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MISOL VA MASALALAR TO‘PLAMI

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ANNOTATSIYA

Ushbu metodik qo'llanma pedagogika oliy ta'lim muassasalari «matematika va informatika» bakalavriat ta'lim yonalishining «Matematik analiz» fani dasturiga mos yozilgan bo'lib, bunda matematik analizning ko'p o'zgaruvchili funksiyaning differensial va integral hisobi bo'limidan misol va masalalar berilgan. Bu uslubiy qo'llanmadan «Fizika va astronomiya» bakalavriat yonalishida ta'lim olayatgan talabalar ham foydalanishi mumkin.

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Mundarija

Kirish	**4**
XI-bob. Ko‘p o‘zgaruvchili funksiyalarning differensial hisobi	**5**
1-§. Ko‘p o‘zgaruvchili funksiyalar	5
2-§. Ko‘p o‘zgaruvchili funksiyalarning limiti va uzluksizligi	6
3-§. Xususiy hosilalar va to‘la differensial	8
4-§. Murakkab funksiyalarni differensiallash	11
5-§. Oshkarmas funksiyalarning hosilalari	12
6-§. Yuqori tartibli hususiy hosilalar va yuqori tartibli differensiallar	13
7-§. Ko‘p argumentli funksiyalarning ekstremumlari. Shartli ekstemumlar	15
XII-bob. Karrali integrallar	**17**
1-§. Ikki karrali integrallar	17
2-§. Qutb koordinatalarida ikki karrali integral	21
3-§ Ikki karrali integralni tatbiqlari	22
4-§. Uch karrali integralni hisoblash	25
5-§.Silindrik va sferik koordinatalarda uch karrali integral	26
6-§. Uch karrali integralning tatbiqlari	27
XIII-bob. Egri chiziqli integrallar	**28**
1-§. Birinchi tur egri chiziqli integrallar	28
2-§. Ikkinchi tur egri chiziqli integrallar	29
3-§ Egri chiziqli integrallarning tatbiqlari	32
Javoblar	34
Adabiyot	45

K i r i s h

Pedagogika oliy ta'lim muassasalari "matematika va informatika" bakalavriat ta'lim yo`nalishi talabalari uchun mo`ljallangan o`zbek tilida matematik analizdan misol va masalalar to`plami mavjud emas. Shu sababli dars o`tkazish uchun bir nechta adabiyotlardan misol va masalalarni tanlashga to`g`ri keladi.

Ushbu metodik qo`llanma pedagogika oliy yurtlari "matematika va informatika" bakalavriat ta'lim yo`nalishi talabalari uchun mavjud fan dasturiga mos ravishda yozilgan bo`lib, ko`p argumentli funksiyalarning differensial va integral hisobi bo`limlarini o`z ichiga oladi.

Har bir bobdan oldin kerakli bo`lgan nazariy ma'lumotlar keltirilgan, deyarli barcha misol va masalalarning javoblari berilgan.

Ushbu qo`llanmani tayyorlashda mavjud bo`lgan misol va masalalar to`plamidan ijodiy foydalanildi.

XI-BOB. KO`P O`ZGARUVCHILI FUNKSIYALARNING DIFFERENSIAL HISOBI

1-§. Ko`p o`zgaruvchili funksiyalar

1. Berilgan doimiy C soni uchun xOy tengsizlikdagi f(x,y)=C tenglikni qanoatlantiruvchi barcha nuqtalar to`plami z=f(x,y) funksiyaning sath chizig`i deyiladi.

2. Berilgan doimiy C soni uchun fazodagi f(x,y,z)=C tenglikni qanoatlantiruvchi barcha nuqtalar to`plami u=f(x,y,z) funksiyaning yuksaklik sirti deyiladi.

1. $f(x,y)=\dfrac{3xy}{x^2+y^2}$ funksiyani $x=-2,\ y=8$ dagi qiymatini toping;
2. $f(x,y)=\sqrt{y^2-2x^2}$ funksiyani $x=-4,\ y=10$ dagi qiymatini toping;
3. $F(x,y)=\dfrac{2x-y}{x-2y}$, $F(3,1)$, $F(1,3)$, $F(-2,-4)$, $F(-4,-2)$, $F(a;a)$, $F(a;-a)$ larni hisoblang;
4. $F(x,y)=\dfrac{x+2y}{x-y}$, $F(2;1)$, $F(-3;-1)$, $F(a;b)$, $(a\neq b)$ larni toping;
5. $f(x,y)=e^{\sin(x+y)}$, $f\left(\dfrac{\pi}{2},\dfrac{\pi}{2}\right)$, $f\left(\dfrac{\pi}{2},2\pi\right)$ larni toping;
6. $f(x,y)=x^y+y^{x-1}$, $f(1,1)$, $f(1,2)$, $f(2;2)$ larni toping;

Quyidagi funksiyalarning aniqlanish sohalarini toping.

7. $z=\dfrac{1}{x-y}$;
8. $z=\dfrac{1}{x+y}$;
9. $z=\sqrt{x+y}$;
10. $z=\sqrt{x-y}$;
11. $z=\sqrt{x+y}-\sqrt{x-y}$;
12. $z=\sqrt{x}+\sqrt{y}$;
13. $z=\sqrt{x^2+y^2-9}$;
14. $z=\sqrt{25-x^2-y^2}$;

15. $z = \ln\left(\frac{x^2}{9} + \frac{y^2}{4} - 1\right);$

16. $z = \ln\left(y^2 - 4x + 8\right);$

17. $z = \frac{1}{2 - x^2 - y^2};$

18. $z = \frac{1}{\sqrt{x}} + \frac{1}{\sqrt{y}};$

19. $z = \arcsin\frac{y-1}{x};$

20. $z = \arcsin\frac{x}{2} + \arccos\frac{y}{2};$

21. $u = \frac{1}{\sqrt{x}} + \frac{1}{\sqrt{y}} + \frac{1}{\sqrt{z}};$

22. $u = \sqrt{R^2 - x^2 - y^2 - z^2} + \sqrt{x^2 + y^2 + z^2 - r^2} \quad (r < R).$

Quyidagi funksiyalarning sath chiziqlarini yasang.

23. $z = x + y;$

24. $z = x^2 + y^2;$

25. $z = x^2 - y^2;$

26. $z = x^2 + y;$

27. $u = \frac{x + y + z}{x - y + z}$ funksiyaning yuksaklik sirtlarini toping;

28. $u = z^2 + y^2 + z^2$ funksiyaning yuksaklik sirtlarini toping.

2-§. Ko`p o`zgaruvchili funksiyaning limiti va uzluksizligi

Agar har bir $\varepsilon > 0$ *uchun shunday* $\delta > 0$ *topilib, koordinatalari* $|x - x_0| < \delta, |y - y_0| < \delta$ *tengsizliklarni qanoatlantiruvchi* $M_0(x_0, y_0)$ *dan boshqa barcha* $M(x, y)$ *nuqtalarda* $|f(x, y) - A| < \varepsilon$ *tengsizlik o`rinli bo`lsa, A soni* $f(x, y)$ *funksiyaning* $M_0(x_0, y_0)$ *nuqtadagi limiti deyiladi va u* $\lim\limits_{\substack{x \to x_0 \\ y \to y_0}} f(x, y) = A$ *ko`rinishda yoziladi.*

Agar $\lim\limits_{\substack{x \to x_0 \\ y \to y_0}} f(x, y) = f(x_0, y_0)$ *tenglik o`rinli bo`lsa,* $f(x, y)$ *funksiya* $M_0(x_0, y_0)$ *nuqtada uzluksiz deyiladi.*

Funksiya limitining ta'rifiga asoslanib tengliklarni isbotlang.

29. a) $\lim\limits_{\substack{x\to 2\\ y\to 3}}(2x+3y)=13$; b) $\lim\limits_{\substack{x\to 2\\ y\to -1}} x^2 y=-4$.

30. a) $\lim\limits_{\substack{x\to 1\\ y\to 1}}(3x-y)=2$; b) $\lim\limits_{\substack{x\to 1\\ y\to -1}}\left(x^2+y^2\right)=2$.

Limitlarni toping

31. $\lim\limits_{\substack{x\to 0\\ y\to 0}}\frac{\sqrt{x^2+y^2+1}-1}{x^2+y^2}$;

32. $\lim\limits_{\substack{x\to 0\\ y\to 0}}\frac{\sin\left(x^2 y\right)}{x\,y}$;

33. $\lim\limits_{\substack{x\to 0\\ y\to 0}}\frac{\sin\left(x^3+y^3\right)}{x^2+y^2}$;

34. $\lim\limits_{\substack{x\to 0\\ y\to 0}}\frac{\sqrt{1+x^2y^2}-1}{x^2+y^2}$;

35. $\lim\limits_{\substack{x\to 2\\ y\to 2}}\frac{x^2-y^2}{x^2+2x-xy-2y}$;

36. $\lim\limits_{\substack{x\to 0\\ y\to 0}}(1+xy)^{\frac{1}{|x|+|y|}}$;

37. $\lim\limits_{\substack{x\to 2\\ y\to 2}}\frac{x+y}{x-y}$ mavjudmi?

38. $\lim\limits_{\substack{x\to 0\\ y\to 0}}\frac{2xy}{x^2+y^2}$ mavjudmi?

$(0;0)$ ***nuqtada quyidagi funksiyalarning uzluksizligini tekshiring***.

39. $f(x,y)=\frac{xy}{\sqrt{xy+1}-1}$, $f(0;0)=2$;

40. $f(x,y)=\frac{\sin(xy)}{x}$, $f(0;0)=0$;

41. $f(x,y)=\frac{tg(xy)}{x}$, $f(0;0)=0$;

42. $f(x,y)=\frac{1-\sqrt{xy+1}}{xy}$, $f(0;0)=-\frac{1}{2}$.

Quyidagi funksiyalarning uzulish nuqtalarini toping.

43. $z=\frac{y-1}{(x+1)^2+y^2}$;

44. $z=\frac{x+y}{x^2+y^2}$;

45. $z=\frac{x+3y}{2y-x}$;

46. $z=\frac{4x-y}{x+y^2}$;

47. $z=\frac{5x+2y^2}{\sqrt{xy}}$;

48. $z=\frac{1}{4-x^2-y^2}$.

3-§. Xususiy hosilalar va to`la differensial

1. $z=f(x,y)$ *funksiyaning* $M(x,y)$ *nuqtadagi to`la orttirmasi* $\Delta z=f(x+\Delta x,y+\Delta y)-f(x,y)$ *ni* $\Delta z=A\cdot\Delta x+B\cdot\Delta y+a\cdot\Delta x+\beta\cdot\Delta y$ *ko`rinishda yozish mumkin bo`lsa,* $z=f(x,y)$ *funksiya* $M(x,y)$ *nuqtada differensiallanuvchi deyiladi. Bu yerda A, B lar* $\Delta x$, $\Delta y$ *larga bog`liq emas,* $\Delta x\to 0,\Delta y\to 0$ *da* $\alpha\to 0,\ \beta\to 0$ *bo`ladi.* $M(x,y)$ *nuqtada differensiallanuvchi* $z=f(x,y)$ *funksiya to`la orttirmasining bosh qismi* $A\Delta x+B\Delta y$ *uning to`la differensiali deyiladi va* $dz=\frac{\partial z}{\partial x}dx+\frac{\partial z}{\partial x}dy$ *formula yordamida topiladi.*

2. $z=f(x,y)$ *funksiyaning* $M(x,y)$ *nuqtadagi l yo`nalishi bo`yicha hosilasi deb* $\frac{\partial f}{\partial l}=\lim\limits_{M'\to M}\frac{f(M')-f(M)}{\rho(M',M)}$ *ga aytiladi (*M' *nuqta M nuqtadan o`tuvchi l to`g`ri chiziqda yotadi,* $\rho(M',M)$ *esa orientirilgan* MM' *kesmaning uzunligi).*

Agar $z=f(x,y)$ *funksiyaning* $M(x,y)$ *nuqtada differensiallanuvchi va l yo`nalish koordinata o`qlari bilan* α *va* β *burchaklari tashkil qilsa, u holda*

$$\frac{\partial f}{\partial l}=\frac{\partial f}{\partial x}\cos\alpha+\frac{\partial f}{\partial y}\cos\beta.$$

3. $\overrightarrow{grad}\,f=\frac{\partial f}{\partial x}\vec{i}+\frac{\partial f}{\partial y}\vec{j}$ *vector* $z=f(x,y)$ *funksiyaning* $M(x,y)$ *nuqtadagi gradienti deyiladi.*

***49-68 misollarda berilgan funksiyalarning xususiy hosilalarini toping*.**

49. $z = x^2 - 3y^2 + 5xy$;

50. $z = x^3 + 6xy^2 - 4y^3 - 2xy$;

51. $z = (5x^2 y - y^3 + 7)^3$;

52. $z = \sqrt{x^2 - y^2}$;

53. $z = \ln(x + \sqrt{x^2 + y^2})$;

54. $z = \ln(x + \ln y)$;

55. $z = e^{x^2 \sin y}$;

56. $z = ye^{-xy}$;

57. $z = arctg \dfrac{x}{y}$;

58. $z = \ln \arcsin(xy)$;

59. $z = y^x$;

60. $z = x^{y^2}$;

61. $f(x, y) = x + y - \sqrt{x^2 + y^2}$ funksiya uchun $f'_x\ (3, 4)$, $f'_y\ (3, 4)$ larni hisoblang;

62. $f(x, y) = \sqrt[3]{x^2 + y^3}$ funksiya uchun $f'_x\ (1,1)$, $f'_y\ (1,1)$ larni hisoblang.

