1-§. IZOKLINALAR. CHIZIQLAR OILASINING DIFFERENSIAL TENGLAMASINI TUZISH

1-14 mashqlarda berilgan tenglamaning yechimini izoklinalar yordamida (tahminiy) tasvirlang.

1. $y' = y - x^2$.

2. $2(y + y') = x + 3$

3. $y' = \frac{x^2+y^2}{2} - 1$.

4. $(y^2 + 1)y' = y - x$

5. $yy' + x = 0$.

6. $xy' = 2y$

7. $xy' + y = 0$.

8. $y' + y = (x - y')^3$

9. $y' = x - e^y$.

10. $y(y' + x) = 1$

11. $y' = \frac{y-3x}{x+3y}$.

12. $y' = \frac{y}{x+y}$

13. $x^2 + y^2y' = 1$.

14. $(x^2 + y^2)y' = 4x$

15. $y' = f(x, y)$ tenglama yechimlarining maksimum yoki minimum nuqtalaridan iborat (x, y) nuqtalarning geometrik o'rni tenglamasini yozing. Maksimum nuqtalar minimum nuqtalardan qanday farqlanadi.

16. Tenglama yechimlari grafiklarining egilish nuqtalarining geometrik o'rni tenglamasini yozing.

a) $y' = y - x^2$;

b) $y' = x - e^x$;

c) $x^2 + y^2y' = 1$;

d) $y' = f(x, y)$

17-29 mashqlarda chiziqlar oilasining differensial tenglamasini tuzing

17. $y = e^{Cx}$.

18. $y = (x - C)^3$.

19. $y = Cx^3$.

20. $y = \sin(x + C)$.

21. $x^2 + Cy^2 = 2y$.

22. $y^2 + Cx = x^3$.

23. $y = C(x - C)^2$.

24. $Cy = \sin Cx$.

25. $y = ax^2 + be^x$.

26. $(x - a)^2 + by^2 = 1$.

27. $\ln y = ax + by$.

28. $y = ax^3 + bx^2 + cx$.

29. $x = ay^2 + by + c$.

30. Markazi $y = 2x$ to'g'ri chiziqda yotuvchi radiusi 1 ga teng aylanalarning diggerensial tenglamasini tuzing.

31. Bir vaqtda $y = 0$ va $y = x$ to'g'ri chiziqlarga urinuvchi va o'qi Oy o'qqa parallel bo'lgan parabolalarning differensial tenglamasini tuzing.

32. Bir vaqtda $y = 0$ va $y = x$ to'g'ri chiziqlarga urinuvchi va $0 \leq y \leq x$ sohada yotuvchi aylanalarning differential tenglamasini tuzing.

33. Koordinatalar bo'shidan o'tuvchi va o'qi Oy o'qqa parallel bo'lgan parabolalarning differensial tenglamasini tuzing.

34. Abstsissa o'qiga urinuvchi barcha aylanalarning differensial tenglamasini tuzing.

35-36 mashqlarda berilgan chiziqlar oilalari yechimidan iborat bo'lgan differensial tenglamalar sistemasini tuzing.

35. $ax + z = b,\ y^2 + z^2 = b^2.$

36. $x^2 + y^2 = z^2 - 2bz,\ y = ax + b.$

37-50 mashqlarda berilgan chiziqlar oilasi bilan berilgan φ burchak ostida kesishuvchi traektoriyalarning differensial tenglamasini tuzing.

37. $y = Cx^4,\ \varphi = 90^0.$

38. $y^2 = x + C,\ \varphi = 90^0.$

39. $x^2 = y + Cx,\ \varphi = 90^0.$

40. $x^2 + y^2 = a^2,\ \varphi = 45^0.$

41. $y = kx,\ \varphi = 60^0.$

42. $3x^2 + y^2 = C,\ \varphi = 30^0.$

43. $y^2 = 2px,\ \varphi = 60^0.$

44. $r = a + \cos\theta,\ \varphi = 90^0.$

45. $r = a\cos^2\theta,\ \varphi = 90^0.$

46. $r = \sin\theta,\ \varphi = 45^0.$

47. $y = x\ln x + Cx,\ \varphi = \operatorname{arctg} 2.$

48. $x^2 + y^2 = 2ax,\ \varphi = 45^0.$

49. $x^2 + C^2 = 2Cy,\ \varphi = 45^0.$

50. $y = Cx + C^3,\ \varphi = 45^0.$

2-§. OZGARUVCHILARI AJRALADIGAN TENGLAMA

51-65 mahqlarda berilgan tenglamani yeching va uning bir nechta integral chiziqlarini ko'rsating. Boshlang'ich shart berilgan bo'lsa, bu shartni qanoatlantiruvchi yechimini toping.

51. $xydx + (x+1)dy = 0.$

52. $\sqrt{y^2+1}dx = xydy.$

53. $(x^2 - 1)y' + 2xy^2 = 0, y(0) = 1.$

54. $y' \operatorname{ctg} x + y = 2, x \to 0, y \to -1.$

55. $y' = 3\sqrt[3]{y^2}, y(2) = 0.$

56. $xy' + y = y^2, y(1) = 0{,}5.$

57. $2x^2yy' + y^2 = 2.$

58. $y' - xy^2 = 2xy.$

59. $e^{-s}\left(1+\frac{ds}{dt}\right)=1.$

60. $y'=10^{x+y}.$

61. $\frac{xdx}{dt}+t=1.$

62. $y'=\cos(y-x).$

63. $y'-y=2x-3.$

64. $(x+2y)y'=1.$

65. $y'=\sqrt{4x+2y-1}.$

66-67 mashqlarda tenglamaning $x\to+\infty$ da ko'rsatilgan shartni qanoatlantiruvchi yechimini toping.

66. $x^2y'-\cos 2y=1, y(+\infty)=\frac{9\pi}{4}.$

67. Tenglamaning $x\to+\infty$ da chegaralangan yechimini toping

$$3y^2y'+16x=2xy^3,$$

68. Oila chiziqlariga orthogonal traektoriyalarni toping

a) $y=Cx^2$; b) $y=Ce^x$; c) $Cx^2+y^2=1.$

69. Tenglamaning har bir integral chizig'i ikkita gorizontal asimptotaga ega ekanligini ko'rsating

$$y'=\sqrt[3]{\frac{y^2+1}{x^4+1}}$$

70. Tenglama integral chiziqlarining koordinatalar boshi atrofidaga holatini tekshiring. Birinchi koordinatalar choragi chegarasining har bir nuqtasidan bu chorak ichida yotuvchi faqat bitta integral chiziq o'tishini ko'rsating.

$$y'=\sqrt{\frac{\ln(1+y)}{\sin x}}$$

3-§. GEOMETRIK VA FIZIK MASALALAR

71. Shunday chiziqlarni topingki, ihtiyoriy nuqtasidan otkazilgan urinma, urinish nuqtasi ordinatasi va abstsissalar o'qi tashkil qilgan uchburchak yuzi o'zgarmas a^2 miqdorga teng bo'lsin.

72. Shunday chiziqlarni topingki ihtiyoriy nuqtasidan otkazilgan urinma, urinish nuqtasi ordinatasi va abstsissalar o'qi tashkil qilgan uchburchak katetlarining yig'indisi o'zgarmas b miqdorga teng bo'lsin.

73. Quyidagi hossaga ega bo'lgan ciziqlarni toping: ihtiyoriy nuqtasidan o'tkazilgan urinma va normalning abstsissalar o'qidan ajratgan kesmasining uzunligi $2a$ ga teng.

74. Shunday chiziqlarni topingki ihtiyoriy nuqtasidan otkazilgan urinmaning abstsissalar o'qini kesib o'tish nuqtasi abstsissasi urinish nuqta abstsissasidan ikki marta kichik bo'lsin.

75. Quyidagi hossaga ega bo'lgan egri ciziqlarni toping: ihtiyoriy nuqtasidan koordinata o'qlariga parallel chiziqlar o'tkazilsa hosil bo'lgan to'gri to'rtburchak yuzini bu egri chiziq 1:2 nisbatda bo'ladi.

76. Shunday chiziqlarni topingki ihtiyoriy nuqtasidan o'tkazilgan urinma shu nuqtaning qutb radiusi va qutb o'qi bilan teng burchak hosil qilsin.

77-79 masalalarda idishga oqib kirayotgan gaz (yoki suyuqlik) darhol butun aralashma bo'yicha tekis tarqaladi deb hisoblanadi.

77. Idishdagi hajmi 20 l bo'lgan aralashmaning 80% i azot, 20% i kislaroddan iborat. Idishga har sekundda 0,1 l azot kiritilmoqda va shuncha aralashma chiqib ketmoqda. Qancha vaqtdan keyin aralashmadagi azot miqdori99% bo'ladi?

78. Bokda hajmi 100 l bo'lgan aralashma bor. Aralashmada 10 kg tuz mavjud. Idishga har minutda 5 l suv kiritilmoqda va shuncha suyuqlik chiqib ketmoqda. Bir soatdan keyin idishda qancha tuz qoladi.

79. Hajmi 200 m^3 bo'lgan hona havosining 0,15% i is gazidan (CO_2) iborat. Tashqaridagi havoning 0,04% i is gazidan iborat. Ventelyator har minutda ichkaridagi $20m^3$ havoni tashqaridagi havoga almashtiradi. Qancha vaqtdan keyin honadagi havoda is gazi miqdori ikki marta kamayadi?

80-82 masalalarda jismning sovish (yoki isish) tezligi jism temperatirasi va jimni o'rab turgan muhit temperaturasi ayirmasiga proporsional deb hisoblanadi.

80. Jism 10 munutda 100^0 dan 60^0 gacha sovidi. Jism atrofidagi temperatura 20^0 ga teng. Qachon jism 25^0 gacha soviydi?

81. Idishda 20^0 temperaturaga ega 1 kg suv bor. Idishga 0,5kg massali solishtirma issiqlik sig'imi 0,2 ga teng va 75^0 temperaturali metal solindi. Qachon suv va metal temperaturalari bir-biridan 1^0ga farq qiladi. Bunda idishning isishi xisobiga issiqlik yo'qolishi xisobga olinmaydi.

82. Temperaturasi$a$ gradus bo'lgan metal parchasi pechkaga joylandi. Bir soat ichida pechkaning temperaturasi a gardusdan b gradusga oshirildi. Metal va pechka temperaturalari orasidagi farq T bo'lganda, metal temperaturasi minutiga kT gratus ko'tariladi. Bir soatdan keyingi metalning temperaturasini toping.

83. Qayiqning tezligi suvning qarshiligi ta'sirida, qayiq tezligiga proporsional ravishda sekinlashadi. Boshlang'ich tezligi 1,5 m/sek bo'lgan qayiqning tezligi 4 sekunddan keyin 1 m/sek ga tushdi. Qachon qayiqning tezligi 1 sm/sek ga tushadi. Qayiq qancha masofani bosib o'tgach to'htaydi.

84-86 masalalarda radioktiv emirilish qonunidan foydalaning: radioktiv moddaning vaqt birligi ichida emiriladigan qismi miqdori bu moddaning qaralyotgan vaqt momentidagi miqdoriga proportsional.

84. 30 kunda radioktiv modda dastlabki miqdorining 50% i emirildi. Qancha vaqt o'tib modda dastlabki miqdorining 1% i qoladi?

85. Tajribalar ko'rsatadiki bir yilda har bir gram radiyning 0.44 mg i emiriladi. Necha yildan keyin mavjud radiyning yarmi emiriladi?

86. Tog' jinsini tarkibi o'rganilganda, unda 100 mg uran va 14 mg qo'rg'oshini mavjudligi aniqlandi. Ma'lumki, $4.5 \cdot 10^9$ yilda uranning yarmi yemiriladi va 238 gr uran to'liq nurlanganda 206 gr qo'rgoshini hosil bo'ladi. Tog' jinsi yoshini aniqlang. Bunda tog' jinsi hosil bo'lgan vaqtda uning tarkibida qo'rg'oshin bo'lmagan deb hisoblang hamda uran va qo'rgoshinning oraliq radioaktiv moddalarini e'tiborga olmag (chunki bunday moddalar urandan ko'ra tezroq emiriladi).

87. Kichik qalinlikdagi suv qatlamida yutilayotgan yoruqlik miqdori suvga tushayotgan yoruqlik miqdoriga va qatlam qalinligiga proportsional. Qalinligi 35 sm bo'lgan suv qatlami unga tushayotgan yoruqlikni yarmini yutadi. Qalinligi 2 m bo'lgan suv qatlami yoruqlikning qancha qismini yutadi?

88-89 masalalarda differentsial tenglamani tuzishda noma'lum funksiya sifatida tezlikni olish qulay. Gravitatsiya (yerni tortish kuchi) tezlanishini $10\ m/cek^2$ deb hisoblang.

88. Parashutchi 1.5 km balandlikdan sakradi va 0.5 km balandlikda parashutini ochdi. U parashutini ochgungacha qancha vaqt pastga tushdi? Ma'lumki insonning normal zichlikka ega havodagi tushish tezligi chegarasi $50\ m/cek$. Havoning qarshiligi tezlikning kvadratiga proportsional. Havo zichligining balandlikka bog'liq o'zgarishini hisobga olmang.

89. Og'irligi 0.4 kg futbol koptogi $20\ m/cek$ tezlik bilan yuqoriga otildi. Havoning qarshiligi tezlikning kvadratiga proportsional va tezlik 1 m/cek bo'lganda 0.48 gr ga teng. Koptokning ko'tarilish vaqtini va eng yuqori ko'talish balandligini toping. Agar havoning qarshiligi hisobga olinmasa natijalar qanday o'zgaradi?

90. 16.3 m balandlikdan koptok boshlang'ich tezliksiz tashlandi. Havoning qarshiligi hisobga olib koptokning tushish vaqtini aniqlang (**89** masalaga qarang). Erga urilgan momentdagi tezligini toping.

91-95 masalalarda idishdan suyuqlik $0{,}6\sqrt{2gh}$ tezlik bilan oqib chiqadi deb hisoblang, bunda $g = 10\ m/sek^2$ – gravitatsiya (yerni tortish kuchi) tezlanishi, h – suvning chiqish teshigidan balandligi.

91. Suv to'la veritikal o'qli silindrik bokning diametri $2R = 1{,}8\ m$, balandiligi $H = 2{,}45\ m$ va uning tubida diametri $2r = 6\ sm$ bo'lgan teshik bor. Qancha vaqt ichida hamma suv bokdan oqib chiqib ketadi? Bunda silindr o'qi vertikal.

92. 91-masalani gorizontal o'qli silindrik bok uchun yeching. Bunda bokning teshigi eng pastki qismida joylashgan.

93. Silindrik bok vertikal joylashtirilgan va tubida teshigi bor. 5 minutda to'la bokdagi suvning yarmi oqib tushadi. Qancha vaqtda hamma suv oqib ketadi?

94. Voronka konus shaklida bo'lib radiusi $R = 6\ sm$, balandligi $H = 10\ sm$, uchi pastga yo'nalgan va uchida diametri $0{,}5\ sm$ li teshik bor. Voronkadagi hamma suv qancha vaqt ichida oqib ketadi?

95. Bok to'g'ri paralellepiped shaklida bo'lib uning o'lchamlari $60sm \times 75sm$, balandligi $80sm$ va uning tubida yuzasi $2{,}5\ sm^2$ li teshik bor. Bokka sekundiga $1{,}8\ l$ suv quyulmoqda. Qancha vaqtda bok to'ladi? Agar bok tubida teshik bo'lmasa u qancha vaqtda to'ladi?

96. Uzunligi $1\ m$ bo'lgan rezina shnur $f\ kg$ kuch ta'sirida kf metrga cho'zildi. Uzunligi l va og'irligi P bo'lgan huddi shunday shnur bir uchidan osib qo'ilsa u o'z og'irligi ta'sirida qancha cho'ziladi?

97. Agar er sirtidagi bosim $1\ kg/sm^2$ va havoning zichligi $0{,}0012\ g/sm^3$ bo'lsa, h balandlikda atmosfera bosimini aniqlang. Bunda Boil-Mariott qonunidan foydalaning: zichlik bosimga proportsional (havo temperaturasining balandlikka bog'liq o'zgarishini hisobga olmang).

98. Daryo kemasini to'htatish uchun kemadan pristanga arqon tashlandi va pristan ishchisi arqonni ustunga uch qator o'radi hamda $10\ kg$ kuch bilan tortib turibdi. Agar arqon va ustun orasidagi ishqalanish koeffitsienti $\frac{1}{3}$ ga teng bo'lsa, u holda kemani qancha kuch to'htatishini toping.

99. Hajmi $v\ m^3$ bo'lgan yopiq binoda suvli idish joylashgan. Agar berilgan temperaturali $1\ m^3$ havoga singayotgan suv bug'i miqdori q_1 va qaralayotgan momentda $1\ m^3$ havoda mavjud bo'lgan suv bug'i miqdori q bo'lsa, suvning bug'lanish tezligi q_1 bilan q orasidagi farqqa proporsional (havoning va suvning harorati hamda bug'lanish yuzasi maydoni o'zgarmas deb hisoblaymiz). Boshlang'ich momentda idishda m_0 gramm suv, $1\ m^3$ havoda q_0 gramm bug' bo'lgan. t vaqtdan keyin idishda qancha suv qoladi?

100. To'liq yoqilg'i zahirasiga ega raketaning massasi M ga, yoqilg'isiz esa m ga teng. Yoqilg'ining yonib tugashi tesligi c ga teng, raketaning boshlang'ich tezligi nolga teng. Raketaning yoqilg'i tugab bo'lgandan keyingi tezligini toping. Bunda yerni tortish kuchini va havoning qarshiligini hisobga olmang (Siolkovskiy formulasi).

4-§. BIR JINSLI TENGLAMA

101-129 tenglamalarni yeching.

101. $(x+2y)dx - xdy = 0$

102. $(x-y)dx + (x+y)dy = 0$

103. $(y^2 - 2xy)dx + x^2dy = 0$

104. $2x^3y' = y(2x^2 - y^2)$

105. $y^2 + x^2y' = xyy'$

106. $(x^2 + y^2)y' = 2xy$

107. $xy' - y = x\,\mathrm{tg}\frac{y}{x}$

108. $xy' = y - xe^{\frac{y}{x}}$

109. $xy' - y = (x+y)\ln\frac{x+y}{x}$

110. $xy' = y\cos\ln\frac{y}{x}$

111. $\left(y + \sqrt{xy}\right)dx = xdy$

112. $xy' = \sqrt{x^2 - y^2} + y$

113. $(2x - 4y + 6)dx + (x + y - 3) = 0$

114. $(2x + y + 1)dx - (4x + 2y - 3)dy = 0$

115. $x - y - 1 + (y - x + 2)y' = 0$

116. $(x + 4y)y' = 2x + 3y - 5$

117. $(y + 2)dx = (2x + y - 4)dy$

118. $y' = 2\left(\frac{y+2}{x+y-1}\right)^2$

119. $(y' + 1)\ln\frac{y+x}{x+3} = \frac{y+x}{x+3}$

120. $y' = \frac{y+2}{x+1} + \mathrm{tg}\frac{y-2x}{x+1}$

121. $x^3(y' - x) = y^2$

122. $2x^2y' = y^3 + xy$

123. $2xdy + (x^2y^4 + 1)ydx = 0$

124. $ydx + x(2xy + 1)dy = 0$

125. $2y' + x = 4\sqrt{y}$

126. $y' = y^2 - \frac{2}{x^2}$

127. $2xy' + y = y^2\sqrt{x - x^2y^2}$

128. $\frac{2}{3}xyy' = \sqrt{x^6 - y^4} + y^2$

129. $2y + (x^2y + 1)xy' = 0$

130. Berilgan oila chiziqlari bilan 45^0 li burchak ostida kesishuvchi traektoriyalarni toping. Bu burchak egri chiziqqa o'tkazilgan urinmadan traektoriya urinmasigacha manfiy yo'nalishda hisoblangan burchakdan iborat.

a) $y = x\ln Cx$; b) $(x-3y)^4 = Cxy^6$.

