THE MINISTRY OF HIGHER AND SECONDARY VOCATIONAL
EDUCATION OF THE REPUBLIC OF UZBEKISTAN

NAVOI STATE MINING INSTITUTE

CHAIR «AUTOMATION AND MANAGEMENT OF TECHNOLOGICAL PROCESSES AND INDUSTRIES»

THE EDUCATIONAL-METHODOLOGICAL

COMPLEX

ON DISCIPLINE

«Algorithmization of computing methods»

NAVOI

The educational-methodological complex is made on the basis of state standard of the higher vocational training of Republic Uzbekistan defining degree

THE SUMMARY

In an educational-methodological complex lecture, practical, laboratory materials, the test questions flowing, intermediate, total control questions in a subject «Algorithmization of computing methods» are resulted. An educational methodical complex «Automation by technological processes and industries» are intended for students of a direction of the bachelor 5 311 000

The educational methodological complex is intended as the textbook for pupils of technical colleges and liceums.

The educational-methodological complex is discussed and approved on faculty meeting «Automation and management of technological processes and Industries» NavSMI №*1* from«*27*» *august* 201*4*

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THE TYPICAL PROGRAM ON DISCIPLINE

ЎЗБЕКИСТОН РЕСПУБЛИКАСИ ОЛИЙ ВА ЎРТА МАХСУС
ТАЪЛИМ ВАЗИРЛИГИ

Рўйхатга олинди
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2012 йил «26» 12

Ўзбекистон Республикаси Олий ва
ўрта махсус таълим вазирининг
2012 йил «26» 12 даги
607-сонли буйруғи билан
тасдиқланган

ҲИСОБЛАШ УСУЛЛАРИНИ АЛГОРИТМЛАШ

фанининг

ЎҚУВ ДАСТУРИ

Билим соҳаси:	300 000 –	Ишлаб чиқариш техник соҳа
Таълим соҳаси:	310 000 –	Муҳандислик иши
Таълим йўналиши:	5311000 –	Технологик жараёнлар ва ишлаб чиқаришни автоматлаштириш ва бошқариш (тармоқлар бўйича)

Тошкент – 2012

Фаннинг ўқув дастури Олий ва ўрта махсус, касб – ҳунар таълими ўқув услубий бирлашмалари фаолиятини Мувофиқлаштирувчи Кенгашнинг 201***2*** йил «***25***» ***12*** даги «***4***» - сон мажлис баёни билан маъқулланган.

Фаннинг ўқув дастури Тошкент давлат техника университетида ишлаб чиқилди.

Тузувчилар: Тошкент Давлат техника университети «Ишлаб чиқариш жараёнларини автоматлаштириш» кафедраси мудири, ЎзР ФА академиги, т.ф.д., проф. Юсупбеков Н.Р.

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Тақризчилар: Тошкент кимё технология институти «Информатика. Автоматлаштириш ва бошқарув» кафедраси профессори, т.ф.д. Артиков А.А.

«Ўзкимёсаноат» ДАК нинг бош мутахассиси т.ф.д., проф. Юсипов М.М.

Фаннинг ўқув дастури Тошкент давлат техника университети Илмий – услубий Кенгашида тавсия қилинган. (201***2*** йил «***29***» ***03*** даги «***4***» сонли баённома).

КИРИШ

5311000 – «Технологик жараёнлар ва ишлаб чиқаришни автоматлаштириш ва бошқариш» (тармоқлар бўйича) йўналиши бўйича бакалаврларни тайёрлаш ўқув режасида «Ҳисоблаш усулларини алгоритмлаш» ўқув фани умумкасбий фанлар туркумига киритилган.

«Ҳисоблаш усулларини алгоритмлаш» фанидан ҳар хил синфдаги математик масалаларнинг тақрибий ечимларининг алгоритмларини назарий асослаш, қуриш ва амалда қўллаш масалалари ўрганилади.

Ўқув фанининг мақсади ва вазифалари

Ўқув фанининг мақсади – тажриба йўли билан тўпланган натижаларни қайта ишлаш, алгебраик, дифференциал ва интеграл тенгламаларни тақрибий ечимини топишда алгоритмларни тузиш учун мантиқий фикрлаш қобилиятини талабаларда шакллантиришдан иборат.

Ўқув фанининг вазифаси – талабаларни тажриба орқали олинган натижаларни қайта ишлаш, алгебраик, дифференциал ва интеграл тенгламаларни тақрибий ечимини топишда алгоритмларни тузиш учун маъқул вариантларни танлашга ўргатишдан иборат.

Фан бўйича талабаларнинг билим, кўникма ва малакасига қўйиладиган талаблар

«Ҳисоблаш усулларини алгоритмлаш» ўқув фанини ўзлаштириш жараёнида амалга ошириладиган масалалар доирасида бакалавр:

– алгебра, дифференциал ва интеграл тенгламаларини ечимини топишда тақрибий ечим усуллари ***ҳақида тасаввурга эга бўлиши***;

– матрица ва детерминант, дифференциал ва интеграл тенгламаларнинг хусусий ечимларини олиш усулларини ***билиши;***

– мустақил равишда тақрибий ечимлар алгоритмларини туза олиш ***кўникмаларига эга бўлиши керак.***

Қўйилган вазифалар ўқиш жараёнида талабаларни маъруза, лаборатория ва амалий машғулотларда фаол иштирок этиши, адабиётлар билан ишлаши билан амалга оширилади.

Фаннинг ўқув режадаги бошқа фанлар билан ўзаро боғлиқлиги ва услубий жиҳатдан узвий кетма-кетлиги

«Ҳисоблаш усулларини алгоритмлаш» фани мутахассислик фани ҳисобланиб, 3-семестрда ўқитилади. Дастурни амалга ошириш ўқув режасида режалаштирилган «Информатика ва ахборот технологиялари» ва «Олий математика» фанларидан етарли билим ва кўникмаларга эга бўлиш талаб этилади.

Фаннинг ишлаб чиқаришдаги ўрни

Кимё саноати корхоналарида ва илмий текшириш институтларида турли ҳисоб ишларини амалга оширишда ҳисоблаш усулларини алгоритмлашдан фойдаланиб, ишлаб чиқариш унумдорлиги ва марадорлигини ошириш бўйича олиб борилаётган ишлар умумий ҳажмнинг анчагина қисмини ташкил қилади.

Шунинг учун ҳам ҳисоблаш усулларини алгоритмлашни ўрганишга алоҳида талаблар қўйилади. Айниқса мураккаб системалар фаолиятини таҳлил қилишда ҳисоблаш усулларини алгоритмлашдан кенг фойдаланилмоқда. Шунинг учун ушбу фан асосий ихтисослик фани ҳисобланиб, технологик жараёнларнинг ажралмас бўғини сифатида қаралади.

Фанни ўқитишда замонавий ахборот ва педогогик технологиялар

Талабаларнинг ҳисоблаш усулларини алгоритмлаш фанини ўзлаштиришлари учун ўқитишнинг илғор ва замонавий усулларидан фойдаланиш, янги информацион-педагогик технологияларни тадбиқ қилиш муҳим аҳамиятга эгадир. Фанни ўзлаштиришда дарслик, ўқув ва услубий қўлланмалар, маъруза матнлари, тарқатма материаллар, электрон материаллар, виртуал стендлар ҳамда намуналар ва макетлардан фойдаланилади. Маъруза, амалий ва лаборатория дарсларида мос равишдаги илғор педагогик технологиялардан фойдаланилади.

Асосий қисм

Фаннинг назарий машғулотлари мазмуни

Илмий ишларнинг самарадорлигини оширишда математик усулларни ва математик моделлаштиришни қўллаш.

Математик тавсиф тенгламаларининг ечиш усуллари:

Алгебраик ва транцендент тенгламаларни тўғри ва итерация усуллари билан ечиш усулларини алгоритмларини тузиш. (Алгебраик ва транцендент тенгламаларни илдизларини ажратиш. Тенг ярмига бўлиш усули. Ватарлар усули. Ньютон усули. Қўшма усул. Итерация усули).

Алгебраик ва транцендент тенгламалар системаларини тўғри ва итерация усуллари билан ечиш усулларини алгоритмлаш (Гаусс усули. Итерацион (Якоби ва Зайдел) усуллари). Итерация усулларининг яқинлашиш жараёни шартларини ўрганиш. Стационар итерацион усуллларининг яқинлашиш жараёнини етарли ва зарурий шартлари.

Интерполяция усулларини алгоритмлаш. Алгебраик кўп ҳадлар билан интерполяциялаш. Яқинлашиш жараёни шартларини ўрганиш.

Дифференциал тенгламаларни тақрибий ечимларини аниқлаш. Эйлер усули.

Интеграл тенгламаларнинг тақрибий ечимлари. Тўртбурчак ва трапеция усуллари. Симпсон формуласи.

Тажриба натижаларини қайта ишлаш. Энг кичик квадратлар усули.

Ночизиқли тенгламаларни тақрибий ечимлари.

Амалий машғулотлар мазмуни, уларни ташкил этиш бўйича кўрсатма ва тавсиялар

Амалий машғулотларда талабалар маърузаларда ўрганилган назарий билимларини бойитадилар ва мустаҳкамлайдилар. Амалий машғулотларни қуйидаги мавзуларда олиб бориш тавсия этилади:

Алгебраик ва транцендент тенгламаларни ечимини тўғри ва итерацион усуллар билан олиш.

Алгебраик ва транцендент тенгламалар системасини Гаусс усулида ечиш.

Интеграл тенгламаларни Симпсон усулида ечиш.

Тажриба натижаларини Ньютон ва Логранж усули билан интерполяциялаш.

Тажриба натижаларини энг кичик квадратлар усули билан аппроксимациялаш.

Ночизиқли эмпирик боғлиқликларни тузиш.

Амалий машғулотларни ташкил этиш бўйича кафедра профессор-ўқитувчилари томонидан кўрсатма ва тавсиялар ишлаб чиқилади. Унда талабалар асосий маъруза мавзулари бўйича олган билим ва кўникмаларини амалий масалалар ечиш орқали янада бойитадилар. Шунингдек, дарслик ва ўқув қўлланмалар асосида талабалар билимларини мустаҳкамлашга эришиш, тарқатма материаллардан фойдаланиш, илмий мақолалар ва тезисларни чоп этиш орқали билимини ошириш, масалалар ечиш, мавзулар бўйича кўргазмали қуроллар тайёрлаш ва бошқалар тавсия этилади.

Лаборатория ишлари мазмуни, уларни ташкил этиш бўйича кўрсатмалар

Лаборатория ишлари талабаларда ҳисоблаш усулларини алгоритмлашнинг қуллаш ва уларнинг атрофлича таҳлил қилиш бўйича амалий кўникма ва малака ҳосил қилади.

Лаборатория ишларининг тавсия этиладиган мавзулари:

1. Алгебраик ва трансендент тенгламаларни оддий итерация ҳамда ватарлар усули билан ечиш.
2. Оддий итерация усули билан чизиқли бўлмаган тенгламалар системасини ечиш
3. Ньютон усули билан алгебраик ва трансцендент тенгламаларни тақрибий ечиш.
4. Чизиқли алгебраик тенгламалар системасини оддий итерация усули билан ечиш.
5. Чизиқли алгебраик тенгламалар системасини Зейдель усули билан ечиш

Мустақил ишни ташкил этишнинг шакли ва мазмуни

Талаба мустақил ишни тайёрлашда муайян фаннинг хусусиятларини ҳисобга олган ҳолда қуйидаги шакллардан фойдаланиши тавсия этилади:

- дарслик ва ўқув қўлланмалар бўйича фанларнинг боблари ва мавзуларини ўрганиш;
- тарқатма материаллар бўйича маърузалар қисмини ўзлаштириш;
- автоматлаштирилган ўргатувчи ва назорат қилувчи тизимлар билан ишлаш;
- махсус адабиётлар бўйича фанлар бўлимлари ёки мавзулари устида ишлаш;
- янги техникаларни, аппаратураларни, жараён ва технологияларни ўрганиш;
- талабаларнинг ўқув – илмий - тадқиқот ишларини бажариш билан боғлиқ бўлган фанлар бўлимлари ва мавзуларни чуқур ўрганиш;
- фаол ва муаммоли ўқитиш услубидан фойдаланиладиган ўқув машғулотлари;
- масофавий (дистанцион) таълим.

Тавсия этилаётган мустақил ишларнинг мавзулари:

Яхлитлаш хатоликларининг тўпланиши.

Алгебраик тенгламалар системасини ечишда Гаусс усулини қўллаш шартлари.

Дифференциал тенгламаларни Адамс усули билан ечиш.

Биринчи тартибли дифференциал тенгламаларни тақрибий интеграллаш усули билан ечиш.

Майдон ва ҳажмларни каррали интеграл ёрдамида ҳисоблаш.

Интерполяция хатоликлари.

Аппроксимация усуллари ва мезонлари.

Дастурнинг информацион-услубий таъминоти

Мазкур фанни ўқитиш жараёнида таълимнинг замонавий методлари, педогогик ва ахборот-коммуникация технологиялари қўлланилиши назарда тутилган:

- ҳисоблаш усулларини алгоритмлашнинг назарий асослари бўлимига тегишли маъруза дарсларида замонавий компютер технологиялари ёрдамида презентацион ва электрон-дидактик технологиялари;

- ҳисоблаш усулларини алгоритмлашнинг бўйича ўтказиладиган амалий машғулотларда ақлий хужум, гуруҳли фикрлаш педагогик технологияларини қўллаш назарда тутилади.

- ҳисоблаш усулларини алгоритмлашнинг махсус бўлимларига тегишли бўлган тажриба машғулотларида кичик гуруҳлар мусобақалари, гуруҳли фикрлаш педогогик технологияларини қўллаш назарда тутилади.

Фойдаланилаётган асосий дарсликлар ва ўқув қўлланмалар рўйхати

Асосий

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2. Гулямов Ш.М., Мухитдинов Д.П. «Алгоритмизация вычислительных методов». Электронная версия курса лекции. –Ташкент: ТГТУ, 2006.
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Қўшимча

1. Юсупбеков Н.Р., Мухитдинов Д.П., Базаров М.Б., Халилов Ж.А. Бошқариш системаларини компьютерли моделлаштириш асослари. Олий ўқув юртлари учун ўқув қўлланма. –Н.: Навоий-Голд-Сервис, 2009.
2. Пытьев Ю.П. «Математические методы интерпретации эксперимента». – М.: В-Ш., 1989.
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4. Интернет манбалари.

MINISTRY HIGHER AND SECONDARY VOCATIONAL EDUCATION
OF REPUBLIC UZBEKISTAN
NAVOI MINING-METALLURGICAL COMPLEX
NAVOI STATE MINING INSTITUTE

POWER-MECHANICAL FACULTY

Chair «Automation and management of technological processes and industries»

THE WORKING CURRICULUM

On subjects

«ALGORITHMIZATION OF COMPUTING METHODS»

Field of knowledge	300 000-Proizvodstvennno - technical sphere
Sphere of education	310 000 – Engineering has put
Formation direction	5 311 000 «Automation by technological processes and industries»

Navoi – 2013

«CONFIRMATION»
Dean PMF
________ prof. S.Z.Bazarova
«____» ______________ 2013 y

THE WORKING CURRICULUM
on disciplines «Algorithmization of computing methods»

The general allocated hours and distribution of hours by kinds of employment

Semestre	Lecture	Laboratory	Practice	The self study work	Academic year project	In total
3	36	18	18	65	-	137
Total	36	18	18	65	-	137

The working program is discussed and confirmed at session № _1_ from «27» _08_ by 2013 of chair «Automation and management of technological processes and industries»

The head of chair **prof. Bazarov M.B.**

The composer **доц. Urinov S.R.**

Reviewers:

The main instrument operator ON NMM	Kim V. I.
The docent of chair "A&CTP&I" NSMI	Ishmamatov M. R.

The working curriculum is confirmed on council session power - mechanical faculty by report №*1* from *August, 28th, 2013*.

INTRODUCTION

Formation is not simply process of reception of the sum of necessary knowledge, but also process of formation of spiritual essence of the person. To the full it concerns and higher education. For this reason education is inseparable from training process.

The course «Algorithmization of computing methods» is discipline in which are put modelling, numerical methods, computer modelling of system management in the industrial enterprises their designing and operation in national economy branch.

The purpose and problems of studying of a course

The subject purpose is – formed students logic abilities of mind for algorithm construction on definition approximate the decision the algebraic, differential both integrated equation and processing of results of experimental data.

Subject problems is – study students a choice corresponding variants for algorithm construction on definition approximate the decision the algebraic, differential both integrated equation and processing of results of experimental data.

Requirements to knowledge, skills and abilities of students

«Algorithmization of computing methods» ўқув фанини ўзлаштириш жараёнида амалга ошириладиган масалалар доирасида бакалавр:

- алгебра, дифференциал ва интеграл тенгламаларини ечимини топишда тақрибий ечим усуллари ***ҳақида тасаввурга эга бўлиши***;
- матрица ва детерминант, дифференциал ва интеграл тенгламаларнинг ҳусусий ечимларини олиш усулларини ***билиши;***
- мустақил равишда тақрибий ечимлар алгоритмларини туза олиш ***кўникмаларига эга бўлиши керак.***

Қўйилган вазифалар ўқиш жараёнида талабаларни маъруза, лаборатория ва амалий машғулотларда фаол иштирок этиши, адабиётлар билан ишлаши билан амалга оширилади.

Communication of discipline with other disciplines in the curriculum and methodical sequence

«Algorithmization of computing methods» фани мутахассислик фани ҳисобланиб, 3-семестрда ўқитилади. Дастурни амалга ошириш ўқув режасида режалаштирилган «Информатика ва ахборот технологиялари» ва «Олий математика» фанларидан етарли билим ва кўникмаларга эга бўлиш талаб этилади.

Discipline role in manufacture

Кимё саноати корхоналарида ва илмий текшириш институтларида турли ҳисоб ишларини амалга оширишда ҳисоблаш усулларини алгоритмлашдан фойдаланиб, ишлаб чиқариш унумдорлиги ва марадорлигини ошириш бўйича олиб борилаётган ишлар умумий ҳажмнинг анчагина қисмини ташкил қилади.

Шунинг учун ҳам ҳисоблаш усулларини алгоритмлашни ўрганишга алоҳида талаблар қўйилади. Айниқса мураккаб системалар фаолиятини таҳлил қилишда ҳисоблаш усулларини алгоритмлашдан кенг фойдаланилмоқда. Шунинг учун ушбу фан асосий ихтисослик фани ҳисобланиб, технологик жараёнларнинг ажралмас бўғини сифатида қаралади.

Role training new modern information and pedagogical technologies

Талабаларнинг ҳисоблаш усулларини алгоритмлаш фанини ўзлаштиришлари учун ўқитишнинг илғор ва замонавий усулларидан фойдаланиш, янги информацион-педагогик технологияларни тадбиқ қилиш муҳим аҳамиятга эгадир. Фанни ўзлаштиришда дарслик, ўқув ва услубий қўлланмалар, маъруза матнлари, тарқатма материаллар, электрон материаллар, виртуал стендлар ҳамда намуналар ва макетлардан фойдаланилади. Маъруза, амалий ва лаборатория дарсларида мос равишдаги илғор педагогик технологиялардан фойдаланилади.

THE BASIC PART

The general allocated hours and distribution of hours by kinds of employment

Lecture	- 36 hour
Laboratory employment	- 18 hour
Practical employment	- 18 hour

The self study works	- 65 hour
In total	**- 137 hour**

The maintenance of discipline of theoretical employment

Introduction. Subject problems – 2 hour.

Introduction. The cores concept about algorithmization of computing methods.

Methods the decision the equation mathematical characteristics - 22 hour.

Algorithmization of the numerical decision of the algebraic and transcendental equations. A method branch of roots and a method half divisions. (2 hour)

Algorithmization of the numerical decision of the algebraic and transcendental equations. A method a chord and Newton's method. (2 hour)

Algorithmization of the numerical decision of the algebraic and transcendental equations. A method of iteration and a method of secants. (2 hour)

Algorithmization of the numerical decision of system of the algebraic and transcendental equations. A method of Gaussa. (2 hour)

Algorithmization of the numerical decision of system of the algebraic and transcendental equations. Iterative methods of Jacoby and Zeidel. (2 hour)

Algorithmization interpolation methods. (2 hour)

Interpolation of functions. (2 hour)

The numerical decision of the differential equations. Euler's method. (2 hour)

The numerical decision of the differential equations. A method of Runge-Kutta and Adams. (2 hour)

Numerical integration. Quadrature formulas of trapezes and rectangles. Simpson's formula. (2 hour)

Numerical integration. The formula of Gaussa. (2 hour)

Root-mean-square approach of functions. (2 hour)

Method of the least squares. (2 hour)

Algorithmizations methods linear programming - 12 hour

Statement of a problem of linear programming. The basic properties the decision of a problem of linear programming. (2 hour)

Geometrical interpretation of a problem of linear programming. (2 hour)

Finding the decision of a problem of linear programming to Simplex methods. (2 hour)

Finding the decision of a problem of linear programming. A method of artificial basis. (2 hour)

Transport problem. Methods initial basic the decision. (2 hour)

Method of potentials for a finding optimum decisions transport problems. (2 hour)

The list of a practical training (18 hour)

The numerical decision of the algebraic and transcendental equations iterative methods. (4 hour)

The numerical decision of system of the linear algebraic equations methods of Gaussa. (2 hour)

Calculation of integrals by the approached methods (2 hour)

Newton's interpolation polynom and Lagrange (2 hour)

Problems of Cochy for the ordinary differential equations. Euler's methods, Runge-Kutta and Adams. (4 hour)

The geometrical decision of a problem of linear programming. (2 hour)

Finding the decision of a problem of linear programming to Simplex methods. (2 hour)

The list of laboratory researches (18 hour)

The numerical decision of the algebraic and transcendental equations iterative methods and to methods the Chord. (4 hour)

The numerical decision of the algebraic and transcendental equations to Newton's methods. (2 hour)

The numerical decision of system of the linear algebraic equations to methods of simple iteration. (2 hour)

The numerical decision of system of the nonlinear algebraic equations to methods of simple iteration. (2 hour)

Problems of Cochy for the ordinary differential equations. Euler's methods, Runge-Kutta and Adams. (4 hour)

The geometrical decision of a problem of linear programming. (2 hour)

Finding the decision of a problem of linear programming to Simplex methods. (2 hour)

The organisation a form and content of self-study works

Талаба мустақил ишни тайёрлашда муайян фаннинг хусусиятларини ҳисобга олган ҳолда қуйидаги шаклардан фойдаланиши тавсия этилади:

- дарслик ва ўқув қўлланмалар бўйича фанлар боблари ва мавзуларини ўрганиш;
- тарқатма материаллар бўйича маърузалар қисмини ўзлаштириш;
- автоматлаштирилган ўргатувчи ва назорат қилувчи тизимлар билан ишлаш;
- махсус адабиётлар бўйича фанлар бўлимлари ёки мавзулари устида ишлаш;
- янги техникаларни, аппаратураларни, жараён ва технологияларни ўрганиш;
- талабаларнинг ўқув-илмий-тадқиқот ишларини бажариш билан боғлиқ бўлган фанлар бўлимлари ва мавзуларини чуқур ўрганиш;
- фаол ва маммоли ўқитиш услубидан фойдаланиладиган ўқув машғулотлари;
- масофавий (дистанцион) таълим.

Тавсия этилаётган мустақил ишларнинг мавзулари:

- Яхлитлаш хатоликларининг тўпланиши.
- Алгебраик тенгламалар системасини ечишда Гаусс усулини қўллаш шартлари.
- Дифференциал тенгламаларни Адамс усули билан ечиш.
- Биринчи тартибли дифференциал тенгламаларни тақрибий интеграллаш усули билан ечиш.
- Майдон ва ҳажмларни каррали интеграл ёрдамида ҳисоблаш.
- Интерполяция хатоликлари.
- Аппроксимация усуллари ва мезонлари.

Information-methodical maintenance of the program

Мазкур фанни ўқитиш жараёнида таълимнинг замонавий методлари, педогогик ва ахборот-коммуникация технологиялари қўлланилиши назарда тутилган:

- ҳисоблаш усулларини алгоритмлашнинг назарий асослари бўлимига тегишли маъруза дарсларида замонавий компютер технологиялари ёрдамида презентацион ва электрон-дидактик технологиялари;

- ҳисоблаш усулларини алгоритмлашнинг бўйича ўтказиладиган амалий машғулотларда ақлий хужум, гуруҳли фикрлаш педагогик технологияларини қўллаш назарда тутилади.

- ҳисоблаш усулларини алгоритмлашнинг махсус бўлимларига тегишли бўлган тажриба машғулотларида кичик гуруҳлар мусобақалари, гуруҳли фикрлаш педогогик технологияларини қўллаш назарда тутилади.

The list the used basic textbooks and educational the grant

The Basic

1. Юсупбеков Н.Р., Мухитдинов Д.П., Базаров М.Б. Электрон ҳисоблаш машиналарини кимё технологиясида қўллаш. Олий ўқув юртлари учун дарслик. –Т.: Фан, 2010.
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4. Самарский А.А., «Введение в численные методы». – М.: Наука,1987.
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The additional

1. I. Podlubny, Fractional Differential Equations, Academic Press, San Diego, 1999.
2. L. Debnath, Int. J. Math. and Math. Sci., 2003, 1(2003)

3. V. Daftardar-Gejji, H. Jafari, J. Math. Anal. Appl., 316, 753(2006)
4. D.D. Ganji, M. Nourollahi, E. Mohseni, Comput. and Math. with Appl., (In press), doi:10.1016/j.camwa.2006.12.078.
   Брандт З. «Статические методы анализа наблюдений». –М.: Мир, 1975.
5. Интернет манбалари. exponenta.ru, edu.uz, ziyonet.uz, nggi.uz, edu.ru

THE PLANNED SCHEDULE ON DISCIPLINE

CALENDAR - THEMATIC PLAN

On discipline: Algorithmization of computing methods
Lecturer: docent. Urinov Sh.R. **Faculty:** PMF
Consultations and a practical training conducts: ______________
Laboratory researches conducts: _Urinov Sh.R_ **The course** II, **Group ______**

№	Type of lecture	Theme and the summary	Otve-deno	The information about executed works		The teacher's signature
				Number	O'clock	
1	Lecture	Introduction. The cores concept about algorithmization of computing methods.	2			
2	Lecture	Algorithmization of the numerical decision of the algebraic and transcendental equations. A method branch of roots and a method half divisions.	2			
3	Lecture	Algorithmization of the numerical decision of the algebraic and transcendental equations. A method a chord and Newton's method.	2			
4	Lecture	Algorithmization of the numerical decision of the algebraic and transcendental equations. A method of iteration and a method of secants.	2			
5	Lecture	Algorithmization of the numerical decision of system of the algebraic and transcendental equations. A method of Gaussa.	2			
6	Lecture	Algorithmization of the numerical decision of system of the algebraic and transcendental equations. Iterative methods of Jacobi and Zeidel.	2			
7	Lecture	Algorithmization interpolation methods. Interpolation functions.	2			
8	Lecture	The numerical decision of the differential equations. Euler's method.	2			
9	Lecture	The numerical decision of the differential equations. A method of Runge-Kutta and Adams.	2			
10	Lecture	Numerical integration. Quadrature formulas of trapezes and rectangles. Simpson's formula.	2			
11	Lecture	Numerical integration. The formula of Gaussa.	2			
12	Lecture	Root-mean-square approach of functions. Method of the least squares.	2			
13-14	Lecture	Statement of a problem of linear programming. The basic properties the decision of a problem of linear programming.	4			
15-16	Lecture	Geometrical interpretation of a problem of linear programming.	4			
17	Lecture	Finding the decision of a problem of linear programming to Simplex methods.	2			
18	Lecture	Finding the decision of a problem of linear programming. A method of artificial basis.	2			
		IN TOTAL	**36**			

The head of chair: **prof. Bazarov M.B.**

The teacher: **docent Urinov S.R.**

CALENDAR - THEMATIC PLAN

On discipline: Algorithmization of computing methods
Lecturer: docent. Urinov Sh.R. **Faculty:** PMF
Consultations and a practical training conducts: ______________
Laboratory researches conducts: _Urinov Sh.R_ **The course** II, **Group ______**

№	Type of lesson	Theme and the summary	Otve-deno	The information about executed works		The teacher's signature
				Number	O'clock	
1	**2**	**3**	**4**	**5**	**6**	**7**
1-2	Practical work	The numerical decision of the algebraic and transcendental equations iterative methods.	4			
3-4	Practical work	Newton's interpolation polynom and Lagrange	4			
5-6	Practical work	Calculation of integrals by the approached methods	4			
7-8	Practical work	Approximation results of experiment with a method of the least square. Creation non-linear empirical connection	4			
9	Practical work	The geometrical decision of a problem of linear programming.	2			
TOTAL:			**18**			
1-2	Laboratory	The numerical decision of the algebraic and transcendental equations iterative methods Chord and Newton.	4			
3-4	Laboratory	The numerical decision of system of the linear algebraic equations methods of Gaussa, simple iteration and Seidel	4			
5-6	Laboratory	The numerical decision of system of the nonlinear algebraic equations to methods of simple iteration.	4			
7-8	Laboratory	Problems of Cochy for the ordinary differential equations. Euler's methods, Runge-Kutta and Adams.	4			
9	Laboratory	Finding the decision of a problem of linear programming to Simplex methods.	2			
Resulto:			**18**			

The head of chair: **prof. Bazarov M.B.**

The teacher: **docent Urinov S.R.**

LECTURE MATERIALS

Lecture №1.

Introduction. The cores concept about algorithmization of computing methods.

The purpose: **Formation of knowledge, skills on studying of bases of algorithmization, the basic properties of algorithm and classification of computing methods.**

The plan:

1. Classification of computing methods.
2. Preparation of problems for the personal computer decision.
3. Properties of algorithm.
4. Classification of algorithms.

Given a lecture course it is written according to the program on discipline «Algorithmization computing methods», studied by students of technical colleges. The lecture course is covered by following sections of the program: on concept linear нормированного spaces; methods of the numerical decision of systems of the linear equations; methods of the numerical decision of the nonlinear equations and systems; root-mean-square approach of functions; interpolation functions; numerical differentiation and integration; the numerical decision of the ordinary differential equations; numerical methods of search of an extremum of functions of one and several variables. In each theme necessary theoretical data (the basic theorems, definitions, formulas, various computing methods etc.) are resulted And also the examples illustrating application of described methods. Besides, there are exercises for the independent decision and answers to them. Appendices contain block diagrammes of computing algorithms and texts of programs for the considered numerical methods on algorithmic languages PASCAL.

The main objective a lecture course — to help development of practical skills in students with application of numerical methods. Each theme contains: computing algorithm; theoretical substantiations of its application; conditions of the termination of computing process; the examples in full or in part executed "manually"; exercises and answers to them; the appendix, in which the considered computing algorithm is presented in the form of the block diagramme and texts of programs on four (sometimes — on five) algorithmic languages.

Authors hope that mastering by numerical methods will be promoted also by a considerable quantity of in detail solved examples, and also exercises for independent work. It is necessary to notice that often various computing algorithms are illustrated by the same examples. Besides, for many examples considered in the book analytical decisions to which it is possible to compare the found numerical decisions are known. Coincidence of the results received in the different ways, is additional, evident argument of applicability of this or that numerical method. At last, the help in practical application of numerical methods will be rendered by appendices to the given book. In them block diagrammes and texts of 95 programs (with comments) on algorithmic languages used in educational practice are resulted. The material stated in appendices can be applied not only at studying of numerical methods, but also as the ready applied programs which work is checked up in program environments of firms BORLAND and MICROSOFT for personal computers.

The present a lecture course is intended for students of the higher technical educational institutions. It can appear also to useful teachers, engineers and the science officers using in the activity computing methods.

Algorithmization basis. The basic properties of algorithm

Process of preparation and the decision of problems on the personal computer is while difficult enough and labour-consuming, demanding performance of variety of stages. Such stages are:

1) problem statement;
2) the mathematical formulation of a problem;
3) a choice of a numerical method of the decision;
4) working out of algorithm of the decision of problems;
5) a program writing;
6) input of the program and the initial data;
7) program debugging;
8) the problem decision on the personal computer;

The given sequence is characteristic for the decision of each problem. However in the course of problem preparation each stage can have more and less expressed character. Performance of stages in the course of problem preparation has character of consecutive approach as problem specification at the subsequent stage leads to necessity of return to the previous and repeated performance of the subsequent stages.

Let's consider more in detail performance of works at each stage in the course of preparation of a problem for the decision.

Problem statement defines the purpose of the decision of a problem, opening its maintenance. The problem is formulated at level of professional concepts, should be correct and clear to the executor (user). Mistake directed by a problem, found out on the subsequent stages, will lead to that work on preparations of a problem for the decision should begin from the very beginning.

At problem statement the ultimate goal is found out and the general approach to the problem decision is developed. It is found out, how many decisions the problem has and whether has them in general. The general properties of the considered physical phenomenon or object are studied, possibilities of the given programming system are analyzed.

The mathematical formulation of a problem carries out formalisation of a problem by its description by means of formulas, defines the list of the initial given and received results, entry conditions, accuracy of calculation. The mathematical model of a solved problem is in essence developed.

Choice of a numerical method of the decision. In some cases the same problem can be solved by means of various numerical methods. The method choice should be defined by many factors, basic of which accuracy of results of the decision, time of the decision for the personal computer and volume of operative memory are. In each specific case as criterion for a choice of a numerical method accept any of the specified criteria or some integrated criterion.

In simple problems the given stage can be absent, as the numerical method is certain by the mathematical formulation of a problem. For example, calculation of the area of a triangle under the formula of Gerona, roots of a quadratic, etc.

Working out of algorithm of the decision of a problem. At the given stage the necessary logic sequence of calculations taking into account the chosen numerical method of the decision and other actions with which help results will be received is established.

Algorithm – some final sequence of instructions (rules) defining process of transformation of the initial and intermediate data as a result of the decision of a problem.

The program writing is carried out on the developed algorithm by means of the programming language.

Input of the program and the initial data is carried out by means of the personal computer keyboard.

Debugging of programs represents process of detection and elimination of syntactic and logic errors.

The problem decision on микро the personal computer is usually spent with a dialogue mode. In this mode the user by means of the personal computer keyboard can carry out input of the program and its updating, program translation (transfer from the programming language on machine), correction syntactic and logic errors at debugging, reception on an exit of results and the auxiliary information necessary for management by work of the personal computer.

Technology OREG.

- **O –** state the **opinion.**
- **R –** produce one **reason** of the opinion.
- **E –** give an **example** for the explanatory of the reason.
- **G– generalise** the opinion.

Question for OREG: **what properties algorithms should possess?**

Use of computers as executors of algorithms shows a number of requirements to algorithms. Unlike people, the computer can carry out only precisely certain operations. Therefore machine algorithms should possess following properties:

1. Step-type behaviour
2. Clearness;
3. Unambiguity
4. Mass character.
5. Productivity.
6. Finiteness
7. Correctness

That the executor has managed to solve the problem set for it, using algorithm, it should be able to follow its each instructions. Differently, he should understand a management essence. That is at algorithm drawing up it is necessary to consider "game rules", i.e. system of instructions (or system of commands) which understands the computer. For example, at the decision of any problem the student used the reference to functions sin x (it is trigonometrical function) and to function of Bessel (it is cylindrical function), but the computer (as well as the reader, probably) does not understand last. It is not provided by founders of the given class of cars. Hence, (as a whole) the car will not understand algorithm. We will speak in this case about "clearness" of algorithm.

As "CLEARNESS" of algorithms understand instructions which are clear to the executor.

Being clear, the algorithm should not contain nevertheless the instructions which sense can be perceived ambiguously. These properties instructions and instructions which are made for people often do not possess. For example: in the recipe of preparation of an omelette resulted above it is told: "to Break in this mix of 3 eggs and all - it it is good to shake up a spoon". At household level to us it is clear that it is a question of three eggs (and what else! - you will tell). But eggs can be both pigeon, and duck, and even ostrich's (all sharply differ on size from each other). Ambiguity here "has obviously crept in". Or type instructions: "to salt to taste","to fill two-three spoons sugar to sand","has received an estimation 4 or 5","to fry to readiness""''dig from a fence till a dinner" cannot to meet in algorithms. It is obvious that clear in certain situations for the person of the instruction of this kind can stump the computer.

Or we will recollect a parable known for all an imperial will. The tsar has ordered subordinated to execute such decree: "to Execute it is impossible to pardon". He has forgotten to put a comma in the decree, and subordinates did not know that by it to do. "It is impossible to execute instructions, to pardon" and "to execute, it is impossible to pardon" set absolutely different actions on which human life depends.

Besides, in algorithms such situations when after performance of the next instruction of algorithm to the executor it is not clear what of them should be carried out on a following step are inadmissible.

UNAMBIGUITY of algorithms is understood as uniqueness of interpretation of rules of performance of actions and an order of their performance.

As we already know, the algorithm sets full sequence of actions which it is necessary to carry out for the problem decision. Thus, as a rule, for performance of these actions them dismember (break) in certain sequence into simple steps. There is an ordered record of set of accurately divided instructions (instructions, commands), forming прерывную (or as speak, discrete) algorithm structure. To execute actions of the following instruction it is possible only having executed actions previous.

Programming is a process of decomposition of a challenge on a number of simple actions.

As STEP-TYPE BEHAVIOUR understand possibility of splitting of algorithm on the separate elementary actions which performance by the person or car does not raise the doubts.

It is very important, that the made algorithm provided the decision not one private problem, and could carry out the decision of a wide class of problems of the given type.

For example. It is necessary to solve a concrete quadratic h4 - 4x+3=0. But after all it is possible to make algorithm of the decision of any quadratic of a kind: $ax^2 + bx +$ with =0.

Really, for a case when = $b^2 - 4ac > 0$, quadratic roots it is possible to find discriminant D under known formulas.

If D <0 the valid roots do not exist. Thus, this algorithm can be used for any square at an alignment. Such algorithm will be

As FINITENESS of algorithms understand end of work of algorithm as a whole for final number of steps.

Still it is necessary to carry **PRODUCTIVITY** to desirable properties of algorithms**, she assumes that performance of algorithms should come to the end with reception of certain results**.

Similar situations in computer science arise, when no actions can be executed. In the mathematician such situations name uncertainty. For example, division of number into a zero, extraction of a square root from a negative number, and concept of infinity vaguely. Therefore, if the algorithm sets infinite sequence of actions in this case it also is considered result uncertain. But it is possible to operate in another way. Namely: to specify the reason of uncertain result. In that case, "it is impossible to divide type explanatories into a zero", "the computer to execute such not in a condition", etc. it is possible to consider as result algorithm performance.

Thus, property of productivity consists that in all "cases it is possible to specify that we understand as result of performance of algorithm.

And last general property of algorithms - their **correctness.** We say that algorithm **CORRECT if its performance sings correct results of the decision of tasks in view.**

Accordingly we say that the algorithm CONTAINS ERRORS if it is possible to specify such admissible initial data or conditions at which performance of algorithm either will not come to the end in general, or will not be received any results, or the received results will appear wrong.

On used structure of management of computing process algorithms classify as follows: linear structure; branching structure; cyclic structure; with structure of the enclosed cycles; the mixed (combined) structure.

For an illustration of algorithms of any structure the simple mathematical formulations of problems accessible to the pupil of any trades are used. For the decision of such problems in many cases it can appear inexpedient use of the personal computer, however consideration of ways of their programming makes sense, as they are a component more challenges.

At the decision any more or less the challenge can take place some the various algorithms leading to reception of result. It is necessary to choose the best from all possible algorithms in sense of some criterion.

Algorithm of linear structure – algorithm in which all actions are carried out consistently one after another. Such order of performance of actions is called as natural.

Algorithm of branching out structure – algorithm in which depending on performance of some logic condition computing process should go on one or other branch.

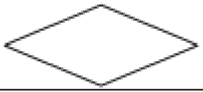
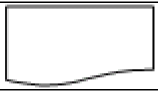
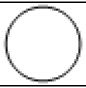
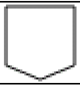
Algorithm of cyclic structure - the algorithm containing repeatedly carried out sites of computing process, named cycles.

Algorithm with structure of the enclosed cycles – the algorithm containing a cycle in which are placed one or other several cycles. There are many ways of record of the algorithms different from each other by presentation, compactness, degree of formalisation and other indicators.

The greatest distribution was received by a graphic way and a so-called algorithmic language of record of the algorithms, focused on the person (pseudo-codes).

Graphic record of algorithm should will be executed according to state standards. (гост 19.002-80"schemes of algorithms and programs. Performance rules»; гост 19.003-80"scheme of algorithms and programs. Designations conditional and graphic»).

The algorithm scheme represents sequence of the blocks ordering
Performance of certain actions, in communication between them.

The name	Symbol (drawing)	Carried out function (explanatory)
1. The block of calculations		Carries out computing action or group of actions
2. The logic block		Choice of a direction of performance of algorithm depending on a condition
3. Input/conclusion blocks		Input or output of the data without dependence from the physical carrier
		Conclusion of the data to the printer
4. The Beginning/end (input/exit)		The beginning or the program end, input or exit in the subroutine
5. The predetermined process		Calculations under the standard or user subroutine
6. The updating block		Performance of the actions changing points of algorithm
7. A connector		Communication instructions between the interrupted lines within one page
8. An interpage connector		Communication instructions between parts of the scheme located on different pages

Rules of construction of block diagrammes:

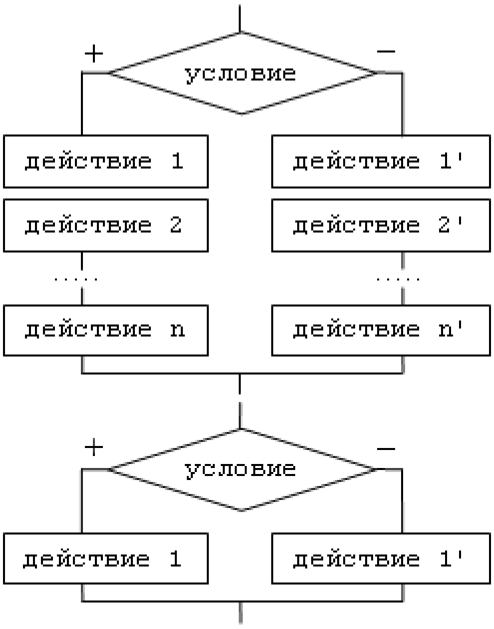
1. The Block diagramme is built in one direction either from top to down, or from left to right
2. All turns of connecting lines are carried out at an angle 90 degrees

Algorithmic design of branching.

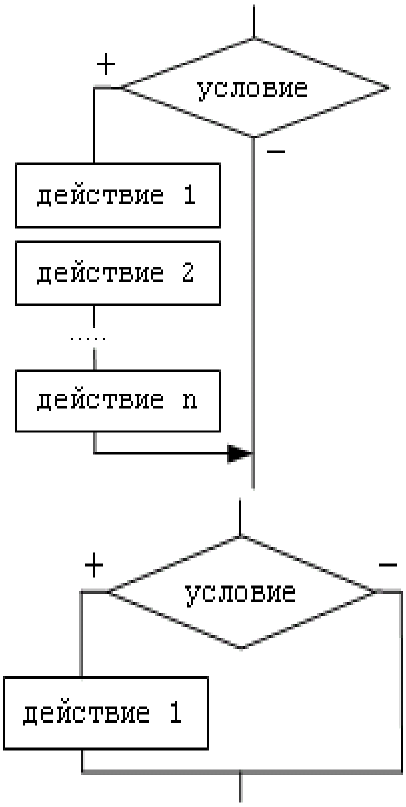
Branching - operating structure, организующая performance only one of two specified actions depending on justice of some condition.

Condition - a question having two variants of the answer: yes or not. Branching record is carried out in two forms: full and incomplete.

The full form:

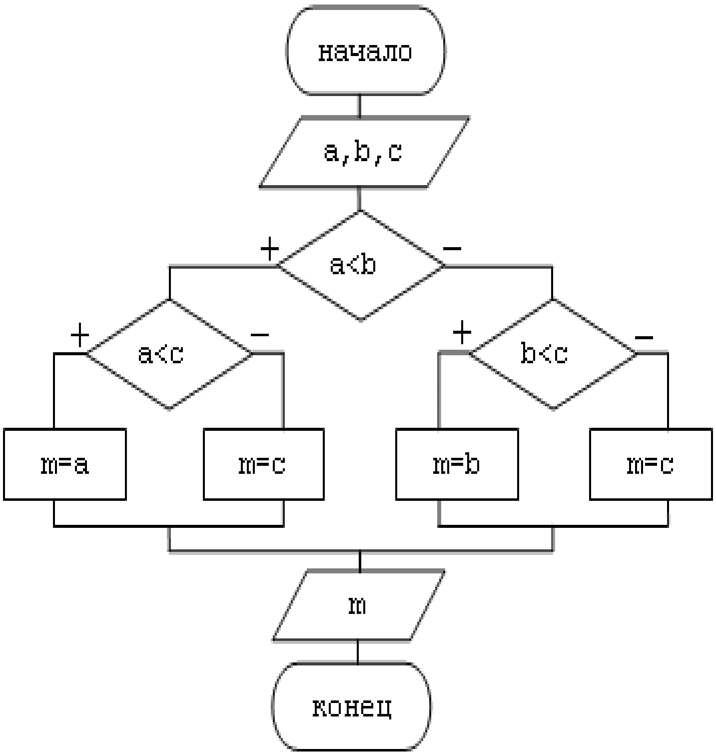


The incomplete form:

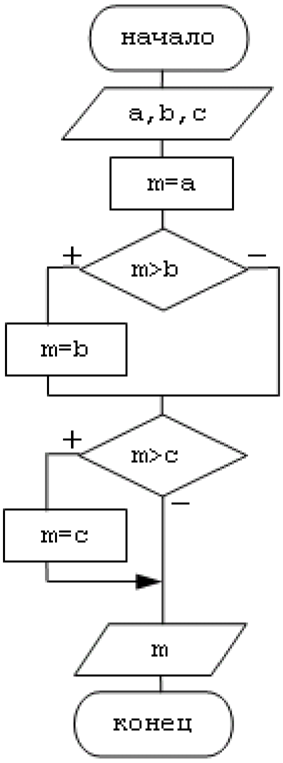


Example: to find least of three numbers.

1 variant of the decision:

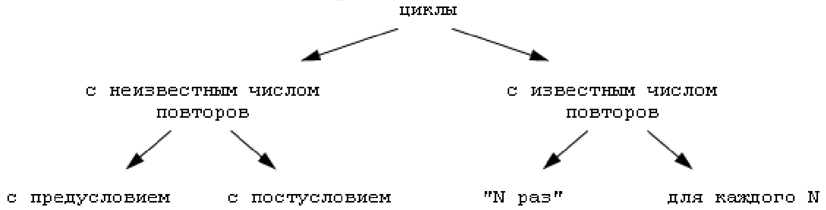


2 variant of the decision:

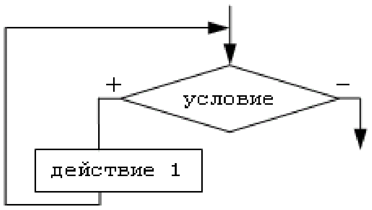


Algorithmic design of a cycle.

Cycle - operating structure, организующая repeated performance of the specified action.

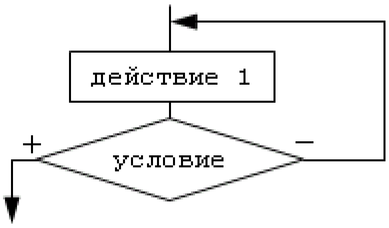


Cycle "while":



Cycle performance "while" begins with condition check, therefore such version of cycles names cycles with a precondition. Transition to action performance is carried out only in the event that the condition is carried out, otherwise there is an exit from a cycle. It is possible to tell that a cycle condition "while" is a condition of an input in a cycle. In that specific case it can appear that action was not carried out never. The cycle condition is necessary for picking up so that actions carried out in a cycle have led to infringement of its validity, differently there will be a cycling.

Cycling - infinite repetition of carried out actions. Cycle "to":



Cycle execution begins with action performance. Thus the cycle body will be realised at least once. After that there is a condition check. Therefore a cycle "to" name a cycle with a postcondition. If the condition is not carried out, there is a return to performance of actions. If the condition is true, the exit from a cycle is carried out. Thus the condition of a cycle "to" is a condition of an exit. For cycling prevention it is necessary to provide the actions leading to the validity of a condition.

Control questions

1. List stages of preparation of problems for the decision on the computer.
2. What properties of algorithm in you know?
3. The basic classification of algorithms.
4. Give definitions of algorithms of branching out and cyclic structure.

Lecture №2.
Algorithmization of the numerical decision of the algebraic and transcendental equations. A method branch of roots and a method half divisions.

The plan:

1. **A method branch of roots**
2. **A method half divisions**

1. Methods of branch of roots

The description of a method of the decision of branch of roots

The numerical decision of the nonlinear equations of a kind

$$F(x) = 0 \quad (2)$$

consists in a finding of values x, satisfying (with the set accuracy) to the given equation and consists of following basic stages:

Branch (isolation, localisation) equation roots.

Specification by means of some computing algorithm of the concrete allocated root with the set accuracy.

The purpose of the first stage is the finding of pieces from a function range of definition in which one root of the solved equation contains only. Are sometimes limited to consideration only any part of the range of definition causing for those or other reasons interest. For realisation of the given stage graphic or analytical ways are used.

At end of the first stage, intervals should be defined, on each of which one root of the equation contains only.

Any iterative method consisting in construction of numerical sequence x_k usually is applied to specification of a root with demanded accuracy (k=0,1,2, …), converging to a required root x the equations.

Analytical way of branch of roots

The analytical way of branch of roots is based on following theorems:

The theorem 1. If function F (x), defining equation F (x) =0, on the piece ends [a; b] accepts values of different signs, i.e.

$$(a)*F(b)<0,$$

that on this piece contains, at least, one root of the equation.

The theorem 2. If function F (x) is strictly monotonous, a root on [a; b] the unique

$$(F'(a)*F'(b)>0).$$

For branch of roots in the analytical way the piece [A; B], drawing 1 on which there are all roots of the equation interesting the calculator. And on a piece [A; B] function F (x) should be defined, continuous and

$$(a)*F(b)<0.$$

Further there are all partial pieces [a; b], containing on one root.

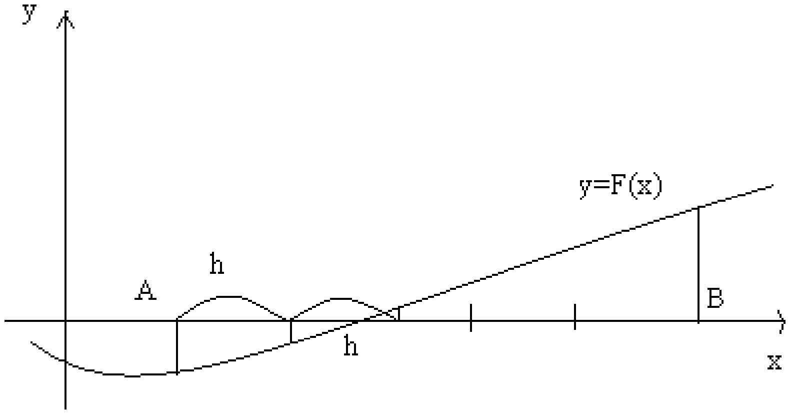
Are calculated value of function F (x), since a point x=A, moving to the right with some step h. If

$$(x)*F(x+h)<0,$$

That on a piece [x; x+h] there is a root and if function F (x) also is strictly monotonous, a root unique. If F (x_k) =0, a x_k-exact root.

Graphic way of branch of roots

The graphic way of branch of roots is based, basically, on visual perception. The branch of roots is made graphically, considering that the valid roots of the equation (1) is there are points of intersection of the schedule of function y=F (x) with an axis of abscisses y=0, it is necessary to construct the function schedule y=F (x) and on axis 0X to note the pieces containing on one root. But it is frequent for simplification of construction of the schedule of function y=F (x) the initial equation (1) replace with the equation equivalent to it $f_1(x) = f_2(x)$. Schedules of functions $y_1=f_1$ (x) and $y_2=f_2$ (x) Further are under construction, and then on axis 0X the pieces localising abscisses of points of intersection of two schedules are marked.



Drawing 1. - a piece Choice

Numerical methods of specification of roots

After the required root of equation F (x) =0 is separated, i.e. the piece [a, b] on which there is only one valid root of the equation is defined, there is an approached value of a root with the set accuracy.

Root specification can be made various methods.

The decision in system MathCad

Problem: to Solve the nonlinear equation $5\sin 2x = \sqrt{1-x}$ (1) numerical method of tangents. We will find and is investigated four roots with accuracy e = 0,000001.

The decision

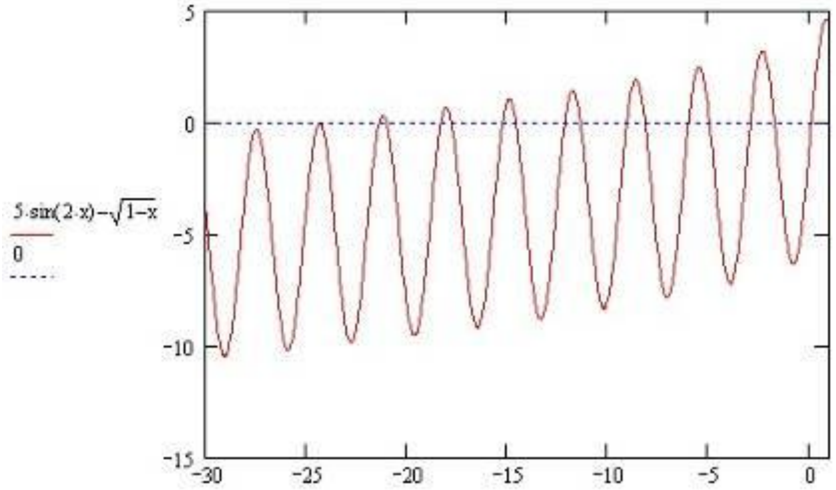
Let's construct in program Mathcad the function schedule

Let's preliminary transfer all to the left part and we will lead to a kind (1) then the equation will become:

$$f(x) := 5 \cdot \sin(2 \cdot x) - \sqrt{1 - x}$$

,

And the schedule of function constructed in program Mathcad, will become presented on drawing 4.



Drawing 4. - the function Schedule in system Mathcad

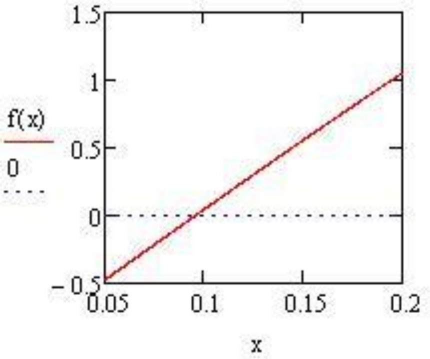
Under the schedule we define quantity and localisations of roots of the equation.

Let's find equation roots

$$5\sin 2x = \sqrt{1-x}$$

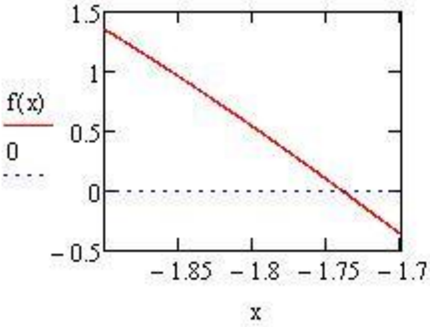
with the set accuracy e = 0,000001

$$f(x) := 5 \cdot \sin(2 \cdot x) - \sqrt{1 - x}$$



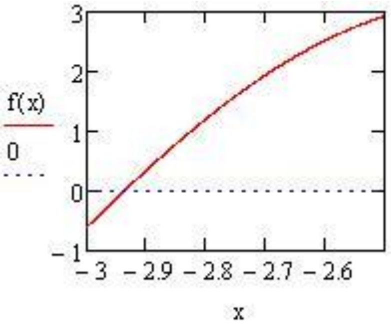
0.05 .. 0.2

root(f(x),x,0.05,0.2) = 0.095679



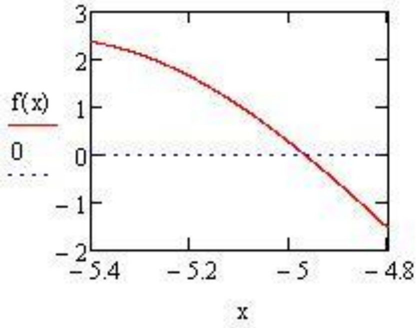
−1.9 .. −1.7

root(f(x),x,−1.9,−1.7) = −1.739493



−3 .. −2.5

root(f(x),x,−3,−2.5) = −2.937543



−5.4 .. −4.8

root(f(x),x,−5.4,−4.8) = −4.967616

2. A method half divisions

Let's consider the equation (1):

$$F(x)=0\,,$$

Where function *F* (x) – is continuous and defined on some piece and $F(a)F(b)<0.$

The last means that function *F* (x) has on a piece at least $[a,b]$one root. We will consider a case, when a root on a piece the unique$[a,b]$.

We halve a piece. If $F\left(\frac{a+b}{2}\right)=0$, is a $\xi=\frac{a+b}{2}$ root of the equation (1). If $F\left(\frac{a+b}{2}\right)\neq 0$, it is considered that half of piece on $[a,b]$which ends function *F* (x) has different signs. New, narrower piece $[a_1,b_1]$again we halve and it is spent on it the same consideration etc. As a result on some step we will receive or exact value of a root of the equation (1), or sequence of the pieces enclosed each other $[a_1,b_1],[a_2,b_2],\dots,[a_n,b_n],\dots,$ such that

$$F(a_n)F(b_n) < 0, \quad (n = 1, 2, \ldots) \qquad (9)$$

and

$$b_n - a_n = \frac{b-a}{2^n}. \qquad (10)$$

The left ends of these pieces form the $a_1, a_2, \ldots, a_n, \ldots$ monotonous (not decreasing) limited sequence, and the right ends - the $b_1, b_2, \ldots, b_n, \ldots$ monotonous (not increasing) limited sequence. Therefore owing to equality (10) there is a general limit

$$\xi = \lim_{n\to\infty} a_n = \lim_{n\to\infty} b_n.$$

Passing in (9) to a limit at $n \to \infty$, owing to a continuity function *F*(x) получим: $[F(\xi)]^2 \le 0.$ From here i.e $F(\xi) = 0,$. is a ξ root of the equation (1).

In practice process (10) is considered finished, if

$$b_n - a_n = \frac{b-a}{2^n} \le \varepsilon, \qquad (11)$$

Where ε – the set accuracy of the decision.

http://math.semestr.ru/optim/secant_method.php Online the decision

Lecture № 3.

Algorithmization of the numerical decision of the algebraic and transcendental equations. Method a chord and Newton's method.

The plan:

1. **A method the Chord**
2. **Newton's method**

1. A method the Chord (a method of proportional parts)

Again we will address to the equation (1):

$$F(x) = 0,$$

Where function F (x) – is continuous and defined on some piece and $[a,b]$ $F(a)F(b) < 0.$ There is faster way of a finding of the isolated root of the equation ξ (1) lying on a piece $[a,b]$. We will assume for definiteness that Instead of $F(a) < 0$ и $F(b) > 0.$ piece division half-and-half $[a,b]$, we will divide it in the relation It $F(a):F(b).$ gives the first approach of a rootуравнения:

$$x_1 = a - \frac{F(a)}{F(b) - F(a)}(b - a). \qquad (12)$$

Then we consider pieces $[a, x_1]$ и $[x_1, b]$. We will choose that from them on which ends function *F* (x) has different signs, we will receive the second approach of a root of the equation etc x_2. until then yet we will not reach inequality performance – the $\left|\frac{x_{n+1} - x_n}{x_n}\right| \le \varepsilon$, где ε set accuracy of the decision. Geometrically this method is equivalent to replacement curve *y*= *F* (x) a chord spent at first through points $A[a, F(a)]$ и $B[b, F(b)]$, and then the chords spent through the ends of received pieces ($[x_1, b], [x_2, b], ..., [x_n, b], ...,$ fig. 2). From here the name – a *method of chords.*

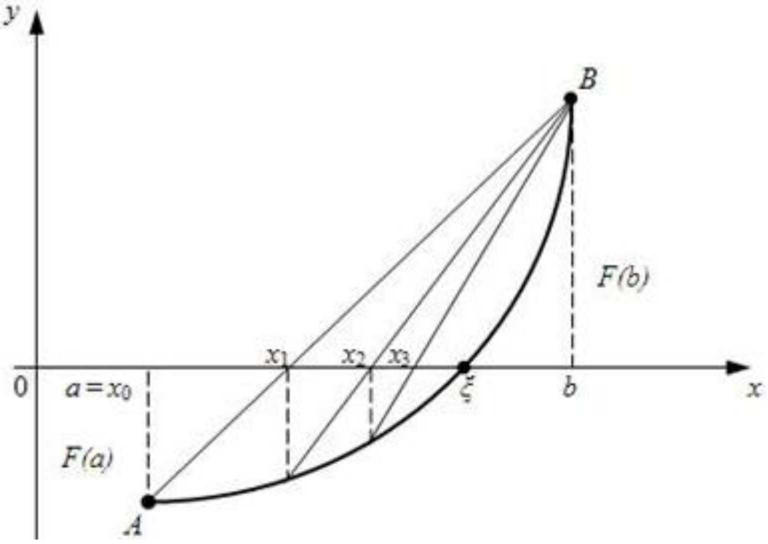
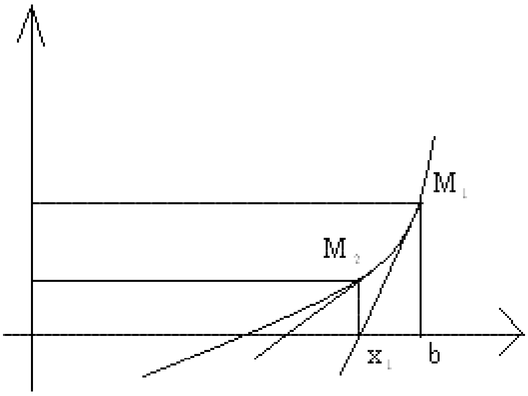


Fig. 2. Geometrical representation of a method of chords.

2. A method of tangents (Newton)

For realisation of the given method, it is necessary to construct initial function y=F (x) and to find values of function on the end of piece F (b). Then to spend a tangent through point M_1. The absciss of a point of intersection of a tangent with axis OX it also is the approached root x_1. Further to find point M_2 (x_1; F (x_1)) to construct the following tangent and to find the second approached root x_2 etc., drawing 2.



Drawing 2. - the Choice of points of a contact

The formula for (n+1) looks like approach:

$$x_{n+1} = x_n - \frac{F(x_n)}{F'(x_n)} \qquad (3)$$

If F (a) *F "(a)> 0, x0=a, otherwise x_0=b.

Iterative process proceeds until it will be revealed that:

$$|F(x_{n+1} \le \varepsilon)| . \quad (4)$$

Advantages of a method: simplicity, speed of convergence. Method lacks: calculation of a derivative and difficulty of a choice of initial position.

At first function analyzes the end and a piece [a; b]. If the condition $f(a)\cdot f''(a) > 0$, the end and a piece $f(a)\cdot f''(a) > 0$ [a; b] also will be the first approach x_1 the equation root, differently the end b a piece [a b] will be the first approach of a root of the equation;. Iterative process which proceeds until Further begins $|f(x1)| > e$. As soon as iterative $|f(x1)| \le e$ process stops, and in x1содержится a required root with necessary approach.

Lecture № 4.

Algorithmization of the numerical decision of the algebraic and transcendental equations. A method of iteration and a method of secants.

The plan:

1. **A method of simple iteration**
2. **A method of secants**

1. A method idle time of iterations (a method consecutive approximation)

It is said that iterative process *converges*, if at performance of consecutive iterations values of the roots turn out, all is closer and closer coming nearer to exact value of a root. Otherwise iterative process is considered *the dispersing*.

Let's copy for convenience the equation (1) in a kind:

$$x = f(x), \qquad (3)$$

That it is possible to receive by replacement: $F(x) = x - f(x)$.

Let – x_0 zero approach, i.e. the initial approached value of a root of the equation (3). Then as the following, 1st, approach we will accept

$$x_1 = f(x_0),$$

The following, 2nd, approach will be

$$x_2 = f(x_1),$$

Etc., as *n* th approach we will accept

$$x_n = f(x_{n-1}). \qquad (4)$$

Here there is a main point: whether comes nearer to the x_n true decision of the equation (3) at unlimited increase *n*? Differently, whether iterative process (4) converges?

