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NIZOMIY NOMIDAGI TOSHKENT DAVLAT
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MATEMATIK TAHLILDAN
MISOL VA MASALALAR TO'PLAMI
1-qism

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ANNOTATSIYA

Ushbu uslubiy qo‘llanma pedagogika oliy o‘quv yurtlari «matematika va informatika» ta’lim yonalishining «Matematik tahlil» fani dasturiga mos yozilgan bo‘lib, bunda matematik tahlilning matematik tahlilga kirish, bir o‘zgaruvchili funksiyaning differentsial va integral hisobi, qatorlar nazariyasi bo‘limlaridan misol va masalalar berilgan. Bu uslubiy qo‘llanmadan «Fizika va astronomiya» bakalavriat yonalishida ta’lim olayatgan talabalar ham foydalanishi mumkin.

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MUNDARIJA

KIRISH

Pedagogika oliy oliy o‘quv yurtlari «matematika va informatika» ta’lim yo‘nalishida bakalavr talabalari uchun mo‘ljallangan o‘zbek tilida «Matematik tahlildan masala va misollar” to‘plami mavjud emas. Shu sababli dars o‘tkazish uchun bir nechta adabiyotlardan misol va masalalarni tanlashga to‘g‘ri keladi.

Ushbu metodik qo‘llanma pedagogika oliy o‘quv yurtlari matematika va informatika yo‘nalishi bakalavr talabalari uchun mavjud fan dasturiga mos ravishda yozilgan bo‘lib, matematik tahlilga kirish, differentsial hisob, integral hisob va qatorlar nazariyasi bo‘limlarini o‘z ichiga oladi.

Metodik qo‘llanmaga matematik tahlilning asosiy tushunchalarini mustahkamlashga, o‘rta maxsus va kasb-hunar ta’limidagi matematikasining matematik tahlilga oid misol va masalalarini yechish ko‘nikmasini shakllantirishga yordam beradigan misol va masalalar kiritilgan.

Har bir bobdan oldin kerakli bo‘lgan nazariy ma’lumotlar keltirilgan, deyarli barcha masala va misollarning javoblari berilgan, ba’zi murakkab misol va masalalar uchun ko‘rsatmalar berilgan.

Ushbu qo‘llanmani tayyorlashda mavjud bo‘lgan misol va masalalar to‘plamlaridan ijodiy foydalanildi.

Mualliflar mazkur misol va masalalar to‘plamidan iborat metodik qo‘llanmani kamchiliklardan holi deb hisoblamaydi, bu to‘plamni mukammallashtirishga qaratilgan har qanday takliflar uchun oldindan minnatdorchilik bildiradi.

I BOB. FUNKSIYA TUSHUNCHASI

1-§. Haqiqiy sonlar. Chegaralangan va chegaralanmagan to‘plamlar.

1. Har qanday cheksiz o‘nli kasr haqiqiy son deyiladi. Davriy cheksiz o‘nli kasr ratsional son deyiladi. Har qanday ratsional sonni $\frac{p}{q}$ (p va q lar butun sonlar va $q\neq 0$) ko‘rinishda yozish mumkin va aksincha.

Davriy bo‘lmagan cheksiz o‘nli kasr irratsional son deyiladi.

2. E bo‘sh bo‘lmagan to‘plam bo‘lsin. Barcha $x\in E$ uchun $x\leq M$ tengsizlikni qanoatlantiruvchi M son mavjud bo‘lsa, E yuqoridan chegaralangan to‘plam deyiladi. M son E to‘plamning yuqori chegarasi deyiladi.

Yuqoridan chegaralangan to‘plam yuqori chegaralarning eng kichigi uning aniq yuqori chegarasi deyiladi va $m^*=supE$ ko‘rinishda belgilanadi. Agar E to‘plam yuqoridan chegaralanmagan bo‘lsa, $supE=+\infty$ deb olinadi.

Teorema. m^* son E to‘plamning yuqori chegarasi bo‘lishi uchun quyidagi shartlarning o‘rinli bo‘lishi zaur va etarlidir.

a) barcha $x\in E$ uchun $x\leq m^*$,

b) m^* dan kichik bo‘lgan har bir a uchun birorta $x'\in E$ topilib, $x'>a$ bo‘ladi.

3. Barcha $x\in E$ uchun $x\geq m$ tengsizlikni qanoatlantiruvchi m son mavjud bo‘lsa, E quyidan chegaralangan to‘plam deyiladi.

Quyidagi chegaralangan to‘plam quyi chegaralarining eng kattasi uning aniq quyi chegarasi deyiladi va $m_*=infE$ ko‘rinishda belgilanadi. E quyidan chegaralanmagan bo‘lsa, $infE=-\infty$ deb olinadi.

Teorema. m_* son E to‘plamning aniq quyi chegarasi bo‘lishi uchun quyidagi ikki shartning o‘rinli bo‘lishi zaur va etarlidir.

a) barcha $x\in E$ lar uchun $x\geq m_*$,

b) m_* dan katta bo‘lgan har bir b son uchun biror $x'\in E$ topilib, $x'<b$ bo‘ladi.

4. Ham yuqoridan, ham quyidan chegaralangan to‘plam chegaralangan to‘plam deyiladi.

Quyidagi cheksiz o‘nli kasring qaysilari ratsional, qaysilari irratsional sonni aniqlaydi? Ratsional sonlarni oddiy kasr ko‘rinishida yozing.

1. 2,13(14). 2. 2,76(11).
3. 3, (74). 4. 0,4212121...,
5. 0,1010010001... 6. 1,320320032...
7. 2,313131... 8. 2,323232...
9. 0,121121112... 10. 0,2020020002...
11. $\sqrt{3}$ sonni aniqlovchi kesimni tuzing.
12. $\sqrt{5}$ sonni aniqlovchi kesimni tuzing.
13. Yig‘indilari ratsional bo‘lgan ikkita har hil irratsional sonlarni ko‘rsating.
14. Ko‘paytmalari ratsional bo‘lgan ikkita har xil irratsional sonlarni ko‘rsating.

Quyidagi sonlarning qaysilari yuqoridan, qaysilari quyidan chegaralanganligini aniqlang.

15. E-[3;5) dagi barcha irratsional sonlar to'plami.

16. E-[a;b) dagi barcha ratsional sonlar to'plami.

17. $\left\{\frac{n^3}{2n^3+3}; n \in N\right\}$.   18. $\left\{\frac{n^2}{n+1}; n \in N\right\}$.

19. $\left\{\frac{m}{n}; m,n \in N, \quad m \le n\right\}$ 20. $\left\{\frac{m}{n}; m,n \in N, \quad m > n\right\}$

21. *E*-[0;4] dagi barcha irratsional sonlar to'plami. Bu to'plamlarning chegaralangan ekanini ko'rsating va *supE*, *infE* larni toping.

22. *E*- barcha musbat irratsional sonlar to'plami bo'lsa, *infE* va *supE* larni toping.

23. Qanday to'plamlar uchun *supE*= *infE* bo'ladi?

24. [0;1] dan boshqa shunday *E* to'plam ko'rsatingki, bu to'plam uchun *infE*=0, *supE*=1 bo'lsin.

2-§. Haqiqiy sonning moduli (absolyut qiymati)

Ta'rifga binoan haqiqiy *a* sonning moduli (absolyut qiymati)

$$|a| = \begin{cases} a, & \text{agar } a \ge 0 \text{ bo'lsa,} \\ -a, & \text{agar } a < 0 \text{ bo'lsa} \end{cases}$$

kabi aniqlanadi.

Haqiqiy sonlarning moduli quyidagi xossalarga ega:

1) $|a| = |-a|$;

2) $|a| \le b$ va $-b \le a \le b$ tengsizliklar teng kuchli;

3) $|a| \ge b$ tengsizlik $a \ge b$ yoki $a \le -b$ ekanini bildiradi;

4) $|a \pm b| \le |a| + |b|$;

5) $|a \pm b| \ge |a| - |b|$;

6) $|a \cdot b| = |a| \cdot |b|$;

7) $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}, \ (b \ne 0)$.

Quyidagi tengsizliklarni yeching.

25. $|x-2| \le 3$.   26. $|x| < 5$.

27. $|x-5| < 4$.   28. $|x+2| \le 4$.

29. $|x+3| > 2$.   30. $|x+1| \ge 3$.

31. $|x+2|+|x-2| \le 10$.   32. $|x+3|+|x-3| \ge 5$.

33. $|x^2-2x-3| > x^2-2x-3$.   34. $|x^2-5x| > x^2-5x$.

Quyidagi tenglamalarni yeching.

35. $|2x+3| = x^2$.   36. $|3x+4| = x+4$.

37. $|x^2-5x+9| = 3$.   38. $\left|\frac{x-1}{x+1}\right| = \frac{x-1}{x+1}$.

39. $|x^2-2x+7|=4$.

40. $\left|\frac{x+2}{x-3}\right| = -\frac{x+2}{x-3}$.

3-§. Funksiya va uning aniqlanish sohasi

1. Elementlari hakiqiy sonlardan iborat bo‘lgan X va Y to‘plamlar berilgan bo‘lsin.

Agar har bir $x \in X$ songa biror qoida yoki qonunga binoan aniq bitta $y \in Y$ son mos qo‘yilgan bo‘lsa, u holda X to‘plamda aniqlangan f funksiya berilgan deyiladi va $y=f(x)$ ko‘rinishda yoziladi.

x erkli o‘zaruvchi yoki argument deyiladi, $f(x)$ funksiyaning x nuqtadagi qiymatini ifodalaydi.

X funksiyaning aniqlanish (mavjudlik) sohasi deyiladi va $D(f)$ ko‘rinishda belgilanadi. Funksiyaning qiymatlar to‘plami $f(X)$ ko‘rinishda belgilanadi.

2. Funksiya analitik usulda berilib, aniqlanish sohasi ko‘rsatilmagan bo‘lsin. Bu holda argumentning analitik ifoda ma’noga (hakiqiy qiymatga) ega bo‘ladigan barcha haqiqiy qiymatlari to‘plami funksiyaning aniqlanish sohasi deb tushuniladi va bu funksiyaning tabiiy aniqlanish sohasi deyiladi.

41. $f(x)=2x^3-3x+4$ funksiya berilgan, $f(-2)$, $f(1)$, $f(a)$ larni toping.

42. $f(x)=2\sin 2x+\cos x$ funksiya berilgan, $f(0),\ f(\frac{\pi}{4}),\ f(a)$ larni toping.

43. $f(x)=\frac{x-2}{x+1}$ funksiya berilgan, $f(0)$, $f(1)$, $f(2)$, $|f(0,5)|$ larni toping. $f(-1)$ mavjudmi?

44. $f(x)=\frac{|x-3|}{x-1}$ funksiya berilgan, $f(0)$, $f(2)$, $f(-2)$, larni toping. $f(1)$ mavjudmi?

45. $f(x)=\begin{cases} 2x, & \text{agar } -1<x<0, \\ 2, & \text{agar } 0\le x<1, \\ x-1, & \text{agar } 1\le x\le 3 \end{cases}$ funksiya berilgan. $f(2)$, $f(0)$, $f(0,5)$, $f(3)$ larni toping. $f(5)$ mavjudmi?

46. $f(x)=\begin{cases} \sin x, & \text{agar } -1\le x<0, \\ 1+x^2, & \text{agar } 0\le x\le 2. \end{cases}$ funksiya berilgan. $f(1),\ f(\frac{\pi}{2}),\ f(-\frac{\pi}{4})$, larni toping. $f(4)$ mavjudmi?

Quyidagi funksiyalarning aniqlanish sohasini toping.

47. $y=x^3-3x+2$.

48. $y=\frac{x}{x-1}$.

49. $y=\frac{2x-1}{x^2-3x+2}$.

50. $y=\frac{1}{x^2+2x-3}$.

51. $y=\sqrt{7-2x}$.

52. $y=\sqrt{4-x^2}$.

53. $y=\sqrt{x^2-4x+3}$.

54. $y=\sqrt{2x^2+x+8}$.

55. $y=\dfrac{2x-3}{\sqrt{x^2+2x-3}}$.

56. $y=\dfrac{x-4}{\sqrt{x^2-x-2}}$.

57. $y=\dfrac{1}{\sqrt{x^2-4x}}$.

58. $y=\dfrac{1}{\sqrt{|x|-x}}$.

59. $y=\lg(3x-4)$.

60. $y=\lg(2x-6)$.

61. $y=\lg(x^2-4x+3)$.

62. $y=\sqrt{\lg\dfrac{5x-x^2}{4}}$.

63. $y=\lg\sin x$.

64. $y=\dfrac{1}{\lg(x-1)}$.

65. $y=\arcsin(3x-4)$.

66. $y=\arcsin(2x-5)$.

67. $y=\arccos\dfrac{2x-3}{5}$.

68. $y=\arcsin\dfrac{1-2x}{4}$.

69. $y=\sqrt{3-x}+\arccos\dfrac{x-2}{3}$.

70. $y=\arcsin\dfrac{x-3}{2}+\lg(2x-3)$.

71. $y=\dfrac{3}{4-x^2}+\lg(x^3-x)$.

72. $y=\sqrt{x^2-3x+2}+\dfrac{1}{\sqrt{3+2x-x^2}}$.

73. $y=\sqrt{\sin x}-\sqrt{9-x^2}$.

74. $y=\dfrac{1}{\sqrt{\cos x}+\sqrt[3]{\sin x}}$.

75.Quyidagi funksiyalarning aniqlanish sohalari ustma-ust tushadimi?

1) $f(x)=\dfrac{1}{x^2-3x+10}$ va $g(x)=\sqrt{(x+5)(x-2)}$;

2) $f(x)=\sqrt{x}\cdot\sqrt{x-5}$ va $g(x)=\sqrt{x(x-5)}$;

3) $f(x)=\dfrac{1}{\sqrt{x}(x-10)(x-0{,}1)}$ va $g(x)=\dfrac{1}{\lg^2 x-1}$;

4) $f(x)=\arcsin(1-x)$ va $g(x)=\sqrt{2x-x^2}$.

76. Quyidagi funksiyalar aynan tengmi?

1) f(x)=x va g(x)=$\dfrac{x^2}{x}$; 2) f(x)=$\lg^2x$ va g(x)=2lgx;

3) f(x)=$(\sqrt{x})^2$ va g(x)=x; 4) f(x)=x^2, $x\in[1;2]$ va g(x)=x^2, $x\in[1;3]$.

4-§. Funksiyaning grafigi

X to‘plamda aniqlangan *y=f(x)* funksiya berilgan bo‘lsin. Funksiyaning *x* ga mos kelgan *f(x)* qiymatini hisoblasak, koordinatalar tekisligidagi $M(x,f(x))$ nuqtaga ega bo‘lamiz. Tekislikdagi nuqtalarning $\{M(x,f(x))/x\in X\}$ to‘plami *y=f(x)* funksiyaning grafigi deyiladi.

Quyidagi funksiyalarning grafigini yasang.

77. $y=2x-3$ 78. y=3x+1 79. $y=x^2$.

80. $y=\frac{x^2}{2}$.   81. $y=x^2+1$.   82. $y=x^2-1$.

83. $y=(x-1)^2$.   84. $y=(x+2)^2$.   85. $y=x^2-2x+3$.

86. $y=x^2-4x$.   87. $y=\frac{x}{x-1}$.   88. $y=\frac{x}{x+2}$.

89. $y=\left|x^2-4x\right|$.   90. $y=x^2-4|x|$.   91. $y=2|x|-x^2-8$.

92. $y=\left|2x^2-8\right|$.

93. $y=\begin{cases} x^2, & agar \quad x\le 0, \\ \frac{x}{3}, & agar \quad x>0. \end{cases}$ 94. $y=\begin{cases} 2, & agar \quad x\le 0, \\ x+2, & agar \quad x>0. \end{cases}$

95. $y=\begin{cases} -x, & agar \quad x\le -1, \\ 1, & agar \quad -1<x<0,. \\ 4^x, & agar \quad x\ge 0. \end{cases}$ 96. $y=\begin{cases} \sin x, & agar \quad x\le 0, \\ 0, & agar \quad 0<x<1, \\ \lg x, & agar \quad x\ge 1. \end{cases}$

5-§. Funksiyalarning kompozitsiyasi. Chegaralangan va chegaralanmagan funksiyalar. Juft va toq funksiyalar.

1. $y=f(u)$ funksiya E to‘plamda, $u=\varphi(x)$ funksiya X to‘plamda berilgan bo‘lib, $u=\varphi(x)$ ning qiymatlar to‘plami $\varphi(X)$ E ning qism to‘plami bo‘lsin. Agar $y=f(u)$ da u ning o‘rniga $\varphi(x)$ ni qo‘ysak, $y=f(\varphi(x))$ funksiyaga ega bo‘lamiz. Bu funksiyani $y=f(u)$ va $u=\varphi(x)$ funksiyalarning kompozitsiyasi yoki murakkab funksiya deyiladi.

2. *y=f(x)* funksiya X to‘plamda berilgan bo‘lsin. Agar shunday M son topilib, barcha $x\in X$ lar uchun $|f(x)|\le M$ tengsizlik o‘rinli bo‘lsa, u holda *y=f(x)* funksiya X to‘plamda chegaalangan funksiya deyiladi.

3. Agar X to‘plamga har bir x son bilan birgalikda $-x$ ham tegishli bo‘lsa, X koordinatalar boshiga nisbatan simmetrik to‘plam deyiladi.

$y=f(x)$ funksiya koordinatalar boshiga nisbatan simmetrik bo‘lgan X to‘plamda berilgan bo‘lsin.

a) agar ixtiyoriy $x\in X$ uchun $f(-x)=f(x)$ tenglik o‘rinli bo‘lsa, *y=f(x)* juft funksiya deyiladi.

b) agar ixtiyoriy $x\in X$ uchun $f(-x)=-f(x)$ tenglik o‘rinli bo‘lsa, *y=f(x)* toq funksiya deyiladi.

97. $y=u^2$, $u=x+1$ funksiyalar berilgan. y ni x orqali ifoda qiling.
98. $y=1-u^2$, $u=\sin x$ funksiyalar berilgan. y ni x orqali ifoda qiling.
99. $y=\sqrt{u+1}$, $u=3^x$ funksiyalar berilgan. y ni x orqali ifoda qiling.
100. $y=\sqrt{1+u^2}$, $u=tgx$ funksiyalar berilgan. y ni x orqali ifoda qiling.
101. $f(x)=x^2$ va $\varphi(x)=2^x$ funksiyalar berilgan. $f(f(x))$, $f(\varphi(x))$, $\varphi(f(x))$, $\varphi(\varphi(x))$ murakkab funksiyalarni toping.

102. $f(x)=x^3$ va $\varphi(x)=3^x$ funksiyalar berilgan. $f(\varphi(x))$ va $\varphi(f(x))$ murakkab funksiyalarni toping.

103. $f(x)=x^3-x$ va $\varphi(x)=\sin 2x$ funksiyalar berilgan. $\varphi(f(1))$, $\varphi(f(2))$, $f(\varphi(\frac{\pi}{2}))$, $f(f(f(1)))$ funksiyalarni toping.

104. $f(x)=\dfrac{5x^2+1}{2-x}$ funksiya uchun $f(3x)$, $f(x^3)$, $3f(x)$, $(f(x))^2$ larni toping.

Quyidagi funksiyalarni juft va toqlikka tekshiring.

105. $f(x)=x^2+1$

106. $f(x)=1-x^2$

107. $f(x)=x^4-2x^2$

108. $f(x)=x^3-x$

109. $f(x)=x^2\cos x$

110. $f(x)=x^2\sin x$

111. $f(x)=3^x$

112. $f(x)=x^3-2$

113. $f(x)=10$

114. $f(x)=|x|$

115. $f(x)=\mathrm{sgn}\, x$

116. $f(x)=[x]$

117. $f(x)=\{x\}$

118. $f(x)=\sin x^2+\sin^2 x$

119. $f(x)=\lg(x+\sqrt{1+x^2})$

120. Ikkita juft funksiyaning ko‘paytmasi juft funksiya bo‘lishini ko‘rsating.

121. Ikkita toq funksiyaning ko‘paytmasi juft funksiya bo‘lishini ko‘rsating.

122. Toq va juft funksiyalarning ko‘paytmasi toq funksiya bo‘lishini ko‘rsating.

123. Agar $\varphi(x)$ juft funksiya bo‘lsa, u holda $f(\varphi(x))$ funksiyaning juft bo‘lishini ko‘rsating.

124. Agar $u=f(x)$ va $x=\varphi(t)$ funksiyalar toq bo‘lsa, u holda $f(\varphi(t))$ funksiyaning toq bo‘lishini ko‘rsating.

125. Agar $f(x)$ toq funksiya bo‘lib, [-5;0] segmentda $f(x)=x^2+3$ formula bilan berilgan bo‘lsa, $f(-2)$, $f(2)$, $f(4)$ larni toping.

Quyidagi funksiyalarning qaysi biri chegaralangan?

126. $f(x)=x^2+2$, [-1;3].

127. $f(x)=\{x\}$, $(-\infty;+\infty)$.

128. $f(x)=x^2$, $(-\infty;0)$.

129. $f(x)=\dfrac{1}{x}$, $(0;+\infty)$.

130. $f(x)=\dfrac{x}{x^2+1}$, $(-\infty;+\infty)$.

131. $f(x)=\dfrac{1}{x^4+1}$, $(-\infty;+\infty)$.

132. $f(x)=\sin ax$, $(-\infty;+\infty)$.

133. $f(x)=\mathrm{tg}\, x$, $(-\frac{\pi}{2};\frac{\pi}{2})$.

6-§. Davriy funksiyalar. Monoton funksiyalar.

1. Agar $l\neq 0$ son uchun har bir $x\in X$ bilan birgalikda $x+l$ va $x-l$ lar ham X to‘plamga tegishli bo‘lsa, X to‘plam l davrli davriy to‘plam deyiladi.

a) Q- ratsional sonlar to‘plami.

b) $X_1=(-\infty;+\infty)$.

c) $X_2=\ldots\cup(-\frac{3\pi}{2},-\frac{\pi}{2})\cup(-\frac{\pi}{2};\frac{\pi}{2})\cup(\frac{\pi}{2};\frac{3\pi}{2})\cup\ldots$

d) $X_3 = \ldots \bigcup [-2,-1] \bigcup [0;1] \bigcup [2;3] \bigcup \ldots$

to‘plamlarning har biri davriy to‘plamga misol bo‘la oladi.

e) $X_4=[0,+\infty)$,

f) $X_5=(-2,2)$,

g) $X_6=[3,6]$

to‘plamlar davriy to‘plam emas.

$y = f(x)$ funksiya l davrli davriy X to‘plamda berilgan bo‘lsin.

Agar ixtiyoriy $x \in X$ lar uchun $f(x+l) = f(x)$ tenglik o‘rinli bo‘lsa, $y=f(x)$ davriy funksiya deyiladi va l uning davri deyiladi. Funksiyaning eng kichik musbat davri (agar u mavjud bo‘lsa) uning asosiy davri deyiladi.

2. $y=f(x)$ funksiya berilgan bo‘lsin. Agar ixtiyoriy $x_1, x_2 \in X$ lar uchun $x_1 < x_2$ dan

a) $f(x_1) < f(x_2)$ kelib chiqsa, $f(x)$ funksiya o‘suvchi,

b) $f(x_1) \le f(x_2)$ kelib chiqsa, $f(x)$ funksiya kamaymovchi,

c) $f(x_1) > f(x_2)$ kelib chiqsa, $f(x)$ funksiya kamayuvchi,

d) $f(x_1) \ge f(x_2)$ kelib chiqsa, $f(x)$ funksiya o‘smovchi deyiladi.

Bu to‘rt tip funksiya bir so‘z bilan monoton funksiya deyiladi.

Quyidagi funksiyalarning qaysilari davriy ekanligini aniqlang. Davriy funksiyalarning asosiy davrini ko‘rsating.

134. $f(x)=\sin 2x$. 135. $f(x)=\cos \pi x$.

136. $f(x)=\sin^2 x$. 137. $f(x)=\{x\}$.

138. $f(x)=5$. 139. $f(x)=\sin(x+1)$.

140. $f(x)=\cos x^2$. 141. $f(x)=\lg(\sin x)$.

142. $f(x)=\sin(\cos x)$. 143. $f(x)=\sin^2 3x$.

144. $D(x)=\begin{cases} 1, & \text{agar } x \text{ ratsional son bo'lsa,} \\ 0, & \text{agar } x \text{ irratsional son bo'lsa} \end{cases}$

Dirixle funksiyasi davriy funksiya bo‘lib, uning asosiy davri yo‘q ekanini ko‘rsating.

145. $f(x)=\{2x\}$ funksiyaning asosiy davrini toping.

Quyidagi funksiyalarning har biri ko‘rsatilgan oraliqlarda qat’iy monoton ekanini ko‘rsating.

146. $f(x)=2x-3$, $(-\infty;+\infty)$. 147. $f(x)=-3x+2$, $(-\infty;+\infty)$.

148. $f(x)=x^2+2x+5$, $(-\infty;-1)$ va $(-1;+\infty)$. 149. $f(x)=\cos x$, $[0;\pi]$.

150. $f(x)=|x|-x$, $(-\infty;0]$. 51. $f(x)=2x-1$, $(-\infty;+\infty)$.

Quyidagi funksiyalarga teskari funksiyalarni toping va ularning grafigini yasang.

152. y=2x. 153. $y = 1-3x$.

154. $y = \frac{1}{x}$. 155. $y = \frac{1}{1-x}$.

156. $y=10^{x+1}$.

157. $y=\log_x 2$.

158. $y=x^2-2x$.

159. $y=1+\lg(x+2)$.

II BOB. LIMITLAR

1-§. Sonli ketma-ketlik limiti

1. Ushbu

$$x_1, x_2, ..., x_n, ... \qquad (1)$$

ketma-ketlik berilgan bo‘lsin. Har bir $\varepsilon > 0$ uchun shunday $N = N(\varepsilon)$ nomer topilib, barcha $n > N$ lar uchun $|x_n - a| < \varepsilon$ tengsizlik o‘rinli bo‘lsa, u holda a soni (1) ketma-ketlikning limiti deyiladi va $\lim\limits_{n\to\infty} x_n = a$ ko‘rinishda yoziladi.

Limitga ega bo‘lgan ketma-ketlik yaqinlashuvchi, limitga ega bo‘lmagan ketma-ketlik uzoqlashuvchi deyiladi.

2. Agar $\lim\limits_{n\to\infty} x_n = 0$ bo‘lsa, u holda x_n cheksiz kichik miqdor (yoki qisqacha cheksiz kichik) deyiladi.

3. Agar istagancha katta $\Delta > 0$ uchun shunday $N=N(\Delta)$ nomer topilib, $n>N$ lar uchun $|x_n|>\Delta$ tengsizlik o‘rinli bo‘lsa, u holda x_n cheksiz katta miqdor (yoki qisqacha cheksiz katta) deyiladi va $\lim\limits_{n\to\infty} x_n = \infty$ ko‘rinishda yoziladi.

Teorema. Agar x_n cheksiz kichik bo‘lsa, u holda $y_n = \dfrac{1}{x_n}$ cheksiz katta bo‘ladi. Agar x_n cheksiz katta bo‘lsa, u holda $y_n = \dfrac{1}{x_n}$ cheksiz kichik bo‘ladi.

4. Teorema. Agar $x_1, x_2, ..., x_n, ...$ va $y_1, y_2, ..., y_n, ...$ ketma-ketliklar yaqinlashuvchi bo‘lsa, u holda

a) $\lim\limits_{n\to\infty}(x_n \pm y_n) = \lim\limits_{n\to\infty} x_n \pm \lim\limits_{n\to\infty} y_n$,

b) $\lim\limits_{n\to\infty}(x_n \cdot y_n) = \lim\limits_{n\to\infty} x_n \cdot \lim\limits_{n\to\infty} y_n$,

c) $\lim\limits_{n\to\infty} \dfrac{x_n}{y_n} = \dfrac{\lim\limits_{n\to\infty} x_n}{\lim\limits_{n\to\infty} y_n}$ $\quad (\lim\limits_{n\to\infty} y_n \neq 0)$ bo‘ladi.

5. Teorema. Agar $\lim\limits_{n\to\infty} x_n = 0$ bo‘lsa va (y_n) chegaralangan bo‘lsa, u holda $\lim\limits_{n\to\infty}(x_n \cdot y_n) = 0$ bo‘ladi.

Ketma-ketlik limiti ta’rifidan foydalanib quyidagilarni isbotlang.

160. $\lim\limits_{n\to\infty} \dfrac{n-1}{n+1} = 1$

161. $\lim\limits_{n\to\infty} \dfrac{4n-1}{2n+1} = 2$

162. $\lim\limits_{n\to\infty} \dfrac{4n+1}{5n+1} = \dfrac{4}{5}$

163. $\lim\limits_{n\to\infty} \dfrac{\sqrt{n^2+n}}{n} = 1$

164. $\lim\limits_{n\to\infty} \dfrac{2n-1}{2-3n} = -\dfrac{2}{3}$ tenglikni isbotlang.

Qaysi n dan boshlab, $\left|\dfrac{2n-1}{2-3n} + \dfrac{2}{3}\right| < 0{,}0001$ tengsizlik o‘rinli bo‘ladi?

165. $\lim\limits_{n\to\infty}\frac{3n-1}{5n+1}=\frac{3}{5}$ tenglikni isbotlang.

Qaysi n dan boshlab, $\left|\frac{3n-1}{5n+1}-\frac{3}{5}\right|<0{,}001$ tengsizlik o‘rinli bo‘ladi?

166. Quyidagi miqdorlarning cheksiz katta ekanligini ko‘rsating.

1) $x_n=8n+1$; 2) $x_n=6n-1$;

3) $x_n=n^x$, $(x>0)$; 4) $x_n=\sqrt{n^3+2}$.

Quyidagi limitlarni toping.

167. $\lim\limits_{n\to\infty}\frac{3n^2+2}{4n^2-1}$. 168. $\lim\limits_{n\to\infty}\frac{3n^3-4}{n^3+6}$.

169. $\lim\limits_{n\to\infty}\frac{2n^3+3}{n^3+n+1}$. 170. $\lim\limits_{n\to\infty}\frac{n^2+n+1}{(n+1)^2}$.

171. $\lim\limits_{n\to\infty}\frac{(n+1)^3}{n^3+1}$. 172. $\lim\limits_{n\to\infty}\frac{(n+1)^4+(n-1)^4}{n^4+10}$.

173. $\lim\limits_{n\to\infty}\frac{\sqrt[3]{n^2+n}}{n+5}$. 174. $\lim\limits_{n\to\infty}\frac{\sqrt[3]{n^3+2n-1}}{n+2}$.

175. $\lim\limits_{n\to\infty}\frac{\sqrt{n^4+n^2+1}}{\sqrt[3]{n^3+2n-1}}$. 176. $\lim\limits_{n\to\infty}\frac{\sqrt{2n^2+1}}{\sqrt[4]{n^4+3n-1}}$.

177. $\lim\limits_{n\to\infty}\frac{n!}{(n+1)!-n!}$. 178. $\lim\limits_{n\to\infty}\frac{(n+1)!+n!}{(n+2)!}$.

179. $\lim\limits_{n\to\infty}\frac{1+\frac{1}{2}+\ldots+\frac{1}{2^n}}{1+\frac{1}{3}+\ldots+\frac{1}{3^n}}$. 180. $\lim\limits_{n\to\infty}\frac{1+2+\ldots+n}{n^2}$.

181. $\lim\limits_{n\to\infty}\left(\frac{\cos n^3}{2n}-\frac{3n}{6n+1}\right)$. 182. $\lim\limits_{n\to\infty}\left(\frac{1}{n}\cos\frac{n\pi}{2}+1\right)$.

183. $\lim\limits_{n\to\infty}\left(\frac{n}{n^2+1}\sin(n+1)+\frac{2n^2}{1-9n^2}\right)$. 184. $\lim\limits_{n\to\infty}\left(\frac{1}{3n}\sin n^2+\frac{2n}{3n+1}\right)$.

185. $\lim\limits_{n\to\infty}\left(\frac{1+2+3+\ldots+n}{n+2}-\frac{n}{2}\right)$. 186. $\lim\limits_{n\to\infty}\frac{1-2+3-4+\ldots-2n}{\sqrt{n^2+1}}$.

187. Monoton ketma-ketliklarning limiti haqidagi teoremalardan foydalanib, quyidagi ketma-ketliklarning limiti mavjudligini ko‘rsating.

1) $x_n = 1 + \frac{1}{2^2} + \ldots + \frac{1}{n^n}$, 2) $x_n = 1 + \frac{1}{2!} + \ldots + \frac{1}{n!}$,

3) $x_n = \frac{1}{2+1} + \frac{1}{2^2+1} + \ldots + \frac{1}{2^n+1}$.

188. Quyidagi ketma-ketliklarning limitlari mavjudligini ko‘rsating va ularni toping.

1) $x_n = \frac{c^n}{n!} (c > 0)$; 2) $x_n = \underbrace{\sqrt{3 + \sqrt{3 + \ldots + \sqrt{3}}}}_{n \text{ ta}}$;

3) $x_n = \underbrace{\sin \sin \ldots \sin 1}_{n \text{ ta}}$

189. $\lim\limits_{n \to \infty} \left(2^n \underbrace{\sqrt{2 - \sqrt{2 + \ldots + \sqrt{2}}}}_{n \text{ ta}} \right)$ ni toping.

2-§. Funksiyaning limiti

1. *y=f(x)* funksiya X to‘plamda berilib, *a* X to‘plamning limit nuqtasi bo‘lsin.

Agar har bir $\varepsilon>0$ uchun shunday $\delta=\delta(\varepsilon)>0$ topilib X to‘plamning $0 < |x - a| < \delta$ tengsizlikni qanoatlantiruvchi barcha x nuqtalarida $|f(x) - A| < \varepsilon$ tengsizlik o‘rinli bo‘lsa, u holda *A f(x)* funksiyaning *x=a* nuqtadagi limiti deyiladi. U $\lim\limits_{x \to a} f(x) = A$ ko‘rinishda yoziladi.

2. Agar har bir $\varepsilon>0$ uchun shunday $\Delta>0$ topilib, $|x|>\Delta$ tengsizlikni qanoatlantiruvchi barcha x larda $|f(x) - B| < \varepsilon$ tengsizlik o‘rinli bo‘lsa, B $f(x)$ funksiyaning $x \to \infty$ dagi limiti deyiladi. U $\lim\limits_{x \to \infty} f(x) = B$ ko‘rinishda yoziladi.

Agar $x>0$ bo‘lsa, $\lim\limits_{x \to +\infty} f(x) = B$, $x<0$ bo‘lsa, $\lim\limits_{x \to -\infty} f(x) = B$ ko‘rinishda yoziladi.

Teorema. Agar *x=a* nuqtada *f(x)* va *g(x)* funksiyalar limitga ega bo‘lsa, u holda:

1. $\lim\limits_{x \to a}(f(x) \pm g(x)) = \lim\limits_{x \to a} f(x) \pm \lim\limits_{x \to a} g(x)$,
2. $\lim\limits_{x \to a}(f(x) \cdot g(x)) = \lim\limits_{x \to a} f(x) \cdot \lim\limits_{x \to a} g(x)$,
3. $\lim\limits_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim\limits_{x \to a} f(x)}{\lim\limits_{x \to a} g(x)} \quad (\lim\limits_{x \to a} g(x) \neq 0)$

tengsizliklar o‘rinli bo‘ladi.

190. Funksiya limiti ta’rifidan foydalanib, quyidagi tengliklarni isbotlang.

1) $\lim\limits_{x \to 3}(3x - 5) = 4$. 2) $\lim\limits_{x \to 1}(4x - 1) = 3$.

3) $\lim\limits_{x\to 2}\frac{x^2-2}{x^2+1}=\frac{2}{5}$.

4) $\lim\limits_{x\to \frac{\pi}{2}}\sin x=1$.

191. $\lim\limits_{x\to 2}(2x-1)=3$ tenglikni isbotlang. δ ning qanday qiymatlarida 0<|x-2|<δ tengsizlikda $|(2x-1)-3|<0{,}01$tengsizlik kelib chiqadi?

193. $\lim\limits_{x\to 3}\frac{x-1}{2(x+1)}=\frac{1}{4}$ tenglikni isbotlang. δ ning qanday qiymatlarida 0<|x-3|<δ tengsizlikda $\left|\frac{x-1}{2(x+1)}-\frac{1}{4}\right|<0{,}01$ tengsizlik kelib chiqadi?

194. Funksiyaning cheksizlikdagi limiti ta'rifidan foydalanib, quyidagi tengliklarni isbotlang.

1) $\lim\limits_{x\to\infty}\frac{x^2}{x^2+1}=1$;

2) $\lim\limits_{x\to\infty}\frac{2x+1}{x+2}=2$;

3) $\lim\limits_{x\to+\infty}(\sqrt{x^2+1}-x)=0$;

4) $\lim\limits_{x\to+\infty}(\sqrt{x+1}-\sqrt{x})=0$.

Quyidagi limitlarni toping.

195. $\lim\limits_{x\to 2}\frac{x^2+5}{x^2+2}$.

196. $\lim\limits_{x\to 1}(5x^3-4x+1)$.

197. $\lim\limits_{x\to 1}\frac{x(x-1)}{x^2-1}$.

198. $\lim\limits_{x\to 5}\frac{x^2-25}{x-5}$.

199. $\lim\limits_{x\to 2}\frac{x^2-5x+6}{x^2-12x+20}$.

200. $\lim\limits_{x\to -1}\frac{x^2-2x+1}{x^3-x}$.

201. $\lim\limits_{x\to -1}\frac{2x^3+2x^2+3x+3}{x^3+x^2+x+1}$.

202. $\lim\limits_{x\to -2}\frac{2x^2+5x+2}{3x^2-x-14}$.

203. $\lim\limits_{x\to 0}\frac{\sqrt{1+x^2}-1}{x}$.

204. $\lim\limits_{x\to 0}\frac{\sqrt{1+2x}-1}{x}$.

205. $\lim\limits_{x\to 0}\frac{\sqrt{x+h}-\sqrt{h}}{x}$.

206. $\lim\limits_{x\to 0}\frac{\sqrt{1+3x^2}-1}{2x^2}$.

207. $\lim\limits_{x\to 0}\frac{\sqrt[3]{1+x^2}-1}{x^2}$.

208. $\lim\limits_{x\to 0}\frac{\sqrt[3]{1-2x}-1}{3x}$.

209. $\lim_{x\to 0}\frac{\sqrt{1+x^2}-1}{\sqrt{16+x^2}-4}$.

210. $\lim_{x\to 0}\frac{\sqrt{9+3x}-3}{\sqrt{25+2x}-5}$.

211. $\lim_{x\to 5}\frac{\sqrt{x-1}-2}{x^2-25}$.

212. $\lim_{x\to 2}\frac{\sqrt{5x-1}-3}{x^2-4}$.

213. $\lim_{x\to\infty}\frac{2x^2-5x+4}{5x^2-2x+3}$.