131. Shunday egri chiziqlarni topingki, ihtiyoriy urinmasining abssissa o'qini kesib o'tish nuqtasi koordinatalar boshi va urinish nuqtasidan baravar uzoqlikda bo'ladi.

132. Shunday egri chiziqlarni topingki, koordinatalar boshidan uning ihtiyoriy urinmasigacha masofa urinish nuqta abssissasiga teng.

133. α va β ning qanday qiymatlarida $y' = ax^\alpha + by^\beta$ tenglama $y = z^m$ almashtirish yordamida bir jinsli tenglamaga keltireiladi?

134. $f(k) = k$ tenglama k_0 ildizga ega bo'lsin. Quyidagilarni ko'rsating:

1) agar $f'(k_0) < 1$ bo'lsa, u holda $y' = f\left(\frac{y}{x}\right)$ tenglamaning hech bir yechimi $y = k_0 x$ to'g'ri chiziqqa koordinatalar boshida urinmaydi;

2) agar $f'(k_0) > 1$ bo'lsa, u holda bu to'g'ri chiziq cheksiz ko'p yechimga urinadi.

135. Quyidagilarni tenglamalarning integral chiziqlarini tahmimiy chizing (tenglamalarni yechmasdan):

a) $y' = \frac{y(2y-x)}{x^2}$; b) $y' = \frac{2y^2-x^2}{xy}$;

c) $y' = \frac{2y^3-x^2y}{2x^2y-x^3}$; d) $xy' = y + \sqrt{y^2 + \frac{y^3}{x}}$

Ko'rsatma. $y = kx$ to'g'ri chiziq va u kesib o'tgan $y' = f\left(\frac{y}{x}\right)$ tenglamaning integral chizig'i orasidagi burchak tangensi $\frac{f(k)-k}{1+kf(k)}$ ga teng (bu tasdiqni asoslang). Integral chiziqlarni tahminiy chizish uchun bu kasr ishorasi k ga qanday bog'liqligini tekshirish kerak.

5 §. BIRINCHI TARTIBLI CHIZIQLI TENGLAMA

136-160 tenglamalarni yeching.

136. $xy' - 2y = 2x^4$

137. $(2x+1)y' = 4x + 2y$

138. $y' + y\,\mathrm{tg}\,x = \sec x$

139. $(xy + e^x)dx - xdy = 0$

140. $x^2y' + xy + 1 = 0.$

141. $y = x(y' - x\cos x)$

142. $2x(x^2 + y)dx = dy$

143. $(xy' - 1)\ln x = 2y$

144. $xy' + (x+1)y = 3x^2e^{-x}$

145. $(x + y^2)dy = ydx$

146. $(2e^y - x)y' = 1$

147. $(\sin^2 y + x\operatorname{ctg} y)y' = 1$

148. $(2x + y)dy = ydx + 4\ln y\, dy$

149. $y' = \frac{y}{3x - y^2}$

150. $(1 - 2xy)y' = y(y - 1)$

151. $y' + 2y = y^2e^x$

152. $(x+1)(y' + y^2) = -y$

153. $y' = y^4\cos x + y\operatorname{tg} x$

154. $xy^2y' = x^2 + y^3$

155. $xydy = (y^2 + x)dx$

156. $xy' - 2x^2\sqrt{y} = 4y$

157. $xy' + 2y + x^5y^3e^x = 0$

158. $2y' - \frac{x}{y} = \frac{xy}{x^2 - 1}$

159. $y'x^3\sin y = xy' - 2y$

160. $(2x^2y\ln y - x)y' = y$

161-166 tenglamalarni o'zgaruvchi almashtirib yoki differensiallash yo'li bilan chiziqli tenglamaga keltiring va integrallang

161. $xdx = (x^2 - 2y + 1)dy$

162. $(x+1)(yy' - 1) = y^2$

163. $x(e^y - y') = 2$

164. $(x^2 - 1)y'\sin y + 2x\cos y = 2x - 2x^3$

165. $y(x) = \int_0^x y(t)dt + x + 1$

166. $\int_0^x (x - t)y(t)dt = 2x + \int_0^x y(t)dt$

167-171 mashqlarda Rikkati tenglamasini hususiy yechimini tanlash usulida toping, Bernulli tenglamasiga keltiring va integrallang

167. $x^2y' + xy + x^2y^2 = 4$

168. $3y' + y^2 + \frac{2}{x^2} = 0$

169. $xy' - (2x+1)y + y^2 = -x^2$

170. $y' - 2xy + y^2 = 5 - x^2$

171. $y' + 2ye^x - y^2 = e^{2x} + e^x$

172. Oila chiziqlariga orthogonal traektoriyalarni toping

$$y^2 = Ce^x + x + 1$$

173. Shunday egri chiziqlarni topingki, ularning ihtiyoriy nuqtasidan o'tkazilgan urinma, urinish nuqta ordinatasi va koordinata o'qlari bilan chegaralangan trapetsiyaning yuzi o'zgarmas miqdor $3a^2$ ga teng.

174. Shunday egri chiziqlarni topingki, ularning ihtiyoriy nuqtasidan o'tkazilgan urinma, urinish nuqtasi bilan koordinatalar boshini tutashtiruvchi kesma va abssissalar o'qi hosil qilgan uchburchak yuzi o'zgarmas miqdor a^2 ga teng.

175. Birinchi bokda 100 l aralashma bo'lib uning tarkibida 10 kg tuz bor. Bok ichiga har minutda 5 l toza suv kiritiladi va shuncha aralashma ikkinchi 100 l hajmli bokka oqib o'tadi. Ikkinchi bokda dastlab toza suv bo'lgan va undan har minutda 5 l aralashma oqib chiqadi. Qachon ikkinchi bokdagi tuz miqdori eng yuqori bo'ladi? Bu miqdorni aniqlang.

176. Δt vaqt ichida (Δt juda kichik miqdor bo'lib yilning ulushidir) har bir gramm radiydan $0{,}00044\Delta t$ grami yemiriladi va $0{,}00043\Delta t$ gramm radon hosil bo'ladi. Δt vaqt ichida har bir gramm radonning $70\Delta t$ grami yemiriladi. Tajriba boshida x_0 miqdorda radiy bor edi. Qaysi vaqtda hosil bo'lgan va hali yemirilmagan radon miqdori eng katta bo'ladi?

177. Birinchi tartibli chiziqli differensial tenglamaning ikkita har hil y_1 va y_2 yechimlari berilgan. Bu tenglamaning umumiy yechimini shu yechimlar orqali ifodalang.

178. Tenglamaning $x \to \frac{\pi}{2}$ da chekli qiymatga ega bo'ladigan yechimini toping

$$y' \sin 2x = 2(y + \cos x)\,.$$

179. $xy' + ay = f(x)$ tenglamada $a = \text{const} > 0$ va $x \to 0$ da $f(x) \to b$ bo'lsin. Bu tenglamaning faqat bitta yechimi $x \to 0$ da chekli qiymatga ega bo'lishini ko'rsating va bu yechimning $x \to 0$ da limitini toping.

180. Oldingi masaladagi tenglamada $a = \text{const} < 0$ va $x \to 0$ da $f(x) \to b$ bo'lsin. Bu tenglamaning barcha yechimlari $x \to 0$ da ayni bir chekli limitga ega bo'lishini ko'rsating va bu limitni toping.

181. $\frac{dx}{dt} + x = f(t)$ tenglamada $-\infty < t < \infty$ da $|f(t)| \le M$ bo'lsin. Bu tenglamaning $-\infty < t < \infty$ da chegaralangan yechimi yagona ekanligini ko'rsating. Bu yechimni toping. Agar $f(t)$ davriy funksiya bo'lsa, u holda topilgan yechim ham davriy bo'lishini ko'rsating.

182. $xy' - (2x^2 + 1)y = x^2$ tenglamaning $x \to +\infty$ da chekli limitga ega bo'ladigan yechimi yagona ekanligini ko'rsating va bu limitni toping. Bu yechimni integral orqali ifodalang.

183. Tenglamaning davriy yechimini toping

$$y' = 2y \cos^2 x - \sin x\,.$$

184. $\frac{dx}{dt} + a(t)x = f(t)$ tenglamada $a(t) \geq c > 0$ va $t \to \infty$ da $f(t) \to 0$ bo'lsin. Bu tenglamaning har bir yechimi $t \to \infty$ da nolga intilishini ko'rsating.

185. Oldingi masaladagi tenglamada $a(t) \geq c > 0$ va $x_0(t)$ funksiya $x_0(0) = b$ boshlang'ich shart bilan aniqlangan yechim bo'lsin. Quyidagi tasdiqni isbotlang: ihtiyoriy $\varepsilon > 0$ uchun shunday $\delta > 0$ topiladiki, $f(t)$ funksiya va b sonni δ dan kichik o'zgartirsak (ya'ni ularni shunday $f_1(t)$ va b_1 songa almashtirsak, bunda $|f_1(t) - f(t)| < \delta, |b_1 - b| < \delta$), u holda $x_0(t)$ yechim $t \geq 0$ da ε dan kichik o'zgaradi. Yechimning bu hususiyati o'zgarmas ta'sir qiluvchi kuch bo'yicha turg'unlik hossasi deb ataladi.

6-§. TO'LIQ DIFFERENSIALLI TENGLAMA. INTEGRALLOVCHI KO'PAYTUVCHI

186-194 mashqlarda berilgan tenglamalarni to'liq differensialli ekanligini tekshiring va ularni integrallang

186. $2xydx + (x^2 - y^2)dy = 0$

187. $(2 - 9xy^2)xdx + (4y^2 - 6x^3)ydy = 0$

188. $e^{-y}dx - (2y + xe^{-y})dy = 0$

189. $\frac{y}{x}dx + (y^3 + \ln x)dy = 0$

190. $\frac{3x^2+y^2}{y^2}dx - \frac{2x^3+5y}{y^3}dy = 0$

191. $2x\left(1 + \sqrt{x^2 - y}\right)dx - \sqrt{x^2 - y}dy = 0$

192. $(1 + y^2 \sin 2x)dx - 2y\cos^2 x\, dy = 0$

193. $3x^2(1 + \ln y)dx = \left(2y - \frac{x^3}{y}\right)dy$

194. $\left(\frac{x}{\sin y} + 2\right)dx + \frac{(x^2+1)\cos y}{\cos 2y-1}dy = 0$

O'zgaruvchi almashtirib yoki biror usulda integrallovchi ko'paytuvchi aniqlab **195-220** tenglamalarni yeching.

195. $(x^2 + y^2 + x)dx + ydy = 0$

196. $(x^2 + y^2 + y)dx - xdy = 0$

197. $ydy = (xdy + ydx)\sqrt{1 + y^2}$

198. $xy^2(xy' + y) = 1$

199. $y^2dx - (xy + x^3)dy = 0$

200. $\left(y - \frac{1}{x}\right)dx + \frac{dy}{y} = 0$

201. $(x^2+3\ln y)ydx = xdy$ **202.** $y^2dx+(xy+\operatorname{tg}xy)dy=0$

203. $y(x+y)dx+(xy+1)dy=0$

204. $y(y^2+1)dx+x(y^2-x+1)dy=0$

205. $(x^2+2x+y)dx=(x-3x^2y)dy$

206. $ydx-xdy=2x^3\operatorname{tg}\frac{y}{x}dx$ **207.** $y^2dx+(e^x-y)dy=0$

208. $xydx=(y^3+x^2y+x^2)dy$

209. $x^2y(ydx+xdy)=2ydx+xdy$

210. $(x^2-y^2+y)dx+x(2y-1)dy=0$

211. $(2x^2y^2+y)dx+(x^3y-x)dy=0$

212. $(2x^2y^3-1)ydx+(4x^2y^3-1)xdy=0$

213. $y(x+y^2)dx+x^2(y-1)dy=0$

214. $(x^2-\sin^2 y)dx+\sin 2y\,dy=0$

215. $x(\ln y+2\ln x-1)dy=2ydx$

216. $(x^2+1)(2xdx+\cos y\,dy)=2x\sin y\,dx$

217. $(2x^3y^2-y)dx+(2x^2y^3-x)dy=0$

218. $x^2y^3+y+(x^3y^2-x)y'=0$

219. $(x^2-y)dx+x(y+1)dy=0$

220. $y^2(ydx-2xdy)=x^3(xy-2ydx)$

7-§. YECHIMNING MAVJUDLIGI VA YAGONALIGI

221. Berilgan tenglamaning berilgan boshlang'ich shartni qanoatlantiruvchi echimiga $y_0,\ y_1,\ y_2$ ketma-ket yaqinlashishlarni quring

a) $y'=x-y^2,\ y(0)=0$ b) $y'=y^2+3x^2-1,\ y(1)=1$

c) $y'=y+e^{y-1},\ y(0)=1$ d) $y'=1+x\sin y,\ y(\pi)=2\pi$

222. Quyida berilgan tenglama va sistema yechimiga ketma-ket yaqinlashuvchi dastalbki ikkita yaqinlashishni quring:

a) $y'=2x+z,\ \ z'=y;\ \ y(1)=1,\ \ z(1)=0.$

b) $\frac{dx}{dt}=y,\ \frac{dy}{dt}=x^2;\ \ x(0)=1,\ \ y(0)=2.$

c) $y'' + y'^2 - 2y = 0;\quad y(0) = 1,\ y'(0) = 0.$

d) $\frac{d^2x}{dt^2} = 3tx;\quad x(1) = 2,\ x'(1) = -1.$

223. Berilgan Koshi masalasinng yechimi mavjud bo'ladigan biror kesmani ko'rsating

a) $y' = x + y^3,\ y(0) = 0.$

b) $y' = 2y^2 - x,\ y(1) = 1.$

c) $\frac{dx}{dt} = t + e^x,\ x(1) = 0.$

d) $\frac{dx}{dt} = y^2,\ \frac{dy}{dt} = x^2,\ x(0) = 1,\ y(0) = 2.$

224. $y' = x - y^2,\ y(0) = 0$ Koshi masalasi yechimiga yechimiga yaqinlashuvchi uchinchi yaqinlashishn toping va $0 \le x \le 0{,}5$ da uning hatoligini baholang.

Ko'rsatma. [1] adabiyotning II bobi 1-§ da yoki [2] adabiyotning 15-§ da yechim mavjudligi isbotlangan. Isbotdagi qoldiq hadni baholang.

225. Tekislikdagi shunday sohani ko'rsatinki, sohaning har bir nuqtasidan tenglamaning faqat bitta integral chizig'i o'tsin. Bunda biror yagonalik teoremasidan foydalaning.

a) $y' = 2xy + y^2$, b) $y' = 2 + \sqrt[3]{y - 2x},$

c) $(x-2)y' = \sqrt{y} - x,$ d) $y' = 1 + \operatorname{tg} y,$

e) $(y-x)y' = y\ln x,$ f) $xy' = y + \sqrt{y^2 - x^2}.$

226. a ning qanday qiymatlarida $y' = |y|^a$ tenglama yechimlarining yagonaligi buziladi va qaysi nuqtalarda buziladi?

227. $y' = f(y)$ tenglama uchun yechim yagonaligining zaririy va yetarli sharti yordamida ([1] adabiyot, III bob, 4-§ 1-punkt yoki [2] adabiyot, 4-§) quyidagi tenglamalarni tekshiring. $f(y)$ funksiya oz ishorasini saqlaydigan sohani ajrating, yechimlarni tahminiy chizing. $y = 0$ da e) va f) tenglamalarning o'ng qismi uzluksizlik bo'yicha qayta aniqlanadi.

a) $y' = \sqrt[3]{y^2},$ b) $y' = y\sqrt[3]{y+1},$

c) $y' = (y-1)\sqrt{y^3},$ d) $y' = \arccos y,$

e) $y' = y\ln y,$ f) $y' = y\ln^2 y$

228. Qanday boshlang'ich shartalarda quyidagi tenglamalar va sistemalarning yechimi mavjud va yagona bo'ladi?

a) $y'' = \operatorname{tg} y + \sqrt[3]{x}$, b) $(x+1)y'' = y + \sqrt{y}$,

c) $(x-y)y'y''' = \ln xy$, d) $y'' - yy''' = \sqrt[5]{y' - x}$,

e) $\frac{dx}{dt} = y^2 + \sqrt[3]{t}$, $\frac{dy}{dt} = \sqrt[3]{x}$,

f) $\frac{dx}{dt} = y^3 + \ln(t+1)$, $x\frac{dy}{dt} = \sqrt[3]{y-t}$.

229. Berilgan tenglamaning ikkita yechimining grafigi tekislikning biror (x_0, y_0) nuqtasida kesishishi mumkinmi?

a) $y' = x + y^2$, b) $y'' = x + y^2$.

230. Berilgan tenglamaning ikkita yechimining grafigi tekislikning biror (x_0, y_0) nuqtasida o'zaro urinishi mumkinmi?

a) $y' = x + y^2$, b) $y'' = x + y^2$,

c) $y''' = x + y^2$.

231. $y^{(n)} = x + y^2$ tenglamaning bir vaqtda ikkita $y(0) = 1$, $y'(0) = 2$ shartni qanoatlantiruvchi yechimlari nechta? $n = 1,\ 2,\ 3$ holatlar alohida qaralsin.

232. $y^{(n)} = f(x,y)$ tenglamaning (f va f_y funksiyalar butun (x,y) nuqtalar tekisligida uzluksiz) (x_0, y_0) nuqtadan belgilangan yo'nalishda o'tuvchi, ya'ni shu nuqtadagi urinmasi Ox o'qi bilan α burchak tashkil qiluvchi yechimlari nechta? $n = 1$, $n = 2$ va $n \geq 3$ holatlar alohida qaralsin.

233. n ning qanday qiymatlarida $y^{(n)} = f(x,y)$ tenglamaning (f va f_y uzluksiz) yechimlari orasida $y_1 = x$, $y_2 = x + x^4$ ikkita funksiya ham bo'lishi mumkin?

234. n ning qanday qiymatlarida $y^{(n)} = f\left(x, y, y', \ldots, y^{(n-1)}\right)$ tenglamaning (f funksiya uzluksiz differensiallanuvchi) yechimlari orasida $y_1 = x$, $y_2 = x + x^4$ ikkita funksiya ham bo'lishi mumkin?

235. $f(x,y)$ funksiya x, y bo'yicha uzluksiz va har bir x uchun y ning o'sishida bu funksiya o'smaydi. Agar $y' = f(x,y)$ tenglamaning ikkita yechimi $y(x_0) = y_0$ boshlang'ich shartni qanoatlantirsa, u holda bu yechimlar $x \geq x_0$ da ustma-ust tushishini isbotlang.