63. $u = \sqrt{x^2 + y^2 - z^2 + 2xz}$;

64. $u = \dfrac{x}{\sqrt{x^2 + y^2 + z^2}}$;

65. $u = e^{x(x^2 + y^2 + z^2)}$;

66. $u = \sin(x^2 + y^2 + z^2)$;

67. $u = x^{\frac{y}{z}}$;

68. $u = x^{y^z}$;

69. $z = \ln(x^2 + y^2)$ funksiya uchun $y\dfrac{\partial z}{\partial x} - x\dfrac{\partial z}{\partial y} = 0$ tenglikning o\`rinli ekanligini ko\`rsating.

70. $z = arctg \dfrac{y}{x}$ funksiya uchun $x\dfrac{\partial z}{\partial x} + y\dfrac{\partial z}{\partial y} = 0$ tenglikning o\`rinli ekanligini ko\`rsating.

71. $u = (x - y)(y - z)(z - x)$ funksiya $\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} + \dfrac{\partial u}{\partial z} = 0$ tenglikni qanotlantirishini ko\`rsating.

72. $u = x + \frac{x-y}{y-z}$ funksiya $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 1$ tenglikni qanotlantirishini ko`rsating.

Funksiyalarning to`la differensiallarini toping.

73. $z = x^2 y^3$;

74. $z = \sqrt{x^2 - y^2}$;

75. $z = x^2 y^4 - x^3 y^3 + x^4 y^2$;

76. $z = xy - x^2 y^3 + x^3 y$;

77. $z = e^{y^2 - xy}$;

78. $z = \cos(xy)$;

79. $z = arctg\,(xy)$;

80. $z = \arcsin\frac{x}{y}$;

81. $u = x^2 yz^4$;

82. $u = \ln\left(x^3 - y^3 + 2z^3\right)$;

83. $u = \frac{y}{xz}$;

84. $u = xy^z$;

85. $f(x,y) = 3x^4 + xy + y^3$ funksiyaning (1, 2) nuqtadagi abtsissa o`qi bilan $135^0$ burchak tashkil qiluvchi yo`nalish bo`yicha hosilasini toping.

86. $f(x,y) = arctg(xy)$ funksiyaning (1, 1) nuqtadagi birinchi chorak burchagi bissektrisasi yo`nalishi bo`yicha hosilasini toping.

87. $f(x,y) = x^3 - 3x^2 y + 3xy^2 + 1$ funksiyaning (3, 1) nuqtadagi shu nuqtadan (6, 5) nuqtaga qaratilgan yo`nalish bo`yicha hosilasini toping.

88. $f(x,y) = x^2 y^2 - xy^3 - 3y - 1$ funksiyaning (2, 1) nuqtadagi shu nuqtadan koordinata boshiga qaratilgan yo`nalish bo`yicha hosilasini toping.

89. $f(x,y) = arctg\frac{y}{x}$ funksiyaning $x^2 + y^2 - 2x = 0$ aylanada yotuvchi $(\frac{1}{2}; \frac{\sqrt{3}}{2})$ nuqtada aylananing shu nuqtadagi yo`nalishi bo`yicha hosilasini toping.

90. $f(x,y) = \frac{y^2}{x}$ funksiyaning $2x^2 + y^2 = 1$ ellipsning ixtiyoriy nuqtasida ellipsning o`sha nuqtadagi normali yo`nalishi bo`yicha olingan hosila nolga tengligini isbotlang.

Funksiyalarning gradientlarini toping

91. $z=\sqrt{4+x^2+y^2}$, $(2,1)$ nuqtada;

92. $z=arctg\dfrac{x}{y}$, $(1;2)$ nuqtada;

93. $z=x^2+y^2$, $(x_0,\, y_o)$ nuqtada;

94. $z=2xy$, $(x_0,\, y_o)$ nuqtada.

4-§. Murakkab funksiyalarni differensiallash

95. $z=e^{x-3y}$, $x=\sin t$, $y=t^2$. $\dfrac{dz}{dt}$ ni toping;

96. $z=\dfrac{y}{x}$ $x=e^t$, $y=1-e^{2t}$. $\dfrac{dz}{dt}$ ni toping;

97. $z=\arcsin(x-y)$, $x=4t^2$, $y=t^3$. $\dfrac{dz}{dt}$ ni toping;

98. $z=e^{x^2+y^2}$, $x=a\cos t$, $y=a\sin t$. $\dfrac{dz}{dt}$ ni toping;

99. $z=\ln\dfrac{x}{y}$, $x=tg^2 t$, $y=ctg^2 t$. $\dfrac{dz}{dt}$ ni toping;

100. $z=\ln\left(e^x+e^y\right)$, $y=x^2$. $\dfrac{dz}{dx}$ ni toping;

101. $z=\arcsin\dfrac{x}{y}$, $y=\sqrt{x^2+1}$. $\dfrac{dz}{dx}$ ni toping;

102. $z=x^2+y^2$, $x=u+v$, $y=u-v$ $\dfrac{\partial z}{\partial u}$, $\dfrac{\partial z}{\partial v}$ larni toping;

103. $z=\ln(x^2+y^2)$, $x=uv$, $y=\dfrac{u}{v}$. $\dfrac{\partial z}{\partial u}$, $\dfrac{\partial z}{\partial v}$ larni toping;

104. $z=x^2y-y^2x$, $x=u\cos v$, $y=u\sin v$. $\dfrac{\partial z}{\partial u}$, $\dfrac{\partial z}{\partial v}$ larni toping;

105. $u=\sin\left(x-3y-2z\right)$, $x=2t^3$, $y=t^2$, $z=-t^4$; $\dfrac{du}{dt}$ ni toping;

106. $u=tg\left(x^2+4y-z\right)$, $y=\sqrt{x}$, $z=-\dfrac{1}{x}$. $\dfrac{du}{dx}$ ni toping.

5-§. Oshkormas funksiyalarning hosilalari

$y = f(x)$ *oshkormas funksiya* $F(x, y) = 0$ *tenglik bilan berilgan bo`lsa, u holda* $\frac{dy}{dx} = \frac{F'_x}{F'_y}$. *Xuddi shu kabi* $\Phi(x, y, z) = 0$ *dan* $\frac{\partial z}{\partial x} = -\frac{\Phi'_x}{\Phi'_z}$, $\frac{\partial z}{\partial y} = -\frac{\Phi'_y}{\Phi'_z}$

107-116 misollarda $\frac{dy}{dx}$ ***ni toping.***

107. $x^3 y - y^3 x = 16$;
108. $x^2 + y^2 + \ln(x^2 + y^2) = 1$;
109. $xy - \ln y = 0$;
110. $ye^x + e^y = 0$;
111. $x^y = y^x$;
112. $x + y = e^{\frac{y}{x}}$;
113. $\frac{x^2}{d^2} - \frac{y^2}{b^2} = 1$;
114. $e^x \sin y = e^y \cos x$;
115. $x^3 + y^3 + 6xy = 0$;
116. $\sin y + e^x - xy^2 = 0$.

117-124 misollarda $\frac{\partial z}{\partial x}$ ***va*** $\frac{\partial z}{\partial y}$ ***larni toping***.

117. $z = \sqrt{xy}$;
118. $x + y + z = e^z$;
119. $x^3 + y^3 + z^3 = 3xyz$;
120. $x^2 + y^2 + z^2 - 8x = 0$;
121. $e^z = \cos x \sin y$;
122. $z^3 + 3xyz = a^3$;
123. $e^Z - xyz = 0$;
124. $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.

6-§. Yuqori tartibli hususiy hosilalar va yuqori tartibli differensiallar

1. Ikki argumentli funksiya uchun Teylor formulasi quyidagicha:

$$\Delta f(x,y) = df(x,y) + \frac{1}{2!}d^2 f(x,y) + ... + \frac{1}{n!}d^n f(x,y) +$$

$$+\frac{1}{(n+1)!}d^{n+1}\cdot f(x+\theta\Delta x, y+\theta\Delta y),\ 0<\theta<1$$

yoki

$$f(x+\Delta x, y+\Delta y) = f(x,y) + \frac{\partial f}{\partial x}\Delta x + \frac{\partial f}{\partial y}\Delta y +$$

$$+\frac{1}{2}\left(\frac{\partial^2 f}{\partial x^2}\Delta x^2 + 2\frac{\partial^2 f}{\partial x \partial y}\Delta x \Delta y + \frac{\partial^2 f}{\partial y^2}\Delta y^2\right) + ... + R_n$$

bu yerda R_n – qoldiq had.

2. Sirtga o`tkazilgan urinma tekislik va normalning tenglamasi.

Agar sirt $F(x,y,z)=0$ tenglama bilan berilgan bo`lsa, uning (x_0, y_0, z_0) nuqtasidagi urinma tekislik tenglamasi

$$F'_x(x_0,y_0,z_0)(x-x_0) + F'_y(x_0,y_0,z_0)\cdot(y-y_0) + F'_z(x_0,y_0,z_0)(z-z_0) = 0,$$

normal tenglamasi

$$\frac{x-x_0}{F'_x(x_0,y_0,z_0)} = \frac{y-y_0}{F'_y(x_0,y_0,z_0)} = \frac{z-z_0}{F'_z(x_0,y_0,z_0)}$$

ko`rinishda bo`ldi.

***125-134 misollarda ko`rsatilgan tartibli hususiy hosilalarni toping*.**

125. $z = x + y + \dfrac{xy}{x-y}$, $\quad \dfrac{\partial^2 z}{\partial x^2}, \dfrac{\partial^2 z}{\partial x \partial y}, \dfrac{\partial^2 z}{\partial y^2}$;

126. $z = xe^y$, $\dfrac{\partial^2 z}{\partial x^2}, \dfrac{\partial^2 z}{\partial x \partial y}, \dfrac{\partial^2 z}{\partial y^2}$;

127. $z = y\ln x$, $\dfrac{\partial^2 z}{\partial x^2}, \dfrac{\partial^2 z}{\partial x \partial y}$;

128. $z = \sqrt{2xy + y^2}$, $\dfrac{\partial^2 z}{\partial x \partial y}$;

129. $z = \ln(y + \sqrt{x^2 + y^2})$, $\dfrac{\partial^2 z}{\partial y^2}$;

130. $z = xe^{xe^y}$, $\dfrac{\partial^2 z}{\partial x^2}, \dfrac{\partial^2 z}{\partial x \partial y}, \dfrac{\partial^2 z}{\partial y^2}$;

131. $z=\ln(x+y)$, $\frac{\partial^3 z}{\partial x^2 \partial y}$;

132. $z=\sin(xy)$, $\frac{\partial^3 z}{\partial x \partial y^2}$;

133. $z=e^{xy^2}$, $\frac{\partial^3 z}{\partial x \partial y^2}$;

134. $u=3^{xyz}$, $\frac{\partial^3 u}{\partial x \partial y \partial z}$;

135. $z=e^x \cos y$, $\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}=0$ tenglikni o`rinli ekanligini ko`rsating.

136. $z=\ln(x^2+y^2)$. $\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}=0$ tenglikni o`rinli ekanligini ko`rsating.

***Quyidagi funksiyalarning ko`rsatilgan tartibli differensiallarini toping*.**

137. $z=x+xy$, $d^2 z$;

138. $z=e^{x+y^2}$, $d^2 z$;

139. $z=\ln(x-y)$, $d^2 z$;

140. $z=\frac{x}{x+y}$, $d^2 z$;

141. $z=x\sin^2 y$, $d^2 z$;

142. $u=x+y+xy$, $d^3 z$;

143. $z=e^{x+y}$, $d^n z$;

144. $x^2+y^2+z^2=2z$, $d^2 z$.

Urinma tekislik va normal tenglamalarini yozing

145. $z=xy$, $(2;1;2)$ nuqtada;

146. $z=x^2+y^2$, $(1;1;2)$ nuqtada;

147. $z=\sin\frac{x}{y}$, $(\pi;1;0)$ nuqtada;

148. $z=arctg\frac{y}{x}$, $\left(1;1;\frac{\pi}{4}\right)$ nuqtada;

149. $x^2+y^2+z^2-1=0$, $(x_0;y_0;z_0)$ nuqtada;

150. $(z^2-x^2)xyz-y^5=5$; $(1;1;2)$ nuqtada;

151. $x^2+2y^2+z^2=1$ ellipsoidning $x-y+2z=0$ tekislikka parallel bo`lgan urinma tekislik tenglamasini tuzing.

152. $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ ellipsoidning koordinata musbat yarim o`qlarida bir hil kesma ajratuvchi urinma tekislik tenglamasini tuzing.