214. $\lim_{x\to\infty}\frac{x^3+x}{x^4-3x^2+1}$.

215. $\lim_{x\to\infty}\frac{10x^4+3x^3+1}{0{,}1x^4+1}$.

216. $\lim_{x\to\infty}\frac{1+x-3x^3}{1+x^2+3x^3}$.

217. $\lim_{x\to\infty}\left(\frac{x^3}{x^2+1}-x\right)$.

218. $\lim_{x\to\infty}\left(\frac{x^4}{x^2+4}-x^2\right)$.

219. $\lim_{x\to+\infty}\frac{\sqrt{x^2+1}+\sqrt{x}}{\sqrt[4]{x^3+x}-x}$.

220. $\lim_{x\to\infty}\frac{\sqrt{x^4-3x+1}}{1-x^2}$.

221. $\lim_{x\to\infty}\frac{\sqrt[3]{x^4+3}-\sqrt[5]{x^3+4}}{\sqrt[3]{x^7+1}}$.

222. $\lim_{x\to\infty}\frac{\sqrt{x^2+1}-\sqrt[5]{x^2+1}}{\sqrt[4]{x^4+1}-\sqrt[5]{x^4+1}}$.

223. $\lim_{x\to+\infty}(\sqrt{x+4}-\sqrt{x})$.

224. $\lim_{x\to+\infty}(\sqrt{x-1}-\sqrt{x})$.

225. $\lim_{x\to+\infty}x(\sqrt{x^2+1}-\sqrt{x^2-1})$.

226. $\lim_{x\to+\infty}x(\sqrt{x^2+1}-x)$.

227. $\lim_{x\to-\infty}(\sqrt{x^2+1}-x)$.

228. $\lim_{x\to-\infty}(\sqrt{x^2+1}+x)$.

Quyidagi misollarni yechishda $\lim_{x\to 0}\frac{\sin x}{x}=1$ tenglikdan foydalaning.

229. $\lim_{x\to 0}\frac{\sin 2x}{3x}$.

230. $\lim_{x\to 0}\frac{\sin 3x}{5x}$.

231. $\lim_{x\to 0}\frac{\sin 6x}{\sin 7x}$.

232. $\lim_{x\to 0}\frac{\sin \alpha x}{\sin \beta x}$.

233. $\lim_{x\to 0}\frac{tg3x}{4x}$.

234. $\lim_{x\to 0}\frac{tgkx}{nx}$.

235. $\lim_{x\to 0}\frac{tg5x}{\sin 4x}$.

236. $\lim_{x\to 0}\frac{4\arcsin x}{3x}$.

237. $\lim_{x\to 0}\frac{arctgx}{3x}$.

238. $\lim_{x\to 0}\frac{2x+\arcsin x}{2x+arctgx}$.

239. $\lim_{x\to 0}\frac{3x+\arcsin x}{\sin x+\arcsin x}$.

240. $\lim_{x\to 0}\frac{1-\cos x}{5x}$.

241. $\lim_{x\to 0}\frac{1-\cos 3x}{x\sin x}$.

242. $\lim_{x\to 0}\frac{1-\cos^3 x}{x\sin 2x}$.

243. $\lim_{x\to 0}\frac{\sin^3\frac{x}{4}}{x^3}$.

244. $\lim_{x\to \frac{\pi}{2}}\frac{1-\sin x}{\left(\frac{\pi}{2}-x\right)^2}$.

245. $\lim_{x\to \frac{\pi}{2}}\left(\frac{\pi}{2}-x\right)tgx$.

246. $\lim_{x\to 1}(1-x)tg\frac{\pi x}{2}$.

247. $\lim_{x\to 0}\frac{\cos\alpha x-\cos\beta x}{x^2}$.

248. $\lim_{x\to \alpha}\frac{\sin^2 x-\sin^2\alpha}{x^2-\alpha^2}$.

249. $\lim_{x\to 0}\frac{1-\sin x-\cos x}{1+\sin x-\cos x}$.

250. $\lim_{x\to \frac{\pi}{2}}\frac{\cos x}{\sqrt[3]{(1-\sin x)^2}}$.

251. $\lim_{x\to \frac{\pi}{2}}\frac{\cos x}{\cos\frac{x}{2}-\sin\frac{x}{2}}$.

252. $\lim_{x\to \frac{\pi}{4}}\frac{1-\sin 2x}{\sin x-\cos x}$.

Quyidagi misollarni yechishda $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x=e$ tenglikdan foydalaning.

253. $\lim_{x\to\infty}\left(\frac{x}{x+1}\right)^x$.

254. $\lim_{x\to\infty}\left(\frac{x+1}{x-1}\right)^x$.

255. $\lim_{x\to\infty}\left(1-\frac{1}{x}\right)^x$.

256. $\lim_{x\to\infty}\left(1+\frac{k}{x}\right)^{mx}$.

257. $\lim_{x\to\infty}\left(1+\frac{5}{x}\right)^{2x}$.

258. $\lim_{x\to\infty}\left(1+\frac{3}{2x}\right)^{-x}$.

259. $\lim_{x\to\infty}\left(\frac{2x+1}{2x-1}\right)^{x}$.

260. $\lim_{x\to\infty}\left(\frac{2x-3}{2x+4}\right)^{x}$

261. $\lim_{x\to\infty}\left(\frac{x^2-2x+1}{x^2-4x+1}\right)^{x}$.

262. $\lim_{x\to\infty}\left(\frac{x+1}{3x-1}\right)^{x}$

263. $\lim_{\alpha\to 0}(1+tg\alpha)^{ctg\alpha}$.

264. $\lim_{x\to 0}(1+\sin 2x)^{\cos ecx}$.

265. $\lim_{x\to +\infty}\left(1+\frac{1}{x}\right)^{x^2}$.

266. $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{\frac{x+1}{x}}$.

267. $\lim_{x\to\infty}\left(x^2\left(1-\cos\frac{1}{x}\right)\right)$.

268. $\lim_{x\to 0}(\cos x)^{\frac{1}{\sin^2 x}}$.

Quyidagi misollarni yechishda $\lim_{x\to 0}\frac{\ln(1+x)}{x}=1$, $\lim_{x\to 0}\frac{a^x-1}{x}=\ln a$, $\lim_{x\to 0}\frac{(1+x)^{\mu}-1}{x}=\mu$ tengliklardan foydalaning.

269. $\lim_{x\to 0}\frac{\ln(1+kx)}{x}$.

270. $\lim_{x\to 0}\frac{\ln(a+x)-\ln a}{x}$.

271. $\lim_{x\to e}\frac{\ln x-1}{x-e}$.

272. $\lim_{x\to\infty}x\ln\frac{x+a}{x}$.

273. $\lim_{x\to 0}\frac{e^{2x}-1}{3x}$.

274. $\lim_{x\to 0}\frac{e^{-x}-1}{\sin x}$.

275. $\lim_{x\to 0}\frac{e^{\sin 2x}-1}{2x}$.

276. $\lim_{x\to 0}\frac{e^{x^2}-1}{1-\cos x}$.

277. $\lim_{x\to 0}\frac{10^{x^2}-1}{x^2+x^3}$.

278. $\lim_{x\to 0}\frac{4^x-1}{tgx}$.

279. $\lim_{x\to 0}\frac{e^{ax}-e^{bx}}{x}$.

280. $\lim_{x\to 0}\frac{e^{\sin 2x}-e^{\sin x}}{x}$.

281. $\lim_{x\to 0}\frac{\sqrt[n]{1+x}-1}{x}$.

282. $\lim_{x\to 0}\frac{\sqrt[4]{1+\sin 3x}-1}{2x}$.

283. $\lim_{x\to 0}\frac{\sqrt[3]{1+2x}-1}{tgx}$.

284. $\lim_{x\to 0}\frac{\sqrt[3]{1+x^2}-\sqrt{1-2x}}{3x}$.

285. $\lim_{x\to 0}\frac{\sqrt[3]{1+x}-\sqrt[4]{1-2x}}{x+x^2}$.

286. $\lim_{x\to 0}\frac{\sqrt[7]{1+\arcsin x}-1}{14x}$.

3-§. Bir tomonli limitlar

1. Agar har bir ε>0 uchun shunday δ=δ(ε)>0 topilib, $a-\delta<x<a\ (a<x<a+\delta)$ tengsizlikni qanoatlantiruvchi x larda $|f(x)-A|<\varepsilon$ tengsizlik o‘rinli bo‘lsa, u holda *A* *f(x)* funksiyaning *x*=*a* dagi chap (o‘ng) limiti deyiladi. Chap limit $f(a-0)=\lim_{x\to a-0}f(x)$, o‘ng limit $f(a+0)=\lim_{x\to a+0}f(x)$ ko‘rinishda belgilanadi.

2. *x*=*a* da funksiya limitga ega bo‘lishi uchun $f(a-0)=f(a+0)$ tenglikning bajarilishi zarur va etarlidir.

Quyidagi funksiyalarning ko‘rsatilgan nuqtalardagi bir tomonli limitlarini toping va grafiklarini yasang.

287. $f(x)=\begin{cases} x+1, & \text{agar } 0<x<1, \\ 3x+1, & \text{agar } 1\le x<3 \end{cases}$ *x*=1 nuqtada.

288. $f(x)=\begin{cases} 2x+1, & \text{agar } 0<x\le 2, \\ 2x-1, & \text{agar } 2<x<3 \end{cases}$ *x*=2 nuqtada.

289. $f(x)=\begin{cases} x^2, & \text{agar } -1\le x<3, \\ 2x+1, & \text{agar } 3\le x<4 \end{cases}$ *x*=3 nuqtada.

290. $f(x)=\begin{cases} 3x, & \text{agar } -1\le x\le 1, \\ 2x, & \text{agar } 1<x\le 3 \end{cases}$ *x*=1 va *x*=2 nuqtalarda.

291. *f(x)=E(x),* *x*=-2, *x*=0, *x*=1 nuqtalarda.

292. *f(x)*={*x*}, *x*=1, *x*=2, *x*=3 nuqtalarda.

293. $f(x)=\frac{3x+1}{x-1}$, *x*=1 nuqtada.

294. $f(x)=\dfrac{\sin x}{x-2}$, x=2 nuqtada.

295. $f(x)=\begin{cases} -\dfrac{1}{x-1}, & \text{agar} \quad x<0, \\ x, & \text{agar} \quad 0\le x<1, \\ 2, & \text{agar} \quad 1\le x\le 2 \end{cases}$ x=0, x=1 nuqtalarda.

296. $f(x)=\begin{cases} -\sin x, & \text{agar} \quad -\dfrac{\pi}{2}<x<0, \\ \sin x, & \text{agar} \quad 0\le x<\dfrac{\pi}{2} \end{cases}$ x=0 nuqtada.

297. $f(x)=\sin\dfrac{1}{x}$ funksiyaning x=0 nuqtada bir tomonli limitlarning mavjud emasligini ko‘rsating.

298. $f(x)=\begin{cases} 3x, & \text{agar} \quad -1\le x<1, \\ 2x+1, & \text{agar} \quad 1<x<2 \end{cases}$ funksiya x=0 nuqtada limitga egami?

4-§. Cheksiz kichiklarni taqqoslash. Ekvivalent cheksiz kichiklar

1. Agar $\lim\limits_{x\to a}\alpha(x)=0$ bo‘lsa, u holda $\alpha(x)$ $x\to a$ da cheksiz kichik funksiya (qisqacha - «cheksiz kichik») deyiladi.

α(x) va *β(x)* lar cheksiz kichiklar bo‘lsin.

2. Agar $\lim\limits_{x\to a}\dfrac{\alpha(x)}{\beta(x)}=0$ bo‘lsa, u holda *α(x)* *β(x)* ga nisbatan yuqori tartibli cheksiz kichik deyiladi va $\alpha(x)=o(\beta(x))$ ko‘rinishda yoziladi.

3. Agar $\lim\limits_{x\to a}\dfrac{\alpha(x)}{\beta(x)}=c$ ($c\neq 0$, $c\neq\infty$) bo‘lsa, u holda *α(x)* va *β(x)* bir xil tartibli cheksiz kichiklar deyiladi. Xususiy holda $\lim\limits_{x\to a}\dfrac{\alpha(x)}{\beta(x)}=1$ bo‘lsa, u holda *α(x)* va *β(x)* lar ekvivalent cheksiz kichiklar deyiladi va $\alpha(x)\sim\beta(x)$ ko‘rinishda yoziladi.

4. Teorema. Agar $x=a$ nuqta atrofida $\alpha(x)$, $\beta(x)$, $\alpha_1(x)$, $\beta_1(x)$ lar cheksiz kichik va $\alpha(x)\sim\alpha_1(x)$, $\beta(x)\sim\beta_1(x)$ bo‘lsa, u holda $\lim\limits_{x\to a}\dfrac{\alpha(x)}{\beta(x)}=\lim\limits_{x\to a}\dfrac{\alpha_1(x)}{\beta_1(x)}$ bo‘ladi.

299. $x\to 0$ da quyidagi cheksiz kichiklarini $\beta(x)=x$ cheksiz kichik bilan taqqoslang.
1) $\alpha(x)=3x$, 2) $\alpha(x)=x^2$, 3) $\alpha(x)=x^5+x^6$, 4) $\alpha(x)=x+\sin x$, 5) $\alpha(x)=\text{tg}x+\sin x$,
6) $\alpha(x)=x\sqrt{1+x^2+x^4}-x^3$, 7) $\alpha(x)=\sqrt{1+\sin x}-1$, 8) $\alpha(x)=x^2+\text{tg}x$.

300. Quyidagi funksiyaning qaysilari ekvivalent cheksiz kichiklar ekanini tekshiring.

1) $\alpha(x) = \sin nx$ va $\beta(x) = nx$, $x \to 0$ da;

2) $\alpha(x) = \mathrm{tg} mx$ va $\beta(x) = mx$, $x \to 0$ da;

3) $\alpha(x) = \sqrt{1+x} - 1$ va $\beta(x) = \frac{1}{2}x$, $x \to 0$ da;

4) $\alpha(x) = x^2 - 1$ va $\beta(x) = 2(x - 1)$, $x \to 1$ da;

5) $\alpha(x) = 1 - \cos x$ va $\beta(\mathrm{x}) = \frac{1}{2}x^2$, $x \to 0$ da;

6) $\alpha(x) = \sqrt{1 + tgx} - 1$ va $\beta(x) = \frac{x}{2}$, $x \to 0$ da;

7) $\alpha(x) = \frac{\sqrt{1 + x^2 + x^3} - 1}{\sin 2x}$ va $\beta(x) = \sin x$, $x \to 0$ da.

Ekvivalent cheksiz kichiklardan foydalanib, quyidagi limitlarni toping.

301. $\lim\limits_{x \to 0} \frac{tg6x}{\sin 3x}$.

302. $\lim\limits_{x \to 0} \frac{\sqrt{1+x} - 1}{\sin 2x}$.

303. $\lim\limits_{x \to 0} \frac{\sqrt{1+tgx} - 1}{\sqrt{1+x+x^2} - 1}$.

304. $\lim\limits_{x \to 0} \frac{\sqrt{1+\sin x} - 1}{x - x^2}$.

305. $\lim\limits_{x \to 0} \frac{1 - \cos x}{x^2 - x^4 + x^6}$.

306. $\lim\limits_{x \to 0} \frac{\sqrt{1+x^2} - 1}{1 - \cos x}$.

III BOB. UZLUKSIZ FUNKSIYALAR

1-§. Uzluksiz funksiyalar. Funksiyalarning uzilish nuqtalari.

1. Agar $\lim\limits_{x \to x_0} f(x) = f(x_0)$ tenglik o‘rinli bo‘lsa, funksiya x_0 nuqtada uzluksiz deyiladi.

2. Argumentning x va x_0 qiymatlari orasidagi x-x_0 ayirma argumentning x_0 nuqtadagi orttirmasi deyiladi va $\Delta x=x-x_0$ orqali belgilanadi. Funksiyaning $x = x_0 + \Delta x$ va x_0 nuqtalardagi qiymatlarining ayirmasi $f(x_0+\Delta x)-f(x_0)$ esa funksiyaning x_0 nuqtadagi orttirmasi deyiladi va $\Delta y= f(x_0+\Delta x)-f(x_0)$ orqali belgilanadi.

Uzluksizlikning ta’rifini yana quyidagicha berish mumkin:

Agar $\lim\limits_{\Delta x \to 0} \Delta y = \lim\limits_{\Delta x \to 0} (f(x_0 + \Delta x) - f(x_0)) = 0$ tenglik o‘rinli bo‘lsa, funksiya x_0 nuqtada uzluksiz deyiladi.

3. Agar $\lim\limits_{x \to x_0} f(x) \neq f(x_0)$ bo‘lsa, u holda $x=x_0$ funksiyaning uzilish nuqtasi deyiladi.

$x=x_0$ funksiyaning uzilish nuqtasi bo‘lsin.

Agar $f(x_0-0)$, $f(x_0+0)$ bir tomonli limitlar mavjud bo‘lsa, u holda $x=x_0$ funksiyaning I tur uzilish nuqtasi deyiladi.

Agar $f(x_0-0)$, $f(x_0+0)$ bir tomonli limitlarning kamida bittasi mavjud bo‘lmasa, u holda $x=x_0$ funksiyaning II tur uzilish nuqtasi deyiladi. II tur uzilish nuqtalar ikki xil bo‘ladi:

a) Agar $f(x_0-0)=f(x_0+0)$ tenglik o‘rinli bo‘lsa, u holda $x=x_0$ da funksiyaning uzluksizligini tiklash mumkin. Buning uchun $f(x_0)=f(x_0-0)= f(x_0+0)$ deb olish kerak.

b) Agar $f(x_0-0)\neq f(x_0+0)$ bo‘lsa, u holda $f(x)$ funksiya $x=x_0$ da sakrashga ega bo‘ladi. Sakrash kattaligi $|f(x_0+0)-f(x_0-0)|$ ga teng bo‘ladi.

Agar $f(x_0-0)=f(x_0)$ $(f(x_0)=f(x_0+0))$ bo‘lsa, $f(x)$ funksiya $x=x_0$ nuqtada chapdan (o‘ngdan) uzluksiz deyiladi.

Quyidagi funksiyalarning ko‘rsatilgan X to‘plamda uzluksizligini ta’rifga binoan isbotlang.

307. $f(x)=x^2-x+3$, $X=(-\infty;+\infty)$.

308. $f(x)=\sin(3x+2)$, $X=(-\infty;+\infty)$.

309. $f(x)= x^2-2x-1$, $X=(-\infty;+\infty)$.

310. $f(x)=\cos(2x-1)$, $X=(-\infty;+\infty)$.

311. $f(x) = \dfrac{1}{x+1}$, $X=(-1;+\infty)$.

Quyidagi funksiyalarning uzulish nuqtalarini toping va ularning turlarini aniqlang. Grafiklarini yasang.

312. $f(x) = \begin{cases} x^2, & \text{agar } x \neq 0, \\ 2, & \text{agar } x = 0 \end{cases}$.

313. $f(x) = \begin{cases} 2x+1, & \text{agar } -1 \le x \le 1, \\ 1-x, & \text{agar } 1 < x < 2 \end{cases}$.

314. $f(x) = \begin{cases} 3x, & \text{agar } -1 \le x \le 1, \\ 2x, & \text{agar } 1 < x \le 3 \end{cases}$.

315. $f(x) = \begin{cases} \dfrac{x^2-1}{x-1}, & \text{agar } x \neq 1, \\ 3, & \text{agar } x = 1 \end{cases}$.

316. $f(x)=\begin{cases}2x-1, & \text{agar} \quad x<0,\\ x, & \text{agar} \quad 1\le x<2,\\ 3, & \text{agar} \quad 2\le x<4\end{cases}$.   317. $f(x)=\frac{1}{x^2}$.

318. 1) $f(x)=x+\frac{1}{x}$,   2) $f(x)=\frac{1}{x^2-1}$,   3) $f(x)=\frac{|x|}{x}$,   4) $f(x)=\frac{1}{\ln|x|}$.

319. Quyidagi funksiyalarning x=0 dagi qiymatini shunday tanlangki, funksiya shu nuqtada uzluksiz bo‘lsin.

1) $f(x)=\frac{\sin x}{x}$,   2) $f(x)=\frac{\sin^2 x}{1-\cos x}$,   3) $f(x)=\frac{\sqrt{1+x}-1}{x}$,

4) $f(x)=\frac{5x^2-3\sin x}{2x}$,   5) $f(x)=\frac{\sqrt{1+2x}-1}{\sin x}$.

2-§. Kesmada uzluksiz funksiyalarning hossalari. Teskari funksiya.

1. Teorema. (Bolsano-Koshi). Agar *f(x)* funksiya [*a,b*] da uzluksiz bo‘lib, kesmaning uchlarida qarama-qarshi ishorali qiymatlarga ega bo‘lsa, u holda *(a,b)* da shunday *c* nuqta mavjudki, *f(c)=0* bo‘ladi.

2. Teorema. (Veyershtrass). Agar *f(x)* funksiya [*a,b*] da uzluksiz bo‘lsa, u holda *f(x)* shu kesmada chegaralangan bo‘ladi.

3. Teorema. Agar *y=f(x)* funksiya *X* oraliqda o‘suvchi (kamayuvchi) va uzluksiz bo‘lsa, u holda qiymatlar to‘plami *f(X)* da unga teskari funksiya mavjud bo‘lib, bu funksiya ham o‘suvchi (kamayuvchi) va uzluksiz bo‘ladi.

320. Quyidagi tenglamalar ko‘rsatilgan kesmalarda yechimga ega ekanini ko‘rsating.

1) $x^3+3x+1=0$, $[-1;0]$; 2) x^4-3x^2+2x-1, $[1;2]$;

3) $x^5-6x^2+3x-7=0$, $[0;2]$; 4) $3\sin^3 x-5\sin x+1=0$, $[0;\frac{\pi}{2}]$;

5) $\cos^4 x+3\cos x+1=0$, $[0;\pi]$.

321. *f(x)* funksiya [0;1] kesmada uzluksiz va faqat ratsional qiymatlarga ega. Agar $f(\frac{1}{2})=3$ bo‘lsa, $f(\frac{\sqrt{2}}{2})$ ni toping.

322. Har qanday toq darajali ko‘phad kamida bitta haqiqiy ildizga ega ekanini isbotlang.

323. *x=asinx+b* (bu erda *a*>0, *b*>0) tenglama berilgan. Bu tenglamaning *a+b* dan katta bo‘lmagan kamida bitta musbat yechimi mavjud ekanini isbotlang.

324. 323 masalada 0<*a*<1, *b*>0 bo‘lsa, tenglama yagona yechimga ega ekanligini isbotlang.

325. [2;3] segmentda uzluksiz va qiymatlar to‘plami f [2;3]=(7;10) bo‘ladigan *f(x)* funksiya mavjudmi?
326. [a;b] segmentda uzluksiz va qiymatlar to‘plami f [a;b]=$(-\infty;+\infty)$ bo‘ladigan *f(x)* fuksiya mavjudmi?
327. Quyidagi funksiyalarga teskari bo‘lgan funksiyalarning mavjudligi va ularning monoton, uzluksiz ekanligini isbotlang.

1) $y=1-3x$; 2) $y=x^{2n+1}$; 3) $y=x^2, x\in[0;+\infty)$;

4) $\mathrm{y}=x+\sin x$; 5) $\mathrm{y}=\cos 2x,\ x\in[0;\frac{\pi}{2}]$.

3-§. Ko‘rsatkichli va logarifmik funksiyalar

1. $y=a^x \ (a>0, a\neq 1)$ - ko‘rsatkichli funksiya deyiladi.

Ko‘rsatkichli funksiyaning aniqlanish sohasi $D(y)=(-\infty;+\infty)$ bo‘lib, *a>1* da o‘suvchi, *a<1* da kamayuvchi bo‘ladi.

2. $y=\log_a x \quad (a>0, a\neq 1)$ logarifmik funksiya deyiladi. Logarifmik funksiyaning aniqlanish sohasi *D(y)=(0;+∞)* bo‘lib, *a>1* da o‘suvchi, *a<1* da kamayuvchi bo‘ladi.

328. Quyidagi funksiyalarning o‘suvchi yoki kamayuvchi ekanligini aniqlang. Grafiklarini yasang.

1) $y=2^x$, 2) $y=(\frac{1}{3})^x$, 3) $y=\frac{1}{8}\cdot 4^{\frac{x}{2}}$, 4) $y=2\cdot 3^x$, 5) $y=3\cdot 2^{-x}$.

329. 1) $y=\ln x$; 2) $y=\log_2 x$; 3) $y=\log_{\frac{1}{2}} x$;

4) $y=\log_3 x$; 5) $y=3\cdot\log_{\frac{1}{4}} x$.

330. Ko‘rsatkichli funksiyalarning xossalaridan foydalanib, quyidagi sonlarni taqqoslang (katta yoki kichikligini aniqlang).

1) $(5{,}6)^{-5}$ va $(5{,}6)^{-3}$; 2) $2^{-6{,}2}$ va $(0{,}25)^{-4{,}3}$;

3) $(0{,}45)^{-1}$ va $(0{,}45)^{-3}$; 4) $(2+\sqrt{3})^2$ va $(\sqrt{2+\sqrt{3}})^{2{,}7}$;

5) $(2-\sqrt{3})^{-2}$ va $(\sqrt{2-\sqrt{3}})^2$; 6) $e^{1{,}2}$ va $e^{-0{,}9}$.

331. Logarifmik funksiyalarning xossalaridan foydalanib, quyidagi sonlarni taqqoslang.

1) $\ln 3$ va $\ln\sqrt{e+1}$; 2) $\log_3 e$ va $\log_3 2{,}7$;

3) $\lg 101$ va $\lg 103$; 4) $\log_{\frac{1}{3}} 7$ va $\log_{\frac{1}{3}} 6$;

5) $\log_{\sqrt{2}} 3$ va $\log_{\sqrt{2}} 1{,}5$; 6) $\log_{\frac{1}{4}} 8$ va $\log_{\frac{1}{4}} 9$.

IV BOB. BIR ARGUMENTLI FUNKSIYALAR UCHUN DIFFERENSIAL HISOB

1-§. Hosila tushunchasi

1. Funksiya orttirmasi Δy ning mos argument orttirmasi Δx ga bo‘lgan nisbatining $\Delta x \to 0$ dagi limiti funksiya hosilasi deyiladi:

$$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{f(x_0+\Delta x)-f(x_0)}{\Delta x}=f'(x_0).$$

Agar bu limit chekli bo‘lsa, u holda *y=f(x)* differensiallanuvchi deyiladi.

2. Funksiya hosilasini topish qoidasi.

y=f(x) funksiya hosilasini topish uchun quyidagi ishlarni bajarish kerak:

1) *f(x)* dagi *x* ni $x+\Delta x$ ga almashtirib, funksiyaning $f(x+\Delta x)=y+\Delta y$ qiymatini topamiz;

2) funksiyaning keyingi qiymatidan oldingi qiymatini ayirib, uning Δy orttirmasini topamiz: $\Delta y=f(x+\Delta x)-f(x)$;

3) funksiya orttirmasi Δy ni Δx ga bo‘lamiz:

$$\frac{\Delta y}{\Delta x}=\frac{f(x+\Delta x)-f(x)}{\Delta x};$$

4) argument orttirmasi $\Delta x\to 0$ da $\frac{\Delta y}{\Delta x}$ nisbatning limitini izlaymiz: $\lim\limits_{\Delta x\to 0}\frac{\Delta y}{\Delta x}$.

3. Geometrik tomondan $f'(x_0)$ hosila *y=f(x)* egri chiziqning $M_0(x_0, f(x_0))$ nuqtasida o‘tkazilgan urinmaning burchak koeffitsientini ifodalaydi: $f'(x_0)=tg\alpha$.

4. Mexanik tomondan $f'(x_0)$ hosila *y=f(x)* harakat qonunining x_0 momentdagi o‘zgarish tezligini ifodalayi.

5. Bir tomonlama hosilalar.

$$f'_+(x_0)=\lim_{\Delta x\to +0}\frac{f(x+\Delta x)-f(x)}{\Delta x},\quad f'_-(x_0)=\lim_{\Delta x\to -0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$$

lar mos holda *f(x)* funksiyaning x_0 nuqtadagi o‘ng va chap hosilalari deyiladi.

Hosila mavjudligining zaruriy va etarli sharti $f'_+(x_0)=f'_-(x_0)$ dan iborat.

Funksiya hosilasini topish amaliga funksiyani differensiallash deyiladi.

6. Differensiallashning asosiy qoidalari.

Agar *c*- o‘zgarmas va *u(x), v(x)* differensiallanuvchi funksiyalar bo‘lsa, u holda

1) $(c)'=0$,

2) $(x)'=1$,

3) $(cu)'=cu'$,

4) $(u\pm v)'=u'\pm v'$,

5) $(uv)'=u'v+uv'$,

6) $\left(\frac{u}{v}\right)'=\frac{u'v-uv'}{v^2}$,

6’) $\left(\frac{c}{v}\right)' = -\frac{cv'}{v^2}$.

7. Murakkab funksiya hosilasi $y'_x = y'_u \cdot u'_x$.

8. Teskari funksiya hosilasi $y'_x = \frac{1}{x'_y}$.

9. Differensiallashning asosiy formulalari.

1) $(u^{\alpha})' = \alpha u^{\alpha-1} \cdot u'$.

2) $(a^u)' = a^u \ln a \cdot u'$, 2’) $(e^u)' = e^u \cdot u'$.

3) $(\log_a u)' = \frac{u'}{u \ln a}$, 3’) $(\ln u)' = \frac{u'}{u}$.

4) $(\sin u)' = \cos u \cdot u'$.

5) $(\cos u)' = -\sin u \cdot u'$.

6) $(tgu)' = \frac{u'}{\cos^2 u}$.

7) $(ctgu)' = -\frac{u'}{\sin^2 u}$.

8) $(\arcsin u)' = \frac{u'}{\sqrt{1-u^2}}$.

9) $(\arccos u)' = -\frac{u'}{\sqrt{1-u^2}}$.

10) $(arctgu)' = \frac{u'}{1+u^2}$.

11) $(arcctgu)' = -\frac{u'}{1+u^2}$.

12) $(shu)' = chu \cdot u'$.

13) $(chu)' = shu \cdot u'$.

14) $(thu)' = \frac{u'}{ch^2 u}$.

15) $(cthu)' = \frac{u'}{sh^2 u}$.

16) $(u^v)' = u^v \ln u \cdot v' + vu^{v-1} \cdot u'$.

Hosila ta’rifidan foydalanib, quyidagi funksiyalarning hosilalarini toping.

332. $y=6x-5$.

333. $y = 9x - x^2$.

334. $y = x^3 + 1$.

335. $y = \cos x$.

336. $y = x^2 + 2x - 1$.

337. $y = \sqrt{2x+1}$.

338. $y=\dfrac{1}{\sqrt{x}}$.

339. $y=tg2x$.

340. $f(x)=2x^2+3x+4$ funksiya berilgan. $f'(2)$ ni toping.

341. $f(x)=\sin 3x$ funksiya berilgan. $f'(\frac{\pi}{3})$ ni toping.

342. $f(x)=\sqrt[3]{x^2}$ funksiya berilgan. $f'(\frac{1}{8})$ ni toping. $f'(0)$ mavjudmi?

343. $f(x)=\sqrt[3]{x-1}$ funksiya berilgan. $f'(0)$, $f'(1)$ larni toping.

344. $f(x)=\sqrt[5]{\sin x}$ funksiya berilgan. $f'(\frac{\pi}{2})$, $f'(0)$ larni toping.

Quyidagi funksiyalarning ko‘rsatilgan nuqtadagi bir tomonli hosilalarini toping.

345. $f(x)=3|x-1|+2, \quad x_0=1$.

346. $f(x)=\sqrt[3]{x^2}, \quad x_0=0$.

347. $f(x)=\begin{cases} x^2\cos\dfrac{1}{x}, \text{ agar } & x\neq 0, \\ 0, \text{ agar } & x=0. \end{cases}$ $x_0=0$.

348. $f(x)=|x-2|x^2, \qquad x_0=2$.

Differensiallash jadvali va qoidalaridan foydalanib quyidagi funksiyalarning hosilalarini toping.

349. $y=5-6x$.

350. $y=11x^3-x^2-4x$.

351. $y=3x^{-3}+2x^{-2}$.

352. $y=\dfrac{3x^2-4x+1}{x^2}$.

353. $y=x^{1/4}-8x^{3/4}$.

354. $y=-\sqrt{x}+\dfrac{1}{x}$.

355. $y=(3x-2)(7x+4)$.

356. $y=\left(6\sqrt[3]{x}-\dfrac{1}{x^2}\right)(7x-3)$.

357. $y=\dfrac{x-1}{5x+1}$.

358. $y=\dfrac{1+\sqrt{x}}{1-\sqrt{x}}$.

359. $y=(1+2x)^{10}$.

360. $y=\left(\dfrac{x+1}{x-1}\right)^2$.

361. $y=4x^2-0{,}5x+3$.

362. $y=x^5-4x^3-x^2$.

363. $y=\frac{2}{x}-\frac{4}{x^2}+\frac{5}{x^3}$.

364. $y=\frac{x^4-7x+3}{x^3}$.

365. $y=x^{3/2}-2x^{\frac{2}{3}}$.

366. $y=3\sqrt{x}-4x\sqrt[4]{x+2}$.

367. $y=(2x+5)(4x+2-3x^2)$.

368. $y=(3x^2-\frac{1}{x^3})(\sqrt[3]{x}+x)$.

369. $y=\frac{3x+3}{3x+7}$.

370. $y=\frac{\sqrt[3]{x-2}}{\sqrt[3]{x+2}}$.

371. $y=(t^4-\frac{1}{t^2}+1)^4$.

372. $y=\left(\frac{1+x^2}{1+x}\right)^5$.

373. $f(x)=\sqrt{x^2+2x+3}$ funksiya berilgan. $f'(1)$ ni toping.

374. $f(x)=5x^2-16\sqrt{x}+7$ funksiya berilgan. $f'(1)$, $f'(4)$, $f'(\frac{1}{4})$ larni toping.

375. $y=\sin x+2\cos x$.

376. $y=\frac{\sin x+\cos x}{x}$.

377. $y=ctg^2 x$.

378. $y=3\cos^2 x-\cos^3 x$.

379. $y=\sin(3x+1)$.

380. $y=\cos(\sin x)$.

381. $y=\sqrt{tg\frac{x}{2}}$.

382. $y=x\arccos x$.

383. $y=\frac{\arcsin x}{\sqrt{1-x^2}}$.

384. $y=\arccos\frac{2x-1}{2}$.

385. $y=\sqrt{1-(arc\cos x)^2}$.

386. $y=x^3\ln x$.

387. $y=\sqrt{\lg x}$.

388. $y=\frac{\ln x}{1+x^2}$.

389. $y=\ln\sin x$.

390. $y=\lg(x^2-1)$.

391. $y=\frac{\cos x}{x}$.

392. $y=\sin^3 x$.

393. $y=\sin x-\frac{1}{3}\sin^3 x$.

394. $y=\cos 2x$.

395. $y=tg\frac{1}{x}$.

396. $y=\sin^2 3x$.

397. $y=ctg\sqrt{1-x^2}$.

398. $y=(arc\sin x)^3$.

399. $y=\frac{x^2}{arc\,tgx}$.

400. $y=\arcsin x^2$.

401. $y = arc\sin\sqrt{\dfrac{1-x}{1+x}}$.

402. $y = \ln^3 x$.

403. $y = x\sin 2x \ln x$.

404. $y = \sqrt{1+\ln^2 x}$.

405. $y = \ln(x^2 - 4x)$.

406. $y = \ln\cos 3x$.

Parametrik ko‘rinishda berilgan quyidagi funksiyalarning y dan x bo‘yicha olingan hosilalarini toping.

407. $\begin{cases} x = t^2, \\ y = 2t. \end{cases}$

408. $\begin{cases} x = \cos t, \\ y = t + \sin t. \end{cases}$

409. $\begin{cases} x = \cos^3 t, \\ y = a\sin^3 t. \end{cases}$

410. $\begin{cases} x = e^t \sin t, \\ y = e^t \cos t. \end{cases}$

411. $\begin{cases} x = \cos t, \\ y = \sin t. \end{cases}$

412. $\begin{cases} x = a\cos\varphi, \\ y = b\sin\varphi. \end{cases}$

413. $\begin{cases} x = a(t - \sin t), \\ y = a(1 - \cos t). \end{cases}$

414. $\begin{cases} x = t(1 - \sin t), \\ y = t\cos t. \end{cases}$

415. $y = xe^x$ funksiya $\dfrac{dy}{dx} = \dfrac{(x+1)y}{x} = 0$ tenglikni qanoatlantirishini ko‘rsating.

416. $y = x^2 \ln x$ funksiya $xy' - 2y = x^2$ tenglikni qanoatlantirishini ko‘rsating.

417. $y = \ln\dfrac{1}{1+x}$ funksiya $xy' + 1 = e^y$ tenglikni qanoatlantirishini ko‘rsating.

418. $sin2x=2sinxcosx$ formuladan differensiallash yordamida $cos2x=cos^2x-sin^2x$ formulani keltirib chiqaring.

419. $cos(x+a)=cosxcosa-sinxsina$ formuladan differensiallash yordamida $sin(x+a)=sinxcosa+sinacosx$ formulani keltirib chiqaring.

420. $1 + x + x^2 + ... + x^4 = \dfrac{x^{n+1} - 1}{x - 1}$ $(x \neq 1)$ ayniyatdan foydalanib, quyidagi yig‘indilarni toping.

1) $1 + 2x + 3x^2 + \ldots + nx^{n-1}$;

2) $2 + 2\cdot 3x + 3\cdot 4x^2 + ... + n(n-1)x^{n-2}$.

2-§. Hosilaning tatbiqi

1. Differensiallanuvchi $y=f(x)$ funksiya grafigining $M_0(x_0, y_0)$ $(y_0 = f(x_0))$ nuqtasida o‘tkazilgan urinma tenglamasi $y - y_0 = f'(x_0)(x - x_0)$ ko‘rinishga, $f'(x_0) \neq 0$ da normal tenglamasi $y - y_0 = -\frac{1}{f'(x_0)}(x - x_0)$ ko‘rinishga ega bo‘ladi.

2. $y=f_1(x)$ va $y=f_2(x)$ funksiyalar grafigining $M_0(x_0, y_0)$ kesishish nuqtasida o‘tkazilgan urinmalar orasidagi φ burchak berilgan ikki egri chiziq orasidagi burchakni ifodalaydi va

$$tg\varphi = \frac{f_2'(x_0) - f_1'(x_0)}{1 + f_1'(x_0) f_2'(x_0)}$$

formula yordamida hisoblanadi.

3. Harakat qonuni $s=f(t)$ bo‘lganda $v = s'(t_0)$, bu harakatning t_0 momentdagi tezligini ifodalaydi.

421. $y = x^2 - 2x + 5$ parabolaning x=0,5 nuqtasida o‘tkazilgan urinmasining burchak koeffitsentini toping.