236. Quyida berilgan tenglamalar va sistemalarning yechimlari koordinatalar boshi atrofida qaysi tartibli hosilallarga ega? ([2] adabiyot 19-§ da yoki [4] adabiyot 6-§ da keltirilgan yechimlarning silliqligi haqidagi teorema)

a) $y' = x + y^{7/3}$, b) $y' = x|x| - y^2$,

c) $y'' = |x^3| + y^{5/3}$, d) $y''' = y - x\sqrt[3]{x}$,

e) $\frac{dx}{dt} = t + y,\ \frac{dy}{dt} = x + t^2|t|$, f) $\frac{dx}{dt} = y^2 + \sqrt[3]{t^4},\ \frac{dy}{dt} = \sqrt[3]{x}$.

237. a ning qanday qiymatlarida tenglamaning har bir yechimini $-\infty < x < +\infty$ intervalgacha davom ettirish mumkin bo'ladi?

a) $y' = |y|^{\boldsymbol{a}}$**,** b) $y' = (y^2 + e^x)^a$,

c) $y' = |y|^{a-1} + \left|x\sqrt[3]{y}\right|^{2a}$, e) $y' = (y^2 + z^2 + 2)^{-a},\ z' = y(1 + z^2)^a$.

238. Quyidagi berilgan tenglamalarning ihtiyoriy $y(x_0) = y_0$ boshlang'ich shartni qanoatlantiruvchi $x_0 \le x < +\infty$ da aniqlangan yechimi mavjudligini isbotlang:

a) $y' = x^3 - y^3$, b) $y' = xy + e^{-y}$.

239. Butun Oxy tekislikda $f(x,y)$ va $f_y(x,y)$ funksiyalar uzluksiz va $f_y(x,y) \le k(x)$ tengsizlik o'rinli bo'lsin, bunda $f(x)$ uzluksiz funksiya. $y' = f(x,y)$ tenglamaning ihtiyoriy $y(x_0) = y_0$ boshlang'ich shartni qanoatlantiruvchi $x_0 \le x < +\infty$ da aniqlangan yechimi mavjudligini isbotlang.

240. $y' = f(x,y)$ vektor tenglama bilan ifodalangan sistema berilgan va bu sistema har bir (x,y) nuqtaning atofida mavjudlik teoremasi shartlarini qanoatlantirsin. $|y| > b$ sohada barcha x larda

$$y \cdot f(x,y) \le k(x)|y|^2$$

tengsizlik o'rinli bo'lsin, bunda $y \cdot f$ – skalayar ko'paytma, $k(x)$ uzluksiz funksiya. Tenglamaning ihtiyoriy $y(x_0) = y_0$ boshlang'ich shartni qanoatlantiruvchi $x_0 \le x < +\infty$ da aniqlangan yechimi mavjudligini isbotlang.

8-§. HOSILAGA NISBATAN YECHILMAGAN TENGLAMA

241-250 masalalarda tenglamaning barcha yechimlarini toping; mahsus yechimini (agar mavjud bo'lsa) ajirating; yechimlarni tasvirlang.

241. $y'^2 - y^2 = 0$ **242.** $8y'^3 = 27y$

243. $(y' + 1)^3 = 27(x + y)^2$ **244.** $y^2\left(y'^2 + 1\right) = 1$

245. $y'^2 - 4y^3 = 0$ **246.** $y'^2 = 4y^3(1 - y)$

247. $xy'^2 = y$ **248.** $yy'^3 + x = 1$

249. $y'^3 + y^2 = yy'(y' + 1)$ **250.** $4(1-y) = (3y-2)^2 y'^2$

251-266 tenglamalar dastlab y' ga nisbatan yechiladi, keyin (2,4,5,6 § larda keltirilgan) oddiy usullarda umumiy yechim qidiriladi. Maxsus yechimlarni (agar mavjud bo'lsa) toping

251. $y'^2 + xy = y^2 + xy'$ **252.** $xy'(xy' + y) = 2y^2$

253. $xy'^2 - 2yy' + x = 0$ **254.** $xy'^2 = y(2y' - 1)$

255. $y'^2 + x = 2y$ **256.** $y'^3 + (x+2)e^y = 0$

257. $y'^2 - 2xy' = 8x^2$ **258.** $(xy' + 3y)^2 = 7x$

259. $y'^2 - 2yy' = y^2(e^x - 1)$ **260.** $y'(2y - y') = y^2 \sin^2 x$

261. $y'^4 + y^2 = y^4$ **262.** $x(y - xy')^2 = xy'^2 - 2yy'$

263. $y(xy' - y)^2 = y - 2xy'$ **264.** $yy'(yy' - 2x) = x^2 - 2y^2$

265. $y'^2 + 4xy' - y^2 - 2x^2y = x^4 - 4x^2$

266. $y(y - 2xy')^2 = 2y'$

267-286 tenglamalarni parameter kiritish usulida yeching

267. $x = y'^3 + y'$ **268.** $x\left(y'^2 - 1\right) = 2y'$

269. $x = y'\sqrt{y'^2 + 1}$ **270.** $y'(x - \ln y') = 1$

271. $y = y'^2 + 2y'^3$ **272.** $y = \ln\left(1 + y'^2\right)$

273. $(y' + 1)^3 = (y' - y)^2$ **274.** $y = (y' - 1)e^{y'}$

275. $y'^4 - y'^2 = y^2$ **276.** $y'^2 - y'^3 = y^2$

277. $y'^4 = 2yy' + y^2$ **278.** $y'^2 - 2xy' = x^2 - 4y$

279. $5y + y'^2 = x(x + y')$ **280.** $x^2y'^2 = xyy' + 1$

281. $y'^3 + y^2 = xyy'$ **282.** $2xy' - y = y' \ln yy'$

283. $y' = e^{\frac{xy'}{y}}$ **284.** $y = xy' - x^2y'^3$

285. $y = 2xy' + y^2y'^3$ **286.** $y(y - 2xy')^3 = y'^2$

Lagranj va Klero tenglamasini yeching (**287-296** masalalar)

287. $y = xy' - y'^2$ **288.** $y + xy' = 4\sqrt{y'}$

289. $y = 2xy' - 4y'^3$ **290.** $y = xy' - (2 + y')$

291. $y'^3 = 3(xy' - y)$

292. $y = xy'^2 - 2y'^3$

293. $xy' - y = \ln y'$

294. $xy'(y' + 2) = y$

295. $2y'^2(y - xy') = 1$

296. $2xy' - y = \ln y'$

297. Agar differensial tenglamaning yechimlari oilasi berilgan bo'lsa, bu tenglamaning mahsus yechimini toping

a) $y = Cx^2 - C^2$

b) $Cy = (x - C)^2$

c) $y = C(x - C)^2$

d) $xy = Cy - C^2$

298. Shunday egri chiziqlarni topingki, har bir urinmasi koordinata o'qlari bilan yuzi $2a^2$ ga teng uchburchak hosil qilsin

299. Shunday egri chiziqlarni topingki, koordinata o'qlaridan ajratgan kesmalari uzunliklariga teskari sonlar kvadratlarining yig'indisi 1 ga teng bo'lsin

300. Koordinata boshidan o'tuvchi shunday egri chiziqlarni topingki, har bir normalining birinchi chorak burchaklari orasidagi kesmasining uzunligi 2 ga teng bo'lsin

9-§. BIRINCHI TARTIBLI TURLI TENGLAMALAR

301-330 tenglamalarni yeching va ularning yechimlarining grafigini yasang

301. $xy' + x^2 + xy - y = 0$

302. $2xy' + y^2 = 1$

303. $(2xy^2 - y)dx + xdy = 0$

304. $(xy' + y)^2 = x^2y'$

305. $y - y' = y^2 + xy'$

306. $(x + 2y^3)y' = y$

307. $y'^3 - y'e^{2x} = 0$

308. $x^2y' = y(y + x)$

309. $(1 - x^2)dy + xydx = 0$

310. $y'^2 + 2(x - 1)y' - 2y = 0$

311. $y + y'\ln^2 y = (x + 2\ln y)y'$

312. $x^2y' - 2xy = 3y$

313. $x + yy' = y^2\left(1 + y'^2\right)$

314. $y = (xy' + 2y)^2$

315. $y' = \frac{1}{x - y^2}$

316. $y'^3 + (3x - 6)y' = 3y$

317. $x - \frac{y}{y'} = \frac{2}{y}$

318. $2y'^3 - 3y'^2 + x = y$

319. $(x + y)^2y' = 1$

320. $2x^3yy' + 3x^2y^2 + 7 = 0$

321. $\frac{dx}{x} = \left(\frac{1}{y} - 2x\right)dy$

322. $xy' = e^y + 2y'$

323. $2(x-y^2)dy = ydx$

324. $x^2y'^2 + y^2 = 2x(2-yy')$

325. $dy + (xy - xy^3)dx = 0$

326. $2x^2y' = y^2(2xy' - y)$

327. $\frac{y-xy'}{x+yy'} = 2$

328. $x(x-1)y' + 2xy = 1$

329. $xy(xy'-y)^2 + 2y' = 0$

330. $(1-x^2)y' - 2xy^2 = xy$

331-420 tenglamalarni yeching.

331. $y' + y = xy^3$

332. $(xy^4 - x)dx + (y + xy)dy = 0$

333. $(\sin x + y)dy + (y\cos x - \ x^2)dx = 0$

334. $3y'^3 - xy' + 1 = 0$

335. $yy' + y^2 \operatorname{ctg} x = \cos x$

336. $(e^y + 2xy)dx + (e^y + x)xdy = 0$

337. $xy'^2 = y - y'$

338. $x(x+1)(y'-1) = y$

339. $y(y - xy') = \sqrt{x^4 + y^4}$

340. $xy' + y = \ln y'$

341. $x^2(dy - dx) = (x+y)ydx$

342. $y' + x\sqrt[3]{y} = 3y$

343. $(x\cos y + \sin 2y)y' = 1$

344. $y'^2 - yy' + e^x = 0$

345. $y' = \frac{x}{y}e^{2x} + y$

346. $(xy' - y)^3 = y'^3 - 1$

347. $(4xy - 3)y' + y^2 = 1$

348. $y'\sqrt{x} = \sqrt{y-x} + \sqrt{x}$

349. $xy' = 2\sqrt{y}\cos x - 2y$

350. $3y'^4 = y' + y$

351. $y^2(y - xy') = x^3y'$

352. $y' = (4x + y - 3)^2$

353. $(\cos x - x\sin x)ydx + (x\cos x - 2y)dy = 0$

354. $x^2y'^2 - 2xyy' = x^2 + 3y^2$

355. $\frac{xy'}{y} + 2xy\ln x + 1 = 0$

356. $xy' = x\sqrt{y - x^2} + 2y$

357. $(1 - x^2y)dx + x^2(y - x)dy = 0$

358. $(2xe^y + y^4)y' = ye^y$

359. $xy'(\ln y - \ln x) = y$

360. $2y' = x + \ln y'$

361. $(2x^2y - 3y^2)y' = 6x^2 - 2xy^2 + 1$

362. $yy' = 4x + 3y - 2$

363. $y^2y' + x^2\sin^3 x = y^3 \operatorname{ctg} x$

364. $2xy' - y = \sin y'$

365. $(x^2y^2 + 1)y + (xy - 1)^2xy' = 0$

366. $y\sin x + y' \cos x = 1$

367. $xdy - ydx = x\sqrt{x^2+y^2}dx$

368. $y^2 + x^2 y'^5 = xy\left(y'^2 + y'^3\right)$

369. $y' = \sqrt[3]{2x - y} + 2$

370. $\left(x - y\cos\frac{y}{x}\right)dx + x\cos\frac{y}{x}dy = 0$

371. $2\left(x^2 y + \sqrt{1 + x^4 y^2}\right)dx + x^3 dy = 0$

372. $\left(y' - x\sqrt{y}\right)(x^2 - 1) = xy$

373. $y'^3 + \left(y'^2 - 2y'\right)x = 3y' - y$

374. $(2x + 3y - 1)dx + (4x + 6y - 5)dy = 0$

375. $(2xy^2 - y)dx + (y^2 + x + y)dy = 0$

376. $y = y'\sqrt{1 + y'^2}$

377. $y^2 = (xyy' + 1)\ln x$

378. $4y = x^2 + y'^2$

379. $2xdy + ydx + xy^2(xdy + ydx) = 0$

380. $xdx + (x^2 \operatorname{ctg} y - 3\cos y)dy = 0$

381. $x^2 y'^2 - 2(xy - 2)y' + y^2 = 0$

382. $xy' + 1 = e^{x-y}$

383. $y' = \operatorname{tg}(y - 2x)$

384. $3x^2 - y = y'\sqrt{x^2 + 1}$

385. $yy' + xy = x^3$

386. $x(x-1)y' + y^3 = xy$

387. $xy' = 2y + \sqrt{1 + y'^2}$

388. $(2x + y + 5)y' = 3x + 6$

389. $y' + \operatorname{tg} y = x\sec y$

390. $y'^4 = 4y(xy' - 2y)^2$

391. $y' = \frac{y^2 - x}{2y(x+1)}$

392. $xy' = x^2 e^{-y} + 2$

393. $y' = 3x + \sqrt{y - x^2}$

394. $xdy - 2ydx + xy^2(2xdy + ydx) = 0$

395. $(x^3 - 2xy^2)dx + 3x^2 ydy = xdy - ydx$

396. $(yy')^3 = 27x(y^2 - 2x^2)$

397. $y' - 8x\sqrt{y} = \frac{4xy}{x^2 - 1}$

398. $[2x - \ln(y+1)]dx - \frac{x+y}{y+1}dy = 0$

399. $xy' = (x^2 + \operatorname{tg} y)\cos^2 y$

400. $x^2(y - xy') = yy'^2$

401. $y' = \frac{3x^2}{x^3 + y + 1}$

402. $y' = \frac{(1+y)^2}{x(y+1) - x^2}$

403. $(y-2xy')^2=4yy'^3$ **404.** $6x^5ydx+(y^4\ln y-3x^6)dy=0$

405. $y'=\frac{1}{2}\sqrt{x}+\sqrt[3]{y}$ **406.** $2xy'+1=y+\frac{x^2}{y-1}$

407. $yy'+x=\frac{1}{2}\left(\frac{x^2+y^2}{x}\right)^2$ **408.** $y'=\left(\frac{3x+y^3-1}{y}\right)^2$

409. $x\sqrt{y^2+1}+1)(y^2+1)dx=xydy$

410. $(x^2+y^2+1)yy'+(x^2+y^2-1)x=0$

411. $y^2(x-1)dx=x(xy+x-2y)dy$

412. $(xy'-y)^2=x^2y^2-x^4$

413. $xyy'-x^2\sqrt{y^2+1}=(x+1)(y^2+1)$

414. $(x^2-1)y'+y^2-2xy+1=0$ **415.** $y'\operatorname{tg}y+4x^3\cos y=2x$

416. $(xy'-y)^2=y'^2-\frac{2yy'}{x}+1$

417. $(x+y)(1-xy)dx+(x+2y)dy=0$

418. $(3xy+x+y)ydx+(4xy+x+2y)xdy=0$

419. $(x^2-1)dx+(x^2y^2+x^3+x)dy=0$

420. $x\left(y'^2+e^{2y}\right)=-2y'$

10-§. TARTIBI KAMAYADIGAN TENGLAMALAR

421-450 tenglamalarni yeching.

421. $x^2y''=y'^2$ **422.** $2xy'y''=y'^2-1$

423. $y^3y''=1$ **424.** $y'^2+2yy''=0$

425. $y''=2yy'$ **426.** $yy''+1=y'^2$

427. $y''(e^x+1)+y'=0$ **428.** $y'''=y''^2$

429. $yy''=y'^2-y'^3$ **430.** $y'''=2(y''-1)\operatorname{ctg}x$

431. $2yy''=y^2+y'^2$ **432.** $y''^3+xy''=2y'$

433. $y''^2+y'=xy''$ **434.** $y''+y'^2=2e^{-y}$

435. $xy'''=y''-xy''$ **436.** $y''^2=y'^2+1$

437. $y'' = e^y$

438. $y'' - xy''' + y'''^3$

439. $2y'(y'' + 2) = xy''^2$

440. $y^4 - y^3y'' = 1$

441. $y'^2 = (3y - 2y')y''$

442. $y''(2y' + x) = 1$

443. $y''^2 - 2y'y''' + 1 = 0$

444. $(1 - x^2)y'' + xy' = 2$

445. $yy'' - 2yy'\ln y = y'^2$

446. $(y' + 2y)y'' = y'^2$

447. $xy'' = y' + x\sin\frac{y'}{x}$

448. $y'''y'^2 = y''^3$

449. $yy'' + y = y'^2$

450. $xy'' = y' + x\left(y'^2 + x^2\right)$

451-454 tenglamalarni karrali integralni oddiy integralga keltirish formulasidan foydalanib yeching ([1] adabiyot IV bob 2-§ ga qarang).

451. $xy^{(4)} = 1$

452. $xy'' = \sin x$

453. $y''' = 2xy''$

454. $xy^{(4)} + y''' = e^x$

455-462 tenglamalarning har ikkala qismini to'liq hosilaga keltirish usuli bilan yeching

455. $yy''' + 3y'y'' = 0$

456. $y'y''' = 2y''^2$

457. $yy'' = y'(y' + 1)$

458. $5y'''^2 - 3y''y^{(4)} = 0$

459. $yy'' + y'^2 = 1$

460. $y'' = xy' + y + 1$

461. $xy'' = 2yy' - y'$

462. $xy'' - y' = x^2yy'$

463-480 tenglamalarni bir jinsli ekanligidan foydalanib tartibini pasaytiring va yeching

463. $xyy'' - xy'^2 = yy'$

464. $yy'' = y'^2 + 15y^2\sqrt{x}$

465. $(x^2 + 1)\left(y'^2 - yy''\right) = xyy'$

466. $xyy'' + xy'^2 = 2yy'$

467. $x^2yy'' = (y - xy')^2$

468. $y'' + \frac{y'}{x} + \frac{y}{x^2} = \frac{y'^2}{y}$

469. $y(xy'' + y') = xy'^2(1 - x)$

470. $x^2yy'' + y'^2 = 0$

471. $x^2\left(y'^2 - 2yy''\right) = y^2$

472. $xyy'' = y'(y + y')$

473. $4x^2y^3y'' = x^2 - y^4$

474. $x^3y'' = (y - xy')(y - xy' - x)$.

475. $\frac{y^2}{x^2} + y'^2 = 3xy'' + \frac{2yy'}{x^2}$

476. $y'' = \left(2xy - \frac{5}{x}\right)y' + 4y^2 - \frac{4y}{x^2}$

477. $x^2\left(2yy'' - y'^2\right) = 1 - 2xyy'$

478. $x^2\left(yy'' - y'^2\right) + xyy' = (2xy' - 3y)\sqrt{x^3}$

479. $x^4\left(y'^2 - 2yy''\right) = 4x^3y' + 1$

480. $yy' + xyy'' - xy'^2 = x^3$

481-500 tenglamalarning tartibini pasaytirib brinchi tartibli tenglamaga keltiring.

