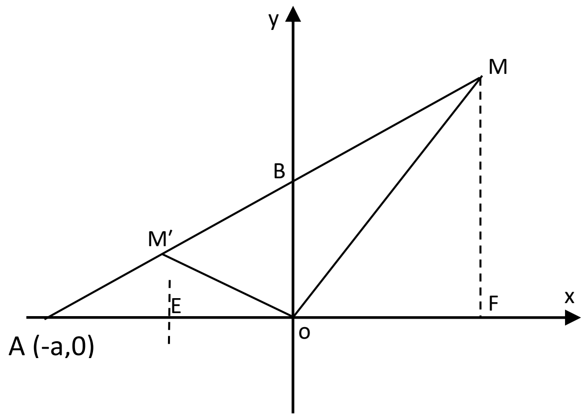


4 - chizma

$d(F_1,M)\cdot d(F_2,M)=b^2$ (1)

$$\sqrt{(x+c)^2+y^2}\cdot\sqrt{(x-c)^2+y^2}=b^2$$

$$\left(x^2+y^2\right)^2-2c^2\left(x^2-y^2\right)=b^4-c^2 \qquad (2)$$



b=c bo`lganda oval **Bernulli lenistikasi** deyiladi. Uning tenglamasi

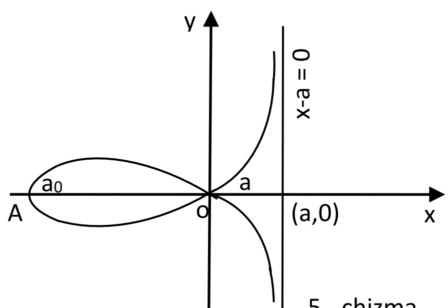
$$\left(x^2+y^2\right)^2-2c^2\left(x^2-y^2\right)=0 \qquad (3)$$

$$x=\rho\cos\varphi,\ \ y=\rho\sin\varphi. \qquad (4)$$

formulalar yordamida dekart koordinataladan qutb koordinatalarga o`tsak, (b>c)

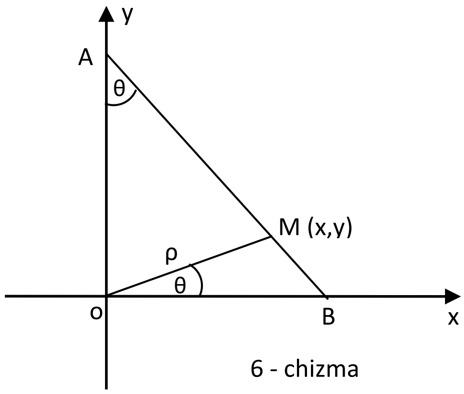
$\rho^2=2c^2\cos 2\varphi$ (5) tenglama kelib chiqadi.

2-masala. **Stroida** tenglamasi tuzilsin.



5 - chizma

A(-a,0) nuqtadan o`tgan ixtiyoriy nur OY o`qini B nuqtada kesadi. AB to`g`ri chiziqdagi B nuqtaning ikkala tomoniga BM=BM´=BO kesmalarni qo`yamiz. AB nurni A nuqta atrofida aylantiramiz , bu vaqtda BO masofa shu bilan birga BM=BM´ kesmalar ham o`zgarib boradi. (5chizma).



6 - chizma

O`zgarib turadigan M va M´ nuqtalarning geometrik o`rniga **strofoida** deyiladi.

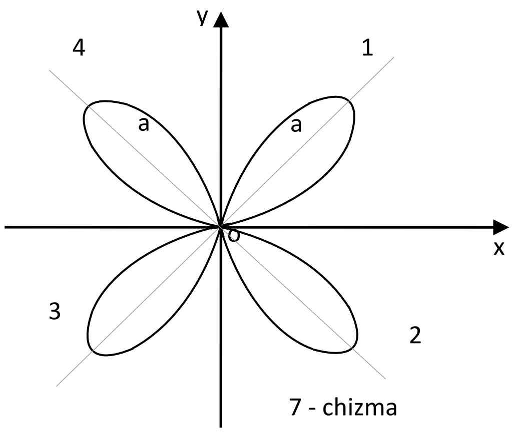
Strofoida tenglamasini chiqaraylik, M nuqtaning koordinatalari x va y bo`lsin. BM=BM´ bo`lgani uchun $|EO| = |OF| = x$ bo`ladi, bunda E va F M´E⊥ox , MF⊥ ox bo`ladi, perpendikulyarning asosi M´OE va OMF uchburchaklar o`xshash $\Rightarrow \frac{EO}{M'E} = \frac{FM}{OF}$

$\frac{x}{M'E} = \frac{y}{x} \Rightarrow M'E = \frac{x^2}{y}$, Δ M´AE , Δ MAF bo`lgani uchun

$$\frac{AE}{AF} = \frac{M'E}{M\,F} \Rightarrow \frac{a-x}{a+x} = \frac{\frac{x^2}{y}}{y} \Rightarrow y^2 = x^2\,\frac{a+x}{a-x}$$

3-masala. To`rt yaproqli gulning qutb koordinatalar bo`yicha tenglamasi tuzilsin.

Uzunligi 2a ga teng AB kesmaning uchlari OX va OY o`qlari bo`ylab sirpansin. Koordinatalar boshi bo`lgan O nuqtadan AB kesmaga perpendikulyar tushuramiz.



7 - chizma

Ta`rif.** Koordinatalar boshidan AB kesmaga tushurilgan perpendikulyar asoslarining geometrik o`rniga **to`rt yaproqli gul deyiladi.

$OM = \rho, \quad \angle MOB = \theta$ belgilaylik, Δ MOA dan

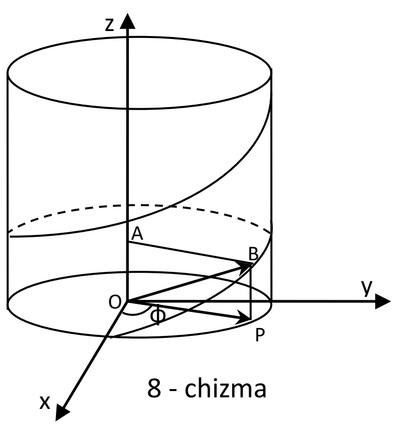
$\frac{MA}{OM} = ctg\theta = MA = \rho\, ctg\theta$ Δ OMB dan

$MB = \rho\, tg\theta$ shartga ko`ra MA+MB=AB=2a, demak $\rho\, ctg\,\theta + \rho\, tg\,\theta = 2a \Rightarrow \rho = a\sin 2\theta \quad \theta' = \theta + 45^0$ desak, $\rho = a\cos 2\theta'$

4. Vint chiziq.

Fazoda OXYZ dekart koordinatalar sistemasi o`rnatilgan bo`lsin. O`zgarmas *a* uzunlikdagi AB kesma OZ o`qqa perpendikulyar bo`lib, uning A uchi shu o`qda yotsin.

**Ta`rif.** (OZ) o`qqa perpendikulyar kesmaning A uchi OZ o`q bo`ylab siljib o`q atrofida shunday aylansaki,



8 - chizma

kesmaning OZ o`qdagi uchi aylanish burchagi $\varphi$ ga proporsional yo`lni bosib o`tsin.kesmaning ikkinchi B uchining iziga **vint chiziq** deyiladi.

Masala. Vint chiziqning parametrik tenglamasi tuzilsin.

AB = a, OA = $\lambda\varphi$ $\overrightarrow{OB} = \overrightarrow{OP} + \overrightarrow{PB} = \overrightarrow{OP} + \overrightarrow{OA}$,

$\overrightarrow{OB} = \vec{r} = a(\vec{i}\cos\varphi + \vec{j}\sin\varphi) + \lambda\varphi\vec{k}$

Xulosa. $\vec{r} = a\cos\varphi\vec{i} + a\sin\varphi\vec{j} + \lambda\varphi\vec{k}$ vint chiziqning vektorli parametrik tenglamasi bo`lib, $x = a\cos\varphi$, $y = a\sin\varphi$, $z = \lambda\varphi$ esa uning koordinata ko`rinishdagi tenglamasidir. Vint chiziq $x^2 + y^2 = a^2$ doiraviy silindrning yon bag`ir chizig`idir.

5. Sikloida.

a radiusli aylana biror to`g`ri chiziq bo`ylab, sirpanmay g`ildirasin. Harakatdagi aylanada ko`rsatilgan nuqtaning iziga **sikloida** deyiladi.

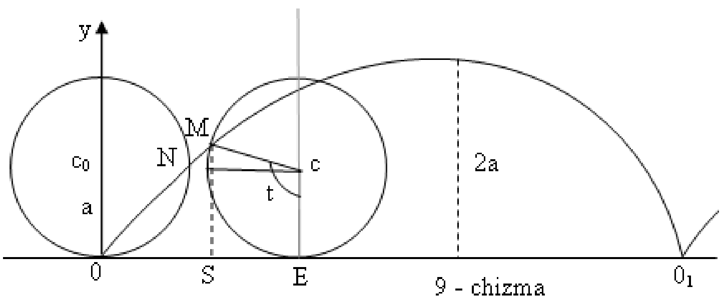
Masala. Sikloidaning parametrik tenglamasi tuzilsin.

Yechish. Masala shartidagi to`g`ri chiziq OX o`q bo`lsin. M nuqta boshlang`ich momentda O vaziyatda bo`lsin. Chizmada t momentdagi vaziyat ko`rsatilgan. Qo`zg`aluvchan aylananing yangi vaziyatdagi markazi C bo`lsin.

$CM = CE = OC_o = a$,

$\angle MCE = t$ belgilaylik, t=0 da $M \equiv 0$, $t = 2\pi$ da $M = O_1$, $d(o, o_1) = 2\pi a$. M nuqtaning OX dagi proyeksiyasi S bo`lsin.

ES=CN,



9 - chizma

$x = OS = OE\text{-}SE = at - a\sin t = a(t - \sin t)$

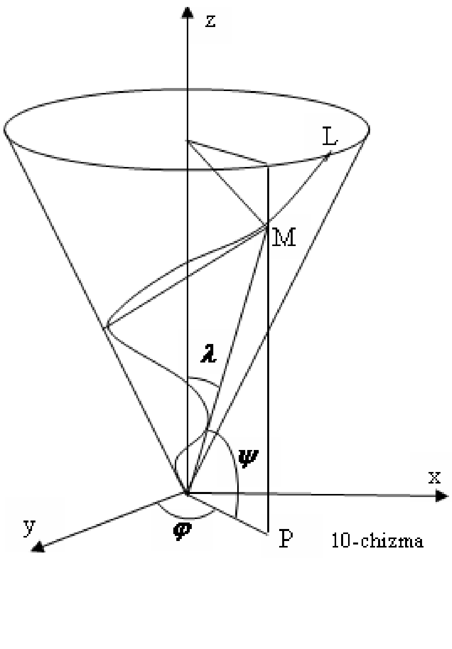
$y = SM = EC + NM = a - a\cos t = a(1 - \cos t)$ $\vec{r} = \overrightarrow{OM} = a\{(t - \sin t) + (1 - \cos t)\}$

6. Fazoda OXYZ dekart koordinatalar sistemasi o`rnatilgan bo`lsin. OZ o`qqa perpendikulyar bo`lmagan OL to`g`ri chiziq OZ atrofida doimiy burchak koeffisenti ω ostida tekis aylansin. M nuqta OL bo`ylab siljisin.

a) Harakatdagi M nuqtaning 0 ga nisbatan masofasi OM ga proporsional ravishda;

b) o`zgarmas tezlik ostida o`zgarsin.

a) xolda M nuqta konus spiralini chizsa; b) xolda konus vint chizig`ini chizadi. Ushbu chiziqlarning parametrik tenglamari tuzilsin.



10-chizma

Yechish. a) Qutb koordinatalar sistemasini o`rnatamiz. M nuqtaning vaziyatini r=d(O,M) masofa, $\psi = POL$-geografik kenglik va geografik uzodlik bo`yicha aniqlanadi. Masala shartiga ko`ra $\psi = \frac{\pi}{2} - \lambda$, bunda $\lambda = Z\overset{\Delta}{O}L$ va $\varphi = \omega t$ $\frac{dr}{dt} = mr$ shartdan r- ni aniqlaymiz. $r = r_0 e^{mt}$ va uni quyidagi formulaga qo`yamiz. $x = r\cos\psi\cos\varphi,\quad y = r\cos\varphi\sin\varphi,\quad z = r\sin\psi$

<u>Xulosa</u>. $x = ae^{k\varphi}\cos\varphi,\ \ y = a e^{k\varphi}\sin\varphi,\ \ z = b e^{k\varphi}$ bunda $k = \frac{m}{\omega},\quad a = r_0\sin\lambda,\ b = r_0\cos\lambda$

b) $x = at\cos t,\quad y = at\sin t,\quad z = bt \ \Rightarrow\ \frac{(x^2+y^2)}{a^2} - \frac{z^2}{b^2} = 0$

7. $x = t$, $y = t^2$, $z = e^t$ fazoviy chiziq qaysi sirtlarning kesishish chizigi bo`lishi mumkin? Javob: $y = x^2$, $z = e^x$ sirtlarning kesishmasi.

8. Chiziq $\begin{cases} x^2 + y^2 + z^2 - R^2 = 0 \\ x + y + z = 0 \end{cases}$ tenglamalar orqali berilgan bo`lsin. Yakobi matritsasi $\begin{pmatrix} 2x & 2y & 2z \\ 1 & 1 & 1 \end{pmatrix}$ ning rangi r = 2. Tenglamalardan birinchisi O(0,0,0) markazli R radiusli sfera, 2- esa O nuqtadan o`tuvchi tekislik. Kesimni aniqlang.

9. Bir aylana 2- aylana bo`ylab tashqaridan sirpanmay g`ildirasa, harakatdagi aylanada ko`rsatilgan nuqtaning chizgan iziga **episikloida** deyiladi.

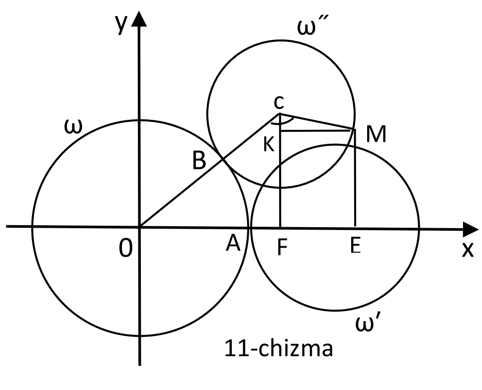
<u>Yechish</u>. $\omega(0,a)$ berilgan aylana, ω′ harakatlanuvchi aylana bo`lsin.OXY dekart koordinatalar sistemasini o`rnataylik. Boshlang`ich momentda ω′ va ω aylanaga $A \in (OX)$ nuqtagada urinsin. t-momentda M vaziyatga ko`tariladi va izlangan chiziqda yotadi. ω aylanada $AB = \angle AOB \cdot OA = a\angle AOB$ (1) ω'' aylanada $BM = BC,\ \angle BCM = mat$ (2) bunda $\angle BCM = t,\ \ MC = CB = ma,\ ma < a,\ m > 0$, ko`ramizki $BM = AB$ (3) $\Rightarrow\ a\angle AOB = mat\ \Rightarrow\ \angle AOB = mt$ (4) M nuqtaning koordinatalarini topamiz. $X = OE = OF + FE$ (5)

$\angle COF : OF = (a + ma)\cos(\angle COF) = a(1+m)\cos mt$ (6)

$FE = KM.\ \triangleright KCM : KM = CM\cdot\sin(\angle KCM) \Rightarrow$

$\angle KCM = \angle BCM - \angle OCF = t - (\frac{\pi}{2} - mt) = t(1+m) - \frac{\pi}{2}$

$FE = KM = ma\sin(t(1+m) - \frac{\pi}{2})$ (7) . (6) va (7) ni (5) ga qo`yamiz.



11-chizma

$x = a[(1+m)\cos mt - m\cos(1+m)t]$ $y = ME = CF - CK$ (8)

$\triangle COF \Rightarrow CF = a(1+m)\sin mt$ (9) $\triangle KCM \Rightarrow CK =$

$= ma\sin(1+m)t$ (10)

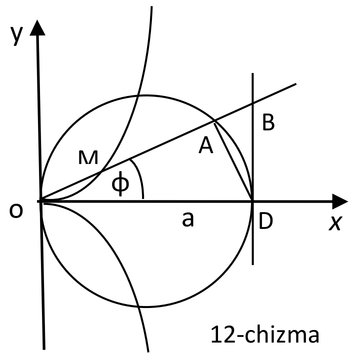
(9) va (10) ni (8) ga qo`yamiz. $y = a[(1+m)\sin mt - m\sin(1+m)t]$

Javob: $x = a[(1+m)\cos mt - m\cos(1+m)t]$,

$y = a[(1+m)\sin mt - m\sin(1+m)t]$

10. $OD = a$ diametrli aylanani $[OB)$ nur orqali kesaylik. (OXY) dekart koordinatalar sistemasini shunday o`rnataylikki, $D \in (OX)$ bo`lsin. D nuqtadan OX ga perpendikulyar d to`g`ri chiziq B nuqtada kesishsin. $[OB)$ nur ω aylana bilan A nuqtada kesishsin.

Ta`rif:** $OM = AB$ tenglikni qanoatlantiruvchi $M$ nuqtalar to`plamiga **Dmokles sissoidasi deyiladi. Ushbu chiziqning parametrik tenglamalari tuzilsin.



12-chizma

Yechish : $OM = AB$, $OD = a$, $\angle BOD = \varphi$ $\triangle OAD$

dan $tg\varphi = \dfrac{AD}{OA} \Rightarrow AD = OAtg\varphi.$ ΔOAD va ΔABD larning

o`xshashligidan $AD = OA \cdot AB$ (2) ΔOAD dan

$OA = OD\cos\varphi = a\cos\varphi$ (3)

$(1),(2),(3) \Rightarrow AB = a\cos\varphi tg^2 = a\dfrac{\sin^2\varphi}{\cos\varphi}$ $OM = r$ belgilasak,

$r = a\dfrac{\sin^2\varphi}{\cos\varphi}$ (4) $x = r\cos\varphi = a\sin^2\varphi$, $y = r\sin\varphi = \dfrac{a\sin^3\varphi}{\cos\varphi}$ $\sin^2\varphi = \dfrac{tg^2\varphi}{1+tg^2\varphi}$. U

holda, $x = a\dfrac{tg^2\varphi}{1+tg^2\varphi}$, $y = a\dfrac{tg^3\varphi}{1+tg^2\varphi}$ (5) $tg\varphi = \varphi$ deb belgilasak,

$x = \dfrac{a\varphi^2}{1+\varphi^2}$, $y = \dfrac{a\varphi^3}{1+\varphi^2}$.

Mustaqil yechish uchun misollar.

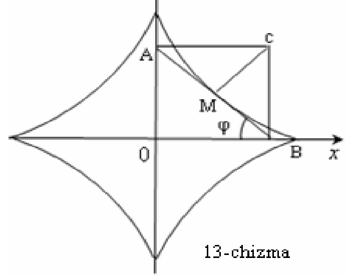
1. Bir aylana ikkinchi aylana bo`ylab ichkaridan sirpanmay g`ildirasa, harakatdagi aylanada ko`rsatilgan nuqtaning chizgan iziga giposikloida deyiladi. Ushbu chiziqning parametrik tenglamalari tuzilsin.

Javob: $x = a[(1-m)\cos mt + m\cos(1-m)t]$,

$y = a[(1-m)\sin mt - m\sin(1-m)t]$

$m = \dfrac{1}{4}$ bo`lganda astroida nomlanuvchi chiziq kelib chiqadi.Uning tenglamasi

$x = a(\frac{3}{4}\cos\frac{1}{4}t + \frac{1}{4}\cos\frac{3}{4}t)$, $y = a(\frac{3}{4}\sin\frac{1}{4}t - \frac{1}{4}\sin\frac{3}{4}t)$



13-chizma

2. Astroidaning oshkormas tenglamasi tuzilsin.

Javob: $x = a\cos^3 t$, $y = a\sin^3 t$, $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$

3. Uzunligi $2a$ ga teng kesmaning uchlari OX va OY o`qlar bo`ylab $(A \in OY)$ sirpansin. $OACB$ to`g`ri to`rtburchakninig C uchidan AB ga

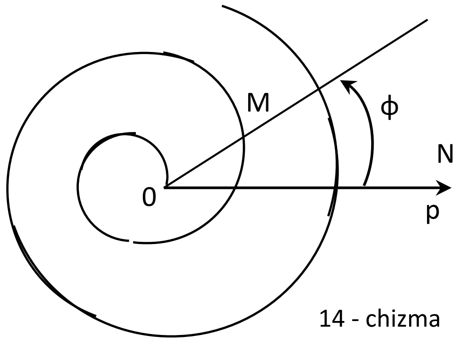
perpendikulyar qilib CM ni tushiraylik. CM perpendikulyar asoslarining geometrik o`rni bo`lgan chiziq (astroida) ning tenglamasi tuzilsin. Javob**:** $x = 2a\cos^3\varphi,\ y = 2a\sin^3\varphi$

<u>Ko`rsatma</u> : $\angle ABO = \varphi -$parametr.

4. M nuqta ON to`g`ri chiziq bo`yalab harakatlansin. $ON$ to`g`ri chiziq esa O nuqta atrofida aylansin.

**Ta`rif:** Qutb atrofida yekis aylanayotgan to`g`ri chiziq ilgarlanma tekis harakat qiluvchi nuqtaning chizgan iziga Arximed spirali deyiladi.

Arximed spiralining qutb kordinatalari bo`yicha tenglamasi tuzilsin. Javob: $r = a\varphi$



14 - chizma

5. a radiusli aylananing ko`rsatilgan nuqtasi orqali o`tuvchi nurni shu nuqta atrofida aylantiraylik. Nurni aylana bilan kesishish nuqtasi A dan ikki tomonga $2b$ uzunlikdagi AM_1 va AM_2 kesmalarni qo`yaylik. $M_1$ va $M_2$ nuqtaning geometrik o`rniga Paskal chanog`i deyiladi. $a = b$ uchun **Kardoida** nomli chiziq hosil bo`ladi. Ushbu chiziqning tenglamalari tuzilsin.

Javob: $r = 2a\cos\varphi \pm 2b \quad (x^2 + y^2 - 2ax)^2 = 4b^2(x^2 + y^2),\ a = b$ bo`lganda kardoida

6. Qanday chiziqning tenglamasi $x = t^2 - t + 1,\ y = t^2 + t - 1$ko`rinishida bo`ladi.

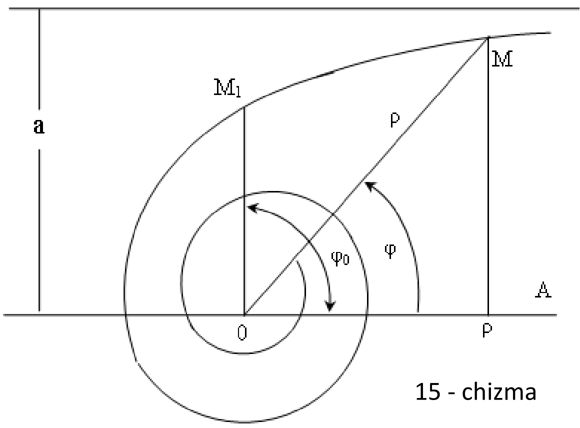
Javob: Parabola.

7.Radiusi $\frac{R}{2}$ ga teng aylananing qutb kordinatalar sistemasidagi tenglamasini tuzing. Javob: $\rho = R\cos\varphi$

**Ko`rsatma:** Diametrning bir uchini koordinatalar boshida tanlab, qutb koordinatalar sistemasini o`rnating.

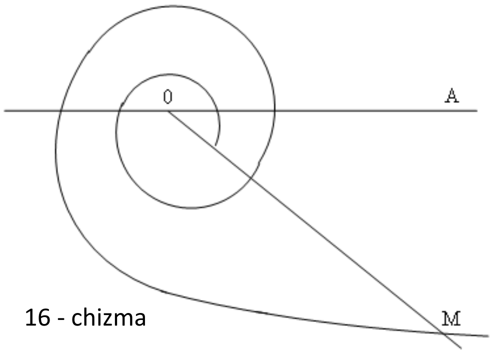
8. $\rho = \frac{a}{\varphi}$ tenglama orqali aniqlangan chiziqqa giperbolik spiral deyiladi. $a -$ musbat son.

Giperbolik spiralni vaziyatini tekshiraylik. $\varphi_0 = \frac{\pi}{2}$ qiymatni olaylik $M_1(\frac{2a}{\pi}, \frac{\pi}{2})$ mos nuqta φ $-$cheksiz o`ssa, $\rho$ $\varphi$ ga teskari proporsional bo`lib, nolga intiladi. U holda o`zgaruvchi $M(\rho,\varphi)$ nuqta qutb atrofida musbat yo`nalishda harakatlanib qutbga cheksiz yaqinlashadi. Agar M_1 nuqta cheksizlikka intilsa , u holda φ $\frac{\pi}{2}$dan boshlab



15 - chizma

kichrayib boradi, ρ esa cheksizlikka intiladi. $M(\rho,\varphi)$ nuqta cheksiz uzoqlashadi. M nuqtaning qutb o`qiga proyeksiyasi $P$ bo`lsin .



16 - chizma

$$PM = \rho\sin\varphi, \quad PM = \rho\sin\varphi = a\frac{\sin\varphi}{\varphi}$$ ma`lumki,

$\frac{\sin\varphi}{\varphi}\xrightarrow[\varphi\to 0]{}1$ u holda $PM\xrightarrow[\varphi\to 0]{}a$ tenglamasi

$$-\rho^2\frac{d\varphi}{d\rho} = a \Rightarrow \frac{d\varphi}{a} = -\frac{d\rho}{\rho^2} \Rightarrow \frac{1}{\rho} = \frac{\varphi}{a}.$$

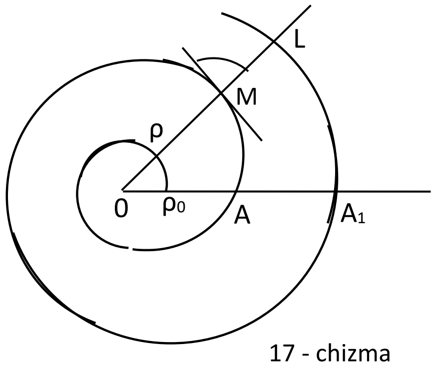
<u>Xulosa</u>. M nuqta spiral bo`ylab cheksizlika intilsa, ushbu nuqta qutb o`qiga parallel va undan a masofada o`tuvchi to`g`ri chiziqqa yaqinlashib boradi. $a$ - manfiy son bo`lsa, $\rho = \frac{a}{\varphi}$ tenglik teskari giperbolik spiralni aniqlaydi.

9. OL to`g`ri chiziq O qutb atrofida doimiy burchak tezligi ω ostida aylanadi. M nuqta OL tog`ri chiziqda $|OM|$ masofaga proporsional tezlik bilan xarakatlanadi. M nuqtaning iziga **logarifmik spiral** deyiladi

Masala. Logarifmik spiralning qutb koordinatalari bo`yicha tenglamasi yozilsin.