422. $y = x^3$ kubik parabolaning $x = 0$, $x = \frac{\sqrt{3}}{3}$ nuqtalarida o‘tkazilgan urinmalarining burchak koeffitsentlarini toping.

423. $y=-x^2+2x-3$ parabolaning qaysi nuqtasidagi urinmasi abssissa o‘qini 45^0 li burchak ostida kesadi.

424. $y=lnx$ egri chiziqning qaysi nuqtasidagi urinmasi $y = 2x - 3$ to‘g‘ri chiziqqa parallel bo‘ladi?

425. $y = x^2 + 3$ parabolaning qaysi nuqtasidagi urinmasi a) $y = 4x - 5$ to‘g‘ri chiziqqa parallel, b) $y = \frac{2}{3}x + 3$ to‘g‘ri chiziqqa perpendikulyar bo‘ladi?

426. Shunday nuqtani topingki $y=x^3-x-1$ va $y = 3x^2 - 4x+1$ egri chiziqlarning shu nuqtadagi urinmalari parallel bo‘lsin.

427. $y=e^x$ egri chiziq ordinata o‘qini qanday burchak ostida kesadi?

428. $y=x^2$ va $y^2=x$ parabolalar qanday burchak ostida kesishadi?

429. $x^2+y^2=8$ aylana va $y^2=2x$ parabola qanday burchak ostida kesishadi?

430. $y=sinx$ va $y=cosx$ $(0 \leq x \leq \pi)$ egri chiziqlar qanday burchak ostida kesishadi?

431. $y=x^4-3$ egri chiziqning $M_0(1;-2)$ nuqtasidagi urinma va norma tenglamalarini tuzing.

432. $y=x^2-3x+5$ egri chiziqning $M_0(2;3)$ nuqtasidagi urinma va norma tenglamalarini tuzing.

433. $y=x^3+2x^2-1$ egri chiziqning $y=2x^2$ parabola bilan kesishish nuqtasidagi urinma va normalining tenglamalarini tuzing.

434. $y=x^2-4$ parabolaning shu parabolaga tegishli bo‘lmagan $M_0(2;-1)$ nuqtadan o‘tuvchi urinmasining tenglamasini tuzing.
435. $xy=a^2$ geperbolaning har bir nuqtasidagi urinmasining koordinata o‘qlari bilan hosil qilgan uchburchaklar yuzlari o‘zgarmas bo‘lishini isbotlang.
436. $x^{\frac{2}{3}}+y^{\frac{2}{3}}=a^{\frac{2}{3}}$ astroida har bir nuqtasidagi urinmasining koordinata o‘qlari orasidagi qismi o‘zgarmas uzunlikka ega bo‘lib, u a ga tengligini isbotlang.
437. $\begin{cases} x=t^2-1, \\ y=t^2+t-3 \end{cases}$ egri chiziqning $M_0(3;-1)$ nuqtasidagi urinma va norma tenglamalarini tuzing.
438. $\begin{cases} x=2\cos t, \\ y=4\sin t \end{cases}$ egri chiziqning $t=\frac{\pi}{4}$ dagi urinma va normal tenglamalarini tuzing.
439. $\begin{cases} x=2t^3-9t^2+12t, \\ y=t^2-t+1 \end{cases}$ egri chiziqning shunday nuqtasini topingki, uning shu nuqtadagi urinmasi ordinata o‘qiga parallel bo‘lsin.
440. $\begin{cases} x=t^2-3t+3, \\ y=t^2-4t+3 \end{cases}$ egri chiziqning an $M_0(1;-1)$ nuqtasidan o‘tuvchi urinma tenglamsini tuzing.
441. Moddiy nuqta $s=\frac{t^3}{3}+2t^2-t$ qonun bo‘yicha to‘g‘ri chiziqli harakat qilayapti. Moddiy nuqtaning t=2 dagi tezligini va tezlanishini toping (s- metrlarda, t- sekuntlarda ifodalanadi).
442. Moddiy nuqta $s=t\ ln(1+t)$ qonun bo‘yicha to‘g‘ri chiziqli harakat qilayapti. Uning t=3 dagi tezligi va tezlanishini toping.
443. 25 kg massali jism $s=ln(1+t^2)$ qonun bo‘yicha to‘g‘ri chiziqli harakat qilmoqda. Harakat boshlanishidan 2s keyin jismning kinetik energiyasi $\frac{mv^2}{2}$ ni toping.
444. Snaryad 200m/s boshlang‘ich tezlik bilan gorizontga nisbatan 45^0 burchak ostida otildi. Uchinchi sekund oxirida snaryad tezligini aniqlang.
445. Agar aylana radiusi 5m/s tezlik bilan o‘zgarsa, aylana uzunligi qanday tezlik bilan o‘zgaradi?
446. Kvadrat tomoni 3 m/s tezlik bilan o‘sib boradi. Kvadrat tomoni 4 sm bo‘lganda uning yuzi qanday tezlik bilan o‘sadi?
447. Moddiy nuqta $y=2x+3$ to‘g‘ri chiziq bo‘yicha harakat qilmoqda. Agar uning absissasi o‘zgarmas $\upsilon=3$ tezlik bilan o‘ssa, ordinatasi qanday tezlik bilan o‘zgaradi?
448. Moddiy nuqta $x^2+y^2=100$ aylananing birinchi kvadratdagi qismi bo‘yicha harakat qilmoqda. Agar uning ordinatasi o‘zgarmas $\upsilon=2$ tezlik bilan o‘ssa,

absissasi qanday tezlik bilan o‘zgaradi? Nuqtaning ordinatasi u=6 bo‘lganda uning absissasining o‘zgarish tezligini toping.

449. Nuqta y=$\sqrt{x}$ parabola boylab harakatlanmoqda. Uning qaysi koordinatasi tezroq o‘zgaradi?

3-§. Funksiyaning differensiali va differensiallanuvchi funksiyalar.

1. Agar $y=f(x)$ funksiyaning Δy orttirmasi $\Delta y=A\cdot\Delta x+o(\Delta x)$ ko‘rinishda yozilishi mumkin bo‘lsa, orttirmaning Δx ga nisbatan chiziqli qismi $A\cdot\Delta x$ funksiyaning differensiali deyiladi va dy yoki $df(x)$ orqali belgilanadi: $dy=A\cdot\Delta x$. Differensial mavjud bo‘lishi uchun chekli hosila $f'(x)$ ning mavjudligi zarur va etarlidir. Bunda $dy=f'(x)dx$ bo‘ladi.

2. Agar $y=f(u)$, $u=\varphi(x)$ murakkab funksiya berilgan bo‘lsa, u holda $dy=f'(u)du$ (differensial formasining invariantligi) bo‘ladi.

3. Agar Δx etarlicha kichik bo‘lsa, u holda Δx ga nisbatan yuqori tartibli cheksiz kichik miqdorlar aniqligi bilan, $\Delta y\approx dy$ taqribiy formula o‘rinli bo‘ladi. $|\Delta y-dy|$ - absolyut hato, $\left|\dfrac{\Delta y-dy}{\Delta y}\right|$ - nisbiy hato bo‘ladi.

450. $y=x^2-x+1$ funksiya berilgan. Agar $x=3$ va $\Delta x=0{,}001$ bo‘lsa, Δy va dy larni toping.

451. $y=x^3-7x^2+8$ funksiya berilgan. $x=5$ va $\Delta x=0{,}01$ bo‘lsa, Δy va dy larni toping.

Quyidagi funksiyalarning differensiallarining toping.

452. $y=\dfrac{1}{x^2}$.
453. $y=\dfrac{x+2}{x-1}$.

454. $y=\ln(1+x^2)$.
455. $y=\sin^2 x$.

456. $y=arctg3x$.
457. $y=e^x\cos x$.

458. $y=5^{x^2}\arccos\dfrac{1}{x}$.
459. $y=\dfrac{tg\sqrt{x}}{\sqrt{x}}$.

460. $y=(1-x-x^2)^3$.
461. $y=tg^2x$.

462. $y=5^{\ln tgx}$.
463. $y=\ln tg(\dfrac{\pi}{2}-\dfrac{x}{4})$.

464. $y=2^{-\frac{1}{\cos x}}$.
465. $y=\sqrt{\arcsin x}+(arctgx)^2$.

466. Quyidagi sonlarni taqribiy hisoblang.

1) $\sqrt[4]{17}$; 2) $0{,}96^3$; 3) $e^{0,2}$; 4) $\lg 10{,}08$;

5) $\sqrt[3]{26{,}97}$; 6) $\ln 1{,}01$; 7) $\cos 32^0$; 8) $\arcsin 0{,}48$.

467. Quyidagi murakkab funksiyalarning differensialini erksiz o‘zgaruvchi va uning differensiali orqali ifodalang.

1) $y=\sqrt[3]{x^2+5x}$, $x=t^3+2t+1$; 2) $y=3^{-\frac{1}{x}}$, $x=\ln tgt$;

3) $s=\cos^2 z, \quad z=\dfrac{t^2-1}{4};$ 4) $y=arctgv, \quad v=\dfrac{1}{tgt};$

5) $y=e^z, \; z=\dfrac{1}{2}\ln t, \; t=2u^2-3u+1.$

468. $f(x)=3x^2-x+2$ funksiya berilgan. Uning barcha $x\in(-\infty;+\infty)$ da differensiallanuvchi ekanligini ko‘rsating.

469. $f(x)=\sqrt{3(x-1)}$ funksiyani x=1 nuqtada differensiallanuvchi emasligini ko‘rsating.

470. $f(x)=\sqrt[3]{(x-2)^2}+x$ funksiyani x=2 nuqtada differensiallanuvchi emasligini ko‘rsating.

471. $y=|\sin x|$ funksiya berilgan. Uni x=0 nuqtada uzluksiz, lekin differensiallanuvchi emasligini ko‘rsating.

472. $f(x)=\begin{cases} x^2\sin\dfrac{1}{x}, & \text{agar } x\neq 0, \\ 0 \;, & \text{agar } x=0 \end{cases}$ funksiya x=0 da differensiallanuvchimi?

473. $f(x)=\begin{cases} \dfrac{\sqrt{x+1}-1}{\sqrt{x}}, & \text{agar } x\neq 0, \\ 0 \;, & \text{agar } x=0 \end{cases}$ funksiya x=0 da differensiallanuvchimi?

4 - §. Yuqori tartibli hosilalar va yuqori tartibli differensiallar.

1. *y=f(x)* funksiya hosilasi y' ning hosilasi berilgan *y=f(x)* funksiyaning ikkinchi tartibli hosilasi deyiladi va y'' orqali belgilanadi.

Umuman, $y^{(n)}=f^{(n)}(x)=(f^{(n-1)}(x))'$; ($n$=2,3, …).

2. $s=f(t)$ harakat qonuniyati berilgan bo‘lsa, $\dfrac{d^2s}{dt^2}$ harakatning tezlanishini ifodalaydi.

3. Agar *u(x)* va *v(x)* *n* marta differensiallanuvchi funksiyalar bo‘lsa, u holda $(c_1u+c_2u)^{(n)}=c_1u^{(n)}+c_2u^{(n)}$ bo‘lib,

$$(uv)^{(n)}=u^{(n)}v+nu^{(n-1)}v'+\frac{n(n-1)}{2!}u^{(n-2)}v''+\ldots+uv^{(n)}=\sum_{k=0}^{n}C_n^k u^{(n-k)}v^{(k)}$$

(Leybnits formulasi) orqali ifodalanadi. Bunda $u^{(0)}=u$, $v^{(0)}=v$; $C_n^k=\dfrac{n(n-1)(n-2)\ldots(n-k+1)}{k!}=\dfrac{n!}{k!(n-k)!}$.

4. Asosiy formulalar:

1) $(a^x)^{(n)} = a^x \ln^n a$, $a>0$; $(e^x)^{(n)}=e^x$,

2) $(x^m)^{(n)} = m(m-1)...(m-n+1)x^{m-n}$.

3) $(\ln x)^{(n)} = (-1)^{n-1}\dfrac{(n-1)!}{x^n}$.

4) $(\sin x)^{(n)} = \sin(x + n\cdot\dfrac{\pi}{2}), \quad n \in N$.

5) $(\cos x)^{(n)} = \cos(x + n\cdot\dfrac{\pi}{2}), \quad n \in N$.

5. Agar $y=f(x)$ funksiya parametrik ko‘rinishda $\begin{cases} x = \varphi(t), \\ y = \psi(t) \end{cases}$, $\alpha \le t \le \beta$, berilgan bo‘lsa, u holda $f''(x) = \dfrac{\psi''(t)\varphi'(t) - \varphi''(t)\psi'(t)}{\varphi'^3(t)}$ bo‘ladi.

6. Yuqori tartibli differensiallar $d^2y, d^3y, ..., d^ny$ bo‘lib, x erkli o‘zgaruvchi bo‘lganda $d^ny = y^{(n)}dx^{(n)}$ bo‘ladi. Agar $y=f(u)$, $u=\varphi(x)$ bo‘lsa, $d^2y = f''(u)du^2 + f'(u)d^2u$ bo‘ladi. Bunda differensial formasining invariantligi buziladi.

Quyidagi funksiyalarning ko‘rsatilgan tartibli hosilalarini toping.

474. $y = x^6 + e^{2x}$, y''. 475. $y = x\ln x$, y^{IV}.

476. $y = (2x+5)^3$, y''. 477. $y = \cos^2 x$, y'''.

478. $y = e^{-x^2}$, y'''. 479. $y = 5^{\sqrt{x}}$, y''.

480. $y = \ln(x + \sqrt{1+x^2})$, y''. 481. $y = \ln tgx$, y''.

482. $f(x)=(x+10)^6$, $f'''(2)$. 483. $f(x)=e^{2x-1}$, $f'''(0)$.

Quyidagi funksiyalarning n – tartibli hosilalarni toping.

484. $y = e^{-x}$. 485. $y = \sin ax$.

486. $y = \cos ax$. 487. $y = \sin^2 ax$.

488. $y = \ln x$. 489. $y = \ln(ax+b)$.

490. $y = shx$. 491. $y = \sin^2 x + \cos^4 x$.

Parametrik ko‘rinishda berilgan funksiyaning ko‘rsatilgan tartibli hosilalarini toping.

492. $\begin{cases} x = 2t^2, \\ y = 3t^3 \end{cases}$ $\frac{d^2y}{dx^2}$.

493. $\begin{cases} x = a\cos t, \\ y = a\sin t \end{cases}$ $\frac{d^2y}{dx^2}$.

494. $\begin{cases} x = \cos^3 t, \\ y = \sin^3 t \end{cases}$ $\frac{d^2y}{dx^2}$.

495. $\begin{cases} x = a\cos^2 t, \\ y = a\sin^2 t \end{cases}$ $\frac{d^2y}{dx^2}$.

496. $\begin{cases} x = \ln t, \\ y = t^2 - 1 \end{cases}$ $\frac{d^2y}{dx^2}$.

497. $\begin{cases} x = arc\sin t, \\ y = \ln(1 - t^2) \end{cases}$ $\frac{d^2y}{dx^2}$.

498. $\begin{cases} x = at^2, \\ y = bt \end{cases}$ $\frac{d^3y}{dx^3}$.

499. $\begin{cases} x = a\cos t, \\ y = a\sin t \end{cases}$ $\frac{d^3y}{dx^3}$.

500. $y = |x^3|$ funksiya berilgan. $y''(0)$ mavjudmi?

501. $f(x) = \begin{cases} x^2 \sin\frac{1}{x}, & \text{agar } x \neq 0, \\ 0 \quad , & \text{agar } x = 0 \end{cases}$ funksiya berilgan. $y''(0)$ mavjudmi?

502 $y = \sqrt{2x - x^2}$ funksiya $y^3 \cdot y'' + 1 = 0$ tenglikni qanoatlantirishini ko'rsating.

503. $y = \cos e^x + \sin e^x$ funksiya $y = \cos e^x + \sin e^x$ tenglikni qanoatlantirishini ko'rsating.

504. $y = x^2 \ln x$ funksiya $xy'' - y' = 2x$ tenglikni qanoatlantirishini ko'rsating.

505. $y = e^{4x} + 2e^{-x}$ funksiya $y''' - 13y' + 12y = 0$ tenglikni qanoatlantirishini ko'rsating.

506. *x=sint*, *y=sinkt* tenglamalar orqali berilgan. *y=f(x)* funksiya $(1 - x^2)y''xy' + k^2y = 0$ munosabatni qanoatlantirishini ko'rsating.

507. *x=3t*^{2}, *y=3t-t*^{3} tenglamalar orqali berilgan *y=f(x)* funksiya $36y''(y - \sqrt{3x}) = x + 3$ munosabatni qanoatlantirishini ko'rsating.

Quyidagi funksiyalarning yuqori tartibli differensiallarini toping.

508. $y = \sqrt[3]{x^2}$, d^2y.

509. $y = x^m$, d^3y.

510. $y = \ln x$, d^3y.

511. $y = \sin^2 x$, d^3y.

512. $y = arctg(\frac{b}{a}tgx)$, d^2y.

513. $y = \sqrt{\ln^2 x - 4}$, d^2y.

514. $y = \ln\frac{1 - x^2}{1 + x^2}$, d^2y.

515. $y = e^{2x}$, d^ny.

V BOB. FUNKSIYALARNI TEKSHIRISH.

1 - §. Differensial hisobning asosiy teoremalari. Lopital qoidasi.

1. Ferma teoremasi. Agar *y=f(x)* funksiya biror [*a,b*] da aniqlangan bo‘lib, uning ichki $c\in(a,b)$ nuqtasida eng katta (kichik) qiymatga ega bo‘lsa vash u nuqtada chekli $f'(c)$ mavjud bo‘lsa, u holda $f'(c)=0$ bo‘ladi.

2. Rol teoremasi. Agar *y=f(x)* funksiya [*a,b*] da uzluksiz va hech bo‘lmaganda *(a,b)* da differensiallanuvchi hamda *f(a)=f(b)* bo‘lsa, u holda shunday $c\in(a,b)$ mavjud bo‘ladiki, $f'(c)=0$ bo‘ladi.

3. Lagranj teoremasi. Agar *y=f(x)* funksiya [*a,b*] da aniqlangan uzluksiz va hech bo‘lmaganda *(a,b)* da differensiallanuvchi bo‘lsa, u holda *(a,b)* da shunday *c* nuqta mavjud bo‘lib,

$$f(b)-f(a)=f'(c)(b-a) \qquad (*)$$

o‘rinli bo‘ladi. (*) ning boshqa ko‘rinishi

$$\Delta f(x)=f'(x+\theta\Delta x)\cdot\Delta x; \quad (0<\theta<1).$$

4. Koshi teoremasi. Agar *f(x)* va $\varphi(x)$ funksiyalar [*a,b*] da aniqlangan uzluksiz va hech bo‘lmaganda *(a,b)* da differensiallanuvchi hamda $\varphi'(x)\neq0$ bo‘lsa, u holda *(a,b)* da shunday *c* nuqta mavjud bo‘ladiki, $\frac{f(b)-f(a)}{\varphi(b)-\varphi(a)}=\frac{f'(c)}{\varphi'(c)}$ o‘rinli bo‘ladi.

5. Lopital qoidasi. Agar *f(x)* va $\varphi(x)$ funksiyalar *a* nuqtaning biror $\bigcup(a,\delta)$ atrofida (*a* dan boshqa) differensiallanuvchi (bunda *a* - son yoki ∞ bo‘lishi mumkin) $\varphi'(x_0)\neq0$ va $\lim\limits_{x\to a}f(x)=\lim\limits_{x\to a}\varphi(x)=0$ yoki $\lim\limits_{x\to a}f(x)=\lim\limits_{x\to a}\varphi(x)=\infty$ bo‘lsa, u holda $\lim\limits_{x\to a}\frac{f(x)}{\varphi(x)}=\lim\limits_{x\to a}\frac{f'(x)}{\varphi'(x)}$ bo‘ladi (o‘ng tomondagi limit mavjud deb qaraladi).

6. $(0\cdot\infty)$ va $(\infty-\infty)$ ko‘rinishdagi aniqmasliklar algebraik shakl o‘zgartirish bilan 3-qoidaga, ya’ni (0/0) yoki (∞/∞) aniqmasliklarga keltiriladi.

7. (1^{∞}), (∞^{0}) va (0^{0}) ko‘rinishdagi aniqmasliklar logarifmlash yoki $[f(x)]^{\varphi(x)}=e^{\varphi(x)\cdot\ln f(x)}$ shakl o‘zgartirish bilan $(0\cdot\infty)$ ko‘rinishga keltiriladi.

516. Quyidagi funksiyalar uchun Roll teoremasi shartlarini bajarishini tekshirib ko‘ring.

1) $f(x)=x^2-3x+5, \quad x\in[1;2]$;

2) $f(x)=x^3-x^2-x+1, \quad x\in[-1;1]$;

3) $f(x)=\sqrt[3]{x^2+9x+14}, \quad x\in[-7;-2]$;

4) $f(x)=\ln\sin x, \quad x\in[\frac{\pi}{6};\frac{5\pi}{6}]$.

517. $f(x)=\sqrt[3]{(x-4)^2}$ funksiya [0;8] segmentda Roll teoremasi shartlarini qanoatlantiradimi? Asoslang.
518. $f(x)=1-\sqrt[3]{x^2}$ funksiya [-1;1] segmentda Roll teoremasi shartlarini qanoatlantiradimi? Asoslang.
519. Agar $a_0x^n+a_1x^{n-1}+...a_{n-1}x=0$ tenglama $x=x_0$ musbat yechimga ega bo‘lsa, u holda $na_0x^{n-1}+(n-1)a_1x^{n-2}+...a_{n-1}=0$ tenglama ham x_0 dan kichik musbat yechimga ega ekanligini isbotlang.
520. $f(x)=1+x^m(x-1)^n \quad (m,n\in N)$ funksiya berilgan. Hosila olmay turib, $f'(x)=0$ tenglama (0;1) intervalda kamida bitta yechimga ega ekanini isbotlang.
521. $x^3-3x+c=0$ tenglama (0;1) intervalda ikkita har xil yechimga ega emasligini ko‘rsating.
522. $f(x)=(x-1)(x-2)(x-3)(x-4)$ funksiyadan hosila olmay turib, $f'(x)=0$ tenglama nechta haqiqiy yechimga ega ekanini aniqlang. Bu yechimlar qaysi intervallarda yotishini ko‘rsating.
523. $x^5+x^4+x^2+10x-5=0$ tenglama faqat bitta musbat yechimga ega ekanligini va bu yechim $(0;\frac{1}{2})$ intervalda yotishini ko‘rsating.
524. Quyidagi funksiyalar uchun Lagranj teoremasi shartlarini bajarilishini tekshirib ko‘ring.

1) $f(x)=2x-x^2, \quad x\in[0;1]$. 2) $f(x)=x^3-4x^2+5x, \; x\in[0;3]$.
3) $f(x)=\sqrt{x}, \quad x\in[0;1]$. 4) $f(x)=\ln x, \quad x\in[1;e]$.
525. Quyidagi funksiyalar [-2;2] segmentda Lagranj teoremasi shartlarini qanoatlantiradimi? Asoslang.
1) $f(x)=\frac{1}{x}$; 2) $f(x)=1-\sqrt[5]{x^4}$.
526. $[x_1; x_2]$ segmentda $f(x)=\sin 3x$ funksiya uchun Lagranj formulasini yozing.
527. $[a;b]$ segmentda $f(x)=x(1-\ln x)$ funksiya uchun Lagranj formulasini yozing.
528. $f(x)=4x^3-5x^2+x-2$ funksiya [0;2] segmentda aniqlangan. Lagranj formulasidan foydalanib c ni toping.
529. Lagranj formulasidan foydalanib, quyidagi tengsizliklarni isbotlang:
1) $\frac{b-a}{b}\le\ln\frac{b}{a}\le\frac{b-a}{a}$, bu yerda $0<a\le b$.
2) $\frac{\beta-2}{\cos^2\alpha}\le tg\alpha-tg\beta\le\frac{\beta-\alpha}{\cos^2\beta}$, bu yerda $0<\alpha\le\beta<\frac{\pi}{2}$.
3) $na^{n-1}(b-a)<b^n-a^n<nb^{n-1}(b-a)$, bu yerda $a<b$.
530. Quyidagi funksiyalar uchun Koshi teoremasi shartlarining bajarilishini tekshirib ko‘ring.
1) $f(x)=x^3$, $\varphi(x)=x^2$, $x\in[1;2]$; 2) $f(x)=\sin x$, $\varphi(x)=\cos x$, $x\in[0;\pi/2]$;
3) $f(x)=\sqrt{x+9}$, $\varphi(x)=\sqrt{x}$, $x\in[1;16]$.

531. $f(x)=x^3$ va $\varphi(x)=x^2+1$ funksiyalar uchun [1;2] segmentda Koshi teoremasini qo‘llab, c ni toping.
532. $f(x)=x^2$ va $\varphi(x)=x^3$ funksiyalar uchun [-1;1] segmentda Koshi teoremasini qo‘llab bo‘ladimi? Asoslang.

Lopital qoydasidan foydalanib, quyidagi limitlarni toping.

533. $\lim\limits_{x\to 0}\dfrac{\sin 6x}{tg2x}$.
534. $\lim\limits_{x\to 3}\dfrac{3-x}{x^3-27}$.

535. $\lim\limits_{x\to 0}\dfrac{\ln(1+x)}{e^x-e^{-x}}$.
536. $\lim\limits_{x\to 0}\dfrac{x^2}{1-\cos 6x}$.

537. $\lim\limits_{x\to +\infty}\dfrac{\pi-2arctgx}{e^{\frac{1}{x}}-1}$.
538. $\lim\limits_{x\to 0}\dfrac{x-\sin x}{x^3}$.

539. $\lim\limits_{x\to +\infty}\dfrac{\ln(1+x)}{x}$.
540. $\lim\limits_{x\to \frac{\pi}{2}}\dfrac{tg3x}{tg5x}$.

541. $\lim\limits_{x\to 1-0}\dfrac{tg\frac{\pi x}{2}}{\ln(1-x)}$.
542. $\lim\limits_{x\to +0} x\cdot e^{\frac{1}{x}}$.

543. $\lim\limits_{x\to 0}(1-\cos x)ctgx$.
544. $\lim\limits_{x\to 0} ctgx\cdot\ln(x+e^x)$.

545. $\lim\limits_{x\to 2}\left(\dfrac{1}{x-2}-\dfrac{12}{x^3-8}\right)$.
546. $\lim\limits_{x\to 1}\left(\dfrac{1}{\ln x}-\dfrac{x}{\ln x}\right)$.

547. $\lim\limits_{x\to 0}\left(ctgx-\dfrac{1}{x}\right)$.
548. $\lim\limits_{x\to \infty}\left(1+\dfrac{1}{5x}\right)^x$.

549. $\lim\limits_{x\to\infty}\sqrt[x]{x^2}$.
550. $\lim\limits_{x\to 0}(\cos x)^{-ctg^2x}$.

551. $\lim\limits_{x\to 0}\left(\dfrac{1}{x}\right)^{tgx}$.
552. $\lim\limits_{x\to 0}(e^x+1)^{\frac{1}{x}}$.

553. $\lim\limits_{x\to \infty}\left(1+\dfrac{1}{x^2}\right)^x$.
554. $\lim\limits_{x\to \infty}\left(1+\dfrac{1}{x}\right)^{x^2}$.

2-§. Funksiyalarning monotonlik sharti. Ekstremumlar.

1. Agar *f(x)* funksiya [*a,b*] da aniqlangan, uzluksiz, aqqalli *(a,b)* da differensiallanuvchi bo‘lsa, u holda

a) $\forall x\in(a,b)$ uchun $f'(x)>0$ ($f'(x)<0$) bo‘lganda, *f(x)* [*a,b*] da o‘suvchi (kamayuvchi) bo‘ladi.

b) $\forall x\in(a,b)$ uchun $f'(x)\geq 0$ ($f'(x)\leq 0$) bo‘lganda, *f(x)* [*a,b*] da kamaymovchi (o‘smovchi) bo‘ladi.

$f'(x)>0$ bo‘lsa,	*f(x)* [*a,b*] da	o‘suvchi

$f'(x)<0$ bo‘lsa,	$f(x)$ $[a,b]$ da	kamayuvchi
$f'(x)\geq 0$ bo‘lsa,	$f(x)$ $[a,b]$ da	kamaymovchi
$f'(x)\leq 0$ bo‘lsa,	$f(x)$ $[a,b]$ da	o‘smovchi

2. Agar $f(x)$ funksiya $[a,b]$ da aniqlangan bo‘lib, x_0 ning $u(x_0,\delta)\subset(a,b)$ (a,b) atrofidagi barcha x nuqtalari uchun $f(x_0)\geq f(x)$ $(f(x_0)\leq f(x))$ o‘rinli bo‘lsa, u holda x_0 nuqta $f(x)$ funksiyaning maksimum (minimum) nuqtasi deyiladi.

Maksimum va minimum nuqtalar ekstremum nuqtalari deyiladi.

3. Ekstremumning zaruriy sharti.

Agar x_0 $f(x)$ funksiyaning ekstremum nuqtasi bo‘lsa, u holda

$f'(x_0)=0$ yoki $f'(x_0)$ mavjud bo‘lmaydi.

$f'(x_0)=0$ bo‘lganda x_0 statsionar nuqta deyiladi.

$f'(x_0)=0$ va chekli xosila mavjud bo‘lmagan nuqtalar kritik nuqtalar deyiladi.

4. Ekstremumning etarli shartlari.

1-qoida. Agar x_0 kritik nuqta va etarli kichik $\delta>0$ uchun $f'(x_0-\delta)>0$, $f'(x_0+\delta)<0$ ($f'(x_0-\delta)<0$, $f'(x_0+\delta)>0$) bo‘lsa, u holda $f(x)$ funksiya x_0 nuqtada maksimum (minimum)ga erishadi.

Agar $f'(x_0-\delta)\cdot f'(x_0+\delta)>0$ bo‘lsa, $f(x)$ funksiya x_0 nuqtada eksttremumga ega bo‘ladi.

$f'(x_0)$	$f'(x_0-\delta)$	$f'(x_0+\delta)$	$f(x)$
$f'(x_0)=0$ yoki mavjudmas	$\pm$	$\mp$	maksimum minimum
$f'(x_0)=0$ yoki mavjudmas	$\pm$	$\pm$	ekstremum mavjud emas

2-qoida. Agar $f'(x_0)=0, f''(x_0)\neq 0$ bo‘lsa, u holda $f''(x)<0$ $(f''(x)>0)$ bo‘lganda $f(x)$ funksiya x_0 nuqtada maksimum (minimum)ga erishadi.

Agar	va	u holda f funksiya x_0 da
$f'(x_0)=0$	$f''(x_0)>0$ bo‘lsa,	minimumga erishadi
$f'(x_0)=0$	$f''(x_0)<0$ bo‘lsa,	maksimumga erishadi

3-qoida. Agar $f'(x_0)=0, f''(x_0)=0,\ldots, f^{(n-1)}(x_0)=0, f^{(n)}(x_0)\neq 0$ bo‘lsa, u holda n juft son bo‘lganda, $f(x)$ x_0 da ekstremumga erishadi. Bunda $f^{(n)}(x_0)<0$ $(f^{(n)}(x_0)>0)$ bo‘lganda maksimum (minimum) bo‘ladi. n - toq son bo‘lganda, ekstremum mavjud bo‘lmaydi.

5. $f(x)$ funksiyaning $[a,b]$ dagi eng katta (kichik) qiymatlarini topish uchun $f(x)$ funksiyaning $x=a$, $x=b$ nuqtalardagi qiymatlari va $[a,b]$ ga tegishli kritik nuqtalardagi qiymatlarini taqqoslab, ularning eng kattasi (kichigi) ni aniqlash kerak.

6. Funksiya parametrik ko‘rinishda berilgan bo‘lsa,

$$\begin{cases} x = \varphi(t), \\ y = \phi(t), a \le t \le b \end{cases}$$

Va shu sohada $\varphi(t)$ hamda $\phi(t)$ larning birinchi va ikkinchi tartibli hosilalari mavjud bo‘lib, $\varphi'(t_0)\neq 0$, $\phi'(t_0)=0$ bo‘lsin. Agar a) $\phi''(t_0)<0$ bo‘lsa, $y=f(x)$ funksiya $x=x_0=\varphi(t_0)$ da maksimum;

b) $\phi''(t_0)>0$ bo‘lsa, $y=f(x)$ funksiya $x=x_0=\varphi(t_0)$ da minimum;

c) $\phi''(t_0)=0$ bo‘lsa, ekstremum «shubhali» bo‘ladi. $\varphi'(t)=0$ ni qanoatlantiruvchi t nuqtalarini alohida tekshirishga to‘g‘ri keladi.

Quyidagi funksiyalarning monotonlik intervallarini toping.

555. $y = -x^2 + 10x + 7$.

556. $y = 4x^2 + 12x + 9$.

557. $y = x^3 - 6x^2 + 9x$.

558. $y=(x-3)^2$.

559. $y = x^4 + 8x^3 + 3$.

560. $y = \dfrac{1}{(x-5)^2}$.

561. $y = \dfrac{2x-3}{x+7}$.

562. $y = 2\sqrt{x} - 3\sqrt[3]{x^2}$.

563. $y = xe^{-5x}$.

564. $y = x^3 \ln x$.

565. $y = \ln(1+x^2)$.

566. $y = \ln(1+x^2)-x$.

567. $y = x + 2\cos x,\ x \in (0;\pi)$.

568. $y = x + \sin x$.

569. $y = \ln(x + \sqrt{1+x^2})$.

570. $y = \dfrac{x}{\ln x}$.

571. $y = x^2 - bx$ funksiya b ning qanday qiymatlarida (-1;1) kamayuvchi bo‘ladi?

572. $y = \cos x + ax + b$ funksiya a ning qanday qiymatlarida $(-\infty; +\infty)$ da o‘suvchi bo‘ladi?

Quyidagi funksiyalarning ekstremumlarini toping.

573. $y=6x - x^2$.

574. $y = 2x^3-3x^2$.

575. $y = (1-x^2)^3$.

576. $y = (x-1)^2(x-2)^3$.

577. $y = \dfrac{x}{9-x^2}$.

578. $y = \dfrac{x+1}{x^2+8}$.

579. $y = \dfrac{x^4+48}{x}$.

580. $y = x - 6\sqrt[3]{x^2}$.

581. $y = \dfrac{x^3}{\sqrt{x^2-6}}$.

582. $y = (x^2-8)e^x$.

583. $y = \sqrt[3]{x^2}e^x$.

584. $y = x\ln x$.

585. $y = \frac{x}{2} - arc\,tgx$.

586. $y = \cos x - \sin x$.

587. $y = x - \ln(1+x)$.

588. $y = x - \ln(1+x^2)$.

589. $y = xe^{x-x^2}$.

590. $y = (x^2 - 2x)\ln x - \frac{3}{2}x^2 + 4x$.

591. *a* va *b* ning har qanday qiymatlarida $y = 3x^4 - 4x^3 + 6x^2 + ax + b$ funksiya faqat birgina ekstremum nuqtasiga ega bo‘ladi. Asoslang.

592. Parametr *a* ning qanday qiymatida $y = x^2 - 4ax - a^4$ funksiya minimumga ega bo‘ladi?

Quyidagi funksiyalarning ko‘rsatilgan oraliqlardagi eng kichik va eng katta qiymatlarini toping.

593. $y = -3x^2 + 4x - 8,\ x \in [0;1]$.

594. $y = -x^3 + 3x^2 - 9x - 7,\ x \in [-4;3]$.

595. $y = \sqrt{25 - x^2},\ x \in [-4;4]$.

596. $y = x + 3\sqrt[3]{x},\ x \in [-8;1]$.

597. $y = \frac{2x^3}{x^2 - 9},\ x \in [4;6]$.

598. $y = \frac{x}{4 + x^2},\ x \in (-\infty;+\infty)$.

599. $y = x\ln^2 x,\ x \in [\frac{1}{e};e]$.

600. $y = \frac{1}{\cos x},\ x \in (\frac{\pi}{2};\frac{3\pi}{2})$.

601. $y = \arccos x^2,\ x \in [-\frac{\sqrt{2}}{2};\frac{\sqrt{2}}{2}]$.

602. $y = \ln x - 2arctgx,\ x \in [1;+\infty)$.

603. $y = 2tgx - tg^2 x,\ x \in [0;\frac{\pi}{2})$.

604. $y = e^{2x} - e^{-2x},\ x \in [-2;1]$.

605. $y = |x|,\ x \in [-1;1]$.

606. $y = [x],\ x \in [0;2]$.

607. $y = \{x\},\ x \in [-1;1]$.

608. $y = \frac{1}{\cos x},\ x \in [-1;1]$.

609. O‘zi bilan kvadratining yig‘indisi eng kichik bo‘ladigan sonni toping.

610. O‘zi bilan unga teskari bo‘lgan sonning yig‘indisi eng kichik bo‘ladigan musbat sonni toping.

611. Radiusi *R* bo‘lgan doiraga ichki chizilgan eng katta yuzli to‘g‘ri to‘rtburchakning tomonini toping.

612. *R* radiusli sharga ichki chizilgan eng katta hajmli silindrning balandligini toping.

613. *R* radiusli sharga ichki chizilgan eng katta hajmli konusning balandligini toping.

614. *R* radiusli sharga ichki chizilgan eng katta yon sirtli silindrning balandligini toping.
615. *R* radiusli sharga tashqi chizilgan eng kichik hajmli konusning balandligini toping.
616. Berilgan silindrga, asos markazi silindr asosining markazi bilan bir xil bo‘lgan konus tashqi chizilgan. Konus asosining radiusi qanday bo‘lganda, uning hajmi eng kichik bo‘ladi?
617. Berilgan konusga ichki chizilgan eng kata hajmli silindrning balandligini toping.
618. $y^2=2rx$ parabolada shunday nuqta topingki, undan $M(r,r)$ nuqtagacha bo‘lgan masofa eng qisqa bo‘lsin.
619. $2x^2+y^2=18$ ellipsda yotuvchi $A(1,4)$ va B(3,0) nuqtalar berilgan. Ellipsda yotuvchi shunday C nuqta topingki, ABC uchburchakning yuzi eng katta bo‘lsin.

3-§. Egri chiziqning qavariqlik va botiqlik intervallari. Burilish nuqta. Asimptotalar

1. Agar (a,b) da $y=f(x)$ egri chiziqning barcha nuqtalari uning har qanday urinmasidan yuqorida (pastda) bo‘lsa, shu intervalda egri chiziq qavariqligi pastga (yuqoriga) qaragan deyiladi.

Qavariqligi pastga (yuqoriga) qaragan egri chiziq qavariq (botiq) egri chiziq ham deyiladi.

Egri chiziqning qavariq qismini botiq qismidan ajratgan nuqta uning burilish nuqtasi deyiladi.