481. $y''\left(3 + yy'^2\right) = y'^4$

482. $y''^2 - y'y''' = \left(\frac{y'}{x}\right)^2$

483. $yy' + 2x^2y'' = xy'^2$

484. $y'^2 + 2xyy'' = 0$

485. $2xy^2(xy'' + y') + 1 = 0$

486. $x\left(y'' + y'^2\right) = y'^2 + y'$

487. $y^2\left(y'y''' - 2y''^2\right) = y'^4$

488. $y(2xy'' + y') = xy'^2 + 1$

489. $y'' + 2yy'^2 = \left(2x + \frac{1}{x}\right)y'$

490. $y'y'' = y''^2 + y'^2y''$

491. $yy'' = y'^2 + 2xy^2$

492. $y''^4 = y'^5 - yy'^3y''$

493. $2yy''' = y'$

494. $y'''y'^2 = 1$

495. $y^2y''' = y'^3$

496. $x^2yy'' + 1 = (1 - y)xy'$

497. $yy'y'' + 2y'^2y'' = 3yy''^2$

498. $\left(y'y'' - 3y''^2\right)y = y'^5$

499. $y^2\left(y'y''' - 2y''^2\right)y = y'^5$

500. $x^2\left(y^2y''' - y'^3\right) = 2y^2y' - 3xyy'^2$

501-505 tenglamalarning berilgan boshlang'ich shartni qanoatlantiruvchi yechimini toping.

501. $yy'' = 2xy'^2; y(2) = 2, y'(2) = 0{,}5$

502. $2y''' - 3y'^2 = 0; y(0) = -3, y'(0) = 1, y''(0) = -1$

503. $x^2y'' - 3xy' = \frac{6y^2}{x^2} - 4y; y(1) = 1, y'(1) = 4$

504. $y''' = 3yy';\ y(0) = -2, y'(0) = 0, y''(0) = 4{,}5$

505. $y'' \cos y + y'^2 \sin y = y';\ y(-1) = \frac{\pi}{6}, y'(-1) = 2$

506. Shunday egri chiziqlarni topingki, ihtiyoriy nuqtasining egrilik radiusi shu nuqta va abtssissa o'qi orasidagi normali uzunligidan ikki marta katta bo'lsin. Ikki holat qaralsin: a) egri chiziq abtssissa o'qiga nisbatan qavariq; b) egri chiziq abtssissa o'qiga nisbatan botiq.

507. Shunday egri chiziqlarni topingki, egrilik radusi urinma va abtssissa o'qi orasidagi burchak kosinusiga teskari proportsional bo'lsin.

508. Chozilmas ipning chetlari mahkamlangan bo'lib, ipning har bir uzunlik birligiga gorizontal tashkil etuvchisi bir hil bo'lgan yuk ta'sir qiladi(osma ko'prik). Ipning muvozanat holatini aniqlang. Ipning og'irligi og'irligi hisobga olinmasin.

509. Chetlari mahkamlab qo'yilgan bir jinsli cho'zilmagan ipning o'z og'irligi ta'siridagi muvozanat holatini aniqlang.

510. $y'' + \sin y = 0$ mayatnik harakati tenglamasi $x \to +\infty$, $y \to \pi$ shartni qanoatlantiruvchi hususiy yechimga egaligini isbotlang.

11-§. O'ZGARMAS KOEFITSIENTLI CHIZIQLI TENGLAMALAR

511-548 tenglamalarni yeching.

511. $y'' + y' - 2y = 0$

512. $y'' + 4y' + 3y = 0$

513. $y'' - 2y' = 0$

514. $2y'' - 5y' + 2y = 0$

515. $y'' - 4y' + 5y = 0$

516. $y'' + 2y' + 10y = 0$

517. $y'' + 4y = 0$

518. $y''' - 8y = 0$

519. $y^{IV} - y = 0$

520. $y^{IV} + 4y = 0$

521. $y^{VI} + 64y = 0$

522. $y'' - 2y' + y = 0$

523. $4y'' + 4y' + y = 0$

524. $y^{V} - 6y^{IV} + 9y''' = 0$

525. $y^{V} - 10y''' + 9' = 0$

526. $y^{IV} + 2y'' + y = 0$

527. $y''' - 3y'' + 3y' - y = 0$

528. $y''' - y'' - y' + y = 0$

529. $y^{IV} - 5y'' + 4y = 0$

530. $y^{V} + 8y''' + 16y' = 0$

531. $y''' - 3y' + 2y = 0$

532. $y^{IV} + 4y'' + 3y = 0$

533. $y'' - 2y' - 3y = e^{4x}$

534. $y'' + y = 4xe^{x}$

535. $y'' - y = 2e^{x} - x^2$

536. $y'' + y' - 2y = 3xe^{x}$

537. $y'' - 3y' = 2y = \sin x$

538. $y'' + y = 4\sin x$

539. $y'' - 5y' + 4y = 4x^2e^{2x}$

540. $y'' - 3y' + 2y = x\cos x$

541. $y'' + 3y' - 4y = e^{-4x} + xe^{-x}$

542. $y'' + 2y' - 3y = x^2e^{x}$

543. $y'' - 4y' + 8y = e^{2x} + \sin 2x$

544. $y'' - 9y = e^{3x}\cos x$

545. $y'' - 2y' + y = 6xe^{x}$

546. $y'' + y = x\sin x$

547. $y'' + 4y' + 4y = xe^{2x}$

548. $y'' - 5y' = 3x^2 + \sin 5x$

549-574 masalalarda berilgan tenglamalarning hususiy yechimi noma'lum koeffitsientlar usulida qanday ko'rinishda izlanishini yozing (koeffitsientlarning qiymatlarini topish talab qilinmaydi).

549. $y'' - 2y' + 2y = e^x + x\cos x$

550. $y'' + 6y' + 10y = 3xe^{-3x} - 2e^{3x}\cos x$

551. $y'' - 8y' + 20y = 5xe^{4x}\sin 2x$

552. $y'' + 7y' + 10y = xe^{-2x}\cos 5x$

553. $y'' - 2y' + 5y = 2xe^x + e^x\sin 2x$

554. $y'' - 2y' + y = 2xe^x + e^x\sin 2x$

555. $y'' - 8y' + 17y = e^{4x}(x^2 - 3x\sin x)$

556. $y''' + y' = \sin x + x\cos x$

557. $y''' - 2y'' + 4y' - 8y = e^{2x}\sin 2x + 2x^2$

558. $y'' - 6y' + 8y = 5xe^{2x} + 2e^{4x}\sin x$

559. $y'' + 2y' + y = x(e^{-x} - \cos x)$

560. $y''' - y'' - y' + y = 3e^x + 5x\sin x$

561. $y'' - 6y' + 13y = x^2e^{3x} - 3\cos 2x$

562. $y'' - 9y = e^{3x}(x^2 + \sin 3x)$

563. $y^{IV} + y'' = 7x - 3\cos x$

564. $y'' + 4y = \cos x\cos 3x$

565. $y''' - 4y'' + 3y' = x^2 + xe^{2x}$

566. $y'' - 4y' + 5y = e^{2x}\sin^2 x$

567. $y'' + 3y' + 2y = e^{-x}\cos^2 x$

568. $y'' - 2y' + 2y = (x + e^x)\sin x$

569. $y^{IV} + 5y'' + 4y = \sin x\cos 2x$

570. $y'' - 3y' + 2y = 2^x$

571. $y'' - y = 4\,\mathrm{sh}\,x$

572. $y'' + 4y' + 3y = \mathrm{ch}\,x$

573. $y'' + 4y = \mathrm{sh}\,x \cdot \sin 2x$

574. $y'' + 2y' + 2y = \mathrm{ch}\,x \cdot \sin x$

575-581 tenglamalarni o'zgarmasni variatsiyalash usulida yeching

575. $y'' - 2y' + y = \frac{e^x}{x}$

576. $y'' + 3y' + 2y = \frac{1}{e^x+1}$

577. $y'' + y = \frac{1}{\sin x}$

578. $y'' + 4y = 2\,\mathrm{tg}\,x$

579. $y'' + 2y' + y = 3e^{-x}\sqrt{x+1}$ **580.** $y'' + y = 2\sec^3 x$

581. $x^3(y'' - y) = x^2 - 2$

582-588 tenglamalarning ko'rsatilgan boshlang'ich shartni qanoatlantiruvchi yechimini toping.

582. $y'' - 2y' + y = 0;\ y(2) = 1, y'(2) = -2$

583. $y'' + y = 4e^x; y(0) = 4, y'(0) = -3$

584. $y'' - 2y' = 2e^x; y(1) = -1, y'(1) = 0$

585. $y'' + 2y' + 2y = xe^{-x}; y(0) = y'(0) = 0$

586. $y''' - y' = 0; y(0) = 3, y'(0) = -1, y''(0) = 1$

587. $y''' - 3y' - 2y = 9e^{2x};\ y(0) = 0, y'(0) = -3, y''(0) = 3$

588. $y^{IV} + y'' = 2\cos x; y(0) = -2, y'(0) = 1, y''(0) = y'''(0) = 0$

589-600 Eyler tenglamalarini yeching

589. $x^2y'' - 4xy' + 6y = 0$ **590.** $x^2y'' - xy' - 3y = 0$

591. $x^3y''' + xy' - y = 0$ **592.** $x^2y''' = 2y'$

593. $x^2y'' - xy' + y = 8x^3$ **594.** $x^2y'' + xy' + 4y = 10x$

595. $x^3y'' - 2xy = 6\ln x$ **596.** $x^2y'' - 3xy' + 5y = 3x^2$

597. $x^2y'' - 6y = 5x^3 + 8x^2$ **598.** $x^2y'' - 2y = \sin\ln x$

599. $(x-2)^2y'' - 3(x-2)y' + 4y = x$

600. $(2x+3)^3y''' + 3(2x+3)y' - 6y = 0$

601-611 tenglamalarni yechishda turli usullardan foydalaning

601. $y'' + 2y' + y = \cos ix$ **602.** $y'' - 2y' + y = xe^x\sin^2 ix$

603. $y'' + 2iy = 8e^x\sin x$ **604.** $y'' + 2iy' - y = 8\cos x$

605. $y''' - 8iy = \cos 2x$ **606.** $y'' - \frac{2y}{x^2} = 3\ln(-x)$

607. $y'' + 2y' + y = xe^x + \frac{1}{xe^x}$

608. $y'' + 2y' + 5y = e^{-x}(\cos^2 x + \operatorname{tg} x)$

609. $x^2y'' - 2y = \frac{3x^2}{x+1}$ **610.** $x^2y'' - xy' + y = \frac{\ln x}{x} + \frac{x}{\ln x}$

611. $y'' + y = f(x)$

612. $f(x)$ funksiyaga qanday shart qo'yilganda **611-**misoldagi tenglamaning barcha yechimlari $x \to +\infty$ da chegaralangan bo'ladi?

613-618 masalalarda berilgan hususiy yechimga ega bo'lgan o'zgarmas koeffisientli chiziqli bir jinsli (mumkin bo'lgan eng kichik tartibli) differensial tenglamani quring.

613. $y_1 = x^2 e^x$ **614.** $y_1 = e^{2x} \cos x$

615. $y_1 = x \sin x$ **616.** $y_1 = xe^x \cos 2x$

617. $y_1 = xe^x,\ y_2 = e^{-x}$ **618.** $y_1 = x,\ y_2 = \sin x$

619. a va b ning qanday qiymatlarida $y'' + ay' + by = 0$ tenglamaning barcha yechimlari $-\infty < x < +\infty$ butun son o'qida chegralangan bo'ladi?

620. a va b ning qanday qiymatlarida $y'' + ay' + by = 0$ tenglamaning barcha yechimlari $x \to +\infty$ da nolga intiladi?

621. a va b ning qanday qiymatlarida $y'' + ay' + by = 0$ tenglama hech bo'lmaganda bitta $x \to +\infty$ da nolga intiluvchi yechimga ega bo'ladi?

622. a va b ning qanday qiymatlarida $y'' + ay' + by = 0$ tenglamaning$y \equiv 0$ yechimidan boshqa barcha yechimlarining absolyut qiymati biror x dan boshlab monoton osuvchi bo'ladi?

623. a va b ning qanday qiymatlarida $y'' + ay' + by = 0$ tenglamaning har bir yechimi cheksiz ko'p x larda nolga aylnadi?

624. a va b ning qanday qiymatlarida $y'' + ay' + by = 0$ tenglamaning barcha yechimlari $x \to +\infty$ da $y = o(e^{-x})$ munosabatni qano-atlantiradi?

625. Berilgan $b > 0$ uchun shunday a ni tanlangki $y'' + ay' + by = 0$ tenglamaning $y(0) = 1,\ y'(0) = 0$ boshlang'ich shartni qanoatlantiruvchi yechimi $x \to +\infty$ da nolga juda tez intilsin.

626. k va ω ning qanday qiymatlarida $y'' + k^2 y = \sin \omega t$ tenglama hech bo'lmaganda bitta davriy yechimga ega bo'ladi?

627. $\ddot{x} + a\dot{x} + bx = \sin \omega t$ tenglamaning davriy yechimini toping va uning amplitudasini ω miqdorga bog'liqligi grafigini chizing.

628. $\ddot{x} + \dot{x} + 4x = e^{i\omega t}$ tenglamaning davriy yechimini toping va kompleks tekislikda bu yechimdagi amplituda ko'paytmasining $\omega \in [0; +\infty)$ dagi ozgarishini tasvirlang.

629. $y'' + ay' + by = f(x)$ tenglama berilgan, bunda $|f(x)| \le m$ $(-\infty < x < \infty)$, tenglamaning harakteristik sonlari $\lambda_2 < \lambda_1 < 0$. Tenglamaning $-\infty < x < \infty$ da chegaralangan yechimini toping. a) tenglamaning qolgan barcha yechimlari chegaralanmaganligini va $x \to \infty$ da bu yechimga yaqinlashishini ko'rsating; b) agar $f(x)$ davriy bo'lsa, u holda bu yechim ham avriy bo'lishini ko'rsating.

Ko'rsatma. O'zgarmasni variatsiyalash usulini qo'llang. Hosil bo'ladigan integrallarning quyi chegarasini shunday tanlash kerakki, natijada integrallar yaqinlashuvchi bo'lsin.

630-632 masalalarda yukning muvozanat holatdan x masofaga chetga chiqishi natijasida unga muvozanat holati tomon yo'nalgan kx kuch ta'sir qiladi deb hisoblansin.

630. Prujinaga osilgan m massali yukning erkin tebranishlari davrini toping, bunda uning harakatiga hech nima qarshilik qilmaydi deb hisoblang.

631. Prujinaning bir uchi mahkamlangan, ikkinchi uchiga m massali yuk osilgan. Yuk v tezlik bilan harakatlanganda qarshilik kuchi hv ga teng bo'ladi. $t = 0$ momentda muvozanat holatda turgan yukka v_0 tezlik berildi. $h^2 < 4\,km$ va $h^2 > 4\,km$ bo'lgan holatlarda yuk harakatini o'rganing.

632. Oldingi masalada shunday qo'shimcha shart berilgan: yukka $f = b\sin\omega t$ davriy tashqi kuch ta'sir qiladi. Ihtiyoriy boshlang'ich shartlarda yukning harakati davriylikka intilashini ko'rsating va bu davriy harakatni (majburiy tebranish) toping.

633. Elastik strerjenning bir uchiga m massa mahkamlangan. Sterjenning ikkinchi uchi shunday tebranadiki, uning qo'zg'alishi vaqtning t momentida $B\sin\omega t$ ga teng. Sterjenda yuzaga keladigan elastiklik kuchi uning chetlari qo'zg'alishining ayirmasiga proporsional. m massa majburiy tebranishining A amplitudasini toping. $A > B$ bo'lishi mumkinmi? (Sterjen massasi va ishqalanishi hisobga olinmasin.)

634. m massali zarra Ox o'qi bo'yicha harakatlanmoqda, bunda $x = 0$ nuqtadan $3mr_0$ ga teng kuch bilan itarilmoqda va $x = 1$ nuqtaga $4mr_1$ ga teng kuch bilan tortilmoqda, bu yerda r_0 va r_1 – zarradandan shu nuqtalargacha masofalar. Zarraning

$$x(0) = 2, \qquad \dot{x}(0) = 0$$

boshlang'ich shartni qanoatlantiruvchi harakatini aniqlang.

635. O'zgarmas to'k manbasiga V kuchlanish, R qarshilik, L samoinduksiya beruvchi moslamalar ketma-ket ulanib elektr zanjiri hosil qilingan va ulagich $t = 0$ momentda ulanadi. Tok kuchini ($t > 0$ da) vaqtga bog'lanishini toping.

636. Oldingi masalada L samoinduksiyani C sigi'imli kondensatorga almashtirib yeching. Zanjir ulanmagunga qadar kondensator zaryadlanmaydi.

637. R qarshilik va C sig'imli kondensator ketma-ket ulangan bo'lib, $t = 0$ momentda zaryad q ga teng. $t = 0$ da zanjir ulanadi. $t > 0$ da zanjirdagi tok kuchini toping.

638. L samoinduksiya, R qarshilik va C sig'imli kondensator ketma-ket ulangan bo'lib, $t = 0$ momentda zaryad q ga teng. Zanjirdagi tok oqimi tebranma harakterga ega bo'lgan holatda, tok kuchini va tebranish chastotasini toping.

639. Tok manbaiga $E = V\sin\omega t$ qonun bo'yicha o'zgaruvchi kuchlanish, R qarshilik va L samoinduksiya ketma-ket ulangan. Zanjirdagi tok kuchini toping(barqaror holat).

640. Tok manbaiga $E = V\sin\omega t$ qonun bo'yicha o'zgaruvchi kuchlanish, R qarshilik, L samoinduksiya va C sig'imli kondensator ketma-ket ulangan. Zanjirdagi tok kuchini toping (barqaror holat). ω chastotaning qanday qiymatida tok kuchi eng katta bo'ladi?

12-§. O'ZGARUVCHI KOEFFITSIENTLI CHIZIQLI TNGLAMALAR

641-662 masalalarda berilgan funksiyalarni chiziqli erkli yoki bog'liqligini tekshiring. Har bir masalada funksiyalar shuday sohada qaraladiki, ularning barchasi bu sohada aniqlangan bo'ladi.

641. $x+2,\ x-2.$

642. $6x+9,\ 8x+12$

643. $\sin x, \cos x.$

644. $1,\ x,\ x^2$

645. $4-x,\ 2x+3,\ 6x+8$

646. $x^2+2,\ 3x^2-1,\ x+4$

647. $x^2-x+3,\ 2x^2+x,\ 2x-4$

648. $e^x,\ e^{2x},\ e^{3x}$

649. $x,\ e^x,\ xe^x$

650. $1,\ \sin^2 x,\ \cos 2x$

651. $\operatorname{sh} x,\ \operatorname{ch} x,\ 2+e^x$

652. $\ln(x^2),\ \ln 3x,\ 7$

653. $x,\ 0,\ e^x$

654. $\operatorname{sh} x,\ \operatorname{ch} x,\ 2e^x-1,\ 3e^x+5$

655. $2^x,\ 3^x,\ 6^x$

656. $\sin x,\ \cos x,\ \sin 2x$

657. $\sin x,\ \sin(x+2),\ \cos(x-5)$

658. $\sqrt{x},\ \sqrt{x+1},\ \sqrt{x+2}$

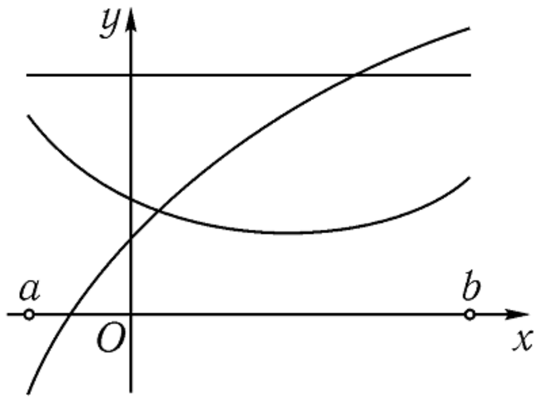
659. $\operatorname{arctg} x,\ \operatorname{arcctg} x,\ 1$

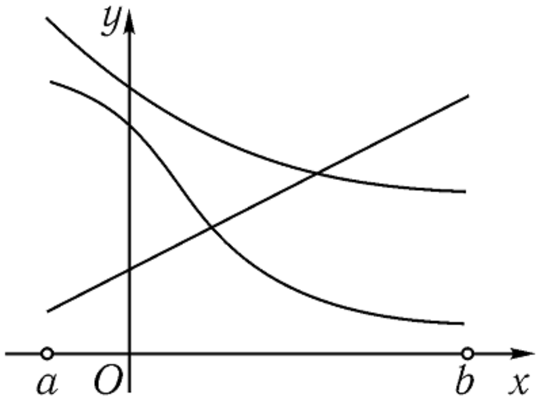
660. $x^2,\ x|x|$

661. $x,\ |x|,\ 2x+\sqrt{4x^2}$

662. $x,\ x^3,\ |x^3|$

663. a) Grafiklari 1-rasmda tasvirlangan funksiyalar $[a,\ b]$ kesmada chiziqli erklimi? b) 2-rasmda tasvirlangan funksiyalarchi?