Conditions convergence of a method of iterations [2]: *if* at *all* values calculated x_n in the course of (4) decisions of a problem:

1) iterative $|f'(x)| < 1$ *process converges;*

2) iterative $|f'(x)| > 1$ *process disperses.*

If the derivative in $f'(x)$ some points on the x_i module is less 1, and in other points – x_j it is more 1 anything the iterative process defined about convergence it is impossible to tell. It can both to converge, and to disperse.

If iterative process disperses, the reason of it often is the unsuccessful choice of zero approach. So, on fig. 1 it is shown that the choice of zero approach essentially influences convergence of iterative process. It directly is connected with, whether there is a zero approach in x_0 area where conditions of convergence of iterative process are satisfied.

2. A method of secants

Secants a method - a method of calculation of zero of continuous functions. Let in [and, b] the zero a continuous function f (x) contains; x_0, x_1 - various points of this piece. Iterative formula C m.:

$$f(x_1) \neq 0 \qquad (1)$$

If the sequence $f(x_1) \neq 0$ converges, it is obligatory to function zero f (x). At presence at f a continuous derivative on [and, b] local convergence C the m. to a simple root will be superlinear. If to strengthen requirements to smoothness f, it is possible to specify an exact order (local) convergence [1]., for $f(x_1) \neq 0$ $f(x_1) \neq 0$ and a such that $f(x_1) \neq 0$,

$$f(x_1) \neq 0$$

Here $f(x_1) \neq 0$

Superlinear convergence C m. for smooth functions - very important circumstance as calculations of derivatives it is not required and on each step is calculated only one new value of function. So, for comparison, in Newton's method, the order (local) convergence k-rogo is equal 2, on each step calculation of value of function and its derivative is required that, as a rule, is not less labour-consuming, than calculation of two values of function.

As convergence C m. depends on smoothness of function and a choice initial приближений, in standard machine subroutines of calculation of zero of continuous functions this method is combined with any method possessing guaranteed convergence, e.g. a method of division of a piece half-and-half. On each step of such combined method the root an is localised in a piece $f(x_1) \neq 0$, on the ends k-rogo function changes a sign (it is supposed that this condition is executed for an initial piece [a, b]). According to a nek-eye the test the next approach gets out or under the formula (1), or under the halving formula. Thus if f(x) - smooth function iterations, since nek-rogo numbers k_0, automatically go on C m. Is possible even more difficult combination of methods, e.g. algorithm zeroin (see [2]), in k-rum, except mentioned above, is used still a method of return square-law interpolation. Sometimes C m. name a method with the iterative formula

$$f(x_1) \neq 0 \quad (2)$$

Other name of a method (2) - a false situation method, or regula falsi. Such method converges only linearly.

At generalisation C the m. on a case of system of the equations is possible a double sight at the iterative formula (1). It is possible to consider that it is received from the formula of a method of Newton by discrete approximation of a derivative. Other possibility - to consider that for f (x) linear interpolation on points is made $f(x_1) \neq 0$ and $f(x_1) \neq 0$ and for the $f(x_1) \neq 0$ zero linear интерполянты is taken. Both interpretations allow to receive a considerable quantity of multidimensional analogues C m.; nek-rye from them (but not all) have the same order (local) convergence $f(x_1) \neq 0$ (see [3]).

Lecture № 5.

Algorithmization of the numerical decision of system of the algebraic and transcendental equations. A method of Gaussa.

The plan:

1. The decision of system of the linear equations a method of Gaussa
2. A method of Gaussa with a choice of the main element
3. An error estimation at the decision of system of the linear equations

1. The decision of system of the linear equations a method of Gaussa

Problems of approximation of function, and also set of other problems of applied mathematics of m of computing physics are reduced to problems about the decision of systems of the linear equations. The most universal method of the decision of system of the linear equations is the method of a consecutive exception of the unknown persons, Gaussa named a method.

For an illustration of sense of a method of Gaussa we will consider system of the linear equations:

$$\begin{cases} 4x_1 - 9x_2 + 2x_3 = 2 \\ 2x_1 - 4x_2 + 4x_3 = 3 \\ -x_1 + 2x_2 + 2x_3 = 1 \end{cases} \qquad \textbf{(1)}$$

This system we will write down in a matrix kind:

$$\begin{pmatrix} 4 & -9 & 2 \\ 2 & -4 & 4 \\ -1 & 2 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} \qquad \textbf{(2)}$$

As it is known, both members of equation it is possible to increase by nonzero number, and also it is possible to subtract another from one equation. Using these properties, we will try to result a matrix of system (2) in a triangular kind, i.e. to a kind, when below the main diagonal all elements – zero. This stage of the decision is called as a forward stroke.

On the forward stroke first step we will increase the first equation on 1/2 and we will subtract from the second then the variable will be excluded from the x_1 second equation. Then, we will increase the first equation on-1/4 and we will subtract from the third then the system (2) will be transformed to kind system:

$$\begin{pmatrix} 4 & -9 & 2 \\ 0 & 0.5 & 3 \\ 0 & -0.25 & 2.5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 1.5 \end{pmatrix} \qquad \textbf{(3)}$$

On the second step of a forward stroke from the third equation it is excluded x_2, i.e. from the third equation it is subtracted the second, increased, on-1/2 that results system (3) in a triangular kind (4)

$$\begin{pmatrix} 4 & -9 & 2 \\ 0 & 0.5 & 3 \\ 0 & 0 & 4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 2.5 \end{pmatrix} \qquad \textbf{(4)}$$

System (4) it is copied in a habitual kind:

$$\begin{cases} 4x_1 - 9x_2 + 2x_3 = 2 \\ 0.5x_2 + 3x_3 = 2 \\ 4x_3 = 2.5 \end{cases} \qquad \textbf{(5)}$$

Now, from system (5) we can find the decision upside-down, i.e. at first we find from the third equation $x_3 = 0.625$, further, substituting in the second equation, we find $x_2 = \frac{2 - 3x_3}{0.5} = 0.25$. Substituting and x_2 in the x_3 first equation of system (5), we find $x_1 = 0.75$. A finding of the decision from (x_1, x_2, x_3) system (5) name reverse motion.

Now, on the basis of the considered example, we will make the general algorithm of a method of Gaussa for system:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n = b_2 \\ \qquad \qquad \cdots \\ a_{n1}x_1 + a_{n2}x_2 + ... + a_{nn}x_n = b_n \end{cases} \quad \textbf{(6)}$$

The method of Gaussa consists of two stages:

The forward stroke – when a matrix of system (6) is led to a triangular kind;

Reverse motion – when unknown persons upside-down, i.e. in sequence are consistently calculated: $x_n, x_{n-1}, x_{n-2}, ..., x_1$.

Forward stroke: for reduction of system (6) to a triangular kind, the equations with nonzero factors at a variable are rearranged x_1 so that they were above, than the equations with zero factors a_{i1}. Further, we subtract the first equation multiplied on a_{21}/a_{11}, from the second equation, we subtract the first equation multiplied on a_{31}/a_{11}, from the third equation etc. in general, we subtract the first equation multiplied on a_{i1}/a_{11}, from *i* - го the equations at $i = \overline{2,n}$, if $a_{i1} \neq 0$. Owing to this procedure, we have nulled all factors at a variable in x_1 each of the equations, since the second, i.e. the system (6) becomes:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n = b_1 \\ \qquad a'_{22}x_2 + ... + a'_{2n}x_n = b'_2 \\ \qquad a'_{32}x_2 + ... + a'_{3n}x_n = b'_3 \\ \qquad \qquad \cdots \\ \qquad a'_{n2}x_2 + ... + a'_{nn}x_n = b'_n \end{cases} \quad \textbf{(7)}$$

Further, we apply тужe the procedure, to the equations of system (7), since the second equation, i.e. the first equation is excluded from "game". Now we try to null factors at a variable x_2, since the third equation etc., yet we will not lead system to a triangular kind. If $\det A \neq 0$, the system is always led (theoretically triangular kind. It is possible to present the general algorithm of a forward stroke in a kind:

$$\begin{cases} k = \overline{1, n-1} \\ i = \overline{k+1, n} \\ l_{ik} \leftarrow \dfrac{a_{ik}}{a_{kk}} \\ b_i \leftarrow b_i - l_{ik}b_k \\ j = \overline{1, n} \\ a_{ij} \leftarrow a_{ij} - l_{ik}a_{kj} \end{cases} \quad \textbf{(8)}$$

Reverse motion: we calculate unknown persons under formulas:

$$\begin{cases} x_n \leftarrow \dfrac{b_n}{a_{nn}} \\ k = n-1, n-2, ..., 1 \\ x_k \leftarrow \dfrac{\left(b_k - \sum\limits_{j=k+1}^{n} a_{kj}x_j\right)}{a_{kk}} \end{cases} \quad \textbf{(9)}$$

The remark: for calculation of a determinant of system it is possible to use the triangular form of the received matrix then the determinant of this matrix is equal to product of diagonal elements, i.e.

$$\det A = \prod_{i=1}^{n} a_{ii} \quad \textbf{(10)}$$

2. A method of Gaussa with a choice of the main element

The method of Gaussa is so universal that for some systems almost "bad" results turn out, various artful ways out therefore are developed. In a case when some factors of a matrix of system are close among themselves, as it is known relative errors strongly increase at subtraction, therefore the classical method of Gaussa gives the big errors. To bypass this difficulty, try to choose in a forward stroke of Gaussa that equation at which the factor at is maximum x_1 and as basic "player" choose this equation, thereby bypassing difficulties of subtraction of close numbers (if it is possible). Further, when it is necessary to null all factors of a variable x_2, except one equation – this special equation again choose that equation at which factor at maximum x_2 etc., yet we will not receive a triangular matrix.

Reverse motion occurs the same as and in a classical method of Gaussa.

3. An error estimation at the decision of system of the linear equations

To estimate errors of calculations of the decision of system of the linear equations, we need to enter concepts of corresponding norms of matrixes.

First of all, we will recollect three most often used norms for a vector $\vec{u}$:

$$\|\vec{u}\|_1 = \sum_{i=1}^{n} |u_i| \qquad \textbf{(11)}$$

$$\|\vec{u}\|_2 = \sqrt{\sum_{i=1}^{n} |u_i|^2} \quad \text{(Euklyde norm)} \qquad \textbf{(12)}$$

$$\|\vec{u}\|_3 = \lim_{p\to\infty} \sqrt[p]{\sum_{i=1}^{n} |u_i|^p} = \max_{1\le i\le n} |u_i| \quad \text{(Chebyshev norm)} \qquad \textbf{(13)}$$

For any norm of vectors it is possible to enter corresponding norm of matrixes:

$$\|A\| = \sup_{u\neq 0} \frac{\|Au\|}{\|u\|} = \sup_{\|u\|=1} \|Au\| \qquad \textbf{(14)}$$

Which is co-ordinated with norm of vectors in the sense that

$$\|Au\| \le \|A\| \cdot \|u\| \qquad \textbf{(15)}$$

It is possible to show that for three norms of a matrix resulted above cases are set A by formulas:

$$\|A\|_1 = \max_{1\le k\le n} \sum_{i=1}^{n} |a_{ik}| \qquad \textbf{(16)}$$

$$\|A\|_2 = \max_{1\le i\le n} \sigma_i \qquad \textbf{(17)}$$

$$\|A\|_T = \max_{1\le i\le n} \sum_{k=1}^{n} |a_{ik}| \qquad \textbf{(18)}$$

Where - σ_i are singular matrix numbers A, i.e. these are positive values of square roots - $\sqrt{\mu_i}$ matrixes ($A^T \cdot A$ which is the is positive-defined matrix, at $\det A \neq 0$).

For material symmetric matrixes - $\sigma_i = |\lambda_i|$ where - λ_i own numbers of a matrix A.

Absolute error of the decision of system:

$$Ax + b \qquad \textbf{(19)}$$

Where - a A system matrix, - the b matrix of the right parts, is estimated by norm:

$$\Delta = \|Ax - b\| \qquad \textbf{(20)}$$

The relative error is estimated under the formula:

$$\delta = \frac{\Delta}{\|\vec{x}\|} \qquad \textbf{(21)}$$

Where $\|\vec{x}\| \neq 0$.

http://matematikam.ru/solve-equations/sistema-gaus.php

Lecture № 6.

Algorithmization of the numerical decision of system of the algebraic and transcendental equations. Iterative methods of Jacoby and Seidel.

The plan:

1. Iterative methods of the decision of systems of the linear equations
2 Method of simple iteration of Jacoby
3. A method of Gaussa-Seidel

1. Iterative methods of the decision of systems of the linear equations

Let's consider system of the linear equations which badly dares methods of Gaussa. We will copy system of the equations in a kind:

$$x = Bx + c \qquad \textbf{(22)}$$

Where - the B set numerical matrix n of th order, - the $c \in R^n$ set constant vector.

2 Method of simple iteration of Jacoby

This method consists in the following: any vector ($x^0 \in R^n$ initial approach) gets out $x^0 \in R^n$ and the iterative sequence of vectors under the formula is under construction:

$$x^{(n)} = Bx^{(n-1)} + c, \; n \in N \qquad \textbf{(23)}$$

Let's result the theorem giving a sufficient condition of convergence of a method of Jacoby.

The theorem. If $\| B \| < 1$, the system of the equations (22) has the unique decision and $x = \xi$ iterations (23) converge to the decision.

It is easy to notice that this theorem is simple generalisation of the theorem of the compressed displays studied by us earlier for single-step iterative process in a general view. All estimations received earlier, are transferred and for system of the equations, a difference only in concepts of corresponding norms. Generalising a method of simple iteration of Jacoby for a case of system of the equations:

$$Ax = b \qquad \textbf{(24)}$$

We build algorithm of the decision:

We copy the equation (24) in a homogeneous kind and it is multiplied by a constant - λ which further we will find from conditions of convergence of iterative process:

$$\lambda \cdot (Ax - b) = 0 \qquad \textbf{(25)}$$

We add to x both parts (25) and it is received:

$$x = x + \lambda(Ax - b) = \varphi(x, \lambda) \qquad \textbf{(26)}$$

We build the iterative formula of Jacoby:

$$x^{(n+1)} = x^{(n)} + \lambda(Ax^{(n)} - b) \qquad \textbf{(27)}$$

Where a constant it is found λ from conditions of convergence of iterative process (27) which in this case looks like:

$$\left\| \varphi'_x (x^{(0)}, \lambda) \right\| < 1 \qquad \textbf{(28)}$$

Where - a $\varphi(x, \lambda)$ vector function from (26) or proceeding from the theorem of the compressed displays $\| I + \lambda A \| < 1$, where *I* - an individual matrix.

Let's consider a numerical **example:**

Let we have system of the equations:

$$\begin{cases} x_1 + 3x_2 + 4x_3 = 1 \\ 2x_1 + 3x_2 - 2x_3 = 2 \\ 3x_1 + 4x_2 + 5x_3 = 3 \end{cases}$$

We copy system in a kind:

$$\begin{cases} \lambda_1(x_1 + 3x_2 + 4x_3 - 1) + x_1 = x_1 \\ \lambda_2(2x_1 + 3x_2 - 2x_3 - 2) + x_2 = x_2 \\ \lambda_3(3x_1 + 4x_2 + 5x_3 - 3) + x_3 = x_3 \end{cases}$$

We make the iterative formula:

$$\begin{cases} x_1^{(n+1)} = x_1^{(n)} + \lambda_1(x_1^{(n)} + 3x_2^{(n)} + 4x_3^{(n)} - 1) \\ x_2^{(n+1)} = x_2^{(n)} + \lambda_2(2x_1^{(n)} + 3x_2^{(n)} - 2x_3^{(n)} - 2) \\ x_3^{(n+1)} = x_3^{(n)} + \lambda_3(3x_1^{(n)} + 4x_2^{(n)} + 5x_3^{(n)} - 3) \end{cases}$$

The factor is chosen λ_i from conditions: i.e $\| E + \lambda A \| < 1$.

$$\begin{cases} m_1 = |1 + \lambda_1| + 3|\lambda_1| + 4|\lambda_1| < 1 \\ m_2 = 2|\lambda_2| + |1 + 3\lambda_2| + 2|\lambda_2| < 1 \\ m_3 = 3|\lambda_3| + 4|\lambda_3| + |1 + 5\lambda_3| < 1 \end{cases}$$

$$\max(m_1, m_2, m_3) < 1.$$

3. A method of Gaussa-Seidel

The set of iterative methods is developed for the decision of linear system of the equations. As the method of simple iteration of Jacoby converges slowly. One of such methods is the method of Gaussa-Seidel.

For a method illustration we will consider a numerical **example:**

$$\begin{cases} 2x - y + z = 5 \\ x + 3y - 2z = 7 \\ x + 2y + 3z = 10 \end{cases} \qquad \textbf{(29)}$$

The equations are copied in such a manner that on the main diagonal there are maximum factors for each equation.

We begin with approach $x = y = z = 0$. Using the first equation, we find for new x value under a x_1 condition $y = z = 0$.

$$x_1 = \frac{5 + y - z}{2} = \frac{5}{2} \qquad \textbf{(30)}$$

Taking this value and $x = x_1 = 2{,}5$ from the $z = 0$ second equation, we find $y_1 = \frac{7 + 2z - x}{3} = \frac{3}{2}$, further from the third equation it is found. z_1 $z_1 = \frac{10 - x - 2y}{3} = \frac{3}{2}$ These three sizes give new approach and it is possible to cycle a loop from the beginning, we receive: etc $x_2 = \frac{5}{2}$ $y_2 = \frac{5}{2}$ $z_2 = \frac{5}{6}$. Iterations proceed before inequality performance $\left\| x^{(i+1)} - x^{(i)} \right\| < \varepsilon$.

The general algorithm of a method of Gaussa-Zejdelja looks like:

Let

$$Ax = b \qquad \textbf{(31)}$$

Where at matrix *A* - all diagonal elements are distinct from zero, i.e. ($a_{ii} \neq 0$ if then $\exists\, a_{ii} = 0$ we rearrange a line so that to achieve a condition $a_{ii} \neq 0$). If i th equation of system (31) to divide on a_{ii}, and then all unknown persons except - x_i to transfer to the right part we will come to equivalent system of a kind:

$$x = Cx + D \qquad \textbf{(32)}$$

where $D = (d_1, d_2, ..., d_n)$, $d_i = \frac{b_i}{a_{ii}}$, $C = (C_{ij})$

$$C_{ij} = \begin{cases} -\frac{a_{ij}}{a_{ii}}, \text{if } i \neq j \\ 0, \text{ if } i = j \end{cases} \qquad \textbf{(33)}$$

The method of Gaussa-Zejdel consists that iterations are made under the formula:

$$x_i^{(k+1)} = \sum_{j=1}^{i-1} C_{ij} x_j^{(k+1)} + \sum_{j=i+1}^{n} C_{ij} x_j^{(k)} + d_i \qquad \textbf{(34)}$$

Where - k iteration number, and $i = \overline{1, n}$.

The remark: for convergence of a method (34) enough performance at least one of conditions:

a)

$$\sum_{j=1,\ j \neq i}^{\mathrm{n}} \left| a_{ij} \right| < \left| a_{ii} \right|, \quad i = \overline{1, n} \qquad \textbf{(35)}$$

- The A symmetric and is positive-defined matrix.

Lecture № 7.
Algorithmization interpolation methods. Interpolation functions.

The plan:

1. Introduction
2. The first interpolation Newton's formula
3. The second interpolation Newton's formula
4. The interpolation formula of Stirlinga
5. An example

1. Introduction

Interpolation – operation of approach of the function set in separate points in some set interval. The elementary problem of interpolation consists in the following. On a piece [*a*, *b*] are set *n* +1 points x_i (*i* = 0, 1, 2, …, *n*), *interpolation* named *in the knots*, and values of some functions $f(x)$ in these points. It is required $f(x_0)=y_0\,, f(x_1)=y_1\,,\ldots, f(x_n)=y_n\,.$ to construct the *interpolating* function accepting $F(x)$ in knots of interpolation the same values, as $f(x)$, i.e. $F(x_0)=y_0\,, F(x_1)=y_1\,,\ldots, F(x_n)=y_n\,.$ Geometrically it means (fig. 1) that it is required to find some curve $y=F(x)$ of the certain type passing through the set set of points $(x_i\,, y_i),\ i=0\,,1\,,2\,,\ldots, n$.

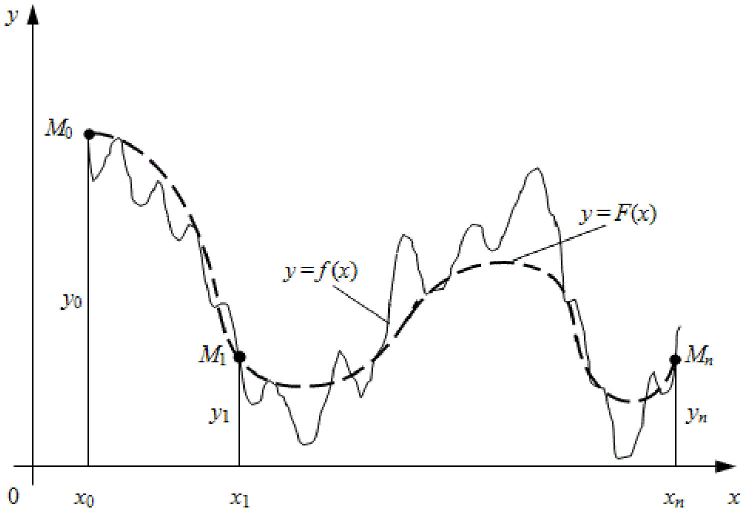


Fig. 1. Geometrical representation of interpolation of function

In such statement the interpolation problem, generally speaking, can have or uncountable set of decisions, or not have at all decisions. However a problem it becomes unequivocal разрешимой if instead of any function to search *F(x)* for a degree polynom $P_n(x)$ not above *n*, satisfying to conditions:

$$P_n(x_0)=y_0\,, P_n(x_1)=y_1\,,\ldots, P_n(x_n)=y_n\,. \qquad (1)$$

Received interpolation formula *y=F(x)* use for the approached calculation of values given функции *f(x)* for those *x* which are distinct from interpolation knots. Such operation is called as function *interpolation f(x).*

2. The first interpolation formule Newton.

Let in equidistant points $x_i=x_0+i\cdot h\ \ (i=0\,,1\,,2\,,\ldots, n)$, where *h* – a *step of interpolation*, preset values for $y_i=f(x_i)$ function *y=f(x)*. It is required to pick up a degree polynom $P_n(x)$ not above *n* satisfying to conditions (1). We will enter final differences for sequence of values $y_i\,,\ i=0\,,1\,,2\,,\ldots, n$:

$$\Delta y_i = y_{i+1}-y_i\,,$$
$$\Delta^2 y_i=\Delta(\Delta y_i)=\Delta y_{i+1}-\Delta y_i\,,$$
$$.\ \ .\ \ .\ \ .\ \ .\ \ .\ \ .\ \ .\ \ .\ \ .\ \ .$$
$$\Delta^n y_i=\Delta(\Delta^{n-1}y_i)=\Delta^{n-1}y_{i+1}-\Delta^{n-1}y_i\,. \qquad (2)$$

Conditions (1) are equivalent to equalities:

$$\Delta^m P_n(x_0)=\Delta^m y_0$$

At *m=0,1,2,…,n*

Lowering the calculations resulted in [1], we will definitively receive the *first interpolation formule Newton*:

$$P_n(x) = y_0 + q\Delta y_0 + \frac{q(q-1)}{2!}\Delta^2 y_0 + \ldots + \frac{q(q-1)\ldots(q-n+1)}{n!}\Delta^n y_0 \,, \qquad (3)$$

Where – $q = \frac{x - x_0}{h}$ number of steps of interpolation from an index point to a x_0 point x.

The formula (3) is expedient for using for function interpolation in an $y=f(x)$ index point vicinity where x_0 q on absolute size it is not enough.

In special cases it is had:

At n = 1 – the formula *of linear interpolation*:

$$P_1(x) = y_0 + q\Delta y_0 \,;$$

at n = 2 – the formula *of square-law* or *parabolic interpolation*:

$$P_2(x) = y_0 + q\Delta y_0 + \frac{q(q-1)}{2}\Delta^2 y_0 \,.$$

3. The second interpolation formule Newton

The first interpolation formule Newton is almost inconvenient for interpolation functions near to the table end. In this case usually apply the *second* interpolation formule Newton:

$$P_n(x) = y_n + q\Delta y_{n-1} + \frac{q(q+1)}{2!}\Delta^2 y_{n-2} + \frac{q(q+1)(q+2)}{3!}\Delta^3 y_{n-3} + \ldots + \frac{q(q+1)\ldots(q+n-1)}{n!}\Delta^n y_0 \,. \qquad (4)$$

The detailed conclusion of the formula (4) is resulted in [1].

Let's notice that if $x<x_0$ and x it is close to x_0 it makes sense to apply the first interpolation formule Newton if $x>x_n$ and x it is close to x_n in this case is more convenient for using the second interpolation formule Newton. In other words, the first interpolation formule Newton is used usually for interpolation *forward*, and the second interpolation formule Newton – for interpolation *back*.

4. The interpolation formule of Stirlinga

The interpolation formule of Stirlinga looks like:

$$\begin{aligned} P_{2n}(x) = {} & y_0 + q\cdot\frac{\Delta y_{-1} + \Delta y_0}{2} + \frac{q^2}{2!}\cdot\Delta^2 y_{-1} + \frac{q(q^2-1)}{3!}\cdot\frac{\Delta^3 y_{-2} + \Delta^3 y_{-1}}{2} + \\ & + \frac{q^2(q^2-1)}{4!}\cdot\Delta^4 y_{-2} + \frac{q(q^2-1)(q^2-2^2)}{5!}\cdot\frac{\Delta^5 y_{-3} + \Delta^5 y_{-2}}{2} + \\ & + \frac{q^2(q^2-1)(q^2-2^2)}{6!}\cdot\Delta^6 y_{-3} + \ldots + \frac{q(q^2-1)(q^2-2^2)(q^2-3^2)\ldots[q^2-(n-1)^2]}{(2n-1)!}\times \\ & \times\frac{\Delta^{2n-1} y_{-n} + \Delta^{2n-1} y_{-(n-1)}}{2} + \frac{q^2(q^2-1)(q^2-2^2)\ldots[q^2-(n-1)^2]}{(2n)!}\Delta^{2n} y_{-n} \,, \end{aligned} \qquad (5)$$

where, as before $q = \frac{x - x_0}{h}$.

There is also a number of others interpolationally formulas: Gauss, Bessel, and so forth the Formula (5) is deduced by Lagrange with use of the first and the second interpolation formulas of Gaussa [1].

5. An example

The table of values of full elliptic integral is set

$$K(\alpha) = \int_0^{\pi/2} \frac{dx}{\sqrt{1 - \sin^2\alpha\cdot\sin^2 x}} \,,$$

To find *K* (78º 30 ’).

Values of full elliptic integral K (α)

α	*K*(*α*)	*ΔK*	*Δ2K*	*Δ3K*	*Δ4K*	*Δ5K*	*Δ6K*
75º	2.76806						
		6461					
76º	2.83267		528				
		6989		84			
77º	2.90256		612		19		
		7601		103		13	

78º	2.97857		715		32		– 5
		8316		135		8	
79º	3.06173		850		40		18
		9166		175		26	
80º	3.15339		1025		66		– 1
		10191		241		25	
81º	3.25530		1266		91		43
		11457		332		68	
82º	3.36987		1598		159		
		13055		491			
83º	3.50042		2089				
		15144					
84º	3.65186						

The decision. According to the table data it is accepted x0 = 78; h=1; x=78º 30 ’, from here q = 0.5. Being limited to differences of the fifth order, under the formula of Stirlinga it is had:

$$K(78^\circ 30') = 2.97857 + 0.5\frac{7601+8316}{2}\cdot 10^{-5} + 0.125\cdot 715\cdot 10^{-5} - 0.0625\frac{103+135}{2}\cdot 10^{-5} -$$

$$-0.0078\cdot 32\cdot 10^{-5} + 0.0117\frac{13+8}{2}\cdot 10^{-5} = 3.019181.$$

Lecture № 8.

The Numerical decision of the differential equations. Euler's method.

The plan:

1. Types of problems for the ordinary differential equations
2. Euler's method

1. Types of problems for the ordinary differential equations

The differential equations arise in many areas of applied mathematics, physics, mechanics, technicians etc. With their help are described almost any problems of dynamics of cars and mechanisms (sections of the dynamic analysis of hydraulic systems, drives and transmissions, control systems see, for example, our site). There is a set of methods of the decision of the differential equations through elementary or special functions. However, more often these methods either are absolutely not applicable, or lead to so difficult decisions that it is easier and more expedient to use the approached numerical methods. The differential equations contain in a large quantity of problems essential nonlinearity, and functions entering into them and factors are set in the form of tables and-or experimental data that actually completely excludes possibility of use of classical methods for their decision and the analysis.

Now there is a set of various numerical methods of the decision of the ordinary differential equations (for example, Euler, Runge-Kutta, Milne, Adams, Gere, etc.) [1 – 6]. We will be limited here to consideration of methods of Euler most widely used in practice and Runge-Kutta. As to other mentioned methods they are in detail stated in the literature, see, for example: [1, 4] – Milne's method, [1, 3, 5] – Adams's method, [5, 6] – Gere's method. We also do not stop here on questions of stability of computing processes, they are in detail shined in the corresponding literature [4, 5, 7].

2. Euler's method

Let's consider the differential equation

$$y' = f(x, y) \qquad (1)$$

With the entry condition

$$y(x_0) = y_0 .$$

Having substituted x_0, y_0 in the equation (1), we will receive value of a derivative in a point x_0 :

$$y'|_{x=x_0} = f(x_0, y_0).$$

At the small Δx takes place:

$$y(x_0 + \Delta x) = y(x_1) = y_0 + \Delta y = y_0 + y'|_{x=x_0} \cdot \Delta x = y_0 + f(x_0, y_0) \cdot \Delta x.$$

Having designated $f(x_0, y_0) = f_0$, we will copy last equality in a kind:

$$y_1 = y_0 + f_0 \cdot \Delta x. \qquad (2)$$

Accepting now (x_1, y_1) for a new starting point, precisely also we will receive:

$$y_2 = y_1 + f_1 \cdot \Delta x.$$

Let's have generally:

$$y_{i+1} = y_i + f_i \cdot \Delta x. \qquad (3)$$

It also is *Euler's method*. The size Δx is called *as integration step*. Using this method, we receive the approached values *y* as the derivative y' actually does not remain to a constant on an interval in length Δx. Therefore we receive an error in definition of value of function *y*, that big, than it is more Δx. Euler's method is the elementary method of numerical integration of the differential equations and systems. Its lacks – small accuracy and regular accumulation of errors.

More exact is *Euler's modified method* or *Euler's method with recalculation*. Its essence that at first under the formula (3) find so-called «rough approach»:

$$\tilde{y}_{i+1} = y_i + f_i \cdot \Delta x ,$$

And then recalculation $\tilde{f}_{i+1} = f(x_{i+1}, \tilde{y}_{i+1})$ receive too approached, but more exact value:

$$y_{i+1} = y_i + \frac{f_i + \tilde{f}_{i+1}}{2} \cdot \Delta x. \qquad (4)$$

Фактически пересчет позволяет учесть, хоть и приблизительно, изменение производной y' на шаге интегрирования Δx, так как учитываются ее значения f_i в начале и $\tilde{f}_{i+1}$ в конце шага (рис. 1), а затем берется их среднее. Метод Эйлера с пересчетом (4) является по существу методом Рунге-Кутта 2-го порядка [2], что станет очевидным из дальнейшего.

Actually recalculation allows to consider, though and approximately, derivative change y' on an integration step as Δx its values f_i in the beginning and $\tilde{f}_{i+1}$ in the end of a step (fig. 1) are considered, and then undertakes their average. Euler's method with recalculation (4) is in essence a method of Runge-Kutta of 2nd order [2] that becomes obvious of further.

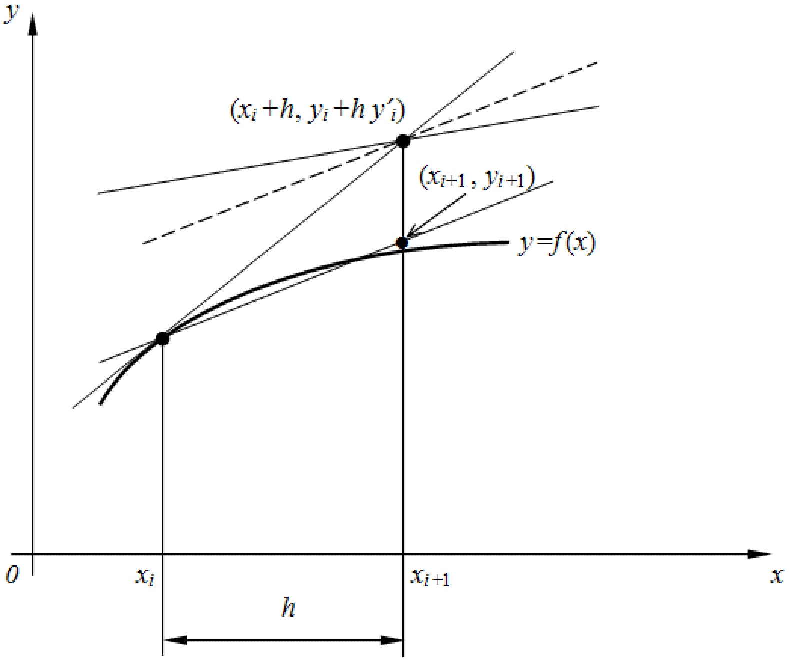


Fig. 1. Geometrical representation of a method of Euler with recalculation.

Lecture № 9.

The numerical decision of the differential equations. A method of Runge-Kutta and Adams.

The plan:

1. Methods of Runge-Kutta
2. Adams's method

1. A method of Runge-Kutta

Again we will consider the differential equation

$$y' = f(x, y) \qquad (1)$$

With the entry condition $y(x_0) = y_0$.

The classical *method of Runge-Kutta* of 4th order is described by the following system of five equalities:

$$y_{i+1} = y_m + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4), \qquad (5)$$

Where

$$k_1 = f(x_i, y_i),$$

$$k_2 = f(x_i + \frac{h}{2}, y_i + \frac{hk_1}{2}),$$

$$k_3 = f(x_i + \frac{h}{2}, y_i + \frac{hk_2}{2}),$$

$$k_4 = f(x_i + h, y_i + hk_3).$$

Strictly speaking, there is not one, and group of methods of Runge-Kutta different from each other rather, i.e. quantity of parameters k_j. In this case we have a method of 4th order which is one of the most put into practice as provides a split-hair accuracy and at the same time differs comparative simplicity. Therefore in most cases it is mentioned in the literature simply as «a method of Runge-Kutta» without instructions of its order.

Example.

To calculate a method of Runge-Kutta integral of the differential equation *y'=x+y* at the entry condition *y(0)=1* on a piece [0, 0.5] with integration step *h=0.1*.

The decision. We will calculate y_1. For this purpose at first it is consistently calculated k_j:

$$k_1 = x_0 + y_0 = 0 + 1 = 1;$$

$$k_2 = x_0 + \frac{h}{2} + y_0 + \frac{hk_1}{2} = (0 + 0.05) + (1 + 0.05) = 1.1;$$

$$k_3 = x_0 + \frac{h}{2} + y_0 + \frac{hk_2}{2} = (0 + 0.05) + (1 + 0.055) = 1.105;$$

$$k_4 = x_0 + h + y_0 + hk_3 = (0 + 0.1) + (1 + 0.1105) = 1.2105.$$

Now we will receive

$$\Delta y_0 = \frac{0.1}{6}(1 + 2 \cdot 1.1 + 2 \cdot 1.105 + 1.2105) = 0.1103$$

And, hence,

$$y_1 = y_0 + \Delta y_0 = 1 + 0.1103 = 1.1103.$$

The subsequent are similarly calculated approach. Results of calculations are tabulated:

Results of numerical integration of the differential equation (1) method of Runge-Kutta of the fourth order

i	*x*	*y*	***k*** = 0.1 (*x* + *y*)	**Δ*y***
0	1	**1**	1	0.1
	0.05	1.05	1.1	0.22
	0.05	1.055	1.105	0.221
	0.1	1.1105	1.210	0.1210

				1/6 · 0.6620=**0.1103**
1	0.1	**1.1103**	1.210	0.1210
	0.15	1.1708	1.321	0.2642
	0.15	1.1763	1.326	0.2652
	0.2	1.2429	1.443	0.1443
				1/6 · 0.7947=**0.1324**
2	0.2	**1.2427**	1.443	0.1443
	0.25	1.3149	1.565	0.3130
	0.25	1.3209	1.571	0.3142
	0.3	1.3998	1.700	0.1700
				1/6 · 0.9415=**0.1569**
3	0.3	**1.3996**	1.700	0.1700
	0.35	1.4846	1.835	0.3670
	0.35	1.4904	1.840	0.3680
	0.4	1.5836	1.984	0.1984
				1/6 · 1.1034=**0.1840**
4	0.4	**1.5836**	1.984	0.1984
	0.45	1.6828	2.133	0.4266
	0.45	1.6902	2.140	0.4280
	0.5	1.7976	2.298	0.2298
				1/6 · 1.2828=**0.2138**
5	0.5	**1.7974**		

So, $y(0.5)$ =1.7974.

For comparison the exact decision of the differential equation (1):

$$y = 2e^{x} - x - 1,$$

Whence $y(0.5) = 2\sqrt{e} - 0.5 - 1 = 1.79744...$

Thus, exact and numerical decisions of the equation (1) have coincided to the fifth decimal sign.

The method of Runge-Kutta also is widely applied to the numerical decision of systems of the ordinary differential equations.

2. Adams's method

Adams's method is applied both to the decision of the simple differential equations, and for their systems.

Problem statement

Adams's method to find the decision of system of the equations on a piece [0; 1] with accuracy ε =$10^{-4.}$

$$\begin{cases} y'(x) = cy(x) - z(x), \\ z'(x) = y(x) - dz(x), \end{cases}$$

$$y(a) = k, \quad z(b) = n$$

Where c, d, k, n – the set constants

The decision of systems of the ordinary differential equations Adams's method

In the given system of the equations will substitute values of factors and entry conditions. We will receive

$$\begin{cases} y' = 2y - z \\ z' = y - 4z \end{cases} \quad y(0) = 3, \quad z(0) = -2$$

Adams's method we will find the decision of this system on the set piece. For this purpose we will calculate a method of Runge-Kutta some initial values of function.

Let's choose a step h and, for brevity, we will enter $x_i = x_0 + ih$ и $y_i = y(x_i)$ $(i = 0, 1, 2, \dots)$

Let's consider numbers:

$$\begin{cases} k_1^{(i)} = hf(x_i, y_i) \\ k_2^{(i)} = hf\left(x_i + \frac{h}{2}, y_i + \frac{k_1^{(i)}}{2}\right) \\ k_3^{(i)} = hf\left(x_i + \frac{h}{2}, y_i + \frac{k_3^{(i)}}{2}\right) \\ k_4^{(i)} = hf(x_i + h, y_i + k_3^{(i)}) \end{cases}$$

According to a method of Runge-Kutta consecutive values y_i are defined under the formula

$$y_{i+1} = y_i + \Delta y_i$$

Where

$$\Delta y_i = \frac{1}{6}\left(k_1^{(i)} + 2 \cdot k_2^{(i)} + 2 \cdot k_3^{(i)} + k_4^{(i)}\right) (i = 0, 1, 2, \dots). \quad (2.1)$$

Having substituted in these formulas initial values we will receive

$x_0 = 0$ $y_0 = 3$ $z_0 = -2$

$x_1 = 0{,}1$ $y_1 = 3{,}3672$ $z_1 = -2{,}1586$

$x_2 = 0{,}2$ $y_2 = 3{,}4944$ $z_2 = -2{,}0867$

$x_3 = 0{,}3$ $y_3 = 3{,}5964$ $z_3 = -1{,}9906$

Further calculation it is continued on Adams's method. All calculations it is written down in tables 2.1 and 2.2.

Table 2.1

k	x_k	y_k	Δy_k	p_k	Δp_k	$\Delta^2 p_k$	$\Delta^3 p_k$	z_k	Δz_k	q_k	Δq_k	$\Delta^2 q_k$	$\Delta^3 q_k$
0	0	3		0,8000	0,0893	-0,0711	0,0636	-2		1,1000	0,1002	-0,1162	0,1040
1	0,1	3,3672		0,8893	0,0183	-0,0075	0,0680	-2,1586		1,2002	-0,0160	-0,0122	-0,3354
2	0,2	3,4944		0,9076	0,0108	0,0605	0,0512	-2,0867		1,1841	-0,0282	-0,3476	0,7024
3	0,3	3,5964	0,9445	0,9183	0,0713	0,1117	-0,1448	-1,9906	1,1757	1,1559	-0,3758	0,3548	-0,6647
4	0,4	4,5409	1,0761	0,9897	0,1831	-0,0330	0,1605	-0,8149	0,3215	0,7801	-0,0210	-0,3099	0,8201
5	0,5	5,6169	1,3300	1,1727	0,1500	0,1275	-0,1562	-0,4934	1,1598	0,7590	-0,3309	0,5102	-0,9910
6	0,6	6,9469	1,3297	1,3227	0,2775	-0,0288	0,2023	0,6664	-0,1157	0,4281	0,1793	-0,4809	1,1396
7	0,7	8,2766	1,8523	1,6003	0,2488	0,1735	-0,2240	0,5507	1,2171	0,6074	-0,3016	0,6587	-1,3700
8	0,8	10,1290	1,9028	1,8490	0,4223	-0,0505		1,7678	-0,4170	0,3058	0,3571	-0,7113	
9	0,9	12,0318	2,6306	2,2713	0,3718			1,3508	1,5432	0,6629	-0,3542		
10	1	14,6623	2,7239	2,6431				2,8940	-0,6786	0,3086			

Table 2.2

k	x	y	y'	z	z'
0	0	3	8	-2	11
1	0,1	3,3672	8,893	-2,1586	12,0016
2	0,2	3,4944	9,0755	-2,0867	11,8412
3	0,3	3,5964	9,1834	-1,9906	11,5588
4	0,4	4,5409	9,8967	-0,8149	7,8005
5	0,5	5,6169	11,7272	-0,4934	7,5905
6	0,6	6,9469	13,2274	0,6664	4,2813
7	0,7	8,2766	16,0025	0,5507	6,0738
8	0,8	10,129	18,4902	1,7678	3,0578
9	0,9	12,0318	22,7128	1,3508	6,6286

(1.3) values received under the formula are necessary for specifying, having calculated them under the formula (1.4). The obtained data we will write down in the table.

Table 2.3

k	x	Δy_k	$\Delta y_k^{кор.}$	Δz_k	$\Delta z_k^{кор.}$
0	0				
1	0,1				
2	0,2				
3	0,3	0,9445	0,946075	1,1757	1,010942

4	0,4	1,0761	1,069808	0,3215	0,710767
5	0,5	1,3300	1,256483	1,1598	0,647071
6	0,6	1,3297	1,444138	-0,1157	0,441063
7	0,7	1,8523	1,733608	1,2171	0,537967
8	0,8	1,9028	2,037263	-0,4170	0,381975
9	0,9	2,6306	2,470742	1,5432	0,602158
10	1	2,7239	2,6431	-0,6786	0,3086

Lecture №10.
Numerical integration. Quadrature formulas of trapezes and rectangles. Simpson's formula.

The plan:

1. **Classification of methods**
2. **A method of trapezes**
3. **Methods of rectangles**
4. **Simpson's method**

1. Classification of methods

It is known that the certain integral of function *f (x)* type $\int_a^b f(x)dx$ (1) numerically represents the area of a curvilinear trapeze limited to curves x=0, y=a, y=b and y = *f (x)* (fig. 1). There are two methods of calculation of this area or certain integral — a method of trapezes (fig. 2) and a method of average rectangles (fig. 3).

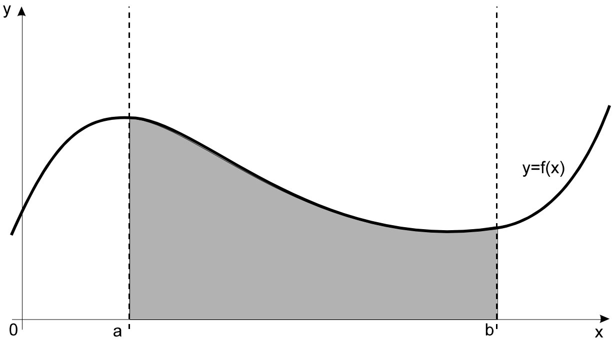


Fig. 1 curvilinear trapeze.

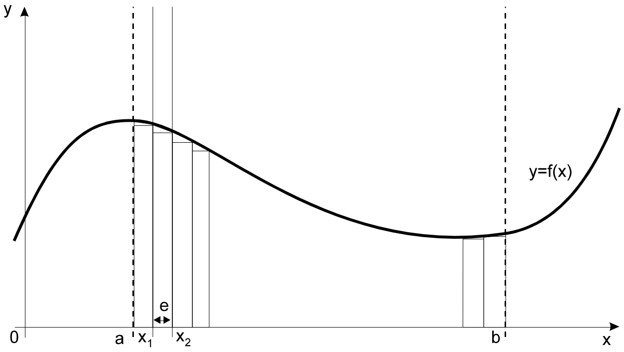


Fig. 2. Method of trapezes.

2. Метод трапеций

The size of certain integral is numerically equal to the area of the figure formed by the schedule of function and an axis of abscisses (geometrical sense of certain integral). Hence, to find it $\int_a^b f(x)dx$ means to estimate the area of the figure limited to perpendiculars, restored to the schedule of subintegral function *f(x)* from points an and b, located on an argument axis x.

We will break an interval [a, b] on n identical sites for the problem decision. The length of each site will be equal *h = (b-a)/n* (fig. 4).

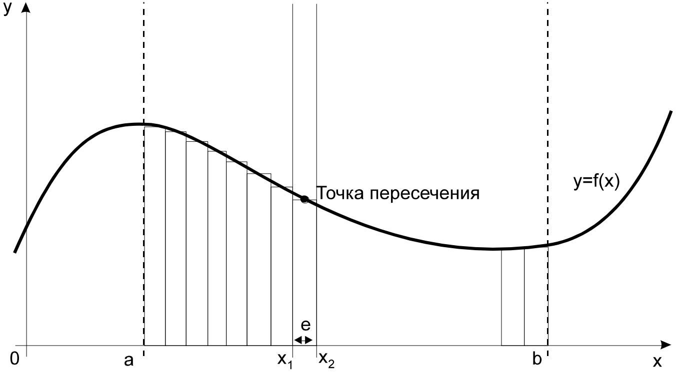


Fig. 3 Method of average rectangles.

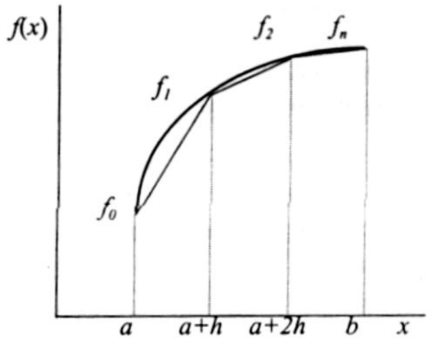


Fig. 4. Interval splitting [a, b] on n identical sites

Let's restore perpendiculars from each point before crossing with the function schedule *f (x)*. If to replace the received curvilinear fragments of the schedule of function with pieces of straight lines, then approximately the figure area and consequently also the size of certain integral is estimated as the area of all received trapezes. We will designate consistently values of subintegral functions on the ends of pieces f_0, f_1, f_2..., f_n also we will count up the area of trapezes

$$S = \frac{f_0 + f_1}{2} \cdot h + \frac{f_1 + f_2}{2} \cdot h + \frac{f_2 + f_3}{2} \cdot h + \ldots + \frac{f_{n-1} + f_n}{2} \cdot h =$$

$$= h\left(\frac{f_0}{2} + \frac{f_1}{2} + \frac{f_1}{2} + \frac{f_2}{2} + \frac{f_2}{2} + \frac{f_3}{2} + \ldots \frac{f_{n-1}}{2} + \frac{f_n}{2}\right) =$$

$$= h\left(\frac{f_0 + f_n}{2} + f_1 + f_2 + \ldots + f_{n-1}\right). \tag{2}$$

Generally the formula of trapezes becomes

$$\int_a^b f(x)dx \approx h\left(\frac{f_0 + f_n}{2} + \sum_{i=2}^{n-1} f_i\right) = \frac{b-a}{n}\left(\frac{f_0 + f_n}{2} + \sum_{i=2}^{n-1} f_i\right), \tag{3}$$

Where f_i - value of subintegral function in points of splitting of an interval (a, b) on equal sites with step *h;* f_0, f_n - values of subintegral function accordingly in points a and b.

The formula of trapezes with constant step:

$$\int_a^b f(x)dx \approx \frac{1}{2} h \sum_{i=0}^{n-1} (y_i + y_{i-1}) = \frac{1}{2} h(y_0 + y_n + 2\sum_{i=1}^{n-1} y_i) \tag{4}$$

3. A method of rectangles

The elementary methods of numerical integration are methods of rectangles. In them subintegral function is replaced with a polynom of zero degree, that is a constant. Similar replacement is ambiguous as the constant can be chosen subintegral function equal to value in any point of an interval of integration. Depending on it methods of rectangles share on: methods of the left, right and average rectangles.

On a method of average rectangles the integral is equal to the sum of the areas of rectangles where the rectangle basis any small size (accuracy), and the height is defined on a point of intersection of the top basis of a rectangle which the function schedule should cross in the middle. Accordingly we receive the formula of the areas for a method of average rectangles:

$$S_b = \sum_a^b \frac{\left| f(x_1) + (fx_2) \right|}{2} \varepsilon \tag{5}$$

The formula of average rectangles with constant step:

$$\int_a^b f(x)dx \approx \frac{1}{2} h \sum_{i=0}^{n-1} f\left(x_i + \frac{h}{2}\right) \tag{6}$$

4. Simpson's (Parabolas) formula

Simpson's rule – one of widest known and applied methods of numerical integration. It is similar to a rule of trapezes as also is based on splitting of the general interval of integration into smaller pieces. However its difference that for area calculation through each three consecutive ordinates of splitting the square parabola is spent. Lowering needless details and calculations we will result a definitive kind *of the formula of Simpson* [3, 4]:

$$I \approx \frac{h}{3}(y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + \ldots + 2y_{n-2} + 4y_{n-1} + y_n). \tag{6}$$

Here n - even number. This formula is much more exact than the formula of trapezes. So, at integration of multinomials of degree not above Simpson's third method gives exact values of integral.

Examples

Let's consider probability integral:

$$I = \int_{-2}^{2} e^{-\frac{x^2}{2}} dx.$$

Exact value of integral of probability to the fifth significant figure equally 2.3925.

Example 1. To calculate integral of probability a method of trapezes with step h = 1.0, 0.5, 0.25.

The decision. Results of calculations are tabulated:

Integration step *h*	Value of integral	The received error: The absolute	The relative, %
1.0	2.3484	– 0.0441	1.843
0.5	2.3813	– 0.0112	0.468
0.25	2.3898	– 0.0027	0.113

Example 2. To calculate integral of probability Simpson's method with step *h* = 1.0, 0.5, 0.25.
The decision. Results of calculations are tabulated:

Integration step *h*	Value of integral	The received error:	
		The absolute	The relative, %
1.0	2.3743	– 0.0182	0.760
0.5	2.3923	– 0.0002	0.008
0.25	2.3926	+ 0.0001	0.004

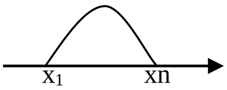
The resulted examples show, how much Simpson's method is more exact than the formula of trapezes.

Example 3.

Application of the formula of average rectangles for the decision of problems of numerical integration (on a calculation example $\int_1^2 (x^2+1)\sin(x-0.5)dx$).

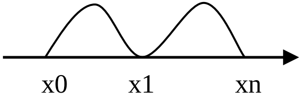
The decision.

$$\int_1^2 (x^2+1)\sin(x-0.5)dx = h\sum_{i=0}^{n-1} f\left(x_i+\frac{h}{2}\right)$$



Let's calculate integral I1 under the formula of a method of average rectangles (6):
h_1=1

I1=hf(x_0+h/2)=((1.5)2+1)sin(1.5-0.5)=2.734

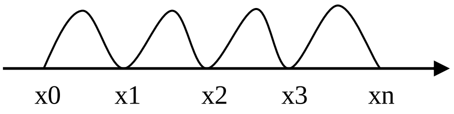


Let's reduce a step twice and we will calculate integral I2 under the formula of a method of average rectangles (6):

h_2=1/2
I2= h(f(x0+h/2)+f(x1+h/2))=1/2 ((1.25)2+1)sin(1.25-0.5)+ ((1.75)2+1)sin(1.75-0.5))=2.8005
Let's calculate criterion for integrals I1 and I2, as I2≥1 the criterion is calculated under the formula:

|(I2-I1)/I2|=0.023746>ε

The received criterion is not carried out, we calculate integral I3, reducing a step twice:



h2=1/4
I3=h(f(x0+h/2)+f(x1+h/2)+f(x2+h/2)+f(x3+h/2))=1/4((1.125)2+1)sin(1.125-0.5)+(1.375)2+1)sin(1.375-0.5)+(1.625)2+1)sin(1.625-0.5)+ (1.875)2+1)sin(1.875-0.5))=2.814

Let's calculate criterion for integrals I2 and I3, as I3≥1 the criterion is calculated under the formula:
|(I3-I2)/I3|=0.004797<ε

The received criterion is carried out, hence, we have calculated the set integral with demanded accuracy.

The answer: $\int_1^2 (x^2+1)\sin(x-0.5)dx =$ 2.814 with accuracy0.01.

http://tgspa.ru/info/education/faculties/ffi/ito/programm/osn_chm/chislennoe_integrirovanie3b_mathcad.htm

Lecture № 11.
Numerical integration. The formula of Gauss.

The plan:

1. The quadrature formula of Gauss

The methods described above use the fixed points of a piece (the ends and the middle) and have a low order of accuracy (0 – methods of the right and left rectangles, 1 – methods of average rectangles and trapezes, 3 – a method of parabolas (Simpson)). If we can choose points in which we calculate values of function *f (x)* it is possible to receive methods of higher order of accuracy at the same quantity of calculations of subintegral function. So for two (as in a method of trapezes) calculations of values of subintegral function, it is possible to receive a method any more 1st, and 3rd order of accuracy:

$$I \approx \frac{b-a}{2}\left(f\left(\frac{a+b}{2}-\frac{b-a}{2\sqrt{3}}\right)+f\left(\frac{a+b}{2}+\frac{b-a}{2\sqrt{3}}\right)\right).$$

Generally, using n points, it is possible to receive a method with accuracy order 2n-1. Values of knots of a method of Gaussa on пточкам are roots of a polynom of Lezhandra of degree n.

Values of knots of a method of Gaussa and their scales are resulted in directories of special functions. The method of Gaussa on five points is most known.

Example 1.

Let's calculate integral $\int_{0.5}^{3}\frac{2x^3}{x^4}dx$ with the method of Gauss.

The decision.

$$I \approx \frac{b-a}{2}\left(f\left(\frac{a+b}{2}-\frac{b-a}{2\sqrt{3}}\right)+f\left(\frac{a+b}{2}+\frac{b-a}{2\sqrt{3}}\right)\right)$$

$$f(x)=\frac{2x^3}{x^4}.$$

$$f1(x)=f\left(\frac{a+b}{2}-\frac{b-a}{2\sqrt{3}}\right)=f\left(\frac{0.5+3}{2}-\frac{3-0.5}{2\sqrt{3}}\right)=f(1.029)=1.94\cdot$$

$$f2(x)=f\left(\frac{a+b}{2}+\frac{b-a}{2\sqrt{3}}\right)=f\left(\frac{0.5+3}{2}+\frac{3-0.5}{2\sqrt{3}}\right)=f(2.47)=0.812$$

$$\int_{0.5}^{3}\frac{2x^3}{x^4}dx=\frac{3-0.5}{2}(1.94+0.812)\approx 3.584\cdot$$

The answer: 3.584.

Example 2.

Let's calculate integral a method $\int_{0.5}^{2.3}\pi\cdot\sin(\pi x)dx$ of Gauss.

The decision.

$$f(x)=\pi\cdot\sin(\pi x).$$

$$f1(x)=f\left(\frac{a+b}{2}-\frac{b-a}{2\sqrt{3}}\right)=f\left(\frac{0.5+2.3}{2}-\frac{2.3-0.5}{2\sqrt{3}}\right)=f(0.88)=-1.156\cdot$$

$$f2(x)=f\left(\frac{a+b}{2}+\frac{b-a}{2\sqrt{3}}\right)=f\left(\frac{0.5+2.3}{2}+\frac{2.3-0.5}{2\sqrt{3}}\right)=f(1.92)=0.781$$

$$\int_{0.5}^{2.3}\pi\cdot\sin(\pi x)dx=\frac{2.3-0.5}{2}(-1.156+0.781)\approx -0.588.$$

The answer: - 0.588.

Lecture № 12.

Root-mean-square approach of functions. A method of the least squares

The plan:

1. Root-mean-square approach of functions

2. A method of the least squares

1. Root-mean-square approach of functions

Let dependence between variables *x* and *y* is set таблично (the skilled data is set). It is required to find function somewhat in the best way describing the data. One of ways of selection of such (approaching) function is the method of the least squares. The method consists in that the sum of squares of deviations of values of required function $\bar{y}_i = \bar{y}(x_i)$ and set таблично y_i was the least:

$$S(c) = (y_1 - \bar{y}_1)^2 + (y_2 - \bar{y}_2)^2 + \ldots + (y_n - \bar{y}_n)^2 \to \min \qquad (6.1)$$

Where *c a* vector –of parametres of required function.

2. A method of the least squares

To construct a method of the least squares two empirical formulas: linear and square-law.

In case of linear function *y=ax+b the* problem is reduced to a finding of parametres *a* and *b* from system of the linear equations

$$\begin{cases} M_{x^2} a + M_x b = M_{xy} \\ M_x a + b = M_y \end{cases} \text{, Where}$$

$$M_{x^2} = \frac{1}{n}\sum_{i=1}^{n} x_i^2, \quad M_x = \frac{1}{n}\sum_{i=1}^{n} x_i, \quad M_{xy} = \frac{1}{n}\sum_{i=1}^{n} x_i y_i, \quad M_y = \frac{1}{n}\sum_{i=1}^{n} y_i$$

а в случае квадратичной зависимости $y = ax^2 + bx + c$ к нахождению параметров a, b и c из системы уравнений:

and in case of square-law dependence $y = ax^2 + bx + c$ to a finding of parameters *a, b* and *c* from system of the equations:

$$\begin{cases} M_{x^4} a + M_{x^3} b + M_{x^2} c = M_{x^2 y} \\ M_{x^3} a + M_{x^2} b + M_x c = M_{xy} \\ M_{x^2} a + M_x b + c = M_y \end{cases} \text{, Where}$$

$$M_{x^4} = \frac{1}{n}\sum_{i=1}^{n} x_i^4, \quad M_{x^3} = \frac{1}{n}\sum_{i=1}^{n} x_i^3, \quad M_{x^2 y} = \frac{1}{n}\sum_{i=1}^{n} x_i^2 y_i$$

To choose from two functions the most suitable. For this purpose to make the table for calculation of the sum of squares of evasion under the formula (6.1). Initial given to take from table 6.

The task 2

To make the program for a finding of approaching functions of the set type with a conclusion of values of their parametres and the sums of squares of evasion corresponding to them. To choose as approaching functions the following: $y = ax + b$, $y = ax^m$, $y = ae^{mx}$. To spend linearization. To define for what kind of function the sum of squares of evasion is the least.

Initial data is placed in table 6.

Approximate fragment of performance of laboratory work

(George E. Forsyth and Michael A. Malcolm and Cleve B. Moler. Computer Methods for Mathematical Computations. Prentice-Hall, Inc., 1977.)

$i := 1..10$

$x_1 := 0.5$

$x_2 := 0.1$

$x_3 := 0.4$

$x_4 := 0.2$

$x_5 := 0.6$

$y_1 := 1.8$

$y_2 := 1.1$

$y_3 := 1.8$

$y_4 := 1.4$

$y_5 := 2.1$

$x_6 := 0.3$

$x_7 := 0.4$

$x_8 := 0.7$

$x_9 := 0.3$

$x_{10} := 0.8$

$y_6 := 1.8$

$y_7 := 1.6$

$y_8 := 2.2$

$y_9 := 1.5$

$y_{10} := 2.3$

$$mx2 := 1\frac{\left[\sum_{i=1}^{10}(x_i)^2\right]}{10} \qquad mx := 1\cdot\frac{\left(\sum_{i=1}^{10}x_i\right)}{10} \qquad mxy := 1\frac{\left(\sum_{i=1}^{10}x_i\cdot y_i\right)}{10} \qquad my := 1\frac{\left(\sum_{i=1}^{10}y_i\right)}{10}$$

$mx2 = 0.229$ $\qquad mx = 0.43$ $\qquad mxy = 0.828$ $\qquad my = 1.76$

Given

$mx2a + mxb = mxy$

$mxa + b = my$

$Find(a,b) \rightarrow$

Table 6

i №		1	2	3	4	5	6	7	8	9	10
1	*x*	0.5	0.1	0.4	0.2	0.6	0.3	0.4	0.7	0.3	0.8
	y	1.8	1.1	1.8	1.4	2.1	1.8	1.6	2.2	1.5	2.3
2	*x*	1.7	1.5	3.7	1.1	6.2	0.3	6.5	3.6	3.8	5.9
	y	1.5	1.4	1.6	1.3	2.1	1.1	2.2	1.8	1.7	2.3
3	*x*	1.7	1.1	1.6	1.2	1.9	1.5	1.8	1.4	1.3	1.0
	y	6.7	5.6	6.7	6.1	7.4	6.9	7.9	5.9	5.6	5.3
4	*x*	1.3	1.2	1.5	1.4	1.9	1.1	2.0	1.6	1.7	1.8
	y	5.5	5.9	6.3	5.8	7.4	5.4	7.6	6.9	6.6	7.5
5	*x*	2.3	1.4	1.0	1.9	1.5	1.8	2.1	1.6	1.7	1.3
	y	5.3	3.9	2.9	5.0	4.0	4.9	5.1	4.5	4.1	3.7
6	*x*	1.8	2.6	2.3	1.3	2.0	2.1	1.1	1.9	1.6	1.5
	y	4.4	6.4	5.3	3.7	4.9	5.6	3.0	5.0	4.3	3.7
7	*x*	1.9	2.1	2.0	2.9	3.0	2.6	2.5	2.7	2.2	2.8
	y	6.6	7.6	6.7	9.2	9.4	7.8	8.4	8.0	7.9	8.7
8	*x*	2.0	1.4	1.0	1.7	1.3	1.6	1.9	1.5	1.2	2.1
	y	7.5	6.1	4.8	7.4	5.7	7.0	7.1	6.8	6.0	8.9
9	*x*	2.0	1.2	1.8	1.9	1.1	1.7	1.6	1.4	1.5	1.3
	y	7.5	5.9	7.0	8.0	5.0	7.4	6.4	6.6	6.3	5.7
10	*x*	1.9	1.1	1.4	2.3	1.7	2.1	1.6	1.5	1.0	1.2
	y	4.7	3.4	3.8	5.2	4.6	5.5	3.9	3.9	3.2	3.5

CONTROL QUESTIONS

1. In what an approach essence таблично the set function on a method of the least squares?
2. Than this method differs from an interpolation method?
3. How the problem of construction of approaching functions in the form of various elementary functions to a case of linear function is reduced?
4. Whether there can be a sum of squares of evasion for any approaching functions equal to zero?
5. What elementary functions are used as approaching functions?
6. How to find parametres for linear and square-law dependence, using a method of the least squares?

Lecture № 13-14

Statement of a problem of linear programming. The basic properties the decision of a problem of linear programming.

The plan:

1. The primary goal of linear programming
2. Examples of the decision of a problem

The primary goal of linear programming in a canonical form is formulated as follows:

To find the non-negative decision of system of restrictions

$$a_{i1}x_1 + a_{i2}x_2 + \ldots + a_{ij}x_j + \ldots + a_{in}x_n + b_i = 0 \left(i = 1,2,\ldots,m\right);$$

$$x_j \geq 0 \left(j = 1,2,\ldots,n\right), \tag{II. 1}$$

Providing a maximum (minimum) of criterion function

$$Z = c_1x_1 + c_2x_2 + \ldots + c_nx_n + Q \rightarrow \max(\min) \tag{II.2}$$

Except a record reduced form can be used partially developed

$$Z = \sum_{j=1}^{n} c_j x_j + Q \rightarrow \max;$$

$$\sum_{j=1}^{n} a_{ij}x_j + b_i = 0 \quad \left(i = 1,2,\ldots,m\right); \; x_j \geq 0 \quad \left(j = 1,2,\ldots,n\right)$$

and matrix forms

$$Z = \dot{C}x + Q \rightarrow \max,$$

$$Ax + B = 0, \quad x \geq 0.$$

All further reasonings will be spent only for the primary goal in a canonical form.

