O‘ZBEKISTON RESPUBLIKASI OLIY VA O‘RTA MAXSUS TA’LIM VAZIRLIGI

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OLIY MATEMATIKADAN MUSTAQIL ISHLAR

O’quv qo’llanma

Toshkent

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A.I.Eshniyozov. Oliy matematikadan mustaqil ishlar. Amaliy mashg'ulotlar uchun o'quv qo'llanma – T., 2019, 252 bet

O'quv qo'llanmada analitik geometriya, limitlar nazariyasi, bir o'zgaruvchi funksiyalarning differensial va integral hisobi va ularning tatbiqlari bo'limlari qaralgan. Har bir mavzu uchun qisqacha nazariy ma'lumotlar, namunaviy misol va masalalar to'liq echimi bilan keltirilgan, mustaqil o'zlashtirish uchun berilgan misol va masalalarning javoblari berilgan.

Oliy o'quv yurtlari talabalari uchun mo'ljallangan.

В пособие рассматривается основные разделы аналитическая геометрия, теория пределов, дифференциальное и интегральное исчисления функций с одной переменных и их применение. По каждой теме кратко излагается основные теоретические сведения, детально разобраны типовые задачи, даются задачи и упражнения для самостоятельной работы с ответами.

Для студентов высших учебных заведений.

In a manual examined basic divisions analytical geometry, theory of limits, differential and integral calculations of functions with one the variables and their application. On every topic briefly expounded basic theoretical information, model tasks are taken apart in detail, tasks and exercises are given for independent work with answers.

For the students of higher educational establishments.

Taqrizchilar :

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SO’Z BOSHI

Hozirgi zamon ilmiy – texnika taraqqiyoti sharoitida Oliy o’quv yurtlarida yuqori malakali mutaxassislar tayyorlash borasida fizika – matematika fanlariga katta e’tibor berilayapti.

Oliy matematika kursi bo’yicha chuqur va har tomonlama bilim egallash uchun faqat asosiy nazariy materialning o’zi yetarli bo’lmasdan, maxsus tanlangan misol va masalalarni yetarlicha yechish ham zarur bo’ladi.

Bunda masala va misollarning aniq matematik ko’yilishi, yechimlarning asoslanganligi va to’laligi, javoblarning to’g’riligi katta ahamiyatga egadir.

E’tiboringizga havola qilinayotgan mazkur qo’llanma Oliy o’quv yurtlarining bakalavr yo’nalishidagi Oliy matematika dasturiga moslab tuzilgan.

Qo’llanmaning har qaysi bobida qisqacha nazariy ma’lumot bayon qilinib, tipik masalalar to’liq yechimlari berilgan. Bulardan tashqari, har qaysi paragraf oxirida mustaqil yechish uchun masalalar va ulaning javoblari keltirilgan. Ana shu misol va masalalarni yechish bilangina – cheklanib qolish yaramaydi, zero ushbu qo’llanma tegishli to’plamlarning o’rnini bosishga da’vo – qilmaydi.

Qo’llanma oliy o’quv yurtlarining barcha bakalar yo’nalishidagi talabalari uchun mo’ljallangan.

Muallif qimmatli metodik maslahatlari uchun UzMU, mexanika – matematika fakulteti professori, fizika – matematika fanlari doktori R.N.G’anixo’jayevga, GulDU fizika – matematika fakulteti “Umumiy matematika” kafedrasi dotsenti, fizika – matematika fanlari nomzodi K.Jomuratovga, hamda “Umumiy matematika” kafedrasi mudiri dotsent, fizika – matematika fanlari nomzodi X. Norjigitovga o’zining chuqur minnatdorchiligini bildiradi.

Muallif.

I BOB. Analitik geometriya

Analitik geometriya bobida siz analitik geometriyaning har xil masalalari: to'g'ri chiziq, tekislik tenglamalari va ularga oid masalalar, vektorlar ustida amallar, vektorlarning skalyar, vektor va aralash ko'paytmalari, geometrik masalalarga ularning tatbiqlari o'rganib olasiz.

Vektorni bazis bo'yicha yoyilmasi

Uchta chiziqli bog'liqmas vektorlar sistemasi $p,\ q,\ r$ berlgan bo'lib, agar ixtiyoriy x vektorni ularning chiziqli kombinatsiya, ya'ni

$$\vec{x} = \alpha \cdot \vec{p} + \beta \cdot \vec{q} + \gamma \cdot \vec{r}$$

shaklida ifodalash mumkin bo'lsa, u holda berilgan sistema bazis deyiladi.

Bu tenglik x vektorning $p,\ q,\ r$ bazis bo'yicha yoyilmasi deyiladi.

$\vec{x}$ vektorning yoyilmasi quyidagi ko'rinishda izlanadi:

$$\vec{x} = \alpha \cdot \vec{p} + \beta \cdot \vec{q} + \gamma \cdot \vec{r}.$$

Bu tenglama α, β, γ larga nisbatan vektor tenglama bo'lib, uch o'zgaruvchili uchta chiziqli tenglamalar sistemasi yordamida quyidagicha yoziladi:

$$\begin{cases} \alpha \cdot p_1 + \beta \cdot q_1 + \gamma \cdot r_1 = x_1 \\ \alpha \cdot p_2 + \beta \cdot q_2 + \gamma \cdot r_2 = x_2 \\ \alpha \cdot p_3 + \beta \cdot q_3 + \gamma \cdot r_3 = x_3 \end{cases}$$

Tenglamalar sistemasini yechib α, β, γ larni topib,

$$\vec{x} = \alpha \cdot \vec{p} + \beta \cdot \vec{q} + \gamma \cdot \vec{r}$$

vektorni ko'rinishini topamiz.

1–masala. x vektorni $p,\ q,\ r$ vektorlar orqali yoyilmasini yozing.

$$x = \{-13,\ 2,\ 18\},$$
$$p = \{1,\ 1,\ 4\},$$
$$q = \{-3,\ 0,\ 2\},$$
$$r = \{1,\ 2,\ -1\}.$$

Echim:

x vektorni $p,\ q,\ r$ vektorlar orqali yoyilmasi: $x = \alpha \cdot p + \beta \cdot q + \gamma \cdot r.$

yoki sistama ko'rinishida

$$\begin{cases} \alpha \cdot p_1 + \beta \cdot q_1 + \gamma \cdot r_1 = x_1 \\ \alpha \cdot p_2 + \beta \cdot q_2 + \gamma \cdot r_2 = x_2 \\ \alpha \cdot p_3 + \beta \cdot q_3 + \gamma \cdot r_3 = x_3 \end{cases}$$

Natijada

$$+\begin{cases} \alpha - 3\beta + \gamma = -13, \\ \alpha + 2\gamma = 2, \qquad |\cdot -2 \\ 4\alpha + 2\beta - \gamma = 18 \end{cases}$$

$$\begin{cases} \alpha - 3\beta + \gamma = -13, \\ -\alpha + 6\gamma = 28, \\ 4\alpha + 2\beta - \gamma = 18 \end{cases}$$

Uchinchi satrga birinchi satrni qo'shib:

$$\begin{cases} \alpha - 3\beta + \gamma = -13, \\ -\alpha + 6\gamma = 28, \\ 5\alpha - \beta = 5 \end{cases}$$

$$+\begin{cases} \alpha - 3\beta + \gamma = -13, \\ -\alpha + 6\gamma = 28, \\ 5\alpha - \beta = 5 \qquad |\cdot 6 \end{cases}$$

$$\begin{cases}\alpha-3\beta+\gamma=-13,\\ 29\alpha=58,\\ 5\alpha-\beta=5\end{cases}$$

$$\begin{cases}\alpha-3\beta+\gamma=-13,\\ \alpha=2,\\ 5\alpha-\beta=5\end{cases}$$

$$\begin{cases}\alpha-3\beta+\gamma=-13,\\ \alpha=2,\\ 5\cdot 2-\beta=5\end{cases}$$

$$\begin{cases}2-3\beta+\gamma=-13,\\ \alpha=2,\\ \beta=5\end{cases}$$

$$\begin{cases}-3\beta+\gamma=-15,\\ \alpha=2,\\ \beta=5\end{cases}$$

$$\begin{cases}-3\cdot 5+\gamma=-15,\\ \alpha=2,\\ \beta=5\end{cases}$$

$$\begin{cases}\alpha=2,\\ \beta=5,\\ \gamma=0.\end{cases}$$

Izlanayotgan yoyilma: $x=2p+5q$ ko'rinishida bo'lar ekan.

1–masala. x vektorni $p,\ q,\ r$ vektorlar orqali yoyilmasini yozing.

1. $x=\{-2,\ 4,\ 7\}$, $p=\{0,\ 1,\ 2\}$, $q=\{1,\ 0,\ 1\}$, $r=\{-1,\ 2,\ 4\}$.

2. $x=\{6,12,-1\}$, $p=\{1,3,0\}$, $q=\{2,-1,1\}$, $r=\{0,-1,2\}$.

3. $x=\{1,-4,4\}$, $p=\{2,1,-1\}$, $q=\{0,3,2\}$, $r=\{1,-1,1\}$.

4. $x=\{-9,5,5\}$, $p=\{4,1,1\}$, $q=\{2,0,-3\}$, $r=\{-1,2,1\}$.

5. $x=\{-5,-5,5\}$, $p=\{-2,0,1\}$, $q=\{1,3,-1\}$, $r=\{0,4,1\}$.

6. $x=\{13,2,7\}$, $p=\{5,1,0\}$, $q=\{2,-1,3\}$, $r=\{1,0,-1\}$.

7. $x=\{-19,-1,7\}$, $p=\{0,1,1\}$, $q=\{-2,0,1\}$, $r=\{3,1,0\}$.

8. $x=\{3,-3,4\}$, $p=\{1,0,2\}$, $q=\{0,1,1\}$, $r=\{2,-1,4\}$.

9. $x=\{3,3,-1\}$, $p=\{3,1,0\}$, $q=\{-1,2,1\}$, $r=\{-1,0,2\}$.

10. $x=\{-1,7,-4\}$, $p=\{-1,2,1\}$, $q=\{2,0,3\}$, $r=\{1,1,-1\}$.

11. $x=\{6,5,-14\}$, $p=\{1,1,4\}$, $q=\{0,-3,2\}$, $r=\{2,1,-1\}$.

12. $x=\{6,-1,7\}$, $p=\{1,-2,0\}$, $q=\{-1,1,3\}$, $r=\{1,0,4\}$.

13. $x=\{5,15,0\}$, $p=\{1,0,5\}$, $q=\{-1,3,2\}$, $r=\{0,-1,1\}$.

14. $x=\{2,-1,11\}$, $p=\{1,1,0\}$, $q=\{0,1,-2\}$, $r=\{1,0,3\}$.

15. $x=\{11,5,-3\}$, $p=\{1,0,2\}$, $q=\{-1,0,1\}$, $r=\{2,5,-3\}$.

16. $x=\{8,0,5\}$, $p=\{2,0,1\}$, $q=\{1,1,0\}$, $r=\{4,1,2\}$.

17. $x=\{3,1,8\}$, $p=\{0,1,3\}$, $q=\{1,2,-1\}$, $r=\{2,0,-1\}$.

18. $x=\{8,1,12\}$, $p=\{1,2,-1\}$, $q=\{3,0,2\}$, $r=\{-1,1,1\}$.

19. $x=\{-9,-8,-3\}$, $p=\{1,4,1\}$, $q=\{-3,2,0\}$, $r=\{1,-1,2\}$.

20. $x=\{-5,9,-13\}$, $p=\{0,1,-2\}$, $q=\{3,-1,1\}$, $r=\{4,1,0\}$.

21. $x=\{-15,5,6\}$, $p=\{0,5,1\}$, $q=\{3,2,-1\}$, $r=\{-1,1,0\}$.

22. $x=\{8,9,4\}$, $p=\{1,0,1\}$, $q=\{0,-2,1\}$, $r=\{1,3,0\}$.

23. $x=\{23,-14,-30\}$, $p=\{2,1,0\}$, $q=\{1,-1,0\}$, $r=\{-3,2,5\}$.

24. $x=\{3,1,3\}$, $p=\{2,1,0\}$, $q=\{1,0,1\}$, $r=\{4,2,1\}$.

25. $x=\{1,7,0\}$, $p=\{0,3,1\}$, $q=\{1,-1,2\}$, $r=\{2,-1,0\}$.

26. $x=\{11,-1,4\}$, $p=\{1,-1,2\}$, $q=\{3,2,0\}$, $r=\{-1,1,1\}$.

27. $x=\{0,-8,9\}$, $p=\{0,-2,1\}$, $q=\{3,1,-1\}$, $r=\{4,0,1\}$.

28. $x=\{8,-7,-13\}$, $p=\{0,1,5\}$, $q=\{3,-1,2\}$, $r=\{-1,0,1\}$.

29. $x=\{2,\ 7,\ 5\}$, $p=\{1,\ 0,\ 1\}$, $q=\{1,-2,0\}$, $r=\{0,\ 3,\ 1\}$.

30. $x=\{-15,-20,-1\}$, $p=\{0,2,1\}$, $q=\{0,1,-1\}$, $r=\{5,-3,2\}$.

Javoblar.1.1 $x=2p-q+r$; **1.2** $x=4p+q-r$; **1.3** $x=-p+3r$; **1.4** $x=-p-q+3r$;

1.5 $x=p-3q+r$;**1.6** $x=3p+q-4r$; **1.7** $x=2p+5q-3r$; **1.8** $x=p-2q+r$;

1.9 $x=-p+q-r$; **1.10** $x=2p-q+3r$; **1.11** $x=-2p-q+4r$;

1.12 $x=-p-3q+4r$; **1.13** $x=4p-q-18r$; **1.14** $x=-3p+2q+5r$;

1.15 $x=3p-6q+r$; **1.16** $x=p-2q+2r$; **1.17** $x=3p-q+2r$;

1.18 $x=-p+4q+2{,}2r$; **1.19** $x=-3p+2q$; **1.20** $x=5p-3q+r$ **1.21** $x=2p-4q+3r$;

1.22 $x=7p-3q+r$; **1.23** $x=13p+15q-6r$;

1.24 $x=-3p+q+2r$; **1.25** $x=2p-q$; **1.26** $x=3p+2q-2r$; **1.27** $x=2p-4q+3r$;

1.28 $x=-4p+3q+r$; **1.29** $x=4p-2q+r$; **1.30** $x=-6p+q+3r$.

Vektorlarning kollinearligi

Bitta to'g'ri chiziqda yoki parallel chiziqlarda yotgan c_1 va c_2 vektorlar kollinear vektorlar deyiladi. Boshqacha aytganda shunday α topilsaki, $\vec{p} = \alpha\vec{q}$ bo'lsa, ya'ni ularning koordinatalari o'zaro proporsional bo'lishi zarur va yetarli bo'ladi.

Demak,

$$\frac{p_1}{q_1} = \frac{p_2}{q_2} = \frac{p_3}{q_3}$$

bo'lsa, kollinear, tenglik bajarilmasa $\vec{p}$ va $\vec{q}$ vektorlar kollinear emas bo'lar ekan.

2–masala. a va b yordamida qurilgan c_1 va c_2 vektorlar kollinearmi?

$a = \{-1, 2, -1\}, \quad b\{2, -7, 1\}, \quad c_1 = 6a - 2b, \quad c_2 = b - 3a.$

$c_1 = 6a - 2b = \{6\cdot(-1) - 2\cdot 2;\ 6\cdot 2 - 2\cdot(-7);\ 1 - 6\cdot(-1) - 2\cdot 1\} =$
$= \{-10,\ 26,\ -8\}.$

$c_2 = b - 3a = \{2 - 3\cdot(-1);\ -7 - 3\cdot 2;\ 1 - 3\cdot(-1)\} = \{5,\ -13,\ 4\}.$

$$\frac{-10}{-5} = \frac{26}{-13} = \frac{-8}{4}, \text{ ya'ni } c_1 = -2c_2.$$

Demak, c_1 va c_2 kollinear ekan.

1. $a = \{1,\ -2,\ 3\},\ b = \{3,\ 0,\ -1\},\ c_1 = 2a + 4b,\ c_2 = 3b - a.$
2. $a = \{1, 0, 1\},\ b = \{-2, 3, 5\},\ c_1 = a + 2b,\ c_2 = 3a - b.$
3. $a = \{-2, 4, 1\},\ b = \{1, -2, 7\},\ c_1 = 5a + 3b,\ c_2 = 2a - b.$
4. $a = \{1, 2, -3\},\ b = \{2, -1, -1\},\ c_1 = 4a + 3b,\ c_2 = 8a - b.$
5. $a = \{3, 5, 4\},\ b = \{5, 9, 7\},\ c_1 = -2a + b,\ c_2 = 3a - 2b.$
6. $a = \{1, 4, -2\},\ b = \{1,\ 1, -1\},\ c_1 = a + b,\ c_2 = 4a + 2b.$
7. $a = \{1, -2,\ 5\},\ b = \{3, -1, 0\},\ c_1 = 4a - 2b,\ c_2 = b - 2a.$
8. $a = \{3, 4, -1\},\ b = \{2, -1,\ 1\},\ c_1 = 6a - 3b,\ c_2 = b - 2a.$

9. $a=\{-2,-3,-2\},\ b=\{1,0,-5\},\ c_1=3a+9b,\ \ c_2=-a-3b.$

10. $a=\{-1,4,\ 2\},\ b=\{3,-2,\ 6\},\ c_1=2a-b,\ \ c_2=3b-6a.$

11. $a=\{5,\ 0,-1\},\ b=\{7,\ 2,\ 3\},\ c_1=2a-b,\ c_2=3b-6a.$

12. $a=\{0,\ 3,-2\},\ b=\{1,-2,\ 1\},\ c_1=5a-2b,\ \ c_2=3a+5b.$

13. $a=\{-2,\ 7,-1\},\ b=\{-3,\ 5,\ 2\},\ c_1=2a+3b,\ \ c_2=3a+2b.$

14. $a=\{3,\ 7,\ 0\},\ b=\{1,-3,\ 4\},\ c_1=4a-2b,\ \ c_2=b-2a.$

15. $a=\{7,\ 9,-2\},\ b=\{5,\ 4,\ 3\},\ c_1=4a-b,\ \ c_2=4b-a.$

16. $a=\{5,\ 0,-2\},\ b=\{6,\ 4,\ 3\},\ c_1=5a-3b,\ c_2=6b-10a.$

17. $a=\{8,\ 3,-1\},\ b=\{4,\ 1,\ 3\},\ c_1=2a-b,\ \ c_2=2b-4a.$

18. $a=\{3,-1,\ 6\},\ b=\{5,\ 7,\ 10\},\ c_1=4a-2b,\ c_2=b-2a.$

19. $a=\{1,-2,\ 4\},\ b=\{7,\ 3,\ 5\},\ c_1=6a-3b,\ c_2=b-2a.$

20. $a=\{3,\ 7,\ 0\},\ b=\{4,\ 6,-1\},\ c_1=3a+2b,\ c_2=5a-7b.$

21. $a=\{2,-1,\ 4\},\ b=\{3,-7,-6\},\ c_1=2a-3b,\ c_2=3a-2b.$

22. $a=\{5,-1,-2\},\ b=\{6,\ 0,\ 7\},\ c_1=3a-2b,\ \ c_2=4b-6a.$

23. $a=\{-9,\ 5,\ 3\},\ b=\{7,\ 1,-2\},\ c_1=2a-b,\ c_2=3a+5b.$

24. $a=\{4,\ 2,\ 9\},\ b=\{0,-1,\ 3\},\ c_1=4b-3a,\ \ c_2=4a-3b.$

25. $a=\{2,-1,\ 6\},\ b=\{-1,\ 3,\ 8\},\ c_1=5a-2b,\ \ c_2=2a-5b.$

26. $a=\{5,\ \ 0,\ \ 8\},\ b=\{-3,\ \ 1,\ \ 7\},\ c_1=3a-4b,\ c_2=12b-9a.$

27. $a=\{-1,\ \ 3,\ \ 4\},\ b=\{2,-1,\ \ 0\},\ c_1=6a-2b,\ \ c_2=b-3a.$

28. $a=\{4,\ \ 2,-7\},\ b=\{5,\ \ 0,-3\},\ c_1=a-3b,\ c_2=6b-2a.$

29. $a=\{2,\ \ 0,-5\},\ b=\{1,-3,\ \ 4\},\ c_1=2a-5b,\ c_2=5a-2b.$

30. $a=\{-1,\ \ 2,\ \ 8\},\ b=\{3,\ 7,-1\},\ c_1=4a-3b,\ c_2=9b-12a.$

Javoblar.2.1 yo’q; **2.2** yo’q; **2.3** yo’q; **2.4** yo’q; **2.5** yo’q; **2.6** yo’q; **2.7** ha; **2.8** ha; **2.9** ha; **2.10** ha; **2.11** ha; **2.12** yo’q; **2.13** yo’q; **2.14** ha; **2.15** yo’q; **2.16** ha; **2.17** ha; **2.18** ha; **2.19** ha; **2.20** yo’q; **2.21** yo’q; **2.22** ha; **2.23** yo’q; **2.24** yo’q; **2.25** yo’q; **2.26** ha; **2.27** ha; **2.28** yo’q; **2.29** yo’q; **2.30** ha .

Vektorlar orasidagi burchak

Ikkita $\overrightarrow{AB}$ va $\overrightarrow{AC}$ vektorning skalyar ko'paytmasi deb, $\left(\overrightarrow{AB},\ \overrightarrow{AC}\right)$ ko'rinishda belgilanuvchi va shu vektorlar uzunliklari ko'paytmasining ular orasidagi burchak kosinusi bilan ko'paytmasiga teng bo'lgan songa aytiladi:

$$\left(\overrightarrow{AB},\ \overrightarrow{AC}\right)=\left|\overrightarrow{AB}\right|\cdot\left|\overrightarrow{AC}\right|\cdot\cos\varphi\,.$$

$A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ va $C(x_3, y_3, z_3)$ berilganda vektorlarning skalyar ko'paytmasi

$$\overrightarrow{AB}=(x_2-x_1, y_2-y_1, z_2-z_1)$$

$$\overrightarrow{AC}=(x_3-x_1, y_3-y_1, z_3-z_1)$$

$$\left(\overrightarrow{AB},\ \overrightarrow{AC}\right)=(x_2-x_1)(x_3-x_1)+(y_2-y_1)(y_3-y_1)+(z_2-z_1)(z_3-z_1)$$

va vektorlarning uzunliklari mos ravishda:

$$\left|\overrightarrow{AB}\right|=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2},$$

$$\left|\overrightarrow{AC}\right|=\sqrt{(x_3-x_1)^2+(y_3-y_1)^2+(z_3-z_1)^2},$$

formulalar bilan topiladi.

$\overrightarrow{AB}$ va $\overrightarrow{AC}$ vektor orasidagi burchak ushbu formula bilan hisoblanadi:

$$\cos\varphi=\frac{\left(\overrightarrow{AB},\ \overrightarrow{AC}\right)}{\left|\overrightarrow{AB}\right|\cdot\left|\overrightarrow{AC}\right|}.$$

3–masala. $\overrightarrow{AB}$ va $\overrightarrow{AC}$ vektorlar orasidagi burchak kosinusini toping.

$$A(1,\ -2,\ 3),$$
$$B(3,\ 4,\ -6),$$
$$C(1,\ 1,\ -1).$$

$$\overline{AB} = \{4,\ 2,\ -3\}, \quad \left|\overline{AB}\right| = \sqrt{4^2 + 2^2 + (-3)^2} = \sqrt{29}.$$

$$\overline{AC} = \{2,\ -1,\ 2\}, \quad \left|\overline{AC}\right| = \sqrt{2^2 + (-1)^2 + (2)^2} = 3.$$

$$\cos(\overline{AB} \wedge \overline{AC}) = \frac{4 \cdot 2 - 2 \cdot 1 - 3 \cdot 2}{3 \cdot \sqrt{29}} = 0.$$

$$(\overline{AB} \wedge \overline{AC}) = \frac{\pi}{2}.$$

$A(1, -2,\ 3)$,	$A(0, -3,\ 6)$,	$A(3,\ 3, -1)$,
1. $B(0, -1,\ 2)$,	**2.** $B(-12, -3, -3)$,	**3.** $B(5,\ 5, -2)$,
$C(3, -4,\ 5)$.	$C(-9, -3, -6)$.	$C(4,\ 1,\ 1)$.
$A(-4, -2,\ 0)$,	$A(5,\ 3, -1)$,	$A(-3, -7, -5)$,
4. $B(-1, -2,\ 4)$,	**5.** $B(5,\ 2,\ 0)$,	**6.** $B(0, -1, -2)$,
$C(3, -2,\ 1)$.	$C(6,\ 4, -1)$.	$C(2,\ 3,\ 0)$.
$A(2, -4,\ 6)$,	$A(0,\ 1, -2)$,	$A(3,\ 3, -1)$,
7. $B(0, -2,\ 4)$,	**8.** $B(3,\ 1,\ 2)$,	**9.** $B(1,\ 5, -2)$,
$C(6, -8,\ 10)$.	$C(4,\ 1,\ 1)$.	$C(4,\ 1,\ 1)$.
$A(2,\ 1, -1)$,	$A(-1, -2,\ 1)$,	$A(6,\ 2, -3)$,
10. $B(6, -1, -4)$,	**11.** $B(-4, -2,\ 5)$,	**12.** $B(6,\ 3, -2)$,
$C(4,\ 2,\ 1)$.	$C(-8, -2,\ 2)$.	$C(7,\ 3, -3)$.
$A(0,\ 0,\ 4)$,	$A(2, -8, -1)$,	$A(3, -6,\ 9)$,
13. $B(-3, -6,\ 1)$,	**14.** $B(4, -6,\ 0)$,	**15.** $B(0, -3,\ 6)$,
$C(-5, -10, -1)$.	$C(-2, -5, -1)$.	$C(9, -12,\ 15)$.
$A(0,\ 2, -4)$,	$A(3,\ 3, -1)$,	$A(-4,\ 3,\ 0)$,
16. $B(8,\ 2,\ 2)$,	**17.** $B(5,\ 1, -2)$,	**18.** $B(0,\ 1,\ 3)$,
$C(6,\ 2,\ 4)$.	$C(4,\ 1,\ 1)$.	$C(-2,\ 4, -2)$.

$A(1,-1,\ 0)$,	$A(7,\ 0,\ 2)$,	$A(2,\ 3,\ 2)$,
19. $B(-2,-1,\ 4)$,	**20.** $B(7,\ 1,\ 3)$,	**21.** $B(-1,-3,-1)$,
$C(8,-1,-1)$.	$C(8,-1,\ 2)$.	$C(-3,-7,-3)$.
$A(2,\ 2,\ 7)$,	$A(-1,\ 2,-3)$,	$A(0,\ 3,-6)$,
22. $B(0,\ 0,\ 6)$,	**23.** $B(0,\ 1,-2)$,	**24.** $B(9,\ 3,\ 6)$,
$C(-2,\ 5,\ 7)$.	$C(-3,\ 4,-5)$.	$C(12,\ 3,\ 3)$.
$A(3,\ 3,-1)$,	$A(-2,\ 1,\ 1)$,	$A(1,\ 4,-1)$,
25. $B(5,\ 1,-2)$,	**26.** $B(2,\ 3,-2)$,	**27.** $B(-2,\ 4,-5)$,
$C(4,\ 1,-3)$.	$C(0,\ 0,\ 3)$.	$C(8,\ 4,\ 0)$.
$A(0,\ 1,\ 0)$,	$A(-4,\ 0,\ 4)$,	$A(-2,\ 4,-6)$,
28. $B(0,\ 2,\ 1)$,	**29.** $B(-1,\ 6,\ 7)$,	**30.** $B(0,\ 2,-4)$,
$C(1,\ 2,\ 0)$.	$C(1,\ 10,\ 9)$.	$C(-6,\ 8,-10)$.

Javoblar. 3.1 π; **3.2** $16^0 15' 37''$; **3.3** $116^0 23' 16''$; **3.4** $\frac{\pi}{4}$; **3.5** $\frac{2\pi}{3}$; **3.6** π; **3.7** π; **3.8** $\arccos 0{,}96$; **3.9** $152^0 44' 2''$; **3.10** $\frac{\pi}{2}$; **3.11** $\frac{\pi}{4}$; **3.12** $\frac{\pi}{3}$; **3.13** 0^0; **3.14** $97^0 39' 44''$; **3.15** π; **3.16** $16^0 15' 37''$; **3.17** $63^0 36' 44''$; **3.18** $\frac{\pi}{2}$; **3.19** 135^0; **3.20** 120^0; **3.21** 0^0; **3.22** $82^0 20' 15''$; **3.23** π; **3.24** $16^0 15' 37''$; **3.25** $27^0 15' 58''$; **3.26** $\frac{\pi}{2}$; **3.27** 135^0; **3.28** $\frac{\pi}{3}$; **3.29** 0^0; **3.30** π.

Parallelogrammning yuzi

a va b vektorlarga qurilgan parallelogramm yuzi

$$S = \left|[\vec{a},\ \vec{b}]\right|$$

shu vektorlarning vektor ko'paytmasidan olingan modulga teng.

$$[\vec{a},\ \vec{b}] = \left[\alpha_1\vec{p} + \alpha_2\vec{q},\ \beta_1\vec{p} + \beta_2\vec{q}\right] = \alpha_1\beta_1[\vec{p},\ \vec{p}] + \alpha_1\beta_2[\vec{p},\ \vec{q}] +$$
$$+ \alpha_2\beta_1[\vec{q},\ \vec{p}] + \alpha_2\beta_2[\vec{q},\ \vec{q}] = (\alpha_1\beta_2 - \alpha_2\beta_1)[\vec{p},\ \vec{q}].$$

Bundan,

$$S = \left|[\vec{a},\ \vec{b}]\right| = (\alpha_1\beta_2 - \alpha_2\beta_1)[\vec{p},\ \vec{q}]\sin\varphi.$$

4–masala. *a* va *b* vektorlarga qurilgan parallelogramm yuzini hisoblang.

$$a = 6p - q,$$
$$b = 5q + p.$$

$$|p| = \frac{1}{2}, \quad |q| = 4, \quad (p\wedge q) = \frac{5\pi}{6}.$$

$$S = |(6p - q)\times(5q + p)| = |6p\times 5q + 6p\times p - 5q\times q - q\times p| =$$
$$= |6p\times 5q + p\times q| = 31|p|\cdot|q|\cdot\sin(p\wedge q) =$$
$$= 31\cdot\frac{1}{2}\cdot 4\cdot\sin\frac{5\pi}{6} = 31\cdot 2\cdot\frac{1}{2} = 31.$$

1. $a = p + 2q, \quad b = 3p - q; \qquad |p| = 1, \qquad |q| = 2, \quad (p\wedge q) = \pi/6.$
2. $a = 3p + q, \quad b = p - 2q; \qquad |p| = 4, \qquad |q| = 1, \quad (p\wedge q) = \pi/4.$
3. $a = p - 3q, \quad b = p + 2q; \qquad |p| = 1/5, \quad |q| = 1, \quad (p\wedge q) = \pi/2.$
4. $a = 3p - 2q, \quad b = p + 5q; \qquad |p| = 4, \qquad |q| = 1/2, \quad (p\wedge q) = 5\pi/6.$
5. $a = p - 2q, \quad b = 2p + q; \qquad |p| = 2, \qquad |q| = 3, \quad (p\wedge q) = 3\pi/4.$
6. $a = p + 3q, \quad b = p - 2q; \qquad |p| = 2, \qquad |q| = 3, \quad (p\wedge q) = \pi/3.$
7. $a = 2p - q, \quad b = p + 3q; \qquad |p| = 3, \qquad |q| = 2, \quad (p\wedge q) = \pi/2.$
8. $a = 4p + q, \quad b = p - q; \qquad |p| = 7, \qquad |q| = 2, \quad (p\wedge q) = \pi/4.$
9. $a = p - 4q, \quad b = 3p + q; \qquad |p| = 1, \qquad |q| = 2, \quad (p\wedge q) = \pi/6.$
10. $a = p + 4q, \quad b = 2p - q; \qquad |p| = 7, \qquad |q| = 2, \quad (p\wedge q) = \pi/3.$
11. $a = 3p + 2q, \quad b = p - q; \qquad |p| = 10, \quad |q| = 1, \quad (p\wedge q) = \pi/2.$
12. $a = 4p - q, \quad b = p + 2q; \qquad |p| = 5, \qquad |q| = 4, \quad (p\wedge q) = \pi/4.$
13. $a = 2p + 3q, \quad b = p - 2q; \qquad |p| = 6, \qquad |q| = 7, \quad (p\wedge q) = \pi/3.$

14. $a = 3p - q, \quad b = p + 2q; \qquad |p| = 3, \quad |q| = 4, \quad (p \wedge q) = \pi / 3.$

15. $a = 2p + 3q, \quad b = p - 2q; \qquad |p| = 2, \quad |q| = 3, \quad (p \wedge q) = \pi / 4.$

16. $a = 2p - 3q, \quad b = 3p + q; \qquad |p| = 4, \quad |q| = 1, \quad (p \wedge q) = \pi / 6.$

17. $a = 5p + q, \quad b = p - 3q; \qquad |p| = 1, \quad |q| = 2, \quad (p \wedge q) = \pi / 3.$

18. $a = 7p - 2q, \quad b = p + 3q; \qquad |p| = 1/2, \; |q| = 2, \; (p \wedge q) = \pi / 2.$

19. $a = 6p - q, \quad b = p + q; \qquad |p| = 3, \quad |q| = 4, \quad (p \wedge q) = \pi / 4.$

20. $a = 10p + q, \quad b = 3p - 2q; \qquad |p| = 4, \quad |q| = 1, \quad (p \wedge q) = \pi / 6.$

21. $a = 6p - q, \quad b = p + 2q; \qquad |p| = 8, \quad |q| = 1/2, \quad (p \wedge q) = \pi / 3.$

22. $a = 3p + 4q, \quad b = q - p; \qquad |p| = 2{,}5, \quad |q| = 2, \quad (p \wedge q) = \pi / 2.$

23. $a = 7p + q, \quad b = p - 3q; \qquad |p| = 3, \quad |q| = 1, \quad (p \wedge q) = 3\pi / 4.$

24. $a = p + 3q, \quad b = 3p - q; \qquad |p| = 3, \quad |q| = 5, \quad (p \wedge q) = 2\pi / 3.$

25. $a = 3p + q, \quad b = p - 3q; \qquad |p| = 7, \quad |q| = 2, \quad (p \wedge q) = \pi / 4.$

26. $a = 5p - q, \quad b = p + q; \qquad |p| = 5, \quad |q| = 3, \quad (p \wedge q) = 5\pi / 6.$

27. $a = 3p - 4q, \quad b = p + 3q; \qquad |p| = 2, \quad |q| = 3, \quad (p \wedge q) = \pi / 4.$

28. $a = 2p + 3q, \quad b = p - 2q; \qquad |p| = 2, \quad |q| = 1, \quad (p \wedge q) = \pi / 3.$

29. $a = 2p - 3q, \quad b = 5p + q; \qquad |p| = 2, \quad |q| = 3, \quad (p \wedge q) = \pi / 2.$

30. $a = 3p + 2q, \quad b = 2p - q; \qquad |p| = 4, \quad |q| = 4, \quad (p \wedge q) = 3\pi / 4.$

Javoblar.4.1 7; **4.2** $14\sqrt{2}$; **4.3** 1; **4.4** 17; **4.5** $15\sqrt{2}$; **4.6** $15\sqrt{3}$; **4.7** 42; **4.8** $35\sqrt{2}$; **4.9** 13; **4.10** $63\sqrt{3}$; **4.11** 50; **4.12** $90\sqrt{2}$; **4.13** $147\sqrt{3}$; **4.14** $42\sqrt{3}$; **4.15** $21\sqrt{2}$; **4.16** 22; **4.17** $16\sqrt{3}$; **4.18** 23; **4.19** $42\sqrt{2}$; **4.20** 46; **4.21** $26\sqrt{3}$; **4.22** 35; **4.23** $33\sqrt{2}$; **4.24** $75\sqrt{3}$; **4.25** $70\sqrt{2}$; **4.26** 45; **4.27** $39\sqrt{2}$; **4.28** $7\sqrt{3}$; **4.29** 102; **4.30** $42\sqrt{2}$.

Vektorlar komplanarligi

Bir tekislikda yoki parallel tekisliklarda yotuchi vektorlarni komplanar vektorlar deyiladi.

Agar $a = (a_1,\ a_2,\ a_3)$, $b = (b_1,\ b_2,\ b_3)$ va $c = (c_1,\ c_2,\ c_3)$ vektorlar komplanar bo'lishi uchun ularning aralash ko'paytmasi nolga teng bo'lishi

zarur va yetarli, ya’ni

$$(\vec{a},\vec{b},\vec{c})=\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}=0.$$

5–masala. a, b va c vektorlar komplanarmi?

$$a=\{7,\ 3,\ 4\},\quad b=\{-1,\ 2,\ -1\},\quad c=\{4,\ 2,\ 4\}.$$

$$(a,b,c)=\begin{vmatrix} 7 & 3 & 4 \\ -1 & -2 & -1 \\ 4 & 2 & 4 \end{vmatrix}=-56-12-8+32+14+12=-18\neq 0\Rightarrow$$

a, b va c vektorlar komplanar emas.

1. $a=\{2,\ 3,\ 1\}$, $b=\{-1,\ 0,-1\}$, $c=\{2,\ 2,\ 2\}$.

2. $a=\{3,\ 2,\ 1\}$, $b=\{2,\ 3,\ 4\}$, $c=\{3,\ 1,-1\}$

3. $a=\{1,\ 5,\ 2\}$, $b=\{-1,\ 1,-1\}$, $c=\{1,\ 1,\ 1\}$.

4. $a=\{1,-1,-3\}$, $b=\{3,\ 2,\ 1\}$, $c=\{2,\ 3,\ 4\}$.

5. $a=\{3,\ 3,\ 1\}$, $b=\{1,-2,\ 1\}$, $c=\{1,\ 1,\ 1\}$.

6. $a=\{3,\ 1,-1\}$, $b=\{-2,-1,\ 0\}$, $c=\{5,\ 2,-1\}$.

7. $a=\{4,\ 3,\ 1\}$, $b=\{1,-2,\ 1\}$, $c=\{2,\ 2,\ 2\}$.

8. $a=\{4,\ 3,\ 1\}$, $b=\{6,\ 7,\ 4\}$, $c=\{2,\ 0,-1\}$.

9. $a=\{3,\ 2,\ 1\}$, $b=\{1,-3,-7\}$, $c=\{1,\ 2,\ 3\}$.

10. $a=\{3,\ 7,\ 2\}$, $b=\{-2,\ 0,-1\}$, $c=\{2,\ 2,\ 1\}$.

11. $a=\{1,-2,\ 6\}$, $b=\{1,\ 0,\ 1\}$, $c=\{2,-6,\ 17\}$.

12. $a=\{6,\ 3,\ 4\}$, $b=\{-1,\ 2,-1\}$, $c=\{2,\ 1,\ 2\}$.

$a=\{2,\ 3,\ 2\}$,	$a=\{5,\ 3,\ 4\}$,	$a=\{3,\ 10,\ 5\}$,
13. $b=\{4,\ 7,\ 5\}$,	14. $b=\{-1,\ 0,-1\}$,	15. $b=\{-2,-2,-3\}$,
$c=\{2,\ 0,-1\}$.	$c=\{4,\ 2,\ 4\}$.	$c=\{2,\ 4,\ 3\}$.
$a=\{-2,-4,-3\}$,	$a=\{3,\ 1,-1\}$,	$a=\{4,\ 2,\ 2\}$,
16. $b=\{4,\ 3,\ 1\}$,	17. $b=\{1,\ 0,-1\}$,	18. $b=\{-3,-3,-3\}$,
$c=\{6,\ 7,\ 4\}$.	$c=\{8,\ 3,-2\}$.	$c=\{2,\ 1,\ 2\}$.
$a=\{4,\ 1,\ 2\}$,	$a=\{5,\ 3,\ 4\}$,	$a=\{3,\ 4,\ 2\}$,
19. $b=\{9,\ 2,\ 5\}$,	20. $b=\{4,\ 3,\ 3\}$,	21. $b=\{1,\ 1,\ 0\}$,
$c=\{1,\ 1,-1\}$.	$c=\{9,\ 5,\ 8\}$.	$c=\{8,\ 11,\ 6\}$.
$a=\{4,-1,-6\}$,	$a=\{3,\ 1,\ 0\}$,	$a=\{3,\ 0,\ 3\}$,
22. $b=\{1,-3,-7\}$,	23. $b=\{-5,-4,-5\}$,	24. $b=\{8,\ 1,\ 6\}$,
$c=\{2,-1,-4\}$.	$c=\{4,\ 2,\ 4\}$.	$c=\{1,\ 1,-1\}$.
$a=\{1,-1,\ 4\}$,	$a=\{6,\ 3,\ 4\}$,	$a=\{4,\ 1,\ 1\}$,
25. $b=\{1,\ 0,\ 3\}$,	26. $b=\{-1,-2,-1\}$,	27. $b=\{-9,-4,-9\}$,
$c=\{1,-3,\ 8\}$.	$c=\{2,\ 1,\ 2\}$.	$c=\{6,\ 2,\ 6\}$.
$a=\{-3,\ 3,\ 3\}$,	$a=\{-7,\ 10,-5\}$,	$a=\{7,\ 4,\ 6\}$,
28. $b=\{-4,\ 7,\ 6\}$,	29 $b=\{0,-2,-1\}$,	30. $b=\{2,\ 1,\ 1\}$,
$c=\{3,\ 0,-1\}$.	$c=\{-2,\ 4,-1\}$.	$c=\{19,\ 11,\ 17\}$.

Javoblar. **5.1** yo'q; **5.2** ha; **5.3** yo'q; **5.4** ha; **5.5** yo'q; **5.6** ha; **5.7** yo'q; **5.8** ha; **5.9** ha; **5.10** yo'q; **5.11** ha; **5.12** yo'q; **5.13** ha; **5.14** yo'q; **5.15** yo'q; **5.16** ha; **5.17** ha; **5.18** yo'q; **5.19** ha; **5.20** yo'q; **5.21** ha; **5.22** ha; **5.23** yo'q; **5.24** ha; **5.25** yo'q; **5.26** yo'q; **5.27** yo'q; **5.28** ha; **5.29** yo'q; **5.30** ha **.**

Tetraedrning balandligi va hajmi

Uchlari $A_1(x_1,\ y_1,\ z_1)$, $A_2(x_2,\ y_2,\ z_2)$, $A_4(x_3,\ y_3,\ z_3)$, $A_4(x_4,\ y_4,\ z_4)$ bo'lgan, hamda A_4 uchidan $A_1A_2A_3$ yog'iga balandlik

tushirilgan tetraedrning hajmini topish masalasini qaraylik.

A_1 uchdan quyidagi vektorlarni o'tkazamiz:

$$\overrightarrow{A_1A_2} = \{x_2 - x_1,\ \ y_2 - y_1,\ \ z_2 - z_1\},$$

$$\overrightarrow{A_1A_3} = \{x_3 - x_1,\ \ y_3 - y_1,\ \ z_3 - z_1\},$$

$$\overrightarrow{A_1A_4} = \{x_4 - x_1,\ y_4 - y_1,\ z_4 - z_1\}.$$

Vektorlarning aralash ko'patmasining geometrik ma'nosidan quyidagiga egamiz:

$$V_t = \frac{1}{6} \cdot V_{pp} = \frac{1}{6}\left|\left(\overrightarrow{A_1A_2},\ \overrightarrow{A_1A_3},\ \overrightarrow{A_1A_4}\right)\right|,$$

bu yerda V_t va V_{pp}–lar mos ravishda $\overrightarrow{A_1A_2},\ \overrightarrow{A_1A_3},\ \overrightarrow{A_1A_4}$ vektorlar yordamida qurilgan tetraedr va parallelepipedning hajmlari.

Ikkinchi tomondan

$$V_t = \frac{1}{3} S_{\Delta A_1A_2A_3} \cdot h,$$

vektor ko'paytmaning geometrik ma'nosidan esa,

$$S_{\Delta A_1A_2A_3} = \frac{1}{2}\left|\left[\overrightarrow{A_1A_2},\ \overrightarrow{A_1A_3}\right]\right|.$$

Demak, tetraedrning balandligi

$$h = \frac{3V_t}{S_{\Delta A_1A_2A_3}} = \frac{\left|\left(\overrightarrow{A_1A_2},\ \overrightarrow{A_1A_3},\ \overrightarrow{A_1A_4}\right)\right|}{\left|\left[\overrightarrow{A_1A_2},\ \overrightarrow{A_1A_3}\right]\right|}$$

ga teng bo'ladi.

6–masala. Uchlari A_1, A_2, A_3, A_4 nuqtalarda yordamida berilgan tetraedrning, A_4 uchidan $A_1A_2A_3$ yog'iga balandlik tushirilgan

tetraedrning hajmini toping.

$$A_1(0,\ -1,\ -1),$$
$$A_2(-2,\ 3,\ 5),$$
$$A_3(1,\ -5,\ -9),$$
$$A_4(-1,\ -6,\ 3).$$

$$\overline{A_1A_2} = \{-2,\ 4,\ 6\},$$
$$\overline{A_1A_3} = \{1,\ -4,\ -8\},$$
$$\overline{A_1A_4} = \{-1,\ -5,\ 4\}.$$

$$V = \frac{1}{6}\left|(\overline{A_1A_2}, \overline{A_1A_3}, \overline{A_1A_4}\right| = \frac{1}{6}\cdot\begin{vmatrix} -2 & 4 & 6 \\ 1 & -4 & -8 \\ -1 & -5 & 4 \end{vmatrix} =$$

$$= \frac{1}{6}\cdot\left|32-30+32-24+80-16\right| = \frac{74}{6}.$$

$$V_{A_1A_2A_3A_4} = \frac{1}{3}S_{A_1A_2A_3}\cdot h \Rightarrow h = \frac{3V}{S}.$$

$$S_{A_1A_2A_3} = \frac{1}{2}\left|\overline{A_1A_2}\times\overline{A_1A_3}\right| = \frac{1}{2}\begin{vmatrix} i & j & k \\ -2 & 4 & 6 \\ 1 & -4 & -8 \end{vmatrix} =$$

$$= \frac{1}{2}\left|-8i-10j+4k\right| = \frac{1}{2}\sqrt{64+100+16} =$$

$$= \frac{1}{2}\sqrt{180} = \sqrt{45}.$$

$$h = \frac{3\cdot 74}{6\cdot\sqrt{45}} = \frac{37}{\sqrt{45}}.$$

1. $A_1(1,\ 3,\ 6),$ $A_2(2,\ 2,\ 1),$ $A_3(-1,\ 0,\ 1),$ $A_4(-4,\ 6,-3).$

2. $A_1(-4,\ 2,\ 6),$ $A_2(2,-3,\ 0),$ $A_3(-10,\ 5,\ 8),$ $A_4(-5,\ 2,-4).$

3. $A_1(7,\ 2,\ 4),$ $A_2(7,-1,-2),$ $A_3(3,\ 3,\ 1),$ $A_4(-4,\ 2,\ 1).$

4. $A_1(2,\ 1,\ 4)$, $A_2(-1,\ 5,-2)$, $A_3(-7,-3,\ 2)$, $A_4(-6,-3,\ 6)$.

5. $A_1(-1,-5,\ 2)$, $A_2(-6,\ 0,-3)$, $A_3(3,\ 6,-3)$, $A_4(-10,\ 6,\ 7)$.

6. $A_1(5,\ 2,\ 0)$, $A_2(2,\ 5,\ 0)$, $A_3(1,\ 2,\ 4)$, $A_4(-1,\ 1,\ 1)$.

7. $A_1(2,-1,-2)$, $A_2(1,\ 2,\ 1)$, $A_3(5,\ 0,-6)$, $A_4(-10,\ 9,-7)$.

8. $A_1(-2,\ 0,-4)$, $A_2(-1,\ 7,\ 1)$, $A_3(4,-8,-4)$, $A_4(1,-4,\ 6)$.

9. $A_1(14,\ 4,\ 5)$, $A_2(-5,-3,\ 2)$, $A_3(-2,-6,-3)$, $A_4(-2,\ 2,-1)$.

10. $A_1(1,\ 2,\ 0)$, $A_2(3,\ 0,-3)$, $A_3(5,\ 2,\ 6)$, $A_4(8,\ 4,-9)$.

11. $A_1(2,-1,\ 2)$, $A_2(1,\ 2,-1)$, $A_3(3,\ 2,\ 1)$, $A_4(-4,\ 2,\ 5)$.

12. $A_1(1,\ 1,\ 2)$, $A_2(-1,\ 1,\ 3)$, $A_3(2,-2,\ 4)$, $A_4(-1,\ 0,-2)$.

13. $A_1(2,\ 3,\ 1)$, $A_2(4,\ 1,-2)$, $A_3(6,\ 3,\ 7)$, $A_4(7,\ 5,-3)$.

14. $A_1(1,\ 1,-1)$, $A_2(2,\ 3,\ 1)$, $A_3(3,\ 2,\ 1)$, $A_4(5,\ 9,-8)$.

15. $A_1(1,\ 5,-7)$, $A_2(-3,\ 6,\ 3)$, $A_3(-2,\ 7,\ 3)$, $A_4(-4,\ 8,-12)$.

16. $A_1(-3,\ 4,-7)$, $A_2(1,\ 5,-4)$, $A_3(-5,-2,\ 0)$, $A_4(2,\ 5,\ 4)$.

17. $A_1(-1,\ 2,-3)$, $A_2(4,-1,\ 0)$, $A_3(2,\ 1,-2)$, $A_4(3,\ 4,\ 5)$.

18. $A_1(4,-1,\ 3)$, $A_2(-2,\ 1,\ 0)$, $A_3(0,-5,\ 1)$, $A_4(3,\ 2,-6)$.

19. $A_1(1,-1,\ 1)$, $A_2(-2,\ 0,\ 3)$, $A_3(2,\ 1,-1)$, $A_4(2,-2,-4)$.

20. $A_1(1,\ 2,\ 0)$, $A_2(1,-1,\ 2)$, $A_3(0,\ 1,-1)$, $A_4(-3,\ 0,\ 1)$.

21. $A_1(1,\ 0,\ 2)$, $A_2(1,\ 2,-1)$, $A_3(2,-2,\ 1)$, $A_4(2,\ 1,\ 0)$.

22. $A_1(1,\ 2,-3)$, $A_2(1,\ 0,\ 1)$, $A_3(-2,-1,\ 6)$, $A_4(0,-5,-4)$.

23. $A_1(3,\ 10,-1)$, $A_2(-2,\ 3,-5)$, $A_3(-6,\ 0,-3)$, $A_4(1,-1,\ 2)$.

24. $A_1(-1,\ 2,\ 4)$, $A_2(-1,-2,-4)$, $A_3(3,\ 0,-1)$, $A_4(7,-3,\ 1)$.

25. $A_1(0,-3,\ 1)$, $A_2(-4,\ 1,\ 2)$, $A_3(2,-1,\ 5)$, $A_4(3,\ 1,-4)$.

26. $A_1(1,\ 3,\ 0)$, $A_2(4,-1,\ 2)$, $A_3(3,\ 0,\ 1)$, $A_4(-4,\ 3,\ 5)$.

27. $A_1(-2,-1,-1)$, $A_2(0,\ 3,\ 2)$, $A_3(3,\ 1,-4)$, $A_4(-4,\ 7,\ 3)$.

28. $A_1(-3,-5,\ 6)$, $A_2(2,\ 1,-4)$, $A_3(0,-3,-1)$, $A_4(-5,\ 2,-8)$.

29. $A_1(2,-4,-3)$, $A_2(5,-6,\ 0)$, $A_3(-1,\ 3,-3)$, $A_4(-10,-8,\ 7)$.

30. $A_1(1,-1,\ 2)$, $A_2(2,\ 1,\ 2)$, $A_3(1,\ 1,\ 4)$, $A_4(6,-3,\ 8)$.

Javoblar. 6.1 $23\frac{1}{3}$; $2\sqrt{14}$; **6.2** $18\frac{2}{3}$; 4; **6.3** $21{,}5$; $\frac{43}{\sqrt{105}}$; **6.4** $\frac{80}{3}$; $\frac{10}{\sqrt{22}}$; **6.5** 190; $2\sqrt{38}$; **6.6** 12; $\frac{18\sqrt{27}}{27}$; **6.7** $46\frac{2}{3}$; $4\sqrt{14}$; **6.8** $83\frac{1}{3}$; $5\sqrt{2}$; **6.9** $112\frac{2}{3}$; $\sqrt{26}$; **6.10** 34; $7\frac{2}{7}$; **6.11** 11; $3\cdot\sqrt{\frac{11}{2}}$; **6.12** $5\frac{5}{6}$; $\sqrt{\frac{35}{2}}$; **6.13** $23\frac{1}{3}$; 5; **6.14** $7\frac{1}{2}$; $\frac{45\sqrt{17}}{17}$; **6.15** $17{,}5$; 7; **6.16** $25\frac{1}{6}$; $\sqrt{\frac{151}{15}}$; **6.17** $6\frac{2}{3}$; $5\sqrt{2}$; **6.18** $45\frac{1}{3}$; $\frac{17\sqrt{5}}{5}$; **6.19** $5{,}5$; $\frac{33}{\sqrt{101}}$; **6.20** $\frac{19}{6}$; $\sqrt{\frac{19}{2}}$; **6.21** $1\frac{1}{6}$; $\sqrt{\frac{7}{11}}$; **6.22** 16; $8\cdot\sqrt{\frac{2}{3}}$; **6.23** $45{,}5$; 7; **6.24** 24; 4; **6.25** $32\frac{1}{3}$; $\sqrt{\frac{97}{2}}$; **6.26** $2{,}5$; $5\cdot\sqrt{\frac{3}{2}}$; **6.27** $23\frac{1}{3}$; $\frac{140}{\sqrt{1021}}$; **6.28** $\frac{191}{6}$; $\sqrt{\frac{191}{3}}$; **6.29** 73; $\frac{438}{\sqrt{747}}$ **6.30** 6; $3\sqrt{6}$.

Nuqtadan tekislikkacha bo’lgan masofa

Izlanayotgan masofani uchlari $M_0(x_0, y_0, z_0)$, $M_1(x_1, y_1, z_1)$, $M_2(x_2, y_2, z_2)$, $M_3(x_3, y_3, z_3)$ berilgan tetraedrning uchi $M_0(x_0, y_0, z_0)$ dan $M_1M_2M_3$ yog’iga tushirilgan balandlik orqali topish mumkin, ya’ni

$$d = \frac{|Ax_0 + By_0 + Cz_0 + D|}{\sqrt{A^2 + B^2 + C^2}}$$

Masalani hal qilishning boshqacha ko’rinishi esa, $M_0(x_0, y_0, z_0)$ nuqtadan tekislikkacha bo’lgan d masofa:

$$d = \frac{\left(\vec{n},\ \overrightarrow{M_1M_0}\right)}{|\vec{n}|},$$

bu yerda $|\vec{n}|$ –tekislikning normal vektori

$$|\vec{n}| = \left[\overrightarrow{M_1M_2},\ \overrightarrow{M_1M_3}\right].$$

$\overrightarrow{M_1M_2} = \{x_2 - x_1,\ y_2 - y_1,\ z_2 - z_1\}$, $\overrightarrow{M_1M_3} = \{x_3 - x_1,\ y_3 - y_1,\ z_3 - z_1\}$, $\overrightarrow{M_1M_0} = \{x_0 - x_1,\ y_0 - y_1,\ z_0 - z_1\}$ vektorlarning koordinatalarini topamiz va

$$|\vec{n}| = \left[\overrightarrow{M_1M_2},\ \overrightarrow{M_1M_3}\right] = \begin{vmatrix} i & j & k \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix}$$

ni topamiz.

7–masala. M_1, M_2, M_3 nuqtalardan o’tuvchi M_0 nuqtadan tekislikkacha bo’lgan masofani toping.

$$M_1(2,\ 3,\ 1),$$
$$M_2(4,\ 1,\ -2),$$
$$M_3(6,\ 3,\ 7),$$
$$M_0(-5,\ -4,\ 8).$$

Uch nuqtadan o'tuvchi tekislik tenglamasi

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0,$$

$$\begin{vmatrix} x-2 & y-3 & z-1 \\ 2 & -2 & -3 \\ 4 & 0 & 6 \end{vmatrix} = 0,$$

$$-12(x-2)-24(y-3)+8(z-1)=0,$$
$$-12x-24y+8z+88=0,$$
$$d=\frac{|Ax_0+By_0+Cz_0+D|}{\sqrt{A^2+B^2+C^2}},$$

$$d=\frac{|-12\cdot(-5)-24\cdot(-4)+8\cdot 8+88|}{\sqrt{(-12)^2+(-24)^2+8^2}}=\frac{308}{\sqrt{784}}=\frac{308}{28}=11.$$

1.
$M_1(-3,\ 4,-7),$
$M_2(1,\ 5,-4),$
$M_3(-5,-2,\ 0),$
$M_0(-12,\ 7,-1).$

2.
$M_1(-1,\ 2,-3),$
$M_2(4,-1,\ 0),$
$M_3(2,\ 1,-2),$
$M_0(1,-6,-5).$

3.
$M_1(-3,-1,\ 1),$
$M_2(-9,\ 1,-2),$
$M_3(3,-5,\ 4),$
$M_0(-7,\ 0,-1).$

4.
$M_1(1,-1,\ 1),$
$M_2(-2,\ 0,\ 3),$
$M_3(2,\ 1,-1),$
$M_0(-2,\ 4,\ 2).$

5.
$M_1(1,\ 2,\ 0),$
$M_2(1,-1,\ 2),$
$M_3(0,\ 1,-1),$
$M_0(2,-1,\ 4).$

6.
$M_1(1,\ 0,\ 2),$
$M_2(1,\ 2,-1),$
$M_3(2,-2,\ 1),$
$M_0(-5,-9,\ 1).$

7. $M_1(1,\ 2,-3)$, $M_2(1,\ 0,\ 1)$, $M_3(-2,-1,\ 6)$, $M_0(3,-2,-9)$.

8. $M_1(3,\ 10,-1)$, $M_2(-2,\ 3,-5)$, $M_3(-6,\ 0,-3)$, $M_0(-6,\ 7,-10)$.

9. $M_1(-1,\ 2,\ 4)$, $M_2(-1,-2,-4)$, $M_3(3,\ 0,-1)$, $M_0(-2,\ 3,\ 5)$.

10. $M_1(0,-3,\ 1)$, $M_2(-4,\ 1,\ 2)$, $M_3(2,-1,\ 5)$, $M_0(-3,\ 4,-5)$.

11. $M_1(1,\ 3,\ 0)$, $M_2(4,-1,\ 2)$, $M_3(3,\ 0,\ 1)$, $M_0(4,\ 3,\ 0)$.

12. $M_1(-2,-1,-1)$, $M_2(0,\ 3,\ 2)$, $M_3(3,\ 1,-4)$, $M_0(-21,\ 20,-16)$.

13. $M_1(-3,-5,\ 6)$, $M_2(2,\ 1,-4)$, $M_3(0,-3,-1)$, $M_0(3,\ 6,\ 68)$.

14. $M_1(2,-4,-3)$, $M_2(5,-6,\ 0)$, $M_3(-1,\ 3,-3)$, $M_0(2,-10,\ 8)$.

15. $M_1(1,-1,\ 2)$, $M_2(2,\ 1,\ 2)$, $M_3(1,\ 1,\ 4)$, $M_0(-3,\ 2,\ 7)$.

16. $M_1(1,\ 3,\ 6)$, $M_2(2,\ 2,\ 1)$, $M_3(-1,\ 0,\ 1)$, $M_0(5,-4,\ 5)$.

17. $M_1(-4,\ 2,\ 6)$, $M_2(2,-3,\ 0)$, $M_3(-10,\ 5,\ 8)$, $M_0(-12,\ 1,\ 8)$.

18. $M_1(7,\ 2,\ 4)$, $M_2(7,-1,-2)$, $M_3(-5,-2,-1)$, $M_0(10,\ 1,\ 8)$.

19. $M_1(2,\ 1,\ 4)$, $M_2(3,\ 5,-2)$, $M_3(-7,-3,\ 2)$, $M_0(-3,\ 1,\ 8)$.

20. $M_1(-1,-5,\ 2)$, $M_2(-6,\ 0,-3)$, $M_3(3,\ 6,-3)$, $M_0(10,-8,-7)$.

21. $M_1(0,-1,-1)$, $M_2(-2,\ 3,\ 5)$, $M_3(1,-5,-9)$, $M_0(-4,-13,\ 6)$.

22. $M_1(5,\ 2,\ 0)$, $M_2(2,\ 5,\ 0)$, $M_3(1,\ 2,\ 4)$, $M_0(-3,-6,-8)$.

23. $M_1(2,-1,-2)$, $M_2(1,\ 2,\ 1)$, $M_3(5,\ 0,-6)$, $M_0(14,-3,\ 7)$.

24. $M_1(-2,\ 0,-4)$, $M_2(-1,\ 7,\ 1)$, $M_3(4,-8,-4)$, $M_0(-6,\ 5,\ 5)$.

25. $M_1(14,\ 4,\ 5)$, $M_2(-5,-3,\ 2)$, $M_3(-2,-6,-3)$, $M_0(-1,-8,\ 7)$.

26. $M_1(1,\ 2,\ 0)$, $M_2(3,\ 0,-3)$, $M_3(5,\ 2,\ 6)$ $M_0(-13,-8,\ 16)$.

27. $M_1(2,-1,\ 2)$, $M_2(1,\ 2,-1)$, $M_3(3,\ 2,\ 1)$, $M_0(-5,\ 3,\ 7)$.

28. $M_1(1,\ 1,\ 2)$, $M_2(-1,\ 1,\ 3)$, $M_3(2,-2,\ 4)$, $M_0(2,\ 3,\ 8)$.

29. $M_1(1,\ 1,-1)$, $M_2(2,\ 3,\ 1)$, $M_3(3,\ 2,\ 1)$, $M_0(-3,-7,\ 6)$.

30. $M_1(1,\ 5,-7)$, $M_2(-3,\ 6,\ 3)$, $M_3(-2,\ 7,\ 3)$, $M_0(1,-1,\ 2)$.

Javoblar. **7.1** $\frac{459}{\sqrt{2265}}$; **7.2** $5\sqrt{2}$; **7.3** 0; **7.4** $\frac{9}{\sqrt{101}}$; **7.5** $\frac{1}{\sqrt{38}}$; **7.6** $\sqrt{77}$; **7.7** $2\sqrt{6}$; **7.8** 7; **7.9** $\frac{5}{9}$; **7.10** $\frac{90}{\sqrt{194}}$; **7.11** $\sqrt{6}$; **7.12** $\frac{1023}{\sqrt{1021}}$; **7.13** $\sqrt{573}$; **7.14** $\frac{73}{\sqrt{83}}$; **7.15** $\sqrt{6}$; **7.16** $2\sqrt{14}$; **7.17** 4; **7.18** 3; **7.19** 4; **7.20** $2\sqrt{38}$; **7.21** $2\sqrt{45}$; **7.22** $8\sqrt{3}$; **7.23** $4\sqrt{14}$; **7.24** $\frac{23\sqrt{2}}{5}$; **7.25** $3\sqrt{\frac{13}{2}}$; **7.26** $19\frac{1}{7}$; **7.27** $2\sqrt{22}$; **7.28** $7\sqrt{\frac{7}{10}}$; **7.29** $\frac{45}{\sqrt{17}}$; **7.30** 7.

Normal vektori berilgan tekislik tenglamasi

8–masala. $\overline{BC}$ vektorga perpendikulyar bo'lgan A nuqtadan o'tuvchi tekislik tenglamasini yozing.

$$A(0,\ -2,\ 8),$$
$$B(4,\ 3,\ 2),$$
$$C(1,\ 4,\ 3).$$
$$\overline{BC}=\{-3,\ 1,\ 1\}.$$

$\overline{BC}$ izlananayotgan tekislikka perpendikulyar bo'lganligidan, uni vektorning normali sifatida olish mumkin, u holda

$$-3(x-0)+(y+2)+(z-8)=0,$$
$$-3x+y+z-6=0.$$

$A(1,\ 0,-2)$,	$A(-1,\ 3,\ 4)$,	$A(4,-2,\ 0)$,
1. $B(2,-1,3)$,	2. $B(-1,\ 5,\ 0)$,	3. $B(1,-1,-5)$,
$C(0,-3,\ 2)$.	$C(2,\ 6,\ 1)$.	$C(-2,\ 1,-3)$.
$A(-8,\ 0,\ 7)$,	$A(7,-5,\ 1)$,	$A(-3,\ 5,-2)$,
4. $B(-3,\ 2,\ 4)$,	5. $B(5,-1,-3)$,	6. $B(-4,\ 0,\ 3)$,
$C(-1,\ 4,\ 5)$.	$C(3,\ 0,-4)$.	$C(-3,\ 2,\ 5)$.
$A(1,-1,\ 8)$,	$A(-2,\ 0,-5)$,	$A(1,\ 9,-4)$,
7. $B(-4,-3,\ 10)$,	8. $B(2,\ 7,-3)$,	9. $B(5,\ 7,\ 1)$,
$C(-1,-1,\ 7)$.	$C(1,\ 10,-1)$.	$C(3,\ 5,\ 0)$.
$A(-7,\ 0,\ 3)$,	$A(0,-3,\ 5)$,	$A(5,-1,\ 2)$,
10. $B(1,-5,-4)$,	11. $B(-7,\ 2,\ 6)$,	12. $B(2,-4,\ 3)$,
$C(2,-3,\ 0)$.	$C(-3,\ 2,\ 4)$.	$C(4,-1,\ 3)$.
$A(-3,\ 7,\ 2)$,	$A(1,-1,\ 5)$,	$A(-10,\ 0,\ 9)$,
13. $B(3,\ 5,\ 1)$,	14. $B(0,\ 7,\ 8)$,	15. $B(12,\ 4,\ 11)$,
$C(4,\ 5,\ 3)$.	$C(-1,\ 3,\ 8)$.	$C(8,\ 5,\ 15)$.
$A(3,-3,-6)$,	$A(2,\ 1,\ 7)$,	$A(-7,\ 1,-4)$,
16. $B(1,\ 9,-5)$,	17. $B(9,\ 0,\ 2)$,	18. $B(8,\ 11,-3)$,
$C(6,\ 6,-4)$.	$C(9,\ 2,\ 3)$.	$C(9,\ 9,-1)$.
$A(1,\ 0,-6)$,	$A(-3,\ 1,\ 0)$,	$A(-4,-2,\ 5)$,
19. $B(-7,\ 2,\ 1)$,	20. $B(6,\ 3,\ 3)$,	21. $B(3,-3,-7)$,
$C(-9,\ 6,\ 1)$.	$C(9,\ 4,-2)$.	$C(9,\ 3,-7)$.

$A(0,-8,10)$,	$A(1,-5,-2)$,	$A(0,7,-9)$,
22. $B(-5,5,7)$,	**23.** $B(6,-2,1)$,	**24.** $B(-1,8,-11)$,
$C(-8,0,4)$.	$C(2,-2,-2)$.	$C(-4,3,-12)$.
$A(-3,-1,7)$,	$A(5,3,1)$,	$A(-1,2,-2)$,
25. $B(0,2,-6)$,	**26.** $B(0,0,-3)$,	**27.** $B(13,14,1)$,
$C(2,3,-5)$.	$C(5,-1,0)$.	$C(14,15,2)$.
$A(7,-5,0)$,	$A(-3,6,4)$,	$A(2,5,-3)$,
28. $B(8,3,-1)$,	**29.** $B(8,-3,5)$,	**30.** $B(7,8,-1)$,
$C(8,5,1)$.	$C(0,-3,7)$.	$C(9,7,4)$.

Javoblar. 8.1 $2x+2y+z=0$; **8.2** $3x+y+z-4=0$; **8.3** $-3x+2y+2z+16=0$; **8.4** $-4x+2y+z-39=0$; **8.5** $-2x+y-z+20=0$; **8.6** $x+2y+2z-9=0$; **8.7** $3x+2y-3z+23=0$; **8.8** $-x+3y+2z+8=0$; **8.9** $-2x-2y-z-16=0$; **8.10** $x+2y+4z-5=0$; **8.11** $2x-z+5=0$; **8.12** $2x+3y-7=0$; **8.13** $x+2z-1=0$; **8.14** $x+4y+3=0$; **8.15** $-4x+y+4z-76=0$; **8.16** $5x-3y+z-18=0$; **8.17** $2y+z-9=0$; **8.18** $x-2y+2z+17=0$; **8.19** $-x+2y+1=0$; **8.20** $3x+y-5z+8=0$; **8.21** $x+y+6=0$; **8.22** $3x+5y+3z+10=0$; **8.23** $4x+3z+2=0$; **8.24** $-3x-5y-z+26=0$; **8.25** $2x+y+z=0$; **8.26** $5x-y+3z-19=0$; **8.27** $x+y+z+1=0$; **8.28** $y+z+5=0$; **8.29** $x+z-1=0$; **8.30** $2x-y+5z+16=0$.

Tekisliklar orasidagi burchak

Fazoda tekisliklar

$$A_1x+B_1y+C_1z+D_1=0 \quad \text{va} \quad A_2x+B_2y+C_2z+D_2=0$$

tenglamalar bilan berilgan bo'lsin. Ular orasidagi φ burchak ushbu formula bilan hisoblanadi:

$$\cos\varphi = \frac{\overrightarrow{n_1}\cdot\overrightarrow{n_2}}{\left|\overrightarrow{n_1}\right|\cdot\left|\overrightarrow{n_2}\right|},$$

bunda $\overrightarrow{n_1} = \{A_1,\ B_1,\ C_1\},\ \overrightarrow{n_2} = \{A_2,\ B_2,\ C_2\}$ – mos ravishda berilgan tekisliklarning normal vektorlari.

9–masala. Tekisliklar orasidagi burchakni toping.

$$6x + 2y - 4z + 17 = 0,$$
$$9x + 3y - 6z - 4 = 0.$$

$\overline{n_1} = \{6,\ 2, -4\}$,

$\overline{n_2} = \{9,\ 3, -6\}$.

$$\cos\varphi = \frac{6\cdot 9 + 2\cdot 3 + (-4)\cdot(-6)}{\sqrt{6^2 + 2^2 + (-4)^2}\cdot\sqrt{9^2 + 3^2 + (-6)^2}} =$$

$$= \frac{84}{\sqrt{56}\cdot\sqrt{126}} = \frac{84}{\sqrt{7056}} = \frac{84}{84} = 1,$$

$\varphi = \arccos 1 = 0.$

1. $x - 3y + 5 = 0,\ 2x - y + 5z - 16 = 0.$
2. $x - 3y + z - 1 = 0,\ x + z - 1 = 0.$
3. $4x - 5y + 3z - 1 = 0,\ x - 4y - z + 9 = 0.$
4. $3x - y + 2z + 15 = 0,\ 5x + 9y - 3z - 1 = 0.$
5. $x - y\sqrt{2} + z - 1 = 0,\ x + y\sqrt{2} - z + 36 = 0.$
6. $3y - z = 0,\ 2y + z = 0.$
7. $6x + 3y - 2z = 0,\ x + 2y + 6z - 12 = 0.$
8. $x + 2y + 2z - 3 = 0,\ 16x + 12y - 15z - 1 = 0.$
9. $2x - y + 5z + 16,\ x + 2y + 3z + 8 = 0.$
10. $2x + 2y + z - 1 = 0,\ x + z - 1 = 0.$
11. $3x + y + z - 4 = 0,\ y + z + 5 = 0.$
12. $3x - 2y - 2z - 16 = 0,\ x + y - 3z - 7 = 0.$
13. $2x + 2y + z + 9 = 0,\ x - y + 3z - 1 = 0.$
14. $x + 2y + 2z - 3 = 0,\ 2x - y + 2z + 5 = 0.$
15. $3x + 2y - 3z - 1 = 0,\ x + y + z - 7 = 0.$

16. $\begin{cases} x-3y-2z-8=0, \\ x+y-z+3=0. \end{cases}$ **17.** $\begin{cases} 3x-2y+3z+23=0, \\ y+z+5=0. \end{cases}$ **18.** $\begin{cases} x+y+3z-7=0, \\ y+z-1=0. \end{cases}$

19. $\begin{cases} x-2y+2z+17=0, \\ x-2y-1=0. \end{cases}$ **20.** $\begin{cases} x-2y-1=0, \\ x+y+6=0. \end{cases}$ **21.** $\begin{cases} 2x-z+5=0, \\ 2x+3y-7=0. \end{cases}$

22. $\begin{cases} 5x+3y+z-18=0, \\ 2y+z-9=0. \end{cases}$ **23.** $\begin{cases} 4x+3z-2=0, \\ x+2y+2z+5=0. \end{cases}$ **24.** $\begin{cases} x+4y-z+1=0, \\ 2x+y+4z-3=0. \end{cases}$

25. $\begin{cases} 2y+z-9=0, \\ x-y+2z-1=0. \end{cases}$ **26.** $\begin{cases} 2x-6y+14z-1=0, \\ 5x-15y+35z-3=0. \end{cases}$ **27.** $\begin{cases} x-y+7x-1=0, \\ 2x-2y-5=0. \end{cases}$

28. $\begin{cases} 3x-y-5=0, \\ 2x+y-3=0. \end{cases}$ **29.** $\begin{cases} x+y+z\sqrt{2}-3=0, \\ x-y+z\sqrt{2}-1=0. \end{cases}$ **30.** $\begin{cases} x+2y-2z-7=0, \\ x+y-35=0. \end{cases}$

Javoblar. 9.1 $\arccos\frac{\sqrt{3}}{6}\approx 73^0 13'17''$; **9.2** $\arccos\sqrt{\frac{2}{11}}\approx 64^0 45'38''$;

9.3 $\arccos 0{,}7\approx 45^0 34'23''$; **9.4** 90^0; **9.5** 60^0; **9.6** 45^0; **9.7** 90^0; **9.8** $\arccos\frac{2}{15}\approx 82^0$;

9.9 $\arccos\sqrt{\frac{15}{28}}\approx 42^0 57'7''$; **9.10** $\arccos\frac{\sqrt{2}}{2}=45^0$; **9.11** $\arccos\sqrt{\frac{2}{11}}\approx 64^0 45'38''$;

9.12 $\arccos\frac{7}{\sqrt{187}}\approx 59^0 12'37''$; **9.13** $\arccos\frac{1}{\sqrt{11}}\approx 72^0 27'6''$; **9.14** $\arccos\frac{4}{9}\approx 63^0 36'44''$;

9.15 $\arccos\frac{2}{\sqrt{66}}\approx 75^0 44'54''$; **9.16** $\arccos 0=90^0$; **9.17** $\arccos\frac{1}{\sqrt{44}}\approx 81^0 19'45''$;

9.18 $\arccos 2\sqrt{\frac{2}{11}}=31^0 28'56''$; **9.19** $\arccos\frac{\sqrt{5}}{3}=41^0 48'37''$;

9.20 $\arccos\frac{3}{\sqrt{10}}=18^0 26'6''$; **9.21** $\arccos\frac{4}{\sqrt{65}}\approx 60^0 15'18''$; **9.22** $\arccos\frac{\sqrt{7}}{5}=58^0 3'7''$;

9.23 $\arccos\frac{2}{3}=48^0 11'23''$; **9.24** $\arccos\frac{\sqrt{2}}{3\sqrt{42}}=84^0 5'4''$; **9.25** 90^0; **9.26** 1;

9.27 $\arccos\sqrt{\frac{2}{102}}=78^0 34'42''$; **9.28** 45^0; **9.29** 60^0 **9.30** 45^0.

Bir xil uzoqlikda yotgan nuqtaning koordinatalari

10–masala. B va C nuqlardan bir xil uzoqlikda yotgan A nuqtaning koordinatalarini toping.

$$A(x,0,0),\quad B(1,2,3),\quad C(2,6,10).$$

$$AB=\sqrt{(1-x)^2+2^2+3^2}=\sqrt{x^2-2x+14},$$

$$AC=\sqrt{(2-x)^2+6^2+10^2}=\sqrt{x^2-4x+140}.$$

Shartga ko'ra $AB=AC$ ekanligidan

$$\sqrt{x^2-2x+14}=\sqrt{x^2-4x+140},$$

$$x^2-2x+14=x^2-4x+140,$$

$$2x=126,$$

$$x=63$$

Demak, $A(63,\ 0,\ 0)$.

1. $A(0,\ 0,\ z)$, $B(5,\ 1,\ 0)$, $C(0,\ 2,\ 3)$.
2. $A(0,\ 0,\ z)$, $B(3,\ 3,\ 1)$, $C(4,\ 1,\ 2)$.
3. $A(0,\ 0,\ z)$, $B(3,\ 1,\ 3)$, $C(1,\ 4,\ 2)$.
4. $A(0,\ 0,\ z)$, $B(-1,-1,-6)$, $C(2,\ 3,\ 5)$.
5. $A(0,\ 0,\ z)$, $B(-13,\ 4,\ 6)$, $C(10,-9,\ 5)$.
6. $A(0,\ 0,\ z)$, $B(-5,-5,\ 6)$, $C(-7,\ 6,\ 2)$.
7. $A(0,\ 0,\ z)$, $B(-18,\ 1,\ 0)$, $C(15,-10,\ 2)$.
8. $A(0,\ 0,\ z)$, $B(10,\ 0,-2)$, $C(9,-2,\ 1)$.
9. $A(0,\ 0,\ z)$, $B(-6,\ 7,\ 5)$, $C(8,-4,\ 3)$.

$A(0,\ 0,\ z)$,	$A(0,\ 0,\ z)$,	$A(0,\ y,\ 0)$,
10. $B(6,-7,\ 1)$,	**11.** $B(7,\ 0,-15)$,	**12.** $B(3,\ 0,\ 3)$,
$C(-1,\ 2,\ 5)$.	$C(2,\ 10,-12)$.	$C(0,\ 2,\ 4)$.
$A(0,\ y,\ 0)$,	$A(0,\ y,\ 0)$,	$A(0,\ y,\ 0)$,
13. $B(1,\ 6,\ 4)$,	**14.** $B(-2,\ 8,\ 10)$,	**15.** $B(-2,-4,\ 6)$,
$C(5,\ 7,\ 1)$.	$C(6,\ 11,-2)$.	$C(7,\ 2,\ 5)$.
$A(0,\ y,\ 0)$,	$A(0,\ y,\ 0)$,	$A(0,\ y,\ 0)$,
16. $B(2,\ 2,\ 4)$,	**17.** $B(0,-4,\ 1)$,	**18.** $B(0,\ 5,-9)$,
$C(0,\ 4,\ 2)$.	$C(1,-3,\ 5)$.	$C(-1,\ 0,\ 5)$.
$A(0,\ y,\ 0)$,	$A(0,\ y,\ 0)$,	$A(0,\ y,\ 0)$,
19. $B(-2,\ 4,-6)$,	**20.** $B(7,\ 3,-4)$,	**21.** $B(0,-2,\ 4)$,
$C(8,\ 5,\ 1)$.	$C(1,\ 5,\ 7)$.	$C(-4,\ 0,\ 4)$.
$A(x,\ 0,\ 0)$,	$A(x,\ 0,\ 0)$,	$A(x,\ 0,\ 0)$,
22. $B(0,\ 1,\ 3)$,	**23.** $B(4,\ 0,\ 5)$,	**24.** $B(8,\ 1,-7)$,
$C(2,\ 0,\ 4)$.	$C(5,\ 4,\ 2)$.	$C(10,-2,\ 1)$.
$A(x,\ 0,\ 0)$,	$A(x,\ 0,\ 0)$,	$A(x,\ 0,\ 0)$,
25. $B(3,\ 5,\ 6)$,	**26.** $B(4,\ 5,-2)$,	**27.** $B(-2,\ 0,\ 6)$,
$C(1,\ 2,\ 3)$.	$C(2,\ 3,\ 4)$.	$C(0,-2,-4)$.
$A(x,\ 0,\ 0)$,	$A(x,\ 0,\ 0)$,	$A(x,\ 0,\ 0)$,
28. $B(1,\ 5,\ 9)$,	**29.** $B(4,\ 6,\ 8)$,	**30.** $B(-2,-4,-6)$,
$C(3,\ 7,\ 11)$.	$C(2,\ 4,\ 6)$.	$C(-1,-2,-3)$.

Javoblar. 10.1 $A\left(0;0;-2\frac{1}{6}\right)$; **10.2** $A(0;0;1)$; **10.3** $A(0;0;-1)$; **10.4** $A(0;0;0)$;

10.5 $A(0;0;7{,}5)$; **10.6** $A\left(0;0;-\frac{3}{8}\right)$; **10.7** $A(0;0;1)$; **10.8** $A(0;0;-3)$; **10.9** $A(0;0;5{,}25)$;

10.10 $A(0;0;-7)$; **10.11** $A\left(0;0;-4\frac{1}{3}\right)$; **10.12** $A\left(0;\ \frac{1}{2};\ 0\right)$; **10.13** $A(0;\ 11;\ 0)$;

10.14 $A\left(0;\ -\frac{7}{6};\ 0\right)$; **10.15** $A\left(0;\ 1\frac{5}{6};\ 0\right)$; **10.16** $A(0;\ -1;\ 0)$; **10.17** $A(0;\ 9;\ 0)$;

10.18 $A(0;\ 8;\ 0)$; **10.19** $A(0;\ 17;\ 0)$; **10.20** $A\left(0;\ \frac{1}{4};\ 0\right)$; **10.21** $A(0;\ 3;\ 0)$;

10.22 $A(2{,}5;\ 0;\ 0)$; **10.23** $A(2;\ 0;\ 0)$; **10.24** $A(-2{,}25;\ 0;\ 0)$; **10.25** $A(14;\ 0;\ 0)$;

10.26 $A(4;\ 0;\ 0)$; **10.27** $A(-5;\ 0;\ 0)$; **10.28** $A(18;\ 0;\ 0)$; **10.29** $A(15;\ 0;\ 0)$;

10.30 $A(-21;\ 0;\ 0)$.