664. $y_1, y_2, ..., y_n$ funksiyalarning Vronskiy determinanti x_0 nuqtada nolga teng, x_1 nuqtada esa nolga teng emas. Bu funksiyalarning $[x_0, x_1]$ kesmada chiziqli bog'liqligi (yoki erkliligi) haqida biror tasdiq aytish mumkinmi?

665. Barcha x larda $y_1, \ \dots, y_n$ funksiyalarning Vronskiy determinant nolga teng. Bu funksiyalar chiziqli bog'liq bo'lishi mumkinmi? Chiziqli erklichi?

666. Agar $y_1, ..., y_n$ funksiyalar a) chiziqli bog'liq b) chiziqli erkli bo'lsa, u holda bu funksiyalarning Vronskiy determinant haqida nima deyish mumkin?

667. $y_1 = x, \ y_2 = x^5, \ y_3 = |x^5|$ funksiyalar $x^2 y'' - 5xy' + 5y = 0$ tenglamani qanoatlantiradi. Ular $(-1, 1)$ intervalda chiziqli bog'liqmi? Javobingizni izohlang.

668. Bizga $y'' + p(x)y' + q(x)y = 0$ (uzluksiz koeffitsientli) tenglama berilgan. Bu tenglamaning ikkita yechimi x ning bita qiymati ustida maksimumga erishsa, u holda bu yechimlar chiziqli bog'liq bo'lishini isbotlang.

669. $y''' + xy = 0$ tenglamaning 4 ta yechimi berilgan va ularning grafigi bitta nuqtada bir-biiga urinadi. Bu yechimlar orasida nechtasi chiziqli erkli?

670. Tenglamalarni yechmasdan qanday intervalda ko'rsatilgan boshlang'ich shartlarni qanoatlantiruvchi yechim mavjud bo'lishini aniqlang, bunda chiziqli tenglama uchun yechim mavjudlik intervali haqidagi tasdiqdan foydalang ([1] adabiyot, V bob 1-§):

a) $(x+1)y'' - 2y = 0, \ y(0) = 0, \ y'(0) = 2;$

b) $y'' + y \operatorname{tg} x = 0, \ y(5) = 1, \ y'(5) = 0.$

671. Koeffitsientlari uzluksiz bo'lgan $y^{(n)} + p_1(x) y^{(n-1)} + \ ... + p_n(x) y = 0$ tenglamaning ikki yechimining grafiklari $x, \ y$ tekislikda a) kesishishi b) bir-biriga urinishi mumkinmi?

672. n ning qanday qiymatlarida **671**-masaladagi tenglama $y = x^3$ hususiy yechimga ega bo'lishi mumkin?

673. Qanday tartibli chiziqli bir jinsli tenglama $(0,1)$ intervalda quyidagi to'rtta hususiy yechimga ega bo'lishi mumkin:

$$y_1 = x^2 - 2x + 2,\ \ y_2 = (x-2)^2,\ \ y_3 = x^2 + x - 1,\ \ y_4 = 1 - x\,?$$

674 – 680 masalalarning har birida berilgan hususiy yechimlarga ega bo'lgan (mumkin qadar eng kichik tartibli) chiziqli bir jinsli differensial tenglamani tuzing.

674. $1,\ \cos x.$

675. $x,\ e^x.$

676. $3x,\ x-2,\ e^x+1.$

677. $x^2-3x,\ 2x^2+9,\ 2x+3.$

678. $x^x, \operatorname{sh} x, \operatorname{ch} x.$

679. $x,\ x^2,\ e^x.$

680. $x,\ x^3,\ |x^3|.$

681 – 701 masalalarda berilgan tenglamaning hususiy yechimidan foydalanib ularning umumiy yechimini toping. Agar masalada hususiy yechim berilmagan bo'lsa, uni tanlash usulida qidiring. Masalan hususiy yechimni $y_1 = e^{ax}$ ko'rsatkichli funksiya ko'rinishida yoki $y_1 = x^n + ax^{n-1} + bx^{n-1} + \ ...$ algebraik ko'phad ko'rinishida qidirish mumkin.

681. $(2x+1)y'' + 4xy' - 4y = 0.$

682. $x^2(x+1)y'' - 2y = 0;\ \ \ y_1 = 1 + \frac{1}{x}.$

683. $xy'' - (2x+1)y' + (x+1)y = 0.$

684. $xy'' + 2y' - xy = 0;\ \ \ y_1 = \frac{e^x}{x}.$

685. $y'' - 2(1+\operatorname{tg} x)y = 0;\ \ \ y_1 = \operatorname{tg} x.$

686. $x(x-1)y'' - xy' + y = 0.$

687. $(e^x+1)y'' - 2y' - e^x y = 0;\ \ \ y_1 = e^x - 1.$

688. $x^2 y'' \ln x - xy' + y = 0.$

689. $y'' - y' \operatorname{tg} x + 2y = 0;\ \ \ y_1 = \sin x.$

690. $(x^2-1)y'' + (x-3)y' - y = 0.$

691. $y'' - (x+1)y' - 2(x-1)y = 0.$

692. $y'' + 4xy' + (4x^2+2)y = 0;\ \ \ y_1 = e^{ax^2}.$

693. $xy'' - (2x+1)y' + 2y = 0.$

694. $x(2x+1)y'' + 2(x+1)y' - 2y = 0.$

695. $x(x+4)y'' - (2x+4)y' + 2y = 0.$

696. $x(x^2+6)y'' - 4(x^2+3)y' + 6xy = 0.$

697. $(x^2+1)y'' - 2y = 0.$

698. $2x(x+2)y'' + (2-x)y' + y = 0.$

699. $xy''' - y'' - xy' + y = 0;\ \ y_1 = x,\ y_1 = e^{ax^2}.$

700. $x^2(2x-1)y''' + (4x-3)xy'' - 2xy' + 2y = 0;\ \ y_1 = x,\ y_1 = \frac{1}{x}.$

701. $(x^2-2x+3)y''' - (x^2+1)y'' + 2xy' - 2y = 0;\ \ y_1 = x,\ y_1 = e^x.$

702, 703 masalalarda bir jinsli bo'lmagan chiziqli differensial tenglamaning umumiy yechimini toping, bunda mos bir jinsli tenglamaning hususiy yechimi ko'phaddan iborat ekanligi ma'lum.

702. $(x+1)xy'' + (x+2)y' - y = x + \frac{1}{x}.$

702. $(2x+1)y'' + (2x-1)y' - 2y = x^2 + x.$

704, 705 masalalarda bir jinsli bo'lmagan chiziqli differensial tenglamaning ikkita hususiy yechimini bilgan holda uning umumiy yechimini toping.

704. $(x^2-1)y'' + 4xy' + 2y = 6x;\ \ y_1 = x,\ y_2 = \frac{x^2+x+1}{x+1}.$

705. $(3x^3+x)y'' + 2y' - 6xy = 4 - 12x^2;\ \ y_1 = 2x,\ y_2 = (x+1)^2.$

704, 705 tenglamalarda izlanayotgan funksiyani $y = a(x)z$ formula bilan almashtirib birinchi tartibli hosila qatnashgan hadni yo'qting.

706. $x^2y'' - 2xy' + (x^2+2)y = 0.$

707. $x^2y'' - 4xy' + (6-x^2)y = 0.$

708. $(1+x^2)y'' + 4xy' + 2y = 0.$

709. $x^2y'' + 2x^2y' + (x^2-2)y = 0.$

710. $xy'' + y' + xy = 0.$

711 – 715 tenglamalarda erkli o'zgaruvchini $t = \varphi(x)$ formula bilan almashtirib birinchi tartibli hosila qatnashgan hadni yo'qting.

711. $xy'' - y' - 4x^3y = 0.$

712. $(1+x^2)y'' + xy' + y = 0.$

713. $x^2(1-x^2)y'' + 2(x-x^3)y' - 2y = 0.$

714. $y'' - y' + e^{4x}y = 0.$

715. $2xy'' + y' + xy = 0.$

716. Ikkinchi tartibli chiziqli bir jinsli bo'lmagan tenglamaning uchta $y_1 = 1$, $y_2 = x,\ y_3 = x^2$ hususiy yechimini bilgan holda uning umumiy yechimini yozing.

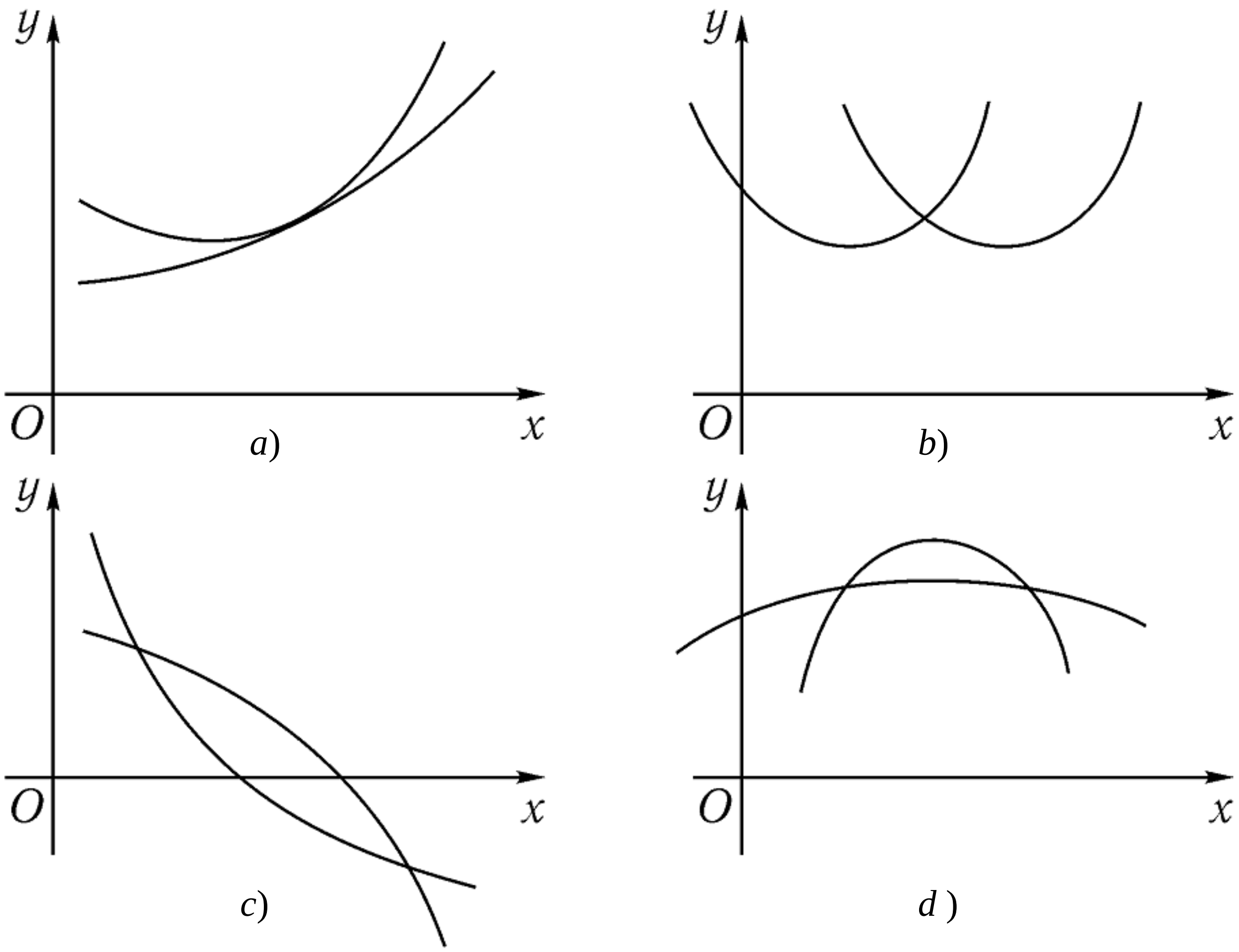
717. Agar $y'' + p(x)y' + q(x)y = 0$ tenglamaning barcha yechimlari $x \to \infty$ da birinchi tartibli hosilasi bilan birgalikda nolga intilishi ma'lum bo'lsa, $p(x)$ funksiya haqida nima aytish mumkin?

Ko'rsatma. Liuvill formulasidan foydalaning.

718. $q(x) < 0$ bo'lgan holatda $y'' + p(x)y' + q(x)y = 0$ tenglamaning yechimlari musbat maksimumga ega bo'la olmasligini isbotlang.

719. $y'' + q(x)y = 0$ tenglama yechimlarining egilish nuqtalari qayerda yotishi mumkin?

720. $y'' + q(x)y = 0$ tenglama ($q(x)$ funksiya uzluksiz) ikkita yechimining grafiklari 3,*a* rasmdagidek, 3,*b* rasmdagidek, 3,*c* rasmdagidek, 3,*d* rasmdagidek joylashishi mumkinmi?



3-rasm

721. $y'' + p(x)y' + q(x)y = 0$ tenglamaning (uzluksiz koeffitsientli) ihtiyoriy ikkita yechimining nisbati lokal maksimum nuqtaga ega bo'la olmasligini isbotlang.

722. $q(x) > 0$ bo'lgan holatda $y'' + q(x)y = 0$ tenglamaning ihtiyoriy $y(x)$ yechimi uchun $\frac{y'(x)}{y(x)}$ nisbat kamayuvchi bo'lishini isbotlang, bu yerda $y(x) \neq 0$.

723. $q(x) \leq 0$ bo'lgan holatda $y'' + q(x)y = 0$ tenglamaning $y(x_0) > 0$, $y'(x_0) > 0$ musbat boshlang'ich shartli barcha yechimlari barcha $x > x_0$ nuqtalarda musbatligicha bo'lib qolishini isbotlang.

724. $y'' - x^2y = 0$ tenglamaning $y(0) = 1$, $y'(0) = 0$ boshlang'ich shartli yechimi deyarli musbat hamda juft funksiya bo'lishini isbotlang.

725. $q(x) \leq 0$ bo'lgan holatda

$$y'' + q(x)y = 0, \qquad y(x_1) = a, \qquad y(x_2) = b$$

chegaraviy masala ihtyoriy a, b va $x_1 \neq x_2$ larda yagona yechimga ega bo'lishini isbotlang. Agar $b = 0$ bolsa, bu yechim monoton funksiya bo'lishini isbotlang.

726. $y'' + my = 0$ tenglamaning ihtiyoriy notrivial yechimining ketma-ket kelgan ikkita noli orasidagi masofani toping, bu yerda $m = \text{const} > 0$. $a \leq x \leq b$ kesmada nechta nol joylashishi mumkin?

727 – **730** masalalarda tenglamaning ihtiyoriy notrivial yechimining berilgan kesmadagi ketma-ket kelgan nollari orasidagi masofani yuqoridan va quyidan baholang. Bunda avval kelgan masalalardagi natijalardan va taqqoslash teoremalaridan foydalaning.

727. $y'' + 2xy = 0,\ \ 20 \leq x \leq 45.$

728. $xy'' + y = 0,\ \ 25 \leq x \leq 100.$

729. $y'' - 2xy' + (x + 1)^2y = 0,\ \ 4 \leq x \leq 19.$

730. $y'' - 2e^xy' + e^{2x}y = 0,\ \ 2 \leq x \leq 6.$

731. $y'' + xy = 0$ tenglamaning ihtiyoriy yechimi $-25 \leq x \leq 25$ kesmada kamida 15 ta nolga ega bo'lishini isbotlang.

732. $y'' + q(x)y = 0$ tenglama yechimining ketma-ket kelgan $x_1, x_2, \ldots$ nollarini o'sish tartibida joylashtirib olaylik, bu yerda $q(x) > 0$; $x_1 \leq x < \infty$ da $q(x)$ funksiya uzluksiz va o'suvchi. Ketma-ket kelgan nollar orasidagi masofa kamayuvchi bo'lishini, ya'ni $x_{n+1} - x_n < x_n - x_{n-1}$ tengsizlikni isbotlang.

733. Oldingi masalada $\lim\limits_{x\to\infty} q(x) = c$ bo'lsin. $\lim\limits_{n\to\infty}(x_{n+1} - x_n) = \dfrac{\pi}{\sqrt{c}}$ limitni isbotlang.

734. Bizga $y'' + q(x)y = 0$ va $z'' + Q(x)z = 0$ tenglamalarning $y(x_0) = z(x_0)$, $y'(x_0) = z'(x_0)$ teng boshlang'ich shartli $y(x)$ va $z(x)$ yechimlari berilgan bo'lib, (x_0, x_1) intervalda $Q(x) > q(x)$, $y(x) > 0$, $z(x) > 0$ tengsizliklar o'rinli bo'lsin. Bu intervalda $\frac{z(x)}{y(x)}$ nisbat kamayuvchiligini isbotlang.

$$\max_{x_n \leq x \leq x_{n+1}} |y(x)| = b_n$$

735. **732**-masalada shart ham o’rinli bo’lsin. $b_1 > b_2 > b_3 >$ ... munosabatni isbotlang.

736. **733**-masalada c limit chekli bo’lsin. **735**-masaladagi belgilashlarni hisobga olib $n \to \infty$ da $b_n \to B > 0$ limitni isbotlang.

737. $\frac{d^2y}{dx^2} \pm \frac{y}{(\psi(x))^4} = 0$ tenglamada $t = \varphi(x)$ formula bilan erkli o’zgaruvchin almashtirib $\frac{d^2y}{dt^2} + b(t)\frac{dy}{dt} \pm y = 0$ ko’rinishdagi tenglamani hosil qiling, keyin $y = a(t)u$ formula bilan noma’lum funksiyani almashtirib birinchi tartibli hosila qatnashgan hadni yo’qoting. (Bu almashtirish Liuvill almashtirish deb ataladi. Ko’p hollarda bu usul bilan $y'' + q(x)y = 0$ tenglama analogik ko’rinishga ega “deyarli o’zgarmas koeffitsientli” ($t > t_0$larda kuchsiz o’zgaruvchi) tennglamaga keltiriladi. Boshqacha aytganda hosil bo’lgan $y'' + Q(t)y = 0$ tenglamaning $Q(t)$ ko’effisienti $t > t_0$larda kuchsiz o’zgaradi. Bu esa yechimning $x \to \infty$ da asimptotik harakterini tekshirish imkonini beradi).