Usually specific targets of linear programming have distinct from initial an appearance, therefore to solve their such problems it is necessary to lead to a canonical form

Let the problem of linear programming with variables and the mixed system from m restrictions is set:

$$Z = c_1x_1 + c_2x_2 + \ldots + c_nx_n + Q \rightarrow \max; \tag{II.3}$$

$$a_{i1}x_1 + a_{i2}x_2 + \ldots + a_{in}x_n + b_i \leq 0 \quad \left(i = 1,2,\ldots,r\right);$$

$$a_{k1}x_1 + a_{k2}x_2 + \ldots + a_{kn}x_n + b_k \geq 0 \quad \left(k = r+1,\ldots,t\right);$$

$$a_{l1}x_1 + a_{l2}x_2 + \ldots + a_{ln}x_n + b_l = 0 \quad \left(l = t+1,\ldots,m\right);$$

$$x_j \geq 0 \quad \left(j = 1,2,\ldots,s \leq n\right). \tag{II.4}$$

For reduction of this problem to a canonical form it is necessary to replace variables, i.e. To exclude those variables which can accept both positive, and negative values. The system of restrictions-inequalities should be replaced by equivalent system of the equations with non-negative variables.

Replacement of inequalities with the equations. Replacement of system of restrictions-inequalities in (II.4) equivalent system of the equations is carried out by introduction of artificial, non-negative variables y,

$$a_{i1}x_1 + a_{i2}x_2 + \ldots + a_{in}x_n + y_i + b_i = 0 \; ;$$

$$a_{k1}x_1 + a_{k2}x_2 + \ldots + a_{kn}x_n - y_k + b_k = 0 \; ;$$

$$y_i \geq 0 \quad \left(i = 1,2,\ldots,r\right); \quad y_k \geq 0 \quad \left(k = r+1,\ldots,t\right). \tag{II.5}$$

Such transformation increases number of variables, without changing a problem being.

Replacement of unlimited variables. Variables which can accept negative values, are expressed through non-negative variables $x_1, x_2, \ldots, x_s$ and $y_1, y_2, \ldots, y_r$. Replacement of variables represents the system decision, concerning a replaced variable, and can be executed with the help жордановых exceptions. For replacement of one variable one step of exceptions is required, therefore to lead problem canonical form is possible only in case a rank of system of more number of unlimited variables.

After replacement the problem dares in new variables. The optimum decision in new variables is substituted in the communication equations therefore the optimum decision in initial variables turns out.

At the decision of economic and technical problems, as a rule, variables can be only positive real numbers. If in a problem any variable by the nature can accept negative values in most cases change of the formulation of conditions allows to get rid of unlimited variables.

Minimisation of form Z. Further the problem of maximisation of form Z will be considered only. If it is necessary to solve a problem of minimisation of the linear form, criterion function factors should be increased on (-

1) and to solve this new problem on a maximum. The required minimum of criterion function turns out multiplication of the found maximum value on (-1), i.e. $Z_{\min} = -\max(-Z)$

Таблица II. 1

Example II.1. THE Colliery works in a complex with concentrating factory. Daily average extraction of mine makes D=3300 t and planned instantaneous ash content coal A = 19,2 %. All coal of mine is transferred to enrichment, therefore every day tasks on quality of coal is corrected for constant maintenance instantaneous ash content processed raw materials. As a result of receipt of party of coal with high instantaneous ash content the concentrating factory demands to lower next days instantaneous ash content extracted coal to 18 %. In this connection it is required to correct daily tasks mining to mine sites so that extraction decrease as a whole, but to mine was minimum. Indicators of work of sites of mine are resulted in III.1.

II. 1

Site number	Daily loading according to plan D_{ni}, t	instantaneous ash content extracted coal A_i, %	The greatest possible loading on a site D_i^{max}, t
1	900	20	1000
2	850	23	920
3	850	18	950
4	700	15	800

Loadings can be increased by a site at the expense of redistribution of empty trolleys and manpower resources. Coal from sites 1 and 2 is transported on the conveyor line having daily productivity no more P_1 = 1850 t and from sites 3 and 4 on line with daily productivity no more P_2=1700 t.

The task in view can be shown to a problem of linear programming with non-negative variables if as variables to accept loadings on faces, and with unlimited variables if for variables corrective amendments of daily tasks of sites are accepted. For more evident illustration of all stages of the decision of problems of linear programming two variants of statement of a problem here will be considered.

Variant 1. If as variables x_i to accept loadings on clearing sites, but a main objective of the decision of a problem - maintenance of the maximum extraction can be described the following criterion function

$$D = x_1 + x_2 + x_3 + x_4 \rightarrow \max.$$

Thus following restrictions should be carried out: on quality of coal

$$A_1x_1 + A_2x_2 + A_3x_3 + A_4x_4 = A_{n\,n}(x_1 + x_2 + x_3 + x_4)$$

On extraction of sites

$$x_1 \le D_1^{\max}, \qquad x_2 \le D_2^{\max},$$

$$x_3 \le D_3^{\max}, \qquad x_4 \le D_4^{\max},$$

But throughput of transport communications

$$x_1 + x_2 \le \Pi_1, \qquad x_3 + x_4 \le \Pi_2$$

On the physical essence loading on a face - size positive, therefore $x_i \ge 0 \ (i = 1, 2, 3, 4)$

After substitution of the initial data and reduction of similar members the problem becomes

$$\begin{array}{l}
D = x_1 + x_2 + x_3 + x_4 \rightarrow \max \\
2x_1 + 5x_2 \quad - 3x_4 = 0, \\
x_1 \qquad\qquad - 1000 \le 0, \\
\quad x_2 \qquad\quad - 920 \le 0. \\
\qquad x_3 \qquad - 950 \le 0 \\
\qquad\quad x_4 \quad - 800 \le 0 \\
x_1 + x_2 \qquad\quad - 1850 \le 0 \\
\qquad x_3 + x_4 \quad - 1700 \le 0 \\
x_i \ge 0 \ (i = 1, 2, 3, 4)
\end{array}$$

For problem reduction to a canonical form it is necessary to replace inequalities with equivalent restrictions-equalities by introduction of auxiliary non-negative variables y_i.

$$\begin{array}{l}
D = x_1 + x_2 + x_3 + x_4 \rightarrow \max \\
2x_1 + 5x_2 \qquad - 3x_4 \qquad\qquad = 0, \\
-x_1 \qquad\qquad\qquad - y_1 + 1000 = 0, \\
\quad -x_2 \qquad\qquad\quad - y_2 \ + 920 = 0, \\
\qquad -x_3 \qquad\qquad - y_3 \ + 950 \ = 0,
\end{array}$$

$$\begin{array}{llllll}
 & & & -x_4 & -y_4 & +800 = 0 \\
-x_1 & -x_2 & & & -y_5 & +1850 = 0 \\
 & & -x_3 & -x_4 & -y_6 & +1700 = 0 \\
\end{array}$$

$$x_i \geq 0 \quad (i = 1,\dots,4)$$

$$y_j \geq 0 \quad (j = 1,\dots,6)$$

Variant 2. We will designate variable updatings of daily tasks on sites through Δ_i then the problem purpose - extraction maximisation - will be reached at

$$\Delta = \Delta_1 + \Delta_2 + \Delta_3 + \Delta_4 \rightarrow \max$$

Thus restrictions should be carried out: on quality

$$\frac{(D_{n1} + D_1)A_1 + (D_{n2} + D_2)A_2 + (D_{n3} + D_3)A_3 + (D_{n4} + D_4)A_4}{(D_{n1} + D_1) + (D_{n2} + D_2) + (D_{n3} + D_3) + (D_{n4} + D_4)} = A_{n\text{я}}$$

On extraction of sites

$$0 \leq D_{n1} + \Delta_1 \leq D_1^{\max},$$

$$0 \leq D_{n2} + \Delta_2 \leq D_2^{\max},$$

$$0 \leq D_{n3} + \Delta_3 \leq D_3^{\max},$$

$$0 \leq D_{n4} + \Delta_4 \leq D_4^{\max},$$

On throughput of transport communications

$$(D_{n1} + \Delta_1) + (D_{n2} + \Delta_2) \leq \Pi_1,$$

$$(D_{n3} + \Delta_3) + (D_{n4} + \Delta_4) \leq \Pi_2.$$

After substitution of the initial data, replacement of bilateral restrictions unilateral and transition to equivalent system of the equations, a problem can lead the kind

$$D = \Delta_1 + \Delta_2 + \Delta_3 + \Delta_4 \rightarrow \max$$

$$D = \Delta_1 + \Delta_2 + \Delta_3 + \Delta_4 \rightarrow \max$$

$$\begin{array}{lllll}
0{,}02\Delta_1 & +0{,}05\Delta_2 & & -0{,}03\Delta_4 & +39{,}5 = 0, \\
-\Delta_1 & & & & -u_1 + 100 = 0, \\
 & -\Delta_2 & & & -u_2 + 70 = 0, \\
 & & -\Delta_3 & & -u_3 + 100 = 0, \\
 & & & -\Delta_4 & -u_4 + 100 = 0, \\
\Delta_1 & & & & -u_5 + 900 = 0, \\
 & \Delta_2 & & & -u_6 + 850 = 0, \\
 & & \Delta_3 & & -u_7 + 850 = 0, \\
 & & & \Delta_4 & -u_8 + 700 = 0, \\
-\Delta_1 & -\Delta_2 & & & -u_9 + 100 = 0, \\
 & & -\Delta_3 & -\Delta_4 & -u_{10} + 150 = 0, \\
\end{array}$$

$$u_j \geq 0 \quad (j = 1, \ \dots, \ 10)$$

Variables Δ can accept both positive, and negative values, therefore for reduction of this problem to a canonical form it is necessary to express them through non-negative variables u_j. This operation will be carried out more low at the further decision of a problem.

The optimum decision of a problem of the linear programming led to a canonical form, the non-negative decision of system of restrictions (II.1), providing a criterion function maximum (II.2) is.

At the decision of system of restrictions there can be three cases:

1. System of restrictions not compatibility and the optimum decision is impossible. Not compatibility systems of restrictions it is caused by the economic and technological reasons. More often not compatibility speaks insufficient quantity of resources because of what restrictions on planned amounts of works cannot be executed. Besides, in planning problems mining works not compatibility systems of restrictions it is often caused by impossibility of performance of requirements to quality of a mineral at the developed industrial situation (the certain maintenance of useful and harmful components in blocks or faces). Restriction revealing because of which all system not compatibility, allows to specify problem statement.

2. The system of restrictions has the unique decision $x_1 = \beta_1 \geq 0;\ x_2 = \beta_2 \geq 0;\ x_n = \beta_n \geq 0$. In this case the problem of linear programming is reduced to the decision of system the linear equation and substitution of this unique decision in criterion function, i.e.

$$Z_{\max} = c_1\beta_1 + c_2\beta_2 + \dots + c_n\beta_n + Q$$

3. The system of restrictions has uncountable set of decisions. From the point of view of maximisation of form Z this case represents the greatest interest and will be considered more low.

Computing procedure of search of the optimum decision of a problem of linear programming is based on following theorems.

The theorem 1. The set of admissible decisions of the primary goal of linear programming is convex.

The theorem 2. The non-negative basic decision of system of linear restrictions (II.1) is a point of set of decisions of the primary goal of linear programming.

The point $\bar{x}$ belonging to set **X,** is called as extreme if it cannot be presented as a convex combination of other points.

As number of the variable equations in system (II.1**)** **$x_j \geq 0$** (j=1, 2..., n) there is more than number of restrictions I (i=l, 2..., n) *THE* system has set of decisions. One of possible decisions of system can be found, if (n-m) any variables to equate to zero. Then the received system from T THE equations with n unknown persons is easy for solving (if a determinant made of factors at unknown persons, does not address in zero, i.e. When lines and columns of a matrix of factors are linearly independent). The decision received thus is called as basic, and making it T variables also are called as basic. The others (n-m) variables are called as not basic or free. In each concrete system of the equations (II.1**)** usually there are some basic decisions with various basic variables.

The theorem 3. The linear form of a problem of linear programming reaches the unique maximum value in an extreme point of set of decisions.

From theorems 2 and 3 it is possible to draw the important conclusion - it is necessary to search for the optimum decision of the primary goal of linear programming among set of admissible basic decisions of system of restrictions.

Lecture № 15-16

Geometrical interpretation of a problem of linear programming.

The plan: 1. Problem statement
2. Geometrical representation.
3. An example of the decision of a problem
4. Geometrical problem interpretation

Better and more visually to present geometrical sense of a problem of linear programming, we will address to the elementary two-dimensional case (when the model includes two variables) and then we will make generalisations at presence n variables.

In case of two variables the model of linear programming has the following appearance

$$Z = c_1 x_1 + c_2 x_2 \rightarrow \max ; \qquad (II.6)$$

$$a_i x_1 + a_i x_2 + b_i \geq 0 \quad (i = 1, \ 2, \ \ldots, \ m); \qquad (II.7)$$

$$x_1 \geq 0; \ \ x_2 \geq 0.$$

Each restriction (II.7) represents a straight line (fig. II.1) which breaks all space (an initial plane) on two semiplanes one of which satisfies to restriction (this area in drawing is shaded).

The system of restrictions according to the theorem 1 represents convex set, and in a considered two-dimensional case - a convex polygon of restrictions (fig. II.2). In special cases the polygon can address in a point (then the decision is unique), a straight line or a piece. If the system of restrictions is inconsistent (несовместна) it is impossible to construct a polygon of restrictions also a problem of linear programming has no decisions. Such case is shown on fig. II.3. Really, there is no point of space which simultaneously would satisfy to restriction y_1 and to restrictions y_2 and y_3.

The polygon of restrictions can be not closed (fig. II.4). In this case, as it will be shown more low, criterion function Z is not limited from above.

In a case п variables each restriction represents (n-l) a-dimensional hyperplane which divides all space into two semispaces. The system of restrictions in this case gives a convex polyhedron of decisions - the general part of the n-dimensional space, satisfying to all restrictions.

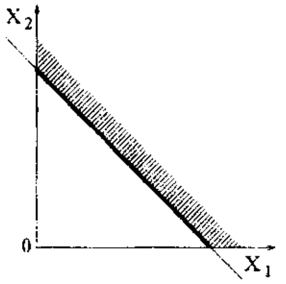


Fig. II.1. Geometrical sense of restriction

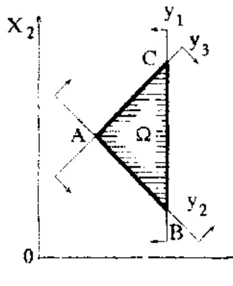


Fig. II.2. Geometrical interpretation of system of restrictions

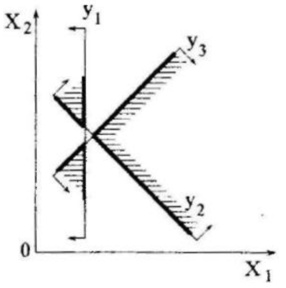


Fig. II.3. Imcompatibility systems of restrictions

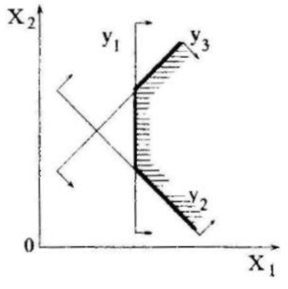


Fig. II.4. Limitlessness of criterion function

In three-dimensional space (n=3) each restriction represents a plane in space. All restrictions, being crossed, form a convex polyhedron which in special cases can be a point, a piece, a beam, a polygon or many-sided unlimited area.

For finding-out of geometrical sense of criterion function we will give to variable Z various numerical values (Z=0, Z=l, Z=2, Z=D).

To these numerical values Z there corresponds sequence of the equations and system of parallel straight lines in space (fig. II.5).

$$c_1 x_1 + c_2 x_2 = 0,$$
$$c_1 x_1 + c_2 x_2 = 1,$$
$$\ldots\ldots\ldots\ldots\ldots$$
$$c_1 x_1 + c_2 x_2 = D.$$

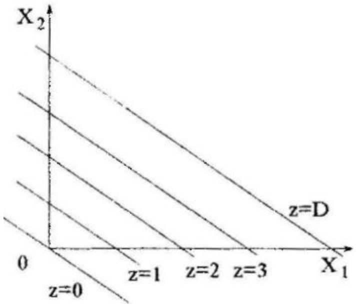


Fig. II.5. Geometrical interpretation of criterion function

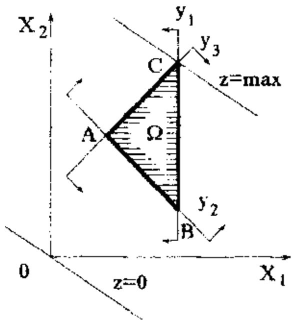


Fig. II.6. Geometrical sense of the optimum decision of a problem of linear programming

The first straight line (Z=0) passes through the beginning of co-ordinates perpendicularly (ортогонально) to the directing vector $C=(C_1C_2)$, the subsequent straight lines are parallel to the first and will defend from it in a direction of a vector With on size 1, 2, D. As a whole variable Z defines evasion of the points lying on a straight line $Z=c_1x_1+c_2x_2$ from a straight line $c_1x_1+c_2x_2=0$, passing through the beginning of co-ordinates. To define evasion of any point from straight line Z=0, it is enough to substitute co-ordinates of this point in the criterion function equation.

In n-dimensional space of the criterion function equal to zero ($Z=c_1x_1+c_2x_2+\ldots+c_jx_j+\ldots+c_nx_n=0$), geometrically there corresponds (n-1) the-dimensional hyperplane passing through the beginning of co-ordinates.

The distance from a point with co-ordinates $x'=(x'_1+x'_2+\ldots+x'_n)$ to a hyperplane is equal

$$R=\frac{a_1x_1'+a_2x_2'+\ldots+a_nx_n'}{\sqrt{a_1^2+a_2^2+\ldots+a_n^2}}$$

or

$$R\sqrt{a_1^2+a_2^2+\ldots+a_n^2}=a_1x_1'+a_2x_2'+\ldots+a_nx_n'$$

From here it is visible that, if in the linear form to substitute point co-ordinates, the distance from a point x' to the corresponding hyperplane Z=0, postponed in the scale equal to norm of a vector of an orthogonal plane will turn out (or in the scale equal to norm of the directing vector).

The scaled distance **y,** equal $R\sqrt{\sum_{j=1}^{n}a_j^2}$, is called as evasion of a point from a plane. As according to the theorem 3 linear form Z reaches the extreme value in an extreme point (top) of a polyhedron of restrictions geometrically the problem of linear programming consists in search of top of a polyhedron of the admissible decisions, having the maximum evasion from a hyperplane expressed by criterion function, equal to zero (fig. II.6).

If a polyhedron of restrictions will not close (fig. II.4 see) evasion is equal to infinity as a straight line parallel to criterion function, it is possible to move as much as necessary upwards, without leaving for area of admissible values of variables.

The graphic method of their decision is based on geometrical interpretation of linear problems. This method can be used effectively at the decision of problems with two (sometimes with three) variable and reduced to them as it is impossible to represent graphically spaces большей dimensions. For the graphic decision of a problem of linear programming it is necessary in accepted system of co-ordinates to construct the equation of all restrictions which set will give a polyhedron of restrictions. Then build the equation of criterion function equal to zero, i.e. Passing through the beginning of co-ordinates, Z=0. After that, moving the direct (plane) corresponding to criterion function, in parallel to itself, find a point of a contact of this direct (plane) with a polygon (polyhedron) of restrictions - the top of a polygon having the maximum evasion from direct (plane), Z=0.

Let's show use graphic метода on a concrete example.

Example II.2. It is required to define annual volumes of extraction of ore on three enterprises (tab. II.2).

Table II.2

Indicators	The enterprise		
	1	2	3
The maximum annual extraction Q_i^{max}, million ***t***.	16,7	17,4	15,9
Annual production rate of structure q_i, million ***t***.	2,5	2,35	2,5
The metal maintenance α_i, %	5,9	6,4	7,7
Has arrived from extraction and processings 1 million t. ores p_i, billion roubl.	9,24	9,6	12,12

In total in work there are 13 structures. The ore extracted on three mines, is processed at one concentrating factory, and the average maintenance of metal in ore should be within 6,4 - 6,6 %.

For operated variables we accept number of the structures, allocated to mines for ore transportation on concentrating factory $-x_1, x_2$ и x_3, and for criterion of an optimality - total profit on extraction and ore processing.

Then criterion function of a problem will become

$$\sum_{i=1}^{3} p_i q_i x_i = 9{,}24\cdot 2{,}5x_1 + 9{,}6\cdot 2{,}35x_2 + 12{,}12\cdot 2{,}5x_3 =$$

$$= 23{,}1x_1 + 22{,}56x_2 + 30{,}3x_3 \to \max.$$

At the decision it is necessary to observe following restrictions:

On extraction of mines

$$q_i x_i \le Q_i^{\max};$$

$$2{,}5x_1 \le 16{,}7; \quad 2{,}35x_2 \le 17{,}4; \quad 2{,}5x_3 \le 15{,}9;$$

on number of structures

$$x_1 + x_2 + x_3 = 13;$$

on quality

$$6{,}4 \le \frac{\sum_{i=1}^{n} q_i\, x_i\, \alpha_i}{\sum_{i=1}^{n} q_i\, x_i} \le 6{,}6$$

On positivity of variables $x_1 \ge 0; \quad x_2 \ge 0; \quad x_3 \ge 0.$

Having substituted values q_i and α in restrictions on quality and having executed necessary transformations, we will receive

$$-1{,}75x_1 - 0{,}47x_2 + 2{,}75x_3 \le 0;$$

$$-1{,}25x_1 \qquad + 3{,}25x_3 \ge 0.$$

Using restriction – equality $x_1 + x_2 + x_3 = 13,$ we will express in criterion function **x₂** through **x₁** and **x₃.** As a result we will receive the following economic-mathematical model with two variables:

$$\begin{aligned}
&23{,}1x_1 + 22{,}6(13 - x_1 - x_3) + 30{,}3x_3 \to \max,\\
&2{,}5x_1 \le 16{,}7,\\
&2{,}35(13 - x_1 - x_3) \le 17{,}4,\\
&2{,}5x_3 \le 15{,}9,\\
&-1{,}75x_1 - 0{,}47(13 - x_1 - x_3) + 2{,}75x_3 \le 0,\\
&-1{,}25x_1 \qquad\qquad + 3{,}25x_3 \ge 0,\\
&\quad x_1 \ge 0, \qquad x_3 \ge 0.
\end{aligned}$$

After transformation it is had

$$\begin{aligned}
&0{,}5x_1 + 7{,}7x_3 \to \max,\\
&2{,}5x_1 \le 16{,}7,\\
&2{,}35x_1 + 2{,}35x_3 \ge 13{,}2,\\
&2{,}5x_3 \le 15{,}9,\\
&-1{,}28x_1 + 3{,}22x_3 \le 6{,}1,\\
&-1{,}25x_1 + 3{,}25x_3 \ge 0,\\
&\quad x_1 \ge 0, \qquad x_3 \ge 0.
\end{aligned}$$

Geometrical interpretation of a problem is resulted on fig. II.7, where $x_1 = 0,\ x_3 = 0$ - axes of co-ordinates. Besides, five more restrictions are constructed, and by short shading and arrows admissible semiplanes are shown. The system of restrictions forms area of admissible decisions - convex polygon ABCD.

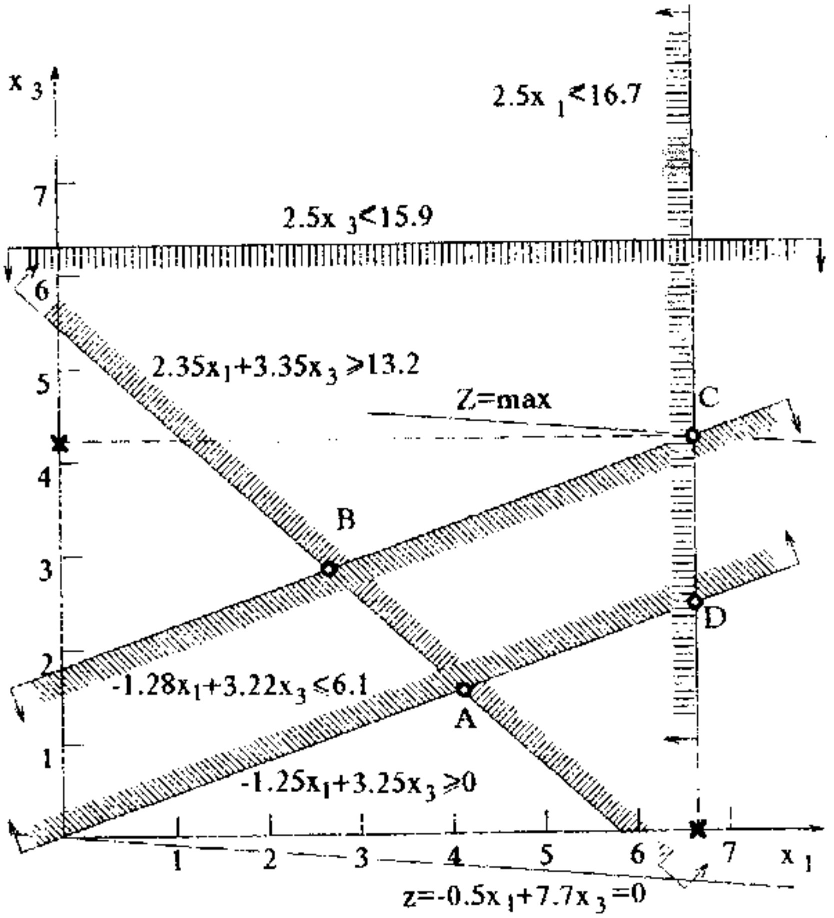


Fig. II.7. Geometrical interpretation of a problem

Through the beginning of co-ordinates there passes a straight line corresponding to the equation of criterion function, equal to zero ($Z = 0,5x_1 + 7,7x_3 = 0$). We move this straight line in parallel to themselves until it will not concern tops of a polygon of the restrictions, having the maximum removal from an initial straight line (Z=0). The top With gives to us x_1, and x_3, turning criterion function in a maximum. Lowering from a point C perpendiculars on co-ordinate axes, we will receive x_1=6,68 and x_3=4,55. Then $x_2 = 13 - x_1 - x_3 = 1,77$.

So the maximum value of criterion function is reached at x_1=6,68, **x_2=1,77** and x_3=4,55. Divisibility of number of structures speaks about necessity of their distribution and movement management on an open cycle.

Lecture №17.

Finding the decision of a problem of linear programming to simplex methods.

The plan: 1. Mathematical bases a simplex of a method of the decision

1. Mathematical bases a simplex of a method of the decision

It is known that if the problem of linear programming has the optimum decision there is at least one optimum basic decision. Thus, by search of basic decisions it is possible to receive the required decision. The number of basic decisions makes N = C_n^k, where n - number of variables, and K=r(A) - number of basic variables. This number very quickly grows at increase in number of variables, therefore in rather small problems continuous search becomes impracticable even by means of the COMPUTER.

The number of touched decisions can be reduced at the expense of an exception of consideration of inadmissible basic decisions. The admissible basic decision or the basic decision represents the basic decision with positive values of basic variables. Hence, to touch only basic decisions, the algorithm of search should answer a following condition: at transition from one decision to another should remain innegativity all variables. Performance of this condition does a problem of search of more foreseeable, but as a whole procedure remains ineffective as transition to another does not guarantee its improvement against one decision. What is quality of the decision? The procedure ultimate goal - achievement of a maximum of linear form Z, therefore can serve as an indicator of quality of the decision level 2 in the given basic decision. Hence, efficiency of procedure of search can be raised sharply if each step improves quality of the decision or to provide growth of linear form Z. On the basis of these reasonings it is possible to formulate the second condition to which the algorithm of the decision of a linear problem should answer: transition from one basic decision to another should provide growth of criterion function Z.

This idea can be realised only in the event that there is some basic decision which gradually improves.

The basic method of the decision of problems of linear programming is the simplex-method in which all process of the decision shares on three stages: search of the initial basic decision, search basic and then the optimum decision.

To search of basic, basic and optimum decisions apply special procedures - ordinary and modified Jordanov's exceptions.

That in system of linear forms *y=Ax* to change in places dependent variable y_r and an independent variable x_S, it is necessary to solve r-e the equation rather x, and to substitute this decision in all other equations of system.

It is obvious that to solve r-e equation rather x, is possible only in the event that $a_{rs} \neq 0$.

Definition. Step ordinary Jordanov's an exception made over system of linear forms y=Ax with the resolving element $a_{rs} \neq 0$, with r-th in the resolving line and s-th a resolving column, name the schematised operation of recalculation of factors in linear forms at change by places dependent variable y_r and independent x_s.

For definition of operations of recalculation of elements of a matrix And in system of linear forms y=Ax at replacement y_r on x_s it is necessary to present a matrix in the form of tab. II.3 and to make corresponding algebraic actions.

Table II.3

	x_1	x_2	...	x_s	...	x_n
y_1	a_{11}	a_{12}	...	a_{1s}	...	a_{1n}
y_2	a_{21}	a_{22}	...	a_{2s}	...	a_{2n}
...	...	...	...	...	...	...
y_r	a_{r1}	a_{r2}	...	a_{rs}	...	a_{rn}
...	...	...	...	...	...	...
y_m	a_{m1}	a_{m2}	...	a_{ms}	...	a_{mn}

In the new table instead of r-th forms the new form from a basic variable x_s which turns out as a result of the decision r-th forms concerning this variable will settle down

$$x_s = -\frac{a_{r1}}{a_{rs}}x_1 - \frac{a_{r2}}{a_{rs}}x_2 + \ldots + \frac{1}{a_{rs}}y_r - \ldots - \frac{a_{rn}}{a_{rs}}x_n$$

Having analysed factors at variables x_j and y_r, may be following conclusions:

1. In the new table on a place of a resolving element a_{rs} should be written down $1/a_{rs}$.
2. Other elements resolving r-th register lines in the new table with a return sign and share on resolving element, i.e. Instead $(-a_{rj}/a_{rs})$ registered
3. In the new table on a place of a resolving column it is necessary to write down elements a_{is} instead of elements
4. Instead of the elements a_{ij} which are not belonging to the resolving line and a column, in the new table elements register bij $=(a_{ij}a_{rs} - a_{is}a_{rj})/a_{rs}$

Thus, for performance of one step Jordanov's exceptions with a resolving element a_{rs} it is necessary to carry out four operations by the rules formulated here and as a result the new system of forms in the form of tab. II-4 will be received.

Table II.4

	x_1	x_2	...	y_r	...	x_n
y_1	b_{11}	b_{12}	...	$\frac{a_{1s}}{a_{rs}}$	...	b_{1n}
...	...	...	...	...	...	...
x_s	$\frac{-a_{r1}}{a_{rs}}$	$\frac{-a_{r2}}{a_{rs}}$	...	$\frac{1}{a_{rs}}$	...	$\frac{-a_{rn}}{a_{rs}}$
...	...	...	...	...	...	...
y_m	b_{m1}	b_{m2}	...	$\frac{a_{ms}}{a_{rs}}$	...	b_{mn}

Modified Jordanovs exceptions. If system of linear forms y=Ax to present in a kind y = (-1)-A(-1)x and in this system to make replacement of a dependent variable y_r on independent x_s, with the help Jordanovs exceptions such procedure is called as modified Jordanov's exceptions.

Procedure modified Jordanov's exceptions is deduced similarly and consists in the following.

1. The system y=Ax is represented in a kind y=(-1)-A(-1)x and is brought in tab. II.5

Table II.5

	$-x_1$	$-x_2$	...	$-x_s$	...	$-x_n$
y_1	α_{11}	α_{12}	...	α_{1s}	...	α_{1n}
...	...	...	...	...	...	...
y_r	α_{r1}	α_{r2}	...	α_{rs}	...	α_{rn}
...	...	...	...	...	...	...
y_m	α_{m1}	α_{m2}	...	α_{ms}	...	α_{mn}

The note. $\alpha_{ij} = -a_{ij}$ $(i = 1,2,\dots,m)$; $(j = 1,2,\dots,n)$

2. Resolving element α_{rs} replace with unit.
3. Other elements of a resolving line remain without changes.
4. A sign at other elements of a resolving column change for the opposite.
5. All elements α_{ij} which are not belonging to the resolving column and a line, replace with elements

$$\beta_{ij} = \alpha_{ij}\alpha_{rs} - \alpha_{is}\alpha_{ri}$$

6. All elements of the new table divide into resolving element α_{rs}

As a result of one step modified Jordanov's exceptions with a resolving element α_{rs} new tab. II.6 turns out.

Table II.6

	$-x_1$	$-x_2$	...	$-y_r$	...	$-x_n$
y_1	$\frac{\beta_{11}}{\alpha_{rs}}$	$\frac{\beta_{12}}{\alpha_{rs}}$	...	$\frac{-\alpha_{1s}}{\alpha_{rs}}$	...	$\frac{\beta_{1n}}{\alpha_{rs}}$
...	...	...	...	...	...	...
x_s	$\frac{\alpha_{r1}}{\alpha_{rs}}$	$\frac{\alpha_{r2}}{\alpha_{rs}}$	...	$\frac{1}{\alpha_{rs}}$	...	$\frac{\alpha_{rn}}{\alpha_{rs}}$
...	...	...	...	...	...	...
y_m	$\frac{\beta_{m1}}{\alpha_{rs}}$	$\frac{\beta_{m2}}{\alpha_{rs}}$	...	$\frac{-\alpha_{ms}}{\alpha_{rs}}$	...	$\frac{\beta_{mn}}{\alpha_{rs}}$

For preservation of monotony of calculations at the decision of various problems only procedure modified Jordanov's exceptions further will be used. Unlike ordinary in modified Jordanov's exceptions the sign varies on opposite at a resolving column, instead of at a line.

Lecture №18.
Finding the decision of a problem of linear programming. A method of artificial basis.

The plan:

1. Search of the initial basic decision

1. Search of the initial basic decision

Let the problem of linear programming from l by variables and the mixed system from *m* restrictions is set:

$$\begin{aligned}
&Z = c_1 x_1 + c_2 x_2 + \ldots + c_n x_n + Q \to \max;\\
&a_{i1} x_1 + a_{i2} x_2 + \ldots + a_{in} x_n + b_i \geq 0 \; (i = 1,\ 2,\ \ldots,\ r);\\
&a_{k1} x_1 + a_{k2} x_2 + \ldots + a_{kn} x_n + b_k = 0 \; (k = r+1,\ \ldots,\ m);\\
&x_j \geq 0 \; (j = 1,\ 2,\ \ldots,\ s < n).
\end{aligned} \tag{II.8}$$

For problem reduction to a canonical form the system of restrictions - inequalities is led to equivalent system of the equations by introduction of artificial, non-negative variables y_i

$$\begin{aligned}
&a_{i1} x_1 + a_{i2} x_2 + \ldots + a_{in} x_n - y_i + b_i = 0 ,\\
&y_i \geq 0 \; (i = 1,\ 2,\ \ldots,\ r)
\end{aligned} \tag{II.9}$$

Also replacement of unlimited variables is made.

After reduction of system of restrictions to system of the linear equations it is necessary to find its common decision. It is obvious that the equations received from inequalities, easily dare concerning artificial variables y_i and the common decision of this part of system of the equations will be

$$\begin{aligned}
&y_i = a_{i1} x_1 + \ldots + a_{in} x_n + b_i,\\
&y_i \geq 0 \; (i = 1,\ 2,\ \ldots,\ r).
\end{aligned} \tag{II.10}$$

For other part of system of the equations the common decision can be received with the help Jordan’s exceptions (or it is established it imcompatibility).

The system decision can be combined with replacement of variables, and for this purpose it is necessary to enter unlimited variables into basis.

After search of the common decision of system the initial basic decision turns out by equating of independent variables with zero.

Thus, reception of the initial basic decision is reduced to following operations. The initial problem is led to a kind (II.10) and registers in a simplex-table (tab. II.9).

Table II.9

	$-x_1$	$-x_2$	…	$-x_n$	1
y_1	$-a_{11}$	$-a_{12}$	…	$-a_{1n}$	b_1
…	…	…	…	…	…
y_r	$-a_{r1}$	$-a_{r2}$	…	$-a_{rn}$	b_r
0	$-a_{r+1,1}$	$-a_{r+1,2}$	…	$-a_{r+1,n}$	b_{r+1}
…	…	…	…	…	…
0	$-a_{m1}$	$-a_{m2}$	…	$-a_{mn}$	b_m
Z	$-c_1$	$-c_2$	…	$-c_n$	Q

In the lines corresponding to restrictions - to inequalities, auxiliary variables register, and in lines with the equations auxiliary variables are equal to zero - (0-variables).

Jordanov’s exceptions unlimited variables $x_{s+1},\ \ldots,\ x_n$ are expressed by consecutive steps through non-negative variables and simultaneously with it 0-variables are translated on table top.

The column under translated on top of the table of a 0-variable is excluded. The equations of communication for unlimited variables are remembered, and corresponding lines do not participate in the further analysis. As a result of transformations the table containing the initial basic decision, has the following appearance (tab. II.10).

At following stages of the decision of a problem the part of the table allocated with a dashed line is analyzed only. In the received basic decision independent variables are equated to zero, and basic variables and form Z appear equal to corresponding free members, i.e.

$$x_1 = 0,\ \ldots,\ x_s = 0;\ y_1 = 0,\ \ldots,\ y_p = 0; \tag{II.11}$$

$$\begin{bmatrix} y_{p+1} \\ \cdot \\ \cdot \\ \cdot \\ y_p \end{bmatrix} = \overline{\beta_1};\ Z = q.$$

Table II.10

		Non-negative, independent variables		
		0,…,0	$-x_1, …, -x_s, -y_1, …, -y_n$	1
	0 . . . 0	A_0	‖0‖	$\overline{\beta_0}$
Unlimited variables	x_{s+1} . . . x_n		A_2	$\overline{\beta_2}$
Non-negative basic variables	y_{p+1} . . . y_p		A_1	$\overline{\beta_1}$
	Z		$\overline{\gamma_1}$	q

If at least one unlimited variable cannot be expressed through non-negative variables because of occurrence of zero in a column of the simplex-table corresponding to it such problem is not led to a canonical form and cannot be solved a simplex-method.

If in a line corresponding to a 0-variable, all elements except a free member are equal to zero, system of restrictions imcompatible.

Practical work №1-2

The numerical decision of the algebraic and transcendental equations iterative methods.

Let's consider the equation

$$f(x)=0 \tag{1.1}$$

Where $f(x)$ defined and continuous on some final or infinite interval $a<x<b$.

Any value x^* turning function $f(x)$ in zero $f(x^*)\equiv 0$, is called as a root of the equation (1.1), and the way of a finding of this value x^* and is the decision of the equation (1.1).

To find roots of the equation of a kind (1.1) precisely it is possible only in rare instances. Besides, often the equation contains the factors known only approximately and therefore, the problem about exact definition of roots of the equation loses meaning. Methods of the numerical decision of the equations of the kind (1.1) are developed, allowing to find the approached values of roots of this equation.

Thus it is necessary to solve two problems:

1) branch of roots, i.e. Search enough small areas, in each of which are concluded only one root of the equation;

2) calculation of roots with the set accuracy.

Let's take advantage of known result of the mathematical analysis: if continuous function accepts on the ends of some interval of value of different signs an interval contains at least one root of the equation.

For allocation of the areas containing one root, it is possible to use, for example, graphic in the way, or moving along a range of definition with some step, to check on the ends of intervals a condition of change of a sign on function.

For the decision of the second problem exists numerous methods from which we will consider four: a method of iterations, a method half divisions, a method of chords, a method of tangents.

The task 1

To make branch of roots: graphically and under the program (accuracy $\varepsilon=10^{-1}$). Individual tasks are resulted in table 1.

The task 2

1. To spend specification of roots by a method half divisions.

As initial approach we will choose $c=(a+b)/2$, then we investigate function on the ends of pieces $[a,c]$ and $[c,b]$. That piece at which value of function on the ends has opposite signs gets out. Process proceeds until the condition $|b-a|<\varepsilon$ will be satisfied. Accuracy ε to accept the equal 10^{-3}.

2. To make specification of roots by a method of simple iteration.

Let roots are separated and $[a,b]$ contains a unique root. The equation (1.1) we will lead to an iterative kind:

$$x=\varphi(x) \tag{1.2}$$

where function $\varphi(x)$ is differentiated on $[a,b]$ and for any. $x\in[a,b]\,|\,\varphi'(x)|<1$. Function $\varphi(x)$ can be picked up in a kind

$$\varphi(x)=x+kf(x), \tag{1.3}$$

Where k is from a condition $|\varphi'(k,x)|=|1+kf'(x)|<1$, for $\forall x\in[a,b]$.

Last condition guarantees convergence of iterative sequence $x_1,x_2,\cdots x_{n-1},x_n\cdots$ to a root ζ. As a condition of the termination of the account we will consider inequality performance

$$|x_n-x_{n-1}|<\frac{\varepsilon(1-q)}{q};\ q=max|\varphi'(x)| \tag{1.4}$$

3. To make specification of roots by a method of chords or tangents (X, K in table 1) with the set accuracy $\varepsilon=10^{-4}$.

The settlement formula for a method of chords:

$$x_{n+1}=\frac{x_0f(x_n)-x_nf(x_0)}{(f(x_n)-f(x_0))},$$

For a method of tangents:

$$x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)},$$

Value x_0 for a method of chords and an index point for a method of tangents gets out of a condition of performance of an inequality $f(x_0)f^{//}(x_0)>0$.

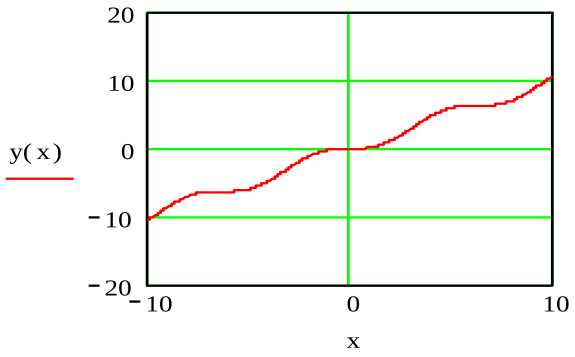
As a result of calculations under these formulas the sequence of the approached values of a root can be received $x_1, x_2, \cdots x_{n-1}, x_n \cdots$. Process of calculations comes to an end at condition performance $|x_n - x_{n-1}| < \varepsilon$ ($\varepsilon = 10^{-5}$). In each case to print quantity of the iterations necessary for achievement of set accuracy.

APPROXIMATE VARIANT OF PERFORMANCE OF WORK ON MATHCAD

1. Definition, construction of tables of values and schedules of functions and branch of roots of the equation *y=x-sinx-0,25*.

We separate roots graphically.
We calculate values of argument and function.

$$y(x) := x - \sin(x) - 0.25$$



$i := 0 .. 10$ $x_i := -5 + i$ $F_i := y(x_i)$

We type i, x_i F_i. More low, x= and nearby we click the mouse, we type F=, also nearby we click the mouse.

x =

	0
0	-5
1	-4
2	-3
3	-2
4	-1
5	0
6	1
7	2
8	3
9	4
10	5

F =

	0
0	-6.209
1	-5.007
2	-3.109
3	-1.341
4	-0.409
5	-0.25
6	-0.091
7	0.841
8	2.609
9	4.507
10	5.709

Given

$x - \sin(x) - 0.25 = 0$

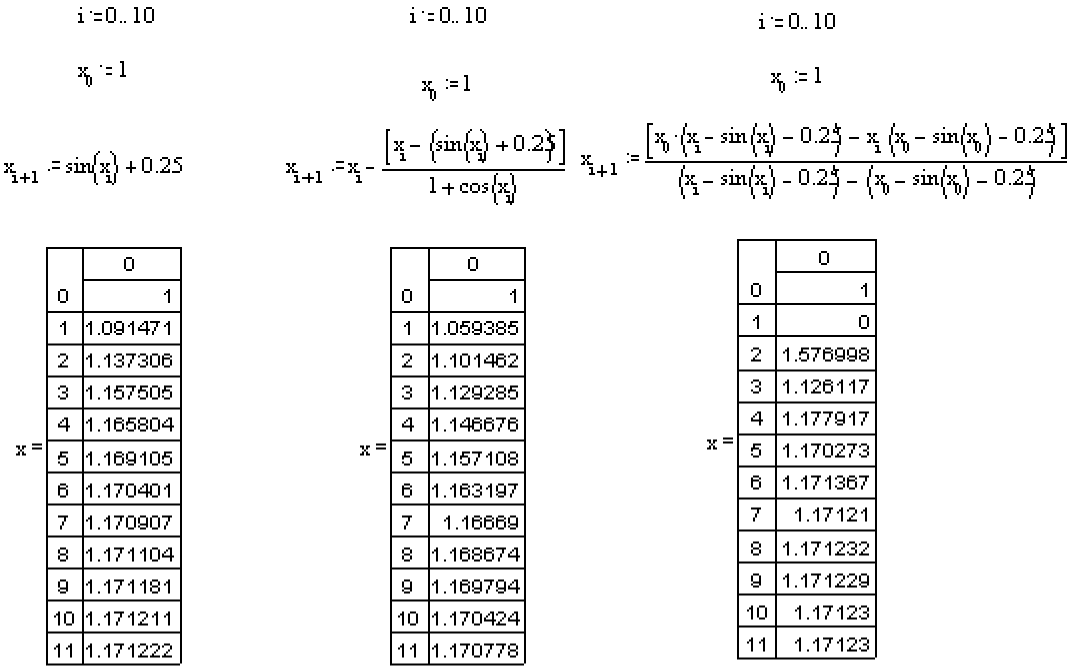
$\mathrm{Find}(x) \rightarrow 1.17122965250166599$

2. The decision with use of operators ***given, find***.

3. The symbolical decision.

$$x - \sin(x) - 0.25 \text{ solve}, x \rightarrow 1.17122965250166599$$

4. At the left the decision a method of iterations, in the middle a method of tangents, on the right a method of chords.



$i := 0..10$

$x_0 := 1$

$x_{i+1} := \sin(x_i) + 0.25$

$x =$

	0
0	1
1	1.091471
2	1.137306
3	1.157505
4	1.165804
5	1.169105
6	1.170401
7	1.170907
8	1.171104
9	1.171181
10	1.171211
11	1.171222

$i := 0..10$

$x_0 := 1$

$$x_{i+1} := x_i - \frac{[x_i - (\sin(x_i) + 0.25)]}{1 + \cos(x_i)}$$

$x =$

	0
0	1
1	1.059385
2	1.101462
3	1.129285
4	1.146676
5	1.157108
6	1.163197
7	1.16669
8	1.168674
9	1.169794
10	1.170424
11	1.170778

$i := 0..10$

$x_0 := 1$

$$x_{i+1} := \frac{[x_0 \cdot (x_i - \sin(x_i) - 0.25) - x_i \cdot (x_0 - \sin(x_0) - 0.25)]}{(x_i - \sin(x_i) - 0.25) - (x_0 - \sin(x_0) - 0.25)}$$

$x =$

	0
0	1
1	0
2	1.576998
3	1.126117
4	1.177917
5	1.170273
6	1.171367
7	1.17121
8	1.171232
9	1.171229
10	1.17123
11	1.17123

Table 1

N	Method	The equation
1	K	$x + x\ln(x+0.5) - 0.5 = 0$
2	K	$x2^x - 1 = 0$
3	X	$x^3 - 2x^2 + x - 3 = 0$
4	K	$x^3 + 12x - 2 = 0$
5	X	$5x - 8\ln(x) - 8 = 0$
6	K	$x^4 + 0.5x^3 - 4x^2 - 3x - 0.5 = 0$
7	X	$x - \sin(x) - 0.25 = 0$
8	K	$x^3 - 6x^2 + 20 = 0$
9	X	$5x^3 + 10x^2 + 5x - 1 = 0$
10	K	$0.1x^2 - x\ln(x) = 0$

Solve the equation with Newton's method

1. $x^3 + 2x^2 + 2 = 0$
2. $x^3 - 2x + 2 = 0$
3. $x^3 + x - 3 = 0$
4. $x^3 - 0{,}2x^2 + 0{,}4x - 1{,}4 = 0$
5. $x^3 + 3x^2 + 12x + 3 = 0$
6. $x^3 - 0{,}1x^2 + +0{,}4x + 1{,}2 = 0$
7. $x^3 - 0{,}2x^2 + 0{,}5x - 1{,}4 = 0$

14. $x^3 - 3x^2 + 9x - 10 = 0$
15. $x^3 + 3x - 1 = 0$
16. $x^3 + 0{,}4x^2 + 0{,}6x - 1{,}6 = 0$
17. $x^3 - 0{,}1x^2 + 0{,}4x - 1{,}4 = 0$
18. $x^3 - 0{,}2x^2 + 0{,}5x - 1 = 0$
19. $x^3 - 3x^2 + 6x - 5 = 0$
20. $x^3 + 2x + 4 = 0$

8. $x^3 - 3x^2 + 12x - 12 = 0$

9. $x^3 + 4x - 6 = 0$

10. $x^3 + 3x^2 + 6x - 1 = 0$

11. $x^3 - 3x^2 + 6x - 2 = 0$

12. $x^3 - 3x^2 + 12x - 9 = 0$

13. $x^3 + 3x + 1 = 0$

21. $x^3 + 0{,}2x^2 + 0{,}5x + 0{,}8 = 0$

22. $x^3 + 0{,}1x^2 + 0{,}4x - 1{,}2 = 0$

23. $x^3 - 0{,}1x^2 + 0{,}4x - 1{,}5 = 0$

24. $x^3 - 0{,}2x^2 + 0{,}3x - 1{,}2 = 0$

25. $x^3 + 0{,}2x^2 + 0{,}5x - 2 = 0$

26. $x^3 + 0{,}2x^2 + 0{,}5x - 1{,}2 = 0$

CONTROL QUESTIONS

1. Stages of the decision of the equation from one unknown person.
2. Ways of branch of roots.
3. How the graphic branch of roots is specified by means of calculations?
4. To give the verbal description of algorithm of a method половинного divisions.
5. Necessary conditions of convergence of a method половинного divisions.
6. Condition of the termination of the account of a method of simple iteration. A method error.
7. The verbal description of algorithm of a method of chords. Graphic representation of a method. Error calculation.
8. The verbal description of algorithm of a method of tangents (Newton). Graphic representation of a method. A condition of a choice of an index point.

Practical work № 3-4.
Newton's interpolation polynom and Lagrange

The plan:

Let function *f(x)* is set as table, or its calculation demands bulky calculations. We will replace approximately function *f(x)* with any function *F(x)* so that the deviation *f(x)* from *F(x)* was in the set area somewhat minimum. Similar replacement is called as function approximation *f(x)*, and function *F(x)* – approximating (approaching) function.

The classical approach to the decision of a problem of construction of approaching function is based on the requirement of strict coincidence of values *f (x)* and *F (x)* in points x_i (*i=0,1,2, … n*), i.e.

$$F(x_0)=y_0,\ F(x_1)=y_1,\ldots,F(x_n)=y_n. \tag{3.1}$$

In this case a finding of the approached function name interpolation (or interpolation), points $x_0, x_1, \cdots x_n$ – interpolation knots.

Often интерполирование it is conducted for the functions set by tables with equidistant values of argument *x*. In this case the table step $h=x_{i+1}-x_i\ (i=0,1,2,\ldots)$ is constant size. For such tables construction интерполяционных formulas (as, however, and calculation under these formulas) considerably becomes simpler.

По заданной таблице значений функции составить формулу интерполяционного многочлена **Лагранжа** (3.2) и построить график $L_2(x)$.. Исходные данные берутся из таблицы 3.1.

The task 1

Under the set table of values of function to make the formula interpolation a multinomial **of Lagrange** (3.2) and to construct the schedule *L₂(x)*. The initial data undertakes from table 3.1.

$$L_2(x)=y_0\frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}+y_1\frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}+y_2\frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} \tag{3.2}$$

Table 3.1.

№	x_0	x_1	x_2	y_0	y_1	y_2
1	2	3	5	4	1	7
2	4	2	3	5	2	8
3	0	2	3	-1	-4	2
4	7	9	13	2	-2	3
5	-3	-1	3	7	-1	4
6	1	2	4	-3	-7	2
7	-2	-1	2	4	9	1
8	2	4	5	9	-3	6
9	-4	-2	0	2	8	5
10	-1	1.5	3	4	-7	1
11	2	4	7	-1	-6	3
12	-9	-7	-4	3	-3	4
13	0	1	4	7	-1	8
14	8	5	0	9	2	4
15	-7	-5	-4	4	-4	5

The task 2

To calculate one value of the set function for intermediate value of argument (*a*) with the help interpolation a multinomial **of Lagrange** (3.3) and to estimate an interpolation error. For task performance the initial data undertakes from table 3.2, 3.3 or 3.4.

$$L_n(x)=\sum_{i=0}^{n} y_i\frac{(x-x_0)\ldots(x-x_{i-1})(x-x_{i+1})\ldots(x-x_n)}{(x_i-x_0)\ldots(x_i-x_{i-1})(x_i-x_{i+1})\ldots(x_i-x_n)} \tag{3.3}$$

For an error $R_n(x)$ inequality is carried out

$$|R_n(x)|\le\frac{M_{n+1}}{(n+1)!}|\Pi_{n+1}(x)|,\qquad x\in[x_o,x_n] \tag{3.4}$$

where $M_{n+1} = \max|f^{(n+1)}(x)|$.

The table 3.2

№ Variant	Valuea	№ tables
1	-2	3.3
2	3.77	3.4
3	0.55	3.3
4	4.83	3.4
5	3.5	3.3
6	5.1	3.4
7	1.75	3.3
8	4.2	3.4
9	-1.55	3.3
10	6.76	3.4

The table 3.3

x	-3.2	-0.8	0.4	2.8	4.0	6.4	7.6
$f(x) = 2.1\sin(0.37x)$	-1.94	-0.61	0.31	1.81	2.09	1.47	0.68

The table 3.4

x	1.3	2.1	3.7	4.5	6.1	7.7	8.5
$f(x) = \lg(x)/x + x^2$	1.777	4.563	13.84	20.39	37.34	59.41	72.4

The table 3.5

x	0.10	0.15	0.20	0.25	0.30	0.35	0.40
$f(x) = \cos(x)$	0.995	0.988	0.980	0.969	0.955	0.939	0.921

The table 3.6

x	0.65	0.70	0.75	0.80	0.85	0.90	0.95
$f(x) = \sin(x)$	0.605	0.644	0.681	0.71	0.75	0.783	0.813

The task 3.

To condense a part of the table set on a piece *[a, b]* functions, using interpolation **Newton's** multinomial (3.5) and to estimate an error of interpolation *D* (the formula (3.6)). Table 3.7 of final differences to count manually on a piece *[a, b]* with step *h*. For task performance the initial data undertakes from tables 3.8, 3.5 and 3.6.

$$P_2(x) = y_0 + t\Delta y_0 + \frac{t(t-1)}{2!}\Delta^2 y_0 + \frac{t(t-1)(t-2)}{3!}\Delta^3 y_0, \qquad (3.5)$$

where $t = \frac{x - x_0}{h}$.

$$D \approx \frac{t(t-1)(t-2)}{3!} f'''(\xi) \quad , \qquad (3.6)$$

where ξ – Some internal point of the least interval containing all knots $x_i\,(i = \overline{0,n})$ and x.

The formula (3.5) is called as the first interpolation **Newton's** formula. If calculated value of a variable is closer to the piece end *[a, b]*, apply Newton's second formula – interpolation back (the formula (3.6)).

$$P_n(x) = y_n + t\Delta y_{n-1} + \frac{t(t+1)}{2!}\Delta^2 y_{n-2} + \frac{t(t-1)(t-2)}{3!} \Delta^3 y_{n-3} \qquad (3.6)$$

где $t = \frac{x - x_n}{h}$ и $D = \frac{t(t+1)(t+2)}{3!} f'''(\xi)$.

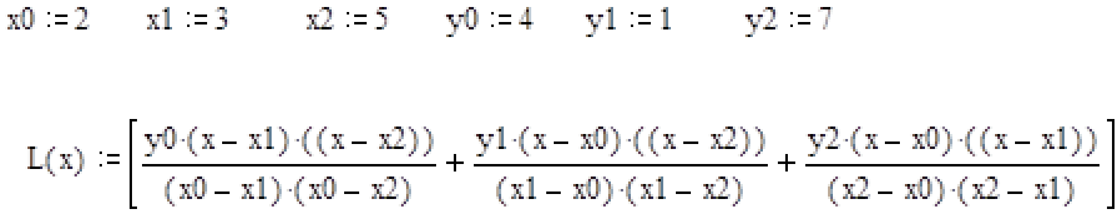
The table3.7

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
x_0	y_0	$\Delta\, y_0 = y_1 - y_0$	$\Delta^2 y_0 = \Delta y_1 - \Delta y_0$	$\Delta^3 y_0 = \Delta^2 y_1 - \Delta^2 y_0$
$x_1 = x_0 + h$	y_1	$\Delta\, y_1 = y_2 - y_1$	$\Delta^2 y_1 = \Delta y_2 - \Delta y_1$	
$x_2 = x_1 + h$	y_2	$\Delta\, y_2 = y_3 - y_2$		
$x_3 = x_2 + h$	y_3			

Table 3.8

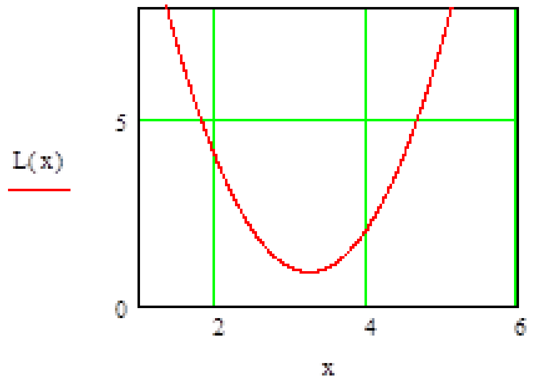
№	a	b	h_0	h	№ tables
1	0.65	0.80	0.05	0.01	3.6
2	0.25	0.40	0.05	0.025	3.5
3	0.75	0.90	0.05	0.01	3.6
4	0.70	0.85	0.05	0.025	3.6
5	0.80	0.95	0.05	0.025	3.6
6	0.1	0.25	0.05	0.025	3.5
7	0.15	0.3	0.05	0.025	3.5
8	0.7	0.85	0.05	0.025	3.6
9	0.2	0.35	0.05	0.01	3.5
10	0.80	0.95	0.05	0.01	3.6

Approximate fragment of performance of work in MathCAD



$x0 := 2 \qquad x1 := 3 \qquad x2 := 5 \qquad y0 := 4 \qquad y1 := 1 \qquad y2 := 7$

$$L(x) := \left[\frac{y0\cdot(x-x1)\cdot((x-x2))}{(x0-x1)\cdot(x0-x2)} + \frac{y1\cdot(x-x0)\cdot((x-x2))}{(x1-x0)\cdot(x1-x2)} + \frac{y2\cdot(x-x0)\cdot((x-x1))}{(x2-x0)\cdot(x2-x1)}\right]$$



$x0 := 2 \qquad x1 := 3 \qquad x2 := 5 \qquad y0 := 4 \qquad y1 := 1 \qquad y2 := 7$

$$L(x) := \left[\frac{4\cdot(x-3)\cdot((x-5))}{(2-3)\cdot(2-5)} + \frac{1\cdot(x-2)\cdot((x-5))}{(3-2)\cdot(3-5)} + \frac{7\cdot(x-2)\cdot((x-3))}{(5-2)\cdot(5-3)}\right]$$

$2\cdot x^2 - 13\cdot x + 22$

CONTROL QUESTIONS

1. In what feature of approach таблично the set function by a method of interpolation?
2. How existence and uniqueness interpolation a multinomial is proved?
3. How interpolation a multinomial degree is connected with quantity of knots of interpolation?
4. How are under construction interpolation multinomials of Lagrange and Newton?
5. In what feature of these two ways of interpolation?
6. How the estimation of an error of a method of interpolation is made by a multinomial of Lagrange?
7. How the method interpolation for specification of tables of functions is used?
8. In what difference between the first and the second interpolation Newton's formulas?

Practical work № 5-6
Calculation of integrals by the approached methods
The plan:

1. **Method of trapezes and Simpson**
2. **Methods of rectangles**
3. **The quadrature formula of Gaussa**

1. A method of trapezes and Simpson

Формулы, используемые для приближенного вычисления однократных интегралов, называют квадратурными формулами. Простой прием построения квадратурных формул состоит в том, что подынтегральная функция *f(x)* заменяется на отрезке [a,b] интерполяционным многочленом, например, многочленом Лагранжа $L_n(x)$; для интеграла имеем приближенное равенство (4.1). Предполагается, что отрезок [a,b] разбит на *n* частей точками (узлами) x_i, наличие которых подразумевается при построении многочлена $L_n(x)$. Для равноотстоящих узлов

The formulas used for approached calculation of unitary integrals, name квадратурными formulas. Simple reception of construction quadrature formulas consists that subintegral function *f(x)* is replaced on a piece [a,b] interpolation with a multinomial, for example, a multinomial of Lagrange $L_n(x)$; for integral it is had the approached equality (4.1). It is supposed that the piece [a,b] is broken on *n* parts by points (knots) x_i which presence is meant at construction of multinomial $L_n(x)$. For equidistant knots $x_i = x_0 + ih,\ h = \dfrac{b-a}{n},\ x_0 = a,\ x_n = b.$

$$\int_a^b f(x)dx \approx \int_a^b L_n(x)\,dx \qquad (4.1)$$

At certain assumptions we receive the formula of trapezes

$$\int_a^b f(x)dx \approx h\left(\frac{y_0 + y_n}{2} + y_1 + y_2 + \ldots + y_{n-1}\right), \qquad (4.2)$$

Where y_i values –of function in interpolation knots.

We have the following estimation of an error of a method of integration under the formula of trapezes (4.2):

$$|R_n| \le M\frac{|b-a|\cdot h^2}{12},\ \text{где}\ M = \max\left|f^{(2)}(x)\right|,\ x \in [a,b]. \qquad (4.3)$$

In many cases of more exact there is Simpson's formula (the formula of parabolas):

$$\int_a^b f(x)dx \approx \frac{2h}{3}\left(\frac{y_0 + y_{2m}}{2} + 2y_1 + y_2 + \ldots + 2y_{2m-1}\right). \qquad (4.4)$$

For Simpson's formula it is had the following estimation of an error:

$$|R_n| \le M\frac{|b-a|\cdot h^4}{180},\ \text{где}\ M = \max\left|f^{(4)}(x)\right|,\ x \in [a,b].$$

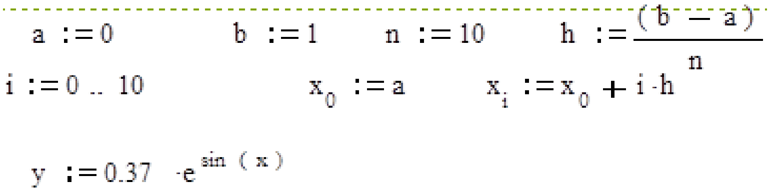
The task 1

To make the program of calculation of integral from the set function on a piece $[a,b]$ under the formula of trapezes with step *h*=0.1 and *h*=0.05. To compare results. To estimate accuracy under the formula (4.3). To compare results. The initial data for task performance undertakes from table 4.

The task 2

To make the program of calculation of integral from the set function on a piece $[a,b]$ under Simpson's formula a method of the repeated account with accuracy $\varepsilon = 10^{-6}$. The initial data for task performance undertakes from table 4.