11–masala. Markazi koordinata boshida bo'lgan k–gomotetiya koeffitsiyenti bo'lsin. A nuqta α tekislikning obrazi(aksi)ga tegishliligini tekshiring.

$$A(1,\ 1,\ 1),$$
$$\alpha: 7x-6y+z-5=0,$$
$$k=-2.$$

Markazi koordinata boshida bo'lgan α tekislik o'xshash akslantirishlarga ko'ra α' tekislikka o'tadi.

$$\alpha: Ax+By+Cz+D=0,$$
$$\alpha': Ax+By+Cz+k\cdot D=0,$$
$$\alpha': 7x-6y+z+10=0.$$
$$A(1,\ 1,\ 1)\Rightarrow 7-6+1+10=0,$$
$$12\neq 0.$$

Shunday qilib, A nuqta α tekislikning obrazi (aksi)ga tegishli bo'lmas ekan.

1. $A(1,\ 2,-1)$, $\alpha: 2x+3y+z-1=0$, $k=2$.

2. $A(2,\ 1,\ 2)$, $\alpha: x-2y+z+1=0$, $k=-2$.

$A(-1,\ 1,\ 1),$

3. $\alpha : 3x - y + 2z + 4 = 0,$

$k = \frac{1}{2}.$

$A(-2,\ 4,\ 1),$

4. $\alpha : 3x + y + 2x + 2 = 0,$

$k = 3.$

$A\left(1,\ \frac{1}{3}, -2\right),$

5. $\alpha : x - 3y + z + 6 = 0,$

$k = \frac{1}{3}.$

$A\left(\frac{1}{2},\ \frac{1}{3},\ 1\right),$

6. $\alpha : 2x - 3y + 3z - 2 = 0,$

$k = 1{,}5.$

$A(2,\ 0, -1),$

7. $\alpha : x - 3y + 5z - 1 = 0,$

$k = -1.$

$A(1, -2,\ 1),$

8. $\alpha : 5x + y - z + 6 = 0,$

$k = 2/3.$

$A(2, -5,\ 4),$

9. $\alpha : 5x + 2y - z + 3 = 0,$

$k = \frac{4}{3}.$

$A(2, -3,\ 1),$

10. $\alpha : x + y - 2z + 2 = 0,$

$k = \frac{5}{2}.$

$A(-2,\ 3, -3),$

11. $\alpha : 3x + 2y - z - 2 = 0,$

$k = \frac{3}{2}.$

$A\left(\frac{1}{4},\ \frac{1}{3},\ 1\right),$

12. $\alpha : 4x - 3y + 5z - 10 = 0,$

$k = \frac{1}{2}.$

$A(0,\ 1, -1),$

13. $\alpha : 6x - 5y + 3z - 4 = 0,$

$k = -\frac{3}{4}.$

$A(2,\ 3, -2),$

14. $\alpha : 3x - 2y + 4z - 6 = 0,$

$k = -\frac{4}{3}.$

15. $A(-2,-1,1)$, $\alpha: x-2y+6z-10=0$, $k=\frac{3}{5}$.

16. $A(5,\ 0,-1)$, $\alpha: 2x-y+3z-1=0$, $k=3$.

17. $A\left(\frac{1}{3},\ 1,\ 1\right)$, $\alpha: 3x-y+5z-6=0$, $k=\frac{5}{6}$.

18. $A(2,\ 5,\ 1)$, $\alpha: 5x-2y+z-3=0$, $k=\frac{1}{3}$.

19. $A(-1,\ 2,\ 3)$, $\alpha: x-3y+z+2=0$, $k=2{,}5$.

20. $A(4,\ 3,\ 1)$, $\alpha: 3x-4y+5z-6=0$, $k=\frac{5}{6}$.

21. $A(3,\ 5,\ 2)$, $\alpha: 5x-3y+z-4=0$, $k=\frac{1}{2}$.

22. $A(4,\ 0,-3)$, $\alpha: 7x-y+3z-1=0$, $k=3$.

23. $A(-1,\ 1,-2)$, $\alpha: 4x-y+3z-6=0$, $k=-\frac{5}{3}$.

24. $A(2,-5,-1)$, $\alpha: 5x+2y-3z-9=0$, $k=\frac{1}{3}$.

25. $A(-3,-2,\ 4)$, $\alpha: 2x-3y+z-5=0$, $k=-\frac{4}{5}$.

26. $A(5,\ 0,-6)$, $\alpha: 6x-y-z+7=0$, $k=\frac{2}{7}$.

$A(1,\ 2,\ 2)$,
27. $\alpha : 3x - z + 5 = 0$,
$k = -\frac{1}{5}$.

$A(3,\ 2,\ 4)$,
28. $\alpha : 2x - 3y + z - 6 = 0$,
$k = \frac{2}{3}$.

$A(7,\ 0, -1)$,
29. $\alpha : x - y - z - 1 = 0$,
$k = 4$.

$A(0,\ 3, -1)$,
30. $\alpha : 2x - y + 3z - 1 = 0$,
$k = 2$.

Javoblar. 11.1 yo'q**; 11.2** ha**; 11.3** ha**; 11.4** yo'q**; 11.5** ha**; 11.6** ha**; 11.7** yo'q**; 11.8** yo'q**; 11.9** ha**; 11.10** yo'q**; 11.11** ha**; 11.12** ha**; 11.13** yo'q**; 11.14** ha**; 11.15** ha**; 11.16** yo'q**; 11.17** ha**; 11.18** ha**; 11.19** yo'q**; 11.20** ha**; 11.21** ha**; 11.22** yo'q**; 11.23** yo'q**; 11.24** ha**; 11.25** yo'q**; 11.26** yo'q**; 11.27** ha**; 11.28** yo'q**; 11.29** yo'q; **11.30** yo'q**.**

To'g'ri chiziqning kanonik tenglamasi

12–masala. To'g'ri chiziqning kanonik tenglamasini yozing.

$$x - 3y + 2z + 2 = 0,$$
$$x + 3y + z + 14 = 0.$$

$$\overline{S} = \overline{n_1} \times \overline{n_2} = \begin{vmatrix} i & j & k \\ 1 & -3 & 2 \\ 1 & 3 & 1 \end{vmatrix} = -9i + j + 6k.$$

$$\overline{S} = \{-9,\ 1,\ 6\}.$$

(x_0, y_0, z_0) to'g'ri chiziqdan o'tuvchi biror nuqtaning koordinasini topamiz.

z ning koordinatasiga $z = 0$ qiymatni beramiz

$$\begin{cases} x - 3y + 2 = 0, \\ x + 3y + 14 = 0 \end{cases} \Rightarrow \begin{cases} x - 3y + 2 = 0, \\ 6y = -12 \end{cases} \Rightarrow \begin{cases} x = -8, \\ y = -2 \end{cases}$$

Shunday qilib, izlanayotgan nuqtaning koordinatasi $(-8,\ -2,\ 0)$.

To'g'ri chiziq tenglamasi

$$\frac{x+8}{-9}=\frac{y+2}{1}=\frac{z}{6}.$$

1. $\begin{cases}2x+y+z-2=0,\\ 2x-y-3z+6=0.\end{cases}$
2. $\begin{cases}x-2y+z-4=0,\\ 2x+2y-z-8=0.\end{cases}$
3. $\begin{cases}x+y+z-2=0,\\ x-y-2z+2=0.\end{cases}$
4. $\begin{cases}2x+3y+z+6=0,\\ x-3y-2z+3=0.\end{cases}$
5. $\begin{cases}3x+y-z-6=0,\\ 3x-y+2z=0.\end{cases}$
6. $\begin{cases}x+5y+2z+11=0,\\ x-y-z-1=0.\end{cases}$
7. $\begin{cases}3x+4y-2z+1=0,\\ 2x-4y+3z+4=0.\end{cases}$
8. $\begin{cases}5x+y-3z+4=0,\\ x-y+2z+2=0.\end{cases}$
9. $\begin{cases}x-y-z-2=0,\\ x-2y+z+4=0.\end{cases}$
10. $\begin{cases}4x+y-3z+2=0,\\ 2x-y+z-8=0.\end{cases}$
11. $\begin{cases}3x+3y-2z-1=0,\\ 2x-3y+z+6=0.\end{cases}$
12. $\begin{cases}6x-7y-4z-2=0,\\ x+7y-z-5=0.\end{cases}$
13. $\begin{cases}8x-y-3z-1=0,\\ x+y+z+10=0.\end{cases}$
14. $\begin{cases}6x-5y-4z+8=0,\\ 6x+5y+3z+4=0.\end{cases}$
15. $\begin{cases}x+5y-z-5=0,\\ 2x-5y+2z+5=0.\end{cases}$
16. $\begin{cases}2x-3y+z+6=0,\\ x-3y-2z+3=0.\end{cases}$
17. $\begin{cases}5x+y+2z+4=0,\\ x-y-3z+2=0.\end{cases}$
18. $\begin{cases}4x+y+z+2=0,\\ 2x-y-3z-8=0.\end{cases}$
19. $\begin{cases}2x+y-3z-2=0,\\ 2x-y+z+6=0.\end{cases}$
20. $\begin{cases}x+5y-z+11=0,\\ x-y+2z-1=0.\end{cases}$
21. $\begin{cases}x+y-2z-2=0,\\ x-y+z+2=0.\end{cases}$
22. $\begin{cases}x-y+z-2=0,\\ x-2y-z+4=0.\end{cases}$
23. $\begin{cases}6x-7y-z-2=0,\\ x+7y-4z-5=0.\end{cases}$
24. $\begin{cases}x+5y+2z-5=0,\\ 2x-5y-z+5=0.\end{cases}$
25. $\begin{cases}x-3y+z+2=0,\\ x+3y+2z+14=0.\end{cases}$
26. $\begin{cases}2x+3y-2z+6=0,\\ x-3y+z+3=0.\end{cases}$
27. $\begin{cases}3x+4y+3z+1=0,\\ 2x-4y-2z+4=0.\end{cases}$

28. $\begin{cases} 3x+3y+z-1=0, \\ 2x-3y-2z+6=0. \end{cases}$ **29.** $\begin{cases} 6x-5y+3z+8=0, \\ 6x+5y-4z+4=0. \end{cases}$ **30.** $\begin{cases} 2x-3y-2z+6=0, \\ x-3y+z+3=0. \end{cases}$

Javoblar. 12.1 $1-x=\frac{y-4}{4}=-\frac{z}{2}$; **12.2** $\frac{x-4}{-1}=\frac{y}{3}=\frac{z}{6}$; **12.3** $\frac{x}{-1}=\frac{y-2}{3}=\frac{z}{-2}$;

12.4 $\frac{x+3}{-3}=\frac{y}{5}=\frac{z}{-9}$; **12.5** $x-1\frac{1}{3}=\frac{y}{-9}=\frac{z+2}{-6}$;**12.6** $\frac{x+6}{-3}=\frac{y+1}{3}=\frac{z}{-6}$;

12.7 $\frac{x+1}{-4}=\frac{y-\frac{1}{2}}{-13}=\frac{z}{-20}$; **12.8** $\frac{x+1}{-1}=\frac{y-1}{-13}=\frac{z}{-6}$;**12.9** $\frac{x-8}{-3}=\frac{y-6}{-2}=\frac{z}{-1}$;

12.10 $\frac{x-1}{-2}=\frac{y+6}{-10}=\frac{z}{-6}$; **12.11** $\frac{x+1}{-3}=\frac{y-\frac{1}{3}}{-7}=\frac{z}{-15}$; **12.12** $\frac{x}{35}=\frac{y-\frac{4}{7}}{2}=\frac{z}{49}$;

12.13 $\frac{x+1}{2}=\frac{y+9}{-11}=\frac{z}{9}$; **12.14** $\frac{x+1}{5}=\frac{y-0,4}{-42}=\frac{z}{60}$; **12.15** $\frac{x}{5}=\frac{y-1}{-4}=\frac{z}{-15}$;

12.16 $\frac{x+3}{9}=\frac{y}{5}=\frac{z}{-3}$; **12.17** $\frac{x+1}{-1}=\frac{y-1}{17}=\frac{z}{-6}$; **12.18** $\frac{x-1}{-2}=\frac{y+6}{14}=\frac{z}{-6}$;

12.19 $\frac{x+2}{-2}=\frac{y-4}{-8}=\frac{z}{-4}$; **12.20** $\frac{x+1}{9}=\frac{y+2}{-3}=\frac{z}{-6}$; **12.21** $\frac{x}{-1}=\frac{y-2}{-3}=\frac{z}{-2}$;

12.22 $\frac{x-8}{3}=\frac{y-6}{2}=\frac{z}{-1}$; **12.23** $\frac{x-1}{35}=\frac{y-\frac{4}{7}}{23}=\frac{z}{49}$; **12.24** $\frac{x}{5}=\frac{y-1}{5}=\frac{z}{-15}$;

12.25 $\frac{x+8}{-9}=\frac{y+2}{-1}=\frac{z}{6}$; **12.26** $\frac{x+3}{-3}=\frac{y}{-4}=\frac{z}{-9}$; **12.27** $\frac{x+1}{4}=\frac{y+\frac{1}{2}}{12}=\frac{z}{-20}$;

12.28 $\frac{x+1}{-3}=\frac{y-\frac{4}{3}}{8}=\frac{z}{-15}$; **12.29** $\frac{x+1}{5}=\frac{y-0,4}{42}=\frac{z}{60}$; **12.30** $\frac{x+3}{9}=\frac{y}{4}=\frac{z}{3}$.

To’g’ri chiziq va tekislikning kesishish nuqtasi

13–masala. To’g’ri chiziq va tekislikning kesishish nuqtasini toping.

$$\frac{x+2}{-1}=\frac{y-1}{1}=\frac{z+3}{2},$$
$$x+2y-z-2=0,$$

$$\begin{cases} x = -t - 2, \\ y = t + 1, \\ z = 2t - 3. \end{cases}$$

Tekislik tenglamasiga olib borib qo'yamiz

$$\begin{aligned} &(-t-2) + 2(t+1) - (2t-3) - 2 = 0, \\ &-t - 2 + 2t + 2 - 2t + 3 - 2 = 0, \\ &-t + 1 = 0, \\ &t = 1. \end{aligned}$$

Shunday qilib, izlanayotgan nuqtaning koordinasi $(-3,\ 2,\ -1)$.

1. $\dfrac{x-2}{-1} = \dfrac{y-3}{-1} = \dfrac{z+1}{4}$, $x + 2y + 3z - 14 = 0$.

2. $\dfrac{x+1}{3} = \dfrac{y-3}{-4} = \dfrac{z+1}{5}$, $x + 2y - 5z + 20 = 0$.

3. $\dfrac{x-1}{-1} = \dfrac{y+5}{4} = \dfrac{z-1}{2}$, $x - 3y + 7z - 24 = 0$.

4. $\dfrac{x-1}{1} = \dfrac{y}{0} = \dfrac{z+3}{2}$, $2x - y + 4z = 0$.

5. $\dfrac{x-5}{1} = \dfrac{y-3}{-1} = \dfrac{z-2}{0}$, $3x + y - 5z - 12 = 0$.

6. $\dfrac{x+1}{-3} = \dfrac{y+2}{2} = \dfrac{z-3}{-2}$, $x + 3y - 5z + 9 = 0$.

7. $\dfrac{x-1}{-2} = \dfrac{y-2}{1} = \dfrac{z+1}{-1}$, $x - 2y + 5z + 17 = 0$.

8. $\dfrac{x-1}{2} = \dfrac{y-2}{0} = \dfrac{z-4}{1}$, $x - 2y + 4z - 19 = 0$.

9. $\dfrac{x+2}{-1} = \dfrac{y-1}{1} = \dfrac{z+4}{-1}$, $2x - y + 3z + 23 = 0$.

10. $\dfrac{x+2}{1} = \dfrac{y-2}{0} = \dfrac{z+3}{0}$, $2x - 3y - 5z - 7 = 0$.

11. $\dfrac{x-1}{2} = \dfrac{y-1}{-1} = \dfrac{z+2}{3}$, $4x + 2y - z - 11 = 0$.

12. $\dfrac{x-1}{1} = \dfrac{y+1}{0} = \dfrac{z-1}{-1}$, $3x - 2y - 4z - 8 = 0$.

13. $\dfrac{x+3}{1} = \dfrac{y-2}{-5} = \dfrac{z+2}{3}$, $5x - y + 4z + 3 = 0$.

14. $\dfrac{x-2}{2} = \dfrac{y-2}{-1} = \dfrac{z-4}{3}$, $x + 3y + 5z - 42 = 0$.

15. $\dfrac{x-3}{-1} = \dfrac{y-4}{5} = \dfrac{z-4}{2}$, $7x + y + 4z - 47 = 0$.

16. $\frac{x+3}{2}=\frac{y-1}{3}=\frac{z-1}{5}$, $2x+3y+7z-52=0$.

17. $\frac{x-3}{2}=\frac{y+1}{3}=\frac{z+3}{2}$, $3x+4y+7z-16=0$.

18. $\frac{x-5}{-2}=\frac{y-2}{0}=\frac{z+4}{-1}$, $2x-5y+4z+24=0$.

19. $\frac{x-1}{8}=\frac{y-8}{-5}=\frac{z+5}{12}$, $x-2y-3z+18=0$.

20. $\frac{x-3}{1}=\frac{y-1}{-1}=\frac{z+5}{0}$, $x+7y+3z+11=0$.

21. $\frac{x-5}{-1}=\frac{y+3}{5}=\frac{z-1}{2}$, $3x+7y-5z-11=0$.

22. $\frac{x-1}{7}=\frac{y-2}{1}=\frac{z-6}{-1}$, $4x+y-6z-5=0$.

23. $\frac{x-3}{1}=\frac{y+2}{-1}=\frac{z-8}{0}$, $5x+9y+4z-25=0$.

24. $\frac{x+1}{-2}=\frac{y}{0}=\frac{z+1}{3}$, $x+4y+13z-23=0$.

25. $\frac{x-1}{6}=\frac{y-3}{1}=\frac{z+5}{3}$, $3x-2y+5z-3=0$.

26. $\frac{x-2}{4}=\frac{y-1}{-3}=\frac{z+3}{-2}$, $3x-y+4z=0$.

27. $\frac{x-1}{2}=\frac{y+2}{-5}=\frac{z-3}{-2}$, $x+2y-5z+16=0$.

28. $\frac{x-1}{1}=\frac{y-3}{0}=\frac{z+2}{-2}$, $3x-7y-2z+7=0$.

29. $\frac{x+3}{0}=\frac{y-2}{-3}=\frac{z+5}{11}$, $5x+7y+9z-32=0$.

30. $\frac{x-7}{3}=\frac{y-3}{1}=\frac{z+1}{-2}$, $2x+y+7z-3=0$.

Javoblar. 13.1 $(1;2;3)$; **13.2** $(2;-1;4)$; **13.3** $(0;-1;3)$; **13.4** $(2;0;-1)$; **13.5** $(7;1;2)$; **13.6** $(-4;0;1)$; **13.7** $(7;-1;-5\frac{1}{5})$; **13.8** $(3;2;5)$; **13.9** $(-3;2;-5)$; **3.10** $(-1;2;-3)$; **13.11** $(3;0;1)$; **13.12** $(2;-1;0)$; **13.13** $(-2;-3;1)$; **13.14** $(4;1;7)$; **13.15** $(2;9;6)$; **13.16** $(-1;4;6)$; **13.17** $(5;2;-1)$; **13.18** $(3;2;-5)$; **13.19** $(9;3;7)$; **13.20** $(4;0;-5)$; **13.21** $(4;2;3)$; **13.22** $(8;3;5)$; **13.23** $(4;-3;8)$; **13.24** $(-3;0;2)$; **13.25** $(7;4;-2)$; **13.26** $(6;-2;-5)$; **13.27** $(3;-7;1)$; **13.28** $(2;3;-4)$; **13.29** $(-3;-1;6)$; **13.30** $(10;4;-3)$.

To’g’ri chiziqqa nisbatan nuqtalarning simmetrikligi

14–masala. To’g’ri chiziqqa nisbatan, M nuqtaga simmetrik bo’lgan M' nuqtani toping.

$$M(3,\ 3,\ 3),$$

$$\frac{x-1}{-1}=\frac{y-1{,}5}{0}=\frac{z-3}{1}.$$

$$-1\cdot(x-3)+0\cdot(y-3)+1\cdot(z-3)=0,$$

$$-x+z=0.$$

To’g’ri chiziq va tekislikning kesishish nuqtasini topamiz

$$\frac{x-1}{-1}=\frac{y-1{,}5}{0}=\frac{z-3}{1}\Rightarrow\begin{cases}x=-t+1,\\ y=1{,}5,\\ z=t+3.\end{cases}$$

$$-(-t+1)+(t+3)=0,$$

$$2t+2=0,$$

$$t=-1.$$

$M_0(2;\ 1{,}5;\ 2)$ – kesishish nuqtasining koordinatasi

Bundan,

$$x_{M_0}=\frac{x_M+x_{M'}}{2}\Rightarrow x_{M'}=2x_{M_0}-x_M=2\cdot 2-3=1,$$

$$y_{M_0}=\frac{y_M+y_{M'}}{2}\Rightarrow y_{M'}=2y_{M_0}-y_M=2\cdot 1{,}5-3=0,$$

$$z_{M_0}=\frac{z_M+z_{M'}}{2}\Rightarrow z_{M'}=2z_{M_0}-z_M=2\cdot 2-3=1.$$

Demak, $M'(1,\ 0,\ 1)$ – izlanayotgan nuqta.

To’g’ri chiziqqa (1–15 variantlar uchun) yoki tekislikka (16–30 variantlar uchun) nisbatan M nuqtaga simmetrik bo’lgan M' nuqtani toping

1. $M(0,-3,-2),$ $\dfrac{x-1}{1}=\dfrac{y+1{,}5}{-1}=\dfrac{z}{1}.$

2. $M(2,-1,\ 1),$ $\dfrac{x-4{,}5}{1}=\dfrac{y+3}{-0{,}5}=\dfrac{z-2}{1}.$

$M(1,\ 1,\ 1),$

3. $\dfrac{x-2}{1}=\dfrac{y+1{,}5}{-2}=\dfrac{z-1}{1}.$

$M(1,\ 2,\ 3),$

4. $\dfrac{x-0{,}5}{0}=\dfrac{y+1{,}5}{-1}=\dfrac{z-1{,}5}{1}.$

$M(1,\ 0,-1),$

5. $\dfrac{x-3{,}5}{2}=\dfrac{y-1{,}5}{2}=\dfrac{z}{0}.$

$M(2,\ 1,\ 0),$

6. $\dfrac{x-2}{0}=\dfrac{y+1{,}5}{-1}=\dfrac{z+0{,}5}{1}.$

$M(-2,-3,\ 0),$

7. $\dfrac{x+0{,}5}{1}=\dfrac{y+1{,}5}{0}=\dfrac{z-0{,}5}{1}.$

$M(-1,\ 0,-1),$

8. $\dfrac{x}{-1}=\dfrac{y-1{,}5}{0}=\dfrac{z-2}{1}.$

$M(0,\ 2,\ 1),$

9. $\dfrac{x-1{,}5}{2}=\dfrac{y}{-1}=\dfrac{z-2}{1}.$

$M(3,-3,-1),$

10. $\dfrac{x-6}{5}=\dfrac{y-3{,}5}{4}=\dfrac{z+0{,}5}{0}.$

$M(-1,\ 2,\ 0),$

11. $\dfrac{x+0{,}5}{1}=\dfrac{y+0{,}75}{-0{,}2}=\dfrac{z-2}{2}.$

$M(2,-2,-3),$

12. $\dfrac{x-1}{-1}=\dfrac{y+0{,}5}{0}=\dfrac{z+1{,}5}{0}.$

$M(-1,\ 0,\ 1),$

13. $\dfrac{x+0{,}5}{0}=\dfrac{y-1}{0}=\dfrac{z-4}{2}.$

$M(0,-3,-2),$

14. $\dfrac{x-0{,}5}{0}=\dfrac{y+1{,}5}{-1}=\dfrac{z-1{,}5}{1}.$

$M(1,\ 0,\ 1),$

15. $4x+6y+4z-25=0.$

$M(-1,\ 0,-1),$

16. $2x+6y-2z+11=0.$

$M(0,\ 2,\ 1),$

17. $2x+4y-3=0.$

$M(2,\ 1,\ 0),$

18. $y+z+2=0.$

$M(-1,\ 2,\ 0),$

19. $4x-5y-z-7=0.$

$M(2,-1,\ 1),$

20. $x-y+2z-2=0.$

21. $M(1,\ 1,\ 1),$ $x+4y+3z+5=0.$

22. $M(1,\ 2,\ 3),$ $2x+10y+10z-1=0.$

23. $M(0,-3,-2),$ $2x+10y+10z-1=0.$

24. $M(1,\ 0,-1),$ $2y+4z-1=0.$

25. $M(3,-3,-1),$ $2x-4y-4z-13=0.$

26. $M(-2,-3,\ 0),$ $x+5y+4=0.$

27. $M(2,-2,-3),$ $y+z+2=0.$

28. $M(-1,\ 0,\ 1),$ $2x+4y-3=0.$

29. $M(3,\ 3,\ 3),$ $8x+6y+8z-22=0.$

30. $M(-2,\ 0,\ 3),$ $2x-2y+10z+1=0.$

Javoblar. 14.1 $M'(1;1;1)$; **14.2** $M'(3;-3;-1)$; **14.3** $M'(1;\ 0;-1)$; **14.4** $M'(0;-3;-2)$; **14.5** $M'(2;-1;\ 1)$; **14.6** $M'(2;-2;-3)$; **14.7** $M'(-1;\ 0;-1)$; **14.8** $M'(3;3;3)$; **14.9** $M'(-1;\ 0;1)$; **14.10** $M'(-1;\ 2;0)$; **14.11** $M'(-2;-3;0)$; **14.12** $M'(2;\ 1;\ 0)$; **14.13** $M'(0;2;1)$; **14.14** $M'(1;2;3)$; **14.15** $M'(3;3;3)$; **14.16** $M'(-2;-3;0)$; **14.17** $M'(-1;\ 0;1)$; **14.18** $M'(2;-2;-3)$; **14.19** $M'(3;-3;-1)$; **14.20** $M'(1;\ 0;-1)$; **14.21** $M'(0;-3;-2)$; **14.22** $M'(0;-3;-2)$; **14.23** $M'(1;2;3)$; **14.24** $M'(1;1;1)$; **14.25** $M'(2;-1;\ 1)$; **14.26** $M'(-1;-2;\ 0)$; **14.27** $M'(2;\ 1;\ 0)$; **14.28** $M'(0;\ 2;\ 1)$; 14.29 $M'(-1;\ 0;-1)$ 14.30 $M'(-3;\ 1;-2)$.

II BOB. Limitlar nazariyasi

Limitlar nazariyasi bo'limida, siz sonli ketma-ketlik tushunchasi, nuqtada son va funksiyaning uzluksizligi, turli limitlarni hisoblashning maxsus yo'llari bilan tanishasiz.

Ta'rif. Agar a nuqtaning ixtiyoriy $(a-\varepsilon,\ a+\varepsilon)$ atrofi $\forall\varepsilon>0$ olinganda ham $\{x_n\}$ ketma-ketlikning biror hadidan boshlab, keyingi

barcha hadlari shu trofga tegishli bo'lsa, a son $\{x_n\}$ ketma – ketlikning limiti deyiladi va

$$\lim_{n\to\infty} x_n = a$$

kabi belgilanadi.

Ketma-ketlikning ti'rifini quyidagicha ta'riflash ham mumkin.

Ta'rif. Agar $\forall \varepsilon > 0$ son olinganda ham shunday natural n_0 son $(n_0 \in N)$ topilsaki, barcha $n > n_0$ lar uchun

$$|x_n - a| < \varepsilon$$

tengsizlik bajarilsa, a son $\{x_n\}$ ketma–ketlikning limiti deyiladi.

1–masala. $\lim_{n\to\infty} a_n = a$ ekanligini isbotlang ($N(\varepsilon)$ ko'rsating).

$$a_n = \frac{3n-2}{2n-1},\ a = \frac{3}{2}.$$

limitning ta'rifiga ko'ra:

$$\forall \varepsilon > 0:\ \exists N(\varepsilon):\ \forall n : n \ge N(\varepsilon):\ |a_n - a| < \varepsilon$$

$$\left|\frac{3n-2}{2n-1} - \frac{3}{2}\right| < \varepsilon;$$

$$\left|\frac{2(3n-2)-3(2n-1)}{2(2n-1)}\right| < \varepsilon;$$

$$\left|\frac{6n-4-6n+3}{2(2n-1)}\right| < \varepsilon;$$

$$\left|\frac{-1}{2(2n-1)}\right| < \varepsilon;$$

$$\left|\frac{1}{2(2n-1)}\right| < \varepsilon;$$

$\forall n \in N:\ \dfrac{1}{2(2n-1)} > 0$ ekanligidan,

$$\frac{1}{2(2n-1)} < \varepsilon;;$$

$$2n-1 > \frac{1}{2\varepsilon};$$

$$n > \frac{1}{2}\left(\frac{1}{2\varepsilon}+1\right);$$

$$N(\varepsilon) = \left[\frac{1}{2}\left(\frac{1}{2\varepsilon}+1\right)\right]+1 = \left[\frac{3}{2}+\frac{1}{4\varepsilon}\right] = \left[\frac{1+6\varepsilon}{4\varepsilon}\right]$$

$\forall n > N(\varepsilon)$ da $|a_n - a| < \varepsilon$ tengsizlik bajarilishidan,

$$\lim_{n\to\infty}\frac{3n-2}{2n-1} = \frac{3}{2}$$

kelib chiqadi.

1. $a_n = \dfrac{4n-1}{2n+1},\ a = 2.$

2. $a_n = \dfrac{7n+4}{2n+1},\ a = \dfrac{7}{2}.$

3. $a_n = \dfrac{2n-5}{3n+1},\ a = \dfrac{2}{3}.$

4. $a_n = \dfrac{7n-1}{n+1},\ a = 7.$

5. $a_n = \dfrac{4n^2+1}{3n^2+2},\ a = \dfrac{4}{3}.$

6. $a_n = \dfrac{9-n^3}{1+2n^3},\ a = -\dfrac{1}{2}.$

7. $a_n = \dfrac{4n-3}{2n+1},\ a = 2.$

8. $a_n = \dfrac{1-2n^2}{2+4n^2},\ a = -\dfrac{1}{2}.$

9. $a_n = -\dfrac{5n}{n+1},\ \ a = -5.$

10. $a_n = \dfrac{n+1}{1-2n},\ \ a = -\dfrac{1}{2}.$

11. $a_n = \dfrac{2n+1}{3n-5},\ \ a = \dfrac{2}{3}.$

12. $a_n = \dfrac{1-2n^2}{n^2+3},\ \ a = -2.$

13. $a_n = \dfrac{3n^2}{2-n^2},\ \ a = -3.$

14. $a_n = \dfrac{n}{3n-1},\ \ a = \dfrac{1}{3}.$

15. $a_n = \dfrac{3n^3}{n^3-1}, \quad a = 3.$ **16.** $a_n = \dfrac{4+2n}{1-3n}, \quad a = -\dfrac{2}{3}.$

17. $a_n = \dfrac{5n+15}{6-n}, \quad a = -5.$ **18.** $a_n = \dfrac{3-n^2}{4+2n^2}, \quad a = -\dfrac{1}{2}.$

19. $a_n = \dfrac{2n-1}{2-3n}, \quad a = -\dfrac{2}{3}.$ **20.** $a_n = \dfrac{3n-1}{5n+1}, \quad a = \dfrac{3}{5}.$

21. $a_n = \dfrac{4n-3}{2n+1}, \quad a = 2.$ **22.** $a_n = \dfrac{1-2n^2}{2+4n^2}, \quad a = -\dfrac{1}{2}.$

23. $a_n = \dfrac{5n+1}{10n-3}, \quad a = \dfrac{1}{2}.$ **24.** $a_n = \dfrac{2-2n}{3+4n}, \quad a = -\dfrac{1}{2}.$

25. $a_n = \dfrac{23-4n}{2-n}, \quad a = 4.$ **26.** $a_n = \dfrac{1+3n}{6-n}, \quad a = -3.$

27. $a_n = \dfrac{2n+3}{n+5}, \quad a = 2.$ **28.** $a_n = \dfrac{3n^2+2}{4n^2-1}, \quad a = \dfrac{3}{4}.$

29. $a_n = \dfrac{2-3n^2}{4+5n^2}, \quad a = -\dfrac{3}{5}.$ **30.** $a_n = \dfrac{2n^3}{n^3-2}, \quad a = 2.$

Javoblar. 1.1 $\left[\dfrac{3+\varepsilon}{2\varepsilon}\right]$; **1.2** $\left[\dfrac{1+2\varepsilon}{4\varepsilon}\right]$; **1.3** $\left[\dfrac{17+6\varepsilon}{9\varepsilon}\right]$; **1.4** $\left[\dfrac{8}{\varepsilon}\right]$; **1.5** $\left[\sqrt{\dfrac{1}{3}\left|\dfrac{5}{3\varepsilon}-2\right|}\right]+1$;

1.6 $\left[\sqrt[3]{\dfrac{1}{2}\left(\dfrac{19}{2\varepsilon}-1\right)}\right]+1$; **1.7** $\left[\dfrac{5+\varepsilon}{2\varepsilon}\right]$; **1.8** $\left[\sqrt{\dfrac{1}{2}\left|\dfrac{1}{\varepsilon}-1\right|}\right]+1$; **1.9** $\left[\dfrac{5}{\varepsilon}\right]$; **1.10** $3\cdot\left[\dfrac{1}{4\varepsilon}+\dfrac{1}{2}\right]$;

1.11 $2+\left[\dfrac{13+6\varepsilon}{9\varepsilon}\right]$; **1.12** $\left[\sqrt{\left|\dfrac{7}{\varepsilon}-3\right|}\right]+1$; **1.13** $\left[\sqrt{\left|\dfrac{6}{\varepsilon}+2\right|}\right]+1$; **1.14** $\left[\dfrac{1+12\varepsilon}{9\varepsilon}\right]$;

1.15 $\left[\sqrt[3]{1+\dfrac{3}{\varepsilon}}\right]+1$; **1.16** $\left[\dfrac{14+12\varepsilon}{9\varepsilon}\right]$; **1.17** $7+\left[\dfrac{45}{\varepsilon}\right]$; **1.18** $\left[\sqrt{\left|\dfrac{5}{2\varepsilon}-2\right|}\right]+1$; **1.19** $\left[\dfrac{1+15\varepsilon}{9\varepsilon}\right]$;

1.20 $\left[\dfrac{8+20\varepsilon}{25\varepsilon}\right]$; **1.21** $\left[\dfrac{5+\varepsilon}{2\varepsilon}\right]$; **1.22** $\left[\sqrt{\dfrac{1}{2}\left|\dfrac{1}{\varepsilon}-1\right|}\right]+1$; **1.23** $\left[\dfrac{5+14\varepsilon}{20\varepsilon}\right]$; **1.24** $\left[\dfrac{7+2\varepsilon}{8\varepsilon}\right]$;

1.25 $3+\left[\frac{15}{\varepsilon}\right]$; **1.26** $7+\left[\frac{19}{\varepsilon}\right]$; **1.27** $\left[\frac{7}{\varepsilon}\right]-4$; **1.28** $\left[\frac{1}{2}\sqrt{\left|\frac{11}{4\varepsilon}+1\right|}\right]+1$; **1.29** $\left[\sqrt{\frac{1}{5}\left|\frac{22}{5\varepsilon}-4\right|}\right]+1$;

1.30 $\left[\sqrt[3]{2+\frac{4}{\varepsilon}}\right]+1$.

Sonli ketma-ketlikning limiti

2–masala. Sonli ketma-ketlikning limitini hisoblang.

$$\lim_{n\to\infty}\frac{(6-n)^2-(6+n)^2}{(6+n)^2-(1-n)^2}=\lim_{n\to\infty}\frac{(36-12n+n^2)-(36+12n+n^2)}{(36+12n+n^2)-(1-2n+n^2)}=$$

$$=\lim_{n\to\infty}\frac{-24n}{14n+35}=\lim_{n\to\infty}\frac{-24}{14+35/n}=-\frac{24}{14}=-\frac{12}{7}.$$

1. $\lim\limits_{n\to\infty}\dfrac{(3-n)^2+(3+n)^2}{(3-n)^2-(3+n)^2}$.

2. $\lim\limits_{n\to\infty}\dfrac{(3-n)^4+(2-n)^4}{(1-n)^4-(1+n)^4}$.

3. $\lim\limits_{n\to\infty}\dfrac{(3-n)^4-(2-n)^4}{(1-n)^3-(1+n)^3}$.

4. $\lim\limits_{n\to\infty}\dfrac{(1-n)^4-(1+n)^4}{(1+n)^3-(1-n)^3}$.

5. $\lim\limits_{n\to\infty}\dfrac{(n+1)^3-(n+1)^2}{(n-1)^3-(n+1)^3}$.

6. $\lim\limits_{n\to\infty}\dfrac{(1+2n)^3-8n^3}{(1+2n)^2+4n^2}$.

7. $\lim\limits_{n\to\infty}\dfrac{(3-4n)^2}{(n-3)^3-(n+3)^3}$.

8. $\lim\limits_{n\to\infty}\dfrac{(3-n)^3}{(n+1)^2-(n+1)^3}$.

9. $\lim\limits_{n\to\infty}\dfrac{(n+1)^2+(n-1)^2-(n+2)^3}{(4-n)^3}$.

10. $\lim\limits_{n\to\infty}\dfrac{2(n+1)^2+(n-2)^3}{n^2+2n-3}$.

11. $\lim\limits_{n\to\infty}\dfrac{(n+1)^3+(n+2)^3}{(n+4)^3+(n+5)^3}$.

12. $\lim\limits_{n\to\infty}\dfrac{(n+3)^3+(n+4)^3}{(n+3)^4-(n+4)^4}$.

13. $\lim\limits_{n\to\infty}\dfrac{(n+1)^4-(n-1)^4}{(n+1)^3+(n-1)^3}$.

14. $\lim\limits_{n\to\infty}\dfrac{8n^3-2n}{(n+1)^4-(n-1)^4}$.

15. $\lim\limits_{n\to\infty}\dfrac{(n+6)^3-(n+1)^3}{(2n+3)^2+(n+4)^2}$.

16. $\lim\limits_{n\to\infty}\dfrac{(2n-3)^3-(n+5)^3}{(3n-1)^3+(2n+3)^3}$.

17. $\lim\limits_{n\to\infty}\dfrac{(n+10)^2+(3n+1)^2}{(n+6)^3-(n+1)^3}$.

18. $\lim\limits_{n\to\infty}\dfrac{(2n+1)^3+(3n+2)^3}{(2n+3)^3-(n-7)^3}$.

19. $\lim\limits_{n\to\infty}\dfrac{(n+7)^3-(n+2)^3}{(3n+2)^2+(4n+1)^2}$.

20. $\lim\limits_{n\to\infty}\dfrac{(2n+1)^3-(2n+3)^3}{(2n+1)^2+(2n+3)^2}$.

21. $\lim\limits_{n\to\infty}\dfrac{n^3-(n-1)^3}{(n+1)^4-n^4}$.

22. $\lim\limits_{n\to\infty}\dfrac{(n+2)^4-(n-2)^4}{(n+5)^2+(n-5)^2}$.

23. $\lim\limits_{n\to\infty}\dfrac{(n+1)^4-(n-1)^4}{(n+1)^3+(n-1)^3}$.

24. $\lim\limits_{n\to\infty}\dfrac{(n+1)^3-(n-1)^3}{(n+1)^2-(n-1)^2}$.

25. $\lim\limits_{n\to\infty}\dfrac{(n+1)^3-(n-1)^3}{(n+1)^2-(n-1)^2}$.

26. $\lim\limits_{n\to\infty}\dfrac{(n+2)^3+(n-2)^3}{n^4+2n^2-1}$.

27. $\lim\limits_{n\to\infty}\dfrac{(n+1)^3+(n-1)^2}{n^3-3n}$.

28. $\lim\limits_{n\to\infty}\dfrac{(n+1)^3+(n-1)^3}{n^3+1}$.

29. $\lim\limits_{n\to\infty}\dfrac{(n+2)^2-(n-2)^2}{(n+3)^2}$.

30. $\lim\limits_{n\to\infty}\dfrac{(2n+1)^2-(n+1)^2}{n^2+n+1}$.

Javoblar. **2.1** $-\infty$; **2.2** $\dfrac{1}{2}$; **2.3** 2; **2.4** -4; **2.5** $-\infty$; **2.6** $1{,}5$; **2.7** $-\dfrac{8}{9}$; **2.8** 1; **2.9** 0; **2.10** $+\infty$;
2.11 1; **2.12** $-\dfrac{1}{2}$; **2.13** 4; **2.14** 1; **2.15** 3; **2.16** $\dfrac{1}{5}$; **2.17** $\dfrac{2}{3}$; **2.18** 5; **2.19** $\dfrac{3}{5}$; **2.20** -3;
2.21 0; **2.22** $+\infty$; **2.23** 4; **2.24** $+\infty$; **2.25** 3; **2.26** 0; **2.27** 2; **2.28** 2; **2.29** 0; **2.30** 3.

Sonli ketma-ketlikning limiti

3–masala. Sonli ketma-ketlikning limitini hisoblang.

$$\lim_{n\to\infty}\frac{\sqrt[3]{n^2-1}+7n^3}{\sqrt[4]{n^{12}+n+1}-n}=\lim_{n\to\infty}\frac{\sqrt[3]{\dfrac{1}{n^7}-\dfrac{1}{n^9}}+7}{\sqrt[4]{1+\dfrac{1}{n^{11}}+\dfrac{1}{n^{12}}}-\dfrac{1}{n^2}}=7.$$

1. $\lim\limits_{n\to\infty}\dfrac{n\sqrt[3]{5n^2}+\sqrt[4]{9n^8+1}}{(n+\sqrt{n})\sqrt{7-n+n^2}}$.

2. $\lim\limits_{n\to\infty}\dfrac{\sqrt{n-1}+\sqrt{n^2+1}}{\sqrt[3]{3n^3+3}+\sqrt[4]{n^5+1}}$.

3. $\lim\limits_{n\to\infty}\dfrac{\sqrt{n^3+1}-\sqrt{n-1}}{\sqrt[3]{n^3+1}-\sqrt{n-1}}$.

4. $\lim\limits_{n\to\infty}\dfrac{\sqrt{3n-1}+\sqrt[3]{125n^3+n}}{\sqrt[3]{n}-n}$.

5. $\lim\limits_{n\to\infty}\dfrac{n\sqrt[5]{n}-\sqrt[3]{27n^6+n^2}}{(n+\sqrt[4]{n})\sqrt{9+n^2}}$.

6. $\lim\limits_{n\to\infty}\dfrac{\sqrt{n+2}-\sqrt{n^2+2}}{\sqrt[4]{4n^4+1}-\sqrt[3]{n^4-1}}$.

7. $\lim\limits_{n\to\infty}\dfrac{\sqrt{n^4+2}+\sqrt{n-2}}{\sqrt[4]{n^4+2}+\sqrt{n-2}}$.

8. $\lim\limits_{n\to\infty}\dfrac{6n^3-\sqrt{n^5+1}}{\sqrt{4n^6+3}-n}$.

9. $\lim\limits_{n\to\infty}\dfrac{\sqrt{5n+2}-\sqrt[5]{8n^3+5}}{\sqrt[4]{n+7}-n}$.

10. $\lim\limits_{n\to\infty}\dfrac{n\sqrt[4]{3n+1}+\sqrt{81n^4-n^2+1}}{(n+\sqrt[3]{n})\sqrt{5-n+n^2}}$.

11. $\lim\limits_{n\to\infty}\dfrac{\sqrt{n+3}-\sqrt{n^2-3}}{\sqrt[3]{n^5-4}-\sqrt[4]{n^4+1}}$.

12. $\lim\limits_{n\to\infty}\dfrac{\sqrt{n^3+3}-\sqrt{n-3}}{\sqrt[5]{n^5+3}+\sqrt{n-3}}$.

13. $\lim\limits_{n\to\infty}\dfrac{\sqrt[3]{n}-9n^2}{3n-\sqrt[4]{9n^8+1}}$.

14. $\lim\limits_{n\to\infty}\dfrac{\sqrt{4n+1}-\sqrt[3]{27n^3+4}}{\sqrt[4]{n}-\sqrt[3]{n^5+n}}$.

15. $\lim\limits_{n\to\infty}\dfrac{n\sqrt[3]{7n}-\sqrt[4]{81n^8-1}}{\left(n+4\sqrt{n}\right)\sqrt{n^2-5}}$.

16. $\lim\limits_{n\to\infty}\dfrac{\sqrt[3]{n^3-7}+\sqrt[3]{n^2+4}}{\sqrt[4]{n^5+5}+\sqrt{n}}$.

17. $\lim\limits_{n\to\infty}\dfrac{\sqrt{n^6+4}+\sqrt{n-4}}{\sqrt[5]{n^6+6}-\sqrt{n-6}}$.

18. $\lim\limits_{n\to\infty}\dfrac{4n^2-\sqrt[4]{n^3}}{\sqrt[3]{n^6+n^3+1}-5n}$.

19. $\lim\limits_{n\to\infty}\dfrac{\sqrt{n+3}-\sqrt[3]{8n^3+3}}{\sqrt[4]{n+4}-\sqrt[5]{n^5+5}}$.

20. $\lim\limits_{n\to\infty}\dfrac{n\sqrt[4]{11n}+\sqrt{25n^4-81}}{\left(n-7\sqrt{n}\right)\sqrt{n^2-n+1}}$.

21. $\lim\limits_{n\to\infty}\dfrac{\sqrt[3]{n^2}-\sqrt{n^2+5}}{\sqrt[5]{n^7}-\sqrt{n+1}}$.

22. $\lim\limits_{n\to\infty}\dfrac{\sqrt{n^7+5}-\sqrt{n-5}}{\sqrt[7]{n^7+5}+\sqrt{n-5}}$.

23. $\lim\limits_{n\to\infty}\dfrac{\sqrt[3]{n^2+2}-5n^2}{n-\sqrt{n^4-n+1}}$.

24. $\lim\limits_{n\to\infty}\dfrac{\sqrt{n+2}-\sqrt[3]{n^3+2}}{\sqrt[7]{n+2}-\sqrt[5]{n^5+2}}$.

25. $\lim\limits_{n\to\infty}\dfrac{n\sqrt{71n}-\sqrt[3]{64n^6+9}}{\left(n-\sqrt[3]{n}\right)\sqrt{11+n^2}}$.

26. $\lim\limits_{n\to\infty}\dfrac{\sqrt{n+6}-\sqrt{n^2-5}}{\sqrt[3]{n^3+3}+\sqrt[4]{n^3+1}}$.

27. $\lim\limits_{n\to\infty}\dfrac{\sqrt{n^8+6}-\sqrt{n-6}}{\sqrt[8]{n^8+6}+\sqrt{n-6}}.$

28. $\lim\limits_{n\to\infty}\dfrac{n^2-\sqrt{n^3+1}}{\sqrt[3]{n^6+2}-n}.$

29. $\lim\limits_{n\to\infty}\dfrac{\sqrt{n+1}-\sqrt[3]{n^3+1}}{\sqrt[4]{n+1}-\sqrt[5]{n^5+1}}.$

30. $\lim\limits_{n\to\infty}\dfrac{n\sqrt[6]{n}+\sqrt[5]{n^{10}+1}}{\left(n+\sqrt[4]{n}\right)\sqrt[3]{n^3-1}}.$

Javoblar. 3.1 $\sqrt{3}$; **3.2** $-\infty$; **3.3** ∞; **3.4** 7; **3.5** 5; **3.6** -3; **3.7** 0; **3.8** ∞; **3.9** 3; **3.10** 2; **3.11** 9; **3.12** 0; **3.13** ∞; **3.14** $3\sqrt{3}$; **3.15** 0; **3.16** -3; **3.17** 0; **3.18** ∞; **3.19** 4; **3.20** 2; **3.21** 5; **3.22** 0; **3.23** ∞; **3.24** 5; **3.25** 1; **3.26** -4; **3.27** -1; **3.28** ∞; **3.29** 1; **3.30** ∞.

Sonli ketma-ketlikning limiti

4-masala. Sonli ketma-ketlikning limitini hisoblang.

$$\lim_{n\to\infty}\frac{\sqrt{(n^2+5)(n^4+2)}-\sqrt{n^6-3n^3+5}}{n}=$$

$$=\lim_{n\to\infty}\frac{\left(\sqrt{(n^2+5)(n^4+2)}-\sqrt{n^6-3n^3+5}\right)\left(\sqrt{(n^2+5)(n^4+2)}+\sqrt{n^6-3n^3+5}\right)}{n\left(\sqrt{(n^2+5)(n^4+2)}+\sqrt{n^6-3n^3+5}\right)}=$$

$$=\lim_{n\to\infty}\frac{5n^4+3n^3+2n^2+5}{n\left(\sqrt{(n^2+5)(n^4+2)}+\sqrt{n^6-3n^3+5}\right)}=$$

$$=\lim_{n\to\infty}\frac{\frac{1}{n^4}\left(5n^4+3n^3+2n^2+5\right)}{\frac{1}{n^4}n\left(\sqrt{(n^2+5)(n^4+2)}+\sqrt{n^6-3n^3+5}\right)}=$$

$$=\lim_{n\to\infty}\frac{5+\frac{3}{n}+\frac{2}{n^2}+\frac{5}{n^4}}{\left(\sqrt{(1+\frac{5}{n^2})(1+\frac{2}{n^4})}+\sqrt{1-\frac{3}{n^3}+\frac{5}{n^6}}\right)}=\frac{5+0+0+0}{\sqrt{(1+0)(1+0)}+\sqrt{1-0+0}}=\frac{5}{2}.$$

1. $\lim\limits_{n\to\infty} n(\sqrt{n^2+1}-\sqrt{n^2-1}).$

2. $\lim\limits_{n\to\infty} n(\sqrt{n(n-2)}-\sqrt{n^2-3}).$

3. $\lim\limits_{n\to\infty}\left(n-\sqrt[3]{n^3-5}\right)n\sqrt{n}.$

4. $\lim\limits_{n\to\infty}(\sqrt{(n^2+1)(n^2-4)}-\sqrt{n^4-9}).$

5. $\lim\limits_{n\to\infty} \dfrac{\sqrt{n^5-8}-n\sqrt{n(n^2+5))}}{\sqrt{n}}$.

6. $\lim\limits_{n\to\infty} (\sqrt{n^2-3n+2}-n)$.

7. $\lim\limits_{n\to\infty} \left(n+\sqrt[3]{4-n^3}\right)$.

8. $\lim\limits_{n\to\infty} (\sqrt{n(n+2)}-\sqrt{n^2-2n+3})$.

9. $\lim\limits_{n\to\infty} (\sqrt{(n+2)(n+1)}-\sqrt{(n-1)(n+3)})$.

10. $\lim\limits_{n\to\infty} n^2(\sqrt{n(n^4-1)}-\sqrt{n^5-8})$.

11. $\lim\limits_{n\to\infty} n(\sqrt[3]{5+8n^3}-2n)$.

12. $\lim\limits_{n\to\infty} n^2(\sqrt[3]{5+n^3}-\sqrt[3]{3+n^3})$.

13. $\lim\limits_{n\to\infty} (\sqrt[3]{(n+2)^2}-\sqrt[3]{(n-3)^2})$.

14. $\lim\limits_{n\to\infty} \dfrac{\sqrt{(n+1)^3}-\sqrt{n(n-1)(n-3)}}{\sqrt{n}}$.

15. $\lim\limits_{n\to\infty} (\sqrt{n^2+3n-2}-\sqrt{n^2-3})$.

16. $\lim\limits_{n\to\infty} \sqrt{n}(\sqrt{n+2}-\sqrt{n-3})$.

17. $\lim\limits_{n\to\infty} \dfrac{\sqrt{n(n^5+9)}-\sqrt{(n^4-1)(n^2+5)}}{n}$.

18. $\lim\limits_{n\to\infty} (\sqrt{n(n+5)}-n)$.

19. $\lim\limits_{n\to\infty} \sqrt{n^3+8}(\sqrt{n^3+2}-\sqrt{n^3-1})$.

20. $\lim\limits_{n\to\infty} \dfrac{\sqrt{\left(n^3+1\right)\left(n^2+3\right)}-\sqrt{n\left(n^4+2\right)}}{2\sqrt{n}}$.

21. $\lim\limits_{n\to\infty}(\sqrt{(n^2+1)(n^2+2)}-\sqrt{(n^2-1)(n^2-2)})$.

22. $\lim\limits_{n\to\infty}\dfrac{\sqrt{(n^5+1)(n^2-1)}-n\sqrt{n(n^4+1)}}{n}$.

23. $\lim\limits_{n\to\infty}\dfrac{\sqrt{(n^4+1)(n^2-1)}-\sqrt{n^6-1}}{n}$.

24. $\lim\limits_{n\to\infty}(n-\sqrt{n(n-1)})$.

25. $\lim\limits_{n\to\infty}n^3(\sqrt[3]{n^2(n^6+4)}-\sqrt[3]{(n^8-1)})$.

26. $\lim\limits_{n\to\infty}(n\sqrt{n}-\sqrt{n(n+1)(n+2)})$.

27. $\lim\limits_{n\to\infty}\sqrt[3]{n}(\sqrt[3]{n^2}-\sqrt[3]{n(n-1)})$.

28. $\lim\limits_{n\to\infty}\sqrt{n+2}(\sqrt{n+3}-\sqrt{n-4})$.

29. $\lim\limits_{n\to\infty}n(\sqrt{n^4+3}-\sqrt{n^4-2})$.

30. $\lim\limits_{n\to\infty}\sqrt{n(n+1)(n+2)}(\sqrt{n^3-3}-\sqrt{n^3-2})$.

Javoblar. 4.1 ∞; **4.2** $-\infty$; **4.3** 0; **4.4** $-\dfrac{3}{2}$; **4.5** $-\dfrac{5}{2}$; **4.6** $-\dfrac{3}{2}$; **4.7** 0; **4.8** 2; **4.9** $\dfrac{1}{2}$; **4.10** $-\infty$; **4.11** 0; **4.12** $\dfrac{2}{3}$; **4.13** 0; **4.14** $\dfrac{7}{2}$; **4.15** $\dfrac{3}{2}$; **4.16** $\dfrac{5}{2}$; **4.17** $-\dfrac{5}{2}$; **4.18** $\dfrac{5}{2}$; **4.19** $\dfrac{3}{2}$; **4.20** $\dfrac{3}{4}$; **4.21** 3; **4.22** $-\infty$; **4.23** $-\dfrac{1}{2}$; **4.24** $\dfrac{1}{2}$; **4.25** 0; **4.26** $-\infty$; **4.27** $\dfrac{1}{3}$; **4.28** $\dfrac{7}{2}$; **4.29** 0; **4.30** $-\dfrac{1}{2}$.

Sonli ketma-ketlikning limiti

5–masala. Sonli ketma-ketlikning limitini hisoblang.

$$\lim_{n\to\infty}\frac{(2n+1)!+(2n+2)!}{(2n+3)!-(2n+2)!}=\lim_{n\to\infty}\frac{(2n+1)!+(2n+2)!}{(2n+3)\cdot(2n+2)!-(2n+2)!}=$$

$$=\lim_{n\to\infty}\frac{(2n+1)!+(2n+2)!}{(2n+2)!\cdot\big((2n+3)-1\big)}=\lim_{n\to\infty}\frac{(2n+1)!+(2n+2)!}{(2n+2)!\cdot(2n+2)}=$$

$$= \lim_{n\to\infty}\left(\frac{(2n+1)!}{(2n+2)!\cdot(2n+2)} + \frac{(2n+2)!}{(2n+2)!\cdot(2n+2)}\right) =$$

$$= \lim_{n\to\infty}\left(\frac{1}{(2n+2)\cdot(2n+2)} + \frac{1}{(2n+2)}\right) = 0 + 0 = 0.$$

1. $\lim\limits_{n\to\infty}(\frac{1}{n^2} + \frac{2}{n^2} + \frac{3}{n^2} + ... + \frac{n-1}{n^2})$.

2. $\lim\limits_{n\to\infty}\frac{(2n+1)!+(2n+2)!}{(2n+3)!}$.

3. $\lim\limits_{n\to\infty}\left(\frac{1+3+5+7+...+(2n-1)}{n+1} - \frac{2n+1}{2}\right)$.

4. $\lim\limits_{n\to\infty}\frac{2^{n+1}+3^{n+1}}{2^n+3^n}$.

5. $\lim\limits_{n\to\infty}\frac{1+2+3+...+n}{\sqrt{9n^4+1}}$.

6. $\lim\limits_{n\to\infty}\frac{1+3+5+...+(2n-1)}{1+2+3+...+n}$.

7. $\lim\limits_{n\to\infty}\left(\frac{1+3+5+...+(2n-1)}{n+3} - n\right)$.

8. $\lim\limits_{n\to\infty}\frac{1+4+7+...+(3n-2)}{\sqrt{5n^4+n+1}}$.

9. $\lim\limits_{n\to\infty}\frac{(n+4)!-(n+2)!}{(n+3)!}$.

10. $\lim\limits_{n\to\infty}\frac{(3n-1)!+(3n+1)!}{(3n!)(n-1)}$.

11. $\lim\limits_{n\to\infty}\frac{2^n-5^{n+1}}{2^{n+1}+5^{n+2}}$.

12. $\lim\limits_{n\to\infty}\dfrac{1+\dfrac{1}{3}+\dfrac{1}{3^2}+\ldots+\dfrac{1}{3^n}}{1+\dfrac{1}{5}+\dfrac{1}{5^2}+\ldots+\dfrac{1}{5^n}}.$

13. $\lim\limits_{n\to\infty}\dfrac{1-3+5-7+9-11+\ldots+(4n-3)-(4n-1)}{\sqrt{n^2+1}+\sqrt{n^2+n+1}}.$

14. $\lim\limits_{n\to\infty}\dfrac{1-2+3-4+\ldots+(2n-1)-2n}{n}.$

15. $\lim\limits_{n\to\infty}\dfrac{\sqrt[3]{n^3+5}-\sqrt{3n^4+2}}{1+3+5+\ldots+(2n-1)}.$

16. $\lim\limits_{n\to\infty}\dfrac{3^n-2^n}{3^{n-1}+2^n}.$

17. $\lim\limits_{n\to\infty}\left(\dfrac{n+2}{1+2+3+\ldots+n}-\dfrac{2}{3}\right).$

18. $\lim\limits_{n\to\infty}(\dfrac{5}{6}+\dfrac{13}{36}+\ldots+\dfrac{3^n+2^n}{6^n}).$

19. $\lim\limits_{n\to\infty}\dfrac{2-5+4-7+\ldots+2n-(2n+3)}{n+3}.$

20. $\lim\limits_{n\to\infty}\dfrac{1+2+\ldots+n}{n-n^2+3}.$

21. $\lim\limits_{n\to\infty}\dfrac{n^2+\sqrt{n}-1}{2+7+12+\ldots+(5n-3)}.$

22. $\lim\limits_{n\to\infty}(\dfrac{3}{4}+\dfrac{5}{16}+\dfrac{9}{64}+\ldots+\dfrac{1+2^n}{4^n}).$

23. $\lim\limits_{n\to\infty}\dfrac{2+4+6+\ldots 2n}{1+3+5+\ldots+(2n-1)}.$

24. $\lim\limits_{n\to\infty}\left(\dfrac{1+5+9+13+\ldots+(4n-3)}{n+1}-\dfrac{4n+1}{2}\right).$

25. $\lim\limits_{n\to\infty}\dfrac{1-2+3-4+...-2n}{\sqrt[3]{n^3+2n+2}}$.

26. $\lim\limits_{n\to\infty}\dfrac{2^n+7^n}{2^n-7^{n-1}}$.

27. $\lim\limits_{n\to\infty}\dfrac{n!+(n+2)!}{(n-1)!+(n+2)!}$.

28. $\lim\limits_{n\to\infty}\dfrac{3+6+9+...+3n}{n^2+4}$.

29. $\lim\limits_{n\to\infty}(\dfrac{7}{10}+\dfrac{29}{100}+...+\dfrac{2^n+5^n}{10^n})$.

30. $\lim\limits_{n\to\infty}\left(\dfrac{2+4+...+2n}{n+3}-n\right)$.

Javoblar. 5.1 $\dfrac{1}{2}$; **5.2** 0; **5.3** $-\dfrac{3}{2}$; **5.4** 3; **5.5** $\dfrac{1}{6}$; **5.6** 2; **5.7** -3; **5.8** $\dfrac{3}{2\sqrt{5}}$; **5.9** ∞; **5.10** 3; **5.11** $-\dfrac{1}{5}$; **5.12** $\dfrac{6}{5}$; **5.13** -1; **5.14** 0; **5.15** $-\sqrt{3}$; **5.16** 3; **5.17** $-\dfrac{2}{3}$; **5.18** ∞; **5.19** -3; **5.20** $-\dfrac{1}{2}$; **5.21** $\dfrac{2}{5}$; **5.22** ∞; **5.23** 1; **5.24** $-\dfrac{7}{2}$; **5.25** -1; **5.26** -7; **5.27** 1; **5.28** $\dfrac{3}{2}$; **5.29** ∞; **5.30** -2.

Sonli ketma-ketlikning limiti

6–masala. Sonli ketma-ketlikning limitini hisoblang.

$$\lim_{n\to\infty}\left(\frac{4n^2+4n-1}{4n^2+2n+3}\right)^{1-2n}=\lim_{n\to\infty}\left(\frac{4n^2+2n+3+2n-4}{4n^2+2n+3}\right)^{1-2n}=$$

$$=\lim_{n\to\infty}\left(1+\frac{2n-4}{4n^2+2n+3}\right)^{1-2n}=\lim_{n\to\infty}\left(1+\frac{1}{\left(\frac{4n^2+2n+3}{2n-4}\right)}\right)^{1-2n}=$$

$$= \lim_{n\to\infty}\left(1+\frac{1}{\left(\frac{4n^2+2n+3}{2n-4}\right)}\right)^{\left(\frac{4n^2+2n+3}{2n-4}\right)\left(\frac{2n-4}{4n^2+2n+3}\right)(1-2n)} =$$

$$= \lim_{n\to\infty}\left(1+\frac{1}{\left(\frac{4n^2+2n+3}{2n-4}\right)}\right)^{\left(\frac{4n^2+2n+3}{2n-4}\right)^{\lim_{n\to\infty}\left(\frac{2n-4}{4n^2+2n+3}\right)(1-2n)}} =$$

$\lim_{t\to 0}\left(1+t\right)^{\frac{1}{t}} = e$ ikkinchi ajoyib limitdan foydalangan holda,

$$e^{\lim_{n\to\infty}\frac{(2n-4)(1-2n)}{4n^2+2n+3}} = e^{\lim_{n\to\infty}\frac{2n-4n^2-4+8n}{4n^2+2n+3}} = e^{\lim_{n\to\infty}\frac{-4n^2+10n-4}{4n^2+2n+3}} =$$

$$= e^{\lim_{n\to\infty}\frac{\frac{1}{n^2}(-4n^2+10n-4)}{\frac{1}{n^2}(4n^2+2n+3)}} = e^{\lim_{n\to\infty}\frac{-4+\frac{10}{n}-\frac{4}{n^2}}{4+\frac{2}{n}+\frac{3}{n^2}}} = e^{\frac{-4+0-0}{4+0+0}} = e^{-1} = \frac{1}{e}.$$

1. $\lim_{n\to\infty}\left(\frac{n+1}{n-1}\right)^{n}$.

2. $\lim_{n\to\infty}\left(\frac{2n+3}{2n+1}\right)^{n+1}$.

3. $\lim_{n\to\infty}\left(\frac{n^2-1}{n^2}\right)^{n^4}$.

4. $\lim_{n\to\infty}\left(\frac{n-1}{n+3}\right)^{n+2}$.

5. $\lim_{n\to\infty}\left(\frac{2n^2+2}{2n^2+1}\right)^{n^2}$.

6. $\lim_{n\to\infty}\left(\frac{3n^2-6n+7}{3n^2+20n-1}\right)^{-n+1}$.

7. $\lim_{n\to\infty}\left(\frac{n^2-3n+6}{n^2+5n+1}\right)^{n/2}$.

8. $\lim_{n\to\infty}\left(\frac{n-10}{n+1}\right)^{3n+1}$.

9. $\lim_{n\to\infty}\left(\frac{6n-7}{6n+4}\right)^{3n+2}$.

10. $\lim_{n\to\infty}\left(\frac{3n^2+4n-1}{3n^2+2n+7}\right)^{2n+5}$.

11. $\lim_{n\to\infty}\left(\frac{n^2+n+1}{n^2+n-1}\right)^{-n^2}$.

12. $\lim_{n\to\infty}\left(\frac{2n^2+5n+7}{2n^2+5n+3}\right)^{n}$.

13. $\lim\limits_{n\to\infty}\left(\dfrac{n-1}{n+1}\right)^{n^2}$.

14. $\lim\limits_{n\to\infty}\left(\dfrac{5n^2+3n-1}{5n^2+3n+3}\right)^{n}$.

15. $\lim\limits_{n\to\infty}\left(\dfrac{3n+1}{3n-1}\right)^{2n+3}$.

16. $\lim\limits_{n\to\infty}\left(\dfrac{2n^2+7n-1}{2n^2+3n-1}\right)^{-n^3}$.

17. $\lim\limits_{n\to\infty}\left(\dfrac{n+3}{n+5}\right)^{n+4}$.

18. $\lim\limits_{n\to\infty}\left(\dfrac{n^3+1}{n^3-1}\right)^{2n-n^3}$.

19. $\lim\limits_{n\to\infty}\left(\dfrac{2n^2+21n-7}{2n^2+18n+9}\right)^{2n+1}$.

20. $\lim\limits_{n\to\infty}\left(\dfrac{10n-3}{10n-1}\right)^{5n}$.

21. $\lim\limits_{n\to\infty}\left(\dfrac{3n^2-5n}{3n^2-5n+7}\right)^{n+1}$.

22. $\lim\limits_{n\to\infty}\left(\dfrac{n+3}{n+1}\right)^{-n^2}$.

23. $\lim\limits_{n\to\infty}\left(\dfrac{n^2-6n+5}{n^2-5n+5}\right)^{3n+2}$.

24. $\lim\limits_{n\to\infty}\left(\dfrac{n+4}{n+2}\right)^{n}$.

25. $\lim\limits_{n\to\infty}\left(\dfrac{7n^2+18n-15}{7n^2+11n+15}\right)^{n+2}$.

26. $\lim\limits_{n\to\infty}\left(\dfrac{2n-1}{2n+1}\right)^{n+1}$.

27. $\lim\limits_{n\to\infty}\left(\dfrac{n^3+n+1}{n^3+2}\right)^{2n^2}$.

28. $\lim\limits_{n\to\infty}\left(\dfrac{13n+3}{13n-10}\right)^{n-3}$.

29. $\lim\limits_{n\to\infty}\left(\dfrac{2n^2+2n+3}{2n^2+2n+1}\right)^{3n^2-7}$.

30. $\lim\limits_{n\to\infty}\left(\dfrac{n+5}{n-7}\right)^{n/6+1}$.

Javoblar. 6.1 e^2; **6.2** e; **6.3** 0; **6.4** e^{-4}; **6.5** $\sqrt{e}$; **6.6** $e^{\frac{26}{3}}$; **6.7** e^{-4}; **6.8** e^{-33}; **6.9** $e^{-\frac{11}{2}}$; **6.10** $e^{\frac{4}{3}}$; **6.11** e^{-2}; **6.12** 1; **6.13** 0; **6.14** $e^{-\frac{4}{5}}$; **6.15** $e^{\frac{4}{3}}$; **6.16** 0; **6.17** e^{-2}; **6.18** e^{-2}; **6.19** e^3; **6.20** $\dfrac{1}{e}$; **6.21** 1; **6.22** 0; **6.23** e^{-3}; **6.24** e^2; **6.25** e; **6.26** $\dfrac{1}{e}$; **6.27** e^2; **6.28** e; **6.29** e^3; **6.30** e^2.

7-masala. Isbotlang ($\delta(\varepsilon)$ toping):

$$\lim_{x\to\frac{1}{3}}\frac{15x^2-2x-1}{x-\frac{1}{3}}=8$$

Koshining funksiya limiti ta'rifiga ko'ra: Agar $\forall\,\varepsilon>0$ son olinganda ham shunday $\exists\,\delta(\varepsilon)>0$ topilsaki, $\forall\,x\in M$ uchun

$$(0<|\,x-a\,|<\delta(\varepsilon))\Rightarrow(|\,f(x)-A|<\varepsilon)$$

tengsizlik bajarilsa, $A\in R$ soni $f(x)$ funksiyaning A nuqtadagi limiti deyiladi:

$$\forall\,\varepsilon>0:\ \exists\,\delta(\varepsilon)>0:\ \forall\,x\in M:\ (0<|\,x-a\,|<\delta(\varepsilon))\Rightarrow(|\,f(x)-A|<\varepsilon).$$

Natijad , $x\neq\dfrac{1}{3}$ da

$$\left|\frac{15x^2-2x-1}{x-\frac{1}{3}}=8\right|=\left|\frac{(15x+3)\left(x-\frac{1}{3}\right)}{x-\frac{1}{3}}-8\right|=$$

$$=|\,15x+3-8\,|=|\,15-x\,|=15\left|x-\frac{1}{3}\right|<\varepsilon$$

yoki

$$\left|x-\frac{1}{3}\right|<\frac{\varepsilon}{15}$$

Bu erda $\delta(\varepsilon)=\dfrac{\varepsilon}{15}.$

Shunday qilib, $x\to\dfrac{1}{3}$ da funksiyaning limiti mavjud va u 8 ga va

$\delta(\varepsilon)=\dfrac{\varepsilon}{15}$ teng.

1. $f(x)=5x^2-1,$ $x_0=6.$

2. $f(x)=4x^2-2,$ $x_0=5.$

3. $f(x)=3x^2-3,$ $x_0=4.$

4. $f(x)=-2x^2-5,$ $x_0=2.$

5. $f(x)=-3x^2-6,$ $x_0=1.$

6. $f(x)=-4x^2-7,$ $x_0=1.$

7. $f(x)=-5x^2-8,$ $x_0=2.$

8. $f(x)=-5x^2-9,$ $x_0=3.$

9. $f(x)=-4x^2+9,$ $x_0=4.$

10. $f(x)=-3x^2+8,$ $x_0=5.$

11. $f(x)=-2x^2+7,$ $x_0=6.$

12. $f(x)=2x^2+6,$ $x_0=7.$

13. $f(x)=3x^2+5,$ $x_0=8.$

14. $f(x)=4x^2+4,$ $x_0=9.$

15. $f(x)=5x^2+3,$ $x_0=8.$

16. $f(x)=5x^2+1,$ $x_0=7.$

17. $f(x)=4x^2-1,$ $x_0=6.$

18. $f(x)=3x^2-2,$ $x_0=5.$

19. $f(x)=2x^2-3,$ $x_0=4.$

20. $f(x)=-2x^2-4,$ $x_0=3.$

21. $f(x)=-3x^2-5,$ $x_0=2.$

22. $f(x)=-4x^2-6,$ $x_0=1.$

23. $f(x)=-5x^2-7,$ $x_0=1.$

24. $f(x)=-4x^2-8,$ $x_0=2.$

25. $f(x)=-3x^2-9,$ $x_0=3.$

26. $f(x)=-2x^2+9,$ $x_0=4.$

27. $f(x)=2x^2+8,$ $x_0=5.$

28. $f(x)=3x^2+7,$ $x_0=6.$

29. $f(x)=4x^2+6,$ $x_0=7.$

30. $f(x)=5x^2+5,$ $x_0=8.$

Javoblar. **7.1** $\frac{\varepsilon}{2}$; **7.2** $\frac{\varepsilon}{5}$; **7.3** $\frac{\varepsilon}{3}$; **7.4** $\frac{\varepsilon}{4}$; **7.5** $\frac{\varepsilon}{6}$; **7.6** $\frac{\varepsilon}{6}$; **7.7** $\frac{\varepsilon}{9}$; **7.8** $\frac{\varepsilon}{3}$; **7.9** $\frac{\varepsilon}{3}$; **7.10** $\frac{\varepsilon}{7}$; **7.11** ε; **7.12** $\frac{\varepsilon}{2}$; **7.13** $\frac{\varepsilon}{6}$; **7.14** $\frac{\varepsilon}{10}$; **7.15** ε; **7.16** ε; **7.17** $\frac{\varepsilon}{6}$; **7.18** $\frac{\varepsilon}{6}$; **7.19** $\frac{\varepsilon}{2}$; **7.20** $\frac{\varepsilon}{5}$; **7.21** $\frac{\varepsilon}{2}$;

7.22 $\frac{\varepsilon}{2}$; **7.23** $\frac{\varepsilon}{2}$; **7.24** ε; **7.25** $\frac{\varepsilon}{3}$; **7.26** $\frac{\varepsilon}{5}$; **7.27** $\frac{\varepsilon}{2}$; **7.28** $\frac{\varepsilon}{3}$; **7.29** $\frac{\varepsilon}{3}$; **7.30** $\frac{\varepsilon}{15}$.

8–masala . $f(x)$ funksiya x_0 nuqtada uzluksiz ekanligi isbotlang ($\delta(\varepsilon)$ni toping.).

$$f(x)=2x^2-4,\quad x_0=3.$$

$$|x-x_0|<\delta(\varepsilon) \text{ da } |f(x)-f(x_0|<\varepsilon,$$

$$|2x^2-4-(2\cdot 9-4)|=|2x^2-18|=2|x^2-9|<\varepsilon,$$

$$|x^2-9|<\varepsilon/2,\qquad |(x-3)(x+3)|<\varepsilon/2\Rightarrow|x-3|<\varepsilon/2\Rightarrow$$

$$|x-x_0|<\delta(\varepsilon)=\tfrac{\varepsilon}{2}\Rightarrow|f(x)-f(x_0)|<\varepsilon \text{ bajariladi.}$$

1. $f(x)=5x^2-1,\ x_0=6.$ **2.** $f(x)=4x^2-2,\ x_0=5.$

3. $f(x)=3x^2-3,\ x_0=4.$ **4.** $f(x)=-2x^2-5,\ x_0=2.$

5. $f(x)=-3x^2-6,\ x_0=1.$ **6.** $f(x)=-4x^2-7,\ x_0=1.$

7. $f(x)=-5x^2-8,\ x_0=2.$ **8.** $f(x)=-5x^2-9,\ x_0=3.$

9. $f(x)=-4x^2+9,\ x_0=4.$ **10.** $f(x)=-3x^2+8, x_0=5.$

11. $f(x)=-2x^2+7, x_0=6.$ **12.** $f(x)=2x^2+6, x_0=7.$

13. $f(x)=3x^2+5, x_0=8.$ **14.** $f(x)=4x^2+4, x_0=9.$

15. $f(x)=5x^2+3, x_0=8.$ **16.** $f(x)=5x^2+1, x_0=7.$

17. $f(x)=4x^2-1, x_0=6.$ **18.** $f(x)=3x^2-2, x_0=5.$

19. $f(x)=2x^2-3, x_0=4.$ **20.** $f(x)=-2x^2-4, x_0=3.$

21. $f(x)=-3x^2-5, x_0=2.$ **22.** $f(x)=-4x^2-6, x_0=1.$

23. $f(x)=-5x^2-7, x_0=1.$ **24.** $f(x)=-4x^2-8, x_0=2.$

25. $f(x)=-3x^2-9, x_0=3.$ **26.** $f(x)=-2x^2+9,\ x_0=4.$

27. $f(x)=2x^2+8,\ x_0=5.$ **28.** $f(x)=3x^2+7,\ x_0=6.$

29. $f(x)=4x^2+6,\ x_0=7.$ **30.** $f(x)=5x^2+5,\ x_0=8.$

Javoblar. 8.1 $\frac{\varepsilon}{5}$; **8.2** $\frac{\varepsilon}{4}$; **8.3** $\frac{\varepsilon}{3}$; **8.4** $\frac{\varepsilon}{2}$; **8.5** $\frac{\varepsilon}{3}$; **8.6** $\frac{\varepsilon}{4}$; **8.7** $\frac{\varepsilon}{5}$; **8.8** $\frac{\varepsilon}{5}$; **8.9** $\frac{\varepsilon}{4}$; **8.10** $\frac{\varepsilon}{3}$; **8.11** $\frac{\varepsilon}{2}$; **8.12** $\frac{\varepsilon}{2}$; **8.13** $\frac{\varepsilon}{3}$; **8.14** $\frac{\varepsilon}{4}$; **8.15** $\frac{\varepsilon}{5}$; **8.16** $\frac{\varepsilon}{5}$; **8.17** $\frac{\varepsilon}{4}$; **8.18** $\frac{\varepsilon}{3}$; **8.19** $\frac{\varepsilon}{2}$; **8.20** $\frac{\varepsilon}{2}$; **8.21** $\frac{\varepsilon}{3}$; **8.22** $\frac{\varepsilon}{4}$; **8.23** $\frac{\varepsilon}{5}$; **8.24** $\frac{\varepsilon}{4}$; **8.25** $\frac{\varepsilon}{3}$; **8.26** $\frac{\varepsilon}{2}$; **8.27** $\frac{\varepsilon}{2}$; **8.28** $\frac{\varepsilon}{3}$; **8.29** $\frac{\varepsilon}{4}$; **8.30** $\frac{\varepsilon}{5}$.

Funksiyaning limiti

9–masala . Funksiyaning limitini hisoblang .

$$\lim_{x\to 3}\frac{x^3-4x^2-3x+18}{x^3-5x^2+3x+9}=\left\{\frac{0}{0}\right\}=\lim_{x\to 3}\frac{(x-3)(x^2-x-6)}{(x-3)(x^2-2x-3)}=$$

$$=\lim_{x\to 3}\frac{x^2-x-6}{x^2-2x-3}=\left\{\frac{0}{0}\right\}=\lim_{x\to 3}\frac{(x-3)(x+2)}{(x-3)(x+1)}=$$

$$=\lim_{x\to 3}\frac{x+2}{x+1}=\frac{3+2}{3+1}=\frac{5}{4}=1\frac{1}{4}.$$

1. $\lim\limits_{x\to -1}\dfrac{(x^3-2x-1)(x+1)}{x^4+4x^2-5}.$

2. $\lim\limits_{x\to -1}\dfrac{x^3-3x-2}{x+x^2}.$

3. $\lim\limits_{x\to -1}\dfrac{(x^2+3x+2)^2}{x^3+2x^2-x-2}.$

4. $\lim\limits_{x\to 1}\dfrac{(2x^2-x-1)^2}{x^3+2x^2-x-2}.$

5. $\lim\limits_{x\to -3}\dfrac{(x^2+2x-3)^2}{x^3+4x^2+3x}.$

6. $\lim\limits_{x\to -1}\dfrac{(x^3-2x-1)(x+1)}{x^4+4x^2-5}.$

7. $\lim\limits_{x\to 0}\dfrac{(1+x^3)-(1+3x)}{x+x^5}.$

8. $\lim\limits_{x\to 1}\dfrac{x^2-2x+1}{2x^2-x-1}.$

9. $\lim\limits_{x\to -1}\dfrac{x^3-3x-2}{x^2-x-2}.$

10. $\lim\limits_{x\to -1}\dfrac{x^3+5x^2+7x+3}{x^3+4x^2+5x+2}.$

11. $\lim\limits_{x\to 1}\dfrac{x^3-3x+2}{x^3-x^2-x+1}.$

12. $\lim\limits_{x\to 1}\dfrac{x^3+x^2-5x+3}{x^3-x^2-x+1}.$

13. $\lim\limits_{x\to -1}\dfrac{x^3+4x^2+5x+2}{x^3-3x-2}.$

14. $\lim\limits_{x\to 1}\dfrac{x^4-1}{2x^4-x^2-1}.$

15. $\lim\limits_{x\to -2}\dfrac{x^3+5x^2+8x+4}{x^3+3x^2-4}.$

16. $\lim\limits_{x\to 2}\dfrac{x^3-5x^2+8x-4}{x^3-3x^2+4}.$

17. $\lim\limits_{x\to 2}\dfrac{x^3-6x^2+12x-8}{x^3-3x^2+4}$.

18. $\lim\limits_{x\to -2}\dfrac{x^3+5x^2+8x+4}{x^3+7x^2+16x+12}$.

19. $\lim\limits_{x\to -1}\dfrac{x^3-3x-2}{(x^2-x-2)^2}$.

20. $\lim\limits_{x\to 2}\dfrac{x^3-3x-2}{x-2}$.

21. $\lim\limits_{x\to -1}\dfrac{x^3-3x-2}{x^2+2x+1}$.