738 – **748** masalalarda Liuvil almashtirishidan (**737**-masalaga qarang) va 4-punktdagi (77-bet) tasdiqdan foydalanib berilgan tenglama yechimlarining $x \to \infty$ da asimptotik harakterini tekshiring.

738. $y'' + x^4y = 0.$

739. $y'' - x^2y = 0.$

740. $y'' + x^2y = 0.$

741. $y'' + e^{2x}y = 0.$

742. $xy'' - y = 0.$

743. $y'' - xy = 0.$

744. $xy'' + 2y' + y = 0.$

745. $y'' - 2(x-1)y' + x^2y = 0.$

746. $y'' + (x^4 + 1)y = 0.$

747. $(x^2 + 1)y'' - y = 0.$

748. $x^2y'' + y\ln^2 x = 0.$

749 – **750** masalalarda Liuvil almashtirishini ikki marta qo’llab berilgan tenglama yechimlarining yanada aniqroq asimptotik ifodalanishini hosil qiling.

749. $y'' - 4x^2y = 0.$

750. $xy'' + y = 0.$

13-§. CHEGARAVIY MASALALAR

751 – **762** tenglamalarning ko’rsatilgan chegaraviy shartni qanoatlantiruvchi yechimini toping.

751. $y'' - y = 2x;\ \ y(0) = 0,\ y(1) = -1.$

752. $y'' + y' = 1;\ \ y'(0) = 0,\ y(1) = 1.$

753. $y'' - y' = 0;\ \ y(0) = -1,\ y'(1) - y(1) = 2.$

754. $y'' + y = 1;\ \ y(0) = 0,\ y\left(\frac{\pi}{2}\right) = 0.$

755. $y'' + y = 1;\ \ y(0) = 0,\ y(\pi) = 0.$

756. $y'' + y = 2x - \pi;\ \ y(0) = 0,\ y(\pi) = 0.$

757. $y'' - y' - 2y = 0;\ \ y'(0) = 2,\ y(+\infty) = 0.$

758. $y'' - y = 1;\ \ y(0) = 0,\ x \to \infty$ da $y(x)$ chegaralangan.

759. $y'' - 2iy = 0;\ \ y(0) = -1,\ y(+\infty) = 0.$

760. $x^2y'' - 6y = 0;\ \ y(1) = 2,\ y(0)$ chegaralangan.

761. $x^2y'' - 2xy' + 2y = 0;\ \ y(1) = 3,\ x \to 0$ da $y(x) = o(x)$.

762. $x^2y'' + 5xy' + 3y = 0;\ \ y'(1) = 3,\ x \to +\infty$ da $y(x) = O(x^{-2})$.

763. a ning qanday qiymatlarida $y'' + ay = 1;\ \ y(0) = 0,\ y(1) = 0$ chegaraviy masala yechimga ega bo'lmaydi?

764 – 779 chegarviy masalalarning har biri uchun Grin funksiyasini quring.

764. $y'' = f(x);\ \ y(0) = 0,\ y(1) = 0.$

765. $y'' + y = f(x);\ \ y'(0) = 0,\ y(\pi) = 0.$

766. $y'' + y' = f(x);\ \ y(0) = 0,\ y'(1) = 0.$

767. $y'' - y = f(x);\ \ y'(0) = 0,\ y'(2) + y(2) = 0.$

768. $y'' + y = f(x);\ \ y(0) = y(\pi),\ y'(0) = y'(\pi).$

769. $x^2y'' + 2xy' = f(x);\ \ y(1) = 0,\ y'(3) = 0.$

770. $xy'' - y' = f(x);\ \ y'(1) = 0,\ y(2) = 0.$

771. $x^2y'' - 2y = f(x);\ \ y(1) = 0,\ y(2) + 2y'(2) = 0.$

772. $y'' = f(x);\ \ y(0) = 0,\ x \to \infty$ da $y(x)$ chegaralangan.

773. $y'' + y' = f(x);\ \ y'(0) = 0,\ y(+\infty) = 0.$

774. $xy'' + y' = f(x);\ \ y(1) = 0,\ x \to \infty$ da $y(x)$ chegaralangan.

775. $y'' + 4y' + 3y = f(x);\ \ y(0) = 0,\ x \to \infty$ da $y(x) = O(e^{-2x})$.

776. $x^2y'' + xy' - y = f(x);\ \ y(1) = 0,\ x \to \infty$ da $y(x)$ chegaralangan.

777. $x^2y'' + 2xy' - 2y = f(x);\ \ y(1) = 0,\ y(0)$ chegaralangan.

778. $y'' - y = f(x),\ x \to \pm\infty$ da $y(x)$ chegaralangan.

779. $x^2y'' - 2y = f(x),\ x \to 0$ da va $x \to +\infty$ da $y(x)$ chegaralangan.

780. a ning qanday qiymatlarida $y'' + ay = f(x);\ y(0) = 0,\ y(1) = 0$ chegaraviy masalaning Grin funksiyasi mavjud?

781. $x^2y'' + 2xy' - 2y = f(x),\ x \to 0$ da va $x \to +\infty$ da $y(x)$ chegaralangan. Ushbu chegaraviy masala yechimini va uning hosilasini yuqoridan va quyidan baholang, bu yerda $0 \le f(x) \le m$.

Ko'rsatma. Yechimni Grin funksiyasi yordamida yozib oling.

782 – 785 masalalarda hos qiymatlarni va hos funksiyalarni toping.

782. $y'' = \lambda y;\ \ y(0) = 0,\ y(l) = 0.$

783. $y'' = \lambda y;\ \ y'(0) = 0,\ y'(l) = 0.$

784. $y'' = \lambda y;\ \ y(0) = 0,\ y'(l) = 0.$

785. $x^2y'' = \lambda y;\ \ y(1) = 0,\ y(a) = 0\ \ (a > 1).$

14-§. O'ZGARMAS KOEFFITSIENTLI CHIZIQLI SISTEMALAR

786 – 812 masalalarda berilgan tenglamalar sistemasini yeching ($\dot{x}$ orqali $\frac{dx}{dt}$ belgilangan va h.k. Ishni soddalashtirish uchun ba'zi masalalarda harakteristik tenglamaning ildizlari ko'rsatilgan).

786. $\begin{cases}\dot{x} = 2x + y,\\ \dot{y} = 3x + 4y.\end{cases}$

787. $\begin{cases}\dot{x} = x - y,\\ \dot{y} = y - 4x.\end{cases}$

788. $\begin{cases}\dot{x} + x - 8y = 0,\\ \dot{y} - x - y = 0.\end{cases}$

789. $\begin{cases}\dot{x} = x + y,\\ \dot{y} = 3y - 2x.\end{cases}$

790. $\begin{cases}\dot{x} = x - 3y,\\ \dot{y} = 3x + y.\end{cases}$

791. $\begin{cases}\dot{x} + x + 5y = 0,\\ \dot{y} - x - y = 0.\end{cases}$

792. $\begin{cases}\dot{x} = 2x + y,\\ \dot{y} = 4y - x.\end{cases}$

793. $\begin{cases}\dot{x} = 3x - y,\\ \dot{y} = 4x - y.\end{cases}$

794. $\begin{cases}\dot{x} = 2y - 3x,\\ \dot{y} = y - 2x.\end{cases}$

795. $\begin{cases}\dot{x} - 5x - 3y = 0,\\ \dot{y} + 3x + y = 0.\end{cases}$

796. $\begin{cases}\dot{x} = x + z - y,\\ \dot{y} = x + y - z,\\ \dot{z} = 2x - y.\end{cases}$

$(\lambda_1 = 1,\ \ \lambda_2 = 2,\ \ \lambda_3 = -1).$

797. $\begin{cases}\dot{x} = x - 2y - z,\\ \dot{y} = y - x + z,\\ \dot{z} = x - z.\end{cases}$

$(\lambda_1 = 0,\ \ \lambda_2 = 2,\ \ \lambda_3 = -1).$

798. $\begin{cases} \dot{x} = 2x - y + z, \\ \dot{y} = x + 2y - z, \\ \dot{z} = x - y + 2z. \end{cases}$

$(\lambda_1 = 1,\ \lambda_2 = 2,\ \lambda_3 = 3).$

799. $\begin{cases} \dot{x} = 3x - y + z, \\ \dot{y} = x + y + z, \\ \dot{z} = 4x - y + 4z. \end{cases}$

$(\lambda_1 = 1,\ \lambda_2 = 2,\ \lambda_3 = 5).$

800. $\begin{cases} \dot{x} = 4y - 2z - 3x, \\ \dot{y} = z + x, \\ \dot{z} = 6x - 6y + 5z. \end{cases}$

$(\lambda_1 = 1,\ \lambda_2 = 2,\ \lambda_3 = -1).$

801. $\begin{cases} \dot{x} = x - y - z, \\ \dot{y} = x + y, \\ \dot{z} = 3x + z. \end{cases}$

$(\lambda_1 = 1,\ \lambda_{2,3} = 1 \pm 2i).$

802. $\begin{cases} \dot{x} = 2x + y, \\ \dot{y} = x + 3y - z, \\ \dot{z} = 2y + 3z - x. \end{cases}$

$(\lambda_1 = 2,\ \lambda_{2,3} = 3 \pm i).$

803. $\begin{cases} \dot{x} = 2x + 2z - y, \\ \dot{y} = x + 2z, \\ \dot{z} = y - 2x - z. \end{cases}$

$(\lambda_1 = 1,\ \lambda_{2,3} = \pm i).$

804. $\begin{cases} \dot{x} = 4x - y - z, \\ \dot{y} = x + 2y - z, \\ \dot{z} = x - y + 2z. \end{cases}$

$(\lambda_1 = 2,\ \lambda_2 = \lambda_3 = 3).$

805. $\begin{cases} \dot{x} = 2x - y - z, \\ \dot{y} = 3x - 2y - 3z, \\ \dot{z} = 2z - x + y. \end{cases}$

$(\lambda_1 = 0,\ \lambda_2 = \lambda_3 = 1).$

806. $\begin{cases} \dot{x} = y - 2x - 2z, \\ \dot{y} = x - 2y + 2z, \\ \dot{z} = 3x - 3y + 5z. \end{cases}$

$(\lambda_1 = 3,\ \lambda_2 = \lambda_3 = -1).$

807. $\begin{cases} \dot{x} = 3x - 2y - z, \\ \dot{y} = 3x - 4y - 3z, \\ \dot{z} = 2x - 4y. \end{cases}$

$(\lambda_1 = -5,\ \lambda_2 = \lambda_3 = 2).$

808. $\begin{cases} \dot{x} = x - y + z, \\ \dot{y} = x + y - z, \\ \dot{z} = 2z - y. \end{cases}$

$(\lambda_1 = 2,\ \lambda_2 = \lambda_3 = 1).$

809. $\begin{cases} \dot{x} = y - 2z - x, \\ \dot{y} = 4x + y, \\ \dot{z} = 2x + y - z. \end{cases}$

$(\lambda_1 = 1,\ \lambda_2 = \lambda_3 = -1).$

810. $\begin{cases} \dot{x} = 2x + y, \\ \dot{y} = 2y + 4z, \\ \dot{z} = x - z. \end{cases}$

$(\lambda_1 = 3,\ \lambda_2 = \lambda_3 = 0).$

811. $\begin{cases} \dot{x} = 2x - y - z, \\ \dot{y} = 2x - y - 2z, \\ \dot{z} = 2z - x + y. \end{cases}$

$(\lambda_1 = \lambda_2 = \lambda_3 = 1).$

812. $\begin{cases} \dot{x} = 4x - y, \\ \dot{y} = 3x + y - z, \\ \dot{z} = x + z. \end{cases}$ $(\lambda_1 = \lambda_2 = \lambda_3 = 2).$

813 – 825 masalalarda normal ko’rinishga keltirilmagan sistemalarni yeching.

813. $\begin{cases} \ddot{x} = 2x - 3y, \\ \ddot{y} = x - 2y. \end{cases}$

814. $\begin{cases} \ddot{x} = 3x + 4y, \\ \ddot{y} = -x - y. \end{cases}$

815. $\begin{cases} \ddot{x} = 2y, \\ \ddot{y} = -2x. \end{cases}$

816. $\begin{cases} \ddot{x} = 3x - y - z, \\ \ddot{y} = -x + 3y - z, \\ \ddot{z} = -x - y + 3z. \end{cases}$

817. $\begin{cases} \dot{x} - 5\dot{y} = 4y - x, \\ 3\dot{x} - 4\dot{y} = 2x - y. \end{cases}$

818. $\begin{cases} \ddot{x} + \dot{x} + \dot{y} - 2y = 0, \\ \dot{x} - \dot{y} + x = 0. \end{cases}$

819. $\begin{cases} \ddot{x} - 2\ddot{y} + \dot{y} + x - 3y = 0, \\ 4\ddot{y} - 2\ddot{x} - \dot{x} - 2x + 5y = 0. \end{cases}$

820. $\begin{cases} \ddot{x} - x + 2\ddot{y} - 2y = 0, \\ \dot{x} - x + \dot{y} + y = 0. \end{cases}$

821. $\begin{cases} \ddot{x} - 2\dot{y} + 2x = 0, \\ 3\dot{x} + \ddot{y} - 8y = 0. \end{cases}$

822. $\begin{cases} \ddot{x} + 3\ddot{y} - x = 0, \\ \dot{x} + 3\dot{y} - 2y = 0. \end{cases}$

823. $\begin{cases} \ddot{x} + 5\dot{x} + 2\dot{y} + y = 0, \\ 3\ddot{x} + 5x + \dot{y} + 3y = 0. \end{cases}$

824. $\begin{cases} \ddot{x} + 4\dot{x} - 2x - 2\dot{y} - y = 0, \\ \ddot{x} - 4\dot{x} - \ddot{y} + 2\dot{y} + 2y = 0. \end{cases}$

825. $\begin{cases} 2\ddot{x} + 2\dot{x} + x + 3\ddot{y} + \dot{y} + y = 0, \\ \ddot{x} + 4\dot{x} - x + 3\ddot{y} + 2\dot{y} - y = 0. \end{cases}$

826 – 845 masalalarda chiziqli bir jinsli bo'lmagan sistemalarni yeching.

826. $\begin{cases} \dot{x} = y + 2e^t, \\ \dot{y} = x + t^2. \end{cases}$

827. $\begin{cases} \dot{x} = y - 5\cos t, \\ \dot{y} = 2x + y. \end{cases}$

828. $\begin{cases} \dot{x} = 3x + 2y + 4e^{5t}, \\ \dot{y} = x + 2y. \end{cases}$

829. $\begin{cases} \dot{x} = 2x - 4y + 4e^{-2t}, \\ \dot{y} = 2x - 2y. \end{cases}$

830. $\begin{cases} \dot{x} = 4x + y - e^{2t}, \\ \dot{y} = y - 2x. \end{cases}$

831. $\begin{cases} \dot{x} = 2y - x + 1, \\ \dot{y} = 3y - 2x. \end{cases}$

832. $\begin{cases} \dot{x} = 5x - 3y + 2e^{3t}, \\ \dot{y} = x + y + 5e^{-t}. \end{cases}$

833. $\begin{cases} \dot{x} = 2x + y + e^t, \\ \dot{y} = -2x + 2t. \end{cases}$

834. $\begin{cases} \dot{x} = x + 2y, \\ \dot{y} = x - 5\sin t. \end{cases}$

835. $\begin{cases} \dot{x} = 2x - 4y, \\ \dot{y} = x - 3y + 3e^t. \end{cases}$

836. $\begin{cases} \dot{x} = 2x - y, \\ \dot{y} = y - 2x + 18t. \end{cases}$

837. $\begin{cases} \dot{x} = x + 2y + 16te^t, \\ \dot{y} = 2x - 2y. \end{cases}$

838. $\begin{cases} \dot{x} = 2x + 4y - 8, \\ \dot{y} = 3x + 6y. \end{cases}$

839. $\begin{cases} \dot{x} = 2x - 3y, \\ \dot{y} = x - 2y + 2\sin t. \end{cases}$

840. $\begin{cases} \dot{x} = x - y + 2\sin t, \\ \dot{y} = 2x - y. \end{cases}$

841. $\begin{cases} \dot{x} = 2x - y, \\ \dot{y} = x + 2e^t. \end{cases}$

842. $\begin{cases} \dot{x} = 4x - 3y + \sin t, \\ \dot{y} = 2x - y - 2\cos t. \end{cases}$

843. $\begin{cases} \dot{x} = 2x + y + 2e^t, \\ \dot{y} = x + 2y - 3e^{4t}. \end{cases}$

844. $\begin{cases}\dot{x} = x - y + 8t, \\ \dot{y} = 5x - y.\end{cases}$

845. $\begin{cases}\dot{x} = 2x - y, \\ \dot{y} = 2y - x - 5e^t \sin t\,.\end{cases}$

846 – 850 masalalarda berilgan sistemani o'zgarmasni variatsiyalash usulida yeching.

846. $\begin{cases}\dot{x} = y + \operatorname{tg}^2 t - 1, \\ \dot{y} = -x + \operatorname{tg} t.\end{cases}$

847. $\begin{cases}\dot{x} = 2y - x, \\ \dot{y} = 4y - 3x + \frac{e^{3t}}{e^{2t}+1}.\end{cases}$

848. $\begin{cases}\dot{x} = -4x - 2y + \frac{2}{e^t-1}, \\ \dot{y} = 6x + 3y - \frac{3}{e^t-1}.\end{cases}$

850. $\begin{cases}\dot{x} = 3x - 2y, \\ \dot{y} = 2x - y + 15e^t\sqrt{t}.\end{cases}$

Vektor ko'rinishda yozilgan **851 – 866** sistemalarni yeching: $\dot{x} = Ax$, bu yerda x – vector, A – berilgan matritsa.

851. $\dot{x} = Ax,\ \ A = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}.$

852. $\dot{x} = Ax,\ \ A = \begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix}.$

853. $\dot{x} = Ax,\ \ A = \begin{pmatrix} 1 & -2 \\ 2 & -3 \end{pmatrix}.$

854. $\dot{x} = Ax,\ \ A = \begin{pmatrix} 3 & -2 \\ 4 & -1 \end{pmatrix}.$

855. $\dot{x} = Ax,\ \ A = \begin{pmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \\ 3 & -1 & -2 \end{pmatrix}.$

856. $\dot{x} = Ax,\ \ A = \begin{pmatrix} 1 & -2 & 2 \\ 1 & 4 & -2 \\ 1 & 5 & -3 \end{pmatrix}.$

857. $\dot{x} = Ax,\ \ A = \begin{pmatrix} -1 & -2 & 2 \\ -2 & -1 & 2 \\ -3 & -2 & 3 \end{pmatrix}.$

858. $\dot{x} = Ax,\ \ A = \begin{pmatrix} -3 & 2 & 2 \\ -3 & -1 & 1 \\ -1 & 2 & 0 \end{pmatrix}.$

859. $\dot{x} = Ax,\ \ A = \begin{pmatrix} 3 & -3 & 1 \\ 3 & -2 & 2 \\ -1 & 2 & 0 \end{pmatrix}.$

860. $\dot{x} = Ax,\ \ A = \begin{pmatrix} 2 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}.$

861. $\dot{x} = Ax,\ \ A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 2 & 2 & 1 \end{pmatrix}.$

862. $\dot{x} = Ax,\ \ A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}.$

863. $\dot{x} = Ax,\ \ A = \begin{pmatrix} -2 & 1 & 2 \\ -1 & 0 & 2 \\ -2 & 0 & 3 \end{pmatrix}.$

864. $\dot{x} = Ax,\ \ A = \begin{pmatrix} 0 & 1 & -1 \\ 1 & 0 & -1 \\ 2 & 2 & -3 \end{pmatrix}.$

865. $\dot{x} = Ax,\ \ A = \begin{pmatrix} 4 & 2 & -2 \\ 1 & 3 & -1 \\ 3 & 3 & -1 \end{pmatrix}.$

866. $\dot{x} = Ax,\ \ A = \begin{pmatrix} 2 & 0 & -1 \\ 1 & -1 & 0 \\ 3 & -1 & -1 \end{pmatrix}.$

867 – 873 masalalarda berilgan matritsaning e^A ko'rsatkichli funksiyasini toping.