Parametr sifatida t vaqtni olib, uni chiziqning A nuqtasidan o`tgan momenti $(\varphi = 0, \rho = \rho_0)$dan boshlab hisoblaymiz. $\varphi = \omega t \Rightarrow \frac{ds}{dt} = m\rho$, $m = const$,

$\frac{d\rho}{\rho} = mdt \Rightarrow \rho = \rho_0 e^{mt}$ $t = \frac{\varphi}{\omega} \Rightarrow \rho = \rho_0 e^{m\frac{\varphi}{\omega}} = \rho_0 e^{\kappa\varphi}$, bunda $k = \frac{m}{\omega}$, $k = 0$ da $\rho = \rho_0$ -aylana hosil bo`ladi. $\varphi$ -o`ssa, ρ -o`sadi, φ -kamaysa, ρ -kamayadi.



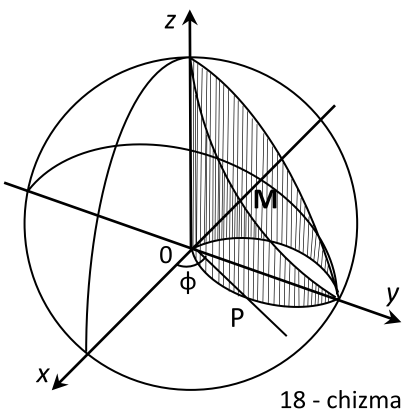
17 - chizma

Logarifmik spiralning dekart koordinatalar sistemasidagi tenglamasi $x = \rho_0 e^{k\varphi}\cos\varphi, \quad y = \rho_0 e^{k\varphi}\sin\varphi.$

10. $\rho = f(\varphi)$ qutb tenglama bilan berilgan chiziqning dekart koordinatalar sistemasidagi tenglamasi yozilsin. Arximed spirali giperbolik spiralning dekart sistemadagi tenglamasi qanday?

Javob: $x = f(y)\cos\varphi$, $y = f'(\varphi)\sin\varphi$, $x = a\varphi\cos\varphi$, $y = a\varphi\sin\varphi$, $x = \frac{a}{\varphi}\cos\varphi$, $y = \frac{a}{\varphi}\sin\varphi$

11. $x^2 + y^2 + z^2 = R^2$ sfera va $x^2 + \left(y - \frac{R}{2}\right)^2 = \frac{R^2}{4}$ silindrning kesishish chizig`iga **Viviani chizig`i** deyiladi. Uning parametrik tenglamalarini yozing.



18 - chizma

Javob: $\angle XOP = U$ belgisak, $X = R\cos u \sin u$, $y = R\sin u$, $z = R\cos u$.

12. $x = t^2\cos t$, $y = t^2\sin t, z = t^2(0 < t < \infty)$ chiziq obrazinig doiraviy konusda yotishini ko`rsating va regurlyarligini isbotlang.

Javob: $x^2 + y^2 - z^2 = 0$

13. $x = 1 + \cos t$, $y = \sin t$, $z = 2\sin\frac{t}{2}$ $(-2\pi \le t \le 2\pi)$ chiziqning regulyarligini sferaga va $(x-1)^2 + y^2 = 1$ silindrik sirtga tegishliligini ko`rsating.

14. Dekart yaprog`i oshkormas tenglamasi $x^3 + y^3 - 3axy = 0$ orqali berilgan. Parametr t-ni, $\frac{y}{x} = t$ tenglik orqali kiriting va parametrik tenglamalarini aniqlang

Javob: $x = \frac{3at}{1+t^3}$, $y = \frac{3at^2}{1+t^3}$.

15. $x = t, y = t \ +1$, $z = (t-1)^3$ chiziqning regulyarligini isbotlang. Ushbu chiziqning XOY va XOZ koordinata tekisliklaridagi proyeksiyalarini aniqlang.

Javob: $y = x^2 + 1$, $z = 0$ va $z = (x-1)^3$, $y = 0$.

16. $x = \frac{t}{1+t^2+t^4}$, $y = \frac{t^2}{1+t^2+t^4}, z = \frac{t^3}{1+t^2+t^4}$ egri chiziqning regulyarligini isbotlang va $x^2 + \left(y - \frac{1}{2}\right)^2 + z^2 = \frac{1}{4}$ sferada yotishini ko`rsating.

17. $x = t^2\cos t$, $y = t^2\sin t$, $z = t^2\ (0 < t < \infty)$ egri chiziqning regulyarligini isbotlang va konus sirtida yotishini ko`rsating. φ_0 -burchakni toping.

18. $x = 1$, $y = t^2$, $z = t^3$ egri chiziqning koordinata tekisliklaridagi proyeksiyalari qanday tenglamaga ega. Javob: (OXY)da $y = x^2$,(OXZ)da $z = x^3$, (OYZ)da $z = y^{\frac{3}{2}}$.

19. Qutbiy tenglamalari orqali berilgan egri chiziqlar obrazlarini tasvirlang

a) $\rho = a\sin k\varphi$,

b) $\rho = a\cos k\varphi$ (a,k-musbat o`zgarmas sonlar).

$|\sin k\varphi| \le 1$, $|\cos k\varphi| \le 1$ ni e’tiborga olinsa gullarni a radiusli aylanaga ichki chizilganligini kelib chiqadi. $\sin k\varphi, \cos k\varphi$ davriy funksiyalar bo`lgani uchun gullar kongurent yaproqlardan iborat bo`lib, eng katta a radiusga nisbatan simmetrik vaziyatga ega bo`lishi kelib chiqadi. Quyidagi hollar bo`lishi mumkin: $\rho = \sin k\varphi$ yoki $\rho = a\cos k\varphi$ egri chiziqlar. a, k musbat o`zgarmas sonlar. $|\sin k\varphi| \le 1$ ,$|\cos k\varphi| \le 1$ bo`lgaligi uchun egri chiziq a radusli aylana ichida joylashadi, $\sin k\varphi$, $\cos k\varphi$ lar davriy funksiyalar bo`lganligi uchun gullar kongruent yaproqlardan iborat bo`lib, eng katta a radiusga nisbatan simmetrik joylashadi.

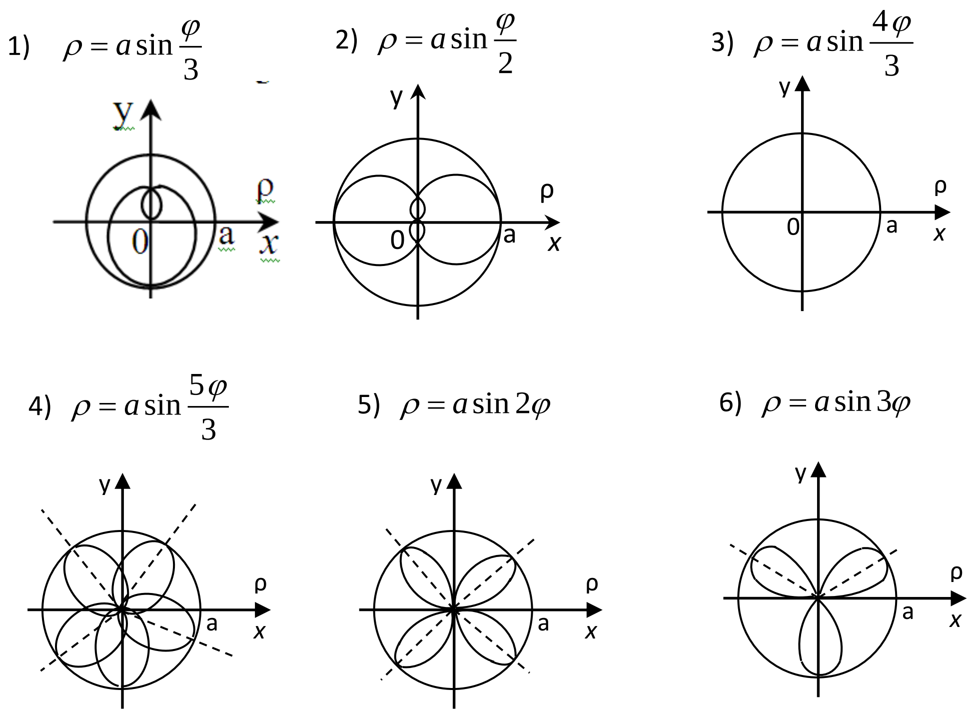
I. k ratsional son.

$$\left.\begin{array}{l} 6-1)\ toq\ \ bo`lsa, \\ 5-2)\ juft\ \ bo`lsa, \end{array}\right\} \text{u holda gul } \begin{cases} k\ \ ta \\ 2k\ ta \end{cases} \text{ yaproqdan iborat.}$$

II. $k-$**ratsional son**, yani $k=\dfrac{m}{n}$;

$$\left.\begin{array}{l} 1,4-\ \ 1)\ m\ va\ n\ sonlarning \\ \qquad ikkalasi\ \ ham \\ \qquad toq\ \ son\ \ bo`lsa, \\ \\ 2,3-\ \ 2)\ m\ va\ n\ sonlardan \\ \qquad biri\ \ juft\ \ bo`lsa, \end{array}\right\} \text{gul } \begin{cases} m\ \ ta \\ 2m\ ta \end{cases} \text{ yaproqdan iborat.}$$

III. k-irratsional bo`lsa, gul qisman bir –birini qoplovchi yaproqlarning cheksiz to`plamidan iborat.



19 - chizma

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1. Wilhelm Klingenberg, A Course in differential geometry,1978 by Springer-Verlag, New York Inc. Printed in the United States of America.
2. M.A.Arstrong, Basic Tpology, Springer, 1998 y.
3.Сборник задач по дифференциал`ной геометрии. Под ред. Феденко А.С. М., 1979

14.Egri chiziqlarning maxsus nuqtalari.

Reja:

1. Maxsus nuqta va tiplari.
2. Misollar

Tayanch tushuncha va iboralar :

Uzluksiz almashtirish, topologik almashtirish, elementar yoy, elementar chiziq, chiziqning chegaraviy nuqtalari, sodda chiziq, umumiy chiziq, regulyar va silliq chiziq.

Egri chiziqning vektorli parametrik tenglamasi

$$\gamma : \vec{r} = \vec{r}(t), \quad t \in \mathrm{I} \qquad \textbf{(1)}$$

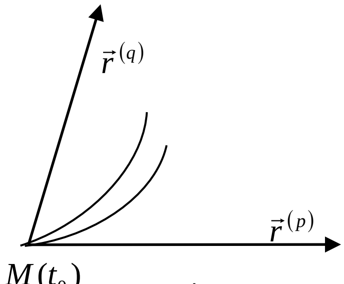
ko`rinishda bo`lsin . M(t_0) $\in \gamma$ nuqtada $\vec{r}_t(t_0) = 0$ bo`lsa M($t_0$) nuqtani **noregulyar nuqta** deb ataymiz. M($t_0$) nuqtada noldan farqli birinchi hosila vektor $\vec{r}^{\,(p)}(t_0)$ bo`lib, unga kolleniar bo`lmagan noldan farqli hosila vektor $\vec{r}^{\,(q)}(t_0)$ bo`lsin. Quyidagi hollar bo`lishi mumkin:

1) P-toq , q-juft; γ chiziq obrazi M(t_0) nuqta atrofida , regulyar nuqta atrofidagi kabi ko`rinishga egadir .

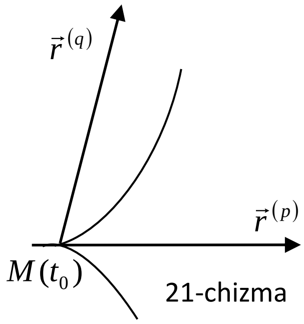
2) P-toq , q-toq; M(t_0) – chiziqning bukilish nuqtasi

3) P-juft , q- toq; M(t_0) – chiziqning 1- tur qaytish nuqtasi (20-chizma)

4) P-juft , q-juft; M(t_0) – chiziqning 2- tur qaytish nuqtasi (21-chizma)



20- chizma



21-chizma

$$F(x, y) = 0 \qquad \textbf{(2)}$$

tenglama orqali berilgan chiziqning $M_0(x_0,y_0)$ nuqtasining maxsus nuqta bo`lish shartlarini keltiraylik:

$$F(x_0, y_0) = 0, \quad F'_x(x_0, y_0) = 0, \quad F'_y(x_0, y_0) = 0 .$$

$F''_{x^2}, F''_{xy}, F''_{y^2}$ hosilalardan biri M_0 nuqtada noldan farqli bo`lsa, $M_0$- maxsus nuqtani **qo`shaloq** yoki **ikki karrali** deyiladi.

$F''_{y^2}(x_0, y_0) \neq 0$ bo`lsin. Egri chiziqning M_0 nuqtadagi urinmalarining burchak koeffisenti

$$F''_{y^2} \cdot k^2 + 2F''_{xy}k + F''_{xx} = 0 \ . \qquad \textbf{(3)}$$

tenglamadan aniqlanadi.

$F''^2_{xy} - F''_{xx} F''_{yy} > 0 \quad \Rightarrow M_0\left(x_0, y_0\right) \in \gamma$ tugun nuqta,

$F''_{xy} - F''_{xx} F''_{yy} < 0 \quad \Rightarrow M_0\left(x_0, y_0\right) \in \gamma$ - ajralgan nuqta,

$F''_{xy} - F''_{xx} F''_{yy} = 0 \quad \Rightarrow M_0\left(x_0, y_0\right) \in \gamma$ – o`z - o`ziga urinish nuqtasi, I- tur yoki II- tur qaytish nuqta bo`lishi mumkin.

$F(x_0, y_0) = F'_x(x_0, y_0) = F'_y(x_0, y_0) = F''_{xx}(x_0, y_0) = F''_{xy}(x_0, y_0) = F''_{y^2}(x_0, y_0) = 0$ uchinchi tartibli hususiy hosilalar orasida noldan farqlilari mavjud bo`lsa, u holda 3 karrali maxsus nuqtalarni aniqlash, tipini ko`rsatish, ushbu maxsus nuqta atrofida egri chiziq tuzulishini tekshirish mumkin.

Masala yechish namunalari

1. $\vec{r} = t^2 \vec{i} + t^3 \vec{j}$ chiziqning maxsus nuqtasini aniqlang va tipini ko`rsating.

Yechish: $r'_t(t) = 2t\vec{i} + 3t^2\vec{j}$, t=0 da $\vec{r}(0) = 0$, $r'_t(0) = 0$

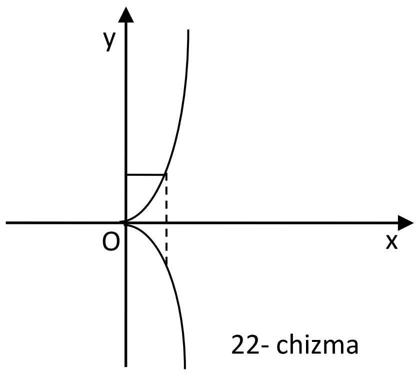
$\vec{r}''_{t^2}(t) = 2\vec{i} + 6t\vec{j}$, $\quad \vec{r}''_{t^2}(0) = 2\vec{i} \neq 0, \quad \vec{r}'''_{t^3}(0) = 6\vec{j} \neq 0$

$\vec{r}''_{t^2}$ va $\vec{r}'''_{t^3}$ vektorlar t=0 nuqtada kollenear emas. O(0,0) – maxsus nuqta, bo`lib egri chiziqning 1-tur qaytish nuqtasidir.

Tenglamani koordinata ko`rinishida yozaylik:

$$\gamma : \begin{cases} x = t^2 \\ y = t^3 \end{cases} \quad \Rightarrow y^2 = x^3$$ yarim kubik parabola ;

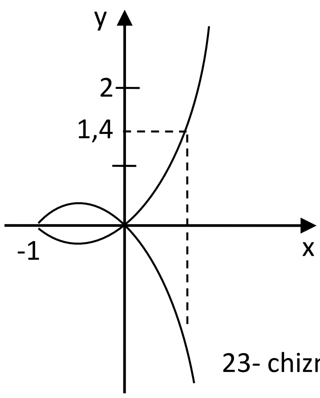
x	4	1	0	1	4	9
y	-8	-1	0	1	8	27
t	-2	-1	0	1	2	3



22- chizma

2. $y^2 - x^2 - x^3 = 0$ chiziqning maxsus nuqtalarini aniqlang va tipini ko`rsating;

Yechish: $$\begin{cases} F(x,y) = y^2 - x^2 - x^3 = 0, \\ F'_x(x,y) = -2x - 3x^2 = 0, \\ F'_y(x,y) = 2y = 0, \qquad \Rightarrow y = 0, \end{cases}$$



23- chizma

$$x^2 + x^3 = 0 \Rightarrow x^2(x+1) = 0 \Rightarrow x = 0,\ x = -1,$$

$$2x + 3x^2 = 0 \Rightarrow x(2+3x) = 0 \Rightarrow x = 0,\ x = -\frac{2}{3},$$

$x = -1$ va $x = -\dfrac{2}{3}$ qiymatlar sistemasini qanoatlantirmaydi.

O (0,0) nuqta sistemani qanoatlantiradi. Demak, maxsus nuqta O(0,0)

$F''_{xx}(0{,}0) = -2$; $F''_{xy}(0{,}0) = 0$; $F''_{y^2}(0{,}0) = 2$; $D = a_{12}^2 - a_{11}a_{22} = 4 > 0$.

Xulosa: O(0,0) – o`z o`zini kesish nuqtasi ya`ni tugun nuqta.

Mustaqil yechish uchun masalalar

1.Quyidagi chiziqlarning mahsus nuqtalari va ularning tipini ko`rsating

1) $x^2 - y^2 - x^4 = 0.$ Javob: 0(0,0)- o`z –o`zini kesish nuqtasi.

2) $x^3 - x^2 - y^2 = 0.$ Javob: 0(0,0) ajralgan nuqta.

3) $x^2 - x^2y^2 + y^2 = 0$ Javob: 0 (0,0) ajralgan nuqta.

4) $x^5 + 5x^4 - 4y^2 = 0$ Javob: 0(0,0) - o`z-o`zini kesish.

5) $ax^2 + x^3 - y^2 = 0.$ Javob: 0(0,0) – maxsus nuqta,

a < 0 da ajralgan nuqta,

a=0 da 1-tur qaytish nuqtasi,

a >0 da tugun nuqta.

6) $x^3 + y^3 - 3xy = 0.$ Javob: 0(0,0)-tugun nuqta.

7) $x^2 + y^2 - x^4 - y^4 = 0.$ Javob: 0(0,0) ajralgan nuqta.

8) $x^2 - x^6 + y^4 = 0.$ Javob: 0(0,0) ajralgan nuqta.

9) $\left(x^2 + y^2\right)^2 - a^2\left(x^2 - y^2\right) = 0.$ Javob: 0(0,0) tugun nuqta.

10) $\left(y - x^2\right)^2 - x^5 = 0.$ Javob: 0(0,0) -2- tur qaytish nuqtasi.

11) $(a + x)y^2 - (a - x)x^2 = 0.$ Javob: 0(0,0) tugun nuqta.

12) $x^4 - 8xy^2 + y^4 = 0.$ Javob: 0(0,0) uch karrali mahsus nuqta.

2. Quyidagi ajoyib chiziqlarning mahsus nuqtalarini aniqlang va tipini ko`rsatining.

1) Diokles sissoidasi: $\left(x^2 + y^2\right)x - 2ay^2 = 0.$

Javob: 0(0,0) - I - tur qaytish nuqtasi.

2) Nikomey konxoidasi : $\left(x^2 + y^2\right)(y - a)^2 - l^2y^2 = 0.$

Javob: l < a da 0(0.0) - ajralgan nuqta.

3) Strofoida : $(2a - x)y^2 = x(x - a)^2.$ Javob: A(0,0)-tugun nuqta.

4) Kardoida: $\left(x^2 + y^2 - 2ax\right)^2 - 4a^2\left(x^2 + y^2\right) = 0.$ Javob: 0(0,0) – I – tur qaytish nuqtasi .

5) Astroida : $x = a\cos^3 t,\ y = a\sin^3 t.$ Javob : 4-ta maxsus nuqta 1-tur qaytish nuqtasi.

6) Traktrissa : $x = a \sin t,\ y = a\left(\cos t + \ln \operatorname{tg}\frac{\mathbf{t}}{\mathbf{2}}\right);$

Javob: A(a,0) – 1-tur qaytish nuqtasi.

7) Bernulli lemniskatasi: $\left(x^2 + y^2\right)^2 - 2a^2\left(x^2 - y^2\right) = 0;$

Javob: O(0,0)- o`z o`zini kesish nuqtasi.

8) Dekart yaprog`i: $x=\frac{3at}{1+t^3}$; $y=\frac{3at^2}{1+t^3}$;

Javob: O(0,0) o`z o`zini kesish nuqtasi.

9) Sikloida : $x=a(t-\sin t)$; $y=a(1-\cos t)$;

Javob: $(2k\pi,0)$, k=1,2, … maxsus nuqtalar. 1-tur qaytish nuqtalar.

10) x=acost, y=asint , z=bt vint chiziq OXY koordinata tekisligiga (OZ) o`q bilan $\theta$ burchak tashkil etuvchi (OYZ) tekislikka parallel to`g`ri chiziqlar orqali proyeksiyalanadi. Vint chiziq proyeksiyasi maxsus nuqtaga egami? Maxsus nuqta $\theta$ ning qaysi qiymatida mavjud bo`lishi mumkin. Tipi qanday?

3. Transendent egri chiziqlarning maxsus nuqtalarini izlang.

1) $y^2=1-e^{-x^2}$; Javob: O(0,0) – ikki karrali nuqta.

2) $y^2=1-e^{-x^3}$; Javob: O(0,0) – qaytish nuqtasi.

3) $y=\frac{x}{1+e^{\frac{1}{x}}}$; Javob: O(0,0) – tig`izlanish nuqtasi

4) $y=arctg\left(\frac{1}{\sin x}\right)$; Javob: $x=k\pi(k=0,\pm1,\pm2...)$ – birinchi tur qaytish nuqtasi.

5) $y^2=\sin\frac{\pi}{x}$; Javob: $x=0$ – ikkinchi tur qaytish nuqtasi.

6) $y^2=\sin x^2$; Javob: $x=0$ – ikki karrali nuqta.

7) $y^2=\sin^3 x$; Javob: $x=k\pi(k=0,\pm1,\pm2...)$ – qaytish nuqtasi.

8) $y=x\ln x$; Javob: O(0,0) – tugash nuqtasi.

Foydalanilgan adabiyotlar

1. Wilhelm Klingenberg, A Course in differential geometry,1978 by Springer-Verlag, New York Inc. Printed in the United States of America.
2. M.A.Arstrong, Basic Tpology, Springer, 1998 y.
3.Сборник задач по дифференциал`ной геометрии. Под ред. Феденко А.С. М., 1979

15. Asimptotalar

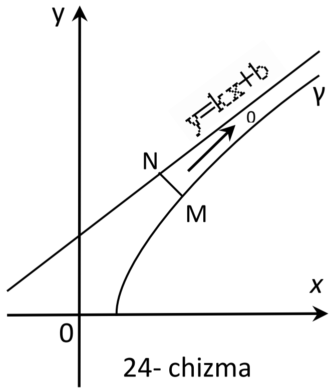
Reja:

1. Asimptota ta’rifi
2. Algebraik chiziq asimptotalari
3. Misollar

Tayanch tushuncha va iboralar: Urinma, nuqtadagi urinma, normal tekislik.

Tekis egri chiziq quyidagi parametrik tenglamalari orqali berilgan bo`lsin:

$$\gamma : x = x(t), y = y(t). \qquad (1)$$



24- chizma

**Ta`rif.** Birorta $\ell$ to`g`ri chiziq mavjud bo`lib , γ chiziq ustidagi M(t) nuqta chiziq bo`ylab cheksiz uzoqlashganda, ushbu nuqtadan $\ell$ to`g`ri chiziqqacha masofa nolga intilsa , u holda $\ell$ to`g`ri chiziqni berilgan γ chiziqning asimptotasi deb ataladi.

$$Ax + By + C = 0 \ , \quad (2)$$

to`g`ri chiziqni (1) chiziqqa asimptota bo`lish sharti:

$$\lim_{t\to T}|Ax(t) + By(t) + C| = 0. \quad (3)$$

Asimptota $y = kx + b$, (4)

ko`rinishdagi tenglamaga ega bo`lsa , koeffitsientlarni quyidagicha topiladi :

$$k = \lim_{t\to T}\frac{y(t)}{x(t)}, \quad b = \lim_{t\to T}[y(t) - kx(t)]. \quad (5)$$

(1) egri chiziq vertikal asimptotaga ega bo`lsa , uning tenglamasi

$$a = \lim_{t\to T} x(t) \ , \quad \lim_{t\to T} y(t) = \infty . \qquad (6)$$

Gorizontal asimptota uchun $\lim_{t\to T} x(t) = \infty, \ b = \lim_{t\to T} y(t).$ (7)

Tekis chiziq $y = f(x),$ (8)

tgenglama orqali berilgan bo`lsa, asimptota uchun $k = \lim_{x\to\infty}\frac{f(x)}{x}, \ b = \lim_{x\to\infty}[f(x) - kx]$

(9)

koeffisientlarni hisoblaymiz.

Algebraik ko`rinishda berilgan tekis chiziq asimptotalarining koeffisiantlarini eng yuqori darajali o`zgaruvchilar koeffisentlarini nolga tenglashtirib tenglamalarni yechish orqali topiladi.

$$A_n(k) = 0, \quad A_{n-1}(b, k) = 0. \qquad (10)$$

Masala yechish namunalari

1 $y = \frac{5}{x^2 - 16}$ egri chiziq asimptotalarini aniqlang.

Yechish: 1) $x = \pm 4$ ga $y = \infty$. ikkita asimptota mavjud bo`lib, tenglamalari $x - 4 = 0$ va $x + 4 = 0$.

2) $x \to \pm\infty$ ga $y \to 0 \Rightarrow$ absissa o`qi $y = 0$ berilgan chiziq asimptotalaridan biridir.

Javob: $x - 4 = 0, \ x + 4 = 0, \ y = 0$.

2. $x=\frac{2t-1}{t^2-1}$, $y=\frac{t^2}{t-1}$ parametrik qurishda berilgan yassi egri chiziq asimptotalarini aniqlang.

Yechish: $t=-1$ da $x=-\infty, y=-\frac{1}{2}\Rightarrow$ asimptota tenglamasi $y+\frac{1}{2}=0,$

$t=1$ da $x=\infty,\ y=\infty\Rightarrow$ ko`ramizki koordinata o`qlariga parallel bo`lmagan asimptota mavjud bo`lib, uni $y=kx+b$ tenglama ko`rinishida izlaymiz.

$$k=\lim_{t\to1}\frac{y(t)}{x(t)}=\lim_{t\to1}\left(\frac{t^2}{t-1}:\frac{2t-1}{t^2-1}\right)=\lim_{t\to1}\frac{t^2(t+1)}{2t-1}=2$$

$$b=\lim_{t\to1}\left[y(t)-kx(t)\right]=\lim_{t\to1}\left[\frac{t^2}{t-1}-2\frac{2t-1}{t^2-1}\right]=\lim_{t\to1}\left[\frac{t^2(t+1)-4t+2}{t^2-1}\right]=\lim_{t\to1}\frac{t^3+t^2-4t+2}{t^2-1}=$$

$$=\lim_{t\to1}\frac{(t-1)(t^2+2t-2)}{(t-1)(t+1)}=\lim_{t\to1}\frac{t^2+2t-2}{t+1}=\frac{1}{2}$$ k va b qiymatlarini $y=kx+b$ tenglamaga qo`yamiz.