To calculate integral in MathCAD from the set function on a piece [a, b] under the formula of trapezes and direct way.



$$a := 0 \qquad b := 1 \qquad n := 10 \qquad h := \frac{(b - a)}{n}$$

$$i := 0\,..\,10 \qquad x_0 := a \qquad x_i := x_0 + i\cdot h$$

$$y := 0.37\cdot e^{\sin(x)}$$

$$s := h \cdot \left(\sum_{i=1}^{n-1} y_i + \frac{y_0 + y_n}{2} \right)$$

$$s = 0.604$$

$$\int_0^1 0.37 \; e^{\sin(x)} \, dx = 0.604$$

Таблица 4

N	Функция	a	b
1	$0.37e^{\sin x}$	0	1
2	$0.5x + x\ln x$	1	2
3	$(x+1.9)\sin(x/3)$	1	2
4	$\frac{1}{x}\ln(x+2)$	2	3
5	$\frac{3\cos x}{2x+1.7}$	0	1
6	$(2x+0.6)\cos(x/2)$	1	2
7	$2.6x^2 \ln x$	1.2	2.2
8	$(x^2+1)\sin(x-0.5)$	1	2
9	$x^2\cos(x/4)$	2	3
10	$\frac{\sin(0.2x-3)}{x^2+1}$	3	4

3. A method of rectangles

The elementary methods of numerical integration are methods of rectangles. In them subintegral function is replaced with a polynom of zero degree, that is a constant. Similar replacement is ambiguous as the constant can be chosen subintegral function equal to value in any point of an interval of integration. Depending on it methods of rectangles share on: methods of the left, right and average rectangles.

On a method of average rectangles the integral is equal to the sum of the areas of rectangles where the rectangle basis any small size (accuracy), and the height is defined on a point of intersection of the top basis of a rectangle which the function schedule should cross in the middle. Accordingly we receive the formula of the areas for a method of average rectangles:

$$S_b = \sum_a^b \frac{|f(x_1) + (fx_2)|}{2} \varepsilon \quad (5)$$

The formula of average rectangles with constant step:

$$\int_a^b f(x)dx \approx \frac{1}{2} h \sum_{i=0}^{n-1} f\left(x_i + \frac{h}{2}\right) \quad (6)$$

5. The quadrature formula of Gaussa

The methods described above use the fixed points of a piece (the ends and the middle) and have a low order of accuracy (0 – methods of the right and left rectangles, 1 – methods of average rectangles and trapezes, 3 – a method of parabolas (Simpson)). If we can choose points in which we calculate values of function *f (x)* it is possible to receive methods of higher order of accuracy at the same quantity of calculations of subintegral function. So for two (as in a method of trapezes) calculations of values of subintegral function, it is possible to receive a method any more 1st, and 3rd order of accuracy:

$$I \approx \frac{b-a}{2}\left(f\left(\frac{a+b}{2} - \frac{b-a}{2\sqrt{3}}\right) + f\left(\frac{a+b}{2} + \frac{b-a}{2\sqrt{3}}\right)\right).$$

Generally, using n points, it is possible to receive a method with accuracy order 2n-1. Values of knots of a method of Gaussa on n points are roots of a polynom of Lezhandra of degree n.

Values of knots of a method of Gaussa and their scales are resulted in directories of special functions. The method of Gaussa on five points is most known.

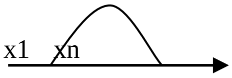
Examples

Example 1.

Application of the formula of average rectangles for the decision of problems of numerical integration (on calculation example $\int_1^2 (x^2+1)\sin(x-0.5)dx$).

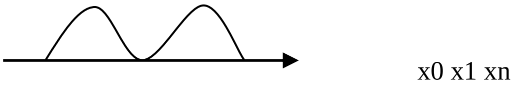
The decision.

$$\int_1^2 (x^2+1)\sin(x-0.5)dx = h\sum_{i=0}^{n-1} f\left(x_i+\frac{h}{2}\right)$$



Let's calculate integral I1 under the formula of a method of average rectangles (6):
h1=1

I1=hf(x0+h/2)=((1.5)2+1)sin(1.5-0.5)=2.734

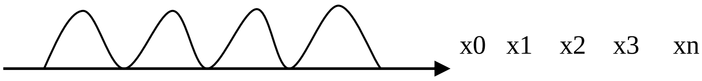


Let's reduce a step twice and we will calculate integral I2 under the formula of a method of average rectangles (6):

h2=1/2
I2= h(f(x0+h/2)+f(x1+ h/2))= (1/2) ((1.25)2+1)sin(1.25-0.5)+ ((1.75)2+1)sin(1.75-0.5))=2.8005
Let's calculate criterion for integrals I1 and I2, as I2≥1 the criterion is calculated under the formula:

$|(I_2-I_1)/I_2|=0.023746>\varepsilon$

The received criterion is not carried out, we calculate integral I3, reducing a step twice:



h2=1/4
I3=h(f(x0+h/2)+f(x1+h/2)+f(x2+h/2)+f(x3+h/2))=(1/4)((1.125)2+1)sin(1.125-0.5)+(1.375)2+1)sin(1.375--0.5)+(1.625)2+1)sin(1.625-0.5)+ (1.875)2+1)sin(1.875-0.5))=2.814

Let's calculate criterion for integrals I2 and I3, as I3≥1 the criterion is calculated under the formula:
$|(I_3-I_2)/I_3|=0.004797<\varepsilon$

The received criterion is carried out, hence, we have calculated the set integral with demanded accuracy.
The answer: $\int_1^2 (x^2+1)\sin(x-0.5)dx =$2.814 with accuracy0.01.

Example 2. We Will calculate integral $\int_{0.5}^{3} \frac{2x^3}{x^4}dx$ method of Gaussa.

The decision.

$$I \approx \frac{b-a}{2}\left(f\left(\frac{a+b}{2}-\frac{b-a}{2\sqrt{3}}\right)+f\left(\frac{a+b}{2}+\frac{b-a}{2\sqrt{3}}\right)\right)$$

$$f(x)=\frac{2x^3}{x^4}\cdot$$

$$f1(x)=f\left(\frac{a+b}{2}-\frac{b-a}{2\sqrt{3}}\right)=f\left(\frac{0.5+3}{2}-\frac{3-0.5}{2\sqrt{3}}\right)=f(1.029)=1.94\cdot$$

$$f2(x)=f\left(\frac{a+b}{2}+\frac{b-a}{2\sqrt{3}}\right)=f\left(\frac{0.5+3}{2}+\frac{3-0.5}{2\sqrt{3}}\right)=f(2.47)=0.812$$

$$\int_{0.5}^{3}\frac{2x^3}{x^4}dx=\frac{3-0.5}{2}(1.94+0.812)\approx 3.584.$$

The answer: 3.584.

Example 3. We Will calculate integral $\int_{0.5}^{2.3}\pi\cdot\sin(\pi x)dx$ method of Gaussa.

The decision.

$$f(x)=\pi\cdot\sin(\pi x).$$

$$f1(x)=f\left(\frac{a+b}{2}-\frac{b-a}{2\sqrt{3}}\right)=f\left(\frac{0.5+2.3}{2}-\frac{2.3-0.5}{2\sqrt{3}}\right)=f(0.88)=-1.156\cdot$$

$$f2(x)=f\left(\frac{a+b}{2}+\frac{b-a}{2\sqrt{3}}\right)=f\left(\frac{0.5+2.3}{2}+\frac{2.3-0.5}{2\sqrt{3}}\right)=f(1.92)=0.781$$

$$\int_{0.5}^{2.3}\pi\cdot\sin(\pi x)dx=\frac{2.3-0.5}{2}(-1.156+0.781)\approx -0.588\cdot$$

The answer: - 0.588.

Exercise

Calculate the set integrals under formulas of rectangles, a trapeze and Simpson if the integration piece is broken on n=2 and n=4 equal parts. To estimate an error of result and сравныть the approached values of integral with the exact.

1. $\int_0^1\frac{dx}{1+x^2}\ \left(\text{З}=\frac{\pi}{4}\approx 0{,}785\right).$

2. $\int_0^1\frac{dx}{1+x}\quad (\text{З}=\ln 2\approx 0{,}693).$

3. $\int_0^{\pi/4}\sin 4x\,dx\quad (\text{З}=0{,}5).$

4. $\int_0^1\frac{dx}{\sqrt{1+x^2}}\quad (\text{З}=\ln(1+\sqrt{2})\approx 0{,}881).$

5. $\int_1^e \ln x\,dx\quad (\text{З}=1).$

6. $\int_0^1\ln(x+1)dx\quad (\text{З}=2\ln 2-1\approx 0{,}386).$

7. $\int_0^{\pi/2}x\cos x\,dx\ \left(\text{З}=\frac{\pi}{2}-1\approx 0{,}571\right).$

8. $\int_0^1\frac{e^x dx}{1+e^{2x}}\quad \left(\text{З}=\operatorname{arctg}e-\frac{\pi}{4}\approx 0{,}433\right).$

9. $\int_0^{\pi}\cos^3 x\,dx\quad (\text{З}=0).$

10. $\int_0^{\pi/4}\frac{dx}{\cos x}\quad (\text{З}=\ln(1+\sqrt{2})\approx 0{,}881).$

11. $\int_0^{\pi/2}\frac{dx}{1+\sin x}\quad (\text{З}=1).$

12. $\int_0^1\operatorname{arctg}x\,dx\ \left(\text{З}=\frac{1}{4}(\pi-2\ln 2)\approx 0{,}438\right).$

13. $\int_0^1\frac{dx}{1+e^x}\quad (\text{З}\approx 0{,}38).$

14. $\int_0^{\sqrt{2}/2}\arcsin x\,dx\ \left(\text{З}=\frac{\sqrt{2}}{8}(\pi+4)-1\approx 0{,}26\right).$

15. $\int_0^{\pi/4}\operatorname{tg}x\,dx\ \left(\text{З}=\frac{1}{2}\ln 2\approx 0{,}346\right).$

16. $\int_{\pi/4}^{\pi/2}\operatorname{ctg}x\,dx\quad \left(\text{З}=\frac{1}{2}\ln 2\approx 0{,}346\right).$

17. $\int_0^1 x\,e^x\,dx\quad (\text{З}=1).$

18. $\int_0^1\sqrt{1+x}\,dx\quad \left(\text{З}=\frac{2}{3}(2\sqrt{2}-1)\approx 1{,}22\right).$

CONTROL QUESTIONS

1. What advantages of the formula of parabolas in comparison with the formula of trapezes and a consequence of that are these advantages?

2. Whether formulas (4.2) are true, (4.4) for is unequal straining knots?

3. In what cases the approached formulas of trapezes and parabolas appear exact?

4. How the step size influences accuracy of numerical integration?

5. In what way it is possible to predict approximate size of a step for achievement of the set accuracy of integration?

6. Whether it is possible to achieve unlimited reduction of an error of integration by consecutive reduction of a step?

Practical work № 7-8.
Approximation results of experiment with a method of the least square. Creation non-linear empirical connection

The plan:

1. Root-mean-square approach of functions

2. A method of the least squares

1. Root-mean-square approach of functions

Let dependence between variables *x* and *y* is set таблично (the skilled data is set). It is required to find function somewhat in the best way describing the data. One of ways of selection of such (approaching) function is the method of the least squares. The method consists in that the sum of squares of deviations of values of required function $\bar{y}_i = \bar{y}(x_i)$ and set таблично y_i was the least:

$$S(c) = (y_1 - \bar{y}_1)^2 + (y_2 - \bar{y}_2)^2 + \ldots + (y_n - \bar{y}_n)^2 \to \min \qquad (6.1)$$

Where *c a* vector –of parametres of required function.

2. A method of the least squares

To construct a method of the least squares two empirical formulas: linear and square-law.

In case of linear function *y=ax+b the* problem is reduced to a finding of parametres *a* and *b* from system of the linear equations

$$\begin{cases} M_{x^2}a + M_x b = M_{xy} \\ M_x a + b = M_y \end{cases}, \text{ Where}$$

$$M_{x^2} = \frac{1}{n}\sum_{i=1}^{n} x_i^{\,2}, \quad M_x = \frac{1}{n}\sum_{i=1}^{n} x_i, \quad M_{xy} = \frac{1}{n}\sum_{i=1}^{n} x_i y_i, \; M_y = \frac{1}{n}\sum_{i=1}^{n} y_i$$

а в случае квадратичной зависимости $y = ax^2 + bx + c$ к нахождению параметров a, b и c из системы уравнений:

and in case of square-law dependence $y = ax^2 + bx + c$ to a finding of parameters *a*, *b* and *c* from system of the equations:

$$\begin{cases} M_{x^4}a + M_{x^3}b + M_{x^2}c = M_{x^2y} \\ M_{x^3}a + M_{x^2}b + M_x c = M_{xy} \\ M_{x^2}a + M_x b + c = M_y \end{cases}, \text{ Where}$$

$$M_{x^4} = \frac{1}{n}\sum_{i=1}^{n} x_i^{\,4}, \quad M_{x^3} = \frac{1}{n}\sum_{i=1}^{n} x_i^{\,3}, \quad M_{x^2y} = \frac{1}{n}\sum_{i=1}^{n} x_i^{\,2} y_i$$

To choose from two functions the most suitable. For this purpose to make the table for calculation of the sum of squares of evasion under the formula (6.1). Initial given to take from table 6.

The task 2

To make the program for a finding of approaching functions of the set type with a conclusion of values of their parametres and the sums of squares of evasion corresponding to them. To choose as approaching functions the following: $y = ax + b$, $y = ax^m$, $y = ae^{mx}$. To spend linearization. To define for what kind of function the sum of squares of evasion is the least.

Initial data is placed in table 6.

Approximate fragment of performance of laboratory work

(George E. Forsyth and Michael A. Malcolm and Cleve B. Moler. Computer Methods for Mathematical Computations. Prentice-Hall, Inc., 1977.)

$i := 1..10$

$x_1 := 0.5$ $y_1 := 1.8$

$x_2 := 0.1$ $y_2 := 1.1$

$x_3 := 0.4$ $y_3 := 1.8$

$x_4 := 0.2$ $y_4 := 1.4$

$x_5 := 0.6$ $y_5 := 2.1$

$x_6 := 0.3$ $y_6 := 1.8$

$x_7 := 0.4$ $y_7 := 1.6$

$x_8 := 0.7$ $y_8 := 2.2$

$x_9 := 0.3$ $y_9 := 1.5$

$x_{10} := 0.8$ $y_{10} := 2.3$

$$mx2 := 1\frac{\left[\sum_{i=1}^{10}\left(x_i\right)^2\right]}{10} \qquad mx := 1\cdot\frac{\left(\sum_{i=1}^{10}x_i\right)}{10} \qquad mxy := 1\frac{\left(\sum_{i=1}^{10}x_i\cdot y_i\right)}{10} \qquad my := 1\frac{\left(\sum_{i=1}^{10}y_i\right)}{10}$$

$mx2 = 0.229$ $mx = 0.43$ $mxy = 0.828$ $my = 1.76$

Given

$mx2a + mxb = mxy$

$mxa + b = my$

$Find(a,b) \rightarrow$

Table 6

i №		1	2	3	4	5	6	7	8	9	10
1	*x*	0.5	0.1	0.4	0.2	0.6	0.3	0.4	0.7	0.3	0.8
	y	1.8	1.1	1.8	1.4	2.1	1.8	1.6	2.2	1.5	2.3
2	*x*	1.7	1.5	3.7	1.1	6.2	0.3	6.5	3.6	3.8	5.9
	y	1.5	1.4	1.6	1.3	2.1	1.1	2.2	1.8	1.7	2.3
3	*x*	1.7	1.1	1.6	1.2	1.9	1.5	1.8	1.4	1.3	1.0
	y	6.7	5.6	6.7	6.1	7.4	6.9	7.9	5.9	5.6	5.3
4	*x*	1.3	1.2	1.5	1.4	1.9	1.1	2.0	1.6	1.7	1.8
	y	5.5	5.9	6.3	5.8	7.4	5.4	7.6	6.9	6.6	7.5
5	*x*	2.3	1.4	1.0	1.9	1.5	1.8	2.1	1.6	1.7	1.3
	y	5.3	3.9	2.9	5.0	4.0	4.9	5.1	4.5	4.1	3.7
6	*x*	1.8	2.6	2.3	1.3	2.0	2.1	1.1	1.9	1.6	1.5
	y	4.4	6.4	5.3	3.7	4.9	5.6	3.0	5.0	4.3	3.7
7	*x*	1.9	2.1	2.0	2.9	3.0	2.6	2.5	2.7	2.2	2.8
	y	6.6	7.6	6.7	9.2	9.4	7.8	8.4	8.0	7.9	8.7
8	*x*	2.0	1.4	1.0	1.7	1.3	1.6	1.9	1.5	1.2	2.1
	y	7.5	6.1	4.8	7.4	5.7	7.0	7.1	6.8	6.0	8.9
9	*x*	2.0	1.2	1.8	1.9	1.1	1.7	1.6	1.4	1.5	1.3
	y	7.5	5.9	7.0	8.0	5.0	7.4	6.4	6.6	6.3	5.7
10	*x*	1.9	1.1	1.4	2.3	1.7	2.1	1.6	1.5	1.0	1.2
	y	4.7	3.4	3.8	5.2	4.6	5.5	3.9	3.9	3.2	3.5

CONTROL QUESTIONS

1. In what an approach essence таблично the set function on a method of the least squares?
2. Than this method differs from an interpolation method?
3. How the problem of construction of approaching functions in the form of various elementary functions to a case of linear function is reduced?
4. Whether there can be a sum of squares of evasion for any approaching functions equal to zero?
5. What elementary functions are used as approaching functions?
6. How to find parametres for linear and square-law dependence, using a method of the least squares?

Practical work № 9.
The geometrical decision of a problem of linear programming

1. Geometrical interpretation of a problem of linear programming
2. Using geometrical interpretation, find decisions of problems

1. Geometrical interpretation of a problem of linear programming

1.29. To find a maximum and a minimum of function $F=x_1+x_2$ under conditions

$$\begin{cases} 2x_1+4x_2\leqslant 16, \\ -4x_1+2x_2\leqslant 8, \\ x_1+3x_2\geqslant 9, \end{cases}$$
$$x_1,\ x_2\geqslant 0.$$

The decision. We will construct a polygon of decisions. For this purpose in inequalities of system of restrictions and conditions nonnegativity variables signs on inequalities we will replace with signs on exact equalities:

$$\begin{cases} 2x_1+4x_2=16, & \text{(I)} \\ 4x_1+2x_2=\ 8, & \text{(II)} \\ x_1+3x_2=\ 9, & \text{(III)} \\ x_1=0, & \text{(IV)} \\ x_2=0. & \text{(V)} \end{cases}$$

Having constructed the received straight lines, will find corresponding semiplanes and their crossing (fig. 1.6).

Apparently from fig. 1.6, a polygon of decisions of a problem is triangle ABC. Co-ordinates of points of this triangle satisfy to a condition nonnegativity and to inequalities of system of restrictions of a problem. Hence, the problem will be solved, if among points of triangle ABC to find such in which function $F=x_1+x_2$ accepts the maximum and minimum values. For a finding of these points we will construct a straight line $x_1+x_2=4$ (number 4 is taken any) and a vector C = (1 ; 1).

Moving the given straight line in parallel to itself in a direction of a vector With, we see that its last general point with a polygon of decisions of a problem is point C. Hence, in this point function F accepts the maximum value. As with - a point of intersection of straight lines I and II its co-ordinates satisfy to the equations of these straight lines:

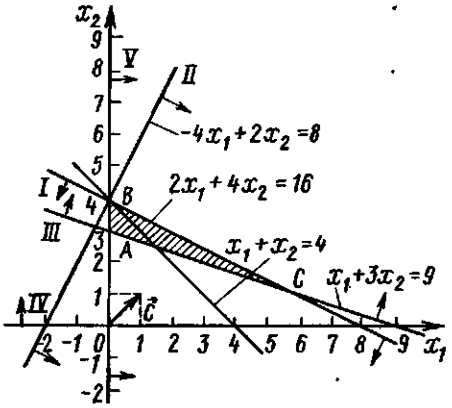


Fig. 1.6

$$\begin{cases} 2x_1+4x_2=16, \\ x_1+3x_2=\ 9. \end{cases}$$

Having solved this system of the equations, we will receive $x_1^*=6,\ x_2^*=1$. Thus, the maximum value of function $F_{max}=7$.

For a finding of the minimum value of criterion function of a problem it is moved a straight line $x_1+x_2=4$ in a direction opposite to a direction of vector C = (1 ; 1). In this case, apparently from fig. 1.6, last general point of a straight line with a polygon of decisions of a problem is A.Sledovatelno's point, in this point function F accepts the minimum value. For definition of co-ordinates of a point And we solve system of the equations

$$\begin{cases} x_1+3x_2=9, \\ x_1=0, \end{cases}$$

whence $x_1^*=0,\ x_2^*=3$.. Substituting the found values of variables in criterion function, we will receive $F_{min} = 3$.

1.30. To find the maximum value of function $F =-16x_1-X_2+x_3 + 5x_4 + 5x_5$ under conditions

$$\begin{cases} 2x_1+\ x_2+x_3=10, \\ -2x_1+3x_2+x_4=\ 6, \\ 2x_1+4x_2-x_5=\ 8, \end{cases}$$

$$x_1,\ x_2,\ x_3,\ x_4,\ x_5\geqslant 0.$$

The decision. Unlike considered above problems in an initial problem of restriction are set in the form of the equations. Thus number of unknown persons equally five. Therefore the given problem should be reduced to a problem in which the number of unknown persons would be equal to two. In the case under consideration it can be made by transition from the initial problem which have been written down in the form of basic, to the Problem which has been written down in the form of standard.

It has been above shown that the initial problem is written down, in the form of the basic for a problem consisting in a finding of the maximum value of function $F = 2x_1+ 3\ x_2$ under conditions

$$\begin{cases} 2x_1+\ x_2\leqslant 10, \\ -2x_1+3x_2\leqslant\ 6, \\ 2x_1+4x_2\geqslant\ 8, \end{cases}$$

$$x_1,\ x_2,\ x_3,\ x_4,\ x_5\geqslant 0.$$

From criterion function of an initial problem variables x_3, x_4, x_5 are excluded by means of substitution of their values from the corresponding equations of system of restrictions.

Let's construct a polygon of decisions of the received problem (fig. 1.7). Apparently from fig. 1.7, the maximum value problem criterion function accepts in a point from crossing of straight lines I and II. Along each of boundary straight lines value of one of the variables, excluded at transition to corresponding inequality, is equal to zero. Therefore in each of tops of the received polygon of decisions of last problem at least two variables of an initial problem accept zero values. So, in

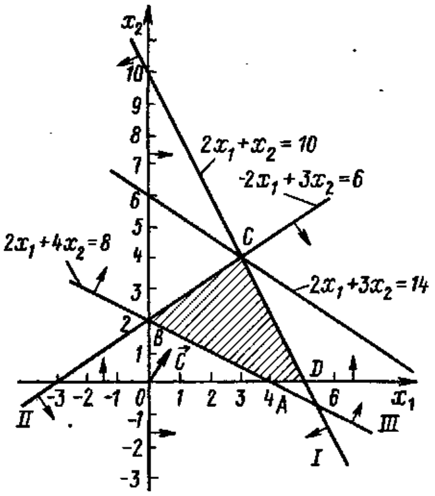


Рис. 1.7

To point C it is had x_3=0 and x_4=0. Substituting these values in the first and second equations of system of restrictions of an initial problem, we receive system of two equations

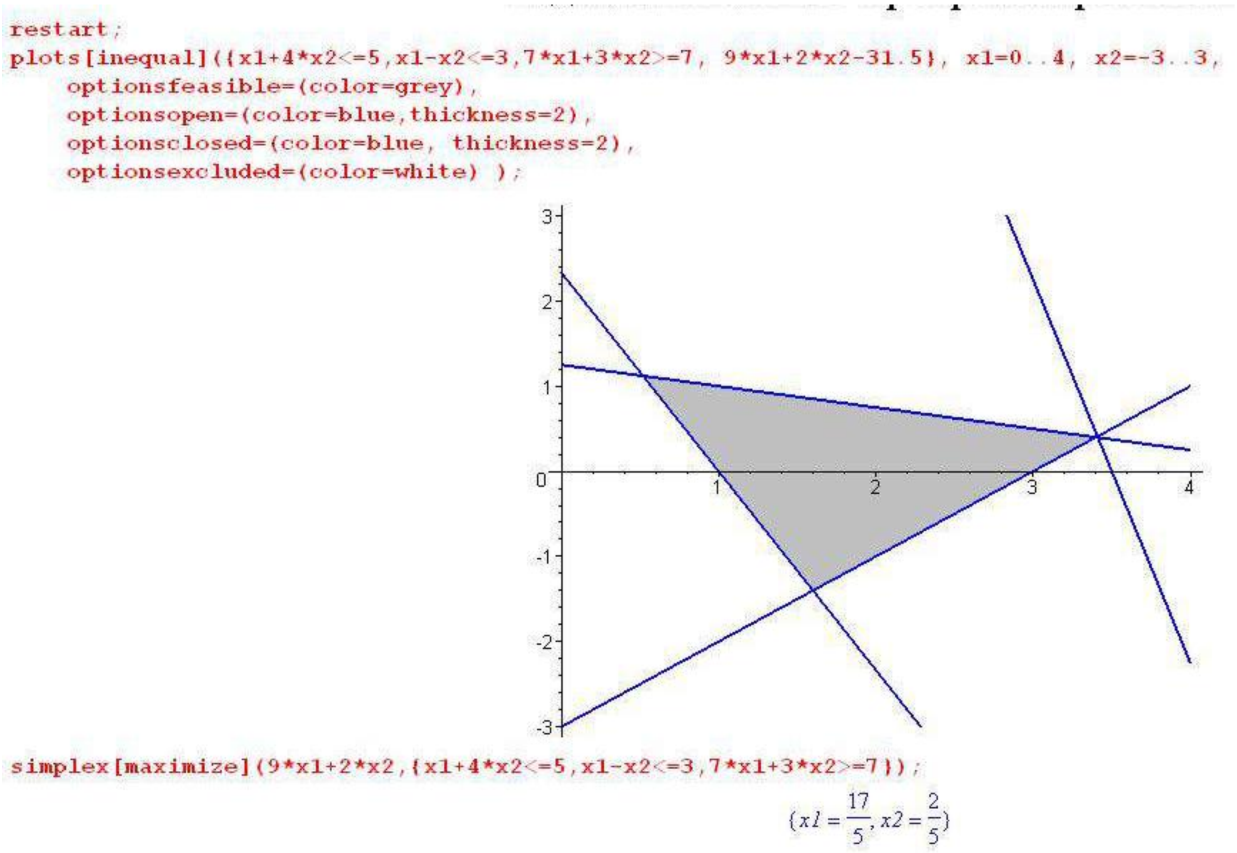
$$\begin{cases} 2x_1+\ x_2=10, \\ -2x_1+3x_2=\ 6, \end{cases}$$

Solving which it is found $x_1^*=3,\ x_2^*=4$.

Substituting the found values x_1 and x_2 in the third equation of system of restrictions of an initial problem, we define value of a variable x_5, equal 14.

Hence, the optimum plan of a considered problem is $X^* = (3;\ 4;\ 0;\ 0;\ 14)$. At this plan value of criterion function is $F_{max} = 18$.

Решение задачи в Maple



```
restart;
plots[inequal]({x1+4*x2<=5,x1-x2<=3,7*x1+3*x2>=7, 9*x1+2*x2-31.5}, x1=0..4, x2=-3..3,
    optionsfeasible=(color=grey),
    optionsopen=(color=blue,thickness=2),
    optionsclosed=(color=blue, thickness=2),
    optionsexcluded=(color=white) );
```

```
simplex[maximize](9*x1+2*x2,{x1+4*x2<=5,x1-x2<=3,7*x1+3*x2>=7});
```

$$\left\{x1 = \frac{17}{5}, x2 = \frac{2}{5}\right\}$$

2. Using geometrical interpretation, find decisions of problems

1.32. $F = x_1 + x_2 \to \max;$

$$\begin{cases} x_1 + 2x_2 \leqslant 14, \\ -5x_1 + 3x_2 \leqslant 15, \\ 4x_1 + 6x_2 \geqslant 24, \\ x_1,\ x_2 \geqslant 0. \end{cases}$$

1.33. $F = x_1 + 2x_2 \to \max;$

$$\begin{cases} 4x_1 - 2x_2 \leqslant 12, \\ -x_1 + 3x_2 \leqslant 6, \\ 2x_1 + 4x_2 \geqslant 16, \\ x_1,\ x_2 \geqslant 0. \end{cases}$$

1.34. $F = -2x_1 + x_2 \to \min;$

$$\begin{cases} 3x_1 - 2x_2 \leqslant 12, \\ -x_1 + 2x_2 \leqslant 8, \\ 2x_1 + 3x_2 \geqslant 6, \\ x_1,\ x_2 \geqslant 0. \end{cases}$$

1.35. $F = -x_1 + 4x_2 + 2x_4 - x_5 \to \max;$

$$\begin{cases} x_1 - 5x_2 + x_3 = 5, \\ -x_1 + x_2 + x_4 = 4, \\ x_1 + x_2 + x_5 = 8. \\ x_1,\ x_2,\ \ldots,\ x_5 \geqslant 0. \end{cases}$$

1.36. $F = -5x_1 + x_2 - x_3 \to \max;$

$$\begin{cases} 3x_1 - x_2 - x_3 = 4, \\ x_1 - x_2 + x_3 - x_4 = 1, \\ 2x_1 + x_2 + 2x_3 + x_5 = 7. \\ x_1,\ x_2,\ \ldots,\ x_5 \geqslant 0. \end{cases}$$

1.37. For manufacture of tables and cases the furniture factory uses necessary resources. Norms of expenses of resources on one product of the given kind, profit on realisation of one product and total of available resources of each kind are resulted in the following table:

Resources	Norms of expenditures of resources on one product		The General an amount of resources
	Table	The Case	
Wood (м3): I aspect	0,2	0,1	40
II aspect	0,1	0,3	60
Labour input (person-hour)	1,2	1,5	371,4
Profit on realisation of one product (rouble.)	6	8	

To define, how many tables and cases the factory should produce, that the profit on their realisation was maximum.

1.38. For manufacture of two kinds of products A and B the turning, milling and grinding equipment is used. Norms of expenses of time for each of equipment types on one product of the given kind are resulted in the table. In it the general fund of working hours of each of equipment types, and also profit on realisation of one product is specified.

Type the equipment	Expenditures of time (machine tool-hour) on working off of one product		The General fund of useful working hours of the equipment (hour)
	A	B	
The milling	10	8	168
The turning	5	10	180
The grinding	6	12	144
Profit on realisation of one product (roub.)	14	18	

To find the plan of release of products A and B, providing the maximum profit on their realisation.

1.39. At furniture factory it is necessary to cut out preparations of three kinds from plywood standard sheets in the quantities accordingly equal of 24, 31 and 18 pieces Each sheet of plywood can be cut on for
Cooking by two ways. The quantity of received preparations at the given way раскроя is resulted in the table. In it the size of a waste which turn out at the given way раскроя one sheet of plywood is specified.

Aspect preform	Amount of preforms (piece) in open on a mode	
	1	2
I	2	6
II	5	4
III	2	3
Magnitude of a waste (sm^3)	12	16

To define, how many sheets of plywood and on what way follow раскроить so that has been received not less the necessary quantity of preparations at the minimum waste.

1.40. On a fur farm silver foxes and polar foxes can be grown up. For maintenance of normal conditions of their cultivation it is used three kinds of forages. The quantity of a forage of each kind which foxes and polar foxes should receive daily, is resulted in the table. In it are specified total of a forage of each kind which can be used a fur farm, and profit on realisation of one skin of a fox and a polar fox.

Feed kind	Quantity of units of a feed which daily should receive		Feed Total
	fox	A polar fox	
I	2	3	180
II	4	1	240
III	6	7	426
Profit on realisation of one glass-paper (roub.)	16	12	

To define, how many foxes and polar foxes should be grown up on a fur farm that the profit on realisation of their skins was maximum.

LABORATORY MATERIALS

Laboratory work №1-2

The numerical decision of the algebraic and transcendental equations by iterative methods Chord and Newton

Let's consider the equation

$$f(x) = 0 \tag{1.1}$$

Where $f(x)$ defined and continuous on some final or infinite interval $a < x < b$.

Any value x^* turning function $f(x)$ in zero $f(x^*) \equiv 0$, is called as a root of the equation (1.1), and the way of a finding of this value x^* and is the decision of the equation (1.1).

To find roots of the equation of a kind (1.1) precisely it is possible only in rare instances. Besides, often the equation contains the factors known only approximately and therefore, the problem about exact definition of roots of the equation loses meaning. Methods of the numerical decision of the equations of the kind (1.1) are developed, allowing to find the approached values of roots of this equation.

Thus it is necessary to solve two problems:

1) branch of roots, i.e. Search enough small areas, in each of which are concluded only one root of the equation;

2) calculation of roots with the set accuracy.

Let's take advantage of known result of the mathematical analysis: if continuous function accepts on the ends of some interval of value of different signs an interval contains at least one root of the equation.

For allocation of the areas containing one root, it is possible to use, for example, graphic in the way, or moving along a range of definition with some step, to check on the ends of intervals a condition of change of a sign on function.

For the decision of the second problem exists numerous methods from which we will consider four: a method **of iterations**, a method half **divisions**, a method **of chords**, a method **of tangents**.

The task 1

To make branch of roots: graphically and under the program (accuracy $\varepsilon = 10^{-1}$). Individual tasks are resulted in table 1.

The task 2

1. To spend specification of roots by a method half divisions.

As initial approach we will choose $c = (a+b)/2$, then we investigate function on the ends of pieces $[a,c]$ and $[c,b]$. That piece at which value of function on the ends has opposite signs gets out. Process proceeds until the condition $|b-a| < \varepsilon$ will be satisfied. Accuracy ε to accept the equal 10^{-3}.

2. To make specification of roots by a method of simple iteration.

Let roots are separated and $[a,b]$ contains a unique root. The equation (1.1) we will lead to an iterative kind:

$$x = \varphi(x) \tag{1.2}$$

where function $\varphi(x)$ is differentiated on $[a,b]$ and for any. $x \in [a,b] \, | \varphi'(x) | < 1$. Function $\varphi(x)$ can be picked up in a kind

$$\varphi(x) = x + kf(x), \tag{1.3}$$

Where k is from a condition $| \varphi'(k,x) | = | 1 + kf'(x) | < 1$, for $\forall x \in [a,b]$.

Last condition guarantees convergence of iterative sequence $x_1, x_2, \cdots x_{n-1}, x_n \cdots$ to a root ζ. As a condition of the termination of the account we will consider inequality performance

$$|x_n - x_{n-1}| < \frac{\varepsilon(1-q)}{q}; \; q = max|\varphi'(x)| \tag{1.4}$$

3. To make specification of roots by a method of chords or tangents (X, K in table 1) with the set accuracy $\varepsilon = 10^{-4}$.

The settlement formula for a method of chords:

$$x_{n+1} = \frac{x_0 f(x_n) - x_n f(x_0)}{(f(x_n) - f(x_0))},$$

For a method of tangents:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)},$$

Value x_0 for a method of chords and an index point for a method of tangents gets out of a condition of performance of an inequality $f(x_0)f^{//}(x_0)>0$.

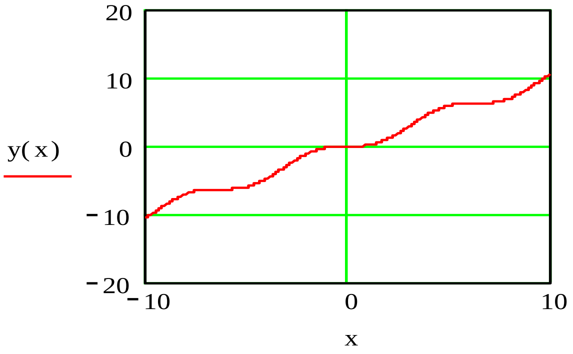
As a result of calculations under these formulas the sequence of the approached values of a root can be received $x_1, x_2, \cdots x_{n-1}, x_n \cdots$. Process of calculations comes to an end at condition performance $|x_n - x_{n-1}| < \varepsilon$ ($\varepsilon = 10^{-5}$). In each case to print quantity of the iterations necessary for achievement of set accuracy.

APPROXIMATE VARIANT OF PERFORMANCE OF WORK ON MATHCAD

1. Definition, construction of tables of values and schedules of functions and branch of roots of the equation *y=x-sinx-0,25.*

We separate roots graphically.
We calculate values of argument and function.

$$y(x) := x - \sin(x) - 0.25$$



$i := 0..\ 10$ $\qquad x_i := -5 + i$ $\qquad F_i := y(x_i)$

We type i, x

We type i, x_i F_i. More low, x= and nearby we click the mouse, we type F=, also nearby we click the mouse.

x =

	0
0	-5
1	-4
2	-3
3	-2
4	-1
5	0
6	1
7	2
8	3
9	4
10	5

F =

	0
0	-6.209
1	-5.007
2	-3.109
3	-1.341
4	-0.409
5	-0.25
6	-0.091
7	0.841
8	2.609
9	4.507
10	5.709

Given

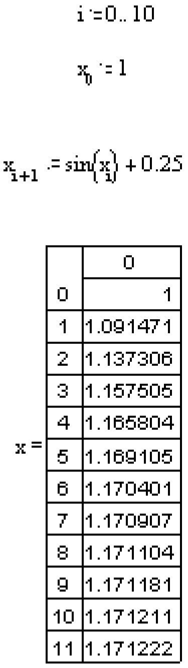
$$x - \sin(x) - 0.25 = 0$$

$$\mathrm{Find}(x) \rightarrow 1.1712296525016659$$

2. The decision with use of operators ***given, find***.
3. The symbolical decision.

$$x - \sin(x) - 0.25 \ \mathrm{solve}, x \rightarrow 1.1712296525016659$$

4. At the left the decision a method of iterations, in the middle a method of tangents, on the right a method of chords.



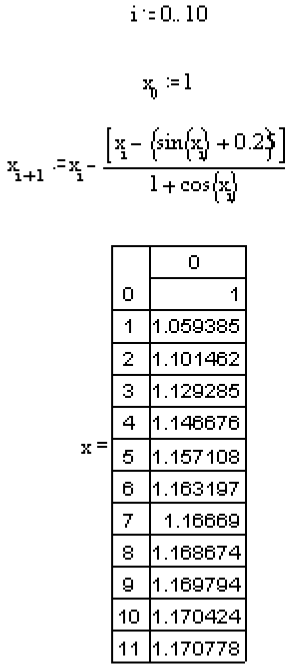
$i := 0..10$

$x_0 := 1$

$x_{i+1} := \sin(x_i) + 0.25$

x =

	0
0	1
1	1.091471
2	1.137306
3	1.157505
4	1.165804
5	1.169105
6	1.170401
7	1.170907
8	1.171104
9	1.171181
10	1.171211
11	1.171222



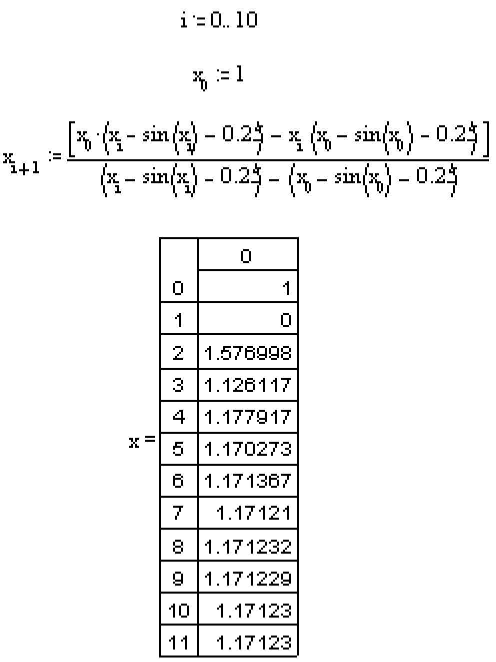
$i := 0..10$

$x_0 := 1$

$$x_{i+1} := x_i - \frac{\left[x_i - \left(\sin(x_i) + 0.25\right)\right]}{1 + \cos(x_i)}$$

x =

	0
0	1
1	1.059385
2	1.101462
3	1.129285
4	1.146676
5	1.157108
6	1.163197
7	1.16669
8	1.168674
9	1.169794
10	1.170424
11	1.170778



$i := 0..10$

$x_0 := 1$

$$x_{i+1} := \frac{\left[x_0 \cdot \left(x_i - \sin(x_i) - 0.25\right) - x_i \cdot \left(x_0 - \sin(x_0) - 0.25\right)\right]}{\left(x_i - \sin(x_i) - 0.25\right) - \left(x_0 - \sin(x_0) - 0.25\right)}$$

x =

	0
0	1
1	0
2	1.576998
3	1.126117
4	1.177917
5	1.170273
6	1.171367
7	1.17121
8	1.171232
9	1.171229
10	1.17123
11	1.17123

Table 1

N	Method	The equation
1	K	$x + x\ln(x+0.5) - 0.5 = 0$
2	K	$x2^x - 1 = 0$
3	X	$x^3 - 2x^2 + x - 3 = 0$
4	K	$x^3 + 12x - 2 = 0$
5	X	$5x - 8\ln(x) - 8 = 0$
6	K	$x^4 + 0.5x^3 - 4x^2 - 3x - 0.5 = 0$
7	X	$x - \sin(x) - 0.25 = 0$
8	K	$x^3 - 6x^2 + 20 = 0$
9	X	$5x^3 + 10x^2 + 5x - 1 = 0$
10	K	$0.1x^2 - x\ln(x) = 0$

Newton's methods

Example 1. To solve the cubic equation $x^3 + x - 10 = 0$ with relative accuracy ε =0,001 **method of tangents** of Newton-Rafsona.

The decision. In this case $F(x) = x^3 + x - 10$. Hence, $F'(x) = 3x^2 + 1$. As zero approach we will accept x_0=3 (Exact value of a rootξ=2). Then under the formula($x_{n+1} = x_n - \frac{F(x_n)}{F'(x_n)}$) received:

$$x_1 = 3 - \frac{20}{28} = 2.285714,$$

$$x_2 = 2.285714 - \frac{4.227400}{16.673465} = 2.032173,$$

Let's check up, whether the set relative accuracy is reached ε:

$$\left|\frac{x_2 - x_1}{x_1}\right| = \left|\frac{2.032173 - 2.285714}{2.285714}\right| \approx 0.110924 > \varepsilon = 0.001$$

Continue iterations:

$$x_3 = 2.032173 - \frac{0.424493}{13.389181} = 2.000469.$$

Again we will check up, whether the set relative accuracy is reached ε:

$$\left|\frac{x_3 - x_2}{x_2}\right| = \left|\frac{2.000469 - 2.032173}{2.032173}\right| \approx 0.015601 > \varepsilon = 0.001$$

The following iteration to within 6 decimal signs gives almost exact value of a root:

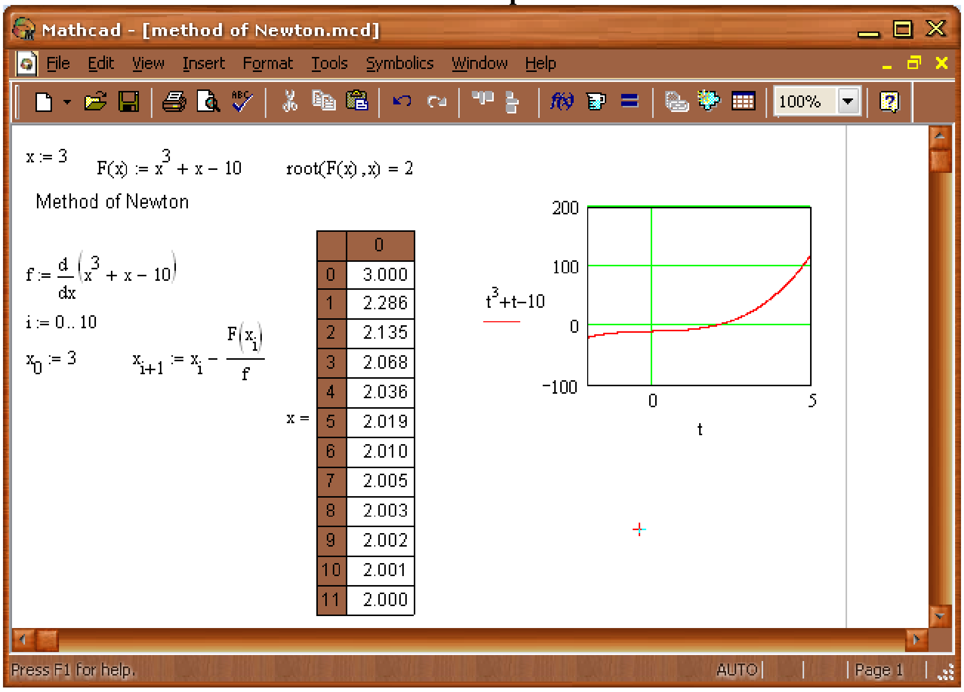
$$x_4 = 2.000469 - \frac{0.006098}{13.005629} = 2.0000001$$

However, here again it is necessary to check up, whether the set relative accuracy is reached ε:

$$\left|\frac{x_4 - x_3}{x_3}\right| = \left|\frac{2.0000001 - 2.000469}{2.000469}\right| \approx 0.000234 < \varepsilon = 0.001$$

The found root of the equation is equal 2.0000001. Thus, computing process has converged for 4 iterations, and we have received a required root with the set relative accuracy ε.

The decision an example in MathCAD



x := 3 $\quad F(x) := x^3 + x - 10 \quad$ root(F(x) ,x) = 2

Method of Newton

$f := \frac{d}{dx}\left(x^3 + x - 10\right)$

$i := 0 .. 10$

$x_0 := 3 \qquad x_{i+1} := x_i - \frac{F(x_i)}{f}$

x =

	0
0	3.000
1	2.286
2	2.135
3	2.068
4	2.036
5	2.019
6	2.010
7	2.005
8	2.003
9	2.002
10	2.001
11	2.000

t^3+t-10

Variants for laboratory works 1,2

Solve the following the equation with accuracy 0,001

1) $x^3 - 9x^2 + 31x + 37 = 0$
2) $\ln x + x + 13 = 0$
3) 1.5cos(x-0.6)+x-2.047=0, [0;π/2]
4) $2x-1.3^x = 0$, [0;10];
5) $\sqrt{\frac{\pi}{2}} e^{0,6x} + \frac{x}{0,36 + x^3} = 0, [-1;1]$
6) x^2+4sinx-1.6280819=0, [0;1];
7) x+lgx=0,5;
8) $x^3+0,4x^2+0,6x-1,6=0$;
9) $x^3 - 9x^2 + 31x + 30 = 0$
10) $\ln x + x - 13 = 0$
11) 1.5cos(x-0.6)+x+2.047=0, [0;π/2]
12) $3x-1.3^x = 0$, [0;10];
13) $x + \frac{x}{0,36 + x^3} = 0, [-1;1]$
14) x^2+4cosx-1.628=0, [0;1];
15) x+lnx=0,5;
16) $x^3+0,4x^2+0,6x-1,6=0$;

Solve the equation with Newton's method

1. $x^3 + 2x^2 + 2 = 0$
2. $x^3 - 2x + 2 = 0$
3. $x^3 + x - 3 = 0$
4. $x^3 - 0,2x^2 + 0,4x - 1,4 = 0$
5. $x^3 + 3x^2 + 12x + 3 = 0$
14. $x^3 - 3x^2 + 9x - 10 = 0$
15. $x^3 + 3x - 1 = 0$
16. $x^3 + 0,4x^2 + 0,6x - 1,6 = 0$
17. $x^3 - 0,1x^2 + 0,4x - 1,4 = 0$
18. $x^3 - 0,2x^2 + 0,5x - 1 = 0$

6. $x^3 - 0{,}1x^2 + +0{,}4x + 1{,}2 = 0$
7. $x^3 - 0{,}2x^2 + 0{,}5x - 1{,}4 = 0$
8. $x^3 - 3x^2 + 12x - 12 = 0$
9. $x^3 + 4x - 6 = 0$
10. $x^3 + 3x^2 + 6x - 1 = 0$
11. $x^3 - 3x^2 + 6x - 2 = 0$
12. $x^3 - 3x^2 + 12x - 9 = 0$
13. $x^3 + 3x + 1 = 0$
19. $x^3 - 3x^2 + 6x - 5 = 0$
20. $x^3 + 2x + 4 = 0$
21. $x^3 + 0{,}2x^2 + 0{,}5x + 0{,}8 = 0$
22. $x^3 + 0{,}1x^2 + 0{,}4x - 1{,}2 = 0$
23. $x^3 - 0{,}1x^2 + 0{,}4x - 1{,}5 = 0$
24. $x^3 - 0{,}2x^2 + 0{,}3x - 1{,}2 = 0$
25. $x^3 + 0{,}2x^2 + 0{,}5x - 2 = 0$
26. $x^3 + 0{,}2x^2 + 0{,}5x - 1{,}2 = 0$

The literature

1. Демидович Б.П., Марон И.А. Основы вычислительной математики. – М.: Наука, 1970. – 664 с.

2. Мак-Кракен Д., Дорн У. Численные методы и программирование на ФОРТРАНе. – М.: Мир, 1977. – 584 с.

CONTROL QUESTIONS

1. Stages of the decision of the equation from one unknown person.
2. Ways of branch of roots.
3. How the graphic branch of roots is specified by means of calculations?
4. To give the verbal description of algorithm of a method половинного divisions.
5. Necessary conditions of convergence of a method половинного divisions.
6. Condition of the termination of the account of a method of simple iteration. A method error.
7. The verbal description of algorithm of a method of chords. Graphic representation of a method. Error calculation.
8. The verbal description of algorithm of a method of tangents (Newton). Graphic representation of a method. A condition of a choice of an index point.

Laboratory work № 3-4
The numerical decision of system of the linear algebraic equations methods of Gaussa, simple iteration and Seidel.

1. Methods of Gaussa

Problems of approximation of function, and also set of other problems of applied mathematics of m of computing physics are reduced to problems about the decision of systems of the linear equations. The most universal method of the decision of system of the linear equations is the method of a consecutive exception of the unknown persons, Gaussa named a method.

For an illustration of sense of a method of Gaussa we will consider system of the linear equations:

$$\begin{cases} 4x_1 - 9x_2 + 2x_3 = 2 \\ 2x_1 - 4x_2 + 4x_3 = 3 \\ -x_1 + 2x_2 + 2x_3 = 1 \end{cases} \qquad \textbf{(1)}$$

This system we will write down in a matrix kind:

$$\begin{pmatrix} 4 & -9 & 2 \\ 2 & -4 & 4 \\ -1 & 2 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} \qquad \textbf{(2)}$$

As it is known, both members of equation it is possible to increase by nonzero number, and also it is possible to subtract another from one equation. Using these properties, we will try to result a matrix of system (2) in a triangular kind, i.e. to a kind, when below the main diagonal all elements – zero. This stage of the decision is called as a forward stroke.

On the forward stroke first step we will increase the first equation on 1/2 and we will subtract from the second then x_1 the variable will be excluded from the second equation. Then, we will increase the first equation on-1/4 and we will subtract from the third then the system (2) will be transformed to kind system:

$$\begin{pmatrix} 4 & -9 & 2 \\ 0 & 0.5 & 3 \\ 0 & -0.25 & 2.5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 1.5 \end{pmatrix} \qquad \textbf{(3)}$$

On the second step of a forward stroke from the third equation it is excluded x_2, i.e. from the third equation it is subtracted the second, increased, on-1/2 that results system (3) in a triangular kind (4)

$$\begin{pmatrix} 4 & -9 & 2 \\ 0 & 0.5 & 3 \\ 0 & 0 & 4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 2.5 \end{pmatrix} \qquad \textbf{(4)}$$

System (4) it is copied in a habitual kind:

$$\begin{cases} 4x_1 - 9x_2 + 2x_3 = 2 \\ 0.5x_2 + 3x_3 = 2 \\ 4x_3 = 2.5 \end{cases} \qquad \textbf{(5)}$$

Now, from system (5) can find the decision upside-down, i.e. at first we find from the third equation x_3=0.625, further, substituting in the second equation, we find x_2=(2-3x_3)/0.5. Substituting x_2 and x_3 in the first equation of system (5), we find x_1=0.75. A decision finding (x_1, x_2, x_3) from system (5) name reverse motion.

Example:

Solve the equation with a method of Gaussa.

$$\begin{cases} x_1 + x_2 - x_3 - x_4 = 0 \\ \qquad x_2 + 2x_3 - x_4 = 2 \\ x_1 - x_2 - x_4 = -1 \\ -x_1 + 3x_2 - 2x_3 = 0 \end{cases}$$

The decision:

$$\begin{cases} x_1+x_2-x_3-x_4=0 \\ \quad x_2+2x_3-x_4=2 \\ \quad 2x_2-x_3=1 \\ \quad 4x_2-3x_3-x_4=0 \end{cases} \Rightarrow \begin{cases} x_1+x_2-x_3-x_4=0 \\ \quad x_2+2x_3-x_4=2 \\ \quad 5x_3-2x_4=2 \\ \quad 11x_3-3x_4=8 \end{cases} \Rightarrow \begin{cases} x_1+x_2-x_3-x_4=0 \\ \quad x_2+2x_3-x_4=2 \\ \quad 5x_3-2x_4=2 \\ \quad -7x_4=-7 \end{cases}$$

$$\Rightarrow \begin{cases} x_1+x_2-x_3-x_4=0 \\ \quad x_2+2x_3-x_4=2 \\ \quad 5x_3-2x_4=2 \\ \quad x_4=1 \end{cases} \Rightarrow \begin{cases} x_1+x_2-x_3-x_4=0 \\ \quad x_2+2x_3-x_4=2 \\ \quad x_3=1 \\ \quad x_4=1 \end{cases} \Rightarrow \begin{cases} x_1+x_2-x_3-x_4=0 \\ \quad x_2=1 \\ \quad x_3=1 \\ \quad x_4=1 \end{cases} \Rightarrow \begin{cases} x_1=1 \\ x_2=1 \\ x_3=1 \\ x_4=1 \end{cases}$$

Example. Solve following systems the equation a method of Gaussa with accuracy 0,001.

$$\begin{cases} 0{,}68x_1+0{,}05x_2-0{,}11x_3+0{,}08x_4=2{,}15 \\ 0{,}21x_1-0{,}13x_2+0{,}27x_3-0{,}8x_4=0{,}44 \\ -0{,}11x_1-0{,}84x_2+0{,}28x_3+0{,}06x_4=-0{,}83 \\ -0{,}08x_1+0{,}15x_2-0{,}5x_3-0{,}12x_4=1{,}16 \end{cases}$$

The decision of systems the equation in MathCAD

Comments. Function ***augment (A, b)*** forms the expanded matrix of system addition to a system matrix on the right a column of the right parts. Function ***rref*** leads the expanded matrix of system to a step kind, carrying out direct and return courses гауссова exceptions. Last column contains the system decision.

$$A := \begin{bmatrix} 1 & 2 & 3 \\ 1 & -3 & 2 \\ 1 & 1 & 1 \end{bmatrix} \qquad b := \begin{bmatrix} 7 \\ 5 \\ 3 \end{bmatrix} \qquad \mathrm{rref}(\mathrm{augment}(A,b)) = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

2. Methods of simple iteration.

Methods of the decision of systems of the linear equations

$$\begin{cases} a_{11}x_1+a_{12}x_2+\ldots+a_{1n}x_n=b_1 \\ a_{21}x_1+a_{22}x_2+\ldots+a_{2n}x_n=b_2 \\ \ldots\ldots \\ a_{m1}x_1+a_{m2}x_2+\ldots+a_{mn}x_n=b_n \end{cases} \qquad (2.1)$$

Or in a vector kind

$$Ax=b \qquad (2.2)$$

It is possible to divide on two basic groups: direct methods and iterative. Direct methods give the exact decision for final number of operations; Kramer's methods and Gaussa concern them, for example. Iterative methods give the decision of system of the equations as a limit consecutive приближений. For iterative methods performance of conditions of convergence and additional transformations of system in equivalent to it is necessary.

The task 1

1. To solve system of the linear equations a **method of Gaussa**. Tasks are resulted in table 2.

The comment. The control of carried out calculations is the important element of the decision of any computing problem. For the forward stroke control use the control sums which represent the sums of factors at unknown persons and a free member for each equation of the set system.

For the control of calculations in the basic part of the scheme of unique division (columns of factors at unknown and free members) over the control sums carry out the same actions, as over other elements of the same line. In the absence of computing errors the control sum for every line in limits influences of errors of a rounding off and their accumulation should coincide with the lower case sum - the second column of the control. The lower case sums represent the sums of all elements from the basic part of this line.

The task 2

To solve system (2.1) method of simple iteration. It is supposed further that a matrix *And* square and невырожденная.

Let's preliminary result system (2.2) in an iterative kind:

$$x=Cx+f \qquad (2.3)$$

For any initial vector x_0 iterative process

$$x^{n+1}=Cx^n+f$$

Converges, if one of conditions is executed

а) $\sum_{j=1}^{n} \left| c_{i,j} \right| = \alpha < 1, \quad 1 \le i \le n,$ (2.4)

б) $\sum_{i=1}^{n} \left| c_{i,j} \right| = \alpha < 1, \quad 1 \le j \le n,$ (2.5)

в) $\sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij}^{2}} = \alpha < 1.$ (2.6)

Process of calculations is finished at condition performance

$$\rho_i (x^{k-1}, x^k) \le \varepsilon (1-\alpha)/\alpha \qquad (2.7)$$

where ρ_i ($i=1,2,3$)– one of the metrics, defined by the left part (2.4) - (2.6) on which convergence, ε the set – accuracy (ε =10^{-4} – has been established).

The task 3

To solve system (2.1) **method of Seidel.**

The method of Zejdel differs from a method of simple iteration by that having found any value for components, we on a following step use it for search following components. Calculations are conducted under the formula

$$x_i^{(k+1)} = -\sum_{j=1}^{i-1} \frac{a_{ij}}{a_{ii}} x_j^{(k+1)} - \sum_{j=i+1}^{n} \frac{a_{ij}}{a_{ii}} x_j^{(k)} + \frac{b_i}{a_{ii}}. \qquad (2.8)$$

Each of conditions (2.4) - (2.6) is sufficient for convergence of iterative process on a **method of Zejdel.** Practically more conveniently following transformation of system (2.2). Домножая both parts (2.2) on A^T, we will receive system equivalent to it

$$CX = d\,,$$

where $C=A^TA$ and $d=A^Tb$. Further, having divided each equation on c_{ij}, we will lead system to a kind (2.8). Similar transformation also guarantees convergence of iterative process.

APPROXIMATE variant of performance of laboratory work

Example. Solve system of the equations

$X_1+2X_2+3X_3=7,$
$X_1-3X_2+2X_3=5,$
$X_1+X_2+X_3=3.$

1. The symbolical decision of systems of the equations

The fragment of a brief with corresponding calculations is resulted more low. Here = - logic equality.

Given

$x1 + 2\cdot x2 + 3\cdot x3 = 7$

$x1 - 3\cdot x2 + 2\cdot x3 = 5$

$x1 + x2 + x3 = 3$

$$\mathrm{Find}(x1, x2, x3) \rightarrow \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$$

2. The decision of system of the linear algebraic equations as matrix equation Ax=b

Order of performance of the task.

1. Establish a mode of automatic calculations.
2. Enter a matrix of system and a matrix-column of the right parts.
3. Calculate the system decision under the formula $x=A^{-1}b$.
4. Check up correctness of the decision multiplication of a matrix of system to a decision vector-column.
5. Find the decision of system by means of function lsolve and compare results.

$$A := \begin{pmatrix} 1 & 2 & 3 \\ 1 & -3 & 2 \\ 1 & 1 & 1 \end{pmatrix} \qquad b := \begin{pmatrix} 7 \\ 5 \\ 3 \end{pmatrix}$$

$$x := A^{-1} \cdot b \qquad x = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} \qquad A \cdot x - b = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

check

Let's solve system by means of function ***lsolve*** **and** will compare result to the decision $x=A^{-1}b$.

$$x := \mathrm{lsolve}(A, b) \qquad x = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$$

3. The decision of linear system a method of Gaussa

Comments. Function ***augment (A, b)*** forms the expanded matrix of system addition to a system matrix on the right a column of the right parts. Function ***rref*** leads the expanded matrix of system to a step kind, carrying out direct and return courses гауссова exceptions. Last column contains the system decision.

$$\mathrm{rref}(\mathrm{augment}(A, b)) = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{pmatrix}$$

4. The decision of system Kramer's method

Order of performance of work.

1. We calculate D a determinant of matrix A.
2. Let's set matrix DX1, replacement of the first column of matrix A, a matrix b. We calculate a determinant of matrix DX1.
3. Let's set matrix DX2, replacement of the second column of matrix A, a matrix b. We calculate a determinant of matrix DX2.
4. Let's set matrix DX3, replacement of the third column of matrix A, a matrix b. We calculate a determinant of matrix DX3.
5. We define the decision of system of the linear equations x_1, x_2, x_3.

$$D := |A| \qquad D = 9$$

$$DX1 := \begin{pmatrix} 7 & 2 & 3 \\ 5 & -3 & 2 \\ 3 & 1 & 1 \end{pmatrix} \qquad DX1 := |DX1| \qquad DX1 = 9$$

$$DX2 := \begin{pmatrix} 1 & 7 & 3 \\ 1 & 5 & 2 \\ 1 & 3 & 1 \end{pmatrix} \qquad DX2 := |DX2| \qquad DX2 = 0$$

$$DX3 := \begin{pmatrix} 1 & 2 & 7 \\ 1 & -3 & 5 \\ 1 & 1 & 3 \end{pmatrix} \qquad DX3 := |DX3| \qquad DX3 = 18$$

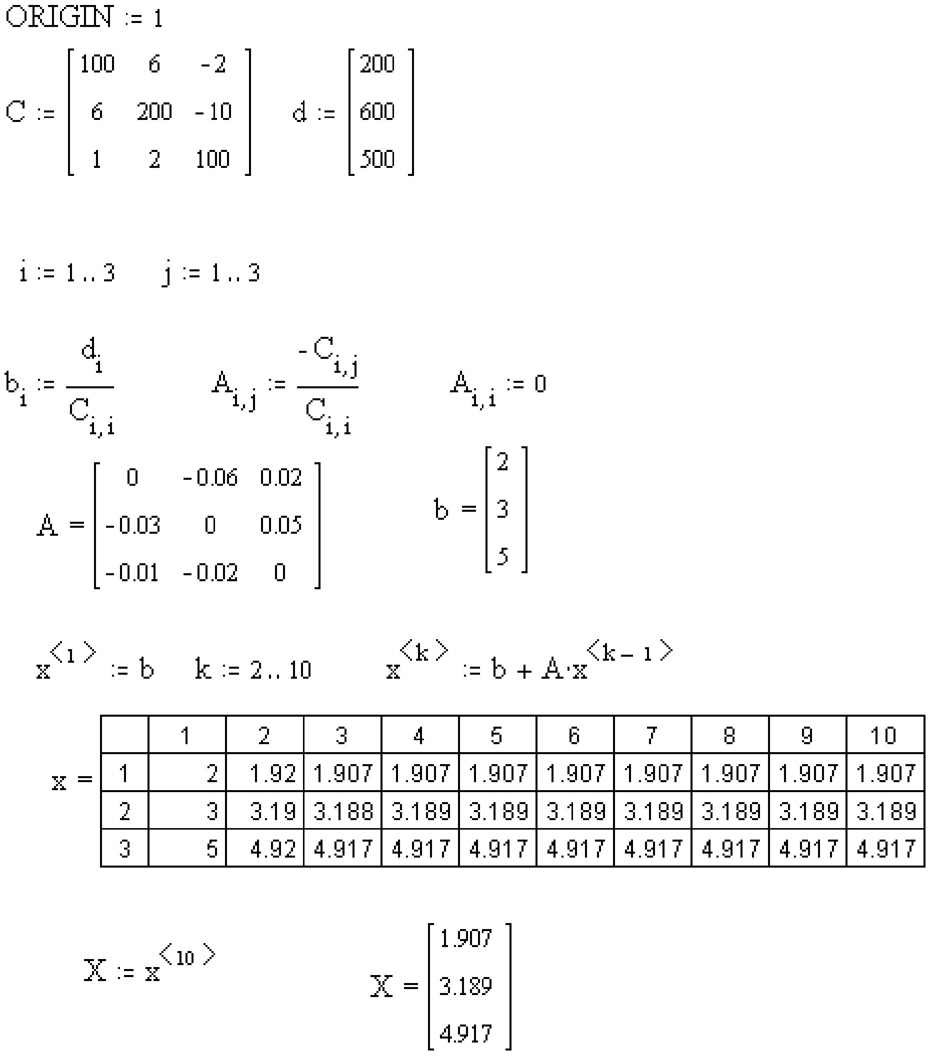
$$x1 := \frac{DX1}{D} \quad x1 = 1 \qquad x2 := \frac{DX2}{D} \quad x2 = 0 \qquad x3 := \frac{DX3}{D} \quad x3 = 2$$

5. The decision of system linear algebraic the equation a method of simple iterations

Order of performance of the task

1. Enter matrixes C and d.

2. Transform initial system Cx=d to a kind x=b+Ax.
3. Define zero approach of the decision.
4. Set quantity of iterations.
5. Calculate consecutive approach.