867. $A = \begin{pmatrix} 3 & 0 \\ 0 & -2 \end{pmatrix}.$

868. $A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$

869. $A = \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}.$

870. $A = \begin{pmatrix} 3 & -1 \\ 2 & 0 \end{pmatrix}.$

871. $A = \begin{pmatrix} -2 & -4 \\ 1 & 2 \end{pmatrix}.$

872. $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix}.$

873. $A = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix}.$

874 va **875** masalalarda e^A matritsani hisoblamasdan $\det e^A$ ni toping.

874. $A = \begin{pmatrix} 1 & 0 & 3 \\ -1 & 2 & 0 \\ 0 & 1 & -1 \end{pmatrix}.$

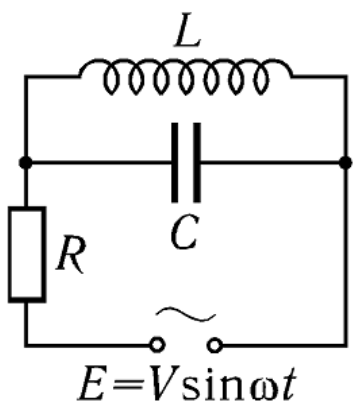
875. $A = \begin{pmatrix} 1 & 4 & 2 \\ 3 & 1 & -1 \\ 2 & 1 & -3 \end{pmatrix}.$

876. m massali jism x, y tekislikda harakatlanmoqda va $(0, 0)$ nuqtaga a^2mr kuch bilan tortilmoqda, bu yerda r – shu nuqtagacha masofa. Jismning $x(0) = d$, $y(0) = 0$, $\dot{x}(0) = 0$, $\dot{y} = v$ boshlang'ich shart ostidagi harakatini va bu harakat traektoriyasini toping.

877. Prujinaning bir uchi qo'zg'almas 0 nuqtaga mahkamlangan, ikkinchi uchiga $3m$ massali yuk osilgan va bu yukka $2m$ massali yuk biriktirilgan yana bir prujina osilgan. Har ikkala yuk 0 nuqtadan o'tuvchi bitta to'g'ri chiziq bo'yicha ta'sirsiz harakatlanmoqda. Har bir prujina a^2mx kuch ta'sirida x miqdorga cho'ziladi. Sistemaning mumkin bo'lgan davriy harakatini toping.

878. Valning uchlariga inersiya moment I_1 va I_2 bo'lgan ikkita shkiv mahkamlangan. Bir shkiv ikkinchisiga nisbatan ihtiyoriy φ burchakka burilganda valning deformatsiyalanishi natijasida $K\varphi$ aylanish momentiga ega elastiklik kuchi yuzaga keladi. Tashqi kuchlarni hisobga olmasdan val tebranishining aylanish chastotasini toping.

879. $E = V\sin\omega t$ kuchlanishli tok manbaiga R qarshilik ulangan. Keyin zanjir ikki tarmoqqa ajratilgan bo'lib, bir tarmoqqa L samoinduksiya, ikkinchisiga esa C sig'im ulangan(rasm). Zanjirdagi R qarshilik orqali o'tgan tok kuchini toping (o'rnatilgan rejim). Qanday ω chastotada tok kuchi eng katta bo'ladi? Eng kichik bo'ladi?



Ko'rsatma. Elektr zanjiri haqidagi masalalarda differensial tenglamalarni tuzish haqida 11-§ ning 5 punkitida bayon qilingan.

880. Har qanday uzluksiz va ω davrli $f(t)$ vektor funksiya olinganda ham vektorli ko'rinishda yozilgan $\dot{x} = Ax + f(t)$ tenglamalar sistemasi davriy yechimga ega bo'lishi uchun A matritsaning hos qiymatlariga qanday shart qo'yish yetarli?

Ko'rsatma. O'zgarmasni variatsiyalash usulini vektor ko'rinishda qo'llang, umumiy yechimni e^{tA} fundamental matritsa, $f(t)$ funksiya va boshlang'ich shartlar orqali ifodalang. Davriylik shartidan foydalaning.

15-§. TURG'UNLIK

881-898 masalalarni turg'unlik tarifidan foydalanaib yechiladi.

881. Turg'unlikning Lyapunov ma'nosidagi ta'rifidan foydalanib berilgan tenglamalarning ko'rsatilgan boshlang'ich shartni qanoatlentiruvchi yechimini turg'unlikka tekshiring

a) $3(t-1)\dot{x} = x,\ \ x(2) = 0.$ b) $\dot{x} = 4x - t^2x,\ \ x(0) = 0.$

c) $\dot{x} = t - x,\ \ x(0) = 1.$ d) $2t\dot{x} = x - x^3,\ \ x(1) = 0.$

882-888. masalalarda berilgan sistemaning traektoriyalarini $(0,0)$ nuqta atrofida tasvirlang va chizmaga asosan nol yechim turg'unligini tekshiring.

882. $\dot{x} = -x, \ \ \dot{y} = -2y.$

883. $\dot{x} = x, \ \ \dot{y} = 2y.$

884. $\dot{x} = -x, \ \ \dot{y} = y.$

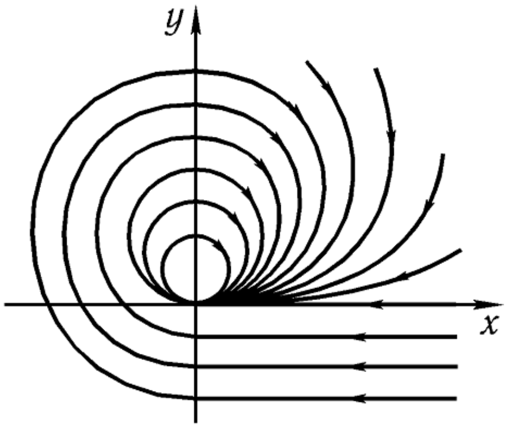
885. $\dot{x} = -y, \ \ \dot{y} = 2x^3.$

886. $\dot{x} = y, \ \ \dot{y} = -\sin x.$

887. $\dot{x} = y, \ \ \dot{y} = x^3(1+y^2).$

888. $\dot{x} = -y\cos x, \ \ \dot{y} = \sin x.$

889. $\frac{dx}{dt} = P(x,y), \ \frac{dy}{dt} = Q(x,y)$ tenglamalar sistemasining traektoriyalari holatlar tekisligida tasvirlangan (5-rasm), bu yerda $P, P_x^{'}, P_y^{'}, Q, Q_x^{'}, Q_y^{'}$ funksiyalar uzluksiz. $t \to +\infty$ da yechimlarning harakteri haqida nima deyish mumkin? Nol yechim asimptotik turg'un bo'ladimi? U Lyapunov ma'nosida turg'un bo'ladimi?



5-rasm

890-892 masalalarda sistemaning umumiy echimi ko'rsatilgan. Bu sistemaning nol yechimi turg'un bo'ladimi?

890. $x = C_1\cos^2 t - C_2e^{-t}, \ y = C_1t^4e^{-t} + 2C_2$.

891. $x = \frac{C_1 - C_2t}{1+t^2}, \ y = (C_1t^3 + C_2)e^{-t}$.

892. $x = (C_1 - C_2t)e^{-t}, \ y = \frac{C_1\sqrt[3]{t}}{\ln(t^2+2)} + C_2$.

893. $\frac{dx}{dt} = a(t)x$ tenglamaning nol yechimi Lyapunov ma'nosida turg'un bo'lishi uchun (bu yerda $a(t)$ funksiya uzluksiz)

$$\overline{\lim_{t\to+\infty}} \int_0^t a(s)ds < +\infty$$

tengsizlik bajarilishi zarur va yetarli ekanligini isbotlang.

894. Agar chiziqli differensial tenglamalar sistemasining biror yechimi Lyapunov ma'nosida turg'un bo'lsa, u holda bu sistemaning barcha yechimlari turg'un bo'lishini isbotlang.

895. Agar chiziqli bir jinsli sistemaning har bir yechimi $t \to +\infty$ da chegaralangan bo'lsa, u holda sistemaning nol yechimi Lyapunov ma'nosida turg'un bo'lishini isbotlang.

896. Agar chiziqli bir jinsli sistemaning har bir yechimi $t \to +\infty$ da nolga intilsa, u holda sistemaning nol yechimi asimptotik turg'un bo'lishini isbotlang.

897. Agar chiziqli bir jinsli sistemaning biror yechimi $t \to +\infty$ da chegaralanmagan bo'lsa, u holda sistemaning nol yechimi noturg'un bo'lishini isbotlang.

898. Agar $t \to +\infty$ da $a_{11}(t) + a_{22}(t) \to b > 0$ munosabat ma'lum bo'lsa, $\dot{x}_1 = a_{11}(t)x_1 + a_{12}(t)x_2,\ \dot{x}_2 = a_{21}(t)x_1 + a_{22}(t)x_2$ sistemaning nol yechimi turg'un bo'ladimi?

Lyapunovning birinchi yaqinlashish bo'yish turg'unlik haqidagi teoremasi yordamida **899-906** masalalarda berilgan sistemaning nol yechimining turg'unligini tekshiring.

899. $\begin{cases} \dot{x} = 2xy - x + y, \\ \dot{y} = 5x^4 + y^3 + 2x - 3y. \end{cases}$

900. $\begin{cases} \dot{x} = x^2 + y^2 - 2x, \\ \dot{y} = 3x^2 - x + 3y. \end{cases}$

901. $\begin{cases} \dot{x} = e^{x+2y} - \cos 3x, \\ \dot{y} = \sqrt{4 + 8x} - 2e^y. \end{cases}$

902. $\begin{cases} \dot{x} = \ln(4y + e^{-3x}), \\ \dot{y} = 2y - 1 + \sqrt[3]{1 - 6x}. \end{cases}$

903. $\begin{cases} \dot{x} = \ln(3e^y - 2\cos x), \\ \dot{y} = 2e^x - \sqrt[3]{8 + 12y}. \end{cases}$

904. $\begin{cases} \dot{x} = \operatorname{tg}(y - x), \\ \dot{y} = 2^y - 2\cos\left(\frac{\pi}{3} - x\right). \end{cases}$

905. $\begin{cases} \dot{x} = \operatorname{tg}(z - y) - 2x, \\ \dot{y} = \sqrt{9 + 12x} - 3e^y, \\ \dot{z} = -3y. \end{cases}$

906. $\begin{cases} \dot{x} = e^x - e^{-3z}, \\ \dot{y} = 4z - 3\sin(x + y), \\ \dot{z} = \ln(1 + z - 3x). \end{cases}$

907-912 masalalarda berilgan sistemaning nol yechimi a va b parametrlarning qanday qiymatlarida turg'un bo'lishini tekshiring.

907. $\begin{cases} \dot{x} = ax - 2y + x^2, \\ \dot{y} = x + y + xy. \end{cases}$

908. $\begin{cases} \dot{x} = ax + y + x^2, \\ \dot{y} = x + ay + y^2. \end{cases}$

909. $\begin{cases} \dot{x} = x + ay + y^2, \\ \dot{y} = bx - 3y - x^2. \end{cases}$

910. $\begin{cases} \dot{x} = y + \sin x, \\ \dot{y} = ax + by. \end{cases}$

911. $\begin{cases} \dot{x} = 2e^{-x} - \sqrt{4 + ay}, \\ \dot{y} = \ln(1 + x + ay). \end{cases}$

912. $\begin{cases} \dot{x} = \ln(e + ax) - e^y, \\ \dot{y} = bx + \operatorname{tg} y. \end{cases}$

913. Berilgan sistemaning $x = -t^2,\ y = t$ yechimini turg'unlikka tekshiring

$$\dot{x} = y^2 - 2ty - 2y - x, \qquad \dot{y} = 2x + 2t^2 + e^{2t-2y}.$$

914. Berilgan sistemaning $x = \cos t,\ y = 2\sin t$ yechimini turg'unlikka tekshiring

$$\begin{cases} \dot{x} = \ln\left(x + 2\sin^2\frac{t}{2}\right) - \frac{y}{2}, \\ \dot{y} = (4 - x^2)\cos t - 2x\sin^2 t - \cos^3 t. \end{cases}$$

915-922 masalalarda berilgan sistemaning barcha muvozanat holatlarini toping va ularni tug'unlikka tekshiring.

915. $\begin{cases} \dot{x} = y - x^2 - x, \\ \dot{y} = 3x - x^2 - y. \end{cases}$

916. $\begin{cases} \dot{x} = (x-1)(y-1), \\ \dot{y} = xy - 2. \end{cases}$

917. $\begin{cases} \dot{x} = y, \\ \dot{y} = \sin(x+y). \end{cases}$

918. $\begin{cases} \dot{x} = \ln(-x+y^2), \\ \dot{y} = x - y - 1. \end{cases}$

919. $\begin{cases} \dot{x} = 3 - \sqrt{4 + x^2 + y}, \\ \dot{y} = \ln(x^2 - 3). \end{cases}$

920. $\begin{cases} \dot{x} = e^y - e^x, \\ \dot{y} = \sqrt{3x + y^2} - 2. \end{cases}$

921. $\begin{cases} \dot{x} = \ln(1 + y + \sin x) \\ \dot{y} = 2 + \sqrt[3]{3\sin x - 8}. \end{cases}$

922. $\begin{cases} \dot{x} = -\sin y, \\ \dot{y} = 2x + \sqrt{1 - 3x - \sin y}. \end{cases}$

923-931 masalalarda berilgan sistemaning nol yechimini turg'unlikka tekshiring, bunda Lyapunov funksiyasini quring va Lyapunov yoki Chetayev teoremasini qo'llang.

923. $\begin{cases} \dot{x} = x^3 - y, \\ \dot{y} = x + y^3. \end{cases}$

924. $\begin{cases} \dot{x} = y - x + xy, \\ \dot{y} = x - y - x^2 - y^3. \end{cases}$

925. $\begin{cases} \dot{x} = 2y^3 - x^5, \\ \dot{y} = -x - y^3 + y^5. \end{cases}$

926. $\begin{cases} \dot{x} = xy - x^3 + y^3, \\ \dot{y} = x^2 - y^3. \end{cases}$

927. $\begin{cases} \dot{x} = y - 3x - x^3, \\ \dot{y} = 6x - 2y. \end{cases}$

928. $\begin{cases} \dot{x} = 2y - x - y^3, \\ \dot{y} = x - 2y. \end{cases}$

929. $\begin{cases} \dot{x} = -x - xy, \\ \dot{y} = y^3 - x^3. \end{cases}$

930. $\begin{cases} \dot{x} = x - y - xy^2, \\ \dot{y} = 2x - y - y^3. \end{cases}$

931. $\begin{cases} \dot{x} = -\operatorname{sgn} x - \operatorname{sgn} y, \\ \dot{y} = \operatorname{sgn} x - \operatorname{sgn} y. \end{cases}$

932-948 masalalarda nol yechimni turg'unlikka tekshiring. Bunda ko'phadning barcha ildizlari manfiy haqiqiy qismga ega bo'lish shartlaridan, masalan Raus-Gurvis shartidan yoki Mihaylov alomatidan foydalaning.

932. $y''' + y'' + y' + 2y = 0.$

933. $y''' + 2y'' + 2y' + 3y = 0.$

934. $y^{\mathrm{IV}} + 2y''' + 4y'' + 3y' + 2y = 0.$

935. $y^{\mathrm{IV}} + 2y''' + 3y'' + 7y' + 2y = 0.$

936. $y^{\mathrm{IV}} + 2y''' + 6y'' + 5y' + 6y = 0.$

937. $y^{\mathrm{IV}} + 8y''' + 14y'' + 36y' + 45y = 0.$

938. $y^{\mathrm{IV}} + 13y''' + 16y'' + 55y' + 76y = 0.$

939. $y^{\mathrm{IV}} + 3y''' + 26y'' + 74y' + 85y = 0.$

940. $y^{\mathrm{IV}} + 3{,}1y''' + 5{,}2y'' + 9{,}8y' + 5{,}8y = 0.$

941. $y^{\mathrm{V}} + 2y^{\mathrm{IV}} + 4y''' + 6y'' + 5y' + 4y = 0.$

942. $y^{\mathrm{V}} + 2y^{\mathrm{IV}} + 5y''' + 6y'' + 5y' + 2y = 0.$

943. $y^{\mathrm{V}} + 3y^{\mathrm{IV}} + 6y''' + 7y'' + 4y' + 4y = 0.$

944. $y^{\mathrm{V}} + 4y^{\mathrm{IV}} + 9y''' + 16y'' + 19y' + 13y = 0.$

945. $y^{\mathrm{V}} + 4y^{\mathrm{IV}} + 16y''' + 25y'' + 13y' + 9y = 0.$

946. $y^{\mathrm{V}} + 3y^{\mathrm{IV}} + 10y''' + 22y'' + 23y' + 12y = 0.$

947. $y^{\mathrm{V}} + 5y^{\mathrm{IV}} + 15y''' + 48y'' + 44y' + 74y = 0.$

948. $y^{\mathrm{V}} + 2y^{\mathrm{IV}} + 14y''' + 36y'' + 23y' + 68y = 0.$

a va b parametrlarning qanday qiymatlarida **949-958** masalalarning nol yechimi asimptotik turg'un bo'ladi.

949. $y''' + ay'' + by' + 2y = 0.$

950. $y''' + 3y'' + ay' + by = 0.$

951. $y^{\mathrm{IV}} + 2y''' + 3y'' + 2y' + ay = 0.$

952. $y^{\mathrm{IV}} + ay''' + y'' + 2y' + y = 0.$

953. $ay^{\mathrm{IV}} + y''' + y'' + y' + by = 0.$

954. $y^{\mathrm{IV}} + y''' + ay'' + y' + by = 0.$

955. $y^{\mathrm{IV}} + ay''' + 4y'' + 2y' + by = 0.$

956. $y^{\mathrm{IV}} + 2y''' + ay'' + by' + y = 0.$

957. $y^{\mathrm{IV}} + ay''' + 4y'' + by' + y = 0.$

958. $y^{\mathrm{IV}} + 2y''' + 4y'' + ay' + by = 0.$

959 va **960** masalalarda davriy koeffitsientli tenglamalar turg'unligini tekshirish uchun monodrom matritsani topish va multiplikatorni hisoblash kerak([5] adabiyotning III bob 15§, 16§lariga qarang).