Quyidagi funksiyalar uchun Teylor formulasini yozing (birinchi va ikkinchi tartibli hosillar bilan cheklaning)

153. $z=\frac{1}{x-y}$;

154. $z=\ln(x+y)$;

155. $z=e^{xy}$;

156. $z=\sin x\cdot\cos y$;

157. $z=arctg\frac{x}{y}$;

158. $z=e^{x}\cos y$.

7-§. Ko`p argumentli funksiyalarning ekstremumlari. Shartli ekstremumlar

$z=f(x,y)$ funksiya $M(x_0,y_0)$ nuqtada ekstremumga ega bo`lib, $f'_x(x_0,y_0)$, $f'_y(x_0,y_0)$ xususiy hosilalar mavjud bo`lsa, u holda $f'_x(x_0,y_0)=0, f'_y(x_0,y_0)=0$ bo`ladi.

$f''_{x^2}(x_0,y_0)=A, f''_{y^2}(x_0,y_0)=C, f''_{xy}(x_0,y_0)=B$ deb olaylik.

Agar $\Delta=A\cdot C-B^2>0$ bo`lsa, $M(x_0,y_0)$ nuqtada funksiya ekstremumga ega bo`lib, A<0 bo`lsa, maksimum, A>0 bo`lsa minimum bo`ladi.

Quyidagi funksiyalarning ekstremum nuqtalarini toping.

159. $z=3x+6y-x^2-xy+y^2$;

160. $z=e^{\frac{x}{2}}\left(x+y^2\right)$;

161. $z=2x^3-xy^2+5x^2+y^2$;

162. $z=3\ln x+xy^2-y^3$;

163. $z=x^2+y^2-8x-2$;

164. $z=3x^2-y^2+4y+5$;

165. $z=x^3+xy^2+6xy$;

166. $z=x^3+8y^3+6xy-1$;

167. $z=\left(x^2+y^2\right)^{\frac{2}{3}}+4$;

168. $z = 1 - x^4 - (y-2)^6$;

169. $e^z - xyz + x^2y^2 = 0$;

170. $3x^2 + 5y^2 + 2z^2 - 2xy = 0$;

171. $z = x^2 + xy + y^2 + \frac{a^3}{x} + \frac{a^3}{y}$ funksiyani $x = y = \frac{a}{\sqrt[3]{3}}$ nuqtada minimumga ega ekanligini tekshiring .

172. $z = x^4 + y^4 - 2x^2 - 4xy - 2y^2$ funksiyani $x = y = \sqrt{2}$ nuqtada minimumga ega ekanligini tekshiring.

Qu`yidagi funksiyalarni ko`rsatilgan yopiq sohalardagi eng kichik va eng katta qiymatlarni toping.

173. $z = x^2y$; $x^2 + y^2 \le 1$ doirada;

174. $z = x^2 - y^2$; $x^2 + y^2 \le 4$ doirada;

175. $z = x^2 + 3y^2 + x - y$; $x \le 0$, $y \ge 0$, $y - x \le 1$ uchburchakda.

176. $z = x^3 + y^3 - 9xy - 25$; $0 \le x \le 5$, $0 \le y \le 5$ kvadratda

177. $z = x^2 + y^2 - xy + x + y$. $x = 0$, $y = 0$ $x + y = -3$ tug`ri chiziqlar bilan chegaralangan uchburchakda;

178. $z = x^3 + 8y^3 - 6xy + 1$ $x = 0$, $x = 2$, $y = -1$, $y = 1$ tug`ri burchakli to`rtburchaktda;

Shartli ekstemumlarni toping.

179. $z = \frac{1}{x} + \frac{1}{y}$, $\frac{1}{x^2} + \frac{1}{y^2} = 1$;

180. $z = x^2 + y^2$. $x + y = 1$;

181. $z = e^{xy}$, $x + y = a$;

182. $z = x^m + y^m$ $(m > 1)$, $x + y = 2$ $(x \ge 0, y \ge 0)$;

183. $A(1;0)$ nuqtadan $4x^2 + 9y^2 = 36$ ellipsgacha bo`lgan eng qisqa masofani toping.

184. $A(-1;5)$ nuqtadan $y^2 = x$ parabolagacha bo`lgan eng qisqa masofani toping.

185. $x - y = 5$ to`g`ri chiziq va $y = x^2$ parabola orasidagi eng qisqa masofani toping.

186. $\frac{x^2}{4} + \frac{y^2}{9} = 1$ ellipsda $3x + y - 9 = 0$ tug`ri chiziqqa eng yaqin va eng uzoq nuqtalarni toping.

XII-BOB. KARRALI INTEGRALLAR

1-§. Ikki karrali integrallar

1. To`g`ri to`rtburchak bo`yicha ikki karrali integral. Agar integrallash sohasi $D=\left\{(x,y)\middle|\begin{matrix} a\le x\le b \\ c\le y\le d\end{matrix}\right\}$ *to`g`ri to`rtburchakdan iborat bo`lsa, u holda ikki karrali integral quyidagi ikki formulaning biri bilan hisoblanadi:*

$$\iint_D f(x,y)dxdy=\int_c^d dy\int_a^b f(x,y)dx, \qquad (1)$$

$$\iint_D f(x,y)dxdy=\int_a^b dx\int_c^d f(x,y)dy. \qquad (2)$$

(1) va (2) tengliklarning o`ng tomonidagi integrallar takroriy integrallar deyiladi. (1) dagi* $\int_a^b f(x,y)\,dx$ *ichki integral deyiladi va [a; b] da y ni o`zgarmas deb x ga nisbatan hisoblab, biror $\varphi(y)$ *natija olinadi. So`ngra* $\int_c^d \varphi(y)\,dy$ *tashqi integral hisoblanadi.*

2. Agar $D=[a,b;c,d]$ *to`g`ri to`rtburchakda integral ostidagi funksiya* $f(x,y)=\varphi(x)\cdot\phi(y)$ *ko`rinishda bo`lsa, u holda*

$$\iint_D f(x,y)dxdy=\int_a^b \varphi(x)dx\int_c^d \phi(y)dy$$

ko`paytma shaklida yoziladi.

3. D soha shuday L yopiq egri chiziq bilan chegaralangaki, Oy o`qiga parallel bo`lgan har bir to`g`ri chiziq uni ko`pi bilan 2 nuqtada kesib o`tadi. D sohani Ox o`qiga proeksiyalasak, [a; b] hosil qilinadi L konturda A(a) va B(b) nuqtalar uni ikki bo`lakka ajratadi: ularning tenglamalari $y=\varphi_1(x)$ *va* $y=\varphi_2(x)$ $\varphi_1(x)\le\varphi_2(x)$ $x\in[a;b]$ *bo`lsin. Bu holda D soha qisqacha:*

$$D:\left\{\begin{matrix} a\le x\le b \\ \varphi_1(x)\le y\le\varphi_2(x)\end{matrix}\right\}$$

ko`rinishda belgilanadi.

Ikki karrali integral takroriy integrallar yordamida

$$\iint_D f(x,y)dx\,dy=\int_a^b dx\int_{\varphi_1(x)}^{\varphi_2(x)} f(x,y)dy \qquad (1)$$

ko`rinishda yoziladi.

4. D sohani chegralovchi L yopiq kontur Ox ga parallel to`g`ri chiziqlar bilan eng ko`pi bilan ikkita umumiy nuqtaga ega bo`lsa, D sohani Oy ga proeksiyalab, [c;d] ni topamiz va $x=\phi_1(y),\ x=\phi_2(y),\ \phi_1(y)\le\phi_2(y)$ *larni hosil qilamiz. Bu holda* $\iint_{(D)} f(x,y)dxdy=\int_c^d dy\int_{\phi_1(y)}^{\phi_2(y)} f(x,y)dx$ *(2) ko`rinishda bo`ladi.*

5. Agar D soha 1 va 2 hollardagi shartlarni bir vaqtda qanoatlantirsa, u holda ikki karrali integral quyidagicha hisoblanadi.

$$\iint_D f(x,y)dxdy=\int_c^d dy\int_{\varphi_1(x)}^{\varphi_2(x)} f(x,y)dy=\int_c^d dy\int_{\phi_1(y)}^{\phi_2(y)} f(x,y)dx,$$

bu tengliklardan takroriy integrallarda integrallash tartibini o`zgartirish mumkinligi kelib chiqadi.

6. Agar soha 1 va 2-hollardagi shartlarni qanoatlantirmasa, u holda D sohani bir necha 1 yoki 2-shartlarni qanoatlantiradigan sohalarga ajratib integralning additivlik xossasidan foydalaniladi.

Takroriy integrallarni hisoblang

187. $\int_1^2 dx\int_0^1 xydy$;

188. $\int_2^3 dx\int_1^2 x^2ydy$;

189. $\int_1^2 dy\int_1^3 \frac{dx}{x^2}$;

190. $\int_2^3 dy\int_1^2 \frac{xdx}{y^2}$;

191. $\int_2^4 dx\int_0^{x^2} xdy$;

192. $\int_0^2 dx\int_0^x x^2dy$;

193. $\int_0^1 dy\int_{y^2}^{y} xdx$;

194. $\int_1^2 dy\int_0^{y^3} \frac{4}{y^3}dx$;

195. $\int_0^2 dx\int_0^x \left(x^2+2xy\right)dy$;

196. $\int_0^2 dy\int_0^{y^2} \left(x+2y\right)dx$;

197. $\int_0^1 dx \int_0^x e^{\frac{y}{x}} dx$;

198. $\int_0^5 dx \int_0^{5-x} \sqrt{4+x+y}\, dy$.

Ikki karrali integralni takroriy integralga keltiring: $\iint_D f(x,y)dxdy$

199. $D: \begin{cases} y^2 = x \\ x = 1 \end{cases}$;

200. $D: \begin{cases} xy = 6 \\ x + y - 7 = 0 \end{cases}$;

201. $D: \begin{cases} y^2 = x \\ y = x \end{cases}$;

202. $D: \begin{cases} y^2 = 4x \\ y \geq 0 \\ x = 4 \end{cases}$;

203. $D - x = 0,\ y = 0,\ x + y = 2$ to`g`ri chiziqlar bilan chegaralangan uchburchak;

204. $D - x^2 + y^2 \leq 1,\ x \geq 0,\ y \geq 0$;

205. $D - x + y \leq 1,\ x - y \leq 1,\ x \geq 0$;

206. $D - y \geq x^2,\ y \leq 4 - x^2$;

207. $D - y = x^2$ va $y = \sqrt{x}$ parabolalar bilan chegaralangan.

208. $D - y = x,\ y = x + 3,\ y = -2x + 1,\ y = -2x + 5$ to`g`ri chiziqlar bilan chegaralangan parallelogramm.

209-216 misollarda integrallash tartibini o`zgartiring.

209. $\int_0^1 dy \int_y^{\sqrt{y}} f(x,y)dx$;

210. $\int_{-1}^1 dx \int_0^{\sqrt{1-x^2}} f(x,y)dy$;

211. $\int_0^1 dx \int_{x^3}^{x^2} f(x,y)dy$;

212. $\int_0^2 dx \int_x^{2x} f(x,y)dy$;

213. $\int_0^1 dy \int_y^{2-y} f(x,y)dx$;

214. $\int_0^1 dx \int_{e^{-x}}^{e^x} f(x,y)dy$;

215. $\int_0^1 dx \int_0^x f(x,y)dy + \int_1^2 dx \int_0^{2-x} f(x,y)dy$;

216. $\int_0^1 dx \int_0^{x^2} f(x,y)dy + \int_1^3 dx \int_0^{\frac{3-x}{2}} f(x,y)dy$.

Ikki karrali integrallarni hisoblang

217. $\iint_D \frac{dxdy}{(x+y)^2}$, $D - x=3,\ x=4,\ y=1,\ y=2$ to\`g\`ri chiziqlar bilan chegaralangan to\`g\`ri burchakli to\`rtburchak.

218. $\iint_D xydxdy$, $D - y=0,\ y=0,\ y=1-x^2$ chiziqlar bilan chegaralangan soha.

219. $\iint_D (x+y)dxdy$, $D - x=0,\ y=0,\ x+y=3$ to\`g\`ri chiziqlar bilan chegaralangan uchburchak.

220. $\iint_D x\sqrt{y}dxdy$, $D - y=1,\ y=x$ va $y=3x$ to\`g\`ri chiziqlar bilan chegaralangan soha.

221. $\iint_D x^3y^2dxdy$, $D - x^2+y^2 \le R$ doira.

222. $\iint_D (x^2+y)dxdy$, $D - y=x^2,\ y^2=x$ parabolalar bilan chegaralangan soha.

223. $\iint_D xy^2dxdy$, $D - y^2=x,\ x=1$ chiziqlar bilan chegaralangan soha.

224. $\iint_D (x+y^2)dxdy$, $D - y^2=x+2,\ y=x$ chiziqlar bilan chegaralangan soha.

225. $\iint_D \sqrt{1-x^2-y^2}dxdy$, $D - x^2+y^2 \le 1$ doiraning birinchi kvadrantdagi qismi.

226. $\iint_D x^2y^2\sqrt{1-x^3-y^3}dxdy$, $D-$ koordinata o\`qlari va $x^3+y^3=1$ chiziq bilan chegaralangan soha.