Og`ma asimptota tenglamasi $y=2x+\frac{1}{2}$ ko`rinishga ega bo`ladi.

Javob: $y+\frac{1}{2}=0$ va $y=2x+\frac{1}{2}$

3. Algebraik metod orqali asimptotalarni toping.

$$x^3-2y(x+1)^2=0$$

1) $x+1=0$ da $y=\infty$ ordinata o`qiga parallel asimptota tenglamasi: $x+1=0$.

2) tenglamaga y=kx+b ni qo`yamiz.

$x^3-2(kx+b)(x+1)^2=0\Rightarrow\quad x^3-2kx^3-4kx^2-2kx-2bx^2-4bx-2b=0$,

$(1-2k)x^3-(4k+2b)x^2-(2k+4b)x-2=0.$ $1-2k=0$ va $4k+2b=0\Rightarrow k=\frac{1}{2}, b=-1$.

Javob: $y=\frac{1}{2}x-1$ va $x+1=0$.

Mustaqil yechishga masalalar

1. Quyidagi chiziqlarning asimptotalarini aniqlang.

1) $y=\frac{2x}{x-1}$; Javob: $y=2,\ x=1$.

2) $y=\frac{1}{2x^2+x-1}$; Javob: $y=0,\ x=-1,\ x=\frac{1}{2}$.

3) $y=\frac{2}{x-3}$; Javob: $x=3,\ y=0$.

4) $y=\frac{a^3}{a^2+x^2}$; Javob: $y=0$.

5) $y=\frac{x^2}{x^2-1}$; Javob: $x=\pm1,\ y=1$.

6) $y=\dfrac{3}{(x-2)^2}$; Javob: $x=2,\ y=0$.

7) $y=\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}-1=0$; Javob: $y=\pm\dfrac{b}{a}x$.

8) $y=\dfrac{1}{x^2-1}$; Javob: $x=\pm1, y=0$.

9) $x^2+y^2-x^2y^2=0$; Javob: $x=y=\pm1$.

10) $xy^2-(x-1)^2=0$; Javob: $x=0$.

2. Quyidagi chiziqlarning og`ma va koordinata o`qlariga parallel asimptotalarini toping.

1) $y=\dfrac{x^2-4x+7}{x}$; Javob: $x=0,\ y=x-4$.

2) $y=\dfrac{x^2}{x-3}$; Javob: $x=3,\ y=x+3$.

3) $y=\dfrac{x^2}{x+2}$; Javob: $x=-2,\ y=x-2$.

4) $y=\dfrac{x^2}{x+4}$; Javob: $y=x+1,\ x-1=0$.

5) $x=\dfrac{y^3}{y^2-4}$; Javob: $y=x,\ y=\pm2.$

6) $y=\dfrac{x}{2x-1}+x$; Javob: $y=x+\dfrac{1}{2}$ va $x=\dfrac{1}{2}$.

7) $x^3-2y\left(x+1\right)^2$; Javob: $y=\dfrac{1}{2}x-1,\ \ x=-1$.

8) $xy^2-y^2-4x=0$; Javob: $y=\pm2,\ x=1$.

9) $xy^2-x^2-2x+\dfrac{5}{4}=0$; Javob: $x=0$.

10) $\left(x^2-y^2\right)\left(x-y\right)=1$; Javob: $y=\pm x$.

11) $y=\dfrac{x^3}{2\left(x+1\right)^2}$; Javob: $x+1=0; x-2y-2=0$.

12) $y=\dfrac{x^3}{x^2-3}$; Javob: $y=x,\ x=\pm\sqrt{3}.$

13) $y^3-4x^2y+2x^2+y^2-5x+y+4=0$; Javob: $y=2x-\dfrac{2}{3},\ y=-2x-\dfrac{3}{4},\ y=\dfrac{1}{2}.$

3. Parametrik ko`rinishda berilgan quyidagi chiziqlarning asimptotalarini aniqlang

1) $x=\frac{3at}{t^3+1},\ \ y=\frac{3at^2}{t^3+1};$ Javob: $x+y+a=0$.

2) $x=\frac{t^2}{t-1}, y=\frac{6}{t^2-1};$ Javob: $x=-\frac{1}{2},\ \ 2x-4y-3=0.$

3) $x=\frac{2t}{(t-1)(t-2)}, y=\frac{t^2}{(t-1)(t-3)};$ Javob: $x=3,\ y=-4,\ y=\frac{1}{4}x-\frac{1}{8}.$

4) $x=\frac{2t}{t^2+1},\ \ y=\frac{t^3}{t^{2+1}};$ Javob: $x=1$.

5) $x=\frac{t^2}{t^2+1},\ \ y=\frac{t(1-t^2)}{1+t^2};$ Javob: $x=1$.

6) $x=\frac{t^2}{1-t},\ \ y=\frac{t^2}{1-t^2};$ Javob: $x=\frac{1}{2},\ y=\frac{1}{2}x-\frac{1}{4},\ \ y=x+1$.

7) $x=\frac{t^2}{1-t^2},\ \ y=\frac{t^3}{1-t^2};$ Javob: $x=-1;\ \ y=\pm(x-\frac{1}{2})$

8) $x=\frac{t}{1-t^2},\ \ y=\frac{t(1-2t^2)}{1-t^2};$ Javob: $x=0,\ x+y-2=0,\ x+y+2=0.$

9) $x=\frac{5t^2}{1+t^5},\ \ y=\frac{5t^2}{1+t^5};$ Javob: $x+y-1=0.$

10) $x=\frac{(t+2)^2}{t+1},\ \ y=\frac{(t-2)^2}{t-1};$ Javob: $2x+9=0,\ x=\frac{9}{2},\ \ x-y-6=0$.

4. Ushbu chiziqlarning asimptotalarini toping.

1) $x^3-y^3+x^2+y^2=0;$ Javob : $y=x+\frac{2}{3}$.

2) $y=x+\frac{1}{x};$ Javob: $x=0,\ \ y=x$.

3) $x(x^2+y^2)-ay^2;$ Javob: $x-a=0$.

4) $x^3-2y(x-1)^2=0;$ Javob: $y=\frac{1}{2}x+1,\ x-1=0.$

5) $x^4-4x^2y^2-6x^2-4y^2=0;$ Javob: $x-2y=0,\ \ x+2y=0.$

6) $x^2-2x^2y^2+y^4-2x=0;$ Javob: $y=\pm x$.

7) $y+x^2y^2-1=0;$ Javob: $x=0,\ y=0.$

8) $x^2-x^2y^2+y^2=0;$ Javob: $x=y=\pm 1.$

5. Qutb koordinatalar bo`yicha berilgan ushbu chiziqlarning asimptotalarini toping.

1) Nikomed konxoidasi $r=\frac{a}{\sin\varphi}+\ell;$ Javob: $y=a$.

2) Diokles sissoidasi $r=\frac{2a\sin^2\varphi}{\cos\varphi}$; Javob: $x=2a$.

3) Giperbolik spiral $r=\frac{a}{\varphi}, 0<\varphi<\infty$; Javob: $y=a$.

Yechish: $x=\frac{a\cos\varphi}{\varphi}$, $y=\frac{a\sin\varphi}{\varphi}$, $r\to\infty \Rightarrow \varphi\to 0$

$$x=\lim_{\varphi\to 0}\frac{a\cos\varphi}{\varphi}=\infty, \quad y=\lim_{\varphi\to 0} a\frac{\sin\varphi}{\varphi}=a$$

Foydalanilgan adabiyotlar

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2. M.A.Arstrong, Basic Tpology, Springer, 1998 y.
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16. Chiziqning urinmasi va normali.

Reja :

1. Urinmaning turlicha tenglamalari
2. Normal tekislik tenglamasi
3. Misollar

Tayanch tushuncha va iboralar: Chiziq urinmasi, chiziq normali , urinma tenglamasi, chiziq normalini tenglamasi.

Tekis chiziq tenglamasiga mos ravishda urinma va normal tenglamalari turlicha bo`lishi mumkin. Chiziq tenglamasi $\vec r=\vec r(t)$ ko`rinishda bo`lsa, urinma tenglamasi

$$\vec R=\vec r+\lambda \vec r_t', \qquad (1)$$

chiziq $x=x(t)$, $y=y(t)$ tenglama orqali berilgan bo`lsa, urinma tenglamasi

$$\frac{X-x}{x_t'}=\frac{Y-y}{y_t'}, \qquad (2)$$

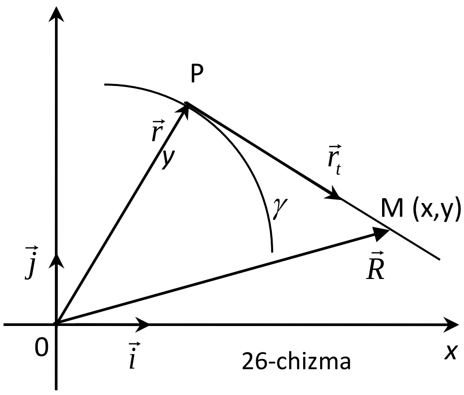
bunda $P(x,y)$-urinish nuqta, $\vec r_t'\{x_t', y_t'\}$-urinmaning yo`naltiruvchi vektori, $M(X,Y)$-urinma to`g`ri chiziqning ixtiyoriy nuqtasi.

Tekis chiziq $y=f(x)$ tenglama orqali berilgan bo`lsa, $P(x,y)$ nuqtadagi urinma tenglamasi

$$Y-y=f_x'(x)(X-x),$$

(3)

ko`rinishda bo`lib, $F(x,y)=0$ oshkormas tenglama orqali berilgan chiziqning $P(x,y)$ nuqtasidagi urinmasi



26-chizma

$$(X-x)F'_x+(Y-y)F'_y=0, \quad (4)$$

ko`rinishdagi tenglamaga ega.

Mos ravishda normal tenglamalarini yozaylik:

$(\vec{R}-\vec{r})\vec{r}\,'_t=0,\ (X-x)x'_t+(Y-y)y'_t=0,\ X-x+f'_x(x)(Y-y)=0$

$$\frac{X-x}{F'_x}=\frac{Y-y}{F'_y}. \qquad (5)$$

Masala yechish namunalari:

1. $\vec{r}=a\vec{e}\varphi$ aylana ixtiyoriy nuqtasidagi urinmasi va normalining tenglamasini yozing, bunda $\vec{e}(\varphi)=\cos\varphi\vec{i}+\sin\varphi\vec{j}$.

Yechish:

$\vec{R}=a\,\vec{e}(\varphi)+\lambda\,a\,\vec{g}(\varphi)$-urinma. $(\vec{R}-a\vec{e}(\varphi))\,\vec{g}(\varphi)=0$ -normal, bunda $\vec{g}(\varphi)=-\sin\varphi\vec{i}+\cos\varphi\vec{j}$ $\quad |\vec{e}(\varphi)|=|\vec{g}(\varphi)|=1.$

2. $x=t^3\,, y=t^2$ egri chiziqning $M(-7,-1)$ nuqtadan o`tuvchi urinmasi va normali tenglamasini aniqlang.

Yechish:

$$\frac{X-t^3}{3t^2}=\frac{Y-t^2}{2t} \Rightarrow \frac{-7-t^3}{3t^2}=\frac{-1-t^2}{2t} \Rightarrow 2(7+t^3)=3t(1+t^2) \Rightarrow t^3+3t-14=0$$

Tenglamaning haqiqiy ildizi $t=2$. Urinish nuqtasi $P(8,4)$. Urinma vektori $\vec{r}_t{}'\{12,4\}$. Urinma tenglamasi $\dfrac{X-8}{12}=\dfrac{Y-4}{4} \Rightarrow X-3Y+4=0$.

Normal tenglamasi $12(X-8)+4(Y-4)=0 \Rightarrow 3X+Y-28=0$.

3. $Y^2=2pX$ parabolaning P(x_0 , y_0) nuqtasidagi urinmasi *a* normalini aniqlang.

Yechish: $Y=y_0+k\left(x-x_0\right)$to`g`ri chiziqning parabolaga urinma bo`lish shartini topaylik: $K^2x^2+2\left(ky_0-k^2x_0-p\right)x+y_0^2-2kx_0y_0+k^2x_0^2=0.$

Kvadrat tenglama diskriminanti nolga teng bo`lsa, kesuvchi urinma vaziyatni oladi. $D=(ky_0-k^2x_0-p)^2-k^2(y_0{}^2-2kx_0y_0+k^2x_0{}^2)=0$,

$$2k^2x_0-2ky_0+p=0 \quad => \quad k_{1,2}=\tfrac{y_0}{2x_0} \quad . \qquad (1)$$

Ikkinchi tomondan $y^2-2px=0$ parabolaning P(x_0,y_0) nuqtasida o`tkazilgan urinmaning $k=y'_x(x_0)$ burchak koeffitsienti: $k=\tfrac{p}{y_0}$ (2)

Urinmaning ordinata o`qida ajratgan kesmasi:

$$b=y_0-p\tfrac{x_0}{y_0} \qquad (3)$$

k va b larni ya`ni (1) va (3) ni $y=kx+b$ tenglamaga qo`yamiz :

$$y=\tfrac{y_0}{2x_0}x+y_0-p\tfrac{x_0}{y_0} \quad => \quad 2x_0y_0y=y_0^2x+2x_0y_0^2-2px_0^2 \quad =>$$

$$2x_0y_0y = 2px_0x + 2x_0(y_0^2 - px_0)\ ,\ y_0^2 = 2px_0 \quad =>$$

$\boxed{y\,y_0 = p(x+x_0)}$ - urinma tenglamasi

$y_0(x-x_0)+p(y-y_0)=0$-normal tenglamasi

4. $x^3+y^3-3axy=0$ chiziqning $A\left(\frac{3a}{2},\frac{3a}{2}\right)$ nuqtasidagi urinmasi va normalini aniqlang.

Yechish: $F'_x\left(\frac{3a}{2},\frac{3a}{2}\right)=\frac{9}{4}a^2;\quad F'_y\left(\frac{3a}{2},\frac{3a}{2}\right)=\frac{9}{4}a^2$

$$\frac{9}{4}a^2\left(x-\frac{3a}{2}\right)+\frac{9}{4}a^2\left(y-\frac{3a}{2}\right)=0 \Rightarrow x+y-3a=0$$

Normal tenglamasi $x-y=0$.

5. $x=t^2-1, y=t^3+1$ chiziqning $2x-y+3=0$ to`g`ri chiziqqa parallel urinmasi va normalining tenglamasini yozing.

Yechish: $x'_t=2t,\ \ y'_t=3t^2,\ k=\frac{y'_t}{x'_t}=\frac{3}{2}t=2 \Rightarrow t=\frac{4}{3},\quad x_0=\frac{7}{9}, y_0=\frac{91}{27}, x'_t=\frac{8}{3}, y'_t=\frac{16}{3}$

Urinma tenglamasi: $\frac{x-\frac{7}{9}}{\frac{8}{3}}=\frac{y-\frac{91}{27}}{\frac{15}{3}} \Rightarrow 2x-y+\frac{49}{27}=0$

Normal tenglamasi: $\frac{8}{3}\left(x-\frac{7}{9}\right)+\frac{15}{3}\left(y-\frac{91}{27}\right)=0 \Rightarrow 27+54y-203=0$

Mustaqil yechish uchun mashq va masalalar.

1. Quyidagi chiziqlarning urinmasi va normali tenglamasini yozing:

a) $x=t^3-2t, y=t^2+1 \quad A(t=1)$ nuqtada

Javob: $2x-y+4=0$ va $x+2y-3=0$.

b) $x=t^2, y=t^3$ chiziqning $P(0,0)$nuqtadagi urinmasining tenglamasini aniqlang;

Javob: $y=0$.

v) $x=\frac{t^2}{t-1}, y=\frac{t}{t^2-1}$ $(-\infty<t<-1,\ -1<t<1,\ 1<t<\infty)$ chiziqning $P\left(4,\frac{2}{3}\right)$ nuqtasidagi urinmasi va normalining tenglamasini aniqlang;

Javob: $y=\frac{2}{3}$ va $x=4$.

2. $x=a(1-\cos t), y=a(1-\sin t),\quad 0\le t<2\pi$ chiziq urinmalari koordinata o`qlari bilan kesishib, urinish nuqtasiga bog`liq bo`lmagan doimiy perimetrli uchburchak tashkil etishi isbotlansin.

3. $x=a\cos^3 t, y=a\sin^3 t$ chiziq urinmasi va normalining tenglamasini yozing ;

Javob: urinma: $2x\sin t + 2y\cos t - a\sin 2t = 0$,

normal: $x\cos t - y\sin t - a\cos 2t = 0$.

4. Quyidagi chiziqlarning urinmasi va normalini tenglamasini aniqlang:

$a)\ y = \sin x,\quad A(0,0),\ B\left(\frac{\pi}{2},1\right).$ nuqtalarda.

Javob : A nuqtada $x = y\ \ va\ \ x = -y$, B nuqtada $y = 1\ \ va\ \ x = \frac{\pi}{2}$.

$b)\ y = x^3,\ \ A(0,0),\ \ B(1,1).$

Javob: A nuqtada $y = 0$ va $x = 0.$

B nuqtada $3x - y - 2 = 0$ va $x + 3y - 4 = 0.$

v) $y = tgx,\ \ A(0,0),\ B\left(\frac{\pi}{4},1\right).$

Javob: A nuqtada $y = x$ va $y = -x.$

B nuqtada $2x - y + 1 - \frac{\pi}{2} = 0$ va $x + 2y - 2 - \frac{\pi}{4} = 0.$

5. $y = x^2$ parabolaga $y = 4x - 5$ to`g`ri chiziqqa parallel ravishda o`tkazilgan urinmasini aniqlang;

Javob: $y = 4x - 4$.

6. $y=x^2 - 2x + 1$ chiziqning (1,0) nuqtasidagi urinmasi va normalini aniqlansin ;

Javob: y=0 va x=0.

7. $y = x^2$ parabolada urinmasi (OX) absissa o`qi bilan 45^o burchak tashkil etuvchi nuqtani toping. Javob: $P\left(\frac{1}{2},\frac{1}{4}\right)$.

8. $y=x^2-6x+5$ parabolada urinmasi x-2y+8=0 to`g`ri chiziqqa perpendikulyar bo`lgan nuqtani toping. Javob: A (2,-3).

9. $A(0,2)$ nuqtadan $x^2 + y^2 = 1$ aylanaga o`tkazilgan urinmalar tenglamalarini yozing.

Javob : $\sqrt{3}x - y + 2 = 0$ va $\sqrt{3}x + y - 2 = 0$.

10. $x = a\left(\ln tg\left(\frac{\varphi}{2}\right) + \cos\varphi\right),\quad y = a\sin\varphi$ $(0 < \varphi < \pi)$ traktrisa urinmalarining urinish nuqtasi bilan (OX) o`q orasidagi chegaralangan kesmaning doimiy uzunlikda bo`lishi ko`rsatilsin.

11. $x^3 + xy^2 - ay^2 = 0$ chiziqning $A(\frac{a}{2},\frac{a}{2})$ nuqtadagi urinmasi va normalining tenglamasini aniqlang. Javob: 2x – y + 4=0 va x + 2y – 3 = 0.

12. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ellips va $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ giperbolaning ixtiyoriy $M_0(x_0,y_0)$ nuqtasidagi urinmasi va normalining tenglamasi mos ravishda $\frac{x_0X}{a^2}+\frac{y_0Y}{b^2}=1$, $\frac{x_0X}{a^2}-\frac{y_0Y}{b^2}=1$ - urinma, $\frac{(X-x_0)a^2}{x_0}-\frac{(Y-y_0)b^2}{y_0}=0$, $\frac{(X-x_0)a^2}{x_0}+\frac{(Y-y_0)b^2}{y_0}=0$ - normal bo`lishini isbotlang.

13. $\left(x^2+y^2\right)^2-2a^2\left(x^2-y^2\right)=c$ chiziqni $M_0(x_0,y_0)$ nuqtasidagi urinmasi va normal tenglamasini yozing.

Javob: $x_0\left(x_0^{\,2}+y_0^{\,2}-a^2\right)(X-x_0)+y_0\left(x_0^{\,2}+y_0^{\,2}+a^2\right)(Y-y_0)=0$ - urinma

$y_0\left(x_0^{\,2}+y_0^{\,2}+a^2\right)(X-x_0)-x_0\left(x_0^{\,2}+y_0^{\,2}-a^2\right)(Y-y_0)=0$ - normal

14. Qutb koordinatalarida berilgan $r=a\varphi$ chiziq urinmasi va normalining tenglamasini yozing

Javob: $(\sin\varphi+\varphi\cos\varphi)x-(\cos\varphi-\varphi\sin\varphi)y-a\varphi^2=0$ - urinma,

$(\cos\varphi-\varphi\sin\varphi)x+(\sin\varphi+\varphi\cos\varphi)y-a\varphi=0$ - normal.

15. $r=2a\cos\varphi$ chiziqning $A\left(\sqrt{2}a,\frac{\pi}{4}\right)$ nuqtasidagi urinma va normalini aniqlang ; Javob: urinma: $y-a=0$, normal : $x-a=0$.

16. $\gamma_1: r=ae^{\varphi}$ va $\gamma_2: r=be^{-\varphi}$ qutbiy tenglamalar orqali berilgan chiziqlarning to`g`ri burchak ostida kesishisini isbotlang .

Isbot : Tenglamalarni koordinatalarda yozaylik :

$$\gamma_1:\begin{cases}x=ae^{\varphi}\cos\varphi\\ y=ae^{\varphi}\sin\varphi\end{cases},\qquad \gamma_2:\begin{cases}x=be^{\varphi}\cos\varphi\\ y=be^{\varphi}\sin\varphi\end{cases}.$$

γ_1 uchun $k=\frac{y'(\varphi)}{x'(\varphi)}=\frac{\sin(\varphi)+\cos(\varphi)}{\cos(\varphi)-\sin(\varphi)}$, (*)

γ_2 uchun $\bar{k}=\frac{\sin(\varphi)-\cos(\varphi)}{\cos(\varphi)+\sin(\varphi)}$, (**)

(*), (**) $\Rightarrow k\bar{k}=-1 \Rightarrow$ kesishish nuqtalarida urinmalari o`zaro perpendikulyar.

17. Quyidagi chiziqlarning kesishish nuqtalari va shu nuqtalarda tashkil etgan burchaklarni toping

a) $y^2=4x$, $x^2=4y$. Javob: $M_1(0,0)$, $M_2(4,4)$; $\varphi_1=\frac{\pi}{2}$, $\varphi_2=arctg\left(\frac{3}{4}\right)$.

b) $x^2+y^2=9$, $x^2+y^2-6x=9$. Javob: $M_1(0,3)$, $M_2(0,-3)$, $\varphi_1=\varphi_2=\frac{\pi}{4}$,

v) $x^2+y^2+2x=7$, $y^2=4x$. Javob: $M_1(1,2)$, $M_2(1,-2)$, $\varphi_1=\varphi_2=\frac{\pi}{2}$.

18.Quyidagi chiziqlarni to`g`ri burchak ostida kesishishini isbotlang.

a) $y=x-x^2$ va $y=x^2-x$;

b) $y^2=2ax+a^2$ va $y^2=-2bx+b^2$;

v) $x^2-y^2=a$ va $xy=b$;

g) $r=a(1+\cos\varphi)$ va $r=a(1-\cos\varphi)$;

19. $x^{\frac{2}{3}}+y^{\frac{2}{3}}=a^{\frac{2}{3}}$ astroida urinmasining koordainata o`qlari bilan kesishib ajratgan kesmasi *a* uzunlikka ega ekanligini isbotlang.

20. $\gamma_1: y=\sin x$ va $\gamma_2: \bar{y}=\mathrm{t}g\bar{x}$ egri chiziqlarning koordinata boshi $O(0,0)$ nuqtadagi o`zaro urinish tartibini aniqlang.

Yechish: $y'_x=\cos$ va $\bar{y}'_{\bar{x}}=\frac{1}{\cos^2\bar{x}}$, $y'_x(0)=\bar{y}'_x(0)=1$,

$y''_{x^2}=-\sin x$ va $\bar{y}''_{\bar{x}^2}=\frac{2\sin x}{\cos^3\bar{x}}$, $y''_{x^2}(0)=\bar{y}''_{\bar{x}^2}(0)=0$,

$y'''_{x^3}=-\cos x$ va $\bar{y}'''_{\bar{x}^3}=-\frac{2\cos^2\bar{x}+6\sin^2\bar{x}}{\cos^4\bar{x}}$, $y'''_{x^3}(0)\neq\bar{y}'''_{\bar{x}^3}(0)$.

Javob: urinish tartibi $k=2$.

21. $y=x^3$ va $y=x\sin x$ chiziqlarning $O(0,0)$ nuqtadagi urinish tartibini aniqlang.

Javob: $k=1$.

22. $y=\sin x$ va $y=x^4-\frac{1}{6}x^2+x$ chiziqlar O(o,o) nuqtada o`zaro uchinchi tartibli urinishini isbotlang.

23. $x^2+y^2=2$ aylana bilan M (1,1) nuqtada urinuvchi $y=x^2+ax+b$ parabola tenglamasini toping. Javob: $y=x^2-3x+3$.

24. $y=x^2$ parabola bilan O(o,o) nuqtada ikkinchi tartibli urinuvchi aylanani aniqlang. Javob: $x^2+y^2-4=0$.

25. y=f(x) chiziq bilan A(o,f(o)) nuqtada n - tartibli urinishga ega bo`lgan $y=a_0+a_1x+a_2x^2+...+a_nx^n$ chiziq tenglamasini yozing.

Javob: $y=f(0)+f'(0)x+f''(0)\frac{x^2}{2!}+...+f^{(n)}(0)\frac{x^n}{n!}$.

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1. Wilhelm Klingenberg, A Course in differential geometry,1978 by Springer-Verlag, New York Inc. Printed in the United States of America.
2. M.A.Arstrong, Basic Tpology, Springer, 1998 y.

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17. Chiziqlar oilasini o`ramasi

Reja:

1. Bir parametrli va ikki parametrli chiziqlar oilasi
2. Misollar

Tayanch tushuncha va iboralar: Bir parametrik chiziq, ikki parametrik chiziq, o`rama, chiziqlar oilasining o`ramasi, maxsus nuqta, diskriminant chiziq

$F(x,y,c)=0$ bir parametrli chiziqlar oilasining o`ramasini $F(x,y,c)=0$ va $F'_c(x,y,c)=0$ tenglamalar sistemasini yechib F'_x va F'_y bir vaqtda nolga aylanmasa diskiriminant chiziq tarkibidan aniqlanadi.