959. $\ddot{x} + p(t)x = 0,\ p(t) = a^2\ \ (0 < t < \pi),\ p(t) = b^2\ \ (\pi < t < 2\pi),$ $p(t + 2\pi) \equiv p(t)$ tenglamaning nol yechimini turg'unlikka tekshiring, bunda parametrlarning qiymat quyida berilgan:

a) $a = 0{,}5,\ b = 0;$ b) $a = 0{,}5,\ b = 1;$

c) $a = 0{,}5,\ b = 1{,}5;$ d) $a = 0{,}75,\ b = 0;$

e) $a = 1,\ b = 0;$ f) $a = 1,\ b = 1{,}5;$

960. a va b ning qanday qiymatlarida koeffitsientlari davriy bo'lgan quyidagi sistemaning nol yechimi turg'un bo'ladi?

$$\begin{pmatrix}\dot{x}\\ \dot{y}\end{pmatrix} = A(t)\begin{pmatrix}x\\ y\end{pmatrix}, \qquad A(t+2) \equiv A(t),$$

bu yerda $0 < t < 1$ da $A(t) = \begin{pmatrix}0 & a\\ 0 & 0\end{pmatrix}$, $1 < t < 2$ da $A(t) = \begin{pmatrix}0 & 0\\ b & 0\end{pmatrix}$.

16-§. MAXSUS NUQTALAR

961-978 masalalarda berilgan tenglamalar va sistemalarning maxsus nuqtalarini tekshiring. (x, y) tekislikda integral chiziqlarning joylashuvini tasvirlang.

961. $y' = \frac{2x+y}{3x+4y}$. **962.** $y' = \frac{x-4y}{2y-3x}$.

963. $y' = \frac{y-2x}{y}$. **964.** $y' = \frac{x+4y}{2x+3y}$.

965. $y' = \frac{x-2y}{3x-4y}$. **966.** $y' = \frac{2x-y}{x-y}$.

967. $y' = \frac{y-2x}{2y-3x}$. **968.** $y' = \frac{4y-2x}{x+y}$.

969. $y' = \frac{y}{x}$. **970.** $y' = \frac{4x-y}{3x-2y}$.

971. $\begin{cases}\dot{x} = 3x,\\ \dot{y} = 2x + y.\end{cases}$ **972.** $\begin{cases}\dot{x} = 2x - y,\\ \dot{y} = x.\end{cases}$

973. $\begin{cases}\dot{x} = x + 3y,\\ \dot{y} = -6x - 5y.\end{cases}$ **974.** $\begin{cases}\dot{x} = x,\\ \dot{y} = 2x - y.\end{cases}$

975. $\begin{cases}\dot{x} = -x - 5y,\\ \dot{y} = 2x + 2y.\end{cases}$ **976.** $\begin{cases}\dot{x} = 3x + y,\\ \dot{y} = y - x.\end{cases}$

977. $\begin{cases}\dot{x} = 3x - 2y,\\ \dot{y} = 4y - 6x.\end{cases}$ **978.** $\begin{cases}\dot{x} = y - 2x,\\ \dot{y} = 2y - 4x.\end{cases}$

979-992 masalalarda berilgan tenglamalar va sistemalarning maxsus nuqtalari toping va tekshiring.

979. $y' = \frac{2y-x}{3x+6}$. **980.** $y' = \frac{2x+y}{x-2y-5}$.

981. $y' = \frac{4y^2-x^2}{2xy-4y-8}$. **982.** $y' = \frac{2y}{x^2-y^2-1}$.

983. $y' = \frac{x^2+y^2-2}{x-y}$.

984. $y' = \frac{y+\sqrt{1+2x^2}}{x+y+1}$.

985. $\begin{cases} \dot{x} = x^2 - y, \\ \dot{y} = \ln(1 - x + x^2) - \ln 3\,. \end{cases}$

986. $\begin{cases} \dot{x} = \ln(2 - y^2)\,, \\ \dot{y} = e^x - e^y. \end{cases}$

987. $\begin{cases} \dot{x} = (2x - y)(x - 2), \\ \dot{y} = xy - 2. \end{cases}$

988. $\begin{cases} \dot{x} = \sqrt{x^2 - y + 2} - 2, \\ \dot{y} = \operatorname{arctg}(x^2 + xy). \end{cases}$

989. $\begin{cases} \dot{x} = x^2 - y, \\ \dot{y} = x^2 - (y - 2)^2. \end{cases}$

990. $\begin{cases} \dot{x} = \ln\frac{y^2-y+1}{3}, \\ \dot{y} = x^2 - y^2. \end{cases}$

991. $\begin{cases} \dot{x} = \ln(1 - y + y^2)\,, \\ \dot{y} = 3 - \sqrt{x^2 + 8y}. \end{cases}$

992. $\begin{cases} \dot{x} = \sqrt{(x - y)^2 + 3} - 2, \\ \dot{y} = e^{y^2-x} - e. \end{cases}$

993-997 masalalarda berilgan tenglamalar integral chiziqlarning koorinatalar boshi atrofida joylashuvini tasvirlang.

Ko’rsatma. 993-997 masalalarda maxsus nuqtalar 16-§ da ko’rsatilgan tiplarga mansub emas. Ularni tekshirish uchun bir nechta izoklinalarni chizish mumkin. Keyin integral chiziqlar maxsus nuqtaga qaysi tomondan yaqinlashayotganini tushuntirish kerak.

993. $y' = \frac{xy}{x+y}$.

994. $y' = \frac{x^2+y^2}{x^2+y}$.

995. $y' = \frac{2xy}{y+x^2}$.

996. $y' = \frac{xy}{y-x^2}$.

997. $y' = \frac{y^2}{y+x^2}$.

998. Agar

$$(ax + by)dx + (mx + ny)dy = 0$$

tenglamaning maxsus nuqtasi markazdan iborat bo’lsa, u holda bu tenglama to’liq differensialli tenglama ekanligini isbotlang. Teskari tasdiq noto’g’riligiga misol keltiring.

999. Agar avvalgi masaladagi tenglama to’liq differensialli bo’lmasa, ammo koordinatalar boshi atrofida uzluksiz bo’lgan integrallovchi ko’paytuvchiga ega bo’lsa, u holda maxsus nuqta (agar $an \neq bm$ bo’lsa) egardan iboratligini isbotlang.

1000. Faraz qilaylik

$$y' = \frac{ax + by + p(x, y)}{cx + dy + q(x, y)} \qquad (1)$$

tenglamada p va q funksiyalar $(0,0)$ nuqtaning biror atrofida aniqlangan va differensiallanuvchi, bu $(0,0)$ nuqta ustida esa $p = p_x^{'} = p_y^{'} = q = q_x^{'} = q_y^{'} = 0$ bo'lsin. Agar (1) tenglamada y n $-y$ almashtirsak tenglama o'zgarmasa va

$$\begin{vmatrix} c-\lambda & d \\ a & b-\lambda \end{vmatrix} = 0$$

tenglamaning ildizlari sof mavhum bo'lsa, u holda $(0,0)$ maxsus nuqta markaz bo'lishini isbotlang.

17-§. HOLATLAR TEKISLIGI

1001-1020 masalalarda berilgan tenglama traektoriyalarini xolatlar tekisligida tasvirlang. Chizmaga ko'ra $t \to +\infty$ da yechimlarning harakteri haqida hulosa chiqaring.

1001. $\ddot{x} + 4x = 0.$ **1002.** $\ddot{x} - x = 0.$

1003. $\ddot{x} - x + x^2 = 0.$ **1004.** $\ddot{x} - 3x^2 = 0.$

1005. $\ddot{x} + 2x^3 = 0.$ **1006.** $\ddot{x} + 2x^3 - 2x = 0.$

1007. $\ddot{x} + e^x - 1 = 0.$ **1008.** $\ddot{x} - 2^x + x + 1 = 0.$

1009. $\ddot{x} - \sin x = 0.$ **1010.** $\ddot{x} + 2\cos x - 1 = 0.$

1011. $\ddot{x} - 4\dot{x} + 3x = 0.$ **1012.** $\ddot{x} + 2\dot{x} + 5x = 0.$

1013. $\ddot{x} - \dot{x} - 2x = 0.$ **1014.** $\ddot{x} + 2\dot{x} + \dot{x}^2 + x = 0.$

1015. $\ddot{x} + \dot{x} + 2x - x^2 = 0.$ **1016.** $\ddot{x} + \dot{x}^2 - x^2 + 1 = 0.$

1017. $\ddot{x} + 2^{\dot{x}} - x^2 = 0.$ **1018.** $\ddot{x} + \sqrt{\dot{x}^2 + x^2} - 1 = 0.$

1019. $\ddot{x} + 5\dot{x} - 4\ln\frac{x^2+1}{2} = 0.$ **1020.** $\ddot{x} + \dot{x} + \mathrm{arctg}(x^2 - 2x) = 0.$

1021-1034 masalalarda berilgan sistema traektoriyalarini holatlar tekisligida tasvirlang va maxsus nuqtalarni tekshiring.

1021. $\begin{cases} \dot{x} = 2x + y^2 - 1, \\ \dot{y} = 6x - y^2 + 1. \end{cases}$ **1022.** $\begin{cases} \dot{x} = y^2 - 4x^2, \\ \dot{y} = 4y - 8. \end{cases}$

1023. $\begin{cases} \dot{x} = 4 - 4x - 2y, \\ \dot{y} = xy. \end{cases}$ **1024.** $\begin{cases} \dot{x} = 1 - x^2 - y^2, \\ \dot{y} = 2x. \end{cases}$

1025. $\begin{cases} \dot{x} = 2 + y - x^2, \\ \dot{y} = 2x(x - y). \end{cases}$

1026. $\begin{cases} \dot{x} = xy - 4, \\ \dot{y} = (x - 4)(y - x). \end{cases}$

1027. $\begin{cases} \dot{x} = 1 - x^2 - y^2, \\ \dot{y} = 2xy. \end{cases}$

1028. $\begin{cases} \dot{x} = 2(x - 1)(y - 2), \\ \dot{y} = y^2 - x^2. \end{cases}$

1029. $\begin{cases} \dot{x} = (x + y)^2 - 1, \\ \dot{y} = 1 - y^2 - x. \end{cases}$

1030. $\begin{cases} \dot{x} = (2x - y)^2 - 9, \\ \dot{y} = 9 - (x - 2y)^2. \end{cases}$

1031. $\begin{cases} \dot{x} = (2x - y)^2 - 9, \\ \dot{y} = (x - 2y)^2 - 9. \end{cases}$

1032. $\begin{cases} \dot{x} = x^2 + y^2 - 6x - 8y, \\ \dot{y} = x(2y - x + 5). \end{cases}$

1033. $\begin{cases} \dot{x} = x^2 - y, \\ \dot{y} = (x - y)(x - y + 2). \end{cases}$

1034. $\begin{cases} \dot{x} = x^2 + y^2 - 5, \\ \dot{y} = (x - 1)(x + 3y - 5). \end{cases}$

1035. Hech bir qarshilik ta’sir etmayotgan mayatnik harakatining tenglamasini keltirib chiqaring. Tenglamadagi barcha o’zgarmaslar 1 ga teng bo’lganda traektoriyalarni holatlar tekisligida tasvirlang. Turli tipdagi traektoriyalarning fizik mohiyatini tushuntirib bering.

1036. Tezligining kvadratiga proporsional qarshilik ta’sir qilayotgan mayatnik tenglamasini keltirib chiqaring. Traektoriyalarni holatlar tekisligida tasvirlang.

Ko’rsatma. 1035 masala uchun hosil qilingan chizmalardan foydalaning.

1037. O’zgarmas kuch ta’sir qilayotgan mayatnik tenglamasini keltirib chiqaring, bunda kuch mayatnik massasining yarmiga teng va doimo mayatnik harakatlanayotgan aylanaga o’tkazilgan urinmaning bitta yo’nalishida yo’nalgan.

l va g o’zgarmaslarni 1 ga teng deb xisoblang va hosil bo’lgan tenglama traektoriyalarini holatlar tekisligida tasvirlang. Mayatninkning qanday harakatlari turli tipdagi traektoriyalarga ega bo’ladi.

1038. m massali yuk prujinaga mahkamlangan. Yuk muvozanat holatidan x masofaga chetlanganda unga prujina muvozanat holatiga yo’nalgan kx kuch bilan ta’sir qiladi. Ishqalanish kuchi $f = \text{const}$ ga teng va u yuk tezligiga qarama-qarshi tomonga yo’nalgan. $t = 0$ da yuk muvozanat holatdan h masofada bo’lib nol tezlikka ega.

Yuk harakatining tenglamasini keltirib chiqaring. $m = 2,\ k = 2,\ f = 1,\ h = 5$ deb hisoblab yuk harakatini holatlar tekisligida tasvirlang.

1039. Holatlar tekisligida uzunligi o'zgarayotgan mayatnikning kichik tebranishini tasvirlang. Bunda mayatnik yuqoriga harakatlanganda uning uzunligi l ga teng, pastga hakatlanganda esa $L > l$ ga teng deb oling. Bir martalik to'liq tebranishda amlpituda necha marta kattalashadi? (Masalan: belanchakning tebranishi)

Ko'rsatma. Kichik tebranishlarda $\sin x \approx x$ deb olish mumkin. Mayatnik uzunligining o'zgarishi (tebranishi) juda tez sodir bo'ladi, bunda mayatnikning chetlanish burchagi va uning o'qqa nisbatan harakatining oniy miqdoriga tebranishlar ta'sir o'tkazmaydi.

Qutb koordinatalarida ifodalangan 1040-1046 sistemalar traektoriyalarini holatlar tekisligida tasvirlang va limit davralarga egaligini tekshiring.

1040. $\frac{dr}{dt} = r(1 - r^2),\ \frac{d\varphi}{dt} = 1.$

1041. $\frac{dr}{dt} = r(r - 1)(r - 2),\ \frac{d\varphi}{dt} = 1.$

1042. $\frac{dr}{dt} = r(1 - r)^2,\ \frac{d\varphi}{dt} = 1.$

1043. $\frac{dr}{dt} = \sin r\,,\ \frac{d\varphi}{dt} = 1.$

1044. $\frac{dr}{dt} = r(|r - 1| - |r - 2| - 2r + 3),\ \frac{d\varphi}{dt} = 1.$

1045. $\frac{dr}{dt} = r \sin\frac{1}{r},\ \frac{d\varphi}{dt} = 1.$

1046. $\frac{dr}{dt} = r(1 - r) \sin\frac{1}{1-r},\ \frac{d\varphi}{dt} = 1.$

1047. Qanday shartlarda

$$\frac{dr}{dt} = f(r), \qquad \frac{d\varphi}{dt} = 1,$$

sistema (bu yerda $f(r)$ uzluksiz funksiya) limit davraga ega bo'ladi? Qanday shartlarda bu davra turg'un bo'ladi? Noturg'un bo'ladi? Yarim turg'un bo'ladi?

1048. a o'zgarmasning qanday qiymatlarida

$$\frac{dr}{dt} = (r - 1)(a + \sin^2 \varphi), \qquad \frac{d\varphi}{dt} = 1,$$

sistema turg'un limit davraga ega bo'ladi? Noturg'un limit davragachi?

1049-1052 tenglamalar uchun izoklinalar yordamida traektoriyalarni holatlar tekisligida tasvirlang va maxsus nuqtalarni tekshiring. Chizmaga asosan $t \to +\infty$ da yechimlarning hususiyati haqida va yopiq traektoriyalarning mavjudligi haqida hulosa chiqaring.

1049. $\ddot{x} + \dot{x}^3 - \dot{x} + x = 0.$

1050. $\ddot{x} + (x^2 - 1)\dot{x} + x = 0.$

1051. $\ddot{x} + \dot{x}\,\mathrm{arctg}\,\dot{x} + x = 0.$

1052. $\ddot{x} + 2^{\dot{x}} - \dot{x} + x = 0.$

1053. $\ddot{x} + 2a\dot{x} - b\,\mathrm{sgn}\,\dot{x} + x = 0$ $(0 < a < 1,\ b > 0)$ tenglama uchun holatlar tekisligida traektoriyalarni quring va limit davra Ox o'qini kesib o'tadigan nuqtalarni toping.

Ko'rsatma. Traektoriyalar Ox o'qi bilan kesishgan ketma-ket nuqtalar abtsissalari orasidagi bog'lanishni toping.

1054. $\ddot{x} + F(\dot{x}) + x = 0$ tenglamada F uzluksiz funksiya va $y > 0$ da $F(y) > 0$, $y < 0$ da $F(y) < 0$. Bu tenglama holatlar tekisligida limit davraga ega bo'lmasligini ko'rsating.

Ko'rsatma. $\frac{d}{dt}(x^2 + y^2)$ to'liq hosilani ishorasini tekshiring.

1055. Agar $f(x,y),\ f_x',\ f_y'$ funksiyalar uzluksiz, $f(0{,}0) < 0$ va $x^2 + y^2 > b^2$ sohada $f(x,y) > 0$ bo'lsa, u holda $\ddot{x} + f(x,\dot{x})\dot{x} + x = 0$ tenglama $x(t) \not\equiv 0$ davriy yechimga ega bo'lishini isbotlang.

Ko'rsatma. Holatlar tekisligida $\frac{d}{dt}(x^2 + y^2)$ to'liq hosilani ishorasini tekshiring. Hech bir traektoriya chiqib ketolmaydigan halqani quring. [3] adabiyotdagi 21 teoremani qo'llang.

18-§. YECHIMNI BOSHLANG'ICH SHARTLARGA VA PARAMETRLARGA BOG'LIQLIGI. DIFFERENSIAL TENGLAMALARNING TAQRIBIY YECHIMLARI

1056. Agar y_0 son 0,01 dan kichik o'zgarsa, $y' = x + \sin y$ tenglamaning $y(0) = y_0 = 0$ boshlang'ich shartni qanoatlantiruvchi yechimi $0 \le x \le 1$ kesmada qancha o'zgarishi mumkinligini baholang?

1057. $\ddot{x} + \sin x = 0$ mayatnik tenglamasining $x(0) = 0,\ \dot{x}(0) = 0$ boshlang'ich shartni qanoatlantiruvchi yechimi $0 \le t \le T$ kesmada qaralmoqda. Agar tenglamaning o'ng qismiga $|\varphi(t)| \le 0{,}1$ tengsizlikni qanoatlantiruvchi $\varphi(t)$ funksiya qo'shilsa, masalaning yechimi qancha o'zgarishi mumkinligini baholang?

1058. $\ddot{x} + \sin x = 0$ tenglamani $\ddot{x} + x = 0$ tenglamaga almashtirib taqribiy yechimini toping. Agar $|x| \le 0{,}25$ da $|x - \sin x| < 0{,}003$ ekanligi ma'lum bo'lsa,

$0 \leq t \leq 2$ kesmadagi $x(0) = 0{,}25$, $\dot{x}(0) = 0$ boshlang'ich shartdan aniqlangan yechim hatoligini baholang.

1059–1063 masalalarda ko'rsatilgan kesmadagi taqribiy yechimning hatoligini baholang.

1059. $y' = \frac{x}{4} - \frac{1}{1+y^2}$, $y(0) = 1$; $\tilde{y} = 1 - \frac{x}{2}$, $|x| \leq \frac{1}{2}$.

1060. $\dot{x} = x - y$, $\dot{y} = tx$, $x(0) = 1$, $y(0) = 0$; $\tilde{x} = 1 + t + \frac{t^2}{2}$, $\tilde{y} = \frac{t^2}{2}$, $|t| \leq 0{,}1$.

1061. $y'' - x^2 y = 0$, $y(0) = 1$, $y'(0) = 0$; $\tilde{y} = e^{x^4/12}$, $|x| \leq 0{,}5$