$\mathrm{ORIGIN} := 1$

$$C := \begin{bmatrix} 100 & 6 & -2 \\ 6 & 200 & -10 \\ 1 & 2 & 100 \end{bmatrix} \qquad d := \begin{bmatrix} 200 \\ 600 \\ 500 \end{bmatrix}$$

$i := 1..3 \qquad j := 1..3$

$$b_i := \frac{d_i}{C_{i,i}} \qquad A_{i,j} := \frac{-C_{i,j}}{C_{i,i}} \qquad A_{i,i} := 0$$

$$A = \begin{bmatrix} 0 & -0.06 & 0.02 \\ -0.03 & 0 & 0.05 \\ -0.01 & -0.02 & 0 \end{bmatrix} \qquad b = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$$

$$x^{\langle 1 \rangle} := b \qquad k := 2..10 \qquad x^{\langle k \rangle} := b + A \cdot x^{\langle k-1 \rangle}$$

x =

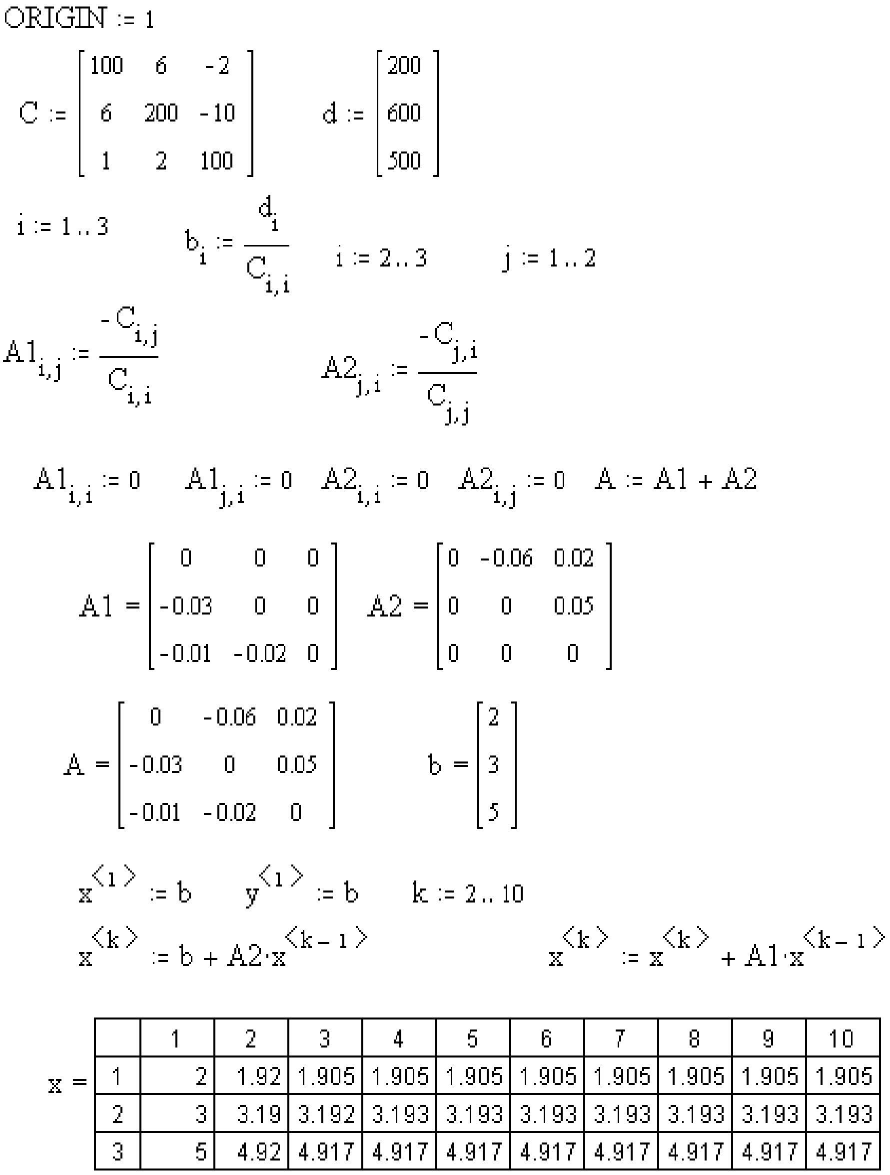
	1	2	3	4	5	6	7	8	9	10
1	2	1.92	1.907	1.907	1.907	1.907	1.907	1.907	1.907	1.907
2	3	3.19	3.188	3.189	3.189	3.189	3.189	3.189	3.189	3.189
3	5	4.92	4.917	4.917	4.917	4.917	4.917	4.917	4.917	4.917

$$X := x^{\langle 10 \rangle} \qquad X = \begin{bmatrix} 1.907 \\ 3.189 \\ 4.917 \end{bmatrix}$$

6. The decision of system of the linear algebraic equations a method of Zejdel

Order of performance of the task

1. Enter matrixes C and d.
2. Transform system Cx=d to a kind x=b+A1x+A2x.
3. Define zero approach of the decision.
4. Set quantity of iterations.
5. Calculate consecutive approach.



ORIGIN := 1

$$C := \begin{bmatrix} 100 & 6 & -2 \\ 6 & 200 & -10 \\ 1 & 2 & 100 \end{bmatrix} \qquad d := \begin{bmatrix} 200 \\ 600 \\ 500 \end{bmatrix}$$

$$i := 1..3 \qquad b_i := \frac{d_i}{C_{i,i}} \qquad i := 2..3 \qquad j := 1..2$$

$$A1_{i,j} := \frac{-C_{i,j}}{C_{i,i}} \qquad A2_{j,i} := \frac{-C_{j,i}}{C_{j,j}}$$

$$A1_{i,i} := 0 \quad A1_{j,i} := 0 \quad A2_{i,i} := 0 \quad A2_{i,j} := 0 \quad A := A1 + A2$$

$$A1 = \begin{bmatrix} 0 & 0 & 0 \\ -0.03 & 0 & 0 \\ -0.01 & -0.02 & 0 \end{bmatrix} \quad A2 = \begin{bmatrix} 0 & -0.06 & 0.02 \\ 0 & 0 & 0.05 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & -0.06 & 0.02 \\ -0.03 & 0 & 0.05 \\ -0.01 & -0.02 & 0 \end{bmatrix} \qquad b = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$$

$$x^{\langle 1 \rangle} := b \qquad y^{\langle 1 \rangle} := b \qquad k := 2..10$$

$$x^{\langle k \rangle} := b + A2 \cdot x^{\langle k-1 \rangle} \qquad x^{\langle k \rangle} := x^{\langle k \rangle} + A1 \cdot x^{\langle k-1 \rangle}$$

x =

	1	2	3	4	5	6	7	8	9	10
1	2	1.92	1.905	1.905	1.905	1.905	1.905	1.905	1.905	1.905
2	3	3.19	3.192	3.193	3.193	3.193	3.193	3.193	3.193	3.193
3	5	4.92	4.917	4.917	4.917	4.917	4.917	4.917	4.917	4.917

table 2

№ вар.	a_{1i}	a_{2i}	a_{3i}	b_{1i}
1	0.35 0.12 - 0.13	0.12 0.71 0.15	- 0.13 0.15 0.63	0.10 0.26 0.38
2	0.71 0.10 - 0.10	0.10 0.34 0.64	0.12 - 0.04 0.56	0.29 0.32 - 0.10
3	0.34 - 0.04 0.06	- 0.04 0.44 0.56	0.10 - 0.12 0.39	0.33 - 0.05 0.28
4	0.10 - 0.04 - 0.43	- 0.04 0.34 0.05	- 0.63 0.05 0.13	- 0.15 0.31 0.37
5	0.63 0.05 0.15	0.05 0.34 0.10	0.15 0.10 0.71	0.34 0.32 0.42
6	1.20 - 0.50 - 0.30	- 0.20 1.70 0.10	0.30 - 1.60 - 1.50	- 0.60 0.30 0.40
	0.30	1.20	- 0.20	- 0.60

7	- 0.10	- 0.20	1.60	0.30
	- 1.50	- 0.30	0.10	0.70
8	0.20	0.44	0.91	0.74
	0.58	- 0.29	0.05	0.02
	0.05	0.34	0.10	0.32
9	6.36	1.75	1.0	41.70
	7.42	19.03	1.75	49.49
	1.77	0.42	6.36	27.67
10	3.11	- 1.66	- 0.60	- 0.92
	- 1.65	3.15	- 0.78	2.57
	0.60	0.78	- 2.97	1.65

Exercises

Method of Gaussa of system of the linear algebraic equations of Ah=b. To compare to the exact decision ξ.

1. $A = \begin{bmatrix} 5 & 0 & 1 \\ 1 & 3 & -1 \\ -3 & 2 & 10 \end{bmatrix}, \quad b = \begin{bmatrix} 11 \\ 4 \\ 6 \end{bmatrix}, \quad \xi = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}.$

2. $A = \begin{bmatrix} 2 & 0 & -1 \\ -1 & 3 & 1 \\ 1 & -1 & 4 \end{bmatrix}, \quad b = \begin{bmatrix} -3 \\ 2 \\ 3 \end{bmatrix}, \quad \xi = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}.$

3. $A = \begin{bmatrix} 2 & 0 & -1 \\ 1 & -3 & 1 \\ 1 & 1 & 3 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}, \quad \xi = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}.$

4. $A = \begin{bmatrix} 5 & 1 & -1 \\ -1 & 3 & 1 \\ 1 & -2 & 4 \end{bmatrix}, \quad b = \begin{bmatrix} -5 \\ 5 \\ 1 \end{bmatrix}, \quad \xi = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}.$

5. $A = \begin{bmatrix} 3 & 1 & -1 \\ -2 & 4 & 1 \\ 1 & 1 & 3 \end{bmatrix}, \quad b = \begin{bmatrix} -1 \\ 5 \\ -3 \end{bmatrix}, \quad \xi = \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}.$

6. $A = \begin{bmatrix} 3 & 1 & -1 \\ 2 & 4 & 1 \\ 1 & -1 & 3 \end{bmatrix}, \quad b = \begin{bmatrix} 6 \\ 9 \\ 4 \end{bmatrix}, \quad \xi = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}.$

7. $A = \begin{bmatrix} 2 & -1 & 0 \\ 2 & 5 & -2 \\ 1 & -1 & 3 \end{bmatrix}, \quad b = \begin{bmatrix} -2 \\ -4 \\ 2 \end{bmatrix}, \quad \xi = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}.$

8. $A = \begin{bmatrix} 3 & -1 & 1 \\ 0 & 2 & -1 \\ -1 & 1 & 5 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 3 \\ -5 \end{bmatrix}, \quad \xi = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}.$

9. $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & 0 \\ 1 & -1 & 5 \end{bmatrix}, \quad b = \begin{bmatrix} 7 \\ 7 \\ 11 \end{bmatrix}, \quad \xi = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}.$

The task I **II.:** Solve systems the equation with a method of Gaussa.

№ 1
$$\begin{cases} 4,4x_1 - 2,5x_2 + 19,2x_3 - 10,8x_4 = 4,3 \\ 5,5x_1 - 9,3x_2 - 14,2x_3 + 13,2x_4 = 6,8 \\ 7,1x_1 - 11,5x_2 + 5,3x_3 - 6,7x_4 = -1,8 \\ 14,2x_1 + 23,4x_2 - 8,8x_3 + 5,3x_4 = 7,2 \end{cases}$$

№ 2
$$\begin{cases} 8,2x_1 - 3,2x_2 + 14,2x_3 + 14,8x_4 = -8,4 \\ 5,6x_1 - 12x_2 + 15x_3 - 6,4x_4 = 4,5 \\ 5,7x_1 + 3,6x_2 - 12,4x_3 - 2,3x_4 = 3,3 \\ 6,8x_1 + 13,2x_2 - 6,3x_3 - 8,7x_4 = 14,3 \end{cases}$$

№ 3
$$\begin{cases} 5,7x_1 - 7,8x_2 - 5,6x_3 - 8,3x_4 = 2,7 \\ 6,6x_1 + 13,1x_2 - 6,3x_3 + 4,3x_4 = -5,5 \\ 14,7x_1 - 2,8x_2 + 5,6x_3 - 12,1x_4 = 8,6 \\ 8,5x_1 + 12,7x_2 - 23,7x_3 + 5,7x_4 = 14,7 \end{cases}$$

№ 4
$$\begin{cases} 3,8x_1 + 14,2x_2 + 6,3x_3 - 15,5x_4 = 2,8 \\ 8,3x_1 - 6,6x_2 + 5,8x_3 + 12,2x_4 = -4,7 \\ 6,4x_1 - 8,5x_2 - 4,3x_3 + 8,8x_4 = 7,7 \\ 17,1x_1 - 8,3x_2 + 14,4x_3 - 7,2x_4 = 13,5 \end{cases}$$

№ 5
$$\begin{cases} 15{,}7x_1 + 6{,}6x_2 - 5{,}7x_3 + 11{,}5x_4 = -2{,}4 \\ 8{,}8x_1 - 6{,}7x_2 + 5{,}5x_3 - 4{,}5x_4 = 5{,}6 \\ 6{,}3x_1 - 5{,}7x_2 - 23{,}4x_3 + 6{,}6x_4 = 7{,}7 \\ 14{,}3x_1 + 8{,}7x_2 - 15{,}7x_3 - 5{,}8x_4 = 23{,}4 \end{cases}$$

№ 6
$$\begin{cases} 4{,}3x_1 - 12{,}1x_2 + 23{,}2x_3 - 14{,}1x_4 = 15{,}5 \\ 2{,}4x_1 - 4{,}4x_2 + 3{,}5x_3 + 5{,}5x_4 = 2{,}5 \\ 5{,}4x_1 + 8{,}3x_2 - 7{,}4x_3 - 12{,}7x_4 = 8{,}6 \\ 6{,}3x_1 - 7{,}6x_2 + 1{,}34x_3 + 3{,}7x_4 = 12{,}1 \end{cases}$$

CONTROL QUESTIONS

1. The method of Gaussa concerns what type - direct or iterative?
2. In what consists a straight line and reverse motion in the scheme of unique division?
3. How it will be organised, the control over calculations in direct and reverse motion?
4. How the iterative sequence for a finding of the decision of system of the linear equations is under construction?
5. How it is formulated sufficient conditions of convergence of iterative process?
6. How these conditions are connected with a choice of the metrics of space?
7. In what difference of iterative process of a method of Zejdel from similar process of a method of simple iteration?

Laboratory work № 5-6

The numerical decision of system of the nonlinear algebraic equations to methods of simple iteration.

The decision of systems of the nonlinear equations

The system of the nonlinear equations is given

$$\begin{cases} f_1(x_1, x_2, x_3, \dots, x_n) = 0, \\ f_2(x_1, x_2, x_3, \dots, x_n) = 0, \\ \dots \\ f_n(x_1, x_2, x_3, \dots, x_n) = 0, \end{cases} \qquad (1)$$

or

$$f_i(x_1, x_2, x_3, \dots, x_n) = 0, i = \overline{1 \dots n}.$$

It is necessary to solve this system, i.e. to find a vector $\bar{X} = [x_1, x_2, x_3, \dots, x_n]$, Satisfying to system *(1)* with accuracy ε.

The vector $\bar{X}$ defines a point in n-dimensional Evklidovom space, i.e. $\bar{X} \in$ to this space and satisfies to all equations of system *(1)*.

Unlike systems of the linear equations for *systems of the nonlinear equations* direct methods of the decision are unknown. At the decision *of systems of the nonlinear equations* iterative methods are used. Efficiency of all iterative methods depends on a choice of initial approach (index point), i.e. a vector $\overline{X^0} = [x_1^0, x_2^0, \dots, x_n^0]$.

The area in which initial approach $\overline{X^0}$ converges to the required decision, is called as area of convergence G. If initial approach $\overline{X^0}$ lies outside of G it is not possible to receive the system decision.

The index point $\overline{X^0}$ choice is in many respects defined by intuition and experience of the expert. [5]

Method of simple iterations

For application of this method the initial system *(1)* should be transformed to a kind

$$\begin{cases} x_1 = \varphi_1(x_1, x_2, x_3, \dots, x_n) = 0, \\ x_2 = \varphi_2(x_1, x_2, x_3, \dots, x_n) = 0, \\ \dots\dots\dots\dots\dots\dots\dots\dots \\ x_n = \varphi_n(x_1, x_2, x_3, \dots, x_n) = 0, \end{cases} \qquad (2)$$

or

$$x_i = \varphi_i(x_1, x_2, x_3, \dots, x_n), i = \overline{1, n}.$$

Further, having chosen initial approach $\overline{X^0} = [x_1^0, x_2^0, \dots, x_n^0]$ and using system *(2)*, we build iterative process of search in the scheme:

$$x_i^k =_i (x_1^{k-1}, x_2^{k-1}, x_3^{k-1}, \dots, x_n^{k-1}),$$

i.e. on each k-th step of search the vector of variables $\bar{X}$ is found, using values of the variables received on a step $(k-1)$.

Iterative process of search stops, as soon as the condition will be satisfied

$$\left| x_j^k - x_j^{k-1} \right| \le \varepsilon, j = \overline{1, n}. \qquad (3)$$

Thus the condition *(3)* should be carried out simultaneously on all variables.

The method of simple iterations is used for the decision of such systems of the nonlinear equations in which the condition of convergence of iterative process of search, namely (3) is satisfied, i.e. the sum of absolute sizes of the private derivative all transformed equations of system *(2)* on j-th variable is less than unit.

$$\sum_{i=1}^{n} \left| \frac{\partial \varphi_i}{\partial x_j} \right| < 1, j = \overline{1, n}.$$

For two system equitions will be represent system (*) in a kind:

$$\begin{cases} x_1 = g_1(x_1, x_2), \\ x_2 = g_2(x_1, x_2). \end{cases} \qquad \mathbf{(5)}$$

It is represented the right members of equation in a kind:

$$\begin{aligned} g_1(x_1, x_2) &= x_1 + \lambda_{11} f_1(x_1, x_2) + \lambda_{12} f_2(x_1, x_2), \\ g_2(x_1, x_2) &= x_2 + \lambda_{21} f_1(x_1, x_2) + \lambda_{22} f_2(x_1, x_2). \end{aligned} \qquad (6)$$

For a method of simple iteration

$$g_1(x_1,x_2) = x_1 + \lambda_{11}f_1(x_1,x_2) + \lambda_{12}f_2(x_1,x_2),$$
$$g_2(x_1,x_2) = x_2 + \lambda_{21}f_1(g_1,x_2) + \lambda_{22}f_2(x_1,x_2). \qquad (7)$$

For a method of Zejdel

For search of factors λ_{ij} solve system

$$\begin{cases} \begin{cases} 1 + \lambda_{11} \left.\frac{\partial f_1}{\partial x_1}\right|_{\mathbf{x}^{(0)}} + \lambda_{12} \left.\frac{\partial f_2}{\partial x_1}\right|_{\mathbf{x}^{(0)}} = 0, \\ \lambda_{11} \left.\frac{\partial f_1}{\partial x_2}\right|_{\mathbf{x}^{(0)}} + \lambda_{12} \left.\frac{\partial f_2}{\partial x_2}\right|_{\mathbf{x}^{(0)}} = 0; \end{cases} \\ \begin{cases} \lambda_{21} \left.\frac{\partial f_1}{\partial x_1}\right|_{\mathbf{x}^{(0)}} + \lambda_{22} \left.\frac{\partial f_2}{\partial x_1}\right|_{\mathbf{x}^{(0)}} = 0, \\ 1 + \lambda_{21} \left.\frac{\partial f_1}{\partial x_1}\right|_{\mathbf{x}^{(0)}} + \lambda_{22} \left.\frac{\partial f_2}{\partial x_1}\right|_{\mathbf{x}^{(0)}} = 0. \end{cases} \end{cases} \qquad (8)$$

Let's use further system *(7)* for search of roots of the equation. In the program **$g_1(x_1,x_2)$=y1** and **$g_2(x_1,x_2)$=y2**.

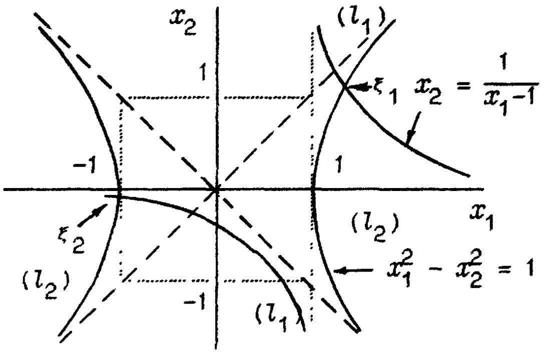
Example. The system of the equations is given

$$\begin{cases} x_2(x_1 - 1) - 1 = 0, (l_1) \\ x_1^2 - x_2^2 - 1 = 0, (l_2) \end{cases} \qquad (1)$$

to find with accuracy $\varepsilon=10^{-3}$ its decision located in the first quarter of a plane $0x_1x_2$.

The decision. The curves defined by the equations (1), are represented on fig. 1. These curves are crossed in two points ξ_1 and ξ_2. Let's result system (1) in a kind convenient for iterations, and we will find the decision system (1) ξ_1 with the set accuracy.

We take as initial value (the graphic decision) $x^{(0)} = \begin{pmatrix}1,5\\1,5\end{pmatrix}$ and for definition of factors λ_{ij} olve system of the equations.



- A curve (l_1) – a hyperbole (two branches) - A curve (l_2) – a hyperbole (two branches)

Fig. 1.

Let's calculate private derivatives of functions.

$$f_1(x_1,x_2) = x_2(x_1 - 1) - 1, \quad f_2(x_1,x_2) = x_1^2 - x_2^2 - 1$$

In a point $x^{(0)}$:

$$\left.\frac{\partial f_1}{\partial x_1}\right|_{\mathbf{x}^{(0)}} = \left.x_2\right|_{\mathbf{x}^{(0)}} = 1,5; \qquad \left.\frac{\partial f_2}{\partial x_1}\right|_{\mathbf{x}^{(0)}} = \left.2x_1\right|_{\mathbf{x}^{(0)}} = 3;$$

$$\left.\frac{\partial f_1}{\partial x_2}\right|_{\mathbf{x}^{(0)}} = \left.(x_1 - 1)\right|_{\mathbf{x}^{(0)}} = 0,5; \qquad \left.\frac{\partial f_2}{\partial x_2}\right|_{\mathbf{x}^{(0)}} = \left.-2x_2\right|_{\mathbf{x}^{(0)}} = -3.$$

Having solved system

$$\begin{cases} \begin{cases} 1 + 1,5\,\lambda_{11} + 3\,\lambda_{12} = 0, \\ 0,5\,\lambda_{11} - 3\,\lambda_{12} = 0; \end{cases} \\ \begin{cases} 1,5\,\lambda_{21} + 3\,\lambda_{22} = 0, \\ 1 + 0,5\,\lambda_{21} - 3\,\lambda_{22} = 0, \end{cases} \end{cases}$$

let's find $\lambda_{11} = -\frac{1}{2}, \quad \lambda_{12} = -\frac{1}{12}, \quad \lambda_{21} = -\frac{1}{2}, \quad \lambda_{22} = \frac{1}{4}.$

Condition $\lambda_{11}\lambda_{22} - \lambda_{12}\lambda_{21} \neq 0$ is executed. The resulted system has the following appearance:

$$\begin{cases} x_1 = g_1(x_1, x_2) = x_1 - \frac{1}{2}\left[x_2(x_1 - 1) - 1\right] - \frac{1}{12}\left[x_1^2 - x_2^2 - 1\right], \\ x_2 = g_2(x_1, x_2) = x_2 - \frac{1}{2}\left[x_2(x_1 - 1) - 1\right] + \frac{1}{4}\left[x_1^2 - x_2^2 - 1\right]. \end{cases} \qquad (2)$$

Using the received representations (2) for functions $g_1(x_1, x_2)$ and $g_2(x_1, x_2)$, will find vectors consecutive approximation. An estimation of an error of each approach we will define in distance between vectors of two consecutive iterations on *m-norm*.

$$d_k = \|\mathbf{x}^{(k)} - \mathbf{x}^{(k-1)}\| = \max\{|x_1^{(k)} - x_1^{(k-1)}|, |x_2^{(k)} - x_2^{(k-1)}|\}.$$

Then will receive

$$\mathbf{x}^{(1)} = \begin{bmatrix} x_1^{(1)} \\ x_2^{(1)} \end{bmatrix} = \begin{bmatrix} g_1(x_1^{(0)}, x_2^{(0)}) \\ g_2(x_1^{(0)}, x_2^{(0)}) \end{bmatrix} = \begin{bmatrix} g_1(1,5;\ 1,5) \\ g_2(1,5;\ 1,5) \end{bmatrix} = \begin{bmatrix} 1,70833 \\ 1,37500 \end{bmatrix},$$

$$d_1 = 0,20833;$$

$$\mathbf{x}^{(2)} = \begin{bmatrix} x_1^{(2)} \\ x_2^{(2)} \end{bmatrix} = \begin{bmatrix} g_1(x_1^{(1)}, x_2^{(1)}) \\ g_2(x_1^{(1)}, x_2^{(1)}) \end{bmatrix} = \begin{bmatrix} g_1(1,70833;1,37500) \\ g_2(1,70833;1,37500) \end{bmatrix} = \begin{bmatrix} 1,71904 \\ 1,39497 \end{bmatrix},$$

$$d_2 = 0,01997;$$

$$\mathbf{x}^{(3)} = \begin{bmatrix} x_1^{(2)} \\ x_2^{(2)} \end{bmatrix} = \begin{bmatrix} g_1(x_1^{(2)}, x_2^{(2)}) \\ g_2(x_1^{(2)}, x_2^{(2)}) \end{bmatrix} = \begin{bmatrix} g_1(1,71904;1,39497) \\ g_2(1,71904;1,39497) \end{bmatrix} = \begin{bmatrix} 1,71676 \\ 1,39574 \end{bmatrix},$$

$$d_3 = 0,00281;$$

$$\mathbf{x}^{(4)} = \begin{bmatrix} x_1^{(3)} \\ x_2^{(3)} \end{bmatrix} = \begin{bmatrix} g_1(x_1^{(3)}, x_2^{(3)}) \\ g_2(x_1^{(3)}, x_2^{(3)}) \end{bmatrix} = \begin{bmatrix} g_1(1,71676;1,39574) \\ g_2(1,71676;1,39574) \end{bmatrix} = \begin{bmatrix} 1,71662 \\ 1,39533 \end{bmatrix},$$

$$d_4 = 0,00041;$$

$$\mathbf{x}^{(5)} = \begin{bmatrix} x_1^{(4)} \\ x_2^{(4)} \end{bmatrix} = \begin{bmatrix} g_1(x_1^{(4)}, x_2^{(4)}) \\ g_2(x_1^{(4)}, x_2^{(4)}) \end{bmatrix} = \begin{bmatrix} g_1(1,71662;1,39533) \\ g_2(1,71662;1,39533) \end{bmatrix} = \begin{bmatrix} 1,71667 \\ 1,39533 \end{bmatrix}.$$

took $\|x^{(5)} - x^{(4)}\| = 0,00005$. As on a condition $\varepsilon=10^{-3}$, that according to an estimation (25b) it is possible to take 5 approach as the decision $\xi = x^{(5)} = \begin{pmatrix} 1,7167 \\ 1,3953 \end{pmatrix}$.

The decision of systems nonlinear the equation in MathCAD.

MathCAD gives the chance to find the decision of system of the equations numerical methods, thus the maximum number of the equations in MathCAD2001i is finished to 200.

For the decision of system of the equations it is necessary to execute following stages.

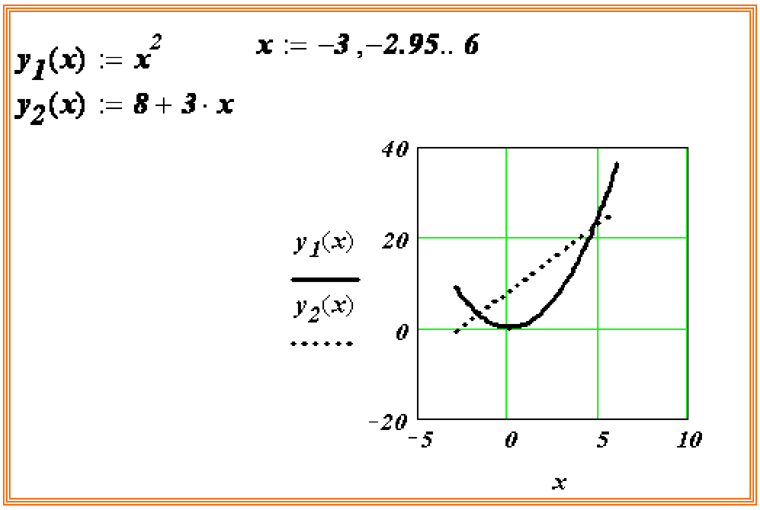
The task of initial approach for all unknown persons entering into system of the equations. At a small number of unknown persons this stage can be executed graphically, as shown in an example.

Example. The system of the equations is given:

$$y = x^2;$$

$$y = \mathbf{8} + \mathbf{3}x.$$

To define initial approach for decisions of this system.



$y_1(x) := x^2$ $\qquad x := -3, -2.95 .. 6$

$y_2(x) := 8 + 3 \cdot x$

It is visible that the system has two decisions: for the first decision as initial approach the point (-2, 2), and for the second decision – a point (5, 20) can be accepted. ¨

Calculation of the decision of system of the equations with the set accuracy. Already known computing block *Given* is for this purpose used.

Function *Find* calculates the decision of system of the equations with the set accuracy, and the call of this function looks like *Find (x),* where *x* – the list of variables on which the decision is searched. Initial values to these variables are set in the block <Entry conditions>. The number of arguments of function should be equal to number of unknown persons.

Following expressions are inadmissible in the decision block:

- Restrictions with a sign ^{1};
- Discrete variable or the expressions containing a discrete variable in any form;
- Blocks of the decision of the equations cannot be enclosed each other, each block can have only one keyword *Given* and a name of function *Find* (or *Minerr*).

Example. Using block *Given*, to calculate all decisions of system of the previous example. To execute check of the found decisions.

x:=-2 *y:=2* Initial approach for the first decision

Given

$y=8+3\cdot x$

$y=x^2$

$S_A:=Find(x,y)$

$$S_A = \begin{pmatrix} -1.702 \\ 2.895 \end{pmatrix}$$ *Projections of the first decision*

x:=5 *y:=20* Initial approach for the second decision

Given

$y=8+3\cdot x$

$y=x^2$

$x>0$ Restriction on positivity of a projection x the second decision

$S_B:=Find(x,y)$

$$S_B = \begin{pmatrix} 4.702 \\ 22.105 \end{pmatrix}$$ *Projections of the second decision*

Example. Using function ***Minerr***, calculate the decision of system of the equations

$$x + y = 0.95;$$

$$(x^2 + 1)^2 + (y^2 + 1)^2 = 5.5.$$

$x := 0 \quad y := 1$

Given

$(x^2 + 1)^2 + (y^2 + 1)^2 = 5.5$

Жирный знак =

$x + y = 0.95$

Жирный знак =

$z := \mathrm{Minerr}(x, y)$

$z = \begin{pmatrix} -0.106 \\ 1.056 \end{pmatrix}$

Найденное решение

Проверка найденного решения

$z_0 + z_1 = 0.95$

$\left\lfloor (z_0)^2 + 1 \right\rfloor^2 + \left\lfloor (z_1)^2 + 1 \right\rfloor^2 = 5.5$

Exercises

Method of iterations to solve systems of the equations with accuracy $\varepsilon = 10^{-2}$.

1\. $\begin{cases} x_1^{2/3} + x_2^{2/3} = 4, \\ x_1^2 - 2x_2 = 0 \end{cases} (x_1 > 0).$

2\. $\begin{cases} x_1^{2/3} + x_2^{2/3} = 4, \\ x_1^2 - 2x_2 = 0 \end{cases} (x_1 < 0).$

3\. $\begin{cases} x_2 - \sqrt{x_1 + 1} = 0, \\ x_1^2 + x_2^2 - 2x_2 = 0 \end{cases} (x_1 > 0).$

4\. $\begin{cases} x_1 \cos x_1 - x_2 = 0, \\ x_1^2 + x_2^2 - 1 = 0 \end{cases} (x_1 > 0).$

5\. $\begin{cases} x_1 \cos x_1 - x_2 = 0, \\ x_1^2 + x_2^2 - 1 = 0 \end{cases} (x_1 < 0).$

6\. $\begin{cases} 2x_1^2 + x_2^2 = 1, \\ x_2 - x_1^{2/3} = 0 \end{cases} (x_2 > 0).$

7\. $\begin{cases} 2x_1^2 + x_2^2 = 1, \\ x_2 + x_1^{2/3} = 0 \end{cases} (x_2 < 0).$

8\. $\begin{cases} x_2 - \sin x_1 = 0, \\ x_1^2 + x_2^2 - 1 = 0 \end{cases} (x_1 > 0).$

9\. $\begin{cases} x_2 - \sin x_1 = 0, \\ x_1^2 + x_2^2 - 1 = 0 \end{cases} (x_1 < 0).$

10\. $\begin{cases} x_1^2 + x_2^2 - 2x_2 = 0, \\ x_2 - e^{-x_1} = 0 \end{cases} (x_1 < 0).$

11\. $\begin{cases} x_1^2 + x_2^2 - 2x_2 = 0, \\ x_2 - e^{-x_1} = 0 \quad (x_1 > 0). \end{cases}$

12\. $\begin{cases} x_1^2 - 2x_1 + x_2^2 - 2x_2 + 1 = 0, \\ x_2 - \sqrt{x_1 + 1} = 0 \quad (x_1 > 0). \end{cases}$

13\. $\begin{cases} x_1^2 + x_2^2 - 2x_1 = 0, \\ x_2 - \ln x_1 = 0 \end{cases} (x_2 > 0).$

14\. $\begin{cases} x_1^2 + 2x_1 + x_2^2 + 2x_2 + 1 = 0, \\ x_2 - \sqrt{x_1 + 1} = -1. \end{cases}$

15\. $\begin{cases} x_1^{2/3} + x_2^{2/3} = 1, \\ x_1^2 + x_2^2 - 2x_1 = 0 \end{cases} (x_2 > 0).$

16\. $\begin{cases} x_1^{2/3} + x_2^{2/3} = 1, \\ x_2^2 + x_1^2 - 2x_1 = 0 \end{cases} (x_2 < 0).$

17\. $\begin{cases} x_1 \sin x_1 - x_2 = 0, \\ x_1^2 + x_2^2 - 1 = 0 \quad (x_1 > 0). \end{cases}$

18\. $\begin{cases} x_1 \sin x_1 - x_2 = 0, \\ x_1^2 + x_2^2 - 1 = 0 \quad (x_1 < 0). \end{cases}$

The note. For the curve image $(x_1^2 + x_2^2)^2 = 2(x_1^2 - x_2^2)$ (lemniscate Bernulli) to take advantage of polar co-ordinates.

http://pers.narod.ru/study/mathcad/07.html#start

Laboratory work № 7-8

Problems of Cochy for the ordinary differential equations. Euler's methods, Runge-Kutta and Adams

The plan:

1. Euler's method and Runge-Kutta.
2. Adams's method

Let the differential equation of the first order is given

$$y' = f(x,y). \qquad (5.1)$$

It is required to find on a piece $[a,b]$ the decision $y(x)$, satisfying to the entry condition

$$y(a) = y_0 \qquad (5.2)$$

Let's assume that conditions of the theorem of existence and uniqueness are executed. For the decision we use **Euler's** method (a method of the first order of accuracy, settlement formulas (5.3)) and a method **of Runge-Kutta** (a method of the fourth order of accuracy, settlement formulas (5.4)) with step *h* and 2*h*. We will notice that results can strongly differ, whereas Euler's method, having only the first order of accuracy, is used, as a rule, for estimated calculations. A rough estimation of an error of a method of Runge-Kutta to calculate ε under the formula (5.5 [2].

$$y_{i+1} = y_i + hf(x_i, y_i), \quad \text{where } h \text{ – a splitting step.} \qquad (5.3)$$

$$y_{i+1} = y_i + \frac{k_1 + 2k_2 + 2k_3 + k_4}{6}, \text{ where} \qquad (5.4)$$

$$k_1 = hf(x_i, y_i),\ k_2 = hf(x_i + \frac{h}{2}, y_i + \frac{k_1}{2}),\ k_3 = hf(x_i + \frac{h}{2}, y_i + \frac{k_2}{2}),$$

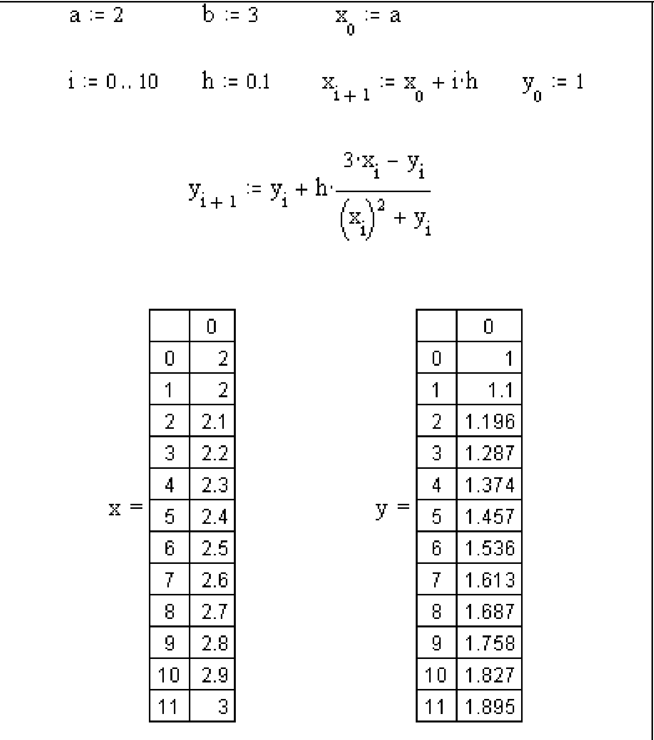
$$k_4 = hf(x_i + h, y_i + k_3).$$

$$\varepsilon = \frac{|y_{2h} - y_h|}{15} \qquad (5.5)$$

APPROXIMATE fragment of performance of work

1. To solve the differential equation y ' =f (x, y) **Euler's method** on a piece [a, b] with step h with the entry condition y (a) =y₀, f (x, y) = (3x-y) / (x²+y), a=2, b=3, h=0.1, y₀=1.

2. To solve the differential equation y' =f (x, y) a method of Runge-Kutta on a piece [a, b] with step h with the entry condition y (a) =y₀.



$a := 2 \quad b := 3 \quad x_0 := a$

$i := 0..10 \quad h := 0.1 \quad x_{i+1} := x_0 + i \cdot h \quad y_0 := 1$

$$y_{i+1} := y_i + h \cdot \frac{3 \cdot x_i - y_i}{(x_i)^2 + y_i}$$

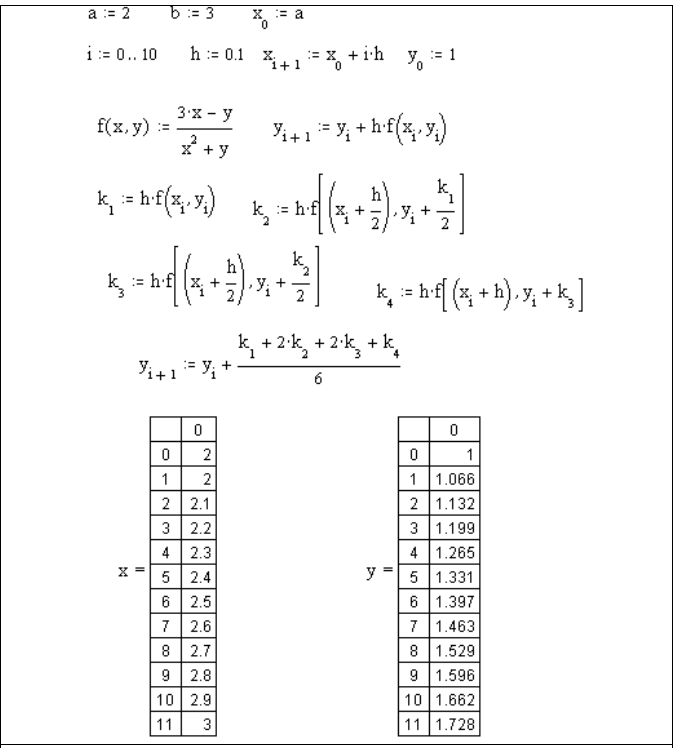
x =

	0
0	2
1	2
2	2.1
3	2.2
4	2.3
5	2.4
6	2.5
7	2.6
8	2.7
9	2.8
10	2.9
11	3

y =

	0
0	1
1	1.1
2	1.196
3	1.287
4	1.374
5	1.457
6	1.536
7	1.613
8	1.687
9	1.758
10	1.827
11	1.895

Euler's method



$a := 2 \quad b := 3 \quad x_0 := a$

$i := 0..10 \quad h := 0.1 \quad x_{i+1} := x_0 + i \cdot h \quad y_0 := 1$

$$f(x,y) := \frac{3 \cdot x - y}{x^2 + y} \qquad y_{i+1} := y_i + h \cdot f(x_i, y_i)$$

$$k_1 := h \cdot f(x_i, y_i) \qquad k_2 := h \cdot f\left[\left(x_i + \frac{h}{2}\right), y_i + \frac{k_1}{2}\right]$$

$$k_3 := h \cdot f\left[\left(x_i + \frac{h}{2}\right), y_i + \frac{k_2}{2}\right] \qquad k_4 := h \cdot f\left[\left(x_i + h\right), y_i + k_3\right]$$

$$y_{i+1} := y_i + \frac{k_1 + 2 \cdot k_2 + 2 \cdot k_3 + k_4}{6}$$

x =

	0
0	2
1	2
2	2.1
3	2.2
4	2.3
5	2.4
6	2.5
7	2.6
8	2.7
9	2.8
10	2.9
11	3

y =

	0
0	1
1	1.066
2	1.132
3	1.199
4	1.265
5	1.331
6	1.397
7	1.463
8	1.529
9	1.596
10	1.662
11	1.728

Method of Runge-Kutta

3. Adams's method

The decision of systems of the ordinary differential equations Adams's method

In the given system of the equations we will substitute values of factors and entry conditions. We will receive

$$\begin{cases} y' = 2y - z \\ z' = y - 4z \end{cases} \quad y(0) = 3, \quad z(0) = -2$$

Adams's method we will find the decision of this system on the set piece. For this purpose we will calculate a method of Runge-Kutta some initial values of function.

Let's choose a step *h* and, for brevity, will enter $x_i = x_0 + ih$ и $y_i = y(x_i)\,(i = 0,1,2,\dots)$

Let's consider numbers:

$$\begin{cases} k_1^{(i)} = hf(x_i, y_i) \\ k_2^{(i)} = hf\left(x_i + \frac{h}{2}, y_i + \frac{k_1^{(i)}}{2}\right) \\ k_3^{(i)} = hf\left(x_i + \frac{h}{2}, y_i + \frac{k_3^{(i)}}{2}\right) \\ k_4^{(i)} = hf(x_i + h, y_i + k_3^{(i)}) \end{cases}$$

According to a method of Runge-Kutta consecutive values y_i are defined under the formula

$$y_{i+1} = y_i + \Delta y_i$$

where

$$\Delta y_i = \frac{1}{6}\left(k_1^{(i)} + 2 \cdot k_2^{(i)} + 2 \cdot k_3^{(i)} + k_4^{(i)}\right)\,(i = 0,1,2,\dots\,). \quad (2.1)$$

Having substituted in these formulas initial values we will receive

$$x_0 = 0 \quad y_0 = 3 \quad z_0 = -2$$
$$x_1 = 0{,}1 \quad y_1 = 3{,}3672 \quad z_1 = -2{,}1586$$
$$x_2 = 0{,}2 \quad y_2 = 3{,}4944 \quad z_2 = -2{,}0867$$
$$x_3 = 0{,}3 \quad y_3 = 3{,}5964 \quad z_3 = -1{,}9906$$

Further calculation it is continued on Adams's method. All calculations it is written down in tables 2.1 and 2.2.

Table 2.1

k	x_k	y_k	Δy_k	p_k	Δp_k	$\Delta^2 p_k$	$\Delta^3 p_k$	z_k	Δz_k	q_k	Δq_k	$\Delta^2 q_k$	$\Delta^3 q_k$
0	0	3		0,8000	0,0893	-0,0711	0,0636	-2		1,1000	0,1002	-0,1162	0,1040
1	0,1	3,3672		0,8893	0,0183	-0,0075	0,0680	-2,1586		1,2002	-0,0160	-0,0122	-0,3354
2	0,2	3,4944		0,9076	0,0108	0,0605	0,0512	-2,0867		1,1841	-0,0282	-0,3476	0,7024
3	0,3	3,5964	0,9445	0,9183	0,0713	0,1117	-0,1448	-1,9906	1,1757	1,1559	-0,3758	0,3548	-0,6647
4	0,4	4,5409	1,0761	0,9897	0,1831	-0,0330	0,1605	-0,8149	0,3215	0,7801	-0,0210	-0,3099	0,8201
5	0,5	5,6169	1,3300	1,1727	0,1500	0,1275	-0,1562	-0,4934	1,1598	0,7590	-0,3309	0,5102	-0,9910
6	0,6	6,9469	1,3297	1,3227	0,2775	-0,0288	0,2023	0,6664	-0,1157	0,4281	0,1793	-0,4809	1,1396
7	0,7	8,2766	1,8523	1,6003	0,2488	0,1735	-0,2240	0,5507	1,2171	0,6074	-0,3016	0,6587	-1,3700
8	0,8	10,1290	1,9028	1,8490	0,4223	-0,0505		1,7678	-0,4170	0,3058	0,3571	-0,7113	
9	0,9	12,0318	2,6306	2,2713	0,3718			1,3508	1,5432	0,6629	-0,3542		
10	1	14,6623	2,7239	2,6431				2,8940	-0,6786	0,3086			

Table 2.2

k	x	y	y'	z	z'
0	0	3	8	-2	11
1	0,1	3,3672	8,893	-2,1586	12,0016
2	0,2	3,4944	9,0755	-2,0867	11,8412
3	0,3	3,5964	9,1834	-1,9906	11,5588
4	0,4	4,5409	9,8967	-0,8149	7,8005
5	0,5	5,6169	11,7272	-0,4934	7,5905
6	0,6	6,9469	13,2274	0,6664	4,2813

7	0,7	8,2766	16,0025	0,5507	6,0738
8	0,8	10,129	18,4902	1,7678	3,0578
9	0,9	12,0318	22,7128	1,3508	6,6286

(1.3) values received under the formula are necessary for specifying, having calculated them under the formula (1.4). The obtained data we will write down in the table.

Table 2.3

k	x	Δy_k	$\Delta y_k^{кор.}$	Δz_k	$\Delta z_k^{кор.}$
0	0				
1	0,1				
2	0,2				
3	0,3	0,9445	0,946075	1,1757	1,010942
4	0,4	1,0761	1,069808	0,3215	0,710767
5	0,5	1,3300	1,256483	1,1598	0,647071
6	0,6	1,3297	1,444138	-0,1157	0,441063
7	0,7	1,8523	1,733608	1,2171	0,537967
8	0,8	1,9028	2,037263	-0,4170	0,381975
9	0,9	2,6306	2,470742	1,5432	0,602158
10	1	2,7239	2,6431	-0,6786	0,3086

The task 1

To write the program of the decision of the differential equation $y\,'=f(x, y)$ Euler's method on a piece [a,b] with step h and *2h* and the entry condition $y(a)=y_0$**.** The Initial data for task performance undertakes from table 5. To compare results.

The task 2

To write the program of the decision of the differential equation $y\,'=f(x, y)$ *a* method of Runge-Kutta on a piece [a,b] with step h and *2h* and the entry condition $y(a)=y_0$. To estimate an error under the formula (5.5). The initial data for task performance undertakes from table 5.

Table 5

N	Функция	a	b	y_0	h
1	$\frac{3x-y}{x^2+y}$	2	3	1	0.1
2	$\frac{2x+y+4}{2y+x}$	3	4	1	0.1
3	$\frac{x^2-y}{2x+y+1}$	0	1	2	0.1
4	$\frac{x^2-y+2}{xy+3x}$	2	3	1	0.1
5	$\frac{3-x-y^2}{2-xy^2}$	1	2	1	0.1
6	$\frac{2-x-y^2x}{3x+y}$	0	1	1	0.1
7	$\frac{1+3xy}{5-x+y^2}$	0	1	2	0.1
8	$\frac{x^2y+2}{2x-y}$	0	1	1	0.1
9	$\frac{x^2+y+2}{2x-y}$	2	3	2	0.1
10	$\frac{xy+4}{2y-xy+1}$	0	1	3	0.1

CONTROL QUESTIONS

1. To check up for the differential equation of a condition of the theorem of existence and uniqueness.
2. The approached methods of the decision of the differential equations are subdivided into what basic groups?
3. In what form it is possible to receive the decision of the differential equation on Euler's method?
4. What geometrical sense of the decision of the differential equation Euler's method?
5. In what form it is possible to receive the decision of the differential equation on a method of Runge-Kutta?
6. What way of an estimation of accuracy is used at the approached integration of the differential equations by Euler's methods and Runge-Kutta?
7. How to calculate an error under the set formula, using a method of double recalculation?

Laboratory work № 9

Finding the decision of a problem of linear programming to Simplex methods

The plan:

1. A simplex method of the decision of a problem of linear programming
2. Examples of the decision of a problem of linear programming with a simplex method

1. A simplex method of the decision of a problem of linear programming

Decisions of any problem of linear programming can be found either a simplex method, or a method of artificial basis. Before to apply one of the specified methods, it is necessary to write down an initial problem in the form of the primary goal of linear programming if it has no such form of record.

2. Examples of the decision of a problem of linear programming with a simplex method

1.41. For manufacturing of various products A, B and C before acceptance uses three various kinds of raw materials. Norms of the expense of raw materials on manufacture of one product of each kind, the price of one product A, B and C, and also total of raw materials of each kind which can be used the enterprise, are resulted in tab. 1.5.

Table 1.5

Raw materials kind	**Norms of expenses of raw materials (kg) on one product**			**Total of raw materials (kg)**
	A	B	C	
I	18	15	12	360
II	6	4	8	192
II	5	3	3	180
The price of one product (rbl.)	9	10	16	

Products A, B and C can be made in any parities (sale is provided), but manufacture is limited by the raw materials of each kind allocated to the enterprise.

To make the plan of manufacture of products at which the total cost of all production made by the enterprise is maximum.

The decision. We will make mathematical model of a problem. Required release of products A we will designate through x, products B - through x_2, products C - through x_3. As there are restrictions on the fund of raw materials of each kind allocated to the enterprise, variables x_1, x_2, x_3 should be satisfy to the following system of inequalities:

$$\begin{cases} 18x_1+15x_2+12x_3\leqslant 360, \\ 6x_1+4x_2+8x_3\leqslant 192, \\ 5x_1+3x_2+3x_3\leqslant 180. \end{cases} \tag{29}$$

The total cost of production made by the enterprise under condition of release x_t products A, x_2 products B and x_3 products C makes

$$F=9x_1+10x_2+16x_3. \tag{30}$$

Under the economic maintenance variables x_1, X_2 and X_3 can accept only non-negative values:

$$x_1,\ x_2,\ x_3\geqslant 0. \tag{31}$$

Thus, we come to the following mathematical problem: among all non-negative decisions of system of inequalities (29) it is required to find such at which function (30) accepts the maximum value.

Let's write down this problem in the form of the primary goal of linear programming. For this purpose we will pass from restrictions-inequalities to restrictions-equalities. We will enter three additional variables therefore restrictions will register in the form of system of the equations

$$\begin{cases} 18x_1+15x_2+12x_3+x_4=360, \\ 6x_1+4x_2+8x_3+x_5=192, \\ 5x_1+3x_2+3x_3+x_6=180. \end{cases}$$

These additional variables on economic sense mean not used at the given plan of manufacture quantity of raw materials of this or that kind. For example, x_4 is not used quantity of raw materials of I kind.

The transformed system of the equations we will write down in the vector form:

$$x_1P_1+x_2P_2+x_3P_3+x_4P_4+x_5P_5+x_6P_6=P_0,$$

where

$$P_1=\begin{pmatrix}18\\6\\5\end{pmatrix};\quad P_2=\begin{pmatrix}15\\4\\3\end{pmatrix};\quad P_3=\begin{pmatrix}12\\8\\3\end{pmatrix};\quad P_4=\begin{pmatrix}1\\0\\0\end{pmatrix};$$

$$P_5=\begin{pmatrix}0\\1\\0\end{pmatrix};\quad P_6=\begin{pmatrix}0\\1\end{pmatrix};\quad P_0=\begin{pmatrix}360\\192\\180\end{pmatrix}.$$

As among vectors $P_1, P_2, P_3, P_4, P_5, P_6$ are three individual vectors, for the given problem it is possible to write down the basic plan directly. That is plan X = (0; 0; 0; 360; 192; 180), defined by system of three-dimensional individual vectors $P_4, P_5, P_6,$ which form basis of three-dimensional vector space.

We make the simplex table for I iteration (tab. 1.6), we count up values $F_0, z_i - c_i$ and the initial basic plan for an optimality is checked:

$$F_0=(C, P_0)=0; \; z_1=(C, P_1)=0; \; z_2=(C, P_2)=0; \; z_3=(C, P_3)=0;$$

$$z_1-c_1=0-9=-9; \; z_2-c_2=0-10=-10; \; z_3-c_3=-16.$$

For basis vectors $z_i-c_i=0.$

The table 1.6

i	Ба-зис	$C_б$	P_0	9	10	16	0	0	0
				P_1	P_2	P_3	P_4	P_5	P_6
1	P_4	0	360	18	15	12	1	0	0
2	P_5	0	192	6	4	⑧	0	1	0
3	P_6	0	180	5	3	3	0	0	1
4			0	−9	−10	−16	0	0	0

Apparently from tab. 1.6, values of all basic variables x_1, x_2, x_3 are equal to zero, and additional variables accept the values according to problem restrictions. These values of variables answer such "plan" at which it is made nothing, the raw materials are not used also value of criterion function to equally zero (i.e. Cost of made production is absent). This plan, of course, is not optimum.

It is visible and from 4th line of tab. 1.6 as in it is available three negative number: $z_1-c_1=-9$, $z_2-c_2=-10$ и $z_3-c_3=$-16. Negative numbers not only testify to possibility of increase in a total cost of made production, but also show, on this sum how many will increase at introduction in the plan of unit of this or that kind of production.

So, number-9 means that at inclusion in the plan of manufacture of one product And the release increase is provided - production for 9 rouble. If to include in the manufacture plan on one product In and With the total cost of produced production will increase accordingly for 10 and 16 rouble. Therefore from the economic point of view the most expedient inclusion in the plan of manufacture of products an is it is necessary to make and on the basis of a formal sign of a simplex method as the maximum negative number on absolute size Δ_i costs in 4th line of a column of vector P_3. Hence, into basis we will enter vector P_3. We define a vector which is subject to an exception of basis. It is for this purpose found θ_0=min (b_i/a_{i3}) for $a_{i3}>0$, i.e. θ_0= min (360/12; 192/8; 180/3) = 192/8.

Найдя число 192/8 = 24, мы тем самым с экономической точки зрения определили, какое количество изделий С предприятие может изготовлять с учетом норм расхода и имеющихся объемов сырья каждого вида. Так как сырья данного вида соответственно имеется 360, 192 и 180 кг, а на одно изделие С требуется затратить сырья каждого вида соответственно 12, 8 и 3 кг, то максимальное число изделий С, которое может быть изготовлено предприятием, равно min(360/12; 192/8; 180/3)=192/8=24, т. е. ограничивающим фактором для производства изделий С является имеющийся объем сырья II вида. С учетом его наличия предприятие может изготовить 24 изделия С. При этом сырье II вида будет полиостью использовано.

Having found number 192/8 = 24, we thereby from the economic point of view have defined, what quantity of products About the enterprise can produce taking into account norms of the expense and available volumes of raw materials of each kind. As raw materials of the given kind accordingly there are 360, 192 and 180 kg, and With it is required to spend for one product of raw materials of each kind accordingly 12, 8 and 3 kg the maximum number of products With which can be made the enterprise, is equal min (360/12; 192/8; 180/3) =192/8=24, i.e. Limiting factor for manufacture of products C is the available volume of raw materials of II kind. Taking into account its presence the enterprise can make 24 products C. At these raw materials of II kind полиостью it will be used.

Hence, vector P_5 is subject to an exception of basis. The column of vector P_3 and 2nd line are directing. We make the table for II iteration (tab. 1.7).

Table 1.7

i	Ба-зис	$C_б$	P_0	9	10	16	0	0	0
				P_1	P_2	P_3	P_4	P_5	P_6
1	P_4	0	72	9	9	0	1	−3/2	0
2	P_3	16	24	3/4	1/2	1	0	1/8	0
3	P_6	0	108	11/4	3/2	0	0	−3/8	1
4			384	3	−2	0	0	2	0

At first we fill a line of the vector again entered into basis, i.e. A line, which number coincides with number of a directing line. Here 2nd line is directing. Elements of this line of tab. 1.7 turn out from corresponding elements of tab. 1.6 their division into a resolving element. Thus in column C_6 we write down factor C_3=16 standing in a column of vector P_3 entered into basis. Then we fill elements of columns for the vectors entering into new basis. In these columns on crossing of lines and columns of the vectors with the same name we put down units, and all other elements it is believed equal to zero.

To definition of other elements of tab. 1.7 it is applied a triangle rule. These elements can be calculated and is direct under recurrent formulas.

Let's calculate the elements of tab. 1.7 standing In a column of vector P_0. The first of them is in 1st line of this column. For its calculation it is found three numbers:

1) the number standing in tab. 1.6 on crossing of a column of vector P_0 and 1st line (360);

2) the number standing in tab. 1.6 on crossing of a column of a vector of Rz and 1st line (12);

3) the number standing in tab. 1.7 on crossing of a column of vector P_0 and 2nd line (24).

Subtracting from the first product of two others, we find a required element: 360-12*24=72; we write down it in 1st line of a column of vector P_0 tab. 1.7.

The second element of a column of vector P_0 tab. 1.7 has been already calculated earlier. For calculation of the third element of a column of vector P_0 also it is found three numbers. The first of them (180) is on crossing of 3rd line and a column of vector P_0 tab. 1.6, the second (3) - on crossing of 3rd line and a column of vector P_3 tab. 1.6, the third (24) - on crossing of 2nd line and a column of vector P_0 tab. 1.8. So, the specified element is 180-24·3=108. Number 108 it is written down in 3rd line of a column of a vector of P_0 of tab. 1.7.

Value F_0 can be found in 4th line of a column of the same vector in two ways:

1) under the formula $F_0=(C, P_0)$, i.e. F_0=0*72+16*24+0*108=384;

2) by a triangle rule; in this case the triangle is formed by numbers 0,-16, 24. This way results besides to result: 0 (-16) *24 = 384.

At definition by a rule of a triangle of elements of a column of a vector of P_0 the third standing in the bottom top of a triangle, all time remains invariable and first two numbers varied only. We will consider it at a finding of elements of a column of vector P_1 tab. 1.7. For calculation of the specified elements first two numbers we take from columns of vectors P_1 and P_3 tab. 1.6, and the third - from tab. 1.7. This number costs on crossing of 2nd line and a column of vector P_1 of last table. As a result we receive values of required elements: 18-12·3/4 =9; 5-3·(3/4)=11/4.

Число z_1-c_1 в 4-й строке столбца вектора P_1 табл. 1.7 можно найти двумя способами:

The number z_1-c_1 can be found tab. 1.7 in 4th line of a column of vector P_1 in two ways:

1) under the formula $z_1-c_1=(C, P_1)-c_1$ had 0*9+16*3/4 + +0*11/4-9=3;

2) by a triangle rule we will receive -9-(-16)*(3/4)=3. Similarly we find elements of a column of vector P_2. Elements of a column of a vector of R it is calculated by a triangle rule. However constructed for definition of these elements triangles look differently.

At calculation of an element of 1st line of the specified column the triangle formed by numbers 0,12 and 1/8 turns out. Hence, the required element is equal 0-12 * (1/8) =-3/2. The element standing in 3rd line of the given column, is equal 0-3*(1/8)=-3/8.

Upon termination of calculation of all elements of tab. 1.7 in it the new basic plan and factors of decomposition of vectors $P\ (j=\overline{1,6})$ through basic vectors P_4, P_3, P_6 and values are received Δ'_j и F'_0.

Apparently from this table, the new basic plan of a problem is plan X = (0; 0; 24; 72; 0; 108). At the given plan of manufacture 24 products are produced With and remains not used 72 kg of raw materials of I kind and 108 kg of raw materials of III kind. Cost of all production made at this plan is equal 384 rbl. the Specified numbers tab. 1.7 are written down in a column of vector P_0. Apparently, the data of this column still represents parametres of a considered problem though they have undergone considerable changes. The data on other columns has changed, and their economic maintenance became more difficult. So, for example, we take the data of a column of vector P_2. Number 1/2 in 2nd line of this column shows, on how many it is necessary to reduce manufacturing of products With if to plan release of one product of Century Numbers 9 and 3/2 in 1st and 3rd lines of vector P_2 show accordingly, how many it is required raw materials I and II kind at inclusion in the plan of manufacture of one product In, and number-2 in 4th line shows that if release of one product will be planned In, it will provide output increase in cost expression for 2 rbl. Differently if to include in the production plan one product In it will demand reduction of release of a product With on 1/2 units and will demand additional expenses of raw materials of I kind of 9 kg and 3/2 kg of raw materials of III kind, and the total cost of produced production according to the new optimum plan will increase for 2 rbl. Thus, numbers 9 and 3/2 act as though as new "norms" of expenses of raw materials I and III kind on manufacturing of one product In (apparently from tab. 1.6, earlier they were equal 15 and 3) that speaks reduction of release of products With.

The same economic sense the data of a column of vector P_1 has also tab. 1.7. The numbers which have been written down in a column of vector P5 have A bit different economic maintenance. Number 1/8 in 2nd line of this column, shows that the increase in volumes of raw materials of II kind at 1 kg would allow to increase release of products C by 1/8 units Simultaneously 3/2 kg of raw materials of I kind and 3/8 kg of raw materials of III kind would be required in addition. The increase in release of products C at 1/8 units will lead to output growth иа 2 rbl.

From stated above the economic maintenance of given tab. 1.7 follows that the plan of a problem found on II iteration is not optimum. It is visible and from 4th line of tab. 1.7 as in a column of vector P_2 of this line there is a negative number-2. Means, it is necessary to enter vector P_0 into basis, i.e. In the new plan it is necessary to provide release of products of B. For definition of possible number of manufacturing of products In it is necessary to consider available quantity of raw materials of each kind, namely: possible release of products In is defined min (b'_i/a'_{i2}) for $a'_{i2}>0,$ i.e. find

$$0_0 = \min\left(\frac{72}{9};\ \frac{24\cdot 2}{1};\ \frac{108\cdot 2}{3}\right) = \frac{72}{9} = 8.$$

Hence, vector P_4, otherwise, release of products B is subject to an exception of basis is limited available the enterprises by raw materials of I kind. Taking into account available volumes of these raw materials the enterprise should make 8 products B. Number 9 is a resolving element, and the column of vector P_2 and 1st line of tab. 1.7 are directing. We make the table for III iteration (tab. 1.8).

Table 1.8

i	Ба-зис	$C_б$	P_0	9	10	16	0	0	0
				P_1	P_2	P_3	P_4	P_5	P_6
1	P_2	10	8	1	1	0	1/9	−1/6	0
2	P_3	16	20	1/4	0	1	−1/18	5/24	0
3	P_6	0	96	5/4	0	0	−1/6	−1/8	1
4			400	5	0	0	2/9	5/3	0

In tab. 1.8 at first we fill elements of 1st line which represents a line of vector P_2 again entered into basis. Elements of this line it is received from elements of 1st line of tab. 1.7 by division of the last into a resolving element (i.e. On 9). Thus in column C_6 of the given line it is written down C_2=10.

Then we fill elements of columns of vectors of basis and by a triangle rule we calculate elements of other columns. As a result in tab. 1.8 new basic plan X = (0 is received; 8; 20; 0; 0; 96) and factors of decomposition of vectors $P_i\ (j=\overline{1,6})$ through basic vectors P_2, P_3, P_6 and corresponding values Δ_j'' и F_0''.

We check, whether the given basic plan is optimum or not. For this purpose we will consider 4th line of tab. 1.8. This line among numbers Δ_j'' are no negative. It means that the found basic plan is optimum and F_{max}=400.