22. $\lim\limits_{x\to 1}\dfrac{x^2-2x+1}{x^3-x^2-x+1}$.

23. $\lim\limits_{x\to 1}\dfrac{x^4-1}{2x^4-x^2-1}$.

24. $\lim\limits_{x\to -1}\dfrac{x^2+3x+2}{x^3+2x^2-x-2}$.

25. $\lim\limits_{x\to 1}\dfrac{2x^2-x-1}{x^3+2x^2-x-2}$.

26. $\lim\limits_{x\to -3}\dfrac{x^2+2x-3}{x^3+4x^2+3x}$.

27. $\lim\limits_{x\to -1}\dfrac{x^3-2x-1}{x^4+2x+1}$.

28. $\lim\limits_{x\to 0}\dfrac{(1+x)^3-(1+3x)}{x^2+x^5}$.

29. $\lim\limits_{x\to 1}\dfrac{x^2-1}{2x^2-x-1}$.

30. $\lim\limits_{x\to -3}\dfrac{x^3+7x^2+15x+9}{x^3+8x^2+21x+18}$.

Javoblar. 9.1 0; **9.2** 0; **9.3** 0; **9.4** 0; **9.5** 0; **9.6** 0; **9.7** 0; **9.8** 0; **9.9** 0; **9.10** 2; **9.11** $\dfrac{3}{2}$; **9.12** 2; **9.13** $-\dfrac{1}{3}$; **9.14** $\dfrac{2}{3}$; **9.15** $\dfrac{1}{3}$; **9.16** $\dfrac{1}{3}$; **9.17** 0; **9.18** -1; **9.19** $-\dfrac{1}{3}$; **9.20** 9; **9.21** -3; **9.22** $\dfrac{1}{2}$; **9.23** $\dfrac{2}{3}$; **9.24** $-\dfrac{1}{2}$; **9.25** $\dfrac{1}{2}$; **9.26** $-\dfrac{2}{3}$; **9.27** $-\dfrac{1}{2}$; **9.28** 3; **9.29** $\dfrac{2}{3}$; **9.30** 2.

Funksiyaning limiti

10–masala. Funksiyaning limitini hisoblang .

$$\lim_{x\to 3}\frac{\sqrt{x+13}-2\sqrt{x+1}}{\sqrt[3]{x^2-9}}=\lim_{x\to 3}\frac{\left(\sqrt{x+13}-2\sqrt{x+1}\right)\left(\sqrt{x+13}+2\sqrt{x+1}\right)}{\sqrt[3]{x^2-9}\left(\sqrt{x+13}+2\sqrt{x+1}\right)}=$$

$$=\lim_{x\to 3}\frac{x+13-4(x+1)}{\sqrt[3]{x^2-9}\left(\sqrt{x+13}+2\sqrt{x+1}\right)}=$$

$$=\lim_{x\to 3}\frac{-3x+9}{\sqrt[3]{(x-3)(x+3)}\left(\sqrt{x+13}+2\sqrt{x+1}\right)}=$$

$$= \lim_{x\to 3} \frac{-3(x-3)}{\sqrt[3]{(x-3)(x+3)}\left(\sqrt{x+13}+2\sqrt{x+1}\right)} =$$

$$= \lim_{x\to 3} \frac{-3\sqrt[3]{(x-3)^3}}{\sqrt[3]{(x-3)(x+3)}\left(\sqrt{x+13}+2\sqrt{x+1}\right)} =$$

$$= \lim_{x\to 3} \frac{-3\sqrt[3]{(x-3)^2}}{\sqrt[3]{(x+3)}\left(\sqrt{x+13}+2\sqrt{x+1}\right)} =$$

$$= \lim_{x\to 3} \frac{-3\sqrt[3]{(3-3)^2}}{\sqrt[3]{(3+3)}\left(\sqrt{3+13}+2\sqrt{3+1}\right)} =$$

$$= \lim_{x\to 3} \frac{-3\sqrt[3]{0^2}}{\sqrt[3]{6}\left(\sqrt{16}+2\sqrt{4}\right)} = 0.$$

1. $\lim\limits_{x\to 4} \dfrac{\sqrt{1+2x}-3}{\sqrt{x}-2}.$

2. $\lim\limits_{x\to -8} \dfrac{\sqrt{1-x}-3}{2+\sqrt[3]{x}}.$

3. $\lim\limits_{x\to 1} \dfrac{\sqrt{x-1}}{\sqrt[3]{x^2-1}}.$

4. $\lim\limits_{x\to 3} \dfrac{\sqrt{x+13}-2\sqrt{x+1}}{x^2-9}.$

5. $\lim\limits_{x\to -2} \dfrac{\sqrt[3]{x-6}+2}{x^3+8}.$

6. $\lim\limits_{x\to 16} \dfrac{\sqrt[4]{x}-2}{\sqrt{x}-4}.$

7. $\lim\limits_{x\to 8} \dfrac{\sqrt{9+2x}-5}{\sqrt[3]{x}-2}.$

8. $\lim\limits_{x\to 0} \dfrac{\sqrt{1-2x+x^2}-(1+x)}{x}.$

9. $\lim\limits_{x\to 0} \dfrac{\sqrt[3]{8+3x+x^2}-2}{x+x^2}.$

10. $\lim\limits_{x\to 0} \dfrac{\sqrt[3]{27+x}-\sqrt[3]{27-x}}{x+2\sqrt[3]{x^4}}.$

11. $\lim\limits_{x\to 1} \dfrac{\sqrt[3]{x}-1}{\sqrt{1+x}-\sqrt{2x}}.$

12. $\lim\limits_{x\to 0} \dfrac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt[3]{1+x}-\sqrt[3]{1-x}}.$

13. $\lim\limits_{x\to 2} \dfrac{\sqrt[3]{4x}-2}{\sqrt{2+x}-\sqrt{2x}}.$

14. $\lim\limits_{x\to 1} \dfrac{\sqrt{x}-1}{x^2-1}.$

15. $\lim\limits_{x\to 3} \dfrac{\sqrt[3]{9x}-3}{\sqrt{3+x}-\sqrt{2x}}.$

16. $\lim\limits_{x\to -2} \dfrac{\sqrt[3]{x-6}+2}{x+2}.$

17. $\lim\limits_{x\to 4} \dfrac{\sqrt[3]{16x}-4}{\sqrt{4+x}-\sqrt{2x}}.$

18. $\lim\limits_{x\to 8} \dfrac{\sqrt{9+2x}-5}{\sqrt[3]{x^2}-4}.$

19. $\lim\limits_{x\to\frac{1}{2}}\dfrac{\sqrt[3]{\frac{x}{4}}-\frac{1}{2}}{\sqrt{\frac{1}{2}+x}-\sqrt{2}x}.$

20. $\lim\limits_{x\to\frac{1}{3}}\dfrac{\sqrt[3]{\frac{x}{9}}-\frac{1}{3}}{\sqrt{\frac{1}{3}+x}-\sqrt{2}x}.$

21. $\lim\limits_{x\to\frac{1}{4}}\dfrac{\sqrt[3]{\frac{x}{16}}-\frac{1}{4}}{\sqrt{\frac{1}{4}+x}-\sqrt{2}x}.$

22. $\lim\limits_{x\to 0}\dfrac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt[7]{x}}.$

23. $\lim\limits_{x\to 0}\dfrac{\sqrt[3]{27+x}-\sqrt[3]{27-x}}{\sqrt[3]{x^2}+\sqrt[5]{x}}.$

24. $\lim\limits_{x\to 0}\dfrac{\sqrt[3]{8+3x-x^2}-2}{\sqrt[3]{x^2+x^3}}.$

25. $\lim\limits_{x\to 0}\dfrac{\sqrt{1+2x+3x^2}-(1+x)}{\sqrt[3]{x}}.$

26. $\lim\limits_{x\to 8}\dfrac{\sqrt{9+2x}-5}{\sqrt[3]{x}-2}.$

27. $\lim\limits_{x\to 16}\dfrac{\sqrt[4]{x}-2}{\sqrt[3]{(\sqrt{x}-4)^2}}.$

28. $\lim\limits_{x\to -2}\dfrac{\sqrt[3]{x-6}+2}{\sqrt[3]{x^3+8}}.$

29. $\lim\limits_{x\to 4}\dfrac{\sqrt{x}-2}{\sqrt[3]{x^2}-16}.$

30. $\lim\limits_{x\to -8}\dfrac{10-x-6\sqrt{1-x}}{2+\sqrt[3]{x}}.$

Javoblar. 10.1 $1\frac{1}{3}$; **10.2** -2; **10.3** 0; **10.4** $-\frac{1}{16}$; **10.5** $\frac{1}{144}$; **10.6** $\frac{1}{4}$; **10.7** $2\frac{2}{5}$; **10.8** -2; **10.9** $\frac{1}{4}$; **10.10** $\frac{2}{27}$; **10.11** $-\frac{2\sqrt{2}}{3}$; **10.12** $\frac{3}{2}$; **10.13** $-1\frac{1}{3}$; **10.14** $\frac{1}{4}$; **10.15** $-\frac{2\sqrt{6}}{3}$; **10.16** $\frac{1}{12}$; **10.17** $-\frac{4\sqrt{2}}{3}$; **10.18** $0{,}6$; **10.19** $-\frac{2}{3}$; **10.20** $-\frac{2}{3}\sqrt{\frac{2}{3}}$; **10.21** $-\frac{2}{3}\sqrt{\frac{1}{2}}$; **10.22** 0; **10.23** 0; **10.24** 0; **10.25** 0; **10.26** $2\frac{2}{5}$; **10.27** 0; **10.28** 0; **10.29** 0; **10.30** 0.

Funksiyaning limiti

11–masala. Funksiyaning limitini hisoblang .

$$\lim_{x\to 0}\frac{\ln(1-3x)}{\sqrt{8x+4}-2}=\left(\frac{0}{0}\right)=\frac{1}{2}\lim_{x\to 0}\frac{\ln(1-3x)}{\sqrt{2x+1}-1}=\frac{1}{2}\lim_{x\to 0}\frac{\frac{-3}{1-3x}}{\frac{1}{\sqrt{2x+1}}}=$$

$$=-\frac{1}{2}\lim_{x\to 0}\frac{3\sqrt{2x+1}}{1-3x}=-\frac{1}{2}\cdot\frac{3}{1}=-\frac{3}{2}.$$

1. $\lim\limits_{x\to 0}\dfrac{\ln(1+\sin x)}{\sin 4(x-\pi)}$.

2. $\lim\limits_{x\to 0}\dfrac{1-\cos 10(x+\pi)}{e^{x^2}-1}$.

3. $\lim\limits_{x\to 0}\dfrac{3x^2-5x}{\sin 3x}$.

4. $\lim\limits_{x\to 0}\dfrac{1-\cos 2x}{\cos 7x-\cos 3x}$.

5. $\lim\limits_{x\to 0}\dfrac{4x}{tg(\pi(2+x))}$.

6. $\lim\limits_{x\to 0}\dfrac{2x}{tg(2\pi(x+1/2))}$.

7. $\lim\limits_{x\to 0}\dfrac{1-\cos^3 x}{4x^2}$.

8. $\lim\limits_{x\to 0}\dfrac{\arcsin 3x}{\sqrt{2+x}-\sqrt{2}}$.

9. $\lim\limits_{x\to 0}\dfrac{2^{x+1}-2}{\ln(1+4x)}$.

10. $\lim\limits_{x\to 0}\dfrac{arctg\,2x}{\sin(2\pi(x+10))}$.

11. $\lim\limits_{x\to 0}\dfrac{\ln(1-7x)}{\sin(\pi(x+7))}$.

12. $\lim\limits_{x\to 0}\dfrac{\cos(x+5\pi/2)tgx}{\arcsin 2x^2}$.

13. $\lim\limits_{x\to 0}\dfrac{1-\sqrt{3x+1}}{\cos(\pi(x+1/2)}$.

14. $\lim\limits_{x\to 0}\dfrac{\sin 7x}{x^2+\pi x}$.

15. $\lim\limits_{x\to 0}\dfrac{\sqrt{4+x}-2}{3arctgx}$.

16. $\lim\limits_{x\to 0}\dfrac{2\sin(\pi(x+1))}{\ln(1+2x)}$.

17. $\lim\limits_{x\to 0}\dfrac{\cos 2x-\cos x}{1-\cos x}$.

18. $\lim\limits_{x\to 0}\dfrac{\sqrt{1+x}-1}{\sin(\pi(x+2))}$.

19. $\lim\limits_{x\to 0}\dfrac{\sin(5(x+\pi))}{e^{3x-1}}$.

20. $\lim\limits_{x\to 0}\dfrac{1-\sqrt{\cos x}}{x\sin x}$.

21. $\lim\limits_{x\to 0}\dfrac{\arcsin 2x}{\ln(e-x)-1}$.

22. $\lim\limits_{x\to 0}\dfrac{e^{4x}-1}{\sin(\pi(x/2+1))}$.

23. $\lim\limits_{x\to 0}\dfrac{1+\cos(x-\pi)}{(e^{3x}-1)^2}$.

24. $\lim\limits_{x\to 0}\dfrac{\sin^2 x-tg^2 x}{x^4}$.

25. $\lim\limits_{x\to 0}\dfrac{\arcsin 2x}{\ln(e-x)-1}$.

26. $\lim\limits_{x\to 0}\dfrac{tgx-\sin x}{x(1-\cos 2x)}$.

27. $\lim\limits_{x\to 0}\dfrac{\ln(x^2+1)}{2-\sqrt{2x^2+4}}$.

28. $\lim\limits_{x\to 0}\dfrac{tg(\pi(1+x/2))}{\ln(x+1)}$.

29. $\lim\limits_{x\to 0}\dfrac{e^{4\pi x}-1}{\sqrt[3]{8+24x}-2}$.

30. $\lim\limits_{x\to 0}\dfrac{x\sin 2x}{1+\cos(x-3\pi)}$.

Javoblar. 11.1 $\dfrac{1}{4}$; **11.2** 50; **11.3** $-1\dfrac{2}{3}$; **11.4** $-\dfrac{1}{10}$; **11.5** $\dfrac{4}{\pi}$; **11.6** $\dfrac{1}{\pi}$; **11.7** $\dfrac{3}{8}$; **11.8** $6\sqrt{2}$; **11.9** $\dfrac{\ln 2}{2}$; **11.10** $\dfrac{1}{\pi}$; **11.11** $\dfrac{7}{\pi}$; **11.12** $-\dfrac{1}{2}$; **11.13** $-\dfrac{3}{2}$; **11.14** $\dfrac{3}{\pi}$; **11.15** $\dfrac{7}{\pi}$; **11.16** $\dfrac{1}{12}$; **11.17** $-\pi$; **11.18** -3; **11.19** $\dfrac{1}{2\pi}$; **11.20** $-1\dfrac{2}{3}$; **11.21** $\dfrac{1}{4}$; **11.22** $-\dfrac{2}{3}$; **11.23** $-\dfrac{8}{\pi}$; **11.24** $\dfrac{1}{18}$; **11.25** -1; **11.26** $-2e$; **11.27** $\dfrac{1}{4}$; **11.28** 2; **11.29** $\dfrac{\pi}{2}$; **11.30** 2π .

Funksiyaning limiti

12–masala. Funksiyaning limitini hisoblang .

$$\lim_{x\to\pi}\frac{\cos 3x-\cos x}{\operatorname{tg}^2 2x}=\lim_{x\to\pi}\frac{-2\sin\frac{3x+x}{2}\sin\frac{3x-x}{2}}{\operatorname{tg}^2 2x}=$$

$$=\lim_{x\to\pi}\frac{-2\sin 2x\sin x}{\operatorname{tg}^2 2x}=\begin{bmatrix}x=y+\pi\Rightarrow y=x-\pi\\ x\to\pi\Rightarrow y\to 0\end{bmatrix}=$$

$$=\lim_{y\to 0}\frac{-2\sin 2(y+\pi)\sin(y+\pi)}{\operatorname{tg}^2 2(y+\pi)}=\lim_{y\to 0}\frac{2\sin(2y+2\pi)\sin y}{\operatorname{tg}^2(2y+2\pi)}=$$

$$=\begin{bmatrix}\sin y\approx y,\ \text{при}\quad y\to 0\\ \sin 2y\approx 2y,\ \text{при}\quad y\to 0\ \ (2y\to 0)\\ tg2y\approx y,\ \text{при}\quad y\to 0\ \ (2y\to 0)\end{bmatrix}=$$

$$=\lim_{y\to 0}\frac{2\cdot 2y\cdot y}{(2y)^2}=\lim_{y\to 0}\frac{4y^2}{4y^2}=\lim_{y\to 0}1=1.$$

1. $\lim\limits_{x\to 1}\dfrac{x^2-1}{\ln x}$.

2. $\lim\limits_{x\to 1}\dfrac{\sqrt{x^2-x+1}-1}{\ln x}$.

3. $\lim\limits_{x\to\pi}\dfrac{1+\cos 3x}{\sin^2 7x}$.

4. $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{1-\sin 2x}{(\pi-4x)^2}$.

5. $\lim\limits_{x\to 1}\dfrac{1+\cos\pi x}{tg^2\pi x}$.

6. $\lim\limits_{x\to\frac{\pi}{2}}\dfrac{tg3x}{tgx}$.

7. $\lim\limits_{x\to\pi}\dfrac{\sin^2 x - tg^2 x}{(x-\pi)^4}$.

8. $\lim\limits_{x\to 1}\dfrac{\sqrt{x^2-x+1}-1}{tg\pi x}$.

9. $\lim\limits_{x\to\pi}\dfrac{\cos 5x-\cos 3x}{\sin^2 x}$.

10. $\lim\limits_{x\to 2\pi}\dfrac{\sin 7x-\sin 3x}{e^{x^2}=e^{4\pi^2}}$.

11. $\lim\limits_{x\to 2}\dfrac{\sin 7\pi x}{\sin 8\pi x}$.

12. $\lim\limits_{x\to 2}\dfrac{\ln(5-2x)}{\sqrt{10-3x}-2}$.

13. $\lim\limits_{x\to 1}\dfrac{\sqrt{x^2-3x+3}-1}{\sin\pi x}$.

14. $\lim\limits_{x\to\pi}\dfrac{x^2-\pi^2}{\sin x}$.

15. $\lim\limits_{x\to 1}\dfrac{3^{5x-3}-3^{2x^2}}{tg\pi x}$.

16. $\lim\limits_{x\to 4}\dfrac{2^x-16}{\sin\pi x}$.

17. $\lim\limits_{x\to\frac{\pi}{2}}\dfrac{\ln 2x-\ln\pi}{\sin(\frac{5x}{2})\cos x}$.

18. $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\ln tgx}{\cos 2x}$.

19. $\lim\limits_{x\to\pi}\dfrac{e^{\pi}-e^{x}}{\sin 5x-\sin 3x}$.

20. $\lim\limits_{x\to 2}\dfrac{\ln(9-2x^2)}{\sin 2\pi x}$.

21. $\lim\limits_{x\to 2}\dfrac{1-2^{4-x^2}}{2(\sqrt{2x-\sqrt{3x^2-5x+2}})}$.

22. $\lim\limits_{x\to 1}\dfrac{\sqrt[3]{x}-1}{\sqrt[4]{x}-1}$.

23. $\lim\limits_{x\to -2}\dfrac{tg\pi x}{x+2}$.

24. $\lim\limits_{x\to\pi}\dfrac{1-\sin(x/2)}{\pi-x}$.

25. $\lim\limits_{x\to\frac{\pi}{3}}\dfrac{1-2\cos x}{\pi-3x}$.

26. $\lim\limits_{x\to 2}\dfrac{arctg(x^2-2x)}{\sin 3\pi x}$.

27. $\lim\limits_{x\to 1}\dfrac{1-x^2}{\sin\pi x}$.

28. $\lim\limits_{x\to 1}\dfrac{\cos(\pi x/2)}{1-\sqrt{x}}$.

29. $\lim\limits_{x\to 1}\dfrac{3-\sqrt{10-x}}{\sin 3\pi x}$.

30. $\lim\limits_{x\to\pi}\dfrac{\sin 5x}{tg3x}$.

Javoblar. **12.1** 2; **12.2** $\dfrac{1}{2}$; **12.3** $\dfrac{9}{98}$; **12.4** $\dfrac{1}{8}$; **12.5** $\dfrac{1}{2}$; **12.6** $\dfrac{1}{3}$; **12.7** -1; **12.8** $\dfrac{1}{2\pi}$; **12.9** 8;
12.10 $\dfrac{1}{\pi\cdot e^{4\pi^2}}$; **12.11** $\dfrac{7}{8}$; **12.12** $2\dfrac{2}{3}$; **12.13** $\dfrac{1}{2\pi}$; **12.14** -2π; **12.15** $\dfrac{9\ln 3}{\pi}$; **12.16** $\dfrac{16\ln 2}{\pi}$;

12.17 $\frac{2\sqrt{2}}{\pi}$; **12.18** -1; **12.19** $\frac{e^{\pi}}{2}$; **12.20** $-\frac{4}{\pi}$; **12.21** $-\frac{8\ln 2}{5}$; **12.22** $\frac{4}{3}$; **12.23** π;

12.24 0; **12.25** $-\frac{\sqrt{3}}{3}$; **12.26** $\frac{2}{3\pi}$; **12.27** $\frac{2}{\pi}$; **12.28** π; **12.29** $-\frac{1}{18\pi}$; **12.30** $-\frac{5}{3}$.

Funksiyaning limiti

13–masala. Funksiyaning limitini hisoblang .

$$\lim_{x\to\pi}\frac{\sin\left(\frac{x^2}{\pi}\right)}{2^{\sqrt{\sin x+1}}-2}=\left[\begin{matrix}x=y+\pi\Rightarrow y=x-\pi\\ x\to\pi\quad\Rightarrow y\to 0\end{matrix}\right]=$$

$$=\lim_{y\to 0}\frac{\sin\left(\frac{(y+\pi)^2}{\pi}\right)}{2^{\sqrt{\sin(y+\pi)+1}}-2}==\lim_{y\to 0}\frac{\sin\left(\frac{y^2+2y\pi+\pi^2}{\pi}\right)}{2^{\sqrt{-\sin y+1}}-2}=$$

$$=\lim_{y\to 0}\frac{\sin\left(y\cdot\frac{y+2\pi}{\pi}+\pi\right)}{2^{\sqrt{-\sin y+1}}-2}=\lim_{y\to 0}\frac{-\sin\left(y\cdot\frac{y+2\pi}{\pi}\right)}{2^{\sqrt{1-\sin y}}-2}=$$

$$=\lim_{y\to 0}\frac{-\sin\left(y\cdot\frac{y+2\pi}{\pi}\right)}{2\left(e^{\ln 2(\sqrt{1-\sin y}-1)}-1\right)}=\left[\begin{matrix}\sin\left(y\cdot\frac{y+2\pi}{\pi}\right)\approx y\cdot\frac{y+2\pi}{\pi}\\ e^{\ln 2(\sqrt{1-\sin y}-1)}-1\approx\ln 2(\sqrt{1-\sin y}-1)\end{matrix}\right]=$$

$$=\lim_{y\to 0}\frac{-y\cdot\frac{y+2\pi}{\pi}}{2\ln 2(\sqrt{1-\sin y}-1)}=\lim_{y\to 0}\frac{-y\cdot\frac{y+2\pi}{\pi}(\sqrt{1-\sin y}+1)}{2\ln 2(\sqrt{1-\sin y}-1)(\sqrt{1-\sin y}+1)}=$$

$$= \lim_{y\to 0} \frac{-y\cdot\frac{y+2\pi}{\pi}(\sqrt{1-\sin y}+1)}{2\ln 2(1-\sin y-1)} = \lim_{y\to 0} \frac{y\cdot\frac{y+2\pi}{\pi}(\sqrt{1-\sin y}+1)}{2\ln 2\sin y} =$$

$$= [\sin y \approx y] = \lim_{y\to 0} \frac{y\cdot\frac{y+2\pi}{\pi}(\sqrt{1-\sin y}+1)}{2\ln 2 y} =$$

$$= \lim_{y\to 0} \frac{\frac{y+2\pi}{\pi}(\sqrt{1-\sin y}+1)}{2\ln 2} = \frac{\frac{0+2\pi}{\pi}(\sqrt{1-\sin 0}+1)}{2\ln 2} =$$

$$= \frac{2(\sqrt{1}+1)}{2\ln 2} = \frac{2}{\ln 2}.$$

1. $\lim\limits_{x\to \pi/2} \dfrac{2^{\cos^x}-1}{\ln\sin x}.$

2. $\lim\limits_{x\to \frac{1}{2}} \dfrac{(2x-1)^2}{e^{\sin \pi x}-e^{-\sin 3\pi x}}.$

3. $\lim\limits_{x\to 2} \dfrac{\ln(x-\sqrt[3]{2x-3})}{\sin(\pi x/2)-\sin((x-1)\pi)}.$

4. $\lim\limits_{x\to 2} \dfrac{tgx-tg2}{\sin\ln(x-1)}.$

5. $\lim\limits_{x\to \frac{\pi}{2}} \dfrac{e^{tg2x}-e^{-\sin 2x}}{\sin x-1}.$

6. $\lim\limits_{x\to \frac{\pi}{6}} \dfrac{\ln\sin 3x}{(6x-\pi)^2}.$

7. $\lim\limits_{x\to 3} \dfrac{\sin(\sqrt{2x^2-3x-5}-\sqrt{1+x}}{\ln(x-1)-\ln(x+1)+\ln 2}.$

8. $\lim\limits_{x\to 2\pi} \dfrac{(x-2\pi)^2}{\lg(\cos x-1)}.$

9. $\lim\limits_{x\to \frac{1}{2}} \dfrac{\ln(4x-1)}{\sqrt{1-\cos\pi x}-1}.$

10. $\lim\limits_{x\to -2} \dfrac{\arcsin\frac{x+2}{2}}{3^{\sqrt{2+x+x^2}}-9}.$

11. $\lim\limits_{x\to 3} \dfrac{2^{\sin\pi x}-1}{\ln(x^3-6x-8)}.$

12. $\lim\limits_{x\to \pi} \dfrac{\ln\cos 2x}{(1-\pi/x)^2}.$

13. $\lim\limits_{x\to 2} \dfrac{tg\ln(3x-5)}{e^{x+3}-e^{x^2+1}}.$

14. $\lim\limits_{x\to 2\pi} \dfrac{\ln\cos x}{3^{\sin 2x}-1}.$

15. $\lim\limits_{x\to 1} \dfrac{\sqrt[3]{1+\ln^2 x}-1}{1+\cos\pi x}.$

16. $\lim\limits_{x\to \pi} \dfrac{\cos\frac{x}{2}}{e^{\sin x}-e^{\sin 4x}}.$

17. $\lim\limits_{x\to 3}\dfrac{\ln(2x-5)}{e^{\sin \pi x}-1}$.

18. $\lim\limits_{x\to \frac{\pi}{3}}\dfrac{e^{\sin^2 6x}-e^{\sin^2 3x}}{\log_3 \cos 6x}$.

19. $\lim\limits_{x\to \frac{\pi}{2}}\dfrac{e^{\sin 2x}-e^{tg\,2x}}{\ln(2x/\pi)}$.

20. $\lim\limits_{x\to -2}\dfrac{tg(e^{x+2}-e^{x^2-4})}{tgx+tg2}$.

21. $\lim\limits_{x\to 1}\dfrac{\sqrt{2^x+7}-\sqrt{2^{x+1}+5}}{x^3-1}$.

22. $\lim\limits_{x\to \pi}\dfrac{\ln(2+\cos x)}{(3^{\sin x}-1)^2}$.

23. $\lim\limits_{x\to \pi}\dfrac{(x^3-\pi^3)\sin 5x}{e^{\sin^2 x}-1}$.

24. $\lim\limits_{x\to -1}\dfrac{tg(x+1)}{e^{\sqrt[3]{x^3-4x^2+6}}-e}$.

25. $\lim\limits_{x\to \pi}\dfrac{\ln\cos 2x}{\ln\cos 4x}$.

26. $\lim\limits_{x\to \frac{\pi}{2}}\dfrac{\ln\sin x}{(2x-\pi)^2}$.

27. $\lim\limits_{x\to a}\dfrac{a^{x^2-a^2}-1}{\lg\ln(x/a)}$.

28. $\lim\limits_{x\to -3}\dfrac{\sin(e^{\frac{\sqrt[3]{1-x^2}}{2}}-e^{\sqrt[3]{x+2}})}{arctg(x+3)}$.

29. $\lim\limits_{x\to a\pi}\dfrac{\ln(\cos(x/a)+2)}{a^{a^2\pi^2/x^2-a\pi/x}-a^{a\pi/x-1}}$.

30. $\lim\limits_{x\to \pi}\dfrac{tg(3^{\pi/x}-3)}{3^{\cos(3x/2)}-1}$.

Javoblar. 13.1 $-2\ln 2$; **13.2** $\dfrac{1}{e\cdot\pi^2}$; **13.3** $\dfrac{2}{3\pi}$; **13.4** $\dfrac{1}{\cos^2(2)}$; **13.5** 0; **13.6** $-\dfrac{1}{8}$; **13.7** 8; **13.8** $\dfrac{8}{\pi}$; **13.9** 1; **13.10** $-\dfrac{\ln 3+1}{27\ln^2 3}$; **13.11** $-\dfrac{\pi\ln 2}{21}$; **13.12** $-2\pi^2$; **13.13** $-e^{-5}$; **13.14** 0; **13.15** $\dfrac{2}{3\pi^2}$; **13.16** $\dfrac{1}{10}$; **13.17** $-\dfrac{2}{\pi}$; **13.18** $-\dfrac{3}{2}\cdot\ln 3$; **13.19** -2π; **13.20** $5\cos^2 2$; **13.21** $-\dfrac{\ln 2}{9}$; **13.22** $\dfrac{1}{2\ln^2 3}$; **13.23** $-15\pi^2$; **13.24** $\dfrac{3}{11e}$; **13.25** $\dfrac{1}{4}$; **13.26** $-\dfrac{1}{8}$; **13.27** $2a^2\cdot\ln a$; **13.28** $-\dfrac{1}{12e}$; **13.29** $\dfrac{\pi^2}{2\ln a}$; **13.30** $-\dfrac{2}{\pi}$.

Funksiyaning limiti

14–masala. Funksiyaning limitini hisoblang .

$$\lim_{x\to 0}\frac{2^{3x}-3^{5x}}{\sin 7x-2x}=\lim_{x\to 0}\frac{(8^x-1)-(243^x-1)}{\sin 7x-2x}=$$

$$=\lim_{x\to 0}\frac{\left((e^{\ln 8})^x-1\right)-\left((e^{\ln 243})^x-1\right)}{\sin 7x-2x}=$$

$$=\lim_{x\to 0}\frac{\left(e^{x\ln 8}-1\right)-\left(e^{x\ln 243}-1\right)}{\sin 7x-2x}=$$

$$=\lim_{x\to 0}\frac{\frac{1}{x}\left(e^{x\ln 8}-1\right)-\left(e^{x\ln 243}-1\right)}{\frac{1}{x}\left(\sin 7x-2x\right)}=$$

$$=\frac{\lim\limits_{x\to 0}\frac{1}{x}\left(e^{x\ln 8}-1\right)-\left(e^{x\ln 243}-1\right)}{\lim\limits_{x\to 0}\frac{1}{x}\left(\sin 7x-2x\right)}=$$

$$=\frac{\lim\limits_{x\to 0}\frac{e^{x\ln 8}-1}{x}-\lim\limits_{x\to 0}\frac{e^{x\ln 243}-1}{x}}{\lim\limits_{x\to 0}\frac{\sin 7x}{x}-\lim\limits_{x\to 0}\frac{2x}{x}}=\left[\begin{array}{l}e^{x\ln 8}-1\approx x\ln 8\\ e^{x\ln 243}-1\approx x\ln 243\\ \sin 7x\approx 7x\end{array}\right]=$$

$$=\frac{\lim\limits_{x\to 0}\frac{x\ln 8}{x}-\lim\limits_{x\to 0}\frac{x\ln 243}{x}}{\lim\limits_{x\to 0}\frac{7x}{x}-\lim\limits_{x\to 0}\frac{2x}{x}}=\frac{\lim\limits_{x\to 0}\ln 8-\lim\limits_{x\to 0}\ln 243}{\lim\limits_{x\to 0}7-\lim\limits_{x\to 0}2}=$$

$$=\frac{\ln 8-\ln 243}{7-2}=\frac{1}{5}\ln\frac{8}{243}=\frac{1}{5}\ln\frac{2^3}{3^5}.$$

1. $\lim\limits_{x\to 0}\dfrac{7^{2x}-5^{3x}}{2x-arctg3x}$. **2.** $\lim\limits_{x\to 0}\dfrac{e^{3x}-e^{-2x}}{2\arcsin x-\sin x}$. **3.** $\lim\limits_{x\to 0}\dfrac{6^{2x}-7^{-2x}}{\sin 3x-2x}$.

4. $\lim\limits_{x\to 0}\dfrac{e^{5x}-e^{3x}}{\sin 2x-\sin x}$. **5.** $\lim\limits_{x\to 0}\dfrac{3^{2x}-5^{3x}}{arctgx+x^3}$. **6.** $\lim\limits_{x\to 0}\dfrac{e^{2x}-e^{3x}}{arctgx-x^2}$.

7. $\lim\limits_{x\to 0}\dfrac{3^{5x}-2^x}{x-\sin 9x}$. **8.** $\lim\limits_{x\to 0}\dfrac{e^{4x}-e^{-2x}}{2arctgx-\sin x}$. **9.** $\lim\limits_{x\to 0}\dfrac{12^x-5^{-3x}}{2\arcsin x-x}$.

10. $\lim\limits_{x\to 0}\dfrac{e^{7x}-e^{-2x}}{\sin x-2x}$. **11.** $\lim\limits_{x\to 0}\dfrac{3^{5x}-2^{7x}}{\arcsin 2x-x}$. **12.** $\lim\limits_{x\to 0}\dfrac{e^{5x}-e^x}{\arcsin x+x^3}$.

13. $\lim\limits_{x\to 0}\dfrac{4^x-2^{7x}}{tg3x-x}$. **14.** $\lim\limits_{x\to 0}\dfrac{e^x-e^{-x}}{tg2x-\sin x}$. **15.** $\lim\limits_{x\to 0}\dfrac{10^{2x}-7^{-x}}{2tgx-arctgx}$.

16. $\lim\limits_{x\to 0}\dfrac{e^{2x}-e^x}{\sin 3x-\sin 5x}$. **17.** $\lim\limits_{x\to 0}\dfrac{7^{3x}-3^{2x}}{tgx+x^3}$. **18.** $\lim\limits_{x\to 0}\dfrac{e^{4x}-e^{2x}}{2tgx-\sin x}$.

19. $\lim\limits_{x\to 0}\dfrac{3^{2x}-7^x}{\arcsin x-5x}$. **20.** $\lim\limits_{x\to 0}\dfrac{e^{2x}-e^{-5x}}{2\sin x-tgx}$. **21.** $\lim\limits_{x\to 0}\dfrac{4^{5x}-9^{-2x}}{\sin x-tgx^3}$.

22. $\lim\limits_{x\to 0}\dfrac{e^{3x}-e^{2x}}{\sin 3x-tg2x}$. **23.** $\lim\limits_{x\to 0}\dfrac{5^{2x}-2^{3x}}{\sin x+\sin x^2}$. **24.** $\lim\limits_{x\to 0}\dfrac{e^x-e^{3x}}{\sin 3x-tg2x}$.

25. $\lim\limits_{x\to 0}\dfrac{9^x-2^{3x}}{arctg2x-7x}$. **26.** $\lim\limits_{x\to 0}\dfrac{e^x-e^{-2x}}{x+\sin x^2}$. **27.** $\lim\limits_{x\to 0}\dfrac{3^{5x}-2^{-7x}}{2x-tgx}$.

28. $\lim\limits_{x\to 0}\dfrac{e^{2x}-e^x}{\sin 2x-\sin x}$. **29.** $\lim\limits_{x\to 0}\dfrac{e^{2x}-e^x}{x+tgx^2}$. **30.** $\lim\limits_{x\to 0}\dfrac{2^{3x}-3^{2x}}{x+\arcsin x^3}$.

Javoblar.14.1 $\ln\dfrac{125}{49}$; **14.2** 5; **14.3** $\ln\left(6^2\cdot 7^2\right)$; **14.4** 2; **14.5** $\ln\dfrac{9}{125}$; **14.6** -1; **14.7** $\dfrac{1}{8}\ln\dfrac{2}{243}$; **14.8** 6; **14.9** $\ln(12\cdot 5^3)$; **14.10** -9; **14.11** $\ln\left(\dfrac{3^5}{2^7}\right)$; **14.12** 4; **14.13** $\ln\sqrt{\dfrac{1}{2^5}}$; **14.14** 2; **14.15** $\ln 700$; **14.16** $-\dfrac{1}{2}$; **14.17** $\ln\dfrac{7^3}{3^2}$; **14.18** 2; **14.19** $\ln\sqrt{\dfrac{7}{9}}$; **14.20** 7; **14.21** $\ln(2^{10}\cdot 9^2)$; **14.22** 1; **14.23** $\ln\dfrac{25}{8}$; **14.24** -2; **14.25** $-\dfrac{1}{5}\ln\dfrac{9}{8}$; **14.26** 3; **14.27** $\ln(3^5\cdot 2^7)$; **14.28** 1; **14.29** 1; **14.30** $\ln\dfrac{2^3}{3^2}$.

Funksiyaning limiti

15–masala. Funksiyaning limitini hisoblang .

$$\lim_{x\to 1}\frac{e^x-e}{\sin(x^2-1)}=\begin{bmatrix}x=y+1\Rightarrow y=x-1\\ x\to 1\Rightarrow y\to 0\end{bmatrix}=$$

$$=\lim_{y\to 0}\frac{e^{y+1}-e}{\sin((y+1)^2-1)}=\lim_{y\to 0}\frac{e(e^y-1)}{\sin(y^2+2y+1-1)}=$$

$$=\lim_{y\to 0}\frac{e(e^y-1)}{\sin(y^2+2y)}=\begin{bmatrix}e^y-1\approx y\\ \sin(y^2+2y)\approx y^2+2y\end{bmatrix}=$$

$$=\lim_{y\to 0}\frac{e\cdot y}{y^2+2y}=\lim_{y\to 0}\frac{e}{y+2}=\frac{e}{0+2}=\frac{e}{2}.$$

1. $\lim\limits_{x\to 0}\dfrac{e^x+e^{-x}-2}{\sin^2 x}.$

2. $\lim\limits_{x\to 0}\dfrac{1+x\sin x-\cos 2x}{\sin^2 x}.$

3. $\lim\limits_{x\to -1}\dfrac{x^3+1}{\sin(x+1)}.$

4. $\lim\limits_{x\to a}\dfrac{tgx-tga}{\ln x-\ln a}.$

5. $\lim\limits_{x\to 0}\dfrac{\sqrt{1+tgx}-\sqrt{1-\sin x}}{x^3}.$

6. $\lim\limits_{x\to 0}\dfrac{e^{ax}-e^{-\beta x}}{\sin ax-\sin \beta x}.$

7. $\lim\limits_{x\to 0}\dfrac{\sqrt{1+x\sin x}-1}{e^{x^2}-1}.$

8. $\lim\limits_{x\to 0}\dfrac{x^2(e^x-e^{-x})}{e^{x^3+1}-e}.$

9. $\lim\limits_{x\to \pi/3}\dfrac{1-2\cos x}{\sin(\pi-3x)}.$

10. $\lim\limits_{x\to 1}\dfrac{1-x^2}{\sin \pi x}.$

11. $\lim\limits_{x\to \pi/4}\dfrac{\sin x-\cos x}{\ln tgx}.$

12. $\lim\limits_{x\to b}\dfrac{a^x-a^b}{x-b}.$

13. $\lim\limits_{x\to 0}\dfrac{1-\cos 2x+tg^2 x}{x\sin 3x}.$

14. $\lim\limits_{x\to 0}\dfrac{\sin 2x-2\sin x}{x\ln\cos 5x}.$

15. $\lim\limits_{h\to 0}\dfrac{\ln(x+h)+\ln(x-h)+2\ln x}{h^2}.$

16. $\lim\limits_{x\to 1}\dfrac{1-x}{\log_2 x}.$

17. $\lim\limits_{x\to 0}\dfrac{e^{\sin 2x}-e^{\sin x}}{tgx}.$

18. $\lim\limits_{x\to 1}\dfrac{2^x-2}{\ln x}.$

19. $\lim\limits_{h\to 0}\dfrac{\sin(x+h)-\sin(x-h)}{h}$.

20. $\lim\limits_{x\to 0}\dfrac{\sqrt{x+2}-\sqrt{2}}{\sin 3x}$.

21. $\lim\limits_{h\to 0}\dfrac{a^{x+h}+a^{x-h}-2a^{x}}{h^{2}}$.

22. $\lim\limits_{x\to 0}\dfrac{1-\sqrt{\cos x}}{1-\cos\sqrt{x}}$.

23. $\lim\limits_{x\to 3}\dfrac{\sqrt[3]{5+x}-2}{\sin \pi x}$.

24. $\lim\limits_{x\to \pi/6}\dfrac{2\sin^{2}x+\sin x-1}{2\sin^{2}x-3\sin x+1}$.

25. $\lim\limits_{x\to 10}\dfrac{\lg x-1}{\sqrt{x-9}-1}$.

26. $\lim\limits_{x\to 0}\dfrac{3^{x+1}-3}{\ln(1+x\sqrt{1+xe^{x}})}$.

27. $\lim\limits_{x\to 0}\dfrac{\sqrt{\cos x}-1}{\sin^{2}2x}$.

28. $\lim\limits_{x\to 0}\dfrac{\sin bx-\sin ax}{\ln(tg(\pi/4+ax))}$.

29. $\lim\limits_{x\to \pi/2}\dfrac{1-\sin^{3}x}{\cos^{2}x}$.

30. $\lim\limits_{x\to 3}\dfrac{\log_{3}x-1}{tg\pi x}$.

Javoblar. **15.1** 1; **15.2** 3; **15.3** 3; **15.4** $\dfrac{a}{\cos^{2}a}$; **15.5** $\dfrac{1}{4}$; **15.6** 1; **15.7** $\dfrac{1}{2}$; **15.8** $\dfrac{2}{e}$; **15.9** $-\dfrac{\sqrt{3}}{3}$; **15.10** $\dfrac{2}{\pi}$; **15.11** $\dfrac{\sqrt{2}}{2}$; **15.12** $a^{b}\cdot\ln a$; **15.13** 1; **15.14** $\dfrac{2}{25}$; **15.15** $-\dfrac{1}{x^{2}}$; **15.16** $-\ln 2$; **15.17** 1; **15.18** $2\ln 2$; **15.19** $2\cos x$; **15.20** $\dfrac{1}{6\sqrt{2}}$; **15.21** 0; **15.22** 0; **15.23** $-\dfrac{1}{12\pi}$; **15.24** -3; **15.25** $\dfrac{1}{5\ln 10}$; **15.26** $3\ln 3$; **15.27** $-\dfrac{1}{16}$; **15.28** $\dfrac{b-a}{2a}$; **15.29** $\dfrac{3}{2}$; **15.30** $\dfrac{1}{3\pi\ln 3}$.

Funksiyaning limiti

16–masala. Funksiyaning limitini hisoblang .

$$\lim_{x\to 0}\left(\frac{1+x^{2}\cdot 2^{x}}{1+x^{2}\cdot 5^{x}}\right)^{\frac{1}{\sin^{3}x}}=\lim_{x\to 0}\left(e^{\ln\frac{1+x^{2}\cdot 2^{x}}{1+x^{2}\cdot 5^{x}}}\right)^{\frac{1}{\sin^{3}x}}=$$

$$=\lim_{x\to 0}e^{\frac{1}{\sin^{3}x}\ln\frac{1+x^{2}\cdot 2^{x}}{1+x^{2}\cdot 5^{x}}}=\exp\left\{\lim_{x\to 0}\frac{1}{\sin^{3}x}\ln\frac{1+x^{2}\cdot 2^{x}}{1+x^{2}\cdot 5^{x}}\right\}=$$

$$= \exp\left\{\lim_{x\to 0}\frac{1}{\sin^3 x}\ln\frac{1+x^2\cdot 5^x - x^2\cdot 5^x + x^2\cdot 2^x}{1+x^2\cdot 5^x}\right\} =$$

$$= \exp\left\{\lim_{x\to 0}\frac{1}{\sin^3 x}\ln\left(1-\frac{-x^2\cdot 5^x + x^2\cdot 2^x}{1+x^2\cdot 5^x}\right)\right\} =$$

$$= \exp\left\{\lim_{x\to 0}\frac{1}{\sin^3 x}\ln\left(1-x^2\,\frac{5^x + 2^x}{1+x^2\cdot 5^x}\right)\right\} =$$

$$= \left[\begin{matrix} e^{x\cdot\ln 5}-1\approx x\cdot\ln 5 \\ e^{x\cdot\ln 2}-1\approx x\cdot\ln 2 \end{matrix}\right] = \exp\left\{\lim_{x\to 0}\frac{1}{\sin^3 x}\ln\left(1-x^2\,\frac{5^x + 2^x}{1+x^2\cdot 5^x}\right)\right\} =$$

$$= \exp\left\{\lim_{x\to 0}-\frac{x\cdot\ln 5}{x\cdot\left(1+x^2\cdot 5^x\right)} + \lim_{x\to 0}\frac{x\cdot\ln 2}{x\cdot\left(1+x^2\cdot 5^x\right)}\right\} =$$

$$= \exp\left\{\lim_{x\to 0}-\frac{\ln 5}{1+x^2\cdot 5^x} + \lim_{x\to 0}\frac{\ln 2}{1+x^2\cdot 5^x}\right\} =$$

$$= \exp\left\{-\frac{\ln 5}{1+0\cdot 5^0} + \frac{\ln 2}{1+0\cdot 5^0}\right\} = \exp\{-\ln 5+\ln 2\} =$$

$$= \exp\left\{-\ln\frac{5}{2}\right\} = \exp\left\{\ln\frac{2}{5}\right\} = e^{\ln\frac{2}{5}} = \frac{2}{5}.$$

1. $\lim\limits_{x\to 0}(1-\ln(1+x^3))^{\frac{3}{(x^2\arcsin x)}}$.

2. $\lim\limits_{x\to 0}(\cos\sqrt{x})^{\frac{1}{x}}$.

3. $\lim\limits_{x\to 0}\left(\dfrac{1+x\cdot 2^x}{1+x\cdot 3^x}\right)^{\frac{1}{x^2}}$.

4. $\lim\limits_{x\to 0}(2-3^{arctg^2\sqrt{x}})^{\frac{2}{\sin x}}$.

5. $\lim\limits_{x\to 0}\left(\dfrac{1+\sin x\cos\alpha x}{1+\sin x\cos\beta x}\right)^{ctg^3 x}$

6. $\lim\limits_{x\to 0}(5-\dfrac{4}{\cos x})^{\frac{1}{\sin^2 3x}}$.

7. $\lim\limits_{x\to 0}(1-\ln(1+\sqrt[3]{x}))^{\frac{x}{\sin^4\sqrt[3]{x}}}$.

8. $\lim\limits_{x\to 0}\left(2-e^{\arcsin^2\sqrt{x}}\right)^{\frac{3}{x}}$.

9. $\lim\limits_{x\to 0}(\cos\pi x)^{\frac{1}{(x\sin\pi x)}}$.

10. $\lim\limits_{x\to 0}(1+\sin^2 3x)^{\frac{1}{\ln\cos x}}$.

11. $\lim\limits_{x\to 0}(tg(\frac{\pi}{4}-x))^{ctgx}$.

12. $\lim\limits_{x\to 0}(1-x\sin^2 x)^{\frac{1}{\ln(1+\pi x^3)}}$.

13. $\lim\limits_{x\to 0}(2-5^{\arcsin x^3})^{\frac{\cos ec^2 x}{x}}$.

14. $\lim\limits_{x\to 0}(2-\cos 3x)^{\frac{1}{\ln(1+x^2)}}$.

15. $\lim\limits_{x\to 0}(2-e^{\sin x})^{ctg\pi x}$.

16. $\lim\limits_{x\to 0}(\cos x)^{\frac{1}{\ln(1+\sin^2 x)}}$.

17. $\lim\limits_{x\to 0}(2-e^{x^2})^{\frac{1}{\ln(1+tg^2(\pi\frac{x}{3}))}}$.

18. $\lim\limits_{x\to 0}(3-2\cos x)^{-\cos ec^2 x}$.

19. $\lim\limits_{x\to 0}(2-3^{\sin^2 x})^{\frac{1}{\ln\cos x}}$.

20. $\lim\limits_{x\to 0}\sqrt[x^2]{2-\cos x}$

21. $\lim\limits_{x\to 0}(6-\frac{5}{\cos x})^{ctg^2 x}$.

22. $\lim\limits_{x\to 0}(3-\frac{2}{\cos x})^{\cos ec^2 x}$.

23. $\lim\limits_{x\to 0}(\frac{1+\sin x\cos 2x}{1+\sin x\cos 3x})^{\frac{1}{\sin x^3}}$.

24. $\lim\limits_{x\to 0}(2-e^{x^2})^{\frac{1}{(1-\cos\pi x)}}$.

25. $\lim\limits_{x\to 0}(1+\ln\frac{1}{3}arctg\sqrt[6]{x})^{\frac{1}{x^3}}$.

26. $\lim\limits_{x\to 0}\frac{1+tgx\cos 2x}{1+tgx\cos 5x}^{\frac{1}{x^3}}$.

27. $\lim\limits_{x\to 0}(\frac{1+x3^x}{1+x7^x})^{\frac{1}{tg^2 x}}$.

28. $\lim\limits_{x\to 0}(1+tg^2 x)^{\frac{1}{\ln(1+3x^2)}}$.

29. $\lim\limits_{x\to 0}(1-\ln\cos x)^{\frac{1}{tg^2 x}}$.

30 $\lim\limits_{x\to 0}(1-\sin^2\frac{x}{2})^{\frac{1}{\ln(1+tg^2 3x)}}$.

Javoblar.16.1 e^{-3}; **16.2** $\frac{1}{\sqrt{e}}$; **16.3** $\frac{2}{3}$; **16.4** $\frac{1}{9}$; **16.5** $e^{\frac{\beta^2-\alpha^2}{2}}$; **16.6** $e^{-\frac{2}{9}}$; **16.7** e^{-1}; **16.8** e^{-3}; **16.9** $e^{-\frac{\pi}{2}}$; **16.10** e^{-18}; **16.11** e^{-2}; **16.12** $e^{-\frac{1}{\pi}}$; **16.13** $\frac{1}{5}$; **16.14** $e^{\frac{9}{2}}$; **16.15** $e^{-\frac{1}{\pi}}$; **16.16** $e^{-\frac{1}{2}}$; **16.17** $e^{-\frac{9}{\pi^2}}$; **16.18** e^{-1}; **16.19** 9; **16.20** $\sqrt{e}$; **16.21** $e^{-\frac{5}{2}}$; **16.22** e^{-1}; **16.23** $e^{\frac{5}{2}}$; **16.24** $e^{-\frac{2}{\pi^2}}$; **16.25** $\frac{1}{3}$; **16.26** $e^{\frac{21}{2}}$; **16.27** $\frac{3}{7}$; **16.28** $e^{\frac{1}{3}}$; **16.29** $e^{\frac{1}{2}}$; **16.30** $e^{-\frac{1}{36}}$.

Funksiyaning limiti

17–masala. Funksiyaning limitini hisoblang .

$$\lim_{x\to 0}\left(\frac{x^3+4}{x^3+9}\right)^{\frac{1}{x+2}}=\lim_{x\to 0}\left(\frac{0^3+4}{0^3+9}\right)^{\frac{1}{0+2}}=\left(\frac{4}{9}\right)^{\frac{1}{2}}=\sqrt{\frac{4}{9}}=\frac{2}{3}.$$

1. $\lim\limits_{x\to 0}\left(\frac{\sin 2x}{x}\right)^{1+x}$. **2.** $\lim\limits_{x\to 0}\left(\frac{2+x}{3-x}\right)^{x}$. **3.** $\lim\limits_{x\to 0}\left(\frac{\sin 4x}{x}\right)^{\frac{2}{x+2}}$.

4. $\lim\limits_{x\to 0}\left(\frac{e^{3x}-1}{x}\right)^{\cos^2(\frac{\pi}{4}+x)}$. **5.** $\lim\limits_{x\to 0}(\cos x)^{x+3}$. **6.** $\lim\limits_{x\to 0}\left(\frac{x^2+4}{x+2}\right)^{x^2+3}$.

7. $\lim\limits_{x\to 0}\left(\frac{\ln(1+x}{6x}\right)^{\frac{x}{x+2}}$. **8.** $\lim\limits_{x\to 0}\left(\frac{tg4x}{x}\right)^{2+x}$. **9.** $\lim\limits_{x\to 0}\left(\frac{e^{x^3}-1}{x^2}\right)^{\frac{8x+3}{1+x}}$.

10. $\lim\limits_{x\to 0}\left(\frac{x+2}{x+4}\right)^{\cos x}$. **11.** $\lim\limits_{x\to 0}\left(\frac{\sin 6x}{2x}\right)^{2+x}$. **12.** $\lim\limits_{x\to 0}\left(\frac{e^{x^2}-1}{x^2}\right)^{\frac{6}{1+x}}$.

13. $\lim\limits_{x\to 0}\left(\frac{\sin 2x}{\sin 3x}\right)^{x^2}$. **14.** $\lim\limits_{x\to 0}\left(tg(x+\frac{\pi}{3})\right)^{x+2}$. **15.** $\lim\limits_{x\to 0}\left(\frac{x^3+8}{3x^2+10}\right)^{x+2}$.

16. $\lim\limits_{x\to 0}(\sin(x+2))^{\frac{3}{3+x}}$. **17.** $\lim\limits_{x\to 0}\left(\frac{2^{2x}-1}{x}\right)^{x+1}$. **18.** $\lim\limits_{x\to 0}\left(\frac{x^4+5}{x+10}\right)^{\frac{4}{x+2}}$.

19. $\lim\limits_{x\to 0}\left(\frac{11x+8}{12x+1}\right)^{\cos^2 x}$. **20.** $\lim\limits_{x\to 0}\left(\frac{x^3+1}{x^3+8}\right)^{\frac{2}{x+1}}$. **21.** $\lim\limits_{x\to 0}\left(\frac{\ln(1+x^2)}{x^2}\right)^{\frac{3}{x+8}}$.

22. $\lim\limits_{x\to 0}(\cos\frac{x}{\pi})^{1+x}$. **23.** $\lim\limits_{x\to 0}\left(\frac{\arcsin x}{x}\right)^{2(x+5)}$. **24.** $\lim\limits_{x\to 0}\left(\frac{arctg3x}{x}\right)^{x+2}$.

25. $\lim\limits_{x\to 0}(e^x+x)^{\cos x^4}$. **26.** $\lim\limits_{x\to 0}\left(\frac{\sin 5x^2}{\sin x}\right)^{\frac{1}{x+6}}$. **27.** $\lim\limits_{x\to 0}\left(tg(\frac{\pi}{4}-x)\right)^{\frac{e^x-1}{x}}$.

28. $\lim\limits_{x\to 0}\left(6-\dfrac{5}{\cos x}\right)^{tg^2 x}$. **29.** $\lim\limits_{x\to 0}\left(\dfrac{1+8x}{2+11x}\right)^{\frac{1}{x^2+1}}$. **30.** $\lim\limits_{x\to 0}\left(\dfrac{\arcsin^2 x}{\arcsin^2 4x}\right)^{2x+1}$.

Javoblar.17.1 2; **17.2** 1; **17.3** 4; **17.4** $\sqrt{3}$; **17.5** 1; **17.6** 8; **17.7** 1; **17.8** 16; **17.9** 0; **17.10** $\dfrac{1}{2}$; **17.11** 9; **17.12** 1; **17.13** 1; **17.14** 3; **17.15** $0{,}64$; **17.16** $\sin 2$; **17.17** $2\ln 2$; **17.18** $\dfrac{1}{4}$; **17.19** 8; **17.20** $\dfrac{1}{64}$; **17.21** 1; **17.22** 1; **17.23** 1; **17.24** 9; **17.25** 1; **17.26** 0; **17.27** 1; **17.28** 1; **17.29** $\dfrac{1}{2}$; **17.30** $\dfrac{1}{16}$.

Funksiyaning limiti

18–masala. Funksiyaning limitini hisoblang .

$$\lim_{x\to 1}\left(\frac{2x-1}{x}\right)^{\frac{\ln(3+2x)}{\ln(2-x)}}=\lim_{x\to 1}\left(e^{\ln\left(\frac{2x-1}{x}\right)}\right)^{\frac{\ln(3+2x)}{\ln(2-x)}}=\lim_{x\to 1}e^{\frac{\ln(3+2x)}{\ln(2-x)}\cdot\ln\left(\frac{2x-1}{x}\right)}=$$

$$=\exp\left\{\lim_{x\to 1}\frac{\ln(3+2x)}{\ln(2-x)}\cdot\ln\left(\frac{2x-1}{x}\right)\right\}=\begin{bmatrix}x=y+1\Rightarrow y=x-1\\ x\to 1\quad\Rightarrow y\to 0\end{bmatrix}=$$

$$=\exp\left\{\lim_{x\to 1}\frac{\ln(3+2(y+1)}{\ln(2-(y+1))}\cdot\ln\left(\frac{2(y+1)-1}{y+1}\right)\right\}=$$

$$=\exp\left\{\lim_{x\to 1}\frac{\ln(5+2y)}{\ln(1-y)}\cdot\ln\left(\frac{2y+1}{y+1}\right)\right\}=$$

$$=\exp\left\{\lim_{x\to 1}\frac{\ln(5+2y)}{\ln(1-y)}\cdot\ln\left(1+\frac{y}{y+1}\right)\right\}=$$

$$=\begin{bmatrix}\ln\left(1+\dfrac{y}{y+1}\right)\approx\dfrac{y}{y+1},\ \text{при } y\to 0\ \left(\dfrac{y}{y+1}\to 0\right)\\ \ln(1-y)\approx -y,\ \ \text{при } y\to 0\ \left(-y\to 0\right)\end{bmatrix}=$$

$$= \exp\left\{\lim_{x\to 1}\frac{\ln(5+2y)}{-y}\cdot\left(\frac{y}{y+1}\right)\right\} = \exp\left\{\lim_{x\to 1}-\frac{\ln(5+2y)}{y+1}\right\} =$$

$$= \exp\left\{-\frac{\ln(5+2\cdot 0)}{0+1}\right\} = \exp\{-\ln 5\} = \exp\left\{\ln\frac{1}{5}\right\} = e^{\ln\frac{1}{5}} = \frac{1}{5}.$$

1. $\lim\limits_{x\to 1}\left(\frac{3x-1}{x+1}\right)^{\frac{1}{\sqrt[3]{x}-1}}$.

2. $\lim\limits_{x\to a}\left(\frac{\sin x}{\sin a}\right)^{\frac{1}{x-a}}$.

3. $\lim\limits_{x\to 1}\left(\frac{2x-1}{x}\right)^{\frac{1}{\sqrt[3]{x}-1}}$.

4. $\lim\limits_{x\to 2}\left(\frac{\cos x}{\cos 2}\right)^{\frac{1}{x-2}}$.

5. $\lim\limits_{x\to 8}\left(\frac{2x-7}{x+1}\right)^{\frac{1}{\sqrt[3]{x}-2}}$.

6. $\lim\limits_{x\to \pi/4}(tgx)^{1/(\cos(3\pi/4-x)}$.

7. $\lim\limits_{x\to 1}\left(\frac{2x-1}{x}\right)^{1/(\sqrt[5]{x}-1)}$.

8. $\lim\limits_{x\to a}(2-x/a)^{tg\frac{\pi x}{2a}}$.

9. $\lim\limits_{x\to 2\pi}(\cos x)^{ctg2x/\sin 3x}$.

10. $\lim\limits_{x\to 2\pi}(\cos x)^{1/\sin^2 2x}$.

11. $\lim\limits_{x\to 3}\left(\frac{6-x}{3}\right)^{tg(\pi x/6)}$.

12. $\lim\limits_{x\to 4\pi}(\cos x)^{ctgx/\sin 4x}$.

13. $\lim\limits_{x\to 1}(3-2x)^{tg(\pi x/2)}$.

14. $\lim\limits_{x\to 4\pi}(\cos x)^{\frac{5}{tg5x\sin 2x}}$.

15. $\lim\limits_{x\to 3}\left(\frac{9-2x}{3}\right)^{tg(\pi x/6)}$.

16. $\lim\limits_{x\to \pi/2}(\sin x)^{6tgx\cdot tg3x}$.

17. $\lim\limits_{x\to 1}(2e^{x-1}-1)^{x/(x-1)}$.

18. $\lim\limits_{x\to \pi/2}\left(tg\frac{x}{2}\right)^{1/(x-\pi/2)}$.

19. $\lim\limits_{x\to 1}(2e^{x-1}-1)^{\frac{3x-1}{x-1}}$.

20. $\lim\limits_{x\to \pi/2}(1+\cos 3x)^{\sec x}$.

21. $\lim\limits_{x\to 2}(2e^{x-2}-1)^{\frac{3x+2}{x-2}}$.

22. $\lim\limits_{x\to 1}\left(\frac{\sin(x-1)}{x-1}\right)^{\frac{\sin(x-1)}{x-1-\sin(x-1)}}$.

23. $\lim\limits_{x\to 1}\left(\dfrac{2-x}{x}\right)^{1/\ln(2-x)}$.

24. $\lim\limits_{x\to \pi/2}(ctg\,\dfrac{x}{2})^{1/\cos x}$.

25. $\lim\limits_{x\to 1}(2-x)^{\frac{\sin(\pi x/2)}{\ln(2-x)}}$.

26. $\lim\limits_{x\to 3}\left(\dfrac{\sin x}{\sin 3}\right)^{1/(x-3)}$.

27. $\lim\limits_{x\to 1}\left(\dfrac{x+1}{2x}\right)^{\frac{\ln(x+2)}{\ln(2-x)}}$.

28. $\lim\limits_{x\to \pi/2}(\sin x)^{\frac{18\sin x}{ctgx}}$.

29. $\lim\limits_{x\to 1}\left(\dfrac{1}{x}\right)^{\frac{\ln(x+1)}{\ln(2-x)}}$.

30. $\lim\limits_{x\to \pi}(\mathrm{ctg}\,\dfrac{x}{4})^{1/\cos(x/2)}$.

Javoblar.18.1 e^3; **18.2** e^{ctga}; **18.3** e^3; **18.4** $e^{-tg\,2}$; **18.5** $e^{\frac{4}{3}}$; **18.6** e^2; **18.7** e^5; **18.8** $e^{\frac{2}{\pi}}$; **18.9** $e^{-\frac{1}{12}}$; **18.10** $e^{-\frac{1}{8}}$; **18.11** $e^{\frac{2}{\pi}}$; **18.12** $e^{-\frac{1}{8}}$; **18.13** $e^{\frac{4}{\pi}}$; **18.14** $e^{-\frac{1}{4}}$; **18.15** $e^{\frac{4}{\pi}}$; **18.16** e^{-1}; **18.17** e^2; **18.18** e; **18.19** e^4; **18.20** e^{-3}; **18.21** e^{16}; **18.22** e^{-1}; **18.23** e^2; **18.24** e; **18.25** e; **18.26** $e^{ctg\,3}$; **18.27** $\sqrt{3}$; **18.28** 1; **18.29** 2; **18.30** e.

Funksiyaning limiti

19–masala. Funksiyaning limitini hisoblang .

$$=\left(\lim_{x\to 1}\frac{e^{2x}-e^2}{x-1}\right)^{1+1}=\left(\lim_{x\to 1}\frac{e^2(e^{2x-2}-1)}{x-1}\right)^2=$$

$$\lim_{x\to 1}\left(\frac{e^{2x}-e^2}{x-1}\right)^{x+1}=\lim_{x\to 1}\left(\frac{e^{2x}-e^2}{x-1}\right)^{\lim\limits_{x\to 1}x+1}=$$

Cheksiz kichiklarning ekvivalentlik munosabatidan foydalanib:

$x\to 1$ $(2x-2\to 0)$ da $e^{2x-2}-1\sim 2x-2$ ega bo'lamiz.

Natijada:

$$=\left(\lim_{x\to 1}\frac{e^2(2x-2)}{x-1}\right)^2=\left(\lim_{x\to 1}2e^2\right)^2=\left(2e^2\right)^2=4e^4.$$

1. $\lim\limits_{x\to e}\left(\dfrac{\ln x-1}{x-e}\right)^{\sin\frac{\pi}{2e}x}$.

2. $\lim\limits_{x\to\pi/4}(tgx)^{ctgx}$.

3. $\lim\limits_{x\to\pi/4}\left(\dfrac{\ln tgx}{1-ctgx}\right)^{1/(x+\pi/4)}$.

4. $\lim\limits_{x\to 2}(\sin x)^{3/(1+x)}$.

5. $\lim\limits_{x\to 2}\left(\dfrac{\sin 3\pi x}{\sin \pi x}\right)^{\sin^2(x-2)}$.

6. $\lim\limits_{x\to\pi/6}(\sin x)^{6x/\pi}$.

7. $\lim\limits_{x\to 3}(2-\dfrac{x}{3})^{\sin \pi x}$.

8. $\lim\limits_{x\to 1}\left(\dfrac{1+x}{2+x}\right)^{(1-x^2)/(1-x)}$.

9. $\lim\limits_{x\to 1}(1+e^x)^{\frac{\sin \pi x}{1-x}}$.

10. $\lim\limits_{x\to 1}\left(\dfrac{tg9\pi x}{\sin 4\pi x}\right)^{x/(x+1)}$.

11. $\lim\limits_{x\to 3}\left(\dfrac{\arcsin(x-3)}{\sin 3\pi x}\right)^{x^2-8}$.

12. $\lim\limits_{x\to\pi/4}(\sin 2x)^{\frac{x^2-\pi^2/16}{x-\pi/4}}$.

13. $\lim\limits_{x\to 1}\left(arctg\,\dfrac{x-3/4}{(x-1)^2}\right)^{x+1}$.

14. $\lim\limits_{x\to\pi}\left(ctg\,\dfrac{x}{4}\right)^{\sin(x-\pi)}$.

15. $\lim\limits_{x\to a}\left(\dfrac{\sin x-\sin a}{x-a}\right)^{x^2/a^2}$.

16. $\lim\limits_{x\to 2}\left(\dfrac{\sqrt{x+2}-2}{x^2-4}\right)^{1/x}$.

17. $\lim\limits_{x\to\pi/4}(\sin x+\cos x)^{1/tgx}$.

18. $\lim\limits_{x\to\pi/8}(tg2x)^{\sin(\pi/8+x)}$.

19. $\lim\limits_{x\to 1}(\arcsin x)^{tg\pi x}$.

20. $\lim\limits_{x\to\pi}(x+\sin x)^{\sin x+x}$.

21. $\lim\limits_{x\to 1}(\ln^2 ex)^{1/(x^2+1)}$.

22. $\lim\limits_{x\to 1}(\sqrt{x}+1)^{\pi/arctgx}$.

23. $\lim\limits_{x\to 1}\left(\dfrac{x^3-1}{x-1}\right)^{1/x^2}$.

24. $\lim\limits_{x\to 1}\left(\dfrac{e^{\sin \pi x}-1}{x-1}\right)^{x^2+1}$.

25. $\lim\limits_{x\to 2}(\cos \pi x)^{tg(x-2)}$.

26. $\lim\limits_{x\to 1/2}(\arcsin x+\arccos x)^{\frac{1}{x}}$.

27. $\lim\limits_{x\to\pi/2}(\cos x+1)^{\sin x}$.

28. $\lim\limits_{x\to 1}(\sqrt[3]{x}+x-1)^{\sin(\pi x/4)}$.

29. $\lim\limits_{x\to 1}\left(\dfrac{x^2+2x-3}{x^2+4x-5}\right)^{1/(2-x)}$.

30. $\lim\limits_{x\to 1}\left(\dfrac{1+\cos\pi x}{tg^2\pi x}\right)^{x^2}$.

Javoblar.19.1 $\dfrac{1}{e}$; **19.2** 1; **19.3** 1; **19.4** $\sin 2$; **19.5** 1; **19.6** $\dfrac{1}{2}$; **19.7** 1; **19.8** $\dfrac{4}{9}$; **19.9** $(1+e)^{\pi}$; **19.10** $\dfrac{3}{2}$; **19.11** $-\dfrac{1}{3\pi}$; **19.12** 1; **19.13** $\dfrac{\pi^2}{4}$; **19.14** 0; **19.15** $\cos a$; **19.16** $\dfrac{1}{4}$; **19.17** $\sqrt{2}$; **19.18** 1; **19.19** 1; **19.20** π^{π}; **19.21** 1; **19.22** 16; **19.23** 3; **19.24** π^2; **19.25** 1; **19.26** $\dfrac{\pi^2}{4}$; **19.27** 1; **19.28** 1; **19.29** $\dfrac{2}{3}$; **19.30** $\dfrac{1}{2}$.

20–masala. Funksiyaning yoki sonli ketma-ketlikning limitini hisoblang.

$$\lim_{n\to\infty}\frac{\sqrt{n^2+3n-1}+\sqrt[3]{2n^2+1}}{n+2\sin n}=$$

$$=\lim_{n\to\infty}\frac{\frac{1}{n}\left(\sqrt{n^2+3n-1}+\sqrt[3]{2n^2+1}\right)}{\frac{1}{n}\left(n+2\sin n\right)}=$$

$$=\lim_{n\to\infty}\frac{\sqrt{1+\frac{3}{n}-\frac{1}{n^2}}+\sqrt[3]{\frac{2}{n}+\frac{1}{n^3}}}{1+2\frac{\sin n}{n}}=\left[\lim_{n\to\infty}\frac{\sin n}{n}=0\right]=$$

$$=\frac{\sqrt{1+0-0}+\sqrt[3]{0+0}}{1+2\cdot 0}=\frac{\sqrt{1}+\sqrt[3]{0}}{1}=1.$$

1. $\lim\limits_{x\to 0}\sqrt{4\cos 3x+x\operatorname{arctg}\frac{1}{x}}$.

2. $\lim\limits_{x\to \pi/2}\sqrt{3\sin x+(2x-\pi)\sin\dfrac{x}{2x-\pi}}$.

3. $\lim\limits_{n\to\infty}\dfrac{2n-\sin n}{\sqrt{n}-\sqrt[3]{n^3-7}}$.

4. $\lim\limits_{x\to 0}\dfrac{tgx\cdot\cos\frac{1}{x}+\lg(2+x)}{\lg(4+x)}$.

5. $\lim\limits_{n\to\infty} \dfrac{e^{1/n} + \sin\dfrac{n}{n^2+1}\cos n}{1+\cos\frac{1}{n}}$.

6. $\lim\limits_{n\to\infty} \dfrac{\sqrt[4]{2+n^5-\sqrt{2n^3+3}}}{(n+\sin n)\sqrt{7n}}$.

7. $\lim\limits_{x\to\pi/4} \dfrac{\sqrt[3]{tgx} + (4x-\pi)\cos\dfrac{x}{4x-\pi}}{\lg(2+tgx)}$.

8. $\lim\limits_{n\to\infty} (\sin\sqrt{n^2+1}\,arctg\,\dfrac{n}{n^2+1})$.

9. $\lim\limits_{n\to\infty} \dfrac{n^2-\sqrt{3n^5-7}}{(n^2-n\cos n+1)\sqrt{n}}$.

10. $\lim\limits_{n\to\infty} \dfrac{3\sin n+\sqrt{n-1}}{n+\sqrt{n+1}}$.

11. $\lim\limits_{n\to\infty} \dfrac{(1-\cos n)\sqrt[3]{n}}{\sqrt{2n+1}-1}$.

12. $\lim\limits_{x\to 0} \ln\left(2+\sqrt{\operatorname{arctg}x\sin\dfrac{1}{x}}\right)$.

13. $\lim\limits_{x\to -2} \sqrt{\dfrac{1+\cos\pi x}{4+(x+2)\sin\dfrac{x}{x+2}}}$.

14. $\lim\limits_{n\to\infty} \dfrac{n}{\sqrt[3]{n^4-3}+\sin n}$.

15. $\lim\limits_{n\to\infty} \dfrac{\sqrt[3]{n^2\cos n}+\sqrt{3n^2+2}}{\sqrt[5]{n^6+1}}$.

16. $\lim\limits_{x\to 0} \dfrac{\sqrt[3]{tgx}\,arctg\,\dfrac{1}{x}+3}{2-\lg(1+\sin x)}$.

17. $\lim\limits_{x\to 0}\sqrt{arctg\,\sin^2\frac{1}{x}+5\cos x}.$

18. $\lim\limits_{x\to 0}\sqrt{4\cos x+\sin\frac{1}{x}\ln(1+x)}.$

19. $\lim\limits_{x\to 0}\sqrt{2\cos^2 x+(e^x-1)\sin\frac{1}{x}}.$

20. $\lim\limits_{x\to 0}\dfrac{2+\ln(e+x\sin\frac{1}{x})}{\cos x+\sin x}.$

21. $\lim\limits_{x\to 0}\ln\left((e^{x^2}-\cos x)\cos(\tfrac{1}{x})+tg(x+\tfrac{\pi}{3})\right).$

22. $\lim\limits_{x\to 0}\dfrac{\cos x+\ln(1+x)\sqrt{2+\cos(1/x)}}{2+e^x}.$

23. $\lim\limits_{x\to 1}\dfrac{\cos 2\pi x}{2+(e^{\sqrt{x-1}}-1)arctg\,\frac{x+2}{x-1}}.$

24. $\lim\limits_{x\to 0}\sqrt{(e^{\sin x}-1)\cos\frac{1}{x}+4\cos x}$

25. $\lim\limits_{x\to 0}\dfrac{\cos(1+x)}{(2+\sin\frac{1}{x})\ln(1+x)+2}.$

26. $\lim\limits_{x\to 2}\sqrt[3]{\lg(x+2)+\sin\sqrt{4-x^2}\cos\frac{x+2}{x-2}}.$

27. $\lim\limits_{x\to \pi/2}\dfrac{2+\cos x\sin\frac{2}{2x-\pi}}{3+2x\sin x}.$

28. $\lim\limits_{x\to 1}tg(\cos x+\sin\frac{x-1}{x+1}\cos\frac{x+1}{x-1}.$

29. $\lim\limits_{x\to 0}\sqrt{x(2+\sin\frac{1}{x})+4\cos x}$

30. $\lim\limits_{x\to 1}\dfrac{\sin x+\sin \pi x\cdot \operatorname{arctg}\dfrac{1+x}{1+\cos x}}{(2+\sin\dfrac{1}{x})\ln(1+x)+2}.$

Javoblar.20.1 2; **20.2** $\sqrt{3}$; **20.3** -2; **20.4** $\dfrac{1}{2}$; **20.5** $\dfrac{1}{2}$; **20.6** $-\dfrac{2}{7}$; **20.7** $\dfrac{1}{\lg(3)}$; **20.8** 0; **20.9** $-\sqrt{3}$; **20.10** 0; **20.11** 0; **20.12** $\ln 2$; **20.13** $\dfrac{\sqrt{2}}{2}$; **20.14** 1; **20.15** 0; **20.16** $\dfrac{3}{2}$; **20.17** $\sqrt{5}$; **20.18** 2; **20.19** $\sqrt{2}$; **20.20** 3; **20.21** $\ln\sqrt{3}$; **20.22** $\dfrac{1}{3}$; **20.23** $\dfrac{1}{2}$; **20.24** 2; **20.25** $\dfrac{\cos 1}{2}$; **20.26** $\sqrt[3]{\lg 4}$; **20.27** $\dfrac{2}{3+\pi}$; **20.28** $tg(\cos 1)$; **20.29** 2; **20.30** $\dfrac{\sin 1}{1+\cos 1}$.

III BOB. Funktsiyaning hosilasi

Bir argumentli funksiyalarni differensiallash bo'limida siz funksiyaning hosilasini topish, differensiallashning asosiy qoidalari, asosiy formulalari, yuqori tartibli hosilalarni hisoblash, shuningdek funksiyalarni umumiy tekshirish masalalari bilan tanishasiz.

Ta'rif. Agar ushbu

$$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{f(x_0+\Delta x)-f(x_0)}{\Delta x}$$

limit mavjud va chekli bo'lsa, u $f(x)$ funksiyaning x_0 dagi hosilasi deyiladi va

$$f'(x_0)=\lim_{\Delta x\to 0}\frac{f(x_0+\Delta x)-f(x_0)}{\Delta x} \qquad (1)$$

kabi belgilanadi.

Agar $x_0+\Delta x=x$ deyilsa, unda $\Delta x=x-x_0$ va $\Delta x\to 0$ da $x\to x_0$ bo'lib, (1) munosabat quyidagi

$$f'(x_0)=\lim_{x\to x_0}\frac{f(x)-f(x_0)}{x-x_0}$$

ko'rinishga keladi.

1–masala. Hosila ta'rifidan foydalanib, $f'(0)$ toping.

$$f(x)=\begin{cases}1-\cos\left(x\sin\frac{1}{x}\right), & x\neq 0;\\ 0, & x=0\end{cases}$$

$x=0$ nuqtada hosilasi

$$f'(0)=\lim_{\Delta x\to 0}\frac{f(0+\Delta x)-f(0)}{\Delta x}$$

Hosila ta'rifidan foydalanib:

$$f'(0)=\lim_{\Delta x\to 0}\frac{f(0+\Delta x)-f(0)}{\Delta x}=\lim_{\Delta x\to 0}\frac{1-\cos(\Delta x\sin\frac{1}{\Delta x})-0}{\Delta x}=$$

$$=\lim_{\Delta x\to 0}\frac{1-\left(1-2\sin^2\left(\frac{1}{2}\cdot\Delta x\sin\frac{1}{\Delta x}\right)\right)}{\Delta x}=\lim_{\Delta x\to 0}\frac{2\sin^2\left(\frac{1}{2}\cdot\Delta x\sin\frac{1}{\Delta x}\right)}{\Delta x}=$$

$$=\left[\sin\left(\frac{1}{2}\cdot\Delta x\sin\frac{1}{\Delta x}\right)\approx\frac{1}{2}\cdot\Delta x\sin\frac{1}{\Delta x}\right]=$$

$$=\lim_{\Delta x\to 0}\frac{2\cdot\left(\frac{1}{2}\cdot\Delta x\sin\frac{1}{\Delta x}\right)^2}{\Delta x}=\lim_{\Delta x\to 0}\frac{1}{2}\cdot\Delta x\sin^2\frac{1}{\Delta x}=$$

$$=\begin{bmatrix}\sin^2\frac{1}{\Delta x}-\text{chegaralangan, u holda}\\ \Delta x\to 0\ \text{da}\quad \Delta x\sin^2\frac{1}{\Delta x}\to 0,\end{bmatrix}=2\cdot 0=0.$$

1. $f(x)=\begin{cases}\operatorname{tg}\left(x^3+x^2\sin\left(\frac{x}{2}\right)\right), & x\neq 0;\\ 0, & x=0\end{cases}$

2. $f(x)=\begin{cases}\arcsin\left(x^2\cos\left(\frac{1}{9x}\right)\right)+\frac{2}{3}x, & x\neq 0;\\ 0, & x=0\end{cases}$

3. $f(x)=\begin{cases}\operatorname{arctg}\left(x\cos\left(\frac{1}{5x}\right)\right), & x\neq 0;\\ 0, & x=0\end{cases}$

4. $f(x)=\begin{cases}\ln\left(1-\sin\left(x^3\sin\frac{1}{x}\right)\right), & x\neq 0;\\ 0, & x=0\end{cases}$

5. $f(x)=\begin{cases}\sin\left(x\sin\frac{3}{x}\right), & x\neq 0;\\ 0, & x=0\end{cases}$

6. $f(x)=\begin{cases}\sqrt{1+\ln\left(1+x^2\sin\frac{1}{x}\right)}-1, & x\neq 0;\\ 0, & x=0\end{cases}$

7. $f(x)=\begin{cases}\sin(e^{x^2\sin\frac{5}{x}}-1)+x, & x\neq 0;\\ 0, & x=0\end{cases}$

8. $f(x)=\begin{cases}x^2\cos\left(\frac{4}{3x}\right)+\frac{x^2}{2}, & x\neq 0;\\ 0, & x=0\end{cases}$.