2. Hosila yordamida qavariqlikni tekshirish.

Agar $a<x<b$ da	u holda $a<x<b$ da
$f''(x)>0$ bo‘lsa,	$y=f(x)$ qavariq
$f''(x)<0$ bo‘lsa,	$y=f(x)$ botiq

3. Funksiya grafigining asimptotalarini topish.

a) Agar $\lim\limits_{x\to a-0} f(x)=\pm\infty$ yoki $\lim\limits_{x\to a+0} f(x)=\pm\infty$ bo‘lsa, $x=a$ vertikal asimptota bo‘ladi.

b) Agar $k=\lim\limits_{x\to\infty}\frac{f(x)}{x}, b=\lim\limits_{x\to\infty}(f(x)-kx)$ lar mavjud bo‘lsa, u holda $y=kx+b$ og‘ma asimptota bo‘ladi.

Quyidagi funksiyalar grafiklarining qavariq va botiqlik intervallari va burilish nuqtalarini toping.

620. $y=x^3-3x^2-9x+9$.
621. $y=x^2+x^4$.
622. $y=\frac{x}{x^2+9}$.
623. $y=x\sqrt[3]{x^2}(x+8)$.

624. $y=\frac{x-5}{x+7}$.

625. $y=\frac{x^3+8}{x}$.

626. $y=5+\sqrt[3]{x-4}$.

627. $y=e^{\sin x},\ x\in[-\frac{\pi}{2};\frac{\pi}{2}]$.

628. $y=\ln(1+x^2)$.

629. $y=\frac{a}{x}\ln\frac{x}{a},\quad (a>0)$.

630. $y=x\,arc\,tgx$.

631. $y=e^{arctgx}$.

632. $y(x^2+a^2)=a^2(a-x)$ egri chiziqning bir to‘g‘ri chiziqda yotuvchi uchta burilish nuqtasi mavjudligini ko‘rsating.

633. $y=\frac{x+1}{x^2+1}$ egri chiziqning bir to‘g‘ri chiziqda yotuvchi uchta burilish nuqtasi mavjudligini ko‘rsating.

634. $x=1$ da $y=x^3+ax^2+1$ egri chiziqning burilish nuqtasi mavjud bo‘lishi uchun a qanday bo‘lishi kerak?

635. a ning qanday qiymatida $y=e^x+ax^3$ egri chiziq burilish nuqtaga ega?

636. a va b larning qanday qiymatlarida $M(1,3)$ nuqta $y=ax^3+bx^2$ egri chiziqning burilish nuqtasi bo‘ladi?

637. $\begin{cases} x=t^2, \\ y=3t+t^3 \end{cases}$ egri chiziqning burilish nuqtalarining toping.

638. $\begin{cases} x=e^t, \\ y=\sin t \end{cases}$ egri chiziqning burilish nuqtalarining toping.

Quyidagi funksiyalar grafiklarining asimptotlarini toping.

639. $y=\frac{1}{x+5}$.

640. $y=\frac{3}{(x-4)^2}$.

641. $y=\frac{2x+1}{x-3}$.

642. $y=\frac{x^3}{4-x^2}$.

643. $y=\sqrt{\frac{x^3}{x-6}}$.

644. $y=\sqrt{1+x^2}+2x$.

645. $y=x+\frac{\sin x}{x}$.

646. $y=xe^x$.

647. $y=xe^{\frac{2}{x}}$.

648. $y=x\ln(e+\frac{1}{x})$.

649. $y=x\,arc\,tgx$.

650. $y=\ln(1-x^2)$.

651. Quyidagi chiziqlarning asimptotalarini toping.

1) $\begin{cases} x = \dfrac{1}{t}, \\ y = \dfrac{t}{1+t} \end{cases}$ 2) $\begin{cases} x = \dfrac{2t}{1-t^2}, \\ y = \dfrac{t^2}{1-t^2} \end{cases}$ 3) $\begin{cases} x = \dfrac{2e^t}{t-1}, \\ y = \dfrac{te^t}{t-1} \end{cases}$

4-§. Funksiyalarni to‘la tekshirish

y=f(x) funksiya berilgan bo‘lib, uning grafigini chizish talab qilinsa, quyidagilarni bajarish kerak:

1. Funksiyaning aniqlanish sohasi, uzilish nuqtalarini topish, uning juft-toqligi va davriyligini tekshirish;
2. Aniqlanish sohasining chegaraviy nuqtalarida funksiyaning holatini, funksiya grafigining koordinata o‘qlari bilan kesishish nuqtalarini topish;
3. Funksiyaning monotonlik oraliqlarini aniqlab, ekstremumini tekshirish, grafikning qavariqligi va burilish nuqtalarini topish;
4. Funksiya grafigining asimptotalarini topish;
5. Yuqoridagi ma’lumotlarga asoslanib, funksiya grafigini yasash.

Quyidagi funksiyalarni to‘la tekshiring va grafiklarini yasang.

652. $y = 2x^4 - x^2 + 1.$

653. $y=x^5-x^3-2x.$

654. $y=36x(x-1)^3.$

655. $y = (x^2-1)^3.$

656. $y = \dfrac{2x+1}{x+5}.$

657. $y = \dfrac{1}{x^2+4}.$

658. $y = \dfrac{x}{x^2-4}.$

659. $y = \dfrac{8}{16-x^2}.$

660. $y = \dfrac{x^2+6}{x^2-1}.$

661. $y = \dfrac{x^2}{x-3}.$

662. $y = \dfrac{x^3}{1-x^2}.$

663. $y = x^2 - \dfrac{8}{x}.$

664. $y = \sqrt{x} - 2x.$

665. $y = 3\sqrt[3]{x^2} - 4\sqrt{x}.$

666. $y=\frac{e^x}{x}$.

667. $y=\frac{x}{e^x}$.

668. $y=e^{-x^2}$.

669. $y=e^{x^2}$.

670. $y=e^{\frac{1}{x}}$.

671. $y=x^2 e^{\frac{1}{x}}$.

672. $y=x^3e^{-4x}$.

673. $y=xe^{-\frac{x^2}{2}}$.

674. $y=\frac{1}{e^x-1}$.

675. $y=x+\frac{\ln x}{x}$.

676. $y=(1+\frac{1}{x})^x$.

677. $y=\ln\cos x$.

678. $y=x+\sin x$.

679. $y=\frac{\sin x}{x}$.

680. $y=x-2\,arc\,tgx$.

681. $y=\sin\frac{1}{x}$.

682. $y=2|x|-x^2$.

683. $y=\sqrt[3]{x^2}-x$.

684. $y=\ln\sin x$.

685. $y=\cos x-\ln\cos x$.

686. $\begin{cases} x=t^3+3t+1, \\ y=t^3-3t+1 \end{cases}$

687. $\begin{cases} x=t^3-3\pi, \\ y=t^3-6arc\,tgt \end{cases}$

688. $\rho=a\sin 3\varphi,\ a>0$.

689. $\rho=atg\varphi,\ a>0$.

690. $\rho=a(1+\cos\varphi),\ a>0$.

691. $\rho=a(1+\sin\varphi),\ a>0$.

5-§. Teylor formulasi.

692. $f(x) = x^4 + 2x^3-3x^2 - 4x+1$ ko‘phadni $x+1$ ning darajalari bo‘yicha yoying.

693. $f(x) = x^4 - 5x^3 + x^2 - 3x + 4$ ko‘phadni $x - 4$ ning darajalari bo‘yicha yoying.

694. $f(x) = (x^2 - 3x + 1)^3$ ko‘phadni x ning darajalari bo‘yicha yoying.

695. $f(x)=\frac{1}{x}$ funksiyani x_0=-1dagi n- tartibli Teylor formulasini yozing.

696. $f(x) = xe^x$ funksiyani x_0=0 dagi n-tartibli Makloren formulasini yozing.

697. $f(x)=\sqrt{x}$ funksiyani x_0=4 dagi n- tartibli Teylor formulasini yozing.

698. $f(x) =x^3 \ln x$ funksiyani x_0=1dagi n- tartibli Teylor formulasini yozing.

699. $f(x) = \sin^2 x$ funksiyani x_0=0 dagi $2n$- tartibli Teylor formulasini yozing.

VI BOB. ANIQMAS INTEGRALLAR

1-§. Asosiy integrallar jadvali

1. Agar *Φ(x)* funksiya biror *D* oraliqda uzluksiz bo‘lib, shu oraliqda *Φ′(x)=f(x)* o‘rinli bo‘lsa, u holda *Φ(x)* funksiya *f(x)* funksiyaning boshlang‘ich funksiyasi deyiladi.

Berilgan *f(x)* funksiyaning biror *D* oraliqdagi barcha boshlang‘ich funksiyalari to‘plami *Φ(x)+C* berilgan funksiyaning *D* oraliqda aniqmas integrali deyiladi va $\int f(x)dx$ orqali belgilanadi. Berilgan funksiyaning aniqmas integralini topish amali integrallash deyiladi.

2. Aniqmas integralning asosiy hossalari:

a) $d(\int f(x)dx) = f(x)dx,$

b) $\int d\Phi(x) = \Phi(x) + C,$

c) $\int kf(x)dx = k\int f(x)dx,$ k- o‘zgarmas,

d) $\int (f_1(x) + f_2(x))dx = \int f_1(x)dx + \int f_2(x)dx.$

3. Asosiy integrallar jadvali.

I. $\int x^{\alpha} dx = \frac{x^{\alpha+1}}{\alpha+1} + C \qquad (\alpha \in R, \alpha \neq -1),$

II. $\int \frac{dx}{x} = \ln|x| + C.$

III. $\int a^x dx = \frac{a^x}{\ln a} + C,\ \int e^x dx = e^x + C.$

IV. $\int \cos x dx = \sin x + C.$

V. $\int \sin x dx = -\cos x + C.$

VI. $\int \frac{dx}{\cos^2 x} = tgx + C.$

VII. $\int \frac{dx}{\sin^2 x} = -ctgx + C.$

VIII. $\int \frac{dx}{x^2 + a^2} = \frac{1}{a} arctg \frac{x}{a} + C.$

IX. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C.$

X. $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln\left|\frac{x-a}{x+a}\right| + C.$

XI. $\int \frac{dx}{\sqrt{x^2 \pm A}} = \ln\left|x + \sqrt{x^2 \pm A}\right| + C.$

XII. $\int \sqrt{x^2 - a^2} dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a}{4}\ln\left|\frac{x-a}{x+a}\right| + C.$

XIII. $\int shxdx = chx + C.$

XIV. $\int chxdx = shx + C.$

XV. $\int \frac{dx}{ch^2 x} = thx + C.$

XVI. $\int \frac{dx}{sh^2 x} = -cthx + C.$

Quyidagi integrallarni toping.

700. $\int (x^4 - 4x^3 + 2x)dx$.

701. $\int (2t^3 + 6t^3)dt$.

702. $\int (x-1)(x+4)dx$.

703. $\int x^2 (x+1)(5x-3)dx$.

704. $\int (\sqrt{x} + \frac{1}{x})dx$.

705. $\int (\frac{1}{x\sqrt{x}} - \frac{1}{x^2})dx$.

706. $\int (\sqrt{x\sqrt{x}} + x\sqrt{x})dx$.

707. $\int \frac{(1+\sqrt{x})^2}{\sqrt{x}} dx$.

708. $\int \frac{(1-x)^3}{x\sqrt{x}} dx$.

709. $\int 2^x dx$.

710. $\int \frac{3 \cdot 2^x - 2 \cdot 3^x}{2^x} dx$.

711. $\int a^x e^x dx$.

712. $\int (2e^x - \sqrt[3]{x^2})dx$.

713. $\int (10^x - 2\sin x)dx$.

714. $\int \frac{1+\cos^2 x}{1+\cos 2x} dx$.

715. $\int \frac{\cos 2x}{\cos^2 x \cdot \sin^2 x} dx$.

716. $\int tg^2 xdx$.

717. $\int ctg^2 xdx$.

718. $\int \sin^2 \frac{x}{2} dx$.

719. $\int \frac{dx}{\cos 2x + \sin^2 x}$.

720. $\int \frac{(1+2x^2)dx}{x^2(1+x^2)}$.

721. $\int \frac{(1+x)^2 dx}{x(1+x^2)}$.

2-§. Integrallashning asosiy usullari.

a) Bevosita integrallash. Berilgan integral asosiy integrallar jadvali (3) da bo‘lmasligi mumkin, lekin ba’zi aynan shakl o‘zgartishlar natijasida jadvaldagi integrallarga keltirish mumkin.

b) O‘zgaruvchini almashtirish formulasi: $\int f(x)dx = \int f(\varphi(t))\varphi'(t)dt$, bunda $x = \varphi(t)$ hosilasi uzluksiz funksiyadir.

c) Bo‘laklab integrallash formulasi: $\int udv = uv - \int vdu$, bunda $u = u(x)$ va $v = v(x)$ differensiallanuvchti funksiyalar.

Integrallash formulalarining invariantligidan foydalanib, quyidagi integrallarni toping.

722. $\int (x+1)^{14}\,dx$.
723. $\int \sqrt{8-2x}\,dx$.
724. $\int \sqrt[5]{(3x+1)^2}\,dx$.
725. $\int x\sqrt{1+x^2}\,dx$.
726. $\int x\sqrt{1-x^2}\,dx$.
727. $\int \frac{xdx}{\sqrt{x^2+1}}$.
728. $\int \frac{x^4 dx}{\sqrt{4+x^5}}$.
729. $\int \sin^3 x \cos x\,dx$.
730. $\int \frac{\sin x}{\cos^2 x}dx$.
731. $\int \frac{\sqrt{\ln x}}{x}dx$.
732. $\int \frac{dx}{x\sqrt{\ln x}}$.
733. $\int \frac{(arc\,tgx)^2\,dx}{1+x^2}$.
734. $\int \frac{dx}{arc\sin^3 x\cdot\sqrt{1-x^2}}$.
735. $\int \cos 3x\,dx$.
736. $\int \sin(2x-1)\,dx$.
737. $\int \cos(1-2x)\,dx$.
738. $\int e^x \sin e^x\,dx$.
739. $\int \frac{dx}{2x-1}$.
740. $\int \frac{xdx}{x^2+1}$.
741. $\int ctg\,dx$.
742. $\int \frac{dx}{x\ln x}$.
743. $\int \frac{\sin 2xdx}{1+\cos^2 x}$.
744. $\int e^{2x}dx$.
745. $\int e^{-3x}dx$.
746. $\int e^{\sin x}\cos xdx$.
747. $\int xe^{x^2}\,dx$.
748. $\int e^{-x^3}x^2dx$.
749. $\int \frac{dx}{\sqrt{1-4x^2}}$.
750. $\int \frac{dx}{\sqrt{1-25x^2}}$.
751. $\int \frac{dx}{\sqrt{4-x^2}}$.
752. $\int \frac{dx}{1+9x^2}$.
753. $\int \frac{xdx}{1+x^4}$.

Bo‘laklab usulidan foydalanib,quyidagi integrallarni toping.

754. $\int x\sin 2xdx$.
755. $\int x\cos xdx$.
756. $\int xe^x dx$.
757. $\int xarctgxdx$.
758. $\int \arccos xdx$.
759. $\int arctg\sqrt{x}dx$.
760. $\int x^2e^{-x}dx$.
761. $\int x^3e^x dx$.
762. $\int x^3\sin xdx$.
763. $\int x^2\cos^2 xdx$.

764. $\int \frac{x^2 dx}{(1+x^2)^2}$.

765. $\int x^2 \ln(1+x)dx$.

O‘zgaruvchini almashtirish usulidan foydalanib, quyidagi integrallarni toping.

766. $\int \frac{dx}{1+\sqrt{x+1}}$.

767. $\int \frac{dx}{x\sqrt{x+1}}$.

768. $\int \frac{x^3 dx}{\sqrt{x-1}}$.

769. $\int \frac{dx}{1+\sqrt{x}}$.

770. $\int \frac{x+1}{x\sqrt{x-2}}dx$.

771. $\int \frac{\sqrt{x}}{x+(x+1)}dx$.

772. $\int \frac{dx}{1+\sqrt[3]{x+1}}$

773. $\int \frac{dx}{3+\sqrt{2x+1}}$.

774. $\int \frac{dx}{\sqrt[3]{x}(\sqrt[3]{x}-1)}$

775. $\int \frac{dx}{\sqrt{1+e^x}}$.

3-§. Ratsional funksiyalarni integrallash

1. Butun ratsional funksiyalarni integrallash bevosita bajariladi:

$$\int P_n(x)dx = \int (a_0x^n + a_1x^{n-1} + ... + a_{n-1}x + a_n)dx = a_0\frac{x^{n+1}}{n+1} + ... + a_nx + C.$$

Kasr-ratsional funksiya $\frac{P_n(x)}{Q_m(x)}$ $(n \geq m)$ bo‘lganda, suratini maxrajga bo‘lish bilan

$$\frac{P_n(x)}{Q_m(x)} = r(x) + \frac{P_k(x)}{Q_m(x)}, \text{ bunda } k < m.$$

$\frac{P_n(x)}{Q_m(x)}$ - noto‘g‘ri kasr-ratsional funksiya,

$r(x)$ - butun ratsional funksiya,

$\frac{P_k(x)}{Q_m(x)}$ - to‘g‘ri kasr-ratsional funksiyaga keltiriladi. To‘g‘ri kasr ratsional funksiya $\frac{P_k(x)}{Q_m(x)}$ ni $Q_m(x) = (x-a)^{\alpha}(x-b)^{\beta}...(x^2+px+q)^{\lambda}$ yoyilmasidan foydalanib elementar kasrlar yig‘indisi shaklida yoziladi:

$$\frac{P_k(x)}{Q_m(x)} = \frac{A_1}{(x-a)^{\alpha}} + \frac{A_2}{(x-a)^{\alpha-1}} + ... + \frac{A_{\alpha}}{x-a} + \frac{B_1}{(x-b)^{\beta}} + \frac{B_2}{(x-b)^{\beta-1}} + ... + \frac{B_{\beta}}{x-b} +$$

$$+ ... + \frac{M_1x+N_1}{(x^2+px+q)^{\lambda}} + \frac{M_2x+N_2}{(x^2+px+q)^{\lambda-1}} + ... + \frac{M_{\lambda}x+N_{\lambda}}{x^2+px+q},$$

bunda

$$A_1, A_2, ..., A_\alpha, B_1, B_2, ..., B_\beta, ..., M_1, M_2, ..., M_\lambda, N_1, N_2, ..., N_\lambda$$

$(\alpha+\beta+...+\lambda=m)$ noma'lum koeffitsientlar bo'lib, ularni koeffitsientlarni solishtirish usuli bilan aniqlanadi.

2. Elementar kasrlar quyidagicha integrallanadi:

I. $\int \frac{Adx}{x-a} = A\ln|x-a|+C.$

II. $\int \frac{Adx}{(x-a)^\alpha} = \frac{A}{(1-\alpha)(x-a)^{\alpha-1}} + C, (\alpha>1).$

III. $\int \frac{Mx+N}{x^2+px+q}dx = \frac{M}{2}\ln(x^2+px+q) + \frac{2N-Mp}{\sqrt{4q-p^2}}arctg\frac{2x-p}{\sqrt{4q-p^2}} + C,$

$(4q-p^2>0).$

IV. $\int \frac{Mx+N}{(x^2+px+q)^k}dx = \frac{M}{2(1-k)(x^2+px+q)^{k-1}} + \frac{2N-Mp}{2}\int\frac{du}{(u^2+a^2)^k}.$

bunda $u = x+\frac{p}{2}$, $a=\frac{1}{2}\sqrt{4p-q^2}$ va rekurrent formula

$$I_k = \int\frac{du}{(u^2+a^2)^k} = \frac{u}{a^2(2k-2)(u^2+a^2)^{k-1}} + \frac{1}{2a^2}\frac{2k-3}{2k-2}\cdot I_{k-1};$$

$(k=2,3,4,...)$ dan foydalaniladi.

Har qanday ratsional funksiyaning integrali elementar funksiya bo'ladi.

Quyidagi sodda kasrlarni integrallang.

776. $\int\frac{dx}{2x+1}.$

777. $\int\frac{dx}{1-3x}.$

778. $\int\frac{dx}{x^2+10x+3}.$

779. $\int\frac{dx}{2x^2+2x+5}.$

780. $\int\frac{x-4}{x^2+x-12}dx.$

781. $\int\frac{6x-1}{x^2-4x+13}dx.$

782. $\int\frac{3-5x}{4x^2+16x-9}dx.$

783. $\int\frac{12x+11}{9x^2-6x+2}dx.$

784. $\int\frac{dx}{(x-2)^3}.$

785. $\int\frac{dx}{(3x-1)^2}.$

786. $\int\frac{x^2dx}{(x^2+2x+2)^2}.$

787. $\int\frac{x^3+x-1}{(x^2+2)^2}dx.$

Kasr-ratsional funksiyalarni integrallarini toping.

788. $\int \frac{3x+8}{(x-2)(x+5)}dx$.

789. $\int \frac{7x+12}{(x-1)(3x+1)}dx$.

790. $\int \frac{5x-10-x^2}{x^2-4x+3}$.

791. $\int \frac{x^2-72}{x(x+4)(x-3)}dx$.

792. $\int \frac{x^4-16x^3+5x+8}{x^3-16x}dx$.

793. $\int \frac{6+8x-x^2}{x^3+3x^2+2x}dx$.

794. $\int \frac{3x+1}{(x+3)^2(x-5)}dx$.

795. $\int \frac{x^2+5x+9}{(x-2)^3}dx$.

796. $\int \frac{x^3-10x+25}{x^2(x-5)^2}dx$.

797. $\int \frac{3x^3-7x^2+6x}{(x-1)^2(1-2x)}dx$.

798. $\int \frac{4x^2-5\mathrm{x}+9}{(x^2-4x+13)(x+1)}dx$.

799. $\int \frac{x^2-7x-6}{(x^2+9)(x-3)}dx$.

800. $\int \frac{5x^4-x^3+4x^2+8}{x^3-8}dx$.

801. $\int \frac{x^3-7x^2-3}{x^2(x^2+4)}dx$.

802. $\int \frac{x^3-12x^2-3x}{(x^2-2x+2)(x^2-1)}dx$.

803. $\int \frac{4x^3+3x^2-17x}{(x^2+2x+2)(x^2+9)}dx$.

804. $\int \frac{x^3+x-1}{(x^2+2)^2}dx$.

805. $\int \frac{x^3+1}{(x^2-4x+5)^2}dx$.

4-§. Ba’zi irratsional funksiyalarni integrallash

1. Ratsionallashtirish usuli.

Ba’zi irratsional funksiyalarni integrallashda o‘zgaruvchini tegishlicha almashtirib, ratsional funksiyaning integraliga keltirish, odatda, ratsionallashtirish usuli deyiladi.

2. Agar integral ostidagi funksiya erkli o‘zgaruvchi x ning kasr darajalarining ratsional funksiyasidan iborat bo‘lsa, ratsionalllashtirish usulidan foydalaniladi:

$\int R(x, x^{\frac{p_1}{q_1}}, x^{\frac{p_2}{q_2}}, ..., x^{\frac{p_k}{q_k}})dx$ bo‘lganda $x = t^n$ almashtirishdan foydalaniladi. Bunda n soni $q_1, q_2, ..., q_k$ larning eng kichik umumiy bo‘linuvchisi, integral ostidagi R esa o‘z argumentlarining ratsional funksiyasini bildiradi.

3. Agar integrla ostida x va kasr-chiziqli $\frac{ax+b}{cx+d}$ funksiyaning kasr darajalarining ratsional funksiyasi bo‘lsa, u holda

$$\int R\left(x, \left(\frac{ax+b}{cx+d}\right)^{\frac{p_1}{q_1}}, ..., \left(\frac{ax+b}{cx+d}\right)^{\frac{p_k}{q_k}} \right)dx \text{ ni } \frac{ax+b}{cx+d} = t^n$$

almashtirish orqali ratsionallashtirish mumkin, bunda ham n soni $q_1, q_2, ..., q_k$ larning eng kichik umumiy bo‘linuvchisi.

4. Eyler almashtirishlari.

$\int R(x_1 \sqrt{ax^2 + bx + c})dx$ (bunda R - o‘z argumentlarining ratsional funksiyasi) ko‘rinishdagi integrallar Eyler almashtirishlari yordamida ratsionallashtirish usuli bilan hisoblanadi.

Eylerning I almashtirishi:

Agar $a > 0$ bo‘lsa, $\sqrt{ax^2 + bx + c} = t \pm x\sqrt{a}$.

Eylerning II almashtirishi:

Agar $c>0$ bo‘lsa, $\sqrt{ax^2 + bx + c} = t \pm x\sqrt{c}$.

Eylerning III almashtirishi:

Agar α $ax^2 + bx + c = 0$ ning haqiqiy ildizlaridan biri bo‘lsa, $\sqrt{ax^2 + bx + c} = (x - \alpha)t$ bo‘ladi.

5. Binomial differensialni integrallash.

$x^m (a + bx^n)^p$ ni integrallash (bunda $m, n, p \in Q$) P.L.Chebishev teoremasiga ko‘ra faqat quyidagi 3 ta holda bajariladi:

I. $p \in Z$ bo‘lsa, $p>0$ da $(a + bx^n)^p$ ni Nyuton binomi formulasi bo‘yicha yoyiladi, $p<0$ da $x = t^n$ deb olinadi, bunda k berilgan m va n ning umumiy maxraji.

II. $p = \frac{r}{s}$ kasr son, lekin $\frac{m+1}{n}$ butun son bo‘lsa, $a + bx^n = t^s$ almashtirish qo‘llaniladi.

III. $p = \frac{r}{s}$, $\frac{m+1}{n}$- kasr sonlar bo‘lib, $p + \frac{m+1}{n}$ butun son bo‘lsa, $a + bx^n = t^s x^n$ almashtirish qo‘llaniladi.

$\int R\left(x, \sqrt[m]{\frac{ax+b}{a_1x+b_1}}, \sqrt[n]{\frac{ax+b}{a_1x+b_1}}, \dots, \sqrt[k]{\frac{ax+b}{a_1x+b_1}} \right)dx$ ko‘rinishdagi integrallar.

806. $\int \frac{dx}{x+\sqrt[3]{x}}$.

807. $\int \frac{\sqrt{x+9}}{x}dx$.

808. $\int \frac{dx}{\sqrt{x}-2\sqrt[3]{x}}$.

809. $\int \frac{1+\sqrt[4]{x}}{x+\sqrt{x}}dx$.

810. $\int \frac{\sqrt[3]{x}-1}{\sqrt{x}+1}dx$.

811. $\int \frac{dx}{(1-\sqrt[3]{x^2})\sqrt{x}}$.

812. $\int \frac{\sqrt[3]{x}+2}{(\sqrt[4]{x}+\sqrt[6]{x})\sqrt[6]{x^5}}dx$.

813. $\int \sqrt{\frac{x}{x+5}}\cdot\frac{dx}{x^2}$.

814. $\int \sqrt{\frac{4x-5}{x+1}}dx.$

815. $\int \sqrt[3]{\frac{x+2}{x-2}}\cdot\frac{dx}{x}$.

2. Eyler almashtrishlari: $\int R(x,\sqrt{ax^2+bx+c})dx$.

816. $\int \frac{dx}{1+\sqrt{x^2+2x+2}}$.

817. $\int \frac{dx}{x\sqrt{2+x-x^2}}$.

818. $\int \frac{dx}{\sqrt{1-x^2}-1}$.

819. $\int \frac{\sqrt{2x+x^2}}{x^2}dx$.

820. $\int \frac{x^2\,dx}{\sqrt{1-2x-x^2}}$.

821. $\int \frac{dx}{(x-2)\sqrt{(7x-x^2-10)^3}}$.

3. Binomial differensiallar: $\int x^m(a+bx^n)^p\,dx$.

822. $\int \sqrt{x}(1+\sqrt[3]{x})^4\,dx$.

823. $\int x^{-1}(1+x^{\frac{1}{3}})^{-3}\,dx$.

824. $\int \frac{dx}{x\sqrt[3]{x^2+1}}$.

825. $\int x^5\sqrt[3]{(1+x^3)^2}\,dx$.

826. $\int \frac{dx}{\sqrt[3]{1+x^3}}$.

827. $\int \frac{dx}{\sqrt[4]{1+x^4}}$.

828. $\int \frac{dx}{x^{11}\sqrt{1+x^4}}$. 829. $\int \sqrt[3]{x(1-x^2)}\,dx$.

5-§. Trigonometrik va giperbolik funksiyalarni integrallash

1. $\int R(\sin x, \cos x)dx$ ni integrallash uchun umumiy usul $t = tg\frac{x}{2}$ almashtirishdan foydalaniladi.

Bunda $\sin x = \frac{2t}{1+t^2}$, $\cos x = \frac{1-t^2}{1+t^2}$, $dx = \frac{2dt}{1+t^2}$ bo‘lib,

$\int R(\sin x, \cos x)dx = \int R\left(\frac{2t}{1+t^2}, \frac{1-t^2}{1+t^2}\right)\frac{dt}{1+t^2}$ bo‘ladi.

2. $\int \sin^m x \cos^n x dx$ uchun

a) *m* va *n* lardan kamida biri toq son, masalan, $n=2k+1$ bo‘lsa, $t=\sin x$, *m* toq bo‘lganda $t=\cos x$ almashtirish qo‘llaniladi.

b) *m* va *n* musbat juft sonlar bo‘lganda, integral ostidagi funksiyaning shaklini o‘zgartirishda

$\cos^2 x = \frac{1+\cos 2x}{2}$, $\sin^2 x = \frac{1-\cos 2x}{2}$, $2\sin x \cos x = \sin 2x$

almashtirishlardan foydalaniladi.

c) *m* va *n* juft sonlar bo‘lib, kamida biri manfiy bo‘lsa, $tgx = t$ $(ctgx = t)$ almashtirishni qo‘llash lozim, bunda $\sin x = \frac{t}{\sqrt{1+t^2}}$, $\cos x = \frac{t}{\sqrt{1+t^2}}$, $dx = \frac{dt}{1+t^2}$ bo‘ladi.

3. $\int sin\, nx\, sin\, mxdx$, $\int sin\, nx\, cos\, mxdx$, $\int cos\, nx\, cos\, mxdx$ integrallar

$sin\, nx\, sin\, mxdx = \frac{1}{2}(cos(n-m)x - cos(n+m)x)$,

$sin\, nx\, cos\, mxdx = \frac{1}{2}(sin(n-m)x + sin(n+m)x)$,

$cos\, nx\, cos\, mxdx = \frac{1}{2}(cos(n-m)x + cos(n+m)x)$

almashtirishlardan foydalanib topiladi.

4. Trigonometrik almashtirishlar.

$\int R(x, \sqrt{a^2-x^2})dx$ uchun $x = a\sin t$,

$\int R(x, \sqrt{a^2+x^2})dx$ uchun $x = atgt$,

$\int R(x, \sqrt{x^2-a^2})dx$ uchun $x = a\sec t$ almashtirishlarni qo‘llab, $\sin x$ va $\cos x$ larning ratsional funksiyalari hosil qilinadi. Bunda ham *R* o‘z argumentlarining ratsional funksiyasi.

5. Giperbolik funksiyalarni integrallash. Bunda quyidagi formulalardan foydalanish lozim:

$$ch^2x - sh^2x = 1, \qquad shxchx = \frac{1}{2}sh2x, \qquad sh^2x = \frac{1}{2}(ch2x - 1),$$

$$ch^2x = \frac{1}{2}(ch2x + 1).$$

Universal almashtirish $th\frac{x}{2} = z$ bo‘lsa, $shx = \frac{2z}{1-z^2}$, $chx = \frac{1+z^2}{1-z^2}$, $dx = \frac{2dz}{1-z^2}$ bo‘ladi, bunda $x = 2Arthz = \ln\frac{1+z}{1-z}$ $(-1 < z < 1)$.

Trigonometrik funksiyalarni integrallang.

830. $\int \frac{dx}{3+5\cos x}.$

861. $\int \frac{\sin^3 x}{4+\cos x}dx.$

832. $\int \frac{\cos^3 x}{\sin^6 x}dx.$

833. $\int \sin^3 x\cos^2 xdx.$

834. $\int \frac{\sin^3 x}{\cos^4 x}dx.$

835. $\int \frac{dx}{\cos x\sin^3 x}.$

836. $\int \frac{dx}{\sin^3 x\cos^4 x}.$

837. $\int \frac{\sin^4 x}{\cos^2 x}dx.$

838. $\int \frac{dx}{\sin^2 x\cos^4 x}.$

839. $\int \frac{dx}{(\sin x+\cos x)^2}.$

840. $\int \frac{dx}{4-5\sin x}.$

841. $\int \frac{dx}{4\sin x+3\cos x+5}.$

842. $\int \frac{1+\sin x}{\sin x(1+\cos x)}dx.$

843. $\int \sin^5 x\,dx.$

844. $\int \cos^7 xdx.$

845. $\int \sin 3x\sin 7xdx$.

846. $\int \sin 2x\sin 9xdx.$

847. $\int \sin 4x\sin 5x\sin 7xdx.$

848. $\int \cos 3x\cos 5xdx.$

849. $\int \sin 2x\sin 3x\cos 5x\,dx.$

VII. BOB. ANIQ INTEGRALLAR

1 -§. Aniq integral tushunchasi va uning sodda xossalari.

1. Agar [*a,b*] da *f(x)* funksiya berilgan bo‘lsa, [*a,b*] ni $a = x_0 < x_1 < ... < x_n = b$ (1) nuqtalar yordamida *n* ta ixtiyoriy bo‘lakka bo‘lib, $\Delta x_k = x_k - x_{k-1}$, $\lambda = \max \Delta x_k$ belgilashlar kiritsak, u holda [*a,b*] ning *T* bo‘linishi berilgan deyiladi.

2. [*a,b*] ning *T* bo‘linishi berilgan bo‘lsin.

Har bir $[x_{k-1}; x_k]$ da bittadan ξ_k nuqta olib, yig‘indi tuzamiz:

$$\sum_{k=1}^{n} f(\xi_k)\Delta x_k = f(\xi_1)\Delta x_1 + f(\xi_2)\Delta x_2 + ... + f(\xi_n)\Delta x_n \qquad (2)$$

(2) *f(x)*funksiyaning [*a,b*] dagi integral yig‘indisi deyiladi.

3. Agar $\lambda(T) \to 0$ da (2) integral yig‘indining chekli *I* limiti mavjud bo‘lsa va bu limit *T* bo‘linishning hosil qilinishi hamda ξ_k larning tanlanishiga bog‘liq bo‘lmasa, u holda *I* berilgan *f(x)* funksiyaning [*a,b*] bo‘yicha olingan aniq integrali deyiladi va

$$I = \int_a^b f(x)dx$$

orqali belgilanadi; bu holda *f(x)* funksiyani [*a,b*] da integrallanuvchi deb aytiladi.

4. [*a,b*] ning *T* bo‘linishida hosil qilinadigan

$$\overline{s} = \sum_{k=1}^{n} M_k \Delta x_k \qquad (4)$$

$$\underline{s} = \sum_{k=1}^{n} m_k \Delta x_k \qquad (5)$$

Darbuning yuqori va quyi yig‘indilari deb aytiladi, bunda

$$M_k = \sup_{x\in[x_{k-1},x_k]} \{f(x)\}, \quad m_k = \inf_{x\in[x_{k-1},x_k]} \{f(x)\}.$$

5. *f(x)*funksiyaning [*a,b*] da integrallanuvchi bo‘lishining zarur va etarli sharti

$\lambda(T) \to 0$ da $\lim(\overline{s} - \underline{s}) = 0$ dan iborat.

6. [*a,b*] da uzluksiz bo‘lgan funksiya integrallanuvchi bo‘ladi.

Aniq integral ta’rifidan foydalanib, quyidagilarni hisoblang. (ya’ni $\int_a^b f(x)dx = \lim\limits_{\lambda\to 0}\sum\limits_{k\to 1}^{n} f(\xi_k)\Delta x_k$ ni) toping.

850. $\int_0^1 xdx$ ([0; 1] kesmani n ta teng bo‘laklarga bo‘ling va $\xi_k = \frac{1}{k}$ deb oling).

851. $\int_0^1 x^2dx$ ([0;1] kesmani n ta teng bo‘laklarga bo‘ling va $\xi_k = \frac{1}{k}$ deb oling).

852. $\int_a^b e^x dx$ ([0; 1] kesmani n ta teng bo‘laklarga bo‘ling va ξ_k deb $[x_{k-1};\, x_k]$ kesmaning chap chetki nuqtasini oling).

853. $\int_a^b \frac{dx}{x^2}$ (kesmani n ta teng bo‘laklarga bo‘ling va ξ_k ni kesma uchlarining o‘rta gemetriyaga qilib oling, ya’ni $\xi_k = \sqrt{x_{k-1}\ x_k}$).

854. $\int_1^2 \frac{dx}{x}$ ([1;2] kesmani geometrii progressiya tashkil qiladigan yordamida *n* ta bo‘lakka bo‘lib, $\xi_k = q^k$ deb oling).

855. Aniq integrallarni xossalaridan foydalanib, ularni hisoblamasdan quyidagi integrallarining qaysinisi katta ekanligini ko‘rsating.

1) $\int_0^1 xdx$ yoki $\int_0^1 x^2dx;$ 2) $\int_1^2 xdx$ yoki $\int_1^2 x^2dx;$

3) $\int_0^{\frac{\pi}{1}} xdx$ yoki $\int_0^{\frac{\pi}{2}} \sin xdx;$ 4) $\int_0^1 e^x dx$ yoki $\int_0^1 e^{x^2} xdx;$

5) $\int_0^1 2^{x^2} dx$ yoki $\int_0^1 2^{x^3} dx;$ 6) $\int_1^2 \ln \text{xdx}$ yoki $\int_1^2 (\ln x)^2 dx;$

7) $\int_3^4 \ln xdx$ yoki $\int_3^4 (\ln x)^2 dx.$

856. Aniq integral xossalaridan foydalanib, quyidagi tengsizliklarni isbotlang.

1) $\frac{1}{6} < \int_0^2 \frac{dx}{10+x} < \frac{1}{5};$ 2) $1 < \int_0^1 e^{x^2} dx < e;$ 3) $\frac{2}{5} < \int_1^2 \frac{xdx}{x^2+1} < \frac{1}{2};$

4) $0 < \int_8^{18} \frac{x+1}{x+2} dx < 9{,}5;$ 5) $\frac{2}{\sqrt[4]{e}} < \int_0^2 e^{x^2-x} dx < 2e^2.$

857. Quyidagi funksiyalardan *x* bo‘yicha hosila oling.

1) $y = \int_0^x \sin t\, dt,$ 2) $y = \int_x^5 \sqrt{1+t^2}\, dt,$ 3) $y = \int_0^{2x} \frac{\sin t}{t} dt,$

4) $y = \int_2^{e^x} \frac{\sin t}{t} dt,$ 5) $y = \int_{x^2}^1 e^{nt}\, dt,$ 6) $y = \int_x^{x^2} \ln^2 t dt.$

858. Parametrik ko‘rinishda berilgan funksiyalardan $\frac{dy}{dx}$ hosilani toping.

$1)\, x=\int_0^t \sin z dz,\quad y=\int_0^t \cos z dz;\qquad 2)\, x=\int_1^{t^2} z\ln z dz,\qquad y=\int_{t^2}^1 z^2\ln z dz$.