Hence, the output plan including manufacturing of 8 products In and 20 product C, is optimum. At the given plan of release of products the raw materials I and II kinds completely are used and remain not used 96 kg of raw materials of III kind, and cost of made production is equal 400 rbl.

The optimum plan of production does not provide manufacturing of products A. Introduction in an output plan of products of kind A would lead to reduction of the specified total cost. It is visible from 4th line of a column of vector P_1 where number 5 shows that at the given plan inclusion in it of release of unit of a product And leads only to reduction of a combined value of cost by 5 rbl.

The decision of the given example a simplex method could be spent, using only one table (tab. 1.9). In this * to the table all three iterations of computing process are written consistently down one for another.

Table 1.9

i	Базис	$C_б$	P_0	9	10	16	0	0	0
				P_1	P_2	P_3	P_4	P_6	P_0
1	P_4	0	360	18	15	12	1	0	0
2	P_5	0	192	6	4	8	0	1	0
3	P_6	0	180	5	3	3	0	0	1
4			0	−9	−10	−16	0	0	0
1	P_4	0	72	9	9	0	1	−3/2	0
2	P_3	16	24	3/4	1/2	1	0	1/8	0
3	P_6	0	108	11/4	3/2	0	0	−3/8	1
4			384	3	−2	0	0	2	0
1	P_2	10	8	1	1	0	1/9	−1/6	0
2	P_3	16	20	1/4	0	1	−1/18	5/24	0
3	P_6	0	96	5/4	0	0	−1/6	−1/8	1
4			400	5	0	0	2/9	5/3	0

1.42. To find a function $F=2x_1-6x_2+5x_5$ maximum under conditions

$$\begin{cases} -2x_1+x_2+x_3+x_5=20, \\ -\ x_1-2x_2+x_4+3x_5=24, \\ 3x_1-x_2-12x_5+x_6=18, \end{cases}$$

$$x_j \geqslant 0\ (j=\overline{1,6}).$$

The decision. System of the equations of a problem we will write down in the vector form:

$$x_1P_1+x_2P_2+x_3P_3+x_4P_4+x_5P_5+x_6P_6=P_0,$$

where

$$P_1=\begin{pmatrix}-2\\-1\\3\end{pmatrix};\quad P_2=\begin{pmatrix}1\\-2\\-1\end{pmatrix};\quad P_3=\begin{pmatrix}1\\0\\0\end{pmatrix};\quad P_4=\begin{pmatrix}0\\1\\0\end{pmatrix};$$

$$P_5=\begin{pmatrix}1\\3\\-12\end{pmatrix};\quad P_6=\begin{pmatrix}0\\0\\1\end{pmatrix};\quad P_0=\begin{pmatrix}20\\24\\18\end{pmatrix}.$$

As among vectors P_1, P_2, P_3, P_4, P_5, P_6 is available three individual vectors for the given problem it is possible to find the basic plan directly. That is plan X = (0; 0; 20; 24; 0; 18). We make the simplex table (tab. 1.10) and it is checked, whether the given basic plan is optimum.

Table 1.10

i	Базис	$C_б$	P_0	2	−6	0	0	5	0
				P_1	P_2	P_3	P_4	P_5	P_6
1	P_3	0	20	−2	1	1	0	1	0
2	P_4	0	24	−1	−2	0	1	3	0
3	P_6	0	18	3	−1	0	0	−12	1
4			0	−2	6	0	0	−5	0

Apparently from tab. 1.10, the initial basic plan is not optimum. Therefore we pass to the new basic plan. It can be made, as in columns of vectors P_1 and P_5 which 4th line contains negative numbers, there are positive elements. For transition to the new basic plan we will enter into basis vector P_5 and we will exclude from basis vector P_4. We make table of II iteration.

Table 1.11

i	Базис	$C_б$	P_0	2	−6	0	0	5	0
				P_1	P_2	P_3	P_4	P_5	P_6
1	P_3	0	12	−5/3	5/3	1	−1/3	0	0
2	P_5	5	8	−1/3	−2/3	0	1/3	1	0
3	P_6	0	114	−1	−9	0	4	0	1
			40	−11/3	8/3	0	5/3	0	0

Apparently from tab. 1.11, the new basic plan of a problem is not optimum as in 4th line of a column of vector P_1 there is a negative number-11/3. As in a column of this vector there are no the positive elements, the given problem has no optimum plan.

Decisions of problem in MathCAD

Criterion function is set **$L=9x_1+2x_2 \rightarrow max$**

Condition:

$x_1+4x_2 \le 5$

$x1-x2 \le 3$

$7x_1+3x_2 \ge 7$

$x_1, x_2 \ge 0$

L(x1,x2):=9·x1+2·x2

x1:=10 x2:=10

Given

x1+4·x2≤5 x1≥0

x1-x2≤3 x2≥0

7·x1+3·x2≥7

$$\begin{pmatrix}x1\\x2\end{pmatrix} := Maximize(L, x1, x2) = \begin{pmatrix}3.4\\0.4\end{pmatrix} \quad L(x1, x2) = 31.4$$

3. Using the considered method, find the decision following problems

1.49. $F=3x_1+2x_3-6x_6\rightarrow\max;$

$$\begin{cases} 2x_1+x_2-3x_3+6x_6=18, \\ -3x_1+2x_3+x_4-2x_6=24, \\ x_1+3x_3+x_5-4x_6=36, \end{cases}$$

$$x_j\geqslant 0 \ (j=\overline{1,6}).$$

1.50. $F=2x_1+3x_2-x_4\rightarrow\max;$

$$\begin{cases} 2x_1-x_2-2x_4+x_5=16, \\ 3x_1+2x_2+x_3-3x_4=18, \\ -x_1+3x_2+4x_4+x_6=24, \end{cases}$$

$$x_j\geqslant 0 \ (j=\overline{1,6}).$$

1.51. $F=8x_2+7x_4+x_6\rightarrow\max;$

$$\begin{cases} \text{\textcircled{x_1}}-2x_2-3x_4-2x_6=12, \\ 4x_2+\text{\textcircled{x_3}}-4x_4-3x_6=12, \\ 5x_2+5x_4+\text{\textcircled{x_5}}+x_6=25, \end{cases}$$

$$x_j\geqslant 0 \ (j=\overline{1,6}).$$

1.52. $F=x_1+3x_2-5x_4\rightarrow\max;$

$$\begin{cases} 2x_1+4x_2+x_3+2x_4=28, \\ -3x_1+5x_2-3x_4+x_5=30, \\ 4x_1-2x_2+8x_4+x_6=32, \end{cases}$$

$$x_i\geqslant 0 \ (i=\overline{1,6}).$$

1.53. $F=3x_1+2x_5-5x_6\rightarrow\max;$

$$\begin{cases} 2x_1+x_2-3x_5+5x_6=34, \\ 4x_1+x_3+2x_5-4x_6=28, \\ -3x_1+x_4-3x_5+6x_6=24, \end{cases}$$

$$x_j\geqslant 0 \ (j=\overline{1,6}).$$

1.54. $F=x_1+2x_2-x_3\rightarrow\max;$

$$\begin{cases} -x_1+4x_2-2x_3\leqslant 6, \\ x_1+x_2+2x_3\geqslant 6, \\ 2x_1-x_2+2x_3=4, \end{cases}$$

$$x_1,\ x_2,\ x_3\geqslant 0.$$

1.55. $F=8x_1-3x_2+x_3+6x_4-5x_5\rightarrow\max;$

$$\begin{cases} 2x_1+4x_2+\text{\textcircled{x_3}}+x_4-2x_5=28, \\ x_1-2x_2+x_4+x_5=31, \\ -x_1+3x_2+5x_3+4x_4-8x_5=118, \end{cases}$$

$$x_1,\ x_2,\ x_3,\ x_4,\ x_5\geqslant 0.$$

1.56. $F=2x_1-3x_2+4x_3+5x_4-x_5+8x_6\rightarrow\max;$

$$\begin{cases} x_1+5x_2-3x_3-4x_4+2x_5+x_6=120, \\ 2x_1+9x_2-5x_3-7x_4+4x_5+2x_6=320, \end{cases}$$

$$x_j\geqslant 0 \ (j=\overline{1,6}).$$

1.57. $F=-3x_1+5x_2-3x_3+x_4+x_5+8x_6\rightarrow\max;$

$$\begin{cases} x_1-3x_2+4x_3+5x_4-6x_5+x_6=60, \\ 7x_1-17x_2+26x_3+31x_4-35x_5+6x_6=420, \end{cases}$$

$$x_j\geqslant 0 \ (j=\overline{1,6}).$$

1.58. $F=5x_1-x_2+8x_3+10x_4-5x_5+x_6\rightarrow\max;$

$$\begin{cases} 2x_1-x_2+3x_4+x_5-x_6=36, \\ -x_1+2x_2+x_3+2x_4+2x_6=20, \\ 3x_2-x_2+2x_3-x_4+3x_5+x_6=30, \end{cases}$$

$$x_j\geqslant 0 \ (j=\overline{1,6}).$$

THEMES OF SELF STUDY WORKS

1&8. Integer linear programming and its application at the decision of problems of planning of mining manufacture

- Features of integer linear problems and methods of their decision
- Use boolean variables at construction of models of integer problems of planning
- Model of planning of placing coal of the concentrating factories
- Model of operational planning of arrangement of the self-propelled equipment on clearing blocks of mine
- Problem model about cutting
- The decision of integer problems a method from sections
- The decision of integer problems a method of branches and borders
- Partial search in problems with boolean variables

2&9. Nonlinear programming and its use in planning and management of mining manufacture

- General characteristic, the basic types and features of problems of nonlinear programming
- Methods of the decision of problems of unconditional optimisation
- Direct methods of the decision of problems of conditional optimisation
- Methods of transformation for the decision of problems of conditional optimisation
- The approached methods of decisions of nonlinear problems

3&10. Dynamic optimising models of planning and management of mining manufacture

- The general statement and geometrical interpretation of dynamic problems of optimisation
- Optimality principle, the basic functional equation and order of the decision of problems a method of dynamic programming
- Problem about definition of an optimum trajectory of moving of system and its decision a method of dynamic programming
- Use of dynamic programming for the decision of static problems of distribution of resources
- Dynamic problem of distribution of resources
- Problem of search of the shortest distances on a network
- Use of dynamic programming by optimisation of alternative counts

4. Network planning and management of realisation of programs

- The basic definitions and stages of network planning and management
- Network representation of programs (network model)
- Calculation of time parametres of network model
- Construction of the planned schedule of realisation of programs
- Planned schedule optimisation on time at the limited resources
- Planned schedule optimisation on expenses
- Management of process of realisation of programs

5. Analytical models of systems of mass service

- Systems of mass service
- Order of the decision of problems of mass service
- Modelling of systems of mass service with refusals
- The opened systems of mass service with expectation
- The closed systems of mass service

6. Statistical modelling of productions

- Problems of modelling of processes and classification of types of interaction of cars and mechanisms
- Modelling of direct interaction of cars and mechanisms
- Interaction modelling through a warehouse
- Statistical modelling of systems of mass service

7. Decision-making in the conditions of uncertainty

- Elements of the theory of statistical decisions
- Choice of criterion of decision-making and definition of rational accuracy of the initial information
- The basic concepts of the theory of games
- Methods of the decision of pair games

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1. Юсупбеков Н.Р., Мухитдинов Д.П., Базаров М.Б., Халилов Ж.А. Бошқариш системаларини компьютерли моделлаштириш асослари. Олий ўқув юртлари учун ўқув қўлланма. –Н.: Навоий-Голд-Сервис, 2009.
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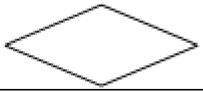
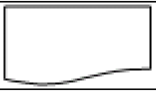
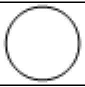
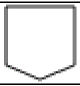
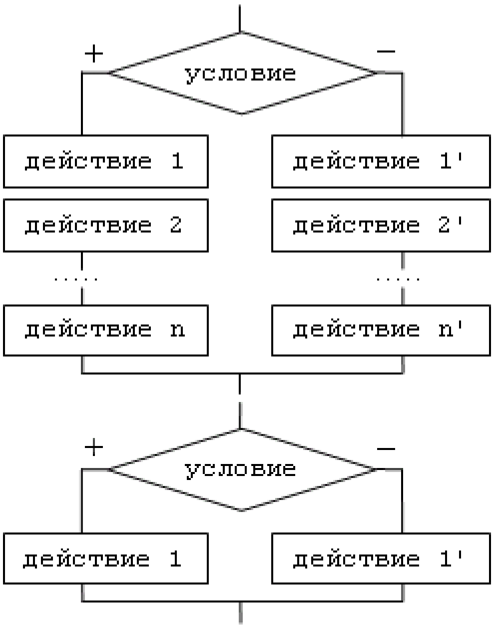
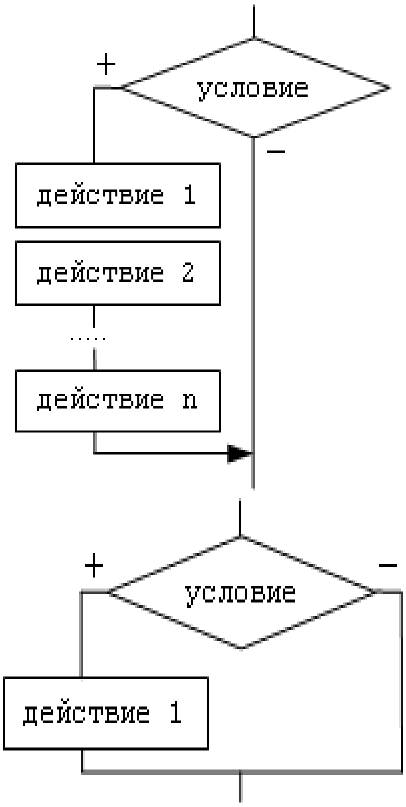
DISTRIBUTING MATERIALS

The name	Symbol (drawing)	Carried out function (explanatory)
1. The block of calculations		Carries out computing action or group of actions
2. The logic block		Choice of a direction of performance of algorithm depending on a condition
3. Input/conclusion blocks		Input or output of the data without dependence from the physical carrier
		Conclusion of the data to the printer
4. The Beginning/end (input/exit)		The beginning or the program end, input or exit in the subroutine
5. The predetermined process		Calculations under the standard or user subroutine
6. The updating block		Performance of the actions changing points of algorithm
7. A connector		Communication instructions between the interrupted lines within one page
8. An interpage connector		Communication instructions between parts of the scheme located on different pages

The full form:



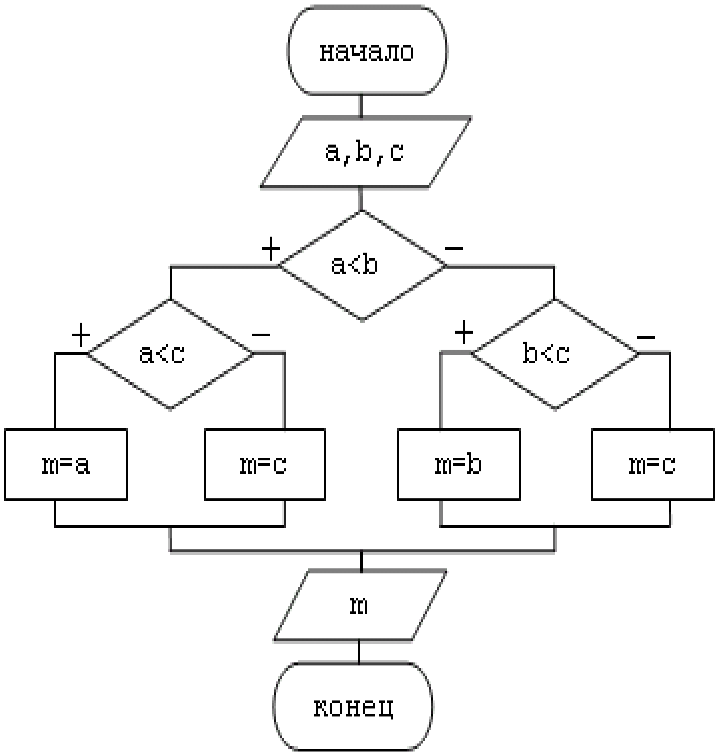
The incomplete form:

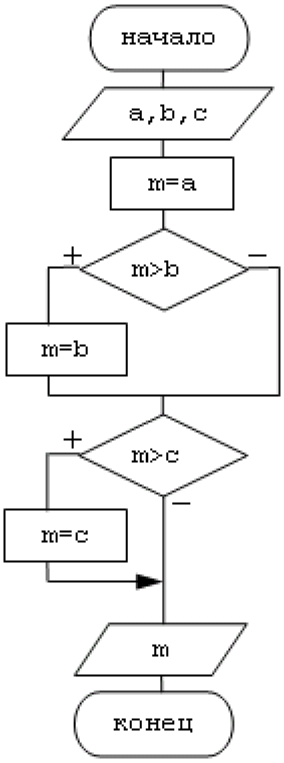


Example: to find least of three numbers.

1 Decision variant:

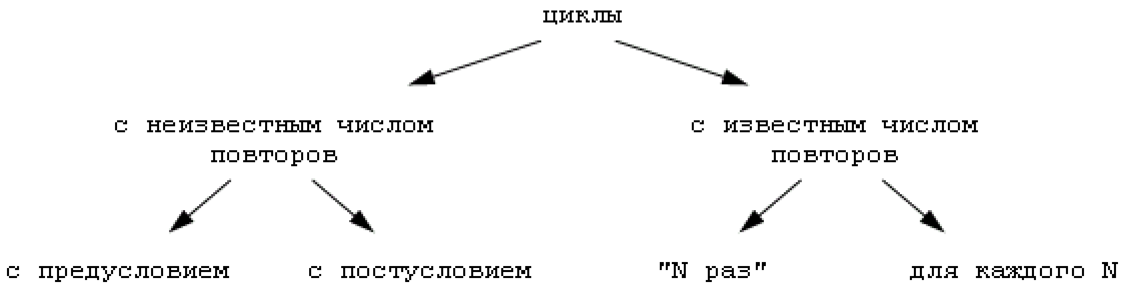
2 variant of the decision:



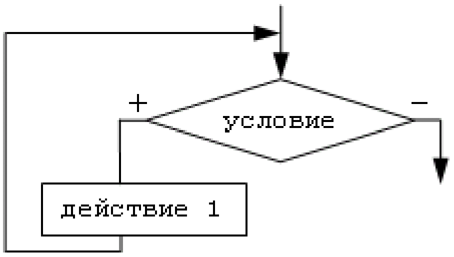


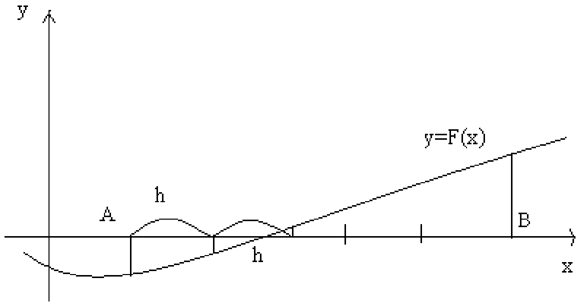
Algorithmic design of a cycle.

Cycle - operating structure, organized repeated performance of the specified action.



Cycle "while":





Rice 1. A piece choice

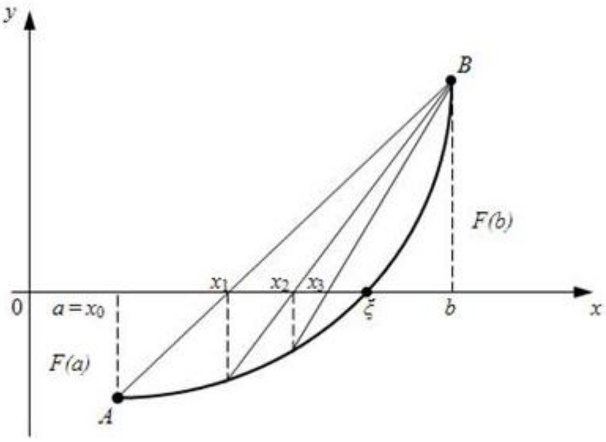


Fig. 2. Geometrical representation of a method of chords.

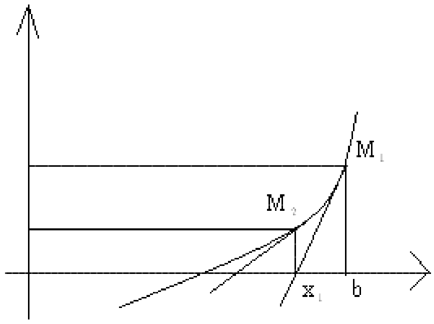


Fig. 2. A choice of points of a contact

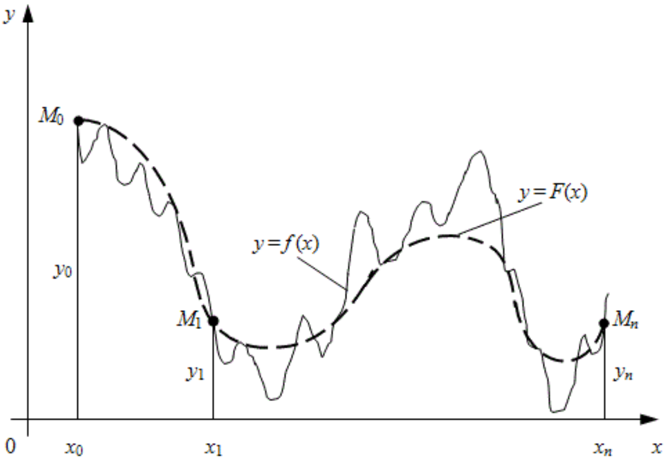


Fig. 1. Geometrical representation of interpolation of function

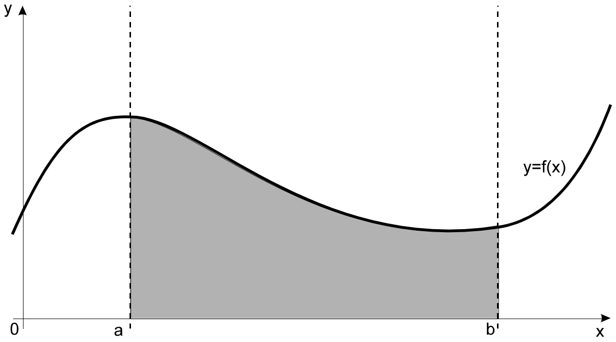


Fig. 1 Curvilinear trapeze.

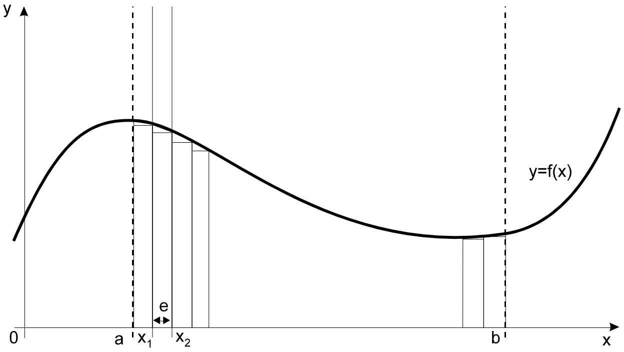


Fig. 2 Method of trapezes.

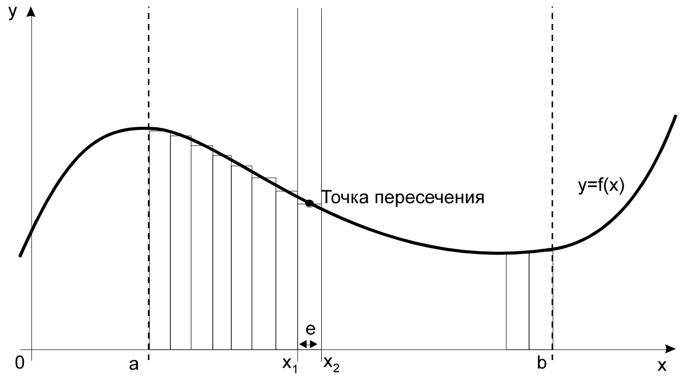


Fig. 3 Method of average rectangles.

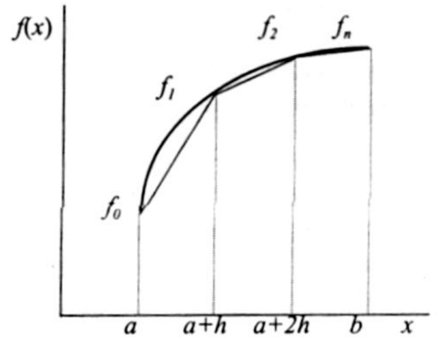


Fig. 4. Interval splitting [a, b] on n identical sites

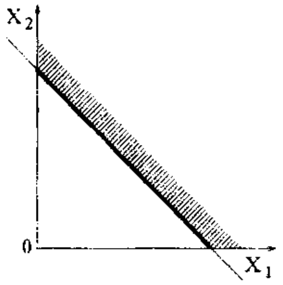


Fig. II.1. Geometrical sense of restriction

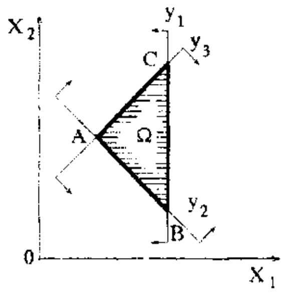


Fig. II.2. Geometrical interpretation of system of restrictions

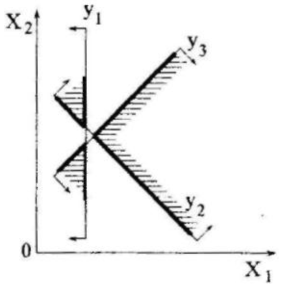


Fig. II.3. Incompatibility systems of restrictions

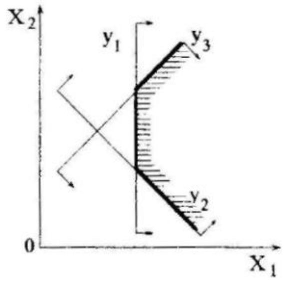


Fig. II.4. Limitlessness of criterion function

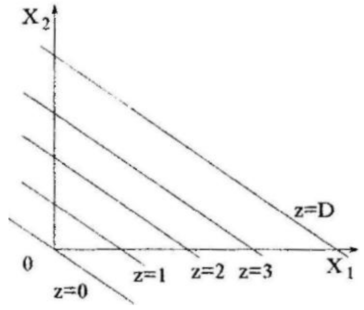


Fig. II.5. Geometrical interpretation of criterion function

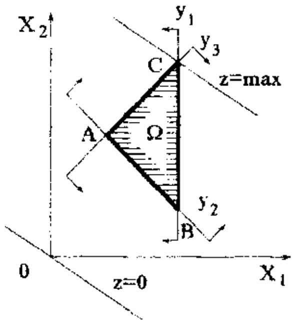


Fig. II.6. Geometrical sense of the optimum decision of a problem of linear programming

QUESTIONS FOR FLOWING, INTERMEDIATE AND TOTAL EXAMINATION

1. Classification of computing methods.
2. Preparation of problems for the personal computer decision.
3. Properties of algorithm.
4. Classification of algorithms.
5. Method branch of roots
6. Method half divisions
7. Method the Chord
8. Newton's method
9. Method of simple iteration
10. Method of secants
11. The decision of system of the linear equations a method of Gaussa
12. Method of Gaussa with a choice of the main element
13. Error estimation at the decision of system of the linear equations
14. Iterative methods of the decision of systems of the linear equations
15. Method of simple iteration of Jacoby
16. Method of Gaussa-Zejdelja
17. The first interpolation Newton's formula
18. The second interpolation Newton's formula
19. The interpolation formula of Stirlinga
20. Types of problems for the ordinary differential equations
21. Euler's method
22. Methods of Runge-Kutta
23. Adams's method
24. Method of trapezes
25. Methods of rectangles
26. Simpson's method
27. The quadrature formula of Gaussa
28. Root-mean-square approach of functions
29. Method of the least squares
30. The primary goal of linear programming
31. Geometrical representation ЛП.
32. Geometrical interpretation of problem ЛП
33. Mathematical bases a simplex of a method of the decision
34. Search of the initial basic decision
35. Features of a transport problem
36. Constructions of basic decision ТЗ
37. Conditions and a method of construction of the optimum decision of a transport problem
38. Algorithm of the decision of a transport problem on a network

VARIANTS TOTAL EXAMINATION

Variant № 1

1. Classification of computing methods.
2. Iterative methods of the decision of systems of the linear equations
3. Simpson's method

Variant № 2

1. Preparation of problems for the personal computer decision.
2. Method of simple iteration of Jacoby
3. The quadrature formula of Gaussa

Variant № 3

1. Root-mean-square approach of functions
2. Method of Gaussa-Zejdel
3. Properties of algorithm.

Variant № 4

1. Classification of algorithms.
2. The first interpolation Newton's formula
3. Method of the least squares

Variant № 5

1. The primary goal of linear programming
2. The second interpolation Newton's formula
3. Method branch of roots

Variant № 6

1. Method half divisions
2. The interpolation formula of Stirling
3. Geometrical representation linear programming.

Variant № 7

1. Geometrical interpretation of problem linear programming
2. Types of problems for the ordinary differential equations
3. Method the Chord

Variant № 8

1. Newton's method
2. Euler's method
3. Mathematical bases a simplex of a method of the decision

Variant № 9

1. Search of the initial basic decision
2. Methods of Runge-Kutta
3. Method of simple iteration

Variant № 10

1. Method of secants
2. Adams's method
3. Features of a transport problem

Variant № 11

1. Constructions of basic decision transport task
2. Method of trapezes
3. The decision of system of the linear equations a method of Gauss

Variant № 12

1. Method of Gauss with a choice of the main element
2. Methods of rectangles
3. Conditions and a method of construction of the optimum decision of a transport problem

Variant № 13

1. Algorithm of the decision of a transport problem on a network
2. Error estimation at the decision of system of the linear equations
3. Classification of computing methods.

Variant № 14

1. Iterative methods of the decision of systems of the linear equations
2. The quadrature formula of Gaussa
3. Algorithm of the decision of a transport problem on a network

Variant № 15

1. Conditions and a method of construction of the optimum decision of a transport problem
2. Simpson's method
3. Error estimation at the decision of system of the linear equations

Variant № 16

1. Method of Gaussa with a choice of the main element
2. Methods of rectangles
3. Constructions of basic decision transport tasks

Variant № 17

1. Features of a transport problem
2. Method of trapezes
3. The decision of system of the linear equations a method of Gaussa

Variant № 18

1. Classification of computing methods.
2. Iterative methods of the decision of systems of the linear equations
3. Simpson's method

Variant № 19

1. Preparation of problems for the personal computer decision.
2. Method of simple iteration of Jakoby
3. The quadrature formula of Gauss

Variant № 20

1. Root-mean-square approach of functions
2. Method of Gaussa-Zeidel
3. Properties of algorithm.

THE BASIC ABSTRACT
(The plan, keywords and word-combinations)

Lecture №1.

Introduction. The cores concept about algorithmization of computing methods.

The plan:

1. Classification of computing methods.
2. Preparation of problems for the personal computer decision.
3. Properties of algorithm.
4. Classification of algorithms.

Lecture №2.

Algorithmization of the numerical decision of the algebraic and transcendental equations. A method branch of roots and a method half divisions.

The plan:

1. A method branch of roots
2. A method half divisions

Lecture № 3.

Algorithmization of the numerical decision of the algebraic and transcendental equations. Method a chord and Newton's method.

The plan:

1. A method the Chord
2. Newton's method

Lecture № 4.

Algorithmization of the numerical decision of the algebraic and transcendental equations. A method of iteration and a method of secants.

The plan:

1. A method of simple iteration
2. A method of secants

Lecture № 5.

Algorithmization of the numerical decision of system of the algebraic and transcendental equations. Method of Gaussa.

The plan:

1. The decision of system of the linear equations a method of Gaussa
2. A method of Gaussa with a choice of the main element
3. An error estimation at the decision of system of the linear equations

Lecture № 6.

Algorithmization of the numerical decision of system of the algebraic and transcendental equations. Iterative methods of Jacoby and Zejdel.

The plan:

1. Iterative methods of the decision of systems of the linear equations
2 Method of simple iteration of Jacoby
3. A method of Gaussa-Zejdel

Lecture № 7.

Algorithmization interpolation methods. Interpolation functions.

The plan:

1. Introduction
2. The first interpolationNewton's formula
3. The second interpolation Newton's formula
4. The interpolation formula of Stirlinga
5. An example

Lecture № 8.

The numerical decision of the differential equations. Euler's method.

The plan:

1. Types of problems for the ordinary differential equations
2. Euler's method

Lecture № 9.

The numerical decision of the differential equations. A method of Runge-Kutta and Adams.

The plan:

1. Methods of Runge-Kutta
2. Adams's method

Lecture №10.

Numerical integration. Quadrature formulas of trapezes and rectangles. Simpson's formula.

The plan:

1. Classification of methods
2. A method of trapezes
3. Methods of rectangles
4. Simpson's method

Lecture № 11.

Numerical integration. The formula of Gaussa.

The plan:

1. The quadrature formula of Gaussa

Lecture № 12.

Root-mean-square approach of functions. A method of the least squares

The plan:

1. Root-mean-square approach of functions
2. A method of the least squares

Lecture № 13

Statement of a problem of linear programming. The basic properties the decision of a problem of linear programming.

The plan:

1. The primary goal of linear programming
2. Examples of the decision of a problem

Lecture № 14

Geometrical interpretation of a problem of linear programming.

The plan:

1. Problem statement
2. Geometrical representation.
3. An example of the decision of a problem
4. Geometrical problem interpretation

Lecture №15.

Finding the decision of a problem of linear programming to simplex methods.

The plan:

1. Mathematical bases a simplex of a method of the decision

Lecture №16.

Finding the decision of a problem of linear programming. A method of artificial basis.

The plan:

1. Search of the initial basic decision

Lecture №17.

Transport problem. Methods initial basic the decision

The plan:

1. Features of a transport problem
2. Constructions of the basic decision

Lecture №18.

Method of potentials for a finding the optimum decision transport problems.

The plan:

1. Conditions and a method of construction of the optimum decision of a transport problem
2. Algorithm of the decision of a transport problem on a network

TESTS ON DISCIPLINE

1. In what method consecutive approach are calculated under the formula

$$x_{n+1} := x_n - \frac{f(x_n)\cdot(a-x_n)}{f(a)-f(x_n)}$$

Method of chords
Method of tangents
Method of division of a piece half-and-half
Method a trapeze

2. The condition of monotonous convergence consecutive приближений in a method of chords is:

Preservation of a sign on the second derivative initial function
Preservation of a sign on the first derivative initial function
Coincidence of signs on the first and second derivatives of initial function
Coincidence of signs by the first

3. What speed of convergence of a method of tangents?

The square-law
The linear
The cubic
The face-to-face

4. What speed of convergence of a method of chords?

The linear
The square-law
The cubic
The face-to-face

5. Criterion of convergence of an iterative method:

Own numbers of a matrix of transition on the module there is less than unit
Own numbers of a matrix of transition on the module there is more than unit
The system matrix - is a matrix with diagonal prevalence
Matrix

6. The formula of Zejdel is an initial formula of a method:

Зейделя
Simple iterations
Relaxations
The reflective

7. The relaxation method converges, if:

The relaxation parametre w lies on an interval (0,2)
The relaxation parametre w on the module is less 2
The relaxation parametre w is not negative
The reflective

8. The number of conditionality of a matrix of system influences on:

Sensitivity decisions to an error of the initial data
Speed of convergence of iterative process
Choice of initial approach
The adaptive

9. By means of a sedate method is:

Maximum on the module own number
The maximum own number
Minimum on the module own number
The extreme

10. By means of a method of rotations:

The matrix is led to a diagonal kind
The matrix is led to a triangular kind
The matrix is transposed
Vector

CRITERIA OF THE ESTIMATION

КИРИШ

Кадрлар тайёрлаш Миллий дастурини амалга оширишнинг янги сифат босқичида олий таълим муассасаларида талабалар билимини баҳолаш ва назорат қилишнинг рейтинг тизимини жорий этишдан мақсад мамлакатимизда таълим сифатини ошириш орқали рақобатбардош юқори малакали мутахассисларни тайёрлашдан иборатдир. Олий ўқув юртларида талабаларнинг билим даражаси асосан рейтинг тизими бўйича баҳоланади. Талабалар билимини рейтинг тизими асосида баҳолаш – талабанинг бутун ўқиш жараёни давомида ўз билимини ошириши учун мунтазам ишлаши ҳамда ўз ижодий фаолиятини такомиллаштиришини рағбатлантиришга қаратилган.

Рейтинг тизими мамлакатимизда юқори малакали мутахассислар тайёрлашнинг сифат кўрсаткичларининг жаҳон андозалари ҳамда халқаро мезонларга мувофиқлигини таъминлашга қаратилган.

«Ҳисоблаш усулларини алгоритмлаш» фани бўйича тайёрланган мазкур услубий кўрсатма Ўзбекистон Республикасининг "Таълим тўғрисида", "Кадрлар тайёрлаш миллий дастури тўғрисида"ги қонунлари ва Ўзбекистон Республикаси Олий ва ўрта махсус таълим вазирлигининг 2005 йил 30 сентябрдаги 217-сонли буйруғи билан тасдиқланган "Олий таълим муассаларида талабалар билимини баҳолашнинг рейтинг тизими тўғрисида мувaққат Низом", 2005 йил 21 февралдаги 34-сонли "Талабалар мустақил ишини ташкил этиш, назорат қилиш ва баҳолаш тартиби тўғрисида Намунавий низом", 2006 йил 18 июлдаги 166-сонли буйруғи билан тасдиқланган дастури асосида ишлаб чиқилган. Ушбу услубий кўрсатма фан ўқитувчилари томонидан «Ҳисоблаш усулларини алгоритмлаш» фанидан талабалар билимини баҳолашда кенг фойдаланишга тавсия этилиб, айни пайтда талабалар учун ҳам мазкур фанни ўзлаштириш жараёнида қандай баллар тўплаш мумкинлиги ҳақида тасаввурга эга бўлиш имконини беради.

«Ҳисоблаш усулларини алгоритмлаш» фани бўйича талабалар билимини баҳолашнинг рейтинг тизими қуйидаги вазифаларнинг бажарилишини кўзда тутади:

1) талабанинг бутун семестр давомида бир меъёрда ва фаол равишда ўқишини таъминлаш;

2) семестр мобайнида талабалар билими, маҳорати, кўникмалари, тасаввур ва тушуниш қобилиятларини объектив назорат қилиш имконини бериш;

3) талаба билимининг сифат кўрсаткичларини ҳаққоний, аниқ ва адолатли баллар орқали баҳолаш;

4) талабаларнинг ўзлаштиришини доимий назоратга олиш билан уларнинг бутун ўқув йили давомида ўз устида бир меъёрда ва фаол равишда ишлашини йўлга қўйиш;

5) талабаларнинг ўқув йили давомидаги давоматини тўлиқ таъминлаш;

6) талабаларнинг мустақил ишлашга бўлган кўникмаларини ривожлантириш;

7) Давлат таълим стандарти, ўқув режалари асосида «Ҳисоблаш усулларини алгоритмлаш» фанинг ўқув дастурлари ва ўқув машғулотларига оид турли услубий қўлланмаларни такомиллаштириш ҳамда бошқа услубий ишларни ўтказишни олдиндан режалаштириш;

8) талабаларда билим олишга интилиш даражасини, профессор-ўқитувчиларда эса ўқитиш масъулиятини ошириш;

9) талабаларни қўшимча ахборот манбаларидан самарали фойдаланишга ундаш ва бошқалар.

«Ҳисоблаш усулларини алгоритмлаш» фанидан тайёрланган ушбу рейтинг тизими бўйича услубий кўрсатма институтнинг барча бакалавриат таълим йўналишининг биринчи босқич талабаларига мўлжалланган.

2. РЕЙТИНГ БАҲОЛАШ ТУРЛАРИ ВА ШАКЛЛАРИ

«Ҳисоблаш усулларини алгоритмлаш» фани барча таълим йўналишлариининг ўқув режаси бўйича 2-босқич 3-семестрига мўлжалланган. Мазкур фанлар бўйича талабаларнинг ўзлаштиришини баҳолаш бутун ўқув семестр давомида мунтазам равишда олиб борилади ҳамда қуйидаги назорат турлари орқали амалга оширилади:

1) жорий баҳолаш – (ЖБ);
2) оралиқ баҳолаш – (ОБ);
3) якуний баҳолаш – (ЯБ).

Жорий баҳолаш (ЖБ)да фаннинг ҳар бир мавзуси бўйича талабанинг билим даражасини аниқлаб бориш назарда тутилади. У одатда маъруза, амалий ёки семинар машғулотлари дарсларида амалга оширилиши мумкин. Талабанинг билим даражасини энг аввало унинг аудиториядаги, яъни дарс ўтиш жараёнидаги фаоллиги, ўтилган мавзуларни ўзлаштириш даражаси белгилаб бериб, у қуйидаги ҳолатлар орқали намоён бўлади:

1) Маърузани сифатли конспектлаштириш даражаси, тинглаш, ўқитувчи томонидан ташкил этилган мавзуга оид баҳс ва мунозараларда фаол иштирок этиш;

2) Амалий ёки семинар машғулотларини конспектлаштириш даражаси,унга тайёргарлик кўриш, мисол-масала, тест ва бошқаларни ишлаб чиқишда фаол қатнашиш ва ҳ.к.

Барча фанлар каби бу фандан талабанинг семестр давомида ўзлаштириш кўрсаткичи 100 баллик тизимда баҳоланади. Шундан ЖБга жами баллнинг 35 бали (унинг 11 балли – мустақил таълимга) ***ажратилган***. Ушбу ЖБ таркибига талабаларнинг маъруза, амалий ва семинар дарсларига фаол қатнашишлари, уй вазифаларини бажаришлари, мисоллар, масалалар, ёзма иш, назорат ишлари, тест ва кейс-стадиларни ечишлари ҳамда мустақил ишлар бўйича топшириқларни бажаришлари натижасида тўплаган баллари киради.

Талабаларнинг фанни ўзлаштиришлари бўйича назорат турлари ичида “оралиқ баҳолаш” (ОБ) муҳим аҳамият касб этади.

Оралиқ баҳолаш (ОБ) да «Ҳисоблаш усулларини алгоритмлаш» фанинг бир неча мавзуларини қамраб олган бўлими ёки қисми бўйича машғулотлар ўтиб бўлингандан сўнг талабанинг билимлари баҳоланади. ОБда талабанинг муайян саволга жавоб бериш ёки муаммони ечиш маҳорати ва қобилияти аниқланади. ОБга жами баллнинг 35%, яъни 35 бали (унинг 11 бали – мустақил таълимга) ажратилган бўлиб, у ёзма иш, назорат иши, тест, оғзаки савол-жавоб ва бошқа кўринишларда ўтказилиши мумкин. Оралиқ назоратни ёзма иш шаклида ўтказилганда 2-5 та саволдан иборат бўлган вариантлар тузиб олинади. Оралиқ назорат тест шаклида ўтказилса, у ҳолда 10 та тест саволидан кам бўлмаган вариантлар тузилади. Агар оралиқ назорат оғзаки савол-жавоб тарзида ўтказилса, у ҳолда 2-5та саволдан иборат бўлган вариантлар тузилиб, улар асосида талабанинг билими баҳоланади. Оралиқ назорат саволлари ҳар бир янги ўқув йили бошида кафедра профессор-ўқитувчилари томонидан тузилиб, кафедра мажлисида муҳокама қилинади ва тасдиқланади.

Ушбу фандан семестр давомида икки марта ОБ назоратини ўтказиш режалаштирилган. Ҳар бир оралиқ назорат бўйича талабанинг билимини мос равишда 17 ва 18 баллардан, жами 35 баллга қадар баҳолаш мумкин. ОБни НДКИда ишлаб чиқилган ўқув жараёни жадвали (графиги) асосида ўтказиш кўзда тутилади.

Ҳар бир профессор-ўқитувчи белгиланган кунларда оралиқ назоратни ўтказиб, талабаларнинг ЖБ ва ОБ бўйича олган балларини тегишли гуруҳ журналига, кафедра ва деканатдаги оралиқ назоратларни қайд қилиш журналларига ёзиб қўйишлари шарт.

Якуний баҳолаш (ЯБ) одатда ўқув семестрининг охирида фаннинг ўтилган барча мавзулари бўйича талаба ўзлаштирган билимни баҳолаш мақсадида ўтказилади. У ёзма иш ёки бошқа шаклларда (оғзаки, тест, ҳимоя ва ҳоказо) ўтказилиши мумкин. Якуний баҳолашга жами баллнинг 30 %, яъни 30 балли ажратилган. Якуний назорат ёзма иш шаклида ўтказиш режалаштирилади.

ЯБ ёзма иши Ўзбекистон Республикаси Олий ва ўрта махсус таълим Вазирлигининг 2005 йил 30 сентябрдаги 217–сонли буйруғи билан тасдиқланган Олий таълим муассасаларида талабалар билимини баҳолашнинг рейтинг тизими тўғрисида муваққат НИЗОМнинг 1-иловасида келтирилган «Рейтинг тизимини яуний баҳолаш босқичида ёзма иш усулини қўллаш тартиби»га биноан ўтказилади.

«Ҳисоблаш усулларини алгоритмлаш» фани бўйича талабалар билимини баҳолашда қуйидаги намунавий мезонларни инобатга олиш тавсия этилади:

Балл	**Баҳо**	**Талабанинг билим даражасини ифодаловчи ҳолатлар**
86 - 100	Аъло	• Introduction. The cores concept about algorithmization of computing methods. • Algorithmization of the numerical decision of the algebraic and transcendental equations. A method branch of roots and a method половинного divisions. • Algorithmization of the numerical decision of the algebraic and transcendental equations. A method a chord and Newton's method. • Algorithmization of the numerical decision of the algebraic and transcendental equations. A method of iteration and a method of secants. • Algorithmization of the numerical decision of system of the algebraic and transcendental equations. A method of Gaussa. • Algorithmization of the numerical decision of system of the algebraic and transcendental equations. Iterative methods of Jakobi and Zejdelja. • Algorithmization интерполяционной methods. • Интерполирование functions. • The numerical decision of the differential equations. Euler's method. • The numerical decision of the differential equations. A method of Runge-Kutta and Adams. • Numerical integration. Kvadraturnye formulas of trapezes and rectangles. Simpson's formula. • Numerical integration. The formula of Gaussa. • Root-mean-square approach of functions. • Method of the least squares. • Statement of a problem of linear programming. The basic properties the decision of a problem of linear programming. • Geometrical interpretation of a problem of linear programming. • Finding the decision of a problem of linear programming to Simplex methods. • Finding the decision of a problem of linear programming. A method of artificial basis. • Transport problem. Methods initial basic the decision. • Method of potentials for a finding optimum decisions transport problems. • таърифини қўллаш билиши ва фанни аъло даражада ўзлаштирган бўлиши зарур.
71 -	Яхши	• Введение. Основные понятие об алгоритмизации вычислительных методов.

85		• Algorithmization of the numerical decision of the algebraic and transcendental equations. A method branch of roots and a method половинного divisions. • Algorithmization of the numerical decision of the algebraic and transcendental equations. A method a chord and Newton's method. • Algorithmization of the numerical decision of the algebraic and transcendental equations. A method of iteration and a method of secants. • Algorithmization of the numerical decision of system of the algebraic and transcendental equations. A method of Gaussa. • Algorithmization of the numerical decision of system of the algebraic and transcendental equations. Iterative methods of Jakobi and Zejdelja. • Algorithmization интерполяционной methods. • Интерполирование functions. • The numerical decision of the differential equations. Euler's method. • The numerical decision of the differential equations. A method of Runge-Kutta and Adams. • Numerical integration. Kvadraturnye formulas of trapezes and rectangles. Simpson's formula. • Numerical integration. The formula of Gaussa. • Root-mean-square approach of functions. • Method of the least squares. • Statement of a problem of linear programming. The basic properties the decision of a problem of linear programming. • Geometrical interpretation of a problem of linear programming. • Finding the decision of a problem of linear programming to Simplex methods. • билиши ва фанни яхши ўзлаштирган бўлиши зарур.
55 – 70	Қониқарли	• Introduction. The cores concept about algorithmization of computing methods. • Algorithmization of the numerical decision of the algebraic and transcendental equations. A method branch of roots and a method половинного divisions. • Algorithmization of the numerical decision of the algebraic and transcendental equations. A method a chord and Newton's method. • Algorithmization of the numerical decision of the algebraic and transcendental equations. A method of iteration and a method of secants. • Algorithmization of the numerical decision of system of the algebraic and transcendental equations. A method of Gaussa. • Algorithmization of the numerical decision of system of the algebraic and transcendental equations. Iterative methods of Jakobi and Zejdelja. • Algorithmization интерполяционной methods. • Интерполирование functions. • The numerical decision of the differential equations. Euler's method. • The numerical decision of the differential equations. A method of Runge-Kutta and Adams. • Numerical integration. Kvadraturnye formulas of trapezes and rectangles. Simpson's formula. • Numerical integration. The formula of Gaussa. • Root-mean-square approach of functions. • Method of the least squares. • Statement of a problem of linear programming. The basic properties the decision of a problem of linear programming. • ларни билиши ва қисман ўзлаштирган бўлиши зарур.
0 – 54 гача	Қониқарсиз	• ўтилган мавзулар бўйича назарий ҳамда амалий билимларни ўзлаштиришнинг паст даражаси; • муҳим топшириқ ва масалалар бўйича хулоса ва қарорларни қабул қила олмаслиги; • фаннинг моҳиятини тушунмаслиги ҳамда унинг асосий қоидаларини баён эта олмаслиги ва ҳ.к.

«Ҳисоблаш усулларини алгоритмлаш» фани бўйича талабанинг ЖБ, ОБ ва ЯБдаги ўзлаштириш кўрсаткичи ҳар бир семестр якунида деканатлар томонидан бериладиган махсус қайдномаларга киритилиши ва уларнинг натижалари кафедра мажлисида таҳлил қилиб борилиши лозим.

3. ТАЛАБАЛАР БИЛИМИНИ БАҲОЛАШ ТАРТИБИ

Талабалар билимининг балларда ифодаланган ўзлаштириши қуйидагича баҳоланади:

86 – 100 балл – “Аъло”;
71 – 85 балл – “Яхши”;
55 – 70 балл – “Қониқарли”;
0 – 54 баллгача – “Қониқарсиз”.

Саралаш бали 55 бални ташкил қилади.

Агар бу фан бўйича талаба бирор баҳолаш турини (ЖБ, ОБ) бўйича ижобий натижага эга бўлмаса, у ҳолда талабанинг қайта ўзлаштирган билимини баҳолаш учун муҳлат одатда навбатдаги шу назорат тури ўтказилгунга қадар белгиланади.

ЖБга ажратилган умумий балл ва ОБга ажратилган умумий баллдан саралаш балини тўплаган талабага якуний баҳолашда иштирок этиш ҳуқуқи берилади. Семестр якунида фан бўйича саралаш балидан кам балл тўплаган талабанинг ўзлаштириши қониқарсиз ҳамда академик қарздор ҳисобланади. Академик қарздор талабаларга семестр тугаганидан кейин қайта ўзлаштириши учун муддат берилади. Кафедранинг тегишли профессор-ўқитувчиларига қарздор талабалар билан ишлаш вазифаси юклатилиб, у махсус график шаклида тузилади.

4. ЯКУНИЙ БАҲОЛАШДА ЁЗМА ИШНИ ЎТКАЗИШ ТАРТИБИ

Талабалар билимини рейтинг тизими бўйича баҳолашнинг ёзма иш усули, талабаларда мустақил фикрлаш ва ўз фикрини ёзма ифодалаш кўникмаларини ривожлантиради.

«Ҳисоблаш усулларини алгоритмлаш» фанидан якуний баҳолаш ёзма иш шаклида ўтказилади. Ёзма иш саволлари ва вариантлари ўқув йилининг бошида кафедра профессор-ўқитувчилари томонидан янгидан тузилиб, кафедра мажлисида муҳокама этилади ва тасдиқланади. Ўтиладиган барча фанлар бўйича хар бир ўқув йили учун якуний баҳолаш бўйича ёзма иш саволлари ва вариантлари «Автоматлаштирилган бошќарув ва информацион технологиялар» кафедрасининг йигилиши куриб чикилади, мухокама килинади ва тасдиқланади.

Ёзма ишнинг ҳар бир варианти бўйича қўйилган саволларнинг мазмуни, қамров даражаси ва аҳамиятлиги даражаси кафедра мудири томонидан текширилиб, унинг имзоси билан тасдиқланади. Ёзма ишни ўтказиш асосан семестрнинг сўнгги иккита ўқув ҳафталарига мўлжалланган бўлиб, у белгиланган ҳафталардаги мазкур фан бўйича ўқув машғулотлари чоғида ўтказилади. Ёзма иш вариантида 3 та назорат саволлари келтирилади. Ёзма ишларни баҳолаш мезонлари якуний баҳолашга ажратилган 30 баллдан келиб чиққан ҳолда ишлаб чиқилади, яъни ҳар бир саволга максимум 10 баллдан тўғри келади. Ёзма иш ўтказилгандан кейин уч кун давомида профессор-ўқитувчилар уни текшириб баҳолайдилар. Ёзма иш ҳажми талабанинг фан бўйича тасаввури, билими, амалий кўникмасини баҳолаш учун етарли бўлиши зарур. Талабаларнинг ёзма ишлари икки йил мобайнида деканатда сақланади.

5. РЕЙТИНГ НАТИЖАЛАРИНИ ҚАЙД ҚИЛИШ ТАРТИБИ

«Ҳисоблаш усулларини алгоритмлаш» фанидан талабанинг билимини баҳолаш турлари орқали тўплаган баллари семестр якунида рейтинг қайдномасига бутун сонлар билан қайд қилинади.

«ҲИСОБЛАШ УСУЛЛАРИНИ АЛГОРИТМЛАШ» фанидан
РЕЙТИНГ ЖАДВАЛИ

	Рейтинг назорати турлари ва уларнинг сони	Рейтинг меъёрлари	
		МИН (семестр)	МАКС(семестр)
	I.ЖОРИЙ БАҲОЛАШ - 35 балл		
1	Амалий машғулотида фаол иштирок этиш ва топшириқларни ва мустақил ишларни (топшириқларни) бажариш	13+6	24+11
	Жами балл:	19	35
	II.ОРАЛИҚ БАҲОЛАШ 35 балл		
2	Ўтилган мавзулар ва мустақил ўрганиши лозим бўлган мавзулар саволлари бўйича ёзма назорат ўтказиш	13+6	24+11
	Жами балл:	19	35
	Жами тўпланган баллар (ЖБ+ОБ)	38	70
	III. ЯКУНИЙ БАҲОЛАШ (30 % = 30 балл)		
3	Ёзма иш шаклида назорат	17	30
	Умумий баллар (ЖБ+ОБ+ЯБ)	55	100

ТАЛАБАНИНГ ФАННИ ЎЗЛАШТИРИШ ДАРАЖАСИ:

Ўзлаштириш баҳоси	Умумий баллар –УБ (I+II+III)	Тўпланган рейтинг (саралаш) баллари	
		ЖБ+ОБ	ЯБ
Аъло	86 -100	60 – 70	16 – 30
Яхши	71 – 85	50 – 59	21 - 26
Қониқарли	55 – 70	39 – 49	16 - 21
Қониқарсиз	55 баллдан кам	38 баллдан кам	17 баллдан кам

Изоҳ: *Жадвал «Технологик жараёнлар ва ишлаб чиқаришни автоматлаштириш ва бошқарув» кафедраси мажлисида тасдиқланган (Баённома №1, 2013 йил «27» август).*

STANDARD DOCUMENTS

ЎЗБЕКИСТОН РЕСПУБЛИКАСИНИНГ ҚОНУНИ
АХБОРОТЛАШТИРИШ ТЎҒРИСИДА

(Ўзбекистон Республикаси Олий Кенгашининг Ахборотномаси, 1993 й., 6-сон, 252-модда; 2001 йил, 1-2-сон, 23-модда)

I БОБ. УМУМИЙ ҚОИДАЛАР

1-модда. Қонуннинг мақсади

Ушбу Қонун ахборот мажмуи фаолиятининг иқтисодий, ҳуқуқий ва ташкилий асосларини, унинг Ўзбекистон Республикасида тутган ўрни ва аҳамиятини белгилайди, ахборот эгалари ва ахборотдан фойдаланувчилар бўлмиш давлат ҳокимияти ва бошқарув органлари, юридик ва жисмоний шахслар ўртасидаги муносабатларни тартибга солиб туради.

2-модда. Қонуннинг амал қилиш соҳаси

Ушбу Қонун давлат органларининг, юридик ва жисмоний шахсларнинг:

ахборотларни тўплаш, жамғариш, қайта ишлаш, узатиш, қўллаш ва рухсат этилмаган танишувдан сақлаш;

ахборот тизимларини, маълумотлар базалари ва банкларини, ахборотларни қайта ишлаш ва узатишнинг бошқа тизимларини яратиш, жорий этиш ва улардан фойдаланиш соҳасидаги муносабатларига нисбатан татбиқ этилади.

Ушбу Қонун бошқа қонунларнинг (оммавий ахборот воситалари тўғрисидаги ҳамда бошқа қонунларнинг) таъсири остидаги ахборотга, ҳужжатлаштирилмаган ахборотга, шунингдек муаллифлик ва патент ҳуқуқи меъёрлари билан тартибга солинадиган муносабатларга тааллуқли эмас.

3-модда. Давлатнинг ахборотлаштириш соҳасидаги сиёсати

Давлатнинг ахборотлаштириш соҳасидаги сиёсатининг асосий йўналишлари Ўзбекистон Республикаси Вазирлар Маҳкамаси республикани ривожлантиришнинг истиқболга мўлжалланган ҳамда реал илмий-техникавий, иқтисодий, ижтимоий ва сиёсий шарт-шароитларни ҳисобга олган ҳолда тасдиқлайдиган Ўзбекистон Республикасининг ахборотлаштириш концепциясида белгиланиб:

давлат ва жамоат органларининг, фуқароларнинг, мулкчилик шаклидан қатъи назар, корхоналар, муассасалар ва ташкилотларнинг (матнда бундан кейин «ташкилотлар» деб юритилади) ахборотга бўлган эҳтиёжини ҳар томонлама қондиришни;

ахборотни бир тартибга солишни, стандартлаштиришни, ягона ахборот майдони яратишни ҳамда республика жаҳон ахборот ҳамжамиятига кириши учун шароит яратишни;

ахборотлаштиришнинг жамият ривожига таъсирини ўрганишни ва баҳолашни назарда тутади.

II БОБ. АХБОРОТ ТИЗИМЛАРИ ВА УЛАРНИНГ МАЖМУИ

4-модда. Ўзбекистон Республикасининг ахборот мажмуи

Республиканинг ахборот мажмуи давлат органлари, юридик ва жисмоний шахсларнинг ахборот тизимларидан ташкил топади.

5-модда. Давлат органларининг ахборот тизимлари

Республика бюджети ҳисобидан вужудга келтирилган ҳамда давлат ҳокимияти ва бошқарув органларининг фаолият кўрсатишини таъминловчи ахборотларга ишлов бериш тизимлари, маълумот базалари ва банклари, эксперт ва ахборот-қидирув тизимлари ҳамда шохобчалари Ўзбекистон Республикаси давлат органларининг ахборот тизимига киради.

6-модда. Ҳудудий ахборот тизимлари

Ҳудудий ахборот тизимлари маҳаллий давлат ҳокимияти ва бошқарув органларининг таҳлил этиш ва бошқариш вазифаларини таъминлаш учун ташкил этилади.

7-модда. Тармоқлар ва ташкилотларнинг ахборот тизимлари

Тармоқлар ва ташкилотларнинг ахборот тизимлари вазирликлар ва идоралар, мулк шаклидан қатъи назар, концернлар, корпорациялар, ишлаб чиқариш бирлашмалари, ташкилотлар ва корхоналарнинг ишлашини таъминловчи ахборотларга ишлов бериш тизимларидан, маълумот базалари ва банкларидан иборатдир.

Автоматлаштирилган кредит-банк ва биржа тизимлари ҳамда пулсиз муомала тизимлари ҳам тармоқ ахборот тизимларига киради.

8-модда. Автоматлаштирилган кредит-банк ва биржа тизимлари

Автоматлаштирилган кредит-банк ва биржа тизимлари ўзаро ҳисоб-китоблар жадал ўтказилишини таъминлаш, кредит-молия операцияларини амалга ошириш, шунингдек биржа фаолиятини, брокерлик ва маклерлик хизматларини автоматлаштириш, солиқ ва аудиторлик фаолиятларини амалга ошириш (бюджетларни, капитал маблағларни, солиқ назоратини шакллантириш) учун тузилади.

9-модда. Пулсиз муомаланинг автоматлаштирилган тизимлари

Пулсиз муомаланинг автоматлаштирилган тизимлари кредит карточкалари ва пулсиз молия ҳужжатларининг бошқа турларидан фойдаланган ҳолда ўзаро ҳисоб-китоблар ўтказишда аҳолига қулайлик яратиш мақсадларида Ўзбекистон Республикаси Жамғарма банки тизими, шунингдек бошқа банклар, манфаатдор вазирликлар ва идоралар асосида ташкил этилади.

10-модда. Ахборот узатиш

Тармоқ, ҳудудий ва давлат ахборот тизимлари ўртасида ахборотлар узатиш зарур рўйхат, маълумотлар таркиби ва ҳажмлари доирасида олдиндан келишган ҳолда амалга оширилади.

11-модда. Хусусий ва давлат тасарруфида бўлмаган бошқа ахборот тизимлари

Жисмоний шахсларнинг (Ўзбекистон Республикаси, бошқа давлатлар фуқароларининг) ахборот тизимлари ўз маблағлари ҳисобига ташкил этилади ва улар томонидан белгиланган тартибда рухсатнома олинган тақдирдагина ишлатилади.

Давлат тасарруфида бўлмаган ахборот тизимлари ўз муассисларининг маблағлари ҳисобига ташкил этилади ва улар томонидан ахборот маҳсулотлари яратиш ва хизматлари ташкил этиш учун фойдаланилади.

12-модда. Алоқа ва маълумотлар узатиш тизимлари

Алоқа ва маълумотлар узатиш тизимлари ахборотлаштиришнинг коммуникациявий асоси ҳисобланади. Мазкур тармоқлар алоқага қўшилиш, маълумотларни қабул қилиш ва узатишга оид халқаро стандартлар ва протоколлар талабларига риоя этиш асосида тузилади, улар эса алоқа тармоқлари тузилмасининг янги турларини яратиш ва ахборот хизматининг янги турларини ташкил этиш имкониятини таъминлайди.

13-модда. Ахборотлаштиришда тизим, дастур ва тармоқ таъминоти бирлиги

Ахборотлаштиришда тизим, дастур ва тармоқ таъминоти бирлиги ахборотлаштириш жараёнларининг давлат томонидан тартибга солиниши принципларига, шунингдек ахборот воситалари ва маҳсуллари ишлаб чиқаришда ҳамда улардан фойдаланишда ягона стандартларга, сифат сертификатларига риоя этилиши устидан назоратни амалга оширувчи давлат бошқарувининг махсус органлари фаолиятига асосланади.

III БОБ. АХБОРОТЛАШТИРИШ ИНФРАСТРУКТУРАСИ ВА САНОАТИ

14-модда. Ахборотлар инфраструктураси

Ўзбекистон Республикасининг ахборотлар инфраструктурасини — ахборотларни қайта ишловчи ва ахборотга оид бошқа хизмат кўрсатувчи, автоматлаштирилган тизимларга сервис хизмати кўрсатувчи; ходимлар ва фойдаланувчиларга ўргатувчи; маслаҳат берувчи ва услубиятга доир ишларни бажарувчи, фойдаланувчиларга ахборот хизмати кўрсатиш сифатини оширишга доир бошқа ёрдамчи фойдали фаолиятни амалга оширувчи мулкчиликнинг барча шаклларидаги илмий ва ишлаб чиқариш тузилмалари ташкил этади.

15-модда. Ахборотлаштириш саноати

Давлат органлари томонидан, шунингдек уставида ахборотлаштириш маҳсулоти ишлаб чиқариш фаолияти билан шуғулланиш назарда тутилган, юридик шахслар, шу йўналишда тадбиркорлик фаолиятини амалга ошираётган жисмоний шахслар томонидан ахборотлаштириш маҳсулоти ишлаб чиқариш — ахборотлаштириш саноатидан иборат иқтисодий фаолият тармоғини ташкил этади.