9. $f(x)=\begin{cases}\operatorname{arctg}(x^3-x^{\frac{3}{2}}\sin\frac{1}{3x}), & x\neq 0;\\ 0, & x=0\end{cases}$

10. $f(x)=\begin{cases}\sin x\cdot\cos\frac{5}{x}, & x\neq 0;\\ 0, & x=0\end{cases}$

11. $f(x)=\begin{cases}x+\arcsin\left(x^2\sin\frac{6}{x}\right), & x\neq 0;\\ 0, & x=0\end{cases}$

12. $f(x)=\begin{cases}\operatorname{tg}\left(2^{x^2\cos\left(\frac{1}{8x}\right)}-1+x\right), & x\neq 0;\\ 0, & x=0\end{cases}$

13. $f(x)=\begin{cases}\operatorname{arctg}x\cdot\sin\frac{7}{x}, & x\neq 0;\\ 0, & x=0\end{cases}$

14. $f(x)=\begin{cases}2x^2+x^2\cos\frac{1}{9x}, & x\neq 0;\\ 0, & x=0\end{cases}$

15. $f(x)=\begin{cases}x^2\cos^2\frac{11}{x}, & x\neq 0;\\ 0, & x=0\end{cases}$

16. $f(x)=\begin{cases}2x^2+x^2\cos\frac{1}{x}, & x\neq 0;\\ 0, & x=0\end{cases}$

17. $f(x)=\begin{cases}\dfrac{\ln(\cos x)}{x}, & x\neq 0;\\ 0, & x=0\end{cases}$

18. $f(x)=\begin{cases}6x+x\sin\frac{1}{x}, & x\neq 0;\\ 0, & x=0\end{cases}$

19. $f(x)=\begin{cases}\dfrac{e^{x^2}-\cos x}{x}, & x\neq 0;\\ 0, & x=0\end{cases}$

20. $f(x)=\begin{cases}e^{x\sin\frac{5}{x}}-1, & x\neq 0;\\ 0, & x=0\end{cases}$

21. $f(x)=\begin{cases}3^{x^2\sin\frac{2}{x}}-1+2x, & x\neq 0;\\ 0, & x=0\end{cases}$

22. $f(x)=\begin{cases}\sqrt{1+\ln\left(1+3x^2\cos\frac{2}{x}\right)}-1, & x\neq 0;\\ 0, & x=0\end{cases}$

23. $f(x)=\begin{cases}e^{x\sin\frac{3}{5x}}-1, & x\neq 0;\\ 0, & x=0\end{cases}$

24. $f(x)=\begin{cases}\dfrac{e^{\operatorname{tg}x}-e^{\sin x}}{x^2}, & x\neq 0;\\ 0, & x=0\end{cases}$

25. $f(x)=\begin{cases}\operatorname{arctg}(\frac{3x}{2}-x^2\sin\frac{1}{x}), & x\neq 0;\\ 0, & x=0\end{cases}$

26. $f(x)=\begin{cases}e^{\sin\left(x^{\frac{3}{2}}\sin\frac{2}{x}\right)}-1+x^2, & x\neq 0;\\ 0, & x=0\end{cases}$

27. $f(x)=\begin{cases}\sqrt[3]{1-2x^3\sin\frac{5}{x}}-1+x, & x\neq 0;\\ 0, & x=0\end{cases}$

28. $f(x)=\begin{cases}x^2e^{|x|}\sin\frac{1}{x^2}, & x\neq 0;\\ 0, & x=0\end{cases}$

29. $f(x)=\begin{cases}\dfrac{\ln\left(1+2x^2+x^3\right)}{x}, & x\neq 0;\\ 0, & x=0\end{cases}$

30. $f(x)=\begin{cases}\dfrac{\cos x-\cos 3x}{x}, & x\neq 0;\\ 0, & x=0\end{cases}$

Javoblar. **1.1** 0; **1.2** $\dfrac{2}{3}$; **1.3** mavjud emas; **1.4** 0; **1.5** mavjud emas;**1.6** 0; **1.7** 1; **1.8** 0; **1.9** 0; **1.10** mavjud emas; **1.11** 1; **1.12** 1; **1.13** mavjud emas; **1.14** 0; **1.15** 0; **1.16** 0; **1.17** $-\dfrac{1}{2}$; **1.18** mavjud emas; **1.19** $1{,}5$; **1.20** mavjud emas; **1.21** -2; **1.22** 0; **1.23** mavjud emas; **1.24** $\ln\sqrt{2}$; **1.25** $\dfrac{3}{2}$; **1.26** 0; **1.27** 1; **1.28** 0; **1.29** 2; **1.30** 4.

Differensiallanuvchi $y=f(x)$ funksiya grafigining $M_0(x_0,y_0)$ $(y_0=f(x_0))$ nuqtasida o'tkazilgan urinma tenglamasi

$$y-y_0=f'(x_0)(x-x_0)$$

ko'rinishga,
$f'(x_0)\neq 0$ da normal tenglamasi

$$y-y_0=-\frac{1}{f'(x_0)}(x-x_0)$$

ko'rinishga ega bo'ladi.

2–masala. Funksiya grafigining abssissasi x_0 bo'lgan nuqtasiga o'tkazilgan urinma tenglamasini tuzing.

$$y=6\sqrt[3]{x}-\frac{16\sqrt[4]{x}}{3},\quad x_0=1.$$

Echim:

$$y' = \left(6\sqrt[3]{x} - \frac{16\sqrt[4]{x}}{3}\right)' = \left(6 \cdot x^{\frac{1}{3}} - \frac{16 \cdot x^{\frac{1}{4}}}{3}\right)' =$$

$$= 6 \cdot \frac{1}{3} \cdot x^{-\frac{2}{3}} - \frac{16}{3} \cdot \frac{1}{4} \cdot x^{-\frac{3}{4}} = 2 \cdot x^{-\frac{2}{3}} - \frac{4}{3} \cdot x^{-\frac{3}{4}}.$$

$$y'_0 = y'_0(x_0) = 2 \cdot 1^{-\frac{2}{3}} - \frac{4}{3} \cdot 1^{-\frac{3}{4}} = 2 - \frac{4}{3} = \frac{2}{3}.$$

y' funksiya x_0 nuqtada hosilaga ega ekanligidan, urinma tenglamasi quyidagi ko'rinishda bo'ladi:

$$y - y_0 = y'_0(x - x_0), \text{ bu yerda } \quad y'_0 = \frac{2}{3}$$

$$y_0 = y(x_0) = 6\sqrt[3]{1} - \frac{16\sqrt[4]{1}}{3} = 6 - \frac{16}{3} = \frac{2}{3}$$

U holda:

$$y - \frac{2}{3} = \frac{2}{3} \cdot (x - 1)$$

$$y = \frac{2}{3} \cdot x - \frac{2}{3} + \frac{2}{3}$$

$$y = \frac{2}{3} \cdot x$$

Shunday qilib, urinma tenglamasi:

$$y = \frac{2}{3} \cdot x.$$

Funksiya grafigining abssissasi x_0 bo'lgan nuqtasiga o'tkazilgan normal (1–12 variantlarda) yoki urinma (13–30 varintlarda) tenglamasini tuzing.

1. $y = \dfrac{4x - x^2}{4}, \ x_0 = 2.$ **2.** $y = 2x^2 + 3x - 1, \ x_0 = -2.$

3. $y = x^2 + 8\sqrt{x} - 32, \ x_0 = 4.$ **4.** $y = x + \sqrt{x^3}, \ x_0 = 1.$

5. $y=\sqrt[3]{x^2}-20,\ x_0=-8.$

6. $y=\dfrac{1+\sqrt{x}}{1-\sqrt{x}},\ x_0=4.$

7. $y=8\sqrt[4]{x}-70,\ x_0=16.$

8. $y=2x^2-3x+1,\ x_0=1.$

9. $y=\dfrac{x^2-3x+6}{x^2},\ x_0=3.$

10. $y=\sqrt{x}-3\sqrt[3]{x},\ x_0=64.$

11. $y=\dfrac{x^3+2}{x^3-2},\ x_0=2.$

12. $y=2x^2+3,\ x_0=-1.$

13. $y=\dfrac{x^{29}+6}{x^4+1},\ x_0=1.$

14. $y=2x+\dfrac{1}{x},\ x_0=1.$

15. $y=\dfrac{-2(x^8+2)}{3(x^4+1)},\ x_0=1.$

16. $y=\dfrac{x^5+1}{x^4+1},\ x_0=1.$

17. $y=\dfrac{x^{16}+9}{1-5x^2},\ x_0=1.$

18. $y=3\left(\sqrt[3]{x}-2\sqrt{x}\right),\ x_0=1.$

19. $y=\dfrac{1}{3x+2},\ x_0=2.$

20. $y=\dfrac{x}{x^2+1},\ x_0=-2.$

21. $y=\dfrac{x^2-3x+3}{x},\ x_0=3.$

22. $y=\dfrac{2x}{x^2+1},\ x_0=1.$

23. $y=-2\left(\sqrt[3]{x}+3\sqrt{x}\right),\ x_0=1.$

24. $y=\dfrac{1+3x^2}{3+x^2},\ x_0=1.$

25. $y=14\sqrt{x}-15\sqrt[3]{x}+2,\ x_0=1.$

26. $y=3\sqrt[4]{x}-\sqrt{x},\ x_0=1.$

27. $y=\dfrac{3x-2x^3}{3},\ x_0=1.$

28. $y=\dfrac{x^2}{10}+3,\ x_0=2.$

29. $y=\dfrac{x^2-2x-3}{4},\ x_0=4.$

30. $y=x-x^3,\ x_0=-1.$

Javoblar. **2.1** $x=2$; **2.2** $y=\dfrac{x}{5}+1\dfrac{2}{5}$; **2.3** $y=-\dfrac{x}{10}+\dfrac{2}{5}$; **2.4** $y=-\dfrac{2x}{5}+2\dfrac{2}{5}$;

2.5 $y=3x+8$; **2.6** $y=-2x+5$; **2.7** $y=-4x+10$; **2.8** $y=-x+1$; **2.9** $y=9x-26\dfrac{1}{3}$; **2.10**

$x=64$; **2.11** $y=\frac{3}{4}x+\frac{1}{6}$; **2.12** $y=-4x+1$; **2.13** $y=7{,}5x-4$; **2.14** $y=x+2$;

2.15 $y=-\frac{2}{3}x-\frac{1}{3}$; **2.16** $y=\frac{x}{2}+\frac{1}{2}$; **2.17** $y=\frac{9}{4}x-\frac{19}{4}$; **2.18** $y=-2x-1$;

2.19 $y=-\frac{3}{64}x+\frac{7}{32}$; **2.20** $y=-\frac{3}{25}x-\frac{16}{25}$; **2.21** $y=x-2$; **2.22** $y=1$;

2.23 $3y+11x+13=0$; **2.24** $y=x$; **2.25** $y=2x+1$; **2.26** $y=\frac{x}{4}+\frac{7}{4}$; **2.27** $y=-x+1\frac{1}{3}$; **2.28** $y=\frac{2}{5}x+\frac{13}{5}$; **2.29** $y=\frac{3}{2}x-\frac{19}{4}$; **2.30** $y=\frac{x}{2}+\frac{1}{2}$.

Funksiyaning differentsiali

Agar $y=f(x)$ funksiyaning Δy orttirmasi $\Delta y=A\cdot\Delta x+o(\Delta x)$ ko'rinishda yozilishi mumkin bo'lsa, ortirmaning Δx ga nisbatan chiziqli qismi $A\cdot\Delta x$ funksiyaning differensiali deyiladi va dy yoki $df(x)$ orqali belgilanadi: $dy=A(x)\Delta x$. Differensila mavjud bo'lishi uchun chekli hosila $f'(x)$ning mavjudligi va yetarlidir. Bunda $dy=f'(x)dx=y'dx$ bo'ladi.

Agar $y=f(u),\ u=\varphi(x)$ murakkab funksiya berilgan bo'lsa, u holda $dy=f'(u)du$ bo'ladi.

3–masala. Differensial dyni toping.

$$y=x\sqrt{x^2-1}+\ln\left|x+\sqrt{x^2-1}\right|.$$

$$dy=y'dx=\left(x\sqrt{x^2-1}+\ln\left|x+\sqrt{x^2-1}\right|\right)'\cdot dx=$$

$$=\left(\sqrt{x^2-1}+x\cdot\frac{1}{2\sqrt{x^2-1}}\cdot 2x+\frac{1}{x+\sqrt{x^2-1}}\cdot\left(1+\frac{1}{2\sqrt{x^2-1}}\cdot 2x\right)\right)dx=$$

$$=\left(\sqrt{x^2-1}+\frac{x^2}{\sqrt{x^2-1}}+\frac{x-\sqrt{x^2-1}}{x^2-\left(x^2-1\right)}\cdot\left(1+\frac{x}{\sqrt{x^2-1}}\right)\right)dx=$$

$$=\left(\sqrt{x^2-1}+\frac{x^2}{\sqrt{x^2-1}}+x-\sqrt{x^2-1}\cdot\left(1+\frac{x}{\sqrt{x^2-1}}\right)\right)dx=$$

$$=\left(\sqrt{x^2-1}+\frac{x^2}{\sqrt{x^2-1}}+x-\sqrt{x^2-1}+\frac{x^2}{\sqrt{x^2-1}}-x\right)dx=\frac{2x^2dx}{\sqrt{x^2-1}}$$

1. $y = x \cdot \arcsin\left(\frac{1}{x}\right) + \ln\left|x + \sqrt{x^2 - 1}\right|, \ x > 0.$

2. $y = \operatorname{tg}\left(2\arccos\sqrt{1 - 2x^2}\right), \quad x > 0.$

3. $y = \sqrt{1 + 2x} - \ln\left|x + \sqrt{1 + 2x}\right|.$

4. $y = x^2 \cdot \operatorname{arctg}\left(\sqrt{x^2 - 1}\right) - \sqrt{x^2 - 1}.$

5. $y = \arccos\left(\frac{1}{\sqrt{1 + 2x^2}}\right), \ x > 0.$

6. $y = x \ln\left|x + \sqrt{x^2 + 3}\right| - \sqrt{x^2 + 3}.$

7. $y = \operatorname{arctg}(\operatorname{sh}x) + (\operatorname{sh}x)\ln(\operatorname{ch}x).$

8. $y = \arccos\left(\frac{x^2 - 1}{x^2\sqrt{2}}\right).$

9. $y = \ln\left(\cos^2 x + \sqrt{1 + \cos^4 x}\right).$

10. $y = \ln\left(x + \sqrt{1 + x^2}\right) - \sqrt{1 + x^2}\operatorname{arctg}x.$

11. $y = \frac{\ln|x|}{1 + x^2} - \frac{1}{2}\ln\frac{x^2}{1 + x^2}.$

12. $y = \ln\left(e^x + \sqrt{e^{2x} - 1}\right) + \arcsin e^x.$

13. $y = x\sqrt{4 - x^2} + a \cdot \arcsin \frac{x}{2}$

14. $y = \ln\left(\operatorname{tg} \frac{x}{2}\right) - \frac{x}{\sin x}.$

15. $y = 2x + \ln\left|\sin x + 2\cos x\right|.$

16. $y = \sqrt{\operatorname{ctg} x} - \frac{\sqrt{\operatorname{tg}^3 x}}{3}.$

17. $y = \ln\left|\frac{x + \sqrt{x^2 + 1}}{2x}\right|.$

18. $y = \sqrt[3]{\frac{x+2}{x-2}}.$

19. $y = \operatorname{arctg} \frac{x^2 - 1}{x}.$

20. $y = \ln\left|x^2 - 1\right| - \frac{1}{x^2 - 1}.$

21. $y = \operatorname{arctg}\left(\operatorname{tg} \frac{x}{2} + 1\right).$

22. $y = \ln\left|2x + 2\sqrt{x^2 + x} + 1\right|.$

23. $y = \ln\left|\cos \sqrt{x}\right| + \sqrt{x}\operatorname{tg}\sqrt{x}.$

24. $y = e^x(\cos 2x + 2\sin 2x).$

25. $y = x(\sin(\ln x) - \cos(\ln x))$

26. $y = \left(\sqrt{x-1} - \frac{1}{2}\right)e^{2\sqrt{x-1}}.$

27. $y = \cos x \cdot \ln(\mathrm{tg}x) - \ln\left(\mathrm{tg}\,\frac{x}{2}\right).$

28. $y = \sqrt{3+x^2} - x\ln\left|x+\sqrt{3+x^2}\right|.$

29. $y = \sqrt{x} - (1+x)\mathrm{arctg}\sqrt{x}$.

30. $y = x \cdot \mathrm{arctg}x - \ln\sqrt{1+x^2}.$

Javoblar. **3.1** $\arcsin\left(\frac{1}{x}\right)dx$; **3.2** $\frac{2\sqrt{2}}{(1-4x^2)^2 \cdot \sqrt{1-2x^2}}dx$; **3.3** $\frac{x-1}{\left(x+\sqrt{1+2x}\right)\sqrt{1+2x}}dx$;

3.4 $2x \cdot arctg\left(\sqrt{x^2-1}\right)dx$; **3.5** $\frac{\sqrt{2}\cdot x}{1+2x^2}dx$; **3.6** $\ln\left|x+\sqrt{x^2+3}\right|dx$;

3.7 $\mathrm{ch}x \cdot (1+\ln(\mathrm{ch}x))dx$; **3.8** $-\frac{2}{x\sqrt{x^4+2x^2-1}}dx$; **3.9** $-\frac{\sin 2x \cdot dx}{\sqrt{1+\cos^4 x}}$;

3.10 $-\frac{x}{\sqrt{1+x^2}} \cdot \mathrm{arctg}xdx$; **3.11** $-\frac{2x \cdot \ln|x|}{(1+x^2)^2}dx$; **3.12** $\left(\frac{e^x}{\sqrt{e^{2x}-1}} + \frac{e^x}{\sqrt{1-e^{2x}}}\right)dx$;

3.13 $\frac{-2x^2+4+2a}{\sqrt{4-x^2}}dx$; **3.14** $\frac{2ctgx}{\sin x}dx$; **3.15** $\frac{5\cos x}{\sin x + 2\cos x}dx$;

3.16 $-\frac{\sqrt{2}}{\cos x \cdot \sqrt{\sin^3 2x}}dx$; **3.17** $-\frac{dx}{x\left(x+\sqrt{x^2+1}\right)}$; **3.18** $\frac{4dx}{3(x-2)\sqrt[3]{(x+2)^2(x-2)}}$;

3.19 $\frac{x^2+1}{x^4-x^2+1}dx$; **3.20** $\frac{2x^3}{(x^2-1)^2}dx$; **3.21** $\frac{dx}{3+2\sin x+\cos x}$; **3.22** $\frac{dx}{\sqrt{x^2+x}}$; **3.23**

$\frac{dx}{2\cos^2\sqrt{x}}$; **3.24** $5e^x \cdot \cos 2xdx$; **3.25** $2\sin(\ln x)dx$; **3.26** $e^{2\sqrt{x-1}}dx$; **3.27**

$-\sin x \cdot \ln tgxdx$; **3.28** $-\ln\left|x+\sqrt{3+x^2}\right|dx$; **3.29** $-arctg\sqrt{x}dx$; **3.30** $arctgxdx$.

Funksiyaning differentsiali

Agar x argumentning ortirmasi $\Delta x = x - x_0$ absolyut qiymati kichik bo'lsa, u holda

$$f(x) = f(x_0 + \Delta x) \approx f(x_0) + f'(x_0) \cdot \Delta x$$

ko'rinishda yoziladi. Bu formuladan funksiyalarning qiymatlarini taqribiy hisoblashlarda foydalaniladi.

4–masala. Differensial yordamida taqribiy hisoblang.

$$y = \frac{1}{\sqrt{2x+1}}, \quad x = 1{,}58.$$

Echim:

Agar x argumentning ortirmasi $\Delta x = x - x_0$ absolyut qiymati kichik bo'lsa, u holda

$$f(x) = f(x_0 + \Delta x) \approx f(x_0) + f'(x_0) \cdot \Delta x$$

$x_0 = 1{,}5$ ni deb olamiz

U holda:

$$\Delta x = 0{,}08$$

Hisoblaymiz:

$$y(1{,}5) = \frac{1}{\sqrt{2 \cdot 1{,}5 + 1}} = \frac{1}{\sqrt{4}} = \frac{1}{2}.$$

$$y' = \left(\frac{1}{\sqrt{2x+1}}\right)' = \left((2x+1)^{-\frac{1}{2}}\right)' = -\frac{1}{2} \cdot (2x+1)^{-\frac{3}{2}} \cdot 2 = -\frac{1}{\sqrt{(2x+1)^3}}.$$

$$y'(1{,}5) = -\frac{1}{\sqrt{(2 \cdot 1{,}5 + 1)^3}} = -\frac{1}{\sqrt{4^3}} = -\frac{1}{8}.$$

Natijada:

$$y(1{,}58) \approx y(1{,}5) + y'(1{,}5)\cdot 0{,}08 = \frac{1}{2} - \frac{1}{8}\cdot 0{,}08 = 0{,}5 - 0{,}01 = 0{,}49.$$

1. $y = \sqrt[3]{x},\ x = 7{,}76.$ **2.** $y = \sqrt[3]{x^3 + 7x},\ x = 1{,}012.$

3. $y = \dfrac{x + \sqrt{5 - x^2}}{2},\ x = 0{,}98.$ **4.** $y = \sqrt[3]{x},\ x = 27{,}54.$

5. $y = \arcsin x,,\ x = 0{,}08.$ **6.** $y = \sqrt[3]{x^2 + 7x + 5},\ x = 0{,}97.$

7. $y = \sqrt[3]{x},\ x = 26{,}46.$ **8.** $y = \sqrt[3]{x^2 + x + 3},\ x = 1{,}97.$

9. $y = x^{11},\ x = 1{,}021.$ **10.** $y = \sqrt[3]{x},\ x = 1{,}21.$

11. $y = x^{21},\ x = 0{,}998.$ **12.** $y = \sqrt[3]{x^2},\ x = 1{,}03.$

13. $y = x^6,\ x = 2{,}01.$ **14.** $y = \sqrt[3]{x},\ x = 8{,}24.$

15. $y = x^7,\ x = 1{,}996.$ **16.** $y = \sqrt[3]{x},\ x = 7{,}64.$

17. $y = \sqrt{4x - 1},\ x = 2{,}56.$ **18.** $y = \dfrac{1}{\sqrt{2x^2 + x + 1}},\ x = 1{,}016.$

19. $y = \sqrt[3]{x},\ x = 8{,}36.$ **20.** $y = \dfrac{1}{\sqrt{x}},\ x = 4{,}16.$

21. $y = x^7,\ x = 2{,}002.$ **22.** $y = \sqrt{4x - 3},\ x = 1{,}78.$

23. $y = \sqrt{x^3},\ x = 0{,}98.$ **24.** $y = x^5,\ x = 2{,}997.$

25. $y = \sqrt[5]{x^2},\ x = 1{,}03.$ **26.** $y = x^4,\ x = 3{,}998.$

27. $y = \sqrt{1 + x + \sin x},\ x = 0{,}01.$ **28.** $y = \sqrt[3]{3x + \cos x},\ x = 0{,}01.$

29. $y = \sqrt[4]{2x - \sin\frac{\pi x}{2}},\ x = 1{,}02.$ **30.** $y = \sqrt{x^2 + 5},\ x = 1{,}97.$

Javoblar. 4.1 $1{,}98$; **4.2** $2{,}01$; **4.3** $1{,}495$; **4.4** $3{,}02$; **4.5** $0{,}08$; **4.6** $1{,}99$; **4.7** $2{,}98$; **4.8** $2{,}975$; **4.9** $1{,}231$; **4.10** $1{,}07$; **4.11** $0{,}958$; **4.12** $1{,}02$; **4.13** $65{,}92$; **4.14** $2{,}02$; **4.15** $126{,}208$; **4.16** $1{,}97$; **4.17** $3{,}04$; **4.18** $0{,}495$; **4.19** $2{,}03$; **4.20** $0{,}49$; **4.21** $128{,}896$; **4.22** $2{,}03$; **4.23** $0{,}98$; **4.24** $241{,}785$; **4.25** $1{,}012$; **4.26** $255{,}488$; **4.27** $1{,}01$; **4.28** $1{,}01$; **4.29** $1{,}01$;

4.30 2,98.

Differensiallashning asosiy qoidalari

Funksiya hosilasini topish amaliga funksiyani differensiallash deyiladi. Differensiallashning asosiy qoidalari:

Agar $u(x),\ v(x)$ differensiallanuvchi funksiyalar bo'lib, $C-const$, ya'ni o'zgarmas bo'lsin.

1. $\left[u(x)\pm v(x)\right]'=u'(x)\pm v'(x).$

2. $\left[Cu(x)\right]'=Cu'(x).$

3. $\left[u(x)\cdot v(x)\right]'=u'(x)\cdot v(x)+u(x)\cdot v'(x).$

4. $\left[\frac{u(x)}{v(x)}\right]'=\frac{u'(x)\cdot v(x)-u(x)\cdot v'(x)}{v^2(x)}$, bu yerda $v(x)\neq 0.$

5–masala. Funksiyaning hosilasini toping.

$$y'=\left(\frac{3x^6+4x^4-x^2-2}{15\sqrt{1+x^2}}\right)'=$$

$$=\frac{\left(3x^6+4x^4-x^2-2\right)'\sqrt{1+x^2}-\left(3x^6+4x^4-x^2-2\right)\left(\sqrt{1+x^2}\right)'}{15(1+x^2)}=$$

$$=\frac{\left(18x^5+16x^3-2x\right)\sqrt{1+x^2}-\left(3x^6+4x^4-x^2-2\right)\cdot\frac{1}{\sqrt{1+x^2}}\cdot 2x}{15(1+x^2)}=$$

$$= \frac{\left(18x^5+16x^3-2x\right)\sqrt{1+x^2}-\left(3x^6+4x^4-x^2-2\right)\cdot\frac{1}{\sqrt{1+x^2}}\cdot 2x}{15(1+x^2)} =$$

$$= \frac{x\left(18x^4+16x^2-2\right)(1+x^2)-\left(3x^6+4x^4-x^2-2\right)\cdot x}{15\sqrt{1+x^2}(1+x^2)} =$$

$$= \frac{x\left(18x^4+16x^2-2+18x^6+16x^4-2x^2-3x^6-4x^4+x^2+2\right)}{15\sqrt{1+x^2}(1+x^2)} =$$

$$= \frac{x\left(15x^6+30x^4+15x^2\right)}{15\sqrt{1+x^2}(1+x^2)} = \frac{15x^3\left(x^4+2x^2+1\right)}{15\sqrt{1+x^2}(1+x^2)} = \frac{x^3\left(x^4+2x^2+1\right)}{\sqrt{1+x^2}(1+x^2)}.$$

1. $y=\dfrac{2(3x^3+4x^2-x-2)}{15\sqrt{1+x}}$.

2. $y=\dfrac{(2x^2-1)\sqrt{1+x^2}}{3x^3}$.

3. $y=\dfrac{x^4-8x^2}{2(x^2-4)}$.

4. $y=\dfrac{2x^2-x-1}{3\sqrt{2+4x}}$.

5. $y=\dfrac{(1+x^8)\sqrt{1+x^8}}{12x^{12}}$.

6. $y=\dfrac{x^2}{2\sqrt{1-3x^4}}$.

7. $y=\dfrac{(x^2-6)\sqrt{(4+x^2)^3}}{120x^5}$.

8. $y=\dfrac{(x^2-8)\sqrt{x^2-8}}{6x^3}$.

9. $y=\dfrac{4+3x^3}{x\cdot\sqrt[3]{(2+x^3)^2}}$.

10. $y=\sqrt[3]{\dfrac{(1+x^{3/4})^2}{x^{3/2}}}$.

11. $y=\dfrac{x^6+x^3-2}{\sqrt{1-x^3}}$.

12. $y=\dfrac{(x^2-2)\sqrt{4+x^2}}{24x^3}$.

13. $y=\dfrac{1+x^2}{2\sqrt{1+2x^2}}$.

14. $y=\dfrac{\sqrt{x-1}\cdot(3x+2)}{4x^2}$.

15. $y=\dfrac{\sqrt{\left(1+x^2\right)^3}}{3x^3}$.

16. $y=\dfrac{x^6+8x^3-128}{\sqrt{8-x^3}}$.

17. $y=\dfrac{\sqrt{2x+3}\cdot(x-2)}{x^2}$.

18. $y=(1-x^2)\sqrt[5]{x^3+\frac{1}{x}}$.

19. $y=\dfrac{(2x^2+3)\sqrt{x^2-3}}{9x^3}$.

20. $y=\dfrac{x-1}{(x^2+5)\sqrt{x^2+5}}$.

21. $y=\dfrac{(2x+1)\sqrt{x^2-x}}{x^2}$.

22. $y=2\cdot\sqrt{\dfrac{1-\sqrt{x}}{1+\sqrt{x}}}$.

23. $y=\dfrac{1}{(x+2)\sqrt{x^2+4x+5}}$.

24. $y=3\cdot\dfrac{\sqrt[3]{x^2+x+1}}{x+1}$.

25. $y=3\cdot\sqrt[3]{\dfrac{x+1}{(x-1)^2}}$.

26. $y=\dfrac{x+7}{6\sqrt{x^2+2x+7}}$.

27. $y=\dfrac{x\sqrt{x+1}}{x^2+x+1}$.

28. $y=\dfrac{x^2+2}{2\sqrt{1-x^4}}$.

29. $y=\dfrac{(x+3)\sqrt{2x-1}}{2x+7}$.

30. $y=\dfrac{3x+\sqrt{x}}{\sqrt{x^2+2}}$.

Javoblar. 5.1 $x\sqrt{1+x}$; **5.2** $\dfrac{1}{x^4\sqrt{1+x^2}}$; **5.3** $x+\dfrac{16}{(x^2-4)^2}$; **5.4** $\dfrac{x}{\sqrt{2+4x}}$; **5.5** $-\dfrac{\sqrt{1+x^8}}{x^{13}}$;

5.6 $\dfrac{x}{(1-3x^4)\sqrt{1-3x^4}}$; **5.7** $\dfrac{\sqrt{4+x^2}}{x^6}$; **5.8** $\dfrac{4\sqrt{x^2-8}}{x^4}$; **5.9** $\dfrac{-8}{x^2(2+x^3)\sqrt[3]{(2+x^3)^2}}$;

5.10 $-\dfrac{1}{2x\cdot\sqrt{x}\cdot\sqrt[3]{1+x^{3/4}}}$; **5.11** $\dfrac{9x^5}{2\sqrt{1-x^3}}$; **5.12** $\dfrac{-x^4+x^3+2x^2-2x+24}{24x^4\sqrt{4+x^2}}$;

5.13 $\dfrac{x^3}{(1+2x^2)\sqrt{1+2x^2}}$; **5.14** $\dfrac{-3x^2+8}{8x^3\sqrt{x-1}}$; **5.15** $-\dfrac{\sqrt{1+x^2}}{x^2}$; **5.16** $\dfrac{9x^5}{2\sqrt{8-x^3}}$;

5.17 $\dfrac{-x^2+3x+12}{x^3\sqrt{2x+3}}$; **5.18** $\dfrac{1}{5\sqrt[5]{\left(x^3+\frac{1}{x}\right)^4}}\cdot\left(-13x^4+3x^2-9-\dfrac{1}{x^2}\right)$; **5.19** $\dfrac{3}{x^4\sqrt{x^2-3}}$;

5.20 $\dfrac{-2x^2+3x+5}{\sqrt{\left(x^2+5\right)^5}}$; **5.21** $\dfrac{3}{x^2\sqrt{x^2-3}}$; **5.22** $-\dfrac{1}{\sqrt{x(1-x)}\cdot(1+\sqrt{x})}$;

5.23 $-\dfrac{2x^2+8x+9}{(x+2)^2\cdot\sqrt{(x^2+4x+5)^3}}$; **5.24** $-\dfrac{x^2+2}{\sqrt[3]{(x^2+x+1)^2}\cdot(x+1)^2}$;

5.25 $-\sqrt[3]{\dfrac{x-1}{(x+1)^2}}\cdot\dfrac{x+3}{(x-1)^2}$; **5.26** $-\dfrac{x}{(x^2+2x+7)\cdot\sqrt{x^2+2x+7}}$;

5.27 $\dfrac{-x^3-x^2+3x+2}{2\sqrt{x+1}\cdot(x^2+x+1)^2}$; **5.28** $-\dfrac{2x^3+x}{(1-x^4)\cdot\sqrt{1-x^4}}$; **5.29** $\dfrac{2x^2+15x+20}{(2x+7)^2\cdot\sqrt{2x-1}}$;

5.30 $\dfrac{12\sqrt{x}+2-x^2}{2\sqrt{x}(x^2+2)\sqrt{x^2+2}}$.

Murakkab funksiyaning hosilasi

Agar $y=f(x)$ bo'lib, $u=\varphi(x)$ bo'lsa, ya'ni y funksiya x argument bilan oraliq argument orqali bog'langan bo'lsa, y ni x ning murakkab funksiyasi deyiladi.

Murakkab funksiyaning hosilasi, uning oraliq argument bo'yicha hosilasini oraliq argumentning erkli argument bo'yicha hosilasiga ko'paytmasiga teng, ya'ni:

$$\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx} \quad \text{yoki} \quad y'=f'(u)\cdot u'(x).$$

6–masala. Funksiyaning hosilasini toping.

$$y=\frac{e^{x^2}}{1+x^2}.$$

$$y'=\left(\frac{e^{x^2}}{1+x^2}\right)'=\frac{\left(e^{x^2}\right)'\left(1+x^2\right)-e^{x^2}\left(1+x^2\right)'}{\left(1+x^2\right)^2}=$$

$$=\frac{e^{x^2}\cdot 2x\cdot\left(1+x^2\right)-e^{x^2}\cdot 2x}{\left(1+x^2\right)^2}=\frac{2x^3\cdot e^{x^2}}{\left(1+x^2\right)^2}.$$

1. $y=x-\ln\left(2+e^x+2\sqrt{e^{2x}+e^x+1}\right)$.

2. $y = \dfrac{e^{2x}\left(2 - \sin 2x - \cos 2x\right)}{8}.$

3. $y = \dfrac{1}{2} \cdot \operatorname{arctg} \dfrac{e^x - 3}{2}.$

4. $y = \dfrac{1}{\ln 4} \ln \dfrac{1 + 2^x}{1 - 2^x}.$

5. $y = 2\sqrt{e^x + 1} + \ln \dfrac{\sqrt{e^x + 1} - 1}{\sqrt{e^x + 1} + 1}.$

6. $y = \dfrac{2}{3} \sqrt{(arctge^x)^3}.$

7. $y = \dfrac{1}{2} \cdot \ln\left(e^{2x} + 1\right) - 2\operatorname{arctg} e^x.$

8. $y = \ln\left(e^x + 1\right) + \dfrac{18e^{2x} + 27e^x + 11}{6(e^x + 1)^3}.$

9. $y = \dfrac{2\left(\sqrt{2^x - 1} - \operatorname{arctg}\sqrt{2^x - 1}\right)}{\ln 2}.$

10. $y = 2(x - 2)\sqrt{1 + e^x} - 2 \ln \dfrac{\sqrt{1 + e^x} - 1}{\sqrt{1 + e^x} + 1}.$

11. $y = \dfrac{e^{\alpha x}\left(\alpha \cdot \sin \beta x - \beta \cdot \cos \beta x\right)}{\alpha^2 + \beta^2}.$

12. $y = \dfrac{e^{\alpha x}\left(\beta \cdot \sin \beta x - \alpha \cdot \cos \beta x\right)}{\alpha^2 + \beta^2}.$

13. $y = e^{ax} \dfrac{1}{2a} + \left(\dfrac{a \cdot \cos 2bx + 2b \cdot \sin 2bx}{2(a^2 + 4b^2)} \right)$

14. $y = x + \dfrac{1}{1 + e^x} - \ln(1 + e^x).$

15. $y = x - 3\ln\left((1 + e^{\frac{x}{6}})\sqrt{1 + e^{\frac{x}{6}}} \right) - 3\operatorname{arctg} e^{\frac{x}{6}}.$

16. $y = x + \dfrac{8}{1+e^{\frac{x}{4}}}$.

17. $y = \ln\left(e^x + \sqrt{e^{2x}-1}\right) + \arcsin e^{-x}$.

18. $y = x - e^{-x}\arcsin e^x - \ln\left(1+\sqrt{1-e^{2x}}\right)$.

19. $y = x - \ln(1+e^x) - 2e^{-\frac{x}{2}} \cdot \operatorname{arctg} e^{\frac{x}{2}} - \left(\operatorname{arctg} e^{-\frac{x}{2}}\right)^2$.

20. $y = \dfrac{e^{x^3}}{1+x^3}$.

21. $y = \dfrac{1}{m\sqrt{ab}}\operatorname{arctg}\left(e^{mx}\cdot\sqrt{\dfrac{a}{b}}\right)$.

22. $y = 3e^{\sqrt[3]{x}}\left(\sqrt[3]{x^2} - 2\sqrt[3]{x} + 2\right)$.

23. $y = \ln\dfrac{\sqrt{1+e^x+e^{2x}} - e^x - 1}{\sqrt{1+e^x+e^{2x}} - e^x + 1}$.

24. $y = e^{\sin x}\left(x - \dfrac{1}{\cos x}\right)$.

25. $y = \dfrac{e^x}{2}\left(\left(x^2-1\right)\cos x + (x-1)^2 \sin x\right)$.

26. $y = \operatorname{arctg}(e^x - e^{-x})$.

27. $y = 3e^{\sqrt[3]{x}}\left(\sqrt[3]{x^5} - 5\sqrt[3]{x^4} + 20x - 60\sqrt[3]{x^2} + 120\sqrt[3]{x} - 120\right)$.

28. $y = -\dfrac{e^{3x}}{3\operatorname{sh}^3 x}$.

29. $y = \arcsin e^{-x} - \sqrt{1-e^{2x}}$.

30. $y = -\dfrac{1}{2}e^{-x^2}\left(x^4 + 2x^2 + 2\right)$.

Javoblar. 6.1 $\dfrac{1}{\sqrt{e^{2x}+e^x+1}}$; **6.2** $e^{2x}\cdot\sin^2 x$; **6.3** $\dfrac{e^x}{e^{2x}-6e^x+13}$; **6.4** $\dfrac{2^x}{1-2^{2x}}$;

6.5 $\sqrt{e^x+1}$; **6.6** $\dfrac{e^x\cdot\sqrt{arctge^x}}{1+e^{2x}}$; **6.7** $\dfrac{e^{2x}-e^x}{1+e^{2x}}$; **6.8** $\dfrac{e^{4x}}{(e^x+1)^4}$; **6.9** $\sqrt{2^x-1}$;

6.10 $\dfrac{2e^x+x\cdot e^{2x}-2}{e^x\sqrt{1+e^x}}$; **6.11** $e^{\alpha x}\cdot\sin\beta x$;

6.12 $\dfrac{e^{\alpha x}(\beta^2\cdot\cos\beta x+2\alpha\beta\sin 3x-\alpha^2\cdot\cos\beta x)}{\alpha^2+\beta^2}$; **6.13** $e^{ax}\cdot\cos^2 bx$; **6.14** $\dfrac{1}{(1+e^x)^2}$;

6.15 $\dfrac{1-e^{x/2}-e^{x/3}-e^{x/6}}{2(1+e^{x/3})(1+e^{x/6})}$; **6.16** $\dfrac{1+e^{x/2}}{\left(1+e^{x/4}\right)^2}$; **6.17** $\sqrt{\dfrac{e^x-1}{e^x+1}}$; **6.18** $e^{-x}\arcsin e^x$;

6.19 $\dfrac{arctge^{x/2}}{e^{x/2}\cdot(1+e^x)}$; **6.20** $\dfrac{3x^5\cdot e^{x^3}}{(1+x^3)^2}$; **6.21** $\dfrac{e^{mx}}{b+a\cdot e^{2mx}}$; **6.22** $e^{\sqrt[3]{x}}$;

6.23 $\dfrac{1}{\sqrt{1+e^x+e^{2x}}}$; **6.24** $e^{\sin x}\cdot\left(x\cdot\cos x-\dfrac{\sin x}{\cos^2 x}\right)$; **6.25** $x^2\cdot e^x\cdot\cos x$;

6.26 $\dfrac{e^{3x}+e^x}{e^{4x}-e^{2x}+1}$; **6.27** $x\cdot e^{\sqrt[3]{x}}$; **6.28** $\dfrac{e^{3x}\cdot(\mathrm{ch}x-\mathrm{sh}x)}{\mathrm{sh}^4 x}$; **6.29** $\dfrac{e^x\sqrt{e^{2x}-1}-\sqrt{e^{-2x}-1}}{\sqrt{1-e^{-2x}}\cdot\sqrt{1-e^{2x}}}$;

6.30 $x^5\cdot e^{-x^2}$.

Funksiyaning hosilasi

7–masala. Funksiyaning hosilasini toping.

$$y=\ln\ln^2\ln^3 x.$$

$$y'=\left(\ln\ln^2\ln^3 x\right)'=\frac{1}{\ln^2\ln^3 x}\cdot 2\ln\ln^3 x\cdot\frac{1}{\ln^3 x}\cdot 3\ln^2 x\cdot\frac{1}{x}=$$

$$=\frac{2}{\ln\ln^3 x}\cdot\frac{3}{\ln x}\cdot\frac{1}{x}=\frac{6}{x\cdot\ln x\cdot\ln\ln^3 x}.$$

1. $y=\sqrt{x}\ln\left(\sqrt{x}+\sqrt{x+a}\right)-\sqrt{x+a}$. **2.** $y=\ln\left(x+\sqrt{a^2+x^2}\right)$.

3. $y=2\sqrt{x}-4\ln\left(2+\sqrt{x}\right)$. **4.** $y=\ln\dfrac{x^2}{\sqrt{1-ax^4}}$.

5. $y=\ln\left(\sqrt{x}+\sqrt{x+1}\right)$. **6.** $y=\ln\dfrac{a^2+x^2}{a^2-x^2}$.

7. $y=\ln^2(x+\cos x).$

8. $y=\ln^3(x+\cos x).$

9. $y=\ln\dfrac{x^2}{1-x^2}.$

10. $y=\ln \operatorname{tg}\left(\frac{\pi}{4}+\frac{x}{2}\right).$

11. $y=\ln\sqrt[4]{\dfrac{1+2x}{1-2x}}.$

12. $y=x+\dfrac{1}{\sqrt{2}}\ln\dfrac{x-\sqrt{2}}{x+\sqrt{2}}+a^{\pi^{\sqrt{2}}}.$

13. $y=\ln\left(\sin\dfrac{2x+4}{x+1}\right).$

14. $y=\log_{16}\log_5 \operatorname{tg}x.$

15. $y=\log_4\log_2 \operatorname{tg}x.$

16. $y=\dfrac{x\left(\cos(\ln x)+\sin(\ln x)\right)}{2}.$

17. $y=\ln\cos\dfrac{2x+3}{x+1}.$

18. $y=\lg\ln(\operatorname{ctg}x).$

19. $y=\log_a\left(\dfrac{1}{\sqrt{1-x^4}}\right).$

20. $y=\dfrac{1}{\sqrt{2}}\ln\left(\sqrt{2}\operatorname{tg}x+\sqrt{1+2\operatorname{tg}^2 x}\right).$

21. $y=\ln\left(\arcsin\sqrt{1-e^{2x}}\right).$

22. $y=\ln\arccos\sqrt{1-e^{4x}}.$

23. $y=\ln\left(bx+\sqrt{a^2+b^2x^2}\right).$

24. $y=\ln\dfrac{\sqrt{x^2+1}+x\sqrt{2}}{\sqrt{x^2+1}-x\sqrt{2}}.$

25. $y=\ln\left(\arccos\dfrac{1}{\sqrt{x}}\right).$

26. $y=\ln\left(e^x+\sqrt{1+e^{2x}}\right).$

27. $y=\ln\dfrac{\sqrt{5}+\operatorname{tg}\frac{x}{2}}{\sqrt{5}-\operatorname{tg}\frac{x}{2}}.$

28. $y=\ln\left(\dfrac{\ln x}{\sin\frac{1}{x}}\right).$

29. $y=\ln\ln\sin\left(1+\frac{1}{x}\right).$

30. $y=\ln\ln^3\ln^2 x.$

Javoblar. 7.1 $\dfrac{1}{2\sqrt{x}}\cdot\ln\left(\sqrt{x}+\sqrt{x+a}\right)$; **7.2** $\dfrac{1}{\sqrt{a^2+x^2}}$; **7.3** $\dfrac{\sqrt{x}}{\sqrt{x}\cdot(2+\sqrt{x})}$; **7.4** $\dfrac{2}{x-ax^5}$;

7.5 $\dfrac{1}{2\sqrt{x^2+x}}$; **7.6** $\dfrac{4a^2x}{a^4-x^4}$; **7.7** $\dfrac{1-\sin x}{x+\cos x}\cdot 2\ln(x+\cos x)$;

7.8 $-\dfrac{3\cdot\sin x\cdot\ln^2(1+\cos x)}{1+\cos x}$; **7.9** $\dfrac{2}{x\cdot(1-x^2)}$; **7.10** $\dfrac{1}{\cos x}$; **7.11** $\dfrac{1}{1-4x^2}$; **7.12** $\dfrac{x^2}{x^2-2}$;

7.13 $-\frac{2}{(x+1)^2}\cdot ctg\,\frac{2x+4}{x+1}$; **7.14** $\frac{1}{\sin 2x\cdot\ln 4\cdot\ln tgx}$; **7.15** $\frac{1}{\sin 2x\cdot\ln 2\cdot\ln tgx}$;

7.16 $\cos(\ln x)$; **7.17** $\frac{tg\,\frac{2x+3}{x+1}}{(x+1)^2}$; **7.18** $-\frac{2}{\ln(ctgx)\cdot\ln 10\cdot\sin 2x}$; **7.19** $\frac{2x^3}{\ln a\cdot(1-x^4)}$;

7.20 $\frac{1}{\cos^2 x\sqrt{1+2tg^2 x}}$; **7.21** $-\frac{e^x}{\sqrt{1-e^{2x}}\cdot\arcsin\sqrt{1-e^{2x}}}$;

7.22 $\frac{2e^{2x}}{\sqrt{1-e^{4x}}\cdot\arccos\sqrt{1-e^{4x}}}$; **7.23** $\frac{b}{\sqrt{a^2+b^2x^2}}$; **7.24** $\frac{2\sqrt{2}}{(1-x^2)\sqrt{x^2+x}}$;

7.25 $\frac{1}{2x\cdot\sqrt{x-1}\cdot\arccos\frac{1}{\sqrt{x}}}$; **7.26** $\frac{e^x}{\sqrt{1+e^{2x}}}$; **7.27** $\frac{\sqrt{5}}{6\cos^2\frac{x}{2}-1}$;

7.28 $\frac{\sin\frac{1}{x}-x\cdot\ln x\cdot\cos\frac{1}{x}}{x\cdot\ln x\cdot\sin\frac{1}{x}}$; **7.29** $-\frac{ctg\left(1+\frac{1}{x}\right)}{x^2\cdot\ln\sin\left(1+\frac{1}{x}\right)}$; **7.30** $\frac{6}{x\cdot\ln x\cdot\ln\ln^2 x}$.

Funksiyaning hosilasi

8–masala. Funksiyaning hosilasini toping.

$$y=tg\sqrt{\cos\left(\frac{1}{3}\right)}+\frac{\sin^2 31x}{31\cos 62x}.$$

$$y'=\left(tg\sqrt{\cos\left(\frac{1}{3}\right)}+\frac{\sin^2 31x}{31\cos 62x}\right)'=\left(\frac{\sin^2 31x}{31\cos 62x}\right)'=$$

$$=\left(\frac{2\sin^2 31x}{62\cos 62x}\right)'=\left(\frac{1-\cos 62x}{62\cos 62x}\right)'=\left(\frac{1}{62\cos 62x}-\frac{1}{62}\right)'=$$

$$=\left(\frac{1}{62\cos 62x}\right)'=-\frac{1}{62\cos^2 62x}\cdot(-\sin 62x)\cdot 62=\frac{\mathrm{tg}62x}{\cos 62x}.$$

1. $y=\sin\sqrt{3}+\frac{1}{3}\cdot\frac{\sin^2 3x}{\cos 6x}$.
2. $y=\cos\ln 2-\frac{1}{3}\cdot\frac{\cos^2 3x}{\sin 6x}$.
3. $y=\mathrm{tg}\,\lg\frac{1}{3}+\frac{1}{4}\cdot\frac{\sin^2 4x}{\cos 8x}$.
4. $y=\frac{\cos(\sin 5)\cdot\sin^2 2x}{2\cos 4x}$.

5. $y=\operatorname{ctg}\sqrt[3]{5}-\frac{1}{8}\cdot\frac{\cos^2 4x}{\sin 8x}$.

6. $y=\frac{\sin(\cos 3)\cdot\cos^2 2x}{4\sin 4x}$.

7. $y=\frac{\cos\ln 7\cdot\sin^2 7x}{7\cos 14x}$.

8. $y=\cos(\operatorname{ctg}2)-\frac{1}{16}\cdot\frac{\cos^2 8x}{\sin 16x}$.

9. $y=\operatorname{ctg}(\cos 2)+\frac{1}{6}\cdot\frac{\sin^2 6x}{\cos 12x}$.

10. $y=\sqrt[3]{\operatorname{ctg}2}-\frac{1}{20}\cdot\frac{\cos^2 10x}{\sin 20x}$.

11. $y=\frac{1}{3}\cdot\cos\left(\operatorname{tg}\frac{1}{2}\right)+\frac{1}{10}\cdot\frac{\sin^2 10x}{\cos 20x}$.

12. $y=\ln\sin\frac{1}{2}-\frac{1}{24}\cdot\frac{\cos^2 12x}{\sin 24x}$.

13. $y=8\sin(\operatorname{ctg}3)+\frac{1}{5}\cdot\frac{\sin^2 5x}{\cos 10x}$.

14. $y=\frac{\sin(\operatorname{ctg}3)\cdot\cos^2 14x}{28\sin 28x}$.

15. $y=\frac{\cos\left(\operatorname{tg}\frac{1}{3}\right)\cdot\sin^2 15x}{15\cos 30x}$.

16. $y=\frac{\sin\left(\operatorname{tg}\frac{1}{7}\right)\cdot\cos^2 16x}{32\sin 32x}$.

17. $y=\frac{\operatorname{ctg}\left(\sin\frac{1}{3}\right)\cdot\sin^2 17x}{17\cos 34x}$.

18. $y=\frac{\sqrt[5]{\operatorname{ctg}2}\cdot\cos^2 18x}{36\sin 36x}$.

19. $y=\frac{\operatorname{tg}(\ln 2)\cdot\sin^2 19x}{19\cos 38x}$.

20. $y=\operatorname{ctg}(\cos 5)-\frac{1}{40}\cdot\frac{\cos^2 20x}{\sin 40x}$.

21. $y=\sqrt{\operatorname{tg}4}+\frac{\sin^2 21x}{21\cos 42x}$.

22. $y=\cos(\ln 13)-\frac{1}{44}\cdot\frac{\cos^2 22x}{\sin 44x}$.

23. $y=\ln\cos\frac{1}{3}+\frac{\sin^2 23x}{23\cos 46x}$.

24. $y=\operatorname{ctg}\left(\sin\frac{1}{13}\right)-\frac{1}{48}\cdot\frac{\cos^2 24x}{\sin 48x}$.

25. $y=\sin(\ln 2)+\frac{\sin^2 25x}{25\cos 50x}$.

26. $y=\sqrt[3]{\cos\sqrt{2}}-\frac{1}{52}\cdot\frac{\cos^2 26x}{\sin 52x}$.

27. $y=\sqrt[7]{\operatorname{tg}(\cos 2)}+\frac{\sin^2 27x}{27\cos 54x}$.

28. $y=\sin\sqrt[3]{\operatorname{tg}2}-\frac{\cos^2 28x}{56\sin 56x}$.

29. $y=\cos^2(\sin 3)+\frac{\sin^2 29x}{29\cos 58x}$.

30. $y=\sin^3(\cos 2)-\frac{\cos^2 30x}{60\sin 60x}$.

Javoblar. 8.1 $\frac{tg6x}{\cos 6x}$; **8.2** $\frac{1}{2\sin^2 3x}$; **8.3** $\frac{tg8x}{\cos 8x}$; **8.4** $\frac{\cos(\sin 5)\cdot tg4x}{\cos 4x}$; **8.5** $\frac{1}{4\sin^2 4x}$;

8.6 $-\frac{\sin(\cos 3)}{4\sin^2 2x}$; **8.7** $\frac{\cos\ln 7\cdot tg14x}{\cos 14x}$; **8.8** $\frac{1}{4\sin^2 8x}$; **8.9** $\frac{tg12x}{\cos 12x}$; **8.10** $\frac{1}{4\sin^2 10x}$;

8.11 $\frac{tg20x}{\cos 20x}$; **8.12** $\frac{1}{4\sin^2 12x}$; **8.13** $\frac{tg10x}{\cos 10x}$; **8.14** $-\frac{\cos(ctg3)}{4\sin^2 14x}$; **8.15** $\frac{\cos\left(tg\frac{1}{3}\right)\cdot tg30x}{\cos 30x}$;

8.16 $-\frac{\sin\left(tg\frac{1}{7}\right)}{4\sin^2 16x}$; **8.17** $\frac{ctg\left(\sin\frac{1}{3}\right)\cdot tg34x}{\cos 34x}$; **8.18** $-\frac{\sqrt[5]{ctg2}}{4\sin^2 18x}$; **8.19** $\frac{tg(\ln 2)\cdot tg38x}{\cos 38x}$;

8.20 $\frac{1}{4\sin^2 20x}$; **8.21** $\frac{tg42x}{\cos 42x}$; **8.22** $\frac{1}{4\sin^2 22x}$; **8.23** $\frac{tg46x}{\cos 46x}$; **8.24** $\frac{1}{4\sin^2 24x}$;

8.25 $\frac{\sin 50x}{\cos^2 50x}$; **8.26** $\frac{1}{2\sin^2 26x}$; **8.27** $tg54x\cdot\sec 54x$; **8.28** $\frac{1}{4\sin^2 28x}$; **8.29** $\frac{tg58x}{\cos 58x}$;

8.30 $\frac{1}{4\sin^2 30x}$.

Funksiyaning hosilasi

9–masala. Funksiyaning hosilasini toping.

$$y'=\left(\operatorname{arctg}\frac{\operatorname{tg}\frac{x}{2}+1}{2}\right)'=\frac{1}{1+\left(\frac{\operatorname{tg}\frac{x}{2}+1}{2}\right)^2}\cdot\left(\frac{\operatorname{tg}\frac{x}{2}+1}{2}\right)'=$$

$$=\frac{1}{\operatorname{tg}^2\frac{x}{2}+2\operatorname{tg}\frac{x}{2}+5}\cdot\frac{1}{2}\cdot\frac{1}{\cos^2\frac{x}{2}}\cdot\frac{1}{2}=$$

$$=\frac{1}{\sin^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}+5\cos^2\frac{x}{2}}=\frac{1}{1+\sin x+4\cos^2\frac{x}{2}}=$$

$$=\frac{1}{\sin x+2\cos x+2}=\frac{1}{\sin x+2\cos x+3}.$$

1. $y=\operatorname{arctg}\frac{\operatorname{tg}x-\operatorname{ctg}x}{\sqrt{2}}$.
2. $y=\arcsin\frac{\sqrt{x}-2}{\sqrt{5x}}$.

3. $y = \frac{2x-1}{4} \cdot \sqrt{2+x-x^2} + \frac{9}{8} \cdot \arcsin \frac{2x-1}{3}.$

4. $y = arctg \frac{\sqrt{1+x^2}-1}{x}.$

5. $y = \arccos \frac{x^2-4}{\sqrt{x^2+16}}.$

6. $y = \sqrt{\frac{2}{3}} \cdot \operatorname{arctg} \frac{3x-1}{\sqrt{6x}}.$

7. $y = \frac{1}{4} \cdot \ln \frac{x-1}{x+1} - \frac{1}{2} \operatorname{arctg} x.$

8. $y = \frac{1}{2} \cdot (x-4)\sqrt{8x-x^2-7} - 9 \arccos \sqrt{\frac{x-1}{6}}.$

9. $y = \frac{(1+x)\operatorname{arctg}\sqrt{x}}{x^2} + \frac{1}{3x\sqrt{x}}.$

10. $y = \frac{x^3}{3} \cdot \arccos x - \frac{2+x^2}{9} \cdot \sqrt{1-x^2}.$

11. $y = \frac{1}{2\sqrt{x}} + \frac{1+x}{2x} \cdot \operatorname{arctg}\sqrt{x}.$

12. $y = \frac{3+x}{2} \cdot \sqrt{x(2-x)} + 3 \arccos \sqrt{\frac{x}{2}}.$

13. $y = \frac{4+x^4}{x^3} \cdot \operatorname{arctg} \frac{x^2}{2} + \frac{4}{x}.$

14. $y = \arcsin \sqrt{\frac{x}{x+1}} + \operatorname{arctg}\sqrt{x}.$

15. $y = \frac{1}{2} \cdot \sqrt{\frac{1}{x^2}-1} - \frac{\arccos x}{2x^2}.$

16. $y = 6 \arcsin \frac{\sqrt{x}}{2} - \frac{6+x}{2} \cdot \sqrt{x(4-x)}.$

17. $y=\frac{x-3}{2}\sqrt{6x-x^2-8}+\arcsin\sqrt{\frac{x}{2}-1}.$

18. $y=\frac{(1+x)\operatorname{arctg}\sqrt{x}-\sqrt{x}}{x}.$

19. $y=\frac{2\sqrt{1-x}\cdot\arcsin\sqrt{x}}{x}+\frac{2}{\sqrt{x}}.$

20. $y=\frac{2x-5}{4}\cdot\sqrt{5x-4-x^2}+\frac{9}{4}\cdot\arcsin\sqrt{\frac{x-1}{3}}.$

21. $y=\operatorname{arctg}x+\frac{5}{6}\cdot\ln\frac{x^2+1}{x^2+4}.$

22. $y=\arcsin\frac{x-2}{(x-1)\sqrt{2}}.$

23. $y=\sqrt{1-x^2}-x\cdot\arcsin\sqrt{1-x^2}.$

24. $y=\sqrt{x}+\frac{1}{3}\cdot\operatorname{arctg}\sqrt{x}+\frac{8}{3}\cdot\operatorname{arctg}\frac{\sqrt{x}}{2}.$

25. $y=\operatorname{arctg}\frac{\sqrt{1-x}}{1-\sqrt{x}}.$

26. $y=(2x^2+6x+5)\operatorname{arctg}\frac{x+1}{x+2}-x.$

27. $y=\frac{x}{2\sqrt{1-4x^2}}\arcsin 2x+\frac{1}{8}\cdot\ln(1-4x^2).$

28. $y=\left(2x^2-x+\frac{1}{2}\right)\operatorname{arctg}\frac{x^2-1}{x\sqrt{3}}-\frac{x^2}{2\sqrt{3}}-\frac{\sqrt{3}}{2}\cdot x.$

29. $y=\left(x-2\sqrt{x}+2\right)\operatorname{arctg}\frac{\sqrt{x}}{\sqrt{x}+2}-\sqrt{x}.$

30. $y=\sqrt{1+2x-x^2}\arcsin\frac{x\sqrt{2}}{1+x}-\sqrt{2}\ln(1+x).$

Javoblar. 9.1 $\frac{\sqrt{2}}{\sin^4 x+\cos^4 x}$; **9.2** $\frac{1}{2x\sqrt{x+\sqrt{x}-1}}$; **9.3** $\sqrt{2+x-x^2}$;

9.4 $\dfrac{-1+\sqrt{1+x^2}}{\left(x^2+(\sqrt{1+x^2}-1)^2\right)\sqrt{1+x^2}}$; **9.5** $-\dfrac{2\sqrt{2}(4+x^2)}{x^4+16}$; **9.6** $\dfrac{3x+1}{\sqrt{x}(9x^2+1)}$; **9.7** $\dfrac{1}{x^4-1}$;

9.8 $\sqrt{8x-x^2-7}$; **9.9** $-\dfrac{(2+x)\cdot arctg\sqrt{x}}{x^3}$; **9.10** $x^2\cdot\arccos x$;

9.11 $-\dfrac{1}{2x^2}\cdot arctg\sqrt{x}$; **9.12** $-\dfrac{x^2}{\sqrt{x(2-x)}}$; **9.13** $\dfrac{x^4-12}{x^4}\cdot\operatorname{arctg}\dfrac{x^2}{2}$; **9.14** $\dfrac{1}{\sqrt{x}(x+1)}$;

9.15 $\dfrac{x+\sqrt{1-x^2}\cdot\arccos x}{x^3\sqrt{1-x^2}}$; **9.16** $\dfrac{x^2-3}{\sqrt{x(4-x)}}$; **9.17** $\sqrt{6x-x^2-8}$;

9.18 $-\dfrac{1}{x^2}\cdot arctg\sqrt{x}+\dfrac{1}{x\sqrt{x}}$; **9.19** $\dfrac{x-2}{x^2\sqrt{1-x}}\cdot\arcsin\sqrt{x}$; **9.20** $\sqrt{5x-4-x^2}$;

9.21 $\dfrac{x^2+9}{(1+x^2)\cdot(x^2+4)}$; **9.22** $\dfrac{1}{(x-1)\sqrt{x^2-2}}$; **9.23** $-\arcsin\sqrt{1-x^2}$;

9.24 $\dfrac{3x^2+16x+32}{6\sqrt{x}(x+1)(x+4)}$; **9.25** $\dfrac{1}{4\sqrt{x(1-x)}}$; **9.26** $(4x+6)arctg\dfrac{x+1}{x+2}$;

9.27 $\dfrac{\arcsin 2x}{2(1-4x^2)\cdot\sqrt{1-4x^2}}$; **9.28** $(4x-1)arctg\dfrac{x^2-1}{x\sqrt{3}}+\dfrac{\sqrt{3}(x^2+1)(3x^2-2x-x^4)}{2(x^4+x^2+1)}$;

9.29 $\left(1+\dfrac{1}{\sqrt{x}}\right)arctg\dfrac{\sqrt{x}}{\sqrt{x}+2}$; **9.30** $\dfrac{1-x}{\sqrt{1+2x-x^2}}\cdot\arcsin\dfrac{x\sqrt{2}}{1+x}$.

Funksiyaning hosilasi

10–masala. Funksiyaning hosilasini toping.

$$y=\frac{2}{3}\cdot\operatorname{cth}x-\frac{\operatorname{ch}x}{3\operatorname{sh}^3x}.$$

$$y'=\left(\frac{2}{3}\cdot\operatorname{cth}x-\frac{\operatorname{ch}x}{3\operatorname{sh}^3x}\right)'=-\frac{2}{3}\cdot\frac{1}{\operatorname{sh}^2x}-\frac{(\operatorname{ch}x)'\cdot\operatorname{sh}^3x-\operatorname{ch}x\cdot(\operatorname{sh}^3x)'}{3\operatorname{sh}^6x}=$$

$$=-\frac{2}{3}\cdot\frac{1}{\operatorname{sh}^2x}-\frac{\operatorname{sh}x\cdot\operatorname{sh}^3x-\operatorname{ch}x\cdot3\operatorname{sh}^2x\cdot\operatorname{ch}x}{3\operatorname{sh}^6x}=-\frac{2\operatorname{sh}^2x}{3\operatorname{sh}^4x}-\frac{\operatorname{sh}^2x-3\operatorname{ch}^2x}{3\operatorname{sh}^4x}=$$

$$=\frac{-2\operatorname{sh}^2x-\operatorname{sh}^2x+3\operatorname{ch}^2x}{3\operatorname{sh}^4x}=\frac{3\operatorname{ch}^2x-3\operatorname{sh}^2x}{3\operatorname{sh}^4x}=\frac{3}{3\operatorname{sh}^4x}=\frac{1}{\operatorname{sh}^4x}.$$

1. $y = \dfrac{1}{4\sqrt{5}} \ln \dfrac{2 + \sqrt{5} \cdot \operatorname{th} x}{2 - \sqrt{5} \cdot \operatorname{th} x}.$

2. $y = \dfrac{\operatorname{sh} x}{4\operatorname{ch}^4 x} + \dfrac{3\operatorname{sh} x}{8\operatorname{ch}^2 x} + \dfrac{3}{8} \operatorname{arctg}(\operatorname{sh} x).$

3. $y = \dfrac{1}{2} \cdot \ln \dfrac{1 + \sqrt{\operatorname{th} x}}{1 - \sqrt{\operatorname{th} x}} + \operatorname{arctg} \sqrt{\operatorname{th} x}.$

4. $y = \dfrac{3}{8\sqrt{2}} \ln \dfrac{\sqrt{2} - \operatorname{th} x}{\sqrt{2} + \operatorname{th} x} - \dfrac{\operatorname{th} x}{4(2 - \operatorname{th}^2 x)}.$

5. $y = \dfrac{1}{2} \operatorname{th} x + \dfrac{1}{4\sqrt{2}} \ln \dfrac{1 + \sqrt{2} \operatorname{th} x}{1 - \sqrt{2} \operatorname{th} x}.$

6. $y = -\dfrac{1}{2} \cdot \ln\left(\operatorname{th} x \dfrac{x}{2} \right) - \dfrac{\operatorname{ch} x}{2\operatorname{sh}^2 x}.$

7. $y = \dfrac{1}{2a\sqrt{1 + a^2}} \ln \dfrac{a + \sqrt{1 + a^2} \operatorname{th} x}{a - \sqrt{1 + a^2} \operatorname{th} x}.$

8. $y = \dfrac{1}{18\sqrt{2}} \ln \dfrac{1 + \sqrt{2} \operatorname{cth} x}{1 - \sqrt{2} \operatorname{cth} x}.$

9. $y = \operatorname{arctg} \dfrac{\sqrt{\operatorname{sh} 2x}}{\operatorname{ch} x - \operatorname{sh} x}.$

10. $y = \dfrac{1}{6} \ln \dfrac{1 - \operatorname{sh} 2x}{1 + \operatorname{sh} 2x}.$

11. $y = \sqrt[4]{\dfrac{1+\operatorname{th}x}{1-\operatorname{th}x}}.$

12. $y = \dfrac{\operatorname{sh}x}{1+\operatorname{ch}x}.$

13. $y = \dfrac{\operatorname{ch}x}{\sqrt{\operatorname{sh}2x}}.$

14. $y = \dfrac{\operatorname{sh}3x}{\sqrt{\operatorname{ch}6x}}.$

15. $y = \dfrac{1+8\operatorname{ch}^2 x \cdot \ln(\operatorname{ch}x)}{2\operatorname{ch}^2 x}.$

16. $y = -\dfrac{12\operatorname{sh}^2 x+1}{3\operatorname{sh}^2 x}.$

17. $y = -\dfrac{\operatorname{sh}x+1}{2\operatorname{ch}^2 x} + \dfrac{3}{2}\cdot \arcsin(\operatorname{th}x).$

18. $y = \dfrac{1}{\sqrt{8}} \arcsin \dfrac{3+\operatorname{ch}x}{1+3\operatorname{ch}x}.$

19. $y = \dfrac{1}{\sqrt{8}} \arcsin \dfrac{4+\sqrt{8}\operatorname{th}\frac{x}{2}}{4-\sqrt{8}\operatorname{th}\frac{x}{2}}.$

20. $y = \dfrac{1}{4}\ln\left|\operatorname{th}\dfrac{x}{2}\right| - \dfrac{1}{4}\cdot \ln \dfrac{3+\operatorname{ch}x}{\operatorname{sh}x}.$

21. $y = -\dfrac{1}{4} \arcsin \dfrac{5+3\operatorname{ch}x}{3+5\operatorname{ch}x}.$

22. $y=\dfrac{1-8\mathrm{ch}^2x}{4\mathrm{ch}^4x}.$

23. $y=\dfrac{2}{\mathrm{sh}x}-\dfrac{1}{3\mathrm{sh}^3x}+\dfrac{\mathrm{sh}x}{2\mathrm{ch}^2x}+\dfrac{5}{2}\mathrm{arctg}(\mathrm{sh}x).$

24. $y=\dfrac{8}{3}\cdot\mathrm{cth}x-\dfrac{1}{3\mathrm{ch}x\cdot\mathrm{sh}^3x}.$

25. $y=\dfrac{1}{2}\cdot\mathrm{arctg}(\mathrm{sh}x)-\dfrac{\mathrm{sh}x}{2\mathrm{ch}^2x}.$

26. $y=\dfrac{3}{2}\cdot\ln\left(\mathrm{th}\,\frac{x}{2}\right)+\mathrm{ch}x-\dfrac{\mathrm{ch}x}{2\mathrm{sh}^2x}.$

27. $y=-\dfrac{\mathrm{sh}x}{2\mathrm{ch}^2x}-\dfrac{1}{\mathrm{sh}x}-\dfrac{3}{2}\mathrm{arctg}(\mathrm{sh}x).$

28. $y=\dfrac{\mathrm{sh}x}{2\mathrm{ch}^2x}+\dfrac{1}{2}\cdot\mathrm{arctg}(\mathrm{sh}x).$

29. $y=\dfrac{1}{2}\left(\dfrac{\mathrm{sh}x}{\mathrm{ch}^2x}+\mathrm{arctg}(\mathrm{sh}x)\right).$

30. $y=-\dfrac{\mathrm{ch}x}{2\mathrm{sh}^2x}-\dfrac{1}{2}\ln\left(\mathrm{th}\,\dfrac{x}{2}\right)$

Javoblar. 10.1 $\dfrac{1}{4-\mathrm{sh}^2x}$; **10.2** $\dfrac{1-3\mathrm{ch}^2x}{4\mathrm{ch}^5x}$; **10.3** $\sqrt{\mathrm{th}x}$; **10.4** $\dfrac{1}{(1+\mathrm{ch}^2x)^2}$;

10.5 $\dfrac{1}{\mathrm{ch}^2x(1-\mathrm{sh}^2x)}$; **10.6** $\dfrac{1}{\mathrm{sh}^3x}$; **10.7** $\dfrac{1}{a^2\cdot\mathrm{ch}^2x+(1+a^2)\cdot\mathrm{sh}^2x}$; **10.8** $\dfrac{1}{9\cdot(1+\mathrm{ch}^2x)}$;

10.9 $\dfrac{\mathrm{ch}x+\mathrm{sh}x}{\sqrt{\mathrm{sh}2x}\cdot\mathrm{ch}2x}$; **10.10** $\dfrac{\mathrm{ch}2x}{\mathrm{sh}^2\,2x+\mathrm{sh}2x-2}$; **10.11** $\dfrac{1}{2\sqrt{\mathrm{ch}x-\mathrm{sh}x}}$; **10.12** $\dfrac{1}{1+\mathrm{ch}x}$;

10.13 $-\dfrac{1}{2\mathrm{sh}x\sqrt{\mathrm{sh}2x}}$; **10.14** $\dfrac{3\mathrm{ch}3x}{\mathrm{ch}6x\cdot\sqrt{\mathrm{ch}6x}}$; **10.15** $\dfrac{\mathrm{sh}x\cdot(4\mathrm{ch}^2x-1)}{\mathrm{ch}^3x}$; **10.16** $-\dfrac{2\mathrm{ch}x}{3\mathrm{sh}^3x}$;

10.17 $\dfrac{\mathrm{ch}2x}{\mathrm{ch}^3x}$; **10.18** $-\dfrac{9\mathrm{sh}x}{8(1+3\mathrm{ch}x)}$; **10.19** $\dfrac{1}{2\left(\mathrm{ch}^2\frac{x}{2}+1\right)}$; **10.20** $\dfrac{1}{2\mathrm{sh}x}$;

10.21 $\dfrac{1}{3+5\mathrm{ch}x}$; **10.22** $\dfrac{4\mathrm{th}^3x}{\mathrm{ch}^2x}$; **10.23** $-\dfrac{2\mathrm{ch}x}{\mathrm{sh}^2x}+\dfrac{\mathrm{ch}x}{\mathrm{sh}^4x}+\dfrac{1-\mathrm{sh}^2x}{2\mathrm{ch}^3x}+\dfrac{5}{2\mathrm{ch}x}$;

10.24 $\dfrac{1-4\mathrm{sh}^3x}{\mathrm{ch}^2x\cdot\mathrm{sh}^4x}$; **10.25** $\dfrac{\mathrm{sh}^2x}{\mathrm{ch}^3x}$; **10.26** $\dfrac{\mathrm{ch}^4x}{\mathrm{sh}^3x}$; **10.27** $\dfrac{\mathrm{ch}x}{\mathrm{sh}^2x}-\dfrac{2-\mathrm{sh}^2x}{\mathrm{ch}^3x}$; **10.28** $\dfrac{1}{\mathrm{ch}^3x}$;

10.29 $\dfrac{1}{\mathrm{ch}^3x}$; **10.30** $\dfrac{1}{\mathrm{sh}^3x}$.

Funksiyaning hosilasi

11–masala. Funksiyaning hosilasini toping.

$$y=x^{e^x}\cdot x^9.$$

$$\ln y=\ln\left(x^{e^x}\cdot x^9\right)=e^x\cdot\ln x+9\cdot\ln x=\ln x\cdot(e^x+9).$$

$$\frac{y'}{y}=\left(\ln x\cdot(e^x+9)\right)'=\frac{1}{x}\cdot(e^x+9)+\ln x\cdot e^x=e^x\left(\ln x+\frac{1}{x}\right)+\frac{9}{x}.$$

$$y'=y\cdot\left(e^x\left(\ln x+\frac{1}{x}\right)+\frac{9}{x}\right)=x^{e^x}\cdot x^9\cdot\left(e^x\left(\ln x+\frac{1}{x}\right)+\frac{9}{x}\right).$$

1. $y=(\mathrm{arctg}x)^{\frac{1}{2}\cdot\ln(\mathrm{arctg}x)}$.

2. $y=\left(\sin\sqrt{x}\right)^{\ln\left(\sin\sqrt{x}\right)}$.

3. $y=(\sin x)^{5e^x}$.

4. $y=(\arcsin x)^{e^x}$.

5. $y=(\ln x)^{3^x}$.

6. $y=x^{\arcsin x}$.

7. $y=(\mathrm{ctg}3x)^{2e^x}$.

8. $y=x^{e^{\mathrm{tg}x}}$.

9. $y=(\mathrm{tg}x)^{4e^x}$.

10. $y=(\cos5x)^{e^x}$.

11. $y=(x\sin x)^{8\ln(x\sin x)}$.

12. $y=(x-5)^{\mathrm{ch}x}$.

13. $y=\left(x^3+4\right)^{\mathrm{tg}x}$.

14. $y=x^{\sin x^3}$.

15. $y=\left(x^2-1\right)^{\mathrm{sh}x}$.

16. $y=\left(x^4+5\right)^{\mathrm{ctg}x}$.

17. $y=(\sin x)^{\frac{5x}{2}}$.

18. $y=\left(x^2+1\right)^{\cos x}$.

19. $y=19^{x^{19}}\cdot x^{19}$.

20. $y=x^{3^x}\cdot 2^x$.

21. $y=\left(\sin\sqrt{x}\right)^{e^{1/x}}$.

22. $y=x^{e^{\mathrm{ctg}x}}$.

23. $y=x^{e^{\cos x}}$.

24. $y=x^{2^x}\cdot 5^x$.

25. $y=x^{e^{\sin x}}$.

26. $y=(\mathrm{tg}x)^{\ln\frac{\mathrm{tg}x}{4}}$.

27. $y=x^{e^{\mathrm{arctg}x}}$.

28. $y=\left(x^8+4\right)^{\mathrm{th}x}$.

29. $y=29^{x^{29}}\cdot x^{29}$.

30. $y=\cos 2x^{\ln\frac{\cos 2x}{4}}$

Javoblar. 11.1 $(arctgx)^{\frac{1}{2}\ln(arctgx)}\cdot\dfrac{\ln(arctgx)}{arctgx\cdot(1+x^2)}$;

11.2 $\left(\sin\sqrt{x}\right)^{\ln\left(\sin\sqrt{x}\right)}\cdot\dfrac{\ln\left(\sin\sqrt{x}\right)ctg\sqrt{x}}{\sqrt{x}}$;

11.3 $5e^x\cdot(\sin x)^{5e^x}\cdot\left(\ln(\sin x)+ctgx\right)$;

11.4 $(\arcsin x)^{e^x}\cdot e^x\cdot\left(\ln\left(\arcsin x\right)+\dfrac{1}{\sqrt{1-x^2}\arcsin x}\right)$;

11.5 $(\ln x)^{3^x}\cdot 3^x\cdot\left((\ln 3\cdot\ln(\ln x)+\dfrac{1}{x\cdot\ln x}\right)$; **11.6** $x^{\arcsin x}\cdot\left(\dfrac{\ln x}{\sqrt{1-x^2}}+\dfrac{\arcsin x}{x}\right)$;

11.7 $2e^x\cdot(ctg3x)^{2e^x}\cdot\left(\ln(ctg3x)-\dfrac{6}{\sin 6x}\right)$; **11.8** $x^{e^{\mathrm{tg}x}}\cdot e^{\mathrm{tg}x}\cdot\left(\dfrac{\ln x}{\cos^2 x}+\dfrac{1}{x}\right)$;

11.9 $\left(\mathrm{tg}x\right)^{4e^x}\cdot 4e^x\cdot\left(\ln(tgx)+\dfrac{2}{\sin 2x}\right)$; **11.10** $\left(\cos 5x\right)^{e^x}\cdot e^x\cdot\left(\ln(\cos 5x)-tg5x\right)$;

11.11 $\dfrac{16(x\sin x)^{8\ln(x\sin x)}\cdot\ln(x\sin x)\cdot(1+x\cdot ctgx)}{x}$;

11.12 $(x-5)^{chx}\cdot\left(\mathrm{sh}x\cdot\ln(x-5)+\dfrac{\mathrm{ch}x}{x-5}\right)$; **11.13** $\left(x^3+4\right)^{tgx}\cdot\left(\dfrac{\ln(x^3+4)}{1+x^2}+\dfrac{3x^2\cdot tgx}{x^3+4}\right)$;

11.14 $x^{\sin x^3}\cdot\left(3x^2\cdot\ln x\cdot\cos x^3+\frac{\sin x^3}{x}\right)$;

11.15 $(x^2-1)^{\text{sh}x}\cdot\left(\text{ch}x\cdot\ln(x^2-1)+\frac{2x\cdot\text{sh}x}{x^2-1}\right)$;

11.16 $(x^4+5)^{ctgx}\cdot\left(\frac{4x^3\cdot ctgx}{x^4+5}-\frac{\ln\left(x^4+5\right)}{\sin^2 x}\right)$; **11.17** $\frac{5}{2}\cdot(\sin x)^{\frac{5x}{2}}\cdot\left(\ln(\sin x)+x\cdot ctgx\right)$;

11.18 $(x^2+1)^{\cos x}\cdot\left(\frac{2x\cdot\cos x}{x^2+1}-\sin x\cdot\ln(x^2+1)\right)$;

11.19 $19^{x^{19}}\cdot x^{19}\cdot 19\left(x^{18}\cdot\ln 19+\frac{1}{x}\right)$; **11.20** $x^{3^x}\cdot 2^x\cdot\left(3^x\cdot\ln 3\cdot\ln(x)+\frac{3^x}{x}+\ln 2\right)$;

11.21 $(\sin\sqrt{x})^{e^{1/x}}\cdot e^{\frac{1}{x}}\cdot\left(\frac{\ln(\sin\sqrt{x})}{x}+\frac{tg\sqrt{x}}{2\sqrt{x}}\right)$; **11.22** $x^{e^{ctgx}}\cdot e^{ctgx}\cdot\left(\frac{1}{x}-\frac{\ln x}{\sin^2 x}\right)$;

11.23 $x^{e^{\cos x}}\cdot e^{\cos x}\cdot\left(\frac{1}{x}-\sin x\cdot\ln x\right)$; **11.24** $x^{2^x}\cdot 5^x\cdot\left(2^x\cdot\ln 2\cdot\ln(x)+\frac{2^x}{x}+\ln 5\right)$;

11.25 $x^{e^{\sin x}}\cdot e^{\sin x}\cdot\left(\cos x\cdot\ln x+\frac{1}{x}\right)$; **11.26** $tgx^{\ln\frac{tgx}{4}}\cdot\frac{\ln(tgx)}{\sin 2x}$;

11.27 $x^{e^{arctgx}}\cdot e^{arctgx}\cdot\left(\frac{\ln x}{1+x^2}+\frac{1}{x}\right)$; **11.28** $(x^8+1)^{\text{th}x}\cdot\left(\frac{\ln(x^8+1)}{\text{ch}^2 x}+\frac{8x^7\cdot\text{th}x}{x^8+1}\right)$;

11.29 $x^{29^x}\cdot 29^x\cdot\left(29^x\cdot\ln 29\cdot\ln(x)+\frac{29^x}{x}+\ln 29\right)$;

11.30 $-\cos 2x^{\ln\frac{\cos 2x}{4}}\cdot tg2x\cdot\ln(\cos 2x)$.