1-misol $(y-c)^2-x^3=0$ chiziqlar oilasining o`ramasi topilsin.

$F'_c(x,y,c)=-2(y-c)=0 \Rightarrow y-c=0$ u holda $x=o$ chiziq diskiriminant chiziq bo`lib o`rama emas. Haqiqatdan ham $F'_x=-3x^2=0$, $F'_y=2(y-c)=0$ u holda $x=0$ va $y-c=0$ OY o`qqa tegishli (O,C) nuqtalar maxsus nuqtalardan tashkil topadi.

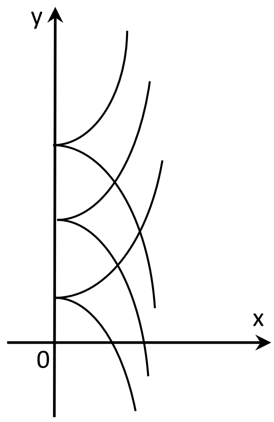
2-misol $x=c^2+u$, $y=c^3+u$ chiziqlar oilasining o`ramasi aniqlansin.

<u>Yechish:</u>

$$y=c^3+x-c^3 \Rightarrow x-y+c^3-c^2=0,\quad F'_c=3c^2-2c=0 \Rightarrow c_1=0,\ c_2=\frac{2}{3},$$

$$c=0 \text{ da } x=y,\qquad c=\frac{2}{3} \text{ da } x-\frac{4}{9}=y-\frac{8}{27}.$$

$x=y$- diskriminant chiziq chunki $F(x,y,0)=0$ va $F'_c=0$ bo`lib $F(x,y,c)=c^3-c^2=0$.



27-chizma

3-misol $x\cos c+y\sin c-p=0$ chiziqlar oilasining o`ramasi aniqlansin.

$$\begin{cases} x\cos c+y\sin c-p=0 \\ -x\sin c+y\cos c=0 \end{cases} \Rightarrow \begin{cases} x=p\cos c \\ y=p\sin c \end{cases} \Rightarrow x^2+y^2=p^2.$$

Javobi : p radiusli aylana to`g`ri chiziqlar oilasining o`ramasidir.

Mustaqil yechish uchun masalalar

1. $y=c^2(x-c)^2$ parabolalar oilasini o`ramasi aniqlasin.

Javob: $y=o$ va $16y-x^4=0$ diskiriminant chiziqlardir.

2. $x^2+(y-6)=16$ aylanalar oilasining o`ramasi aniqlansin.

Javob: $x=\pm 4$.

3. $y^2-(x-a)^3=0$ yarim kubik parabulalar oilasini o`ramasi aniqlansin.

Javob: $xy = \pm\frac{1}{2}\sqrt{c}$ ikita giperbola .

4. Bosh diametrlari umumiy va yuzlari teng ellipslar oilasining o`ramasi aniqlansin.

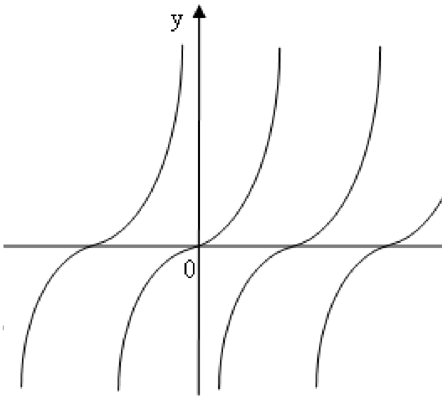
Javob: $xy = \pm\frac{1}{2}\sqrt{c}$ ikkita giperbola.

5. $x = x(t), y = y(t)$ tekis chiziq normallar oilasining o`ramasi aniqlansin.

Javob: $x = x(t) - \frac{(x'^2 + y'^2)y'}{y''x' - x''y'}$, $y = y(t) - \frac{(x'^2 + y'^2)x'}{x''y' - x'y''}$.

6. $(x-c)^2 + (y-c)^2 = c^2$ aylanalar oilasining o`ramasi aniqlansin.

Javob: x=0, y=0 .



28-chizma

7. $y = (x-c)^3$ oilaning o`ramasi aniqlansin.

Javob: $y = 0$.

8. $y^2 - (x-c)^3 = 0$ chiziqlar oilasining o`ramasini toping.

Javob: $y = 0$ diskriminant chiziq.

9. $(1-c^2)x + 2(y-a) = 0$ oilaning o`ramasi qanday chiziq

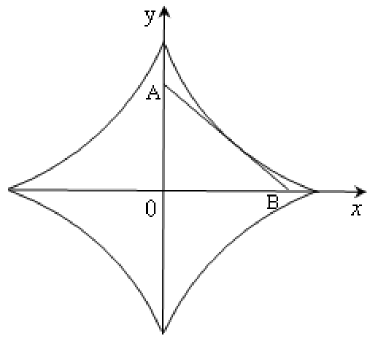
Javob: $(x - \frac{a}{2})^2 + y^2 = \frac{a^2}{4}$ aylana.

10. $c^2(x-a) - cy - a = 0$ chiziqlar oilasining o`ramasi topilsin.

Javob: $y^2 + 4a(x-a) = 0$ parabola.

11. $x^2 + y^2 = R^2$ aylana $Ax + By + C = 0$ to`g`ri chiziqlar oilasining o`ramasi bo`lishi uchun A,B,C koeffitsientlar o`zaro qanday munosabatda bo`ladi ?

Javob: $(A^2 + B^2)R^2 = C^2$.



29-chizma

12. O`zgarmas $a$ uzunlikdagi AB kesmaning uchlari koordinat o`qlariga tegishli bo`lib, shu o`qlar bo`ylab sirpanadi. AB kesma tegishli bo`lgan to`g`ri chiziqlar oilasi o`ramasining tenglamasini aniqlang. Javob: $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ astroida.

13. (OX) o`qqa koordinatalar boshida urinuvchi aylanalar oilasini tog`ri burchak ostida kesib o`tuvchi egri chiziqlar oilasi tenglamasini yozing. Javob: $(x-c)^2 + y^2 = c^2$.

14. $y^2 = 2px$ parabolalar oilasini to`g`ri burchak ostida kesib o`tuvchi chiziqlar oilasining tenglamasi qanday ? Javob: $x^2 + \frac{y^2}{2} = c$.

15. Koordinata o`qlari bilan kesishib , o`zgarmas S yuza tashkil etuvchi to`g`ri chiziqlar oilasining o`ramasini aniqlang.

Javob: $xy = \pm \frac{s}{2}$ giperbolalar.

16. To`g`ri chiziq kesmasi o`zgarmas $a$ uzunlikda bo`lib uchlari (ox) va (oy) o`qlari bo`ylab sirpanadi. Ushbu kesmalar tegishli bo`lgan m chiziqlar oilasini tenglamasini aniqlang.

Javob: $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ - astroida.

17. $\frac{x^2}{p} + \frac{y^2}{q} = 1, \quad p + q = 1$ chiziqlar oilasini o`ramasini toping?

Javob: $x \neq y = \pm 1$ 4-ta to`g`ri chiziqlar.

18. $F(x, y, \alpha, \beta) = 0$ chiziqlar oilasi uchun $\varphi(\alpha, \beta) = 0$ shart bajarilsa oila o`ramasi tegishli nuqtalar qanoatlantiruvchi shartlarni yozing.

Javob: $F(x, y, \alpha, \beta) = 0$, $\varphi(\alpha, \beta) = 0$, $\frac{D(F, \varphi)}{D(\alpha, \beta)} = 0$.

19. $y^2 = 2px$ parabolar p-parametrga bog`liq oila tashkil etadi . U $(ox)$ ga parallel bo`lib uchlari $y^2 = 2qx$ parabolalar oilasining o`ramasini aniqlang?

Javob: $y^2 = 2(p+q)x$.

20. $y^2 = 2px$ parabolalar faqat vatarlari diametrlar bo`lgan aylanalar oilasini qaraymiz .

Ushbu aylanalar oilasining o`ramasini toping?

Javob: O`rama $\left(x - \frac{3p}{4}\right)^2 + y^2 = \left(\frac{3p}{4}\right)^2$, aylana va parabolaning $x = -\frac{p}{2}$ direktirissasidan iboratdir.

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1. Wilhelm Klingenberg, A Course in differential geometry,1978 by Springer-Verlag, New York Inc. Printed in the United States of America.
2. M.A.Arstrong, Basic Tpology, Springer, 1998 y.
3.Сборник задач по дифференциал`ной геометрии. Под ред. Феденко А.С. М., 1979.

18-19. Yopishma tekislik, bosh normal va binormal. Tabiiy uchyoqlik . Frene reperi.

Tayanch tushuncha va iboralar: Bir parametrik chiziq, ikki parametrik chiziq, o`rama, chiziqlar oilasining o`ramasi, maxsus nuqta.

Fazoviy regulyar egri chiziq tenglamasi $\gamma : \bar{r} = \bar{r}(t)$, $t \in [a, b]$, (1)

yoki $x = x(t), y = y(t), z = z(t)$ (2) bo`lsin . $P(t) \in \gamma$ nuqtasida uchyoqli burchak o`rnataylik .

Uchyoqli burchakning qirralari uchun berilgan γ chiziqning P nuqtadagi urinmsi, binormali va bosh normali olinsa, yoq tekisliklari uchun normal tekislik, yopishma tekislik, to`g`rilovchi tekislik olinadi. Urinmaning yo`naltiruvchu vektori $\vec{r}'_t(t)$, binormal to`g`ri chiziqning yo`naltiruvchi vektori $\vec{b} = \left[\vec{r}', \vec{r}''_{t^2}\right]$, bosh normal tog`ri chiziqning yo`naltiruvchi vektori esa $\vec{n} = \left[\vec{b}\vec{r}'_t\right]$.

Urinmaning yo`naltiruvchi ort vektori $\vec{\tau} = \dfrac{\vec{r}'_t}{|\vec{r}_t|}$, (3)

binormalning yo`naltiruvchi ort vektori $\vec{\beta} = \dfrac{\left[\vec{r}'_t \vec{r}''_{t^2}\right]}{\left\|\left[\vec{r}'_t \vec{r}''_{t^2}\right]\right\|}$, (4)

bosh normalning ort vektori $\vec{\nu} = \dfrac{\left[\left[\vec{r}'_t \vec{r}''_{t^2}\right]\vec{r}'_t\right]}{\left\|\left[\left[\vec{r}' \vec{r}''_{t^2}\right]\vec{r}'_t\right]\right\|}$, (5)

ko`rinishda ifodalanadi .

Urinma tenglamasi : $\vec{R} = \vec{r} + \lambda \vec{r}'$ yoki $\dfrac{X-x}{x'_t(t_0)} = \dfrac{Y-y}{y'_t(t_0)} = \dfrac{Z-z}{z'_t(t_0)} = \lambda$.

(6)

Binormal tenglamasi : $\overline{R} = \overline{r} + \lambda\left[\vec{r}'\vec{r}''_{t^2}\right]$

yoki $\dfrac{X-x}{\begin{vmatrix} y'_t & z'_t \\ y''_{t^2} & z''_{t^2}\end{vmatrix}_P} = \dfrac{Y-y}{\begin{vmatrix} z'_t & x'_t \\ z''_{t^2} & x''_{t^2}\end{vmatrix}_P} = \dfrac{Z-z}{\begin{vmatrix} x'_t & y'_t \\ x''_{t^2} & y''_{t^2}\end{vmatrix}_P} = \lambda$. (7)

Bosh normal tenglamasi : $\vec{R} = \vec{r} + \lambda\left[\left[\vec{r}'_t \vec{r}''_{t^2}\right]\vec{r}'_t\right]$ yoki

$$\frac{X-x}{\begin{vmatrix} b_2 & b_3 \\ y'_t & z'_t\end{vmatrix}_P} = \frac{Y-y}{\begin{vmatrix} b_3 & b_1 \\ z'_t & x'_t\end{vmatrix}_P} = \frac{Z-z}{\begin{vmatrix} b_1 & b_2 \\ x'_t & y'_t\end{vmatrix}_P} = 0, \quad (8)$$

bunda $b_1 = \begin{vmatrix} y'_t & z'_t \\ y''_{t^2} & z''_{t^2}\end{vmatrix}_P$, $b_2 = \begin{vmatrix} z'_t & x'_t \\ z''_{t^2} & x''_{t^2}\end{vmatrix}_P$, $b_3 = \begin{vmatrix} x'_t & y'_t \\ x''_{t^2} & y''_{t^2}\end{vmatrix}$. (9)

Yopishma tekislik tenglamasi : $\left(\left(\vec{R} - \vec{r}\right)\vec{r}'_t \vec{r}''_{t^2}\right) = 0$ yoki

$$\begin{vmatrix} X-x & Y-y & Z-z \\ x'_t & y'_t & z'_t \\ x''_{t^2} & y''_{t^2} & z''_{t^2}\end{vmatrix}_P = 0. \quad (10)$$

Normal tekislik tenglamasi:

$\left(\overline{R} - \overline{r}\right)\vec{r}'_t = 0$ yoki $(X-x)x'_t + (Y-y)y'_t + (Z-z)z'_t = 0$. (11)

To`g`rilovchi tekislik tenglamasi :

$$\left(\vec{R}-\vec{r}\right)\left[\left[\vec{r}'\vec{r}''_{t^2}\right]\vec{r}'_t\right]=0 \quad \text{yoki} \quad \begin{vmatrix} X-x & Y-y & Z-z \\ b_1 & b_2 & b_3 \\ x'_t & y'_t & z'_t \end{vmatrix}_P = 0. \qquad (12)$$

Masala yechish namunalari :

1. $x=a\cos(t), y=a\sin(t), z=bt$ vint chiziqning $P(a,0,0)$ nuqtadagi Frene reperi qirqlari va yoqlarining tenglamalarini aniqlang .

Yechish: $\vec{r}'_t(P)\{o,a,b\}, \vec{r}''_{t^2}(P)\{-a,0,0\}$, bunda $t=o$

1) $\dfrac{X-a}{0}=\dfrac{y}{a}=\dfrac{z}{b}$ - urinma tenglamasi.

2) $ay+bz=0$ - normal tekislik tenglamasi.

3) $\begin{vmatrix} X-a & Y & Z \\ 0 & a & b \\ -a & 0 & 0 \end{vmatrix}=0; az-by=0$ -yopishma tekislik tenglamasi.

4) $\left[\vec{r}'_t\vec{r}''_{t^2}\right]=\vec{b}\{o,-b,a\}$, $\dfrac{x-a}{0}=\dfrac{y}{-b}=\dfrac{z}{a}$ - binormal tenglamasi.

6) $\begin{vmatrix} X-a & Y & Z \\ 0 & -b & a \\ o & a & b \end{vmatrix}=o$ yoki $x-a=0$ to`g`rilovchi tekislik tenglamasi.

2. $x=\dfrac{2}{t},\ y=\ln t,\ z=-t^2$ - egri chiziqda binormal $x-y+8z+2=0$ tekislikka parallel bo`lgan nuqtani aniqlang.

Yechish: $\vec{r}'_t\left\{-\dfrac{2}{t^2},\dfrac{1}{t},-2t\right\},\ \vec{r}''_{t^2}\left\{\dfrac{4}{t^2},-\dfrac{1}{t^2},-2\right\}$, $\vec{b}=\left[\vec{r}'_t\,\vec{r}''_{t^2}\right]\left\{-\dfrac{4}{t},-\dfrac{12}{t^2},-\dfrac{2}{t^4}\right\}$,

$\left[\vec{r}'_t\,\vec{r}''_{t^2}\right]\perp\vec{N}\{1,-1,8\}\Rightarrow t^3-3t^2+4=0\Rightarrow(t-2)^2(t+1)=0\Rightarrow t_{1,2}=2,\ t_3=-1,$

$t=2$ da $M_0(1,\ln 2,-4)$ - izlangan nuqta.

3. $x=t,\ y=t^2,\ z=\dfrac{2}{3}t^2$ chiziqning barcha binormallari birorta $\vec{P}\{p_1,p_2,p_3\}$ vektor bilan φ_0 o`zgarmas burchak tashkil etishini isbotlang va φ_0 bur-chakni aniqlang .

Yechish: $\vec{r}'_t=\{1,2t,2t^2\},\ \vec{r}''_{t^2}\{0,2,4t\},\ \vec{b}=\left[\vec{r}'_t\vec{r}''_{t^2}\right]\{2t^2,-2t,1\}$.

$$\cos\varphi_0=\frac{\left(\vec{b}\vec{r}'_t\right)}{\left|\vec{b}\right|\left|\vec{r}'_t\right|}=\frac{2t^2p_1-2tp_2+p_3}{\sqrt{p_1^2+p_2^2+p_3^2}\left(2t^2+1\right)}\Rightarrow\vec{p}\{1,0,1\} \quad \text{va} \quad \varphi_0=45^0.$$

4. $x=a\cos t,\ y=a\sin t,\ z=bt$ vint chiziq uchun Frene formulalarini yozing.

Yechish: $\vec{\tau}=\vec{\tau}(S)=\dfrac{a\vec{e}(\varphi)+b\vec{k}}{\sqrt{a^2+b^2}}$, $\quad \vec{\nu}=\dfrac{\ddot{\vec{r}}(S)}{\left|\ddot{\vec{r}}(S)\right|}=-\vec{e}(\varphi)$, $\quad \vec{\beta}=\dfrac{a\vec{k}_1-b\vec{g}(\varphi)}{\sqrt{a^2+b^2}}$,

$k_1=\left|\ddot{\vec{r}}(s)\right|=\dfrac{a}{\sqrt{a^2+b^2}}$, $\quad k_2=-\dfrac{b}{a^2+b^2}$, $\quad \dfrac{d\vec{\tau}}{dS}=\dfrac{a}{a^2+b^2}\vec{\nu}(S)$,

$\dfrac{d\vec{\nu}(S)}{dS}=-\dfrac{a}{a^2+b^2}\vec{\tau}(S)+\dfrac{b}{a^2+b^2}\vec{\beta}(S)$, $\quad \dfrac{d\vec{\beta}}{dS}=-\dfrac{b}{a^2+b^2}\vec{\nu}(S)$.

Mustaqil yechish uchun masalalar

1. $x=t,\ y=\sin t,\ z=-\cos t$ chiziqning barcha bosh normallarini OYZ koordinata tekisligiga parallel vaziyatda ekanligi isbotlansin.

2. $x=y^2,\ z=x^2$ chiziqning (1,1,1,) nuqtasidagi bosh va binormallarning tenglamasi yozilsin.

Javob: $\dfrac{x-1}{6}=\dfrac{y-1}{-8}=1-z$ - binormal,

$\dfrac{x-1}{31}=\dfrac{y-1}{26}=\dfrac{z-1}{-22`}$ - bosh normali.

3. $x=\dfrac{t^2}{2},\ y=\dfrac{2t^3}{3},\ z=\dfrac{t^4}{2}$ chiziqning $\left(\dfrac{1}{2},\dfrac{2}{3},\dfrac{1}{2}\right)$ nuqtasidagi to`g`rilovchi tekislikning tenglamasi yozilsin.

Javob: $6x+3y-6z-2=0$.

4. $x=t,\ y=t^2,\ z=e^t$ chiziqning (0,0,1) nuqtasidagi bosh normalining tenglamasini yozing. Javob: $x=\dfrac{y}{-4}=\dfrac{z-1}{-1}$.

5. $x=\cos t,\ y=\sin t,\ z=2\sin\dfrac{t}{2}$ chiziqning P(1,0,0) nuqtadagi yopishma tekisligi va binormal to`g`ri chizig`ining tenglamasini yozing.

Javob: yopishma tekislik tenglamasi: $z-y=0$,

binormal tenglamasi: $\begin{cases}x-1=0\\ y+z=0\end{cases}$.

6. $x=t,\ y=t^2,\ z=t^3$ chiziqning $M_0\left(2,-\dfrac{1}{3},-6\right)$ nuqtadan o`tuvchi yopishma tekisliklari tenglamasini yozing.

Javob: 1.

$3x+3y+z+1=0$; 2. $3x-3y+z-1=0$; 3. $108x-18y+z-216=0$.

7. $x=a\cos t,\ y=a\sin t,\ z=bt$ vint chiziqning ixtiyoriy nuqtasida Frene reperi elelmentlarini toping.

Javob: Urinma: $\dfrac{x-a\cos t}{-a\sin t}=\dfrac{y-a\sin t}{a\cos t}=\dfrac{z-bt}{b}$,

normal tekisligi: $(a\sin t)x-(a\cos t)y-bz+b^2t=0$, binormal: $\frac{x-a\cos t}{a\sin t}=\frac{y-a\sin t}{b\cos t}=\frac{z-bt}{a}$,

yopishma tekislik: $(b\sin t)x-(b\cos t)y+az-abt=0$,

bosh normal: $\frac{x-a\cos t}{\cos t}=\frac{y-a\sin t}{\sin t}$, $z=bt$,

to`g`rilovchi tekislik: $x\cos t+y\sin t-a=0$.

8. $x=a\cos t,\ y=a\sin t,\ z=bt$ vint chiziqqa tegishli bo`lgan silindr tenglamasini yozing va chiziqning barcha nuqtalarida bosh normal to`g`ri chiziqni silindr o`qiga perpendikulyarligini isbotlang.

9. $x=a\cos t,\ y=a\sin t,\ z=bt$ vint chiziq barcha bosh normallari yotgan sirt tenglamasini aniqlang. Javob: $y=(x-1)tg\frac{z}{b}$.

10. $x=a(t-\sin t),\ y=a(1-\cos t),\ z=4a\sin\frac{t}{2}$ chiziqning har bir nuqtasidan bosh normal to`g`ri chiziqlarga uzunligi $d=a\sqrt{1+\sin^2\frac{t}{2}}$ bo`lgan kongurent kesmalar qo`yilgan. Kesmaning ikkinchi uchi qanday chiziq tashkil etadi? Tenglamasini yozing.

Javob: $y-a=0$. tekislikda sinusoida tenglamasi $x=at,\ y=a,\ z=3a\sin\frac{t}{2}$. $(-\infty<t<\infty)$

11. $x=\cos t,\ y=t,\ z=\sin t$ chiziq bosh normallarining musbat yo`nalishga chiziqning har qaysi bir nuqtasidan uzunligi $l=\frac{5}{\sin t}$ bo`lgan kesmalar qo`yilgan bo`lsin. Ushbu kesmalar ikkinchi uchlari tashkil etgan chiziq tenglamasini yozing.

Javob: $x=\cos t-5ctgt,\ y=t,\ z=\sin t-5$.

12. $x=\cos\alpha\cos t,\ y=\cos\alpha\sin t,\ z=t\sin\alpha$ $(\alpha=\cos t)$ egri chiziqning har qaysi bir nuqtasidan binormal to`g`ri chiziqlarga birlik kesmalar qo`yilgan bo`lsin. Yangi chiziq binormallari, berilgan chiziq binormallari bilan θ burchak ostida kesishadi. θ- burchakni kosinusini toping. Javob: $\cos\theta=\cos\alpha\sqrt{1+\sin^2\alpha}$.

13. $x=t\sin t,\ y=t\cos t,\ z=te^t$ chiziqning $O(o,o,o,)$ nuqtadagi urinmasi, bosh normali va binormalini ort vektorlari aniqlansin.

Javob: $\vec{\tau}=\frac{\vec{j}+\vec{k}}{\sqrt{2}},\ \vec{v}=\frac{2\vec{i}-\vec{j}+\vec{k}}{\sqrt{6}},\ \vec{\beta}=\frac{\vec{i}+\vec{j}-\vec{k}}{\sqrt{3}}$.

14. $x=\cos^2 t,\ y=\sin^3 t,\ z=\cos 2t$ chiziqning ixtiyoriy nuqtasining urinmasi bosh normali va binormalining ort vektori aniqlansin

Javob: $\vec{\tau}=\frac{3}{5}\cos t\vec{i}+\frac{3}{5}\sin t\vec{j}-\frac{4}{5}\vec{k}$, $\vec{v}=\sin t\vec{i}+\cos t\vec{j}$, $\vec{\beta}=\frac{4}{5}\cos t\vec{i}-\frac{4}{5}\sin t\vec{j}-\frac{3}{5}\vec{k}$.

15. $x=t$, $y=t^2$, $z=t^3$ chiziqning $O(o,o,o)$ nuqtasidagi $\vec{\tau},\vec{v},\vec{\beta}$ vektorlarini $\vec{i},\vec{j},\vec{k}$ ortvektorlari bilan ustma-ust tushishi isbotlansin.

16. $x=\cos t+t\sin t, y=\sin t-t\cos t, t=e^{\frac{-t^2}{2}}$ chiziqning barcha binormallarining (OZ) o`q bilan kesishishi isbotlansin.

17. $x=3t,\ y=3t^2,\ z=2t^3$ chiziq berilgan. Ushbu chiziqning urinma va binormali tashkil etgan burchaklardan birining bissektirisasi γ chiziqning ixtiyoriy nuqtasida yo`nalishini o`zgartirmasligini isbotlang.

18. $2a^2y=x^3,\ \ 2xz=a^2$ chiziqda binormali (OY) o`q bilan kesishadigan nuqtani aniqlansin.

**Ko`rsatma**: $x=\pm\frac{a}{\sqrt[4]{2}}$ , bo`lgan ikkita nuqta.

19. $x=\cos^3 t, y=\sin^3 t, z=\cos 2t$ chiziqning ixtiyoriy nuqtasidagi yopishma tekisligi tenglamasini aniqlang.

Javob: $4\cos tx-4\sin ty-3z-\cos 2t=0$.

20. $x=\sin t, y=\cos t, z=tgt$ chiziqning $t=\frac{\pi}{4}$ nuqtasidagi kanonik reperining ortvektorlari aniqlansin.

Javob: $\vec{\tau}=\frac{1}{\sqrt{10}}\left(\vec{i}-\vec{j}+2\sqrt{2}\vec{k}\right)$, $\vec{v}=\frac{1}{\sqrt{82}}\left(-7\vec{i}+\vec{j}+4\sqrt{2}\vec{k}\right)$, $\vec{\beta}=\frac{1}{\sqrt{42}}\left(-2\vec{i}-6\vec{j}-\sqrt{2}\vec{k}\right)$.

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20. Chiziqning yoy uzunligi

Tayanch tushuncha va iboralar: Yoy uzunligi, tabiiy parametrlash, parametr.

Chiziq tenglamalari:

γ: $\begin{cases} x=x(t) \\ y=y(t) \end{cases}$ bo`lsa S=$\int_{t_1}^{t_2}\sqrt{x'^2(t)+y'^2(t)}dt$.

γ: y=f(x) bo`lsa S=$\int_{x_1}^{x_2}\sqrt{1+f'^2(x)}dx$.

γ: r=r(θ) bo`lsa $\int_{\varphi_1}^{\varphi_2}\sqrt{r^2(t)+r'^2(\varphi)}d\varphi$.