2-§. Qutb koordinatalarida ikki karrali integral

1. Agar uzluksiz differensiallanuvchi $x=\varphi(u,\upsilon),\ y=\varphi(u,\upsilon)$ *funksiyalar D sohani S sohaga o`zaro bir qiymatli aks ettirsa, u holda o`zgaruvchilarni almashtirish formulasi*

$\iint\limits_D f(x,y)dxdy=\iint\limits_S f[\varphi(u,\upsilon),\ \phi(u,\ \upsilon)]|j|dud\upsilon$ *bo`ladi.*

Bunda $J(u,\upsilon)$ *yakobian quyidagicha hisoblanadi:*

$$J=\begin{vmatrix}\frac{\partial x}{\partial u} & \frac{\partial x}{\partial \upsilon}\\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial \upsilon}\end{vmatrix}.$$

2. $x=\rho\cos\varphi,\ y=\rho\sin\varphi$ *formulalar yordamida qutb koordinatalar sistemasiga o`tiladigan bo`lsa, yuqoridagi formulaga asosan:*

$$\iint\limits_D f(x,y)\,dxdy=\iint\limits_S f(\rho\cos\varphi,\rho\sin\varphi)\rho\,d\rho\,d\varphi$$

bo`ladi, bunda $|J|=\rho$.

3. Qutb koordinatlar sistemasida ikki karrali integralni takroriy integrallarga aylantirish quyidagicha bo`ladi:

a) agar S soha qutb o`qi bilan* $\varphi_1=a,\varphi_2=\beta$ *burchaklar tashkil qilgan nurlar va* $\rho=\rho_1(\varphi),\ \rho=\rho_2(\varphi),(\rho_1<\rho_2)$ *egri chiziqlar bilan chegaralangan bo`lsa,

$$\iint\limits_S f(\rho\cos\varphi,\ \rho\sin\varphi)\,\rho\,d\rho d\varphi=\int\limits_a^\beta d\varphi\int\limits_{\rho_1}^{\rho_2} f(\rho\cos\varphi,\rho\sin\varphi)\,\rho\,d\,\rho$$

bo`ladi.

b) Agar S soha koordinatalar boshi 0 nuqtani o`z ichiga olsa,

$$\iint\limits_S f(\rho\cos\varphi,\ \rho\sin\varphi)\,\rho\,d\rho\,d\varphi=\int\limits_0^{2\pi} d\varphi\int\limits_0^{\rho(\varphi)} f(\rho\cos\varphi,\rho\sin\varphi)\,\rho\,d\,\rho$$

bo`ladi.

$\iint\limits_D f(x,y)dxdy$ ***integralda qutb koordinatalariga o`ting***.

227. $D-x^2+y^2\le R^2$ doira.

228. $D-x^2+y^2\le ax$ doira.

229. $D-x^2+y^2\le by$ doira

230. $D-x^2+y^2=4x,\ x^2+y^2=8x$ aylanalar; $y=x$ va $y=2x$ to`g`ri

chiziqlar bilan chegaralangan soha.

231. $D - x^2 + y^2 \le ax$ va $x^2 + y^2 \le by$ doiralarning umumiy qismi.

232. $D - y = x,\ y = 0,\ x = 1$ to`g`ri chiziqlar bilan chegaralangan uchburchak.

Qutb koordinatalariga o`tib qo`yidagi integrallarni hisoblang.

233. $\iint\limits_D \sqrt{x^2 + y^2}\,dxdy,\quad D - x^2 + y^2 \le 9$ doiraning 1-kvadrantdagi qismi.

234. $\iint\limits_D ydxdy,\quad D - x^2 + y^2 \le 1$ doiraning yuqori yarim qismi.

235. $\iint\limits_D \frac{dxdy}{\sqrt{x^2 + y^2}},\quad D - x^2 + y^2 \ge 1,\quad x^2 + y^2 \le 4$ halqa.

236. $\iint\limits_D \frac{dxdy}{1 + x^2 + y^2},\quad D - x^2 + y^2 \le 1,\quad y \ge 0$ yarim doira.

237. $\iint\limits_D (x^2 + y^2)dxdy,\quad D - x^2 + y^2 \le 4x$ doira.

238. $\int\limits_0^R dx \int\limits_0^{\sqrt{R^2 - x^2}} \ln(1 + x^2 + y^2)dy$.

3-§. Ikki karrali integralning tatbiqlari

1. D sohaning yuzi ikki karrali integral yordamida $S_D = \iint\limits_D d\sigma$ *formula bo`yicha hisoblanadi.*

To`g`ri burchakli koordinatalarda yuz elementi $d\sigma = dxdy$ *bo`lgani uchun* $S_D = \iint\limits_D dxdy$ *bo`ladi. Egri chiziqli koordinatala sistemasida esa* $d\sigma = |J(u,\upsilon)|\,dud\upsilon$ *bo`lgani uchun* $S_D = \iint\limits_D |J(u,\upsilon)|\,dud\upsilon$ *bo`ladi. Xususiy holda qutb koordinatalar sistemasida* $|J| = \rho$ *bo`lib,* $S_D = \iint\limits_D \rho\, d\rho\, d\rho$ *bo`ladi.*

2. Quyidan z=0 tekisligi, yuqoridan $z = f(x,y) \ge 0$ *uzluksiz sirt, yon yoqlardan yasovchisi Oz o`qiga parallel, yo`naltiruvchisi D sohaning konturidan iborat bo`lgan silindrik sirt bilan chegaralangan jismning hajmi*

$$V = \iint\limits_D f(x,y)\,dxdy$$

formula bilan hisoblanadi.

3. Qutb koordinatalar sistemasida jism hajmi uchun ushbu

$$V=\iint_D f(\rho\cos\varphi,\ \rho\sin\varphi)\,\rho\,d\rho\,d\varphi$$

formul o`rinlidir.

4. a) Agar sirt $z=f(x,y)$ *tenglama berilib, uning* Oxy *tekisligiga proeksiyasi D bo`lsa, u holda sirt yuzi* $S=\iint_D\sqrt{1+\left(\frac{\partial f}{\partial x}\right)^2+\left(\frac{\partial f}{\partial y}\right)^2}\,dxdy$ *formula yordamida topiladi.*

239-250 masalalarda berilgan chiziqlar bilan chegaralangan tekis figuralarning yuzalarini hisoblang.

239. $y=0$, $y=4$, $y=-x$, $y=\frac{x-1}{2}$;

240. $y=\frac{9}{x}$, $y=x$, $x=6$;

241. $y^2=-x,\ x=-4$;

242. $y=x^2$, $x+y=6$;

243. $y^2=2x,\ y=-x$;

244. $y^2=\frac{b^2}{a}x$, $y=\frac{b}{a}x$;

245. $y=\sqrt{x},\ y=2\sqrt{x},\ x=4$;

246. $x^2+y^2-2ax=0$ va $x^2+y^2-2ay=0$ (Qutb koordinatalariga o`ting);

247. $x^2+y^2-2ax=0,\quad x^2+y^2-ax=0$;

248. $x^2+y^2=r^2$, $x^2+y^2-2ry=0$;

249. $\rho=3\cos\varphi$;

250. $\rho=a(1+\cos\varphi)$.

251-266 misollarda ko`rsatilgan sirtlar bilan chegaralangan jismlarning hajmlarini hisoblang.

251. $3x+2y+z-6=0,\ x=0,\ y=0,\ z=0$;

252. $y=\sqrt{x},\ y=2\sqrt{x},\ x+z=4,\ z=0$;

253. $z=x^2+y^2, x+y=3,\ x=0,\ y=0,\ z=0$;

254. $z=x^2+y^2+1,\ \ x=4,\ y=4,\ x=0,\ y=0,\ z=0$;

255. $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1,\ \ x=0,\ \ y=0,\ \ z=0$;

256. $z = x^2 + y^2,\ z = 0,\quad y = 1,\ y = 2x,\ y = 6 - x$;

257. $y = x^2,\ y + z = 2,\ z = 0$;

258. $x^2 + y^2 = 1,\quad x + y + z = 3,\quad z = 0$;

259. $z = x^2 - y^2,\quad x = 1,\ y = 0,\quad z = 0$;

260. $x^2 + y^2 = 9,\quad x^2 + z^2 = 9$;

261. $z = 2 - \sqrt{x^2 + y^2},\quad z = 0$ (Qutb koordinatasiga o`ting).

262. $x^2 + y^2 = 9,\quad z = 5x,\quad z = 0$;

263. $z = 1 - x^2 - y^2,\quad y = x,\quad y = x\sqrt{3},\quad z = 0$ (I oktantdagi qismi).

264. $z = x^2 + y^2,\quad x^2 + y^2 = 2x,\quad z = 0$;

265. $z = 4 - x^2 - y^2,\ z = \dfrac{2 + x^2 + y^2}{2}$;

266. $x^2 + y^2 + z - 4 = 0,\quad z = 0$.

267-274 masalalarda berilgan sirt bo`laklarining yuzini hisoblang.

267. $6x + 3y + 2z = 12$ tengsizlikning I oktantdagi bo`lagi.

268. $x^2 + y^2 = R^2$ tsilindrning $z = 0,\ z = H$ tekisliklar orasidagi bo`lagi.

269. $x^2 + y^2 = R^2$ tsilindrning $z = 0,\ z = kx$ tekisliklar orasidagi bo`lagi.

270. $x^2 + z^2 = 9$ tsilindrning $x^2 + y^2 = 9$ tsilindr bilan kesilgan bo`lagi.

271. $x^2 + y^2 = 2z$ paraboloidning $x^2 + y^2 = 3$ tsilindr ichida joylashgan bo`lagi.

272. $z = \sqrt{x^2 + y^2}$ konusning $x^2 + y^2 = 2x$ tsilindr ichida joylashgan bo`lagi.

273. $z^2 = x^2 + y^2$ konusning $z^2 = 2py$ tsilindr bilan kesilgan bo`lagi.

274. $x^2 + y^2 + z^2 = 9$ sferaning $x^2 + y^2 = 4$ tsilindr bilan kesilgan bo`lagi.

Qo`yidagi bir jinisli tekis figuraning og`irlik markazini koordinatalarini toping.

275. $D - y = x^2$ parabola va $x + y = 2$ to`g`ri chiziqlar bilan chegaralangan soha.

276. $y = \sqrt{R^2 - x^2},\ y \geq 0$;

277. $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1,\quad y \geq 0$;

278. $D - y = \sqrt{2x - x^2}$ va $y = 0$ chiziqlar bilan chegaralangan soha.

4-§. Uch karrali integralni hisoblash

1. Uch o`lchovli fazoda T jism berilgan bo`lib, unda $u = f(x, y, z)$ *funksiya aniqlangan bo`lsin.*

Agar T jism quyidagi tengsizliklar bilan aniqlangan bo`lsa,

$$T = \left\{ (x, y\, z) \left| \begin{array}{l} a \le x \le b \\ \varphi_1(x) \le y \le \varphi_2(x) \\ \phi_1(x, y) \le z \le \phi_2(x, y) \end{array} \right. \right\},$$

U holda uch karrali integral

$$\iiint\limits_T f(x, y, z)dxdydz = \int_a^b dx \int_{\varphi_1(x)}^{\varphi_2(x)} dy \int_{\phi_1(x,y)}^{\phi_2(x,y)} f(x, y, z)dz$$

formula yordamida hisoblanadi.

Takroriy integrallarni hisoblang.

279. $\int_0^2 dx \int_0^3 dy \int_0^4 (x + y + z)dz$;

280. $\int_{-1}^1 dx \int_{x^2}^1 dy \int_0^2 (z + 4)dz$;

281. $\int_0^2 dx \int_0^x dy \int_0^y xyzdz$;

282. $\int_0^{\frac{1}{2}} dx \int_x^{2x} dy \int_0^{\sqrt{1-x^2-y^2}} zdz$.

283-292 misollarda uch karrali integrallarni hisoblang.

283. $\iiint\limits_G (x^2 + y^2 + z^2)dxdydz, \quad x = 0, \quad x = a, \quad y = 0, \quad y = b, \quad z = 0, \quad z = c$;

284. $\iiint\limits_G ydxdydz, \quad x = 0, \quad x = 2, \quad y = 0, \quad y = 1, \quad z = 0, \quad z = 1 - y$;

285. $\iiint\limits_G xz^2dxdydz, \quad x = \sqrt{2y - y^2}, \quad x = 2, \quad y = 0, \quad y = 2, \quad z = 0, \quad z = 3$;

286. $\iiint\limits_G (2x + 3y - z)dxdydz, \quad x = 0, \quad y = 0, \quad x + y = 3, \quad z = 0, \quad z = 4$;

287. $\iiint\limits_G xyzdxdydz, \quad x = 0, \quad y = 0, \quad z = 0, \quad x + y + z = 1$;

288. $\iiint\limits_G xy^2z^3dxdydz, \quad x = 1, \quad y = x, \quad z = 0, \quad z = xy$;

289. $* \iiint\limits_G z^2 dxdydz, \quad z = x^2 + y^2, \quad z = 2, \quad z = 6$;

290. $\iiint\limits_G z^3 dxdydz, \quad z = 4 - x^2 - y^2, \quad z = 0, \quad z = 3$ (289-misolning javobiga qarang).