Funksiyaning hosilasi

12–masala. Funksiyaning hosilasini toping.

$$y=\arcsin(e^{-2x})+\ln\left(e^{2x}+\sqrt{e^{4x}-1}\right).$$

$$y' = \left(\arcsin(e^{-2x}) + \ln\left(e^{2x} + \sqrt{e^{4x}-1}\right)\right)' =$$

$$= \frac{1}{\sqrt{1-\left(e^{-2x}\right)^2}} + \frac{1}{e^{2x}+\sqrt{e^{4x}-1}} \cdot \left(e^{2x}\cdot 2 + \frac{1}{2\sqrt{e^{4x}-1}} \cdot e^{4x} \cdot 4 \right) =$$

$$= \frac{1}{\sqrt{1-e^{-4x}}} + \frac{1}{e^{2x}+\sqrt{e^{4x}-1}} \cdot 2e^{2x} \cdot \left(\frac{\sqrt{e^{4x}-1}}{\sqrt{e^{4x}-1}} + \frac{e^{2x}}{\sqrt{e^{4x}-1}} \right) =$$

$$= \frac{e^{2x}}{\sqrt{e^{4x}-1}} + \frac{2e^{2x}}{e^{2x}+\sqrt{e^{4x}-1}} \cdot \frac{\sqrt{e^{4x}-1}+e^{2x}}{\sqrt{e^{4x}-1}} =$$

$$= \frac{e^{2x}}{\sqrt{e^{4x}-1}} + \frac{e^{2x}}{\sqrt{e^{4x}-1}} \frac{3e^{2x}}{\sqrt{e^{4x}-1}}.$$

1. $y = \frac{1}{24}\left(x^2+8\right)\sqrt{x^2-4} + \frac{x^2}{16}\arcsin\frac{2}{x}, \quad x>0.$

2. $y = \frac{4x+1}{16x^2+8x+3} + \frac{1}{\sqrt{2}} \cdot \operatorname{arctg} x \frac{4x+1}{\sqrt{2}}.$

3. $y = 2x - \ln\left(1+\sqrt{1-e^{4x}}\right) - e^{-2x}\cdot\arcsin(e^{2x}).$

4. $y = \sqrt{9x^2-12x+5}\cdot\operatorname{arctg}(3x-2) - \ln\left(3x-2+\sqrt{9x^2-12x+5}\right).$

5. $y = \frac{2}{x-1}\cdot\sqrt{2x-x^2} + \ln\frac{1+\sqrt{2x-x^2}}{x-1}.$

6. $y = \frac{x^4}{81}\cdot\arcsin\frac{3}{x} + \frac{1}{81}(x^2+18)\sqrt{x^2-9}, \ x>0.$

7. $y = \frac{1}{\sqrt{2}}\cdot\operatorname{arctg}\frac{3x-1}{\sqrt{2}} + \frac{1}{3}\cdot\frac{3x-1}{3x^2-2x+1}.$

8. $y = 3x - \ln\left(1+\sqrt{1-e^{6x}}\right) - e^{-3x}\cdot\arcsin(e^{3x}).$

9. $y = \ln\left(4x-1+\sqrt{16x^2-8x+2}\right) - \sqrt{16x^2-8x+2}\cdot\operatorname{arctg}(4x-1).$

10. $y = \ln\frac{1+2\sqrt{-x-x^2}}{2x+1} + \frac{4}{2x+1}\cdot\sqrt{-x-x^2}.$

11. $y=(2x+3)^4\cdot\arcsin\dfrac{1}{2x+3}+\dfrac{2}{3}\cdot(4x^2+12x+11)\cdot\sqrt{x^2+3x+2},\quad 2x+3>0.$

12. $y=\dfrac{x+2}{x^2+4x+6}+\dfrac{1}{\sqrt{2}}\cdot\operatorname{arctg}\dfrac{x+2}{\sqrt{2}}.$

13. $y=5x-\ln\left(1+\sqrt{1-e^{10x}}\right)-e^{-5x}\cdot\arcsin(e^{5x}).$

14. $y=\sqrt{x^2-8x+17}\cdot\operatorname{arctg}(x-4)-\ln\left(x-4+\sqrt{x^2-8x+17}\right).$

15. $y=\ln\dfrac{1+\sqrt{-3+4x-x^2}}{2-x}+\dfrac{2}{2-x}\cdot\sqrt{-3+4x-x^2}.$

16. $y=(3x^2-4x+2)\cdot\sqrt{9x^2-12x+3}+(3x-2)^4\cdot\arcsin\dfrac{1}{3x-2},\quad 3x-2>0.$

17. $y=\dfrac{1}{\sqrt{2}}\cdot\operatorname{arctg}\dfrac{x-1}{\sqrt{2}}+\dfrac{x-1}{x^2-2x+3}.$

18. $y=\ln\left(e^{5x}+\sqrt{e^{10x}-1}\right)+\arcsin(e^{-5x}).$

19. $y=\ln\left(2x-3+\sqrt{4x^2-12x+10}\right)+\sqrt{4x^2-12x+10}\cdot\operatorname{arctg}(2x-3).$

20. $y=\ln\dfrac{1+\sqrt{-3-4x-x^2}}{-x-2}-\dfrac{2}{x+2}\cdot\sqrt{-3-4x-x^2}.$

21. $y=\dfrac{2}{3}\cdot(4x^2-4x+3)\cdot\sqrt{x^2-x}+(2x-1)^4\cdot\arcsin\dfrac{1}{2x-1},\quad 2x-1>0.$

22. $y=\dfrac{2x-1}{4x^2-4x+3}+\dfrac{1}{\sqrt{2}}\cdot\operatorname{arctg}\dfrac{2x-1}{\sqrt{2}}.$

23. $y=\arcsin(e^{-4x})+\ln\left(e^{4x}+\sqrt{e^{8x}-1}\right).$

24. $y=\ln\left(5x+\sqrt{25x^2+1}\right)-\sqrt{25x^2+1}\cdot\operatorname{arctg}5x.$

25. $y=\dfrac{2}{3x-2}\cdot\sqrt{-3+12x-9x^2}+\ln\dfrac{1+\sqrt{-3+12x-9x^2}}{3x-2}.$

26. $y=(3x+1)^4\cdot\arcsin\dfrac{1}{3x+1}+(3x^2+2x+1)\cdot\sqrt{9x^2+6x},\quad 3x+1>0.$

27. $y=\dfrac{1}{\sqrt{2}}\cdot\operatorname{arctg}\dfrac{2x+1}{\sqrt{2}}+\dfrac{2x+1}{4x^2+4x+3}.$

28. $y=\ln\left(e^{3x}+\sqrt{e^{6x}-1}\right)+\arcsin(e^{-3x}).$

29. $y=\sqrt{49x^2+1}\cdot\operatorname{arctg}7x-\ln\left(7x+\sqrt{49x^2+1}\right)$

30. $y=\frac{1}{x}\cdot\sqrt{1-4x^2}+\ln\frac{1+\sqrt{1+4x^2}}{2x}.$

Javoblar. 12.1 $\frac{x^3-x}{8\sqrt{x^2-4}}+\frac{8}{x}\cdot\arcsin\frac{2}{x}$; **12.2** $\frac{16}{\left(16x^2+8x+3\right)^2}$;

12.3 $2e^{-2x}\cdot\arcsin\left(e^{2x}\right)$;**12.4** $\frac{(9x-6)\cdot arctg(3x-2)}{\sqrt{9x^2-12x+5}}$; **12.5** $\frac{2x^2-7x+3}{(x-1)^2\sqrt{2x-x^2}}$;

12.6 $\frac{4x^3}{81}\cdot\arcsin\frac{3}{x}+\frac{x\cdot(x^2-1)}{27\sqrt{x^2-9}}$; **12.7** $\frac{4}{3\left(3x^2-2x+1\right)^2}$; **12.8** $3e^{-3x}\cdot\arcsin\left(e^{3x}\right)$;

12.9 $\frac{4(1-4x)}{\sqrt{16x^2-8x+2}}\cdot arctg(4x-1)$; **12.10** $-\frac{2x+3}{\sqrt{-x-x^2}\cdot(2x+1)^2}$;

12.11 $8(2x+3)^3\cdot\arcsin\frac{1}{2x+3}$; **12.12** $\frac{4}{(x^2+4x+6)^2}$; **12.13** $5e^{-5x}\cdot\arcsin(e^{5x})$;

12.14 $\frac{x-4}{\sqrt{x^2-8x+17}}\cdot arctg(x-4)$; **12.15** $\frac{4-x}{(2-x)^2\cdot\sqrt{-3+4x-x^2}}$;

12.16 $12(3x-2)^3\cdot\arcsin\frac{1}{3x-2}$; **12.17** $\frac{4}{(x^2-2x+3)^2}$; **12.18** $5e^{5x}\cdot\sqrt{1-e^{-10x}}$;

12.19 $\frac{4x-6}{\sqrt{4x^2-12x+10}}\cdot arctg(2x-3)$;

12.20 $\frac{2x^2+8x+9+\sqrt{-3-4x-x^2}}{(x+2)\cdot\sqrt{-3-4x-x^2}\cdot\left(1+\sqrt{-3-4x-x^2}\right)}$; **12.21** $8(2x-1)\cdot\arcsin\frac{1}{2x-1}$;

12.22 $\frac{8}{\left(4x^2-4x+3\right)^2}$; **12.23** $4\sqrt{\frac{e^{4x}-1}{e^{4x}+1}}$; **12.24** $\frac{25x\cdot arctg5x}{\sqrt{25x^2+1}}$;

12.25 $\frac{3-9x}{\sqrt{-3+12x-9x^2}\cdot(3x-2)}$;

12.26 $12(3x+1)^3\cdot\arcsin\frac{1}{3x+1}+(3x+1)\cdot\frac{18x^2}{\sqrt{9x^2+6x}}$; **12.27** $\frac{8}{\left(4x^2+4x+3\right)^2}$;

12.28 $3\sqrt{\frac{e^{3x}-1}{e^{3x}+1}}$; **12.29** $\frac{7\cdot arctg7x}{2\sqrt{49x^2+1}}$; **12.30** $-\frac{1}{x^2\sqrt{1-4x^2}}-\frac{1}{x\sqrt{1+4x^2}}$.

Funksiyaning hosilasi

13–masala. Funksiyaning hosilasini toping.

$$y=\frac{\arcsin x}{\sqrt{1-x^2}}+\frac{1}{2}\ln\frac{1-x}{1+x}.$$

$$y'=\left(\frac{\arcsin x}{\sqrt{1-x^2}}+\frac{1}{2}\ln\frac{1-x}{1+x}\right)'=\frac{\left(\arcsin x\right)'\cdot\sqrt{1-x^2}-\arcsin x\left(\sqrt{1-x^2}\right)'}{1-x^2}+$$

$$+\frac{1}{2}\cdot\frac{1+x}{1-x}\cdot\frac{-1\cdot(1+x)-(1-x)\cdot1}{(1+x)^2}=$$

$$=\frac{\frac{1}{\sqrt{1-x^2}}\cdot\sqrt{1-x^2}-\arcsin x\cdot\frac{1}{2\sqrt{1-x^2}}\cdot2x}{1-x^2}+$$

$$+\frac{1}{2}\cdot\frac{1}{1-x}\cdot\frac{-2}{1+x}=\frac{1-\frac{x\cdot\arcsin x}{\sqrt{1-x^2}}}{1-x^2}-\frac{1}{1-x^2}=\frac{x\cdot\arcsin x}{\left(1-x^2\right)\sqrt{1-x^2}}.$$

1. $y=\frac{x\cdot\arcsin x}{\sqrt{1-x^2}}+\ln\sqrt{1-x^2}$.

2. $y=4\ln\frac{x}{1+\sqrt{1-4x^2}}-\frac{\sqrt{1-4x^2}}{x^2}$.

3. $y=x(2x^2+5)\sqrt{x^2+1}+3\ln\left(x+\sqrt{x^2+1}\right)$.

4. $y=x^3\arcsin x+\frac{x^2+2}{3}\sqrt{1-x^2}$.

5. $y = 3\arcsin\dfrac{3}{4x+1} + 2\sqrt{4x^2+2x-2},\quad 4x+1>0.$

6. $y = \sqrt{1+x^2}\operatorname{arctg}x - \ln\left(x+\sqrt{1+x^2}\right).$

7. $y = 2\arcsin\dfrac{2}{3x+4} + \sqrt{9x^2+24x+12},\quad 3x+4>0.$

8. $y = x(2x^2+1)\sqrt{x^2+1} - \ln\left(x+\sqrt{x^2+1}\right).$

9. $y = \ln\left(x+\sqrt{x^2+1}\right) - \dfrac{\sqrt{x^2+1}}{x}.$

10. $y = \sqrt{1-3x-2x^2} + \dfrac{3}{2\sqrt{2}}\arcsin\dfrac{4x+3}{\sqrt{17}}.$

11. $y = \sqrt{(4+x)(1+x)} + 3\ln\left(\sqrt{4+x}+\sqrt{1+x}\right).$

12. $y = \ln\dfrac{\sqrt{x^2-x+1}}{x} + \sqrt{3}\operatorname{arctg}\dfrac{2x-1}{3}.$

13. $y = \dfrac{1}{12}\ln\dfrac{x^4-x^2+1}{\left(x^2+1\right)^2} - \dfrac{1}{2\sqrt{3}}\operatorname{arctg}\dfrac{\sqrt{3}}{2x^2-1}.$

14. $y = 4\arcsin\dfrac{4}{2x+3} + \sqrt{4x^2+12x-7},\quad 2x+3>0.$

15. $y = 2\arcsin\dfrac{2}{3x+1} + \sqrt{9x^2+6x-3},\quad 3x+1>0.$

16. $y = (2+3x)\sqrt{x-1} - \dfrac{3}{2}\operatorname{arctg}\sqrt{x-1}.$

17. $y = \dfrac{1}{3}(x-2)\sqrt{x+1} + \ln\left(\sqrt{x+1}+1\right).$

18. $y = \sqrt{x^2+1} - \dfrac{1}{2}\ln\dfrac{\sqrt{x^2+1}-x}{\sqrt{x^2+1}+1}.$

19. $y = \sqrt[3]{\dfrac{x-1}{x+1}} - \dfrac{1}{2}\left(\dfrac{1}{2} + \dfrac{1}{x^2-1}\right)\operatorname{arctg}x.$

20. $y = x\ln\left(\sqrt{1-x}+\sqrt{1+x}\right)+\frac{1}{2}(\arcsin x - x).$

21. $y = \operatorname{arctg}\sqrt{x^2-1} - \frac{\ln x}{\sqrt{x^2-1}}.$

22. $y = 3\arcsin\frac{3}{x+2} + \sqrt{x^2+4x-5}, \quad x+2>0.$

23. $y = \sqrt{(3-x)(2+x)} + 5\arcsin\sqrt{\frac{x+2}{5}}.$

24. $y = x(\arcsin x)^2 + 2\sqrt{1-x^2}\arcsin x - 2x.$

25. $y = \frac{\sqrt{1-x^2}}{x} + \arcsin x.$

26. $y = x^2\arccos x - \frac{x^2+2}{3}\sqrt{1-x^2}.$

27. $y = \frac{\sqrt{x^2+2}}{x^2} - \frac{1}{\sqrt{2}}\ln\frac{\sqrt{2}+\sqrt{x^2+2}}{x}.$

28. $y = \frac{x}{4}(10-x^2)\sqrt{4-x^2} + 6\arcsin\frac{x}{2}.$

29. $y = \arcsin\frac{1}{2x+3} + 2\sqrt{x^2+3x+2}, \quad 2x+3>0.$

30. $y = x\cdot\arcsin\sqrt{\frac{x}{x+1}} - \sqrt{x} + \operatorname{arctg}\sqrt{x}.$

Javoblar. 13.1 $\frac{\arcsin x}{\sqrt{(1-x^2)^3}}$; **13.2** $\frac{2}{x^3\sqrt{1-4x^2}}$; **13.3** $8\sqrt{(x^2+1)^3}$; **13.4** $3x^2\arcsin x$;

13.5 $\frac{7\cdot(4x+1)}{2\sqrt{4x^2+2x-2}}$; **13.6** $\frac{x\cdot arctgx}{\sqrt{1+x^2}}$; **13.7** $\frac{8(3x+4)}{\sqrt{9x^2+24x+12}}$; **13.8** $8x^2\sqrt{x^2+1}$;

13.9 $\frac{\sqrt{x^2+1}}{x^2}$; **13.10** $-\frac{2x}{\sqrt{1-3x-2x^2}}$; **13.11** $\sqrt{\frac{4+x}{1+x}}$; **13.12** $\frac{2x-1}{x\cdot(x^2-x+1)}$;

13.13 $\frac{x^3}{(x^4-x^2+1)\cdot(x^2+1)}$; **13.14** $\frac{2\sqrt{4x^2+12x-7}}{2x+3}$; **13.15** $\frac{3\sqrt{9x^2+6x-3}}{3x+1}$;

13.16 $\frac{18x^2-8x-3}{4x\sqrt{x-1}}$; **13.17** $\frac{3x\sqrt{x+1}+3x-\sqrt{x+1}+2}{6\sqrt{x+1}\cdot\left(\sqrt{x+1}+1\right)}$; **13.18** $\frac{2\sqrt{x^2+1}+x+2}{2\left(\sqrt{x^2+1}+1\right)\cdot\sqrt{x^2+1}}$;

13.19 $\frac{5x^2+17}{12(x^4-1)}+\frac{x\cdot arctgx}{(x^2-1)^2}$; **13.20** $\ln\left(\sqrt{1-x}+\sqrt{1+x}\right)$; **13.21** $\frac{x\cdot\ln x}{\sqrt{(x^2-1)^3}}$; **13.22**

$\frac{\sqrt{x^2+4x-5}}{x+2}$; **13.23** $\sqrt{\frac{3-x}{2+x}}$; **13.24** $(\arcsin x)^2$; **13.25** $-\frac{\sqrt{1-x^2}}{x^2}$; **13.26**

$2x\cdot\arccos x-x^2\cdot\sqrt{\frac{1-x}{1+x}}$; **13.27** $-\frac{4}{x^3\cdot\sqrt{x^2+2}}$; **13.28** $\sqrt{(4-x^2)^3}$; **13.29**

$\frac{4\sqrt{x^2+3x+2}}{2x+3}$; **13.30** $\arcsin\sqrt{\frac{x}{x+1}}$.

Funksiyaning hosilasi

14–masala. Funksiyaning hosilasini toping.

$$y=\sqrt{\frac{\mathrm{tg}x+\sqrt{2\mathrm{tg}x}+1}{\mathrm{tg}x-\sqrt{2\mathrm{tg}x}+1}}.$$

$$y'=\left(\sqrt{\frac{\mathrm{tg}x+\sqrt{2\mathrm{tg}x}+1}{\mathrm{tg}x-\sqrt{2\mathrm{tg}x}+1}}\right)'=\frac{1}{2}\sqrt{\frac{\mathrm{tg}x+\sqrt{2\mathrm{tg}x}+1}{\mathrm{tg}x-\sqrt{2\mathrm{tg}x}+1}}\cdot\left(\frac{\mathrm{tg}x+\sqrt{2\mathrm{tg}x}+1}{\mathrm{tg}x-\sqrt{2\mathrm{tg}x}+1}\right)'=$$

$$=\frac{1}{2}\sqrt{\frac{\mathrm{tg}x+\sqrt{2\mathrm{tg}x}+1}{\mathrm{tg}x-\sqrt{2\mathrm{tg}x}+1}}\cdot\frac{\left(\frac{1}{\cos^2 x}+\frac{1}{2\sqrt{2\mathrm{tg}x}}\cdot\frac{2}{\cos^2 x}\right)(\mathrm{tg}x-\sqrt{2\mathrm{tg}x}+1)-(\mathrm{tg}x-\sqrt{2\mathrm{tg}x}+1)\left(\frac{1}{\cos^2 x}+\frac{1}{2\sqrt{2\mathrm{tg}x}}\cdot\frac{2}{\cos^2 x}\right)}{(\mathrm{tg}x-\sqrt{2\mathrm{tg}x}+1)^2}=$$

$$=\frac{1}{2}\sqrt{\frac{\mathrm{tg}x+\sqrt{2\mathrm{tg}x}+1}{\mathrm{tg}x-\sqrt{2\mathrm{tg}x}+1}}\cdot\frac{\left(1+\frac{1}{\sqrt{2\mathrm{tg}x}}\right)(\mathrm{tg}x-\sqrt{2\mathrm{tg}x}+1)-(\mathrm{tg}x-\sqrt{2\mathrm{tg}x}+1)\left(1-\frac{1}{\sqrt{2\mathrm{tg}x}}\right)}{\cos^2 x\cdot(\mathrm{tg}x-\sqrt{2\mathrm{tg}x}+1)^2}=$$

$$=\frac{1}{2}\sqrt{\frac{\mathrm{tg}x+\sqrt{2\mathrm{tg}x}+1}{\mathrm{tg}x-\sqrt{2\mathrm{tg}x}+1}}\cdot\frac{\mathrm{tg}x-\sqrt{2\mathrm{tg}x}+1+\frac{\sqrt{\mathrm{tg}x}}{\sqrt{2}}-1+\frac{1}{\sqrt{2\mathrm{tg}x}}-\mathrm{tg}x-\sqrt{2\mathrm{tg}x}-1+\frac{\sqrt{\mathrm{tg}x}}{\sqrt{2}}+1+\frac{1}{\sqrt{2\mathrm{tg}x}}}{\cos^2 x\cdot(\mathrm{tg}x-\sqrt{2\mathrm{tg}x}+1)^2}=$$

$$=\frac{1}{2}\sqrt{\frac{\mathrm{tg}x+\sqrt{2\mathrm{tg}x}+1}{\mathrm{tg}x-\sqrt{2\mathrm{tg}x}+1}}\cdot\frac{\sqrt{2\mathrm{tg}x}-2\sqrt{2\mathrm{tg}x}+\frac{\sqrt{2}}{\sqrt{\mathrm{tg}x}}}{\cos^2 x\cdot(\mathrm{tg}x-\sqrt{2\mathrm{tg}x}+1)^2}=$$

$$=\frac{1}{2}\sqrt{\frac{\mathrm{tg}x+\sqrt{2\mathrm{tg}x}+1}{\mathrm{tg}x-\sqrt{2\mathrm{tg}x}+1}}\cdot\frac{\frac{\sqrt{2}}{\sqrt{\mathrm{tg}x}}-\sqrt{2\mathrm{tg}x}}{\cos^2 x\cdot(\mathrm{tg}x-\sqrt{2\mathrm{tg}x}+1)^2}=$$

$$= \frac{1}{\sqrt{2}} \sqrt{\frac{\operatorname{tg}x - \sqrt{2\operatorname{tg}x} + 1}{\left(\operatorname{tg}x + \sqrt{2\operatorname{tg}x} + 1\right)\left(\operatorname{tg}x - \sqrt{2\operatorname{tg}x} + 1\right)^2}} \cdot \frac{\frac{1}{\sqrt{\operatorname{tg}x}} - \sqrt{\operatorname{tg}x}}{\cos^2 x \cdot (\operatorname{tg}x - \sqrt{2\operatorname{tg}x} + 1)} =$$

$$= \frac{1}{\sqrt{2}} \sqrt{\frac{1}{\operatorname{tg}^2 x + 1}} \cdot \frac{\frac{1}{\sqrt{\operatorname{tg}x}} - \sqrt{\operatorname{tg}x}}{\cos^2 x \cdot (\operatorname{tg}x - \sqrt{2\operatorname{tg}x} + 1)} =$$

$$= \frac{1}{\sqrt{2}} \cos x \cdot \frac{\frac{1}{\sqrt{\operatorname{tg}x}} - \sqrt{\operatorname{tg}x}}{\cos^2 x \cdot (\operatorname{tg}x - \sqrt{2\operatorname{tg}x} + 1)} =$$

$$= \frac{1 - \operatorname{tg}x}{\sqrt{2\operatorname{tg}x} \cdot \left(\sin x - \sqrt{\sin 2x} + \cos x\right)}.$$

1. $y = \dfrac{1}{\sin \alpha} \ln(\operatorname{tg}x + \operatorname{ctg}\alpha).$

2. $y = x \cdot \cos\alpha + \sin\alpha \cdot \ln \sin(x - \alpha).$

3. $y = \dfrac{1}{2\sqrt{2}} \left(\sin(\ln x) - \left(\sqrt{2} - 1\right) \cdot \cos(\ln x)\right) x^{\sqrt{2}+1}.$

4. $y = \operatorname{arctg}\left(\dfrac{\cos x}{\sqrt[4]{\cos 2x}}\right).$

5. $y = 3\dfrac{\sin x}{\cos^2 x} + 2\dfrac{\sin x}{\cos^4 x}.$

6. $y = (a^2 + b^2)^{-\frac{1}{2}} \cdot \arcsin\left(\dfrac{\sqrt{a^2 + b^2} \cdot \sin x}{b}\right).$

7. $y = \dfrac{7^x \left(3\sin 3x + \cos 3x \cdot \ln 7\right)}{9 + \ln^2 7}.$

8. $y = \ln \dfrac{\sin x}{\cos x + \sqrt{\cos 2x}}.$

9. $y = \dfrac{1}{a(1 + a^2)} \left(\operatorname{arctg}(a \cos x) + a \ln\left(\operatorname{tg}\frac{x}{2}\right)\right).$

10. $y = -\dfrac{1}{3\sin^3 x} - \dfrac{1}{\sin x} + \dfrac{1}{2} \ln \dfrac{1 + \sin x}{1 - \sin x}.$

11. $y = (1+x^2)e^{\operatorname{arctg} x}$.

12. $y = \dfrac{\operatorname{ctg} x + x}{1 - x \cdot \operatorname{ctg} x}$.

13. $y = \dfrac{1}{2\sin\frac{\alpha}{2}} \cdot \operatorname{arctg} \dfrac{2x\sin\frac{\alpha}{2}}{1-x^2}$.

14. $y = \operatorname{arctg} \dfrac{\sqrt{\sqrt{x^4+1}-x^2}}{x}, \quad x > 0.$

15. $y = \dfrac{6^x(\sin 4x \cdot \ln 6 - 4\cos 4x)}{16 + \ln^2 6}$.

16. $y = \operatorname{arctg} \dfrac{\sqrt{2\operatorname{tg} x}}{1 - \operatorname{tg} x}$.

17. $y = \operatorname{arctg} \dfrac{2\sin x}{\sqrt{9\cos^2 x - 4}}$.

18. $y = \dfrac{5^x(2\sin 2x + \cos 2x \cdot \ln 5)}{4 + \ln^2 5}$.

19. $y = \ln \dfrac{\sqrt{2} + \operatorname{th} x}{\sqrt{2} - \operatorname{th} x}$.

20. $y = \dfrac{3^x(4\sin 4x + \ln 3 \cdot \cos 4x)}{16 + \ln^2 3}$.

21. $y = \dfrac{4^x(\ln 4 \cdot \sin 4x - 4\cos 4x)}{16 + \ln^2 4}$.

22. $y = \dfrac{\cos x}{\sin^2 x} - 2\cos x - 3\ln\left(\operatorname{tg}\frac{x}{2}\right)$.

23. $y = \dfrac{5^x(\sin 3x \cdot \ln 5 - 3\cos 3x)}{9 + \ln^2 5}$.

24. $y = x - \ln(1+e^x) - 2e^{-\frac{x}{2}}\operatorname{arctg} e^{\frac{x}{2}}$.

25. $y = \dfrac{2^x(\sin x + \cos x \cdot \ln 2)}{1 + \ln^2 2}$.

26. $y=\dfrac{\ln(\operatorname{ctg}x+\operatorname{ctg}\alpha)}{\sin\alpha}$.

27. $y=2\dfrac{\cos x}{\sin^4 x}+3\dfrac{\cos x}{\sin^2 x}$.

28. $y=\dfrac{\cos x}{3(2+\sin x)}+\dfrac{4}{3\sqrt{3}}\operatorname{arctg}\dfrac{2\operatorname{tg}\left(\frac{x}{2}\right)+1}{\sqrt{3}}$.

29. $y=\dfrac{3^x(\ln 3\cdot\sin 2x-2\cos 2x)}{\ln^2 3+4}$.

30. $y=\dfrac{1}{2}\ln\dfrac{1+\cos x}{1-\cos x}-\dfrac{1}{\cos x}-\dfrac{1}{3\cos^3 x}$.

Javoblar. 14.1 $\dfrac{1}{\cos x\cdot\cos(\alpha-x)}$; **14.2** $\dfrac{\sin x}{\sin(x-\alpha)}$;

14.4 $-\dfrac{\sin^3 x}{\left(\sqrt{\cos 2x}+\cos^2 x\right)\cdot\sqrt[4]{(\cos 2x)^3}}$;

14.5 $\dfrac{3+3\sin^2 x}{\cos^3 x}+\dfrac{2-6\sin^2 x}{\cos^5 x}$; **14.6** $\dfrac{\cos x}{\sqrt{b^2\cdot\cos^2 x-a^2\cdot\sin^2 x}}$; **14.7** $7^x\cdot\cos 3x$;

14.8 $\dfrac{1}{\sin x\cdot\sqrt{\cos 2x}}$; **14.9** $\dfrac{\cos x\cdot ctgx}{1+a^2\cdot\cos^2 x}$; **14.10** $\dfrac{1}{\cos x\cdot\sin^4 x}$; **14.11** $(2x+1)\cdot e^{\operatorname{arctg}x}$;

14.12 $-\dfrac{x^2}{(\sin x-x\cdot\cos x)^2}$; **14.13** $\dfrac{1+x^2}{(1-x^2)^2+4x^2\cdot\sin^2\frac{\alpha}{2}}$;

14.14 $-\dfrac{1}{\left(x^4+1\right)\cdot\sqrt{\sqrt{x^4+1}-x^2}}$; **14.15** $6^x\sin 4x$; **14.16** $\dfrac{1-tgx+\sqrt{2tgx}}{\sqrt{2tgx}}$;

14.17 $\dfrac{2}{\cos x\sqrt{9\cos^2 x-4}}$; **14.18** $5^x\cdot\cos 2x$; **14.19** $\dfrac{2\sqrt{2}}{\operatorname{ch}^2 x+1}$; **14.20** $3^x\cdot\cos 4x$;

14.21 $4^x\sin 4x$; **14.22** $-\dfrac{2+3\sin^2 x}{\sin^3 x}$; **14.23** $5^x\sin 3x$; **14.24** $x\cdot e^{-\frac{x}{2}}\cdot arctge^{\frac{x}{2}}$;

14.25 $2^x\cdot\cos x$; **14.26** $-\dfrac{1}{\sin(\alpha+x)}$; **14.27** $3\cos ecx-8\cos ec^5 x$; **14.28** $\dfrac{2\sin x+7}{3(2+\sin x)^2}$;

14.29 $3^x\sin 2x$; **14.30** $-\dfrac{1}{\sin x\cdot\cos^4 x}$.

Parametrik berilgan funksiyalarning hosilalari

Agar x ning funksiyasi y ushbu

$$\begin{cases} x = \varphi(t), \\ y = \psi(t) \end{cases}$$

parametrik tenglamalar bilan berilgan bo'lsa, u holda y ning x bo'yicha hosilasi y'_x

$$y'_x = \frac{y'_t}{x'_t}$$

tenglik bilan aniqlanadi.

15–masala. y'_x funksiyaning hosilasini toping.

$$\begin{cases} x = \ln\left(t + \sqrt{1+t^2}\right) \\ y = \sqrt{1+t^2} - \ln\frac{1+\sqrt{1+t^2}}{t} \end{cases}$$

$$x'_t = \left(\ln\left(t + \sqrt{1+t^2}\right)\right)' = \frac{1}{t+\sqrt{1+t^2}} \cdot \left(1 + \frac{1}{2\sqrt{1+t^2}} \cdot 2t\right) =$$

$$= \frac{1}{t+\sqrt{1+t^2}} \cdot \frac{\sqrt{1+t^2}+t}{\sqrt{1+t^2}} = \frac{1}{\sqrt{1+t^2}}.$$

$$y'_t = \left(\sqrt{1+t^2} - \ln\frac{1+\sqrt{1+t^2}}{t}\right)' = \frac{1}{2\sqrt{1+t^2}}\cdot 2t - \frac{1}{1+\sqrt{1+t^2}}\cdot\left(\frac{1+\sqrt{1+t^2}}{t}\right)' =$$

$$= \frac{t}{\sqrt{1+t^2}} - \frac{t}{1+\sqrt{1+t^2}}\cdot\frac{\frac{1}{2\sqrt{1+t^2}}\cdot 2t\cdot t - \left(1+\sqrt{1+t^2}\right)}{t^2} =$$

$$= \frac{t}{\sqrt{1+t^2}} - \frac{t}{1+\sqrt{1+t^2}}\cdot\frac{t^2-\sqrt{1+t^2}-1-t^2}{t^2\sqrt{1+t^2}} =$$

$$= \frac{t}{\sqrt{1+t^2}} + \frac{1}{1+\sqrt{1+t^2}}\cdot\frac{\sqrt{1+t^2}+1}{t\sqrt{1+t^2}} = \frac{t^2+1}{t\sqrt{1+t^2}}.$$

Natijada:

$$y'_x = \frac{y'_t}{x'_t} = \left(\frac{1}{\sqrt{1+t^2}}\right)/\left(\frac{t^2+1}{t\sqrt{1+t^2}}\right) = \frac{t}{t^2+1}.$$

1. $y = \begin{cases} x = \frac{3t^2+1}{t^3} \\ y = \sin\left(\frac{t^3}{3}+t\right) \end{cases}$

2. $\begin{cases} x = \sqrt{1-t^2} \\ y = \operatorname{tg}\sqrt{1+t} \end{cases}$

3. $\begin{cases} x = \sqrt{2t-t^2} \\ y = \frac{1}{\sqrt[3]{(1-t)^2}} \end{cases}$

4. $\begin{cases} x = \arcsin(\sin t) \\ y = \arccos(\cos t) \end{cases}$

5. $\begin{cases} x = \ln(t+\sqrt{t^2+1}) \\ y = t\sqrt{t^2+1} \end{cases}$

6. $\begin{cases} x = \sqrt{2t-t^2} \\ y = \arcsin(t-1) \end{cases}$

7. $\begin{cases} x = \operatorname{ctg}(2e^t) \\ y = \ln(\operatorname{tg} e^t) \end{cases}$

8. $\begin{cases} x = \ln(\operatorname{ct\,g}\, t) \\ y = \frac{1}{\cos^2 t} \end{cases}$

9. $\begin{cases} x = \operatorname{arctg} e^{\frac{t}{2}} \\ y = \sqrt{e^t+1} \end{cases}$

10. $\begin{cases} x = \ln\sqrt{\frac{1-t}{1+t}} \\ y = \sqrt{1-t^2} \end{cases}$

11. $\begin{cases} x = \ln \frac{1}{\sqrt{1-t^4}} \\ y = \arcsin \frac{1-t^2}{1+t^2} \end{cases}$

12. $\begin{cases} x = \sqrt{1-t^2} \\ y = \frac{t}{\sqrt{1-t^2}} \end{cases}$

13. $\begin{cases} x = \arcsin\left(\sqrt{1-t^2}\right) \\ y = (\arccos t)^2 \end{cases}$

14. $\begin{cases} x = \frac{t}{\sqrt{1-t^2}} \\ y = \ln \frac{1+\sqrt{1-t^2}}{t} \end{cases}$

15. $\begin{cases} x = (1+\cos^2 t)^2 \\ y = \dfrac{\cos t}{\sin^2 t} \end{cases}$

16. $\begin{cases} x = \ln \frac{1-t}{1+t} \\ y = \sqrt{1-t^2} \end{cases}$

17. $\begin{cases} x = \arccos \frac{1}{t} \\ y = \sqrt{1-t^2} + \arcsin \frac{1}{t} \end{cases}$

18. $\begin{cases} x = \frac{1}{\ln t} \\ \ln \frac{1+\sqrt{1-t^2}}{t} \end{cases}$

19. $\begin{cases} x = \arcsin \sqrt{t} \\ y = \sqrt{1+\sqrt{t}} \end{cases}$

20. $\begin{cases} x = (\arcsin t)^2 \\ y = \frac{t}{\sqrt{1-t^2}} \end{cases}$

21. $\begin{cases} x = t\sqrt{t^2+1} \\ y = \ln \frac{1+\sqrt{1+t^2}}{t} \end{cases}$

22. $\begin{cases} x = \operatorname{arctg} t \\ y = \ln \frac{\sqrt{1+t^2}}{t+1} \end{cases}$

23. $\begin{cases} x = \ln\left(1-t^2\right) \\ y = \arcsin \sqrt{1-t^2} \end{cases}$

24. $\begin{cases} x = \operatorname{arctg} \frac{t+1}{t-1} \\ y = \arcsin \sqrt{1-t^2} \end{cases}$

25. $\begin{cases} x = \ln \sqrt{\frac{1-\sin t}{1+\sin t}} \\ y = \frac{1}{2}\operatorname{tg}^2 t + \ln \cos t \end{cases}$

26. $\begin{cases} x = \sqrt{t-t^2} - \operatorname{arctg} \sqrt{\frac{1-t}{t}} \\ y = \sqrt{t} - \sqrt{1-t} \cdot \arcsin \sqrt{t} \end{cases}$

27. $\begin{cases} x = \ln(\operatorname{tg} t) \\ y = \dfrac{1}{\sin^2 t} \end{cases}$

28. $\begin{cases} x = \frac{t^2 \ln t}{1-t^2} + \ln \sqrt{1-t^2} \\ y = \frac{t}{\sqrt{1-t^2}} \arcsin t + \ln \sqrt{1-t^2} \end{cases}$

29. $\begin{cases} x = e^{\sec^2 t} \\ y = \operatorname{tg} t \cdot \ln \cos t + \operatorname{tg} t - t \end{cases}$

30. $\begin{cases} x = \frac{t}{\sqrt{1-t^2}} \cdot \arcsin t + \ln \sqrt{1-t^2} \\ y = \frac{t}{\sqrt{1-t^2}} \end{cases}$

Javoblar. 15.1 $-t^4\cdot\cos\left(\frac{t^3}{3}+t\right)$; **15.2** $\frac{\sqrt{1-t}}{2t\cdot\cos^2\left(\sqrt{1+t}\right)}$; **15.3** $\frac{2\sqrt{2t-t^2}}{3(1-t^2)\cdot\sqrt[3]{(1-t)^2}}$;

15.4 1; **15.5** $2t^2+1$; **15.6** $\frac{1}{(1-t)}$; **15.7** $-\sin(2e^t)$; **15.8** $-2tg^2t$; **15.9** $\sqrt{e^{2t}+e^t}$;

15.10 $t\cdot\sqrt{1-t^2}$; **15.11** $\frac{t^2-1}{t^3}$; **15.12** $\frac{1}{t\cdot(t^2-1)}$; **15.13** $\frac{2\arccos t\cdot\sqrt{1-t^2}}{\sqrt{1-t^2}}$; **15.14** $\frac{t^2-1}{t}$;

15.15 $\frac{1}{4\sin^4 t\cdot\cos t}$; **15.16** $\frac{t\cdot\sqrt{1-t^2}}{2}$; **15.17** t^2-1; **15.18** $\frac{\ln^2 t}{\sqrt{1-t^2}}$; **15.19** $2\sqrt{\frac{1-t}{1+\sqrt{t}}}$;

15.20 $\frac{1}{(1-t^2)\cdot 2\arcsin t}$; **15.21** $-\frac{1}{t\cdot(2t^2+1)}$; **15.22** $\frac{t-1}{t+1}$; **15.23** $\frac{\sqrt{1-t^2}}{2t}$;

15.24 $\frac{t^2+1}{\sqrt{1-t^2}}$; **15.25** $\frac{\sin t\cdot\cos t-1}{\cos t}$; **15.26** $\frac{\sqrt{t}\cdot\arcsin\sqrt{t}}{2\cdot(1-t)}$; **15.27** $-2ctg^2t$;

15.28 $\frac{\arcsin t\cdot\sqrt{1-t^2}}{2t\cdot\ln t}$; **15.29** $\frac{1}{2}\cdot ctgt\cdot\ln\cos t\cdot e^{-\sec^2 t}$; **15.30** $\frac{1}{\arcsin t}$.

Egri chiziqqa nuqtadan o'tkazilgan urinma va normal tenglamasi

$y=f(x)$ egri chiziqning $M_0(x_0, f(x_0))$ nuqtasidan o'tkazilgan urinma tenglamasi:

$$y-f(x_0)=f'(x_0)(x-x_0)$$

$y=f(x)$ egri chiziqning $M_0(x_0, f(x_0))$ nuqtasidan o'tkazilgan normal(perpendikulyar) tenglamasi:

$$y-y_0=-\frac{1}{f'(x_0)}(x-x_0)\quad \left(f'(x_0)\neq 0\right).$$

16–masala. Funksiya grafigining $t=t_0$ parametrning qiymatiga mos kelgan egri chiziqning nuqtasiga o'tkazilgan urinma va normal tenglamasini tuzing.

$$\begin{cases} x=2e^t \\ y=e^{-t}, \quad t_0=0 \end{cases}$$

Echim:

$t_0 = 0$ ekanligidan, u holda

$$x_0 = 2e^0 = 2$$
$$y_0 = e^{-0} = 1$$

Hosilalarni topamiz:

$$x'_t = \left(2e^t\right)' = 2e^t$$
$$y'_t = \left(e^{-t}\right)' = -e^{-t}$$
$$y'_x = \frac{y'_t}{x'_t} = \frac{-e^{-t}}{2e^t} = -\frac{1}{2e^{2t}}$$

u holda

$$y'_0 = -\frac{1}{2e^{2\cdot 0}} = -\frac{1}{2}.$$

urinma tenglamasi:

$$y - y_0 = y'_0(x - x_0)$$
$$y - 1 = -\frac{1}{2}\cdot(x-2)$$
$$y = -\frac{1}{2}\cdot x + 2$$

normal tenglamasi:

$$y - y_0 = -\frac{1}{y'_0}(x - x_0)$$
$$y - 1 = -\frac{1}{\left(-\frac{1}{2}\right)}(x-2)$$
$$y = 2x - 3.$$

1. $\begin{cases} x = a\sin^3 t \\ y = a\cos^3 t, \quad t_0 = \dfrac{\pi}{3} \end{cases}$

2. $\begin{cases} x = \sqrt{3}\cdot\cos t \\ y = \sin t, \quad t_0 = \dfrac{\pi}{3} \end{cases}$

3. $\begin{cases} x = a(t - \sin t) \\ y = a(1 - \cos t), \quad t_0 = \dfrac{\pi}{3} \end{cases}$

4. $\begin{cases} x = 2t - t^2 \\ y = 3t - t^3, \quad t_0 = 1 \end{cases}$

5. $\begin{cases} x = \frac{2t+t^2}{1+t^3} \\ y = \frac{2t-t^2}{1+t^3}, \quad t_0 = 1 \end{cases}$

6. $\begin{cases} x = \arcsin \frac{t}{\sqrt{1+t^2}} \\ y = \arccos \frac{t}{\sqrt{1+t^2}}, \quad t_0 = -1 \end{cases}$

7. $\begin{cases} x = t(t\cdot\cos t - 2\sin t) \\ y = t(t\cdot\sin t + 2\cos t), \quad t_0 = \dfrac{\pi}{4} \end{cases}$

8. $\begin{cases} x = \frac{3at}{1+t^2} \\ y = \frac{3at^2}{1+t^2}, \quad t_0 = 2 \end{cases}$

9. $\begin{cases} x = 2\ln(\operatorname{ctg} t) + \operatorname{ctg} t \\ y = \operatorname{tg} t + \operatorname{ctg} t, \quad t_0 = \dfrac{\pi}{4} \end{cases}$

10. $\begin{cases} x = \frac{1}{2}\cdot t^2 - \frac{1}{4}\cdot t^4 \\ y = \frac{1}{2}\cdot t^2 + \frac{1}{3}\cdot t^3, \quad t_0 = 0 \end{cases}$

11. $\begin{cases} x = a\cdot t\cdot\cos t \\ y = a\cdot t\cdot\sin t, \quad t_0 = \dfrac{\pi}{2} \end{cases}$

12. $\begin{cases} x = \sin t \\ y = \cos t, \quad t_0 = \dfrac{\pi}{6} \end{cases}$

13. $\begin{cases} x = \arcsin \frac{t}{\sqrt{1+t^2}} \\ y = \arccos \frac{1}{\sqrt{1+t^2}}, \quad t_0 = 1 \end{cases}$

14. $\begin{cases} x = \frac{1+\ln t}{t^2} \\ y = \frac{3+2\ln t}{t}, \quad t_0 = 1 \end{cases}$

15. $\begin{cases} x = \frac{1+t}{t^2} \\ y = \frac{3}{2t^2} + \frac{2}{t}, \quad t_0 = 2 \end{cases}$

16. $\begin{cases} x = a\cdot\sin^3 t \\ y = a\cdot\cos^3 t, \quad t_0 = \dfrac{\pi}{6} \end{cases}$

17. $\begin{cases} x = a(t\cdot\sin t + \cos t) \\ y = a(\sin t - t\cdot\cos t), \quad t_0 = \dfrac{\pi}{4} \end{cases}$

18. $\begin{cases} x = \frac{t+1}{t} \\ y = \frac{t-1}{t}, \quad t_0 = -1 \end{cases}$

19. $\begin{cases} x = 1 - t^2 \\ y = 1 - t^3, \quad t_0 = 2 \end{cases}$

20. $\begin{cases} x = \ln(1 + t^2) \\ y = t - \operatorname{arctg} t, \quad t_0 = 1 \end{cases}$

21. $\begin{cases} x = t(1-\sin t) \\ y = t\cdot\cos t, \quad t_0 = 0 \end{cases}$

22. $\begin{cases} x = \frac{1+t^3}{t^2-1} \\ y = \frac{t}{t^2-1}, \quad t_0 = 2 \end{cases}$

23. $\begin{cases} x = 3\cos t \\ y = 4\sin t, \quad t_0 = \dfrac{\pi}{4} \end{cases}$

24. $\begin{cases} x = t - t^4 \\ y = t^2 - t^3, \quad t_0 = 1 \end{cases}$

25. $\begin{cases} x = t^3 + 1 \\ y = t^2 + t + 1, \quad t_0 = 1 \end{cases}$

26. $\begin{cases} x = 2\cos t \\ y = \sin t, \quad t_0 = -\dfrac{\pi}{3} \end{cases}$

27. $\begin{cases} x = 2\mathrm{tg}t \\ y = 2\sin^2 t + \sin 2t, \quad t_0 = \dfrac{\pi}{4} \end{cases}$

28. $\begin{cases} x = t^3 + 1 \\ y = t^2, \quad t_0 = -2 \end{cases}$

29. $\begin{cases} x = \sin t \\ y = a^t, \quad t_0 = 0 \end{cases}$

30. $\begin{cases} x = \sin t \\ y = \cos 2t, \quad t_0 = \dfrac{\pi}{6} \end{cases}$

Javoblar. **16.1** $y = -\frac{x}{\sqrt{3}} + \frac{a}{2};\ y = \sqrt{3}x - a$; **16.2** $y = 3x - \sqrt{3}$; **16.3** $y = -\frac{x}{\sqrt{3}} + \frac{a\cdot\pi}{3\sqrt{3}}$;

16.4 $y = 3x - 1;\ y = -\frac{x}{3} + 2\frac{1}{3}$; **16.5** $y = 3x - 4;\ y = -\frac{x}{3} + 1$; **16.6** $y = 2x + \frac{3\pi}{4}$;

$y = -\frac{x}{2} + \frac{\pi}{8}$; **16.7** $y = -x + \frac{\pi^2\cdot\sqrt{2}}{16};\ y = x + \frac{\pi\cdot\sqrt{2}}{2}$; **16.8** $y = -\frac{4}{3}\cdot x - 4a;$

$y = \frac{3}{4}\cdot x + \frac{3a}{2}$; **16.9** $y = 2;\ x = 1$; **16.10** $y = x;\ y = -x$; **16.11** $y = -\frac{2x}{\pi} + \frac{a\cdot\pi}{2};$

$y = -\frac{\pi\cdot x}{2} + \frac{a\cdot\pi}{2}$; **16.12** $y = -\frac{1}{\sqrt{3}}x + \frac{2}{\sqrt{3}};\ y = \sqrt{3}\cdot x$; **16.13** $y = 2x - \frac{\pi}{4};\ y = -\frac{x}{2} + \frac{3\pi}{8};$

16.14 $y = x + 2;\ y = -x + 4$; **16.15** $y = \frac{7x}{4} + 2\frac{17}{48};\ y = -\frac{4x}{7} + 4\frac{2}{21}$; **16.16**

$y = -\sqrt{3}x + \frac{4\sqrt{3}a}{8};\ y = \frac{x}{\sqrt{3}} + \frac{a}{\sqrt{3}}$; **16.17** $y = x + \frac{\sqrt{2}\cdot a\cdot\pi}{4};\ y = -x + \sqrt{2}\cdot a$;

16.18 $y = -x + 2;\ y = x + 2$; **16.19** $y = \frac{11x}{4} + \frac{9}{4};\ y = -\frac{4x}{11} - \frac{78}{11}$;

16.20 $y=\frac{x}{2}+\frac{4-\pi-\ln 4}{4}$; $y=-2x+\frac{4-\pi+\ln 16}{2}$; **16.21** $y=0;\ x=0$;

16.22 $x=3;\ y=\frac{2}{3}$; **16.23** $y=-\frac{4}{3}x+4\sqrt{2};\ y=\frac{3}{4}x+\frac{7\sqrt{2}}{8}$; **16.24** $y=\frac{x}{3};\ y=-3x$;

16.25 $y=x+1;\ y=-x+5$; **16.26** $y=\frac{\sqrt{3}}{6}x-\frac{2\sqrt{3}}{3};\ y=-2\sqrt{3}\cdot x+\frac{3\sqrt{3}}{2}$;

16.27 $y=-2x+6;\ y=\frac{1}{2}x+1$; **16.28** $y=-\frac{1}{3}x+7;\ y=3x-23$;

16.29 $y=x\cdot\ln a+1;\ y=-\frac{x}{\ln a}+1$; **16.30** $y=-2x+1{,}5;\ y=\frac{1}{2}x+0{,}25$.

Yuqori tartibli hosilalar

Birinchi tartibli hosiladan olingan hosila, ya'ni

$$(y')'=(f'(x))'\ \text{ yoki }\ y''=f''(x)$$

$y=f(x)$ funksiyaning ikkinchi tartibli hosilasi deyiladi.

Ikkinchi tartibli hosilaning hosilasiga uchinchi tartibli hosila deyiladi va y''', $f'''(x)$, $\frac{d^3y}{dx^3}$ belgilarning biri bilan belgilanadi.

Umuman, $y=f(x)$ funksiyaning $n-$tartibli hosilasi deb, uning $(n-1)-$tartibli hosilasining hosilasiga aytiladi va $y^{(n)}$, $f^{(n)}(x)$, $\frac{d^ny}{dx^n}$ belgilarning biri bilan belgilanadi.

17–masala. Funksiyaning $n-$tartibli hosilasini toping.

$$y=3^{2x+5}.$$

Echim:

$$y'=\left(3^{2x+5}\right)'=3^{2x+5}\cdot\ln 3\cdot 2.$$

$$y''=\left(3^{2x+5}\cdot\ln 3\cdot 2\right)'=3^{2x+5}\cdot\ln^2 3\cdot 2^2.$$

Shunday qilib,

$$y^{(n)} = 3^{2x+5} \cdot \ln^n 3 \cdot 2^n = 2^n \cdot \ln^n 3 \cdot 3^{2x+5}.$$

1. $y = x \cdot e^{ax}$.

2. $y = \sin 2x + \cos(x+1)$.

3. $y = \sqrt[5]{e^{7x-1}}$.

4. $y = \dfrac{4x+7}{2x+3}$.

5. $y = \lg(5x+2)$.

6. $y = a^{3x}$.

7. $y = \dfrac{x}{2(3x+2)}$.

8. $y = \lg(x+4)$.

9. $y = \sqrt{x}$.

10. $y = \dfrac{2x+5}{13(3x+1)}$.

11. $y = 2^{3x+5}$.

12. $y = \sin(x+1) + \cos 2x$.

13. $y = \sqrt[3]{e^{2x+1}}$.

14. $y = \dfrac{4+15x}{5x+1}$.

15. $y = \lg(3x+1)$.

16. $y = 7^{5x}$.

17. $y = \dfrac{x}{9(4x+9)}$.

18. $y = \lg(1+x)$.

19. $y = \dfrac{4}{x}$.

20. $y = \dfrac{5x+1}{13(2x+3)}$.

21. $y = a^{2x+3}$.

22. $y = \sin(3x+1) + \cos 5x$.

23. $y = \sqrt{e^{3x+1}}$.

24. $y = \dfrac{11+12x}{6x+5}$.

25. $y = \lg(2x+7)$.

26. $y = 2^{kx}$.

27. $y = \dfrac{x}{x+1}$.

28. $y = \log_3(x+5)$.

29. $y = \dfrac{1+x}{1-x}$.

30. $y = \dfrac{7x+1}{17(4x+3)}$.

Javoblar. 17.1 $(n+ax) \cdot e^{ax} \cdot a^{n-1}$; **17.2** $2^n \sin\left(\dfrac{\pi}{2}n + 2x\right) + \cos\left(\dfrac{\pi}{2}n + x + 1\right)$;

17.3 $\left(\frac{7}{5}\right)^n \cdot \sqrt[5]{e^{7x-1}}$; **17.4** $\frac{(-1)^n \cdot 2^n \cdot n!}{(2x+3)^{n+1}}$; **17.5** $\frac{(-1)^{n-1} \cdot (n-1)! \cdot 5^n}{\ln 10 \cdot (5x+2)^n}$; **17.6** $a^{3x} \cdot 3^n \ln^n a$;

17.7 $\frac{(-1)^{n-1} \cdot n! \cdot 3^{n-1}}{(3x+2)^{n+1}}$; **17.8** $\frac{(-1)^{n-1} \cdot (n-1)!}{\ln 10 \cdot (x+4)^n}$; **17.9** $\frac{(-1)^{n-1} \cdot \prod_{k=1}^{n-1}(2k-1)}{2^{(n+1)} \cdot x^{n-1} \cdot \sqrt{x}}$;

17.10 $\frac{(-1)^n \cdot n! \cdot 3^{n-1}}{(3x+1)^{n+1}}$; **17.11** $2^{3x+5} \cdot 3^n \cdot \ln^n 2$;

17.12 $\sin\left(\frac{3\pi}{2} \cdot n + x + 1\right) + 2^n \cdot \cos\left(\frac{3\pi}{2} \cdot n + 2x\right)$;

17.13 $\left(\frac{2}{3}\right)^n \cdot \sqrt[3]{e^{2x+1}}$; **17.14** $\frac{(-1)^n \cdot n! \cdot 5^n}{(5x+1)^{n+1}}$; **17.15** $\frac{(-1)^{n-1} \cdot (n-1)! \cdot 3^n}{\ln 10 \cdot (3x+1)^n}$; **17.16** $5^n \ln(7)^n \cdot 7^{5x}$;

17.17 $\frac{(-1)^{n-1} \cdot n! \cdot 4^{n-1}}{(4x+9)^{n+1}}$; **17.18** $\frac{(-1)^{n-1} \cdot (n-1)!}{\ln 10 \cdot (1+x)^n}$; **17.19** $\frac{4 \cdot (-1)^n \cdot n!}{x^n}$; **17.20** $\frac{(-1)^{n-1} \cdot n! \cdot 2^{n-1}}{(2x+3)^{n+1}}$;

17.21 $a^{2x+3} \cdot 2^n \cdot \ln^n a$; **17.22** $3^n \cdot \sin\left(\frac{3\pi}{2} \cdot n + 3x + 1\right) + 5^n \cdot \cos\left(\frac{3\pi}{2} \cdot n + 5x\right)$;

17.23 $\left(\frac{3}{2}\right)^n \cdot \sqrt{e^{3x+1}}$; **17.24** $(-1)^n \cdot n! \cdot 6^n \cdot (6x+5)^{-n-1}$;

17.25 $(-1)^{n-1} \cdot \frac{2^n \cdot (n-1)!}{\ln 10} \cdot (2x+7)^{-n}$; **17.26** $2^{kx} \cdot k^n \ln^n 2$; **17.27** $(-1)^{k+2} \cdot \frac{(k+1)! \cdot}{(x+1)^{k+2}}$;

17.28 $\frac{(-1)^{n-1} \cdot (n-1)!}{\ln 3 \cdot (x+5)^n}$; **17.29** $\frac{2 \cdot (k+1)!}{(1-x)^{k+2}}$; **17.30** $\frac{(-1)^{n-1} \cdot n! \cdot 4^{n-1}}{(4x+3)^{n+1}}$.

Yuqori tartibli hosilalar

Birinchi tartibli hosiladan olingan hosila, ya'ni

$$(y')' = (f'(x))' \quad \text{yoki} \quad y'' = f''(x)$$

$y = f(x)$ funksiyaning ikkinchi tartibli hosilasi deyiladi.

Ikkinchi tartibli hosilaning hosilasiga uchinchi tartibli hosila deyiladi va y''', $f'''(x)$, $\frac{d^3y}{dx^3}$ belgilarning biri bilan belgilanadi.

Umuman, $y = f(x)$ funksiyaning $n-$tartibli hosilasi deb, uning

$(n-1)-$tartibli hosilasining hosilasiga aytiladi va $y^{(n)}$, $f^{(n)}(x)$, $\frac{d^n y}{dx^n}$ belgilarning biri bilan belgilanadi.

Ikkita funksiya ko'paytmasining $n-$tartibli hosilasi ushbu

$$(u\cdot v)^{(n)} = u^{(n)}v + nu^{(n-1)}v' + \frac{n(n-1)}{2!}u^{(n-2)}v'' + \ldots +$$
$$+ \frac{n(n-1)\ldots(n-k+1)}{k!}u^{(n-k)}v^{(k)} + \ldots + nu'v^{(n-1)} + uv^{(n)}$$

formuladan foydalanib topiladi. Bu formula Leybnits formulasi deyiladi. Xususan,

$$(u\cdot v)' = u'v + u\cdot v'.$$

18–masala. Funksiyalarning talab qilingan tartibli hosilasini toping.

$$y = \left(x^3+3\right)e^{4x+3}, \quad y^{IV} = ?$$

Echim:

$$y' = \left(\left(x^3+3\right)e^{4x+3}\right)' = 3x^2e^{4x+3} + \left(x^3+3\right)e^{4x+3}\cdot 4 =$$
$$= (4x^3+3x^2+12)\cdot e^{4x+3}.$$

$$y'' = \left((4x^3+3x^2+12)\cdot e^{4x+3}\right)' =$$
$$= (12x^2+6x)\cdot e^{4x+3} + (4x^3+3x^2+12)\cdot e^{4x+3}\cdot 4 =$$
$$= (16x^3+24x^2+6x+48)\cdot e^{4x+3}.$$

$$y''' = \left((16x^3+24x^2+6x+48)\cdot e^{4x+3}\right)' =$$
$$= (48x^2+48x+6)\cdot e^{4x+3} + (16x^3+24x^2+6x+48)\cdot e^{4x+3}\cdot 4 =$$
$$= (64x^3+144x^2+72x+198)\cdot e^{4x+3}.$$

$$y^{IV} = \left((64x^3+144x^2+72x+198)\cdot e^{4x+3}\right)' =$$
$$= (192x^2+288x+72)\cdot e^{4x+3} + (64x^3+144x^2+72x+198)\cdot e^{4x+3}\cdot 4 =$$
$$= (256x^3+768x^2+576x+864)\cdot e^{4x+3}.$$

1. $y=(2x^2-7)\ln(x-1),\quad y^{\mathrm{V}}=?$

2. $y=(3-x^2)\ln^2 x,\quad y'''=?$

3. $y=x\cdot\cos x^2,\quad y'''=?$

4. $y=\dfrac{\ln(x-1)}{\sqrt{x-1}},\quad y'''=?$

5. $y=\dfrac{\log_2 x}{x^3},\quad y'''=?$

6. $y=(4x^3+5)e^{2x+1},\quad y^{\mathrm{V}}=?$

7. $y=x^2\cdot\sin(5x-3),\quad y'''=?$

8. $y=\dfrac{\ln x}{x^2},\quad y^{\mathrm{IV}}=?$

9. $y=(2x+3)\ln^2 x,\quad y'''=?$

10. $y=(1+x^2)\operatorname{arctg}x,\quad y'''=?$

11. $y=\dfrac{\ln x}{x^3},\quad y^{\mathrm{IV}}=?$

12. $y=(4x+3)\cdot 2^{-x},\quad y^{\mathrm{V}}=?$

13. $y=e^{1-2x}\cdot\sin(2+3x),\quad y^{\mathrm{IV}}=?$

14. $y=\dfrac{\ln(3+x)}{3+x},\ y'''=?.$

15. $y=(2x^3+1)\cos x,\quad y^{\mathrm{V}}=?.$

16. $y=(x^2+3)\ln(x-3),\quad y^{\mathrm{IV}}=?.$

17. $y=(1-x-x^2)e^{\frac{x-1}{2}},\ y^{\mathrm{IV}}=?.$

18. $y=\dfrac{1}{x}\cdot\sin 2x,\quad y'''=?.$

19. $y=(x+7)\ln(x+4),\quad y^{\mathrm{V}}=?.$

20. $y=(3x-7)\cdot 3^{-x},\quad y^{\mathrm{IV}}=?.$

21. $y=\dfrac{\ln(2x+5)}{2x+5},\quad y'''=?.$

22. $y=e^{\frac{x}{2}}\cdot\sin 2x,\quad y^{\mathrm{IV}}=?.$

23. $y=\dfrac{\ln x}{x^5},\ y'''=?.$

24. $y=x\ln(1-3x),\quad y^{\mathrm{IV}}=?.$

25. $y=(x^2+3x+1)e^{3x+2},\quad y^{\mathrm{V}}=?.$

26. $y=(5x-8)e^{-x},\quad y^{\mathrm{IV}}=?.$

27. $y=\dfrac{\ln(x-2)}{x-2},\quad y^{\mathrm{V}}=?.$

28. $y=e^{-x}\cdot(\cos 2x-3\sin 2x),\ y^{\mathrm{IV}}=?.$

29. $y=(5x-1)\cdot\ln^2 x,\quad y'''=?.$

30. $y=\dfrac{\log_3 x}{x^2},\quad y^{\mathrm{IV}}=?.$

Javoblar. 18.1 $\dfrac{8\cdot(x^2-5x-11)}{(x-1)^5}$; **18.2** $\dfrac{-4\cdot\ln x}{x}+\dfrac{6\ln x-7x^2-15}{x^3}$;

18.3 $-24x^2\cdot\cos x^2+(8x^4-6)\cdot\sin x^2$; **18.4** $\dfrac{46-15\ln(x-1)}{8\sqrt{(x-1)^7}}$; **18.5** $\dfrac{47-60\ln x}{\ln 2\cdot x^6}$;

18.6 $32(4x^3+30x^2+60x+35)e^{2x+1}$;

18.7 $-150x\cdot\sin(5x-3)+(30-125x^2)\cdot\cos(5x-3)$; **18.8** $\dfrac{-154+120\ln x}{x^6}$;

18.9 $\dfrac{4\ln x\cdot(3-x)-18}{x^3}$; **18.10** $\dfrac{4}{(1+x^2)^2}$; **18.11** $\dfrac{-342+360\ln x}{x^7}$;

18.12 $\left(-\ln^5 2\cdot(4x+3)+20\ln^4 2\right)\cdot 2^{-x}$;

18.13 $-122e^{1-2x}\cdot\sin(2+3x)-597e^{1-2x}\cdot\cos(2+3x)$; **18.14** $\dfrac{11-6\ln(3+x)}{(3+x)^4}$;

18.15 $(30x^2-120)\cos x-(2x^3-120x+1)\sin x$; **18.16** $\dfrac{-2x^2+24x-126}{(x-3)^4}$;

18.17 $-\frac{1}{16}\cdot(55+17x+x^2)e^{\frac{x-1}{2}}$; **18.18** $\frac{12x^2-6}{x^4}\cdot\sin 2x+\frac{12-8x^2}{x^3}\cdot\cos 2x$;

18.19 $\frac{-120x+1680}{(x+4)^7}$; **18.20** $(7\ln 3-12-3\ln 3\cdot x)\cdot\ln^3 3\cdot 3^{-x}$;

18.21 $\frac{88-48\ln(2x+5)}{(2x+5)^4}$; **18.22** $\frac{161}{16}\cdot e^{\frac{x}{2}}\cdot\sin 2x-15\cdot e^{\frac{x}{2}}\cdot\cos 2x$; **18.23** $\frac{107-210\ln x}{x^8}$;

18.24 $-\frac{54(4-3x)}{(1-3x)^4}$; **18.25** $3^3\cdot(9x^2+57x+35)e^{3x+2}$;

18.26 $2^{-3}\cdot\ln^3 2\cdot(5\ln 2\cdot x-8\ln 2-20)$; **18.27** $\frac{274-120\ln(x-2)}{(x-2)^6}$;

18.28 $-e^{-x}\cdot(79\cos 2x+3\sin 2x)$; **18.29** $\frac{6-2(5x+2)\ln x}{x^3}$; **18.30** $\frac{-154+120\ln x}{x^6\cdot\ln 3}$.

Parametrik ko’rinishda berilgan funksiyaning hosilasi.

Agar $y=f(x)$ funksiya rarametrik ko’rinishda berilgan bo’lsa, ya’ni

$$\begin{cases} x=\varphi(t), \\ y=\psi(t), \end{cases} \qquad \alpha\le t\le\beta,$$

bo’lsa, u holda

$$f'(x)=\frac{\psi'(t)}{\varphi'(t)}$$

va

$$f''(x)=\frac{\psi''(t)\varphi'(t)-\varphi''(t)\psi'(t)}{\varphi'^3(t)}$$

bo’ladi.

19–masala. Parametrik ko’rinishda berilgan funksiyaning y''_{xx} hosilasini toping.

$$\begin{cases} x = \ln t \\ y = \operatorname{arctg} t \end{cases}$$

Echim:

$$x'_t = (\ln t)' = \frac{1}{t}$$

$$y'_t = (\operatorname{arctg} t)' = \frac{1}{t^2+1}$$

Natijada:

$$y'_x = \frac{y'_t}{x'_t} = \frac{\frac{1}{t^2+1}}{\frac{1}{t}} = \frac{t}{t^2+1}$$

$$\left(y'_x\right)' = \left(\frac{t}{t^2+1}\right)' = \frac{1\cdot(1+t^2)-t\cdot 2t}{(1+t^2)^2} = \frac{1-t^2}{(1+t^2)^2}$$

U holda:

$$y''_{xx} = \frac{\left(y'_x\right)'_t}{x'_t} = \frac{\frac{1-t^2}{(1+t^2)^2}}{\frac{1}{t}} = \frac{t\cdot(1-t^2)}{(1+t^2)^2}.$$

1. $\begin{cases} x = \cos 2t \\ y = 2\sec^2 t \end{cases}$ **2.** $\begin{cases} x = \sqrt{1-t^2} \\ y = \dfrac{1}{t} \end{cases}$ **3.** $\begin{cases} x = e^t \cos t \\ y = e^t \sin t \end{cases}$

4. $\begin{cases} x = \operatorname{sh}^2 t \\ y = \dfrac{1}{\operatorname{ch}^2 t} \end{cases}$ **5.** $\begin{cases} x = t + \sin t \\ y = 2 - \cos t \end{cases}$ **6.** $\begin{cases} x = \dfrac{1}{t} \\ y = \dfrac{1}{1+t^2} \end{cases}$

7. $\begin{cases} x = \sqrt{t} \\ y = \dfrac{1}{\sqrt{1-t}} \end{cases}$ **8.** $\begin{cases} x = \sin t \\ y = \sec t \end{cases}$ **9.** $\begin{cases} x = \operatorname{tg} t \\ y = \dfrac{1}{\sin 2t} \end{cases}$

10. $\begin{cases} x=\sqrt{t-1} \\ y=\dfrac{t}{\sqrt{1-t}} \end{cases}$ **11.** $\begin{cases} x=\sqrt{t} \\ y=\sqrt[3]{t-1} \end{cases}$ **12.** $\begin{cases} x=\dfrac{\cos t}{1+2\cos t} \\ y=\dfrac{\sin t}{1+2\cos t} \end{cases}$

13. $\begin{cases} x=\sqrt{t^3-1} \\ y=\ln t \end{cases}$ **14.** $\begin{cases} x=\mathrm{sh}^2 t \\ y=\mathrm{th}^2 t \end{cases}$ **15.** $\begin{cases} x=\sqrt{t-1} \\ y=\dfrac{1}{\sqrt{t}} \end{cases}$

16. $\begin{cases} x=\cos^2 t \\ y=\mathrm{tg}^2 t \end{cases}$ **17.** $\begin{cases} x=\sqrt{t-3} \\ y=\ln(t-2) \end{cases}$ **18.** $\begin{cases} x=\sin t \\ \ln(\cos t) \end{cases}$

19. $\begin{cases} x=t+\sin t \\ y=2+\cos t \end{cases}$ **20.** $\begin{cases} x=t-\sin t \\ y=2-\cos t \end{cases}$ **21.** $\begin{cases} x=\cos t \\ y=\ln(\sin t) \end{cases}$

22. $\begin{cases} x=\cos t+t\cdot\sin t \\ y=\sin t-t\cdot\cos t \end{cases}$ **23.** $\begin{cases} x=e^t \\ y=\arcsin t \end{cases}$ **24.** $\begin{cases} x=\cos t \\ y=\sin^4\left(\frac{t}{2}\right) \end{cases}$

25. $\begin{cases} x=\mathrm{ch}t \\ y=\sqrt[3]{\mathrm{sh}^2 t} \end{cases}$ **26.** $\begin{cases} x=\mathrm{arctg}t \\ y=\dfrac{t^2}{2} \end{cases}$ **27.** $\begin{cases} x=2(t-\sin t) \\ y=4(2+\cos t) \end{cases}$

28. $\begin{cases} x=\sin t-t\cdot\cos t \\ y=\cos t+t\cdot\sin t \end{cases}$ **29.** $\begin{cases} x=\dfrac{1}{t^2} \\ y=\dfrac{1}{t^2+1} \end{cases}$ **30.** $\begin{cases} x=\cos t+\sin t \\ y=\sin 2t \end{cases}$

Javoblar. 19.1 $\dfrac{1}{\cos^6 t}$; **19.2** $\dfrac{3-2t^2}{t^5}$; **19.3** $\dfrac{2}{e^t\cdot(\cos t-\sin t)^3}$; **19.4** $\dfrac{2}{\mathrm{ch}^6 t}$;

19.5 $\dfrac{1}{(1+\cos t)^2}$; **19.6** $\dfrac{2(t^2-3)\cdot t^4}{(1+t^2)^3}$; **19.7** $(1+2t)\sqrt{1-t}$; **19.8** $\dfrac{1+2\sin^2 t}{\cos^5 t}$;

19.9 $-\dfrac{2\cdot\cos^3 t}{\sin t\cdot\cos 2t}$; **19.10** $\dfrac{2}{\sqrt{(1-t)^3}}$; **19.11** $-\dfrac{2(t+3)}{9\sqrt[3]{(t-1)^5}}$; **19.12** $-\dfrac{(1+2\cos t)^3}{\sin^3 t}$;

19.13 $\dfrac{2(2-t^3)}{3t^6}$; **19.14** $\dfrac{2-6\mathrm{sh}^2 t}{\mathrm{ch}^6 t}$; **19.15** $\dfrac{(2t-3)\sqrt{t}}{t^3}$; **19.16** $\dfrac{2}{\cos^6 t}$;

19.17 $-\frac{2t}{(t-2)^2}$; **19.18** $-\frac{1+\sin^2 t}{\cos^4 t}$; **19.19** $-\frac{1}{(1+\cos t)^2}$; **19.20** $-\frac{1}{(1-\cos t)^2}$;

19.21 $-\frac{1+\cos^2 t}{\sin^4 t}$; **19.22** $\frac{1}{t\cdot\cos^3 t}$; **19.23** $\frac{t^2+t-1}{e^{2t}\cdot\sqrt{(1-t^2)^3}}$;

19.24 $\frac{\cos^2(t/2)+1}{4\cos^3(t/2)}$; **19.25** $-\frac{2(3+\mathrm{ch}^2 t}{9\mathrm{sh}^4 t}$; **19.26** $\frac{1+3t^2}{1+t}$; **19.27** $\frac{1}{(1-\cos(t))^2}\,2$;

19.28 $-\frac{1}{t\cdot\sin^3 t}$; **19.29** $-\frac{2t^6}{(1+t^2)^3}$; **19.30** 2.

20–masala. y funksiya berilgan tenglamaning yechimi bo'lishini ko'rsating.

$$y=-\sqrt{x^4-x^2}$$
$$x\cdot y\cdot y'-y^2=x^4 \qquad (1)$$

Echim:

$$y'=\left(-\sqrt{x^4-x^2}\right)'=-\frac{1}{2\sqrt{x^4-x^2}}\cdot(4x^3-2x)=\frac{x-2x^3}{\sqrt{x^4-x^2}}$$

(1) tenglamaga qo'yamiz:

$$x\cdot(-\sqrt{x^4-x^2})\cdot\frac{x-2x^3}{\sqrt{x^4-x^2}}-(-\sqrt{x^4-x^2})^2=x^4.$$

Soddalashtiramiz:

$$x\cdot(2x^3-x)-(x^4-x^2)=x^4$$
$$2x^4-x^2-x^4+x^2=x^4.$$
$$x^4=x^4.$$

Tenglik o'rinli. y funksiya berilgan (1) tenglamaning yechimi bo'ladi.

1. $y = x \cdot e^{-\frac{x^2}{2}}$

$x \cdot y' = \left(1 - x^2\right) y \quad (1)$

2. $y = \dfrac{\sin x}{x}$

$x \cdot y' + y = \cos x \quad (1)$

3. $y = 5 \cdot e^{-2x} + \dfrac{e^x}{3}$

$y' + 2y = e^x \quad (1)$

4. $y = 2 + c \cdot \sqrt{1 - x^2}$

$\left(1 - x^2\right) \cdot y' + xy = 2x \quad (1)$

5. $y = x \cdot \sqrt{1 - x^2}$

$y \cdot y' = x - 2x^3 \quad (1)$

6. $y = \dfrac{c}{\cos x}$

$y' - \operatorname{tg} x \cdot y = 0 \quad (1)$

7. $y = -\dfrac{1}{3x + c}$

$y' = 3y^2 \quad (1)$

8 $y = \ln(c + e^x)$

$y' = e^{x-y} \quad (1)$

9. $y = \sqrt{x^2 - c \cdot x}$

$\left(x^2 + y^2\right) dx - 2 \cdot x \cdot y \cdot dy = 0 \quad (1)$

10. $y = x \cdot (c - \ln x)$

$(x - y) dx + x \cdot dy = 0 \quad (1)$

11. $y = e^{\operatorname{tg} \frac{x}{2}}$

$y' \sin x = y \ln y \quad (1)$

12. $y = \dfrac{1 + x}{1 - x}$

$y' = \dfrac{1 + y^2}{1 + x^2} \quad (1)$

13. $y = \dfrac{b + x}{1 + bx}$

$y - x \cdot y' = b\left(1 + x^2 \cdot y'\right) \quad (1)$

14. $y = \sqrt{2 + 3x - 3x^2}$

$y \cdot y' = \dfrac{1 - 2x}{y} \quad (1)$

15. $y = \sqrt{\ln\left(\dfrac{1 + e^x}{2}\right)^2 + 1}$

$(1 + e^x) \cdot y \cdot y' = e^x \quad (1)$

16. $y = \operatorname{tg} x (\ln 3x)$

$(1 + y^2) dx = x \cdot dy \quad (1)$

17. $y = -\sqrt{\dfrac{2}{x^2} - 1}$

$1 + y^2 + x \cdot y \cdot y' = 0 \quad (1)$

18. $y = \sqrt[3]{x - \ln x - 1}$

$\ln x + y^3 - 3 \cdot x \cdot y^2 \cdot y' = 0 \quad (1)$

19. $y = a + \dfrac{7x}{ax+1}$

$y - x \cdot y' = a\left(1 + x^2 \cdot y'\right)$ (1)

20. $y = a \cdot \operatorname{tg}\sqrt{\dfrac{a}{x} - 1}$

$a^2 + y^2 + 2x\sqrt{ax - x^2} \cdot y' = 0$ (1)

21. $y = \sqrt[4]{\sqrt{x} + \sqrt{x+1}}$

$8 \cdot x \cdot y' - y = \dfrac{-1}{y^3\sqrt{x+1}}$ (1)

22. $y = (x+1) \cdot e^{x^2}$

$y' - 2xy = 2 \cdot x \cdot e^{x^2}$ (1)

23. $y = \dfrac{2}{x^3+1} + \dfrac{1}{x}$

$x \cdot (x^3+1) \cdot y' = \left(2x^3 - 1\right)y = \dfrac{x^3 - 2}{x}$ (1)

24. $y = e^{x+x^2} + 2e^x$

$y' - y = 2xe^{x+x^2}$ (1)

25. $y = -x \cdot \cos x + 3x$

$x \cdot y' = y + x^2 \sin x$ (1)

26. $y = \dfrac{1}{\sqrt{\sin x + x}}$

$2\sin x \cdot y' + y\cos x == y^3(x \cdot \cos x - \sin x)$ (1)

27. $y = \dfrac{x}{x-1} + x^2$

$x \cdot (x-1) \cdot y' + y = x^2(2x-1)$ (1)

28. $y = \dfrac{x}{\cos x}$

$y' - y \cdot \operatorname{tg} x = \sec x$ (1)

29. $y = (x+1)^n \cdot (e^x - 1)$

$y' - \dfrac{n \cdot y}{x+1} = e^x(x+1)^n$ (1)

30. $y = 2\dfrac{\sin x}{x} + \cos x$

$x \cdot \sin x \cdot y' + (\sin x - x \cdot \cos x)y = \sin x \cdot \cos x - x$ (1)

IV BOB. Grafiklar

Funktsiyaning grafigi

1–masala. Birinchi tartibli hosila yordamida funksiyaning grafigini yasang.

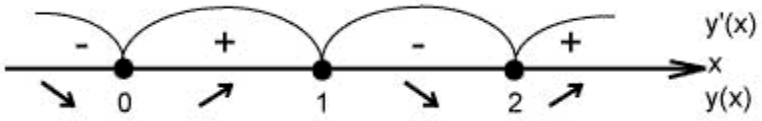
$$y = x^2(x-2)^2.$$

1) $D(y) = (-\infty;+\infty)$.

2) Funksiya juft ham, toq ham emas.

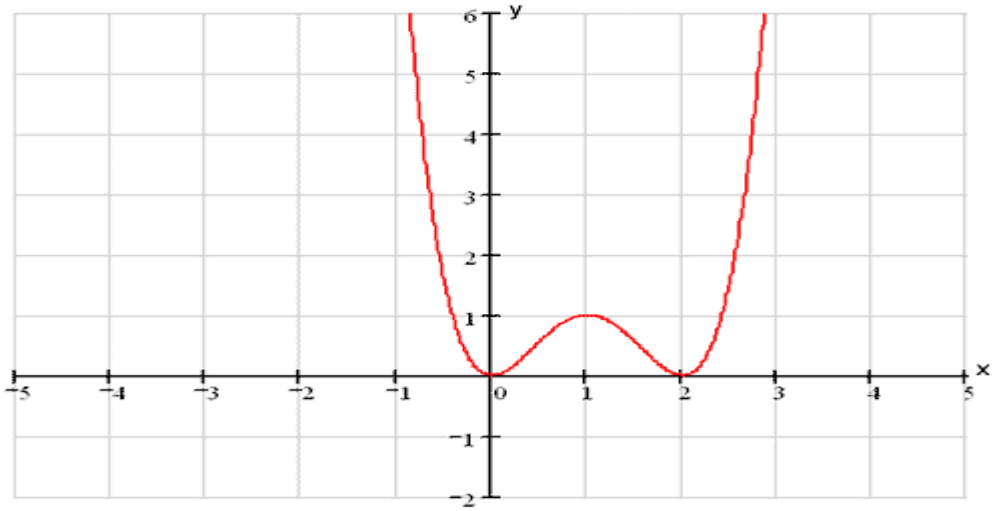
3) $y' = 2x(x-2)^2 + 2x^2(x-2) = 4x(x-2)(x-1)$.

$y' = 0$ da, $\begin{cases} x = 2, \\ x = 1, \\ x = 0 \end{cases}$



(0;0)- minimum nuqta,
(2;0)- minimum nuqta ,
(1;1)- maksimum nuqta .

Функциянинг графиги



1. $y = 2x^3 - 9x^2 + 12x - 9$.

2. $y = 3x - x^3$.

3. $y = (x^3 - 9x^2)/4 + 6x - 9..$

4. $y = 2 - 3x^2 - x^3$.

5. $y = (x+1)^2(x-1)^2$.

6. $y = 2x^3 - 3x^2 - 4$.

7. $y = 3x^2 - 2 - x^3$.

8. $y = (x-1)^2(x-3)^2$.

9. $y=(x^3+3x^2)/4-5.$ **10.** $y=6x-8x^3.$

11. $y=16x^2(x-1)^2.$ **12.** $y=2x^3+3x^2-5.$

13. $y=2-12x^2-8x^3.$ **14.** $y=(2x+1)^2(2x-1)^2.$

15. $y=2x^3+9x^2+12x.$ **16.** $y=12x^2-8x^3-2.$

17. $y=(2x-1)^2(2x-3)^2.$ **18.** $y=27(x^3-x^2)/4-4.$

19. $y=x(12-x^2)/8.$ **20.** $y=x^2(x-4)^2/16.$

21. $y=27(x^3+x^2)/4-5.$ **22.** $y=(16-6x^2-x^3)/8.$

23. $y=-(x^2-4)^2/16.$ **24.** $y=16x^3-36x^2+24x-9.$

25. $y=(6x^2-x^3-16)/8.$ **26.** $y=-(x-2)^2(x-6)^2/16.$

27. $y=16x^3-12x^2-4.$ **28.** $y=(11+9x-3x^2-x^3)/8.$

29. $y=-(x+1)^2(x-3)^2/16.$ **30.** $y=16x^3+12x^2-5.$

2–masala. Birinchi tartibli hosila yordamida funksiyaning grafigini yasang.