Masala yechish na`munalari

1. $x = a(t - \sin t),\ \ y = a(1 - \cos t),\ \ z = 4a\cos\left(\frac{t}{2}\right)$ chiziq bir o`ramining yoy uzunligi hisoblansin.

Yechish: $x'(t) = a(1 - \cos t),\ y'(t) = a\sin t,\ z'(t) = -2a\sin\frac{t}{2}$,

$$S = \int_0^{2\pi} \sqrt{a^2\left[(1-\cos t)^2 + \sin^2 t + 4\sin^2\frac{t}{2}\right]}\, dt = 2a\int_0^{\pi}\sqrt{2(1-\cos t) + 4\sin^2\frac{t}{2}}\, dt =$$

$$= 2a\int_0^{\pi}\sqrt{8\sin^2\frac{t}{2}}dt = 4\sqrt{2}a\int_0^{\pi}\sin\frac{t}{2}\, dt = 8\sqrt{2}a(-\cos\frac{t}{2})\Big|_0^{\pi} = 8\sqrt{2}\, a(\cos\frac{\pi}{2} + \cos 0) = 8\sqrt{2}\, a$$

2. $x = a\, cht,\ y = a\, sht,\ z = at$ chiziqning $M_0(t=0)$ va $M(t)$ nuqtalari orasidagi yoy uzunligi hisoblansin.

Yechish:

$$S = \int_0^t \sqrt{x'^2(t) + y'^2(t) + z'^2(t)}dt = a\int_0^t \sqrt{sh^2 t + ch^2 t + 1}\ dt = a\int_0^t \sqrt{2}chtdt = \sqrt{2}\ a\, sht\Big|_0^t = \sqrt{2}\, a\, sht\ .$$

3. r=a(1+cosφ) kordoidaning yoy uzunligini hisoblang.

Yechish:

$$S = \int_{\varphi_1}^{\varphi_2} \sqrt{r^2 + r'^2(\varphi)}\, d\varphi = 2\int_0^{\pi}\sqrt{a^2(1+\cos\varphi)^2 + a^2\sin^2\varphi}\, d\varphi = 2a\int_0^{\pi}\sqrt{2 + 2\cos\varphi}\, d\varphi =$$

$$= 2\sqrt{2}a\int_0^{\pi}\sqrt{2\cos^2\frac{\varphi}{2}}\, d\varphi = 4a\int_0^{\pi}\cos\frac{\varphi}{2}d\varphi = 8a\int_0^{\pi}\cos\frac{\varphi}{2}d(\frac{\varphi}{2}) = 8a\sin\frac{\varphi}{2}\Big|_0^{\pi} = 8a\ .$$

Mustaqil yechish uchun masalalar

1. y=a$ch\frac{x}{a}$ chiziqning $M_1(x_1)$, $M_2(x_2)$ nuqtalari orasidagi yoy uzunligi hisoblansin. Javob: $S = \left(sh\frac{x_2}{a} -, sh\frac{x_1}{a}\right)$.

2. y=$\ln\frac{e^x + 1}{e^x - 1}$ chiziqning $M_1(x_1)$, $M_2(x_2)$ nuqtalari orasidagi yoy uzunligi hisoblansin. Javob: $S = \ln\left|\frac{shx_2}{shx_1}\right|$.

3. x=a (cost-t sint), y=a(sint-t cost) chiziqning $M_1(x_1)$, $M_2(x_2)$ nuqtalari orasidagi yoy uzunligi hisoblansin. Javob: $S = a\frac{(t_2^2 - t_1^2)}{2}$.

4. $\begin{cases} x = a(\ln tg(\frac{t}{2}) + \cos t) \\ y = a\sin t \end{cases}$ chiziqning $M_1(x_1)$, $M_2(x_2)$ nuqtalari orasidagi yoy uzunligi hisoblansin. Javob: $S = a(\ln\sin t_2 - \ln\sin t_1),\ 0 < t_1,\ t_2 \le \frac{\pi}{2}$.

5. y= ln cosx chiziqning $x_1=0$, $x_2=\frac{\pi}{3}$ nuqtalari orasidagi yoy uzunligi hisoblansin.

Javob: $S = \ln tg(\frac{5\pi}{12})$.

6. $x=8a\ t^3$, $y=3a(2t^2-t^4)$ chiziqning $t_1=0$, $t_2=\sqrt{2}$ nuqtalari orasidagi yoy uzunligi hisoblansin. Javob: S = 24 a.

7. $x=a\cos^3 t$, $y=a\sin^3 t$ astroida to`la uzunigi hisoblansin.

Javob: S=6a.

8. x=a(t-sin t), y=a(1-cos t) sikloidaning $0 \le t \le \frac{\pi}{2}$ orasidagi yoy uzunligini hisoblang. Javob: S=8a.

9. $\vec{r} = a\cos t\,\vec{i} + a\sin t\,\vec{j} + bt\,\vec{k}$ vint chiziqning $M_1(t=0)$, $M_2(t=2\pi)$ nuqtalari orasidagi yoy uzunligi hisoblansin. Javob: $S = 2\pi\sqrt{a^2+b^2}$.

10. $x=e^t\cos t$, $y=e^t\sin t$, $z=e^t$ chiziqning $M_1(t=0)$, $M_2(t=2\pi)$ nuqtalari orasidagi yoy uzunligi hisoblansin. Javob: $S=\sqrt{3}(e^{\pi}-1)$.

11. x=3a cos t , y=3a sin t , z=4at chiziqning XOY tekislik bilan kesishish nuqtasi t=0 dan ixtiyoroy M(t) nuqtasigacha yoy uzunligi aniqlansin. Javob: S=5at.

12. $y = \frac{1}{3a^2}x^3$, $z = \frac{a^2}{2x}$ chiziqning $y = \frac{a}{3}$ va y=9a tekisliklar orasidagi yoy uzunligi hisoblansin; javob: S=9a.

13. $x=\cos^3 t$, $y=\sin^3 t$ va z=cos2t , $0 < t \le 2\pi$ yopiq chiziqning yoy uzunligini hisoblang. Javob: S=10.

14. $r = a\,sin^3\frac{\varphi}{3}$ chiziqning bir arkasi to`la uzunligi aniqlansin.

Ko`rsatma: $s = \int_{\varphi_1}^{\varphi_2}\sqrt{r^2 + r_{\varphi}'^2}\,d\varphi$; $\varphi_1=0$; $\varphi_2=3\pi$; Javob: $s = \frac{3}{2}\pi a$.

15. $r = a\,cos^4\frac{\varphi}{4}$ chiziqning bir arkasi to`la uzunligi aniqlansin.

Javob: $s = \frac{16a}{3}$.

16. $x = R\cos t,\ y = R\sin t$ aylana uzunligi aniqlansin.

Javob: $C=2\pi R$.

17. $x=y^2$ chiziqning $a \le t \le b$ kesma uzunligi aniqlansin.

18. Quyidagi chiziqlarning $M_1(x_1)$, $M_2(x_2)$ nuqtalari orasidagi yoy uzunligini toping ;

1) $y = x^{\frac{3}{2}}$, Javob: $S = \frac{1}{27}\left[(4+9x)^{\frac{3}{2}} - (4+9x_1)^{\frac{3}{2}}\right]$.

2) $y = \ln x$, Javob: $S = \sqrt{1+x_2^2} - \ln\left|\frac{x_2}{x_1}\right| + \ln\frac{1+\sqrt{1+x_1^2}}{1+\sqrt{1+x_2^2}}$.

3) $y = \ln\cos x,\ x_1 = 0,\ x_2 = \frac{\pi}{3}$, Javob: $S = \ln tg\left(\frac{5\pi}{12}\right)$.

19. $r = a\varphi$ Arximed spiralini bir o`ramining yoy uzunligini hisoblang.

Javob:

$$S = \frac{s}{2}\left[2\pi\sqrt{1+4\pi^2} + \ln\left(2\pi + \sqrt{1+4\pi^2}\right)\right].$$

20. Aylananing tabiiy parametrlashtirilgan tenglamasini yozing.

Javob: $x = R\cos(\frac{s}{r})$, $y = R\sin(\frac{s}{r})$.

21. $y = ach\left(\frac{x}{a}\right)$ xalqa chiziqning tabiiy parametrlashtirilgan tenglamasini yozing.

Javob: $x = arsh(\frac{5}{a})$, $y = \sqrt{a^2 + s^2}$.

22. r=a(1+cosφ) tenglama orqali chiziqning tabiiy parametrlashtirilgan tenglama yozing. Javob: $s^2 + 9R^2 = 16a^2$.

23.Egri chiziq yoy uzunligi differensialining silindrik koordinatalari bo`yicha ifodasini aniqlang.

**Ko`rsatma:** Nuqtaning dekart va silindrik koordinatalari orasidagi $x = r\cos\varphi$, $y = r\sin\varphi$, z=z. Bog`lanish formulalaridan foydalaning.

Javob: $ds^2 = dr^2 + r^2 d\varphi^2 + dz^2$.

24. Egri chiziq yoy uzunligi differentsialining sferik koordinatalari ifodasini aniqlang.

Ko`rsatma: Quyidagi formulalardan foydalaning $x = \rho\sin\theta\cos\varphi$ $y = \rho\sin\theta\sin\varphi$, $z = \rho\cos\theta$;

Javob: $ds^2 = d\rho^2 + \rho^2 d\theta^2 + \rho^2\sin^2\theta + d\varphi^2$.

Foydalanilgan adabiyotlar

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2. M.A.Arstrong, Basic Tpology, Springer, 1998 y.

3.Сборник задач по дифференциал`ной геометрии. Под ред. Феденко А.С. М., 1979

21-23.Chiziqning egriligi va buralishi.

Egrilikni hisoblash formulalari

a) Tekis chiziq uchun $k_1 = \dfrac{|x_t' y_t'' - x_t'' y_t'|}{(x_t'^2 + y_t'^2)^{\frac{3}{2}}}$; $k_1 = \dfrac{|y''_{x^2}(x)|}{(1 + y_x'^2(x))^{\frac{3}{2}}}$; $k_1 = \dfrac{|r^2 + 2r_\varphi'^2 + r\, r''_{\varphi^2}|}{(r^2 + r_\varphi'^2)^{\frac{3}{2}}}$;

b) Egrilik fazoviy chiziq uchun $k_1 = \dfrac{\sqrt{\begin{vmatrix} y' & z' \\ y'' & z'' \end{vmatrix}_P^2 + \begin{vmatrix} z' & x' \\ z'' & x'' \end{vmatrix}_P^2 + \begin{vmatrix} x' & y' \\ x'' & y'' \end{vmatrix}_P^2}}{(x'^2 + y'^2 + z'^2)_P^{\frac{3}{2}}}$;

v) Buralishni hisoblash formulasi $k_2 = \dfrac{\begin{vmatrix} x' & y' & z''' \\ x'' & y'' & z'' \\ x''' & y''' & z''' \end{vmatrix}_P}{\begin{vmatrix} y' & z' \\ y'' & z'' \end{vmatrix}_P^2 + \begin{vmatrix} z' & x' \\ z'' & x'' \end{vmatrix}_P^2 + \begin{vmatrix} x' & y' \\ x'' & y'' \end{vmatrix}_P^2}$;

Chiziq tabiiy parametrlashtirilgan bo`lsa,

$$k_2 = \frac{\left(\dot{\vec r}_S\, \ddot{\vec r}_{S^2}\, \dddot{\vec r}_{S^3}\right)}{\left|\ddot{\vec r}_{s^2}\right|^2} \quad \text{yoki} \quad k_2 = \frac{\begin{vmatrix} \dot x & \dot y & \dot z \\ \ddot x & \ddot y & \ddot z \\ \dddot x & \dddot y & \dddot z \end{vmatrix}_P}{\ddot x_{s^2}^2 + \ddot y_{s^2}^2 + \ddot z_{s^2}^2};$$

Masala yechish na`munalari

1. x=*a*cht , y=*a*sht , z=at uchun k_1=? k_2=?

<u>Yechish:</u>

$$k_1 = \frac{\sqrt{\begin{vmatrix} a\,cht & a \\ a\,sht & 0 \end{vmatrix}^2 + \begin{vmatrix} a & a\,sht \\ 0 & a\,cht \end{vmatrix}^2 + \begin{vmatrix} a\,sht & a\,cht \\ a\,cht & a\,sht \end{vmatrix}^2}}{\left(a^2 sh^2 t + a^2 ch^2 t + a^2\right)^{\frac{3}{2}}} = \frac{\sqrt{a^4\left(sh^2 t + ch^2 t + \left(sh^2 t - ch^2 t\right)^2\right)}}{a^3\left(sh^2 t + ch^2 t + 1\right)^{\frac{3}{2}}} =$$

$$= \frac{a^2\sqrt{sh^2 t + ch^2 t + 1}}{a^3(sh^2 t + ch^2 t + 1)} = \frac{\sqrt{2}cht}{a(2ch^2 t)^{\frac{3}{2}}} = \frac{1}{2ach^2 t};$$

$$\begin{vmatrix} asht & acht & a \\ acht & asht & 0 \\ asht & acht & 0 \end{vmatrix} = a^3(ch^2 t - sh^2 t) = a^3; \quad k_2 = \frac{a^3}{2a^4 ch^2 t} = \frac{1}{2ach^2 t};$$

Xulosa: $k_1 = k_2 = \dfrac{1}{2ach^2 t}$;

2. $x + shx = \sin y + y$, $z + e^z = x + ln(1 + x) + 1$ oshkormas ko`rinishda berilgan chiziqning O(0,0,0) nuqtadagi egriligi aniqlansin

Yechish: $1 + chx = (\cos y + 1)y'$, $y' = \dfrac{1 + chx}{\cos y + 1}$ O nuqtada $y'(0) = 1$,

$$y''_{x^2} = \frac{shx(1+\cos y) + (1+chx)\sin y\, y'}{(1+\cos y)^2} = 0,\ (1+e^z)\,z' = 1 + \frac{1}{1+x},\ \ z'_x = \frac{x+2}{(x+1)(1+e^z)},$$

O nuqtada $z'_x(0) = 1$,

$$z''_{x^2} = \frac{(x+1)(1+e^z) - (x+2)\left[(1+e^z) + (x+1)e^z z'\right]}{(x+1)^2(1+e^z)^2} = \frac{2 - 2(2+1)}{4} = -1.$$

$$k_1 = \frac{\sqrt{\begin{vmatrix} 1 & 1 \\ 0 & -1 \end{vmatrix}^2 + \begin{vmatrix} 1 & 1 \\ -1 & 0 \end{vmatrix}^2 + \begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix}^2}}{(1+1+1)^{\frac{3}{2}}} = \frac{\sqrt{2}}{\sqrt{27}} = \frac{\sqrt{6}}{9}$$

javob: $k_1 = \dfrac{\sqrt{6}}{9}$;

3. $r = a(1 + \cos\varphi)$ kordoida egriligini hisoblang.

Yechish. $r'_\varphi = -a\sin\varphi$, $\bar{r}''_{\varphi^2} = -a\cos t$

$$k_1 = \frac{\left|r^2 + 2r'_\varphi - r''_{\varphi^2}\right|}{\left(r' + r'^2_\varphi\right)^{\frac{3}{2}}} = \frac{\left|a^2(1+\cos\varphi)^2 + 2a^2\sin^2\varphi + a^2(1+\cos\varphi)\cos\varphi\right|}{(a^2(1+\cos\varphi)^2 + a^2\sin^2\varphi)^{\frac{3}{2}}} =$$

$$= \frac{1}{a}\frac{\left(1 + 3\cos\varphi + \cos^2\varphi\right) + 2\sin^2\varphi + \cos^2\varphi}{\left(2 + 2\cos\varphi\right)^{\frac{3}{2}}} = \frac{3(1+\cos\varphi)}{2\sqrt{2}\,a(1+\cos\varphi)^{\frac{3}{2}}} = \frac{3}{2\sqrt{2}\,a(1+\cos\varphi)^{\frac{1}{2}}} =$$

$$= \frac{3}{2\sqrt{2}\,a\left(2\cos^2\dfrac{\varphi}{2}\right)^{\frac{1}{2}}} = \frac{3}{4a\left(\cos^2\dfrac{\varphi}{2}\right)^{\frac{1}{2}}} = \frac{3}{4a\left(\cos\dfrac{\varphi}{2}\right)};$$

Mustaqil yechish uchun masalalar

1. Quyidagi tekis chiziqlarni egriligi hisoblansin.

a) y=sin x , Javob : $k_1 = \dfrac{|\sin x|}{\left(1 + \cos^2 x\right)^{\frac{3}{2}}}$.

b) $y = a\,ch\left(\dfrac{x}{a}\right)$, Javob : $k_1 = \dfrac{a}{y^2}$.

v) $y^2 = 2px$, Javob : $k_1 = \dfrac{p^2}{\left(y^2 + p^2\right)^{\frac{3}{2}}}$.

g) $x=t^2,\ y=t^3,$ Javob : $k_1=\dfrac{6}{t\left(4+9t^2\right)^{\frac{3}{2}}}$.

d) $x=a\cos t,\ y=b\sin t$, Javob: $k_1=\dfrac{ab}{\left(a^2\sin^2 t+b^2\cos^2 t\right)^{\frac{3}{2}}}$.

e) $x=a\,cht,\ y=b\,sht,$ Javob : $k_1=\dfrac{ab}{\left(a^2sh^2t+b^2ch^2t\right)^{\frac{3}{2}}}$.

j) $\begin{cases} x=a(t-\sin t)\\ y=a(1-\cos t)\end{cases}$, Javob : $k_1=\dfrac{1}{4a\left|\sin\frac{t}{2}\right|}$.

z) $\begin{cases} x=a\cos^3 t\\ y=a\sin^3 t\end{cases}$, Javob : $k_1=\dfrac{2}{3a|\sin 2t|}$.

2. $r=a(1+\cos\varphi)$ kardoidaning egriligi hisoblansin. Javob : $k_1=\dfrac{3}{4a\left|\cos\frac{\varphi}{2}\right|}$.

3. F(x,y)=0 oshkormas ko`rinishda berilgan chiziqning egriligi hisoblash formulasi yozilsin. Javob : $k_1=\dfrac{\mathrm{mod}\begin{vmatrix} F''_{XX} & F''_{XY} & F'_X\\ F''_{XY} & F''_{YY} & F'_Y\\ F_X & F_Y & 0\end{vmatrix}P}{\left(F'^2_X+F'^2_Y\right)^{\frac{3}{2}}P}$.

4. Ellipsni egriligi hisoblansin. $\begin{cases} x=a\cos t\\ y=b\sin t\end{cases}$,

$$k_1=\frac{\left(ab\sin^2 t+ab\cos^2 t\right)}{\left(a^2\sin^2 t+b^2t\right)^{\frac{3}{2}}}=\frac{ab}{\left(a^2\sin^2 t+b^2\cos\ \ t\right)^{\frac{3}{2}}}.$$

$k_1=\dfrac{|ab|}{\left(a^2\sin^2 t+b^2\cos^2 t\right)^{\frac{3}{2}}}$, uchlarida , $k_1=\dfrac{b}{a^2}$ va $k_1=\dfrac{a}{b^2}$.

5. R radiusli aylana egriligi hisoblansin. Javob : $k_1=\dfrac{1}{R}$.

6. $y=x^4$ chiziqning O(0,0) nuqtadagi egriligi hisoblansin. javob : $k_1=0$.

7. $x=a\left(\ln tg\left(\dfrac{t}{2}\right)+\cos t\right),\ y=a\sin t$ chiziqning egrilik radiusi hisoblansin.

Javob : $p=\dfrac{1}{k_1}=arctgt$.

8. $y=e^x$ chiziqda egriligi ekstremal qiymatga erishadigan nuqtani ko`rsating.

Javob : $\left(-\dfrac{1}{2}\ln 2,\dfrac{1}{\sqrt{2}}\right)$.

9. $r = a\sin^3\left(\frac{4}{3}\right)$ chiziqda egriligi ekstremal qiymatga erishadigan nuqtani ko`rsating.

Javob : $A\left(\frac{3\pi}{2}, a\right)$ va B(0,0).

10. $x = e^t,\ y = e^{-t},\ z = \sqrt{2}t$ chiziqninng egriligi va buralishini hisoblang.

Javob : $k_1 = -k_2 = \frac{\sqrt{2}}{\left(e^t + e^{-t}\right)^2}$.

11. $x = 2t,\ y = \ln t,\ z = t^2$ chiziqning egriligi va buralishi ko`rsatilsin.

Javob : $k_1 = -k_2 = \frac{2t}{\left(1 + 2t^2\right)^2}$;

12. Vint chiziqning egriligi va buralishini ko`rsating.

Javob: $k_1 = \frac{a}{a^2 + b^2}, k_2 = \frac{b}{a^2 + b^2}$.

13. $x = t\cos t,\ \ y = t\sin t,\ \ z = at$ konus vint chizig`ining O(0,0,0) nuqtadagi egriligi hisoblansin.

Yechish: $x' = \cos t - t\sin t$, $x'(0) = 1$, $y' = \sin t + t\cos t$, $y'(0) = 0$, $x'' = -2\sin t - t\cos t$, $x''(0) = 0$, $y'' = 2\cos t - t\sin t$, $y''(0) = 2$, $z' = a$, $z'' = (0) = 0$,

$$k_1 = \frac{\sqrt{\begin{pmatrix} 0 & 1 \\ 2 & 0 \end{pmatrix}^2 + \begin{pmatrix} a & 1 \\ 0 & 0 \end{pmatrix}^2 + \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}^2}}{1} = \sqrt{8} = 2\sqrt{2}\ .$$

Javob : $k_1 = 2\sqrt{2}$.

14 $x = a(2\cos t - \cos 2t)$, $y = a(2\sin t - \sin 2t)$ chiziqning $t = \frac{\pi}{2}$ nuqtadagi egriligini aniqlang.

Javob : $k_1 = \frac{a\pi}{2}$.

15. $x = \sin t$, $y = \cos t$, $z = tgt$ chiiqning $t_0 = \frac{\pi}{4}$ nuqtadagi burilishini aniqlang.

Javob : $k_1 = -\frac{6}{7}$.

16. $x = t^2 - 1$, $y = t^2 + 2$, $z = t^3$ chiziqning tekis chiziq ekanligini isbotlang. Tekislik tenglamasi qanday?

17. $y = \frac{x^3}{3}$, $z = \frac{1}{2x}$ chiziqni $\left(1, \frac{1}{3}, \frac{1}{2}\right)$ nuqtasidagi burilishi aniqlansin.

Javob : $k_1 = \frac{8}{9}$.

18. $x = \cos t$ $y = \sin t$ $z = \frac{1}{2}\sin 2t$ chiziqning qaysi nuqtalarida burilish musbat?

Javob: $-\frac{\pi}{2} < t < \frac{\pi}{4}$.

19. $\begin{matrix} x^2 - y^2 + z^2 = 1 \\ y - 2x + z = 1 \end{matrix}$ chiziqning $M_0(1,1,1)$ koordinatalar nuqtasidagi burilishi topilsin. Javob: $k_2 = 1$.

20. $x = t$, $y = t^3$, $z = t^2 - 4$ chiziqning t_0=1 nuqtadagi burilishi topilsin.

Javob: $k_2 = -\frac{3}{91}$.

21. $x = 3t - t^3$, $y = 3t^2$, $z = 3t + t^3$ chiziqning egriligi va burilishi hisoblansin.

Javob: $k_1 = k_2 = \frac{1}{3(t^2+1)^2}$.

22. Giperbolik vint chiziqning tabiiy tenglamasini yozing. $\gamma: x = a\,cht,\ y = a\,sht,\ z = at.$

Javob: $k_1 = k_2 = \frac{a}{2a^2 + s^2}$.

23. Sikloidaning tabiiy tenglamasini yozing. Javob: $s^2 + a^2 = 16a^2$, $k_2 = 0$.

24. Traktrissaning tabiiy tenglamasini yozing.

Javob: $p = a\sqrt{\frac{2s}{la} - 1}$, $k_2 = 0$, $t = \frac{1}{p}$.

25. Xalqa chiziqning tabiiy tenglamasini yozing.

Javob: $p = \frac{1}{k_1} = a + \frac{s^2}{a}$, k_2=0.

Foydalanilgan adabiyotlar

1. Wilhelm Klingenberg, A Course in differential geometry,1978 by Springer-Verlag, New York Inc. Printed in the United States of America.
2. M.A.Arstrong, Basic Tpology, Springer, 1998 y.
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24. Tekis chiziqning Evolyuta va Evolventasi.

Tekis egri chiziq tenglamasi: x=x(t), y=y(t). (1)

Berilgan chiziq egrilik markazlarining geometrik o`rni Evolyuta bo`lib, uning tenglamasi

$$X = x - y'_t \frac{x'^2_t + y'^2_t}{\begin{vmatrix} x'_t & y'_t \\ x''_{t^2} & y''_{t^2} \end{vmatrix}}, \qquad Y = y + x'_t \frac{x'^2_t + y'^2_t}{\begin{vmatrix} x'_t & y'_t \\ x''_t & y''_t \end{vmatrix}}. \qquad (2)$$

Evolventa tenglamasi: $\overline{p} = \overline{r}(s) + (\lambda - s)\overline{\tau}(s)$. (3)

Masala yechish namunalari

1. F(x,y)=0 oshkormas tenglama orqali berilgan tekis chiziq evolyutasining tenglamasini yozing.

Yechish: Normalar oilasining tenglamasini yozaylik

$$F_y'(X-x)-F_y'(Y-y)=0 \qquad (*)$$

x-ni parametr uchun olib F(x,y)=0 tenglamani differensiallaymiz

$$F_x'+F_y'y_x'=0 \qquad (**)$$

(*)ni differensiallab (**)-ni e'tiborga olamiz

$$(F_y'F_{xy}''-F_x'F_{yy}'')(X-x)+(F_x'F_{xy}''-F_y'F_{xx}'')(Y-y)=F_x'^2+F_y'^2,$$

$$X=x-\frac{F_x'(F_x'^2+F_y'^2)}{\Delta}, \quad Y=y-\frac{F_x'(F_x'^2+F_y'^2)}{\Delta}, \quad \text{bunda} \quad \Delta=\begin{vmatrix} F_{xx}'' & F_{xy}'' & F_x' \\ F_{yx}'' & F_{yy}'' & F_y' \\ F_x' & F_y' & 0 \end{vmatrix}.$$

2. $x=a\cos t,\ y=b\sin t$ ellips evolyutasining tenglamasini yozing.

Yechish: $x_t'=-a\sin t,\quad y_t'=b\cos t,\quad x_{t^2}''=-a\cos t,\ y_{t^2}''=-b\sin t,$

$$x_t'^2+y_t'^2=a^2\sin^2 t+b^2\cos^2 t=(a^2-b^2)\sin^2 t+b^2,$$

$$\begin{vmatrix} x_t' & y_t' \\ x_{t^2}'' & y_{t^2}'' \end{vmatrix}=x_t'y_{t^2}''-y_t'x_{t^2}''=ab\sin^2 t+ab\cos^2 t=ab,$$

$$X=a\cos t-b\cos t\,\frac{(a^2-b^2)\sin^2 t+b^2}{ab}=\frac{a^2-b^2}{a}\cos^3 t,$$

$$Y=b\sin t-a\sin t\,\frac{(a^2-b^2)\sin^2 t+b^2}{ab}=\frac{b^2-a^2}{b}\sin^3 t.$$

3. $\rho=a\varphi$ Arximed spiralining evolyutasini aniqlang .