291. $\iiint\limits_G \frac{1}{z} dxdydz, \ (x-1)^2 + y^2 = 1 \quad z = 3, \quad z = 6$ (289-misolning javobiga qarang).

292. $\iiint\limits_G \frac{1}{z^2} dxdydz, \quad x^2 + y^2 + z^2 = 9, \quad z = 1, \quad z = 2$ (289-misolning javobiga qarang).

5-§. Silindrik va sferik koordinatalarda uch karrali integral

1. $x = \rho\cos\varphi, \ y = \rho\sin\varphi, z = z$ *silindrik koordinatalarga o`tib, uch karrali integralni hisoblash mumkin:*

$$(0 \le \rho < +\infty, \ 0 \le \varphi \le 2\pi, \ -\infty < z < +\infty)$$

$$\iiint\limits_G f(x,y,z)dxdydz = \iiint\limits_G f(\rho\cos\varphi, \rho\sin\varphi, z)\,\rho\, d\rho\, d\varphi\, dz.$$

2. $x = \rho\cos\varphi, \ \sin\theta, y = \rho\sin\varphi\sin\theta, z = \rho\cos\theta$ *sferik koordinatalarga o`tib, uch karrali integralni hisoblash mumkin:*

$$(0 \le \rho < +\infty, \ 0 \le \varphi \le 2\pi, \ 0 \le \theta \le \pi)$$

$$\iiint\limits_G f(x,y,z)dxdydz = \iiint\limits_G f(\rho\cos\varphi\,\sin\theta, \ \rho\sin\varphi\sin\theta, \ \rho\cos\theta)\rho^2\sin\theta\, d\rho\, d\varphi\, d\theta.$$

Silindrlik koordinatalarga o`tib uch karrali integralni hisoblang.

293. $\iiint\limits_G (x^2 + y^2) dxdydz; \ G - z = \sqrt{9 - x^2 + y^2}$ yarim sfera va $z = 0$ tengsizlik bilan chegaralangan soha.

294. $\iiint\limits_G z\sqrt{x^2 + y^2}\, dxdydz, \ G - \ x^2 + y^2 = 2x$ tsilindr va $z = 0, \ z = 3$ teksliklar bilan chegaralangan soha.

295. $\iiint\limits_G zdxdydz; \ G - z^2 = x^2 + y^2$ konus va $z = 2$ tekisizlik bilan chegaralangan soha.

296. $\iiint\limits_G (x^2 + y^2)dxdydz; \ G - z = \frac{1}{2}(x^2 + y^2)$ aylanma paraboloid va $z = 2$ tengsizlik bilan chegaralangan soha.

Sferik koordinatalarga o`tib uch karrali integralni hisoblang.

297. $\iiint\limits_G x^2\, dxdydz; \ G - x^2 + y^2 + z^2 \le R^2$ shar.

298. $\iiint\limits_G \left(x^2+y^2+z^2\right)dxdydz;\ \ G - x^2+y^2+z^2 \le R^2$ shar.

299. $\iiint\limits_G z\,dxdydz;\ \ G - x^2+y^2+z^2=4$ sfera va $z=\sqrt{x^2+y^2}$ konus bilan chegaralangan soha (Konusning ichki qismi).

300. $\iiint\limits_G \dfrac{dxdydz}{\sqrt{x^2+y^2+z^2}};\ \ G - x^2+y^2+z^2=4$ va $x^2+y^2+z^2=16$ sferalar bilan chegaralangan soha.

6-§. Uch karrali integralning tatbiqlari

Agar G da $f(x,y,z)=1$ *bo`lsa, u holda G jismning hajmi* $V=\iiint\limits_G dxdydz$ *formula yordamida hisoblanadi.*

Agar G jismning zichligi ρ bo'lsa, og`irlik markazining koordinatalari quyidagicha aniqlanadi:

$$\bar{x}=\frac{\iiint\limits_G \rho\, x dx\, dydz}{\iiint\limits_G \rho\, dx\, dydz},\qquad \bar{y}=\frac{\iiint\limits_G \rho\, y\, dx\, dy\, dz}{\iiint\limits_G \rho\, dx\, dy\, dz},\qquad \bar{z}=\frac{\iiint\limits_G \rho\, z\, dx\, dy\, dz}{\iiint\limits_G \rho\ dx\, dy\, dz}$$

***301-312 masalalarda uch karrali integrallar yordamida ko`rsatilgan sirtlar bilan chegaralangan jisimlarning hajmlarini hisoblang*.**

301. $x=0,\ y=1,\ y=3,\ z=0,\ x+2z=3$;

302. $y=4-x^2,\ y=x^2+2,\ z=-1,\ z=2$;

303. $y^2=\dfrac{x}{2},\ x+2y+z=4,\ y=0\ \ z=0$;

304. $x^2+y^2=z+1,\ \ z=3$;

305. $x^2+y^2=1,\ y+z=2$;

306. $z=x^2+y^2,\ z=\sqrt{x^2+y^2}$;

307. $x^2+y^2=9-2z,\ x^2+y^2=1,\ z=0$ (tsilindrning tashqarisida).

308. $3z=10-x^2-y^2,\ \ z=\sqrt{x^2+y^2}$;

309. $x^2+y^2+z^2=4,\ \ x^2+y^2=2x$ (tsilindrning ichi).

310. $x^2+y^2+z^2=4,\ \ \ x^2+y^2=3x$ (paraboloidning ichi).

311. $\left(x^2+y^2+z^2\right)^2=a^3x,\qquad a>0$;

312. $\left(x^2+y^2+z^2\right)^2=axyz,\qquad a>0$.

***313-316 masalalarda bir jinsli jismlarning og`irlik markazlarining koordinatalarini toping*.**

313. $z = 9 - x^2 - y^2$ paraboloid va $z = 0$ tekislik bilan chegaralangan jism.

314. $z^2 = xy,\ \ x = 5,\ \ y = 5,\ z = 0$ sirtlar bilan chegaralangan jism.

315. $x^2 + y^2 + z^2 = 3$ sfera va $x^2 + y^2 = 2z$ paraboloid bilan chegaralangan jism.

316. $x^2 + y^2 + z^2 = 16$ sfera va $z = \sqrt{x^2 + y^2}$ konus bilan chegaralangan jism.

XIII-BOB. EGRI CHIZIQLI INTEGRALLAR

1-§. Birinchi tur egri chiziqli integrallar (yoy uzinligi bo`yicha olingan egri chiziqli integrallar)

1. Tekislikdagi to`g`rilanuvchi sodda egri chiziqda "nuqta funksiyasi" $f(M) = f(x, y)$, egri chiziqni $A_i A_{i+1}$ bo`laklarga bo`lish usuli $T = \{A_0, A_1, ..., A_n\}$ berilgan bo`lsin. Har bir $A_i\, A_{i+1}$ bo`laklarda ixtiyoriy $M_i(\xi_i, \eta_i)$ nuqta tanlab,

$$S_T(f) = \sum_{i=0}^{n-1} f(M_i)\Delta s_i = \sum_{i=0}^{n-1} f(\xi_i, \eta_i)\Delta s_i \qquad (1)$$

integral yig`indini tuzamiz, bu yerda Δs_i bilan $A_i\, A_{i+1}$ yoy uzunligi belgilangan.

Agar $\lambda(T) = \max \Delta s_i \to 0$ da (1) integral yig`indining bo`lish usuli T ga va M_i nuqtalarning tanlab olinishiga bog`liq bo`lmagan chekli limiti mavjud bo`lsa, bu limit f(x,y) funksiyadan G egri chiziq bo`yicha olingan birinchi tur egri chiziqli integral deyiladi va quyidagicha belgilanadi:

$$I = \int_{\Gamma} f(x, y)\, ds \qquad (2)$$

bu yerda ds – yoy differensali.

2. Oddiy aniq integralga keltirish.

AB chiziq $x = \varphi(t),\ y = \psi(t),\ (\alpha \le t \le \beta)$ parametrik tenglamalar bilan berilgan bo`lsin, bu yerda $\varphi(t)$ va $\psi(t)$ hosilalari bilan birga uzluksiz bo`lgan funksiyalar. U holda (2) egri chiziqli integral quyidagicha hisoblanadi: $\int_{AB} f(x, y) ds = \int_{\alpha}^{\beta} f(\varphi(t), \psi(t))\sqrt{(\varphi'(t))^2 + (\psi'(t))^2}\, dt$.

Agar AB chiziq oshkor $y = g(x),\ a \le x \le b$ *tenglama bilan berilgan bo`lsa, (2) integral* $\int\limits_{AB} f(x,y)ds = \int\limits_a^b f(x, g(x))\sqrt{1+(g'(x))^2}dx$ *ko`rinishini oladi.*

Agar AB chiziq qutb koordinatalari sistemasida $\rho = g(\theta)\ (\theta_0 \le \theta \le \theta_1)$ *tenglama bilan berilgan bo`lsa, (2) integral* $\int\limits_{AB} f(x,y)ds = \int\limits_{\theta_0}^{\theta_1} f(\rho\cos\theta, \rho\sin\theta)\sqrt{\rho^2 + \rho'^2}d\theta$ *formula bo`yicha hisoblaymiz.*

Quyidagi egri chiziqli integrallarni hisoblang.

317. $\int\limits_L \frac{ds}{x-y},\ L - y = \frac{x}{2} - 2$ to`g`ri chiziqning $A(0;-2)$ va $B(4;0)$ nuqtalar orasidagi kesma.

318. $\int\limits_L xyds,\ L-$uchlari $A(0;0)$, $B(4;0)$, $C(4;2)$ va $D(0;2)$ nuqtalarda bo`lgan to`g`ri to`rtburchakning konturi.

319. $\int\limits_L \left(x^2+y^2\right)^n ds,\ L - x = a\cos t,\ \ y = a\sin t$ aylana;

320. $\int\limits_L \sqrt{2y}ds,\ L - x = a(t - \sin t,)\ \ y = a(1-\cos t)$ sikloidaning birinchi arki;

321. $\int\limits_L yds,\ L - y^2 = 2px$ parabolaning $x^2 = py$ parabola bilan kesilgan yoyi;

322. $\int\limits_L y^2 ds,\ L - x = a(t-\sin t,),\ \ y = a(1-\cos t)$ sikloidaning birinchi arki.

323. $\int\limits_L (x-y)ds,\ L - x^2 + y^2 = ax$ aylana.

324. $\int\limits_L x\sqrt{x^2-y^2}ds,\ L - \rho = a\sqrt{\cos 2\varphi}$ lemniskataning o`ng yaprog`i.

2-§. Ikkinchi tur egri chiziqli integral.
(Koordinatalar bo`yicha egri chiziqli integral)

1. Sodda AB egri chiziqda $P(M) = P(x,y)$ *va* $Q(M) = Q(x,y)$ *funksiyalar va bu egri chiziqni* $A_i\,A_{i+1}$ *bo`laklarga ajratish usuli* $T = \{A_0, A_1, ..., A_n\}$ *berilgan bo`lsin. Har bir* $A_i\,A_{i+1}$ *bo`laklarda ixtiyoriy* $M_i(\xi_i, \eta_i)$ *nuqta tanlab olib,*

$$S_T(P) = \sum_{i=0}^{n-1} P(\xi_i,\ \eta_i)\Delta x_i$$

$$S_T(Q) = \sum_{i=0}^{n-1} Q(\xi_i,\ \eta_i)\Delta y_i$$

integral yig`indilarni tuzamiz, bu yerda* $\Delta x_i$ *va* $\Delta y_i$ *lar bilan mos ravishda* $A_i\,A_{i+1}$ *yoyning x va u o`qlaridagi proeksiyalari belgilangan.

Agar $\lambda(T)=\max A_iA_{i+1}\to 0$ *da* S_T *(P) va* $S_T(Q)$ *yig`indilarning limitlari mavjud bo`lsa, u holda bu limitlar P(x,y) va Q(x,y) funksiyalardan olingan ikkinchi tur egri chiziqli integrallar deyiladi va mos ravishda* $\int_{AB} P(x,y)dx,\ \int_{AB} Q(x,y)dy$ *belgilanadi.*

$\int_{AB} P(x,y)dx+\int_{AB} Q(x,y)dy$ *yig`indini Ikkinchi tur egri chiziqli integrallarning umumiy ko`rinishi deb atash va* $\int_{AB} P(x,y)dx+Q(x,y)\,dy$ *kabi yozish qabul qilingan.*

2. Oddiy aniq integralga keltirish.

Agar AB egri chiziq

$x=\varphi(t),\ y=\phi(t),\ (a\le t\le\beta)$

parametrik tenglamalar bilan berilsa, u holda ikkinchi tur egri chiziqli integral

$$\int_{AB} P(x,y)dx+Q(x,y)dy=\int_a^b(P(\varphi(t),\phi(t))\varphi'(t)+Q(\varphi(t),\phi(t))\phi'(t))dt \qquad (1)$$

formula bo`yicha hisoblanadi.