859. x ning qaysi qiymatida $f(x)=\int_0^x te^{-t^2}dt$ funksiya ekstremumga ega? U nimaga teng?

2-§. Aniq integralning asosiy xossalari va uni hisoblash

1. Aniq integralning asosiy xossalari:

a) $\int_a^b f(x)dx=-\int_b^a f(x)dx$;

b) $\int_a^a f(x)dx=0$;

c) $\int_a^b f(x)dx=\int_a^c f(x)dx+\int_c^b f(x)dx$;

d) $\int_a^b [f(x)\pm g(x)]dx=\int_a^b f(x)dx\pm\int_a^b g(x)dx$;

e) $\int_a^b kf(x)dx=k\int_a^b f(x)dx$, k- o‘zgarmas.

2. Agar $[a,b]$ da $m\le f(x)\le M$ bo‘lsa, $m(b-a)\le\int_a^b f(x)dx\le M(b-a)$ bo‘ladi va $f(c)=\frac{\int_a^b f(x)dx}{b-a}$ funksiyaning $[a,b]$ dagi o‘rta qiymati deyiladi.

3. Nyuton-Leybnits formulasi:

$$\int_a^b f(x)dx=F(x)\Big|_a^b=F(b)-F(a),$$

bunda $F(x)$ berilgan *f(x)* ning boshlang‘ich funksiyalaridan biri ya’ni $F'(x)=f(x)$.

4. Bo‘laklab integrallash: $\int_a^b udv=uv|_a^b-\int_a^b vdu$, bunda $u=u(x)$ va $v=v(x)$ funksiyalar $[a,b]$ da uzluksiz differensiallanuvchi.

5. O‘rniga qo‘yish usuli:

$$\int_a^b f(x)dx=\int_\alpha^\beta f[\varphi(t)]\varphi'(t)dt$$

bunda $f(x)$ funksiya $[a,b]$ da uzluksiz, $x=\varphi(t)$ funksiya $[\alpha,\beta]$ da hosilasi bilan birga uzluksiz, $a=\varphi(\alpha)$, $b=\varphi(\beta)$.

Nyuton-Leybnits formulasi yordamida integrallarni hisoblang.

860. $\int_2^3 x^2dx$. 861. $\int_1^{16}\sqrt{x}dx$. 862. $\int_{1/3}^1\frac{dx}{x^2}$.

863. $\int_1^8(4x-\frac{1}{3\sqrt[3]{x^2}})dx$. 864. $\int_0^{\frac{\pi}{6}}\sin 3xdx$. 865. $\int_0^1\frac{dx}{1+x^2}$.

866. $\int_{-\pi}^{\pi}\sin^2\frac{x}{2}dx$. 867. $\int_0^{\frac{\pi}{4}}\frac{x^2}{x^2+1}dx$. 868. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\sin 2x\sin 7xdx$.

869. $\int_0^{6\sqrt{3}}\frac{dx}{x^2+36}$. 870. $\int_4^{4\sqrt{3}}\frac{dx}{\sqrt{64-x^2}}$. 871. $\int_3^4\frac{dx}{25-x^2}$.

872. $\int_0^2\frac{xdx}{x^4-9}$. 873. $\int_{\frac{\pi}{12}}^{\frac{\pi}{4}}ctg2xdx$. 874. $\int_e^{e^2}\frac{\sqrt{\ln x}}{x}dx$.

O‘zgaruvchilarni almashtirish usulini qo‘llab quyidagi integrallarni hisoblang.

875. $\int_1^4\frac{dx}{(1+2x)^2}$. 876. $\int_0^5\frac{xdx}{\sqrt{1+3x}}$. 877. $\int_{\frac{2\sqrt{3}}{3}}^2\frac{dx}{\sqrt{16-3x^2}}$.

878. $\int_{-\frac{1}{2}}^{\frac{1}{2}}\frac{3^x}{1+9^x}dx$. 879. $\int_1^3\frac{\sqrt{x}}{x+1}dx$. 880. $\int_4^{25}\frac{dx}{\sqrt{x}-1}$.

881. $\int_0^{\sqrt{3}/2}\sqrt{1-x^2}dx$. 882. $\int_0^2\sqrt{(4-x^2)^3}dx$. 883. $\int_0^{\frac{\pi}{2}}\cos x\, e^{\sin x}dx$.

Bo‘laklab integrallash usulini qo‘llab, quyidagi integrallarni hisoblang.

884. $\int_0^1 xe^{-x}dx$. 885. $\int_0^{\frac{\pi}{2}}x\cos xdx$. 886. $\int_0^{\frac{\pi}{2}}(x-1)\cos xdx$.

887. $\int_0^1 arctgxdx$. 888. $\int_{-1}^0\arccos xdx$. 889. $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\frac{xdx}{\sin^2 x}$.

890. $\int\limits_{-1}^{0}(2x+3)e^{-x}dx$.

891. $\int\limits_{-2}^{2}(1-x)\sin \pi x dx$.

892. $\int\limits_{0}^{e-1}\ln(1+x)dx$.

893. $\int\limits_{1}^{2}x\log_2 x dx$.

894. $f(x)=\sqrt{x}+\dfrac{1}{\sqrt{x}}$ funksiyaning [1;4] segmentdagi o‘rta qiymatini hisoblang.

895. $f(x)=\dfrac{1}{x^2+x}$ funksiyaning [1;1,5] segmentdagi o‘rta qiymatini hisoblang.

896. f(x)=$\sin^2$x funksiyaning [1;π] segmentdagi o‘rta qiymatini hisoblang.

897. $f(x)=\dfrac{2}{e^x+1}$ funksiyaning [1;2] segmentdagi o‘rta qiymatini hisoblang.

3-§. Xosmas integrallar

A. Chegaralanmagan kesmada aniqlangan funksiyaning xosmas integrali

1. *f(x)* funksiya *x* ning barcha $x \geq a$ qiymatlarida aniqlangan va [*a;n*] da integrallanuvchi bo‘lsa, u holda $\lim\limits_{n\to+\infty}\int\limits_{a}^{n}f(x)dx$ limit *f(x)* funksiyaning *a* dan $+\infty$ gacha olingan xosmas integrali deyiladi va $\int\limits_{a}^{+\infty}f(x)dx$ orqali belgilanadi. Shunga o‘xshash

$$\lim_{n\to-\infty}\int\limits_{n}^{b}f(x)dx=\int\limits_{-\infty}^{b}f(x)dx,$$

$$\lim_{\alpha\to-\infty}\int\limits_{\alpha}^{c}f(x)dx+\lim_{\beta\to+\infty}\int\limits_{c}^{\beta}f(x)dx=\int\limits_{-\infty}^{+\infty}f(x)dx.$$

Agar ko‘rsatilgan limitlar mavjud va chekli bo‘lsa, xosmas integrallar yaqinlashuvchi, aks holda uzoqlashuvchi deyiladi.

2. Solishtirish alomati.

f(x) va *g(x)* funksiyalar *x≥a* da aniqlangan va [*a;n*] da integrallanuvchi. Agar $\forall x \geq a$ uchun $0\leq f(x)\leq g(x)$ bo‘lsa, $\int\limits_{a}^{+\infty}g(x)dx$ ning yaqinlashishidan $\int\limits_{a}^{+\infty}f(x)dx$ ning ham yaqinlashishi kelib chiqadi va $\int\limits_{a}^{+\infty}f(x)dx\leq\int\limits_{a}^{+\infty}g(x)dx$, $\int\limits_{a}^{+\infty}f(x)dx$ ning uzoqlashishidan $\int\limits_{a}^{+\infty}g(x)dx$ ning ham uzoqlashuvchi bo‘lishi kelib chiqadi.

3. Absolyut yaqinlashish alomati

f(x) funksiya $\forall x \geq a$ uchun aniqlangan bolsin. Agar $\int\limits_a^{+\infty} |f(x)|\,dx$ yaqinlashuvchi bo‘lsa, $\int\limits_a^{+\infty} f(x)dx$ ham yaqinlashuvchi bo‘ladi va u absolyut yaqinlashuvchi deyiladi. Agar $\int\limits_a^{+\infty} f(x)dx$ yaqinlashuvchi, $\int\limits_a^{+\infty} |f(x)|\,dx$ uzoqlashuvchi bo‘lsa, u holda $\int\limits_a^{+\infty} f(x)dx$ shartli yaqinlashuvchi deyiladi.

B. Chegaralanmagan funksiyalarning xosmas integrallari

1. *f(x)* funksiya [*a;b*] da aniqlangan va [*a;b-ε*) (ε>0) da integrallanuvchi, lekin (*b-ε;b*) da chegaralanmagan bo‘lsa, u holda

$$\int\limits_a^b f(x)dx = \lim_{\varepsilon \to 0+} \int\limits_a^{b-\varepsilon} f(x)dx$$

deb olinadi. Agar bu limit mavjud va chekli bo‘lsa, xosmas integral yaqinlashuvchi, aks holda uzoqlashuvchi deyiladi. Shunga o‘xshash

$$\int\limits_a^b f(x)dx = \lim_{\varepsilon \to 0+} \int\limits_{a+\varepsilon}^{b} f(x)dx$$

va

$$\int\limits_a^b f(x)dx = \lim_{\varepsilon \to 0+} \int\limits_a^{c-\varepsilon} f(x)dx + \lim_{\delta \to 0+} \int\limits_{c+\delta}^{b} f(x)dx$$

ta’riflanadi, oxirgida *c* nuqta atrofida *f(x)* funksiya chegaralanmagan.

2. Agar [*a,b*] da f(x)>0, α<1 va $\lim\limits_{x \to b-0} f(x)(b-x)^{\alpha} < \infty$ bo‘lsa, $\int\limits_a^b f(x)dx$ yaqinlashuvchi, $f(x) > 0$, $\alpha \geq 1$ va $\lim\limits_{x \to b-0} f(x)(b-x)^{\alpha} > 0$ bo‘lsa, $\int\limits_a^b f(x)dx$ uzoqlashuvchi bo‘ladi.

3. Agar $f(x)$ va $\varphi(x)$ funksiyalarning [*a;c*] ning *c* nuqtasida uzilishga ega va $\varphi(x) \geq f(x) \geq 0$ bo‘lsa, u holda $\int\limits_a^c \varphi(x)dx$ ning yaqinlashuvchi bo‘lishidan $\int\limits_a^c f(x)dx$ ning ham yaqinlashuvchi ekani kelib chiqadi.

a) Chegaralari cheksiz bo‘lgan integrallar

Quyidagi integrallarning qaysilari yaqinlashuvchi, qaysilari uzoqlashuvchi.

898. $\int\limits_1^{+\infty} \frac{dx}{x}$. 899. $\int\limits_0^{+\infty} \frac{dx}{x^2+4}$. 900. $\int\limits_0^{+\infty} xe^{-x^2} dx$. 901. $\int\limits_1^{+\infty} \frac{dx}{\sqrt{x}}$.

902. $\int\limits_{0}^{+\infty} \frac{xdx}{1+x^2}$. 903. $\int\limits_{-\infty}^{+\infty} \frac{dx}{1+x^2}$. 904. $\int\limits_{1}^{+\infty} \frac{dx}{x+x^3}$ 905. $\int\limits_{1}^{+\infty} \frac{dx}{x^{\frac{3}{2}}}$.

906. $\int\limits_{0}^{+\infty} \cos xdx$. 907. $\int\limits_{0}^{+\infty} x\sin xdx$.

b) Chegaralanmagan funksiyaning integrali.
Quyidagi integrallarning qaysilari yaqinlashuvga, qaysilari uzoqlashuvchi?

908. $\int\limits_{0}^{1} \frac{dx}{x}$. 909. $\int\limits_{0}^{1} \frac{dx}{\sqrt{x}}$. 910. $\int\limits_{-1}^{0} \frac{dx}{\sqrt[3]{x}}$. 911. $\int\limits_{0}^{1} \frac{dx}{\sqrt{1-x^2}}$.

912. $\int\limits_{0}^{\frac{\pi}{2}} \frac{dx}{\sin x}$. 913. $\int\limits_{0}^{1} \frac{dx}{\sqrt{1-x}}$. 914. $\int\limits_{-1}^{0} \frac{e^{\frac{1}{x}}dx}{x^2}$. 915. $\int\limits_{-1}^{1} \frac{dx}{x^2}$.

916. $\int\limits_{1}^{e} \frac{dx}{x\sqrt{\ln x}}$. 917. $\int\limits_{1}^{2} \frac{dx}{x\ln x}$. 918. $\int\limits_{0}^{1} x\ln xdx$. 919. $\int\limits_{-1}^{1} \frac{x+1}{\sqrt[5]{x^3}}dx$.

920. Taqqoslash teoremalaridan foydalanib, quyidagi integrallarni yaqinlashishlikka tekshiring.

1) $\int\limits_{0}^{1} \frac{\sqrt{x}dx}{\sqrt{1-x^4}}$. 2) $\int\limits_{0}^{2} \frac{dx}{\ln x}$.

3) $\int\limits_{1}^{2} \frac{dx}{\sqrt{(x-1)(2-x)}}$. 4) $\int\limits_{0}^{1} \frac{x^2dx}{\sqrt[3]{(1-x^5)^2}}$.

VIII BOB. ANIQ INTEGRALNING TATBIQLARI

1-§. Tekis (yassi) figuralar yuzlarini hisoblash

1. Dekart koordinatalar sistemasida yuzlarni hisoblash.

a) $y=f(x)$ $(f(x)\geq 0)$ egri chiziq, $x=a$ va $x=b$ to‘g‘ri chiziqlar hamda Ox o‘qining $[a;b]$ kesmasi yuilan chegaralangan egri chiziqli trapetsiyaning yuzi.

$$S=\int_a^b f(x)dx$$

formula orqali hisoblanadi.

b) $y=f(x)$ va $y=g(x)$ $(f(x)\leq g(x), x\in[a,b])$ egri chiziqlar hamda $x=a$ va $x=b$ to‘g‘ri chiziqlar bilan chegaralangan figuraning yuzi

$$S=\int_a^b (g(x)-f(x))dx$$

formula orqali hisoblanadi.

2. Parametrik ko‘rinishda berilgan egri chiziqlar bilan chegaralangan figura yuzini hisoblash.

Agar figura $x=x(t)$, $y=y(t)$ $(0\leq t\leq T)$ parametrik ko‘rinishda berilgan berilgan yopiq egri chiziq bilan chegaralangan bo‘lsa, uning yuzi

$$S=-\int_0^T y(t)x'(t)dx \quad \text{yoki} \quad S=\int_0^T x(t)y'(t)dt$$

formulalarning biri bilan hisoblanadi.

Bu ikki formulani birlashtirib

$$S=\frac{1}{2}\int_0^T (x(t)y'(t)-y(t)x'(t))dt$$

ni hosil qilamiz.

3. Qutb koordinatalar sistemasida berilgan uzluksiz egri chiziq $\rho=\rho(\theta)$ va $\theta=\alpha$, $\theta=\beta(\alpha<\beta)$ nurlar bilan chegaralangan figuraning yuzi

$$S=\frac{1}{2}\int_\alpha^\beta \rho^2 d\theta$$

formula orqali hisoblanadi.

921. $y=x^2$ parabola, $y=0$, $x=-1$ va $x=4$ to‘g‘ri chiziqlar bilan chegaralangan figuraning yuzini toping.

922. $y=\frac{1}{x}$ giperbola, $x=1$, $x=3$ va $y=0$ to‘g‘ri chiziqlar bilan chegaralangan figura yuzini toping.

923. $y=6x-x^2$ parabola, $x=-1$, $x=3$ to‘g‘ri chiziqlar va absissa o‘qi bilan chegaralangan figuraning yuzini toping.

924. $y=x^2-4x+3$ parabola, koordinata o‘qlari va $x=4$ to‘g‘ri chiziq bilan chegaralangan figuraning yuzini toping.

925. $y=2x$, $y=5x$, $x=2$ va $x=6$ to‘g‘ri chiziqlar bilan chegaralangan figuraning yuzini toping.

926. $y=\frac{3}{x}$ giperbola va $x+y-4=0$ to‘g‘ri chiziqlar bilan chegaralangan figura yuzini toping.
927. $y=3x-x^2$ parabola va $5x-y-8=0$ to‘g‘ri chiziqlar bilan chegaralangan figura yuzini toping.
928. $y^2=16x$ parabola va $y=x$ to‘g‘ri chiziqlar bilan chegaralangan figura yuzini toping.
929. $y=8x-x^2$ va $y=x^2+18x-12$ parabolalar bilan chegaralangan figura yuzini toping.
930. $x^2+y^2=16$ aylana va $x+y-4=0$ to‘g‘ri chiziq bilan chegaralangan figuraning yuzlarini toping.
931. $x^2+y^2=8$ aylana va $y^2=2x$ parabola bilan chegaralangan figuraning yuzlarini toping.
932. $y^2=2x+1$ parabola va $x-y-1=0$ to‘g‘ri chiziq bilan chegaralangan figuraning yuzini toping.
933. $x=y^2(y-1)$ chiziq va ordinata o‘qi bilan chegaralangan figuraning yuzini toping.
934. $x=a\cos t$, $y=b\sin t$ ellips bilan chegaralangan figuraning yuzini toping.
935. $x=a(t-\sin t)$, $y=a(1-\cos t)$ sikloidaning bir arkasi va absissa o‘qi bilan chegaralangan figuraning yuzini toping.
936. $x=a\cos^3 t$, $y=a\sin^3 t$ astroida bilan chegaralangan figuraning yuzini toping.
937. $x=2a\cos t-a\sin 2t$, $y=2a\sin t-a\sin 2t$ kardioida bilan chegaralangan figuraning yuzini toping.
938. $\rho = a\sin 2\varphi$ chiziq bilan chegaralangan figura yuzini toping.
939. $\rho^2 = a(1+\cos\varphi)$ kardioida bilan chegaralangan figuraning yuzini toping.
940. $\rho^2 = a^2\cos 2\varphi$ Bernulli lemniskatasi bilan chegaralangan figuraning yuzini toping.
941. $\rho = 2a(2+\cos\varphi)$ Paskal chig‘anog‘i bilan chegaralangan figuraning yuzini toping.

2-§. Yoy uzunligini hisoblash

1. Dekart to‘g‘ri burchakli koordinatalarida yoy uzunligi. Agar $y=f(x)$ $[a,b]$ da silliq egri chiziq (ya’ni $f'(x)$ uzluksiz bo‘lgan) bo‘lsa, u holda uning yoyi uzunligi

$$L=\int_a^b \sqrt{1+y'^2}\,dx$$

formula orqali hisoblanadi. Bunda a va b yoy uchlarining abssissalari ($a<b$).

2. Parametrik ko‘rinishda berilgan egri chiziq yoyining uzunligi.

Agar egri chiziq $x=x(t)$, $y=y(t)$, $t_1\le t\le t_2$ ko‘rinishda berilgan bo‘lib, $x'(t)$, $y'(t)$ uzluksiz funksiyalar bo‘lsa, u holda egri chiziq yoyining uzunligi

$$L=\int_{t_1}^{t_2} \sqrt{x'^2(t)+y'^2(t)}\,dt$$

formula orqali hisoblanadi. Bunda t_1 va t_2 lar t parametrining yoy uchlariga mos qiymatlari ($t_1<t_2$).

3. Qutb koordinatalar sistemasida berilgan silliq egri chiziq $\rho=\rho(\theta)$, $\alpha\le\theta\le\beta$ yoyining uzunligi

$$L=\int_{\alpha}^{\beta}\sqrt{\rho^2+\rho'^2}d\theta$$

orqali hisoblanadi. Bunda α va β- qutb burchagi θ ning yoy uchlaridagi qiymatlari ($\alpha<\beta$).

942. $y^2=x^3$ egri chiziqning $A(0:0)$ va $B(5,\ 5\sqrt{5})$ nuqtalari orasidagi yoyi uzunligini toping.

943. $y=lncosx$ egri chiziqning $x=\frac{\pi}{3}$ dan $x=\frac{\pi}{2}$ gacha bo‘lgan yoyi uzunligini toping.

944. $y=ad\frac{x}{a}$ zanjir chiziqning x=0 dan $x=b$ gacha bo‘lgan yoyi uzunligini toping.

945. $y^2=2px$ parabolaning x=0 dan $M(x,y)$ nuqtagacha bo‘lgan yoyi uzunligini toping.

946. $x^2+y^2=R^2$ aylana uzunligini toping.

947. $x^{\frac{2}{3}}+y^{\frac{2}{2}}=a^{\frac{2}{3}}$ astroidaning uzunligini toping.

948 y=lnx egri chiziqning ($\sqrt{3};\ln\sqrt{3}$) nuqtasidan ($\sqrt{8};\ln\sqrt{8}$)nuqtasigacha bolgan yoyi uzunligini toping.

949. $x=a\cos^3 t$, $y=a\sin^3 t$ astroidaning uzunligini toping.

950. $x=a(t-\sin t)$, $y=a(1-\cos t)$ sikloidaning bir arkasi yoyi uzunligini toping.

951. $\rho\varphi=1$ giperbolik spiralning $\varphi=\frac{3}{4}$ dan $\varphi=\frac{4}{3}$ gacha bo‘lgan yoyi uzunligini toping.

952. $\rho=a(1+\cos\varphi)$ kardioida uzunligini toping.

953. $\rho=5\sin\varphi$ aylana uzunligini toping.

3-§. Hajmlarni hisoblash

1. Ma’lum ko‘ndalang kesimlari bo‘yicha jism hajmini hisoblash.

Agar jismning Ox o‘qiga perpendikulyar tekisliklar bilan kesishmasida hosil bo‘lgan kesim yuzi $S(x)$ berilgan bo‘lsa, u holda bu jism hajmi

$$V=\int_a^b S(x)dx$$

orqali hisoblanadi, bunda a va b lar x ning o‘zgarish chegaralari bo‘lib, $S(x)$ funksiya $[a,b]$ da aniqlangan va uzluksiz deb qaraladi.

2. To‘g‘ri burchakli koordinatalar sistemasida aylanma jism hajmi

a) Ox o‘qi atrofida $a\le x\le b$, $0\le y\le f(x)$ tekis figurani aylantirish natijasida hosil bo‘ladigan jism hajmi

$$V=\pi\int_a^b f^2(x)dx$$

orqali hisoblanadi.

b) Oy o‘qi atrofida $c \le y \le d$, $0 \le x \le \varphi(x)$ tekis figurani aylantirish natijasida hosil bo‘ladigan jism hajmi

$$V = \pi \int_a^b \varphi^2(y)dy$$

orqali hisoblanadi.

c) Oy o‘qi atrofida $a \le x \le b$, $0 \le y \le y(x)$ ni aylantirish natijasida hosil bo‘ladigan jism hajmi

$$V = 2\pi \int_a^b xy(x)dx$$

orqali hisoblanadi.

3. Parametrik usulda berilgan $x = x(t)$, $y = y(t)$, $\alpha \le t \le \beta$ egri chiziqni Ox o‘qi atrofida aylantirish natijasida hosil bo‘ladigan jism hajmi

$$V = \pi \int_\alpha^\beta (y(t))^2 x'(t)dt$$

orqali hisoblanadi.

4. Qutb koordinatalar sistemasida berilgan $\rho = \rho(\theta)$ egri chiziq $\rho = \alpha$, $\rho = \beta$ radius-vektorlar bilan chegaralangan tekis figurani qutb o‘qi atrofida aylantirish natijasida hosil bo‘ladigan jism hajmi

$$V = \pi \int_\alpha^\beta \rho^3 \sin^3 \theta d\theta$$

orqali hisoblanadi.

$0 \le \alpha \le \theta \le \beta \le \pi$ bo‘lganda $V = \frac{2\pi}{3} \int_\alpha^\beta \rho^3(\theta) \sin\theta d\theta$ bo‘ladi.

954. $y=x/2$, $x=4$, $x=6$ to‘g‘ri chiziqlar va absissa o‘qi bilan chegaralangan transsiyani 0_x o‘qi atrofida aylantirishdan hosil bo‘ladigan jismning hajmini toping.

955. $y = \frac{x^2}{4}$ parabola $y=1$, $y=5$ to‘g‘ri chiziqlar bilan chegeralangan figurani ordinata o‘qi atrofida aylantirishdan hosil bo‘ladigan jismning hajmini toping.

956. $y^2=4x$ parabola va $x=4$ to‘g‘ri chiziqlar bilan chegeralangan figurani ordinata o‘qi atrofida aylantirishdan hosil bo‘ladigan jismning hajmini toping.

957. $y=x^2-4$ parabola va abssissa o‘qi bilan chegaralangan transsiyani Ox o‘qi atrofida aylantirishdan hosil bo‘ladigan jismning hajmini toping.

958. $x^2-y^2=4$ giperbola va $y=-2$, $y=2$ to‘g‘ri chiziqlar bilan chegaralangan transsiyani 0_x o‘qi atrofida aylantirishdan hosil bo‘ladigan jismning hajmini toping.

959. $y=2x-x^2$ parabola va abssissa o‘qi bilan chegaralangan figurani ordinata o‘qi atrofida aylantirishdan hosil bo‘ladigan jismning hajmini toping.

960. $x^2+y^2=18$ aylana va $y = \frac{x^2}{3}$ parabolalar bilan chegaralangan figuralarni absissa o‘qi atrofida aylantirishdan hosil bo‘lgan jismlarning hajmlariini toping.

961. $x^2+(y-b)^2=r^2$ $(b>r)$ aylana bilan chegaralangan doiraning absissa o‘qi atrofida aylantirishdan hosil bo‘lgan jismni hajmini toping.
962. $y=3-x^2$ va $y=x^2+1$ parabolalar bilan chegaralangan figurani absissa o‘qi atrofida aylantirishdan hosil bo‘lgan jismning hajmini toping.
963. $x^{\frac{2}{3}}+y^{\frac{2}{3}}=a^{\frac{2}{3}}$ astroidani absissa o‘qi atrofida aylantirishdan hosil bo‘lgan jismning hajmini toping.
964. $x=a\cos t$, $y=b\sin t$ ellips bilan chegaralangan figurani absissa o‘qi atrofida aylantirishdan hosil bo‘lgan jismni hajmini toping.
965. $x=a(t-\sin t)$, $y=a(1-\cos t)$ sikloidaning bir arkasi va absissa o‘qi bilan chegaralangan figurani Ox o‘qi atrofida aylantirishdan hosil bo‘lgan jismning hajmini toping.
966. $x=a\cos^3 t$, $y=a\sin^3 t$ astroidani bilan chegaralangan figurani absissa o‘qi atrofida aylantirishdan hosil bo‘lgan jismning hajmini toping.
967. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ellips bilan chegaralangan figurani absissa va ordinata o‘qilari atroflarida aylantirishdan hosil bo‘ladigan jismlarning hajmlarini toping.

4-§. Aylanma jism sirtining yuzi

1. To‘g‘ri burchakli koordinatalar sistemasida $y=f(x)$ silliq egri chiziq yoyini $(a \le x \le b)$ Ox o‘qi atrofida aylantirish natijasida hosil bo‘ladigan jism sirtining yuzi

$$S=2\pi\int_a^b y\sqrt{1+y'^2}\,dx$$

orqali hisoblanadi.

2. Parametrik ko‘rinishda berilgan egri chiziq uchun $\begin{cases} x=x(t), \\ y=y(t) \end{cases}$ $(t_1 \le t \le t_2)$ sirt yuzi $S=2\pi\int_{t_1}^{t_2} y(t)\sqrt{x'^2+y'^2}\,dt$ orqali hisoblanadi.

3. Qutb koordinatalar sistemasida silliq egri chiziq $\rho=\rho(\theta)$ $(\alpha \le \theta \le \beta)$ berilgan bo‘lsa, uni qutb o‘qi atrofida aylantirishdan hosil bo‘lgan jism sirtining yuzi

$$S=2\pi\int_\alpha^\beta \rho\sin\theta\sqrt{\rho^2+\rho'^2}\,d\theta$$

formula orqali hisoblanadi.
968. $y=x^3$ kubik parabolaning $x=0$ dan $x=\frac{1}{2}$ gacha bo‘lgan yoyini absissa o‘qi atrofida aylantirishdan hosil bo‘ladigan sirtning yuzini toping.
969. $y=3x$ to‘g‘ri chiziqning $x=1$ dan $x=3$ gacha bo‘lgan qismini absissa o‘qi atrofida aylantirishidan hosil bo‘ladigan sirt yuzini toping.

970. $y^2=2x$ parabolaning $x=0$ dan $x=2$ gacha bo‘lgan yoyini absissa o‘qi atrofida aylantirishdan hosil bo‘ladigan sirt yuzini toping.

971. $y=\cos x$ kosinusoidaning $x=-\frac{\pi}{2}$ dan $x=\frac{\pi}{2}$ gacha bo‘lgan yoyining absissa o‘qi atrofida aylantirishdan hosil bo‘ladigan sirt yuzini toping.

972. $x=r\cos t$, $y=r\sin t$ aylana o‘zi diametri atrofida aylatirishdan hosil bo‘ladigan sferaning sirti yuzini toping.

973. $x=a\cos^3 t$, $y=a\sin^3 t$ asroidani absissa o‘qi atrofida aylantirishdan hosil bo‘ladigan sirt yuzini toping.

974. $x=a(t-\sin t)$, $y=a(1-\cos t)$ sikloidaning bir arkasini absissa o‘qi atrofida aylantirishdan hosil bo‘ladigan sirt yuzini toping.

975. $x^2+(y-b)^2=r^2$ $(b>r)$ aylanani absissa o‘qi atrofida aylantirishdan hosil bo‘ladigan sirt yuzini toping.

976. $\rho = a(1+\cos\varphi)$ kardioidani qutb o‘qi atrofida aylantirishdan hosil bo‘ladigan sirt yuzini toping.

977. $\rho = 2\sin\varphi$ aylana qutb o‘qi atrofida aylantirishdan hosil bo‘ladigan sirt yuzini toping.

978. $\rho^2 = a^2\cos 2\varphi$ Bernulli lemniskatasini qutb o‘qi atrofida aylantirishdan hosil bo‘ladigan sirt yuzini toping.

5-§. Yassi egri chiziq va figuralarning og‘irlik markazi. Gyulden teoremalari

1. Agar massa $y=f(x)$ egri chiziq yoyi bo‘yicha tekis taqsimlangan (zichlik ρ=1) bo‘lsa, u holda $y = f(x) \ge 0$, $(a \le x \le b)$ egri chiziqning Ox va Oy o‘qlarga nisbatan statik momentlari mos holda

$$M_x = \int_a^b y\,dl, \quad M_y = \int_a^b x\,dl,$$

bo‘ladi, bunda $dl = \sqrt{dx^2 + dy^2}$.

Shu yoy og‘irlik markazining koordinatalari esa

$$\bar{x} = \frac{1}{L}\int_a^b x\,dl, \quad \bar{y} = \frac{1}{L}\int_a^b y\,dl$$

bo‘ladi, bunda $dl = \sqrt{dx^2 + dy^2}$ va L - yoy uzunligi.

2. $y=f(x)$ silliq egri chiziq, $x=a$, $x=b$ to‘g‘ri chiziqlar va Oy o‘qi bilan chegaralangan egri chiziqli trapetsiyaning Ox va Oy o‘qlariga nisbatan statik momentlari mos holda

$$M_x = \frac{1}{2}\int_a^b y^2 dx, \quad M_y = \int_a^b xy\,dx$$

bo‘ladi. Shu figura og‘irlik markazining koordinatalari esa

$$\bar{x} = \frac{1}{S}\int_a^b x\,dS = \frac{1}{S}\int_a^b xy\,dx,$$

$$\bar{y}=\frac{1}{2S}\int_a^b ydS=\frac{1}{2S}\int_a^b y^2dx$$

bo‘ladi, bunda S- jism sirtining yuzi.

3. *y=f(x)* $(a\le x\le b)$ silliq egri chiziq yoyini *Ox* o‘qi atrofida aylantirishdan hosil bo‘lgan sirt yuzi shu yoy uzunligi bilan uning og‘irlik markazi chizgan aylana uzunligining ko‘paytmasiga teng (Gyuldenning 1-teoremasi):

$$2\pi\bar{y}\cdot L=2\pi\int_a^b\sqrt{1+y'^2}\,dx.$$

4. *y=f(x)* egri chiziq, *x=a*, *x=b* to‘g‘ri chiziqlar va O*x* o‘qi bilan chegaralangan tekis figurani O*x* o‘qi atrofida aylantirishdan hosil bo‘lgan figuraning hajmi berilgan figura yuzi bilan uning og‘irlik markazi chizgan aylana uzunligining ko‘paytmasiga teng (Gyuldenning 2-teoremasi):

$$2\pi\bar{y}\cdot S=\pi\int_a^b f^2(x)dx.$$

979. *y*=cos*x* kosinusoidaning $\left(-\frac{\pi}{2}\le x\le\frac{\pi}{2}\right)$ Ox o‘qiga nisbatan statik momentini toping.

980. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ $(y\ge 0)$ yarim ellipsning O*x* o‘qiga nisbatan statik momentini toping.

981. $x^2+y^2=r^2$ $(y\ge 0)$ yarim aylananing og‘irlik markazi koordinatalarini toping.

982. $x^{\frac{2}{3}}+y^{\frac{2}{3}}=a^{\frac{2}{3}}$ astroida yoyining birinchi kvadrat bo‘lagining og‘irlik markazi koordinatalarini toping.

983. $y=ch\frac{x}{a}$ zanjir chiziqning absissalari $x_1=-a$ va $x_2=a$ bo‘lgan nuqtalari orasidagi yoyi og‘irlik markazi koordinatalarini toping.

984. *x*=*a*(*t*-sin*t*), *y*=*a*(1-cos*t*) sikloida (*t*=0 dan *t*=2π) yoyining og‘irlik markazi koordinatalarini toping.

985. $y=\cos x$ $\left(-\frac{\pi}{2}\le x\le\frac{\pi}{2}\right)$ kosinusoida va absissa o‘qi bilan chegarlangan figuraning og‘irlik markazi koordinatalarini toping.

986. $x^2+y^2=r^2$ $(y\ge 0)$ yarim doiraning og‘irlik markazi koordinatalarini toping.

987. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ellips bilan chegaralangan figuraning birinchi kvadratdagi bo‘lagining og‘irlik markazi koordinatalarini toping.

988. *x=a(t-sint)*, *y=a(1-cost)* (*t*=0 dan *t*=2π gacha) sikloida va absissa o‘qi bilan chegaralangan figuraning og‘irlik markazi koordinatalarini toping.

989. Tomoni *a* bo‘lgan muntazam oltiburchak o‘zining biror tomoni atrofida aylatirilgan. Gulden teoremasi yordamida: a) hosil bo‘lgan jism hajmini toping; b) jism sirti yuzini toping.

990. $x=a(t-\sin t)$, $y=a(1-\cos t)$ sikloidaning birinchi arkasi va absissa bilan chegarlangan figura ordinata o‘qi atrofida aylantirilgan. Hosil bo‘lgan jism hajmini va sirt yuzini toping.

IX BOB. SONLI QATORLAR

1-§. Asosiy tushunchalar

Agar $u_1, u_2, u_3, ..., u_n, ...$- sonlar ketma-ketligi bo‘lsa, u holda

$$u_1 + u_2 + u_3 + ... + u_n + ... \qquad (1)$$

sonli qator deyiladi. $u_1, u_2, u_3, ..., u_n, ...$ -qator hadlari, u_n- qatorning umumiy hadi deyiladi. Ko‘p hollarda (1) qisqacha $\sum_{n=1}^{\infty} u_n$ orqali belgilanadi.

1. Qatorning birinchi n ta hadi yig‘indisi uning xususiy yig‘indisi deyiladi: $S_n = u_1 + u_2 + u_3 + ... + u_n$.

Agar $\lim_{n\to\infty} S_n = S$ chekli bo‘lsa, (1) qator yaqinlashuvchi qator deyiladi va S (1) qatorning yig‘indisi deyiladi. Agar $\lim_{n\to\infty} S_n$ cheksiz bo‘lsa yoki mavjud bo‘lmasa, u holda (1) qator uzoqlashuvchi qator deyiladi.

2. $u_{n+1} + u_{n+2} + ... + u_{n+k} + ... \qquad (2)$

ko‘rinishdagi qator (1) ning qoldig‘i deyiladi va qisqacha $r_n = \sum_{k=n+1}^{\infty} u_k$ orqali belgilanadi.

Bunda:

a) Agar (1) qator yaqinlashuvchi bo‘lsa, u holda (2) qator ham yaqinlashuvchi bo‘ladi va aksincha;

b) Agar (1) qator yaqinlashuvchi bo‘lsa, u holda $au_1 + au_2 + ... + au_n + ...$ qator ham yaqinlashuvchi bo‘ladi va yig‘indisi aS ga teng bo‘ladi;

c) Agar $S = u_1 + u_2 + u_3 + ... + u_n + ...$ va $\sigma = v_1 + v_2 + v_3 + ... + v_n + ...$ qatorlar yaqinlashuvchi bo‘lsa, u holda $(u_1 + v_1) + (u_2 + v_2) + ... + (u_n + v_n) + ...$ qator yaqinlashuvchi bo‘ladi va yig‘indisi $S + \sigma$ ga teng bo‘ladi;

d) Agar (1) qator yaqinlashuvchi bo‘lsa, u holda $\lim_{n\to\infty} u_n = 0$ bo‘ladi (qator yaqinlashishining sharti).

Agar $\lim_{n\to\infty} u_n \neq 0$ bo‘lsa, u holda qator uzoqlashuvchi bo‘ladi.

Quyidagi qatorlarning n-hadini yozing.

991. $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + ...$

992. $\frac{1}{2} - \frac{2}{3} + \frac{3}{4} + ...$

993. $1 + \frac{2}{2} + \frac{3}{4} + \frac{4}{8} + ...$

994. $1 - \frac{1}{2^2} + \frac{1}{3^3} - \frac{1}{4^4} + ...$

995. $1 - 1 + 1 - 1 + ...$

996. $\frac{\sqrt{2}}{1\cdot 2} - \frac{\sqrt{3}}{1\cdot 2\cdot 3} + \frac{\sqrt{4}}{1\cdot 2\cdot 3\cdot 4} - ... .$

Quyidagi misollarda umumiy hadi bo‘yicha birinchi beshta hadini yozing.

997. $a_n = \dfrac{1+n}{1+n^2}$. 998. $a_n = \dfrac{1}{3n-1}$. 999. $a_n = \dfrac{\sqrt{n+1}}{n}$.

1000. $a_n = \dfrac{3n+2}{n^2+4}$. 1001. $a_n = \dfrac{(-1)^n(n-1)}{2^{n+3}}$. 1002. $a_n = \dfrac{2+(-1)^n}{n^2+4}$.

Quyidagi qatorlar uchun qismiy yig‘indisi S_n ni yozing va uning yig‘indisini toping.