Yechish:

$$X=a\varphi\cos\varphi-a(\sin\varphi+\varphi\cos\varphi)\frac{a^2(1+\varphi^2)}{a^2(2+\varphi^2)}=a\frac{\varphi\cos\varphi-\sin\varphi+\varphi^2\sin\varphi}{2+\varphi^2}=a\frac{\varphi\cos\varphi-\sin\varphi(1-\varphi^2)}{2+\varphi^2},$$

$x=a\varphi\cos\varphi,\quad y=a\varphi\sin\varphi,\quad x_\varphi'=a(\cos\varphi-\varphi\sin\varphi),\quad y_\varphi'=a(\sin\varphi+\varphi\cos\varphi),\quad x_{\varphi^2}''=-a(2\sin\varphi+\varphi\cos\varphi),$

$y_{\varphi^2}''=a(2\cos\varphi-\varphi\sin\varphi),\qquad x_\varphi'^2+y_\varphi'^2=a^2(1+\varphi^2),\qquad x_\varphi'y_{\varphi^2}''-x_{\varphi^2}''y_\varphi'=a^2(2+\varphi^2),$

$$Y=a\varphi\sin\varphi+a(\cos\varphi-\varphi\sin\varphi)\frac{a(1+\varphi^2)}{a(2+\varphi^2)}=a\frac{\varphi\sin\varphi+(1+\varphi^2)\cos\varphi}{2+\varphi^2}.$$

Javob:

$$X=a\frac{\varphi\cos\varphi-\sin\varphi(1-\varphi^2)}{2+\varphi^2},\ Y=a\frac{\varphi\sin\varphi+(1+\varphi^2)\cos\varphi}{2+\varphi^2}.$$

Mustaqil yechishga masalalar

1. Quyidagi chiziqlarning evolyutasini aniqlang.

1) $x = acht,\ y = bsht$;   Javob: $X = \frac{a^2+b^2}{a} ch^3 t,\quad Y = \frac{a^2+b^2}{b} sh^3 t$.

2) $y = x^2$;   Javob: $X = -4x^3,\quad Y = 3x^2 + \frac{1}{2}$.

3) $y = \ln x$;   Javob: $X = 2x + \frac{1}{x},\quad Y = \ln x - x^2 - 1$.

4) $y = \sin x$;   Javob: $X = x + \cos x \frac{1+\cos^2 x}{\sin x},\quad Y = -\frac{2\cos^2 x}{\sin x}$.

5) $y = tgx$;   Javob: $X = x - \frac{1+\cos^4 x}{\cos^2 x \sin 2x},\quad Y = tgx + \frac{1+\cos^4 x}{\sin 2x}$.

2. $x = a\left(\ln tg(\frac{t}{2}) + \cos t\right),\quad y = a\sin t$ traktrisaning evolyutasi topilsin.

Javob: $X = a\ln tg\left(\frac{t}{2}\right),\quad Y = \frac{a}{\sin t}$ yoki $Y = ach\left(\frac{X}{a}\right)$.

3. $r = a(1+\cos\varphi)$ chiziq evolyutasi xalqa chiziq qanday chiziq?

Javob: $X = \frac{a}{3}(\cos\varphi - \cos^2\varphi + 2),\quad Y = \frac{a}{3}(1-\cos\varphi)\sin\varphi$

Kardoidal

4. Sikloida evolyutasining sikloida ekanligini isbotlang.
5. Astroida evolyutasining astroida ekanligini isbotlang.
6. $y^2 = 2px$ parobala evolyutasini aniqlang.

Javob: $27py^2 = 8(x-p)^3$ yarim kubik parabola

7. $x^2 + y^2 = R^2$ aylana evolyutasini aniqlang.

Javob:

$x = R(\cos\theta + (\theta - c)\sin\theta),\quad y = R(\sin\theta - (\theta - c)\cos\theta)$

8. $Y = ach\left(\frac{x}{a}\right)$ xalqa chiziqning uchidan o`tuvchi evolventasi tenglamasini yozing.   Javob: $X = a\left(\ln tg\left(\frac{t}{2}\right) + \cos t\right),\quad Y = a\sin t$ traktrissa

9. $x = t,\ y = \frac{t^2}{4}$ parobala evolventasini aniqlang.

Javob : $X = \frac{t}{2} + \frac{2}{\sqrt{t^2+4}}\left(C - \ln\left(t + \sqrt{t^2+4}\right)\right),\quad Y = \frac{t}{\sqrt{t^2+4}}\left(C - \ln\left(t+\sqrt{t^2+4}\right)\right)$.

10. $\vec{r} = a\vec{e}(\varphi) + b\varphi\vec{k}$ vint chiziq evolyutasi tenglamasini yozing.

Javob: $\vec{R} = a\vec{e}(\varphi) + a(\varphi_0 - \varphi)\vec{g}(\varphi) + b\varphi_0\vec{k}$ bunda

$$\vec{e}(\varphi) = \cos\varphi\vec{i} + \sin\varphi\vec{j},\quad \vec{g}(\varphi) = \sin\varphi\vec{i} + \cos\varphi\vec{j}.$$

Foydalanilgan adabiyotlar

1. Wilhelm Klingenberg, A Course in differential geometry,1978 by Springer-Verlag, New York Inc. Printed in the United States of America.

2. M.A.Arstrong, Basic Tpology, Springer, 1998 y.
3.Сборник задач по дифференциал`ной геометрии. Под ред. Феденко А.С. М., 1979

25. Sirt tenglamalari.

Tayanch tushuncha va iboralar: Vektorli parametrik tenglama, oshkor tenglama, oshkormas tenglama, Tor, psevdosfera, siilindrik sirt, Katenoid, giperbolik paraboloid, parabolik silindr.

1. $\bar{r} = \bar{r}(u,v)$ vektorli parametrik tenglamasi.

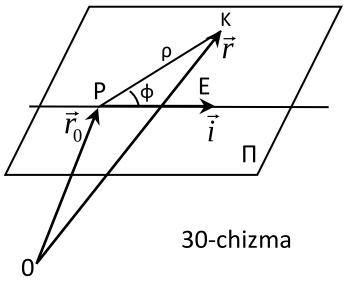
2. $x=x(u,v)$, $y=y(u,v)$, $z=z(u,v)$. Sirtning koordinatalar bo`yicha parametrik tenglamalari.

3. $Z=f(x,y)$ – sirtning oshkor tenglamasi.

4. $F(x,y,z)=0$. Sirtning oshkormas tenglamasi.

Masala yechish na`munalari.

1. Tekislikning vektorli parametrik tenglamasi yozilsin.



30-chizma

Yechish: П tekislikda ixtiyoriy K nuqta olaylik, $K(\varphi,\rho)$ qutb koordinatalarga ega bo`lsin.

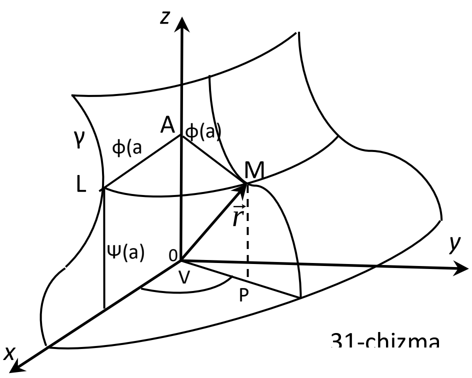
$$\angle EPK = \varphi,\ d(P,K) = \rho,\ \overline{OP} = \vec{r}_0 \ \text{ bo'lsa, } \vec{r} = \overline{OK} = \overline{OP} + \overline{PK} \Rightarrow$$

$$\vec{r} = \vec{r}_0 + \rho(\cos\varphi\,\vec{i} + \sin\varphi\,\vec{j}) = \vec{r}_0 + \rho\,\vec{e}(\varphi).$$

Javob: $\vec{r} = \vec{r}_0 + \rho\vec{e}(\varphi)$, ρ,φ - parametrlar.

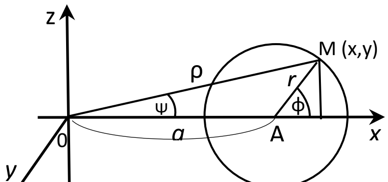
2. Aylanma sirtning parametrik tenglamasi tuzilsin.

Yechish: $\gamma : x = \varphi(u),\ z = \psi(u)$ tenglamalar orqali berilgan chiziqning (OZ) o`q atrofida aylanishidan hosil bo`lgan sirt aylanma sirt deyiladi. $x = \varphi(u) > 0$ bo`lsin, ya`ni aylanayotgan (profil) chiziq γ aylanish o`qi bilan kesishmasin (maxsus nuqta qaralmaydi). Egri chiziqli koordinatalar $\angle$XOP=V va $\gamma$ ning u parametri bo`lsin.



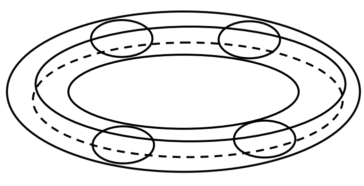
31-chizma

$L \in \gamma$ nuqta (OZ) o`q atrofida aylanib aylana chizadi. Aylana markazi (OZ) o`qga tegishli bo`lib radiusi esa d(A,L)=d(A,M)=d(O,P)=$\varphi(u)$ . U=const, v=t- paralellar oilasi. u=t, v=const- meridianlar oilasi sirtning koordinat chiziqlaridir. $\bar{r} = \varphi(u)(\cos v\bar{i} + \sin v\bar{j}) + \psi(u)\bar{k}$. koordinata ko`rinishida: $x = \varphi(u)\cos v,\ y = \varphi(u)\sin v,\ z = \psi(u)$.



32-chizma

3. Tor. Tor deb $(x-a)^2 + y^2 = r^2$ aylana uninh tekisligida yotgan va OZ o`q atrofida aylantirishdan hosil


33-chizma

bo`lgan sirtga aytiladi. OZ o`q aylana markazidan *a* masofada turadi va $a>r$.

Aylananing parametrik tenglamalari $x = a + r\cos\varphi,\ y = 0,\ z = r\sin\varphi,\ 0 \le \varphi < 2\pi$. Aylanish jarayonida aylananing M nuqtasi tor sirtiga tegishli chiziq chizadi. $< XOM = \psi,\ 0 \le \psi < 2\pi,\ \psi$ - OZ o`q atrofida burish burchagidir. $\rho = \sqrt{x^2 + y^2} = a + r\cos\varphi$ o`zgarmaydi. Torning parametrik tenglamalari $x = \rho\cos\psi,\ y = \rho\sin\psi,\ z = r\sin\varphi$ yoki $x = (a + r\cos\varphi)\cos\psi,\ y = (a + r\cos\varphi)\sin\psi,\ z = r\sin\varphi$.

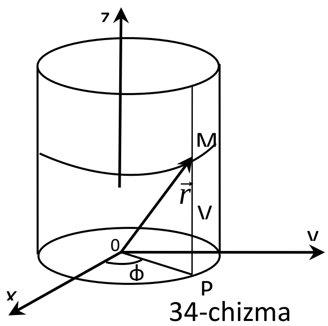
4. Silindrik sirt yo`naltuvchisi $x^2 + y^2 - ay = 0,\ z = 0$ bo`lib yasovchilari $\bar{a}\{1, m, n\}$ vektorga parallel bo`lgan silindrik sirt tenglamasi tuzilsin.

Yechish: Yo`naltuvchi tenglamasini $x^2 + (y - \frac{a}{2})^2 = \frac{a^2}{4}$ ko`rinishga keltiramiz.

$x = \frac{a}{2}\cos t,\ y = \frac{a}{2}(1 + \sin t),\ z = 0$ almashtirish o`tkazamiz. $M_0(\frac{a}{2}\cos t,\ \frac{a}{2}(1 + \sin t), 0)$ yo`naltuvchi nuqtasi. M(x,y,z) silindrik sirtning ixtiyoriy nuqtasi bo`lsin, $\overline{M_0M}$ *va* $\bar{a}$ kollinear vektorlardir.
$\overline{M_0M} = \lambda\ \bar{a},\ \Rightarrow\ x = \frac{a}{2}\cos t + \lambda l,\quad y = \frac{a}{2}(1 + \sin t) + \lambda m,\quad z = \lambda n,\quad \lambda = \frac{z}{n}$
orqali $x = \frac{a}{2}\cos t + \frac{l}{n}z,\quad y = \frac{a}{2}(1 + \sin t) + \frac{m}{n}z$. Formulalarni hosil qilamiz. Ko`ramizki, $x - \frac{l}{n}z = \frac{a}{2}\cos t,\quad y - \frac{m}{n}z = \frac{a}{2}(1 + \sin t)$. U holda, $(x - \frac{l}{n}z)^2 + (y - \frac{m}{n}z)^2 = \frac{a^2}{2}(1 + \sin t)$ kelib chiqadi, $y = \frac{a}{2}(1 + a\sin t)$ ga ko`ra $(nx - lz)^2 + (ny - mz)^2 = na(ny - mz)$ kelib chiqadi.

Javob: $\varphi : (nx - lz)^2 + (ny - mz)^2 = na(ny - mz)$.

Mustaqil yechish uchun masalalar

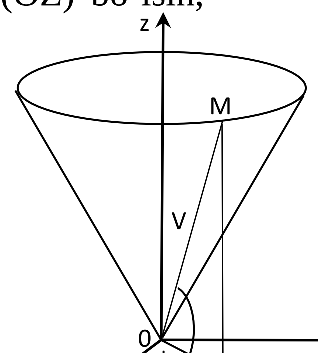


34-chizma

1. Doiraviy silindrning parametrik tenglamasini yozing.

Javob: $\vec{r} = a\,\vec{e}(\varphi) + V\,\vec{k}$ - vektorli tenglamasi, $x = a\cos\varphi,\ y = a\sin\varphi,\ z = V$ koordinatalar bo`yicha tenglamasi.

2. Doiraviy konusning uchi koordinatalar boshida, o`qi esa (OZ) bo`lsin, parametrik tenglamasi yozilsin.

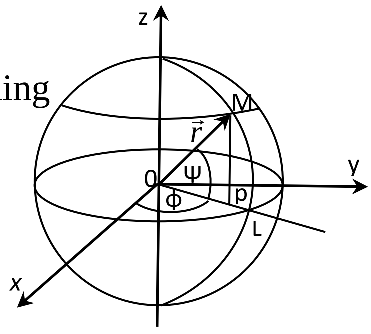


Javob: $\vec{r} = \mathrm{v}\{\cos\alpha\vec{e}(\varphi) + \sin\alpha\vec{k}\},\ x = \mathrm{v}\cos\alpha\cos\varphi,$
$y = \mathrm{v}\cos\alpha\sin\varphi,\ z = \sin\alpha\,\mathrm{v}$.

3. a radiusli sferaning parametrik tenglamasini yozing

Javob: $\vec{r} = a\left[\cos\psi\,\vec{e}(\varphi) + \sin\psi\,\vec{k}\right]$, $x = a\cos\psi\cos\varphi$, $y = a\cos\psi\sin\varphi$,

$z = a\sin\psi$.

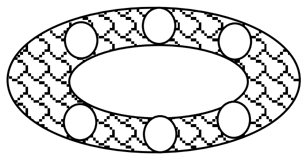


36-chizma

4. a radiusli sferaning parametrik tenglamalarini yozing. Torning parametrik tenglamalari yozilsin.

Javob: $x = (a + r\cos\varphi)\cos\psi$, $y = (a + r\cos\varphi)\sin\psi$, $z = r\sin\varphi$.

5. Traktrissasini asimptotasi atrofida aylanishidan hosil bo`lgan sirtga **psevdosfera** deyiladi. Psevdosferaning parametrik tenglamalari yozilsin.

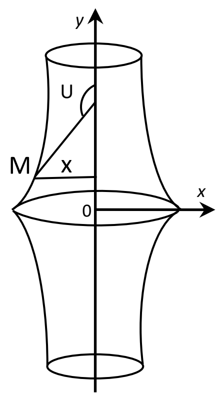

37-chizma

Javob: $x = a\sin u\cos v$, $y = a\sin u\sin v$, $z = a(\ln tg\frac{u}{2} + \cos u)$.

6. **Gelikoid** .(OZ) o`qqa tik AB kesmaning shu o`q atrofida aylanishidan, shuningdek, aylanish burchagiga proporsional tezlik bilan (OZ) bo`ylab siljishidan hosil bo`lgan sirtni to`g`ri **Gelikoid** deyiladi.

Tenglamasi tuzilsin Javob: $x = u\cos v$, $y = u\sin v$, $z = av$

yoki $y = xtg\frac{z}{a}$.



38-chizma

7. Katenoid . $x = ach\left(\frac{u}{a}\right)$, $z = u$ xalqa chiziqning (OZ) o`q atrofida aylanishidan hosil bo`lgan sirt **Katenoid** deyiladi. Katenoidning tenglamasi yozilsin.

Javob: $x = ach\left(\frac{u}{a}\right)\cos v$, $y = ach\left(\frac{u}{a}\right)\sin v$, $z = u$.

8 **Giperbolik paraboloid** $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 2z$ ning parametrik tenglamasi yozilsin .

Javob :

$x = a(u + v)$, $y = b(v - u)$, $z = 2uv$.

9 **Giperbolik va parabolik silindrlarning** parametrik tenglamalari yozilsin.

Javob : **Giperbolik silindr tenglamasi**

$x = achu$, $y = bshu$, $z = v$.

Parabolik slindr tenglamasi

$x = u$, $y = u^2$, $z = v$.

10 Yasovchilari $\vec{a}\{1,2,3\}$ vekorga parallel bo`lib , yo`naltiruvchisi $\gamma : x = u$,

$y = u^2, \quad z = u^3$ **bo`lgan** slindrik sirtning parametrik tenglamalari yozilsin.

Javob: $x = u + v, \quad y = u^2 + 2v, \quad z = u^3 + 3v$.

11 Yo`naltiruvchisi $\gamma : x = \cos u, \quad y = \sin u, \quad z = 0$ bo`lib ,to`g`ri chiziqli

Yasovchilari $\vec{a}\{-1,3,-2\}$ vektorga parallel bo`lgan silindrik sirtning oshkormas tenglamasi yozilsin.

Javob: $x = \cos u - v, \quad y = \sin u, \quad z = -2v \;\Rightarrow \left(x - \dfrac{y}{2}\right)^2 + \left(y - \dfrac{3}{2}z\right)^2 = 1$.

12. Yo`naltiruvchisi $x^2 + y^2 = ay, \quad z = 0$ bo`lib , yasovchilari esa $\vec{a}(l, m, n)$ ga parallel bo`lgan silindrik sirtning tenglamasini yozing.

Javob: $(nx - lz)^2 + (ny - mz)^2 = an(ny - mz)$;

13 $x = 3u + v^2 + 1, \quad y = 2u + v^2 - 1, \quad z = -u + 2v$ sirtni slindrik ekanini ko`rsating.

14 M(a,b,c) nuqtadan o`tib $\gamma : y^2 = 2px, \quad z = 0$ parabola bilan kesishvchi to`g`ri chiziqlar tashkil etgan Konus sirtning tenglamasini yozing.

Javob: $(bz - cy)^2 = 2p(z - c)(az - cx)$.

15. Uchi M(-1,0,0) nuqtada bo`lib ,$2y^2 + z^2 = 4x$ paraboloidga tashqi chizilgan

Konusning tenglamasi yozilsin. Javob: $(x + 1)^2 = 2y^2 + z^2$.

16 $x = x_0 + a\cos u \cos v, \quad y = y_0 + b\cos u \sin v, \quad z = z_0 + c\sin u$ parmetrik berilgan sirtning oshkormas tenglamasini yozing.

Javob: $\dfrac{(x - x_0)^2}{a^2} + \dfrac{(y - y_0)^2}{b^2} + \dfrac{(z - z_0)^2}{c^2} = 1$

ellipsoid.

17 $x = u\cos v, \quad y = u\sin v, \quad z = u^2$ tenglamalar qanday sirtni ifodalaydi.

Javob: $z = x^2 + y^2$ ko`rinishdagi aylanma paraboloid.

18 Vint chiziqning bosh va binormallari tashkil etgan sirtning tenglamasi yozilsin.

Javob: $x = a(1 - u)\cos v, y = a(1 - u)\sin v, \quad z = bv$ to`g`ri Gelikoid.

19 $y = xtgz$ sirtning normallari $y = x, \quad z = \dfrac{\pi}{4}$ normal bo`ylab kesishib, xosil bo`lgan sirtni aniqlang. Javob: Giperbolik paraboloid.

20 $z = pxy$ sirtning parametrik tenglamasini yozing.

Javob : $x = u, \quad y = v, \quad z = puv.$

Foydalanilgan adabiyotlar

1. Wilhelm Klingenberg, A Course in differential geometry,1978 by Springer-Verlag, New York Inc. Printed in the United States of America.
2. M.A.Arstrong, Basic Tpology, Springer, 1998 y.
3.Сборник задач по дифференциал`ной геометрии. Под ред. Феденко А.С. М., 1979

26. Sirtning urinma tekisligi va normali.

Sirt urinma tekisligining tenglamalari:

$$\Omega: \ \vec r = \vec r(u,v) \ \text{uchun} \ ((\vec R - \vec r)\vec r_u \vec r_v) = 0 . \quad (1)$$

$$\Omega: \ \begin{matrix} x = x(u,v) \\ y = y(u,v) \\ z = z(u,v) \end{matrix} \ \text{uchun} \ \begin{vmatrix} X-x & Y-y & Z-z \\ x'_u & y'_u & z'_u \\ x'_v & y' & z'_v \end{vmatrix} = 0 . \quad (2)$$

$$\Omega: z = z(x,y) \ \text{uchun} \ Z - z = p(X-x) + q(Y-y), \ p = \frac{\partial z}{\partial x}, \ q = \frac{\partial z}{\partial y} . \quad (3)$$

$$\Omega: F(x,y,z) = 0 \ \text{uchun} \ (X-x)F'_x + (Y-y)F'_y + (Z-z)F'_z = 0 . \quad (4)$$

Normal tenglamalari:

$$\vec R = \vec r + \lambda[\vec r_u \vec r_v] . \quad (5)$$

$$\frac{X-x}{\begin{vmatrix} y_u & z_u \\ y_v & z_v \end{vmatrix}} = \frac{Y-y}{\begin{vmatrix} z_u & x_u \\ z_v & x_v \end{vmatrix}} = \frac{Z-z}{\begin{vmatrix} x_u & y_u \\ x_v & y_v \end{vmatrix}} . \quad (6)$$

$$\frac{X-x}{-p} = \frac{Y-y}{-q} = Z - z . \quad (7)$$

$$\frac{X-x}{F'_x} = \frac{Y-y}{F'_y} = \frac{Z-z}{F'_z} . \quad (8)$$

Masala yechish namunalari:

1. $x = u + v, \ y = u - v, \ z = uv$ sirtning $M(u = 2, v = 1)$ nuqtasidagi urinma tekisligi va normali aniqlansin.

Yechish: $x_0 = 3, \ y_0 = 1, \ z_0 = 2. \quad x'_u = 1, \ y'_u = 1, \ z'_u = v, \quad x'_v = 1, \ y'_v = -1, \ z'_v = u.$

M nuqtada $\vec r_u'\{1,1,1\}, \ \vec r_v'\{1,-1,2\}$.

Urinma tekislik:

$$\begin{vmatrix} x-3 & y-1 & z-2 \\ 1 & 1 & 1 \\ 1 & -1 & 2 \end{vmatrix} = 0 \quad \Rightarrow 2(x-3) - (z-2) + (y-1) - (z-2) + (x-3) - 2(y-1) = 0 ,$$

$$3x - y - 2z - 4 = 0 .$$

Normal tenglamasi: $\dfrac{x-3}{3} = \dfrac{y-1}{-1} = \dfrac{z-2}{-2}$.

Javob: $3x - y - 2z - 4 = 0, \quad \dfrac{x-3}{3} = \dfrac{y-1}{-1} = \dfrac{z-2}{-2}.$

2. $z = x\varphi\left(\frac{y}{x}\right)$ sirt urinma tekisligini koordinatalar boshidan o`tishi isbotlansin.

Isboti: $x = u,\ \ y = v,\ \ z = u\varphi\left(\frac{v}{u}\right),\ \ x'_u = 1,\ \ y'_u = 0,\ \ z'_u = \varphi\left(\frac{v}{u}\right) - \left(\frac{v}{u}\right)\varphi'\left(\frac{v}{u}\right),$

$$x'_v = 0,\ \ y'_v = 1,\ \ z'_v = u\varphi'\left(\frac{v}{u}\right)\frac{1}{u} = \varphi'\left(\frac{v}{u}\right),\qquad \begin{vmatrix} x-u & y-v & z-u\varphi\left(\frac{v}{u}\right) \\ 1 & 0 & \varphi\left(\frac{v}{u}\right) - \frac{v}{u}\varphi'\left(\frac{v}{u}\right) \\ 0 & 1 & \varphi'\left(\frac{v}{u}\right) \end{vmatrix} = 0.$$

$$z - u\varphi\left(\frac{v}{u}\right) - \left[\varphi\left(\frac{v}{u}\right) - \frac{v}{u}\varphi'\left(\frac{v}{u}\right)\right](x-u) - \varphi'\left(\frac{v}{u}\right)(y-v) = 0,\ \ z - \left[\varphi\left(\frac{v}{u}\right) - \frac{v}{u}\varphi'\left(\frac{v}{u}\right)\right]x - \varphi'\left(\frac{v}{u}\right)y = 0 \ \Rightarrow$$

$O(0,0,0)$ tenglamani qanoatlantiradi.

3. $x^2 + 2y^2 - z^2 = 2$ sirtning $x + y + z = 0$ tekislikka parallel urinma tekisligi va normali aniqlansin.

Yechish:

$$\left.\frac{\partial F}{\partial x}\right|_{M_0} = 2x_0,\quad \left.\frac{\partial F}{\partial y}\right|_{M_0} = 4y_0,\quad \left.\frac{\partial F}{\partial z}\right|_{M_0} = -2z_0.$$

Urinma tekislik: $2x_0(x - x_0) + 4y_0(y - y_0) - 2z_0(z - z_0) = 0$.

$2x_0X + 4y_0Y - 2z_0Z - 4 = 0.$ Bu tenglamani $x + y + z = 0$ tenglama bilan solishtiramiz. $2x_0 = 4y_0 = -2z_0 \ \Rightarrow\ y_0 = \frac{1}{2}x_0,\ \ z_0 = -x_0$. (*)

(*) ni $x_0^2 + 2y_0^2 - z_0^2 - 2 = 0$ ga qo`yamiz.

$$x_0^2 + \frac{1}{2}x_0^2 - x_0^2 = 2 \quad\Rightarrow\quad x_0^2 = 4 \ \Rightarrow\ x_0 = \pm 2,$$

$M'_0(2,1,-2),\ \ M''_0(-2,-1,2)$. Urinma tekislik tenglamalari:

$x + y + z = 1$ va $x + y + z = -1$.