Agar egri chiziq $y=f(x)(a\le x\le b)$ *tenglama bilan berilsa, (1) formula*

$$\int_{AB} P(x,y)dx+Q(x,y)dy=\int_a^b(P(xf(x))+Q(x,f(x))f'(x))dx \qquad (2)$$

ko`rinishni oladi.

Agar $\vec{F}(x,y)=\{P(x,y),\,Q(x,y)\}$ *- kuch maydoni bo`lsa, bu kuchning moddiy nuqtani egri chiziq bo`ylab siljitishda bajargan ishi W ikkinchi tur egri chiziqli integral bilan ifodalanadi:*

$$W=\int_{AB} P(x,y)\,dx+Q(x,y)\,dy.$$

3. Agar P(x,y) va Q(x,y) funksiyalar uchun

$$\frac{\partial Q}{\partial x}=\frac{\partial P}{\partial y} \qquad (1)$$

shart bajarilsa, u holda $P(x,y)dx+Q(x,y)dy$ *ifoda biror u(x,y) funksiyaning to`la differensiali bo`ladi va* $\int_{AB}(x,y)dx+Q(x,y)dy$ *integral integrallash yo`liga bog`liq bo`lmaydi, faqat A va V nuqtalarning berilishi bilan bir qiymatli aniqlanadi.*

To`la differensiali bo`yicha funksiyaning o`zi

$$u(x,y)=\int_{x_0}^{x} P(t,y_0)\,dt+\int_{y_0}^{y} Q(x,t)\,dt+C$$

yoki

$$u(x,y)=\int_{x_0}^{x}P(t,y)\,dt+\int_{y_0}^{y}Q(x_0,t)\,dt+C$$

formula orqali topiladi.

4. Ikki karrali va egri chiziqli integrallarni bog`lovchi

$$\iint_D\left(\frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}\right)dxdy=\oint_\Gamma Pdx+Q\,dy$$

formula Grin formulasi deyilib, bu formuladan foydalanib, D sohaning yuzini quyidagicha ifodalash mumkin:

$$S=\oint_\Gamma xdy=-\oint_\Gamma ydx=\frac{1}{2}\oint_\Gamma xdy-ydx,$$

bu yerda G – D sohaning chegarasi.

Integrallarni hisoblang

325. $\int_L\left(x-\frac{1}{y}\right)dy,\ L-y=x^2$ parabolaning $(1;1)$ nuqtadan $(2;4)$ nuqtagacha bo`lgan yoyi.

326. $\int_L xydx,\quad L-y=\sin x$ sinusoidaning $x=\pi$ dan $x=0$ gacha bo`lgan yoyi.

327. $\int_L xds,\ L-x=2,\ y=x,\ y=0$ to`g`ri yaiziqlardan tashkil topgan uchburchakning konturi (Musbat yo`nalishi bo`yicha).

328. $\int_L(x^2-y)dx,\ L-x=0,\ y=0,\ x=1,\ y=2$ to`g`ri chiziqlardan tashkil topgan to`g`ri to`rtburchak perimetri (musbat yo`nalish bo`yicha).

329. $\int_L(xy-y^2)dx+xdy,\quad L-$quyidagi chiziqlarning $(0;0)$ nuqtadan $(1;2)$ nuqtagacha bo`lgan qismi; a) $y=2x,$ b) $y=2x^2$, c) $y=2\sqrt{x}$.

330. $\int_L 2xydx+x^2dy,\quad L-$ quyidagi chiziqlarning $(0;0)$ nuqtadan $(1;1)$ nuqtagacha bo`lgan qismi: a) $y=x^2,$ b) $y=x^3$, c) $y^2=x$.

331. $\int_L ydx+xdy,\quad L-x=R\cos t,\ y=R\sin t$ aylananing $t=0$ dan $t=\frac{\pi}{2}$ gacha bo`lgan yoyi.

332. $\int_L ydx-xdy,\quad L-\ x=a\cos t,\ y=b\sin t$ ellips (musbat yo`nalishi bo`yicha).

333. $\int_L\frac{y^2dx-x^2dy}{x^2+y^2},\quad L-\ x=a\cos t,\ y=a\sin t$ aylananing $t=0$ dan $t=\pi$ gacha bo`lgan yoyi (yarim aylana).

334. $\int\limits_L (2a-y)dx-(a-y)dy, \quad L-\ x=a(t-\sin t),\ \ y=a(1-\cos t)$ sikloidaning birinchi arki ($t=0$ dan $t=2\pi$ gacha).

To`la differensiali bo`yicha funksiyani o`zini toping.

335. $dz=\left(3x^2y-y^3\right)dx+\left(x^3-3y^2x\right)dy$;

336. $dz=\left(\dfrac{1}{y}-\dfrac{y}{x^2}\right)dx+\left(\dfrac{1}{x}-\dfrac{x}{y^2}\right)dy$;

337. $dz=x^2dx+y^2dy$;

338. $dz=4\left(x^2-y^2\right)(xdx-ydy)$;

339. $dz=\dfrac{xdy-ydx}{x^2+y^2}$;

340. $dz=e^{xy}\left((1+xy)dx+x^2dy\right)$;

341. $dz=xy\left(xy^3dx+\dfrac{4}{3}x^2y^2dy\right)$;

342. $dz=\sin(x+y)(dx+dy)$.

Quyidagi to`la differensiallardan olingan integrallarni hisoblang.

343. $\int\limits_{(-1;2)}^{(2,3)} ydx+xdy$;

344. $\int\limits_{(0;0)}^{(2;1)} 2x\,ydx+x^2dy$;

345. $\int\limits_{(0;0)}^{(1;1)} (x+y)(dxtdy)$;

346. $\int\limits_{(3;4)}^{(5,12)} \dfrac{xdx+ydy}{x^2+y^2}$ (koordinatalar boshi integrallash yo`liga tegishli emas).

3-§. Egri chiziqli integrallarning tatbiqlari

Birinchi tur egri chiziqli integralning tatbiqlari

347. Har bir nuqtadagi zichligi shu nuqtaning ordinatasiga teng bo`lgan $x=a\cos t$, $y=b\sin t$ ellipsining birinchi kvadrantda yotuvchi bo`lagining massasini hisoblang.

348. Har bir nuqtadagi zichligi $\mu=(x,y)=|y|$ bo`lgan $y^2=2px\ \ (0\le x\le \frac{p}{2})$

parabola yoyining massasini toping.

349. Bir jinsli $x = a(t - \sin t)$, $y = a(1 - \cos t)$ $(0 \le t \le \pi)$ sikloida yoyining og`irlik markazi koordinatalarini toping.

350. Bir jinsli $x = a\cos^3 t$, $y = a\sin^3 t$, $(0 \le t \le \pi)$ astroida yoyining og`irlik markazi koordinatalarini toping.

***Ikkinchi tur egri chiziqli integral yordamida yopiq egri chiziqlar bilan chegaralangan tekis figura yuzalarini hisoblang*.**

351. $x = a\cos t$, $y = b\sin t$, ellips bilan.

352. $x = a\cos^3 t$, $y = a\sin^3 t$ astroida bilan.

353. $x = 2r\cos t - r\cos 2t$, $y = 2r\sin t - r\sin 2t$ kardioida bilan.

354. $(x^2 + y^2)^2 = 2a^2(x^2 - y^2)$ Bernulli lemniskatasi bilan.

JAVOBLAR

1. $-\frac{12}{17}$;
2. 6;
3. 5; $\frac{1}{5}$; 0; mavjud emas; -1; 1.
4. 4; 2,5; $\frac{a+2b}{a-b}$;
5. 1; e.
6. $2; 2; 6$.
7. Tekislikning $x-y=0$ to`g`ri chiziqdan tashqari barcha nuqtalar to`plami.
8. Tengsizlikning $x+y=0$ to`g`ri chiziqdan tashqari barcha nuqtalar to`plami.
9. $y=-x$ to`g`ri chiziq va tekislikning bu to`g`ri chiziqdan yuqoridagi qismidan iborat.
10. Tekislikning $y=x$ to`g`ri chiziq va undan pastagi barcha nuqtalari.
11. Tekislikning birinchi va to`rtinchi chorak bissektrisalari orasidagi qismi. Bisektrisalarning o`zlari ham sohaga tegishli.
12. $x \ge 0,\ \ y \ge 0$ (1-kvadrant).
13. $x^2 + y^2 \ge 3^2$.
14. $x^2 + y^2 \le 5^2$.
15. $\frac{x^2}{3^2} + \frac{y^2}{2^2} > 1$.
16. $x < \frac{y^2}{4} + 2$.
17. $x^2 + y^2 \ne \left(\sqrt{2}\right)^2$ Tekislikning $x^2 + y^2 = 2$ aylanada yotmagan barcha nuqtalari to`plami.
18. $x > 0,\ y > 0$.
19. $\begin{cases} y \ge x+1 \\ y \le 1-x \\ x < 0 \end{cases}$.
20. $\begin{cases} -2 \le x \le 2 \\ -2 \le y \le 2 \end{cases}$ kvadrat.
21. Fazodagi uchala koordinatalari musbat bo`lgan barcha nuqtalar to`plamidan iborat.

22. $r^2 \le x^2 + y^2 + z^2 \le R^2$ (fazoning $x^2 + y^2 + z^2 = r^2$, $x^2 + y^2 + z^2 = R^2$ sferalar orasidagi qismi. Sferalarning o`zlari həm sohaga tegishli).

23. $z = C$ desak, $x + y = C$. Demak, sath chiziqlari to`g`ri chiziqlar oilasidan iborat.

24. $x^2 + y^2 = c$ (aylanalar oilasi).

25. $x^2 - y^2 = c$ (giperbolalar oilasi).

26. $y + x^2 = c$ (parabolalar oilasi).

27. $(1-c)x + (1+c)y + (1-c)z = 0$ (tekisliklar oilasi).

28. $x^2 + y^2 + z^2 = c$ sferalar oilasi.

31. $\frac{1}{2}$. 32. 0 33. 0 34. 0 35. 1 36. 1

37. Mavjud emas. 38. Mavjud emas. 43. $(-1;\ 0)$

44. $(0;\ 0)$. 45. $y = \frac{x}{2}$ to`g`ri chiziq. 46. $y^2 = -x$ parabola.

47. Tekislikdagi koordinitalari $xy \ge 0$ tengsizlikni qanoatlantiruvchi barcha $M(x, y)$ nuqtalar to`plami. II va IV kvadrantdagi barcha nuqtalar to`plami.

48. $x^2 + y^2 = 4$ aylana. 49. $z'_x = 2x + 5y;\ z'_y = 5x - 6y;$

50. $z'_x = 3x^2 + 6y^2 - 2y;\ z'_y = 12xy - 12y^2 - 2x;$

51. $z'_x = 30xy(5x^2y - y^3 + 7)^2;\ z'_y = 3(5x^2 - 3y^2)(5x^2y - y^3 + 7)^2;$

52. $z'_x = \frac{x}{\sqrt{x^2 - y^2}};\ z'_y = -\frac{y}{\sqrt{x^2 - y^2}};$

53. $z'_x = \frac{1}{\sqrt{x^2 + y^2}};\ z'_y = \frac{y}{x\sqrt{x^2 + y^2} + x^2 + y^2};$

54. $z'_x = \frac{1}{x + \ln y};\ z'_y = \frac{1}{y(x + \ln y)}.$

55. $z'_x = 2x\sin y e^{x^2 \sin y};\ z'_y = x^2 \cos y e^{x^2 \sin y};$

56. $z'_x = -y^2 e^{-xy};\ z'_y = (1 - xy)e^{-xy};$

57. $z'_x = \frac{y}{x^2 + y^2},\ z'_y = -\frac{x}{x^2 + y^2};$

58. $z'_x = \frac{y}{\arcsin(xy)\cdot\sqrt{1 - x^2y^2}};\ z'_y = \frac{x}{\arcsin(xy)\cdot\sqrt{1 - x^2y^2}};$

59. $z'_x = y^x \ln y;\ z'_y = xy^{x-1};$

60. $z'_x = y^2 x^{y^2 - 1};\ z'_y = 2yx^{y^2}\ln x;$

61. $f'_x(3,4)=\frac{2}{5}$; $f'_y(3,4)=\frac{1}{5}$;

62. $f'_x(1,1)=\frac{\sqrt[3]{2}}{3}$; $f'_y(1,1)=\frac{\sqrt[3]{2}}{2}$;

63. $u'_x=\frac{x+z}{\sqrt{x^2+y^2-z^2+2xz}}$; $u'_y=\frac{y}{\sqrt{x^2+y^2-z^2+2xz}}$;

$u'_z=\frac{x-z}{\sqrt{x^2+y^2-z^2+2xz}}$;

64. $u'_x=\frac{y^2+z^2}{\left(x^2+y^2+z^2\right)^{\frac{3}{2}}}$; $u'_y=-\frac{y}{\left(x^2+y^2+z^2\right)^{\frac{3}{2}}}$; $u'_z=-\frac{z}{\left(x^2+y^2+z^2\right)^{\frac{3}{2}}}$;

65. $u'_x=\left(3x^2+y^2+z^2\right)\cdot e^{x\left(x^2+y^2+z^2\right)}$; $u'_y=2xy\cdot e^{x\left(x^2+y^2+z^2\right)}$; $u'_z=2xze^{x\left(x^2+y^2+z^2\right)}$;

66. $u'_x=2x\cos\left(x^2+y^2+z^2\right)$; $u'_y=2y\cos\left(x^2+y^2+z^2\right)$;

$u'_z=2z\cos\left(x^2+y^2+z^2\right)$;

67. $u'_x=\frac{y}{z}x^{\frac{y-z}{z}}$; $u'_y=\frac{\ln x}{z}x^{\frac{y}{z}}$; $u'_x=-\frac{y\ln x}{z^2}\cdot x^{\frac{y}{z}}$.