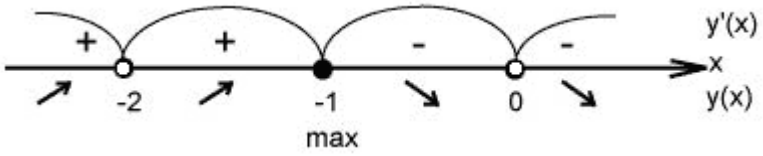
$$y=1-\sqrt[3]{x^2+2x}.$$

1) $D(y)=(-\infty;+\infty).$

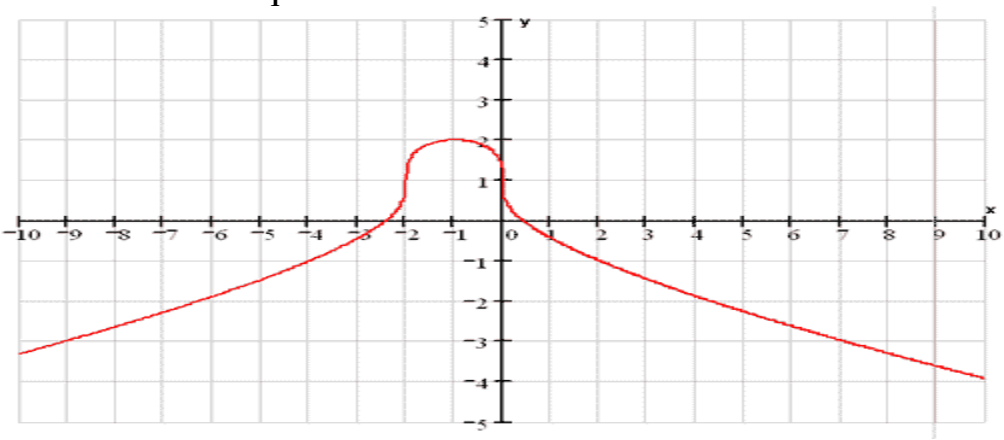
2) Funksiya juft ham, toq ham emas.

3) $y'=-\dfrac{2x+2}{3\sqrt[3]{(x^2+2x)^2}}.$

$y'=0$ da, $x=1;$ $x=0$ va $x=-2$ nuqtalarda y' mavjud emas.



(–1; 2) – maksimum nuqta .



1. $y = 2x - 3\sqrt[3]{x^2}$.

2. $y = \dfrac{12\sqrt[3]{6(x-2)^2}}{x^2+8}$.

3. $y = -\dfrac{12\sqrt[3]{6(x-1)^2}}{x^2+2x+9}$.

4. $y = 1 - \sqrt[3]{x^2+2x}$.

5. $y = 2x + 6 - 3\sqrt[3]{(x+3)^2}$.

6. $y = \dfrac{6\sqrt[3]{6(x-3)^2}}{x^2-2x+9}$.

7. $y = 1 - \sqrt[3]{x^2+4x+3}$.

8. $y = 3\sqrt[3]{(x-3)^2} - 2x + 6$.

9. $y = -\dfrac{6\sqrt[3]{6x^2}}{x^2+4x+12}$.

10. $y = 4x + 8 - 6\sqrt[3]{(x+2)^2}$.

11. $y = \dfrac{3\sqrt[3]{6(x-4)^2}}{x^2-4x+12}$.

12. $y = \sqrt[3]{x(x+2)}$.

13. $y = \sqrt[3]{x^2+4x+3}$.

14. $y = -\dfrac{3\sqrt[3]{6(x+1)^2}}{x^2+6x+17}$.

15. $y = 6\sqrt[3]{(x-2)^2} - 4x + 8$.

16. $y = \dfrac{3\sqrt[3]{6(x-5)^2}}{x^2-6x+17}$.

17. $y = 2 + \sqrt[3]{8x(x+2)}$.

18. $y = 6x - 6 - 9\sqrt[3]{(x-1)^2}$.

19. $y = \sqrt[3]{x^2+6x+8}$.

20. $y = \sqrt[3]{4x(x-1)}$.

21. $y = -\dfrac{3\sqrt[3]{6(x+2)^2}}{x^2+8x+24}$.

22. $y = \sqrt[3]{x(x-2)}$.

23. $y = 1 - \sqrt[3]{x^2-4x+3}$.

24. $y = 9\sqrt[3]{(x+1)^2} - 6x - 6$.

25. $y = \dfrac{6\sqrt[3]{6(x+3)^2}}{x^2+10x+33}$

26. $y = 8x - 16 - 12\sqrt[3]{(x-2)^2}$.

27. $y = -\dfrac{6\sqrt[3]{6(x-6)^2}}{x^2-8x+24}$.

28. $y = 12\sqrt[3]{(x+2)^2} - 8x - 16$.

29. $y = \dfrac{3\sqrt[3]{6(x-1)^2}}{2(x^2+2x+9)}$.

30. $y = 3\sqrt[3]{(x+4)^2} - 2x - 8$.

Funksiyaning eng katta va eng kichik qiymatlari

3–masala. Berilgan kesmada funksiyaning eng katta va eng kichik

qiymatlarini toping.

$$y = 2x^2 + \frac{108}{x} - 59, \quad [2,\ 4].$$

$x \neq 0$.

$y' = 4x - \frac{108}{x^2} = \frac{4x^3 - 108}{x^2}$.

$y' = 0$ da, $x = 3 \in [2;\ 4]$;

$x = 0 \notin [2;\ 4]$ da y' mavjud emas.

$$y(2) = 3,$$
$$y(3) = -5,$$
$$y(4) = 0.$$

$\max\limits_{[2;4]} y = y(2) = 3;$

$\min\limits_{[2;4]} y = y(3) = -5;$

1. $y = x^2 + \frac{16}{x} - 16,\ [1,\ 4].$ **2.** $y = 4 - x - \frac{4}{x^2},\ [1,\ 4].$

3. $y = \sqrt[3]{2(x-2)^2(8-x)} - 1,\ [0,\ 6].$ **4.** $y = \frac{2(x^2+3)}{x^2 - 2x + 5},\ [-3,\ 3].$

5. $y = 2\sqrt{x} - x,\ [0,\ 4].$ **6.** $y = 1 + \sqrt[3]{2(x-1)^2(x-7)},\ [-1,\ 5].$

7. $y = x - 4\sqrt{x} + 5,\ [1,\ 9].$ **8.** $y = \frac{10x}{1+x^2},\ [0,\ 3].$

9. $y = \sqrt[3]{2(x+1)^2(5-x)} - 2,\ [-3,\ 3].$ **10.** $y = 3 - x - \frac{4}{(x+2)^2},\ [-1,\ 2].$

11. $y = \sqrt[3]{2x^2(x-3)},\ [-1,\ 6].$ **12.** $y = \frac{2(-x^2+7x-7)}{x^2-2x+2},\ [1,\ 4].$

13. $y = x - 4\sqrt{x+2} + 8,\ [-1,\ 7].$ **14.** $y = \sqrt[3]{2(x-2)^2(5-x)},\ [1,\ 5].$

15. $y = \frac{4x}{4+x^2},\ [-4,\ 2].$ **16.** $y = -\frac{x^2}{2} + \frac{8}{x} + 8,\ [-4,\ -1].$

17. $y = \sqrt[3]{2x^2(x-6)},\ [-2,\ 4].$ **18.** $y = \frac{-2x(2x+3)}{x^2+4x+5},\ [1,\ 4].$

19. $y=-\dfrac{2(x^2+3)}{x^2+2x+5},\ [-5,\ 1]$. **20.** $y=\sqrt[3]{2(x-1)^2(x-4)},\ [0,\ 4]$.

21. $y=x^2-2x+\dfrac{16}{x-1}-13,\ [2,\ 5]$. **22.** $y=2\sqrt{x-1}-x+2,\ [1,\ 5]$.

23. $y=\sqrt[3]{2(x+2)^2(1-x)},\ [-3,4]$. **24.** $y=-\dfrac{x^2}{2}+2x+\dfrac{8}{x-2}+5,\ [-2,\ 1]$.

25. $y=8x+\dfrac{4}{x^2}-15,\ \left[\dfrac{1}{2},\ 2\right]$. **26.** $y=\sqrt[3]{2(x+2)^2(x-4)}+3,\ [-4,\ 2]$.

27. $y=x^2+4x+\dfrac{16}{x+2}-9,\ [-1,\ 2]$. **28.** $y=\dfrac{4}{x^2}-8x-15,\ \left[-2,\ -\dfrac{1}{2}\right]$.

29. $y=\sqrt[3]{2(x+1)^2(x-2)},\ [-2,\ 5]$. **30.** $y=\dfrac{10x+10}{x^2+2x+2},\ [-1,\ 2]$.

Javoblar. 3.1 $f_{\max}=y(1)=y(4)=4;\ f_{\min}=y(-2)=-4$; **3.2** $f_{\max}=y(2)=1;$ $f_{\min}=y(1)=-1$; **3.3** $f_{\max}=y(0)=y(6)=3;\ f_{\min}=y(2)=-1$; **3.4** $f_{\max}=y(-1)=1;\ f_{\min}=y(3)=3$; **3.5** $f_{\max}=y(1)=1;\ f_{\min}=y(4)=0$; **3.6** $f_{\max}=y(1)=1;\ f_{\min}=y(5)=-3$; **3.7** $f_{\max}=y(1)=y(9)=2;\ f_{\min}=y(4)=1$; **3.8** $f_{\max}=y(1)=5;\ f_{\min}=y(0)=0$; **3.9** $f_{\max}=y(-3)=y(3)=2;\ f_{\min}=y(-1)=-2$; **3.10** $f_{\max}=y(0)=2;\ f_{\min}=y(-1)=0$; **3.11** $f_{\max}=y(6)=6;\ f_{\min}=y(-1)=y(2)=-2$; **3.12** $f_{\max}=y(2)=3;\ f_{\min}=y(1)=-2$; **3.13** $f_{\max}=y(-1)=y(7)=3;\ f_{\min}=y(2)=2$; **3.14** $f_{\max}=y(1)=y(4)=2;$ $f_{\min}=y(2)=y(5)=0$; **3.15** $f_{\max}=y(2)=1;\ f_{\min}=y(-2)=-1$; **3.16** $f_{\max}=y(-2)=2;\ f_{\min}=y(-4)=-2$; **3.17** $f_{\max}=y(0)=0;$ $f_{\min}=y(-2)=y(4)=-4$; **3.18** $f_{\max}=y(1)=-1;\ f_{\min}=y(4)=-\dfrac{88}{37}$; **3.19** $f_{\max}=y(1)=-1;\ f_{\min}=y(-3)=-3$; **3.20** $f_{\max}=y(1)=y(4)=0;\ f_{\min}=y(0)=-2$; **3.21** $f_{\max}=y(5)=6;\ f_{\min}=y(3)=-2$; **3.22** $f_{\max}=y(2)=2;\ f_{\min}=y(1)=y(5)=1$; **3.23** $f_{\max}=y(-3)=y(0)=2;\ f_{\min}=y(4)=-6$; **3.24** $f_{\max}=y(0)=1;$ $f_{\min}=y(-2)=-3$; **3.25** $f_{\max}=y\left(\dfrac{1}{2}\right)=5;\ f_{\min}=y(1)=-3$; **3.26** $f_{\max}=y(-2)=3;$ $f_{\min}=y(2)=y(-4)=-1$; **3.27** $f_{\max}=y(2)=7;\ f_{\min}=y(0)=-1$; **3.28** $f_{\max}=y\left(-\dfrac{1}{2}\right)=5;\ f_{\min}=y(-1)=-3$; **3.29** $f_{\max}=y(5)=6;$ $f_{\min}=y(-2)=y(1)=-2$; **3.30** $f_{\max}=y(-1)=0;\ f_{\min}=y(0)=-5$.

4–masala. Yuqori tartibli hosilalar yordamida berilgan nuqta atrofida funksiyani tekshiring.

$$y = 4x - x^2 - 2\cos(x-2),$$
$$x_0 = 2.$$

$$y' = 4 - 2x + 2\sin(x-2), \qquad y'(2) = 0;$$
$$y'' = -2 + 2\cos(x-2), \qquad y''(2) = 0;$$
$$y''' = -2\sin(x-2), \qquad y'''(2) = 0;$$
$$y^{\mathrm{IV}} = -2\cos(x-2), \qquad y^{\mathrm{IV}}(2) = -2.$$

$y^{\mathrm{IV}} \neq 0$ ekanligidan $x_0 = 2$ nuqtada maksimumga erishadi.

1. $y = x^2 - 4x - (x-2)\ln(x-1),$ $x_0 = 2.$

2. $y = 6e^{x-2} - x^3 + 3x^2 - 6x,$ $x_0 = 2.$

3. $y = 2\ln(x+1) - 2x + x^2 + 1,$ $x_0 = 0.$

4. $y = 2x - x^2 - 2\cos(x-1),$ $x_0 = 1.$

5. $y = \cos^2(x+1) + x^2 + 2x,$ $x_0 = -1.$

6. $y = 2\ln x + x^2 - 4x + 3,$ $x_0 = 1.$

7. $y = 1 - 2x - x^2 - 2\cos(x+1),$ $x_0 = -1.$

8. $y = x^2 + 6x + 8 - 2e^{x+2},$ $x_0 = -2.$

9. $y = 4x + x^2 - 2e^{x+1},$ $x_0 = -1.$

10. $y = (x+1)\sin(x+1) - 2x - x^2,$ $x_0 = -1.$

11. $y = 6e^{x-1} - 3x - x^3,$ $x_0 = 1.$

12. $y = 2x + x^2 - (x+1)\ln(2+x),$ $x_0 = -1.$

13. $y = \sin^2(x+1) - 2x - x^2,$ $x_0 = -1.$

14. $y = x^2 + 4x + \cos^2(x+2),$ $x_0 = -2.$

15. $y = x^2 + 2\ln(x+2),$ $x_0 = -1.$

16. $y = 4x - x^2 + (x-2)\sin(x-2),$ $x_0 = 2.$

17. $y = 6e^x - x^3 - 3x^2 - 6x - 5,$ $x_0 = 0.$

18. $y = x^2 - 2x - 2e^{x-2},$ $x_0 = 2.$

19. $y = \sin^2(x+2) - x^2 - 4x - 4,$ $x_0 = -2.$

20. $y = \cos^2(x-1) + x^2 - 2x,$ $x_0 = 1.$

21. $y = x^2 - 2x - (x-1)\ln x,$ $x_0 = 1.$

22. $y = (x-1)\sin(x-1) + 2x - x^2,$ $x_0 = 1.$

23. $y = x^2 - 4x + \cos^2(x-2),$ $x_0 = 2.$

24. $y = x^4 + 4x^3 + 12x^2 + 24(x + 1 - e^x),$ $x_0 = 0.$

25. $y = \sin^2(x-2) - x^2 + 4x - 4,$ $x_0 = 2.$

26. $y = 6e^{x+1} - x^3 - 6x^2 - 15x - 16,$ $x_0 = -1.$

27. $y = \sin x + \mathrm{sh}x - 2x,$ $x_0 = 0.$

28. $y = \sin^2(x-1) - x^2 + 2x,$ $x_0 = 1.$

29. $y = \cos x + \mathrm{ch}x,$ $x_0 = 0.$

30. $y = x^2 - 2e^{x-1},$ $x_0 = 1.$

Funksiyaning aimptotalari

5–masala. Quyidagi funksiyaning aimptotalarini toping va grafigini yasang.

$$y = \frac{17 - x^2}{4x - 5}$$

1) $D(y) = \left(-\infty;\ \frac{5}{4}\right) \cup \left(\frac{5}{4};\ +\infty\right).$

2) Funksiya juft ham, toq ham emas.

3. a) $\lim\limits_{x\to\frac{5}{4}-0}\dfrac{17-x^2}{4x-5}=-\infty,$

$\lim\limits_{x\to\frac{5}{4}+0}\dfrac{17-x^2}{4x-5}=+\infty,$

$x=\dfrac{5}{4}$–vertikal asimptota.

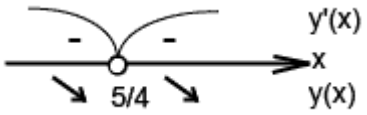
б) $k=\lim\limits_{x\to\infty}\dfrac{f(x)}{x}=\lim\limits_{x\to\infty}\dfrac{17-x^2}{x(4x-5)}=-\dfrac{1}{4}.$

$b=\lim\limits_{x\to\infty}(f(x)-kx)=\lim\limits_{x\to\infty}\left(\dfrac{17-x^2}{4x-5}+\dfrac{x}{4}\right)=\lim\limits_{x\to\infty}\dfrac{68-5x}{16x-20}=-\dfrac{5}{16}.$

Demak, $y=-\dfrac{1}{4}x-\dfrac{5}{16}$–og'ma asimptota.

4) $y'=\dfrac{-2x(4x-5)-4(17-x^2)}{(4x-5)^2}=-\dfrac{4x^2+10x+68}{(4x-5)^2}.$

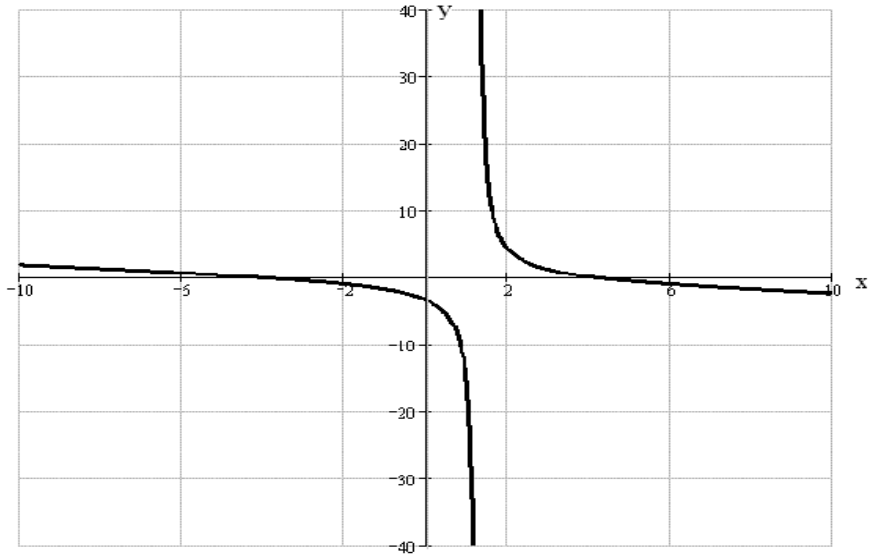
$x=\dfrac{5}{4}$ da y' mavjud emas.



5) O'qlar bilan kesishish nuqtalarini topamiz:

$x=0$ da $y=-\dfrac{17}{5}.$

$y=0$ da $x=\pm4{,}12.$



1. $y=\dfrac{x^2+1}{\sqrt{4x^2-3}}$.

2. $y=\dfrac{x^3-4x}{3x^2-4}$.

3. $y=\dfrac{4x^2+9}{4x+8}$.

4. $y=\dfrac{4x^3+3x^2-8x-2}{2-3x^2}$.

5. $y=\dfrac{x^2-3}{\sqrt{3x^2-2}}$.

6. $y=\dfrac{2x^2-6}{x-2}$.

7. $y=\dfrac{2x^3+2x^2-3x-1}{2-4x^2}$.

8. $y=\dfrac{x^3-5x}{5-3x^2}$.

9. $y=\dfrac{x^2-6x+4}{3x-2}$.

10. $y=\dfrac{2-x^2}{\sqrt{9x^2-4}}$.

11. $y=\dfrac{4x^3-3x}{4x^2-1}$.

12. $y=\dfrac{3x^2-7}{2x+1}$.

13. $y=\dfrac{x^2+16}{\sqrt{9x^2-8}}$.

14. $y=\dfrac{x^3+3x^2-2x-2}{2-3x^2}$.

15. $y=\dfrac{21-x^2}{7x+9}$.

16. $y=\dfrac{2x^2-1}{\sqrt{x^2-2}}$.

17. $y=\dfrac{2x^3-3x^2-2x+1}{1-3x^2}$.

18. $y=\dfrac{x^2-11}{4x-3}$.

19. $y=\dfrac{2x^2-9}{\sqrt{x^2-1}}$.

20. $y=\dfrac{x^3-2x^2-3x+2}{1-x^2}$.

21. $y=\dfrac{x^2+2x-1}{2x+1}$.

22. $y=\dfrac{x^3+x^2-3x-1}{2x^2-2}$.

23. $y=\dfrac{x^2+6x+9}{x+4}$.

24. $y=\dfrac{3x^2-10}{\sqrt{4x^2-1}}$.

25. $y=\dfrac{x^2-2x+2}{x+3}$.

26. $y=\dfrac{2x^3+2x^2-9x-3}{2x^2-3}$.

27. $y=\dfrac{3x^2-10}{3-2x}$.

28. $y=\dfrac{-x^2-4x+13}{4x+3}$.

29. $y=\dfrac{-8-x^2}{\sqrt{x^2-4}}$.

30. $y=\dfrac{9-10x^2}{\sqrt{4x^2-1}}$.

Funksiyalarni tekshirish va grafiklarini chizish

6–masala. Quyidagi funksiyani tekshiring va ularni grafigini yasang.

$$y=\frac{x^2-3x+3}{x-1}.$$

1) $D(y)=(-\infty;\ 1)\cup(1;\ +\infty)$.

2) Funksiya juft ham, toq ham emas.

3. a) $\lim\limits_{x\to 1-0}\dfrac{x^2-3x+3}{x-1}=-\infty$,

$\lim\limits_{x\to 1+0}\dfrac{x^2-3x+3}{x-1}=+\infty$,

$x=1$–vertikal asimptota.

б) $k=\lim\limits_{x\to\infty}\dfrac{f(x)}{x}=\lim\limits_{x\to\infty}\dfrac{x^2-3x+3}{x(x-1)}=1.$

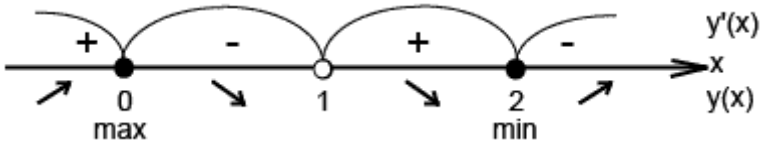
$$b=\lim_{x\to\infty}(f(x)-kx)=\lim_{x\to\infty}\left(\frac{x^2-3x+3}{x-1}-x\right)=\lim_{x\to\infty}\frac{-2x+3}{x-1}=-2.$$

Demak, $y=x-2$–og'ma asimptota.

4) $y'=\dfrac{-(2x-3)(x-1)-(x^2-3x+3)}{(x-1)^2}=\dfrac{x(x-2)}{(x-1)^2}.$

$\begin{cases}x=0,\\ x=1\end{cases}$ da $y'=0$

$x=1$ da y' mavjud emas.

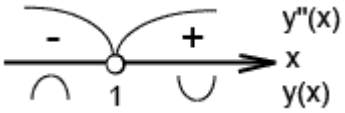


$(0;\ -3)$–funksiyaning maksimum nuqtasi

$(2;\ 1)$– funksiyaning minimum nuqtasi.

5) $y''=\dfrac{(2x-2)(x-1)^2-2(x-1)(x^2-2x)}{(x-1)^4}=\dfrac{2}{(x-1)^3},$

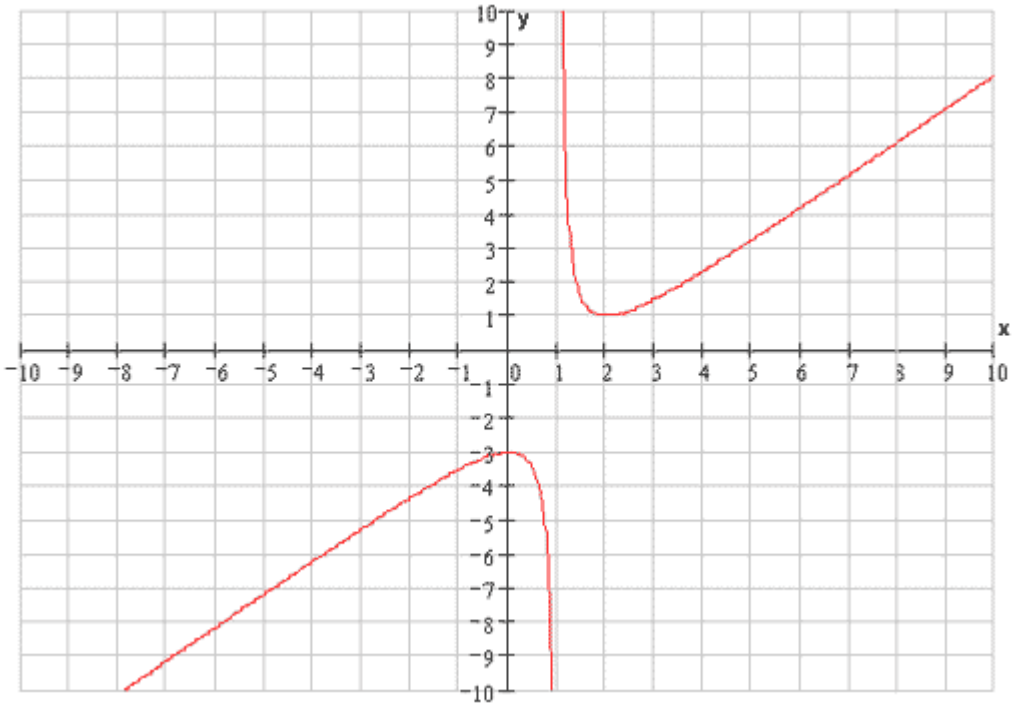
$x=1$ da y' mavjud emas.



6) O'qlar bilan kesishish nuqtalarini topamiz:

$x=0$ da $y=-3.$

$y=0$ da kvadrat tenglama ildizlarga ega emas. Demak funksiyaning grafigi Ox oqi bilan kesishmaydi.



1. $y=\dfrac{x^3+4}{x^2}$.

2. $y=\dfrac{x^2-x+1}{x-1}$.

3. $y=\dfrac{2}{x^2+2x}$.

4. $y=\dfrac{4x^2}{3+x^2}$.

5. $y=\dfrac{12x}{9+x^2}$.

6. $y=\dfrac{4-x^3}{x^2}$.

7. $y=\dfrac{x^2-4x+1}{x-4}$.

8. $y=\dfrac{2x^3+1}{x^2}$.

9. $y=\dfrac{(x-1)^2}{x^2}$.

10. $y=\dfrac{x^2}{(x-1)^2}$.

11. $y=\left(1+\dfrac{1}{x}\right)^2$.

12. $y=\dfrac{12-3x^2}{x^2+12}$.

13. $y=\dfrac{9+6x-3x^2}{x^2-2x+13}$.

14. $y=\dfrac{-8x}{x^2+4}$.

15. $y=\left(\dfrac{x-1}{x+1}\right)^2$.

16. $y=\dfrac{3x^4+1}{x^3}$.

17. $y=\dfrac{4x}{(x+1)^2}$.

18. $y=\dfrac{8(x-1)}{(x+1)^2}$.

19. $y=\dfrac{1-2x^3}{x^2}$.

20. $y=\dfrac{4}{x^2+2x-3}$.

21. $y=\dfrac{4}{3+2x-x^2}$.

22. $y=\dfrac{x^2+2x-7}{x^2+2x-3}$.

23. $y=\dfrac{1}{x^4-1}$.

24. $y=-\left(\dfrac{x}{x+2}\right)^2$.

25. $y=\dfrac{x^3-32}{x^2}$.

26. $y=\dfrac{4(x+1)^2}{x^2+2x+4}$.

27. $y=\dfrac{3x-2}{x^3}$.

28. $y=\dfrac{x^2-6x+9}{(x-1)^2}$. **29.** $y=\dfrac{x^3-27x+54}{x^3}$. **30.** $y=\dfrac{x^3-4}{x^2}$.

Funksiyalarni tekshirish va grafiklarini chizish

7–masala. Quyidagi funksiyani tekshiring va ularni grafigini yasang.

$$y=\frac{e^{2(x+1)}}{2(x+1)}.$$

1) $D(y)=(-\infty;\ 1)\cup(1;\ +\infty)$.

2) Funksiya juft ham, toq ham emas.

3. a) $\lim\limits_{x\to -1-0}\dfrac{e^{2(x+1)}}{2(x+1)}=-\infty$,

$\lim\limits_{x\to -1+0}\dfrac{e^{2(x+1)}}{2(x+1)}=+\infty$,

$x=-1$– vertikal asimptota.

б) $k=\lim\limits_{x\to-\infty}\dfrac{f(x)}{x}=\lim\limits_{x\to-\infty}\dfrac{e^{2(x+1)}}{2(x+1)x}=-\infty$,

$k=\lim\limits_{x\to+\infty}\dfrac{f(x)}{x}=\lim\limits_{x\to+\infty}\dfrac{e^{2(x+1)}}{2(x+1)x}=0.$

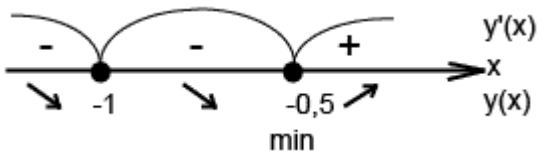
$b=\lim\limits_{x\to+\infty}\dfrac{e^{2(x+1)}}{2(x+1)}=0..$

Demak, $y=0$–gorizpntal asimptota.

4) $y'=\dfrac{1}{2}\cdot\dfrac{2(x+1)e^{e(x+1)}-e^{2(x+1)}}{(x+1)^2}=\dfrac{(2x+1)e^{2(x+1)}}{2(x+1)^2}$.

$x=-0{,}5$ da $y'=0$,

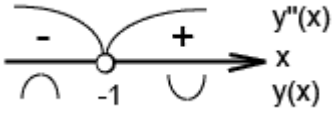
$x=-1$ da y' mavjud emas.



$\left(-\frac{1}{2};e\right)$– funksiyaning minimum nuqtasi.

5) $y''=\frac{(2x^2-2x+1)e^{2(x+1)}}{(x+1)^3}$,

$x=-1$ da y'' mavjud emas.



6) O’qlar bilan kesishish nuqtalarini topamiz:

$x=0$ da $y=\frac{e^2}{2}$.

$y=0$ da kvadrat tenglama ildizlarga ega emas. Demak funksiyaning grafigi Ox oqi bilan kesishmaydi.

1. $y=(2x+3)e^{-2(x+1)}$.

2. $y=3\ln\frac{x}{x-3}-1$.

3. $y=(3-x)e^{x-2}$.

4. $y=\frac{e^{2-x}}{2-x}$.

5. $y=\ln\frac{x}{x+2}+1$.

6. $y=(x-2)e^{3-x}$.

7. $y=\frac{e^{2(x-1)}}{2(x-1)}$.

8. $y=3-3\ln\frac{x}{x+4}$.

9. $y=-(2x+1)e^{2(+1)}$.

10. $y=\frac{e^{2(x+2)}}{2(x+2)}$.

11. $y=\ln\frac{x}{x-2}-2$.

12. $y=(2x+5)e^{-2(x+2)}$.

13. $y=\frac{e^{3-x}}{3-x}$.

14. $y=2\ln\frac{x}{x+1}-1$.

15. $y=(4-x)e^{x-3}$.

16. $y=-\dfrac{e^{-2(x+2)}}{2(x+2)}$.

17. $y=2\ln\dfrac{x+3}{x}-3$.

18. $y=(2x-1)e^{2(1-x)}$.

19. $y=-\dfrac{e^{-(x+2)}}{x+2}$.

20. $y=2\ln\dfrac{x}{x-4}-3$.

21. $y=-(x+1)e^{(x+2)}$.

22. $y=-\dfrac{e^{x+3}}{x+3}$.

23. $y=\ln\dfrac{x}{x+5}-1$.

24. $y=-(2x+3)e^{2(x+2)}$.

25. $y=-\dfrac{e^{-2(x-1)}}{2(x-1)}$.

26. $y=\ln\dfrac{x-5}{x}+2$.

27. $y=(x+4)e^{-(x+3)}$.

28. $y=-\dfrac{e^{x-3}}{x-3}$.

29. $y=\ln\dfrac{x+6}{x}-1$.

30. $y=2\ln\dfrac{x-1}{x}+1$.

Funksiyalarni tekshirish va grafiklarini chizish

8–masala. Quyidagi funksiyani tekshiring va ularni grafigini yasang.

$$y=\sqrt[3]{(2-x)(x^2-4x+1)}.$$

1) $D(y)=(-\infty;+\infty)$.

2) Funksiya juft ham, toq ham emas.

3. a) vertikal asimptotalari yo’q.

б) $k=\lim\limits_{x\to\infty}\dfrac{f(x)}{x}=\lim\limits_{x\to\infty}\dfrac{\sqrt[3]{(2-x)(x^2-4x+1)}}{x}=-1,$

$$b=\lim_{x\to+\infty}(\sqrt[3]{(2-x)(x^2-4x+1)}-x)=$$

$$=\lim_{x\to\infty}\frac{(2-x)(x^2-4x+1)-x^3}{\sqrt[3]{(2-x)^2(x^2-4x+1)^2}+x\sqrt[3]{(2-x)(x^2-4x+1)}+x^2}= \quad .$$

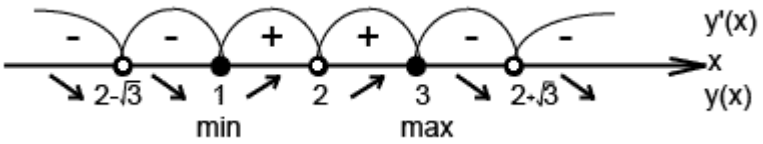
$$=\lim_{x\to\infty}\frac{2-9x+6x^2-2x^3}{\sqrt[3]{(2-x)^2(x^2-4x+1)^2}+x\sqrt[3]{(2-x)(x^2-4x+1)}+x^2}=\frac{-2}{-1}=2.$$

Demak, $y=-x+2$ –og'ma asimptota.

4) $y'=\frac{1}{3}\cdot\frac{-9+12x-3x^2}{\sqrt[3]{(2-x)^2(x^2-4x+1)^2}}=\frac{-x^2+4x-3}{\sqrt[3]{(2-x)^2(x^2-4x+1)^2}}.$

$\begin{cases}x=1,\\ x=3\end{cases}$ da $y'=0,$

$\begin{cases}x=2,\\ x=2\pm\sqrt{3}.\end{cases}$, da y' mavjud emas.



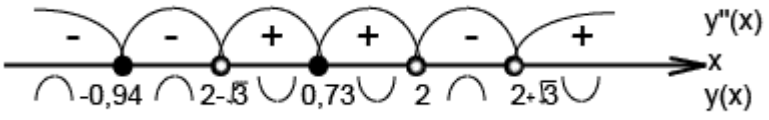
$\left(1;\ -\sqrt[3]{2}\right)$– funksiyaning minimum nuqtasi

$\left(3;\ \sqrt[3]{2}\right)$–funksiyaning maksimum nuqtasi .

5) $y''=\frac{-(4x^4-16x^3+14x^2-8x+10)}{\sqrt[3]{(2-x)^5(x^2-4x+1)^5}},$

$\begin{cases}x=-0{,}94;\\ x=0{,}73\end{cases}$ ad $y''=0,$

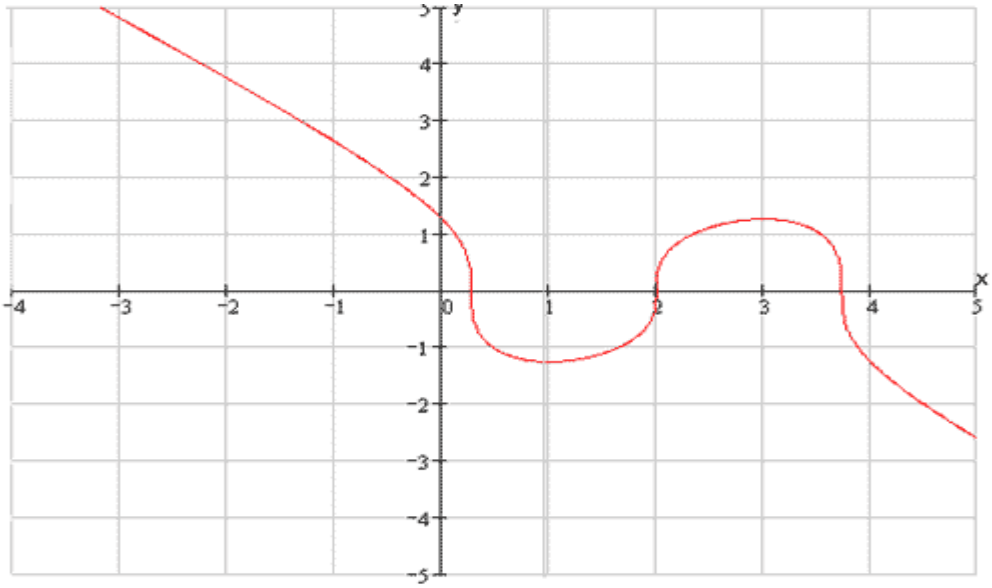
$\begin{cases}x=2,\\ x=2\pm\sqrt{3}.\end{cases}$, da y'' mavjud emas.



6) O'qlar bilan kesishish nuqtalarini topamiz:

$x=0$ da $y=\sqrt[3]{2}$.

$y=0$ da $\begin{cases} x=2, \\ x=2\pm\sqrt{3}. \end{cases}$



1. $y=-\sqrt[3]{(x+3)(x^3+6x+6)}$.
2. $y=\sqrt[3]{(x+2)(x^2+4x+1)}$.
3. $y=\sqrt[3]{(x+1)(x^2+2x-2)}$.
4. $y=\sqrt[3]{(x-1)(x^2-2x-2)}$.
5. $y=\sqrt[3]{(x-3)(x^2-6x+6)}$.
6. $y=\sqrt[3]{(x^2-4x+3)^2}$.
7. $y=\sqrt[3]{x^2(x+2)^2}$.
8. $y=\sqrt[3]{x^2(x-2)^2}$.
9. $y=\sqrt[3]{(x^2-2x-3)^2}$.
10. $y=\sqrt[3]{x^2(x+4)^2}$.
11. $y=\sqrt[3]{x^2(x-4)^2}$.
12. $y=\sqrt[3]{(x+3)x^2}$.
13. $y=\sqrt[3]{(x-1)(x+2)^2}$.
14. $y=\sqrt[3]{(x-1)^2}-\sqrt[3]{x^2}$.
15. $y=\sqrt[3]{(x+6)x^2}$.
16. $y=\sqrt[3]{(x-4)(x+2)^2}$.
17. $y=\sqrt[3]{(x-1)^2}-\sqrt[3]{(x-2)^2}$.
18. $y=\sqrt[3]{(x+1)(x-2)^2}$.
19. $y=\sqrt[3]{(x-3)x^2}$.
20. $y=\sqrt[3]{(x-2)^2}-\sqrt[3]{(x-3)^2}$.
21. $y=\sqrt[3]{(x+2)(x-4)^2}$.
22. $y=\sqrt[3]{(x-6)x^2}$.
23. $y=\sqrt[3]{x^2}-\sqrt[3]{(x-1)^2}$.
24. $y=\sqrt[3]{x(x-3)^2}$.
25. $y=\sqrt[3]{x(x+3)^2}$.
26. $y=\sqrt[3]{(x+2)^2}-\sqrt[3]{(x+3)^2}$.

27. $y=\sqrt[3]{x(x-6)^2}$.

28. $y=\sqrt[3]{x(x+6)^2}$.

29. $y=\sqrt[3]{(x+1)^2}-\sqrt[3]{(x+2)^2}$.

30. $y=\sqrt[3]{x(x-1)^2}$.

Funksiyalarni tekshirish va grafiklarini chizish

9–masala. Quyidagi funksiyani tekshiring va ularni grafigini yasang.

$$y=e^{\sin x+\cos x}.$$

1) $D(y)=(-\infty;+\infty)$.

2) Funksiya juft ham, toq ham emas.

3. a) – vertikal asimptotalari yo'q.

б) og'ma asimptotalari yo'q .

4) davriy funksiya

$$T=-\frac{\pi}{4}+\pi n,\ n\in Z.$$

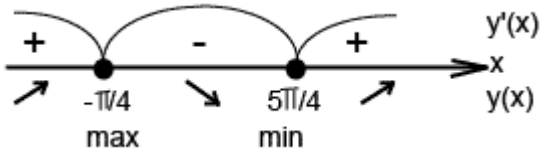
5) $y=e^{\sin x+\cos x}$.

$$y=e^{\sqrt{2}\cos(x-\frac{\pi}{4})},$$

$$y'=-\sqrt{2}\sin(x-\frac{\pi}{4})e^{\sqrt{2}\cos(x-\frac{\pi}{4})}$$

$y'=0$, u holda $\sin(x-\frac{\pi}{4})=0,$

$$x=\frac{\pi}{4}+\pi k,\ \ k\in Z\,.$$



6) $y'' = -\sqrt{2}\cos\left(x-\frac{\pi}{4}\right)e^{\sqrt{2}\cos\left(x-\frac{\pi}{4}\right)} +$

$$+\sqrt{2}\sin\left(x-\frac{\pi}{4}\right)\cdot\sqrt{2}\sin\left(x-\frac{\pi}{4}\right)e^{\sqrt{2}\cos\left(x-\frac{\pi}{4}\right)} =$$

$$=\sqrt{2}e^{\sqrt{2}\cos\left(x-\frac{\pi}{4}\right)}\left(-\cos\left(x-\frac{\pi}{4}\right)+\sqrt{2}\sin^2\left(x-\frac{\pi}{4}\right)\right)=$$

$$=\sqrt{2}e^{\sqrt{2}\cos\left(x-\frac{\pi}{4}\right)}\left(\sqrt{2}-\cos\left(x-\frac{\pi}{4}\right)-\sqrt{2}\cos^2\left(x-\frac{\pi}{4}\right)\right).$$

$y''=0$ при $x=\pm\frac{\pi}{4}+\frac{\pi}{4}+2\pi n,\ n\in Z.$,

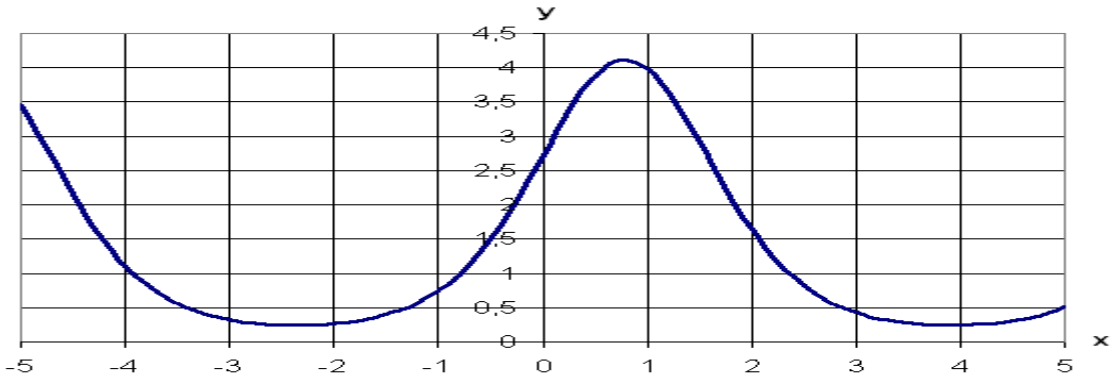
$$\left[\begin{array}{l} x=\frac{\pi}{4}+\frac{\pi}{4}+2\pi n,\ n\in Z. \\ x=-\frac{\pi}{4}+\frac{\pi}{4}+2\pi k,\ \ k\in Z. \end{array}\right. \qquad \left[\begin{array}{l} x=\frac{\pi}{2}+2\pi n,\ n\in Z. \\ x=2\pi k,\ k\in Z. \end{array}\right.$$

$x\in\left(\frac{\pi}{2}+2\pi n;\ 2\pi k\right)$ da funksiya botiq, chunki $y''>0$.

$x\in\left(2\pi k;\ \frac{\pi}{2}+2\pi n\right)$ da funksiya qavariq, chunki $y''<0$.

Egilish nuqtasi:

$$(2\pi k;e),\left(\frac{\pi}{2}+2\pi n;e^{\frac{\sqrt{2}}{2}}\right).$$



1. $y = \operatorname{arctg}\left(\frac{(\sin x + \cos x}{\sqrt{2}}\right).$

2. $y = \ln(\cos x + \sin x).$

3. $y = \left(\frac{1}{\sin x + \cos x}\right).$

4. $y = e^{\sqrt{2}\sin x}.$

5. $y = \operatorname{arctg}\sin x.$

6. $y = \ln(\sqrt{2\sin x}).$

7. $y = \left(\frac{1}{\sin x - \cos x}\right).$

8. $y = e^{\sin x - \cos x}.$

9. $y = \operatorname{arctg}\left(\frac{\sin x - \cos x}{\sqrt{2}}\right).$

10. $y = \ln(\sin x - \cos x).$

11. $y = \frac{1}{(\sin x + \cos x)^2}.$

12. $y = e^{-\sqrt{2}\cos x}.$

13. $y = -\operatorname{arctg}\cos x.$

14. $y = \ln(-\sqrt{2\cos x}).$

15. $y = \frac{1}{(\sin x - \cos x)^2}.$

16. $y = e^{-\sin x - \cos x}.$

17. $y = \sqrt[3]{\sin x}.$

18. $y = \ln(-\sin x - \cos x).$

19. $y = \sqrt{\frac{\sin x - \cos x}{\sqrt{2}}}.$

20. $y = e^{-\sqrt{2}\sin x}$

21. $y = \sqrt[3]{\cos x}$

22. $y = \ln(-\sqrt{2}\sin x).$

23. $y = \sqrt{\cos x}.$

24. $y = e^{\cos x - \sin x}.$

25. $y = \sqrt[3]{\frac{\sin x + \cos x}{\sqrt{2}}}.$

26. $y = \ln(\cos x - \sin x).$

27. $y = \sqrt{\sin x}.$

28. $y = e^{\sqrt{2}\cos x}$

29. $y = \sqrt{\frac{\sin x + \cos x}{\sqrt{2}}}.$

30. $y = \ln(\sqrt{2\cos x}).$

V BOB. ANIQMAS INTEGRAL

ANIQMAS INTEGRALNING TA'RIFI VA XOSSALARI

Berilgan $F(x)$ funksiyani differensiallashda uning $f(x)=F'(x)$ hbsilasini topish talab qilinadi. Masalan: $F(x)=x^3$, $f(x)=3x^2$. Teskari masalani ko'raylik: berilgan $f(x)$ hosilasi bo'yicha shunday $F(x)$ funksiyani topingki, uning hosilasi $f(x)$ ga teng, ya'ni $F'(x)=f(x)$ bo'lsin.

1- ta'rif. *Hosilasi* $f(x)$ *ga teng bo'lgan* $F(x)$ *funksiya* $f(x)$ *funksiyaning boshlang'ich. funksiyasi (boshlang'ichi) deyiladi.*

1- misol. Berilgan: $f(x)=3x^2$. $F(x)$ boshlang'ich funksiyani toping.

Echish. $F(x)=x^3$, chunki

$$F'(x)=(x^3)'=3x^2.$$

2- misol. Berilgan: $f(x)=\frac{1}{2\sqrt{x}}$. $F(x)$ ni toping.

Echish. $F(x)=\sqrt{x}$, chunki $F'(x)=(\sqrt{x})'=\frac{1}{2\sqrt{x}}$.

Ravshanki, agar $F(x)$ funksiya $f(x)$ funksiyaning boshlang'ichi bo'lsa, u holda $F(x)+C$ ko'rinishdagi istalgan funksiya ham (bu yerda *S* — ixtiyoriy o'zgarmas) $f(x)$ ning boshlang'ich funksiyasi bo'ladi, chunki $[F(x)+C]'=f(x)$.

Masalan, agar $f(x)=x^2$ bo'lsa, u holda $F(x)=\frac{x^3}{3}+2$; $F(x)=\frac{x^3}{3}-5$; $F(x)=\frac{x^3}{3}+\ln 6$, chunki $\left(\frac{x^3}{3}+2\right)'=\left(\frac{x^3}{3}-5\right)'=\left(\frac{x^3}{3}+\ln 6\right)'=x^2$.

2- ta'rif. *Agar* $F(x)$ *funksiya* $f(x)$ *funksiyaning boshlang'ichi bo'lsa, u holda* $F(x)+C$ *ifoda* $f(x)$ *futsksiyaning aniqmas integrali deyiladi:* va quyidagicha belgilanadi:

$$\int f(x)dx.$$

Shunday qilib, ta’rifga ko’ra agar $F'(x)=f(x)$ bo’lsa, $\int f(x)dx=F(x)+C.$

Bu yerda $f(x)$ — integral ostidagi funksiya, $f(x)dx$ — integral ostidagi ifoda, $\int$ — integral belgisi, x — integrallash o’zgaruvchisi.

$f(x)$ ning boshlang’ich funksiyasini topish amali funksiyani *integrallash* (integral olish) deyiladi.

Aniqmas integralning xossalari

Agar $F'(x)=f(x)$ bo’lsa, u holda

1. $\left(\int f(x)dx\right)'=f(x).$
2. $d\left(\int f(x)dx\right)=f(x)dx.$
3. $\int dF(x)=F(x)+C.$
4. $\int Af(x)dx=A\int f(x)dx,$ bu yerda $A=const.$
5. $\int[f_1(x)\pm f_2(x)]dx=\int f_1(x)dx\pm\int f_2(x)dx.$

Bu xossalarning to’g’riligi differensiallash orqali tekshiriladi.

Integrallashning asosiy usullarini qarab chiqishdan avval asosiy integrallar jadvalini jiddiy kengaytiradigan bir muhim integrallash qoidasini ko’rib chiqamiz. Agar $\int f(x)dx=F(x)+C$ va $z=\varphi(x)$ bo’lsa, u

holda

$$\int f(z)dx = F(z) + C. \qquad (1)$$

Bu qoida. integrallash formulasining ko'rinishi integrallash o'zgaruvchisining xarakteriga bog'liq emasligini bildiradi. Bu qoidaning to'g'riligi (1) tenglikning har ikki tomonini differensiallash orqali oson tekshiriladi. Jumladan,

1. $\int f(ax)dx = \frac{1}{a}\int f(ax)d(ax) = \frac{1}{a}F(ax) + C.$
2. $\int f(ax \pm b)dx = \frac{1}{a}\int f(ax \pm b)d(ax \pm b) = \frac{1}{a}F(ax \pm b) + C.$

Masalan:

$$\int \sin 3x dx = \frac{1}{3}\int \sin 3x d(3x) = -\frac{1}{3}\cos 3x + C.$$

$$\int e^{\frac{x}{2}}dx = 2\int e^{\frac{x}{2}}d\left(\frac{x}{2}\right) = 2e^{\frac{x}{2}} + C.$$

$$\int \frac{dx}{3x-5} = \frac{1}{3}\int \frac{d(3x-5)}{3x-5} = \frac{1}{3}\ln|3x-5| + C.$$

INTEGRALLASHNING ASOSIY USULLARI

Quyidagilar integrallashning asosiy usullari hisoblanadi:

1. Yoyib integrallash usuli.
2. Bevosita integrallash usuli.
3. O'rniga qo'yish usuli.
4. Bo'laklab integrallash usuli.

Yoyish (integral ostidagi ifodani yoyib integrallash) usuli

Agar $f(x) = f_1(x) \pm f_2(x)$ bo'lsa, u holda 1- § dagi 5- xossaga ko'ra yozish mumkin:

$$\int f(x)dx = \int [f_1(x) \pm f_2(x)]dx = \int f_1(x)dx \pm \int f_2(x)dx.$$

1. $\int (2-3\sqrt{x})^3 dx = \int (8-36\sqrt{x}dx+54xdx-27\sqrt{x^3})dx =$

$= 8\int dx - 36\int \sqrt{x}dx + 54\int dx - 27\int \sqrt[3]{x}dx =$

$= 8x - 36\cdot\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + 54\cdot\frac{x^2}{2} - 27\cdot\frac{x^{\frac{5}{2}}}{\frac{5}{2}} + C =$

$= 8x - 24x\sqrt{x} + 27x^2 - \frac{54}{5}x^2\sqrt{x} + C.$

Shuni qayd qilib o'taymizki, har qaysi qo'shiluvchini integrallagandan so'ng ixtiyoriy o'zgarmasni yozish shart emas, chunki bu o'zgarmaslarning yig'indisi yana o'zgarmas bo'lib, uni biz eng oxirida yozishimiz yetarli.

$$\int \frac{x^3-2x^2+3x+1}{x^2}dx = \int\left(x-2+\frac{3}{x}+\frac{1}{x^2}\right)dx =$$

2. $= \int xdx - 2\int dx + 3\int \frac{dx}{x} + \int \frac{dx}{x^2} =$

$$= \frac{x^2}{2} - 2x + 3\ln|x| - \frac{1}{x} + C.$$

3. $\int \cos^2\frac{x}{2}dx = \frac{1}{2}\int(1+\cos)dx = \frac{1}{2}\int dx + \frac{1}{2}\int \cos xdx = \frac{1}{2}(x+\sin x) + C.$

4. $\int ctg^2 xdx = \int\left(\frac{1}{\sin^2 x}-1\right)dx = \int \frac{1}{\sin^2 x}dx - \int dx = -ctgx - x + C.$

Bu misollardan ko'rinadiki, yoyish usuli bilan integrallaganimizda integral ostidagi funksiyani elementar matematika vositalari yordamida shunday qo'shiluvchilarga yoyganimizda ulardan olingan integral jadvaldagi integraldan iborat bo'lsin.

Bevosita integrallash usuli

Bu usul asosida (1) qoida yotadi. Unga ko'ra aniqmas integrallarni

hisoblaganda integrallash o'zgaruvchisi x erkli o'zgaruvchi yoki $z=\varphi(x)$ funksiyadan iborat bo'lishidan qat'i nazar 1- § da bayon qilingan 1 – 5 xossalarni va I — XVI jadval integrallarni tatbiq qilish mumkin.

Bu usulni tadbig'ini quyidagi misollarda ko'rsatamiz.

1. $\int \frac{dx}{3-x} = -\int \frac{d(3-x)}{3-x} = -\ln|3-x| + C.$

2. $\int \frac{dx}{\cos^2 7x} = \frac{1}{7}\int \frac{d(7x)}{\cos^2 7x} = \frac{1}{7} tg\,7x + C.$

3. $\int (2-3x)^5 dx = -\frac{1}{3}\int (2-3x)^5 d(2-3x) =$

$$= -\frac{1}{3}\cdot\frac{(2-3x)^6}{6} + C = -\frac{1}{18}(2-3x)^6 + C.$$

4. $\int \frac{dx}{1+9x^2} = \frac{1}{3}\int \frac{d(3x)}{1+(3x)^2} = \frac{1}{3} arctg\,3x + C.$

Ko'pchilik hollarda dastlab integral ostidagi funksiyani yoyib olib, so'ngra bevosita integrallashni tatbiq qilishga to'g'ri keladi.

1. $\int tg^2(ax)dx = \int\left(\frac{1}{\cos^2(ax)} - 1\right)dx = \frac{1}{a} tg(ax) - x + C.$

2. $\int \frac{2x-1}{2x+3}dx = \int \frac{2x+3-4}{2x+3}dx = \int\left(1 - \frac{4}{2x+3}\right)dx =$

$$= x - 4\int \frac{dx}{2x+3} = x - 2\ln|2x+3| + C.$$

3. $\int \frac{x^2+3}{x-2}dx = \int \frac{x^2-4+7}{x-2}dx = \int\left(x+2+\frac{7}{x-2}\right)dx =$

$$= \frac{(x+2)^2}{2} + 7\ln|x-2| + C.$$

Integral ostidagi kasrning surati maxrajining differensalidan iborat bo'lsa, integral maxrajning logarifmiga teng bo'ladi.

4. $\int \frac{(x^3-1)dx}{x^4-4x+1} = \frac{1}{4}\int \frac{d(x^4-4x+1)}{x^4-4x+1} = \frac{1}{4}\ln\left|x^4-4x+1\right| + C.$

5. $\int \frac{dx}{x(1+\ln x)} = \int \frac{d(1+\ln x)}{1+\ln x} = \ln\left|1+\ln x\right| + C.$

6. $\int \frac{x^3-1}{x+1}dx = \int \frac{x^3+1-2}{x+1}dx = \int \frac{(x+1)(x^2-x+1)-2}{x+1}dx =$

$$= \int\left(\frac{x^2-x+1}{x+1} - \frac{2}{x+1}\right)dx = \frac{x^3}{3} - \frac{x^2}{3} + x - 2\ln|x+1| + C.$$

1–masala. Aniqmas integralni toping.

$$\int \frac{1-\cos x}{(x-\sin x)^2}dx = \left|\begin{array}{c} x-\sin x = t \\ (1-\cos x)dx = dt \end{array}\right| = \int \frac{dt}{t^2} = -t^{-1} + C = -\frac{1}{x-\sin x} + C.$$

1. $\int (4-3x)e^{-3x}dx.$ **2.** $\int \operatorname{arctg}\sqrt{4x-1}dx.$ **3.** $\int (3x+4)e^{3x}dx.$

4. $\int (4x-2)\cos 2xdx.$ **5.** $\int (4-16x)\sin 4xdx.$ **6.** $\int (5x-2)e^{3x}dx.$

7. $\int (1-6x)e^{2x}dx.$ **8.** $\int \ln(x^2+4)dx.$ **9.** $\int (2-4x)\sin 2xdx.$

10. $\int \operatorname{arctg}\sqrt{6x-1}dx.$ **11.** $\int (4x-3)e^{-2x}dx.$ **12.** $\int (2-9x)e^{-3x}dx.$

13. $\int \operatorname{arctg}\sqrt{2x-1}dx.$ **14.** $\int \operatorname{arctg}\sqrt{3x-1}dx.$ **15.** $\int \operatorname{arctg}\sqrt{5x-1}dx.$

16. $\int (5x+6)\cos 2xdx.$ **17.** $\int (3x-2)\cos 5xdx.$ **18.** $\int (x\sqrt{2}-3)\cos 2xdx.$

19. $\int (4x+7)\cos 3xdx.$ **20.** $\int (2x-5)\cos 4xdx.$ **21.** $\int (8-3x)\cos 5xdx.$

22. $\int (x+5)\sin 3xdx.$ **23.** $\int (2-3x)\sin 2xdx.$ **24.** $\int (4x+3)\sin 5xdx.$

25. $\int (7x-10)\sin 4xdx.$ **26.** $\int (\sqrt{2}-8x)\sin 3xdx.$ **27.** $\int \frac{x}{\cos^2 x}dx.$

28. $\int \frac{x}{\sin^2 x}dx.$ **29.** $\int x\sin^2 xdx.$ **30.** $\int \frac{x\cos x}{\sin^3 x}dx.$

Javoblar. 1.1 $-\ln\left|\frac{1+\sqrt{x^2+1}}{x}\right|+C$; **1.2** $\ln|x|+\frac{1}{2}\ln^2 x+C$; **1.3** $-\arcsin\frac{1}{|x|}+C$;

1.4 $\frac{x^2}{2}+\ln^2 x+C$; **1.5** $\frac{1}{2}\ln\left|x^2+\frac{1}{2}+\sqrt{x^4+x^2+1}\right|+C$;

1.6 $-\frac{1}{4}\cdot(\arccos x)^4+\arccos x+C$; **1.7** $-\frac{\ln^2\cos x}{2}+C$; **1.8** $\frac{1}{2\cos^2(x+1)}+C$;

1.9 $\frac{1}{2}\cdot\ln\left(x^2+1\right)+\frac{1}{2\left(x^2+1\right)}+C$; **1.10** $-\frac{1}{x-\sin x}+C$; **1.11** $-\frac{1}{x\cdot\sin x}+C$;

1.12 $\frac{1}{4}\cdot\ln\left|x^4+1\right|+\frac{1}{2}\cdot arctgx^2+C$; **1.13** $\frac{1}{2}\cdot\ln\left|x^2-\frac{1}{2}+\sqrt{x^4-x^2-1}\right|+C$;

1.14 $\frac{3}{5}\cdot\sqrt[3]{(x-1)^5}+\frac{3}{2}\cdot\sqrt[3]{(x-1)^2}+C$; **1.15** $\ln\left(x-1\right)+\frac{1}{2}\ln^2(x-1)+C$;

1.16 $-\frac{1}{12}\cdot\frac{1}{(x^3+3x+1)^4}+C$; **1.17** $\frac{1}{2}\left(4arctg^2x-\ln(1+x^2)\right)+C$;

1.18 $\frac{1}{2}x^2-2\ln(x^2+4)+C$; **1.19** $\frac{1}{2}\cdot\ln\left|x^2+2\sin x\right|+C$;

1.20 $-\frac{1}{2\cdot(2\sin x-3\cos x)^2}+C$; **1.21** $\ln\left|1+4x^2\right|-\frac{1}{4}\cdot arctg^2 2x+C$;

1.22 $-\frac{1}{\sqrt{x}+x}+C$; **1.23** $\frac{1}{2}\cdot arctgx^2+C$; **1.24** $\sqrt{x^2+1}-\ln\left|\frac{1+\sqrt{x^2+1}}{x}\right|+C$;

1.25 $\sqrt{x^2+1}-\frac{1}{2}\ln\left|\frac{\sqrt{x^2+1}-1}{\sqrt{x^2+1}+1}\right|+C$; **1.26** $\frac{1}{2}\left(arctg^2x+\ln(1+x^2)\right)+C$;

1.27 $\frac{1}{2}\cdot\ln\left|1+x^2\right|-\frac{(arctgx)^5}{5}+C$; **1.28** $\frac{1}{2}\left(x^2-\ln(x^2+1)\right)+C$;

1.29 $\frac{1}{3}\cdot(\arcsin x)^3+\arcsin x+C$; **1.30** $2arctg\sqrt{x}-\ln|x+1|+C$.

Bo’laklab integrallash usuli.

Bo’laklab integrallash usuli quyidagi formula orqali topiladi:

$$\int udv = uv - \int vdu, \qquad (2)$$

bunda, $u(x)$, $v(x)$ lar uzluksiz differensiallanuvchi funksiyalar.

(2) formula bo’laklab integrallash formulasi deyiladi.

Bo’laklab integrallashning mohiyati shundan iboratki, berilgan integralni hisoblashda integral ostidagi $f(x)dx$ ifodani $u \cdot dv$ ko’paytma shaklida tasvirlab va (2) formulani tatbiq qilib, berilgan $\int udv$ integralni $\int vdu$ jadval itegrali yoki osongina olinadigan integral bilan almashtiriladi.

2–masala. Aniqmas integralni toping.

$$\int \ln(4x^2+1)dx = \begin{vmatrix} u = \ln(4x^2+1) & dv = dx \\ du = \dfrac{8x}{4x^2+1} & v = x \end{vmatrix} = x\ln(4x^2+1) - 8\int \frac{x^2}{4x^2+1}dx =$$

$$= x\ln(4x^2+1) - 2\int\left(1 - \frac{1}{4x^2+1}\right)dx = x\ln(4x^2+1) - 2\left(x - \frac{1}{2}\operatorname{arctg}2x\right) + C =$$

$$= x\ln(4x^2+1) + \operatorname{arctg}2x - 2x + C.$$

Maple 7 dasturi yordamida

```
Int(ln(4*x^2+1),x)=int(ln(4*x^2+1),x);
```

$$\int \ln(4x^2+1)\,dx = x\ln(4x^2+1) - 2x + \arctan(2x)$$

tekshirib ko’ring.

1. $\int \frac{dx}{x\sqrt{x^2+1}}$.
2. $\int \frac{1+\ln x}{x}dx$.
3. $\int \frac{dx}{x\sqrt{x^2-1}}$.
4. $\int \frac{x^2+\ln x^2}{x}dx$.
5. $\int \frac{x}{\sqrt{x^4+x^2+1}}dx$.
6. $\int \frac{\arccos^3 x - 1}{\sqrt{1-x^2}}dx$.

7. $\int \mathrm{tg}x \cdot \ln\cos x dx.$ **8.** $\int \frac{\mathrm{tg}(x+1)}{\cos^2(x+1)}dx.$ **9.** $\int \frac{x^3}{(x^2+1)^2}dx.$

10. $\int \frac{\sin x - \cos x}{(\cos x + \sin x)^5}dx.$ **11.** $\int \frac{x\cdot\cos x + \sin x}{(x\cdot\sin x)^2}dx.$ **12.** $\int \frac{x^3+x}{x^4+1}dx.$

13. $\int \frac{x}{\sqrt{x^4-x^2-1}}dx.$ **14.** $\int \frac{x}{\sqrt{x^2-1}}dx.$ **15.** $\int \frac{1+\ln(x-1)}{x-1}dx.$

16. $\int \frac{(x^2+1)dx}{(x^3+3x+1)^5}.$ **17.** $\int \frac{4\mathrm{arctg}x - x}{1+x^2}dx.$ **18.** $\int \frac{x^3}{x^2+4}dx.$

19. $\int \frac{x+\cos x}{x^2+2\sin x}dx.$ **20.** $\int \frac{2\cos x + 3\sin x}{(2\sin x - 3\cos x)^3}dx.$ **21.** $\int \frac{8x - \mathrm{arctg}2x}{1+4x^2}dx.$

22. $\int \frac{\frac{1}{2\sqrt{x}}+1}{(\sqrt{x}+x)^2}dx.$ **23.** $\int \frac{x}{x^4+1}dx.$ **24.** $\int \frac{x+\frac{1}{x}}{\sqrt{x^2+1}}dx.$

25. $\int \frac{x-\frac{1}{x}}{\sqrt{x^2+1}}dx.$ **26.** $\int \frac{\mathrm{arctg}x + x}{1+x^2}dx.$ **27.** $\int \frac{x-(\mathrm{arctg}x)^4}{1+x^2}dx.$

28. $\int \frac{x^3}{x^2+1}dx.$ **29.** $\int \frac{(\arcsin x)^2+1}{\sqrt{1-x^2}}dx.$ **30.** $\int \frac{1-\sqrt{x}}{\sqrt{x}(x+1)}dx.$

Javoblar. 2.1 $(x-1)e^{-3x}+C$; **2.2** $x\cdot arctg\sqrt{4x-1} - \frac{1}{4}\cdot\sqrt{4x-1}+C$;

2.3 $(x+1)e^{3x}+C$; **2.4** $(2x-1)\sin 2x + \cos 2x + C$;

2.5 $(4x-1)\cos 4x - \sin 4x + C$; **2.6** $\frac{1}{9}\cdot(15x-11)e^{3x}+C$;

2.7 $(2-3x)e^{2x}+C$; **2.8** $x\ln(x^2+4) - 2x + 4arctg\left(\frac{x}{2}\right)+C$;

2.9 $(2x-1)\cos 2x - \sin 2x + C$; **2.10** $x\cdot arctg\sqrt{6x-1} - \frac{1}{6}\cdot\sqrt{6x-1}+C$;

2.11 $\frac{1}{2}\cdot(1-4x)e^{-2x}+C$; **2.12** $\frac{1}{3}\cdot(1+9x)e^{-3x}+C$;

2.13 $x\cdot arctg\sqrt{2x-1} - \frac{1}{2}\cdot\sqrt{2x-1}+C$; **2.14** $x\cdot arctg\sqrt{3x-1} - \frac{1}{3}\cdot\sqrt{3x-1}+C$;

2.15 $x \cdot arctg\sqrt{5x-1} - \frac{1}{5} \cdot \sqrt{5x-1} + C$; **2.16** $\frac{1}{2}(5x+6)\sin 2x + \frac{5}{4}\cos 2x + C$;

2.17 $\frac{1}{5}(3x-2)\sin 5x + \frac{3}{25}\cos 5x + C$; **2.18** $\frac{1}{2}(x\sqrt{2}-3)\sin 2x + \frac{\sqrt{2}}{4}\cos 2x + C$;

2.19 $\frac{1}{3}(4x+7)\sin 3x + \frac{4}{9}\cos 3x + C$; **2.20** $\frac{1}{4}(2x-5)\sin 4x + \frac{1}{8}\cos 4x + C$;

2.21 $\frac{1}{5}(8-3x)\sin 5x - \frac{3}{25}\cos 5x + C$; **2.22** $-\frac{1}{3}(x+5)\cos 3x + \frac{1}{9}\sin 3x + C$;

2.23 $\frac{1}{2}(3x-2)\cos 2x - \frac{3}{4}\sin 2x + C$; **2.24** $-\frac{1}{5}(4x+3)\cos 5x + \frac{4}{25}\sin 5x + C$;

2.25 $-\frac{7}{4}x\cos 4x + \frac{7}{16}\sin 4x + \frac{5}{2}\cos 4x + C$; **2.26** $\frac{1}{3}(8x-\sqrt{2})\cos 3x - \frac{8}{9}\sin 3x + C$;

2.27 $x \cdot tgx + \ln|\cos x| + C$; **2.28** $-x \cdot ctgx + \ln|\sin x| + C$;

2.29 $-\frac{x}{5} \cdot \sin 2x - \frac{1}{8} \cdot \cos 2x + \frac{x^2}{4} + C$; **2.30** $-\frac{x + \cos x \cdot \sin x}{2\sin^2 x} + C$

Kvadrat uchhadni o’z ichiga olgan funksiyalarning integrallari

Kvadrat uchhadni o’z ichiga olgan

$$\int \frac{Ax+B}{ax^2+bx+c}dx; \quad \int \frac{Ax+B}{\sqrt{ax^2+bx+c}}dx; \quad \int \sqrt{ax^2+bx+c}\,dx$$

funksiyalarni integrallash jadvalidagi fomulalarga keltirib integrallash uchun avvalo kvadrat uchhaddan to’liq kvadratni ajratib olish kerak bo’ladi. Bu holda ax^2+bx+c kvadrat uchhad quyidagi ko’rinishga keladi:

$$ax^2+bx+c = a\left(x^2+\frac{b}{a}x+\frac{c}{a}\right) = a\left(\left(x+\frac{b}{a}\right)x^2+\frac{b}{a}+\frac{c}{a}-\frac{b^2}{4a^2}\right) =$$

$$= a\left(\left(x+\frac{b}{2a}\right)^2 \pm k^2\right).$$

So’ngra almashtirishlar yo’li bilan yuqoridagi integrallarni integrallash jadvalidagi formulalarga keltirish mumkin.

Masalan,

$$\int \frac{dx}{x^2+5x+7} = \int \frac{dx}{x^2+2\cdot\frac{5}{2}x+\frac{25}{4}-\frac{25}{4}+7} =$$

1. $$= \int \frac{dx}{\left(x+\frac{5}{2}\right)^2+\frac{3}{4}} = \frac{2}{\sqrt{3}} arctg \frac{x+\frac{5}{2}}{\frac{\sqrt{3}}{2}} + C =$$

$$= \frac{2}{\sqrt{3}} arctg \frac{2x+5}{\sqrt{3}} + C.$$

$$\int \frac{3x+1}{\sqrt{-x^2+x+2}} dx = -\frac{3}{2}\int \frac{-2x-\frac{3}{2}}{\sqrt{2+x-x^2}} dx =$$

2. $$= -\frac{3}{2}\int \frac{1-2x-\frac{5}{2}}{\sqrt{2+x-x^2}} dx = -\frac{3}{2}\int \frac{d(2+x-x^2)}{\sqrt{2+x-x^2}} + \frac{5}{3}\int \frac{d\left(x-\frac{1}{2}\right)}{\sqrt{\frac{9}{4}-\left(x-\frac{1}{2}\right)^2}} =$$

$$= -3\sqrt{2+x-x^2} + \frac{5}{2}\arcsin\frac{2x-1}{3} + C.$$

3. $$\int \sqrt{x^2-2x-1}dx = \int \sqrt{(x-1)^2-2}dx =$$

$$= \frac{1}{2}(x-1)\sqrt{x^2-2x-1} - \ln\left|x-1+\sqrt{x^2-2x-1}\right| + C.$$

3–masala. $\int \frac{1}{x^2+x-2} dx$ аниқмас интегрални ҳисоблайлик:

$$\int \frac{1}{x^2+x-2} dx = \int \frac{1}{(x-1)(x+2)} dx = \frac{1}{3}\left(\int \frac{dx}{x-1} - \int \frac{dx}{x+2}\right) =$$

$$= \frac{1}{3}\ln|x-1| - \frac{1}{3}\ln|x+2| + C = \frac{1}{3}\ln\left|\frac{x-1}{x+2}\right| + C.$$

Maple 7 dasturi yordamida

```
> Int(1/(x^2+x-2),x)=int(1/(x^2+x-2),x);
```

$$\int \frac{1}{x^2+x-2}\,dx = -\frac{1}{3}\ln(x+2)+\frac{1}{3}\ln(x-1)$$

tekshirib ko'ring.

1. $\int \frac{dx}{x^2+x+5}$ **2.** $\int \frac{dx}{\sqrt{6-4x-2x^2}}$ **3.** $\int \sqrt{x^2+8x+25}dx$

4. $\int \frac{x+2}{x^2+2x+3}dx$ **5.** $\int \frac{dx}{\sqrt{5-4x-x^2}}$ **6.** $\int \sqrt{8+2x-x^2}dx$

7. $\int \frac{5x-7}{x^2+3x+8}dx$ **8.** $\int \frac{dx}{\sqrt{2+3x-2x^2}}$ **9.** $\int \frac{x+7}{x^2+11x+42}dx$

10. $\int \frac{dx}{x^2-x+14}$ **11.** $\int \frac{dx}{x^2+2x+6}$ **12.** $\int \frac{x}{3x^2-8x+9}dx$

13. $\int \frac{7x+4}{x^2+x+9}dx$ **14.** $\int \frac{dx}{\sqrt{3x^2-6x+9}}$ **15.** $\int \sqrt{x^2+4x+13}dx$

16. $\int \frac{3x-11}{x^2+8x+18}dx$ **17.** $\int \frac{7x-8}{x^2+5x+17}dx$ **18.** $\int \frac{2x-3}{x^2+x+5}dx$

19. $\int \frac{x+2}{x^2-2x-3}dx$ **20.** $\int \frac{dx}{\sqrt{x^2-6x+3}}$ **21.** $\int \sqrt{5+4x-x^2}dx$

22. $\int \frac{x+2}{x^2+2x-3}dx$ **23.** $\int \frac{3x+4}{x^2+7x+14}dx$ **24.** $\int \frac{x}{3x^2+4x+5}dx$

25. $\int \frac{dx}{x^2+4x-12}$ **26.** $\int \frac{dx}{\sqrt{3x^2-6x+12}}$ **27.** $\int \frac{x-3}{x^2-9x+23}dx$

28. $\int \frac{dx}{x^2-6x+34}$ **29.** $\int \frac{x+7}{x^2+11x+42}dx$ **30.** $\int \frac{dx}{x^2+6x+34}$

Javoblar. 3.1 $\frac{2}{\sqrt{19}}\operatorname{arctg}\frac{2x+1}{\sqrt{19}}+C$. **3.2** $\frac{1}{\sqrt{2}}\arcsin\frac{x+1}{2}+C$.

3.3 $\frac{x+4}{2}\sqrt{x^2+8x+25}+\frac{9}{2}\ln\left|x+4+\sqrt{x^2+8x+25}\right|+C$.

3.4 $\frac{1}{2}\ln\left(x^2+2x+3\right)+\frac{1}{\sqrt{2}}\operatorname{arctg}\frac{x+1}{\sqrt{2}}+C.$ **3.5** $\arcsin\frac{x+2}{3}+C.$

3.6 $\frac{x-1}{2}\sqrt{8+2x-x^2}+\frac{9}{2}\arcsin\frac{x-1}{2}+C.$

3.7 $\frac{5}{2}\ln\left(x^2+3x+8\right)-\frac{29}{\sqrt{23}}\operatorname{arctg}\frac{2x+3}{\sqrt{23}}+C.$ **3.8** $\frac{1}{\sqrt{2}}\arcsin\frac{4x-3}{5}+C.$

3.9 $\frac{1}{2}\ln\left(x^2+11x+42\right)+\frac{3}{\sqrt{47}}\operatorname{arctg}\frac{2x+11}{\sqrt{47}}+C.$ **3.10** $\frac{2}{\sqrt{55}}\operatorname{arctg}\frac{2x-1}{\sqrt{55}}+C.$

3.11 $\frac{1}{\sqrt{5}}\operatorname{arctg}\frac{x+1}{\sqrt{5}}+C.$ **3.12** $\frac{1}{6}\ln\left(3x^2-8x+9\right)+\frac{4}{\sqrt{11}}\operatorname{arctg}\frac{6x-8}{\sqrt{11}}+C.$

3.13 $\frac{7}{2}\ln\left(x^2+x+9\right)+\frac{1}{\sqrt{35}}\operatorname{arctg}\frac{2x+1}{\sqrt{35}}+C.$ **3.14** $\frac{1}{\sqrt{3}}\ln\left|x-1+\sqrt{3x^2-6x+9}\right|+C.$

3.15 $\frac{x+2}{2}\sqrt{x^2+4x+13}+\frac{9}{2}\ln\left|x+2+\sqrt{x^2+4x+13}\right|+C.$

3.16 $\frac{3}{2}\ln\left(x^2+8x+18\right)-\frac{23}{\sqrt{2}}\operatorname{arctg}\frac{x+4}{\sqrt{2}}+C.$

3.17 $\frac{7}{2}\ln\left(x^2+5x+17\right)-\frac{51}{\sqrt{43}}\operatorname{arctg}\frac{2x+5}{\sqrt{43}}+C.$

3.18 $\ln\left(x^2+x+5\right)-\frac{8}{\sqrt{19}}\operatorname{arctg}\frac{2x+1}{\sqrt{19}}+C.$

3.19 $-\frac{1}{4}\ln|x+1|+\frac{5}{4}\ln|x-3|+C.$ **3.20** $\ln\left|x-3+\sqrt{x^2-6x+3}\right|+C.$

3.21 $\frac{x-2}{2}\sqrt{5+4x-x^2}+\frac{9}{2}\arcsin\frac{x-2}{2}+C.$ **3.22** $\frac{1}{4}\ln|x+3|+\frac{3}{4}\ln|x-1|+C.$

3.23 $\frac{3}{2}\ln\left(x^2+7x+14\right)-\frac{13}{\sqrt{7}}\operatorname{arctg}\frac{2x+7}{\sqrt{7}}+C.$

3.24 $\frac{1}{6}\ln\left(3x^2+4x+5\right)-\frac{2}{\sqrt{11}}\operatorname{arctg}\frac{6x+4}{\sqrt{11}}+C.$ **3.25** $\frac{1}{8}\ln\left|\frac{x-2}{x+6}\right|+C$

3.26 $\frac{1}{\sqrt{3}}\ln\left|x-1+\sqrt{3x^2-6x+12}\right|+C.$

3.27 $\frac{1}{2}\ln\left(x^2-9x+23\right)+\frac{3}{\sqrt{11}}\operatorname{arctg}\frac{2x-9}{\sqrt{11}}+C.$

3.28 $\frac{1}{5}\operatorname{arctg}\frac{x-3}{5}+C.$ **3.29** $\frac{1}{2}\ln\left(x^2+11x+42\right)+\frac{3}{\sqrt{47}}\operatorname{arctg}\frac{2x+11}{\sqrt{47}}+C.$

3.30 $\frac{1}{5}\operatorname{arctg}\frac{x+3}{5}+C.$

Ratsional kasrlarni integrallash.

Ratsional kasr deb,

$$\frac{Q_m(x)}{P_n(x)}=\frac{b_0x^m+b_1x^{m-1}+...+b_m}{a_0x^n+a_1x^{n-1}+...+a_n},$$

Ko'rinishdagi kasrga aytiladi, bu yerda $P_n(x)$ va $Q_m(x)$– darajalari mos ravishda n va m ga teng bo'lgan x ga nisbatan butun ko'phadlar.

Agar $n\geq m$ bo'lsa, ratsional kasr noto'g'ri, $n<m$ bo'lsa, to'g'ri kasr deyiladi.

Har qanday noto'g'ri kasrning suratini maxrajiga bo'lish natijasida butun qismini ajratib, uni biror ko'phad va to'g'ri kasr yig'indisi shaklida yozish mumkin:

$$\frac{Q_m(x)}{P_n(x)}=q(x)+\frac{Q_k(x)}{P_n(x)},\quad k<n.$$

Masalan, $\frac{x^4+4}{x^2+3x-1}$ noto'g'ri kasrning suratini maxrajiga bo'g'sak, quyidagiga ega bo'lamiz:

$$\frac{x^4+4}{x^2+3x-1}=x^2-3x+10+\frac{-3x+14}{x^2+3x-1}.$$

4–masala. Aniqmas integralni toping.

$$\int\frac{x^3-3x^2-12}{(x-4)(x-3)(x-2)}dx.$$

Kasrni bo'lamiz

$$\begin{array}{l|l} x^3-3x^2-12 & \dfrac{x^3-9x^2+26x-24}{1} \\ \underline{x^3-9x^2+26x-24} & \\ 6x^2-26x+12 & \end{array}$$

$$\int\frac{x^3-3x^2-12}{(x-4)(x-3)(x-2)}dx=\int\left(1+\frac{6x^2-26x+12}{(x-4)(x-3)(x-2)}\right)dx$$

$\dfrac{6x^2-26x+12}{(x-4)(x-3)(x-2)}$ to'g'ri kasrni sodda ratsional kasrlar yig'indisi ko'rinishida yozamiz:

$$\frac{6x^2-26x+12}{(x-4)(x-3)(x-2)}=\frac{A}{x-4}+\frac{B}{x-3}+\frac{C}{x-2}=$$
$$=\frac{A(x-3)(x-2)+B(x-4)(x-2)+C(x-4)(x-3)}{(x-4)(x-3)(x-2)}.$$

$$A(x-3)(x-2)+B(x-4)(x-2)+C(x-4)(x-3)=6x^2-26x+12.$$

$A,\ B,\ C$ noma'lum koeffitsiyentlarni topish uchun maxrajni nolga aylantiradigan son qiymatlarni x ning o'rniga qo'yish bilan topamiz.