Normal esa M'_0 da $$\frac{x-2}{4} = \frac{y-1}{4} = \frac{z+2}{4}.$$

Mustaqil yechish uchun masalalar:

Quyidagi sirtlarning urinma tekisligi va normali tenglamalarini yozing.

1) $z = xy$, (0,0,0) nuqtada. Javob : Urinma tekisligi: $z = 0$,
Normali: (oz) o`q.

2) $z = xy$, (2,1,2) nuqtada. Javob : Urinma tekisligi : $z = x + 2y - 2$,
Normali: $\frac{x-2}{-1} = \frac{y-1}{-2} = \frac{z-2}{1}$.

3) $z=x^2+y^2$, (1,1,2) nuqtada. Javob : Urinma tekisligi : $z=2x+2y-2$.

Normali: $\frac{x-1}{2}=\frac{y-1}{2}=\frac{z-2}{-1}$.

4) $z=\sin\frac{x}{y}$, (π,1,0) nuqtada. Javob : Urinma tekisligi : $z=-x+\pi y$,

Normali: $\frac{x-\pi}{1}=\frac{y-1}{-\pi}=\frac{z}{1}$.

5. $x^2+y^2+z^2-1=0$. Javob : Urinma tekisligi : $Xx+Yy+Zz=1$,

Normali: $\frac{X-x}{x}=\frac{Y-y}{y}=\frac{Z-z}{z}$.

6. $(z^2-x^2)xyz-y^5=5$, (1,1,2) nuqtada.

Javob : Urinma tekisligi : $2x+y+11z=25$,

Normali: $\frac{x-1}{2}=\frac{y-1}{1}=\frac{z-2}{11}$.

7. $e^z-z+xy-3=0$, (2,1,0) nuqtada.

Javob : Urinma tekisligi : $x+2y=4$,

Normali: $\frac{x-2}{1}=\frac{y-1}{2}=\frac{z}{0}$.

8. $x^2+2y^2+3z^2=6$, (1,-1,1) nuqtada.

Javob : Urinma tekisligi : $x-2y+3z=6$,

Normali: $\frac{x-1}{1}=\frac{y+1}{-2}=\frac{z-1}{3}$.

9. $2z=x^2-y^2$, (3,1,4) nuqtada.

Javob : Urinma tekisligi : $3x-y-z=4$,

Normali: $\frac{x-3}{-3}=\frac{y-1}{1}=\frac{z-4}{1}$.

10. $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$, Javob : Urinma tekisligi : $\frac{xx_0}{a^2}+\frac{yy_0}{b^2}-\frac{zz_0}{c^2}=1$,

Normali: $\frac{x-x_0}{\frac{x_0}{a^2}}=\frac{y-y_0}{\frac{y_0}{b^2}}=\frac{z-z_0}{-\frac{z_0}{c^2}}$.

11. $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$, (a,b,c) nuqtada.

Javob: Urinma tekisligi: $\frac{x}{a}+\frac{y}{b}-\frac{z}{c}=1$,

Normali: $\frac{x-a}{\frac{1}{a}}=\frac{y-b}{\frac{1}{b}}=\frac{z-c}{-\frac{1}{c}}$.

12 $xy+xz+z^2=1$ - sirtning $x+2z-y=0$ tekislikka parallel qilib o`tkazilgan urinma tekisligini toping. Javob: $x-y+2z-\frac{5}{2}=0$.

13 $z=2x^2+y^2$ elliptik paraboloidning M(1,-1,3) nuqtasida urinma tekislik va normalining tenglamasini toping. Javob: Urinma tekisligi :, $4x-2y-z-3=0$.

Normali: $\frac{x-1}{4}=\frac{y+1}{-2}=\frac{z-1}{-1}$.

14 $x^2+y^2+z^2=676$ sferada urinma tekisligi $3x-12y+4z=0$ tekislikka parallel bo`lgan nuqtalarni toping. Javob: (6,-24,8) va (-6,24,-8).

15 $4x^2+4y^2+z^2=4$ ellipsoidning $12x-3y+2z=0$ tekislikka parallel urinma tekisligini toping. Javob: $12x-3y+2z=\pm 13$.

16. $ax^2+by^2+cz^2=1$ sirtning $M_0(x_0,y_0,z_0)$ nuqtasidagi urinma tekisligini

$$ax_0x+by_0y+cz_0z=1$$

ko`rinishda yozish mumkinligini isbotlansin.

17. $x=a\cos u\cos v,\ y=a\cos u\sin v,\ z=a\sin u$ sferaning $a,0,\ 0$ nuqtasidagi urinma tekisligi va normalining tenglamasi yozilsin.

Javob: $x-a=0$. Normal $y=z=0$.

18. $z=x\varphi(\frac{y}{x})$ sirtning barcha urinma tekisliklarining $O(0,0,0)$ nuqtadan o`tishi isbotlansin. (namunada berilgan).

19. $x^2+y^2+z^2=\alpha x,\ x^2+y^2++z^2=\beta y,\ x^2+y^2+z^2=\gamma z$ uchta sirtlarning to`g`ri burchak ostida kesishishini isbotlansin.

20. $x=\varphi(u)\cos v,\ y=\varphi(u)\sin v,\ z=\varphi(u)$ sirtning barcha normallari (OZ) o`q bilan kesishishini isbotlang.

21. $xy^2+z^3=12$ sirtning $M_0(1,2,2)$ nuqtasidagi urinma tekisligi va normalining tenglamasi yozilsin. Javob: $x+y+3z-9=0,\ x-1=y-2=\frac{z-2}{3}$.

22. To`g`ri gelikoidning ixtiyoriy nuqtasidagi urinma tekisligi va normalining tenglamasi yozilsin.

Javob: $a\sin vx-a\cos vy+uz-auv=0,$

$$\frac{x-u\cos v}{a\sin v}=\frac{y-u\sin v}{-a\cos v}=\frac{z-av}{u}.$$

23. $xyz-a^3=0$ sirtning ixtiyoriy nuqtasidagi urinma tekisligi va normali aniqlansin. Javob: $y_0z_0x+x_0z_0y+x_0y_0z-3a^3=0$,

$$\frac{x-x_0}{y_0z_0}=\frac{y-y_0}{x_0z_0}=\frac{z-z_0}{x_0y_0}.$$

24. $xyz-1=0$ sirtning $x+y+z=1$ tekislikka parallel urinma tekisligi aniqlansin.

Javob: $x+y+z-3=0$.

25. $z=x\sin^2\dfrac{y}{x},\ x\neq 0$ sirtning barcha urinma tekisliklari fazoning biror fiksirlangan nuqtasidan o`tishi isbotlansin.

**<u>Ko`rsatma</u>:** Tekislik tenglamasida ozod hadning nolga tengligini ko`rsating.

26. $\vec r=a\vec e(\varphi)+v\vec k$ to`g`ri doiraviy slindr urinma tekisligi tenglamasini yozing.

Javob: $\vec e(\varphi)\vec\rho-a=0.$ Bunda $\vec\rho$ - urinma tekislik ixtiyoriy nuqtasining radius-vektori.

27. Sfera urinma tekisligining tenglamasi yozilsin.

<u>Yechish:</u>

$\vec r=a\{\vec e(u)\cos v+\vec k\sin v\},\quad \vec r'_u=a\vec g(u)\cos v,\quad \vec r'_v=a\{-\vec e(u)\sin v+\vec k\cos v\},\ \vec N=[\vec r'_u\,\vec r'_v]=a\cos v\vec r.$

Urinma tekislik $\vec r(\vec\rho-\vec r)=0$ tenglamaga ega, bunda $\vec r$ - urinish nuqtaning radius vektori, $\vec\rho$ esa urinma tekislik ixtiyoriy nuqtasining radius vektori.

28. $\vec r=\{\vec e(u)\cos\alpha+\vec k\sin\alpha\}v$ doiraviy konusning urinma tekisligi tenglamasini aniqlang. Javob: $(b\vec e(u)-a\vec k)\vec\rho=0.$

29. $x=2u-v,\ y=u^2+v^2,\ z=u^3-v^3$ sirtning $M(3,5,7)$ nuqtasidagi urinma tekisligining tenglamasini yozing. Javob: $18x+3y-4z-41=0$.

30. $x=u,\ y=u^2-2uv,\ z=u^3-3u^2v$ sirtning $M(1,3,4)$ nuqtasidagi urinma tekisligi va normalini aniqlang. Javob: $6x+3y-2z-7=0$ va $\dfrac{x-1}{6}=\dfrac{y-3}{3}=\dfrac{z-2}{-2}$.

31. Quyidagi sirtlar uchun urinma tekislik va normal tenglamasini yozing:

a) $z=x^3+y^3,\ M(1,2,9)$ nuqtada;

b) $x^2+y^2+z^2=169,\ M(3,4,12)$ nuqtada;

c) $x^2-2y^2-3z^2-4=0,\ M(3,1,-1)$ nuqtada.

Javoblar:

a) $3x+12y-z-18=0,\ \dfrac{x-1}{3}=\dfrac{y-2}{12}=\dfrac{z-9}{-1}$.

b) $3x+4y+12z-169=0,\ \dfrac{x-3}{3}=\dfrac{y-4}{4}=\dfrac{z-1}{12}$.

c) $3x-2y+3z-4=0,\ \dfrac{x-3}{3}=\dfrac{y-1}{-2}=\dfrac{z-6}{3}$.

32. $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ ellipsoidning $M(x_0, y_0, z_0)$ nuqtasidagi urinma tekisligi tenglamasini yozing. Javob: $\frac{x_0x}{a^2}+\frac{y_0y}{b^2}+\frac{z_0z}{c^2}=1$.

33. $xyz=a^3$ sirtning urinma tekisliklari koordinat o`qlari bilan kesishib doimiy hajmga ega bo`lgan tetraedr tashkil etishi isbotlansin.

34 Konusning ixtiyoriy nuqtasidagi urunma tekisligi uning uchi orqali o`tishini isbotlang.

35 $z=x^3+y^3$ sirtning $M(\alpha,-\alpha,0)$ nuqtasidagi urunma tekisliklarining dasta tashkil etishi isbotlansin.

36 Quyidagi sirtlar oilasining juft-jufti bilan ortogonalini isbotlansin;

a) $4x^2+y^2+z^2=\lambda, y=\mu z, y^2+z^2=\nu e^x$;

b) $x^2+y^2+z^2=\lambda, x^2+y^2+z^2=\mu y, x^2+y^2+z^2=\nu z$;

v) $xy=\lambda z^2, x^2+y^2+z^2=\mu, x^2+y^2+z^2=\nu(x^2-y^2)$;

37 $\sqrt{x}+\sqrt{y}+\sqrt{z}=a,\ (x>0, y>0,\ z>0)$; Sirtning urunma tekisliklari kordinata o`qlari bilan kesishib ajratgan kesmalar yig`indisi sirtdagi urunish nuqtalarga bog`liq bo`lishligi isbotlansin.

38 $x=U,\ y=V,\ z=UV^2$ va $x=U^*,\ y=U^* * V^*,\ z=V^*$, Sirtlarning mos nuqtalardagi urunma tekisliklari o`zaro parallel bo`lsa, ushbu nuqtalar orasidagi moslikni aniqlang; Javob: $U^*=\frac{1}{2UV},\quad V^*=-\frac{V}{2U},\ UV\neq 0.$

39 $x=U+V, y=U-V, z=UV+3$ Sirtning $O(0,0,0)$ nuqta orqali o`tuchi, normalini aniqlang. Javob: $M_1(0,2,2), M_2(0,-2,2), M_3(0,0,3)$ nuqtalardagi normalar.

40 $x^2+3y^2-2xz+z^2-2=0$ Sirtning barcha urunma tekisliklari biror vektorga parallel bo`lsa, ushbu vektorni aniqlang.

<u>Ko`rsatma</u>: $\vec{p}=\vec{i}+\vec{k}$.

41 $Z=x+(y-z)\sin(y-z)$ Sirtning barcha urunma tekisliklari biror vektorga parallel bo`lsa, ushbu vektorni aniqlang. Javob: $\overline{p}=\overline{i}+\overline{j}+\overline{k}$.

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27-30. Sirtning birinchi kvadratik formasi
Sirt ustidagi chiziqning yoy uzunligi;
Sirtdagi chiziqlar tashkil etgan burchak.
Kerakli formulalar:

$$\varphi_1 = ds^2 = Edu^2 + 2F\,du\,dv + Gdv^2, \quad (1)$$

$$S = \int \sqrt{Eu'^2 + 2Fu'v' + Gv'^2}\,dt, \quad (2)$$

$$\cos\theta = \frac{Edu\,\delta u + F(du\,\delta v + dv\delta u) + Gdv\delta u}{(Edu^2 + 2Fdudv + Gdv^2)^{\frac{1}{2}}(E\delta u^2 + 2F\delta u\,\delta v + G\delta v^2)^{\frac{1}{2}}}, \quad (3)$$

$$S = \iint_{(\hat{O})} \sqrt{EG - F^2}\,dudv, \quad (4)$$

$$E = x_u'^2 + y_u'^2 + z_u'^2, F = x_u' x_v' + y_u' y_v' + z_u' z_v', G = x_v'^2 + y_v'^2 + z_v'^2, (5)$$

Masalalar yechish namunalari

1 $x = \varphi(u)\cos v,\ y = \varphi(u)\sin v,\ z = \psi(u)$ aylanma sirtning birinchi kvadratik formasini aniqlang:

Yechish:

$$x_u' = \varphi_u' \cos v, y =_v' \varphi_u'(u)\sin v, z_u' = \psi'(u);$$

$$x'_v = -\psi(u)\sin v, y_v' = \varphi(u)\cos v, z_v' = 0;$$

$$E = x_u'^2 + y_u'^2 + z_u'^2 = \varphi_u'^2(u) + \psi_u'^2(u), F = 0;$$

$$G = x_v'^2 + y_v'^2 + z_v'^2 = \varphi^2(u);$$

$$\varphi_1 = ds^2 = (\varphi'^2(u) + \psi'^2(u))dU^2 + \varphi^2(u)dV^2.$$

2 Sirtning birinchi kvadratik formasi

$dS^2 = dU^2 + sh^2u\,dV^2$ bo\`lib, sirt ustida $\gamma: U - V = 0$ chiziq berilgan . Bu chiziq yoyining uzunligi topilsin.

Yechish: $E = 1, F = 0, G = sh^2U, dU = dV.$

$$S = \int_{u_1}^{u_2} \sqrt{dU^2 + sh^2UdV^2} = \int_{U_1}^{U_2} \sqrt{1 + sh^2U}\,dU = \int_{U_1}^{U_2} chudu = |shu|\Big|_{U_1}^{U_2} = |shU_2 - shU_1|.$$

Javob: S=$|shU_2 - shU_1|$.

3 $x = ach\left(\frac{u}{a}\right)\cos v,\ y = ach(\left(\frac{u}{a}\right)\sin v,\ Z = u$ katenoid ustida $U \pm V = 0$ chiziqlar berilgan. Ular orasidagi burchakni toping.

Yechish: Katenoidning birinchi kvadratik formasi

$$ds^2 = ch^2\left(\frac{u}{a}\right)du^2 + a^2ch^2\left(\frac{u}{a}\right)dv^2\ ,\ E = ch^2\left(\frac{u}{a}\right),\ F = 0.\ G = a^2ch^2\left(\frac{u}{a}\right)\ ,\ du = dv,\ \delta u = -\delta v\,,$$

$$\cos\theta = \frac{ch^2u\,du\,\delta u - a^2ch^2u\,du\,\delta u}{\left(ch^2u + a^2ch^2u\right)^{\frac{1}{2}} du\left(ch^2u + a^2ch^2u\right)^{\frac{1}{2}} \delta u} = \frac{du\,\delta u\left(ch^2u - a^2ch^2u\right)}{du\,\delta u\,ch^2u\left(1+a^2\right)};$$

$$\cos\theta = \frac{1-a^2}{1+a^2}; \qquad \text{Javob: } \cos\theta = \frac{1-a^2}{1+a^2}.$$

4. To`g`ri gelikoid $x = u\cos v, y = u\sin v, z = av$ ustuda $u = 0,\ u = a,\ v = 0,\ v = 1$ chiziqlar bilan chegaralangan to`rtburchakni yuzini hisoblang.

<u>Yechish:</u> $ds^2 = du^2 + \left(u^2 + a^2\right)dv^2 \Rightarrow E = 1, F = 0, G = u^2 + a^2$,

$$S = \iint\limits_{(\phi)} \sqrt{EG - F^2}\,du\,dv = \int_0^1 dv \int_0^a \sqrt{u^2 + a^2}\,du = \frac{a^2}{2}\left[\sqrt{2} + \ln\left(1 + \sqrt{2}\right)\right].$$

Isboti:

$$\int_0^a \sqrt{u^2 + a^2}\,du = \left| \begin{matrix} u = a\,tgt; t_1 = 0 & \Rightarrow u = 0 \\ du = \dfrac{a}{\cos t}dt; & t_2 = \dfrac{\pi}{4} \Rightarrow u = a \end{matrix} \right| = \int_0^{\frac{\pi}{4}} \frac{a^2dt}{\cos^3 t} = \int_0^{\frac{\pi}{4}} \frac{a^2 d\sin t}{\cos^4 t} = \left| \begin{matrix} \sin t & = x \\ x_1 = 0 & x_2 = \dfrac{\sqrt{2}}{2} \end{matrix} \right| =$$

$$a^2 \int_0^{\frac{\sqrt{2}}{2}} \frac{dx}{\left(1 - x^2\right)} = a^2 \int_0^{\frac{\sqrt{2}}{2}} \frac{1}{4}\left(\frac{1}{1-x} + \frac{1}{1+x}\right)^2 dx = \frac{a^2}{4}\left[\int_0^{\frac{\sqrt{2}}{2}} \frac{dx}{\left(1-x\right)^2} + 2\int_0^{\frac{\sqrt{2}}{2}} \frac{dx}{\left(1-x\right)^2} + \int_0^{\frac{\sqrt{2}}{2}} \frac{dx}{\left(1+x\right)^2}\right] =$$

$$\frac{a^2}{4}\left(-\int_0^{\frac{\sqrt{2}}{2}} \frac{d\left(1-x\right)}{\left(1-x\right)^2} + \int_0^{\frac{\sqrt{2}}{2}} \left[\frac{d\left(1+x\right)}{1+x} - \frac{d\left(1-x\right)}{1-x}\right] + \int_0^{\frac{\sqrt{2}}{2}} \frac{d\left(1+x\right)}{\left(1+x\right)^2}\right) = \frac{a^2}{4}\left(\ln\frac{2+\sqrt{2}}{2-\sqrt{2}} + \frac{2}{2-\sqrt{2}} - \frac{2}{2+\sqrt{2}}\right) =$$

$$= a^2\left[\frac{1}{2}\ln\left(1+\sqrt{2}\right) + \frac{\sqrt{2}}{2}\right]. \qquad \text{Javob: } S = \int_0^1 dv \int_0^a \sqrt{n^2 + a^2} = \frac{a^2}{2}\left[\sqrt{2} + \ln\left(1+\sqrt{2}\right)\right].$$

Mustaqil yechish uchun masalalar :

1. Quyudagi sirtlarning birinchi kvadratik formasi hisoblansin

a) Tekislik $x = u,\ y = v,\ z = 0$. Javob: $ds^2 = du^2 + dv^2$.

b) Tekislik $x = \rho\cos\varphi,\ y = \rho\sin\varphi,\ z = 0.$ Javob : $ds^2 = d\rho^2 + \rho^2 d\varphi^2$.

v) Sfera $x = a\cos u\cos v,\ y = a\cos u\sin v,\ z = a\sin u$ Javob: $ds^2 = a^2du^2 + a^2\cos^2 u dv^2$.

g) Psevdosfera $x = a\sin u\cos v,\ y = a\sin u\sin v,\ z = f\left(u\right) = a\left(\ln tg\left(\frac{u}{2}\right) + \cos u\right)$.

Javob: $ds^2 = a^2 ctg^2 u du^2 + a^2\sin^2 u dv^2$.

2. $x = u\cos v, y = u\sin v, z = av$ gelikoidning φ_1 formasi hisoblansin

Javob: $ds^2 = du^2 + \left(u^2 + a^2\right)dv^2$.

3. $z = f(x,y)$ sirtning φ_1 formasi hisoblansin.

Javob :

$ds^2 = (1+p^2)dx^2 + 2pqdxdy + (1+q^2)dy^2,\ p = f'(x), q = f'(y)$.

4. Slindrning chiziqli elementini aniqlang.

Javob: $ds^2 = a^2du^2 + dv^2$.

5. Konusning chiziqli elementini aniqlang.

Javob $ds^2 = dv^2 + v^2\cos^2\alpha\, du^2$.

6. To`g`ri doiraviy konusning yasovchilari $V = Ae^{mu}$ egri chiziq bilan kesishib θ burchak tashkil etadi , $\cos\theta$ ni aniqlang.

Yechish: $ds^2 = dv^2 + (v\cos\alpha)^2 du^2$ yasovchilar uchun $\delta u = 0, \delta v \neq 0$ berilgan chiziq uchun $dv = Ame^{mu}du = mvdu$, $\cos\theta = \dfrac{dv\delta v}{\sqrt{dv^2 + (v\cos\alpha)^2 du^2}\sqrt{\delta v^2}} = \dfrac{m}{\sqrt{m^2 + \cos^2\alpha}} = const$.

7. Torning chiziqli elementi topilsin.

Javob:

$\vec{r} = (a + b\cos v)\vec{e}(u) + b\vec{k}\sin v, \quad ds^2 = b^2dv^2 + (a + b\cos v)du^2$.

8. $x = u\cos v,\ y = u\sin v,\ z = av$ gelikoidda koordinat chiziqlar tashkil etgan burchakni teng ikkiga ajratuvchi chiziqlar tenglamasi aniqlansin.

Javob: $\ln\left(u + \sqrt{u^2 + a^2}\right) \pm V = const$.

9. Sirtnig chiziqli elementi $ds^2 = du^2 + (u^2 + a^2)dv^2$ ko`rinishda sirtga tegishli $\gamma_1 : u = \frac{1}{2}av^2,\ \gamma_2 : u = -\frac{1}{2}av^2,\ \gamma_3 : v = 1$ chiziqlar kesishib xosil qilingan egri uchburchak peremetrini aniqlang. Javob : $\frac{10}{3}a$.

10. $x = \frac{u}{2}\left(\sqrt{3}\cos v + \sin v\right),\ y = \frac{u}{2}\left(\sqrt{3}\sin v - \cos v\right),\ z = av$ sirtdagi $\gamma : u = \frac{1}{2}av^2$ chiziqqa tegishli $A(u = 0, v = 0), B(u = 2a, v = 2)$ nuqtalar orasidagi yoy uzunligini aniqlang. Javob : $\frac{10}{3}a$.

11. $x = u(u+v),\quad y = 3(u - v),\quad z = 2uv$ sirt ustudagi $\gamma_1 : u + v = 0, \gamma_2 : u - v$ chiziqlar kesishib tashkil etgan burchajni aniqlang : Javob : $\varphi = \frac{\pi}{2}$.

12. Chiziqli elementi $ds^2 = du^2 + (u^2 + a^2)dv^2$ bo`lgan sirtdagi $u = \frac{1}{2}av^2$, $u = -\frac{1}{2}av^2$ chiziqlar tashkil etgan θ burchakni aniqlang. Javob: $\theta = 0°$.

13. $u = u_0,\ v = v_0$ chiziqlar $x = u,\ y = v,\ z = uv$ sirtda yotib θ burchak ostida keshadi. θ burchak topilsin. Javob: $\cos\theta = \dfrac{u_0 v_0}{\sqrt{1 + u_0^2}\sqrt{1 + v_0^2}}$.

14. Chiziqli elementi $ds^2 = du^2 + dv^2$ bo`lgan sirtda $v = 2u$ va $v = -2u$ chiziqlar tashkil etgan burchakni aniqlang. Javob: $\cos\alpha = -\frac{3}{5}$.

15. $x = u\cos v,\ \ y = u\sin v,\ \ z = u^2$ sirtdagi $v = u + 1$ va $v = 3 - v$ chiziqlar tashkil etgan burchakni aniqlang. Javob: $\cos\alpha = \frac{2}{3}$

16. Chiziqli elementi $ds^2 = du^2 + \cos^2 u dv^2$ bo`lgan sirtda $u = v$ chiziqning $M_1(u_1, v_1),\ \ M_2(u_2, v_2)$ nuqtalari orasidagi yoy uzunligini hisoblang.

Javob: $S = 2\sqrt{2}(\cos\frac{u_1}{2} - \cos\frac{u_2}{2})$.

17. Chiziqli elementi $ds^2 = du^2 + (u^2 + a^2)dv^2$ bo`lgan sirtga tegishli $u = \pm av,\ v = 1$ chiziqlar tashkil etgan egri uchburchak yuzini aniqlang.

Javob: $S = a^2[\frac{2}{3} - \frac{\sqrt{2}}{3} + \ln(1 + \sqrt{2})]$.

18. To`g`ri gelikoid va katenoidlarning o`zaro izometrik munosabatda ekanini isbotlang.

19. $\sqrt{E}du + \sqrt{G}dv = 0,\ \ \sqrt{E}du - \sqrt{G}dv = 0$ differensial tenglamalar orqali aniqlangan chiziqlar oilasining ortogonalligini isbotlang.

20. Gelikoiddagi $du^2 - (a^2 + u^2)dv^2 = 0$ differensial tenglamalar orqali aniqlangan chiziqlar oilasining ortogonal sistema tashkil etishini isbotlang.

21. Sfera yuzini aniqlang.

Yechish: $ds^2 = a^2(dv^2 + \cos^2 v du^2);\ E = a^2\cos^2 v,\ F = 0,\ G = a^2$

$$S = \iint\limits_{\phi} \sqrt{EG - F^2}dudv = \int\limits_0^{2\pi} du \int\limits_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{a^4\cos^2 v}dv = -a^2\int\limits_0^{2\pi} du \int\limits_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos v dv = a^2\int\limits_0^{2\pi} du \sin v \Bigg|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = 4a^2\pi$$

Javob: $S = 4a^2\pi$.

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31-34. Ikkinchi kvadratik forma. Sirt egriligi.

Tayanch tushuncha va iboralar: Ikkinchi kvadratik forma, sirt egriligi, bosh egrilik, o`rta egrilik, to`la egrilik, Dyupen indikatrissasi.