68. $u'_x=y^z x^{y^z-1}$; $u'_y=zy^{z-1}x^{y^z}\ln x$; $u'_x=y^z x^{y^z}\ln x\ln y$;

73. $dz=2xy^3dx+3x^2y^2dy$;

74. $dz=\frac{xdx-ydy}{\sqrt{x^2-y^2}}$;

75. $dz=\left(2xy^4-3x^2y^3+4x^3y^2\right)dx+\left(4x^2y^3-3x^3y^2+2x^4y\right)dy$;

76. $dz=\left(y-2xy^3+3x^2y\right)dx+\left(x-3x^2y^2+x^3\right)dy$;

77. $dz=-ye^{y^2-xy}dx+(2y-x)e^{y^2-xy}dy$;

78. $dz=-\sin(xy)(ydx+xdy)$;

79. $dz=\frac{ydx+xdy}{1+x^2y^2}$;

80. $dz=\frac{ydx-xdy}{y\sqrt{y^2-x^2}}$;

81. $du=2xyz^4dx+x^2z^4dy+4x^2yz^3dz$;

82. $du=\frac{3x^2dx-3y^2dy+6z^2dz}{x^3-y^3+2z^3}$;

83. $du=-\frac{y}{x^2z}dx+\frac{1}{xz}dy-\frac{y}{xz^2}dz$;

84 $du=y^z dx+xzy^{z-1}dy+xy^z\ln ydz$;

85. $-\frac{\sqrt{2}}{2}$; 86. $\frac{\sqrt{2}}{2}$; 87. 0;

88. $-\sqrt{5}$; 89. $-\frac{1}{2}$; 91. $\frac{1}{3}\left(2\bar{i}+\bar{j}\right)$;

92. $\frac{1}{5}\left(2\bar{i}-\bar{j}\right)$; 93. $2\left(x_0\bar{i}+y_0\bar{j}\right)$; 94. $\left(y_0\bar{i}+x_0\bar{j}\right)$;

95. $e^{\sin t-3t^2}\left(\cos t-6t\right)$; 96. $-\left(e^t+e^{-t}\right)$; 97. $\frac{8t-3t^2}{\sqrt{1-(4t^2-t^3)^2}}$;

98. 0; 99. $\frac{8}{\sin 2t}$; 100. $-\frac{e^x+2xe^{x^2}}{e^x+e^{x^2}}$;

101. $\frac{1}{x^2+1}$; 102. $\frac{\partial z}{\partial u}=4u,\ \frac{\partial z}{\partial v}=4v$;

103. $\frac{\partial z}{\partial u}=\frac{2}{u};\ \frac{\partial z}{\partial v}=\frac{2\left(v^4-1\right)}{\vartheta\left(v^4+1\right)}$;

104 $\frac{\partial z}{\partial u}=-3u^2\sin v\cos v\left(\cos v-\sin v\right)$; $\frac{\partial z}{\partial \vartheta}=u^3\left(\sin v+\cos v\right)\left(1-3\sin u\cos v\right)$;

105. $\left(8t^3+6t^2-6t\right)\cos\left(2t^3-3t^2+2t^4\right)$;

106. $\frac{2x+\frac{2}{\sqrt{x}}+\frac{1}{x^2}}{\cos^2\left(x^2+4\sqrt{x}+\frac{1}{x}\right)}$; 107. $\frac{3x^2y-y^3}{3xy^2-x^3}$; 108. $-\frac{x}{y}$;

109. $\frac{y^2}{1-xy}$; 110. $\frac{y}{y-1}$; 111. $\frac{yx^{y-1}-y^x\ln y}{xy^{x-1}-x^y\ln x}$;

112. $\frac{x^2+xy+y^2}{xy}$; 113. $\frac{b^2x}{a^2y}$; 114. $\frac{e^x\sin y+e^y\sin x}{e^y\cos x-e^x\cos y}$;

115. $-\frac{x^2+2y}{y^2+2x}$; 116. $\frac{e^x-y^2}{2xy-\cos y}$; 117. $\frac{\partial z}{\partial x}=\frac{1}{2}\sqrt{\frac{y}{x}}$; $\frac{\partial z}{\partial y}=\frac{1}{2}\sqrt{\frac{x}{y}}$;

118. $\frac{\partial z}{\partial x}=\frac{\partial z}{\partial y}=\frac{1}{x+y+z-1}$; 119. $\frac{\partial z}{\partial x}=\frac{yz-x^2}{z^2-xy};\ \frac{\partial z}{\partial y}=\frac{xz-y^2}{z^2-xy}$;

120. $\frac{\partial z}{\partial x}=\frac{4-x}{z}$; $\frac{\partial z}{\partial y}=-\frac{y}{x}$; 121. $\frac{\partial z}{\partial x}=-tgx;\ \frac{\partial z}{\partial y}=-tgy$;

122. $\frac{\partial z}{\partial x}=-\frac{yz}{z^2+xy};\ \frac{\partial z}{\partial y}=-\frac{xz}{z^2+xy}$;

123. $\frac{\partial z}{\partial x}=\frac{yz}{e^z-xy};\ \frac{\partial z}{\partial y}=\frac{xz}{e^z-xy}$; 124. $\frac{\partial z}{\partial x}=-\frac{c^2x}{a^2z};\ \frac{\partial z}{\partial y}=-\frac{c^2y}{b^2z}$;

125. $\frac{\partial^2 z}{\partial x^2}=\frac{2y^2}{(x-y)^3};\ \frac{\partial^2 z}{\partial x\partial y}=-\frac{2xy}{(x-y)^3};\ \frac{\partial^2 z}{\partial y^2}=\frac{2x^2}{(x-y)^3}$;

126. $\frac{\partial^2 z}{\partial x^2}=0;\ \frac{\partial^2 z}{\partial x\partial y}=e^y;\ \frac{\partial^2 z}{\partial y^2}=xe^y;$

127. $\frac{\partial^2 z}{\partial x^2}=-\frac{y}{x^2};\ \frac{\partial^2 z}{\partial x\partial y}=\frac{1}{x};$

128. $\frac{\partial^2 z}{\partial x\partial y}=\frac{xy}{\left(2xy+y^2\right)^{\frac{3}{2}}};$ 129. $\frac{\partial^2 z}{\partial y^2}=-\frac{y}{\left(x^2+y^2\right)^{\frac{3}{2}}};$

130. $\frac{\partial^2 z}{\partial x^2}=e^{xe^y+2y};$ $\frac{\partial^2 z}{\partial x\partial y}=e^{xe^y+y}(1+xe^y);$ $\frac{\partial^2 z}{\partial y^2}=x\left(1+xe^y\right)e^{xe^y+y};$

131. $\frac{\partial^3 z}{\partial x^2\partial y}=\frac{2}{(x+y)^3};$ 132. $\frac{\partial^3 z}{\partial x\partial y^2}=-x^2y\cos(xy)-2x\sin(xy);$

133. $\frac{\partial^3 z}{\partial x\partial y^2}=2(1+5xy^2+2x^2y^4)e^{xy^2};$

134. $\frac{\partial^3 z}{\partial x\partial y\partial z}=\left(1-x^2y^2z^2\ln^2 3+3xyz\ln 3\right)3^{xyz}\ln 3;$

137. $2dxdy;$ 138. $e^{x+y^2}\left(dx^2+4ydxdy+\left(2+4y^2\right)dy^2\right).;$

139. $-\frac{dx^2-2dxdy+dy^2}{(x-y)^2};$ 140. $\frac{2}{(x+y)^3}\left(xdy^2+(x-y)dxdy-ydx^2\right).;$

141. $2\sin 2ydxdy+2x\cos 2y\,dy^2;$

142. 0; 143. $e^{x+y}\left(dx^n+n\,dx^{n-1}dy+...+dy^n\right);$

144. $-\frac{1}{(z-1)^3}\left(x^2+z^2-2z+1\right)dx^2+2xydxdy+\left(y^2+z^2-2z+1\right)dy^2\big).;$

145. $z=x+2y-2;\ \frac{x-2}{-1}=\frac{y-1}{-2}=\frac{z-2}{1};$

146. $z=2x+2y-2;\ \frac{x-1}{-2}=\frac{y-1}{-2}=\frac{z-2}{1};$

147. $z=\pi y-x;\ \frac{x-\pi}{1}=\frac{y-1}{-\pi}=\frac{z}{1};$

148. $x-y+2z-\frac{\pi}{2}=0;\ \frac{x-1}{1}=\frac{y-1}{-1}=\frac{z-\frac{\pi}{2}}{2};$

149. $x_0x+y_0y+z_0z=1;\ \frac{x-x_0}{x_0}=\frac{y-y_0}{y_0}=\frac{z-z_0}{z_0};$

150. $2x+y+11z=25;\ \frac{x-1}{2}=\frac{y-1}{1}=\frac{z-2}{11};$

151. $x-y+2z=\sqrt{\frac{11}{2}}$ va $x-y+2z=-\sqrt{\frac{11}{2}};$

152. $x+y+z=\sqrt{a^2+b^2+c^2}$;

153. $\Delta z=\frac{\Delta y-\Delta x}{(x-y)^2}+\frac{(\Delta x-\Delta y)^2}{(x-y)^3}+R_2$;

154. $\Delta z=\frac{1}{x+y}(\Delta x+\Delta y)-\frac{1}{2(x+y)^2}(\Delta x+\Delta y)^2+R_2$;

155. $\Delta z=e^{xy}(y\Delta x+x\Delta y)+\frac{1}{2}e^{xy}\left(y^2\Delta x^2+2(1+xy)\Delta x\Delta y+x^2\Delta y^2\right)+R_2$;

156. $\Delta z=\cos x\cos y\Delta x-\sin x\sin y\Delta y-$
$\frac{1}{2}\left(\sin x\cos y\,\Delta x^2+2\cos x\sin y\Delta x\Delta y+\sin x\cos y\Delta y^2\right)+R_2$;

157. $\Delta z=\frac{1}{x^2+y^2}(y\Delta x-x\Delta y)+$
$\frac{1}{2(x^2+y^2)^2}\left(-2xy\Delta x^2+2(x^2-y^2)\Delta x\Delta y+2xy\Delta y^2\right)+R_2$;

158. $\Delta z=e^x\left(\cos y\Delta x-\sin y\Delta y+\frac{1}{2}\left(\cos y\Delta x^2-\sin y\Delta x\Delta y-\cos y\ \Delta y^2\right)\right)+R_2$;

159. Ekstremum mavjud emas. 160. $(-2;0)$ nuqtada minimumga ega.
161. $(0;0)$ minimum nuqtasi. 162. Ekstremum yo`q.
163. $(4,0)$ minimum nuqtasi. 164. ekstemum yo`q.
165. $(\sqrt{3};-3)$ minimum nuqtasi. 166. $\left(-1;-\frac{1}{2}\right)$ maksimum nuqtasi.
167. $(0;0)$ minimum nuqtasi. 168. $(0;2)$ maksimum nuqtasi.
169. Ekstremum yo`q. 170. Ekstremum yo`q.
173. Eng kichik qiymat $-\frac{2}{3\sqrt{3}}$; eng katta qiymat $\frac{2}{3\sqrt{3}}$;
174. Eng kishik qiymat -4, eng katta qiymat 4.
175. Eng kishik qiymat $-\frac{1}{3}$, eng katta qiymat. 2;
176. Eng kishik qiymat -52 , eng katta qiymat 100;
177. Eng kishik qiymat -1 , eng katta qiymat 6;
178. Eng kishik qiymat -11, eng katta qiymat $9+4\sqrt{2}$;
179. Minimum $-\sqrt{2}$, maksimum $\sqrt{2}$;
180. Minimum $\frac{1}{2}$; 181. Maksimum $e^{\frac{a^2}{4}}$;
182. Minimum 2; 183. $\frac{4}{\sqrt{5}}$;

184. $\sqrt{20}$; 185. $\frac{19\sqrt{2}}{8}$;

186. $\left(\frac{4}{\sqrt{5}},\frac{3}{\sqrt{5}}\right)$ va $\left(-\frac{4}{\sqrt{5}},-\frac{3}{\sqrt{5}}\right)$;

187. $\frac{3}{4}$; 188. $\frac{19}{3}$; 189. $\frac{2}{3}$;

190. $\frac{1}{4}$; 191. 60; 192. 64;

193. $-\frac{1}{15}$; 194. 4; 195.8;