Odatda, bu usulni xususiy qiymatlar usuli deyiladi.

$x=4$ da, $2A=4\Rightarrow A=2;$

$x=3$ da, $-B=-12\Rightarrow B=12;$

$x=2$ da, $2C=-16\Rightarrow C=-8;$

Bundan

$$\int\left(1+\frac{6x^2-26x+12}{(x-4)(x-3)(x-2)}\right)dx=\int\left(1+\frac{2}{x-4}+\frac{12}{x-3}-\frac{8}{x-2}\right)dx=$$

$$= x + 2\ln|x-4| + 12\ln|x-3| - 8\ln|x-2| + C.$$

Maple 7 dasturi yordamida

```
> Int((1+(6*x^2-26*x+12)/((x-4)*(x-3)*(x-2)),x))=int((1+(6*x^2-26*x+12)/((x-4)*(x-3)*(x-2)),x));
```

$$\int 1 + \frac{6x^2 - 26x + 12}{(x-4)(x-3)(x-2)}\,dx = x + 2\ln(x-4) + 12\ln(x-3) - 8\ln(x-2)$$

tekshirib ko'ring.

1. $\int \frac{x^3+1}{x^2-x}dx.$

2. $\int \frac{3x^3+1}{x^2-1}dx.$

3. $\int \frac{x^3-17}{x^2-4x+3}dx.$

4. $\int \frac{2x^3+5}{x^2-x-2}dx.$

5. $\int \frac{2x^3-1}{x^2+x-6}dx.$

6. $\int \frac{3x^3+25}{x^2+3x+2}dx.$

7. $\int \frac{x^3+2x^2+3}{(x-1)(x-2)(x-3)}dx.$

8. $\int \frac{3x^3+2x^2+1}{(x+2)(x-2)(x-1)}dx.$

9. $\int \frac{x^3}{(x-1)(x+1)(x+2)}dx.$

10. $\int \frac{x^3-3x^2-12}{(x-4)(x-3)\cdot x}dx.$

11. $\int \frac{4x^3+x^2+2}{x(x-1)(x-2)}dx.$

12. $\int \frac{3x^2-2}{x^3-x}dx.$

13. $\int \frac{x^3-3x^2-12}{(x-4)(x-2)x}dx.$

14. $\int \frac{x^5-x^3+1}{x^2-x}dx.$

15. $\int \frac{x^5+3x^3-1}{x^2+x}dx.$

16. $\int \frac{2x^5-8x^3+3}{x^2-2x}dx.$

17. $\int \frac{3x^5-12x^3-7}{x^2+2x}dx.$

18. $\int \frac{-x^5+9x^3+4}{x^2+3x}dx.$

19. $\int \frac{-x^5+25x^3+1}{x^2+5x}dx.$

20. $\int \frac{x^3-5x^2+5x+23}{(x-1)(x+1)(x-5)}dx.$

21. $\int \frac{x^5+2x^4-2x^3+5x^2-7x+9}{(x-3)(x-1)x}\,dx.$

22. $\int \frac{2x^4-5x^2-8x-8}{x(x-2)(x+2)}\,dx.$

23. $\int \frac{4x^4+2x^2-x-3}{x(x-1)(x+1)}\,dx.$

24. $\int \frac{3x^4+3x^3-5x^2+2}{x(x-1)(x+2)}\,dx.$

25. $\int \frac{2x^4+2x^3-41x^2+20}{x(x-4)(x+5)}\,dx.$

26. $\int \frac{x^5-x^4-6x^3+13x+6}{x(x-3)(x+2)}\,dx.$

27. $\int \frac{3x^3-x^2-12x-2}{x(x+1)(x-2)}\,dx.$

28. $\int \frac{2x^4+2x^3-3x^2+2x-9}{x(x-1)(x+3)}\,dx.$

29. $\int \frac{2x^3-x^2-7x-12}{x(x-3)(x+1)}\,dx.$

30. $\int \frac{2x^3-40x-8}{x(x+4)(x-2)}\,dx.$

Javoblar. 4.1 $\frac{x^2}{2}+x-\ln|x|+2\ln|x-1|+C;$

4.2 $\frac{3x^2}{2}+2\ln|x-1|+\ln|x+1|+C;$

4.3 $\frac{x^2}{2}+4x+8\ln|x-1|+5\ln|x-3|+C;$

4.4 $x^2+2x+7\ln|x-2|-\ln|x+1|+C;$

4.5 $x^2-2x+11\cdot\ln|x+3|+3\cdot\ln|x-2|+C;$

4.6 $\frac{3x^2}{2}-9x+22\ln|x+1|-\ln|x+2|+C;$

4.7 $x+3\ln|x-1|-19\ln|x-2|+24\ln|x-3|+C;$

4.8 $3x-1{,}25\ln|x+2|8{,}25\ln|x-2|-2\ln|x-1|+C;$

4.9 $x+\frac{1}{6}\cdot\ln|x-1|+\frac{1}{2}\cdot\ln|x+1|-\frac{3}{8}\cdot\ln|x+2|+C;$

4.10 $x+\ln|x-4|+4\cdot\ln|x-3|-\ln|x|+C;$

4.11 $4x+\ln|x|-7\cdot\ln|x-1|-19\cdot\ln|x-2|+C;$

4.12 $3x+2\cdot\ln|x|+\frac{1}{2}\cdot\ln|x-1|-\frac{5}{2}\cdot\ln|x+1|+C;$

4.13 $x+\frac{1}{2}\cdot\ln|x-4|+4\cdot\ln|x-2|-\frac{3}{2}\cdot\ln|x|+C;$

4.14 $\frac{x^4}{4}+\frac{x^3}{3}-\ln|x|+\ln|x-1|+C;$

4.15 $\frac{x^4}{4}-\frac{x^3}{3}+2x^2-4x-\ln|x|+5\ln|x+1|+C;$

4.16 $\frac{x^4}{2}+\frac{4x^3}{3}-\frac{3}{2}\ln|x|+\frac{3}{2}\ln|x-2|+C;$

4.17 $\frac{3x^4}{4}-2x^3-\frac{7}{2}\ln|x|+\frac{7}{2}\ln|x+2|+C;$

4.18 $-\frac{x^4}{4}+x^3+\frac{4}{3}\ln|x|-\frac{4}{3}\ln|x+3|+C;$

4.19 $-\frac{x^4}{4}+\frac{5x^3}{3}+\frac{1}{5}\ln|x|-\frac{1}{5}\ln|x+5|+C;$

4.20 $x-3\cdot\ln|x-1|+\ln|x+1|+2\cdot\ln|x-5|+C;$

4.21 $\frac{x^3}{3}+x+4\cdot\ln|x+3|+2\cdot\ln|x-1|-3\cdot\ln|x|+C;$

4.22 $x^2+2\cdot\ln|x|-\frac{3}{2}\cdot\ln|x-2|-\frac{5}{2}\cdot\ln|x+2|+C;$

4.23 $2x^2+3\cdot\ln|x|+\ln|x-1|+2\ln|x+1|+C;$

4.24 $\frac{3x^2}{2}-\ln|x|+\ln|x-1|+\ln|x+2|+C;$

4.25 $x^2-\ln|x|+\frac{1}{9}\cdot\ln|x-4|-\frac{1}{9}\cdot\ln|x+5|+C;$

4.26 $\frac{x^3}{3}-\ln|x|+3\cdot\ln|x-3|-2\cdot\ln|x+2|+C;$

4.27 $3x+\ln|x|+2\cdot\ln|x+1|-\ln|x-2|+C;$

4.28 $x^2-2x+3\cdot\ln|x|-\frac{3}{2}\cdot\ln|x-1|+\frac{11}{2}\cdot\ln|x+3|+C;$

4.29 $2x+4\cdot\ln|x|+\ln|x-3|-2\cdot\ln|x+1|+C;$

4.30 $2x+\ln|x|+\ln|x+4|-6\cdot\ln|x-2|+C.$

Noma’lum koeffitsiyentlar usuli.

Agar integral ostidagi kasr to’g’ri $(n > m)$ bo’lsa, quyidagicha ish tutamiz:

1. $Q_m(x)$ maxrajni ko’paytuvchilarga yoyamiz. Aytaylik,

$$Q_m(x) = (x-a)\cdot(x-b)^{\alpha}\cdot(x^2+p_1x+q_1)\cdot(x^2+p_2x+q_2)^{\beta}$$

bo’lsin, bu yerda a – sodda haqiqiy ildiz, b – karraligi α bo’lgan haqiqiy ildiz, $x^2+p_1x+q_1$ va $x^2+p_2x+q_2$ – okmpleks qo’shma ildizlarga ega bo’lgan uchhadlar.

Alebradagi teoremaga ko’ra to’g’ri ratsional $\dfrac{Q_m(x)}{P_n(x)}$ kasrni quyidagicha yozish mumkin:

$$\frac{Q_m(x)}{P_n(x)} = \frac{A}{x-a} + \frac{B_1}{x-b} + \frac{B_2}{(x-b)^2} + \ldots + \frac{B_\alpha}{(x-b)^\alpha} + \frac{Mx+N}{x^2+p_1x+q_1} +$$
$$+ \frac{M_1x+N_1}{x^2+p_\gamma x+q_\gamma} + \frac{M_2x+N_2}{\left(x^2+p_\gamma x+q_\gamma\right)^2} + \ldots + \frac{M_\beta x+N_\beta}{\left(x^2+p_\gamma x+q_\gamma\right)^\beta}.$$

Masalan, agar $Q_m(x) = (x-1)(x+2)^3(x^2+2x+2)(x^2+3x+5)^2$ bo’lsa, u holda yoyilma quyidagicha bo’ladi:

$$\frac{Q_m(x)}{P_n(x)} = \frac{A}{x-1} + \frac{B_1}{x+2} + \frac{B_2}{(x+2)^2} + \frac{B_3}{(x+2)^3} + \frac{Mx+N}{x^2+2x+2} +$$
$$+ \frac{M_1x+N_1}{x^2+3x+5} + \frac{M_2x+N_2}{(x^2+3x+5)^2}.$$

Bu yerda $A, B_1, B_2, B_3, M, N, M_1, N_1, M_2, N_2$ – hozircha noma’lum koeffitsiyentlar. Bu koeffitsiyentlarni topish uchun yoyilmaning o’ng

tomonini umumiy maxrajga keltiramiz va chap hamda o'ng tomondagi suratlarni aynan tenglaymiz. Hosil bo'lgan ayniyatda x ning o'ngdagi va chapdagi bir xil darajalari oldidagi koeffitsiyentlarni tenglab, noma'lum koeffitsiyentlarni topish uchun tenglamalar sistemasi tuziladi. Tenglamalar sistemasini yechish bilan noma'lum koeffitsiyentlar topiladi. Bu usulni odatda *noma'lum koeffitsiyentlar usuli* deyiladi.

5–masala. Aniqmas integralni toping.

$$\int \frac{x^3+6x^2+13x+9}{(x+1)(x+2)^3}dx.$$

$\frac{x^3+6x^2+13x+9}{(x+1)(x+2)^3}$ to'g'ri kasrni sodda ratsional kasrlar yig'indisi ko'rinishida yozamiz:

$$\frac{x^3+6x^2+13x+9}{(x+1)(x+2)^3}=\frac{A}{x+1}+\frac{B_1}{x+2}+\frac{B_2}{(x+2)^2}+\frac{B_3}{(x+2)^3}=$$

$$=\frac{A(x+2)^3+B_1(x+1)(x+2)^2+B_2(x+1)(x+2)+B_3(x+1)}{(x+1)(x+2)^3}.$$

$$A(x+2)^3+B_1(x+1)(x+2)^2+B_2(x+1)(x+2)+B_3(x+1)=x^3+6x^2+13x+9$$

$x=-1$ da, $A=1$;

$x=-2$ da, $-B_3=-1\Rightarrow B_3=1$;

x ning bir xil darajalari oldidagi koeffitsiyentlarni tenglaymiz:

x^3: $\quad A+B_1=1\Rightarrow B_1=0$;

x^0: $\quad 8A+4B_1+2B_2+B_3=9\Rightarrow B_2=0$;

Demak,

$$\frac{x^3+6x^2+13x+9}{(x+1)(x+2)^3}=\frac{1}{x+1}+\frac{1}{(x+2)^3}.$$

Bundan:

$$\int\left(\frac{1}{x+1}+\frac{1}{(x+2)^3}\right)dx=\ln|x+1|-\frac{1}{2(x+2)^2}+C.$$

Maple 7 dasturi yordamida

```
Int((x^3+6*x^2+13*x+9)/((x+1)*(x+2)^3),x)=int((x^3+6*x^2+13*x+9)/((x+1)*(x+2)^3),x);
```

$$\int\frac{x^3+6x^2+13x+9}{(x+1)(x+2)^3}dx=\ln(x+1)-\frac{1}{2}\frac{1}{(x+2)^2}$$

tekshirib ko'ring.

1. $\int\frac{x^3+6x^2+13x+8}{x(x+2)^3}dx.$

2. $\int\frac{x^3-6x^2+13x-6}{(x+2)(x-2)^3}dx.$

3. $\int\frac{x^3+6x^2+14x+10}{(x+1)(x+2)^3}dx.$

4. $\int\frac{x^3-6x^2+11x-10}{(x+2)(x-2)^3}dx.$

5. $\int\frac{x^3+6x^2+11x+7}{(x+1)(x+2)^3}dx.$

6. $\int\frac{2x^3+6x^2+7x+1}{(x-1)(x+1)^3}dx.$

7. $\int\frac{x^3+6x^2+10x+10}{(x-1)(x+2)^3}dx.$

8. $\int\frac{2x^3+6x^2+7x+2}{x(x+1)^3}dx.$

9. $\int\frac{x^3-6x^2+13x-8}{x(x-2)^3}dx.$

10. $\int\frac{x^3-6x^2+13x-7}{(x+1)(x-2)^3}dx.$

11. $\int\frac{x^3-6x^2+14x-6}{(x+1)(x-2)^3}dx.$

12. $\int\frac{x^3-6x^2+10x-10}{(x+1)(x-2)^3}dx.$

13. $\int\frac{x^3+x+2}{(x+2)x^3}dx.$

14. $\int\frac{3x^3+9x^2+10x+2}{(x-1)(x+1)^3}dx.$

15. $\int \frac{2x^3+x+1}{(x+1)x^3}\,dx.$

16. $\int \frac{2x^3+6x^2+7x+4}{(x+2)(x+1)^3}\,dx.$

17. $\int \frac{2x^3+6x^2+5x}{(x+2)(x+1)^3}\,dx.$

18. $\int \frac{2x^3+6x^2+7x}{(x-2)(x+1)^3}\,dx.$

19. $\int \frac{2x^3+6x^2+5x+4}{(x-2)(x+1)^3}\,dx.$

20. $\int \frac{x^3+6x^2+4x+24}{(x-2)(x+2)^3}\,dx.$

21. $\int \frac{x^3+6x^2+14x+4}{(x-2)(x+2)^3}\,dx.$

22. $\int \frac{x^3+6x^2+18x-4}{(x-2)(x+2)^3}\,dx.$

23. $\int \frac{x^3+6x^2+10x+12}{(x-2)(x+2)^3}\,dx.$

24. $\int \frac{x^3-6x^2+14x-4}{(x+2)(x-2)^3}\,dx.$

25. $\int \frac{x^3+6x^2+15x+2}{(x-2)(x+2)^3}\,dx.$

26. $\int \frac{2x^3-6x^2+7x-4}{(x-2)(x-1)^3}\,dx.$

27. $\int \frac{2x^3-6x^2+7x}{(x+2)(x-1)^3}\,dx.$

28. $\int \frac{x^3+6x^2-10x+52}{(x-2)(x+2)^3}\,dx.$

29. $\int \frac{x^3-6x^2+13x-6}{(x-2)(x+2)^3}\,dx.$

30. $\int \frac{x^3+6x^2+13x+6}{(x-2)(x+2)^3}\,dx.$

Javoblar. 5.1 $\ln|x|-\frac{1}{2(x+2)^2}+C$; **5.2** $\ln|x+2|-\frac{1}{2(x-2)^2}+C$;

5.3 $\ln|x+1|-\frac{1}{(x+2)^2}+C$; **5.4** $\ln|x+2|+\frac{1}{2(x-2)^2}+C$;

5.5 $\ln|x+1|+\frac{1}{2(x+2)^2}+C$; **5.6** $2\ln|x-1|-\frac{1}{2(x+1)^2}+C$;

5.7 $\ln|x-1|+\frac{1}{(x+2)^2}+C$; **5.8** $2\ln|x|-\frac{1}{2(x+1)^2}+C$;

5.9 $\ln|x|-\frac{1}{2(x-2)^2}+C$; **5.10** $\ln|x+1|-\frac{1}{2(x-2)^2}+C$;

5.11 $\ln|x+1|-\frac{1}{(x-2)^2}+C$; **5.12** $\ln|x+1|+\frac{1}{(x-2)^2}+C$;

5.13 $\ln|x+2|-\frac{1}{2x^2}+C$; **5.14** $3\ln|x-1|-\frac{1}{2(x+1)^2}+C$;

5.15 $2\ln|x+1|-\frac{1}{2x^2}+C$; **5.16** $2\ln|x+2|-\frac{1}{2(x+1)^2}+C$;

5.17 $2\ln|x+2|+\frac{1}{2(x+1)^2}+C$; **5.18** $2\ln|x-2|-\frac{1}{2(x+1)^2}+C$;

5.19 $2\ln|x-2|+\frac{1}{2(x+1)^2}+C$; **5.20** $\ln|x-2|+\frac{4}{(x+2)^2}+C$;

5.21 $2\ln|x-2|-\frac{1}{2(x+1)^2}+C$; **5.22** $\ln|x-2|-\frac{3}{(x+2)^2}+C$;

5.23 $\ln|x-2|+\frac{1}{(x+2)^2}+C$; **5.24** $\ln|x+2|-\frac{1}{(x-2)^2}+C$;

5.25 $\ln|x-2|-\frac{3}{2(x+2)^2}+C$; **5.26** $2\cdot\ln|x-2|-\frac{1}{2(x-1)^2}+C$;

5.27 $2\cdot\ln|x+2|-\frac{1}{2(x-1)^2}+C$; **5.28** $\ln|x-2|+\frac{11}{(x+2)^2}+C$;

5.29 $\frac{1}{16}\cdot\ln|x-2|+\frac{15}{16}\cdot\ln|x+2|+\frac{33x+34}{4(x+2)^2}+C$;

5.30 $\ln|x-2|-\frac{1}{2(x+2)^2}+C$.

Maxrajining ildizlari kompleks va karrali.

6–masala. Aniqmas integralni toping.

$$\int\frac{x^3+5x^2+12x+4}{(x+2)^2(x^2+4)}dx.$$

$\frac{x^3+5x^2+12x+4}{(x+2)^2(x^2+4)}$ to'g'ri kasrni sodda ratsional kasrlar yig'indisi

ko'rinishida yozamiz:

$$\frac{x^3+5x^2+12x+4}{(x+2)^2(x^2+4)}=\frac{A}{x+2}+\frac{B}{(x+2)^2}+\frac{Cx+D}{x^2+4}=$$

$$=\frac{A(x+2)(x^2+4)+B(x^2+4)+(Cx+D)(x+2)^2}{(x+2)^2(x^2+4)}.$$

$$A(x+2)(x^2+4)+B(x^2+4)+(Cx+D)(x^2+4x+4)=x^3+5x^2+12x+4.$$

$x=-2$ da, $8B=-8\Rightarrow B=-1;$

x ning bir xil darajalari oldidagi koeffitsiyentlarni tenglaymiz:

$x^3:$ $\quad A+C=1\Rightarrow A=0;$

$x:$ $\quad 4A+4C+4D=12\Rightarrow C=1;$

$x^0:$ $\quad 8A+4B+4D=4\Rightarrow D=2;$

Bundan:

$$\int\left(-\frac{1}{(x+2)^2}+\frac{x+2}{x^2+4}\right)dx=\frac{1}{x+2}+\frac{1}{2}\int\left(\frac{2x}{x^2+4}\right)+2\int\frac{dx}{x^2+4}=.$$

$$=\frac{1}{x+2}+\frac{1}{2}\ln\left|x^2+4\right|+arctg\,\frac{x}{2}+C.$$

Maple 7 dasturi yordamida

```
Int((x^3+5*x^2+12*x+4)/((x+2)^2*(x^2+4)),x)=int((x^3+5*x^2+12*x+4)/((x+2)^2*(x^2+4)),x);
```

$$\int\frac{x^3+5x^2+12x+4}{(x+2)^2(x^2+4)}dx=\frac{1}{x+2}+\frac{1}{2}\ln(x^2+4)+\arctan\left(\frac{1}{2}x\right)$$

tekshirib ko'ring.

1. $\int\frac{x^3+4x^2+4x+2}{(x+1)^2(x^2+x+1)}dx.$

2. $\int\frac{x^3+4x^2+3x+2}{(x+1)^2(x^2+1)}dx.$

3. $\int \frac{2x^3+7x^2+7x-1}{(x+2)^2(x^2+x+1)}\,dx.$

4. $\int \frac{2x^3+4x^2+2x-1}{(x+1)^2(x^2+2x+2)}\,dx.$

5. $\int \frac{x^3+6x^2+9x+6}{(x+1)^2(x^2+2x+2)}\,dx.$

6. $\int \frac{2x^3+11x^2+16x+10}{(x+2)^2(x^2+2x+3)}\,dx.$

7. $\int \frac{3x^3+6x^2+5x-1}{(x+1)^2(x^2+2)}\,dx.$

8. $\int \frac{x^3+9x^2+21x+21}{(x+3)^2(x^2+3)}\,dx.$

9. $\int \frac{x^3+6x^2+8x+8}{(x+2)^2(x^2+4)}\,dx.$

10. $\int \frac{2x^3-4x^2-16x-12}{(x-1)^2(x^2+4x+5)}\,dx.$

11. $\int \frac{-3x^3+13x^2-13x+1}{(x-2)^2(x^2-x+1)}\,dx.$

12. $\int \frac{x^3+2x^2+10x}{(x+1)^2(x^2-x+1)}\,dx.$

13. $\int \frac{3x^3+x+46}{(x-1)^2(x^2+9)}\,dx.$

14. $\int \frac{4x^3+24x^2+20x-28}{(x+3)^2(x^2+2x+2)}\,dx.$

15. $\int \frac{2x^3+3x^2+3x+2}{(x^2+x+1)(x^2+1)}\,dx.$

16. $\int \frac{x^3+x+1}{(x^2+x+1)(x^2+1)}\,dx.$

17. $\int \frac{x^2+x+3}{(x^2+x+1)(x^2+1)}\,dx.$

18. $\int \frac{2x^3+4x^2+2x+2}{(x^2+x+1)(x^2+x+2)}\,dx.$

19. $\int \frac{2x^3+7x^2+7x+9}{(x^2+x+1)(x^2+x+2)}\,dx.$

20. $\int \frac{4x^2+3x+4}{(x^2+1)(x^2+x+1)}\,dx.$

21. $\int \frac{3x^3+4x^2+6x}{(x^2+2)(x^2+2x+2)}\,dx.$

22. $\int \frac{2x^2-x+1}{(x^2-x+1)(x^2+1)}\,dx.$

23. $\int \frac{x^3+x^2+1}{(x^2+1)(x^2-x+1)}\,dx.$

24. $\int \frac{x^3+x+1}{(x^2+1)(x^2-x+1)}\,dx.$

25. $\int \frac{2x^3+2x+1}{(x^2+1)(x^2-x+1)}\,dx.$

26. $\int \frac{x^3+2x^2+x+1}{(x^2+1)(x^2+x+1)}\,dx.$

27. $\int \frac{x+4}{(x^2+2)(x^2+x+2)}\,dx.$ **28.** $\int \frac{2x^3+2x^2+2x+1}{(x^2+1)(x^2+x+1)}\,dx.$

29. $\int \frac{3x^3+7x^2+12x+6}{(x^2+x+3)(x^2+2x+3)}\,dx.$ **30.** $\int \frac{2x^3+3x^2+3x+2}{(x^2+1)(x^2+x+1)}\,dx.$

Javoblar. 6.1 $-\frac{1}{x+1}+\frac{1}{2}\cdot\ln(x^2+x+1)+\frac{1}{\sqrt{3}}\cdot arctg\left(\frac{2x+1}{\sqrt{3}}\right)+C;$

6.2 $-\frac{1}{x+1}+\frac{1}{2}\cdot\ln\left|x^2+1\right|+arctgx+C;$

6.3 $\frac{1}{x+2}+\ln(x^2+x+1)-\frac{2}{\sqrt{3}}\cdot arctg\left(\frac{2x+1}{\sqrt{3}}\right)+C;$

6.4 $\frac{1}{x+1}+\ln(x^2+2x+2)-arctg(x+1)+C;$

6.5 $-\frac{2}{x+1}+\frac{1}{2}\ln(x^2+2x+2)+arctg(x+1)+C;$

6.6 $-\frac{2}{x+2}+\ln(x^2+2x+3)-\frac{1}{\sqrt{2}}\cdot arctg\left(\frac{x+1}{\sqrt{2}}\right)+C;$

6.7 $\frac{1}{x+1}+\frac{3}{2}\ln(x^2+2)+\frac{1}{\sqrt{2}}\cdot arctg\frac{x}{\sqrt{2}}+C;$

6.8 $-\frac{1}{x+1}+\frac{1}{2}\ln\left|x^2+3\right|+\frac{2}{\sqrt{3}}\cdot arctg\left(\frac{x}{\sqrt{3}}\right)+C;$

6.9 $-\frac{1}{x+2}+\frac{1}{2}\ln(x^2+2)+\frac{1}{2}\cdot arctg\frac{x}{2}+C;$

6.6 $\frac{3}{x-1}+\ln(x^2+4x+5)-arctg(x+2)+C;$

6.11 $-\frac{1}{x-2}-\frac{3}{2}\cdot\ln(x^2-x+1)-\sqrt{3}\cdot arctg\left(\frac{2x-1}{\sqrt{3}}\right)+C;$

6.12 $\frac{3}{x+1}+\frac{1}{2}\cdot\ln(x^2-x+1)+\frac{7}{\sqrt{3}}\cdot arctg\left(\frac{2x-1}{\sqrt{3}}\right)+C;$

6.13 $-\frac{5}{x-1}+\frac{3}{2}\cdot\ln(x^2+9)+\frac{1}{3}\cdot arctg\left(\frac{x}{3}\right)+C;$

6.14 $-\frac{4}{x+3}+2\ln(x^2+2x+2)-8arctg(x+1)+C$;

6.15 $\frac{1}{2}\cdot\ln(x^2+x+1)+\frac{1}{\sqrt{3}}\cdot arctg\left(\frac{2x+1}{\sqrt{3}}\right)+\frac{1}{2}\cdot\ln(x^2+1)+arctgx+C$;

6.16 $\ln(x^2+x+1)-\frac{1}{2}\cdot\ln(x^2+1)+C$;

6.17 $\ln(x^2+x+1)+\frac{2}{\sqrt{3}}\cdot arctg\left(\frac{2x+1}{\sqrt{3}}\right)-\ln(x^2+1)+arctgx+C$;

6.18 $-\ln(x^2+x+1)+\frac{2}{\sqrt{3}}\cdot arctg\left(\frac{2x+1}{\sqrt{3}}\right)+2\ln(x^2+x+2)+C$;

6.19 $\frac{8}{\sqrt{3}}\cdot arctg\left(\frac{2x+1}{\sqrt{3}}\right)+\ln(x^2+x+2)+C$;

6.20 $3\cdot arctgx+\frac{2}{\sqrt{3}}\cdot arctg\left(\frac{2x+1}{\sqrt{3}}\right)+C$;

6.21 $\ln(x^2+2)+\frac{1}{2}\cdot\ln(x^2+2x+2)-arctg(x+1)+C$;

6.22 $\frac{1}{2}\cdot\ln(x^2-x+1)+\frac{1}{\sqrt{3}}\cdot arctg\left(\frac{2x-1}{\sqrt{3}}\right)-\frac{1}{2}\cdot\ln(x^2+1)+arctgx+C$;

6.23 $\frac{1}{2}\cdot\ln(x^2-x+1)+\frac{1}{\sqrt{3}}\cdot arctg\left(\frac{2x-1}{\sqrt{3}}\right)+arctgx+C$;

6.24 $\frac{2}{\sqrt{3}}\cdot arctg\left(\frac{2x-1}{\sqrt{3}}\right)+\frac{1}{2}\ln(x^2+1)+C$;

6.25 $\frac{1}{2}\cdot\ln(x^2-x+1)-\frac{1}{2}\cdot\ln(x^2+1)+\sqrt{3}arctg\left(\frac{2x-1}{\sqrt{3}}\right)+C$;

6.26 $\frac{2}{\sqrt{3}}\cdot arctg\left(\frac{2x+1}{\sqrt{3}}\right)+\frac{1}{2}\cdot\ln(x^2+1)+C$;

6.27 $\ln(x^2+x+2)-\ln(x^2+2)+\frac{1}{\sqrt{2}}\cdot arctg\left(\frac{x}{\sqrt{2}}\right)+C$;

6.28 $\frac{1}{2}\cdot\ln(x^2+x+1)+\frac{1}{\sqrt{3}}\cdot arctg\left(\frac{2x+1}{\sqrt{3}}\right)+\frac{1}{2}\cdot\ln(x^2+1)+C$;

6.29 $\ln(x^2+x+3)+\frac{1}{2}\cdot\ln(x^2+x+3)+C$;

6.30 $\frac{1}{2}\cdot\ln(x^2+x+1)+\frac{1}{\sqrt{3}}\cdot arctg\left(\frac{2x+1}{\sqrt{3}}\right)+\frac{1}{2}\cdot\ln(x^2+1)+arctgx+C.$

$\int x^m(a+bx^n)^p\,dx$ **differensial binomlari integrali,**

bu yerda $a,\ b$ – o'zgarmas sonlar, $m,\ n$ va p –ratsional sonlar.

$\int x^m(a+bx^n)^p\,dx$ ko'rinishdagi integrallarni hisoblash $m,\ n$ va p ratsional sonlarga bog'liqligini rus matematigi P.L.Chebishev ko'rsatgan va uchta holdagina elementar funksiyalar orqali ifodalanadi.

1. Agar p –butun son bo'lsa, u holda integral $x=t^s$ o'rniga qo'yish yordamida (bunda s – kasrlar maxrajining m va n ning eng kichik umumiy karralisi) ratsional funksiya integraliga keltiriladi.

$\int\sqrt[3]{x}(2+\sqrt{x})^2\,dx$ integralda $p=2$ – butun son. Hisoblaymiz

$$\int x^{\frac{1}{3}}(x+x^{\frac{1}{2}})dx=\left|\begin{matrix}m=\frac{1}{3};\ \ n=\frac{1}{2}\\ x=t^6,\ dx=6t^5dt\end{matrix}\right|=\int t^2(2+t^3)^2 6t^5dt=$$

$$=6\int(4t^7+4t^{10}+t^{13})dt=6\left(\frac{1}{2}t^8+\frac{4}{11}t^{11}+\frac{1}{14}t^{14}\right)+C=$$

$$=\left|t=\sqrt[6]{x}\right|=3\sqrt[3]{x^4}+\frac{24}{11}\sqrt[6]{x^{11}}+\frac{3}{7}\sqrt[3]{x^7}+C.$$

2. Agar $\frac{m+1}{n}$ – butun son bo'lsa, u holda integral $a+bx^n=t^s$ o'rniga qo'yish bilan ratsional funksiya integrallanadi, bunda s son p kasrning maxraji.

$\int x^5\sqrt[3]{(1+x^3)^2}\,dx$ integralda $m=5,\ n=3,\ p=\frac{2}{3};\ \frac{m+1}{n}=\frac{5+1}{3}=2$ – butun son. $s=3$, ya'ni p kasrning maxraji.

Tegishli o'rniga qo'yishdan, $1+x^3=t^3,\ x=(t^3-1)^{\frac{1}{3}},\ dx=t^2(t^3-1)^{-\frac{2}{3}}dt.$

Demak,

$$\int (t^3-1)^{\frac{5}{3}}\cdot t^2\cdot t^2(t^3-1)^{-\frac{2}{3}}dt=\int (t^3-1)t^4dt=\int (t^7-t^4)dt=$$

$$=\frac{t^8}{8}-\frac{t^5}{5}+C=t^5\left(\frac{t^3}{8}-\frac{1}{5}\right)+C.$$

x o'zgaruvchiga qaytib, uzil-kesil topamiz:

$$\int x^5\sqrt[3]{(1+x^3)^2}dx=(1+x^3)^{\frac{5}{3}}\left(\frac{1+x^3}{8}-\frac{1}{5}\right)+C.$$

3. $\frac{m+1}{n}+p$ –butun son bo'lganda $ax^{-n}+b=t^s$ o'rniga qo'yish bilan, bu yerda s son p kasrning maxraji.

7–masala. Aniqmas integralni toping.

$$\int\frac{\sqrt[5]{1+\sqrt[3]{x}}}{x\cdot\sqrt[5]{x^2}}.$$

Berilgan integralni $\int\frac{\sqrt[5]{1+\sqrt[3]{x}}}{x\cdot\sqrt[5]{x^2}}=\int x^{-\frac{7}{5}}(1+x^{\frac{1}{3}})^{\frac{1}{5}}dx$ ko'rinishda yozib olamiz. Integral ositidagi ifoda $x^m(a+bx^n)^p$ ekanligidan

$$m=-\frac{7}{5},\ \ n=\frac{1}{3},\ \ \ p=\frac{1}{5},\ \frac{m+1}{n}+p=-1.$$

$x^{-\frac{1}{3}}+1=t^5$ o'rniga qo'yishdan foydalanamiz.

$x=\left(t^5-1\right)^{-3},\ \ dx=-3(t^5-1)^{-4}5t^4dt$ ga egamiz.

$$\int x^{-\frac{7}{5}}(1+x^{\frac{1}{3}})^{\frac{1}{5}}dx=\int (t^5-1)^{\frac{21}{5}}\left(1+(t^5-1)^{-1}\right)^{\frac{1}{5}}\frac{(-15)t^4}{(t^5-1)^4}dt=$$

$$=-\frac{15}{6}t^6+C=-\frac{5}{2}\left(x^{-\frac{1}{3}}+1\right)^{\frac{6}{5}}+C=-\frac{5}{2}\left(\frac{1+\sqrt[3]{x}}{\sqrt[3]{x}}\right)^{\frac{5}{6}}+C.$$

1. $\int \frac{\sqrt{1+\sqrt{x}}}{x\cdot\sqrt[4]{x^3}}\,dx.$ **2.** $\int \frac{\sqrt[3]{1+\sqrt{x}}}{x\cdot\sqrt[3]{x^2}}\,dx.$ **3.** $\int \frac{\sqrt{1+\sqrt[3]{x}}}{x\cdot\sqrt{x}}\,dx.$

4. $\int \frac{\sqrt[3]{1+\sqrt[3]{x}}}{x\cdot\sqrt[9]{x^4}}\,dx.$ **5.** $\int \frac{\sqrt[3]{1+\sqrt[3]{x^2}}}{x\cdot\sqrt[9]{x^8}}\,dx.$ **6.** $\int \frac{\sqrt[3]{(1+\sqrt[3]{x})^2}}{x\cdot\sqrt[9]{x^5}}\,dx.$

7. $\int \frac{\sqrt[3]{(1+\sqrt[3]{x^2})^2}}{x^2\cdot\sqrt[9]{x}}\,dx.$ **8.** $\int \frac{\sqrt{1+\sqrt[3]{x^2}}}{x^2}\,dx.$ **9.** $\int \frac{\sqrt{1+x}}{x^2\cdot\sqrt{x}}\,dx.$

10. $\int \frac{\sqrt[4]{(1+\sqrt{x})^3}}{x\cdot\sqrt[8]{x^7}}\,dx.$ **11.** $\int \frac{\sqrt[4]{(1+\sqrt[3]{x})^3}}{x\cdot\sqrt[12]{x^7}}\,dx.$ **12.** $\int \frac{\sqrt[4]{(1+\sqrt[3]{x^2})^3}}{x^2\cdot\sqrt[6]{x}}\,dx.$

13. $\int \frac{\sqrt{1+\sqrt[4]{x^3}}}{x^2\cdot\sqrt[8]{x}}\,dx.$ **14.** $\int \frac{\sqrt[3]{1+\sqrt[4]{x^3}}}{x^2}\,dx.$ **15.** $\int \frac{\sqrt[3]{(1+\sqrt[4]{x^3})^2}}{x^2\cdot\sqrt[4]{x}}\,dx.$

16. $\int \frac{\sqrt[5]{(1+\sqrt{x})^4}}{x\cdot\sqrt[10]{x^9}}\,dx.$ **17.** $\int \frac{\sqrt[5]{(1+\sqrt[3]{x})^4}}{x\cdot\sqrt[5]{x^3}}\,dx.$ **18.** $\int \frac{\sqrt[5]{(1+\sqrt[3]{x^2})^4}}{x^2\cdot\sqrt[5]{x}}\,dx.$

19. $\int \frac{\sqrt[5]{(1+\sqrt[4]{x^3})^4}}{x^2\cdot\sqrt[20]{x^7}}\,dx.$ **20.** $\int \frac{\sqrt[5]{(1+\sqrt[5]{x^4})}}{x^2\cdot\sqrt[25]{x^{11}}}\,dx.$ **21.** $\int \frac{\sqrt{1+\sqrt[5]{x^4}}}{x^2\cdot\sqrt[5]{x}}\,dx.$

22. $\int \frac{\sqrt[3]{1+\sqrt[5]{x^4}}}{x^2\cdot\sqrt[15]{x}}\,dx.$ **23.** $\int \frac{\sqrt[3]{(1+\sqrt[5]{x^4})^2}}{x^2\cdot\sqrt[3]{x}}\,dx.$ **24.** $\int \frac{\sqrt[4]{(1+\sqrt[5]{x^4})^3}}{x^2\cdot\sqrt[5]{x^2}}\,dx.$

25. $\int \frac{\sqrt[3]{1+\sqrt[4]{x}}}{x\cdot\sqrt[3]{x}}\,dx.$ **26.** $\int \frac{\sqrt[3]{(1+\sqrt[4]{x})^2}}{x\cdot\sqrt[12]{x^5}}\,dx.$ **27.** $\int \frac{\sqrt[4]{1+\sqrt[3]{x}}}{x\cdot\sqrt[12]{x^5}}\,dx.$

28. $\int \frac{\sqrt[4]{1+\sqrt[3]{x^2}}}{x\cdot\sqrt[6]{x^5}}\,dx.$ **29.** $\int \frac{\sqrt[3]{1+\sqrt[5]{x}}}{x\cdot\sqrt[15]{x^4}}\,dx.$ **30.** $\int \frac{\sqrt[3]{(1+\sqrt{x})^2}}{x\cdot\sqrt[6]{x^5}}$

Javoblar. 7.1 $-\frac{4\sqrt{(1+\sqrt{x})^3}}{3\sqrt[4]{x}}+C$; **7.2** $-\frac{3\sqrt[3]{(1+\sqrt{x})^4}}{2\sqrt[3]{x^2}}+C$; **7.3** $-2\sqrt{\frac{(1+\sqrt[3]{x})^3}{x}}+C$;

7.4 $-\frac{9}{4}\left(\sqrt[3]{\frac{1+\sqrt[3]{x}}{\sqrt[3]{x}}}\right)^4+C$; **7.5** $-\frac{9}{8}\left(\sqrt[3]{\frac{1+\sqrt[3]{x^2}}{\sqrt[3]{x^2}}}\right)^4+C$; **7.6** $-\frac{9}{5}\left(\sqrt[3]{\frac{1+\sqrt[3]{x}}{\sqrt[3]{x}}}\right)^5+C$;

7.7 $-\frac{9}{10}\left(\sqrt[3]{\frac{1+\sqrt[3]{x^2}}{\sqrt[3]{x^2}}}\right)^5+C$; **7.8** $\frac{\sqrt{\left(1+\sqrt[3]{x^2}\right)^3}}{x}+C$; **7.9** $-\frac{2}{3}\left(\sqrt{\frac{1+x}{x}}\right)^3+C$;

7.10 $-\frac{8}{7}\left(\sqrt[4]{\frac{1+\sqrt{x}}{\sqrt{x}}}\right)^7+C$; **7.11** $-\frac{12}{7}\sqrt[4]{\left(\frac{1+\sqrt[3]{x}}{\sqrt[3]{x}}\right)^7}+C$; **7.12** $-\frac{6\cdot\sqrt[4]{\left(1+\sqrt[3]{x^2}\right)^7}}{7\sqrt[6]{x^7}}+C$;

7.13 $-\frac{8}{9}\sqrt[4]{\left(\frac{1+\sqrt[4]{x^3}}{\sqrt[4]{x^3}}\right)^3}+C$; **7.14** $-\frac{\sqrt[3]{\left(1+\sqrt[4]{x^3}\right)^4}}{x}+C$; **7.15** $-\frac{4}{5}\left(x^{-\frac{3}{4}}+1\right)^{\frac{5}{3}}+C$;

7.16 $-\frac{10}{9}\left(\sqrt[5]{\frac{1+\sqrt{x}}{\sqrt{x}}}\right)^9+C$; **7.17** $-\frac{5}{3}\left(\sqrt[5]{\frac{1+\sqrt[3]{x}}{\sqrt[3]{x}}}\right)^9+C$; **7.18** $-\frac{5}{6}\left(\sqrt[5]{\frac{1+\sqrt[3]{x^2}}{\sqrt[3]{x^2}}}\right)^9+C$;

7.19 $-\frac{20}{27}\left(\sqrt[5]{\frac{1+\sqrt[4]{x^3}}{\sqrt[4]{x^3}}}\right)^9+C$; **7.20** $-\frac{25}{36}\left(\sqrt[5]{\frac{1+\sqrt[5]{x^4}}{\sqrt[5]{x^4}}}\right)^9+C$;

7.21 $-\frac{5}{6}\cdot\frac{\sqrt{\left(1+\sqrt[5]{x^4}\right)^3}}{x\cdot\sqrt[5]{x}}+C$; **7.22** $-\frac{15\left(1+\sqrt[5]{x^4}\right)^{\frac{4}{3}}}{16\cdot\sqrt[15]{x^{16}}}+C$; **7.23** $-\frac{3\left(1+\sqrt[5]{x^4}\right)^{\frac{5}{3}}}{4\sqrt[3]{x^4}}+C$;

7.24 $-\frac{5}{7}\left(x^{-\frac{4}{5}}+1\right)^{\frac{7}{4}}+C$; **7.25** $-3\left(x^{-\frac{1}{4}}+1\right)^{\frac{4}{3}}+C$; **7.26** $-\frac{12}{5}\sqrt[3]{\left(\frac{1+\sqrt[4]{x}}{\sqrt[4]{x}}\right)^5}+C$;

7.27 $-\frac{12}{5}\left(\sqrt[4]{\frac{1+\sqrt[3]{x}}{\sqrt[3]{x}}}\right)^5+C$; **7.28** $-\frac{6\cdot\sqrt[4]{\left(1+\sqrt[3]{x^2}\right)^5}}{5\sqrt[6]{x^5}}+C$; **7.29** $-\frac{15\left(1+\sqrt{x}\right)^{\frac{4}{3}}}{4\sqrt[15]{x^4}}+C$;

7.30 $-\frac{6}{5}\sqrt[3]{\left(1+\frac{1}{\sqrt{x}}\right)^5}+C$.

VI BOB. Aniq integral.

Aniq integral mavzusida Nyuton–Leybnits formulasi bilan tanishasiz va uni aniq integrallarni yechishdagi tatbiqi, hamda boshlang’ich funksiyalarni topishda qo’llaysiz. Undan tashqari geometrik masalalarni aniq integrallar yordamida yassi figuralarning yuzi, egri chiziq yoyi

uzunligi va jismning hajmini topish bilan bog'liq masalalarni o'zlashtirasiz.

Agar $F(x)$ funksiya $[a,b]$ da uzluksiz $f(x)$ funksiyaning boshlang'ichi bo'lsa, u holda quyidagi formula o'rinlidir:

$$\int_a^b f(x)dx = F(x)\Big|_a^b = F(b)-F(a). \quad (1)$$

Nyuton–leybnits formulasi deb ataluvchi (1) formula aniqmas integral bilan aniq integral o'rtasidagi bog'lanishni ifodalaydi.

Aniq integralda o'zgaruvchini almashtirish.

Bu holda ushbu formula o'rinli bo'ladi:

$$\int_a^b f(x)dx = \int_{\varepsilon}^{\beta} f[\varphi(t)]\cdot\varphi'(t)dt,$$

bu yerda $\varphi(t)$ va $\varphi'(t)$ lar, $[\alpha,\ \beta]$ kesmada uzluksiz funksiyalar, $a=\varphi(\alpha)$, $b=\varphi(\beta)$.

Masalan, $\int_3^8 \frac{xdx}{\sqrt{1+x}}$ integralni topish talab etilsin. $\sqrt{1+x}=t$ belgilash kiritamiz, u holda $1+x=t^2,\ dx=2tdt$. Yangi o'zgaruvchi t ning o'zgarish chegaralarini topamiz: x o'zgaruvchi $[3,\ 8]$ intervalda o'zgaradi. $x=3$ da $t=\sqrt{1+3}=2;\ x=8$ da $t=\sqrt{1+8}=3$.

Demak,

$$\int_3^8 \frac{xdx}{\sqrt{1+x}} = \int_2^3 \frac{(t^2-1)2tdt}{t} = 2\int_2^3 (t^2-1)dt = 2\left(\frac{t^3}{3}-t\right)\Big|_2^3 = \frac{32}{3}.$$

1–masala. Aniq integralni hisoblang.

$$\int_0^{1/2} \frac{8x - arctg\,2x}{1+4x^2}dx = \int_0^{1/2} \frac{8x}{1+4x^2}\,dx - \int_0^{1/2} \operatorname{arctg} 2x\, d(\operatorname{arctg} 2x) =$$

$$= \ln\left|1+4x^2\right|\Big|_0^{1/2} - \frac{1}{2}\operatorname{arctg}^2 2x\Big|_0^{1/2} = \ln 2 - 0 - \frac{1}{2}\cdot\frac{\pi^2}{16} + 0 = \ln 2 - \frac{\pi^2}{32}.$$

1. $\int_{e+1}^{e^2+1} \frac{1+\ln(x-1)}{x-1}dx.$

2. $\int_0^1 \frac{(x^2+1)dx}{(x^3+3x+1)^2}.$

3. $\int_0^1 \frac{4\operatorname{arctg} x - x}{1+x^2}dx.$

4. $\int_0^2 \frac{x^3 dx}{x^2+4}.$

5. $\int_{\pi}^{2\pi} \frac{x+\cos x}{x^2+2\sin x}dx.$

6. $\int_0^{\pi/4} \frac{2\cos x + 3\sin x}{(2\sin x - 3\cos x)^3}dx.$

7. $\int_1^4 \frac{\frac{1}{2\sqrt{x}}+1}{(\sqrt{x}+x)^2}dx.$

8. $\int_0^1 \frac{xdx}{x^4+1}dx.$

9. $\int_{\sqrt{3}}^{\sqrt{8}} \frac{x+\frac{1}{x}}{\sqrt{x^2+1}}dx.$

10. $\int_{\sqrt{3}}^{\sqrt{8}} \frac{x-\frac{1}{x}}{\sqrt{x^2+1}}dx.$

11. $\int_0^{\sqrt{3}} \frac{\operatorname{arctg} x + x}{1+x^2}dx.$

12. $\int_0^{\sqrt{3}} \frac{x-(\operatorname{arctg} x)^4}{1+x^2}dx.$

13. $\int_0^1 \frac{x^3}{x^2+1}dx.$

14. $\int_0^{\sin 1} \frac{(\arcsin x)^2+1}{\sqrt{1-x^2}}dx.$

15. $\int_1^3 \frac{1-\sqrt{x}}{\sqrt{x}(x+1)}dx.$

16. $\int_{\sqrt{3}}^{\sqrt{8}} \frac{dx}{x\sqrt{x^2+1}}.$

17. $\int_1^e \frac{1+\ln x}{x}dx.$

18. $\int_{\sqrt{2}}^2 \frac{dx}{x\sqrt{x^2-1}}.$

19. $\int_1^e \frac{x^2+\ln x^2}{x}dx.$

20. $\int_0^1 \frac{x}{\sqrt{x^4+x^2+1}}\,dx.$

21. $\int_0^1 \frac{x^3 dx}{(x^2+1)^2}$.

22. $\int_0^{\pi/4} \mathrm{tg}x \ln\cos x dx$.

23. $\int_{-1}^{0} \frac{\mathrm{tg}(x+1)}{\cos^2(x+1)} dx$.

24. $\int_0^{1/\sqrt{2}} \frac{(\arccos x)^3 - 1}{\sqrt{1-x^2}} dx$.

25. $\int_{\pi}^{2\pi} \frac{1-\cos x}{(x-\sin x)^2} dx$.

26. $\int_0^{\pi/4} \frac{\sin x - \cos x}{(x-\sin x)^2} dx$.

27. $\int_{\pi/4}^{\pi/2} \frac{x\cos x + \sin x}{(x\sin x)^2} dx$.

28. $\int_0^1 \frac{x^3 + x}{x^4+1} dx$.

29. $\int_{\sqrt{2}}^{\sqrt{3}} \frac{xdx}{\sqrt{x^4 - x^2 - 1}}$.

30. $\int_2^9 \frac{xdx}{\sqrt[3]{x-1}}$.

Javoblar. **1.1** $2\frac{1}{2}$; **1.2** $\frac{4}{15}$; **1.3** $\frac{\pi^2 - 4\cdot\ln 2}{8}$; **1.4** $2 - 2\ln 2$; **1.5** $\ln 2$; **1.6** $-\frac{17}{18}$; **1.7** $\frac{1}{3}$; **1.8** $\frac{\pi}{8}$;

1.9 $1 + \ln\sqrt{\frac{3}{2}}$; **1.10** $1 + \ln\sqrt{\frac{3}{2}}$; **1.11** $\frac{\pi^2}{18} + \ln 2$; **1.12** $\ln 2 - \frac{\pi^5}{5\cdot 3^5}$; **1.13** $\frac{1-\ln 2}{2}$; **1.14** $\frac{4}{3}$;

1.15 $\frac{\pi}{6} - \ln 2$; **1.16** $\ln\sqrt{\frac{3}{2}}$; **1.17** $\frac{3}{2}$; **1.18** $\frac{\pi}{12}$; **1.19** $\frac{e^2+1}{2}$; **1.20** $\ln\sqrt{\frac{3+2\sqrt{3}}{3}}$;

1.21 $\frac{\ln 4 - 1}{4}$; **1.22** $-\frac{1}{2}\cdot\ln^2\frac{\sqrt{2}}{2}$; **1.23** $\frac{tg^2 1}{2}$; **1.24** $\frac{15\pi^4}{2^{10}} - \frac{\pi}{4}$; **1.25** $\frac{1}{2\pi}$; **1.26** $-\frac{3}{16}$;

1.27 $\frac{4\sqrt{2}-2}{\pi}$; **1.28** $\frac{\ln 4 + \pi}{8}$; **1.29** $\ln\sqrt{\frac{5+2\sqrt{5}}{5}}$; **1.30** 23,1.

Bo’laklab integrallash.

Agar $u(x)$ va $v(x)$ lar $[a,b]$ kesmada differensiallanuvchi funksiyalar bo’lsa, u holda ushbu formula o’rinlidir:

$$\int_a^b udv = uv\Big|_a^b - \int_a^b vdu\,.$$

2–masala. Aniq integralni hisoblang.

$$\int_{-2}^{0}(x^2-4)\cos 3x dx = \left|\begin{matrix} u = x^2-4 & dv = \cos 3x dx \\ du = 2x dx & v = \frac{1}{3}\sin 3x \end{matrix}\right| =$$

$$= \frac{1}{3}(x^2-4)\sin 3x\Big|_{-2}^{0} - \frac{2}{3}\int_{-2}^{0} x\sin 3x dx = \left|\begin{matrix} u = x & dv = \sin 3x dx \\ du = dx & v = -\frac{1}{3}\cos 3x \end{matrix}\right| =$$

$$= -\frac{2}{3}\left(-\frac{1}{3}x\cos 3x\Big|_{-2}^{0} + \frac{1}{3}\int_{-2}^{0}\cos 3x dx\right) =$$

$$-\frac{2}{3}\left(-\frac{2}{3}\cos 6 + \frac{1}{9}\sin 3x\Big|_{-2}^{0}\right) = \frac{4}{9}\cos 6 - \frac{2}{27}\sin 6.$$

1. $\int_{-2}^{0}(x^2+5x+6)\cos 2x dx.$

2. $\int_{-1}^{0}(x^2+4x+3)\cos x dx.$

3. $\int_{-2}^{0}(x+2)^2\cos 3x dx.$

4. $\int_{-4}^{0}(x^2+7x+12)\cos x dx.$

5. $\int_{0}^{\pi}(2x^2+4x+7)\cos 2x dx.$

6. $\int_{0}^{\pi}(9x^2+9x+11)\cos 3x dx.$

7. $\int_{0}^{\pi}(8x^2+16x+17)\cos 4x dx.$

8. $\int_{0}^{2\pi}(3x^2+5)\cos 2x dx.$

9. $\int_{0}^{2\pi}(2x^2-15)\cos 3x dx.$

10. $\int_{0}^{2\pi}(3-7x^2)\cos 2x dx.$

11. $\int_{0}^{2\pi}(1-8x^2)\cos 4x dx.$

12. $\int_{-1}^{0}(x^2+2x+1)\sin 3x dx.$

13. $\int_{0}^{3}(x^2-2x)\sin 2x dx.$

14. $\int_{0}^{\pi}(x^2-3x+2)\sin x dx.$

15. $\int_0^{\pi/2}(x^2-5x+6)\sin 3x\,dx.$ **16.** $\int_{-3}^{0}(x^2+6x+9)\sin 2x\,dx.$

17. $\int_0^{\pi/4}(x^2+17{,}5)\sin 2x\,dx.$ **18.** $\int_0^{\pi/2}(1-5x^2)\sin x\,dx.$

19. $\int_{\pi/4}^{3}(3x-x^2)\sin 2x\,dx.$ **20.** $\int_1^2 x\ln^2 x\,dx.$

21. $\int_1^{e^2}\frac{\ln^2 x\,dx}{\sqrt{x}}.$ **22.** $\int_1^8\frac{\ln^2 x\,dx}{\sqrt[3]{x^2}}.$

23. $\int_0^1(x+1)\ln^2(x+1)dx.$ **24.** $\int_2^3(x-1)^3\ln^2(x-1)dx.$

25. $\int_{-1}^{0}(x+2)^3\ln^2(x+2)dx.$ **26.** $\int_0^2(x+1)^2\ln^2(x+1)dx.$

27. $\int_1^e\sqrt{x}\ln^2 x\,dx.$ **28.** $\int_{-1}^{1}x^2e^{-x/2}dx.$

29. $\int_0^1 x^2e^{3x}dx.$ **30.** $\int_{-2}^{0}(x^2+2)e^{x/2}dx.$

Javoblar. **2.1** $\frac{5-\cos 4-\sin 4}{4}$; **2.2** $4-2\cos 1-2\sin 1$; **2.3** $\frac{12-2\sin 6}{27}$;

2.4 $7+\cos 4-2\sin 4$; **2.5** π; **2.6** $-2\pi-2$; **2.7** π; **2.8** 2π; **2.9** $\frac{8\pi}{9}$; **2.10** -7π; **2.11** -2π;

2.12 $-\frac{7+2\cos 3}{27}$; **2.13** $\frac{3\sin 6+\cos 6-1}{4}$; **2.14** $\pi^2-3\pi$; **2.15** $\frac{67-3\pi}{27}$; **2.16** $-\frac{17+\cos 6}{4}$;

2.17 $\frac{17}{2}+\frac{\pi}{8}$; **2.18** $11-5\pi$; **2.19** $\frac{\pi-6+2\cos 6-6\sin 6}{8}$; **2.20** $2\ln^2 2-2\ln 2+\frac{3}{4}$;

2.21 $8e-16$; **2.22** $6\ln^2 8-36\ln 8+54$; **2.23** $2\ln^2 2-2\ln 2+\frac{3}{4}$;

2.24 $4\ln^2 2-2\ln 2+\frac{15}{32}$; **2.25** $4\ln^2 2-2\ln 2+\frac{15}{32}$; **2.26** $9\ln^2 3-6\ln 3+1\frac{25}{27}$;

2.27 $\frac{10e\sqrt{e}-16}{27}$; **2.28** $-\frac{26}{\sqrt{e}}+10\sqrt{e}$; **2.29** $\frac{5e^3-2}{27}$; **2.30** $20-\frac{44}{e}$.

$R(\sin x,\ \cos x)dx$ ko'rinishdagi integrallar

$R(\sin x,\ \cos x)dx$ ko'rinishdagi integralar ($R-\sin x$ va $\cos x$ larga nisbatan ratsional funksiya) $tg\frac{x}{2}=t$ almashtirish yordamida ratsional funksiyalarning integrallariga keltiriladi.

$$\sin x=\frac{2tg\frac{x}{2}}{1+tg^2\frac{x}{2}}=\frac{2t}{1+t^2};$$

$$\sin x=\frac{1-tg^2\frac{x}{2}}{1+tg^2\frac{x}{2}}=\frac{1-t^2}{1+t^2};$$

$$x=2\operatorname{arctg}t;\quad dx=\frac{2}{1+t^2}dt.$$

$R(\sin x,\ \cos x)dx=\int R\left(\frac{2t}{1+t^2},\frac{1-t^2}{1+t^2}\right)\cdot\frac{2dt}{1+t^2}$ ko'rinishga keladi. Bunday almashtirish *universal almashtirish* deyiladi.

3–masala. Aniq integralni hisoblang.

$$\int_0^{\pi/2}\frac{\cos x-\sin x}{(1+\sin x)^2}dx=\left|\begin{matrix} tg\frac{x}{2}=t & \cos x=\frac{1-t^2}{1+t^2}\\ dx=\frac{2}{1+t^2}dt & \sin x=\frac{2t}{1+t^2}\end{matrix}\right|=\int_0^1\frac{\frac{1-t^2}{1+t^2}-\frac{2t}{1+t^2}}{\left(1+\frac{2t}{1+t^2}\right)^2}\cdot\frac{2dt}{1+t^2}=$$

$$=\int_0^1\frac{2(1-2t-t^2)}{(1+t)^4}dt.$$

$\frac{2(1-2t-t^2)}{(1+t^4)}$ to'g'ri kasrni sodda ratsional kasrlar yig'indisi ko'rinishida yozamiz:

$$\frac{2-4t-2t^2)}{(1+t)^4}=\frac{A}{1+t}+\frac{B}{(1+t)^2}+\frac{C}{(1+t^3)}+\frac{D}{(1+t)^4}=$$

$$=\frac{A(1+t)^3+B(1+t)^2+C(1+t)+D}{(1+t)^4}.$$

$$A(1+t)^3+B(1+t)^2+C(1+t)+D=2-4t-2t^2.$$

$t=-1$ da, $D=4$;

x ning bir xil darajalari oldidagi koeffitsiyentlarni tenglaymiz:

$t^3:\quad A=0;$

$t^2:\quad 3A+B=-2\Rightarrow B=-2;$

$t:\quad 3A+2B+C=-4\Rightarrow C=0;$

Demak,

$$\int_0^1\left(\frac{4}{(1+t)^4}-\frac{2}{(1+t)^2}\right)dt=\left(-\frac{4}{3(1+t)^3}+\frac{2}{1+t}\right)\Bigg|_0^1=-\frac{4}{3\cdot 8}+1+\frac{4}{3}-2=\frac{1}{6}.$$

1. $\int\limits_{\pi/2}^{2\operatorname{arctg}2}\frac{dx}{\sin^2 x(1-\cos x)}.$

2. $\int\limits_0^{\pi/2}\frac{\cos x dx}{2+\cos x}.$

3. $\int\limits_{\pi/2}^{2\operatorname{arctg}2}\frac{dx}{\sin^2 x(1+\cos x)}.$

4. $\int\limits_{2\operatorname{arctg}\frac{1}{2}}^{\pi/2}\frac{\cos x dx}{(1-\cos x)^3}.$

5. $\int\limits_{2\operatorname{arctg}2}^{2\operatorname{arctg}3}\frac{dx}{\cos x(1-\cos x)}.$

6. $\int\limits_{2\operatorname{arctg}\frac{1}{3}}^{2\operatorname{arctg}\frac{1}{2}}\frac{dx}{\sin x(1-\sin x)}.$

7. $\int\limits_{2\operatorname{arctg}\frac{1}{2}}^{\pi/2}\frac{dx}{(1+\sin x-\cos x)^2}.$

8. $\int\limits_0^{\pi/2}\frac{\cos x dx}{5+4\cos x}.$

9. $\int_0^{2\pi/3} \frac{1+\sin x}{1+\cos x+\sin x}\,dx.$

10. $\int_{\pi/3}^{\pi/2} \frac{\cos x dx}{1+\sin x-\cos x}.$

11. $\int_0^{\pi/2} \frac{(1+\cos x)dx}{1+\cos x+\sin x}.$

12. $\int_0^{\pi/2} \frac{\sin x dx}{1+\cos x+\sin x}.$

13. $\int_0^{2\operatorname{arctg}\frac{1}{2}} \frac{1+\sin x}{(1-\sin x)^2}\,dx.$

14. $\int_0^{\frac{\pi}{2}} \frac{\cos x}{1+\cos x+\sin x}\,dx.$

15. $\int_0^{2\operatorname{arctg}\frac{1}{3}} \frac{\cos x dx}{(1+\cos x)(1-\sin x)}.$

16. $\int_{-2\pi/3}^{0} \frac{\cos x dx}{1+\cos x-\sin x}.$

17. $\int_{-\pi/2}^{0} \frac{\cos x dx}{(1+\cos x-\sin x)^2}.$

18. $\int_0^{\pi/2} \frac{\cos x dx}{(1+\cos x+\sin x)^2}.$

19. $\int_{00}^{2\operatorname{arctg}\frac{1}{2}} \frac{(1-\sin x)dx}{\cos x(1+\cos x)}.$

20. $\int_0^{\pi/2} \frac{\sin x dx}{(1+\sin x)^2}.$

21. $\int_0^{\pi/2} \frac{\sin x dx}{(1+\sin x+\cos x)^2}.$

22. $\int_{-\pi/2}^{0} \frac{\sin x dx}{(1+\cos x-\sin x)^2}.$

23. $\int_{-2\pi/3}^{0} \frac{\cos^2 x dx}{(1+\cos x-\sin x)^2}.$

24. $\int_0^{\pi/2} \frac{\sin^2 x dx}{(1+\cos x+\sin x)^2}.$

25. $\int_0^{2\pi/3} \frac{\cos^2 x dx}{(1+\cos x+\sin x)^2}.$

26. $\int_{\pi/2}^{2\operatorname{arctg}2} \frac{dx}{\sin x(1+\sin x)}.$

27. $\int_0^{\pi/2} \frac{dx}{(1+\sin x+\cos x)^2}.$

28. $\int_0^{\pi/2} \frac{\sin x dx}{2+\sin x}.$

29. $\int_0^{\pi/4} \frac{dx}{\cos x(1+\cos x)}.$

30. $\int_0^{\pi/2} \frac{\sin x dx}{5+3\sin x}.$

Javoblar. **3.1** $\frac{55}{96}$; **3.2** $\frac{(9-4\sqrt{3})\pi}{18}$; **3.3** $1\frac{5}{24}$; **3.4** -4; **3.5** $\frac{1}{6}+\ln 2-\ln 3$; **3.6** $\ln 3-\ln 2+1$;

3.7 $\frac{2}{3}-\ln\frac{3}{2}$; **3.8** $\frac{\pi}{8}-\frac{5}{6}\cdot arctg\frac{1}{3}$; **3.9** $\frac{\pi}{3}+\ln 2$; **3.10** $\frac{1}{2}\cdot\ln 2-\frac{\pi}{12}$; **3.11** $\frac{1}{2}\cdot\ln 2+\frac{\pi}{4}$;

3.12 $-\frac{1}{2}\cdot\ln 2+\frac{\pi}{4}$; **3.13** $\frac{26}{3}$; **3.14** $\frac{\pi}{4}-\frac{1}{2}\ln 2$; **3.15** $-\frac{1}{3}-2\ln\frac{2}{3}$; **3.16** $\frac{\pi}{3}-\ln 2$;

3.17 $-\frac{1}{2}+\ln 2$; **3.18** $-\frac{1}{2}+\ln 2$; **3.19** $-\frac{1}{2}+2\ln\frac{3}{2}$; **3.20** $\frac{1}{3}$; **3.21** $\ln 2-\frac{1}{2}$; **3.22** $\frac{1}{2}-\ln 2$;

3.23 $\frac{\sqrt{3}}{2}-\ln 2$; **3.24** $\frac{1}{2}-\frac{1}{2}\ln 2$; **3.25** $\frac{\sqrt{3}}{2}-\ln 2$; **3.26** $\ln 2-\frac{1}{3}$;

3.27 $1-\ln 2$; **3.28** $\frac{\pi}{2}-\frac{2\pi}{3\sqrt{3}}$; **3.29** $\frac{\sqrt{2}-2}{\sqrt{2}}-\ln(\sqrt{2}-1)$; **3.30** $\frac{\pi-5arctg2+arctg\frac{3}{4}}{6}$.

$\int tg^m x dx$ va $\int tg^m x dx$ (bu yerda m – butun musbat son) ko'rinishdagi integrallarda mos ravishda

$$tgt=t, \qquad dx=\frac{dt}{1+t^2}$$

$$ctgt=t, \qquad dx=-\frac{dt}{1+t^2}$$

$$\sin 2x=\frac{2tgx}{1+tg^2x}=\frac{2t}{1+t^2}$$

o'rniga qo'yish orqali hisoblanadi.

4–masala. Aniq integralni hisoblang.

$$\int_{\pi/4}^{arctg3}\frac{dx}{(3\mathrm{tg}x+5)\sin 2x}=\left|\begin{matrix}\mathrm{tg}x=t\\ dx=\frac{dt}{1+t^2}\end{matrix}\quad \sin 2x=\frac{2t}{1+t^2}\right|=$$

$$=\int_1^3\frac{dt}{(3t+5)\frac{2t}{1+t^2}(1+t^2)}=\frac{1}{2}\int_1^3\frac{dt}{t(3t+5)}.$$

$$\frac{1}{t(3t+5)}=\frac{A}{t}+\frac{B}{3t+5}=\frac{A(3t+5)+Bt}{t(3t+5)},$$

$$A(3t+5)+Bt=1.$$

$t=0$ da , $A=\frac{1}{5}$;

$t=-\frac{5}{3}$ da , $B=-\frac{3}{5}$;

Shunday qilib,

$$\frac{1}{10}\int_1^3\left(\frac{1}{t}-\frac{3}{3t+5}\right)dt=\frac{1}{10}\left(\ln|t|-\ln|3t+5|\right)\Big|_1^3=\frac{1}{10}(\ln 3-\ln 14-0+\ln 8)=$$

$$=\frac{1}{10}\ln\frac{24}{14}=\frac{1}{10}\ln\frac{12}{7}.$$

1. $\int\limits_{\arccos(4/\sqrt{17})}^{\pi/4}\frac{2\operatorname{ctg}x+1}{(2\sin x+\cos x)^2}dx.$ **2.** $\int\limits_0^{\arccos(4/\sqrt{17})}\frac{3+2tgx}{2\sin^2 x+3\cos^2 x-1}dx.$

3. $\int\limits_{\pi/4}^{\operatorname{arctg}3}\frac{4tgx-5}{1-\sin 2x+4\cos^2 x}dx.$ **4.** $\int\limits_0^{\operatorname{arctg}\frac{1}{3}}\frac{(8+\operatorname{tg}x)}{18\sin^2 x+2\cos^2 x}dx.$

5. $\int\limits_0^{\arccos\sqrt{2/3}}\frac{tgx+2}{\sin^2 x+2\cos^2 x-3}dx.$ **6.** $\int\limits_{\arcsin(1/\sqrt{37})}^{\pi/4}\frac{6\operatorname{tg}xdx}{3\sin 2x+5\cos^2 x}.$

7. $\int\limits_0^{\pi/4}\frac{2\operatorname{tg}^2x-11\operatorname{tg}x-22}{4-\operatorname{tg}x}dx.$ **8.** $\int\limits_{-arctg(1/3)}^{0}\frac{3tgx+1}{2\sin 2x-5\cos 2x+1}dx.$

9. $\int\limits_{\pi/4}^{arctg3}\frac{1+ctgx}{(\sin x+2\cos x)^2}dx.$ **10.** $\int\limits_{\pi/4}^{\arccos(1/\sqrt{3})}\frac{tgx}{\sin^2 x-5\cos^2 x+4}dx.$

11. $\int\limits_0^{\pi/4}\frac{6\sin^2 x}{3\cos 2x-4}dx.$ **12.** $\int\limits_0^{arctg3}\frac{4+tgx}{2\sin^2 x+18\cos^2 x}dx.$

13. $\int_0^{arctg2} \frac{12+tgx}{3\sin^2 x+12\cos^2 x}dx.$ **14.** $\int_0^{\operatorname{arctg}(2/3)} \frac{6+tgx}{9\sin^2 x+4\cos^2 x}dx.$

15. $\int_0^{\arcsin\sqrt{3/7}} \frac{tg^2 xdx}{3\sin^2 x+4\cos^2 x-7}.$ **16.** $\int_0^{\pi/4} \frac{7+3tgx}{(\sin x+2\cos x)^2}dx.$

17. $\int_{\arcsin(2/\sqrt{5})}^{\arcsin(3/\sqrt{10})} \frac{2tgx+5}{(5-tgx)\sin 2x}dx.$ **18.** $\int_{-\arccos(1/\sqrt{10})}^{0} \frac{3tg^2 x-50}{2tgx+7}dx.$

19. $\int_0^{\pi/4} \frac{5tgx+2}{2\sin 2x+5}dx.$ **20.** $\int_{\pi/4}^{\arcsin(2/\sqrt{5})} \frac{4tgx-5}{4\cos^2 x-\sin 2x+1}dx.$

21. $\int_0^{\arcsin\sqrt{7/8}} \frac{6\sin^2 xdx}{4+3\cos 2x}.$ **22.** $\int_{-\arccos(1/\sqrt{5})}^{0} \frac{11-3tgx}{tgx+3}dx.$

23. $\int_0^{\arcsin(3/\sqrt{10})} \frac{2tgx-5}{(4\cos x-\sin x)^2}dx.$ **24.** $\int_{\pi/4}^{\arccos(1/\sqrt{26})} \frac{36dx}{(6-tgx)\sin 2x}.$

25. $\int_0^{\pi/4} \frac{4-7tgx}{2+3tgx}dx.$ **26.** $\int_{-\arcsin(2/\sqrt{5})}^{\pi/4} \frac{2-tgx}{(\sin x+3\cos x)^2}dx.$

27. $\int_{\pi/4}^{\arcsin\sqrt{2/3}} \frac{8tgxdx}{3\cos^2 x+8\sin 2x-7}.$ **28.** $\int_{\arccos(1/\sqrt{10})}^{\arccos(1/\sqrt{26})} \frac{12dx}{(6+5tgx)\sin 2x}.$

29. $\int_0^{\pi/3} \frac{tg^2 x}{4+3\cos 2x}dx.$ **30.** $\int_0^{\arccos(1/\sqrt{6})} \frac{3tg^2 x-1}{tg^2 x+5}dx.$

Javoblar. **4.1** $2\ln 2-\frac{1}{2}$; **4.2** $\frac{3}{\sqrt{2}}arctg\frac{1}{4\sqrt{2}}+\ln\frac{33}{32}$; **4.3** $2\ln 2-\frac{\pi}{8}$; **4.4** $\frac{\pi}{3}+\frac{\ln 2}{36}$;

4.5 $-\frac{\ln 2+\sqrt{2}\pi}{4}$; **4.6** $\frac{5}{6}\ln\frac{6\cdot e}{11}$; **4.7** $2\ln 3-6\ln 2-5\frac{\pi}{4}$; **4.8** $\frac{1}{4}\ln\frac{6}{7}$; **4.9** $\frac{1}{4}\ln\frac{9}{5}+\frac{1}{15}$;

4.10 $\frac{1}{10}\ln\frac{9}{4}$; **4.11** $-\frac{\pi}{4}+\frac{1}{\sqrt{7}}arctg\sqrt{7}$; **4.12** $\frac{\pi}{6}+\frac{\ln 2}{4}$; **4.13** $\frac{\pi}{2}+\frac{\ln 2}{6}$; **4.14** $\frac{\pi}{4}+\frac{\ln 2}{18}$;

4.15 $-\frac{\sqrt{3}}{8}+\frac{\sqrt{3}\pi}{32}$; **4.16** $3\ln\frac{3}{2}+\frac{1}{6}$; **4.17** $2\ln\frac{3}{2}$; **4.18** $-\frac{1}{2}\ln 7-\ln 10-7arctg3$;

4.19 $\frac{1}{2}\ln\frac{14}{5}$; **4.20** $2\ln\frac{5}{4}-\frac{1}{2}arctg\frac{1}{2}$; **4.21** $\frac{\sqrt{7}\pi}{4}-arctg\sqrt{7}$; **4.22** $\ln 45+3arctg2$;

4.23 $\frac{9}{4}-\ln 16$; **4.24** $\frac{\ln 5}{6}$; **4.25** $2\ln 5-3\ln 2-\frac{\pi}{4}$; **4.26** $\frac{15}{4}-\ln 4$;

4.27 $\frac{4}{21}\cdot\ln\left|\frac{7\sqrt{2}-2}{5}\right|-\frac{4}{3}\cdot\ln\left|2-\sqrt{2}\right|$; **4.28** $\ln\frac{105}{93}$; **4.29** $\sqrt{3}-\sqrt{7}\cdot arctg\sqrt{\frac{3}{7}}$;

4.30 $\frac{\pi}{\sqrt{5}}-arctg\sqrt{5}$.

$\int \sin^m x\cdot\cos^n x dx$ ko'rinishdagi integrallar,

bu yerda m va n –butun sonlar.

1. Agar m va n sonlarning hech bo'lmaganda bittasi toq musbat son, masalan, $m=2k+1$ bo'lsa bo'lsa, u holda quyidagicha yo'l tutamiz:

$$\int \sin^{2k+1} x\cdot\cos^n x dx=\int \sin^{2k} x\cdot\cos^n x\cdot\sin x dx=$$
$$=-\int(1-\cos^2 x)^k\cdot\cos^n x d(\cos x).$$

Masalan,

$$\int \sin^5 x\cdot\cos^2 x dx=\int \sin^4 x\cdot\cos^2 x\cdot\sin x dx=$$
$$=-\int(1-\cos^2 x)^2\cdot\cos^2 x d(\cos x)=$$
$$=-\frac{1}{3}\cos^3 x+\frac{2}{5}\cos^5 x+\frac{1}{7}\cos^7 x+C.$$

Agar m va n sonlardan biri toq musbat son, boshqasi istalgan haqiqiy son bo'lsa ham xuddi yuqoridek yo'l tutamiz.

2. Agar m va n juft musbat sonlar bo'lsa, integralni

$$\sin x\cdot\cos x=\frac{1}{2}\sin 2x,\quad \sin^2 x=\frac{1-\cos 2x}{2},\qquad \cos^2 x=\frac{1+\cos 2x}{2}$$

Trigonometrik formulalar yordamida hisoblaymiz.

Masalan,

$$\int \sin^2 x \cos^4 x dx = \int (\sin x \cdot \cos x)^2 \cos^2 x dx =$$

$$= \int \frac{\sin^2 2x}{4} \cdot \frac{1+\cos 2x}{2} dx = \frac{1}{8} \int (\sin^2 2x + \sin^2 2x \cdot \cos 2x) dx =$$

$$= \frac{1}{8} \int \frac{1-\cos 4x}{2} dx + \frac{1}{8 \cdot 2} \int \sin^2 2x d(\sin 2x) =$$

$$= \frac{1}{16} \left(x - \frac{\sin 4x}{4} \right) + \frac{1}{48} \sin^3 2x + C =$$

$$= \frac{1}{16} \left(x - \frac{\sin 4x}{4} + \frac{\sin^3 2x}{3} \right) + C.$$

3. Agar m va n juft-toqligi bir xil bo'lgan butun manfiy sonlar bo'lsa, integral

$$1 + tg^2 x = \frac{1}{\cos^2 x}, \quad 1 - ctg^2 x = \frac{1}{\sin^2 x} \text{ yoki } \frac{1}{\sin^2 x} = \frac{1+tg^2 x}{tg^2 x}$$

formulalar yordamila hisoblanadi.

$$\int \frac{dx}{\cos^4 x} = \int \frac{1}{\cos^2 x} \cdot \frac{dx}{\cos^2 x} = \int (1+tg^2 x)^2 d(tgx) =$$

$$= tgx + \frac{tg^3 x}{3} + C.$$

5–masala. Aniq integralni hisoblang.

$$\int_0^{\pi} 2^4 \cos^8 \left(\frac{x}{2} \right) dx = \left| \cos^2 \left(\frac{x}{2} \right) = \frac{1}{2}(1+\cos x) \right| =$$

$$= \int_0^{\pi} (1+\cos x)^4 dx = \int_0^{\pi} (1+2\cos x + \cos^2 x)^2 dx =$$

$$= \int_0^{\pi} (1+3\cos x + 6\cos^2 x + 4\cos^3 x + \cos^4 x) dx =$$

$$= \int_0^{\pi} \left(\frac{35}{8} + 3\cos x + \frac{7}{2}\cos 2x + \frac{1}{8}\cos 4x \right) dx + 4\int_0^{\pi} (1 - \sin^2 x)\cos x dx =$$

$$= \left(\frac{35}{8}x + 3\sin x + \frac{7}{4}\sin 2x + \frac{1}{32}\sin 4x \right)\Big|_0^{\pi} + 4\int_0^{\pi} (1 - \sin^2 x) d(\sin x) =$$

$$= \frac{35}{8}\pi + 4(\sin x - \frac{1}{3}\sin^3 x)\Big|_0^{\pi} = \frac{35}{8}\pi.$$

1. $\int_{\pi/2}^{\pi} 2^8 \sin^8 x dx.$

2. $\int_0^{\pi} 2^4 \sin^6 x \cos^2 x dx.$

3. $\int_0^{2\pi} \sin^4 x \cos^4 x dx.$

4. $\int_0^{2\pi} \sin^2\left(\frac{x}{4}\right)\cos^6\left(\frac{x}{4}\right) dx.$

5. $\int_{-\pi/2}^{0} 2^8 \sin^8 x dx.$

6. $\int_{\pi/2}^{\pi} 2^4 \sin^6 x \cos^2 x dx.$

7. $\int_0^{\pi} 2^4 \sin^4 x \cos^4 x dx.$

8. $\int_0^{2\pi} \sin^2 x \cos^6 x dx.$

9. $\int_0^{2\pi} \cos^8\left(\frac{x}{4}\right) dx.$

10. $\int_0^{\pi} 2^4 \sin^8\left(\frac{x}{2}\right) dx.$

11. $\int_{-\pi}^{0} 2^8 \sin^6 x \cos^2 x dx.$

12. $\int_{\pi/2}^{2\pi} 2^8 \sin^4 x \cos^4 x dx.$

13. $\int_0^{\pi} 2^4 \sin^2 x \cos^6 x dx.$

14. $\int_0^{2\pi} \cos^8 x dx.$

15. $\int_0^{2\pi} \sin^8 \frac{x}{4} dx.$

16. $\int_0^{\pi} 2^4 \sin^6\left(\frac{x}{2}\right)\cos^2\left(\frac{x}{2}\right) dx.$

17. $\int_{-\pi/2}^{0} 2^8 \sin^4 x \cos^4 x dx.$

18. $\int_{\pi/2}^{\pi} 2^8 \sin^2 x \cos^6 x dx.$

19. $\int_0^{\pi} 2^4 \cos^8 x dx.$

20. $\int_0^{2\pi} \sin^8 x dx.$

21. $\int_0^{2\pi} \sin^6\left(\frac{x}{4}\right)\cos^2\left(\frac{x}{4}\right)dx.$ **22.** $\int_0^{\pi} 2^4 \sin^4\left(\frac{x}{2}\right)\cos^4\left(\frac{x}{2}\right)dx.$

23. $\int_{-\pi/2}^{0} 2^8 \sin^2 x\cos^6 x dx.$ **24.** $\int_{\pi/2}^{\pi} 2^8 \cos^8 x dx.$

25. $\int_0^{\pi} 2^4 \sin^8 x dx.$ **26.** $\int_0^{2\pi} \sin^6 x\cos^2 x dx.$

27. $\int_0^{2\pi} \sin^4\left(\frac{x}{4}\right)\cos^4\left(\frac{x}{4}\right)dx.$ **28.** $\int_0^{\pi} 2^4 \sin^2\left(\frac{x}{2}\right)\cos^6\left(\frac{x}{2}\right)dx.$

29. $\int_{-\pi/2}^{0} 2^8 \cos^8 x dx.$ **30.** $\int_0^{2\pi} \sin^4 3x\cos^4 3x dx.$

Javoblar. 5.1 35π; **5.2** $\frac{5\pi}{8}$; **5.3** $\frac{3\pi}{64}$; **5.4** $\frac{5\pi}{64}$; **5.5** 35π; **5.6** $\frac{5\pi}{16}$; **5.7** $\frac{3\pi}{4}$; **5.8** $\frac{5\pi}{2^6}$;

5.9 $\frac{35\pi}{64}$; **5.10** $\frac{35\pi}{8}$; **5.11** 10π; **5.12** 9π; **5.13** $\frac{5\pi}{8}$; **5.14** $\frac{35\pi}{64}$; **5.15** $\frac{35\pi}{64}$;

5.16 $\frac{5\pi}{8}$; **5.17** 3π; **5.18** 5π; **5.19** $\frac{35\pi}{8}$; **5.20** $\frac{35\pi}{64}$; **5.21** $\frac{5\pi}{64}$; **5.22** $\frac{3\pi}{8}$;

5.23 5π; **5.24** 105π; **5.25** $\frac{35\pi}{8}$; **5.26** $\frac{5\pi}{64}$; **5.27** $\frac{3\pi}{64}$; **5.28** $\frac{5\pi}{8}$; **5.29** 35π; **5.30** $\frac{3\pi}{64}$.