1003. $1 + \dfrac{1}{3} + \dfrac{1}{9} + ... + \dfrac{1}{3^{n-1}} + ...$

1004. $1 - \dfrac{1}{2} + \dfrac{1}{4} - ... + \dfrac{(-1)^{n-1}}{2^{n-1}} + ...$

1005. $\dfrac{1}{1\cdot 2} + \dfrac{1}{2\cdot 3} + ... + \dfrac{1}{n(n+1)} + ...$

1006. $\dfrac{1}{1\cdot 3} + \dfrac{1}{3\cdot 5} + ... + \dfrac{1}{(2n-1)(2n+1)} + ...$

1007. $\dfrac{1}{1\cdot 4} + \dfrac{1}{2\cdot 5} + ... + \dfrac{1}{n(n+3)} + ...$

1008. $\dfrac{1}{3^2} + \dfrac{1}{15^2} + ... + \dfrac{1}{(4n^2-1)^2} + ...$

1009. $\dfrac{3}{1\cdot 4} + \dfrac{5}{4\cdot 9} + ... + \dfrac{2n+1}{n^2(n+1)^2} + ...$

1010. $\dfrac{1}{1\cdot 2\cdot 3} + \dfrac{1}{2\cdot 3\cdot 4} + ... + \dfrac{1}{n(n+1)(n+2)} + ...$

1011. $\dfrac{1}{1\cdot 4\cdot 7} + \dfrac{1}{2\cdot 5\cdot 8} + ... + \dfrac{1}{n(n+3)(n+6)} + ...$

1012. $arctg\dfrac{1}{2} + arctg\dfrac{1}{8} + \ldots + arctg\dfrac{1}{2n^2} + \ldots$ (ko‘rsatma avval $S_n = \dfrac{n}{n+1}$ tenglikni isbotlang)

Qator yaqinlashishining zaruriy shartidan foydalanib quyidagi qatorlarning uzoqlashuvchi ekanligini ko‘rsating.

1013. $\dfrac{1}{9} + \dfrac{2}{19} + \dfrac{3}{29} + ... + \dfrac{n}{10n-1} + ...$ 1014. $\dfrac{4}{7} + \dfrac{7}{12} + \dfrac{10}{17} + ... + \dfrac{3n+1}{5n+2} + ...$

1015. $\sqrt{2} + \sqrt{\dfrac{3}{2}} + ... + \sqrt{\dfrac{n+1}{n}} + ...$ 1016. $\sum\limits_{n=1}^{\infty} \cos\dfrac{1}{2n}$. 1017. $\sum\limits_{n=1}^{\infty} (1+\dfrac{1}{n})^n$.

2-§. Musbat hadli qatorlar

Ushbu

$$a_1 + a_2 + ... + a_n + ..., \qquad (1)$$

$$b_1 + b_2 + ... + b_n + ... \qquad (2)$$

musbat hadli qatorlar berilgan bo‘lsin, ya’ni $a_n \geq 0, b_n \geq 0 \quad \forall n \in N$.

1) Solishtirish alomati. Agar $\forall n \in N$ uchun $a_n \leq b_n$ o‘rinli bo‘lib, (2) qator yaqinlashuvchi bo‘lsa, (1) qator ham yaqinlashuvchi bo‘ladi va (1) qator uzoqlashuvchi bo‘lganda (2) qator ham uzoqlashuvchi bo‘ladi.

Bu alomat n ning biror $n_0 \in N$ qiymatidan boshlab barcha $n > n_0$ uchun $a_n \leq b_n$ bo‘lganda ham o‘rinlidir.

2) 2-solishtirish alomati. Agar chekli $\lim\limits_{n\to\infty}\frac{a_n}{b_n} = k \neq 0$ mavjud bo‘lsa, u holda (1) va (2) qatorlar bir vaqtda yo yaqinlashuvchi, yo uzoqlashuvchi bo‘ladi.

3) Koshi alomati. Agar (1) uchun $\lim\limits_{n\to\infty}\sqrt[n]{a_n} = l$ mavjud bo‘lib, $l < 1$ bo‘lsa, u holda (1) qator yaqinlashuvchi, $l > 1$ da esa uzoqlashuvchi bo‘ladi.

4) Dalamber alomati. Agar (1) uchun $\lim\limits_{n\to\infty}\frac{a_{n+1}}{a_n} = l$ mavjud bo‘lib, $l < 1$ bo‘lsa, qator yaqinlashuvchi, $l > 1$ da esa uzoqlashuvchi bo‘ladi.

5) Koshining integral alomati. Agar *f(x)* funksiya $[1;+\infty)$ da uzluksiz, kamayuvchi, musbat bo‘lsa, u holda $\sum\limits_{n=1}^{\infty} f(n)$ qator va $\int\limits_{1}^{+\infty} f(x)dx$ xosmas integral bir vaqtda yo yaqinlashuvchi, yo uzoqlashuvchi bo‘ladi.

Solishtirish alomatlaridan foydalanib quyidagi qatorlarning yaqinlashishi yoki uzoqlashishini ko‘rsating.

1018. $\frac{1}{4} + \frac{1}{2\cdot 4^2} + \frac{1}{3\cdot 4^3} + ... + \frac{1}{n4^n} + ...$.

1019. $\frac{1}{\ln 2} + \frac{1}{\ln 3} + \frac{1}{\ln 4} + ... + \frac{1}{\ln n} + ...$.

1020. $\frac{1}{2} + \frac{1}{5} + \frac{1}{8} + ... + \frac{1}{3n-1} + ...$.

1021. $\frac{1}{\sqrt[4]{3}} + \frac{1}{\sqrt[4]{4}} + ... + \frac{1}{\sqrt[4]{n+2}} + ...$.

1022. $\frac{1}{8} + \frac{1}{7^2+1} + \frac{1}{7^3+1} + ... + \frac{1}{7^n+1} + ...$.

1023. $\sin\frac{\pi}{2} + \sin\frac{\pi}{4} + ... + \sin\frac{\pi}{2^n} + ...$.

1024. $\frac{2}{3} + \frac{3}{8} + ... + \frac{n+1}{(n+2)n} + ...$.

1025. $tg\frac{\pi}{4} + tg\frac{\pi}{8} + ... + tg\frac{\pi}{4n} + ...$.

1026. $\sum\limits_{n=1}^{\infty}\left(\frac{1+n}{1+n^2}\right)^2$.

1027. $\sum\limits_{n=1}^{\infty}\frac{1}{\sqrt{n^2+2n}}$

1028. $\sum\limits_{n=1}^{\infty}(\sqrt{n} - \sqrt{n-1})$

1029. $\sum\limits_{n=1}^{\infty}\frac{1}{n}(\sqrt{n+1} - \sqrt{n-1})$

Dalamber alomatidan foydalanib, qatorlarni yaqinlashishlikka tekshiring.

1030. $\frac{1}{3}+\frac{2}{3^2}+\frac{3}{3^2}+...+\frac{n}{3^n}...$. 1031. $1+\frac{3}{1}+\frac{3^2}{12}+...+\frac{3^n}{n!}...$.

1032. $\frac{1}{3!}+\frac{1}{5!}+\frac{1}{7!}+...+\frac{1}{(2n+1)!}...$. 1033. $\frac{1}{\sqrt{2}}+\frac{3}{(\sqrt{2})^2}+\frac{5}{(\sqrt{2})^3}+\frac{2n-1}{(\sqrt{2})^n}+...$.

1034. $\sin\frac{\pi}{2}+4\cdot\sin\frac{\pi}{4}+...+n^2\sin\frac{\pi}{2^n}+...$.

1035. $tg\frac{\pi}{4}+2tg\frac{\pi}{8}+...+ntg\frac{\pi}{2^{n+1}}+...$.

1036. $\frac{1}{2}+\frac{1\cdot 3}{2\cdot 5}+\frac{1\cdot 3\cdot 5}{2\cdot 5\cdot 8}+...+\frac{1\cdot 3\cdot 5\cdot ...\cdot(2n+1)}{2\cdot 5\cdot 8\cdot ...\cdot(3n+2)}+...$.

1037. $\frac{2}{1}+\frac{2^2\cdot 1\cdot 2}{2^2}+\frac{2^3\cdot 1\cdot 2\cdot 3}{3^3}+...+\frac{2^n\cdot n!}{n^n}+...$.

Koshi alomatidan foydalanib, qatorlarni yaqinlashishlikka tekshiring.

1038. $\frac{1}{3}+\left(\frac{2}{5}\right)^2+\left(\frac{3}{7}\right)^3+...+\left(\frac{n}{2n+1}\right)^n+...$.

1039. $\frac{1}{2}+\left(\frac{3}{3}\right)^2+\left(\frac{5}{4}\right)^3+...+\left(\frac{2n-1}{n+1}\right)^n+...$.

1040. $\frac{1}{\ln 2}+\frac{1}{\ln^2 3}+...+\frac{1}{\ln^n(n+1)}+...$.

1041. $\arcsin 1+\arcsin^2\frac{1}{2}+...+\arcsin^n\frac{1}{n}+...$.

1042. $\frac{1}{2}+\left(\frac{2}{3}\right)^4+\left(\frac{3}{4}\right)^9+...+\left(\frac{n}{n+1}\right)^{n^2}+...$.

1043. $\frac{1}{2^2}+\frac{4}{3^3}+...+\frac{2^{n-1}}{n^n}+...$.

Koshining integral alomatidan foydalanib, qatorlarni yaqinlashishlikka tekshiring.

1044. $\frac{1}{6}+\frac{2}{9}+\frac{3}{14}+...+\frac{n}{n^2+5}+...$.

1045. $\frac{1}{\sqrt{5}}+\frac{1}{3}+\frac{1}{\sqrt{13}}+...+\frac{1}{\sqrt{4n+1}}+...$.

1046. $\frac{1}{2\sqrt{2}}+\frac{1}{3\sqrt{3}}+...+\frac{1}{n\sqrt{n}}+...$.

1047. $\frac{1}{2\ln 2}+\frac{1}{3\ln 3}+\frac{1}{4\ln 4}+...+\frac{1}{n\ln n}+...$.

1048. $\frac{1}{2\sqrt{\ln 2}}+\frac{1}{3\sqrt{\ln 3}}+\frac{1}{4\sqrt{\ln 4}}+...+\frac{1}{n\sqrt{\ln n}}+...$.

1049. $1+\left(\frac{1+2}{1+2}\right)^2+...+\left(\frac{1+n}{1+n^2}\right)^2+...$.

1050. $1+\left(\frac{1+4}{1+27}\right)^2+...+\left(\frac{1+n^2}{1+n^3}\right)^2+...$.

1051. $\frac{1}{2\ln 2\ln\ln 2}+\frac{1}{3\ln 3\ln\ln 3}+...+\frac{1}{(n+1)\ln(n+1)\ln\ln(n+1)}+...$.

3-§. Ixtiyoriy hadli qatorlar

1. Har qanday ikki qo‘shni hadi qarama-qarshi ishorali qiymatlarga ega bo‘lgan qator ishora almashinuvchi deyiladi. Ishora almashinuvchi qator

$a_1-a_2+a_3-...+(-1)^{n-1}a_n+...;$ (1) $(a_n>0, n\in N)$

kabi ifodalaniladi.

Leybnits alomati. Agar (1) da $\forall n\in N$ uchun $a_n\geq a_{n+1}$ (2) tengsizlik o‘rinli bo‘lib, $\lim\limits_{n\to\infty}a_n=0$ (3) bo‘lsa, u holda (1) qator yaqinlashuvchi bo‘ladi. Agar $r_n=S-S_n$ bo‘lsa, u holda $|r_n|<a_{n+1}$ bo‘ladi, ya’ni qator yig‘indisi S ni uning xususiy yig‘indisi S_n bilan almashtirganda xato birinchi tashlangan had a_{n+1} ning modulidan katta bo‘lmaydi.

2. Ixtiyoriy ishorali hadlarga ega qator

$a_1+a_2+a_3+...+a_n+...$ (4)

berilgan va

$|a_1|+|a_2|+|a_3|+...+|a_n|+...$ (5)

modullardan tuzilgan qator bo‘lsin.

Agar (5) qator yaqinlashuvchi bo‘lsa, u holda (4) qator ham yaqinlashuvchi bo‘ladi va (4) qator absolyut yaqinlashuvchi deyiladi.

Agar (4) qator yaqinlashuvchi, (5) qator esa uzoqlashuvchi bo‘lsa, u holda (4) qator shartli yaqinlashuvchi deyiladi.

3. Agar (4) qator absolyut yaqinlashuvchi bo‘lsa, uning hadlari o‘rnini almashtirganda Yana absolyut yaqinlashuvchi qator hosil bo‘ladi va yig‘indisi o‘zgarmaydi.

Agar (4) qator shartli yaqinlashuvchi bo‘lsa, har qanday B son uchun qator hadlarining o‘rnini tegishlicha almashtirganda, uning yig‘indisi xudi B sondan iborat bo‘ladi. (Riman teoremasi)

4. Ushbu

$a_1+a_2+a_3+...+a_n+...=A$ (4’)

va

$$b_1 + b_2 + b_3 + \ldots + b_n + \ldots = B \qquad (6)$$

qatorlar uchun

$$a_1 b_1 + (a_1 b_2 + a_2 b_1) + (a_1 b_3 + a_2 b_2 + a_3 b_1) + \ldots + (a_1 b_n + \ldots + a_n b_1) + \ldots . \qquad (7)$$

qator (4) va (6) ning ko‘paytmasi deyiladi.

(4) va (6) absolyut yaqinlashuvchi bo‘lganda, (7) ham yaqinlashuvchi bo‘ladi va (7) ning yig‘indisi $C = A \cdot B$ bo‘ladi.

Quyidagi ishora navbatlanuvchi qatorlarni Leybnets alomati yordamida yaqinlashishga tekshiring.

1052. $1 - \frac{1}{2} + \frac{1}{5} - \ldots + \frac{(-1)^n}{n^2+1} + \ldots .$ 1053. $1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \ldots + \frac{(-1)^{n-1}}{\sqrt{n}} + \ldots .$

1054. $\frac{1}{3} - \frac{2}{5} + \frac{3}{7} - \ldots + \frac{2(-1)^{n-1}}{2n+1} + \ldots .$ 1055. $\frac{\ln 2}{2} - \frac{\ln 3}{3} + \frac{\ln 4}{4} - \ldots + \frac{\ln(n+1)}{n+1} + \ldots .$

1056. $1 - \frac{2!}{2^2} + \frac{3!}{3^3} - \ldots + \frac{(-1)^{n-1} n!}{n!} + \ldots .$ 1057. $\frac{1}{3} - \frac{2}{5} + \frac{3}{7} - \ldots + \frac{(-1)^{n-1} n}{2n+1} + \ldots .$

1058. $\sqrt{\frac{1}{101}} - \sqrt{\frac{2}{202}} + \sqrt{\frac{3}{301}} - \ldots + (-1)^{n-1}\sqrt{\frac{n}{100n+1}} + \ldots .$

1059. $\sum_{n=1}^{\infty} (-1)^{n-1} \left(\frac{2n+1}{3n+1} \right)^n .$

1060. $1 - \frac{1}{2^2} + \frac{1}{3^3} - \frac{1}{4^4} + \ldots + \frac{(-1)^{n-1}}{n^n} + \ldots$ qatorning yig‘indisini 0,001 aniqlikda hisoblash uchun nechta hadini olish kerak?

1061. $\frac{1}{3} - \frac{1}{3^2} + \frac{1}{3^3} - \frac{1}{3^4} + \ldots + \frac{(-1)^{n-1}}{3^n} + \ldots$ qator yig‘indisini 0,01 aniqlikda hisoblang.

Quyidagi qatorlarni absolyut va shartli yaqinlashishga tekshiring.

1062. $1 - \frac{1}{2!} + \frac{1}{3!} - \ldots + (-1)^{n-1}\frac{1}{n!} + \ldots .$ 1063. $1 - \frac{1}{\sqrt[3]{2}} + \frac{1}{\sqrt[3]{3}} - \ldots + (-1)^{n-1}\frac{1}{\sqrt[3]{n}} + \ldots .$

1064. $1 - \frac{3}{2} + \frac{5}{2^2} - \ldots + (-1)^n \frac{2n+1}{2^n} + \ldots .$ 1065. $-\frac{1}{4^2} + \frac{1}{7^2} - \frac{1}{10^2} + \ldots + (-1)^n \frac{1}{(3n+1)^2} + \ldots .$

1066. $\frac{1}{\ln 2} - \frac{1}{\ln 3} + \frac{1}{\ln 4} - \ldots + (-1)^{n-1}\frac{1}{\ln(n+1)} + \ldots .$

1067. $\frac{1}{2\ln^2 2} - \frac{1}{3\ln^2 3} + \frac{1}{4\ln^2 4} - \ldots + (-1)^{n-1}\frac{1}{(n+1)\ln^2(n+1)} + \ldots .$

1068. $\frac{\sin 1}{2} + \frac{\sin 2}{2^2} + \frac{\sin 3}{2^3} + \ldots + \frac{\sin n}{2^n} + \ldots .$

1069. $\frac{\cos 1}{1} + \frac{\cos 2}{2^3} + \frac{\cos 3}{3^3} + \ldots + \frac{\cos n}{n^3} + \ldots$

1070. $\frac{\sin\alpha}{1}+\frac{\sin 2\alpha}{2^2}+...+\frac{\sin n\alpha}{n^2}+...$

1071. $-\frac{1}{1}+\frac{1}{2-\ln 2}-+\frac{1}{3-\ln 3}+...+(-1)^n\frac{1}{n-\ln n}+...$

1072. Agar $\sum_{n=1}^{\infty}a_n{}^2$ ва $\sum_{n=1}^{\infty}b_n{}^2$ qatorlar yaqinlashuvchi bo‘lsa, u holda $\sum_{n=1}^{\infty}a_nb_n$ qatorning yaqinlashishini ko‘rsating.

1073. Agar $\sum_{n=1}^{\infty}a_n$ qator absolyut yaqinlashuvchi bo‘lsa, u holda $\sum_{n=1}^{\infty}\frac{n+1}{n}a_n$ qatorning absolyut yaqinlashishini tekshiring.

Quyidagi qatorlar uchun Leybnits alomatini qo‘llab bo‘lmasligiga ishonch hosil qiling. Ularning qaysilari uzoqlashuvchi, qaysilari shartli yaqinlashuvchi, qaysilari absolyut yaqinlashuvchi?

1074. $\frac{1}{\sqrt{2}-1}-\frac{1}{\sqrt{2}+1}+\frac{1}{\sqrt{3}-1}-\frac{1}{\sqrt{3}+1}+...+\frac{1}{\sqrt{k}+1}-\frac{1}{\sqrt{k}-1}+...$

1075. $1-\frac{1}{3}+\frac{1}{2}+\frac{1}{3^3}+\frac{1}{2^2}-\frac{1}{3^5}+...+\frac{1}{2^{k-1}}-\frac{1}{3^{2k-1}}+...$

1076. $1-\frac{1}{3}+\frac{1}{3}-\frac{1}{3^2}+\frac{1}{5}-\frac{1}{3^3}+...+\frac{1}{2k-1}-\frac{1}{3^k}+...$

1077. $\frac{1}{3}-1+\frac{1}{7}-\frac{1}{5}+\frac{1}{11}-\frac{1}{9}+...+\frac{1}{4k-1}-\frac{1}{4k-3}+...$

X BOB. FUNKSIONAL QATORLAR

1-§. Funksional qatorning yaqinlashish sohasi. Tekis yaqinlashish.

1. $u_1(x), u_2(x), ..., u_n(x), ...$ funksiyalar biror E sohada aniqlangan bo‘lsin.

$u_1(x)+u_2(x)+...+u_n(x)+...$ (1)

funksional qator $x = x_0 \in E$ da sonli qator $u_1(x_0)+u_2(x_0)+...+u_n(x_0)+...$ (2) ga aylanadi.

Agar (2) yaqinlashuvchi bo‘lsa, u holda (1) funksional qator x_0 da yaqinlashadi yoki x_0 (1) ning yaqinlashish nuqtasi deyiladi. (1) qatorning barcha yaqinlashish nuqtalarining to‘plami (1) ning yaqinlashish sohasi deyiladi va $S(x)=\lim\limits_{n\to\infty} S_n(x)=\sum\limits_{n=1}^{\infty} u_n(x)$ - qatorning yig‘indisi deyiladi. $R_n(x)=S(x)-S_n(x)$ - qatorning qoldig‘i deyiladi.

2. Agar $\forall \varepsilon>0 \quad \exists N \ n>N$ va $\forall x \in E \Rightarrow |u_{n+1}(x)+...+u_{n+m}(x)|<\varepsilon$ bo‘lsa, u holda $\sum\limits_{n=1}^{\infty} u_n(x)$ funksional qator E da tekis yaqinlashuvchi bo‘ladi va aksincha.

Veyershtrass alomati.

Agar berilgan $\sum\limits_{n=1}^{\infty} u_n(x)$ funksional qator uchun $\forall n \in N$ $\forall x \in E \Rightarrow |u_n(x)| \le c_n$ ni qanoatlantiradigan yaqinlashuvchi sonli qator $\sum\limits_{n=1}^{\infty} c_n$ mavjud bo‘lsa, u holda $\sum\limits_{n=1}^{\infty} u_n(x)$ qator E da tekis yaqinlashuvchi va har bir $x \in E$ nuqtada absolyut yaqinlashuvchi bo‘ladi.

3. Tekis yaqinlashuvchi qatorlarning asosiy xossalari:

a) hadlari uzluksiz funksiyalardan tuzilgan tekis yaqinlashuvchi qatorning yig‘indisi o‘sha sohada uzluksiz funksiya bo‘ladi.

b) uzluksiz funksiyalardan tuzilgan tekis yaqinlashuvchi qatorni hadlab integrallash mumkin, ya’ni $\sum\limits_{n=1}^{\infty}\int\limits_a^b u_n(x)dx = \int\limits_a^b \left(\sum\limits_{n=1}^{\infty} u_n(x)\right)dx$.

c) Agar $\sum\limits_{n=1}^{\infty} u_n(x)$ qatorning har bir hadi J oraliqda uzluksiz differensiallanuvchi va J da yaqinlashuvchi, $\sum\limits_{n=1}^{\infty} u'_n(x)$ qator J da tekis yaqinlashuvchi bo‘lsa, u holda $\sum\limits_{n=1}^{\infty} u_n(x)$ qatorni hadlab differensiallash mumkin, ya’ni $\sum\limits_{n=1}^{\infty} u'_n(x) = \left(\sum\limits_{n=1}^{\infty} u_n(x)\right)'$.

Quyidagi qatorlarning yaqinlashish sohalarini toping.

1078. $1+x^2+x^4+...+x^{2n-2}+....$

1079. $1-2x+4x^2-...+(-1)^{n-1}2^{n-1}\cdot x^{n-1}+...$.

1080. $1+\frac{1}{x}+\frac{1}{x^2}+...+\frac{1}{x^{n-1}}+...$.

1081. $\frac{2x^2}{3}+\frac{4x^2}{8}+...+\frac{2^{n-1}x^n}{n^2-1}+...$.

1082. $\sum_{n=1}^{\infty}\frac{x^n}{1+x^{2n}}$.

1083. $\sum_{n=1}^{\infty}\frac{x^{2n-2}}{1+x^{2n-1}}$.

1084. $\sum_{n=1}^{\infty}\frac{1}{n^x}$.

1085. $\sum_{n=1}^{\infty}\frac{\ln n}{n^x}$.

1086. $\sum_{n=1}^{\infty}\frac{1}{n(1+x)^n}$.

1087. $\sum_{n=1}^{\infty}\frac{1}{n}\sin\frac{x}{n}$.

Tekis yaqinlashishga tekshiring.

1088. $\sum_{n=1}^{\infty}ax^{n-1}, \quad x\in[-q;q], \quad 0<q<1$.

1089. $1+x+\frac{x^2}{2!}+...+\frac{x^{n-1}}{(n-1)!}+...$ ixtiyoriy chekli intervalda.

1090. $\sin x+\frac{\sin 2x}{2^3}+\frac{\sin 3x}{3^3}+...+\frac{\sin nx}{n^3}+..., \ x\in(-\infty;+\infty)$.

1091. $\cos x+\frac{\cos 2x}{2!}+\frac{\cos 3x}{3!}+...+\frac{\cos nx}{n!}+..., \qquad x\in(-\infty;\infty)$.

1092. $\sum_{n=1}^{\infty}\frac{1}{n^2(1+(nx)^2)}, \quad x\in(-\infty;\infty)$.

1093. $\sum_{n=1}^{\infty}\frac{1}{2^{n-1}\sqrt{1+nx}} \quad x\in[0;+\infty)$.

1094. $\sum_{n=1}^{\infty}x^n(1-x)$ qator [0;1] segmentda yaqinlashuvchi, lekin tekis yaqinlashuvchi emasligini ko‘rsating.

1095. $\sum_{n=1}^{\infty}ax^{n-1}$ qator (-1;1) intervalda yaqinlashuvchi, lekin tekis yaqinlashuvchi emasligini ko‘rsating.

1096. $\sum_{n=1}^{\infty}\frac{\cos^n x}{n!}$ qatorni hadlab integrallash mumkinmi?

1097. $\sum_{n=1}^{\infty}\frac{\sin nx}{n^3}$ qatorni hadlab differensiallash mumkinmi?

Ushbu qatorlarning yig‘indilari $(-\infty;+\infty)$ intervalda uzluksiz ekanligini ko‘rsating.

1098. $\sum_{n=1}^{\infty}\frac{x^n}{2^n(1+x^{2n})}$.

1099. $\sum_{n=1}^{\infty}\frac{1}{n^2+n^4x^2}$.

Differensiallash yordamida ushbu qatorlarning yig‘indilarini toping.

1100. $\frac{x^3}{3}+\frac{x^7}{7}+...+\frac{x^{4n-1}}{4n-1}+...$

1101. $x+\frac{x^5}{5}+...+\frac{x^{4n-3}}{4n-3}+...$

1102. $\frac{x^2}{1\cdot 2}+\frac{x^3}{2\cdot 3}+...+\frac{x^{n+1}}{n(n+1)}+...$

Integrallash yordamida ushbu qatorlarning yig‘indisini toping.

1103. $1+2x+3x^2+...+nx^{n-1}+...$, $|x|<1$.

1104. $1+3x+...+\frac{n(n+1)}{2}x^{n-1}+...,$ $|x|<1$.

2-§. Darajali qatorlar

1. $a_0+a_1(x-x_0)+a_2(x-x_0)^2+...+a_n(x-x_0)^n+...$ (1)

ko‘rinishdagi qator darajali qator deyiladi.

$x_0=0$ da $a_0+a_1x+a_2x^2+...+a_nx^n+...$ (2)

qator x ning darajalari bo‘yicha yoyilgan qator deyiladi.

(2) ning yaqinlashish radiusi deb shunday R songa aytiladiki, $|x|<R$ da (2) qator yaqinlashuvchi va $|x|>R$ da uzoqlashuvchi bo‘ladi. $(-R;R)$ esa (2) qatorning yaqinlashish intervali deyiladi.

(2) qatorning yaqinlashish radiusi Koshi-Alamar formulalari yordamida hisoblanadi:

$$R=\overline{\lim_{n\to\infty}}\left|\frac{a_n}{a_{n+1}}\right|,\qquad R=\frac{1}{\overline{\lim_{n\to\infty}}\sqrt[n]{|a_n|}}.$$

(1) qatorning yaqinlashish intervali $(x_0-R;x_0+R)$ bo‘ladi.

2. a) $(-R;R)$ yaqinlashish intervalida joylashgan har qanday $[a,b]$ da (2) qator tekis yaqinlashuvchi bo‘ladi.

b) Yaqinlashish intervalida (2) qator yig‘indisi uzluksiz funksiya bo‘ladi.

c) (2) qatorni yaqinlashish intervalida joylashgan har qanday kesmada hadlab integrallash mumkin:

$$\int_{-r}^{r}\sum_{n=0}^{\infty}a_n x^n dx=\sum_{n=0}^{\infty}a_n\int_{-r}^{r}x^n dx,\quad \forall[-r;r]\subset(-R;R).$$

d) $a_0+a_1x+a_2x^2+...+a_nx^n+...$ (2) va $a_1+2a_2x+...+na_nx^{n-1}+...$ (2') qatorlar bir hil yaqinlashish radiuslariga ega.

(2) qatorni $\forall[-r;r]\subset(-R;R)$ da hadlab differensiallash mumkin:

$$\left(\sum_{n=0}^{\infty}a_nx^n\right)'=\sum_{n=0}^{\infty}na_nx^{n-1}.$$

Quyidagi darajali qatorlarning yaqinlashish radiuslari, yaqinlashish intervallari va yaqinlashish sohalarini toping.

1105. $10x+100x^2+...+10^n x^n+...$

1106. $x-\frac{x^2}{2}+...+(-1)^{n-1}\frac{x^n}{n}+...$

1107. $1-4x+4^2x^2+...+(-4)^{n-1}x^{n-1}+...$

1108. $x-\frac{x^2}{3\cdot 3!}+...+\frac{x^n}{(2n-1)\cdot(2n-1)!}+...$

1109. $1+3x+...+(n-1)3^{n-1}x^{n-1}+...$

1110. $\frac{x}{2\cdot 4^2\ln 2}+\frac{x^2}{3\cdot 4^3\ln 3}+...+\frac{x^{n-1}}{n4^n\ln n}+...$

1111. $\frac{2}{3}x+\left(\frac{3}{5}\right)^2+\left(\frac{4}{7}\right)^3x^3+...+\left(\frac{n+1}{2n+1}\right)^n x^n+...$

1112. $(x-2)+\frac{(x-2)^2}{\sqrt{2}}+\frac{(x-2)^3}{\sqrt{3}}+...+\frac{(x-2)^n}{\sqrt{n}}+...$

1113. $(x-1)+\frac{(x-1)^2}{2^2}+\frac{(x-1)^3}{3^2}+...+\frac{(x-1)^n}{n^2}+...$

1114. $\frac{x+4}{2\cdot 3}+\frac{2(x+4)^2}{3\cdot 3^2}+...+\frac{n(x+4)^n}{(n+1)3^n}+...$

1115. $\frac{x-3}{2}+\frac{(x-3)^2}{2^2\sqrt{2}}+\frac{(x-3)^3}{2^3\sqrt{3}}+...+\frac{(x-3)^n}{2^n\sqrt{n}}+...$

1116. $1+2x^2+...+2^{n-1}x^{2(n-1)}+...$

1117. $x+\frac{x^3}{3\cdot 3!}+...+\frac{x^{2n-1}}{(2n-1)(2n-1)!}+...$

3-§. Teylor qatori

1. Agar $f(x)$ funksiya x_0 nuqtaning biror $u(x_0,\delta)$ atrofida istalgan marta differensiallanuvchi va shu oraliqda $|f^n(x)|<M$ tengsizlik o‘rinli bo‘lib, M soni n ga bog‘liq bo‘lmasa, u holda $f(x)$ funksiya o‘zining Teylor qatoriga yoyiladi:

$$f(x)=f(x_0)+\frac{f'(x_0)}{1!}(x-x_0)+\frac{f''(x_0)}{2!}(x-x_0)^2+...+\frac{f^{(n)}(x_0)}{n!}(x-x_0)^n+... \quad (1)$$

$x_0=0$ da

$$f(x)=f(0)+\frac{f'(0)}{1!}x+\frac{f''(0)}{2!}x^2+...+\frac{f^{(n)}(0)}{n!}x^n+... \qquad (2)$$

(2) qator Makloren qatori deb yuritiladi.

2. Quyida ba’zi funksiyalarning Makloren qatoriga yoyilmalarini keltiramiz:

$$e^x=1+\frac{x}{1!}+\frac{x^2}{2!}+...+\frac{x^n}{n!}+..., \quad x\in(-\infty;+\infty).$$

$$\sin x=\frac{x}{1!}-\frac{x^2}{3!}+...+(-1)^{n-1}\frac{x^{2n-1}}{(2n-1)!}+..., \quad x\in(-\infty;+\infty).$$

$$\cos x=1-\frac{x^2}{2!}+\frac{x^4}{4!}+...+(-1)^{n-1}\frac{x^{2n}}{2n!}+..., \quad x\in(-\infty;+\infty).$$

$$shx=\frac{x}{1!}+\frac{x^3}{3!}+\frac{x^5}{5!}+...+\frac{x^{2n-1}}{(2n-1)!}+..., \quad x\in(-\infty;+\infty).$$

$$chx=1+\frac{x^2}{2!}+\frac{x^4}{4!}+...+\frac{x^{2n}}{2n!}+..., \quad x\in(-\infty;+\infty).$$

$$(1+x)^\alpha=1+\frac{\alpha}{1!}x+\frac{\alpha(\alpha-1)}{2!}x^2+...+\frac{\alpha(\alpha-1)...(\alpha-n+1)}{n!}x^n+...; \quad x\in(-1;1)$$

$$\ln(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-...+(-1)^{n-1}\frac{x^n}{n}+...; \quad x\in(-1;1).$$

Ushbu funksiyalarni Makloren qatorlariga yoying.

1118. $f(x)=e^{6x}$.

1119. $f(x)=e^{-x}$.

1120. $f(x)=\sin 3x$.

1121. $f(x)=\cos\frac{x}{3}$.

1122. $f(x)=\ln(1+4x)$.

1123. $f(x)=\ln(1-\frac{x}{3})$.

1124. $f(x)=(1+x)^{-2}$.

1125. $f(x)=\frac{1}{1-x^3}$.

1126. $f(x)=\sqrt{1+x^3}$.

1127. $f(x)=\frac{1}{\sqrt{1-x^2}}$.

Ushbu funksiyalarni Teylor qatoriga yoying.

1128. $f(x)=\ln x, \quad x_0=1$.

1129. $f(x)=\frac{1}{x}, \quad x_0=-2$.

1130. $f(x)=e^{-3x}, \quad x_0=-4$.

1131. $f(x)=\cos x, \quad x_0=\frac{\pi}{4}$.

Qatorlarni differensiallash va integrallash yordamida ushbu funksiyalarni Makloren qatoriga yoying.

1132. *f(x)=arcsinx.* 1133. *f(x)=arctgx.*

1134. $f(x)=\ln(x+\sqrt{1+x^2})$. 1135. *f(x)=(1+x)ln(1+x).*

4-§. Taqribiy hisoblashlar

Agar berilgan $f(x)$ funksiya qatorga yoyilgan bo‘lsa, bu funksiyaning biror $x=x_0$ nuqtadagi taqribiy qiymatini topish uchun: 1) qatorning birinchi *n* ta hadi yig‘indisi $S_n=\sum_{k=1}^{n}a_k x_0^k$ topiladi; 2) qator yig‘indisining aniq qiymati S bilan S_n orasidagi $S-S_n$ farq, ya’ni R_n qoldiq baholanadi.

Taqribiy hisoblashlarni bajaring.

1136. $\sqrt[3]{e}$ ni 0,01 aniqlikda hisoblang.
1137. sin 12^0 ni 0,001 aniqlikda hisoblang.
1138. cos 1^0 ni 0,001 aniqlikda hisoblang.
1139. $\sqrt[3]{30}$ ni 0,001 aniqlikda hisoblang.
1140. $\sqrt[3]{1{,}06}$ ni 0,0001 aniqlikda hisoblang.
1141. $\sqrt[4]{80}$ ni 0,001 aniqlikda hisoblang.

Quyidagi integrallarni 0,001 aniqlikda hisoblang.

1142. $\int_0^1 e^{-x^2}dx$. 1143. $\int_0^1 \frac{\sin x}{x}dx$.

1144. $\int_{0,1}^{0,2} \frac{e^{-x}}{x^3}dx$. 1145. $\int_0^{0,8} x^{10}\sin x dx$.

1146. $\int_0^{0,5} \frac{e^x-1}{x}dx$. 1147. $\int_0^{1/2} \frac{arctgx}{x}dx$.

Quyidagi integrallarni taqribiy hisoblang va hatolikni baholang. (Buning uchun integral ostidagi funksiyalarning Makloren qatoriga yoyilmasining ko‘rsatilgan sondagi hadlarini oling).

1148. $\int_0^{1/4} e^{-x^2}dx$ (3 ta had). 1149. $\int_{0,1}^{1} \frac{e^x}{x}dx$ (6 ta had).

1150. $\int_0^{1/2}\frac{dx}{\sqrt{1+x^4}}$ (2 ta had). 1151. $\int_0^{\sqrt{3}/3}x^3 arctgxdx$ (2 ta had).

5-§. Furye qatori.

1. Funksiyani uzunligi 2π bo‘lgan kesmada Furye qatoriga yoyish.

f(x) funksiyaning [-π,π] kesmadagi Furye qatori deb quyidagi ko‘rinishdagi qatorga aytiladi:

$$f(x) \sim \frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos nx + b_n \sin nx \qquad (1)$$

Bu qatorning koeffitsientlari *f(x)* funksiyaning Furye koeffitsientlari deyiladi va quyidagi formulalar yordamida hisoblanadi:

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nxdx \qquad (2) \qquad n=0,\ 1,\ 2,\ ...$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nxdx \qquad (3) \qquad n=1,\ 2,\ 3,\ \dots$$

f(x) funksiyaga mos Furye qatori yaqinlashuvchi bo‘lishining etarli shartini Dirixle teoremasi ta’minlaydi.

Teorema. Agar *f(x)* funksiya (-π,π) kesmada bo‘lakli silliq bo‘lsa, u holda *f(x)* funksiyaning Furye qatori shu kesmaning har bir nuqtasida yaqinlashuvchi bo‘lib, bu qatorning yig‘indisi (-π,π) oraliqda funksiya uzluksiz bo‘lgan nuqtalarda *f(x)* ga, uzilish nuqtalarida $\frac{1}{2}(f(x-0)+f(x+0))$ ga, oraliqning uchlarida esa $\frac{1}{2}(f(\pi-0)+f(\pi+0))$ ga teng bo‘ladi.

2. Juft va toq funksiyalar uchun Furye qatorlari.

Agar *f(x)* funksiya (-π,π) oraliqda juft bo‘lsa, u holda Furye koeffitsientlari quyidagi formulalar yordamida hisoblanadi:

$$a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)dx, \quad a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos nxdx, \quad b_n = 0 \qquad (4)$$

Demak, juft funksiyaning Furye qatorida ozod had va kosinuslar qatnashadi, bu qator kosinuslar qatori deyiladi.

Agar *f(x)* funksiya (-π,π) oraliqda toq bo‘lsa, u holda Furye koeffitsientlari quyidagi formulalar yordamida hisoblanadi:

$$a_0 = 0, a_n = 0, b_n = \frac{2}{\pi}\int_0^{\pi} f(x)\sin nxdx\,. \qquad (5)$$

Bu holda funksiyaning Furye qatorida faqat sinuslar qatnashadi, qator esa sinuslar qatori deyiladi.

3. Yarim davrida berilgan funksiyaning Furye qatori. Agar *f(x)* funksiya (0,π) oraliqda berilsa, u holda bu funksiyani kosinuslar yoki sinuslar qatoriga yoyish mumkin. Buning uchun funksiyani (-π,0) oraliqga mos ravishda juft yoki

toq tarzda davom ettirish lozim. Hosil bo‘lgan funksiyaning Furye koeffitsientlari juft funksiya uchun (4), toq funksiya uchun (5) formulalar yordamida aniqlanadi.