16-модда. Ахборот мажмуининг техника базаси

Ўзбекистон Республикаси ахборот мажмуининг техника базаси замонавий компьютер техникасини, дастурий маҳсулларни, коммуникация ва алоқа воситаларини ўз ичига олади. Техника базаси рухсатномалар, шартномалар ҳамда битимлар асосида республикада чиқариладиган ва республикага олиб келинадиган дастурий-аппарат воситалари негизида вужудга келтирилади.

17-модда. Ахборотлар, ахборотлаштириш маҳсулотлари ва ахборот хизматлари бозори

Ахборотлар, ахборотлаштириш маҳсулотлари ва ахборот хизматлари бозори ушбу Қонуннинг қоидалари ҳисобга олинган ҳолда шакллантирилади. Мулкчилик шаклидан қатъи назар, юридик шахслар, шунингдек жисмоний шахслар ахборотлар, ахборотлаштириш маҳсулотлари ва ахборот хизматлари бозорида тенг мавқели шериклар сифатида қатнашадилар.

IV БОБ. АХБОРОТЛАШТИРИШ СОҲАСИДАГИ БОШҚАРУВ

18-модда. Ахборотлаштириш соҳасидаги давлат бошқарув органлари

Ахборотлаштириш соҳасидаги бошқарувни Ўзбекистон Республикаси Фан ва техника давлат қўмитаси амалга оширади. Ахборотлаштирииш маҳсуллари ва тизимларини ҳуқуқий жиҳатдан муҳофаза қилиш учун махсус хизматлар — Дастурий маҳсуллар давлат реестри, Маълумот базалари давлат реестри ва Ахборот тизимлари давлат реестри ташкил этилади. Давлат органлари, юридик ва жисмоний шахслар фаолияти натижасида ҳосил қилинган ва Давлат реестрларида қайд этилган дастурий маҳсуллар ва маълумотлар базаларининг жамламаси Дастурий-ахборот маҳсулотлари миллий фондини ташкил этади.

19-модда. Давлат бошқарув органларининг ахборотлаштириш соҳасидаги ваколатлари ва масъулияти

Ахборотлаштириш соҳасидаги давлат бошқарув органларининг ваколатларига:

ахборотлаштириш соҳасида давлат сиёсатининг асосларини ишлаб чиқиш, давлатнинг, юридик ва жисмоний шахсларнинг ахборотлаштириш захираларини ҳосил этиш ҳамда улардан фойдаланиш ишларини мувофиқлаштириб бориш, субъектларнинг ахборотлаштириш соҳасидаги муносабатларга тааллуқли ҳуқуқлари ва кафолатларини ҳимоя қилиш;

Давлат бошқарув органлари томонидан ҳужжатларни бир хиллаштириш тизимининг таркиби, давлат ва жамоат фаолиятининг барча соҳаларида тўпланадиган ҳамда ишлов бериладиган ахборотларга, шунингдек одамларнинг ҳуқуқлари ва манфаатлари муҳофаза ҳамда ҳимоя этилишини таъминлаш мақсадида фойдаланиладиган хусусий шахслар тўғрисидаги ахборотларга доир классификаторлар, стандартлар белгиланади.

20-модда. Дастурий маҳсулларни экспертизадан ўтказиш ва сертификациялаш

Ахборотлаштириш маҳсулларининг рақобат қобилиятини таъминлаш ва унинг сифатига давлат таъсирини кучайтириш, шунингдек ички бозорни ҳимоя қилиш мақсадида ана шундай маҳсуллар экспертизадан ўтказилади ва сертификацияланади.

21-модда. Ахборотлаштириш соҳасидаги фаолиятни рағбатлантириш ва давлат томонидан тартибга солиб бориш

Давлат бошқарув органлари ахборот технологияси, ахборотлаштириш саноати яратувчиларини иқтисодий жиҳатдан қўллаб-қувватлайдилар, илмий тадқиқотлар ва ишлаб чиқаришнинг устувор йўналишлари ривожлантирилишини рағбатлантирадилар, ахборот маҳсулларининг рақобат қобилиятини оширишга кўмаклашадилар, мутлақо янги ечимларни патентлашни ва ахборот технологияларини ўзлаштиришни таъминлайдилар.

V БОБ. АХБОРОТЛАР ВА АХБОРОТ ТИЗИМЛАРИНИНГ ҲУҚУҚИЙ РЕЖИМИ

22-модда. Ахборотдан фойдаланишнинг ҳуқуқий режими

Давлат органлари, юридик ва жисмоний шахслар Ўзбекистон Республикасининг қонунларида белгилаб берилган ҳуқуқлари ва мажбуриятларига мувофиқ ҳолда ахборотлаштириш соҳасида ҳуқуқий муносабатларнинг субъектлари сифатида иш кўрадилар.

23-модда. Ахборотларга нисбатан мулкчилик ҳуқуқи

Ахборот давлат органларининг, юридик ва жисмоний шахсларнинг фаолият маҳсули сифатида моддий ёки интеллектуал мулк объекти бўлиши мумкин. Давлат органлари, юридик ва жисмоний шахслар ахборотларга нисбатан Ўзбекистон Республикаси қонунлари билан белгиланадиган мулкий ҳуқуққа эгадирлар.

24-модда. Ахборотга нисбатан мулкдорлик ҳуқуқи субъектлари

Давлат ўзининг ҳокимият ва бошқарув органлари тимсолида, юридик ва жисмоний шахслар ахборотга нисбатан мулкий ҳуқуқ субъекти бўлишлари мумкин.

25-модда. Хусусий шахсларга доир ахборотларни қайта ишлаш

Хусусий шахсларга доир ахборотларни қайта ишлашнинг тизимлари аҳолининг талаб-эҳтиёжлари ва манфаатларидаги ўзгаришларни, фуқароларнинг ижтимоий фикрларини ўрганиш, жиноий ҳаракатларга қарши кураш, Ўзбекистон Республикасининг давлат сирларини, иқтисодиётга оид сирларини ва бошқа сирларини қўриқлаш учун зарур бўлган маълумотларни умумлаштириш ва таҳлил этиш, давлатни ижтимоий-иқтисодий ривожлантиришни бошқариш ҳамда унинг истиқлолини таъминлаш учун зарур бўлган бошқа маълумотлар олиш мақсадида давлат ва жамоат ташкилотлари, бошқа ташкилотлар томонидан вужудга келтирилади.

26-модда. Юридик ва жисмоний шахсларнинг ўзларига доир ахборотлар билан танишуви

Юридик ва жисмоний шахслар ахборотнинг тўлиқ ва ишончли бўлишини таъминлаш мақсадида ўзларига доир ахборотлар билан танишиш, уларга аниқликлар киритиш, ана шу ахборотдан ким ва қандай мақсадда фойдаланаётганини билиш ҳуқуқига эгадирлар.

27-модда. Ахборотнинг эгаси ва ундан фойдаланувчи ўртасидаги муносабатлар

Ахборотнинг эгаси ва у ваколат берган шахслар ахборотларни қайта ишлаш ҳамда улардан фойдаланишнинг амалдаги қонунларга зид келмайдиган режими ва қоидаларини белгилайдилар.

28-модда. Ахборот эгасининг жавобгарлиги

Ахборот эгаси атайин нотўғри, чала, муддатни бузиб ахборот берганлик учун фойдаланувчи олдида жавобгар бўлади, шу туфайли фойдаланувчига етказилган зарарни Ўзбекистон Республикаси қонунларига мувофиқ қоплайди.

29-модда. Ахборотлаштириш соҳасидаги муносабатлар субъектларининг ҳуқуқларини ҳимоя қилиш

Ахборотга ва ахборот маҳсулига доир низолар ҳамда уларга эгалик қилиш ҳуқуқлари қонунлар асосида ҳал этилади.

30-модда. Шахсий ахборотларни ва хусусий шахсларга доир ахборотларни ҳимоя қилиш

Шартномага асосан автоматлаштирилган тизимга киритилган шахсий ахборотлар ва хусусий шахсларга доир ахборотлардан фойдаланишнинг белгиланган қоидаларини бузганлик ҳоллари суд томонидан аниқланади.

31-модда. ЭҲМ учун яратилган дастурга муаллифлик ҳуқуқи

Ижодий фаолияти натижасида ЭҲМ учун дастур яратган жисмоний шахс унинг муаллифи деб эътироф этилади. Башарти, ЭҲМ учун дастур икки ёки ундан ортиқ жисмоний шахснинг биргаликдаги ижодий фаолияти натижасида яратилган бўлса, дастур ҳар бири мустақил аҳамиятга эга қисмлардан иборатми-йўқми ёки унинг бўлиниш-бўлинмаслигидан қатъи назар, бу шахслардан ҳар бири бундай дастурнинг муаллифи деб эътироф этилади.

32-модда. ЭҲМ учун яратилган дастур ва бошқа дастурий-ахборот маҳсулларига бўлган мулкий ҳуқуқ

Ўз маблағи ҳисобига дастур ёки бошқа дастурий-ахборот маҳсуллари яратган ёки ана шу ҳуқуқни дастур муаллифи ёхуд бошқа мулкдордан қонуний асосда олган юридик ёки жисмоний шахс ЭҲМ учун яратилган дастур ва бошқа дастурий-ахборот маҳсулларининг эгаси ҳисобланади.

33-модда. Ахборотлаштириш соҳасидаги низоларни қараб чиқиш тартиби

Ахборотлаштириш соҳасидаги суд тасарруфига кирмаган низоларни қараб чиқиш учун ахборотлаштиришни бошқарувчи давлат органлари ҳузурида Ўзбекистон Республикаси қонунлари асосида иш кўрувчи муваққат ва доимий комиссиялар тузилиши мумкин.

VI БОБ. АХБОРОТЛАШТИРИШ СОҲАСИДА ХАЛҚАРО ҲАМКОРЛИК

34-модда. Давлатлараро муносабатлар

Ахборотлаштириш соҳасидаги давлатлараро муносабатлар икки томонлама ва кўп томонлама битимлар, юридик шахсларнинг ўзаро яхлит, биргаликдаги, жамоа, дастурий ва техникавий жиҳатдан ўзаро бир бутун ахборот тизимлари, шунингдек ахборотлаштиришнинг бошқа масалалари бўйича тузадиган биргаликдаги илмий-техника дастурлари, шартномалари ва мажбуриятлари асосида таркиб топади. Ахборотлаштириш соҳасидаги халқаро ҳамкорлик халқаро шартномалар ва битимлар асосида амалга оширилади.

35-модда. Халқаро коммуникация тармоқларига қўшилиш

Давлат ҳокимияти ва бошқарув органлари, юридик ва жисмоний шахслар шартномалар асосида ўз ахборот тизимларини халқаро ахборот тармоқларига қўшишга ҳақлидирлар. Чекланган тарзда ахборотга ишлов берувчи ахборот тизимларини халқаро ахборот тармоқларига қўшилишига фақат зарур ҳимоя чора-тадбирлари кўрилганидан кейингина йўл қўйилади. Юридик ва жисмоний шахсларга қарашли ахборот тизимларининг ахборотлар тармоқларига ғайриқонуний равишда қўшилиши, худди шунингдек улардан ғайриқонуний йўл билан ахборотлар олиши Ўзбекистон Республикаси қонунларига ҳамда халқаро ҳуқуқ меъёрларига мувофиқ жавобгарликка тортишга сабаб бўлади.

Ўзбекистон Республикасининг Президенти И. КАРИМОВ

Ташкент ш., 1993 йил 7 май, 868–XII-сон

Вазирлар Маҳкамасининг 2002 йил 6 июндаги 200-сон қарорига 2-ИЛОВА

Компьютерлаштириш ва ахборот-коммуникация технологияларини ривожлантириш бўйича Мувофиқлаштирувчи Кенгаш тўғрисида низом

I. Умумий қоидалар
II. Асосий вазифалари
III. Асосий функциялари
IV. Мувофиқлаштирувчи Кенгашнинг ваколатлари
V. Мувофиқлаштирувчи Кенгашнинг таркиби ва тузилмаси
VI. Мувофиқлаштирувчи Кенгашнинг ишини ташкил этиш

I. УМУМИЙ ҚОИДАЛАР

1. Мазкур Низом Ўзбекистон Республикаси Президентининг "Компьютерлаштиришни янада ривожлантириш ва ахборот-коммуникация технологияларини жорий этиш тўғрисида" 2002 йил 30 майдаги ПФ-3080сон Фармонига мувофиқ ташкил этилган Компьютерлаштириш ва ахборот-коммуникация технологияларини ривожлантириш бўйича Мувофиқлаштирувчи Кенгашнинг (кейинги ўринларда Мувофиқлаштирувчи Кенгаш деб аталади) фаолиятинитартибга солади.
2. Мувофиқлаштирувчи Кенгаш Ўзбекистон Республикасида компьютерлаштириш ва ахборот-коммуникация технологияларини ривожлантириш соҳасидаги юқори мувофиқлаштирувчи орган ҳисобланади.
3. Мувофиқлаштирувчи Кенгаш ўз фаолиятини Ўзбекистон Республикасининг Конституцияси ва қонунлари, Ўзбекистон Республикаси Президентининг Фармонлари ва фармойишлари, Ўзбекистон Республикаси Вазирлар Маҳкамасининг қарорлари ва мазкур Низом асосида амалга оширади.

II. АСОСИЙ ВАЗИФАЛАРИ

Қуйидагилар Мувофиқлаштирувчи Кенгашнинг асосий вазифалари ҳисобланади:

4. компьютерлаштириш ва ахборот-коммуникация технологияларини ривожлантиришнинг замонавий жаҳон тенденцияларига ва мамлакатни ижтимоий-иқтисодий ривожлантириш стратегиясига мувофиқ келувчи устувор йўналишларини белгилаш;
5. компьютерлаштириш ва ахборот-коммуникация технологияларини жадал ривожлантириш учун қулай шарт-шароитлар ва иқтисодий рағбатлантириш омиллари яратиш бўйича Ҳукуматга таклифлар киритиш;
6. компьютерлаштириш ва ахборот-коммуникация технологияларини ривожлантириш соҳасига оид дастурлар, лойиҳалар ва бошқа норматив- ҳуқуқий ҳужжатларнинг ишлаб чиқилиши ҳамда экспертизадан ўтказилишини ташкил этиш;
7. ахборот-коммуникация технологияларини ривожлантириш дастурларини бажаришда, миллий ахборот инфратузилмасини шакллантириш ва ривожлантиришда давлат бошқарув органлари, хусусий сектор ҳамда жамоат ташкилотларининг келишилган сиёсат юритишлари ва биргаликда иштирок этишларини таъминлаш;
8. ахборот-коммуникация технологиялари соҳасида рақобат муҳитини шакллантиришга кўмаклашиш, инновация бизнесини, шу жумладан мамлакатимизнинг ўзининг дастурий воситалари ва компьютер техникасини ишлаб чиқиш ҳамда ишлаб чиқаришни қўллаб-қувватлаш, иқтисодиётнинг барча соҳалари ва тармоқлари компьютерлаштирилиши учун шарт-шароитлар яратиш;
9. ахборот-коммуникация технологиялари соҳасида халқаро ҳамкорликни ривожлантиришга, ахборот-коммуникация технологиялари инфратузилмасини ривожлантиришга хорижий инвестициялар, ҳомийлик маблағлари ва грантларни жалб этишга, таълим муассасаларининг ахборот тармоқларидан фойдаланиш имкониятларини кенгайтиришга кўмаклашиш;
10. ахборот-коммуникация технологиялари соҳасида малакали кадрлар тайёрлаш ва уларни қайта тайёрлаш ишларини, шу жумладан мутахассисларнинг чет элда ўқишини мувофиқлаштириш;

11. ахборот-коммуникация технологиялари соҳасида ахборот хавфсизлиги тизимларини янада ривожлантиришни ташкил этиш.

III. АСОСИЙ ФУНКЦИЯЛАРИ

Мувофиқлаштирувчи Кенгаш юкланган вазифаларга мувофиқ қуйидаги функцияларни бажаради:

12. Ўзбекистон Республикаси Ҳукуматига ахборот-коммуникация технологияларини ривожлантиришнинг устувор йўналишларини ва уларни ривожлантириш учун қулай шарт-шароитлар яратиш чора-тадбирларини белгилаш юзасидан таклифлар киритади;
13. компьютерлаштириш ва ахборот-коммуникация технологияларини ривожлантириш дастурлари амалга оширилишини ташкил этади;
14. ахборот-коммуникация технологияларини ривожлантириш соҳасига оид дастурлар, қонун лойиҳалари ва бошқа норматив-ҳуқуқий ҳужжатларнинг ишлаб чиқилишини ташкил этади ҳамда уларни экспертизадан ўтказади;
15. давлат бошқарув органлари, хусусий сектор ҳамда жамоат ташкилотларининг компьютерлаштириш ва ахборот-коммуникация технологияларини ривожлантириш дастурларини бажариш, миллий ахборот инфратузилмасини шакллантириш ва ривожлантириш борасида келишилган сиёсат юритишларини ва биргаликда иштирок этишларини мувофиқлаштиради;
16. Кенгаш мажлисларида Ўзбекистон Республикаси Ҳукумати қарорларининг, ахборот-коммуникация технологияларини ривожлантириш соҳасига оид дастурлар ва тадбирларнинг бажарилишини кўриб чиқади;
17. ахборот-коммуникация технологияларини ривожлантириш масалаларига оид Ҳукумат қарорларининг, Кенгаш қарорларининг давлат бошқарув органлари, субъектлар томонидан бажарилишини назорат қилади;

IV. МУВОФИҚЛАШТИРУВЧИ КЕНГАШНИНГ ВАКОЛАТЛАРИ

Мувофиқлаштирувчи Кенгаш:

18. компьютерлаштириш ва ахборот-коммуникация технологияларини ривожлантириш дастурларини, миллий ахборот инфратузилмасини шакллантириш ва ривожлантириш учун давлат бошқаруви органларини, хўжалик юритувчи субъектлар ва жамоат ташкилотларини жалб этиш;
19. ўз ваколатлари доирасида барча вазирликлар, идоралар, хўжалик бирлашмалари, корхоналар ва ташкилотлар томонидан бажарилиши мажбурий бўлган қарорлар қабул қилиш;

V. МУВОФИҚЛАШТИРУВЧИ КЕНГАШНИНГ ТАРКИБИ ВА ТУЗИЛМАСИ

20. Мувофиқлаштирувчи Кенгашга Кенгаш Раиси бошчилик қилади.
21. Мувофиқлаштирувчи Кенгаш таркибига бошқарув ва ахборот- коммуникация технологиялари соҳасида раҳбарлар ва етакчи мутахассислардан бўлган раис ўринбосарлари ва кенгаш аъзолари киради. Мувофиқлаштирувчи Кенгашнинг шахсий таркиби Ўзбекистон Республикаси Президентининг Фармони билан тасдиқланади.

VI. МУВОФИҚЛАШТИРУВЧИ КЕНГАШНИНГ ИШИНИ ТАШКИЛ ЭТИШ

22. Мувофиқлаштирувчи Кенгашнинг раиси Мувофиқлаштирувчи Кенгашнинг фаолиятига раҳбарлик қилади ва унга юкланган вазифаларнинг бажарилиши учун жавоб беради.
23. Мувофиқлаштирувчи Кенгашнинг раиси ўз ўрнида бўлмаган ҳолларда унинг функцияларини раис ўринбосарларидан бири бажаради.
24. Мувофиқлаштирувчи Кенгашнинг фаолияти тенг ҳуқуқлилик ва қарор қабул қилиш вақтида коллегиаллик принципларига асосланади.
25. Мувофиқлаштирувчи Кенгашнинг мажлиси Мувофиқлаштирувчи Кенгаш аъзоларининг оддий кўпчилиги иштирок этаётган бўлса ваколатли ҳисобланади.
26. Мувофиқлаштирувчи Кенгашнинг мажлиси қарорлари Мувофиқлаштирувчи Кенгашнинг Раиси томонидан тасдиқланадиган протоколлар билан расмийлаштирилади.

GLOSSARY

Absolute error	The size equal to a difference between true value of number and its approached value, received as a result of calculation or measurement
Absolute deviation	The deviation equal to the maximum value of absolute size of a difference between approximating and initial functions on the given piece
Adaptive (adapting) algorithms	The algorithms capable automatically to adapt to character of change of function
Adequacy of mathematical model	The basic requirement shown to mathematical model of the considered phenomenon, consisting that the model should precisely enough (within the limits of admissible errors) to reflect characteristic features of the phenomenon
Approximation	Function approach at which $f (x)$ it is required to replace the given function approximately with some function $\varphi (\upsilon)$ that, That a deviation (somewhat) $\varphi(x)$ from $f(x)$ in the set area was the least
Approximating function	Function with which initial function at approximation is replaced
Global interpolation	Interpolation, which interpolating function $\varphi(x)$ is under construction at once for all considered interval of change x
The problem put correctly	Problem in which for any values of the initial data from some class its decision exists, it is unique and it is steady under the initial data
Significant figures	All figures of the given number, since the first nonzero figure
Integrated (or continuous) approximation	Approximation at approach construction on continuous set of points
Интерполирование	Type of dot approximation, at which *interpolating function* $\varphi(x)$, Initial function $f (x)$ accepts in the set points xi *the* same values yi, as
Iteration	Repeated repetition of process consecutive approximate
Square-law (parabolic) interpolation	Interpolation at which in quality интерполяционной functions on a piece [xi-1, xi+1] the square trinomial is accepted
Correct numerical algorithm (method)	The numerical algorithm (method) having the unique numerical decision at any values of the initial data, and also in case of stability of this decision concerning errors of the initial data
Piece (local) interpolation	Interpolation, at which interpolating function $\varphi(x)$ Is under construction separately for different parts of a considered interval of change$x$
Kusochno-linear (or linear) interpolation	The elementary and often used kind of local interpolation at which the set points incorporate rectilinear pieces, and function comes nearer a broken line with tops in the given points
Method of splines	One of methods of numerical integration, especially effective at strictly limited number of knots
Ineradicable errors	Errors, которыене can be reduced the calculator neither prior to the beginning of the problem decision, nor in the course of its decision
Certain integral from function $f (x)$ on a piece $\varphi (x)$	Limit of the integrated sum at such unlimited increase in number of points of splitting at which the length of greatest of elementary pieces aspires to zero
Relative error	The relation of an absolute error to the approached value of number
Parabolic (square-law) interpolation	Interpolation, at which in quality интерполяционной functions on a piece [xi-1, xi+1] the square trinomial is accepted
Error of approximation of a derivative	The size characterising a deviation of approached value derivative of its true value
Error of restriction of the function received by means of a number	Error arising because of the account of only limited number of members of a number
Error of roundings off	The error connected with limitation of a digit grid of the computer
Adapting (adaptive) algorithms	The algorithms capable automatically to adapt to character of change of function
Function derivative y = $f(x)$	Limit of the relation of an increment of function Δy to an argument increment Δx at aspiration Δx to zero
Spline-function	In special way constructed multinomial of the third degree
Convergence of a numerical method	Aspiration of values of the decision of discrete model of a problem to corresponding values of the decision of an initial problem at aspiration to zero of parametre of digitization
Dot approximation	Approximation at which approach is under construction on the set discrete set of points $\{xi\}$
Steady problem (on initial	Problem, in which small increment of initial size Δx leads to a small increment

parameter x)	of required size Δy
Function odd concerning a point x_0	Function for which $f(x\text{-}x0) = -f(x0\text{-}x)$
Function even concerning a point x_0	Function for which $f(x\text{-}x0) = f(x0\text{-}x)$
Numerical methods	Methods of the decision of the difficult mathematical problems, allowing to reduce the problem decision to performance of final number of arithmetic actions over numbers; thus results turn out in the form of numerical values
Step	Difference between the next values of argument
Extrapolation	интерполирование, applied to the approached function evaluation out of a considered piece ($x<x0$, $x>xn$)

TECHNOLOGY TRAINING AND THE TECHNOLOGICAL CARD

TECHNOLOGY TRAINING AND THE TECHNOLOGICAL CARD ON LECTURE

Theme №1	**Introduction. The cores concept about algorithmization of computing methods**

TECHNOLOGY OF TRAINING
(Carrying out) of lecture employment

Employment time - 2 hours (80 minutes)	Quantity of students: 40-50
Mode of study	Introduction-thematic lecture
The lecture plan	– Classification of computing methods. – Preparation of problems for the personal computer decision. – Properties of algorithm. – Classification of algorithms.

The purpose of educational employment: **preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a lecture material to watch mastering at pupils of this knowledge, to form and develop at them skills.**

***Problems of the teacher*:**	***Results of educational activity:***
• To acquaint students with a lecture material, the basic directions of activity on a considered material and the curriculum; • Logically consistently, it is given reason and clearly to state thoughts, correctly to build oral and written speech; • Studying of the basic concepts and the terms applied within the limits of a lecture material; • To make training more clear and accessible, to interest students, and the same to motivate them for the further training and self-education; • Maintenance of perception, judgement and primary storing of communications and relations in object of studying; • Establishment of correctness and sensibleness of mastering of a new teaching material, revealing of incorrect representations and their correction; • Maintenance of mastering of new knowledge and ways of actions; • Formation of complete representation of knowledge on a theme; • To show a distributing material, to give talks and to give practical tasks.	• Mastering of new knowledge and ways of actions; • Primary check of understanding; • Fastening of knowledge and ways of actions; • Generalisation and ordering of knowledge. • Careful studying and the all-round analysis of a lecture material and increase of an educational level which would provide the decision of the put problem; • Increase of a degree of quality of knowledge through introduction of innovative technologies; • Level monitoring обученности pupils on steps, classes, subjects, it is concrete on each student, for the purpose of revealing of the real reasons influencing progress, dynamics of conformity of level of teaching to educational standards. • Monitoring of professional skill of teachers. • To continue activity on the organisation of interaction of participants of educational space; • To create conditions, to generalise an advanced experience and to motivate students; • Increase scientific информативности in a field of knowledge of a subject matter and related subjects.
Training methods	**Lecture – visualisation, conversation**
Technics of training	**Blitz – the interrogation, focusing questions**
Modes of study	**Collective, face-to-face**
Tutorials	**Projector, supply with information, visual materials, educational - methodical grants**
Training conditions	**The audience provided with tutorials**
Monitoring and estimation of knowledge	**The oral control: a question-answer**

TECHNOLOGICAL CARD
Carrying out of lecture employment on a theme:
«Introduction. The cores concept about algorithmization of computing methods»

Stages, time	*The activity maintenance*	
	The teacher	*Students*
1 stage. Introduction *(15 minutes)*	**1.1. Informs a theme, the purpose, planned results of educational employment and the plan of its carrying out.** **1.2. For the purpose of actualisation of knowledge of students asks focusing questions.**	**1.1. Listen.** **1.2. Answer questions.**
2 stage. Basic (information) **(55 minutes)**	2.1. Consistently states a material of lecture on plan questions. – Classification of computing methods. – Preparation of problems for the personal computer decision. – Properties of algorithm. – Classification of algorithms.	**2.1. Listen, discuss the maintenance of schemes and tables, visual materials, specify, ask questions.** **Write down the main thing.**
3 stage. The final *(10 minutes)*	**3.1. Spends a blitz – interrogation on a theme of lecture employment. Does the total conclusion.** **3.2. Gives the task for independent work.**	**3.1. Answer questions.** **3.2. Listen, write down.**

Theme №2	**Algorithmization of the numerical decision of the algebraic and transcendental equations. A method branch of roots and a method half divisions.**

TECHNOLOGY OF TRAINING
(Carrying out) of lecture employment

Employment time - 2 hours (80 minutes)	Quantity of students: 40-50
Mode of study	Introduction-thematic lecture
The lecture plan	– A method branch of roots – A method half divisions
The purpose of educational employment: **preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a lecture material to watch mastering at pupils of this knowledge, to form and develop at them skills.**	
***Problems of the teacher*:**	***Results of educational activity:***
• To acquaint students with a lecture material, the basic directions of activity on a considered material and the curriculum; • Logically consistently, it is given reason and clearly to state thoughts, correctly to build oral and written speech; • Studying of the basic concepts and the terms applied within the limits of a lecture material; • To make training more clear and accessible, to interest students, and the same to motivate them for the further training and self-education; • Maintenance of perception, judgement and primary storing of communications and relations in object of studying; • Establishment of correctness and sensibleness of mastering of a new teaching material, revealing of incorrect representations and their correction; • Maintenance of mastering of new knowledge and ways of actions; • Formation of complete representation of knowledge on a theme; • To show a distributing material, to give talks and to give practical tasks.	• Mastering of new knowledge and ways of actions; • Primary check of understanding; • Fastening of knowledge and ways of actions; • Generalisation and ordering of knowledge. • Careful studying and the all-round analysis of a lecture material and increase of an educational level which would provide the decision of the put problem; • Increase of a degree of quality of knowledge through introduction of innovative technologies; • Level monitoring обученности pupils on steps, classes, subjects, it is concrete on each student, for the purpose of revealing of the real reasons influencing progress, dynamics of conformity of level of teaching to educational standards. • Monitoring of professional skill of teachers. • To continue activity on the organisation of interaction of participants of educational space; • To create conditions, to generalise an advanced experience and to motivate students; • Increase scientific информативности in a field of knowledge of a subject matter and related subjects.
Training methods	**Lecture – visualisation, conversation**
Technics of training	**Blitz – the interrogation, focusing questions**
Modes of study	**Collective, face-to-face**
Tutorials	**Projector, supply with information, visual materials, educational - methodical grants**
Training conditions	**The audience provided with tutorials**
Monitoring and estimation of knowledge	**The oral control: a question-answer**

TECHNOLOGICAL CARD
Carrying out of lecture employment on a theme:
Algorithmization of the numerical decision of the algebraic and transcendental equations. A method branch of roots and a method half divisions.

Stages, time	*The activity maintenance*	
	The teacher	*Students*
1 stage. Introduction ***(15 minutes)***	**1.1. Informs a theme, the purpose, planned results of educational employment and the plan of its carrying out.** **1.2. For the purpose of actualisation of knowledge of students asks focusing questions.**	**1.1. Listen.** **1.2. Answer questions.**
2 stage. Basic (information) **(55 minutes)**	2.1. Consistently states a material of lecture on plan questions. – A method branch of roots – A method half divisions –	**2.1. Listen, discuss the maintenance of schemes and tables, visual materials, specify, ask questions.** **Write down the main thing.**
3 stage. The final ***(10 minutes)***	**3.1. Spends a blitz – interrogation on a theme of lecture employment. Does the total conclusion.** **3.2. Gives the task for independent work.**	**3.1. Answer questions.** **3.2. Listen, write down.**

Theme №3	**Algorithmization of the numerical decision of the algebraic and transcendental equations. Method a chord and Newton's method**

TECHNOLOGY OF TRAINING
(Carrying out) of lecture employment

Employment time - 2 hours (80 minutes)	Quantity of students: 40-50
Mode of study	Introduction-thematic lecture
The lecture plan	– A method the Chord – Newton's method –
The purpose of educational employment: **preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a lecture material to watch mastering at pupils of this knowledge, to form and develop at them skills.**	
***Problems of the teacher*:** • To acquaint students with a lecture material, the basic directions of activity on a considered material and the curriculum; • Logically consistently, it is given reason and clearly to state thoughts, correctly to build oral and written speech; • Studying of the basic concepts and the terms applied within the limits of a lecture material; • To make training more clear and accessible, to interest students, and the same to motivate them for the further training and self-education; • Maintenance of perception, judgement and primary storing of communications and relations in object of studying; • Establishment of correctness and sensibleness of mastering of a new teaching material, revealing of incorrect representations and their correction; • Maintenance of mastering of new knowledge and ways of actions; • Formation of complete representation of knowledge on a theme; • To show a distributing material, to give talks and to give practical tasks.	***Results of educational activity:*** • Mastering of new knowledge and ways of actions; • Primary check of understanding; • Fastening of knowledge and ways of actions; • Generalisation and ordering of knowledge. • Careful studying and the all-round analysis of a lecture material and increase of an educational level which would provide the decision of the put problem; • Increase of a degree of quality of knowledge through introduction of innovative technologies; • Level monitoring обученности pupils on steps, classes, subjects, it is concrete on each student, for the purpose of revealing of the real reasons influencing progress, dynamics of conformity of level of teaching to educational standards. • Monitoring of professional skill of teachers. • To continue activity on the organisation of interaction of participants of educational space; • To create conditions, to generalise an advanced experience and to motivate students; • Increase scientific информативности in a field of knowledge of a subject matter and related subjects.
Training methods	**Lecture – visualisation, conversation**
Technics of training	**Blitz – the interrogation, focusing questions**
Modes of study	**Collective, face-to-face**
Tutorials	**Projector, supply with information, visual materials, educational - methodical grants**
Training conditions	**The audience provided with tutorials**
Monitoring and estimation of knowledge	**The oral control: a question-answer**

TECHNOLOGICAL CARD
Carrying out of lecture employment on a theme:
Algorithmization of the numerical decision of the algebraic and transcendental equations. Method a chord and Newton's method

Stages, time	*The activity maintenance*	
	The teacher	*Students*
1 stage. Introduction ***(15 minutes)***	**1.1. Informs a theme, the purpose, planned results of educational employment and the plan of its carrying out.** **1.2. For the purpose of actualisation of knowledge of students asks focusing questions.**	**1.1. Listen.** **1.2. Answer questions.**
2 stage. Basic (information) **(55 minutes)**	2.1. Consistently states a material of lecture on plan questions. – A method the Chord – Newton's method	**2.1. Listen, discuss the maintenance of schemes and tables, visual materials, specify, ask questions. Write down the main thing.**
3 stage. The final ***(10 minutes)***	**3.1. Spends a blitz – interrogation on a theme of lecture employment. Does the total conclusion.** **3.2. Gives the task for independent work.**	**3.1. Answer questions.** **3.2. Listen, write down.**

Theme №4	**Algorithmization of the numerical decision of the algebraic and transcendental equations. A method of iteration and a method of secants**

TECHNOLOGY OF TRAINING

(Carrying out) of lecture employment

Employment time - 2 hours (80 minutes)	Quantity of students: 40-50
Mode of study	Introduction-thematic lecture
The lecture plan	– A method of simple iteration – A method of secants
The purpose of educational employment: **preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a lecture material to watch mastering at pupils of this knowledge, to form and develop at them skills.**	
***Problems of the teacher*:**	***Results of educational activity:***
• To acquaint students with a lecture material, the basic directions of activity on a considered material and the curriculum; • Logically consistently, it is given reason and clearly to state thoughts, correctly to build oral and written speech; • Studying of the basic concepts and the terms applied within the limits of a lecture material; • To make training more clear and accessible, to interest students, and the same to motivate them for the further training and self-education; • Maintenance of perception, judgement and primary storing of communications and relations in object of studying; • Establishment of correctness and sensibleness of mastering of a new teaching material, revealing of incorrect representations and their correction; • Maintenance of mastering of new knowledge and ways of actions; • Formation of complete representation of knowledge on a theme; • To show a distributing material, to give talks and to give practical tasks.	• Mastering of new knowledge and ways of actions; • Primary check of understanding; • Fastening of knowledge and ways of actions; • Generalisation and ordering of knowledge. • Careful studying and the all-round analysis of a lecture material and increase of an educational level which would provide the decision of the put problem; • Increase of a degree of quality of knowledge through introduction of innovative technologies; • Level monitoring обученности pupils on steps, classes, subjects, it is concrete on each student, for the purpose of revealing of the real reasons influencing progress, dynamics of conformity of level of teaching to educational standards. • Monitoring of professional skill of teachers. • To continue activity on the organisation of interaction of participants of educational space; • To create conditions, to generalise an advanced experience and to motivate students; • Increase scientific информативности in a field of knowledge of a subject matter and related subjects.
Training methods	**Lecture – visualisation, conversation**
Technics of training	**Blitz – the interrogation, focusing questions**
Modes of study	**Collective, face-to-face**
Tutorials	**Projector, supply with information, visual materials, educational - methodical grants**
Training conditions	**The audience provided with tutorials**
Monitoring and estimation of knowledge	**The oral control: a question-answer**

TECHNOLOGICAL CARD

Carrying out of lecture employment on a theme:

Algorithmization of the numerical decision of the algebraic and transcendental equations. A method of iteration and a method of secants

Stages, time	*The activity maintenance*	
	The teacher	*Students*
1 stage. Introduction ***(15 minutes)***	**1.1. Informs a theme, the purpose, planned results of educational employment and the plan of its carrying out.** **1.2. For the purpose of actualisation of knowledge of students asks focusing questions.**	**1.1. Listen.** **1.2. Answer questions.**
2 stage. Basic (information) **(55 minutes)**	2.1. Consistently states a material of lecture on plan questions. – A method of simple iteration – A method of secants	**2.1. Listen, discuss the maintenance of schemes and tables, visual materials, specify, ask questions.** **Write down the main thing.**
3 stage. The final ***(10 minutes)***	**3.1. Spends a blitz – interrogation on a theme of lecture employment. Does the total conclusion.** **3.2. Gives the task for independent work.**	**3.1. Answer questions.** **3.2. Listen, write down.**

Theme №5	**Algorithmization of the numerical decision of system of the algebraic and transcendental equations. Method of Gaussa**

TECHNOLOGY OF TRAINING
(Carrying out) of lecture employment

Employment time - 2 hours (80 minutes)	Quantity of students: 40-50
Mode of study	Introduction-thematic lecture
The lecture plan	– The decision of system of the linear equations a method of Gaussa – A method of Gaussa with a choice of the main element – An error estimation at the decision of system of the linear equations
The purpose of educational employment: **preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a lecture material to watch mastering at pupils of this knowledge, to form and develop at them skills.**	
***Problems of the teacher*:**	***Results of educational activity:***
• To acquaint students with a lecture material, the basic directions of activity on a considered material and the curriculum; • Logically consistently, it is given reason and clearly to state thoughts, correctly to build oral and written speech; • Studying of the basic concepts and the terms applied within the limits of a lecture material; • To make training more clear and accessible, to interest students, and the same to motivate them for the further training and self-education; • Maintenance of perception, judgement and primary storing of communications and relations in object of studying; • Establishment of correctness and sensibleness of mastering of a new teaching material, revealing of incorrect representations and their correction; • Maintenance of mastering of new knowledge and ways of actions; • Formation of complete representation of knowledge on a theme; • To show a distributing material, to give talks and to give practical tasks.	• Mastering of new knowledge and ways of actions; • Primary check of understanding; • Fastening of knowledge and ways of actions; • Generalisation and ordering of knowledge. • Careful studying and the all-round analysis of a lecture material and increase of an educational level which would provide the decision of the put problem; • Increase of a degree of quality of knowledge through introduction of innovative technologies; • Level monitoring обученности pupils on steps, classes, subjects, it is concrete on each student, for the purpose of revealing of the real reasons influencing progress, dynamics of conformity of level of teaching to educational standards. • Monitoring of professional skill of teachers. • To continue activity on the organisation of interaction of participants of educational space; • To create conditions, to generalise an advanced experience and to motivate students; • Increase scientific информативности in a field of knowledge of a subject matter and related subjects.
Training methods	**Lecture – visualisation, conversation**
Technics of training	**Blitz – the interrogation, focusing questions**
Modes of study	**Collective, face-to-face**
Tutorials	**Projector, supply with information, visual materials, educational - methodical grants**
Training conditions	**The audience provided with tutorials**
Monitoring and estimation of knowledge	**The oral control: a question-answer**

TECHNOLOGICAL CARD
Carrying out of lecture employment on a theme:
Algorithmization of the numerical decision of system of the algebraic and transcendental equations. Method of Gaussa

Stages, time	*The activity maintenance*	
	The teacher	*Students*
1 stage. Introduction ***(15 minutes)***	**1.1. Informs a theme, the purpose, planned results of educational employment and the plan of its carrying out.** **1.2. For the purpose of actualisation of knowledge of students asks focusing questions.**	**1.1. Listen.** **1.2. Answer questions.**
2 stage. Basic (information) **(55 minutes)**	2.1. Consistently states a material of lecture on plan questions. – The decision of system of the linear equations a method of Gaussa – A method of Gaussa with a choice of the main element – An error estimation at the decision of system of the linear equations	**2.1. Listen, discuss the maintenance of schemes and tables, visual materials, specify, ask questions.** **Write down the main thing.**
3 stage. The final ***(10 minutes)***	**3.1.Spends a blitz – interrogation on a theme of lecture employment. Does the total conclusion.** **3.2. Gives the task for independent work.**	**3.1. Answer questions.** **3.2. Listen, write down.**

Theme №6	**Algorithmization of the numerical decision of system of the algebraic and transcendental equations. Iterative methods of Jacoby and Zejdel**

TECHNOLOGY OF TRAINING
(Carrying out) of lecture employment

Employment time - 2 hours (80 minutes)	Quantity of students: 40-50
Mode of study	Introduction-thematic lecture
The lecture plan	– Iterative methods of the decision of systems of the linear equations – Method of simple iteration of Jacoby – A method of Gaussa-Zejdel
The purpose of educational employment: **preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a lecture material to watch mastering at pupils of this knowledge, to form and develop at them skills.**	

***Problems of the teacher*:**	***Results of educational activity:***
• To acquaint students with a lecture material, the basic directions of activity on a considered material and the curriculum; • Logically consistently, it is given reason and clearly to state thoughts, correctly to build oral and written speech; • Studying of the basic concepts and the terms applied within the limits of a lecture material; • To make training more clear and accessible, to interest students, and the same to motivate them for the further training and self-education; • Maintenance of perception, judgement and primary storing of communications and relations in object of studying; • Establishment of correctness and sensibleness of mastering of a new teaching material, revealing of incorrect representations and their correction; • Maintenance of mastering of new knowledge and ways of actions; • Formation of complete representation of knowledge on a theme; • To show a distributing material, to give talks and to give practical tasks.	• Mastering of new knowledge and ways of actions; • Primary check of understanding; • Fastening of knowledge and ways of actions; • Generalisation and ordering of knowledge. • Careful studying and the all-round analysis of a lecture material and increase of an educational level which would provide the decision of the put problem; • Increase of a degree of quality of knowledge through introduction of innovative technologies; • Level monitoring обученности pupils on steps, classes, subjects, it is concrete on each student, for the purpose of revealing of the real reasons influencing progress, dynamics of conformity of level of teaching to educational standards. • Monitoring of professional skill of teachers. • To continue activity on the organisation of interaction of participants of educational space; • To create conditions, to generalise an advanced experience and to motivate students; • Increase scientific информативности in a field of knowledge of a subject matter and related subjects.
Training methods	**Lecture – visualisation, conversation**
Technics of training	**Blitz – the interrogation, focusing questions**
Modes of study	**Collective, face-to-face**
Tutorials	**Projector, supply with information, visual materials, educational - methodical grants**
Training conditions	**The audience provided with tutorials**
Monitoring and estimation of knowledge	**The oral control: a question-answer**

TECHNOLOGICAL CARD
Carrying out of lecture employment on a theme:
Algorithmization of the numerical decision of system of the algebraic and transcendental equations. Iterative methods of Jacoby and Zejdel

Stages, time	*The activity maintenance*	
	The teacher	*Students*
1 stage. Introduction ***(15 minutes)***	**1.1. Informs a theme, the purpose, planned results of educational employment and the plan of its carrying out.** **1.2. For the purpose of actualisation of knowledge of students asks focusing questions.**	**1.1. Listen.** **1.2. Answer questions.**
2 stage. Basic (information) **(55 minutes)**	2.1. Consistently states a material of lecture on plan questions. – Iterative methods of the decision of systems of the linear equations – Method of simple iteration of Jacoby – A method of Gaussa-Zejdel	**2.1. Listen, discuss the maintenance of schemes and tables, visual materials, specify, ask questions.** **Write down the main thing.**
3 stage. The final ***(10 minutes)***	**3.1. Spends a blitz – interrogation on a theme of lecture employment. Does the total conclusion.** **3.2. Gives the task for independent work.**	**3.1. Answer questions.** **3.2. Listen, write down.**

Theme №7	**Algorithmization interpolation methods. Interpolation functions**

TECHNOLOGY OF TRAINING
(Carrying out) of lecture employment

Employment time - 2 hours (80 minutes)	Quantity of students: 40-50
Mode of study	Introduction-thematic lecture
The lecture plan	– Introduction – The first interpolationNewton's formula – The second interpolation Newton's formula – The interpolation formula of Stirlinga – An example

The purpose of educational employment: **preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a lecture material to watch mastering at pupils of this knowledge, to form and develop at them skills.**

***Problems of the teacher*:**	***Results of educational activity:***
• To acquaint students with a lecture material, the basic directions of activity on a considered material and the curriculum; • Logically consistently, it is given reason and clearly to state thoughts, correctly to build oral and written speech; • Studying of the basic concepts and the terms applied within the limits of a lecture material; • To make training more clear and accessible, to interest students, and the same to motivate them for the further training and self-education; • Maintenance of perception, judgement and primary storing of communications and relations in object of studying; • Establishment of correctness and sensibleness of mastering of a new teaching material, revealing of incorrect representations and their correction; • Maintenance of mastering of new knowledge and ways of actions; • Formation of complete representation of knowledge on a theme; • To show a distributing material, to give talks and to give practical tasks.	• Mastering of new knowledge and ways of actions; • Primary check of understanding; • Fastening of knowledge and ways of actions; • Generalisation and ordering of knowledge. • Careful studying and the all-round analysis of a lecture material and increase of an educational level which would provide the decision of the put problem; • Increase of a degree of quality of knowledge through introduction of innovative technologies; • Level monitoring обученности pupils on steps, classes, subjects, it is concrete on each student, for the purpose of revealing of the real reasons influencing progress, dynamics of conformity of level of teaching to educational standards. • Monitoring of professional skill of teachers. • To continue activity on the organisation of interaction of participants of educational space; • To create conditions, to generalise an advanced experience and to motivate students; • Increase scientific информативности in a field of knowledge of a subject matter and related subjects.
Training methods	**Lecture – visualisation, conversation**
Technics of training	**Blitz – the interrogation, focusing questions**
Modes of study	**Collective, face-to-face**
Tutorials	**Projector, supply with information, visual materials, educational - methodical grants**
Training conditions	**The audience provided with tutorials**
Monitoring and estimation of knowledge	**The oral control: a question-answer**

TECHNOLOGICAL CARD
Carrying out of lecture employment on a theme:
Algorithmization interpolation methods. Interpolation functions

Stages, time	*The activity maintenance*	
	The teacher	*Students*
1 stage. Introduction *(15 minutes)*	**1.1. Informs a theme, the purpose, planned results of educational employment and the plan of its carrying out.** **1.2. For the purpose of actualisation of knowledge of students asks focusing questions.**	**1.1. Listen.** **1.2. Answer questions.**
2 stage. Basic (information) **(55 minutes)**	2.1. Consistently states a material of lecture on plan questions. – Introduction – The first interpolationNewton's formula – The second interpolation Newton's formula – The interpolation formula of Stirlinga – An example	**2.1. Listen, discuss the maintenance of schemes and tables, visual materials, specify, ask questions.** **Write down the main thing.**
3 stage. The final *(10 minutes)*	**3.1. Spends a blitz – interrogation on a theme of lecture employment. Does the total conclusion.** **3.2. Gives the task for independent work.**	**3.1. Answer questions.** **3.2. Listen, write down.**

Theme №8	**The numerical decision of the differential equations. Euler's method**

TECHNOLOGY OF TRAINING
(Carrying out) of lecture employment

Employment time - 2 hours (80 minutes)	Quantity of students: 40-50
Mode of study	Introduction-thematic lecture
The lecture plan	– Types of problems for the ordinary differential equations – Euler's method
The purpose of educational employment: **preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a lecture material to watch mastering at pupils of this knowledge, to form and develop at them skills.**	
***Problems of the teacher*:**	***Results of educational activity:***
• To acquaint students with a lecture material, the basic directions of activity on a considered material and the curriculum; • Logically consistently, it is given reason and clearly to state thoughts, correctly to build oral and written speech; • Studying of the basic concepts and the terms applied within the limits of a lecture material; • To make training more clear and accessible, to interest students, and the same to motivate them for the further training and self-education; • Maintenance of perception, judgement and primary storing of communications and relations in object of studying; • Establishment of correctness and sensibleness of mastering of a new teaching material, revealing of incorrect representations and their correction; • Maintenance of mastering of new knowledge and ways of actions; • Formation of complete representation of knowledge on a theme; • To show a distributing material, to give talks and to give practical tasks.	• Mastering of new knowledge and ways of actions; • Primary check of understanding; • Fastening of knowledge and ways of actions; • Generalisation and ordering of knowledge. • Careful studying and the all-round analysis of a lecture material and increase of an educational level which would provide the decision of the put problem; • Increase of a degree of quality of knowledge through introduction of innovative technologies; • Level monitoring обученности pupils on steps, classes, subjects, it is concrete on each student, for the purpose of revealing of the real reasons influencing progress, dynamics of conformity of level of teaching to educational standards. • Monitoring of professional skill of teachers. • To continue activity on the organisation of interaction of participants of educational space; • To create conditions, to generalise an advanced experience and to motivate students; • Increase scientific информативности in a field of knowledge of a subject matter and related subjects.
Training methods	**Lecture – visualisation, conversation**
Technics of training	**Blitz – the interrogation, focusing questions**
Modes of study	**Collective, face-to-face**
Tutorials	**Projector, supply with information, visual materials, educational - methodical grants**
Training conditions	**The audience provided with tutorials**
Monitoring and estimation of knowledge	**The oral control: a question-answer**

TECHNOLOGICAL CARD
Carrying out of lecture employment on a theme:
The numerical decision of the differential equations. Euler's method

Stages, time	*The activity maintenance*	
	The teacher	*Students*
1 stage. Introduction *(15 minutes)*	**1.1. Informs a theme, the purpose, planned results of educational employment and the plan of its carrying out.** **1.2. For the purpose of actualisation of knowledge of students asks focusing questions.**	**1.1. Listen.** **1.2. Answer questions.**
2 stage. Basic (information) **(55 minutes)**	2.1. Consistently states a material of lecture on plan questions. – Types of problems for the ordinary differential equations – Euler's method	**2.1. Listen, discuss the maintenance of schemes and tables, visual materials, specify, ask questions.** **Write down the main thing.**
3 stage. The final *(10 minutes)*	**3.1. Spends a blitz – interrogation on a theme of lecture employment. Does the total conclusion.** **3.2. Gives the task for independent work.**	**3.1. Answer questions.** **3.2. Listen, write down.**

Theme №9	**The numerical decision of the differential equations. A method of Runge-Kutta and Adams**

TECHNOLOGY OF TRAINING
(Carrying out) of lecture employment

Employment time - 2 hours (80 minutes)	Quantity of students: 40-50
Mode of study	Introduction-thematic lecture
The lecture plan	– Methods of Runge-Kutta – Adams's method
The purpose of educational employment: **preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a lecture material to watch mastering at pupils of this knowledge, to form and develop at them skills.**	
***Problems of the teacher*:** • To acquaint students with a lecture material, the basic directions of activity on a considered material and the curriculum; • Logically consistently, it is given reason and clearly to state thoughts, correctly to build oral and written speech; • Studying of the basic concepts and the terms applied within the limits of a lecture material; • To make training more clear and accessible, to interest students, and the same to motivate them for the further training and self-education; • Maintenance of perception, judgement and primary storing of communications and relations in object of studying; • Establishment of correctness and sensibleness of mastering of a new teaching material, revealing of incorrect representations and their correction; • Maintenance of mastering of new knowledge and ways of actions; • Formation of complete representation of knowledge on a theme; • To show a distributing material, to give talks and to give practical tasks.	***Results of educational activity:*** • Mastering of new knowledge and ways of actions; • Primary check of understanding; • Fastening of knowledge and ways of actions; • Generalisation and ordering of knowledge. • Careful studying and the all-round analysis of a lecture material and increase of an educational level which would provide the decision of the put problem; • Increase of a degree of quality of knowledge through introduction of innovative technologies; • Level monitoring обученности pupils on steps, classes, subjects, it is concrete on each student, for the purpose of revealing of the real reasons influencing progress, dynamics of conformity of level of teaching to educational standards. • Monitoring of professional skill of teachers. • To continue activity on the organisation of interaction of participants of educational space; • To create conditions, to generalise an advanced experience and to motivate students; • Increase scientific информативности in a field of knowledge of a subject matter and related subjects.
Training methods	**Lecture – visualisation, conversation**
Technics of training	**Blitz – the interrogation, focusing questions**
Modes of study	**Collective, face-to-face**
Tutorials	**Projector, supply with information, visual materials, educational - methodical grants**
Training conditions	**The audience provided with tutorials**
Monitoring and estimation of knowledge	**The oral control: a question-answer**

TECHNOLOGICAL CARD
Carrying out of lecture employment on a theme:
The numerical decision of the differential equations. A method of Runge-Kutta and Adams

Stages, time	*The activity maintenance*	
	The teacher	*Students*
1 stage. Introduction ***(15 minutes)***	**1.1. Informs a theme, the purpose, planned results of educational employment and the plan of its carrying out.** **1.2. For the purpose of actualisation of knowledge of students asks focusing questions.**	**1.1. Listen.** **1.2. Answer questions.**
2 stage. Basic (information) **(55 minutes)**	2.1. Consistently states a material of lecture on plan questions. – Methods of Runge-Kutta – Adams's method	**2.1. Listen, discuss the maintenance of schemes and tables, visual materials, specify, ask questions.** **Write down the main thing.**
3 stage. The final ***(10 minutes)***	**3.1. Spends a blitz – interrogation on a theme of lecture employment. Does the total conclusion.** **3.2. Gives the task for independent work.**	**3.1. Answer questions.** **3.2. Listen, write down.**

Theme №10	**Numerical integration. Quadrature formulas of trapezes and rectangles. Simpson's formula**

TECHNOLOGY OF TRAINING
(Carrying out) of lecture employment

Employment time - 2 hours (80 minutes)	Quantity of students: 40-50
Mode of study	Introduction-thematic lecture
The lecture plan	– Classification of methods – A method of trapezes – Methods of rectangles – Simpson's method
The purpose of educational employment: **preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a lecture material to watch mastering at pupils of this knowledge, to form and develop at them skills.**	

***Problems of the teacher*:**	***Results of educational activity:***
• To acquaint students with a lecture material, the basic directions of activity on a considered material and the curriculum; • Logically consistently, it is given reason and clearly to state thoughts, correctly to build oral and written speech; • Studying of the basic concepts and the terms applied within the limits of a lecture material; • To make training more clear and accessible, to interest students, and the same to motivate them for the further training and self-education; • Maintenance of perception, judgement and primary storing of communications and relations in object of studying; • Establishment of correctness and sensibleness of mastering of a new teaching material, revealing of incorrect representations and their correction; • Maintenance of mastering of new knowledge and ways of actions; • Formation of complete representation of knowledge on a theme; • To show a distributing material, to give talks and to give practical tasks.	• Mastering of new knowledge and ways of actions; • Primary check of understanding; • Fastening of knowledge and ways of actions; • Generalisation and ordering of knowledge. • Careful studying and the all-round analysis of a lecture material and increase of an educational level which would provide the decision of the put problem; • Increase of a degree of quality of knowledge through introduction of innovative technologies; • Level monitoring обученности pupils on steps, classes, subjects, it is concrete on each student, for the purpose of revealing of the real reasons influencing progress, dynamics of conformity of level of teaching to educational standards. • Monitoring of professional skill of teachers. • To continue activity on the organisation of interaction of participants of educational space; • To create conditions, to generalise an advanced experience and to motivate students; • Increase scientific информативности in a field of knowledge of a subject matter and related subjects.
Training methods	**Lecture – visualisation, conversation**
Technics of training	**Blitz – the interrogation, focusing questions**
Modes of study	**Collective, face-to-face**
Tutorials	**Projector, supply with information, visual materials, educational - methodical grants**
Training conditions	**The audience provided with tutorials**
Monitoring and estimation of knowledge	**The oral control: a question-answer**

TECHNOLOGICAL CARD
Carrying out of lecture employment on a theme:
Numerical integration. Quadrature formulas of trapezes and rectangles. Simpson's formula

Stages, time	*The activity maintenance*	
	The teacher	*Students*
1 stage. Introduction ***(15 minutes)***	**1.1. Informs a theme, the purpose, planned results of educational employment and the plan of its carrying out.** **1.2. For the purpose of actualisation of knowledge of students asks focusing questions.**	**1.1. Listen.** **1.2. Answer questions.**
2 stage. Basic (information) **(55 minutes)**	2.1. Consistently states a material of lecture on plan questions. – Classification of methods – A method of trapezes – Methods of rectangles – Simpson's method	**2.1. Listen, discuss the maintenance of schemes and tables, visual materials, specify, ask questions.** **Write down the main thing.**
3 stage. The final ***(10 minutes)***	**3.1. Spends a blitz – interrogation on a theme of lecture employment. Does the total conclusion.** **3.2. Gives the task for independent work.**	**3.1. Answer questions.** **3.2. Listen, write down.**

Theme №11	**Numerical integration. The formula of Gaussa**

TECHNOLOGY OF TRAINING
(Carrying out) of lecture employment

Employment time - 2 hours (80 minutes)	Quantity of students: 40-50
Mode of study	Introduction-thematic lecture
The lecture plan	– The quadrature formula of Gaussa
The purpose of educational employment: **preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a lecture material to watch mastering at pupils of this knowledge, to form and develop at them skills.**	
***Problems of the teacher*:** • To acquaint students with a lecture material, the basic directions of activity on a considered material and the curriculum; • Logically consistently, it is given reason and clearly to state thoughts, correctly to build oral and written speech; • Studying of the basic concepts and the terms applied within the limits of a lecture material; • To make training more clear and accessible, to interest students, and the same to motivate them for the further training and self-education; • Maintenance of perception, judgement and primary storing of communications and relations in object of studying; • Establishment of correctness and sensibleness of mastering of a new teaching material, revealing of incorrect representations and their correction; • Maintenance of mastering of new knowledge and ways of actions; • Formation of complete representation of knowledge on a theme; • To show a distributing material, to give talks and to give practical tasks.	***Results of educational activity:*** • Mastering of new knowledge and ways of actions; • Primary check of understanding; • Fastening of knowledge and ways of actions; • Generalisation and ordering of knowledge. • Careful studying and the all-round analysis of a lecture material and increase of an educational level which would provide the decision of the put problem; • Increase of a degree of quality of knowledge through introduction of innovative technologies; • Level monitoring обученности pupils on steps, classes, subjects, it is concrete on each student, for the purpose of revealing of the real reasons influencing progress, dynamics of conformity of level of teaching to educational standards. • Monitoring of professional skill of teachers. • To continue activity on the organisation of interaction of participants of educational space; • To create conditions, to generalise an advanced experience and to motivate students; • Increase scientific информативности in a field of knowledge of a subject matter and related subjects.
Training methods	**Lecture – visualisation, conversation**
Technics of training	**Blitz – the interrogation, focusing questions**
Modes of study	**Collective, face-to-face**
Tutorials	**Projector, supply with information, visual materials, educational - methodical grants**
Training conditions	**The audience provided with tutorials**
Monitoring and estimation of knowledge	**The oral control: a question-answer**

TECHNOLOGICAL CARD
Carrying out of lecture employment on a theme:
Numerical integration. The formula of Gaussa

Stages, time	*The activity maintenance*	
	The teacher	*Students*
1 stage. Introduction ***(15 minutes)***	**1.1. Informs a theme, the purpose, planned results of educational employment and the plan of its carrying out.** **1.2. For the purpose of actualisation of knowledge of students asks focusing questions.**	**1.1. Listen.** **1.2. Answer questions.**
2 stage. Basic (information) **(55 minutes)**	2.1. Consistently states a material of lecture on plan questions. – The quadrature formula of Gaussa	**2.1. Listen, discuss the maintenance of schemes and tables, visual materials, specify, ask questions.** **Write down the main thing.**
3 stage. The final ***(10 minutes)***	**3.1. Spends a blitz – interrogation on a theme of lecture employment. Does the total conclusion.** **3.2. Gives the task for independent work.**	**3.1. Answer questions.** **3.2. Listen, write down.**

Theme №12	**Root-mean-square approach of functions. A method of the least squares**

TECHNOLOGY OF TRAINING
(Carrying out) of lecture employment

Employment time - 2 hours (80 minutes)	Quantity of students: 40-50
Mode of study	Introduction-thematic lecture
The lecture plan	– Root-mean-square approach of functions – A method of the least squares
The purpose of educational employment: **preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a lecture material to watch mastering at pupils of this knowledge, to form and develop at them skills.**	
***Problems of the teacher*:**	***Results of educational activity:***
• To acquaint students with a lecture material, the basic directions of activity on a considered material and the curriculum; • Logically consistently, it is given reason and clearly to state thoughts, correctly to build oral and written speech; • Studying of the basic concepts and the terms applied within the limits of a lecture material; • To make training more clear and accessible, to interest students, and the same to motivate them for the further training and self-education; • Maintenance of perception, judgement and primary storing of communications and relations in object of studying; • Establishment of correctness and sensibleness of mastering of a new teaching material, revealing of incorrect representations and their correction; • Maintenance of mastering of new knowledge and ways of actions; • Formation of complete representation of knowledge on a theme; • To show a distributing material, to give talks and to give practical tasks.	• Mastering of new knowledge and ways of actions; • Primary check of understanding; • Fastening of knowledge and ways of actions; • Generalisation and ordering of knowledge. • Careful studying and the all-round analysis of a lecture material and increase of an educational level which would provide the decision of the put problem; • Increase of a degree of quality of knowledge through introduction of innovative technologies; • Level monitoring обученности pupils on steps, classes, subjects, it is concrete on each student, for the purpose of revealing of the real reasons influencing progress, dynamics of conformity of level of teaching to educational standards. • Monitoring of professional skill of teachers. • To continue activity on the organisation of interaction of participants of educational space; • To create conditions, to generalise an advanced experience and to motivate students; • Increase scientific информативности in a field of knowledge of a subject matter and related subjects.
Training methods	**Lecture – visualisation, conversation**
Technics of training	**Blitz – the interrogation, focusing questions**
Modes of study	**Collective, face-to-face**
Tutorials	**Projector, supply with information, visual materials, educational - methodical grants**
Training conditions	**The audience provided with tutorials**
Monitoring and estimation of knowledge	**The oral control: a question-answer**

TECHNOLOGICAL CARD
Carrying out of lecture employment on a theme:
Root-mean-square approach of functions. A method of the least squares

Stages, time	*The activity maintenance*	
	The teacher	*Students*
1 stage. Introduction ***(15 minutes)***	**1.1. Informs a theme, the purpose, planned results of educational employment and the plan of its carrying out.** **1.2. For the purpose of actualisation of knowledge of students asks focusing questions.**	**1.1. Listen.** **1.2. Answer questions.**
2 stage. Basic (information) **(55 minutes)**	2.1. Consistently states a material of lecture on plan questions. – Root-mean-square approach of functions – A method of the least squares	**2.1. Listen, discuss the maintenance of schemes and tables, visual materials, specify, ask questions.** **Write down the main thing.**
3 stage. The final ***(10 minutes)***	**3.1. Spends a blitz – interrogation on a theme of lecture employment. Does the total conclusion.** **3.2. Gives the task for independent work.**	**3.1. Answer questions.** **3.2. Listen, write down.**

Theme №13	**Statement of a problem of linear programming. The basic properties the decision of a problem of linear programming**

TECHNOLOGY OF TRAINING
(Carrying out) of lecture employment

Employment time - 2 hours (80 minutes)	Quantity of students: 40-50
Mode of study	Introduction-thematic lecture
The lecture plan	– The primary goal of linear programming – Examples of the decision of a problem
The purpose of educational employment: **preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a lecture material to watch mastering at pupils of this knowledge, to form and develop at them skills.**	
***Problems of the teacher*:**	***Results of educational activity:***
• To acquaint students with a lecture material, the basic directions of activity on a considered material and the curriculum; • Logically consistently, it is given reason and clearly to state thoughts, correctly to build oral and written speech; • Studying of the basic concepts and the terms applied within the limits of a lecture material; • To make training more clear and accessible, to interest students, and the same to motivate them for the further training and self-education; • Maintenance of perception, judgement and primary storing of communications and relations in object of studying; • Establishment of correctness and sensibleness of mastering of a new teaching material, revealing of incorrect representations and their correction; • Maintenance of mastering of new knowledge and ways of actions; • Formation of complete representation of knowledge on a theme; • To show a distributing material, to give talks and to give practical tasks.	• Mastering of new knowledge and ways of actions; • Primary check of understanding; • Fastening of knowledge and ways of actions; • Generalisation and ordering of knowledge. • Careful studying and the all-round analysis of a lecture material and increase of an educational level which would provide the decision of the put problem; • Increase of a degree of quality of knowledge through introduction of innovative technologies; • Level monitoring обученности pupils on steps, classes, subjects, it is concrete on each student, for the purpose of revealing of the real reasons influencing progress, dynamics of conformity of level of teaching to educational standards. • Monitoring of professional skill of teachers. • To continue activity on the organisation of interaction of participants of educational space; • To create conditions, to generalise an advanced experience and to motivate students; • Increase scientific информативности in a field of knowledge of a subject matter and related subjects.
Training methods	**Lecture – visualisation, conversation**
Technics of training	**Blitz – the interrogation, focusing questions**
Modes of study	**Collective, face-to-face**
Tutorials	**Projector, supply with information, visual materials, educational - methodical grants**
Training conditions	**The audience provided with tutorials**
Monitoring and estimation of knowledge	**The oral control: a question-answer**