196. $-\frac{56}{5}$; 197. $\frac{e-1}{2}$; 198. $\frac{506}{15}$;

199. $\int_0^1 dx \cdot \int_{-\sqrt{x}}^{\sqrt{x}} f(x,y)dy = \int_{-1}^{1} dy \int_{y^2}^{1} f(x,y)dx$;

200. $\int_1^6 dx \int_{\frac{6}{x}}^{1-x} f(x,y)dy = \int_1^6 dy \int_{6/y}^{7-y} f(x,y)dx$;

201. $\int_0^1 dx \int_x^{\sqrt{x}} f(x,y)dy = \int_0^1 dy \int_{y^2}^{y} f(x,y)dx$;

202. $\int_0^4 dx \int_0^{2\sqrt{x}} f(x,y)dy = \int_0^4 dy \int_{\frac{y^2}{4}}^{4} f(x,y)dx$;

203. $\int_0^2 dx \int_0^{2-x} f(x,y)dy = \int_0^2 dy \int_0^{2-y} f(x,y)dx$;

204. $\int_0^1 dx \int_0^{\sqrt{1-x}^2} f(x,y)dy = \int_0^1 dy \int_0^{\sqrt{1-y^2}} f(x,y)dx$;

205. $\int_0^1 dx \int_{x-1}^{1-x} f(x,y)dy = \int_{-1}^{0} dy \int_0^{y+1} f(x,y)dx + \int_0^1 dy \int_0^{1-y} f(x,y)dx$;

206. $\int_{-\sqrt{2}}^{\sqrt{2}} dx \int_{x^2}^{4-x^2} f(x,y)dy$;

207. $\int_0^1 dx \int_{x^2}^{\sqrt{x}} f(x,y)dy = \int_0^1 dy \int_{\sqrt{x}}^{x^2} f(x,y)dx$;

208. $\int_{-\frac{2}{3}}^{\frac{1}{3}} dx \int_{1-2x}^{x+3} f(x,y)dy + \int_{\frac{1}{3}}^{\frac{2}{3}} dx \int_x^{x+3} f(x,y)dy + \int_{\frac{2}{3}}^{\frac{5}{3}} dx \int_x^{5-2x} f(x,y)dy$;

209. $\int_0^1 dx \int_{x^2}^{x} f(x,y)dy$; 210. $\int_0^1 dy \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} f(x,y)dx$;

211. $\int\limits_0^1 dy \int\limits_{\sqrt{y}}^{\sqrt[3]{y}} f(x,y)dx$;   212. $\int\limits_0^2 dy \int\limits_{\frac{y}{2}}^{y} f(x,y)dx + \int\limits_2^4 dy \int\limits_{\frac{y}{2}}^{2} f(x,y)dx$;

213. $\int\limits_0^1 dx \int\limits_0^x f(x,y)dy + \int\limits_1^2 dx \int\limits_0^{2-x} f(x,y)dy$;

214. $\int\limits_{\frac{1}{e}}^1 dy \int\limits_{-\ln y}^{1} f(x,y)dx + \int\limits_1^e dy \int\limits_{\ln y}^{1} f(x,y)dx$;

215. $\int\limits_0^1 dy \int\limits_y^{2-y} f(x,y)dx$;   216. $\int\limits_0^1 dy \int\limits_{\sqrt{y}}^{3-2y} f(x,y)dx$;

217. $\ln\frac{25}{24}$;   218. 0;   219. 9;

220. $\frac{8}{63}$;   221. 0;   222. $\frac{33}{144}$;

223. $\frac{4}{21}$;   224. $\frac{27}{20}$;   225. $\frac{\pi}{6}$;

226. $\frac{4}{135}$; 227. $\int\limits_0^{2\pi} d\varphi \int\limits_0^R f(\rho\cos\varphi, \rho\sin\varphi)\rho\, d\rho$;

228. $\int\limits_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\varphi \int\limits_0^{a\cos\varphi} f(\rho\cos\varphi, \rho\sin\varphi)\rho\, d\rho$;

229. $\int\limits_0^{\pi} d\varphi \int\limits_0^{b\sin\varphi} f(\rho\cos\varphi, \rho\sin\varphi)\rho\, d\rho$;

230. $\int\limits_{\frac{\pi}{4}}^{arctg\,2} d\varphi \int\limits_{4\cos\varphi}^{8\cos\varphi} f(\rho\cos\varphi, \rho\sin\varphi)\rho\, d\rho$;

231. $\int\limits_0^{arctg\frac{a}{b}} d\varphi \int\limits_0^{b\sin\varphi} f(\rho\cos\varphi, \rho\sin\varphi)\rho\, d\rho + \int\limits_{arctg\frac{a}{b}}^{\frac{\pi}{2}} d\varphi \int\limits_0^{a\cos\varphi} f(\rho\cos\varphi, \rho\sin\varphi)\rho\, d\rho$;

232. $\int\limits_0^{\frac{\pi}{\pi}} d\varphi \int\limits_0^{\frac{1}{\cos\varphi}} f(\rho\cos\varphi, \rho\sin\varphi)\rho\, d\rho$;

233. $\frac{9\pi}{2}$;   234. $\frac{2}{3}$;   235. 2π;

236. $\frac{\pi\ln 2}{2}$;   237. 24π;   238. $\frac{\pi}{4}\left(\left(1+R^2\right)\ln\left(1+R^2\right)-R^2\right)$;

239. 28;   240. $13{,}5-9\ln 2$;   241. $\frac{2}{3}$;

242. $\frac{125}{6}$;   243. $\frac{2}{3}$;   244. $\frac{ab}{6}$;

245. $\frac{16}{3}$; 246. $a^2\left(\frac{\pi}{2}-1\right)$; 247. $\frac{3}{4}\pi a^2$;

248. $\frac{r^2}{2}\left(\frac{\pi}{3}+\frac{\sqrt{3}}{2}\right)$; 249. $\frac{9\pi}{4}$; 250. $\frac{3\pi a^2}{2}$;

251. 6; 252. $\frac{128}{15}$; 253. 13,5;

254. $16\frac{1}{5}$; 255. $\frac{abc}{6}$; 256. $78\frac{15}{32}$;

257. $\frac{32\sqrt{2}}{15}$; 258. 3π; 259. $\frac{1}{6}$;

260. 144; 261. $\frac{8\pi}{3}$; 262. 90;

263. $\frac{\pi}{48}$; 264. $\frac{3\pi}{2}$; 265. 3π;

266. 8π; 267. 14; 268. $2\pi RH$;

270. 72; 271. $\frac{14\pi}{3}$; 272. $\sqrt{2}\pi$.

273. $2\sqrt{2}\pi\rho^2$; 274. $12\pi\left(3-\sqrt{5}\right)$; 275 $\bar{x}=-\frac{1}{2}, \bar{y}=\frac{8}{5}$;

276. $\bar{x}=0,\ \bar{y}=\frac{4R}{3\pi}$; 277. $\bar{x}=0,\ \bar{y}=\frac{4b}{3\pi}$; 278. $\bar{x}=1, \bar{y}=\frac{4}{3\pi}$;

279. 108; 280. $\frac{40}{3}$; 281. $\frac{4}{3}$;

282. $\frac{7}{192}$; 283. $\frac{abc\left(a^2+b^2+c^2\right)}{3}$;

284. $\frac{1}{3}$; 285. 30; 286. 54;

287. $\frac{1}{720}$; 288. $\frac{1}{364}$;

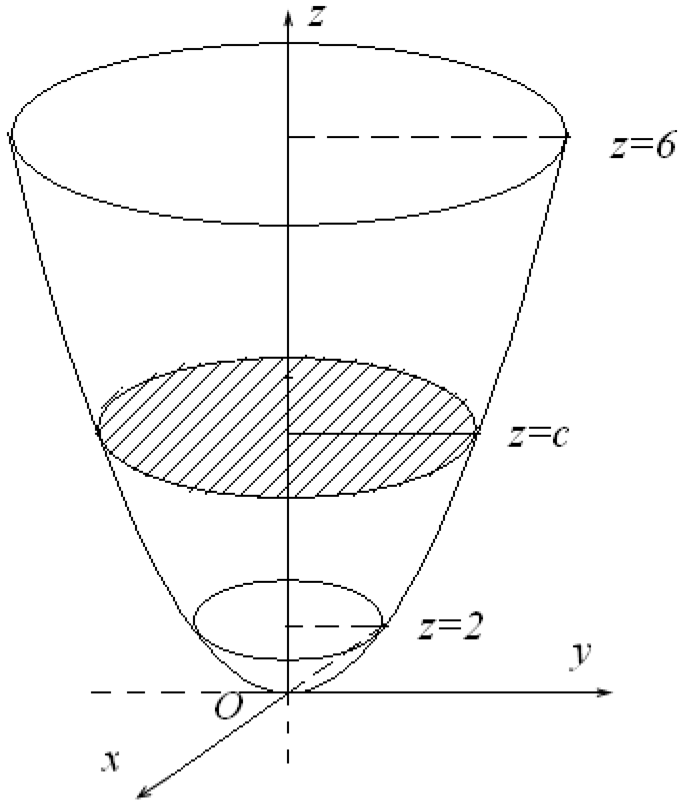
289* 320 π. Berilgan integralni quyidagicha yozib olamiz:

$\iiint z^2 dxdydz = \int z^2 dz \iint\limits_D dxdy$

Bu yerda $\iint\limits_D dxdy$ D soha bo`yicha olingan integral bo`lib, D soha G bilan z=c tekislikning kesishishidan hosil bo`lgan radiusi $\sqrt{z}$ bo`lgan doiradan iborat. $\iint\limits_D dxdy$ integral D sohaning yuzasini ifodalaydi. Shuning uchun

$\iint\limits_D dxdy = \pi z$.

Demak, $\iiint\limits_G z^2 dxdydz = \int\limits_2^6 z^2 dz \underbrace{\iint\limits_D dxdy}_{\pi z} = \pi \int\limits_2^6 z^3 dz = \frac{\pi}{4} z^4 \Big|_2^6 = \pi_1 \frac{6^4 - 2^4}{4} = 320\pi$.



290. $\frac{162\pi}{5}$; 291. $\pi \ln 2$; 292. $\frac{7\pi}{2}$; 293. $\frac{324\pi}{5}$

294. 16; 295. 8π; 296. $\frac{16\pi}{3}$;

297. $\frac{4\pi R^5}{5}$; 298. $\frac{4}{5}\pi R^5$; 299. 2π

300. 24π; 301. 4,5 kub. birl. 302. 8;

303. 3,4; 304. 8π; 305. 2π;

306. $\frac{\pi}{6}$; 307. 16; 308. $\frac{16\pi}{3}$;

309. $\frac{16(3\pi - 4)}{9}$; 310. $\frac{19\pi}{6}$; 311. $\frac{1}{3}\pi a^3$;

312. $\frac{a^3}{360}$; 313. $\bar{x} = 0, \bar{y} = 0, \bar{z} = 3$;

314. $\bar{x} = 3, \bar{y} = 3, \bar{z} = \frac{45}{32}$; 315. $\bar{x} = 0, \bar{y} = 0, \bar{z} = \frac{5(6\sqrt{3} + 5)}{83}$;

316. $\bar{x} = 0, \bar{y} = 0, \bar{z} = \frac{6 + 3\sqrt{2}}{4}$; 317. $\sqrt{5} \ln 2$;

318. 24; 319. $2\pi a^{2n+1}$;

320. $4\pi a\sqrt{a}$; 321. $\frac{P^2}{3}\left(5\sqrt{5}-1\right)$;

322. $\frac{256}{15}a^3$; 323. $\frac{\pi a^2}{2}$; 324. $\frac{2a^3}{3}$;

325. $\frac{14}{23}-\ln 4$; 326. $-\pi$; 327. 2;

328. 2; 329. a) $\frac{1}{3}$; b) $\frac{31}{30}$c) $-\frac{8}{15}$;

330. a) 0; b) 0; c) 0; 331. 0; 332. -2πab ;

333. $\frac{4a}{3}$; 334. πa^2; 335. $z = x^3y - y^3x + C$;

336. $z = \frac{y}{x} + \frac{x}{y} + C$; 337. $z = \frac{x^3 + y^3}{3} + C$;

338. $z = \left(x^2 - y^2\right)^2 + C$; 39. $z = arctg\frac{y}{x} + C$;

340. $z = xe^{xy} + C$; 341. $z = \frac{x^3y^4}{3} + C$;

342. $z = C - \cos\left(x + y\right)$; 343. 8; 344. 4;

345. 2; 346. $\ln\frac{13}{5}$; 347. $\frac{b^2}{2} + \frac{a^2b}{\sqrt{a^2+b^2}}\arcsin\frac{\sqrt{a^2-b^2}}{a}$;

348. $\frac{2}{3}\rho^2\left(2\sqrt{2}-1\right)$; 349. $\overline{x} = \frac{4}{3}a,\ \overline{y} = \frac{4}{3}a$; 350. $\overline{x} = \frac{2}{5}a,\ \overline{y} = \frac{2}{5}a$;

351. πab; 352. $\frac{3}{8}\pi a^2$; 353. 6πa^2; 354. 2a^2;

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