4. Uzunligi $2l$ bo‘lgan kesmada funksiyani Furye qatoriga yoyish. Agar $[-l,l]$ kesmada berilgan *f(x)* funksiya uchun Dirixle teoremasi shartlari bajarilsa, u holda bu funksiya quyidagi ko‘rinishdagi Furye qatoriga yoyiladi:

$$\frac{a_0}{2}+\sum_{n=1}^{\infty}\left(a_n\cos\frac{n\pi x}{l}+b_n\sin\frac{n\pi x}{l}\right),$$

bu erda Furye koeffitsientlari ushbu formulalar yordamida topiladi:

$$a_0=\frac{1}{l}\int_{-l}^{l}f(x)dx,\quad a_n=\frac{1}{l}\int_{-l}^{l}f(x)\cos\frac{n\pi x}{l}dx,\quad b_n=\frac{1}{l}\int_{-l}^{l}f(x)\sin\frac{n\pi x}{l}dx\ ,\ n\in N.$$

Agar *f(x)* funksiya $[-l,l]$ kesmada juft bo‘lsa, u holda Furye koeffitsientlari quyidagi formulalar yordamida hisoblanadi:

$$a_0=\frac{2}{l}\int_{0}^{l}f(x)dx,\quad a_n=\frac{2}{l}\int_{0}^{l}f(x)\cos\frac{n\pi x}{l}dx,\quad b_n=0,\ n\in N.$$

Agar *f(x)* funksiya $[-l,l]$ kesmada toq bo‘lsa, u holda Furye koeffitsientlari quyidagi formulalar yordamida hisoblanadi:

$$b_n=\frac{2}{l}\int_{0}^{l}f(x)\sin\frac{n\pi x}{l}dx,\ n\in N.$$

Quyidagi funksiyalarni $(-\pi;\pi)$ intervalda Furye qatoriga yoying.

1152. $f(x)=-\frac{x}{2}$.

1153. $f(x)=\begin{cases}-1, & agar\ -\pi<x<0,\\ 1, & agar\ 0\le x<\pi.\end{cases}$

1154. $f(x)=\begin{cases}\pi+x, & agar\ -\pi<x<0,\\ \pi-x, & agar\ 0\le x<\pi.\end{cases}$

1155. $f(x)=\begin{cases}1, & agar\ -\pi<x<0,\\ x, & agar\ 0\le x<\pi.\end{cases}$

1156. $f(x)=\begin{cases}-2x, & agar\ -\pi<x<0,\\ 3x, & agar\ 0\le x<\pi.\end{cases}$

1157. $y=x^2$ funksiya $(-\pi;\pi)$ intervalda Furye qatoriga yoying. Yoyilmadan foydalanib $1-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+...+(-1)^{n-1}\frac{1}{n^2}+...$ qatorning yig‘indisini toping.

1158. $y=|x|$ funksiyani $(-\pi;\pi)$ intervalda Furye qatoriga yoying. Yoyilmadan foydalanib $1+\frac{1}{3^2}+\frac{1}{5^2}+...+\frac{1}{(2n-1)^2}+...$ qatorning yig‘indisini toping.

1159. *f(x)=x* funksiyani $(0;\pi)$ intervalda kosinuslar bo‘yicha Furye qatoriga yoying.

1160. $f(x)=1$ funksiyani $(0;\pi)$ intervalda sinuslar bo‘yicha Furye qatoriga yoying.

Quyidagi funksiyalarni ko‘rsatilgan intervallarda Furye qatoriga yoying.

1161. $f(x)=3x, \quad x\in(-2;2)$.

1162. $f(x)=|x|, \quad x\in(-4;\,4)$.

1163. $f(x)=3-x, \quad x\in(-2;2)$.

1164. $f(x)=\begin{cases}0, & \text{agar } -4<x<0,\\ x, & \text{agar } 0\le x<4.\end{cases}$

1165. $f(x)=\dfrac{x^2}{9}, \quad x\in(-3;\,3)$.

Javoblar

1. ratsional son, $2\frac{1301}{9900}$. 2. ratsional son, $2\frac{1507}{1980}$. 3. ratsional son, $3\frac{74}{99}$. 4. ratsional son, $\frac{131}{330}$. 5. irratsional son. 6. irratsional son. 7. ratsional son, $2\frac{31}{99}$. 8. ratsional son, $2\frac{32}{99}$ 9. irratsional son. 10. irratsional son. 13.Bunday sonlar ko‘p, shunday sonlardan ikkitasi: a=1-$\sqrt{2}$, b=4+$\sqrt{2}$. 14.Bunday sonlar ko‘p, shunday sonlardan ikkitasi: a=$\sqrt{2}$-1, b=$\sqrt{2}$+1. 15.Yuqoridan va quyidan chegaralangan. 16. Quyidan chegaralangan. 17.Yuqoridan va quyidan chegaralangan. 18. Quyidan chegaralangan, yuqoridan chegaralanmagan. 19. Yuqoridan va quyidan chegaralangan. 20. Quyidan chegaralangan, yuqoridan chegaralanmagan. 21.supE=4, infE=0. 22. infE=0, supE=+∞. 23. Bir elementli to‘plamlar uchun. 24. Bunday to‘plamlar ko‘p, shunday to‘plamlardan bittasi {0;1}. 25. x∈[-1;5]. 26. x∈(-5;5). 27.x∈(1;9). 28. x∈[-6;2]. 29. x∈(-∞;-5)∪(-1;+∞). 30.x∈(-∞;-4]∪[2;+∞). 31.x∈[-5;5]. 32. *x*∈(-∞;+∞). 33. *x*∈(-1;3). 34. *x*∈(0;5). 35. {-1;3}. 36.{0}. 37. {2;3}. 38. *x*∈(-∞;-1]∪[1;+∞). 39. Yechimga ega emas. 40. *x*∈[-2;3). 41. f(-2)=-6; f(1)=3; f(*a*)=$2a^3$-3*a*+4. 42. f(0)=1; f(π/4)=2+$\frac{\sqrt{2}}{2}$; f(α)=2sin2α+cosα. 43. f(0)=-2; f(1)=-0,5; f(2)=0; |f(0,5)|=1; f(-1) mavjud emas. 44. f(0)=-3; f(2)=1; f(-2)=-5/3; f(1) mavjud emas. 45. f(2)=1; f(π/2)=1+$\frac{\pi^2}{4}$; f(-π/4)= -$\frac{\sqrt{2}}{2}$; f(4) mavjud emas. 47. (-∞;+∞). 48. (-∞;1)∪(1;+∞). 49. (-∞;1)∪(1;2)∪(2;+∞). 50. (-∞;-3)∪(-3;1)∪(1;+∞). 51. (-∞;3,5]. 52. [-2;2]. 53. (-∞;1)∪(3;+∞). 54. (-∞;+∞). 55. (-∞;1)∪(3;+∞). 56. (-∞;-1)∪(2;+∞). 57. (-∞;0)∪(4;+∞). 58.(-∞;0). 59. (4/3;+∞). 60. (3;+∞). 61. (-∞;1)∪(3;+∞). 62. [1;4]. 63. x∈(2kπ; (2k+1)π), k∈Z. 64. (1;2)∪(2;+∞). 65. [1;5/3]. 66.[2;3]. 67. [-1;4]. 68. [-1,5;2,5]. 69. [-1;3]. 70.(1,5;5]. 71.(-1;0)∪(1;2)∪(2;+∞). 72. (-1;1]∪[2;3). 73. [0;3]. 74. x∈(-π/2+2kπ;2kπ)∪(2kπ;π/2+2kπ), k∈Z. 75. 1) yo‘q; 2) yo‘q; 3) yo‘q; 4) yo‘q. 76. 1) yo‘q; 2) yo‘q; 3) yo‘q; 4) yo‘q. 97.y=$(x+1)^2$. 98. y=$\cos^2 x$. 99. $y=\sqrt{3^x+1}$. 100. y=1/|cos*x*|. 101. f(f(*x*))=x^4; f(φ(*x*))=4^x; φ(f(x))=2^{x^2}; φ(φ(x))=2^{2^x}. 102. f(φ(x))=27^x; φ(f(x))=3^{x^3}. 103. φ(f(1))=0; φ(f(2))=sin12x; f(φ(π/2))=0; f(f(f(1)))=0. 104. f(3x)=$\frac{45x+1}{2-3x}$; f(x^3)=$\frac{5x^6+1}{2-x^3}$; 3f(x)= $\frac{3(5x^2+1)}{2-x}$; $(f(x))^2$=$\left(\frac{5x^2+1}{2-x}\right)^2$. 105. juft. 106. juft. 107. juft. 108. toq. 109.juft. 110.toq. 111.juft hamb toq ham emas. 112. juft hamb toq ham emas. 113.juft. 114.juft. 115.toq. 116. juft hamb toq ham emas. 117. juft hamb toq ham emas. 118.juft. 119.toq. 126. chegaralangan. 127. chegaralangan. 128. chegaralanmagan. 129. chegaralanmagan. 130. chegaralangan. 131. chegaralangan. 132. chegaralangan. 133. chegaralanmagan. 134.davriy, π. 135. davriy, 2. 136.davriy, π. 137. davriy, 1. 138. davriy, asosiy davri mavjud

emas. 139.davriy, 2π. 140.davriy emas. 141.davriy, 2π. 142.davriy, 2π. 143. davriy, $\pi/3$. 145. 0,5. 152. y=0,5x. 153.y=(1-x)/3. 154. y=1/x. 155. y=1-1/x. 156.y=lgx-1. 157.$y=\sqrt[x]{2}$. 158. $x\in(-\infty;1]$ da $y=1-\sqrt{1+x}$, $x\in[1;+\infty)$ da $y=1+\sqrt{1+x}$. 159. $y=10^{x-1}-2$. 167. 0,75. 168. 3. 169. 2. 170. 1. 171. 1. 172. 2. 173. 0. 174. 1. 175. ∞. 176. $\sqrt{2}$. 177.0. 178.0. 179. 4/3. 180. 0,5. 181.-0,5. 182.1. 183. 2/9. 184. 2/3. 185.-0,5. 186.-1. 188.1) 0; 2) $\frac{\sqrt{13}+1}{2}$; 3) 0. 189. π. 195. 9/4. 196. 2. 197. 0,5. 198. 10. 199.1/8. 200. 0. 201. 2,5. 202. 3/13. 204. 1. 205. $\frac{1}{2\sqrt{h}}$. 206. 0,75. 207. 1/3. 208. 2/9. 209. 4. 210. 2,5. 211. 1/40. 212. 5/24. 213. 0,4. 214. 0. 215. 100. 216. -1. 217. 0. 218. -4. 219. -1. 220. -1. 221. 0. 222.1. 223. 0. 224. 0. 225. 1. 226. 0,5. 227. 0. 228. 0. 229. 2/3. 230. 3/5. 231. 7/7. 232. α/β. 233. 0,75. 234.m/n. 235. 1,25. 236. 4/3. 237. 2/3. 238. 1. 239. 3. 240. 0. 241. 4,5. 242. 0,75. 243. 1/64. 244. 0,5. 245. 1. 246. $2/\pi$. 247. $0,5(\beta^2-\alpha^2)$. 248. $\sin2\alpha/2\alpha$. 249.-1. 250. ∞. 251. $\sqrt{2}$. 252. 0. 253. 1/e. 254. e^2. 255. 1/e. 256. e^{mk}. 257.e^{10}. 258. $e^{-1,5}$. 259. e. 260. $e^{-3,5}$. 261. e^2. 262. 0. 263. e. 264. e^2. 265. ∞. 266. 1. 267. 0,5. 268. $e^{-0,5}$. 269. k. 270. 1/a. 271. 1/e. 272. a. 273. 2/3. 274. -1. 275. 1. 276. 2. 277. ln10. 278. ln4. 279. a-b. 280.1. 281. 1/n. 282. 3/8. 283. 2/3. 284. 1/3. 285. 5/6. 286. 1/98. 287. f(1-0)=2; f(1+0)=4. 288. f(2-0)=5; f(2+0)=3. 289. f(3-0)=9; f(3+0)=7. 290. f(1-0)=3, f(1+0)=2; f(2-0)=4; f(2+0)=4. 291. f(-2-0)=-3; f(-2+0)=-2; f(-0)=-1; f(+0)=0; f(1-0)=0, f(1+0)=1. 292. f(1-0)=1; f(1+0)=0; f(2-0)=1; f(2+0)=1; f(3-0)=1, f(3+0)=0. 293. $f(1-0)=-\infty$; $f(1+0)=\infty$. 294. $f(2-0)=-\infty$; $f(2+0)=+\infty$. 295. f(-0)=1; f(+0)=1. 296. f(1-0)=1; f(1+0)=2.
297. f(-0)=0; f(+0)=0. 299. 1) Bir xil tartibli; 2) yuqori tartibli; 3) yuqori tartibli; 4) bir xil tartibli; 5) bir xil tartibli; 6) yuqori tartibli; 7) bir xil tartibli; 8) ekvivalent. 300. 1) ekvivalent; 2) ekvivalent; 3) ekvivalent; 4) ekvivalent; 5) ekvivalent; 6) ekvivalent; 7) ekvivalent emas. 301. 2. 302. 0,25. 303. 1. 304. 0,5. 305. 0,5. 306. 1. 312. x=0, uzluksizlikni tiklash mumkin bo‘lgan uzilish. 313. x=1, sakrashga ega, sakrash kattaligi 3. 314. x=1, sakrashgaega, sakrash kattaligi 1. 315. x=1, uzluksizlikni tiklash mumkin bo‘lgan uzluksizlik. 316. x=1, sakrashga ega, sakrash kattaligi 2; x=2, sakrashga ega, sakrash kattaligi 1. 317. x=0, ikkinchi tur uzulish nuqta. 318. 1) x=0, ikkinchi tur uzulish nuqta; 2) $x=\pm1$, ikkinchi tur uzulish nuqtalar; 3) x=0, sakrashga ega, sakrash kattaligi 2; 4) x=0, uzluksizlikni tiklash mumkin bo‘lgan uzulish, 5) $x=\pm1$ ikkinchi tur uzulish nuqtalari. 319. 1) f(0)=1; 2) f(0)=2; 3) f(0)=0,5; 4) f(0)=-1,5; 5) f(0)=1. 321. 3. 325. mavjud emas. Asoslang. 326. mavjud emas. Asoslang. 327. 1) y=1-3x funksiya $(-\infty;+\infty)$ oraliqda uzluksiz va kamayuvchi. Shu sababli uning qiymatlar to‘plami $(-\infty;+\infty)$ oraliqda unga teskari funksiya mavjud, u ham uzluksiz va kamayuvchi. Qolgan misollar ham shu kabi isbotlanadi. 328. 1) o‘suvchi; 2) kamayuvchi; 3) o‘suvchi; 4) o‘suvchi; 5) kamayuvchi. 329. 1) o‘suvchi; 2) o‘suvchi; 3) kamayuvchi; 4) o‘suvchi; 5) kamayuvchi. 340. 11. 341. -3. 342. f’(1/8)=4/3, f’(0) mavjud emas. 343. f’(0)=1/3, $f'(1)=+\infty$. 344. $f'(\pi/2)=0$, $f'(0)=+\infty$. 345. $f'_-(1)=-3$, $f'_+(1)=3$. 346. $f'_-(0)=-\infty$, $f'_+(0)=+\infty$. 347. $f'_-(0)=0$, $f'_+(0)=0$. 348. $f'_-(2)=-4$, $f'_+(2)=4$. 349. -6. 350. $33x^2-2x-4$.

351. $-9x^4-4x^{-3}$. 352. $(4x-2)/x^3$. 353. $\frac{1-24\sqrt{x}}{4\sqrt[4]{x^3}}$. 354. $-\frac{x\sqrt{x}+2}{2x^2}$. 355. 42x-2. 356. $56\sqrt[3]{x}+7/x^2-6/\sqrt[3]{x^2}-6/x^3$. 357. $6/(5x+1)^2$. 358. $1/(\sqrt{x}(1-\sqrt{x})^2)$. 359. $20(1+2x)^9$. 360. $-4(x+1)/(x-1)^3$. 361. 8x-0,5. 362. $5x^4-12x^2-2x$. 363. $-2/x^2+8/x^3-15/x^4$. 364. $1+14/x^3-9/x^4$. 365. $3\sqrt{x}/2-4/(3\sqrt[3]{x})$. 366. $3/(2\sqrt{x})-(5x+8)/\sqrt[4]{(x+2)^3}$. 367. $24-14x-18x^2$. 368. $(6x+3/x^4)(\sqrt[3]{x}+x)+(3x^2-1/x^3)(1/(3\sqrt[3]{x^2})+1)$. 369. $12/(3x+7)^2$. 370. $\frac{\sqrt[3]{(x+2)^2}-\sqrt[3]{(x-2)^2}}{3(x+2)\sqrt[3]{x-2}}$. 371. $4(t^4-t^{-2}+1)^3(4t^3+2t^{-3})$. 372. $5(1+x^2)^4(x^2+x-1)/(1+x)^6$. 373. $2/\sqrt{6}$. 374. f'(1)=2, f'(4)=36, f'(1/4)=-13,5.
375. cosx-2sinx. 376. $((x-1)\cos x-(x+10\sin x)/x^2$. 377. $-2\cos x/\sin^3 x$. 378. 1,5sin2x(cosx-2). 379. 3cos(3x+1). 380. -cosx·sin(sinx). 381. $\frac{1}{2\sqrt{2\sin x}\cdot\cos(x/2)}$.
382. $\arccos x-x/\sqrt{1-x^2}$. 383. $1-\arcsin x/\sqrt{1-x^2}$. 384. $-2/\sqrt{3+4x-4x^2}$. 385. $\arccos x/\sqrt{(1-x^2)(1-(\arccos x)^2)}$. 386. $x^2(3\ln x+1)$. 387. $1/(2x\ln 10\sqrt{\lg x})$. 388. $(1+x^2-2x\ln x)/(x(1+x^2)^2)$. 389. ctgx. 390. $2x/((x^2-1)\ln 10)$. 391. $-(x\sin x+\cos x)/x^2$. 392. $3\sin^2 x\cos x$. 393. $\cos^3 x$. 394. -2sin2x. 395. $-1/(x^2\cos^2(1/x))$. 396. 3sin6x. 397. $x/(\sqrt{1-x^2}\sin^2\sqrt{1-x^2})$. 398. $3(\arcsin x)^2/\sqrt{1-x^2}$.
399. $(2x(1+x^2)\mathrm{arctg}x-x^2)/((1+x^2)(\mathrm{arctg}x)^2)$. 400. $2x/\sqrt{1-x^4}$.
401. $-1/((1+x)\sqrt{2x(1-x)})$. 402. $(3\ln^2 x)/x$. 403. (lnx+1)sin2x+2xcos2xlnx. 404. $\ln x/(x\sqrt{1+\ln^2 x})$. 405. $2(x-2)/(x^2-4x)$. 406. -3tg3x. 407. 1/t. 408. –(1+cost)/sint. 409. –tgt. 410. (1-tgt)/(1+tgt). 411. –ctgt. 412. -bctgφ/a. 413. ctg(t/2).
414. (cost-tsint)/(1-sint-tcost). 421. k=-1. 422. $k_1=0$, $k_2=1$. 423. x=0,5. 424. x=-0,5. 425. a) x=2; b) x=-0,75. 426. x=1. 427. 45^0. 428. A(0;0) nuqtada π/2 burchak ostida, B(1;1) nuqtada arctg0,75 burchak ostida kesishadi.
429. arctg3. 430. $\mathrm{arctg}2\sqrt{2}$. 431. y-4x+6=0 – urinma tenglamasi, 4y+x+7=0 – normal tenglamasi. 432. y-x-1=0 – urinma tenglamasi, y+x-5=0 – normal tenglamasi. 433. y-7x+5=0 – urinma tenglamasi, 7y+x-15=0 – normal tenglamasi. 434. y-2x+5=0, y-6x+13=0. 435. $s=a^2$. 437. 4y-3x+13=0 – urinma tenglamasi, 3y+4x-9=0 – normal tenglamasi. 438. $y+2x-4\sqrt{2}=0$ – urinma tenglamasi, $2y-x-3\sqrt{2}=0$ – normal tenglamasi. 439. A(5;1), B(4;3). 440. y+1=0. 443. 2ln2+0,75; 5/6. 444. 8 J. 445. $1-29{,}4/100\sqrt{2}$ m/s. 446. 10π m/s. 447. $18m^2/s$. 448. 6. 449. (8;6) nuqtada -1,5; (-8;6) nuqtada 1,5. 450. Δy=0,005001; dy=0,005. 451. Δy=0,610801; dy=0,05. 452. $dy=-2x^{-3}dx$. 453. $-3(x-1)^{-2}dx$. 454. $dy=(2x/(1+x^2))\cdot dx$. 455. dy=sin2xdx. 456. $3dx/(1+9x^2)$. 457. $dy=e^x(\cos x-\sin x)dx$.
458. $dy=5^{x^2}(2\ln 5\cdot x\arccos\frac{1}{x}+\frac{1}{x\sqrt{x^2-1}})dx$. 459. $dy=\frac{2\sqrt{x}-\sin 2\sqrt{x}}{4\sqrt{x}\cos^2\sqrt{x}}dx$.
460. $dy=-3(1-x-x^2)^2(1+2x)dx$. 461. $dy=2\sin x dx/\cos^3 x$. 462. $dy=5^{\ln tgx}\cdot 2\ln 5\cdot dx/\sin 2x$. 463. dy=-dx/sin(x/2). 464. $dy=2^{1/\cos x}\cdot\sin x\cdot\ln 2\cdot dx/\cos^2 x$. 465.

$dy=(0,5/\sqrt{(1-x^2)\arcsin x}+2\mathrm{arctg}x/(1+x^2))dx$. 466. 1) 2,03; 2) 0,88; 3) 1,2; 4) 1,003; 5) 2,999; 6) 0,01; 7) 0,849; 8) 0,5. 467. 1) $dy=\frac{(2t^3+4t+7)(3t^2+2)dt}{3\sqrt[3]{(t^3+2t+1)(t^3+2t+6)^2}}$; 2) $dy=\frac{2\ln 3}{3^{\frac{1}{\ln tgt}}\cdot\ln^2 tgt}\cdot\frac{dt}{\sin 2t}$; 3) $ds=-0,5t\sin(0,5(t^2-1))dt$; 4) $dy=-dt$; 5) $dy=\frac{(4u-3)du}{2\sqrt{2u^2-3u+1}}$. 474. $y''=30x^4+4e^{2x}$. 475. $y^{IV}=2/x^3$. 476. $y''=24(2x+5)$. 477. $y'''=4xe^{-x^2}(3-2x^2)$. 479. $y''=5x\ln 5(1-\sqrt{x})/(4x)$. 480. $y''=-x/\sqrt{(x^2+1)^3}$.
481. $-4\cos 2x/\sin^2 2x$. 482. $f'''(2)=2488320$. 483. $f'''(0)=8/e$. 484. $y^{(n)}=(-1)^n e^{-x}$. 485. $y^{(n)}=a^n\sin(ax+n\pi/2)$. 486. $y^{(n)}=a^n\cos(ax+n\pi/2)$. 487. $y^{(n)}=2^{n-1}a^n\sin(2ax+(n-1)\pi/2)$. 488. $y^{(n)}=(-1)^{n-1}(n-1)!x^{-n}$. 489. $y^{(n)}=(-1)^{n-1}(n-1)!a^n(ax+b)^{-n}$. 490. $y^{(n)}=\mathrm{sh}x$, agar n-juft bo‘lsa, $y^{(n)}=\mathrm{ch}x$, agar n- toq bo‘lsa. 491. $y^{(n)}=4^{n-1}\cos(4x+n\pi/2)$. 492. $-4/(81t^4)$. 493. $-1/(a\sin^3 t)$. 494. $1/(3\sin t\cos^4 t)$. 495. 0. 496. $4t^2$. 497. $2/(t^2-1)$. 498. $3b/(8a^3t^5)$.
499. $-3\cos t/(a^2\sin 5t)$. 500. mavjud, toping. 508. $d^2y=-2dx^2/(9x\sqrt[3]{x})$.
509. $d^3y=m(m-1)(m-2)x^{m-3}dx^3$. 510. $d^3y=2dx^3/x^3$. 511. $d^3y=-4\sin 2x dx^3$.
512. $d^2y=\frac{ab(a^2-b^2)\sin 2x dx^2}{(a^2\cos^2 x+b^2\sin^2 x)^2}$. 513. $d^2y=\frac{4\ln x-4-\ln^3 x}{x^2\sqrt{(\ln^2 x-4)^3}}dx^2$. 514. $d^2y=-\frac{4(1+3x^4)}{(x^4-1)^2}dx^2$. 515. $d^ny=2^ne^{2x}dx^n$. 526. $\sin 3x_2-\sin 3x_1=3(x_2-x_1)\cos c$, $x_1<c<x_2$.
527. $b(1-\ln b)-a(1-\ln a)=(a-b)\ln c$, $a<c<b$. 528. $c=(5+\sqrt{97})/12$. 531. 14/9. 533. 3. 534. -1/18. 535. 0,5. 536. 1/18. 539. 0. 540. 5/3. 541. ∞. 542. ∞. 543. 0. 544. 2. 545. 0,5. 546. -1. 547. 0. 548. e^5. 549. 1. 550. $\sqrt{e}$. 551. 1. 552. e. 553. 1. 554. +∞.
555. (-∞;5) da o‘sadi, (5;+∞) da kamayadi. 556. (-∞;-1,5) da kamayadi, (-1,5;+∞) da o‘sadi. 557. (-∞;1) va (3;+∞) da o‘sadi, (1;3) da kamayadi. 558. (-∞;3) da kamayadi, (3;+∞) da o‘sadi. 559. (-∞;-6) da kamayadi, (-6;+∞) da o‘sadi.
560. (-∞;5) da o‘sadi, (5;+∞) da kamayadi. 561. (-∞;-7) va (-7;+∞) da o‘sadi. 562. (0;1/64) da o‘sadi, (1/64;+∞) da kamayadi. 563. (-∞;0,2) da o‘sadi, (0,2;+∞) da kamayadi. 564. $(0;\ 1/\sqrt[3]{e})$ da kamayadi, $(1/\sqrt[3]{e},\ +\infty)$ o‘sadi. 565. (-∞;0) da kamayadi, (0;+∞) da o‘sadi. 566. (-∞;+∞) da o‘sadi. 567. (0;π/6) da va (5π/6;π) da o‘sadi, (π/6; 5π/6) da kamayadi. 568. (-∞;+∞) da o‘sadi. 569. (-∞;+∞) da o‘sadi.
570. (0;1) va (1;e) da kamayadi, (e;+∞) da o‘sadi. 571. b≥2. 572. a≥1. 573. $y_{max}(3)=9$. 574. $y_{max}(0)=0$, $y_{min}(1)=1$. 575. $y_{max}(0)=1$. 576. $y_{max}(1)=0$, $y_{min}(1,4)=-0,03456$. 577. ekstremum yoq. 578. $y_{max}(2)=1,4$; $y_{min}(-4)=-1/8$. 579. $y_{max}(-2)=-32$, $y_{min}(2)=32$. 580. $y_{max}(0)=0$, $y_{min}(64)=-32$. 581. $y_{max}(-3)=-9\sqrt{3}$, $y_{min}(3)=9\sqrt{3}$. 582. $y_{max}(-4)=8e^{-4}$, $y_{min}(2)=-4e^2$. 583. $y_{min}(0)=0$, $y_{max}(2/3)=\sqrt[3]{12}/3\sqrt[3]{e^2}$.
584. $y_{min}(1/e)=-1/e$. 585. $y_{max}(-1)=(\pi-2)/4$, $y_{min}(1)=(2-\pi)/2$.
586. $y_{max}(-\pi/4+2\pi n)=\sqrt{2}$, $y_{min}(3\pi/4+2\pi n)=-\sqrt{2}$. 587. $y_{min}(0)=0$. 588. ekstremum yoq. 589. $y_{max}(1)=1$, $y_{min}(-0,5)=-1/(2\sqrt[4]{e^3})$. 590. $y_{max}(1)=2,5$; $y_{min}(e)=e(4-e)/2$. 592.

a=0. 593. -8; -20/3. 594. -12; 20. 595. 3; 5. 596. -14; 4. 597. $9\sqrt{3}$; 128/7. 598. -1/4; 1/4. 599. 0; e. 600. eng kichik qiymati yo‘q; eng katta qiymat -1. 601. π/3; π/2. 602. -π/2 eng kichik qiymat, eng katta qiymat mavjud emas. 603. Eng kichik qiymat yo‘q, eng katta qiymat 1. 604. $e^{-4}-e^{4}$; $e^{2}-e^{-2}$. 605. 0; 1. 606. 0; 2. 607. Eng kichik qiymat 0; eng katta qiymat yo‘q. 608. 1; 1/cos1. 609. -0,5. 610. 1. 611. $\sqrt{2}R$. 612. $2R\sqrt{3}/3$. 613. 4R/3. 614. $\sqrt{2}R$. 615. 4R. 616. konus asosining radiusi silindr asosining radiusidan 1,5 marta katta bo‘lishi kerak. 617. $H=\sqrt[3]{4}\,h/3$. 618. $(p/\sqrt[3]{2}$; $\sqrt[3]{2}\,p)$. 619. $C(-\sqrt{6};-\sqrt{6})$. 620. (-∞;1) da qavariq, (1;+∞) da botiq, A(1;2) burilish nuqta.

621. (-∞;+∞) da botiq, burilish nuqtasi yo‘q. 622. $(-\infty;-3\sqrt{3})$ va $(0;3\sqrt{3})$ da qavariq, $(-3\sqrt{3};0)$ va $(3\sqrt{3};+\infty)$ da botiq; $A(-3\sqrt{3};-\sqrt{3}/12)$, B(0;0), $C(3\sqrt{3};\sqrt{3}/12)$ burilish nuqtalari. 623. (-∞;-2) va (0;+∞) da botiq, (-2;0)da qavariq; $A(-2;-12\sqrt[3]{4})$, B(0;0) burilish nuqtalar. 624. (-∞;-7) da botiq, (-7;+∞) da qavariq, burilish nuqtasi yo‘q. 625. (-∞;-2) va (0;+∞) da botiq, (-2;0) da qavariq; A(-2;0) burilish nuqta.

626. (-∞;4) da qavariq, (4;+∞) da botiq; A(4;5) burilish nuqta.

627. $(-\pi/2, \arcsin\frac{\sqrt{5}-1}{2})$ botiq, $(\arcsin\frac{\sqrt{5}-1}{2}, \pi/2)$ da qavariq; $A(\arcsin\frac{\sqrt{5}-1}{2}; e^{\frac{\sqrt{5}-1}{2}})$ burilish nuqtasi. 628. (-∞;-1) va (1;+∞) da qavariq, (-1;1) da botiq; $A(-1,\sqrt{2})$, $B(1,\sqrt{2})$ burilish nuqtalari. 629. $(0; ae^{1,5})$ da qavavriq, $(ae^{1,5}; +\infty)$ da botiq, $A(ae^{1,5};1,5e^{-1,5})$ burilish nuqta. 630. (-∞;+∞) da botiq. 631. (-∞; 0,5) da botiq, (0,5; +∞) da qavariq, $A(0,5; e^{\mathrm{arctg}0,5})$ burilish nuqta. 634. a=-3. 635. a≤-e/6 va a>0 da. 636. a=-1,5; b=4,5. 637. A(1;4), B(1;-4). 638. t=3π/4+kπ, k∈Z bo‘ladigan nuqtalar burilish nuqta. 639. x=-5 vertikal asimptota, y=0 gorizontal asimptota. 640. x=4; y=0. 641. x=3; y=2. 642. x=-2, x=2, y=-x. 643. x=6, y=x+3, y=-x-3. 644. y=3x, y=x. 645. y=x. 646. y=0. 647. x=0, y=x+2. 648. x=-1/e; yx+1/e. 649. y=πx/2-1, y=-πx/2-1. 650. x=-1, x=1. 651. 1) x=-1, y=0; 2) y=0,5x-0,5;
y=-0,5x-0,5; 3) y=0,5x+e. 652. D(y)=(-∞;+∞); grafik ordinatalar o‘qiga nisbatan simmetrik; $y_{max}(0)=1$, $y_{min}(\pm0,5)=7/8$; burilish nuqtalari - $A(-\sqrt{3}/6; 67/72)$ va $B(\sqrt{3}/6;67/72)$; asimptotalari yo‘q. 653. D(y)=(-∞;+∞); grafik koordinata boshiga nisbatan simmetrik; $y_{max}(-1)=2$, $y_{min}(1)=-2$; burilish nuqtalari - A(0; 0), $B(0,1\sqrt{30}; -0,22\sqrt{30})$ va $S(-0,1\sqrt{30}; 0,22\sqrt{30})$; asimptotalari yo‘q. 654. D(y)=(-∞;+∞); $y_{min}(0,25)=-3\frac{51}{64}$; burilish nuqtalari - A(1; 0), B(0,5; -2,25); asimptotalari yo‘q. 655. D(y)=(-∞;+∞); grafik ordinatalar o‘qiga nisbatan simmetrik; $y_{min}(0)=-1$; burilish nuqtalari - A(-1; 0), B(1;0), $S(-\sqrt{5}/5;-64/125)$ va $D(\sqrt{5}/5;-64/125)$; asimptotalari yo‘q. 656. D(y)=(-∞;-5)∪(5;+∞); ekstremumlari yo‘q; burilish nuqtalari yo‘q; asimptotalari - x=-5, y=2. 657. D(y)=(-∞;+∞); grafik ordinatalar o‘qiga nisbatan simmetrik; $y_{max}(0)=0,25$; burilish nuqtalari - $A(-2\sqrt{3}/3; 3/16)$, $B(2\sqrt{3}/3; 3/16)$; asimptotasi y=0. 658. D(y)=(-∞; -2)∪(-2;2)∪(2;+∞); grafik koordinata boshiga nisbatan simmetrik; ekstremumlari yo‘q; burilish nuqta -

A(0; 0); asimptotalari x=-2, x=2, y=0. 659. D(y)=(-∞; -4)∪(-4;4)∪(4;+∞); grafik ordinata o'qiga nisbatan simmetrik; $y_{min}(0)=0,5$; burilish nuqtalari yo'q; asimptotalari x=-4, x=4, y=0. 660. D(y)=(-∞; -1)∪(-1;1)∪(1;+∞); grafik ordinata o'qiga nisbatan simmetrik; $y_{min}(0)=-6$; burilish nuqtalari yo'q; asimptotalari x=-1, x=1, y=1. 661. D(y)=(-∞; 3)∪(3;+∞); $y_{max}(0)=0$, $y_{min}(6)=12$; burilish nuqtalari yo'q; asimptotalari x=3, y=x+3. 662. D(y)=(-∞; -1)∪(-1;1)∪(1;+∞); grafik koordinata boshiga nisbatan simmetrik; $y_{max}(\sqrt{3})=-3\sqrt{3}/2$, $y_{min}(-\sqrt{3})=3\sqrt{3}/2$; burilish nuqtalari yo'q; asimptotalari x=-1, x=1, y=-x. 663. D(y)=(-∞;0) ∪(0;+∞); $y_{min}(-\sqrt[3]{4})=6\sqrt[3]{2}$; burilish nuqtasi - A(2; 0); asimptotasi x=0. 664. D(y)=[0,+∞), $y_{max}(1/16)=1/8$; burilish nuqtalari yo'q; asimptotalari yo'q. 665. D(y)=[0,+∞), $y_{min}(1)=-1$; burilish nuqtasi A(729/64; 729/16); asimptotalari yo'q.

666. D(y)=(-∞;0) ∪(0;+∞); $y_{min}(1)=e$; burilish nuqtasi yo'q; asimptotasi y=0. 667. D(y)=(-∞; +∞); $y_{max}(1)=1/e$; burilish nuqtasi yo'q; asimptotasi y=0. 668.

D(y)=(-∞;+∞); grafik ordinata o'qiga nisbatan simmetrik; $y_{max}(0)=1$; burilish nuqtalari $A(-1/\sqrt{2}; e^{-0,5})$, $B(1/\sqrt{2}; e^{-0,5})$; asimptotasi y=0. 669. D(y)=(-∞;+∞); grafik ordinata o'qiga nisbatan simmetrik; $y_{min}(0)=1$; burilish nuqtalari yo'q; asimptotalari yo'q. 670. D(y)=(-∞;0)∪(0;+∞); ekstremumlari yo'q; burilish nuqta- $A(-0,5;e^{-2})$; asimptotalari x=0, y=1. 671. D(y)=(-∞;0)∪(0;+∞); $y_{min}(0,5)=e^2/4$; burilish nuqtalari yo'q; asimptotasi x=0. 672. D(y)=(-∞;+∞); $y_{max}(0,75)=(0,75/e)^3$; burilish nuqtalari-A(0;0), $B((3-\sqrt{3})/4, ((3-\sqrt{3})/4)^3e^{(\sqrt{3}-3)})$, $C((3+\sqrt{3})/4, (3+\sqrt{3})/4)^3e^{(-\sqrt{3}-3)})$; asimptotasi y=0. 673. D(y)=(-∞;+∞); grafik koordinata boshiga nisbatan simmetrik; $y_{max}(1)=1/\sqrt{e}$, $y_{min}(-1)=-1/\sqrt{e}$; burilish nuqtalari-A(0;0), $B(\sqrt{3};\sqrt{3}e^{-1,5})$, $C(-\sqrt{3};-\sqrt{3}e^{-1,5})$; asimptotalari y=0, y=x. 674. D(y)=(-∞;0) ∪(0;+∞); ekstremumlar yo'q; burilish nuqtalari yo'q; asimptotalari x=0, y=0, y=-1. 675. D(y)=(0;+∞); ekstremumlar yo'q; burilish nuqtasi $A(e^{1,5}; e^{1,5}+1,5e^{-1,5})$; asimptotalari x=0, y=x. 676. D(y)=(-∞;-1) ∪(0;+∞); ekstremumlar yo'q; asimptotalari x=-1, y=e. 677. (-π/2+2kπ;π/2+2kπ), k∈Z intervallarda aniqlangan; grafik ordinata o'qiga nisbatan simmetrik; $y_{max}(2k\pi)=0$; burilish nuqtalari yo'q; asimptotalari x=π/2+kπ, k∈Z. 678. D(y)=(-∞; +∞); grafik koordinata boshiga nisbatan simmetrik; $y_{max}(\sqrt{3})=-3\sqrt{3}/2$, $y_{min}(-\sqrt{3})=3\sqrt{3}/2$; burilish nuqtalari $A_k(kx,kx)$, k∈Z; asimptotalari yo'q. 679. D(y)=(-∞;0)∪(0;+∞); grafik ordinata o'qiga nisbatan simmetrik; abstsissasi tgx=x tenglamani qanoatlantiruvchi nuqtalarda ekstremumlar mavjud; burilish nuqtalari- $A_k(k\pi,0)$, k∈Z; asimptotasi y=0. 680. D(y)=(-∞; +∞); grafik koordinata boshiga nisbatan simmetrik; $y_{max}(-1)=-1+\pi/2$, $y_{min}(1)=1-\pi/2$; burilish nuqtasi A(0;0); asimptotasi y=x+π. 681. D(y)=(-∞;0)∪(0; +∞); grafik koordinata boshiga nisbatan simmetrik; $y_{max}(2/(\pi+2k\pi))=1$, k=0, ±2, ±4, …, $y_{min}(2/(\pi+2k\pi)=-1$, k=±1, ±3, … ; asimptotasi y=0. 682. D(y)=(-∞;+∞); grafik ordinata o'qiga nisbatan simmetrik; $y_{max}(-1)=1$, $y_{max}(1)=1$, $y_{min}(0)=0$; burilish nuqtalari yo'q; asimptotalari yo'q.