TECHNOLOGICAL CARD
Carrying out of lecture employment on a theme:
Statement of a problem of linear programming. The basic properties the decision of a problem of linear programming

Stages, time	*The activity maintenance*	
	The teacher	*Students*
1 stage. Introduction ***(15 minutes)***	**1.1. Informs a theme, the purpose, planned results of educational employment and the plan of its carrying out.** **1.2. For the purpose of actualisation of knowledge of students asks focusing questions.**	**1.1. Listen.** **1.2. Answer questions.**
2 stage. Basic (information) **(55 minutes)**	2.1. Consistently states a material of lecture on plan questions. – The primary goal of linear programming – Examples of the decision of a problem	**2.1. Listen, discuss the maintenance of schemes and tables, visual materials, specify, ask questions.** **Write down the main thing.**
3 stage. The final ***(10 minutes)***	**3.1. Spends a blitz – interrogation on a theme of lecture employment. Does the total conclusion.** **3.2. Gives the task for independent work.**	**3.1. Answer questions.** **3.2. Listen, write down.**

Theme №14	**Geometrical interpretation of a problem of linear programming**

TECHNOLOGY OF TRAINING
(Carrying out) of lecture employment

Employment time - 2 hours (80 minutes)	Quantity of students: 40-50
Mode of study	Introduction-thematic lecture
The lecture plan	– Problem statement – Geometrical representation. – An example of the decision of a problem – Geometrical problem interpretation
The purpose of educational employment: **preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a lecture material to watch mastering at pupils of this knowledge, to form and develop at them skills.**	
***Problems of the teacher*:**	***Results of educational activity:***
• To acquaint students with a lecture material, the basic directions of activity on a considered material and the curriculum; • Logically consistently, it is given reason and clearly to state thoughts, correctly to build oral and written speech; • Studying of the basic concepts and the terms applied within the limits of a lecture material; • To make training more clear and accessible, to interest students, and the same to motivate them for the further training and self-education; • Maintenance of perception, judgement and primary storing of communications and relations in object of studying; • Establishment of correctness and sensibleness of mastering of a new teaching material, revealing of incorrect representations and their correction; • Maintenance of mastering of new knowledge and ways of actions; • Formation of complete representation of knowledge on a theme; • To show a distributing material, to give talks and to give practical tasks.	• Mastering of new knowledge and ways of actions; • Primary check of understanding; • Fastening of knowledge and ways of actions; • Generalisation and ordering of knowledge. • Careful studying and the all-round analysis of a lecture material and increase of an educational level which would provide the decision of the put problem; • Increase of a degree of quality of knowledge through introduction of innovative technologies; • Level monitoring обученности pupils on steps, classes, subjects, it is concrete on each student, for the purpose of revealing of the real reasons influencing progress, dynamics of conformity of level of teaching to educational standards. • Monitoring of professional skill of teachers. • To continue activity on the organisation of interaction of participants of educational space; • To create conditions, to generalise an advanced experience and to motivate students; • Increase scientific информативности in a field of knowledge of a subject matter and related subjects.
Training methods	**Lecture – visualisation, conversation**
Technics of training	**Blitz – the interrogation, focusing questions**
Modes of study	**Collective, face-to-face**
Tutorials	**Projector, supply with information, visual materials, educational - methodical grants**
Training conditions	**The audience provided with tutorials**
Monitoring and estimation of knowledge	**The oral control: a question-answer**

TECHNOLOGICAL CARD
Carrying out of lecture employment on a theme:
Geometrical interpretation of a problem of linear programming

Stages, time	*The activity maintenance*	
	The teacher	*Students*
1 stage. Introduction ***(15 minutes)***	**1.1. Informs a theme, the purpose, planned results of educational employment and the plan of its carrying out.** **1.2. For the purpose of actualisation of knowledge of students asks focusing questions.**	**1.1. Listen.** **1.2. Answer questions.**
2 stage. Basic (information) **(55 minutes)**	2.1. Consistently states a material of lecture on plan questions. – Problem statement – Geometrical representation. – An example of the decision of a problem – Geometrical problem interpretation	**2.1. Listen, discuss the maintenance of schemes and tables, visual materials, specify, ask questions.** **Write down the main thing.**
3 stage. The final ***(10 minutes)***	**3.1. Spends a blitz – interrogation on a theme of lecture employment. Does the total conclusion.** **3.2. Gives the task for independent work.**	**3.1. Answer questions.** **3.2. Listen, write down.**

Theme №15	**Finding the decision of a problem of linear programming to simplex methods**

TECHNOLOGY OF TRAINING
(Carrying out) of lecture employment

Employment time - 2 hours (80 minutes)	Quantity of students: 40-50
Mode of study	Introduction-thematic lecture
The lecture plan	– Mathematical bases a simplex of a method of the decision

***The purpose of educational employment:* preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a lecture material to watch mastering at pupils of this knowledge, to form and develop at them skills.**	
***Problems of the teacher*:** • To acquaint students with a lecture material, the basic directions of activity on a considered material and the curriculum; • Logically consistently, it is given reason and clearly to state thoughts, correctly to build oral and written speech; • Studying of the basic concepts and the terms applied within the limits of a lecture material; • To make training more clear and accessible, to interest students, and the same to motivate them for the further training and self-education; • Maintenance of perception, judgement and primary storing of communications and relations in object of studying; • Establishment of correctness and sensibleness of mastering of a new teaching material, revealing of incorrect representations and their correction; • Maintenance of mastering of new knowledge and ways of actions; • Formation of complete representation of knowledge on a theme; • To show a distributing material, to give talks and to give practical tasks.	***Results of educational activity:*** • Mastering of new knowledge and ways of actions; • Primary check of understanding; • Fastening of knowledge and ways of actions; • Generalisation and ordering of knowledge. • Careful studying and the all-round analysis of a lecture material and increase of an educational level which would provide the decision of the put problem; • Increase of a degree of quality of knowledge through introduction of innovative technologies; • Level monitoring обученности pupils on steps, classes, subjects, it is concrete on each student, for the purpose of revealing of the real reasons influencing progress, dynamics of conformity of level of teaching to educational standards. • Monitoring of professional skill of teachers. • To continue activity on the organisation of interaction of participants of educational space; • To create conditions, to generalise an advanced experience and to motivate students; • Increase scientific информативности in a field of knowledge of a subject matter and related subjects.
Training methods	**Lecture – visualisation, conversation**
Technics of training	**Blitz – the interrogation, focusing questions**
Modes of study	**Collective, face-to-face**
Tutorials	**Projector, supply with information, visual materials, educational - methodical grants**
Training conditions	**The audience provided with tutorials**
Monitoring and estimation of knowledge	**The oral control: a question-answer**

TECHNOLOGICAL CARD
Carrying out of lecture employment on a theme:
Finding the decision of a problem of linear programming to simplex methods

Stages, time	*The activity maintenance*	
	The teacher	*Students*
1 stage. Introduction ***(15 minutes)***	**1.1. Informs a theme, the purpose, planned results of educational employment and the plan of its carrying out.** **1.2. For the purpose of actualisation of knowledge of students asks focusing questions.**	**1.1. Listen.** **1.2. Answer questions.**
2 stage. Basic (information) **(55 minutes)**	2.1. Consistently states a material of lecture on plan questions. – Mathematical bases a simplex of a method of the decision	**2.1. Listen, discuss the maintenance of schemes and tables, visual materials, specify, ask questions.** **Write down the main thing.**
3 stage. The final ***(10 minutes)***	**3.1. Spends a blitz – interrogation on a theme of lecture employment. Does the total conclusion.** **3.2. Gives the task for independent work.**	**3.1. Answer questions.** **3.2. Listen, write down.**

Theme №16	**Finding the decision of a problem of linear programming. A method of artificial basis**

TECHNOLOGY OF TRAINING
(Carrying out) of lecture employment

<table>
<tr><td><i>Employment time -</i>
2 hours (80 minutes)</td><td>Quantity of students: 40-50</td></tr>
<tr><td><i>Mode of study</i></td><td>Introduction-thematic lecture</td></tr>
<tr><td><i>The lecture plan</i></td><td>– Search of the initial basic decision</td></tr>
<tr><td colspan="2"><i>The purpose of educational employment:</i> preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a lecture material to watch mastering at pupils of this knowledge, to form and develop at them skills.</td></tr>
<tr><td><i>Problems of the teacher</i>:
• To acquaint students with a lecture material, the basic directions of activity on a considered material and the curriculum;
• Logically consistently, it is given reason and clearly to state thoughts, correctly to build oral and written speech;
• Studying of the basic concepts and the terms applied within the limits of a lecture material;
• To make training more clear and accessible, to interest students, and the same to motivate them for the further training and self-education;
• Maintenance of perception, judgement and primary storing of communications and relations in object of studying;
• Establishment of correctness and sensibleness of mastering of a new teaching material, revealing of incorrect representations and their correction;
• Maintenance of mastering of new knowledge and ways of actions;
• Formation of complete representation of knowledge on a theme;
• To show a distributing material, to give talks and to give practical tasks.</td><td><i>Results of educational activity:</i>
• Mastering of new knowledge and ways of actions;
• Primary check of understanding;
• Fastening of knowledge and ways of actions;
• Generalisation and ordering of knowledge.
• Careful studying and the all-round analysis of a lecture material and increase of an educational level which would provide the decision of the put problem;
• Increase of a degree of quality of knowledge through introduction of innovative technologies;
• Level monitoring обученности pupils on steps, classes, subjects, it is concrete on each student, for the purpose of revealing of the real reasons influencing progress, dynamics of conformity of level of teaching to educational standards.
• Monitoring of professional skill of teachers.
• To continue activity on the organisation of interaction of participants of educational space;
• To create conditions, to generalise an advanced experience and to motivate students;
• Increase scientific информативности in a field of knowledge of a subject matter and related subjects.</td></tr>
<tr><td><i>Training methods</i></td><td>Lecture – visualisation, conversation</td></tr>
<tr><td><i>Technics of training</i></td><td>Blitz – the interrogation, focusing questions</td></tr>
<tr><td><i>Modes of study</i></td><td>Collective, face-to-face</td></tr>
<tr><td><i>Tutorials</i></td><td>Projector, supply with information, visual materials, educational - methodical grants</td></tr>
<tr><td><i>Training conditions</i></td><td>The audience provided with tutorials</td></tr>
<tr><td><i>Monitoring and estimation of knowledge</i></td><td>The oral control: a question-answer</td></tr>
</table>

TECHNOLOGICAL CARD
Carrying out of lecture employment on a theme:
Finding the decision of a problem of linear programming. A method of artificial basis

Stages, time	*The activity maintenance*	
	The teacher	*Students*
1 stage. Introduction ***(15 minutes)***	**1.1. Informs a theme, the purpose, planned results of educational employment and the plan of its carrying out.** **1.2. For the purpose of actualisation of knowledge of students asks focusing questions.**	**1.1. Listen.** **1.2. Answer questions.**
2 stage. Basic (information) **(55 minutes)**	2.1. Consistently states a material of lecture on plan questions. – Search of the initial basic decision	**2.1. Listen, discuss the maintenance of schemes and tables, visual materials, specify, ask questions.** **Write down the main thing.**
3 stage. The final ***(10 minutes)***	**3.1. Spends a blitz – interrogation on a theme of lecture employment. Does the total conclusion.** **3.2. Gives the task for independent work.**	**3.1. Answer questions.** **3.2. Listen, write down.**

Theme №17	**Transport problem. Methods initial basic the decision**

TECHNOLOGY OF TRAINING
(Carrying out) of lecture employment

Employment time - 2 hours (80 minutes)	Quantity of students: 40-50
Mode of study	Introduction-thematic lecture
The lecture plan	– Features of a transport problem – Constructions of the basic decision
The purpose of educational employment: **preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a lecture material to watch mastering at pupils of this knowledge, to form and develop at them skills.**	
***Problems of the teacher*:**	***Results of educational activity:***
• To acquaint students with a lecture material, the basic directions of activity on a considered material and the curriculum; • Logically consistently, it is given reason and clearly to state thoughts, correctly to build oral and written speech; • Studying of the basic concepts and the terms applied within the limits of a lecture material; • To make training more clear and accessible, to interest students, and the same to motivate them for the further training and self-education; • Maintenance of perception, judgement and primary storing of communications and relations in object of studying; • Establishment of correctness and sensibleness of mastering of a new teaching material, revealing of incorrect representations and their correction; • Maintenance of mastering of new knowledge and ways of actions; • Formation of complete representation of knowledge on a theme; • To show a distributing material, to give talks and to give practical tasks.	• Mastering of new knowledge and ways of actions; • Primary check of understanding; • Fastening of knowledge and ways of actions; • Generalisation and ordering of knowledge. • Careful studying and the all-round analysis of a lecture material and increase of an educational level which would provide the decision of the put problem; • Increase of a degree of quality of knowledge through introduction of innovative technologies; • Level monitoring обученности pupils on steps, classes, subjects, it is concrete on each student, for the purpose of revealing of the real reasons influencing progress, dynamics of conformity of level of teaching to educational standards. • Monitoring of professional skill of teachers. • To continue activity on the organisation of interaction of participants of educational space; • To create conditions, to generalise an advanced experience and to motivate students; • Increase scientific информативности in a field of knowledge of a subject matter and related subjects.
Training methods	**Lecture – visualisation, conversation**
Technics of training	**Blitz – the interrogation, focusing questions**
Modes of study	**Collective, face-to-face**
Tutorials	**Projector, supply with information, visual materials, educational - methodical grants**
Training conditions	**The audience provided with tutorials**
Monitoring and estimation of knowledge	**The oral control: a question-answer**

TECHNOLOGICAL CARD
Carrying out of lecture employment on a theme:
Transport problem. Methods initial basic the decision

Stages, time	*The activity maintenance*	
	The teacher	*Students*
1 stage. Introduction ***(15 minutes)***	**1.1. Informs a theme, the purpose, planned results of educational employment and the plan of its carrying out.** **1.2. For the purpose of actualisation of knowledge of students asks focusing questions.**	**1.1. Listen.** **1.2. Answer questions.**
2 stage. Basic (information) **(55 minutes)**	2.1. Consistently states a material of lecture on plan questions. – Features of a transport problem – Constructions of the basic decision	**2.1. Listen, discuss the maintenance of schemes and tables, visual materials, specify, ask questions.** **Write down the main thing.**
3 stage. The final ***(10 minutes)***	**3.1. Spends a blitz – interrogation on a theme of lecture employment. Does the total conclusion.** **3.2. Gives the task for independent work.**	**3.1. Answer questions.** **3.2. Listen, write down.**

Theme №18	**Method of potentials for a finding the optimum decision transport problems**

TECHNOLOGY OF TRAINING
(Carrying out) of lecture employment

Employment time - 2 hours (80 minutes)	Quantity of students: 40-50
Mode of study	Introduction-thematic lecture
The lecture plan	– Conditions and a method of construction of the optimum decision of a transport problem – Algorithm of the decision of a transport problem on a network
The purpose of educational employment: **preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a lecture material to watch mastering at pupils of this knowledge, to form and develop at them skills.**	
***Problems of the teacher*:**	***Results of educational activity:***
• To acquaint students with a lecture material, the basic directions of activity on a considered material and the curriculum; • Logically consistently, it is given reason and clearly to state thoughts, correctly to build oral and written speech; • Studying of the basic concepts and the terms applied within the limits of a lecture material; • To make training more clear and accessible, to interest students, and the same to motivate them for the further training and self-education; • Maintenance of perception, judgement and primary storing of communications and relations in object of studying; • Establishment of correctness and sensibleness of mastering of a new teaching material, revealing of incorrect representations and their correction; • Maintenance of mastering of new knowledge and ways of actions; • Formation of complete representation of knowledge on a theme; • To show a distributing material, to give talks and to give practical tasks.	• Mastering of new knowledge and ways of actions; • Primary check of understanding; • Fastening of knowledge and ways of actions; • Generalisation and ordering of knowledge. • Careful studying and the all-round analysis of a lecture material and increase of an educational level which would provide the decision of the put problem; • Increase of a degree of quality of knowledge through introduction of innovative technologies; • Level monitoring обученности pupils on steps, classes, subjects, it is concrete on each student, for the purpose of revealing of the real reasons influencing progress, dynamics of conformity of level of teaching to educational standards. • Monitoring of professional skill of teachers. • To continue activity on the organisation of interaction of participants of educational space; • To create conditions, to generalise an advanced experience and to motivate students; • Increase scientific информативности in a field of knowledge of a subject matter and related subjects.
Training methods	**Lecture – visualisation, conversation**
Technics of training	**Blitz – the interrogation, focusing questions**
Modes of study	**Collective, face-to-face**
Tutorials	**Projector, supply with information, visual materials, educational - methodical grants**
Training conditions	**The audience provided with tutorials**
Monitoring and estimation of knowledge	**The oral control: a question-answer**

TECHNOLOGICAL CARD
Carrying out of lecture employment on a theme:
Method of potentials for a finding the optimum decision transport problems

Stages, time	*The activity maintenance*	
	The teacher	*Students*
1 stage. Introduction ***(15 minutes)***	**1.1. Informs a theme, the purpose, planned results of educational employment and the plan of its carrying out.** **1.2. For the purpose of actualisation of knowledge of students asks focusing questions.**	**1.1. Listen.** **1.2. Answer questions.**
2 stage. Basic (information) **(55 minutes)**	2.1. Consistently states a material of lecture on plan questions. – Conditions and a method of construction of the optimum decision of a transport problem – Algorithm of the decision of a transport problem on a network	**2.1. Listen, discuss the maintenance of schemes and tables, visual materials, specify, ask questions.** **Write down the main thing.**
3 stage. The final ***(10 minutes)***	**3.1. Spends a blitz – interrogation on a theme of lecture employment. Does the total conclusion.** **3.2. Gives the task for independent work.**	**3.1. Answer questions.** **3.2. Listen, write down.**

TECHNOLOGY TRAINING AND THE TECHNOLOGICAL CARD ON PRACTICE

Theme № 1-2	**The numerical decision of the algebraic and transcendental equations iterative methods.**

TECHNOLOGY OF TRAINING
(Carrying out) of practical employment

Employment time - 4 hours (160 minutes)	Quantity of students: 15-20
Mode of study	Introduction - thematic lecture
The plan practical employment	– Iteration method – Method of Chords – Method half divisions
The purpose of educational employment: **preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a practical material to watch mastering at pupils of this knowledge, to form and develop at them skills.**	
***Problems of the teacher*:**	***Results of educational activity:***
• To acquaint students with a practical material, the basic directions of activity on a considered material and the curriculum; • Logically consistently, it is given reason and clearly to state thoughts, correctly to build oral and written speech; • Studying of the basic concepts and the terms applied in frameworks практичкеского of a material; • To make training more clear and accessible, to interest students, and the same to motivate them for the further training and self-education; • Maintenance of perception, judgement and primary storing of communications and relations in object of studying; • Establishment of correctness and sensibleness of mastering of a new teaching material, revealing of incorrect representations and their correction; • Maintenance of mastering of new knowledge and ways of actions; • Formation of complete representation of knowledge on a theme; • To show a distributing material, to give talks and to give practical tasks.	• Mastering of new knowledge and ways of actions; • Primary check of understanding; • Fastening of knowledge and ways of actions; • Generalisation and ordering of knowledge. • Careful studying and the all-round analysis of a lecture material and increase of an educational level which would provide the decision of the put problem; • Increase of a degree of quality of knowledge through introduction of innovative technologies; • Level monitoring обученности pupils on steps, classes, subjects, it is concrete on each student, for the purpose of revealing of the real reasons influencing progress, dynamics of conformity of level of teaching to educational standards. • Monitoring of professional skill of teachers. • To continue activity on the organisation of interaction of participants of educational space; • To create conditions, to generalise an advanced experience and to motivate students; • Increase scientific информативности in a field of knowledge of a subject matter and related subjects.
Training methods	Practice – visualisation, conversation
Technics of training	Blitz – the interrogation, focusing questions
Modes of study	Collective, face-to-face
Tutorials	Projector, supply with information, visual materials, uchebno - methodical grants
Training conditions	The audience provided with tutorials
Monitoring and estimation of knowledge	The oral control: a question-answer

TECHNOLOGICAL CARD
Carrying out of practical employment on a theme:
The numerical decision of the algebraic and transcendental equations iterative methods.

Stages, time	*The activity maintenance*	
	The teacher	*Students*
1 stage. Introduction ***(15 minutes)***	**1.1. Informs a theme, the purpose, planned results of educational employment and the plan of its carrying out.** **1.2. For the purpose of actualisation of knowledge of students asks focusing questions.**	**1.1. Listen.** **1.2. Answer questions.**
2 stage. Basic (information) **(135 minutes)**	2.1. Consistently states a material of practice concerning the plan. – Iteration method – Method of Chords – Method half divisions	**2.1. Listen, discuss the maintenance of schemes and tables, visual materials, specify, ask questions.** **Write down the main thing.**
3 stage. The final ***(10 minutes)***	**3.1. Spends a blitz – interrogation on a theme of practical employment. Does the total conclusion.** **3.2. Gives the task for independent work.**	**3.1. Answer questions.** **3.2. Listen, write down.**

Theme № 3	**The numerical decision of system of the linear algebraic equations methods of Gaussa**

TECHNOLOGY OF TRAINING
(Carrying out) of practical employment

Employment time - 2 hours (80 minutes)	Quantity of students: 15-20
Mode of study	Introduction - thematic lecture
The plan practical employment	– The theory – The numerical decision of system of the linear algebraic equations methods of Gaussa
The purpose of educational employment: **preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a practical material to watch mastering at pupils of this knowledge, to form and develop at them skills.**	
***Problems of the teacher*:** • To acquaint students with a practical material, the basic directions of activity on a considered material and the curriculum; • Logically consistently, it is given reason and clearly to state thoughts, correctly to build oral and written speech; • Studying of the basic concepts and the terms applied in frameworks практичкеского of a material; • To make training more clear and accessible, to interest students, and the same to motivate them for the further training and self-education; • Maintenance of perception, judgement and primary storing of communications and relations in object of studying; • Establishment of correctness and sensibleness of mastering of a new teaching material, revealing of incorrect representations and their correction; • Maintenance of mastering of new knowledge and ways of actions; • Formation of complete representation of knowledge on a theme; • To show a distributing material, to give talks and to give practical tasks.	***Results of educational activity:*** • Mastering of new knowledge and ways of actions; • Primary check of understanding; • Fastening of knowledge and ways of actions; • Generalisation and ordering of knowledge. • Careful studying and the all-round analysis of a lecture material and increase of an educational level which would provide the decision of the put problem; • Increase of a degree of quality of knowledge through introduction of innovative technologies; • Level monitoring обученности pupils on steps, classes, subjects, it is concrete on each student, for the purpose of revealing of the real reasons influencing progress, dynamics of conformity of level of teaching to educational standards. • Monitoring of professional skill of teachers. • To continue activity on the organisation of interaction of participants of educational space; • To create conditions, to generalise an advanced experience and to motivate students; • Increase scientific информативности in a field of knowledge of a subject matter and related subjects.
Training methods	Practice – visualisation, conversation
Technics of training	Blitz – the interrogation, focusing questions
Modes of study	Collective, face-to-face
Tutorials	Projector, supply with information, visual materials, uchebno - methodical grants
Training conditions	The audience provided with tutorials
Monitoring and estimation of knowledge	The oral control: a question-answer

TECHNOLOGICAL CARD
Carrying out of practical employment on a theme:
The numerical decision of system of the linear algebraic equations methods of Gaussa

Stages, time	*The activity maintenance*	
	The teacher	*Students*
1 stage. Introduction ***(15 minutes)***	**1.1. Informs a theme, the purpose, planned results of educational employment and the plan of its carrying out.** **1.2. For the purpose of actualisation of knowledge of students asks focusing questions.**	**1.1. Listen.** **1.2. Answer questions.**
2 stage. Basic (information) **(55 minutes)**	2.1. Consistently states a material of practice concerning the plan. – The theory – The numerical decision of system of the linear algebraic equations methods of Gaussa	**2.1. Listen, discuss the maintenance of schemes and tables, visual materials, specify, ask questions.** **Write down the main thing.**
3 stage. The final ***(10 minutes)***	**3.1. Spends a blitz – interrogation on a theme of practical employment. Does the total conclusion.** **3.2. Gives the task for independent work.**	**3.1. Answer questions.** **3.2. Listen, write down.**

Theme № 4	**Calculation of integrals by the approached methods**

TECHNOLOGY OF TRAINING
(Carrying out) of practical employment

Employment time - 2 hours (80 minutes)	Quantity of students: 15-20
Mode of study	Introduction - thematic lecture
The plan practical employment	– Method of trapezes – Methods of rectangles – Simpson's method – The quadrature formula of Gaussa
The purpose of educational employment: **preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a practical material to watch mastering at pupils of this knowledge, to form and develop at them skills.**	
***Problems of the teacher*:** • To acquaint students with a practical material, the basic directions of activity on a considered material and the curriculum; • Logically consistently, it is given reason and clearly to state thoughts, correctly to build oral and written speech; • Studying of the basic concepts and the terms applied in frameworks практичкеского of a material; • To make training more clear and accessible, to interest students, and the same to motivate them for the further training and self-education; • Maintenance of perception, judgement and primary storing of communications and relations in object of studying; • Establishment of correctness and sensibleness of mastering of a new teaching material, revealing of incorrect representations and their correction; • Maintenance of mastering of new knowledge and ways of actions; • Formation of complete representation of knowledge on a theme; • To show a distributing material, to give talks and to give practical tasks.	***Results of educational activity:*** • Mastering of new knowledge and ways of actions; • Primary check of understanding; • Fastening of knowledge and ways of actions; • Generalisation and ordering of knowledge. • Careful studying and the all-round analysis of a lecture material and increase of an educational level which would provide the decision of the put problem; • Increase of a degree of quality of knowledge through introduction of innovative technologies; • Level monitoring обученности pupils on steps, classes, subjects, it is concrete on each student, for the purpose of revealing of the real reasons influencing progress, dynamics of conformity of level of teaching to educational standards. • Monitoring of professional skill of teachers. • To continue activity on the organisation of interaction of participants of educational space; • To create conditions, to generalise an advanced experience and to motivate students; • Increase scientific информативности in a field of knowledge of a subject matter and related subjects.
Training methods	Practice – visualisation, conversation
Technics of training	Blitz – the interrogation, focusing questions
Modes of study	Collective, face-to-face
Tutorials	Projector, supply with information, visual materials, uchebno - methodical grants
Training conditions	The audience provided with tutorials
Monitoring and estimation of knowledge	The oral control: a question-answer

TECHNOLOGICAL CARD
Carrying out of practical employment on a theme:
Calculation of integrals by the approached methods

Stages, time	*The activity maintenance*	
	The teacher	*Students*
1 stage. Introduction ***(15 minutes)***	**1.1. Informs a theme, the purpose, planned results of educational employment and the plan of its carrying out.** **1.2. For the purpose of actualisation of knowledge of students asks focusing questions.**	**1.1. Listen.** **1.2. Answer questions.**
2 stage. Basic (information) **(55 minutes)**	2.1. Consistently states a material of practice concerning the plan. – Method of trapezes – Methods of rectangles – Simpson's method – The quadrature formula of Gaussa	**2.1. Listen, discuss the maintenance of schemes and tables, visual materials, specify, ask questions.** **Write down the main thing.**
3 stage. The final ***(10 minutes)***	**3.1. Spends a blitz – interrogation on a theme of practical employment. Does the total conclusion.** **3.2. Gives the task for independent work.**	**3.1. Answer questions.** **3.2. Listen, write down.**

Theme № 5	Interpolation polynom Newton and Lagrange

TECHNOLOGY OF TRAINING
(Carrying out) of practical employment

Employment time - 2 hours (80 minutes)	Quantity of students: 15-20
Mode of study	Introduction - thematic lecture
The plan practical employment	– Interpolation polynom of Newton – Interpolation polynom of Lagrange

The purpose of educational employment: **preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a practical material to watch mastering at pupils of this knowledge, to form and develop at them skills.**	
***Problems of the teacher*:**	***Results of educational activity:***
• To acquaint students with a practical material, the basic directions of activity on a considered material and the curriculum; • Logically consistently, it is given reason and clearly to state thoughts, correctly to build oral and written speech; • Studying of the basic concepts and the terms applied in frameworks практичкеского of a material; • To make training more clear and accessible, to interest students, and the same to motivate them for the further training and self-education; • Maintenance of perception, judgement and primary storing of communications and relations in object of studying; • Establishment of correctness and sensibleness of mastering of a new teaching material, revealing of incorrect representations and their correction; • Maintenance of mastering of new knowledge and ways of actions; • Formation of complete representation of knowledge on a theme; • To show a distributing material, to give talks and to give practical tasks.	• Mastering of new knowledge and ways of actions; • Primary check of understanding; • Fastening of knowledge and ways of actions; • Generalisation and ordering of knowledge. • Careful studying and the all-round analysis of a lecture material and increase of an educational level which would provide the decision of the put problem; • Increase of a degree of quality of knowledge through introduction of innovative technologies; • Level monitoring обученности pupils on steps, classes, subjects, it is concrete on each student, for the purpose of revealing of the real reasons influencing progress, dynamics of conformity of level of teaching to educational standards. • Monitoring of professional skill of teachers. • To continue activity on the organisation of interaction of participants of educational space; • To create conditions, to generalise an advanced experience and to motivate students; • Increase scientific информативности in a field of knowledge of a subject matter and related subjects.
Training methods	Practice – visualisation, conversation
Technics of training	Blitz – the interrogation, focusing questions
Modes of study	Collective, face-to-face
Tutorials	Projector, supply with information, visual materials, uchebno - methodical grants
Training conditions	The audience provided with tutorials
Monitoring and estimation of knowledge	The oral control: a question-answer

TECHNOLOGICAL CARD
Carrying out of practical employment on a theme:
Interpolation polynom Newton and Lagrange

Stages, time	*The activity maintenance*	
	The teacher	*Students*
1 stage. Introduction *(15 minutes)*	**1.1. Informs a theme, the purpose, planned results of educational employment and the plan of its carrying out.** **1.2. For the purpose of actualisation of knowledge of students asks focusing questions.**	**1.1. Listen.** **1.2. Answer questions.**
2 stage. Basic (information) **(55 minutes)**	2.1. Consistently states a material of practice concerning the plan. – Interpolation polynom of Newton – Interpolation polynom of Lagrange	**2.1. Listen, discuss the maintenance of schemes and tables, visual materials, specify, ask questions.** **Write down the main thing.**
3 stage. The final *(10 minutes)*	**3.1. Spends a blitz – interrogation on a theme of practical employment. Does the total conclusion.** **3.2. Gives the task for independent work.**	**3.1. Answer questions.** **3.2. Listen, write down.**

Theme № 6-7	**Problems of Cochy for the ordinary differential equations. Euler's methods, Runge-Kutta and Adams**

TECHNOLOGY OF TRAINING
(Carrying out) of practical employment

Employment time - 4 hours (160 minutes)	Quantity of students: 15-20
Mode of study	Introduction - thematic lecture
The plan practical employment	– Problems of Cochy for the ordinary differential equations. – Euler's method – Method of Runge-Kutta – Adams's method
The purpose of educational employment: **preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a practical material to watch mastering at pupils of this knowledge, to form and develop at them skills.**	
***Problems of the teacher*:** • To acquaint students with a practical material, the basic directions of activity on a considered material and the curriculum; • Logically consistently, it is given reason and clearly to state thoughts, correctly to build oral and written speech; • Studying of the basic concepts and the terms applied in frameworks практичкеского of a material; • To make training more clear and accessible, to interest students, and the same to motivate them for the further training and self-education; • Maintenance of perception, judgement and primary storing of communications and relations in object of studying; • Establishment of correctness and sensibleness of mastering of a new teaching material, revealing of incorrect representations and their correction; • Maintenance of mastering of new knowledge and ways of actions; • Formation of complete representation of knowledge on a theme; • To show a distributing material, to give talks and to give practical tasks.	***Results of educational activity:*** • Mastering of new knowledge and ways of actions; • Primary check of understanding; • Fastening of knowledge and ways of actions; • Generalisation and ordering of knowledge. • Careful studying and the all-round analysis of a lecture material and increase of an educational level which would provide the decision of the put problem; • Increase of a degree of quality of knowledge through introduction of innovative technologies; • Level monitoring обученности pupils on steps, classes, subjects, it is concrete on each student, for the purpose of revealing of the real reasons influencing progress, dynamics of conformity of level of teaching to educational standards. • Monitoring of professional skill of teachers. • To continue activity on the organisation of interaction of participants of educational space; • To create conditions, to generalise an advanced experience and to motivate students; • Increase scientific информативности in a field of knowledge of a subject matter and related subjects.
Training methods	Practice – visualisation, conversation
Technics of training	Blitz – the interrogation, focusing questions
Modes of study	Collective, face-to-face
Tutorials	Projector, supply with information, visual materials, uchebno - methodical grants
Training conditions	The audience provided with tutorials
Monitoring and estimation of knowledge	The oral control: a question-answer

TECHNOLOGICAL CARD
Carrying out of practical employment on a theme:
Problems of Cochy for the ordinary differential equations. Euler's methods, Runge-Kutta and Adams

Stages, time	*The activity maintenance*	
	The teacher	*Students*
1 stage. Introduction ***(15 minutes)***	**1.1. Informs a theme, the purpose, planned results of educational employment and the plan of its carrying out.** **1.2. For the purpose of actualisation of knowledge of students asks focusing questions.**	**1.1. Listen.** **1.2. Answer questions.**
2 stage. Basic (information) **(135 minutes)**	2.1. Consistently states a material of practice concerning the plan. – Problems of Cochy for the ordinary differential equations. – Euler's method – Method of Runge-Kutta – Adams's method	**2.1. Listen, discuss the maintenance of schemes and tables, visual materials, specify, ask questions.** **Write down the main thing.**
3 stage. The final ***(10 minutes)***	**3.1. Spends a blitz – interrogation on a theme of practical employment. Does the total conclusion.** **3.2. Gives the task for independent work.**	**3.1. Answer questions.** **3.2. Listen, write down.**

Theme № 8	**The geometrical decision of a problem of linear programming**

TECHNOLOGY OF TRAINING
(Carrying out) of practical employment

Employment time - 2 hours (80 minutes)	Quantity of students: 15-20
Mode of study	Introduction - thematic lecture
The plan practical employment	– Geometrical interpretation of a problem of linear programming – Using geometrical interpretation, find decisions of problems
The purpose of educational employment: **preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a practical material to watch mastering at pupils of this knowledge, to form and develop at them skills.**	
***Problems of the teacher*:**	***Results of educational activity:***
• To acquaint students with a practical material, the basic directions of activity on a considered material and the curriculum; • Logically consistently, it is given reason and clearly to state thoughts, correctly to build oral and written speech; • Studying of the basic concepts and the terms applied in frameworks практичкеского of a material; • To make training more clear and accessible, to interest students, and the same to motivate them for the further training and self-education; • Maintenance of perception, judgement and primary storing of communications and relations in object of studying; • Establishment of correctness and sensibleness of mastering of a new teaching material, revealing of incorrect representations and their correction; • Maintenance of mastering of new knowledge and ways of actions; • Formation of complete representation of knowledge on a theme; • To show a distributing material, to give talks and to give practical tasks.	• Mastering of new knowledge and ways of actions; • Primary check of understanding; • Fastening of knowledge and ways of actions; • Generalisation and ordering of knowledge. • Careful studying and the all-round analysis of a lecture material and increase of an educational level which would provide the decision of the put problem; • Increase of a degree of quality of knowledge through introduction of innovative technologies; • Level monitoring обученности pupils on steps, classes, subjects, it is concrete on each student, for the purpose of revealing of the real reasons influencing progress, dynamics of conformity of level of teaching to educational standards. • Monitoring of professional skill of teachers. • To continue activity on the organisation of interaction of participants of educational space; • To create conditions, to generalise an advanced experience and to motivate students; • Increase scientific информативности in a field of knowledge of a subject matter and related subjects.
Training methods	Practice – visualisation, conversation
Technics of training	Blitz – the interrogation, focusing questions
Modes of study	Collective, face-to-face
Tutorials	Projector, supply with information, visual materials, uchebno - methodical grants
Training conditions	The audience provided with tutorials
Monitoring and estimation of knowledge	The oral control: a question-answer

TECHNOLOGICAL CARD
Carrying out of practical employment on a theme:
The geometrical decision of a problem of linear programming

Stages, time	*The activity maintenance*	
	The teacher	*Students*
1 stage. Introduction ***(15 minutes)***	**1.1. Informs a theme, the purpose, planned results of educational employment and the plan of its carrying out.** **1.2. For the purpose of actualisation of knowledge of students asks focusing questions.**	**1.1. Listen.** **1.2. Answer questions.**
2 stage. Basic (information) **(55 minutes)**	2.1. Consistently states a material of practice concerning the plan. – Geometrical interpretation of a problem of linear programming – Using geometrical interpretation, find decisions of problems	**2.1. Listen, discuss the maintenance of schemes and tables, visual materials, specify, ask questions.** **Write down the main thing.**
3 stage. The final ***(10 minutes)***	**3.1. Spends a blitz – interrogation on a theme of practical employment. Does the total conclusion.** **3.2. Gives the task for independent work.**	**3.1. Answer questions.** **3.2. Listen, write down.**

Theme № 9	**Finding the decision of a problem of linear programming to Simplex methods**

TECHNOLOGY OF TRAINING
(Carrying out) of practical employment

Employment time - 2 hours (80 minutes)	Quantity of students: 15-20
Mode of study	Introduction - thematic lecture
The plan practical employment	– Simplex method of the decision of a problem of linear programming – Examples of the decision of a problem of linear programming with a simplex method
***The purpose of educational employment:* preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on a practical material to watch mastering at pupils of this knowledge, to form and develop at them skills.**	
***Problems of the teacher*:**	***Results of educational activity:***
• To acquaint students with a practical material, the basic directions of activity on a considered material and the curriculum; • Logically consistently, it is given reason and clearly to state thoughts, correctly to build oral and written speech; • Studying of the basic concepts and the terms applied in frameworks практичкеского of a material; • To make training more clear and accessible, to interest students, and the same to motivate them for the further training and self-education; • Maintenance of perception, judgement and primary storing of communications and relations in object of studying; • Establishment of correctness and sensibleness of mastering of a new teaching material, revealing of incorrect representations and their correction; • Maintenance of mastering of new knowledge and ways of actions; • Formation of complete representation of knowledge on a theme; • To show a distributing material, to give talks and to give practical tasks.	• Mastering of new knowledge and ways of actions; • Primary check of understanding; • Fastening of knowledge and ways of actions; • Generalisation and ordering of knowledge. • Careful studying and the all-round analysis of a lecture material and increase of an educational level which would provide the decision of the put problem; • Increase of a degree of quality of knowledge through introduction of innovative technologies; • Level monitoring обученности pupils on steps, classes, subjects, it is concrete on each student, for the purpose of revealing of the real reasons influencing progress, dynamics of conformity of level of teaching to educational standards. • Monitoring of professional skill of teachers. • To continue activity on the organisation of interaction of participants of educational space; • To create conditions, to generalise an advanced experience and to motivate students; • Increase scientific информативности in a field of knowledge of a subject matter and related subjects.
Training methods	Practice – visualisation, conversation
Technics of training	Blitz – the interrogation, focusing questions
Modes of study	Collective, face-to-face
Tutorials	Projector, supply with information, visual materials, uchebno - methodical grants
Training conditions	The audience provided with tutorials
Monitoring and estimation of knowledge	The oral control: a question-answer

TECHNOLOGICAL CARD
Carrying out of practical employment on a theme:
Finding the decision of a problem of linear programming to Simplex methods

Stages, time	*The activity maintenance*	
	The teacher	*Students*
1 stage. Introduction ***(15 minutes)***	**1.1. Informs a theme, the purpose, planned results of educational employment and the plan of its carrying out.** **1.2. For the purpose of actualisation of knowledge of students asks focusing questions.**	**1.1. Listen.** **1.2. Answer questions.**
2 stage. Basic (information) **(55 minutes)**	2.1. Consistently states a material of practice concerning the plan. – Simplex method of the decision of a problem of linear programming – Examples of the decision of a problem of linear programming with a simplex method	**2.1. Listen, discuss the maintenance of schemes and tables, visual materials, specify, ask questions.** **Write down the main thing.**
3 stage. The final ***(10 minutes)***	**3.1. Spends a blitz – interrogation on a theme of practical employment. Does the total conclusion.** **3.2. Gives the task for independent work.**	**3.1. Answer questions.** **3.2. Listen, write down.**

TECHNOLOGY TRAINING AND THE TECHNOLOGICAL CARD ON LABORATORY

Theme № 1	**The numerical decision of the algebraic and transcendental equations iterative methods and to methods the Chord**

TECHNOLOGY OF TRAINING
(Carrying out) laboratory work

Employment time - 4 hours (160 minutes)	Quantity of students: 15-20
Mode of study	Introduction - thematic lecture
The plan of laboratory works	– Iteration method – Method of Chords – Method half divisions
The purpose of educational employment: **preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on laboratory work to watch mastering at pupils of this knowledge, to form and develop at them skills.**	
***Problems of the teacher*:** • To acquaint students with a laboratory material and the device, the basic directions of activity on a considered material and the curriculum; • Logically consistently, it is given reason and clearly to state thoughts, correctly to build oral and written speech; • Studying of the basic concepts and the terms applied in frameworks практичкеского of a material; • To make training more clear and accessible, to interest students, and the same to motivate them for the further training and self-education; • Maintenance of perception, judgement and primary storing of communications and relations in object of studying; • Establishment of correctness and sensibleness of mastering of a new teaching material, revealing of incorrect representations and their correction; • Maintenance of mastering of new knowledge and ways of actions; • Formation of complete representation of knowledge on a theme; • To show a distributing material, to give talks and to give laboratory works.	***Results of educational activity:*** • Mastering of new knowledge and ways of actions; • Primary check of understanding; • Fastening of knowledge and ways of actions; • Generalisation and ordering of knowledge. • Careful studying and the all-round analysis of a laboratory material and increase of an educational level which would provide the decision of the put problem; • Increase of a degree of quality of knowledge through introduction of innovative technologies; • Level monitoring обученности pupils on steps, classes, subjects, it is concrete on each student, for the purpose of revealing of the real reasons influencing progress, dynamics of conformity of level of teaching to educational standards. • Monitoring of professional skill of teachers. • To continue activity on the organisation of interaction of participants of educational space; • To create conditions, to generalise an advanced experience and to motivate students; • Increase scientific информативности in a field of knowledge of a subject matter and related subjects.
Training methods	**Laboratory – visualisation, conversation**
Technics of training	**Blitz – the interrogation, focusing questions**
Modes of study	**Collective, face-to-face**
Tutorials	**Visual materials, uchebno-methodical grants, laboratory installations and devices**
Training conditions	**The audience provided with tutorials**
Monitoring and estimation of knowledge	**The oral control: a question-answer**

TECHNOLOGICAL CARD
Carrying out laboratory work on a theme:
The numerical decision of the algebraic and transcendental equations iterative methods and to methods the Chord

Stages, time	*The activity maintenance*	
	The teacher	*Students*
1 stage. Introduction ***(15 minutes)***	**1.1. Laboratory work and the plan of its carrying out informs a theme, the purpose, planned results.** **1.2. For the purpose of actualisation of knowledge of students asks focusing questions.**	**1.1. Listen.** **1.2. Answer questions.**
2 stage. Basic (information) **(135 minutes)**	2.1. Consistently states a material laboratory concerning the plan. – Iteration method – Method of Chords – Method half divisions	**2.1. Listen, discuss the maintenance of schemes and tables, specify, ask questions, carry out laboratory work.** **Write down the main thing results.**
3 stage. The final ***(10 minutes)***	**3.1. Spends a blitz – interrogation on a theme laboratory work. Does the total conclusion.** **3.2. Gives the task for independent work.**	**3.1. Answer questions.** **3.2. Listen, write down.**

Theme № 3	The numerical decision of the algebraic and transcendental equations to Newton's methods

TECHNOLOGY OF TRAINING
(Carrying out) laboratory work

Employment time - 2 hours (80 minutes)	Quantity of students: 15-20
Mode of study	Introduction - thematic lecture
The plan of laboratory works	– The theory – Method of secants – Control variants

The purpose of educational employment: **preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on laboratory work to watch mastering at pupils of this knowledge, to form and develop at them skills.**	
***Problems of the teacher*:**	***Results of educational activity:***
• To acquaint students with a laboratory material and the device, the basic directions of activity on a considered material and the curriculum; • Logically consistently, it is given reason and clearly to state thoughts, correctly to build oral and written speech; • Studying of the basic concepts and the terms applied in frameworks практичкеского of a material; • To make training more clear and accessible, to interest students, and the same to motivate them for the further training and self-education; • Maintenance of perception, judgement and primary storing of communications and relations in object of studying; • Establishment of correctness and sensibleness of mastering of a new teaching material, revealing of incorrect representations and their correction; • Maintenance of mastering of new knowledge and ways of actions; • Formation of complete representation of knowledge on a theme; • To show a distributing material, to give talks and to give laboratory works.	• Mastering of new knowledge and ways of actions; • Primary check of understanding; • Fastening of knowledge and ways of actions; • Generalisation and ordering of knowledge. • Careful studying and the all-round analysis of a laboratory material and increase of an educational level which would provide the decision of the put problem; • Increase of a degree of quality of knowledge through introduction of innovative technologies; • Level monitoring обученности pupils on steps, classes, subjects, it is concrete on each student, for the purpose of revealing of the real reasons influencing progress, dynamics of conformity of level of teaching to educational standards. • Monitoring of professional skill of teachers. • To continue activity on the organisation of interaction of participants of educational space; • To create conditions, to generalise an advanced experience and to motivate students; • Increase scientific информативности in a field of knowledge of a subject matter and related subjects.
Training methods	**Laboratory – visualisation, conversation**
Technics of training	**Blitz – the interrogation, focusing questions**
Modes of study	**Collective, face-to-face**
Tutorials	**Visual materials, uchebno-methodical grants, laboratory installations and devices**
Training conditions	**The audience provided with tutorials**
Monitoring and estimation of knowledge	**The oral control: a question-answer**

TECHNOLOGICAL CARD
Carrying out laboratory work on a theme:
The numerical decision of the algebraic and transcendental equations to Newton's methods

Stages, time	*The activity maintenance*	
	The teacher	*Students*
1 stage. Introduction ***(15 minutes)***	**1.1. Laboratory work and the plan of its carrying out informs a theme, the purpose, planned results.** **1.2. For the purpose of actualisation of knowledge of students asks focusing questions.**	**1.1. Listen.** **1.2. Answer questions.**
2 stage. Basic (information) **(55 minutes)**	2.1. Consistently states a material laboratory concerning the plan. – The theory – Method of secants – Control variants	**2.1. Listen, discuss the maintenance of schemes and tables, specify, ask questions, carry out laboratory work.** **Write down the main thing results.**
3 stage. The final ***(10 minutes)***	**3.1. Spends a blitz – interrogation on a theme laboratory work. Does the total conclusion.** **3.2. Gives the task for independent work.**	**3.1. Answer questions.** **3.2. Listen, write down.**

Theme № 4	**The numerical decision of system of the linear algebraic equations to methods of simple iteration.**

TECHNOLOGY OF TRAINING
(Carrying out) laboratory work

Employment time - 2 hours (80 minutes)	Quantity of students: 15-20
Mode of study	Introduction - thematic lecture
The plan of laboratory works	– Iterative methods of the decision of systems of the linear equations – Method of simple iteration of Jacoby – Method of Gaussa-Zejdel – Exercises
The purpose of educational employment: **preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on laboratory work to watch mastering at pupils of this knowledge, to form and develop at them skills.**	
***Problems of the teacher*:**	***Results of educational activity:***
• To acquaint students with a laboratory material and the device, the basic directions of activity on a considered material and the curriculum; • Logically consistently, it is given reason and clearly to state thoughts, correctly to build oral and written speech; • Studying of the basic concepts and the terms applied in frameworks практичкеского of a material; • To make training more clear and accessible, to interest students, and the same to motivate them for the further training and self-education; • Maintenance of perception, judgement and primary storing of communications and relations in object of studying; • Establishment of correctness and sensibleness of mastering of a new teaching material, revealing of incorrect representations and their correction; • Maintenance of mastering of new knowledge and ways of actions; • Formation of complete representation of knowledge on a theme; • To show a distributing material, to give talks and to give laboratory works.	• Mastering of new knowledge and ways of actions; • Primary check of understanding; • Fastening of knowledge and ways of actions; • Generalisation and ordering of knowledge. • Careful studying and the all-round analysis of a laboratory material and increase of an educational level which would provide the decision of the put problem; • Increase of a degree of quality of knowledge through introduction of innovative technologies; • Level monitoring обученности pupils on steps, classes, subjects, it is concrete on each student, for the purpose of revealing of the real reasons influencing progress, dynamics of conformity of level of teaching to educational standards. • Monitoring of professional skill of teachers. • To continue activity on the organisation of interaction of participants of educational space; • To create conditions, to generalise an advanced experience and to motivate students; • Increase scientific информативности in a field of knowledge of a subject matter and related subjects.
Training methods	**Laboratory – visualisation, conversation**
Technics of training	**Blitz – the interrogation, focusing questions**
Modes of study	**Collective, face-to-face**
Tutorials	**Visual materials, uchebno-methodical grants, laboratory installations and devices**
Training conditions	**The audience provided with tutorials**
Monitoring and estimation of knowledge	**The oral control: a question-answer**

TECHNOLOGICAL CARD
Carrying out laboratory work on a theme:
The numerical decision of system of the linear algebraic equations to methods of simple iteration.

Stages, time	*The activity maintenance*	
	The teacher	*Students*
1 stage. Introduction ***(15 minutes)***	**1.1. Laboratory work and the plan of its carrying out informs a theme, the purpose, planned results.** **1.2. For the purpose of actualisation of knowledge of students asks focusing questions.**	**1.1. Listen.** **1.2. Answer questions.**
2 stage. Basic (information) **(55 minutes)**	2.1. Consistently states a material laboratory concerning the plan. – Iterative methods of the decision of systems of the linear equations – Method of simple iteration of Jacoby – Method of Gaussa-Zejdel – Exercises	**2.1. Listen, discuss the maintenance of schemes and tables, specify, ask questions, carry out laboratory work.** **Write down the main thing results.**
3 stage. The final ***(10 minutes)***	**3.1. Spends a blitz – interrogation on a theme laboratory work. Does the total conclusion.** **3.2. Gives the task for independent work.**	**3.1. Answer questions.** **3.2. Listen, write down.**

Theme №5	**The numerical decision of system of the nonlinear algebraic equations to methods of simple iteration.**

TECHNOLOGY OF TRAINING
(Carrying out) laboratory work

Employment time - 2 hours (80 minutes)	Quantity of students: 15-20
Mode of study	Introduction - thematic lecture
The plan of laboratory works	– The theory – Order of performance of work. – The numerical decision of system of the nonlinear algebraic equations to methods of simple iteration.
The purpose of educational employment: **preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on laboratory work to watch mastering at pupils of this knowledge, to form and develop at them skills.**	
***Problems of the teacher*:**	***Results of educational activity:***
• To acquaint students with a laboratory material and the device, the basic directions of activity on a considered material and the curriculum; • Logically consistently, it is given reason and clearly to state thoughts, correctly to build oral and written speech; • Studying of the basic concepts and the terms applied in frameworks практичкеского of a material; • To make training more clear and accessible, to interest students, and the same to motivate them for the further training and self-education; • Maintenance of perception, judgement and primary storing of communications and relations in object of studying; • Establishment of correctness and sensibleness of mastering of a new teaching material, revealing of incorrect representations and their correction; • Maintenance of mastering of new knowledge and ways of actions; • Formation of complete representation of knowledge on a theme; • To show a distributing material, to give talks and to give laboratory works.	• Mastering of new knowledge and ways of actions; • Primary check of understanding; • Fastening of knowledge and ways of actions; • Generalisation and ordering of knowledge. • Careful studying and the all-round analysis of a laboratory material and increase of an educational level which would provide the decision of the put problem; • Increase of a degree of quality of knowledge through introduction of innovative technologies; • Level monitoring обученности pupils on steps, classes, subjects, it is concrete on each student, for the purpose of revealing of the real reasons influencing progress, dynamics of conformity of level of teaching to educational standards. • Monitoring of professional skill of teachers. • To continue activity on the organisation of interaction of participants of educational space; • To create conditions, to generalise an advanced experience and to motivate students; • Increase scientific информативности in a field of knowledge of a subject matter and related subjects.
Training methods	**Laboratory – visualisation, conversation**
Technics of training	**Blitz – the interrogation, focusing questions**
Modes of study	**Collective, face-to-face**
Tutorials	**Visual materials, uchebno-methodical grants, laboratory installations and devices**
Training conditions	**The audience provided with tutorials**
Monitoring and estimation of knowledge	**The oral control: a question-answer**

TECHNOLOGICAL CARD
Carrying out laboratory work on a theme:
The numerical decision of system of the nonlinear algebraic equations to methods of simple iteration.

Stages, time	*The activity maintenance*	
	The teacher	*Students*
1 stage. Introduction ***(15 minutes)***	**1.1. Laboratory work and the plan of its carrying out informs a theme, the purpose, planned results.** **1.2. For the purpose of actualisation of knowledge of students asks focusing questions.**	**1.1. Listen.** **1.2. Answer questions.**
2 stage. Basic (information) **(55 minutes)**	2.1. Consistently states a material laboratory concerning the plan. – The theory – Order of performance of work. – The numerical decision of system of the nonlinear algebraic equations to methods of simple iteration.	**2.1. Listen, discuss the maintenance of schemes and tables, specify, ask questions, carry out laboratory work.** **Write down the main thing results.**
3 stage. The final ***(10 minutes)***	**3.1. Spends a blitz – interrogation on a theme laboratory work. Does the total conclusion.** **3.2. Gives the task for independent work.**	**3.1. Answer questions.** **3.2. Listen, write down.**

Theme № 6-7	Problems of Cochy for the ordinary differential equations. Euler's methods, Runge-Kutta and Adams

TECHNOLOGY OF TRAINING
(Carrying out) laboratory work

Employment time - 4 hours (160 minutes)	Quantity of students: 15-20
Mode of study	Introduction - thematic lecture
The plan of laboratory works	– Problems of Cochy for the ordinary differential equations. – Euler's method – Method of Runge-Kutta – Adams's method

The purpose of educational employment: **preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on laboratory work to watch mastering at pupils of this knowledge, to form and develop at them skills.**

***Problems of the teacher*:**	***Results of educational activity:***
• To acquaint students with a laboratory material and the device, the basic directions of activity on a considered material and the curriculum; • Logically consistently, it is given reason and clearly to state thoughts, correctly to build oral and written speech; • Studying of the basic concepts and the terms applied in frameworks практичкеского of a material; • To make training more clear and accessible, to interest students, and the same to motivate them for the further training and self-education; • Maintenance of perception, judgement and primary storing of communications and relations in object of studying; • Establishment of correctness and sensibleness of mastering of a new teaching material, revealing of incorrect representations and their correction; • Maintenance of mastering of new knowledge and ways of actions; • Formation of complete representation of knowledge on a theme; • To show a distributing material, to give talks and to give laboratory works.	• Mastering of new knowledge and ways of actions; • Primary check of understanding; • Fastening of knowledge and ways of actions; • Generalisation and ordering of knowledge. • Careful studying and the all-round analysis of a laboratory material and increase of an educational level which would provide the decision of the put problem; • Increase of a degree of quality of knowledge through introduction of innovative technologies; • Level monitoring обученности pupils on steps, classes, subjects, it is concrete on each student, for the purpose of revealing of the real reasons influencing progress, dynamics of conformity of level of teaching to educational standards. • Monitoring of professional skill of teachers. • To continue activity on the organisation of interaction of participants of educational space; • To create conditions, to generalise an advanced experience and to motivate students; • Increase scientific информативности in a field of knowledge of a subject matter and related subjects.
Training methods	**Laboratory – visualisation, conversation**
Technics of training	**Blitz – the interrogation, focusing questions**
Modes of study	**Collective, face-to-face**
Tutorials	**Visual materials, uchebno-methodical grants, laboratory installations and devices**
Training conditions	**The audience provided with tutorials**
Monitoring and estimation of knowledge	**The oral control: a question-answer**

TECHNOLOGICAL CARD
Carrying out laboratory work on a theme:
Problems of Cochy for the ordinary differential equations. Euler's methods, Runge-Kutta and Adams

Stages, time	*The activity maintenance*	
	The teacher	*Students*
1 stage. Introduction *(15 minutes)*	**1.1. Laboratory work and the plan of its carrying out informs a theme, the purpose, planned results.** **1.2. For the purpose of actualisation of knowledge of students asks focusing questions.**	**1.1. Listen.** **1.2. Answer questions.**
2 stage. Basic (information) **(135 minutes)**	2.1. Consistently states a material laboratory concerning the plan. – Problems of Cochy for the ordinary differential equations. – Euler's method – Method of Runge-Kutta – Adams's method	**2.1. Listen, discuss the maintenance of schemes and tables, specify, ask questions, carry out laboratory work.** **Write down the main thing results.**
3 stage. The final *(10 minutes)*	**3.1. Spends a blitz – interrogation on a theme laboratory work. Does the total conclusion.** **3.2. Gives the task for independent work.**	**3.1. Answer questions.** **3.2. Listen, write down.**

Theme № 8	The geometrical decision of a problem of linear programming

TECHNOLOGY OF TRAINING
(Carrying out) laboratory work

Employment time - 2 hours (80 minutes)	Quantity of students: 15-20
Mode of study	Introduction - thematic lecture
The plan of laboratory works	– Geometrical interpretation of a problem of linear programming – Using geometrical interpretation, find decisions of problems
The purpose of educational employment: **preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on laboratory work to watch mastering at pupils of this knowledge, to form and develop at them skills.**	
***Problems of the teacher*:**	***Results of educational activity:***
• To acquaint students with a laboratory material and the device, the basic directions of activity on a considered material and the curriculum; • Logically consistently, it is given reason and clearly to state thoughts, correctly to build oral and written speech; • Studying of the basic concepts and the terms applied in frameworks практичкеского of a material; • To make training more clear and accessible, to interest students, and the same to motivate them for the further training and self-education; • Maintenance of perception, judgement and primary storing of communications and relations in object of studying; • Establishment of correctness and sensibleness of mastering of a new teaching material, revealing of incorrect representations and their correction; • Maintenance of mastering of new knowledge and ways of actions; • Formation of complete representation of knowledge on a theme; • To show a distributing material, to give talks and to give laboratory works.	• Mastering of new knowledge and ways of actions; • Primary check of understanding; • Fastening of knowledge and ways of actions; • Generalisation and ordering of knowledge. • Careful studying and the all-round analysis of a laboratory material and increase of an educational level which would provide the decision of the put problem; • Increase of a degree of quality of knowledge through introduction of innovative technologies; • Level monitoring обученности pupils on steps, classes, subjects, it is concrete on each student, for the purpose of revealing of the real reasons influencing progress, dynamics of conformity of level of teaching to educational standards. • Monitoring of professional skill of teachers. • To continue activity on the organisation of interaction of participants of educational space; • To create conditions, to generalise an advanced experience and to motivate students; • Increase scientific информативности in a field of knowledge of a subject matter and related subjects.
Training methods	**Laboratory – visualisation, conversation**
Technics of training	**Blitz – the interrogation, focusing questions**
Modes of study	**Collective, face-to-face**
Tutorials	**Visual materials, uchebno-methodical grants, laboratory installations and devices**
Training conditions	**The audience provided with tutorials**
Monitoring and estimation of knowledge	**The oral control: a question-answer**

TECHNOLOGICAL CARD
Carrying out laboratory work on a theme:
The geometrical decision of a problem of linear programming

Stages, time	*The activity maintenance*	
	The teacher	*Students*
1 stage. Introduction ***(15 minutes)***	**1.1. Laboratory work and the plan of its carrying out informs a theme, the purpose, planned results.** **1.2. For the purpose of actualisation of knowledge of students asks focusing questions.**	**1.1. Listen.** **1.2. Answer questions.**
2 stage. Basic (information) **(55 minutes)**	2.1. Consistently states a material laboratory concerning the plan. – Geometrical interpretation of a problem of linear programming – Using geometrical interpretation, find decisions of problems	**2.1. Listen, discuss the maintenance of schemes and tables, specify, ask questions, carry out laboratory work.** **Write down the main thing results.**
3 stage. The final ***(10 minutes)***	**3.1. Spends a blitz – interrogation on a theme laboratory work. Does the total conclusion.** **3.2. Gives the task for independent work.**	**3.1. Answer questions.** **3.2. Listen, write down.**

Theme № 9	**Finding the decision of a problem of linear programming to Simplex methods**

TECHNOLOGY OF TRAINING
(Carrying out) laboratory work

Employment time - 2 hours (80 minutes)	Quantity of students: 15-20
Mode of study	Introduction - thematic lecture
The plan of laboratory works	– Simplex method of the decision of a problem of linear programming – Examples of the decision of a problem of linear programming with a simplex method
The purpose of educational employment: **preparation of students for work on employment, the organisation of educational process by transfer to students of knowledge, skills on laboratory work to watch mastering at pupils of this knowledge, to form and develop at them skills.**	
***Problems of the teacher*:**	***Results of educational activity:***
• To acquaint students with a laboratory material and the device, the basic directions of activity on a considered material and the curriculum; • Logically consistently, it is given reason and clearly to state thoughts, correctly to build oral and written speech; • Studying of the basic concepts and the terms applied in frameworks практичкеского of a material; • To make training more clear and accessible, to interest students, and the same to motivate them for the further training and self-education; • Maintenance of perception, judgement and primary storing of communications and relations in object of studying; • Establishment of correctness and sensibleness of mastering of a new teaching material, revealing of incorrect representations and their correction; • Maintenance of mastering of new knowledge and ways of actions; • Formation of complete representation of knowledge on a theme; • To show a distributing material, to give talks and to give laboratory works.	• Mastering of new knowledge and ways of actions; • Primary check of understanding; • Fastening of knowledge and ways of actions; • Generalisation and ordering of knowledge. • Careful studying and the all-round analysis of a laboratory material and increase of an educational level which would provide the decision of the put problem; • Increase of a degree of quality of knowledge through introduction of innovative technologies; • Level monitoring обученности pupils on steps, classes, subjects, it is concrete on each student, for the purpose of revealing of the real reasons influencing progress, dynamics of conformity of level of teaching to educational standards. • Monitoring of professional skill of teachers. • To continue activity on the organisation of interaction of participants of educational space; • To create conditions, to generalise an advanced experience and to motivate students; • Increase scientific информативности in a field of knowledge of a subject matter and related subjects.
Training methods	**Laboratory – visualisation, conversation**
Technics of training	**Blitz – the interrogation, focusing questions**
Modes of study	**Collective, face-to-face**
Tutorials	**Visual materials, uchebno-methodical grants, laboratory installations and devices**
Training conditions	**The audience provided with tutorials**
Monitoring and estimation of knowledge	**The oral control: a question-answer**

TECHNOLOGICAL CARD
Carrying out laboratory work on a theme:
Finding the decision of a problem of linear programming to Simplex methods

Stages, time	*The activity maintenance*	
	The teacher	*Students*
1 stage. Introduction ***(15 minutes)***	**1.1. Laboratory work and the plan of its carrying out informs a theme, the purpose, planned results.** **1.2. For the purpose of actualisation of knowledge of students asks focusing questions.**	**1.1. Listen.** **1.2. Answer questions.**
2 stage. Basic (information) **(55 minutes)**	2.1. Consistently states a material laboratory concerning the plan. – Simplex method of the decision of a problem of linear programming – Examples of the decision of a problem of linear programming with a simplex method	**2.1. Listen, discuss the maintenance of schemes and tables, specify, ask questions, carry out laboratory work.** **Write down the main thing results.**
3 stage. The final ***(10 minutes)***	**3.1. Spends a blitz – interrogation on a theme laboratory work. Does the total conclusion.** **3.2. Gives the task for independent work.**	**3.1. Answer questions.** **3.2. Listen, write down.**

«ALGORITHMIZATION OF COMPUTING METHODS»

Educational-Methodological Complex