Asosiy formulalar: $\varphi_2 = Ldu^2 + 2Mdudv + Ndv^2$. (1)

$$L=\frac{(\vec r''_{uu}\vec r'_u\vec r'_v)}{\sqrt{EG-F^2}},\quad M=\frac{(\vec r''_{uv}\vec r'_u\vec r'_v)}{\sqrt{EG-F^2}},\quad N=\frac{(\vec r''_{vv}\vec r'_u\vec r'_v)}{\sqrt{EG-F^2}}. \qquad (2)$$

Dyupen indikatrisasi: $Ldu^2+2Mdudv+Ndv^2=\pm 1.$ (3)

Eyler formulasi: $k_n=k_n^1\cos^2\varphi+k_n^2\sin^2\varphi.$ (4)

Bosh egriliklar uchun formula:

$$(EG-F^2)K_n^2-(LG-2FM+NE)K_n+(LN-M^2)=0. \qquad (5)$$

O`rta egrilik: $$H=\frac{LG-2FM+NE}{2(EG-F^2)}. \qquad (6)$$

To`la egrilik: $$K=K_n^1K_n^2=\frac{LN-M^2}{EG-F^2}. \qquad (7)$$

Masala yechish namunalari:

1. Psevdosferaning ikkinchi kvadratik formasi aniqlansin**.**

$x=a\sin u\cos v,\quad y=a\sin u\sin v,\quad z=a(\ln tg(\frac{u}{2})+\cos u).$

Ma`lumki, $ds^2=a^2ctg^2udu^2+a^2\sin^2udv^2.$

$E=a^2ctg^2u,\quad F=0,\quad G=a^2\sin^2u.$

$\sqrt{EG-F^2}=\sqrt{a^4\cos^2u}=a^2\cos u$

$$x_{uu}=-a\sin u\cos v,\quad y_{uu}=-a\sin u\sin v,\quad z''_{uu}=a\frac{-2\cos u\sin^2u-\cos^3u}{\sin^2u},$$

$$z'_u=a(\frac{1}{\sin u}-\sin u)=a\frac{\cos^2u}{\sin u},$$

$$L=\frac{1}{a^2\cos u}\begin{vmatrix}-a\sin u\cos v & -a\sin u\sin v & -\frac{a\cos u}{\sin^2u}(1+\sin^2u)\\ a\cos u\cos v & a\cos u\sin v & a\frac{\cos^2u}{\sin u}\\ -a\sin u\sin v & a\sin u\cos v & 0\end{vmatrix}=$$

$$=\frac{-a^3\cos u\sin u}{a^2\cos u}\begin{vmatrix}\sin u\cos v & \sin u\sin v & \frac{\cos u}{\sin^2u}(1+\sin^2u)\\ \cos v & \sin v & \frac{\cos u}{\sin u}\\ -\sin v & \cos v & 0\end{vmatrix}=$$

$$=-a\sin u\left(\frac{\cos u\cos^2v\left(1+\sin^2u\right)}{\sin^2u}-\cos u\sin^2v+\frac{\sin^2v\cos u}{\sin^2u}\left(1+\sin^2u\right)-\cos^2v\cos u\right)=-a\,ctg\,u,$$

$$M=0,\ N=\frac{1}{a^2\cos u}\begin{vmatrix}-a\sin u\cos v & -a\sin u\sin v & 0\\ a\cos u\cos v & a\cos u\sin v & a\dfrac{\cos^2 u}{\sin u}\\ -a\sin u\sin v & a\sin u\cos v & 0\end{vmatrix}=\frac{-a^3\sin^2 u\cos u}{a^2\cos u}\begin{vmatrix}\cos v & \sin v & 0\\ \cos v & \sin v & ctgu\\ -\sin v & \cos v & 0\end{vmatrix}=$$

$$=-a\sin^2 u\left(-\sin^2 v\,ctgu-\cos^2 v\,ctgu\right)=a\sin^2 u\ ctgu.$$

$$\varphi_2=-actgu\left(du^2-\sin^2 u dv^2\right).$$

2. $x=u,\quad y=v,\quad z=u^2+v^2$ sirt ustidagi $u-v^3=0$ chiziqning $A(u=1, v=1)$ nuqtadagi normal egriligi hisoblansin.

Yechish :

$$E=x_u'^2+y_u'^2+z_u'^2=1+4u^2;\quad E|_A=5$$

$$F=x_u' x_v'+y_u' y_v'+z_u' z_v'=4uv;\ F|_A=4$$

$$G=x_v'^2+y_v'^2+z_v'^2=1+4v^2;\quad G|_A=5$$

$$ds^2=5du^2+8dudv+5dv^2,\ \sqrt{EG-F^2}=\sqrt{25-16}=\sqrt{9}=3,$$

$$L=\frac{1}{3}\begin{vmatrix}0 & 0 & 2\\ 1 & 0 & 2u\\ 0 & 1 & 2v\end{vmatrix}=\frac{2}{3};\quad M=\frac{1}{3}\begin{vmatrix}0 & 0 & 0\\ . & . & .\\ . & . & .\end{vmatrix}=0;\quad N=\frac{1}{3}\begin{vmatrix}0 & 0 & 2\\ 1 & 0 & 2u\\ 0 & 1 & 2v\end{vmatrix}=\frac{2}{3},$$

$$R_n=\frac{Ldu^2+2Mdudv+Ndv^2}{Edu^2+2Fdudv+Gdv^2}=\frac{2}{3}\frac{du^2+dv^2}{5du^2+8dudv+5dv^2},\quad du=3v^2dv=3dv,$$

$$K_n=\frac{2}{3}\frac{10dv^2}{45dv^2+24dv^2+5dv^2}=\frac{20}{222}=\frac{10}{111.}.$$

3. Psevdosferaning o`rta va to`la egriligi hisoblansin.

Yechish : $E=a^2ctg^2u,\ F=0,\ G=a^2\sin^2 u,\ L=-actgu,\ M=0,\ N=actgu\sin^2 u$,

$$H=\frac{LG-2FM+NE}{2(EG-F^2)}=\frac{-a^3\cos u\sin u+a^3\dfrac{\cos^3 u}{\sin u}}{2a^4\cos^2 u}=\frac{1}{2a}\left(\frac{\cos^3 a-\cos u\sin^2 u}{\sin u\cos^2 u}\right)=\frac{1}{2a}(ctgu-tgu)$$

$$K=\frac{LN-M^2}{EG-F^2}=\frac{-a^2ctg^2u\sin^2 u}{a^4ctg^2u\sin^2 u}=-\frac{1}{a^2}.$$

Xulosa: Psevdosfera manfiy o`zgarmas egrilikka ega bo`lgan sirt.

Mustaqil yechish uchun masalalar

1 . Quyidagi sirtlarning ikkinchi kvadratik formasi hisoblansin.

a) $x=R\cos u\cos v,\ y=R\cos u\sin v,\ z=R\sin u$ - sfera.

Javob: $\varphi_2=R(du^2+\cos^2 u dv^2).$

b) $x=f(u)\cos v,\ y=f(u)\sin v,\ z=\varphi(u)$ - aylanma sirt.

Javob: $\varphi_2=\dfrac{1}{\sqrt{f'^2+\varphi'^2}}\left[\left(f'\varphi''-f''\varphi'\right)du^2+f\varphi' dv^2\right].$

v) $x=achu\cos v,\ y=achu\sin v,\ z=cshu$ bir pallali aylana giperboloid.

Javob: $\varphi_2 = \frac{-ac}{\sqrt{a^2 sh^2 u + c^2 ch^2 u}}\left(du^2 - ch^2 u dv^2\right)$.

g) $x = a\, shu \cos v,\ y = a\, shu \sin v,\ z = c\, chu$ ikki pallali aylanma giperboloid.

Javob: $\varphi_2 = \frac{ac}{\sqrt{a^2 ch^2 u + c^2 sh^2 u}}\left(du^2 + sh^2 u dv^2\right)$.

d) $x = u\cos v,\ y = u\sin v,\ z = u^2$ aylanma paraboloid.

Javob: $\varphi_2 = \frac{2}{\sqrt{1+4u^2}}\left(du^2 + u^2 dv^2\right)$.

e) $x = R\cos v,\ y = R\sin v,\ z = u$ doiraviy silindr.

Javob: $\varphi_2 = R dv^2$.

ℓ) $x = u\cos v,\ y = u\sin v,\ z = ku$ (uchsiz doiraviy konus).

Javob: $\varphi_2 = \frac{ku}{\sqrt{1+k^2}} dv^2$.

m) $x = ach\left(\frac{u}{a}\right)\cos v,\ y = ach\left(\frac{u}{a}\right)\sin v,\ z = u$ katenoid.

Javob: $\varphi_2 = -\frac{1}{a}\left(du^2 - a^2 dv^2\right)$.

2. $x = u\cos v, y = u\sin v, z = av$ to\`g\`ri gelikoidning φ_2 formasi hisoblansin.

Javob: $\varphi_2 = -\frac{2a\,du\,dv}{\sqrt{u^2 + a^2}}$.

3. Tekislik uchun $\varphi_2 = 0$ ni isbotlang .

4. To\`g\`ri gelikoidning bosh yo\`nalishlari va bosh egriliklarini hisoblang.

Javob: $\frac{du}{dv} = \pm\sqrt{a^2+u^2},\ K_1 = -K_2 = \frac{a}{u^2+a^2}$.

5. $x = \cos v - (u+v)\sin v,\ y = \sin v + (u+v)\cos v,\ z = u + 2v$ sirtning bosh egriliklari aniqlansin.

Javob : $.k_1 = 0,\ k_2 = -\frac{1}{\sqrt{2}(u+v)}$.

6. $Z = \frac{1}{2}\left(ax^2 + by^2\right)$ Paraboloidning $O(0,0,0)$ nuqtadagi $dx : dy$ yo\`nalish bo\`yicha normal egriligi aniqlansin . Javob: $k_n = \frac{a\,dx^2 + b\,dy^2}{dx^2 + dy^2}$.

7. $x = u, y = v, z = uv$ giperbolik paraboloidning ixtiyoriy nuqtasidagi bosh egriliklari aniqlansin. Javob: $k_1 = \frac{-uv + \sqrt{(1+u^2)(1+v^2)}}{(1+u^2+v^2)^{\frac{3}{2}}}$,

$$k_2 = \frac{-uv - \sqrt{(1+u^2)(1+v^2)}}{(1+u^2+v^2)^{\frac{3}{2}}}.$$

8. $\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$ Ikki pallali giperpoloidning uchlaridagi bosh egriliklarni hisoblang. Javob: $k_1 = \frac{a}{b^2}, \quad k_2 = \frac{a}{c^2}$.

9. $z = xy$ sirtning $M(1,1,1)$ nuqtasidagi bosh egriliklarni aniqlang.

Javob: $k_1 = \frac{\sqrt{3}}{9}$, $k_2 = -\frac{\sqrt{3}}{3}$.

10. $\frac{x^2}{P} + \frac{y^2}{q} = 2z$ sirtning $M_0(0,0,0)$ nuqtasidagi bosh egriliklarni aniqlang.

Javob: $k_1 = \frac{1}{P}, \quad k_2 = \frac{1}{q}$.

11. $x = u^2 + v^2, y = u^2 - v^2, z = uv$ sirt $P(2,0,1)$ nuqtasidagi bosh egriliklarni aniqlang.

Javob : $k_1 = \frac{1}{2\sqrt{5}}, \quad k_2 = 0$.

12. $z = 2x^2 + 4.5y^2$ sirtning $O(0,0,0)$ nuqtasidagi Dyupen indikatrisasini aniqlang.

Javob: $4x^2 + 9y^2 = 1$.

13. $\frac{x^2}{P} + \frac{y^2}{q} = 2z$ paraboloidning to`la egriligi aniqlansin.

Javob: $k^{-1} = pq\left(1 + \frac{x^2}{P^2} + \frac{y^2}{q^2}\right)^2$.

14. $ds^2 = du^2 + e^{2u}dr^2$ sirtning to`la egriligi aniqlansin .

Javob: $k = -1$.

15. a radiusli doiraviy silindrning to`la va o`rta egriligi aniqlansin.

Javob: $k = 0, \quad H = -\frac{1}{2a}$.

16. $r = \rho\vec{e}(v) + f(\rho)\vec{k}$ aylanma sirtning to`liq egriligi fo`rmulasini yozing.

Javob: $\frac{f'(\rho) f''(\rho)}{\rho(1+f'^2)^2} = k$.

17.Sferani to`la va o`rta egriligi aniqlansin.

Javob: $k = \frac{1}{a^2}, \quad 2H = \frac{2}{a}$.

18. Katenoid o`rta egriligi nol ekani aniqlansin .

19. $z = x^2 - y^2$ giperbolik paraboloid ixtiyoriy nuqtasidagi to`la va o`rta egriligini hisoblang. Javob:

$$k = -\frac{4}{\left(1+4x^2+4y^2\right)^2}, \quad H = \frac{4\left(y^2 - x^2\right)}{\left(1+4x^2+4y^2\right)^{\frac{3}{2}}}.$$

20. $z = \ln\cos x - \ln\cos y$ sirt o`rta egriligi nol ekani isbotlansin.

21. Sirt φ_1 va φ_2 formalarining koeffisentlari proporsional $L = \lambda E,\ M = \lambda F,\ N = \lambda G$ bo`lsa, $H^2 = K$ tenglikni isbotlang.

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35-36.Sirtdagi ajoyib chiziqlar.

Asosiy formulalar

1. $\vec{P}\{p_1, p_2\}$ va $\vec{q}\{q_1, q_2\}$, yo`nalishlarning sirt urinma tekisligida qo`shmalik sharti

$$LP_1q_1 + M\left(q_1p_2 + q_2p_1\right) + N\,p_2q_2 = 0. \qquad (1)$$

2. Asimptotik chiziqlar tenglamasi.

$$Ldu^2 + 2Mdudv + Ndv^2 = 0. \qquad (2)$$

3. Egrilik chiziqlar tenglamasi;

$$\begin{vmatrix} dv^2 & -du\,dv & du^2 \\ E & F & G \\ L & M & N \end{vmatrix} = 0\ . \qquad (3)$$

4. Giodezik egrilik formulasi: $k_g^1 = \left(\vec{m}\ \dot{\vec{r}}\ \ddot{\vec{r}}\right), \vec{m} = \dfrac{\left[\vec{r}_u\ \vec{r}_v\right]}{\sqrt{EG - F^2}}$. (4)

Giodezik buralish formulasi: $k_g^2 = (\dot{\vec{r}}\ \dot{\vec{m}}\ \vec{m})$. (5)

5. Giodezik chiziq tenglamasi :

$$\frac{d^2u}{dv^2} + \frac{\partial_v E}{2G}\left(\frac{du}{dv}\right)^3 + \left(\frac{\partial_u E}{2E} - \frac{\partial_u G}{G}\right)\left(\frac{du}{dv}\right)^2 + \left(\frac{\partial_v E}{E} - \frac{\partial_v G}{2G}\right)\frac{du}{dv} - \frac{\partial_u G}{2E} = 0. \qquad (6)$$

Masalalar yechish namunalari

1. $x = u\cos v$, $y = u\sin v$, $z = av$ gelikoidga tegishli $v^2du^2 - u^2dv^2 = 0$ chiziqlarning qo`shma to`r tashkil etishi isbotlansin.

Isbot. $P(u,v)du^2 + Q(u,v)dudv + R(u,v)dv^2 = 0$ differensial tenglama orqali sirtga tegishli ikkita chiziqlar oilasining qo`shma bo`lishi sharti. $LR - MQ + NP = 0$ **(*)**

Gelikoidning φ_1 va φ_2 formalari :

$$\varphi_1:\ ds^2 = du^2 + (u^2 + a^2)dv^2,\quad \varphi_2 = -\frac{2a\,du\,dv}{\sqrt{u^2 + a^2}},$$

E=1, F=0, $G = (u^2 + a^2),\ L = N = 0,\ M = \frac{2a}{\sqrt{u^2 + a^2}}$,

$P(u,v) = v^2,\ Q(u,v) = 0,\ R(u,v) = -u^2$ bo`lganidan (*) shart bajariladi. $u = \pm$ cv gelikoid ustidagi chiziqlar oilasi.

2. To`g`ri gelikoid ustidagi asimptotik chiziqlar aniqlansin .

Yechish: $L = N = 0, M = -\frac{2a}{\sqrt{u^2 + a^2}}$, $dudv = 0$,

a) $du = 0 \Rightarrow u = const$.

b) $dv = 0 \Rightarrow v = const$.

$x = c\cos v,\ x = c\sin v,\ z = av$ birinchi oila vint chiziqlar.

Ikkinchi oila $v = const.$ to`g`ri chiziqli yasovchilar $x = u\cos\alpha_0,\ y = u\sin\alpha_0,\ Z = a\alpha_0$.

3. To`g`ri gelikoid ustidagi egrilik chiziqlarni aniqlang.

Gelikoid tenglamasi: $x = u\cos v,\ y = u\sin v,\ z = av$.

Egrilik chiziqlarning differensial tenglamasi: $(a^2 + u^2)dv^2 - du^2 = 0$.

$$(dv + \frac{du}{\sqrt{a^2 + u^2}})(dv - \frac{du}{\sqrt{a^2 + u^2}}) = 0 \Rightarrow v = \pm\ln(u + \sqrt{u^2 + a^2}) + c .$$

Egrilik chiziqlar ikkita oila tashkil etadi.

Mustaqil yechish uchun masalalar

1. $\vec{r} = \vec{r}_1(u) + \vec{r}_2(v)$ ko`chish sirtining koordinat chiziqlari qo`shma to`r tashkil etadi. Isbotlansin.

2. $\frac{x^2}{a} + \frac{y^2}{b} = 2z$ elliptik parabaloidni $x + y = c$ tekisliklar bilan kesaylik , kesimdagi chiziqlar oilasi bilan qo`shma to`r tashkil etuvchi chiziqlar oilasi aniqlansin. Javob: $\frac{x}{a} - \frac{y}{b} = c_1$.

3. xyz=1 sirtning M(1,1,1) nuqtasida $\bar{a}\{1,-2,1\}$ yo`nalishga qo`shma yo`nalishni aniqlang . Javob : $\bar{b}\{1,0,-1\}$.

4. Sirtda bir parametrli chiziqlar oilasi $A(u,v)du+B(u,v)dv=0$ differensial tenglama orqali berilgan .Ushbu oilaga qo`shma chiziqlar oilasining differensialtenglamasi yozilsin.

Javob: $(LB-MA)du+(MB-NA)dv=0$.

5. $x=u\cos v,\ y=u\sin v,\ z=u+v$ qiyshiq gelikoida $u+v=c$ chiziqlar oilasiga qo`shma chiziqlar oilasini aniqlang. Javob: $v=arctgu+C_1$.

6. Har qanday sirtda cheksiz ko`p qo`shma turlar mavjudligi isbotlansin.

Isboti: Sirtda $\frac{dv}{du}=f(u,v)$ (1) differensial tenglama orqali aniqlangan chiziqlar oilasini qaraylik. Ushbu oila bilan qo`shma to`r tashkil etuvchi ikkinchi chiziqlar oilasini izlaymiz. Uning yo`nalishi $dv:du$ nisbat orqali aniqlansin. Yo`nalishlarning qo`shmalik sharti bo`lgan $L+M\left(\frac{\delta v}{\delta u}+f(u,v)\right)+Nf(u,v)\frac{\delta u}{\delta v}=0$ yoki

$$\left(Nf(u,v)+M\right)\frac{\delta v}{\delta u}+Mf(u,v)+L=0. \quad (2)$$

Agar $\begin{cases} Nf(u,v)+M=0 \\ Mf(u,v)+L=0 \end{cases}$. (3)

Holni qaramasak (2) differensial tenglamani bir qiymatli yechish mumkin. Shu yechimlar izlangan qo`shma chiziqlar oilasidan iborat bo`ladi. (3) tengliklar sirtdagi parabolik nuqtalarda bajariladi. (3) tenglik birgalikda bo`lishi uchun $\delta=\begin{vmatrix} N & M \\ m & L \end{vmatrix}=0$ bajarilishi kerak, ya'ni $LN-M^2=0$.

Xulosa:

$LN-M^2\neq 0$ bo`lsa berilgan chiziqlar oilasiga qo`shma chiziqlar oilasi mavjud va bunday oilalar cheksiz ko`p. Parabolik egilgan sirtlarda esa har qanday oilaga qo`shma chiziqlar oilasi ushbu sirtlarning asimptotik chiziqlaridir. Sirtdagi qo`shma to`rni koordinata chiziqlar uchun tanlansa $\varphi_2=Ldu^2+Ndv^2,\quad M=0$.

7. $z=Ax^2+By^2$ paraboloid koordinat chiziqlarining qo`shma to`r tashkil etishini isbotlansin.

8. $\vec{r}=\mathrm{v}\vec{e}(u)+e^{\mathrm{v}}\vec{k}$ aylanma sirtda v-$\mathrm{u}=const$ chiziqlar oilasi bilan qo`shma to`r tashkil etuvchi chiziqlar oilasini aniqlang. Javob: $\mathrm{v}=e^{-U}$ va $\mathrm{v}=Ce^{U}$.

9. $r=\mathrm{v}\,\vec{e}(u)+\ln\mathrm{v}\,\vec{k}$ aylanma sirtning asimptotik chiziqlarini aniqlang.

Javob: $\text{v} = Ce^{U}, \quad \text{v} = Ce^{-U}$.

10. Quyidagi shartlardan biri o`rinli bo`lsa, sirt ustidagi chiziqning asimptotik chiziqdan iborat ekanligini isbotlang:

a) chiziqning ixtiyoriy nuqtasidagi urunmasi asimptotik yo`nalishga ega.

b) Xar bir nuqtasidagi normal egrilik nolga teng.

c) Sirt ustidagi chiziqning ixtiyoriy nuqtasidagi yopishma tekisligi, sirtning urunmasi tekislik bilan ustma – ust tushadi.

11. Psedosfera ustidagi asimptotik chiziqlarni aniqlang.

Javob: $\ln tg\left(\frac{u}{2}\right) \pm V = C$.

12. $x = chu\cos v, \quad y = chu\sin v, \; z = u$ katenoid ustidagi asimptotik chiziqlarni aniqlang. Javob: $U \pm V = Const$.

13. Bir pallali giperboloidning asimptotik chiziqlarini ko`rsating.

Javob: To`g`ri chiziqli yasovchilar.

14. $x = au, \quad y = bv, \quad z = u^2 v$ sirtning asimptotik chizig`ini aniqlang.

Javob: $U = C_1, \quad UV^2 = C_2$.

15. $x = u, \quad y = u\text{v}, \quad z = u + v^3$ sirtning asimptotik chizig`ini aniqlang

Javob: $V = C_1, \quad U = C_2 V$.

16. $z = \ln\cos x - \ln\cos y$ sirtning asimptotik chiziqlar to`rining ortogonaligini ko`rsating.

17. $x = au, \quad y = b\text{v}, \quad z = u\text{v}$ paraboloidning asimptotik chizig`ini aniqlang.

Javob : $U = Const, \quad V = Cosnt$

18. $x = \cos u + u\sin v, \quad y = -\sin v + u\cos v, \quad z = v$ sirtning asimptotik chiziqlarini ko`rsating. Javob: 1) $\text{v} = \text{v}_0$ 2) $\text{v} = 2(U + C)$.

19. $z = \frac{x}{y} + \frac{y}{x}$ sirtning asimptotik chiziqlarini aniqlang,

Javob : $x = c_1 y, \quad \frac{1}{x^2} - \frac{1}{y^2} = c_2$,

20. $x = u, \; y = v, \; z = uv$ sirtning egrilik chiziqlarini aniqlang.

Javob: $x + \sqrt{x^2+1} = c_1\left(y + \sqrt{y^2+1}\right), \; (x + \sqrt{x^2+1})\left(y + \sqrt{y^2+1}\right) = c_2$.

21. $x = v\cos u, \; y = v\sin u, \; z = 2v$ konusning egrilik chizigini aniqlang.

Javob: $u = const,$ to`g`ri chiziqlar, $v = const$ aylanishlar.

22. Ixtiyoriy silindrik sirtning egrilik chizig`ini aniqlang.

Javob: to`g`ri chiziqli yasovchilar va ularni ortogonal kesib o`tuvchi chiziqlar.

23. $x = u^2 + v^2, y = u^2 - v^2, z = v$ sirtning egrilik chiziqlari aniqlansin.

Javob : Koordinat chiziqlari.

24. Ixtiyoriy aylanma sirtning egrilik chiziqlari aniqlang.

Javob : paralellar va meridianlar.

25. Tekislikda va sferada ixtiyoriy chiziqning egrilik chiziq ekani isbotlansin .

26. R radiusli sfera ustudagi r radiusini aylananing geodezik egriligi aniqlansin.

$$\text{Javob: } k_g' = \frac{\sqrt{R^2 - r^2}}{Rr}.$$

27. Gelikoid ustidagi $u = const$ vint chiziqning geodezik egriligini aniqlang.

$$\text{Javob: } k_g' = \frac{|c|}{c^2 + a^2}.$$

28. Sirtning geodezik chizig`i quyudagi xossalardan aqalli bittasi orqali to`la xarakterlanishini isbotlang.

a) Har bir nuqtasida sirt normali chiziqning bosh normali bilan ustma-ust tushadi.

b) Sirt normali chiziq yopishma tekisligida joylashadi.

v) Xar bir nuqtasida geodezik egrilik $k_g' = 0$.

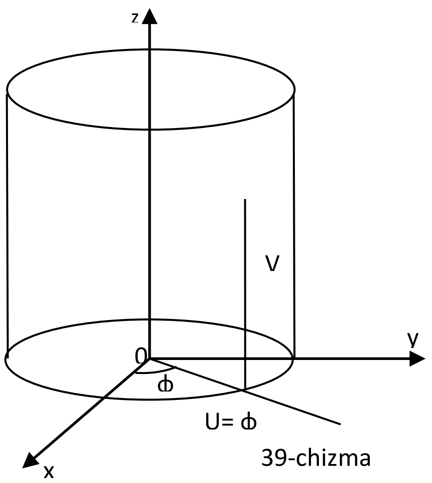
g) Chiziq egriligi va normal egrilik uchun $|k| = k_n$.

d) To`g`rilovchi tekislik sirt urinma tekisligi bilan ustma-ust tushadi.

29. Sirtdagi xar qanday tog`ri chiziq geodezik chiziq ekanini isbotlang.

30. Aylanma sirt meredianining geodezik chiziq ekanini isbotlang.

31.To`g`ri doiraviy silindr sirtidagi geodezik chiziqlar oilasi aniqlansin.



39-chizma

Yechish: Silindrda yo`naltiruvchi $x^2 + y^2 = R^2$.

Yasovchilari (oz)ga parallel

Parametrlashtirish quyidagicha $\begin{cases} x = R\cos\frac{u}{R} \\ y = R\sin\frac{u}{R} \\ z = v \end{cases}$ bunda

$u = \varphi R, \quad (0 \le \varphi \le 2R)$, $\varphi_1 = du^2 + dv^2$.

Geodezik chiziq tenglamasi: $V''_{t^2}U'_t - U''_{t^2}V'_t = 0$,

(U, V) tekislikda ushbu tenglama yechimlari $u = at + b, \quad v = ct + d$.

Silindrik sirtdagi geodezik chiziqlar $\begin{cases} x = R\cos\dfrac{at+b}{R} \\ y = R\sin\dfrac{at+b}{R} \\ z = ct + d \end{cases}$

Bu chiziqlar fazoda vint chiziqlardir

Xulosa : Silindr sirtidagi geodezik chiziqlar vint chiziqlardir .

Foydalanilgan adabiyotlar

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