683. D(y)=(-∞;+∞); $y_{max}(8/27)=4/27$, $y_{min}(0)=0$; burilish nuqtalari yo'q; asimptotalari yo'q. 684. (2kπ; (2k+1)π), k∈Z intervallarda aniqlangan; $y_{max}(\pi/2+2k\pi)=0$, k∈Z; burilish nuqtalari yo'q; asimptotalari x=kπ, k∈Z. 685.

$(3\pi/2+2k\pi;\ 3\pi/2+(2k+1)\pi)$, $k\in Z$ intervallarda aniqlangan; $y_{min}(2k\pi)=1$, $k\in Z$; burilish nuqtalari yo‘q; asimptotalari $x=\pi/2+k\pi$, $k\in Z$. 686. $D(y)=(-\infty;+\infty)$; $y_{max}(-3)=3$, $y_{min}(5)=-1$; burilish nuqtasi A(1;1); asimptotasi yo‘q.
687. $D(y)=(-\infty;+\infty)$; $y_{max}(-1-3\pi)=-1+3\pi/2$, $y_{min}(1-3\pi)=1-3\pi/2$; burilish nuqtasi $A(-3\pi;0)$; asimptotalari $y=x$, $y=x+6\pi$. 688. Uch yoproqli yopiq atirgul. $D(\rho)=[0;\pi/3]\cup[2\pi/3;\pi]\cup[4\pi/3;5\pi/3]$; $\varphi=\pi/6$, $\varphi=5\pi/6$ va $\varphi=3\pi/2$ larda ekstremumlar mavjud. 689. $D(\rho)=[0;\pi/2]\cup[\pi;3\pi/2]$; grafik qutbga nisbatan simmetrik, asimptotalari $x=a$, $x=-a$; (bu erda abstsissa o‘qi qutb o‘qidan iborat, ordinata o‘qi qutbdan o‘tuvchi va qutb o‘qiga perpendikulyar to‘g‘ri chiziqdan iborat deb qaraladi). 690. $D(\rho)=[0;2\pi]$; $\rho_{max}(0)=2a$, $\rho_{min}(\pi)=0$; chiziq qutb o‘qiga nisbatan simmetrik. Bunday chiziq kardioida nomi bilan ataladi. 691. $D(\rho)=[0;2\pi]$; $\rho_{max}(\pi/2)=2a$, $\rho_{min}(3\pi/2)=0$; chiziq ordinata o‘qiga nisbatan simmetrik. 692. $f(x)=1+4(x+1)-3(x+1)^2-2(x+1)^3+(x+1)^4$. 693. $f(x)=-56+21(x-4)+37(x-4)^2+11(x-4)^3+(x-4)^4$. 694. $f(x)=1-9x+30x^2-45x^3+30x^4-9x^5+x^6$. 695. $f(x)=-1-(x+1)-(x+1)^2-\ldots-(x+1)^n+(-1)^n\dfrac{(x+1)^{n+1}}{(-1+\theta(x+1))^{n+2}}$, bu erda $0<\theta<1$.

696. $f(x)=x+\dfrac{x^2}{1!}+\dfrac{x^3}{2!}+\ldots+\dfrac{x^n}{(n-1)!}+\dfrac{x^{n+1}}{(n+1)!}(\theta x+n+1)e^{\theta x}$, bu erda $0<\theta<1$. 697. $f(x)=2+\dfrac{x-4}{4}-\dfrac{(x-4)^2}{64}+\dfrac{(x-4)^3}{512}-\ldots+(-1)^{n-1}\dfrac{(2n-2)!(x-4)^n}{n!(n-1)!2^{4n-2}}+\dfrac{(-1)^n(2n)!(x-4)^{n+1}}{2^{2n+1}n!(n+1)!\sqrt{(4+\theta(x-4))^{2n+1}}}$, bu erda $0<\theta<1$.

698. $f(x)=(x-1)+\dfrac{5}{2!}(x-1)^2+\dfrac{1}{3!}(x-1)^3+$
$+\dfrac{6(x-1)^4}{4!}+\ldots+\dfrac{(-1)^n6(x-1)^n}{(n-3)(n-2)(n-1)n}+\dfrac{(-1)^{n+1}6(x-1)^{n+1}}{(n-2)(n-1)n(n+1)(1+\theta(x-1))^{n-2}}$, bu erda $0<\theta<1$.

699. $f(x)=\dfrac{2x^2}{2!}-\dfrac{2^3x^4}{4!}+\dfrac{2^5x^6}{6!}-\ldots+(-1)^{n-1}\dfrac{2^{2n-1}x^{2n}}{(2n)!}+(-1)^n\dfrac{2^{2n}x^{2n+1}}{(2n+1)!}\sin 2\theta$, bu erda $0<\theta<1$.

700. $\dfrac{1}{5}x^5-x^4+x^2+C$. 701. $\dfrac{1}{2}t^4+2t^3+C$. 702. $\dfrac{x^3}{3}+\dfrac{3x^2}{2}-4x+C$.

703. $x^5+\dfrac{1}{2}x^4-x^3+C$. 704. $\dfrac{3}{2}x\sqrt{x}+\ln|x|+C$. 705. $-\dfrac{2}{\sqrt{x}}+\dfrac{1}{x}+C$.

706. $\dfrac{4}{7}x\sqrt[4]{x^3}+\dfrac{2}{3}x^2\sqrt{x}+C$. 707. $2\sqrt{x}(1+\sqrt{x}+\dfrac{x}{3})+C$.

708. $-\dfrac{2}{\sqrt{x}}(1+3x-x^2-\dfrac{x^3}{5})+C$. 709. $\dfrac{2^ч}{\ln 2}+C$. 710. $3x-2\cdot 1{,}5^x\log_{1,5}e+C$.

711. $\frac{a^x e^x}{\ln a + 1} + C$. 712. $2e^x - \frac{3}{5}x^{\frac{5}{3}} + C$. 713. $\frac{10^x}{\ln 10} + 2\cos x + C$.

714. $0{,}5tgx + 0{,}5x + C$. 715. $-ctgx - tgx + C$. 716 $tgx - x + C$. 717. $-ctgx - x + C$.

718. $0{,}5x - 0{,}5\sin x + C$. 719. $tgx + C$. 720. $-\frac{1}{x} + arctgx + C$.

721. $\ln|x| + 2arctgx + C$. 722. $\frac{1}{15}(x+1)^{15} + C$. 723. $-\frac{1}{3}(8-2x)\sqrt{8-2x} + C$.

724. $\frac{5}{21}(3x+1)\sqrt[5]{(3x+1)^2} + C$. 725. $\frac{1}{3}(1+x^2)\sqrt{1+x^2} + C$.

726. $-\frac{1}{3}(1-x^2)\sqrt{1-x^2} + C$. 727. $\sqrt{1+x^2} + C$. 728. $\frac{1}{5}\sqrt{4+x^5} + C$

729. $\frac{\sin^4 x}{4} + C$. 730. $-\frac{1}{\cos x} + C$. 731. $\frac{2}{3}\ln x\sqrt{\ln x} + C$. 732. $2\sqrt{\ln x} + C$.

733. $\frac{arctg^3 x}{3} + C$. 734. $-\frac{1}{2\arcsin^2 x} + C$. 735. $\frac{1}{3}\sin 3x + C$.

736. $-0{,}5\cos(2x-1) + C$. 737. $-0{,}5\sin(1-2x) + C$. 738. $-\cos e^x + C$.

739. $0{,}5\ln|2x-1| + C$. 740. $0{,}5\ln(x^2+1) + C$. 741. $\ln|\sin x| + C$. 742. $\ln|\ln x| + C$.

743. $\ln(1+\cos^2 x) + C$. 744. $0{,}5e^{2x} + C$. 745. $-\frac{1}{3}e^{-3x} + C$. 746. $e^{\sin x} + C$.

747. $0{,}5e^{x^2} + C$. 748. $-\frac{1}{3}e^{-x^3} + C$. 749. $0{,}5\arcsin 2x + C$. 750. $0{,}2\arcsin 5x + C$.

751. $\arcsin\frac{x}{2} + C$. 752. $\frac{1}{3}arctg3x + C$. 753. $0{,}5arctgx^2 + C$.

754. $0{,}25\sin 2x - 0{,}5x\cos 2x + C$. 755. $x\sin x + \cos x + C$. 756. $(x-1)e^x + C$.

757. $\frac{x^2+1}{2}arctgx - \frac{x}{2} + C$. 758. $x\arccos x - \sqrt{1-x^2} + C$.

759. $(x+1)arctg\sqrt{x} - \sqrt{x} + C$. 760. $C - (x^2+2x+2)e^{-x}$.

761. $(x^3 - 3x^2 + 6x - 6)e^x + C$. 762. $C - (x^3 - 6x)\cos x + (3x^2 - 6)\sin x$.

763. $\frac{1}{6}x^3 + \frac{1}{8}(2x^2 - 1)\sin 2x + \frac{1}{4}x\cos 2x + C$. 764. $C - \frac{x}{2(1+x^2)} + 0{,}5arctgx$.

765. $\frac{1}{3}(x^2+1)\ln(1+x) - \frac{x^3}{9} + \frac{x^2}{6} - \frac{x}{3} + C$. 766. $2(\sqrt{x+1} - \ln(1+\sqrt{x+1}) + C$.

767. $\ln\left|\frac{\sqrt{x+1}-1}{\sqrt{x+1}+1}\right| + C$. 768. $\frac{2\sqrt{x-1}}{35}(5x^3 + 6x^2 + 8x + 16) + C$.

769. $2(\sqrt{x} - \ln(1+\sqrt{x})) + C$. 770. $2\sqrt{x-2} + \sqrt{2}arctg\sqrt{\frac{x-2}{2}} + C$.

771. $2arctg\sqrt{x} + C$. 772. $\frac{3}{2}(x+1)^{\frac{2}{3}} - 3(x+1)^{\frac{1}{3}} + 3\ln|1+\sqrt[3]{x+1}| + C$.

773. $\sqrt{2x+1}-3\ln(3+\sqrt{2x+1})+C$. 774. $3\sqrt[3]{x}+3\ln|\sqrt[3]{x}-1|+C$.

775. $\ln\dfrac{\sqrt{1+e^x}-1}{\sqrt{1+e^x}+1}+C$. 776. $0{,}5\ln|2x+1|+C$. 777. $-\dfrac{1}{3}\ln|1-3x|+C$.

778. $\dfrac{1}{3}arctg(x+5)+C$. 779. $\dfrac{1}{3}arctg\dfrac{2x+1}{3}+C$.

780. $\dfrac{1}{2}\ln|x^2+x-12|+\dfrac{9}{14}\ln\left|\dfrac{x+4}{x-3}\right|+C$. 781. $\ln(x^2-4x+13)+\dfrac{1}{9}arctg\dfrac{x-2}{3}+C$.

782. $-\dfrac{5}{8}\ln|4x^2+16x-9|+\dfrac{7}{5}\ln\left|\dfrac{2x-1}{2x+9}\right|+C$.

783. $\dfrac{2}{3}\ln(9x^2-6x+2)+5arctg(3x-1)+C$. 784. $-\dfrac{1}{2(x-2)^2}+C$.

785. $-\dfrac{1}{3(3x-1)}+C$. 786. $\dfrac{1}{x^2+2x+2}+2arctg\dfrac{x-1}{2}+C$.

787. $\dfrac{2-x}{4(x^2+2)}+\dfrac{1}{2}\ln(x^2+2)-\dfrac{1}{4\sqrt{2}}arctg\dfrac{x}{\sqrt{2}}+C$.

788. $2\ln|x-2|+\ln|x+5|+C$. 789. $4\ln|x-1|-\dfrac{5}{3}\ln|3x+1|+C$

790. $-x+3\ln|x-1|-2\ln|x-3|+C$. 791. $6\ln|x|-2\ln|x+4|-3\ln|x-3|+C$.

792. $\dfrac{x^2}{2}+\dfrac{7}{8}\ln|x-4|-\dfrac{3}{8}\ln|x+4|-\dfrac{1}{2}\ln|x|+C$.

793. $3\ln|x|+3\ln|x+1|-7\ln|x+2|+C$. 794. $\dfrac{1}{4}\ln|x-5|-\dfrac{1}{4}\ln|x+3|-\dfrac{1}{x+3}+C$.

795. $\ln|x-2|-\dfrac{9}{x-2}-\dfrac{11}{(x-2)^2}+C$. 796. $\ln|x-5|-\dfrac{1}{x}-\dfrac{4}{x-5}+C$.

797. $-\dfrac{3x}{2}+\dfrac{2}{x-1}+3\ln|x-1|-\dfrac{13}{4}\ln|1-2x|+C$.

798. $\ln|x+1|+\dfrac{3}{4}\ln|x^2-4x+13|-\dfrac{1}{3}arctg\dfrac{x-2}{3}+C$.

799. $\ln(x^2+9)-\ln|x-3|-\dfrac{1}{3}arctg\dfrac{x}{3}+C$.

800. $\dfrac{5x^2}{2}-x+8\ln|x-2|-2\ln(x^2+2x+4)+\dfrac{20}{\sqrt{3}}arctg\dfrac{x+1}{\sqrt{3}}+C$.

801. $\dfrac{1}{2}\ln(x^2+4)-\dfrac{9}{2}arctg\dfrac{x}{2}-\dfrac{2}{x}+C$.

802. $\ln|x+1|-7\ln|x-1|+3{,}5\ln(x^2-2x+2)+3arctg(x+1)+C$.

803. $2{,}5\ln(x^2+9)-3arctg\dfrac{x}{3}-0{,}5\ln(x^2+2x+2)+3arctg(x+1)+C$.

804. $\frac{2-x}{4(x^2+2)}+\frac{1}{2}\ln(x^2+2)-\frac{1}{4\sqrt{2}}arctg\frac{x}{\sqrt{2}}+C$.

805. $\frac{3x-17}{2(x^2-4x+5)}+\frac{1}{2}\ln(x^2-4x+5)+\frac{15}{2}arctg(x-2)+C$.

806. $1{,}5\ln(\sqrt[3]{x^2}+1)+C$. 807. $2\sqrt{x+9}+3\ln\left|\frac{\sqrt{x+9}-3}{\sqrt{x+9}+3}\right|+C$.

808. $2\sqrt{x}+6\sqrt[3]{x}+24\sqrt[6]{x}+48\ln|\sqrt[6]{x}-2|+C$.

809. $4\sqrt[4]{x}+2\ln(1+\sqrt{x})-4arctg\sqrt[4]{x}+C$.

810. $1{,}2\sqrt[6]{x^5}-2\sqrt{x}-3\sqrt[3]{x}+3\ln(\sqrt[3]{x}-\sqrt[6]{x}+1)+2\sqrt{3}arctg\frac{2\sqrt[6]{x}-1}{\sqrt{3}}+C$.

811. $1{,}5\ln\left|\frac{1+\sqrt[6]{x}}{1-\sqrt[6]{x}}\right|-3arctg\sqrt[6]{x}+C$.

812. $4\sqrt[4]{x}-6\sqrt[6]{x}+12\sqrt[12]{x}+2\ln|x|-36\ln(\sqrt[12]{x}+1)+C$.

813. $-\frac{2}{5}\sqrt{\frac{x+5}{x}}+C$. 814. $\frac{9}{4}\ln\left|\frac{t-2}{t+2}\right|-\frac{9t}{t^2-4}+C, t=\sqrt{(4x-5)/(x+1)}$.

815. $(1-0{,}5x)\sqrt{1-x^2}-1{,}5\arcsin x+C$.

816. $\ln|1+x+\sqrt{x^2+2x+2}|+\frac{2}{x+2+\sqrt{x^2+2x+2}}+C$.

817. $C-\frac{1}{\sqrt{2}}\ln\left|\frac{\sqrt{x+2-x^2}+\sqrt{2}}{x}+\frac{1}{2\sqrt{2}}\right|$. 818. $\frac{1-\sqrt{1-x^2}}{x}+2arctg\sqrt{\frac{1+x}{1-x}}+C$.

819. $\ln|x+1+\sqrt{2x+x^2}|-\frac{4}{x+\sqrt{2x+x^2}}+C$.

820. $\frac{3-x}{2}\cdot\sqrt{1-2x-x^2}+2\arcsin\frac{x+1}{\sqrt{2}}+C$.

821. $\frac{2}{27}(\frac{1}{t}-2t-\frac{t^3}{3})+C,\quad t=\frac{\sqrt{7x-x^2-10}}{x-2}$.

822. $\frac{2}{3}x\sqrt{x}+\frac{24}{11}x\sqrt[6]{x^5}+\frac{36}{13}x^2\sqrt[6]{x}+\frac{8}{5}x^2\sqrt{x}+\frac{6}{17}x^2\sqrt[6]{x^5}+C$.

823. $3\left(\ln\left|\frac{\sqrt[3]{x}}{1+\sqrt[3]{x}}\right|+\frac{2\sqrt[3]{x}+3}{291+\sqrt[3]{x})^2}\right)+C$. 824. $0{,}5\ln(\sqrt[3]{x^2+1}-1)$-

$-0{,}25\ln(\sqrt[3]{(x^2+1)^2}+\sqrt[3]{x^2+1}+1)+0{,}5\sqrt{3}arctg\frac{2\sqrt[3]{x^2+1}+1}{\sqrt{3}}+C$

825. $\frac{1}{8}\sqrt[3]{(1+x^3)^8}-\frac{1}{5}\sqrt[3]{(1+x^3)^5}+C$.

826. $\frac{1}{6}\ln\frac{t^2+t+1}{(t-1)^2}-\frac{1}{\sqrt{3}}arctg\frac{2t+1}{\sqrt{3}}+C,\quad t=\frac{\sqrt[3]{x^3+1}}{x}$.

827. $\frac{1}{4}\ln\frac{\sqrt[4]{1+x^4}+x}{\sqrt[4]{1+x^4}-x}-\frac{1}{2}arctg\frac{\sqrt[4]{1+x^4}}{x}+C$.

828. $C-\frac{1}{10}\sqrt{\left(\frac{1+x^4}{x^4}\right)^5}+\frac{1}{3}\sqrt{\left(\frac{1+x^4}{x^4}\right)^3}-\frac{1}{2}\sqrt{\frac{1+x^4}{x^4}}$.

829. $\frac{t}{2(t^3+1)}-\frac{1}{6}\ln\frac{t+1}{\sqrt{t^2-t+1}}-\frac{1}{2\sqrt{3}}arctg\frac{2t-1}{\sqrt{3}}+C,\ t=\sqrt[3]{\frac{1-x^2}{x^2}}$.

830. $\frac{1}{2}\ln\left|\frac{tg\frac{x}{2}+2}{tg\frac{x}{2}-2}\right|+C$. 831. $0{,}5\cos^2 x-4\cos x+15\ln(4+\cos x)+C$.

832. $\frac{1}{3\sin^3 x}-\frac{1}{5\sin^5 x}+C$. 833. $\frac{1}{15}\cos^3 x(3\cos^2 x-5)+C$.

834. $\frac{1}{3\cos^3 x}-\frac{1}{\cos x}+C$. 835. $\ln|tgx|-\frac{1}{2\sin^2 x}+C$.

836. $\frac{1}{3\cos^3 x}+\frac{2}{\cos x}-\frac{\cos x}{2\sin^2 x}+\frac{5}{2}\ln|tg\frac{x}{2}|+C$. 837. $tgx+\frac{1}{4}\sin 2x-\frac{3}{2}x+C$.

838. $\frac{1}{3}tg^3x+2tgx-ctgx+C$. 839. $C-\frac{1}{1+tgx}$. 840. $\frac{1}{3}\ln\left|\frac{tg\frac{x}{2}-2}{tg\frac{x}{2}-1}\right|+C$.

841. $-\frac{1}{tg\frac{x}{2}+2}+C$. 842. $tg\frac{x}{2}+\frac{1}{4}tg^2\frac{x}{2}+\frac{1}{2}\ln|tg\frac{x}{2}|+C$.

843. $-\cos x+\frac{2}{3}\cos^3 x+\frac{1}{5}\cos^5 x+C$. 844. $\sin x-\sin^3 x+\frac{3}{5}\sin^5 x-\frac{1}{7}\sin^7 x+C$.

845. $\frac{1}{8}\cos 4x-\frac{1}{20}\cos 10x+C$. 846. $\frac{1}{14}\sin 7x-\frac{1}{22}\sin 11x+C$.

847. $\frac{1}{4}(\frac{1}{16}\sin 16x+\frac{1}{10}\sin 10x+\frac{1}{6}\sin 6x+x)+C$.

848. $\frac{1}{4}(\frac{1}{16}\cos 16x-\frac{1}{2}\cos 2x-\frac{1}{6}\cos 6x-\frac{1}{8}\cos 8x)+C$.

849. $\frac{1}{4}\left(\frac{\sin 6x}{6}+\frac{\sin 4x}{4}-\frac{\sin 10x}{10}-x\right)+C$.

857. 1) $\sin x$; 2) $-\sqrt{1+x^2}$; 3) $\frac{\sin 2x}{x}$; 4) $\sin e^x$; 5) $-2xe^{nx^2}$; 6) $2\ln^2 2x-\ln^2 x$.

858. 1) $\frac{dy}{dx}=ctgt$ 2) $\frac{dy}{dx}=-t^2$. 859. $y_{\max}=y(0,5)=\frac{1}{2e^4}$. 860. $\frac{19}{3}$. 861. 42. 862. 2. 863. 125. 864. 1. 865. π/4. 866. π. 867. π/4-1. 868.4/45. 869. π/72. 870. π/6. 871. $0,1\ln\frac{9}{4}$. 871. –ln7/12. 873. ln3/4. 874. $\frac{2(2\sqrt{2}-1)}{3}$. 875. 1/9. 876. 4. 877. $\frac{\pi}{6\sqrt{3}}$. 878. $\frac{\pi}{6\ln 3}$. 879. $2\sqrt{3}-2-\frac{\pi}{6}$. 880. 6+4ln2. 881. $\frac{\pi}{6}+\frac{\sqrt{3}}{8}$. 882.3π. 883.e-1. 884. 2/e-1. 885. π/2-1. 886. π/2-2. 887. $\frac{\pi-2\ln 2}{4}$. 888.π-1. 889. $\frac{\pi\sqrt{3}}{6}+\ln 2$. 890. 3e-5. 891.4/π. 892. 1. 893. $2-\frac{3}{4\ln 2}$. 894. 20/9. 890. $2\ln\frac{6}{5}$. 895. 1/2. 897. $2+\ln\frac{2}{e^2+1}$. 898.uzoqlashuvchi. 899. π/4. 900. 0,5. 901. uzoqlashuvchi. 902. uzoqlashuvchi. 903. π/2. 904. 0,5ln2. 905. 2. 906. uzoqlashuvchi. 907. uzoqlashuvchi. 908. Uzoqlashuvchi. 909. 2. 910.-1,5. 911.π/2. 912. uzoqlashuvchi. 913. 2. 914. 1/e. 915. uzoqlashuvchi. 916. 2. 917. uzoqlashuvchi. 918. -0,25. 919. 10/7. 920. 1) yaqinlashuvchi; 2) yaqinlashuvchi; 3) yaqinlashuvchi. 921. 80/3. 922. ln3. 923. 64/3. 924. 4. 925. 48. 926. 4-ln3. 927. 36. 928. 128/3. 929. 343/3. 930. 4π-8. 931. 2π+4/3 va 6π-4/3. 932. 16/3. 933. 1/12. 934. *πab*.935. 3πa^2. 936. 3πa^2/8. 937. 6πa^2. 938. πa^2/4. 939. 1,5πa^2.940. a^2. 941. 18πa^2. 942. 335/27. 943. 0,5ln3. 944. $ash\frac{b}{a}$. 945. $\frac{y}{2p}\sqrt{y^2+p^2}+\frac{p}{2}\ln\frac{y+\sqrt{y^2+p^2}}{p}$. 946. 2πR. 947. 6*a*.948. $1+\frac{1}{2}\ln\frac{3}{2}$. 949. 6*a*. 950. 8a. 951. $\ln\frac{3}{2}+\frac{5}{12}$. 952. 8a. 953. 5π. 954. 38π/3. 955. 48π. 956. 91,6π. 957. $34\frac{2}{15}\cdot\pi$. 958. 64π/3. 959. 8π/3. 960. 39,6π. 961. 2πr^2b. 962. 32π/3. 963. 32πa^3/105. 964. $\frac{2}{3}\pi^2a^2(\pi^2-6)$. 965. $5\pi^2a^3$. 966. 32πa^3/105. 967. 4πab/3. 968. 61π/1728. 969. $24\pi\sqrt{10}$. 970. $\frac{2\pi}{3}(5\sqrt{5}-1)$. 971. $\pi(\sqrt{2}+\ln(1+\sqrt{2}))$. 972. $4\pi r^2$. 973. 12πa^2/5. 974. $\frac{64}{3}\pi a^2$. 975. $4\pi a^2b$.976. 32πa^2/5. 977. 4π^2. 978. 2πa^2(2-√2). 979. $\sqrt{2}+\ln(1+\sqrt{2})$. 980. $\frac{b^2}{2}+\frac{a^2b}{2\sqrt{a^2-b^2}}\arcsin\frac{\sqrt{a^2-b^2}}{a}$. 981. (0;2a/π). 982. (29/5;29/5). 983. (0; $a\frac{e^4+4e^2-1}{4e(e^2-1)}$). 984. (πa;4a/3). 985. (0;π/8). 986. (0;$\frac{4a}{3\pi}$).

987. $(\frac{4a}{3\pi};\frac{4b}{3\pi})$. 988. ($\pi$a;5a/6). 989. a) $9\pi a^3/2$, b) $6\sqrt{3}\pi a^2$. 990. $6\pi a^3$; $16\pi^2 a^2$. 991. $a_n=\frac{1}{n^2}$. 992. $a_n=(-1)^{n-1}\frac{n}{n+1}$. 993. $a_n=\frac{n}{2^{n-1}}$. 994. $a_n=\frac{(-1)^{n-1}}{n^n}$.
995. $a_n=(-1)^{n-1}$. 996. $a_n=\frac{\sqrt{n+1}}{(n+1)!}$. 997. a_1=1; a_2=3/5; a_3=2/5; a_4=5/17; a_5=3/13.
998. a_1=1/2; a_2=1/5; a_3=1/8; a_4=1/11; a_5=1/14. 999. a_1=√2; a_2=√3/2; a_3=2/3; a_4=√5/4; a_5=√6/5. 1000. a_1=1; a_2=1; a_3=11/13; a_4=7/10; a_5=17/29. 1001. a_1=0; a_2=1/32; a_3=-1/32; a_4=3/128; a_5=1/64. 1002. a_1=1/5; a_2=3/8; a_3=1/13; a_4=3/20; a_5=1/29. 1003. $S_n=\frac{3}{2}\left(1-\frac{1}{3^n}\right)$, S=1,5. 1004. $S_n=\frac{2}{3}\left(1-(-1)^n\cdot\frac{1}{2^n}\right)$, S=2/3.
1005. $S_n=\frac{n}{n+1}$, S=1. 1006. $S_n=\frac{n}{2n+1}$, S=0,5. 1007. $S_n=\frac{1}{3}\left(1+\frac{1}{2}+\frac{1}{3}-\frac{1}{n+1}-\frac{1}{n+2}-\frac{1}{n+3}\right)$, S=11/18. 1008. $S_n=\frac{1}{8}\left(1-\frac{1}{(2n+1)^2}\right)$, S=0,125. 1009. $S_n=1-\frac{1}{(n+1)^2}$, S=1. 1010. $S_n=\frac{1}{2}\left(\frac{1}{2}-\frac{1}{(n+1)(n+2)}\right)$, S=0,25.
1011. $S_n=\frac{1}{18}\left(\frac{1}{n}-\frac{2}{n+3}+\frac{1}{n+6}\right)$, S=1/18. 1012. S=$\pi$/4.
1018.Yaqinlashuvchi. 1019.uzoqlashuvchi. 1020. uzoqlashuvchi.
1021.uzoqlashuvchi. 1022.Yaqinlashuvchi. 1023.Yaqinlashuvchi.
1024.uzoqlashuvchi. 1025.uzoqlashuvchi. 1026.Yaqinlashuvchi.
1027. uzoqlashuvchi. 1028. uzoqlashuvchi. 1029. Yaqinlashuvchi.
1030. Yaqinlashuvchi.1031. Yaqinlashuvchi. 1032. Yaqinlashuvchi.
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1039.uzoqlashuvchi. 1040. Yaqinlashuvchi. 1041. Yaqinlashuvchi.
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1045. uzoqlashuvchi. 1046. Yaqinlashuvchi. 1047. uzoqlashuvchi.
1048. uzoqlashuvchi. 1049. Yaqinlashuvchi. 1050. Yaqinlashuvchi.
1051. uzoqlashuvchi. 1052. Absolyut yaqinlashuvchi. 1053.shartli yaqinlashuvchi.
1054. uzoqlashuvchi. 1055. shartli yaqinlashuvchi. 1056. uzoqlashuvchi.
1057. shartli yaqinlashuvchi. 1058. uzoqlashuvchi. 1059. Absolyut yaqinlashuvchi.
1060. 5 ta. 1061. 5 ta. 1062. Absolyut yaqinlashuvchi. 1063. shartli yaqinlashuvchi. 1064. Absolyut yaqinlashuvchi. 1065. Absolyut yaqinlashuvchi.
1066. shartli yaqinlashuvchi. 1067. Absolyut yaqinlashuvchi. 1068. Absolyut yaqinlashuvchi. 1069. Absolyut yaqinlashuvchi. 1070. Absolyut yaqinlashuvchi.
1071. shartli yaqinlashuvchi. 1074. uzoqlashuvchi. 1075. absolyut yaqinlashuvchi.
1076. uzoqlashuvchi. 1078. $-1<x<1$. 1079. $-0{,}5<x<0{,}5$. 1080. $|x|>1$.

1081. -0,5≤x≤0,5. 1082. x≠±1. 1083. -1<x<1. 1084. x>1. 1085. x>1. 1086. x≤-2, x>0. 1087.-∞<x<+∞. 1096. Mumkin. 1097. Mumkin. 1100. $\frac{1}{4}\ln\frac{1+x}{1-x}-\frac{1}{2}arctgx$.

1101. $\frac{1}{4}\ln\frac{1+x}{1-x}+\frac{1}{2}arctgx$. 1102. $x+(1-x)\ln(1-x); \quad |x|<1$.

1103. $1+2x+3x^2+...+nx^{n-1}+...$, $\quad |x|<1$. $\frac{1}{(1-x)^2}$.

1104. $1+3x+...+\frac{n(n+1)}{2}x^{n-1}+...,\quad |x|<1$. $\frac{1}{(x-1)^3}$.

1105. R=0,1; (-0,1; 0,1). 1106. R=1; (-1; 1]. 1107. R=0,25; (-0,25; 0,25).
1108. R=∞; (-∞;+∞). 1109. R=1/3; (-1/3;1/3). 1110. R=4; [-4;4).
1111. R=2; (-2;2). 1112. R=1; [1;3). 1113. R=1; [-1;1]. 1114. R=3; (-7;-1).
1115. R=2; [1;5). 1116. R=0,5; (-0,5;0,5). 1117. R=∞; (-∞;+∞).

1118. $e^{6x}=1+\frac{6x}{1!}+\frac{36x^2}{2!}+...+\frac{6^n x^n}{n!}+...,\quad x\in(-\infty;+\infty)$.

1119. $e^{x}=1-\frac{x}{1!}+\frac{x^2}{2!}-...(-1)^n\frac{x^n}{n!}+...,\quad x\in(-\infty;+\infty)$.

1120. $\sin 3x=\frac{3x}{1!}-\frac{27x^3}{3!}+...+(-1)^{n-1}\frac{3^{2n-1}\cdot x^{2n-1}}{(2n-1)!}+...,\quad x\in(-\infty;+\infty)$.

1121. $\cos\frac{x}{3}=1-\frac{x^2}{9\cdot 2!}+\frac{x^4}{81\cdot 4!}+...+(-1)^{n-1}\frac{x^{2n}}{3^{2n}\cdot(2n)!}+...,\quad x\in(-\infty;+\infty)$.

1122. $\ln(1+4x)=4x-\frac{16x^2}{2}+\frac{64x^3}{3}-...+(-1)^{n-1}\frac{4^n\cdot x^n}{n}+...;\quad x\in(-0,25;\,0,25]$.

1123. $\ln(1-\frac{x}{3})=-\frac{x}{3}-\frac{x^2}{9\cdot 2}-\frac{x^3}{27\cdot 3}-...-\frac{x^n}{3^n\cdot n}-...;\quad x\in[-3;3)$.

1124. $\frac{1}{(1+x)^2}=\sum_{n=0}^{\infty}(-1)^n(n+1)x^n;\ x\in(-1;1)$. 1125. $\frac{1}{1-x^3}=\sum_{n=0}^{\infty}x^{3n};\ x\in(-1;1)$.

1126. $\sqrt{1+x^3}=1+\frac{x^3}{2}-\frac{x^6}{2^2\cdot 2!}+\frac{3x^9}{2^3\cdot 3!}-...;\ x\in(-1;1)$.

1127. $\frac{1}{\sqrt{1-x^2}}=1+\frac{x^2}{2}+\frac{3x^4}{2^2\cdot 2!}+\frac{3\cdot 5x^6}{2^3\cdot 3!}+...;\ x\in(-1;1)$.

1128. $\ln x=\sum_{n=1}^{\infty}(-1)^{n-1}\frac{(x-1)^n}{n}$. 1129. $\frac{1}{x}=-\sum_{n=0}^{\infty}\frac{(x+2)^n}{2^{n+1}}$.

1130. $e^{-3x}=e^{12}\sum_{n=0}^{\infty}\frac{(-3)^n(x+4)^n}{n!}$.

1131. $\cos x=\frac{1}{\sqrt{2}}\left(1-\left(x-\frac{\pi}{4}\right)-\frac{1}{2!}\left(x-\frac{\pi}{4}\right)^2+\frac{1}{3!}\left(x-\frac{\pi}{4}\right)^3+\frac{1}{4!}\left(x-\frac{\pi}{4}\right)^4-...\right)$.

1132. $\arcsin x = x + \frac{1}{2}\cdot\frac{x^3}{3} + \frac{1\cdot 3}{2\cdot 4}\cdot\frac{x^5}{5} + ...; x \in [-1;1]$.

1133. $arctgx = x - \frac{x^3}{3!} + \frac{x^5}{5!} - ... + (-1)^{n-1}\frac{x^{2n-1}}{(2n-1)!} + ..., \quad x \in (-1;1)$

1134. $\ln(x + \sqrt{1+x^2}) = x - \frac{1}{2}\cdot\frac{x^3}{3} + \frac{1\cdot 3}{2\cdot 4}\cdot\frac{x^5}{5} - ...; -1 \le x \le 1$.

1135. $(1+x)\ln(1+x) = x + \frac{x^2}{1\cdot 2} - \frac{x^3}{2\cdot 3} + \frac{x^4}{3\cdot 4} - ...; x \in (-1;1]$.

1136. 1,39. 1137. 0,208. 1138. 1,000 1139. 3,107. 1140. 1,0196. 1141. 2,991.
1142. 0,747. 1143. 0,946. 1144. 32,831. 1145.0,006. 1146. 0,5633. 1147. 0,487.
1148. 0,24488; 0,00001 aniqlikda. 1149. 3,518; 0,001 aniqlikda.
1150. 0,4971; 0,0001 aniqlikda. 1151. 0,012; 0,001 aniqlikda.

1152. $f(x) \sim \sum_{n=1}^{\infty}(-1)^n\frac{\sin nx}{n}; \quad S(\pm\pi) = 0$.

1153. $f(x) \sim \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{\sin(2n-1)x}{2n-1}; \quad S(\pm\pi) = 0$, S(0)=0

1154. $f(x) \sim \frac{\pi}{2} + \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{\cos(2n-1)x}{(2n-1)^2}; \quad S(\pm\pi) = 0$.

1155. $f(x) \sim \frac{\pi}{4} + \frac{1}{2} - \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\cos(2n-1)x}{(2n-1)^2} + \frac{\pi-2}{\pi}\sum_{n=1}^{\infty}\frac{\sin(2n-1)x}{2n-1} - \sum_{n=1}^{\infty}\frac{\sin 2nx}{2n}$,

$S(\pm\pi) = \frac{\pi+1}{2}, S(0) = 0,5$.

1156. $f(x) \sim \frac{5\pi}{4} - \frac{10}{\pi}\sum_{n=1}^{\infty}\frac{\cos(2n-1)x}{(2n-1)^2} + \sum_{n=1}^{\infty}(-1)^{n-1}\frac{\sin nx}{n}; \quad S(\pm\pi) = \frac{5\pi}{2}$.

1157. $\frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}\frac{(-1)^n\cos nx}{n^2}, \ \frac{\pi^2}{12}$. 1158. $\frac{\pi}{2} - \frac{4}{\pi}\sum_{n=0}^{\infty}\frac{\cos(2n+1)x}{(2n+1)^2}, \ \frac{\pi^2}{8}$.

1159. $f(x) \sim \frac{\pi}{2} - \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{\cos(2n-1)x}{(2n-1)^2}$. 1160. $f(x) \sim \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{\sin(2n-1)x}{2n-1}$.

1161. $f(x) = \frac{12}{\pi}\sum_{n=1}^{\infty}(-1)^{n-1}\frac{\sin(n\pi x/2)}{n}$. 1162. $f(x) = 2 - \frac{16}{\pi^2}\sum_{n=1}^{\infty}\frac{\cos((2n-1)\pi x/4)}{(2n-1)^2}$.

1163. $f(x) = 3 - \frac{4}{\pi}\sum_{n=1}^{\infty}(-1)^{n-1}\frac{\sin(n\pi x/2)}{n}$. 1164. $f(x) = 1 - \frac{8}{\pi^2}\sum_{n=1}^{\infty}\frac{\cos((2n-1)\pi x/4)}{(2n-1)^2}$

$+ \frac{4}{\pi}\sum_{n=1}^{\infty}(-1)^{n-1}\frac{\sin(n\pi x/4)}{n}$. 1165. $f(x) = \frac{1}{3} - \frac{4}{\pi^2}\sum_{n=1}^{\infty}(-1)^{n-1}\frac{\cos(n\pi x/3)}{n^2}$.

ADABIYOT

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