$\int R\left[x, \left(\frac{ax+b}{cx+d}\right)^{\frac{p_1}{q_1}}, \left(\frac{ax+b}{cx+d}\right)^{\frac{p_2}{q_2}}, ...\right]dx$ **ko'rinishdagi integrallar**, bu yerda $p_1, q_1, p_2, q_2, ...$ – butun sonlar. Agar barcha $q_1, q_2, ...$ maxrajlarning eng kichik karralisi k bo'lsa, u holda ushbu integral $\frac{ax+b}{cx+d} = t^k$ o'rniga qo'yish yordamida ratsional funksiyadan olingan integrallarga keltiriladi.

6–masala. Aniq integralni hisoblang.

$$\int_6^9 \sqrt{\frac{9-2x}{2x-21}}dx = \left| \begin{array}{l} \dfrac{9-2x}{2x-21} = t^2 \\ dx = \dfrac{12t}{(t^2+1)}dt \end{array} \right| = 12\int t\frac{t}{(t^2+1)^2}dt =$$

$$= 12\int \frac{t^2}{(t^2+1)^2} = \left| \begin{array}{l} t = tga \\ dt = \dfrac{da}{\cos^2 a} \end{array} \right| = 12\int tg^2 \cos^2 ada =$$

$$= 12\int \sin^2 ada = 6\int (1-\cos 2a)da = 6arctgT - 3\sin(2arctgt) =$$

$$= \left(6artctg\sqrt{\frac{9-2x}{2x-21}} - 3\sin\left(2arctg\sqrt{\frac{9-2x}{2x-21}} \right) \right)\Bigg|_6^9 =$$

$$= 6arctg\sqrt{3} - 3\sin(2arctg\sqrt{3}) - 6arctg\frac{1}{3} + 3\sin(2arctg\frac{1}{\sqrt{3}}) =$$

$$= 2\pi - 3\sin\frac{2\pi}{3} - -\pi + 3\sin\frac{\pi}{3} = \pi - 3\frac{\sqrt{3}}{2} + 3\frac{\sqrt{3}}{2} = \pi.$$

1. $\int_0^1 \frac{4\sqrt{1-x} - \sqrt{3x+1}}{\left(\sqrt{3x+1} + 4\sqrt{1-x}\right)(3x+1)^2}dx.$

2. $\int_1^{64} \frac{1 - \sqrt[6]{x} + 2\sqrt[3]{x}}{x + 2\sqrt{x^3} + \sqrt[3]{x^4}}dx.$

3. $\int_{-14/15}^{-7/8} \frac{6\sqrt{x+2}}{(x+2)^2\sqrt{x+1}}dx.$

4. $\int_0^5 e^{\sqrt{\frac{5-x}{5+x}}} \frac{dx}{(5+x)\sqrt{25-x^2}}.$

5. $\int_8^{12} \sqrt{\frac{6-x}{x-14}}dx.$

6. $\int_0^1 e^{\sqrt{\frac{1-x}{1+x}}} \frac{dx}{(1+x)\sqrt{1-x^2}}.$

7. $\int_{5/2}^{10/3} \frac{\sqrt{x+2} + \sqrt{x-2}}{\left(\sqrt{x+2} - \sqrt{x-2}\right)(x-2)^2}dx.$

8. $\int_1^8 \frac{5\sqrt{x+24}}{(x+24)^2 \cdot \sqrt{x}}dx.$

9. $\int_1^2 \frac{x + \sqrt{3x-2} - 10}{\sqrt{3x-2} + 7}dx.$

10. $\int_6^{10} \sqrt{\frac{4-x}{x-12}}dx.$

11. $\int_0^2 \frac{(4\sqrt{2-x} - \sqrt{2x+2})}{(\sqrt{2x+2} + 4\sqrt{2-x})(2x+2)^2}dx.$

12. $\int_{-1/2}^0 \frac{xdx}{2 + \sqrt{2x+1}}.$

13. $\int_0^4 e^{\sqrt{\frac{4-x}{4+x}}} \frac{dx}{(4+x)\sqrt{16-x^2}}$.

14. $\int_{1/8}^1 \frac{15\sqrt{x+3}}{(x+3)^2\sqrt{x}}dx$.

15. $\int_{-5/3}^1 \frac{\sqrt[3]{3x+5}+2}{1+\sqrt[3]{3x+5}}dx$.

16. $\int_2^3 \sqrt{\frac{3-2x}{2x-7}}dx$.

17. $\int_0^7 \frac{\sqrt{x+25}dx}{(x+25)^2\sqrt{x+1}}$.

18. $\int_0^2 \frac{(4\sqrt{2-x}-\sqrt{3x+2})dx}{(\sqrt{3x+2}+4\sqrt{2-x})(3x+2)^2}$.

19. $\int_0^2 e^{\sqrt{\frac{2-x}{2+x}}} \frac{dx}{(2+x)\sqrt{4-x^2}}$.

20. $\int_3^5 \sqrt{\frac{2-x}{x-6}}dx$.

21. $\int_{1/24}^{1/3} \frac{5\sqrt{x+1}}{(x+1)^2\sqrt{x}}dx$.

22. $\int_9^{15} \sqrt{\frac{6-x}{x-18}}dx$.

23. $\int_0^1 \frac{(4\sqrt{1-x}-\sqrt{2x+1})dx}{(\sqrt{2x+1}+4\sqrt{1-x})(2x+1)^2}dx$.

24. $\int_1^{64} \frac{(2+\sqrt[3]{x})dx}{(\sqrt[6]{x}+2\sqrt[3]{x}+\sqrt{x})\sqrt{x}}$.

25. $\int_{16/15}^{4/3} \frac{4\sqrt{x}}{x^2\sqrt{x-1}}dx$.

26. $\int_0^6 \frac{e^{\sqrt{(6-x)/(6+x)}}dx}{(6+x)\sqrt{36-x^2}}$.

27. $\int_1^{64} \frac{6-\sqrt{x}+\sqrt[4]{x}}{\sqrt{x^3}-7x-6\sqrt[4]{x^3}}dx$.

28. $\int_0^1 \frac{(4\sqrt{1-x}-\sqrt{x+1})dx}{(\sqrt{x+1}+4\sqrt{1-x})(x+1)^2}$.

29. $\int_0^3 \frac{e^{\sqrt{(3-x)/(3+x)}}dx}{(3+x)\sqrt{9-x^2}}$.

30. $\int_0^2 \frac{(4\sqrt{2-x}-\sqrt{x+2})dx}{(\sqrt{x+2}+4\sqrt{2-x})(x+2)^2}$.

Javoblar. **6.1** $\frac{1}{16}\ln 5$; **6.2** $6\ln\frac{4}{3}$; **6.3** 1; **6.4** $\frac{e-1}{5}$; **6.5** $\frac{4\pi}{3}$; **6.6** $e-1$; **6.7** $\frac{1}{2}+\ln 2$;

6.8 $\frac{1}{8}$; **6.9** $-\frac{22}{27}$; **6.10** $\frac{4\pi}{3}$; **6.11** $\frac{1}{24}\ln 5$; **6.12** $\frac{7}{6}-3\ln\frac{3}{2}$; **6.13** $\frac{1}{4}\cdot(e-1)$; **6.14** 3;

6.15 $\frac{8}{3}+\ln 3$; **6.16** $\frac{\pi}{3}$; **6.17** $\frac{1}{40}$; **6.18** $\frac{1}{32}\ln 5$; **6.19** $\frac{e-1}{2}$; **6.20** $\frac{2\pi}{3}$; **6.21** 3;

6.22 2π; **6.23** $\frac{1}{12}\ln 5$; **6.24** $30\ln\frac{3}{2}-6$; **6.25** 2; **6.26** $\frac{e-1}{6}$; **6.27** $4\ln\left(\frac{2}{2\sqrt{2}+1}\right)$; **6.28** $\frac{1}{8}\ln 5$;

6.29 $\frac{e-1}{3}$; **6.30** $\frac{1}{4}\ln 5$.**6.1** $\frac{1}{16}\ln 5$; **6.2** $6\ln\frac{4}{3}$; **6.3** 1; **6.4** $\frac{e-1}{5}$; **6.5** $\frac{4\pi}{3}$; **6.6** $e-1$; **6.7** $\frac{1}{2}+\ln 2$;

6.8 $\frac{1}{8}$; **6.9** $-\frac{22}{27}$; **6.10** $\frac{4\pi}{3}$; **6.11** $\frac{1}{24}\ln 5$; **6.12** $\frac{7}{6}-3\ln\frac{3}{2}$; **6.13** $\frac{1}{4}\cdot(e-1)$; **6.14** 3;

6.15 $\frac{8}{3}+\ln 3$; **6.16** $\frac{\pi}{3}$; **6.17** $\frac{1}{40}$; **6.18** $\frac{1}{32}\ln 5$; **6.19** $\frac{e-1}{2}$; **6.20** $\frac{2\pi}{3}$; **6.21** 3;

6.22 2π; **6.23** $\frac{1}{12}\ln 5$; **6.24** $30\ln\frac{3}{2}-6$; **6.25** 2; **6.26** $\frac{e-1}{6}$; **6.27** $4\ln\left(\frac{2}{2\sqrt{2}+1}\right)$; **6.28** $\frac{1}{8}\ln 5$;

6.29 $\frac{e-1}{3}$; **6.30** $\frac{1}{4}\ln 5$.

$R\left(x,\sqrt{a^2\pm x^2}\right)$ va $R\left(x,\sqrt{x^2-a^2}\right)$ ko'rinishdagi integrallar.

1. $\int_{\alpha}^{\beta} R\left(x,\sqrt{a^2-x^2}\right)dx;$

2. $\int_{\alpha}^{\beta} R\left(x,\sqrt{a^2+x^2}\right)dx;$

3. $\int_{\alpha}^{\beta} R\left(x,\sqrt{x^2-a^2}\right)dx;$

bu yerda $R-$ratsional funksiya.

Agar a) $x=a\sin t$ yoki $x=a\cos t$

b) $x=atgt$ yoki $x=actgt$

c) $x=a\sec t$ yoki $x=a\cos ect$

trigonometrik o'rniga qo'yishlardan foydanilsa, bu integrallar $\int R(\sin t,\cos t)dt$ ko'rinishdagi integrallarga keltiriladi.

7–masala. Aniq integralni hisoblang.

$$\int_0^3 \frac{dx}{(9+x^2)^{3/2}} = \left| \begin{array}{c} x = 3tgt \\ dx = \dfrac{3dt}{\cos^2 t} \end{array} \right| = \int_0^{\pi/4} \frac{3dt}{(9+9tg^2 t)^{3/2}\cos^2 t} =$$

$$= \frac{3}{27}\int_0^{\pi/4} \frac{\cos^3 t}{\cos^2 t}\,dt = \frac{3}{27}\int_0^{\pi/4} \cos t dt = \frac{3}{27}\sin t\Big|_0^{\pi/4} = \frac{\sqrt{2}}{18}.$$

1. $\int_0^{16} \sqrt{256-x^2}\,dx.$

2. $\int_0^1 x^2\sqrt{1-x^2}\,dx.$

3. $\int_0^5 \frac{dx}{(25+x^2)\sqrt{25+x^2}}.$

4. $\int_0^{\sqrt{5}/2} \frac{dx}{\sqrt{(5-x^2)^3}}.$

5. $\int_1^2 \frac{\sqrt{x^2-1}}{x^4}dx.$

6. $\int_0^{\sqrt{2}/2} \frac{x^4 dx}{\sqrt{(1-x^2)^3}}.$

7. $\int_0^{\sqrt{3}} \frac{dx}{\sqrt{(4-x^2)^3}}.$

8. $\int_0^1 \frac{x^4 dx}{(2-x^2)^{3/2}}.$

9. $\int_0^2 \frac{x^2 dx}{\sqrt{16-x^2}}.$

10. $\int_0^2 \sqrt{4-x^2}\,dx.$

11. $\int_0^4 \frac{dx}{(16+x^2)^{3/2}}.$

12. $\int_0^4 x^2\sqrt{16-x^2}\,dx.$

13. $\int_0^{5/2} \frac{x^2 dx}{\sqrt{25-x^2}}dx.$

14. $\int_0^5 x^2\sqrt{25-x^2}\,dx.$

15. $\int_0^4 \sqrt{16-x^2}\,dx.$

16. $\int_0^{4\sqrt{3}} \frac{dx}{\sqrt{(64-x^2)^3}}.$

17. $\int_{\sqrt{2}}^{2\sqrt{2}} \frac{\sqrt{x^2-2}}{x^4}dx.$

18. $\int_0^{2\sqrt{2}} \frac{x^4 dx}{(16-x^2)\sqrt{16-x^2}}.$

19. $\int_{-3}^3 x^2\sqrt{9-x^2}\,dx.$

20. $\int_1^{\sqrt{3}} \frac{dx}{\sqrt{(1+x^2)^3}}.$

21. $\int_0^2 \frac{dx}{\sqrt{(16-x^2)^3}}$.

22. $\int_0^2 \frac{x^4 dx}{\sqrt{(8-x^2)^3}}$.

23. $\int_3^6 \frac{x^2-9}{x^4} dx$.

24. $\int_0^1 \sqrt{4-x^2}\, dx$.

25. $\int_2^4 \frac{\sqrt{x^2-4}}{x^4} dx$.

26. $\int_0^2 \frac{dx}{(4+x^2)\sqrt{4+x^2}}$.

27. $\int_0^{\sqrt{2}} \frac{x^4 dx}{(4-x^2)^{3/2}}$.

28. $\int_0^{1/\sqrt{2}} \frac{dx}{(1-x^2)\sqrt{1-x^2}}$.

29. $\int_0^1 \frac{x^2 dx}{\sqrt{4-x^2}}$.

30. $\int_0^{3/2} \frac{x^2 dx}{\sqrt{9-x^2}}$.

Javoblar. **7.1** 64π; **7.2** $\frac{\pi}{16}$; **7.3** $\frac{\sqrt{2}}{50}$; **7.4** $\frac{\sqrt{3}}{15}$; **7.5** $\frac{\sqrt{3}}{8}$; **7.6** $\frac{5}{4}-\frac{3\pi}{8}$; **7.7** $\frac{\sqrt{3}}{4}$; **7.8** $\frac{5}{2}-\frac{3\pi}{4}$; **7.9** $\frac{4\pi}{3}-2\sqrt{3}$; **7.10** π; **7.11** $\frac{\sqrt{2}}{32}$; **7.12** 16π; **7.13** $\frac{25\pi}{12}-\frac{25\sqrt{3}}{8}$; **7.14** $\frac{625\pi}{16}$; **7.15** 4π; **7.16** $\frac{\sqrt{3}}{64}$; **7.17** $\frac{\sqrt{3}}{16}$; **7.18** $20-6\pi$; **7.19** $\frac{81\pi}{8}$; **7.20** $\frac{\sqrt{3}-\sqrt{2}}{2}$; **7.21** $\frac{\sqrt{3}}{48}$; **7.22** $10-3\pi$; **7.23** $\frac{\sqrt{3}}{72}$; **7.24** $\frac{\pi}{3}+\frac{\sqrt{3}}{2}$; **7.25** $\frac{\sqrt{3}}{32}$; **7.26** $\frac{\sqrt{2}}{8}$; **7.27** $5-\frac{3\pi}{2}$; **7.28** 1; **7.29** $\frac{\pi}{3}-\frac{\sqrt{3}}{2}$; **7.30** $\frac{3\pi}{4}-\frac{9\sqrt{3}}{8}$.

Yassi figuralar yuzlarini hisoblash

1. Uzluksiz $y=f(x)$ $(f(x)\ge 0)$ egri chiziq, $x=a$, $x=b$ to'g'ri chiziqlar hamda Ox o'qning $[a,\ b]$ kesmasi bilan chegaralangan egri chiziqli trapetsiyaning yuzi

$$S=\int_a^b f(x)dx$$

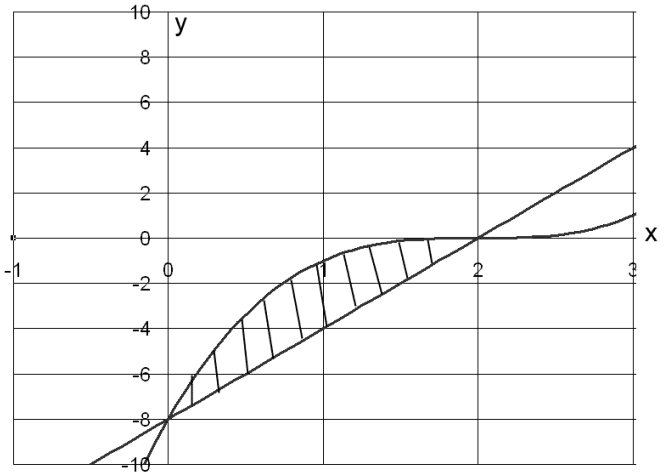
formula bilan hisoblanadi.

2. Uzluksiz $x=\varphi(y)$ $(\varphi(y)\ge 0)$ egri chiziq, $y=c,\ y=d$ to'g'ri chiziqlar hamda Oy o'qning $[c,\ d]$ kesmasi bilan chegaralangan egri chiziqli trapetsiyaning yuzi

$$S=\int_c^d \varphi(y)dy$$

formula bilan hisoblanadi.

8–masala. $y=(x-2)^3,\ y=4x-8$ chiziqlar bilan chegaralangan figuraning yuzini toping.



$$S=2\int_0^2(4x-8-(x-2)^3)dx=2\int_0^2(4x-8-x^3+6x^2-12x+8)dx=$$

$$=2\int_0^2(6x^2-x^3-8x)dx=2(2x^3-\frac{1}{4}x^4-4x^2)\Big|_0^2=4\cdot 2^3-\frac{1}{2}\cdot 2^4-8\cdot 2^2=8.$$

1. $y=x\sqrt{9-x^2},\quad y=0,\quad (0\le x\le 3).$

2. $y=4-x^2,\quad y=x^2-2x.$

3. $y=\sin x\cos^2 x,\quad y=0,\quad (0\le x\le \frac{\pi}{2}).$

4. $y=\sqrt{4-x^2},\quad y=0,\quad x=0,\quad x=1.$

5. $y = x^2\sqrt{4-x^2}, \quad y = 0, \quad (0 \le x \le 2).$

6. $y = \cos x \sin^2 x, \quad y = 0, \quad (0 \le x \le \frac{\pi}{2}).$

7. $y = \sqrt{e^x - 1}, \quad y = 0, \quad 0 \le x \le \ln 2.$

8. $y = \dfrac{1}{x\sqrt{1+\ln x}}, \quad y = 0, \quad x = 1, \quad x = e^3.$

9. $y = \arccos x,\ y = 0,\ x = 0.$

10. $y = (x+1)^2, \quad y^2 = x+1.$

11. $y = 2x - x^2 + 3, \quad y = x^2 - 4x + 3.$

12. $y = x\sqrt{36-x^2}, \quad y = 0, \quad (0 \le x \le 6).$

13. $y = \arccos y, \quad x = 0, \quad y = 0.$

14. $y = x\,arctgx, \quad y = 0, \quad x = \sqrt{3}.$

15. $y = x^2\sqrt{8-x^2}, \quad y = 0 \quad (0 \le x \le 2\sqrt{2}).$

16. $y = \sqrt{e^y - 1}, \quad x = 0, \quad y = \ln 2.$

17. $y = x\sqrt{4-x^2}, \quad y = 0 \quad (0 \le x \le 2).$

18. $y = \dfrac{x}{1+\sqrt{x}}, \quad y = 0, \quad x = 1.$

19. $y = \dfrac{1}{1+\cos x}, \quad y = 0, \quad x = \frac{\pi}{2}, \quad x = -\frac{\pi}{2}.$

20. $x = (y-2)^3, \quad x = 4y - 8.$

21. $y = \cos^5 x \sin 2x, \quad y = 0, \quad (0 \le x \le \frac{\pi}{2}).$

22. $y = \dfrac{x}{(x^2+1)^2}, \quad y = 0, \quad x = 1.$

23. $x = 4 - y^2, \quad x = y^2 - 2y.$

24. $x=\dfrac{1}{y\sqrt{1+\ln y}},\quad x=0,\quad y=1,\quad y=e^3.$

25. $y=\dfrac{e^{1/x}}{x^2},\quad y=0,\quad x=2,\quad x=1.$

26. $y=x^2\sqrt{16-x^2},\quad y=0\quad (0\le x\le 4).$

27. $x=\sqrt{4-y^2},\quad x=0,\quad y=0,\quad y=1.$

28. $y=(x-1)^2,\quad y^2=x-1.$

29. $y=x^2\cos x,\quad y=0,\quad (0\le x\le \pi/2).$

30. $x=4-(y-1)^2,\quad x=y^2-4y+3.$

Javoblar. **8.1** 9; **8.2** 9; **8.3** $\dfrac{1}{3}$; **8.4** $\dfrac{\pi}{3}+\dfrac{\sqrt{3}}{2}$; **8.5** π; **8.6** $\dfrac{1}{3}$; **8.7** $\approx 0{,}43$; **8.8** 2; **8.9** 1; **8.10** $\dfrac{1}{3}$;

8.11 9; **8.12** 72; **8.13** 1; **8.14** $\dfrac{\pi}{\sqrt{3}}-\ln 2$; **8.15** 4π; **8.16** $\approx 0{,}429$; **8.17** $\dfrac{8}{3}$; **8.18** $\dfrac{5}{3}-2\ln 2$; **8.19** 2;

8.20 8; **8.21** $\dfrac{2}{7}$; **8.22** $\dfrac{1}{4}$; **8.23** 9; **8.24** 2; **8.25** $e-\sqrt{e}$; **8.26** 16π; **8.27** $\dfrac{\sqrt{3}}{2}+\dfrac{\pi}{3}$; **8.28** $\dfrac{1}{3}$;

8.29 $\dfrac{\pi^2}{4}-2$; **8.30** 9.

Yassi figuralar yuzlarini hisoblash

3. Uzluksiz $y=f_1(x)$ va $y=f_2(x)$ $\left(f_1(x)\le f_2(x)\right)$ egri chiziqlar hamda , $x=a,\ x=b$ $(a<b)$ to'g'ri chiziqlar bilan chegaralangan figuraning yuzi

$$S=\int_a^b\left(f_2(x)-f_1(x)\right)dx$$

formula bilan hisoblanadi.

4. Uzluksiz $y=\varphi_1(x)$ va $y=\varphi_2(x)$ $\left(\varphi_1(x)\le\varphi_2(x)\right)$ egri chiziqlar

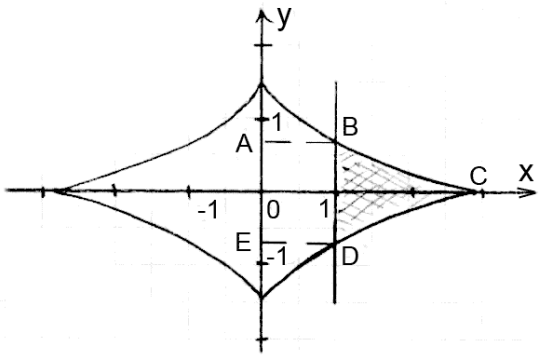
hamda , $y=c,\ y=d\ \ (c<d)$ to'g'ri chiziqlar bilan chegaralangan figuraning yuzi

$$S=\int_c^d\left(\varphi_2(x)-\varphi_1(x)\right)dx$$

formula bilan hisoblanadi.

9–masala. Quyidagi chiziqlar bilan chegaralangan shaklning yuzasini hisoblang.

$$\begin{cases}x=2\sqrt{2}\cos^3 t,\\ y=\sqrt{2}\sin^3 t,\end{cases}$$
$$x=1(x\ge 1).$$



$$S=\int_\alpha^\beta y(t)x'(t)dt.$$

$$2\sqrt{2}\cos^3 t\ge 1\Rightarrow t\in\left[-\frac{\pi}{4}+2\pi n;\frac{\pi}{4}+2\pi n\right].$$

$$S=S_{ABCDE}-S_{ABDE}=\int_{-\pi/4}^{\pi/4}\sqrt{2}\sin^3 t\cdot 6\sqrt{2}\cos^2 t\cdot(-\sin t)dt=1\cdot 1=$$

$$12\int\sin^4 t\cos^2 tdt-1=12\int_{-\pi/4}^{\pi/4}\frac{1}{8}(\cos 4t-4\cos 2t+3)\cdot\frac{1}{2}(1+\cos 2t)dt-1=$$

$$= \frac{12}{16}\int_{\pi/4}^{-\pi/4}(-\cos 4t - \frac{1}{2}\cos 2t + \frac{1}{2}\cos 6t + 1)dt - 1 =$$

$$= \frac{12}{16}\left(-\frac{1}{4}\sin 4t - \frac{1}{4}\sin 2t + \frac{1}{12}\sin 6t + t\right)\Bigg|_{\pi/4}^{-\pi/4} - 1 = 1{,}7.$$

1. $\begin{cases} x = 4\sqrt{2}\cos^3 t, \\ y = 2\sqrt{2}\sin^3 t, \end{cases}$ $x = 2 (x \geq 2).$

2. $\begin{cases} x = \sqrt{2}\cos t, \\ y = 2\sqrt{2}\sin t, \end{cases}$ $y = 2 (y \geq 2).$

3. $\begin{cases} x = 4(t - \sin t), \\ y = 4(1 - \cos t), \end{cases}$ $y = 4 (0 < x < 8\pi, y \geq 4).$

4. $\begin{cases} x = 16\cos^3 t, \\ y = 2\sin^3 t, \end{cases}$ $x = 2 (x \geq 2).$

5. $\begin{cases} x = 2\cos t, \\ y = 6\sin t, \end{cases}$ $y = 3 (y \geq 3).$

6. $\begin{cases} x = 2(t - \sin t), \\ y = 2(1 - \cos t), \end{cases}$ $y = 3 (0 < x < 4\pi, y \geq 3).$

7. $\begin{cases} x = 16\cos^3 t, \\ y = \sin^3 t, \end{cases}$ $x = 6\sqrt{3} (x \geq 6\sqrt{3}).$

8. $\begin{cases} x = 6\cos t, \\ y = 2\sin t, \end{cases}$ $y = \sqrt{3} (y \geq \sqrt{3}).$

9. $\begin{cases} x = 3(t - \sin t), \\ y = 3(1 - \cos t), \end{cases}$ $y = 3 (0 < x < 6\pi, y \geq 3).$

10. $\begin{cases} x = 8\sqrt{2}\cos^3 t, \\ y = \sqrt{2}\sin^3 t, \end{cases}$ $x = 4 (x \geq 4).$

11. $\begin{cases} x = 2\sqrt{2}\cos t, \\ y = 3\sqrt{2}\sin t, \end{cases}$ $y = 3 (y \geq 3).$

12. $\begin{cases} x = 6(t - \sin t), \\ y = 6(t - \cos t), \end{cases}$ $y = 9 (0 < x < 12\pi, y \geq 9).$

13. $\begin{cases} x = 32\cos^3 t, \\ y = \sin^3 t, \end{cases}$ $x = 4 (x \geq 4).$

14. $\begin{cases} x = 3\cos t, \\ y = 8\sin t, \end{cases}$ $y = 4 (y \geq 4).$

15. $\begin{cases} x = 6(t - \sin t), \\ y = 6(1 - \cos t), \end{cases}$
$y = 6 (0 < x < 12\pi, y \geq 6).$

16. $\begin{cases} x = 8\cos^3 t, \\ y = 4\sin^3 t, \end{cases}$
$x = 3\sqrt{3} (x \geq 3\sqrt{3}).$

17. $\begin{cases} x = 6\cos^3 t, \\ y = 4\sin^3 t, \end{cases}$
$y = 2\sqrt{3} (y \geq 2\sqrt{3}).$

18. $\begin{cases} x = 10(t - \sin t), \\ y = 10(1 - \cos t), \end{cases}$
$y = 15 (0 < x < 20\pi, y \geq 15).$

19. $\begin{cases} x = \sqrt{2}\cos t, \\ y = 4\sqrt{2}\sin t, \end{cases}$
$y = 4 (y \geq 4).$

20. $\begin{cases} x = t - \sin t, \\ y = 1 - \cos t, \end{cases}$
$y = 1 (0 < x < 2\pi, y \geq 1).$

21. $\begin{cases} x = 8\cos^3 t, \\ y = 8\sin^3 t, \end{cases}$
$x = 1 (x \geq 1).$

22. $\begin{cases} x = 9\cos t, \\ y = 4\sin t, \end{cases}$
$y = 2 (y \geq 2).$

23. $\begin{cases} x = 8(t - \sin t), \\ y = 8(1 - \cos t), \end{cases}$
$y = 12 (0 < x < 16\pi, y \geq 12).$

24. $\begin{cases} x = 24\cos^3 t, \\ y = 2\sin^3 t, \end{cases}$
$x = 9\sqrt{3} (x \geq 9\sqrt{3}).$

25. $\begin{cases} x = 3\cos t, \\ y = 8\sin t, \end{cases}$
$x = 4\sqrt{3} (y \geq 4\sqrt{3}).$

26. $\begin{cases} x = 2(t - \sin t), \\ y = 2(1 - \cos t), \end{cases}$
$y = 2 (0 < x < 4\pi, y \geq 2).$

27. $\begin{cases} x = 4\sqrt{2}\cos^3 t, \\ y = \sqrt{2}\sin^3 t, \end{cases}$
$x = 2 (x \geq 2).$

28. $\begin{cases} x = 2\sqrt{2}\cos t, \\ y = 5\sqrt{2}\sin t, \end{cases}$
$y = 5 (y \geq 5).$

29. $\begin{cases} x = 4(t - \sin t), \\ y = 4(1 - \cos t), \end{cases}$
$y = 6 (0 < x < 8\pi, y \geq 6).$

30. $\begin{cases} x = 32\cos^3 t, \\ y = 3\sin^3 t, \end{cases}$
$x = 12\sqrt{3} (x \geq 12\sqrt{3}).$

Javoblar. **9.1** $\frac{3}{2}\pi - 2$; **9.2** $\pi - 2$; **9.3** $24\pi + 64$; **9.4** 4π; **9.5** $4\pi - 3\sqrt{3}$; **9.6** $2\sqrt{3}$;

9.7 π; **9.8** $\approx 1{,}12$; **9.9** $\frac{27}{2}\pi + 36$; **9.10** $\frac{3}{2}\pi + 2$; **9.11** $3\pi - 6$; **9.12** $36\pi + 81\sqrt{3}$;

9.13 $4\pi + 3\sqrt{3}$; **9.14** $8\pi - 6\sqrt{3}$; **9.15** $18\pi + 72$; **9.16** $2\pi - 3\sqrt{3}$; **9.17** $4\pi - 6\sqrt{3}$;

9.18 $100\pi + 225\sqrt{3}$; **9.19** $2\pi - 4$; **9.20** $\frac{\pi}{2} + 2$; **9.21** 8π; **9.22** $12\pi - 9\sqrt{3}$;

9.23 $48\sqrt{3}$; **9.24** $3\pi - \frac{9\sqrt{3}}{2}$; **9.25** $2{,}174$; **9.26** $2\pi + 8$; **9.27** $\frac{3\pi}{4}$; **9.28** $5\pi - 10$;

9.29 $16\pi + 36\sqrt{3}$; **9.30** $6\pi - 9\sqrt{3}$.

Figuraning yuzini qutb koordinalar sistemasida hisoblash

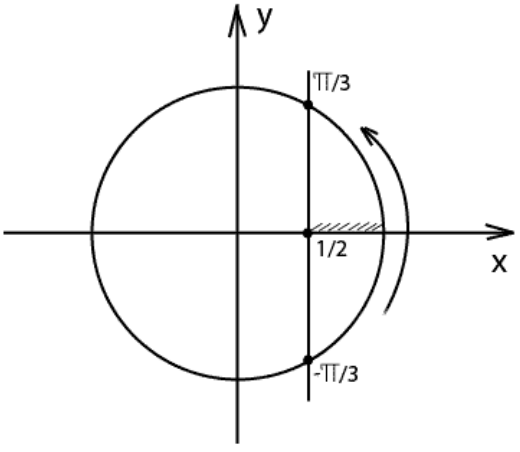
Qutb koordinatalar sistemasida berilgan uzluksiz $r = r(\varphi)$ egri chiziq va $\varphi = \alpha, \varphi = \beta$ $(\alpha < \beta)$ nurlar bilan chegaralangan figuraning yuzi

$$S = \frac{1}{2}\int_{\alpha}^{\beta} r^2(\varphi)d\varphi$$

formula bilan hisoblanadi.

10–masala. Tenglamalari qutb koordinatalari sistemasida berilgan chiziqlar bilan chegaralangan shaklning yuzasini hisoblang.

$$r = 4\cos\varphi, \quad r = 2 \ (r \geq 2).$$



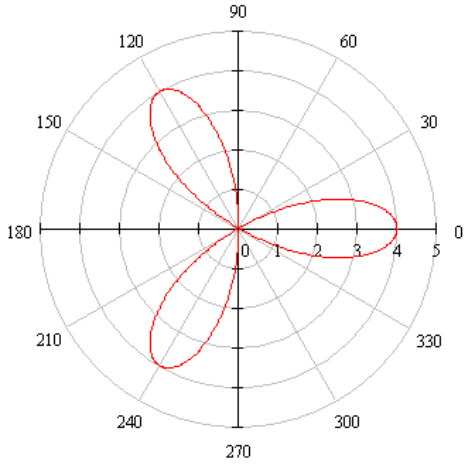
$$4\cos 3\varphi \geq 2,$$

$$\cos 3\varphi \geq \frac{1}{2}.$$

Demak

$$-\frac{\pi}{3}+2\pi n \leq 3\varphi \leq \frac{\pi}{3}+2\pi n, n \in Z$$

$$-\frac{\pi}{9}+\frac{2\pi n}{3} \leq \varphi \leq \frac{\pi}{9}+\frac{2\pi n}{3}, n \in Z,$$



$$S=\frac{1}{2}\int_{\alpha}^{\beta} r^2(\varphi)d\varphi,$$

$$S=6\cdot\frac{1}{2}\int_{-\pi/3}^{0} 16\cos^2 3\varphi d\varphi = 24\int_{-\pi/3}^{0}(1+6\cos\varphi)d\varphi =$$

$$=24(\varphi+\frac{1}{6}\sin 6\varphi)\Big|_{-\pi/3}^{0} = 24(0+0+\frac{\pi}{3}+\frac{1}{6}\cdot 0)=8\pi.$$

1. $r=\cos 2\varphi.$

2. $r=\sqrt{3}\cos\varphi, \quad r=\sin\varphi \quad (0\leq\varphi\leq\pi/2).$

3. $r=4\sin 3\varphi, \quad r=2 \quad (r\geq 2).$

4. $r=2\cos\varphi, \quad r=2\sqrt{3}\sin\varphi \quad (0\leq\varphi\leq\pi/2).$

5. $r=\sin 3\varphi.$

6. $r = 6\sin 3\varphi, \quad r = 3 \ (r \geq 3).$

7. $r = \cos 3\varphi.$

8. $r = \cos\varphi, \quad r = \sqrt{2}\sin(\varphi - \pi/4) \quad (-\pi/4 \leq \varphi \leq \pi/2).$

9. $r = \sin\varphi, \quad r = \sqrt{2}\cos(\varphi - \pi/4) \quad (0 \leq \varphi \leq 3\pi/4).$

10. $r = 6\cos 3\varphi, \quad r = 3 \quad (r \geq 3).$

11. $r = \frac{1}{2} + \sin\varphi.$

12. $r = \cos\varphi, \quad r = \sin\varphi \quad (0 \leq \varphi \leq \pi/2).$

13. $r = \sqrt{2}\cos(\varphi - \pi/4), \quad r = \sqrt{2}\sin(\varphi - \pi/4) \quad (\pi/4 \leq \varphi \leq 3\pi/4).$

14. $r = \cos\varphi, \quad r = 2\cos\varphi.$

15. $r = \sin\varphi, \quad r = 2\sin\varphi.$

16. $r = 1 + \sqrt{2}\cos\varphi.$

17. $r = \frac{1}{2} + \cos\varphi.$

18. $r = 1 + \sqrt{2}\sin\varphi.$

19. $r = \frac{5}{2}\sin\varphi, \quad r = \frac{3}{2}\sin\varphi.$

20. $r = \frac{3}{2}\cos\varphi, \quad r = \frac{5}{2}\cos\varphi.$

21. $r = 4\cos 4\varphi.$

22. $r = \sin 6\varphi.$

23. $r = 2\cos\varphi, \quad r = 3\cos\varphi.$

24. $r = \cos\varphi + \sin\varphi.$

25. $r = 2\sin 4\varphi.$

26. $r = 2\cos 6\varphi.$

27. $r = \cos\varphi - \sin\varphi.$

28. $r = 3\sin\varphi, \quad r = 5\sin\varphi.$

29. $r = 2\sin\varphi, \quad r = 4\sin\varphi.$

30. $r = 6\sin\varphi, \quad r = 4\sin\varphi.$

Javoblar. 10.1 $\frac{\pi}{2}$; **10.2** $\frac{5\pi}{24} - \frac{\sqrt{3}}{4}$; **10.3** $\frac{4\pi}{3} + 2\sqrt{3}$; **10.4** $\frac{5\pi}{6} - \sqrt{3} \approx 0{,}88$; **10.5** $\frac{\pi}{4}$;

10.6 $3\pi + \frac{9\sqrt{3}}{2}$; **10.7** $\frac{\pi}{4} \approx 0{,}78$; **10.8** $\frac{\pi}{4} - \frac{1}{4}$; **10.9** $\frac{\pi - 1}{4}$; **10.10** $\frac{9\pi}{2}$; **10.11** $\frac{3\pi}{4}$; **10.12** $\frac{1}{2}$;

10.13 $\frac{\pi + 2}{4}$; **10.14** $\frac{3\pi}{4}$; **10.15** $\frac{3\pi}{4}$; **10.16** $11{,}94$; **10.17** $\frac{3\pi}{4}$; **10.18** 2π; **10.19** π; **10.20** π;

10.21 8π; **10.22** $\frac{\pi}{4}$; **10.23** $\frac{5\pi}{4}$; **10.24** $\frac{\pi}{2}$; **10.25** $\frac{\pi}{2}$; **10.26** 2π; **10.27** $\frac{\pi}{2}$; **10.28** 4π; **10.29** 3π;

10.30 5π.

Yoy uzunligini hisoblash

Agar $y = f(x)$ funksiya $[a,\ b]$ da silliq egri chiziq (ya'ni $f'(x)$ uzluksiz) bo'lsa, u holda uning yoyi uzunligi

$$l = \int_a^b \sqrt{1 + y'^2}\, dx$$

formula bilan hisoblanadi. Bunda a va b yoy uchlarining abssissalaridir $(a < b)$.

Agar egri chiziq $x = \varphi(y) \quad (c \le y \le d)$ ko'rinishda berilagn bo'lsa, yoy uzunligi

$$l = \int_c^d \sqrt{1 + x'^2}\, dy$$

formula bilan hisoblanadi.

11–masala. Tenglamalari to'g'ri burchakli koordinatalar sistemasida berilgan egri chiziq yoyining uzunligini hisoblang.

$$y=\sqrt{1-x^2}+\arcsin x,\quad 0\le x\le \frac{8}{9}.$$

$$y'=-\frac{x}{\sqrt{1-x^2}}-\frac{1}{\sqrt{1-x^2}}=\frac{-1-x}{\sqrt{1-x^2}}.$$

$$l=\int_a^b\sqrt{1+(y')^2}\,dx,$$

$$l=\int_0^{8/9}\sqrt{1+\frac{(x+1)^2}{1-x^2}}dx=\int_0^{8/9}\sqrt{\frac{1-x^2+x^2+2x+1}{1-x^2}}dx=\int_0^{8/9}\sqrt{\frac{2+2x}{1-x^2}}dx=$$

$$=\int_0^{8/9}\sqrt{\frac{2}{1-x}}dx=\sqrt{2}\int_0^{8/9}\frac{dx}{\sqrt{1-x}}=\int_0^{8/9}\sqrt{\frac{2}{1-x}}\Bigg|_0^{8/9}=$$

$$=-2(\sqrt{2/9}-\sqrt{2})=-2\sqrt{2/9}+2\sqrt{2}=\frac{4\sqrt{2}}{3}.$$

1. $y=\ln x,\quad \sqrt{3}\le x\le\sqrt{15}.$

2. $y=\frac{x^2}{4}-\frac{\ln x}{2},\quad 1\le x\le 2.$

3. $y=\sqrt{1-x^2}+\arcsin x,\quad 0\le x\le\frac{7}{9}.$

4. $y=\ln\frac{5}{2x},\quad \sqrt{3}\le x\le\sqrt{8}.$

5. $y=-\ln\cos x,\quad 0\le x\le\pi/6.$

6. $y=e^x+6,\quad \ln\sqrt{8}\le x\le\ln\sqrt{15}.$

7. $y=2+\arcsin\sqrt{x}+\sqrt{x-x^2},\quad \frac{1}{4}\le x\le 1.$

8. $y=\ln(x^2-1),\quad 2\le x\le 3.$

9. $y=\ln(1-x^2),\quad 0\le x\le\frac{1}{4}.$

10. $y = 2 + \operatorname{ch} x, \quad 0 \le x \le 1.$

11. $y = 1 - \ln \cos x, \quad 0 \le x \le \pi / 6.$

12. $y = e^x + 13, \quad \ln \sqrt{15} \le x \le \ln \sqrt{24}.$

13. $y = -\arccos \sqrt{x} + \sqrt{x - x^2}, \quad 0 \le x \le \frac{1}{4}.$

14. $y = 2 - e^x, \quad \ln \sqrt{3} \le x \le \ln \sqrt{8}.$

15. $y = \arcsin x - \sqrt{1 - x^2}, \quad 0 \le x \le \frac{15}{16}.$

16. $y = 1 - \ln \sin x, \quad \pi/3 \le x \le \pi/2.$

17. $y = 1 - \ln(x^2 - 1), \quad 3 \le x \le 4.$

18. $y = \sqrt{x - x^2} - \arccos \sqrt{x} + 5, \quad \frac{1}{9} \le x \le 1.$

19. $y = -\arccos x + \sqrt{1 - x^2} + 1, \quad 0 \le x \le \frac{9}{16}.$

20. $y = \ln \sin x, \quad \pi/3 \le x \le \pi/2.$

21. $y = \ln 7 - \ln x, \quad \sqrt{3} \le x \le \sqrt{8}.$

22. $y = ch x + 3, \quad 0 \le x \le 1.$

23. $y = 1 + \arcsin x - \sqrt{1 - x^2}, \quad 0 \le x \le \frac{3}{4}.$

24. $y = \ln \cos x + 2, \quad 0 \le x \le \pi/6.$

25. $y = e^x + 26, \quad \ln \sqrt{8} \le x \le \ln \sqrt{24}.$

26. $y = \frac{e^x + e^{-x}}{2} + 3, \quad 0 \le x \le 2.$

27. $y = \arccos \sqrt{x} - \sqrt{x - x^2} + 4, \quad 0 \le x \le \frac{1}{2}.$

28. $y = \frac{e^{2x} + e^{-2x} + 3}{3}, \quad 0 \le x \le 2.$

29. $y = e^x + e, \quad \ln\sqrt{3} \le x \le \ln\sqrt{15}.$

30. $y = \dfrac{1 - e^x - e^{-x}}{2}, \quad 0 \le x \le 3.$

Javoblar. 11.1 $\frac{1}{2}\ln\frac{9}{5} + 2$; **11.2** $\frac{3}{4} + \frac{1}{2}\ln 2$; **11.3** $\frac{2\sqrt{2}}{3}$; **11.4** $1 + \frac{1}{2}\ln\frac{3}{2}$; **11.5** $\ln\sqrt{3}$;

11.6 $1 + \frac{1}{2}\ln\frac{6}{5}$; **11.7** $1 + \ln\frac{3}{2}$; **11.8** 1; **11.9** $\ln\frac{5}{3} - \frac{1}{4}$; **11.10** $\mathrm{sh}1$; **11.11** $\ln\sqrt{3}$; **11.12** $1 + \frac{1}{2}\ln\frac{10}{9}$;

11.13 1; **11.14** $1 + \frac{1}{2}\ln\frac{3}{2}$; **11.15** $\frac{3}{\sqrt{2}}$; **11.16** $\frac{\ln 3}{2}$; **11.17** $1 + \ln\frac{6}{5}$; **11.18** $\frac{4}{3}$; **11.19** $\frac{1}{\sqrt{2}}$; **11.20** $\frac{1}{2}\ln 3$;

11.21 $1 + \frac{1}{2}\ln\frac{3}{2}$; **11.22** $\mathrm{sh}1$; **11.23** $\sqrt{2}$; **11.24** $\ln\sqrt{3}$; **11.25** $2 + \frac{1}{2}\ln\frac{4}{3}$; **11.26** $\frac{1}{2}\left(e^2 - e^{-2}\right)$;

11.27 $\sqrt{2}$; **11.28** $\frac{1}{2}\left(e^4 - e^{-4}\right)$; **11.29** $2 + \frac{1}{2}\ln\frac{9}{5}$; **11.30** $\frac{1}{2}\left(e^3 - e^{-3}\right)$.

Parametrik ko’rinishda berilgan egri chiziq yoyining uzunligi

Agar egri chiziq

$$\left.\begin{aligned} x &= \varphi(t) \\ y &= \psi(t) \end{aligned}\right\}, \quad \alpha \le t \le \beta$$

ko’rinishida berilgan bo’lib, $\varphi'(t)$, $\psi(t)$ uzluksiz funksiyalar bo’lsa, u holda egri chiziq yoyining uzunligi

$$l = \int_{\alpha}^{\beta} \sqrt{\left(\varphi'(t)\right)^2 + \left(\psi'(t)\right)^2}\, dt$$

Formula bilan hisoblanadi. Bunda α va β lar t parametrning yoy uchlariga mos qiymatlaridir $(\alpha < \beta)$.

12–masala. Parametrik ko’rinishda berilgan egri chiziq yoyining uzunligini hisoblang.

$$\begin{cases} x = 4(\cos t + t\sin t), \\ y = 4(\sin t - t\cos t), \end{cases}$$
$$0 \le t \le 2\pi.$$

$$x' = 4(-\sin t + \sin t + t\cos t) = 4t\cos t,$$
$$y' = 4(\cos t - \cos t + t\sin t) = 4t\sin t.$$

$$l = \int_a^b \sqrt{(x_t')^2 + (y_t')}dt,$$

$$l = \int_0^{2\pi} \sqrt{16t^2\cos^2 t + 16t^2\sin^2 t}dt = \int_0^{2\pi} 4tdt = 2t^2\Big|_0^{2\pi} = 2\cdot(2\pi)^2 = 8\pi^2.$$

1. $\begin{cases} x = 5(t - \sin t), \\ y = 5(1 - \cos t), \end{cases}$
$0 \le t \le \pi.$

2. $\begin{cases} x = 3(2\cos t - \cos 2t), \\ y = 3(2\sin t - \sin 2t), \end{cases}$
$0 \le t \le 2\pi.$

3. $\begin{cases} x = (t^2 - 2)\sin t + 2t\cos t, \\ y = (2 - t^2)\cos t + 2t\sin t, \end{cases}$
$0 \le t \le \pi.$

4. $\begin{cases} x = 10\cos^3 t, \\ y = 10\sin^3 t, \end{cases}$
$0 \le t \le \pi/2.$

5. $\begin{cases} x = e^t(\cos t + \sin t), \\ y = e^t(\cos t - \sin t), \end{cases}$
$0 \le t \le \pi.$

6. $\begin{cases} x = 3(t - \sin t), \\ y = 3(t - \cos t), \end{cases}$
$\pi \le t \le 2\pi.$

7. $\begin{cases} x = \frac{1}{2}\cos t - \frac{1}{4}\cos 2t, \\ y = \frac{1}{2}\sin t - \frac{1}{4}\sin 2t, \end{cases}$
$\pi/2 \le t \le 2\pi/3.$

8. $\begin{cases} x = 3(\cos t + t\sin t), \\ y = 3(\sin t - t\cos t), \end{cases}$
$0 \le t \le \pi/3.$

9. $\begin{cases} x = (t^2 - 2)\sin t + 2t\cos t, \\ y = (2 - t^2)\cos t + 2t\sin t, \end{cases}$
$0 \le t \le \pi/3.$

10. $\begin{cases} x = 6\cos^3 t, \\ y = 6\sin^3 t, \end{cases}$
$0 \le t \le \pi/3.$

11. $\begin{cases} x = e^t(\cos t + \sin t), \\ x = e^t(\cos t - \sin t), \end{cases}$
$\pi/2 \le t \le \pi.$

12. $\begin{cases} x = 2{,}5(t - \sin t), \\ y = 2{,}5(1 - \cos t), \end{cases}$
$\pi/2 \le t \le \pi.$

13. $\begin{cases} x = 3{,}5(2\cos t - \cos 2t), \\ y = 3{,}5(2\sin t - \sin 2t), \end{cases}$
$0 \le t \le \pi/2.$

14. $\begin{cases} x = 6(\cos t + t\sin t), \\ y = 6(\sin t - t\cos t), \end{cases}$
$0 \le t \le \pi.$

15. $\begin{cases} x = (t^2 - 2)\sin t + 2t\cos t, \\ y = (2 - t^2)\cos t + 2t\sin t, \end{cases}$
$0 \le t \le \pi/2.$

16. $\begin{cases} x = 8\cos^3 t, \\ y = 8\sin^3 t, \end{cases}$
$0 \le t \le \pi/6.$

17. $\begin{cases} x = e^t(\cos t + \sin t), \\ y = e^t(\cos t - \sin t), \end{cases}$
$0 \le t \le 2\pi.$

18. $\begin{cases} x = 4(t - \sin t), \\ y = 4(1 - \cos t), \end{cases}$
$\pi/2 \le t \le 2\pi/3.$

19. $\begin{cases} x = 2(2\cos t - \cos 2t), \\ y = 2(2\sin t - \sin 2t), \end{cases}$
$0 \le t \le \pi/3.$

20. $\begin{cases} x = 8(\cos t + t\sin t), \\ y = 8(\sin t - t\cos t), \end{cases}$
$0 \le t \le \pi/4.$

21. $\begin{cases} x = (t^2 - 2)\sin t + 2t\cos t, \\ y = (2 - t^2)\cos t + 2t\sin t, \end{cases}$
$0 \le t \le 2\pi.$

22. $\begin{cases} x = 4\cos^3 t, \\ y = 4\sin^3 t, \end{cases}$
$\pi/6 \le t \le \pi/4.$

23. $\begin{cases} x = e^t(\cos t + \sin t), \\ y = e^t(\cos t - \sin t), \end{cases}$
$0 \le t \le 3\pi/2.$

24. $\begin{cases} x = 2(t - \sin t), \\ y = 2(1 - \cos t), \end{cases}$
$0 \le t \le \pi/2.$

25. $\begin{cases} x = 4(2\cos t - \cos 2t), \\ y = 4(2\sin t - \sin 2t), \end{cases}$
$0 \le t \le \pi.$

26. $\begin{cases} x = 2(\cos t + t\sin t), \\ y = 2(\sin t - t\cos t), \end{cases}$
$0 \le t \le \pi/2.$

27. $\begin{cases} x = (t^2 - 2)\sin t + 2t\cos t, \\ y = (2 - t^2)\cos t + 2t\sin t, \end{cases}$
$0 \le t \le 3\pi.$

28. $\begin{cases} x = 2\cos^3 t, \\ y = 2\sin^3 t, \end{cases}$
$0 \le t \le \pi/4.$

29. $\begin{cases} x = e^t(\cos t + \sin t), \\ y = e^t(\cos t - \sin t), \end{cases}$ $\pi/6 \le t \le \pi/4.$

30. $\begin{cases} x = (t^2 - 2)\sin t + 2t\cos t, \\ y = (2 - t^2)\cos t + 2t\sin t, \end{cases}$ $0 \le t \le \pi.$

Javoblar. 12.1 20; **12.2** 48; **12.3** $\frac{\pi^3}{3}$; **12.4** 15; **12.5** $2(e^{\pi} - 1)$; **12.6** 12; **12.7** $\sqrt{2} - 1$; **12.8** $\frac{\pi^2}{6}$; **12.9** $\frac{\pi^3}{81}$; **12.10** $\frac{27}{4}$; **12.11** $2(e^{\pi} - e^{\pi/2})$; **12.12** $5\sqrt{2}$; **12.13** $14(2 - \sqrt{2})$; **12.14** $3\pi^2$; **12.15** $\frac{\pi^3}{24}$; **12.16** 3; **12.17** $2 \cdot (e^{2\pi} - 1)$; **12.18** $8(\sqrt{2} - 1)$; **12.19** $8(2 - \sqrt{3})$; **12.20** $\frac{\pi^2}{4}$; **12.21** $\frac{8\pi^3}{3}$; **12.22** $+\infty$; **12.23** $2(e^{3\pi/2} - 1)$; **12.24** $4(2 - \sqrt{2})$; **12.25** 32; **12.26** $\frac{\pi^2}{4}$; **12.27** $9\pi^3$; **12.28** $\frac{3}{2}$; **12.29** $2(e^{\pi/4} - e^{\pi/6})$; **12.30** $\frac{\pi^3}{3}$.

Qutb koordinatalari sistemasida berilgan egri chiziq yoyining uzunligi

Qutb koordinalar sistemasida berilgan silliq egri chiziq $\rho = f(\varphi)$, $\varphi_0 \le \varphi \le \varphi_1$ yoyining uzunligi

$$l = \int_{\varphi_0}^{\varphi_1} \sqrt{\rho^2 + \rho'^2}\, d\varphi$$

orqali hisoblanadi. Bunda φ_0 va φ_1–qutb burchagi φ ning yoy uchlaridagi qiymatlari $(\varphi_0 < \varphi_1)$.

13–masala. Tenglamalari qutb koordinatalari sistemasida berilgan egri chiziq yoyining uzunligini hisoblang.

$$\rho = 2e^{4\varphi/3},$$
$$-\pi/2 \le \varphi \le \pi/2.$$

$$L = \int_{\alpha}^{\beta} \sqrt{\rho^2 + (\rho')^2}\, d\varphi;$$

$$\rho' = \frac{8}{3} e^{4\varphi/3}.$$

$$L = \int_{-\pi/2}^{\pi/2} \sqrt{4e^{8\varphi/3} + \frac{64}{9} e^{8\varphi/3}}\, d\varphi = \int_{-\pi/2}^{\pi/2} \sqrt{\frac{100}{9} e^{8\varphi/3}}\, d\varphi = \int_{-\pi/2}^{\pi/2} e^{4\varphi/3} d\varphi =$$

$$= \frac{10}{3} \cdot \frac{3}{4} e^{4\varphi/3} \Big|_{-\pi/2}^{\pi/2} = \frac{5}{2} \left(e^{\frac{2\pi}{3}} - e^{-\frac{2\pi}{3}} \right) = 5 \cdot \operatorname{sh} \frac{2\pi}{3}$$

1. $\rho = 3e^{3\varphi/4}$, $-\pi/2 \le \varphi \le \pi/2$.

2. $\rho = \sqrt{2} e^{\varphi}$, $-\pi/2 \le \varphi \le \pi/2$.

3. $\rho = 5e^{5\varphi/12}$, $-\pi/2 \le \varphi \le \pi/2$.

4. $\rho = 6e^{12\varphi/5}$, $-\pi/2 \le \varphi \le \pi/2$.

5. $\rho = 3e^{3\varphi/4}$, $0 \le \varphi \le \pi/3$.

6. $\rho = 4e^{4\varphi/3}$, $0 \le \varphi \le \pi/3$.

7. $\rho = \sqrt{2} e^{\varphi}$, $0 \le \varphi \le \pi/3$.

8. $\rho = 5e^{5\varphi/12}$, $0 \le \varphi \le \pi/3$.

9. $\rho = 12e^{12\varphi/5}$, $0 \le \varphi \le \pi/3$.

10. $\rho = 1 - \sin\varphi$, $-\pi/2 \le \varphi \le -\pi/6$.

11. $\rho = 2(1 - \cos\varphi)$, $-\pi \le \varphi \le -\pi/2$.

12. $\rho = 3(1 + \sin\varphi)$, $-\pi/6 \le \varphi \le 0$.

13. $\rho = 4(1 - \sin\varphi)$, $0 \le \varphi \le \pi/6$.

14. $\rho = 5(1 - \cos\varphi)$, $-\pi/3 \le \varphi \le 0$.

15. $\rho = 6(1 + \sin\varphi)$, $-\pi/2 \le \varphi \le 0$.

16. $\rho = 7(1 - \sin\varphi)$, $-\pi/6 \le \varphi \le \pi/6$.

17. $\rho = 8(1 - \cos\varphi)$, $-2\pi/3 \le \varphi \le 0$.

18. $\rho = 2\varphi$, $0 \le \varphi \le 3/4$.

19. $\rho = 2\varphi,\ 0 \le \varphi \le 4/3.$ **20.** $\rho = 2\varphi,\ 0 \le \varphi \le \frac{5}{12}.$ **21.** $\rho = 2\varphi,\ 0 \le \varphi \le \frac{12}{5}.$

22. $\rho = 4\varphi,\ 0 \le \varphi \le 3/4.$ **23.** $\rho = 3\varphi,\ 0 \le \varphi \le 4/3.$ **24.** $\rho = 5\varphi,\ 0 \le \varphi \le \frac{12}{5}.$

25. $\rho = 2\cos\varphi,\ 0 \le \varphi \le \pi/6.$ **26.** $\rho = 8\cos\varphi,\ 0 \le \varphi \le \pi/4.$ **27.** $\rho = 6\cos\varphi,\ 0 \le \varphi \le \pi/3.$

28. $\rho = 2\sin\varphi,\ 0 \le \varphi \le \pi/6.$ **29.** $\rho = 8\sin\varphi,\ 0 \le \varphi \le \pi/4.$ **30.** $\rho = 6\sin\varphi,\ 0 \le \varphi \le \pi/3.$

Javoblar. **13.1** $10 \cdot \mathrm{sh}\frac{3\pi}{8}$; **13.2** $4 \cdot \mathrm{sh}\frac{\pi}{2}$; **13.3** $26 \cdot \mathrm{sh}\frac{5\pi}{24}$; **13.4** $13 \cdot \mathrm{sh}\frac{6\pi}{5}$; **13.5** $5 \cdot (e^{\pi/4} - 1)$;

13.6 $\frac{5}{3} \cdot (e^{4\pi/9} - 1)$; **13.7** $2 \cdot (e^{\pi/3} - 1)$; **13.8** $13 \cdot (e^{5\pi/36} - 1)$; **13.9** $13 \cdot (e^{4\pi/5} - 1)$;

13.10 2; **13.11** $-4\sqrt{2}$; **13.12** $6(\sqrt{3} - \sqrt{2})$; **13.13** $8(\sqrt{3} - \sqrt{2})$; **13.14** $20\left(1 - \sqrt{\frac{3}{4}}\right)$;

13.15 $12(2 - \sqrt{2})$; **13.16** $10{,}249$; **13.17** 16; **13.18** $\frac{\varphi}{2}\sqrt{\varphi^2 + 1} + \frac{1}{2}\ln\left|\varphi + \sqrt{\varphi^2 + 1}\right|$;

13.19 $\frac{20}{9} + \ln 3$; **13.20** $\frac{65}{144} + \ln\frac{3}{2}$; **13.21** $\frac{156}{25} + 5$; **13.22** $\frac{15}{8} + \ln 4$; **13.23** $\frac{10}{3} + \frac{3}{2}\ln 3$;

13.24 $\frac{78}{5} + \frac{5}{2}\ln 5$; **13.25** $\frac{\pi}{3}$; **13.26** 2π; **13.27** 2π; **13.28** $\frac{\pi}{3}$; **13.29** 2π; **13.30** 2π.

Aylanma jism sirining yuzi

1. $y = f(x)$ $(a \le x \le b)$ silliq egri chiziq yoyining Ox o'qi atrofida aylanishidan hosil bo'lgan jism siritining yuzi

$$S = 2\pi \int_a^b y\sqrt{1+y'^2}\,dx$$

formula bilan hisoblanadi.

2. Agar silliq egri chiziq

$$\left.\begin{aligned} x &= \varphi(t) \\ y &= \psi(t) \end{aligned}\right\}, \qquad \alpha \le t \le \beta$$

Parametrik ko'rinishda berilgan bo'lsa, sirt yuzi

$$S = 2\pi \int_\alpha^\beta y(t)\sqrt{x'^2 + y'^2}\,dt$$

formula bilan hisoblanadi.

3. Agar silliq egri chiziq qutb koordinalar sistemasida

$$\rho = f(x), \qquad \varphi_0 \le \varphi \le \varphi_1$$

ko'rinishda berilgan bo'lsa, uning qutb o'qi atrofida aylanishidan hosil bo'lgan jism sirtining yuzi

$$S = 2\pi \int_{\varphi_0}^{\varphi_1} \rho \sin\varphi \sqrt{\rho^2 + \rho'^2}\,d\varphi$$

formula bilan hisoblanadi.

Jismlarning hajmini ularning ko'ndalang kesimlari bo'yicha hisobalash

Agar ko'ndalang kesim yuzi S ni Ox o'qqa perpendikulyar tekislik orqali x ning funksiyasi sifatida $S = S(x)$ kabi ifodalash mumkin bo'lsa, u holda hajm differensiali uchun asosi S, balandligi dx bo'lgan silindrning hajmi olinadi, ya'ni

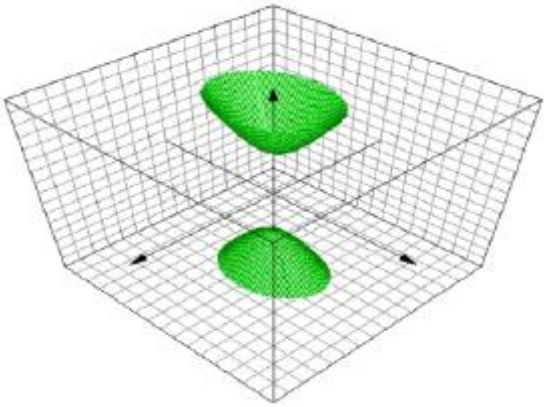
$$dV = S(x)dx.$$

$[a,\ b]$ intervalda integral olib, berilgan jismning hajmini hisoblash uchun formula hosil qilamiz:

$$V = \int_a^b S(x)dx\,.$$

14–masala. Quyidagi sirtlar bilan chegaralangan jismning hajmini toping.

$$\frac{x^2}{9} + \frac{y^2}{16} - \frac{z^2}{64} = -1,\ z = 16.$$



Jismning ko'ndalang kesimida $z = const$ ellips hosil bo'ladi:

$$\frac{x^2}{9} + \frac{y^2}{16} = \frac{z^2}{64} - 1.$$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ko'rinishidagiellipsning yuzi $\pi \cdot a \cdot b$ ga teng.

Ellipsning radiusini topamiz:

$$\frac{x^2}{9 \cdot \frac{z^2 - 64}{64}} + \frac{y^2}{16 \cdot \frac{z^2 - 64}{64}} = 1$$

$$a=\frac{3}{8}\sqrt{z^2-64};\qquad a=\frac{1}{2}\sqrt{z^2-64}$$

$$S=\pi ab=\pi\cdot\frac{3}{8}\cdot\sqrt{z^2-64}=\frac{3\pi}{8}\cdot(z^2-64).$$

$$V=\int_8^{16}S(z)dz=\frac{3\pi}{16}\int_8^{16}(z^2-64)dz=\frac{3\pi}{16}\left(\frac{z^3}{3}-64z\right)\Bigg|_8^{16}=$$

$$=\frac{3\pi}{16}\left(\frac{16^3}{3}-64\cdot16-\frac{8^3}{3}+64\cdot8\right)=$$

$$=\frac{\pi}{16}\left(3\cdot\frac{4096}{3}-3\cdot1024-3\cdot\frac{512}{3}+3\cdot512\right)=$$

$$=\frac{\pi}{16}\left(4096-3072-512+1536\right)=128\pi.$$

1. $\frac{x^2}{9}+y^2=1,\quad z=y,\quad z=0\quad (y\ge0).$

2. $z=x^2+4y^2,\quad z=2.$

3. $\frac{x^2}{9}+\frac{y^2}{4}-z^2=1,\quad z=0,\quad z=3.$

4. $\frac{x^2}{9}+\frac{y^2}{4}-\frac{z^2}{36}=-1,\quad z=12.$

5. $\frac{x^2}{16}+\frac{y^2}{9}+\frac{z^2}{4}=-1,\quad z=1,\quad z=0.$

6. $x^2+y^2=9,\quad z=y,\quad z=0\quad (y\ge0).$

7. $z=x^2+9y^2,\quad z=3.$

8. $\frac{x^2}{4}+y^2-z^2=1,\quad z=0,\quad z=3.$

9. $\frac{x^2}{16}+\frac{y^2}{9}+\frac{z^2}{64}=1,\ z=4,\ z=0.$

10. $\frac{x^2}{16}+\frac{y^2}{9}+\frac{z^2}{16}=1,\quad z=2,\quad z=0.$

11. $\frac{x^2}{3}+\frac{y^2}{4}=1,\quad z=y\sqrt{3},\quad z=0\quad (y\geq 0).$

12. $z=2x^2+8y^2,\quad z=4.$

13. $\frac{x^2}{81}+\frac{y^2}{25}-z^2=1,\quad z=0,\quad z=2.$

14. $\frac{x^2}{4}+\frac{y^2}{9}-\frac{z^2}{36}=-1,\quad z=12.$

15. $\frac{x^2}{16}+\frac{y^2}{9}+\frac{z^2}{36}=1,\quad z=3,\quad z=0.$

16. $\frac{x^2}{3}+\frac{y^2}{16}=1,\quad z=y\sqrt{3},\quad z=0\quad (y\geq 0).$

17. $z=x^2+5y^2,\quad z=5.$

18. $\frac{x^2}{9}+\frac{y^2}{4}-z^2=1,\quad z=0,\quad z=4.$

19. $\frac{x^2}{9}+\frac{y^2}{25}-\frac{z^2}{100}=-1,\quad z=20.$

20. $\frac{x^2}{27}+\frac{y^2}{25}=1,\quad z=\frac{y}{\sqrt{3}},\quad z=0\quad (y\geq 0).$

21. $z=4x^2+9y^2,\quad z=6.$

22. $x^2+\frac{y^2}{4}-z^2=1,\quad z=0,\quad z=3.$

23. $\frac{x^2}{25}+\frac{y^2}{9}-\frac{z^2}{100}=-1,\quad z=20.$

24. $\frac{x^2}{16}+\frac{y^2}{9}+\frac{z^2}{100}=1,\quad z=5,\quad z=0.$

25. $\frac{x^2}{27}+y^2=1,\quad z=\frac{y}{\sqrt{3}},\quad z=0\quad (y\ge 0).$

26. $z=2x^2+18y^2,\quad z=6.$

27. $\frac{x^2}{25}+\frac{y^2}{9}-z^2=1,\quad z=0,\quad z=2.$

28. $\frac{x^2}{16}+\frac{y^2}{9}-\frac{z^2}{64}=-1,\quad z=16.$

29. $\frac{x^2}{16}+\frac{y^2}{9}+\frac{z^2}{144}=1,\quad z=6,\quad z=0.$

30. $\frac{x^2}{16}+\frac{y^2}{9}+\frac{z^2}{196}=1,\quad z=7,\quad z=0.$

Javoblar. **14.1** 2; **14.2** π; **14.3** 72π; **14.4** 48π; **14.5** 11π; **14.6** 26; **14.7** $\frac{3\pi}{2}$; **14.8** 12π; **14.9** 44π; **14.10** 22π; **14.11** 32; **14.12** 2π; **14.13** 210π; **14.14** 48π; **14.15** 33π; **14.16** 32; **14.17** $\frac{5\pi\sqrt{5}}{2}$; **14.18** 152π; **14.19** 200π; **14.20** 1250; **14.21** 3π; **14.22** 24π; **14.23** 200π; **14.24** 55π; **14.25** 2; **14.26** 3π; **14.27** 70π; **14.28** 128π; **14.29** 66π; **14.30** 77π.

To’g’ri burchakli dekart koordinatalar sistemasida aylanma jism hajmi

1. $y = f(x)$ egri chiziq Ox o’q va $x = a,\ x = b$ to’g’ri chiziqlar bilan chegaralangan egri chiziqli trapetsiyaning Ox o’q atrofida aylanishidan hosil bo’lgan jism hajmi

$$V = \pi \int_a^b (f(x))^2 dx = \pi \int_a^b y^2 dx$$

formula bilan hisoblandi.

2. $x = f(y)$ egri chiziq Oy o’q va $y = c,\ y = d$ to’g’ri chiziqlar bilan chegaralangan egri chiziqli trapetsiyaning Oy o’q atrofida aylanishidan hosil bo’lgan jism hajmi

$$V = \pi \int_c^d (\varphi(y))^2 dy = \pi \int_c^d x^2 dy$$

formula bilan hisoblandi.

3. $y = f(x)$ egri chiziq Ox o’q va $x = a,\ y = b$ to’g’ri chiziqlar bilan chegaralangan egri chiziqli trapetsiyaning Oy o’q atrofida aylanishidan hosil bo’lgan jism hajmi

$$V = 2\pi \int_a^b xy dx$$

formula bilan hisoblandi.

4. Umumiy holda, $y = f_1(x),\ y = f_2(x)\ (0 \le f_1(x) \le f_2(x))$ egri chiziqlar va $x = a,\ y = b$ to’g’ri chiziqlar bilan chegaralangan egri chiziqlar bilan chegaralangan figuraning Ox va Oy o’qlari atrofida

aylanishidan hosil bo’lgan jism hajmi mos ravishda quyidagi

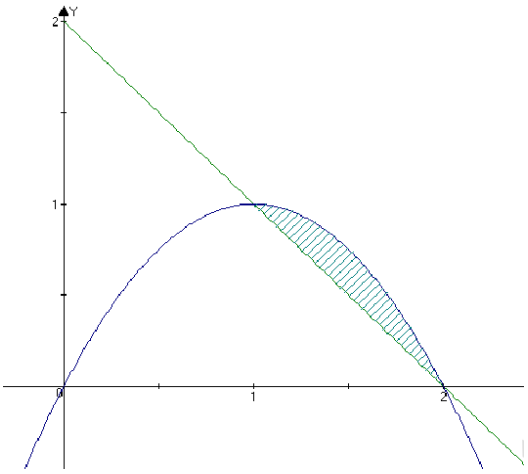
$$V_x = \pi \int_a^b (f_2^2(x) - f_1^2(x))dx$$

$$V_y = 2\pi \int_a^b x(f_2(x) - f_1(x))dx$$

formula bilan hisoblandi.

15–masala. Quyidagi chiziqlar bilan chegaralangan shaklni Ox o’qi atrofida aylantirishdan hosil bo’lgan jismning hajmi toping.

$$y = 2x - x^2,\ y = -x + 2.$$



Echim:

Ox o’q atrofida aylantirishdan hosil bo’lgan figuraning hajmi:

$$V = \pi \int_a^b y^2 dx.$$

Natijada:

$$y = 2x - x^2 = -x + 2 \Rightarrow x_1 = 1;\ x_2 = 2.$$

Sirtning hajmi:

$$V = V_1 - V_2,$$

bunda

$$V_1 = \pi \cdot \int_1^2 (2x - x^2)^2 dx = \pi \cdot \int_1^2 (4x^2 - 4x^3 + x^4) dx = \pi \cdot \left(4\frac{x^3}{3} - 4\frac{x^4}{4} + \frac{x^5}{5} \right) \Bigg|_1^2 =$$

$$= \pi \cdot \left(\left(4\frac{8}{3} - 4\frac{16}{4} + \frac{32}{5} \right) - \left(4\frac{1}{3} - 4\frac{1}{4} + \frac{1}{5} \right) \right) = \pi \cdot \left(\frac{32}{3} - 16 + \frac{32}{3} - \frac{4}{3} + 1 - \frac{1}{5} \right) =$$

$$= \pi \cdot \left(\frac{28}{3} - 15 + \frac{31}{5} \right) = \pi \cdot \frac{28 \cdot 5 - 15 \cdot 15 + 31 \cdot 3}{15} = \pi \cdot \frac{8}{15}$$

$$V_2 = \pi \cdot \int_1^2 (-x + 2)^2 dx = \pi \cdot \int_1^2 (4 - 4x + x^2) dx = \pi \cdot \left(4x - 4\frac{x^2}{2} + \frac{x^3}{3} \right) \Bigg|_1^2 =$$

$$= \pi \cdot \left(\left(8 - 8 + \frac{8}{3} \right) - \left(4 - 2 + \frac{1}{3} \right) \right) = \pi \cdot \left(\frac{8}{3} - 2 - \frac{1}{3} \right) = \frac{1}{3}\pi .$$

Demak,

$$V = V_1 - V_2 = \pi \cdot \frac{8}{15} - \pi \cdot \frac{1}{3} = \pi \cdot \frac{8-5}{15} = \frac{3}{15}\pi = \frac{1}{5}\pi.$$

15–masala. Quyidagi chiziqlar bilan chegaralangan shaklni (1-16 variantlarda) *Ox* o'qi atrofida, (17-30 variantlarda) *Oy* o'qi atrofida aylantirishdan hosil bo'lgan jismning hajmi toping.

1. $y = -x^2 + 5x - 6, \quad y = 0.$
2. $2x - x^2 - y = 0, \quad 2x^2 - 4x + y = 0.$
3. $y = 3\sin x, \quad y = \sin x, \quad 0 \le x \le \pi.$
4. $y = 5\cos x, \quad y = \cos x, \quad x = 0, \quad x \ge 0.$
5. $y = \sin^2 x, \quad x = \pi/2, \quad y = 0.$

6. $x = \sqrt[3]{y-2},\quad x=1,\quad y=1.$

7. $y = xe^{x},\quad y=0,\quad x=1.$

8. $y = 2x - x^{2},\quad y=-x+2,\quad x=0.$

9. $y = e^{1-x},\quad y=0,\quad x=0,\quad x=1.$

10. $y = x^{2},\quad y^{2}-x=0.$

11. $x^{2}+(y-2)^{2}=1.$

12. $y = 1-x^{2},\quad x=0,\quad x=\sqrt{y-2},\quad x=1$

13. $y = x^{2},\quad y=1,\quad x=2.$

14. $y = x^{3},\quad y=\sqrt{x}.$

15. $y = \sin\dfrac{\pi x}{2},\quad y=x^{2}.$

16. $y = \arccos\dfrac{x}{3},\quad y=\arccos x,\quad y=0.$

17. $y = \arcsin\dfrac{x}{5},\quad y=\arcsin x,\quad y=\dfrac{\pi}{2}.$

18. $y = x^{2},\quad x=2,\quad y=0.$

19. $y = x^{2}+1,\quad y=x,\quad x=0,\quad x=1.$

20. $y = \sqrt{x-1},\quad y=0,\quad y=1,\quad x=0{,}5.$

21. $y = \ln x,\quad x=2,\quad y=0.$

22. $y = (x-1)^{2},\quad y=1.$

23. $y^{2} = x-2,\quad y=0,\quad y=x^{3},\quad y=1.$

24. $y = x^{3},\quad y=x^{2}.$

25. $y = \arccos\dfrac{x}{5},\quad y=\arccos\dfrac{x}{3},\quad y=0.$

26. $y = \arcsin x,\quad y=\arccos x,\quad y=0.$

27. $y = x^2 - 2x + 1, \quad x = 2, \quad y = 0.$

28. $y = x^3, \quad y = x.$

29. $y = \arccos x, \quad y = \arcsin x, \quad x = 0.$

30. $y = (x-1)^2, \quad x = 0, \quad x = 2, \quad y = 0.$

Javoblar. **15.1** $\frac{\pi}{30}$; **15.2** $\frac{16\pi}{5}$; **15.3** $4\pi^2$; **15.4** 4π; **15.5** $\frac{3\pi^2}{16}$; **15.6** $\frac{44\pi}{7}$; **15.7** $\pi \cdot \frac{e^2-1}{4}$;

15.8 $\frac{9\pi}{5}$; **15.9** $2\pi \cdot (e^2-1)$; **15.10** $\frac{3\pi}{10}$; **15.11** $4\pi^2$; **15.12** 5π; **15.13** $\frac{26\pi}{5}$; **15.14** $\frac{5\pi}{14}$;

15.15 $0{,}3\pi$; **15.16** $19{,}739$; **15.17** $6\pi^2$; **15.18** 8π; **15.19** $\frac{5\pi}{6}$; **15.20** $\frac{97\pi}{60}$;

15.21 $\pi \cdot \left(4\ln 2 - \frac{3}{2}\right)$; **15.22** $\frac{8\pi}{3}$; **15.23** $\frac{24\pi}{5}$; **15.24** $\frac{\pi}{10}$; **15.25** $4\pi^2$; **15.26** $\frac{\pi}{2}$;

15.27 $\frac{7\pi}{6}$; **15.28** $\frac{4\pi}{15}$; **15.29** $\frac{\pi}{2} + \frac{\pi^2}{4}$; **15.30** $\frac{4\pi}{3}$.

Elementar funksiyalarning hosilalari jadvali

$u = u(x), \quad v = v(x)$ funksiyalar differensiallanuvchi funksiyalar bo'lsin.

1. $y = C,\ y' = 0.$
2. $y = u + v + w,\ y' = u' + v' + w'.$
3. $y = Cu,\ y' = Cu'.$
4. $y = uv,\ y' = uv'.$
5. $y = u^n,\ y' = nu^{n-1}u'.$
6. $y = \frac{u}{v},\ y' = \frac{u'v - uv'}{v^2}.$
7. $y = a^u,\ y' = a^u \ln a \cdot u'.$
8. $y = e^u,\ y' = e^u u'.$
9. $y = \ln u,\ y' = \frac{u'}{u}. u > 0$
10. $y = \log_a u,\ y' = \frac{u'}{u}\log_a e. u > 0$
11. $y = \sin u,\ y' = u' \cos u.$
12. $y = \cos u,\ y' = -u' \sin u.$
13. $y = tg\, u,\ y' = \frac{u'}{\cos^2 x}.$
14. $y = ctg\, u,\ y' = -\frac{u'}{\sin^2 u}$
15. $y = \arcsin u,\ y' = \frac{u'}{\sqrt{1-u^2}}$

16. $y = \arcsin u,\ y' = -\dfrac{u'}{\sqrt{1-u^2}}.$

17. $y = arctg\, u,\ y' = \dfrac{u'}{1+u^2}.$

18. $y = arcctg\, u,\ y' = -\dfrac{u'}{1+u^2}.$

19. $y = f(u),\ u = u(x),\ y' = f'_u(u)\cdot u'_x$

20. $x = x(t),\ y = y(t),\ y'_x = \dfrac{y'_t}{x'_t}.$

Asosiy integrallar jadvali

I. $\int x^m dx = \dfrac{x^{m+1}}{m+1} + C,\ (m \neq -1).$

II. $\int \dfrac{dx}{x} = \ln|x| + C.$

III. $\int \sin x dx = -\cos x + C.$

IV. $\int \cos x dx = \sin x + C.$

V. $\int a^x dx = \dfrac{a^x}{\ln a} + C.$

VI. $\int e^x dx = e^x + C.$

VII. $\int \dfrac{dx}{\cos^2 x} = tgx + C.$

VIII. $\int \dfrac{dx}{\sin^2 x} = -ctgx + C.$

IX. $\int \dfrac{dx}{1+x^2} = arctgx + C.$

X. $\int \dfrac{dx}{\sqrt{1-x^2}} = \arcsin x + C.$

XI. $\int tgxdx = -\ln|\cos x| + C.$

XII. $\int ctgxdx = \ln|\sin x| + C.$

XIII. $\int \frac{dx}{a^2 + x^2} = \frac{1}{a} arctg\frac{x}{a} + C.$

XIV. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin\frac{x}{a} + C.$

XV. $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left|x + \sqrt{x^2 \pm a^2}\right| + C.$

XVI. $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a}\ln\left|\frac{a + x}{a - x}\right| + C.$

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